NATIONAL INSTITUTE OF TCHNOLOGY, JAMSHEDPUR

SPRING SEMESTER: 2023-24

Department of Mathematics

MID SEMESTER EXAMINATION FOR MCA 2023 BATCH, MARCH 2023

Course Code: MA3201 Semester: 2nd Course Title: Numerical Methods
Date: 21-03-2024 (A-Shift, Time: 10.00 To 12.00 AM) Day: Thursday
Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad
Duration: 02 Hours Max. Marks: 30

Instructions:
(a) Answer to all the **five** *(5)* questions.
(b) Marks of the questions are indicated in the right-hand margin.
(c) Missing data, if any, may be assumed suitably.

1. Find the condition of convergence of the sequence of approximations obtained by applying Newton Raphson method for finding the solution of the equation: $f(x) = 0$. Can we apply the Newton Raphson method for finding the root of the equation $3x - \log_{10} x = 6$ in $[2, 3]$? If yes, find the root correct up to three places of decimal. [6]

2. If $x = \alpha$ be the exact root of the equation $f(x) = 0$. Find the value of μ in the iteration formula: $x_{n+1} = (\mu x_n + x_n^{-2} + 1)/(\mu + 1)$ to ensure the fastest possible rate of convergence. [3]

3. (a) Find the 3^{rd} difference of the Polynomial: $f(x) = 5x^4 - x^3 + 2x^2 - 30x + 9$ with $h = 2$. [3]
   (b) Using interpolation process, find the value of x for which $y = 7$ with the help of following table: [3]

x	1	3	4
y	4	12	19

   (c) Find the missing term in the following table using interpolation process: [3]

x	0	1	2	3	4
$f(x)$	1	3	9	---	81

4. Define the spline function of degree n and explain the process of quadratic spline fitting. [4]
5. Setup the Gauss-Seidel iteration scheme in matrix form for solving the 3x3 system: [8]

$$2x - y = 7; \quad -x + 2y - z = 1; \quad -y + 2z = 1.$$

Perform first three iterations by taking $x = y = z = 0$ as the initial approximations to the solution.

Check whether the scheme is convergent or not. Also find the rate of convergence of the scheme.

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR

Department of Mathematics

MID SEMESTER EXAMINATION FOR MCA 2022 BATCH, FEBRUARY 2023

Course Code: MA3201 Semester: 2nd Course Title: Numerical Methods

Date: 20-02-2023 (Time: 3.00 To 5.00) Day: Monday

Course Instructor (Name of the Faculty):Dr. Hari Shankar Prasad

Duration: 02 Hours Max. Marks: 30

Instructions:

(a) Answer to all the **Three** *(3)* questions.

(b) Marks of the questions are indicated in the right-hand margin.

(c) Missing data, if any, may be assumed suitably.

1. (a) Find the condition of convergence of sequence of approximations obtained in Fixed-Point Iteration method. Also obtain the root of the equation $\mathbf{3x - \log_{10} x = 6}$ using this Fixed-Point Iteration method correct up to three places of decimal. [2+3]

   (b) Obtain the condition of convergence of Newton Raphson method and show that Newton-Raphson method is of linear rate of convergence when applied to find the root of multiplicity greater than one. [2+3]

2. (a) Define the factorial polynomial of degree six (6) and show that the 3^{rd} difference of this 6^{th} degree polynomial is a polynomial of degree three (3). [5]

   (b) What do you mean by interpolation? Derive Newton forward difference interpolation formula. [1+4]

3. (a) Find the root of the equation $g(x) = \mathbf{0}$ using Newton Raphson method where $g(x) = \frac{f(x)}{[f(x)]'}$ **with** $f(x) = \boldsymbol{cosx} - xe^x$.

   [5]

   (b) Solve the following system of equations by taking $x = y = z = 0$ as initial approximation in Gauss-Seidel iteration method. Perform five (5) iterations for the solution. [5]

$$\begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 16 \\ -6 \end{bmatrix}.$$

MID SEM - 2023

NATIONAL INSTITUTE OF TCHNOLOGY, JAMSHEDPUR

SPRING SEMESTER: 2023-24

Department of Mathematics

MID SEMESTER EXAMINATION FOR M. Sc 2023 BATCH, MARCH 2023

Course Code: MA3202 Semester: 2nd Course Title: Numerical Analysis & Lab.

Date: 18-03-2024 (Time: 10.00 To 12.00 AM) Day: Monday

Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad

Duration: 02 Hours Max. Marks: 30

Instructions:

(a) Answer to all the **Four** *(4)* questions.

