No. of Pages: 1 Roll No.

National Institute of Technology, Jamshedpur

First Semester (Mid Sem) Examination October, 2023

Branch: MCA (1st Year-2023 Batch) Full Marks : 30

Subject: Mathematical Foundation of Computer Applications Time: 2 hours

Subject Code: MA3101

Answer all questions. The figures on the right hand margin indicate marks.

1. (a) Justify with Cantor's diagonal argument whether the set $(2,3)$ is countable or not. (5)

   (b) Prove that $2^{\aleph_0} = c$, where $\aleph_0$ and c represents the cardinality of the sets $\mathbb{N}$ and $\mathbb{R}$ respectively. (5)

2. (a) Use induction principle to prove that (5)

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1}n^2 = (-1)^{n-1}\frac{n(n+1)}{2}$$

   for all positive integer n.

   (b) Solve the recurrence relation $a_n + 3a_{n-1} - 4a_{n-2} = n2^n$, where $n \geq 2$ and the initial conditions are $a_0 = 1$ and $a_1 = 1$. (5)

3. (a) Let D_{30} denote the set of all positive divisors of 30. Prove that $(D_{30}, \preceq)$ is a poset where $a \preceq b$ iff $a|\ b$. Draw the Hasse diagram of the poset $(D_{30}, \preceq)$. (5)

   If $A = \{2, 10, 15\} \subset D_{30}$, find the least upper bound (LUB) and the greatest lower bound (GLB) of A, if they exists.

   A poset $(A, \preceq)$ is called a lattice if for every pair of elements of A, the LUB and GLB exists. Justify whether $(D_{30}, \preceq)$ is a lattice or not.

   (b) Four points are placed in the interior of a unit circle $x^2 + y^2 = 1$. Show that there always exists two points such that the distance between them is $< \sqrt{3}$. (5)

************* All the Best *************

MID-SEM 2023

No. of Pages: 1 Roll No. 2022PGCSCA103

National Institute of Technology, Jamshedpur

First Semester (Mid Sem) Examination October, 2022

Branch: MCA (1st Year-2022 Batch)
Subject: Mathematical Foundation of Computer Applications
Subject Code: MA3101

Full Marks : 30
Time: 2 hours

Answer all questions. The figures on the right hand margin indicate marks.

1. (a) Let the terms of the Fibonnaci sequence are $f_1, f_2, f_3, \cdots, f_n, \cdots$ where $f_1 = 1$ and $f_2 = 1$. Use mathematical induction to prove that for all integers $n \geq 3$, (5)

$$f_n \geq \left(\frac{1+\sqrt{5}}{2}\right)^{n-2}$$

(b) What is the Cantor's diagonal argument? Using it, show that the interval $(1, 2)$ is not a countable set. (5)

2. (a) Find the solution of the following recurrence relation for any arbitrary initial conditions. (5)

$$a_n + a_{n-1} = 6a_{n-2} + (3+4n)2^n, \quad n \geq 1.$$

(b) In each of the following cases, determine whether the given relation is an equivalence relation or not on the set $\mathbb{Z}$. If so, then find all the equivalence classes. (2×2.5)

(i) $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} |$ where$|a - b| \leq 4\}$

(ii) $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} |$ where $a - b$ is a multiple of 7$\}$

3. (a) Let $S = \{1, 2, 3\}$ be a set and T be the set of all proper non-empty subsets of S. Does the pair $(T, \subseteq)$ represent a poset? If so, draw its Hasse diagram. Justify whether the structure $(T, \subseteq)$ represents a Lattice or not with example. (5)

(b) If four points are placed in the interior of a unit circle (not from the boundary) $x^2 + y^2 = 1$, show that there are always exists two points such that the distance between them is less than $\sqrt{3}$. (5)

************* **All the Best** *************

MID-SEM 2022

No. of Pages: 1 Roll No. ____________________

National Institute of Technology, Jamshedpur

First Semester (Mid Sem) Examination November, 2021

Branch: MCA (1st Year-2021 Batch) Full Marks : 30
Subject: Mathematical Foundation of Computer Applications Time: 2 hours
Subject Code: MA3101 Course Instructor: Dr. Snehasis Kundu

Answer all questions. The figures on the right hand margin indicate marks.
Take N = 'last three digits of your roll number' in all questions if required.

