NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR
SPRING SEMESTER: 2022-23
Department of Mathematics
END SEMESTER EXAMINATION, APRIL 2023

Code: MA3201 Semester: 2nd Course Title: Numerical Methods
: 27-04-2023 Branch: MCA Day: Thursday
Course Instructor (Name of the Faculty): Dr. Hari Shankar Prasad
Duration: 03 Hours (A-Shift, Time: 9.30 AM To 12.30 PM) Max. Marks: 50

Instructions:
(a) Answer to all the **Six** *(6)* questions.
(b) Marks of the questions are indicated in the right-hand margin.
(c) Missing data, if any, may be assumed suitably.

1. (a) Define the rate of convergence of the iterative methods for finding the root of an algebraic and transcendental equations and, find the rate of convergence of Regula-Falsi method. [5]
   (b) Define the Orthogonality and Orthonormality of a set of functions $g_k(x)$, $k = 1,2,3,\ldots\ldots,n$ with respect to the weight function $\boldsymbol{w(x)}$ which are valid over an interval $[a, b]$, where n is a positive integer. Show that $g_k(x) = \frac{sinkx}{\sqrt{\pi}}$, $k = 1,2,3,\ldots..$ are orthonormal set of functions on $-\pi \le x \le \pi$.. [5]

2. (a) Write the Gram-Schmidt orthogonalization process for generating orthogonal polynomials which are defined over an interval $[a, b]$ with respect to the weight function $\boldsymbol{w(x)}$. [5]
   (b) Obtain the least square approximation of second degree for the function $\boldsymbol{f(x) = sinx}$ on $[\,0, \frac{\pi}{2}\,]$ with respect to the weight function $\boldsymbol{w(x) = 1}$. [5]

3. (a) Obtain the natural cubic spline approximation for the function $\boldsymbol{y(x)}$ defined by the data:

x	0	1	2	3
$y(x)$	1	2	33	244

and hence determine the value of $\boldsymbol{y(2.5)}$ and $\boldsymbol{y'(2.5)}$. [5]
   (b) Derive Newton divided difference interpolation polynomial and hence obtain the missing values in the following table: [5]

x	0	1	2	3
$f(x)$	1	3	.....	55

4. The following table gives the velocity **v** of a particle at time **t:**

t (seconds)	0	2	4	6	8	10	12
v(m/sec.)	4	6	16	34	60	94	136

Find the distance moved by the particle in **12** seconds using Simpson's **1/3rd** and 3/8th rule's of Integration respectively. [6]

5. Define the initial value problems and find the value of $\mathbf{y(0.1)}$ and $\mathbf{y(0.2)}$ using the Runge-Kutta method of order four with $\boldsymbol{h = 0.1}$, given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$. [6]

6. **Given that** $\frac{dy}{dx} = \frac{1}{2}(y - x^2), y(0) = 1, h = 0.2$, compute the values **of** $\boldsymbol{y(0.2)}$, $\boldsymbol{y(0.4)}$, ***and*** $\boldsymbol{y(0.6)}$ using Taylor's series method of order three first, and then, evaluate the value of $\boldsymbol{y(0.8)}$ using Milne's Predictor-Corrector method. [8]

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