The following is the complete derivation process of $\mathcal{L}_{ELBO}^{1}$ and $\mathcal{L}_{ELBO}^{2}$.

$$
\begin{aligned}
& logP_{\theta,\varphi,\phi}(Y_C|G) \\
& = log[\int_{\bar{Y}} P_{\theta,\varphi,\phi}(Y_C,\bar{Y}|G)dY] \\
& = log[\int_{\bar{Y}} P_{\varphi}(Y_C|\bar{Y},G)P_{\theta}(\bar{Y}|G)d\bar{Y}] \\
& = log[\int_{\bar{Y}} Q_{\phi}(\bar{Y}|Y_C,G)\frac{P_{\varphi}(Y_C|\bar{Y},G)P_{\theta}(\bar{Y}|G)}{Q_{\phi}(\bar{Y}|Y_C,G)}d\bar{Y}] \\
& \geq \int_{\bar{Y}} Q_{\phi}(\bar{Y}|Y_C,G)log\frac{P_{\varphi}(Y_C|\bar{Y},G)P_{\theta}(\bar{Y}|G)}{Q_{\phi}(\bar{Y}|Y_C,G)}d\bar{Y} \\
& = E_{Q_{\phi}(\bar{Y}|Y_C,G))}logP_{\varphi}(Y_C|\bar{Y},G) - KL(Q_{\phi}(\bar{Y}|Y_C,G)||P_{\theta}(\bar{Y}|G)) \\
& = \mathcal{L}_{ELBO}^{1}
\end{aligned}
$$

$$
\begin{aligned}
& logP_{\theta,\varphi,\phi}(Y_C|G,Y_N) \\
& = log[\int_{\bar{Y}} P_{\theta,\varphi,\phi}(Y_C,\bar{Y}|G,Y_N)dY] \\
& = log[\int_{\bar{Y}} P_{\varphi}(Y_C|\bar{Y},G,Y_N)P_{\theta}(\bar{Y}|G,Y_N)d\bar{Y}] \\
& = log[\int_{\bar{Y}} Q_{\phi}(\bar{Y}|Y_C,G,Y_N)\frac{P_{\varphi}(Y_C|\bar{Y},G,Y_N)P_{\theta}(\bar{Y}|G,Y_N)}{Q_{\phi}(\bar{Y}|Y_C,G,Y_N)}d\bar{Y}] \\
& \geq \int_{\bar{Y}} Q_{\phi}(\bar{Y}|Y_C,G,Y_N)log\frac{P_{\varphi}(Y_C|\bar{Y},G,Y_N)P_{\theta}(\bar{Y}|G,Y_N)}{Q_{\phi}(\bar{Y}|Y_C,G,Y_N)}d\bar{Y} \\
& = E_{Q_{\phi}(\bar{Y}|Y_C,G,Y_N))}logP_{\varphi}(Y_C|\bar{Y},G,Y_N) - KL(Q_{\phi}(\bar{Y}|Y_C,G,Y_N)||P_{\theta}(\bar{Y}|G,Y_N)) \\
& = \mathcal{L}_{ELBO}^{2}
\end{aligned}
$$