1. Use mathematical induction to prove that for all integers $n \geq 1$, (5)

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n-1}n^2 = (-1)^{n-1}\frac{n(n+1)}{2}$$

2. Find the solution of the following recurrence relation for $n \geq 2$, with initial conditions $a_0 = 1$ and $a_1 = N$. (7)

$$\sqrt{a_n} = 2\sqrt{a_{n-1}} + 3\sqrt{a_{n-2}} + N3^n$$

3. (a) Let R_1 and R_2 be relations on a set A represented by the matrices (2+2+2)

$$M_{R_1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Find the matrices that represent (i) $R_1 \cap R_2$; (ii) $R_1 \cup R_2$; and (iii) $R_2 \circ R_1$.

(b) Find all the equivalence relations on the set $A = \{1, 2, 3\}$. (5)

4. Show that if 7 points are chosen inside or on the unit circle $x^2 + y^2 = 1$, then there is a pair of points which are at a distance at most one. (5)

5. The exponential generating function of a sequence $\{a_n\}$ is defined as $G_e(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!}x^n$. (2)
Find the exponential generating function for the sequence

$$b_n = \begin{cases} N & \text{when } n = 0, \\ N^n & \text{when } n \geq 1 \end{cases}$$

MID-SEM 2021

************* **All the Best** *************

No. of Pages: 2 Student Roll No. ____________________

National Institute of Technology, Jamshedpur
First Semester (End Sem) Examination January 2022

Branch: MCA (1st Year) Full Marks : 50
Subject: Mathematical Foundation of Computer Application Time: 3 hours
Subject Code: MA3101 Course Instructor: Dr. Snehasis Kundu

Answer all questions. The figures on the right hand margin indicate marks.
Take N = 'last three digits of your roll number' in all questions if required.

1. (a) Consider the poset $(S_{60}, \preceq)$ where set $S_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ and the partial order relation '$\preceq$' defined as the division relation on S_{60}. Draw the Hasse diagram of the poset. (5)

   (b) Find the complements of 2, 10 and 12 in above Hasse diagram. Find the Supremum and infimum of the subset $A = \{3, 6, 15\}$ in the poset $(S_{60}, \preceq)$ above. (3+2)

2. (a) Find the Adjacency matrix $A(G)$ and the Incidence matrix $M(G)$ for the graph in the **Figure 1** with $\{1, 2, 3, 4, 5\}$ and $\{e_1, e_2, ..., e_8, e_9\}$ as vertex and edge listing respectively. (5)

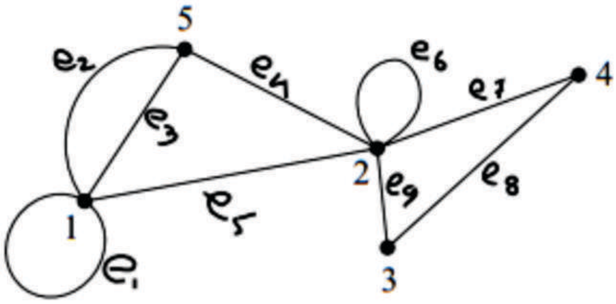

Figure 1

   (b) Find the shortest path from a to z in the graph G in **Figure 2** using Dijkstra's Algorithm. (5)

