The following is the complete derivation process of $\mathcal{L}_{ELBO}^1$, and $\mathcal{L}_{ELBO}^2$ can be derived similarly.

$logP_{\theta,\varphi}(Y_C|G)$

$= log[\int_Y P_{\theta,\varphi}(Y_C, Y|G)dY]$

$= log[\int_Y P_\varphi(Y_C|Y, G)P_\theta(Y|G)dY]$

$= log[\int_Y Q_\phi(Y|Y_C, G)\frac{P_\varphi(Y_C|Y,G)P_\theta(Y|G)}{Q_\phi(Y|Y_C,G)}dY]$

$\geq \int_Y Q_\phi(Y|Y_C, G)log\frac{P_\varphi(Y_C|Y,G)P_\theta(Y|G)}{Q_\phi(Y|Y_C,G)}dY$

$= E_{Q_\phi(Y|Y_C,G)}logP_\varphi(Y_C|Y, G) - KL(Q_\phi(Y|Y_C, G)||P_\theta(Y|G))$

$= \mathcal{L}_{ELBO}^1$