NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR.
MID SEMESTER EXAMINATION (OCTOBER -2023)

SEMESTER : 1st **BATCH : 2023**

BRANCH: MCA **FULL MARKS : 30**

TIME: 2 hrs. **CREDIT: 04**

COURSE INSTRUCTOR: Prof. TARNI MANDAL **COURSE CODE: MA3103**

Subject: Probability and Statistical Computing

INSTRUCTION:

1) Attempt all the questions .
2) Marks of the question and part their off are indicated in the right hand margin.
3) Missing data, if any, may be assumed suitably.
4) Before attempting the question paper be sure that you have got the correct question paper.

SL.No. **Marks.**

1.a) A company has two plants to manufacture scooters. Plant I manufacture 70% of scooters and plant II manufacture30%. At plant I, 80% of the scooters are rated standard quality and at plant II, 90% of the scooters are rated standard quality. A scooter is chosen at random and is found to be of standard quality . What is the chance that it has come from plant II? [5]

(b) The elementary probability law of a continuous random variable x is $f(x) = y_0 . e^{-b(x-a)}$, $a \leq x \leq \infty$, where a,b and y_0 are constants. Show that $y_0 = b = 1/\sigma$ and $a = m - \sigma$, where m and σ are respectively the mean and standard deviation of the distribution. Also show that $\beta_1 = 4$ and $\beta_2 = 9$. [4]

2. (a) To prove that Recurrence relation for the Moments of the Binomial Distribution is [5]
$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d\mu_r}{dp}\right)$. Find the mean and variance.

(b) If x is a Poisson variate with mean λ, show that $\frac{x-\lambda}{\sqrt{\lambda}}$ is a variable with mean zero and variance unity. Find the M.G.F. for this variable and show that it approaches $e^{\frac{t^2}{2}}$ as $\lambda \to \infty$. [5]

3. (a) If X is a normally distributed and the mean of x is 12 and S.D. is 4. [3]
Find out the probability of the following:
i) $x \geq 20$, ii) $x \leq 20$ and iii) $0 \leq x \leq 12$
Given $P(0 \leq x \leq 2) = 0.4772$, $P(0 \leq x \leq 3) = 0.49865$,

b) If z is a standard normal variate and χ^2 is an independent chi-square variate divided by its n degree of freedom obtain the distribution of

$$t = \frac{z}{\sqrt{\frac{\chi^2}{n}}}$$

. Find the mean and variance of the distribution.

4 (a) If x and y are independent Gamma vitiates with parameter μ and ν respectively, [5]
Show that $U = x + y$, $z = \frac{x}{y}$, are independent and that U is a $\gamma(\mu+\nu)$ and Z is a $\beta_2(\mu, \nu)$

(b) From the following table regarding the colour of eyes of fathers and sons, test if the colour of son's eyes is associated with that of the father. [3]

Eye colour of son

Eye colour of father	Not Light	Light
Not Light	230	148
Light	151	471

Given $\chi^2 = 3.84$ at $\nu = 1$ at 5 % level of significance mean.

MID-SEM 2023

Roll No. : 2022PGCSCA103

National Institute of Technology Jamshedpur
Department of Mathematics
Mid-Semester Examination, Oct 18, 2022

MCA 1st Semester
Time Limit: 02 Hours
Maximum Marks: 30
MA3103 : Probability and Statistical Computing
Instructor: Dr. Mahendra Kumar Gupta

Instructions: All questions are compulsory. All parts of a question should preferably be answered at one place. Assume missing data suitably.

1. In a certain factory, machines I, II, and III are all producing springs of the same length. Machines I, II, and III produce 1%, 4%, and 2% defective springs, respectively. Of the total production of springs in the factory, Machine I produces 30%, Machine II produces 25%, and Machine III produces 45%.

(a) If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective.

(b) Given that the selected spring is defective, find the conditional probability that it was produced by Machine II. [2+3]

2. Let X have the following pdf

$$f(x) = \begin{cases} x & 0 < x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Then what is cdf (cumulative distribution function) of X. Compute the value of $P(0.3 < X \le 1.5)$ through cdf. [5]

3. (a) In an exam, there are n choices for each question and there is 10% negative marking. Expectation of marks of answering a question randomly is 0. Then compute the value of n.