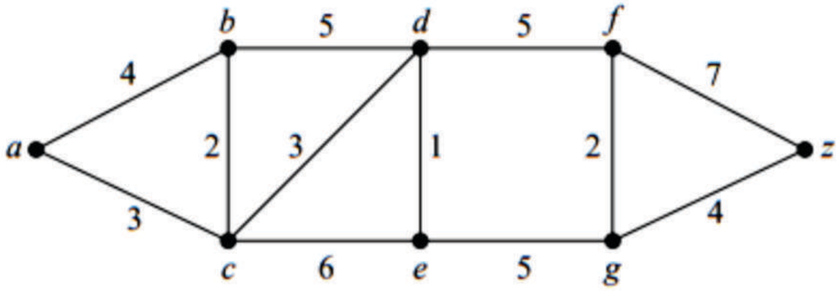

Figure 2

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3. (a) Find the minimal spanning tree of graph G in **Figure 3** using a suitable algorithm. Here $M = \left\lceil \frac{N}{10} \right\rceil$ (Box function). Write minimal weight. (5)

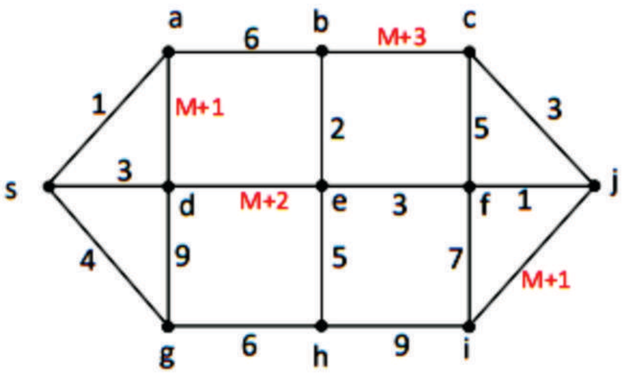

Figure 3

(b) Let G be a graph with 4 vertices $\{v_1, v_2, v_3, v_4\}$ and A be the adjacency matrix. Then find the number of the different $v_2 - v_4$ walks in G of length 4. (5)

$$A = (m_{ij}) = \begin{pmatrix} 1 & 0 & 1 & N \\ 0 & 2 & 0 & 1 \\ 1 & 0 & N & 3 \\ N & 1 & 3 & 1 \end{pmatrix}$$

4. (a) Use suitable colouring algorithm to colour the graph in **Figure 4** starting from the vertex u_1. (5)

(b) Justify whether the graphs G and G' in **Figure 5** are isomorphic or not. If so, find the bijection mapping. (5)

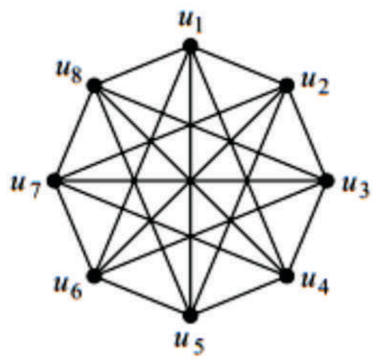

Figure 4

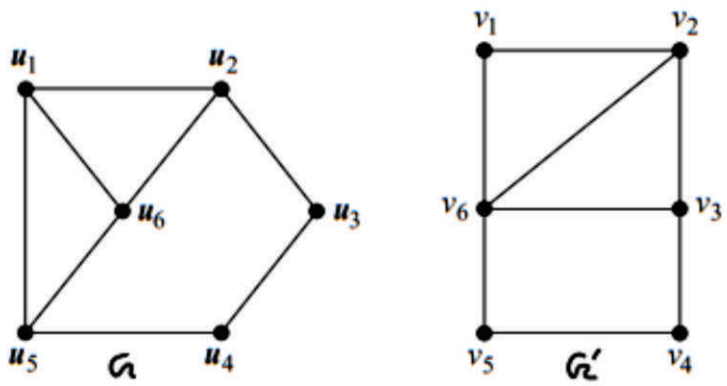

Figure 5

5. (a) Let G be a simple graph with N vertices and e edges. Give an upper bound for $2e$ with proper justification. (5)

(b) Construct a self-complementary graph of N vertices with justification. (5)

************* **All the Best** *************

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