(b) Marks of the questions are indicated in the right-hand margin.

(c) Missing data, if any, may be assumed suitably.

1. Find the condition of convergence of the sequence of approximations obtained by applying Fixed Point Iteration method for finding the solution of the equation: $f(x) = 0$. Can we apply the Fixed Point Iteration method for finding the root of the equation $3x - \log_{10} x = 6$ in $[2, 3]$? If yes, find the root correct up to three places of decimal. [6]

2. We wish to compute the root of the equation $e^{-x} - 3\log_e x = 0$ using a method with the formula $x_{n+1} = x_n - (3\log_e x_n - e^{-x_n})/p$. Find the rate of convergence of the method and Show that the rapid convergence is achieved when the value of p is near 3. [6]

3. (a) Find the 2^{nd} difference of the Polynomial: $f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$ with $h = 2$. [2]

(b) Using interpolation process, find the value of x for which $y = 7$ with the help of following table: [2]

x	1	3	4
y	4	12	19

(c) Find the missing term in the following table using interpolation process: [2]

x	0	1	2	3	4
$f(x)$	1	3	9	----	81

4. (a) Define the spline function of degree n and explain the process of quadratic spline fitting. [6]

(b) Solve the following: [1.5x4]

(i) If $\delta = \Delta E^{-\frac{1}{2}}$ then prove that $E = (\frac{\Delta}{\delta})^2$ (ii) Prove that $\mu\delta = \frac{\Delta}{2} + \frac{\Delta E^{-1}}{2}$

(iii) Taking the interval of differencing $h = 1$, find $(\Delta + \nabla)^2 f(x)$ where $f(x) = x^2 + x$.

(iv) If $\Delta f(x) = x^3 + 3x^2 + 5x + 12$ then find $f(x)$.

END

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR
SPRING SEMESTER: 2023-24
Department of Mathematics
END SEMESTER EXAMINATION, APRIL 2024

Course Code: MA3201 Semester: 2nd Course Title: Numerical Methods
Date: 08-05-2024 Branch: MCA Day: Wednesday
Duration: 03 Hours (B-Shift, Time: 04.00 PM To 07.00 PM) Max. Marks: 50
Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad

Instructions:
(a) Answer to all the **Six (6)** questions.
(b) Marks of the questions are indicated in the right-hand margin.
(c) Missing data, if any, may be assumed suitably.

1. (a) Derive Gauss backward interpolation formula and hence find the number of students getting marks between 60 and 70 from the following data: [5]

Marks	Below 40	40 - 60	60 - 80	80 - 100	100 - 120
Number of Students	250	120	100	70	50

(b) Write the process of Regula-Falsi for finding the root of algebraic and transcendental equation and obtain the Rate of convergence of this method. [5]

2. (a) Obtain the Simpson's $\frac{1}{3}rd$ rule of integration by deriving Newton-Cotes quadrature formula and find the error associated with the Simpson's $\frac{1}{3}rd$ rule of integration in evaluating the integral $\int_0^1 (1+\frac{sinx}{x})\,dx$ with five(5) ordinates. [5]

(b) Define the Orthogonality and Orthonormality of a set of functions $g_k(x),\ k = 1,2,3,\ldots\ldots,n$ with respect to the weight function $w(x)$ which are valid over an interval $[a,b]$, where n is a positive integer. Show that $g_k(x) = \frac{sinkx}{\sqrt{\pi}},\ k = 1,2,3,\ldots\ldots$ are orthogonal set of functions on $-\pi \le x \le \pi$.. [5]

3. (a) Write the Gram-Schmidt orthogonalization process for generating orthogonal polynomials which are defined over an interval $[a,b]$ with respect to the weight function $w(x)$. [5]

(b) Obtain an approximation in the sense of the principal of least squares in the form of polynomial of degree two(2) for the function $f(x) = \frac{1}{1+x^2}$ which is valid in the interval $[-1,\ 1]$. [5]

4. (a) Obtain the natural cubic spline approximation for the function $y(x)$ defined by the data:

x	0	1	2	3
$y(x)$	1	2	33	244

and hence determine the value of $y(2.5)$ and $y'(2.5)$. [5]

(b) The following table gives the velocity v of a particle at time t:

t (seconds)	0	2	4	6	8	10	12
v(m/sec.)	4	6	16	34	60	94	136

Find the distance moved by the particle in **12** seconds using Simpson's 1/3rd and 3/8th rule's of Integration respectively. [5]