(b) In a particular section of NET exam, a question of 5 marks has 4 choices and more than one options can be true. All possible outcomes are equally-likely. If there is no negative marking, what is the expectation of marks if that question is answered randomly? [2+3]

4. (a) Let X have the pdf $f(x) = 1,\ 0 < x < 1$ and 0 otherwise. Let $Y = e^X$. Then what is the pdf of Y.

(b) Let X be a Random variable with pmf $p_x(-2) = 1/5$, $p_x(-1) = 1/6$, $p_x(0) = 1/5$, $p_x(1) = 1/15$ and $p_x(2) = 11/30$, zero elsewhere. Let $Y = X^2$. Then what is pmf of Y

[3+2]

5. Let X_1 and X_2 be discrete random variables. Suppose the conditional pmf of X_1 given X_2 and the marginal pmf of X_2 are given by

$$p(x_1|x_2) = \binom{x_2}{x_1}\left(\frac{1}{2}\right)^{x_2}, \quad x_1 = 0, 1, 2 \ldots x_2$$

$$p(x_2) = \frac{2}{3}\left(\frac{1}{3}\right)^{x_2 - 1}, \quad x_2 = 1, 2, 3 \ldots$$

then what is the conditional mgf of X_1 *i.e.* $E(e^{tX_1}|x_2)$. [5]

6. Let X and Y have the joint pdf $f(x, y) = 6(1 - x - y)$, $x + y < 1$, $0 < x$, $0 < y$, zero elsewhere. Compute $P(2X + 3Y < 1)$ and $E(XY + 2X^2)$ [5]

Good Luck

MID-SEM 2022

Roll No. : ....

National Institute of Technology Jamshedpur
Department of Mathematics

Mid-Semester Examination, Nov 30, 2021

MCA 1st Semester | MA3103 : Probability and Statistical Computing
Time Limit: 02 Hours | Maximum Marks: 30 | Instructor: Dr. Mahendra Kumar Gupta

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Instructions: All questions are compulsory. All parts of a question should preferably be answered at one place. Assume missing data suitably.

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1. (a) A group of 6 friends A, B, C, D, E, F enter a row of seats in a concert hall. What is the probability that the A will sit next to B if all possible seating arrangements are equally likely?

(b) In part (a), suppose these six people go to a restaurant after the concert and sit at a round table. What is the probability that the A will sit next to B? [3+3]

2. A die is cast independently until the first x appears $\{x=$ (your roll number mod 6)+1$\}$. If the casting stops on an odd number of times, Bob wins; otherwise, Joe wins.

(a) Assuming the die is fair, what is the probability that Bob wins?

(b) If die is not fair, let p denote the probability of x. Show that the game favors Bob, for all p, $0 < p < 1$? [3+3]

3. (a) A die is cast once. If outcome is even then an integer number is chosen randomly from 1 to 11. If output is odd then a number is chosen randomly either 0 or 1. What is the probability that total of both is even?

(b) Probability of getting head on a biased coin is $\frac{2}{3}$ in one toss. The coin is tossed 5 times. Out of 5, getting head on exactly x times is $\frac{80}{243}$. What is the value of x? [3+3]

4. (a) State and prove Bay's Theorem.

(b) A person answers each of two multiple choice questions at random. If there are four possible choices on each question, what is the conditional probability that both answers are correct given that at least one is correct? [3+3]

5. (a) A bowl contains <your roll number+2> chips, of which 3 are marked \$2 each and rests are marked \$5 each. Let a person choose, at random and without replacement, three chips from this bowl. If the person is to receive the sum of the resulting amounts, find his expectation.

(b) Show that the moment generating function of the random variable X having the pdf $f(x) = 1/3$, $-1 < x < 2$, zero elsewhere, is

$$M(t) = \begin{cases} \frac{e^{2t}-e^{-t}}{3t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

[3+3]

Good Luck

MID-SEM 2021