NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR

Department of Mathematics

MID SEMESTER EXAMINATION FOR MCA 2022 BATCH, FEBRUARY 2023

Course Code: MA3201 Semester: 2nd Course Title: Numerical Methods

Date: 20-02-2023 (Time: 3.00 To 5.00) Day: Monday

Course Instructor (Name of the Faculty):Dr. Hari Shankar Prasad

Duration: 02 Hours Max. Marks: 30

Instructions:

(a) Answer to all the **Three** *(3)* questions.

(b) Marks of the questions are indicated in the right-hand margin.

(c) Missing data, if any, may be assumed suitably.

1. (a) Find the condition of convergence of sequence of approximations obtained in Fixed-Point Iteration method. Also obtain the root of the equation $\mathbf{3x - \log_{10} x = 6}$ using this Fixed-Point Iteration method correct up to three places of decimal. [2+3]

   (b) Obtain the condition of convergence of Newton Raphson method and show that Newton-Raphson method is of linear rate of convergence when applied to find the root of multiplicity greater than one. [2+3]

2. (a) Define the factorial polynomial of degree six (6) and show that the 3^{rd} difference of this 6^{th}degree polynomial is a polynomial of degree three (3). [5]

   (b) What do you mean by interpolation? Derive Newton forward difference interpolation formula. [1+4]

3. (a) Find the root of the equation $g(x) = 0$ using Newton Raphson method where $g(x) = \frac{f(x)}{[f(x)]'}$ with $f(x) = cosx - xe^x$. [5]

   (b) Solve the following system of equations by taking $x = y = z = 0$ as initial approximation in Gauss-Seidel iteration method. Perform five (5) iterations for the solution. [5]

$$\begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 16 \\ -6 \end{bmatrix}.$$

NATIONAL INSTITUTE OF TCHNOLOGY, JAMSHEDPUR

SPRING SEMESTER: 2023-24

Department of Mathematics

MID SEMESTER EXAMINATION FOR M. Sc 2023 BATCH, MARCH 2023

Course Code: MA3202 Semester: 2nd Course Title: Numerical Analysis & Lab.

Date: 18-03-2024 (Time: 10.00 To 12.00 AM) Day: Monday

Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad

Duration: 02 Hours Max. Marks: 30

Instructions:

(a) Answer to all the **Four** *(4)* questions.

(b) Marks of the questions are indicated in the right-hand margin.

(c) Missing data, if any, may be assumed suitably.

1. Find the condition of convergence of the sequence of approximations obtained by applying Fixed Point Iteration method for finding the solution of the equation: $f(x) = 0$. Can we apply the Fixed Point Iteration method for finding the root of the equation $3x - \log_{10} x = 6$ in $[2, 3]$? If yes, find the root correct up to three places of decimal. [6]

2. We wish to compute the root of the equation $e^{-x} - 3\log_e x = 0$ using a method with the formula $x_{n+1} = x_n - (3\log_e x_n - e^{-x_n})/p$. Find the rate of convergence of the method and Show that the rapid convergence is achieved when the value of p is near 3. [6]

3. (a) Find the 2^{nd} difference of the Polynomial: $f(x) = x^4 - 12x^3 + 42x^2 - 30x + 9$ with $h = 2$. [2]

   (b) Using interpolation process, find the value of x for which $y = 7$ with the help of following table: [2]

x	1	3	4
y	4	12	19

   (c) Find the missing term in the following table using interpolation process: [2]

x	0	1	2	3	4
$f(x)$	1	3	9	----	81

4. (a) Define the spline function of degree n and explain the process of quadratic spline fitting. [6]

   (b) Solve the following: [1.5x4]

   (i) If $\delta = \Delta E^{-\frac{1}{2}}$ then prove that $E = (\frac{\Delta}{\delta})^2$ (ii) Prove that $\mu\delta = \frac{\Delta}{2} + \frac{\Delta E^{-1}}{2}$

   (iii) Taking the interval of differencing $h = 1$, find $(\Delta + \nabla)^2 f(x)$ where $f(x) = x^2 + x$.

   (iv) If $\Delta f(x) = x^3 + 3x^2 + 5x + 12$ then find $f(x)$.

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