5. (a) Define the initial value problem and find the value of $y(-0.1)$, and $y(0.1)$ using the Taylor's series method of order four with $h = 0.1$, given that $\frac{dy}{dx} = x^2 + y^2 - 2,\ y(0) = 1$. [4]

(b) Given that $\frac{dy}{dx} = (x - y^2),\ y(0) = 1$, first compute the values of $y(0.1)$, $y(0.2)$, and $y(0.3)$ using the Runge-Kutta method of order four with $h = 0.1$ and then, evaluate the value of $y(0.4)$ by Adam-Bashforth-Moulton Predictor-Corrector method of order four. [6]

END

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR
SPRING SEMESTER: 2022-23
Department of Mathematics
END SEMESTER EXAMINATION, APRIL 2023

Code: MA3201 Semester: 2nd Course Title: Numerical Methods
: 27-04-2023 Branch: MCA Day: Thursday
ourse Instructor (Name of the Faculty): Dr. Hari Shankar Prasad
Duration: 03 Hours (A-Shift, Time: 9.30 AM To 12.30 PM) Max. Marks: 50

Instructions:
(a) Answer to all the **Six** *(6)* questions.
(b) Marks of the questions are indicated in the right-hand margin.
(c) Missing data, if any, may be assumed suitably.

1. (a) Define the rate of convergence of the iterative methods for finding the root of an algebraic and transcendental equations and, find the rate of convergence of Regula-Falsi method. [5]
   (b) Define the Orthogonality and Orthonormality of a set of functions $g_k(x)$, $k = 1,2,3,\ldots\ldots,n$ with respect to the weight function $\boldsymbol{w(x)}$ which are valid over an interval $[a, b]$, where n is a positive integer. Show that $g_k(x) = \frac{sinkx}{\sqrt{\pi}}$, $k = 1,2,3,\ldots..$ are orthonormal set of functions on $-\pi \le x \le \pi$.. [5]

2. (a) Write the Gram-Schmidt orthogonalization process for generating orthogonal polynomials which are defined over an interval $[a, b]$ with respect to the weight function $\boldsymbol{w(x)}$. [5]
   (b) Obtain the least square approximation of second degree for the function $\boldsymbol{f(x) = sinx}$ on $[\,0, \frac{\pi}{2}\,]$ with respect to the weight function $\boldsymbol{w(x) = 1}$. [5]

3. (a) Obtain the natural cubic spline approximation for the function $\boldsymbol{y(x)}$ defined by the data:

x	0	1	2	3
$y(x)$	1	2	33	244

and hence determine the value of $\boldsymbol{y(2.5)}$ and $\boldsymbol{y'(2.5)}$. [5]
   (b) Derive Newton divided difference interpolation polynomial and hence obtain the missing values in the following table: [5]

x	0	1	2	3
$f(x)$	1	3	.....	55

4. The following table gives the velocity **v** of a particle at time **t:**

t (seconds)	0	2	4	6	8	10	12
v(m/sec.)	4	6	16	34	60	94	136

Find the distance moved by the particle in **12** seconds using Simpson's **1/3rd** and 3/8th rule's of Integration respectively. [6]

5. Define the initial value problems and find the value of $\mathbf{y(0.1)}$ and $\mathbf{y(0.2)}$ using the Runge-Kutta method of order four with $\boldsymbol{h = 0.1}$, given that $\frac{dy}{dx} = \boldsymbol{xy + y^2}, \boldsymbol{y(0) = 1}$. [6]

6. **Given that** $\frac{dy}{dx} = \frac{1}{2}(y - x^2), y(0) = 1, h = 0.2$, compute the values **of** $\boldsymbol{y(0.2)}$, $\boldsymbol{y(0.4)}$, ***and*** $\boldsymbol{y(0.6)}$ using Taylor's series method of order three first, and then, evaluate the value of $\boldsymbol{y(0.8)}$ using Milne's Predictor-Corrector method. [8]

END

END SEM - 2023

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR
SPRING SEMESTER: 2021-22
Department of Mathematics
END SEMESTER EXAMINATION FOR MCA 2021 BATCH, MAY 2022

Course Code: MA3201 Semester: 2nd Course Title: Numerical Methods
Date: 13-05-2022 (Time: 9.00 To 12.00 AM) Day: Friday
Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad
Duration: 03 Hours Max. Marks: 50

Instructions: Answer to all the *Five (5)* questions. Marks of the questions are indicated in the right-hand margin.

