

The following is the complete derivation process of \mathcal{L}_{ELBO}^1 and \mathcal{L}_{ELBO}^2 .

$$\begin{aligned}
& \log P_{\theta, \varphi}(Y_C | G) \\
&= \log \left[\int_{\bar{Y}} P_{\theta, \varphi}(Y_C, \bar{Y} | G) d\bar{Y} \right] \\
&= \log \left[\int_{\bar{Y}} P_{\varphi}(Y_C | \bar{Y}, G) P_{\theta}(\bar{Y} | G) d\bar{Y} \right] \\
&= \log \left[\int_{\bar{Y}} Q_{\phi}(\bar{Y} | Y_C, G) \frac{P_{\varphi}(Y_C | \bar{Y}, G) P_{\theta}(\bar{Y} | G)}{Q_{\phi}(\bar{Y} | Y_C, G)} d\bar{Y} \right] \\
&\geq \int_{\bar{Y}} Q_{\phi}(\bar{Y} | Y_C, G) \log \frac{P_{\varphi}(Y_C | \bar{Y}, G) P_{\theta}(\bar{Y} | G)}{Q_{\phi}(\bar{Y} | Y_C, G)} d\bar{Y} \\
&= E_{Q_{\phi}(\bar{Y} | Y_C, G)} \log P_{\varphi}(Y_C | \bar{Y}, G) - KL(Q_{\phi}(\bar{Y} | Y_C, G) || P_{\theta}(\bar{Y} | G)) \\
&= \mathcal{L}_{ELBO}^1
\end{aligned}$$

$$\begin{aligned}
& \log P_{\theta, \varphi}(Y_C | G, Y_N) \\
&= \log \left[\int_{\bar{Y}} P_{\theta, \varphi}(Y_C, \bar{Y} | G, Y_N) d\bar{Y} \right] \\
&= \log \left[\int_{\bar{Y}} P_{\varphi}(Y_C | \bar{Y}, G, Y_N) P_{\theta}(\bar{Y} | G, Y_N) d\bar{Y} \right] \\
&= \log \left[\int_{\bar{Y}} Q_{\phi}(\bar{Y} | Y_C, G, Y_N) \frac{P_{\varphi}(Y_C | \bar{Y}, G, Y_N) P_{\theta}(\bar{Y} | G, Y_N)}{Q_{\phi}(\bar{Y} | Y_C, G, Y_N)} d\bar{Y} \right] \\
&\geq \int_{\bar{Y}} Q_{\phi}(\bar{Y} | Y_C, G, Y_N) \log \frac{P_{\varphi}(Y_C | \bar{Y}, G, Y_N) P_{\theta}(\bar{Y} | G, Y_N)}{Q_{\phi}(\bar{Y} | Y_C, G, Y_N)} d\bar{Y} \\
&= E_{Q_{\phi}(\bar{Y} | Y_C, G, Y_N)} \log P_{\varphi}(Y_C | \bar{Y}, G, Y_N) - KL(Q_{\phi}(\bar{Y} | Y_C, G, Y_N) || P_{\theta}(\bar{Y} | G, Y_N)) \\
&= \mathcal{L}_{ELBO}^2
\end{aligned}$$