1. (a) Derive Newton forward interpolation formula and hence find the number of men getting wages between Rs. 10 and 15 from the following data: [6]

Wages in rupees	0 - 10	10 - 20	20 - 30	30 - 40
Number of men	09	30	35	42

(b) Use Lagrange's interpolation to find the root of the equation $f(x) = 0$ given that $f(30) = -30, f(34) = -13, f(38) = 3$ and $f(42) = 180$. [4]

2. (a) The velocity V of a particle at distance S from a point on its path is given by the table: [5]

S ft.	0	10	20	30	40	50	60
V ft./sec.	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson 3/8- rule of integration. Compare the result with Simpson 1/3 rule.

(b) Derive the formula for the error associated with the Simpson 1/3 rule of integration, and using this formula find the error associated with the evaluation of the integral $\int_4^{5.2} \log x \, dx$. [4+1]

3. (a) What do you mean by spline interpolation? Write the algorithm of Cubic spline interpolation process. [2+4]

(b) Using Gauss-Seidel method, solve the system:
$54x + y + z = 110; 2x + 15y + 6z = 72; -x + 6y + 27z = 85.$
Perform first four iterations by taking $x = y = z = 0$ as the initial approximations. [4]

4. Using Gram-Schmidt orthogonalization process, compute the first three polynomials which are orthogonal on [-1, 1] with respect to the weight function $w(x)=1/\sqrt{x^2+1}$ and using these polynomials obtain the least square approximation of second degree for $f(x) = x$ on [-1, 1]. [8]

5. Solve the initial value problems (IVPs): [3x4]

(i) $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ using Taylor series method of order three for $y(0.1)$ and $y(0.2)$ with $h = 0.1$.

(ii) $\frac{dy}{dx} = -x\,y^2$, $y(0) = 2$ by Euler's predictor-corrector method for $y(0.2)$ in steps of 0.2, correct to three decimal places.

(iii) $\frac{dy}{dx} = \sqrt{x^2 + y}$; $y(0) = 0.8$ for $y(0.2)$ and $y(0.4)$, using Runge-Kutta 4th order method with $h = 0.2$.

END

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR
Department of Mathematics
END SEMESTER EXAMINATION: 2018-19

Course Code: MH 32102 Course Title: Computer Oriented Numerical Techniques
Semester: II Date: 13-05-2019 Day: MONDAY
CLOSED BOOK
Course Instructor/Instructor in-charge (Name of the Faculty): Dr. Hari Shankar Prasad
Duration: 03 hour Max. Marks: 50

Instructions:
(a) Answer to all the *Five (5)* questions.
(b) Marks of the questions and part thereof are indicated in the right hand margin.
(c) Missing data, if any, may be assumed suitably.
(d) Before attempting the questions paper be sure that you have got the correct question paper.

1. (a) Derive Gauss forward difference interpolation formula and using it, estimate the number of students who obtained marks between 40 and 45, with the help of following table: [3+3]

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

(b) Explain the method of Regula–Falsi for finding the root of algebraic and transcendental equations. [4]

2. (a) The velocity v (km/min.) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows:

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes using Simpson's rule of integration. [5]

(b) Using Gram-Schmidt orthogonalization process, compute the first three orthogonal polynomials which are orthogonal on [-1, 1] with respect to the weight function $w(x) = 1/\sqrt{1-x^2}$. Using these polynomials obtain the least square approximation of $f(x) = x^4$ on [-1, 1]. [5]

3. (a) Estimate the error associated with Composite Simpson's $\frac{1}{3}rd$ rule of integration. [5]

(b) Can we find a real root of the equation $x^3 + x^2 - 1 = 0$ in the interval [0, 1] using Fixed Point Iteration method ?. If yes, find the root correct up to three places of decimal. [5]

4. (a) Derive Newton-Cotes Quadrature formula and hence obtain the Composite Simpson's $\frac{1}{3}rd$ and $\frac{3}{8}th$ rule of integration. [3+1+1]

(b) Write the process of fitting cubic spline to the given set of data points $(x_i, y_i), i = 0,1,2,\ldots,N$. [5]

5. (a) Employ Taylor's series method to obtain approximate value of $y(0.1)$ and $y(0.2)$ with step size $h = 0.1$, for the initial value problem: $dy/dx = x + y; y(0) = 1$. [5]

(b) Using Runge-Kutta method of order four with $h = 0.1$, solve the initial value problem $\frac{dy}{dx} = x + y^2, y(0) = 1$ to get the value of $y(0.1)$ and $y(0.2)$. [5]

END SEM - 2019