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PROBABILITY DISTRIBUTION

Sol^n: 12 Given: $m = 1.5$.

To find: $P(X=0)$, $P(X>2)$

Solution: Let X denote no. of cars which are hired out per day

By Poisson's distribution,

$$P(X) = \frac{e^{-m} \cdot m^x}{x!}$$

$$P(x) = \frac{e^{-1.5}(1.5)^x}{x!}$$

$$\therefore P(0) = \frac{e^{-1.5}(1.5)^0}{0!} = \frac{1}{e^{1.5}} = 0.223$$

$$\therefore P(x>2) = 1 - P(x \leq 2)$$

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[0.223 + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$$

$$= 1 - [0.223 + 0.335 + 0.251]$$

$$= 1 - [0.809]$$

$$P(x>2) = 0.191$$

Proportion of days on which neither car is used $= 0.223 = 22.3\%$

Proportion of days on which some demand is refused $= 0.191 = 19.1\%$

Sol^n: 14) Given: $p = \frac{2}{5}$

To find: $P(x \geq 4)$

Solution: By Poisson's distribution,

$$P(x) = \frac{e^{-m} \cdot m^x}{x!}$$

$$P(x \geq 4) = 1 - P(x<4)$$
$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 1 - \left[\frac{e^{-m}.m^0}{0!} + \frac{e^{-m}.m^1}{1!} + \frac{e^{-m}.m^2}{2!} + \frac{e^{-m}.m^3}{3!}\right]$$
$$= 1 - \left[e^{-m} + me^{-m} + \frac{m^2}{2}e^{-m} + \frac{m^3 e^{-m}}{6}\right]$$

$$m = n \times p$$
$$m = 20 \times \frac{2}{5} = 4$$

$$= 1 - \left[e^{-8} + 8e^{-8} + \frac{\cancel{64}}{2}e^{-8} + \frac{\cancel{(8)^3}}{6}e^{-8}\right]$$
$$= 1 - [\cancel{0.531}]\ 0.042$$
$$P(x \geq 4) = \cancel{0.469}\ 0.958$$

The probability that at least 4 particles are recorded in a two minute period is 0.958.

Sol^n: 20) Given: $P(X=3) = \frac{1}{6}$; $P(X=2) = \frac{1}{3}$

To find: $P(X=0)$

Solution: By Poisson's distribution

$$P(X) = \frac{e^{-m}.m^x}{x!}$$

$$P(3) = \frac{e^{-m}.m^3}{6} = \frac{m^3 e^{-m}}{6} \quad \dots (i)$$

$$P(2) = \frac{e^{-m}m^2}{2} = \frac{m^2 e^{-m}}{2} \quad \dots (ii)$$

By Recurrence Relation of Probabilites,

$$P(3) = \frac{m}{3} P(2)$$

$$\frac{1}{6} = \frac{m}{3} \times \frac{1}{3} \quad \therefore \frac{1}{6} = \frac{m}{9} \quad \therefore m = \frac{3}{2}$$

$$P(0) = \frac{e^{-\frac{3}{2}}\left(\frac{3}{2}\right)^0}{0!} = e^{-\frac{3}{2}} = 0.223$$

1:17

x	f	xf	$P(X=x) = [e^{-m}.m^x/x!]$	$P(x)\Sigma xf$
0	142	0	0.367	140.8
1	156	156	0.367	146.8
2	69	138	0.183	73.2
3	27	81	0.061	24.4
4	5	20	0.015	6
5	1	5	0.003	1.2
	$\Sigma f = 400$	$\Sigma xf = 400$		$\Sigma P(X)\Sigma f(x) = 398.4$

$$\therefore m = \frac{\Sigma xf}{\Sigma f} = \frac{400}{400} = 1$$

Sol^n:2) Given: $P(X=1) = 2P(X=2)$

To find: $P(X=3)$

Solution: By Poisson distribution

$$P(X) = \frac{e^{-m}.m^x}{x!}$$

$$P(1) = e^{-m}.m \quad \dots (i)$$

$$P(2) = \frac{e^{-m}.m^2}{2} \quad \dots (ii)$$

$$\therefore me^{-m} = \frac{2m^2e^{-m}}{2}$$

$$\therefore m = m^2$$

$$\therefore m = 1$$

$$P(3) = \frac{e^{-1}(1)^3}{3!} = \frac{e^{-1}}{6} = 0.061$$

$P(X=3) = 0.061$

Solⁿ: 10) Given: $m = 100$, $\sigma = 2$

To find: $P(98 \le x \le 102)$

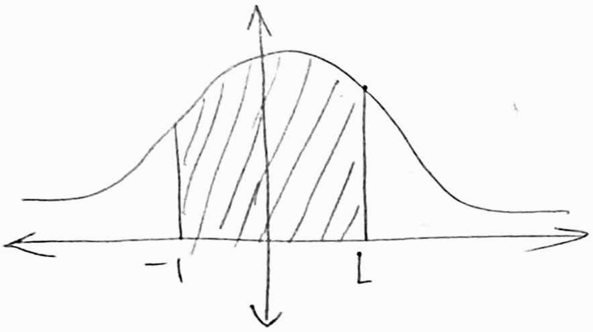
Solution: Let x be resistance of resist in ohms

$\therefore Z = \frac{x-m}{\sigma}$

$\therefore Z = \frac{98-100}{2} = \frac{-2}{2} = -1$

$\therefore Z = \frac{102-100}{2} = \frac{2}{2} = 1$

$\therefore P(98 \le x \le 102) = P(-1 \le Z \le 1)$



= Area between $z=0$ and $z=-1$
+ Area between $z=0$ and $z=1$
= 2 (Area between $z=0$ and $z=1$)
$\because$ curve is symmetric

$= 2 \times 0.3413$
$= 0.6826$

$\therefore$ 68.26% of resistors will have the resistance between 98 and 102 ohms.

Solⁿ: 11) Given: $n = 2000$, $m = 1000$, $\sigma = 200$

To find: $P(X \le 700)$.

Solution: Let x be the burning hours of the lamp

(i) By ~~Pois~~ Normal Standard Curve,

$Z = \frac{x-m}{\sigma}$

$\therefore Z = \frac{700-1000}{200} = \frac{-300}{200} = -1.5$

$\therefore P(x \le 700) = P(Z \le -1.5)$
$= 0.5 -$ Area between $z=0$ and $z=1.5$ ... curve is Symmetric

$= 0.5 - 0.4332$

$= 0.0668 \approx 0.067$

$\therefore$ 134 lamps will be expected to fail in first 700 hours.

(ii) X when 10% of the lamps have failed.

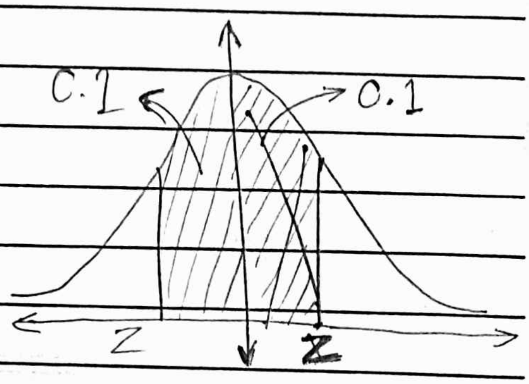
$P(X) = 0.1$

$\therefore Z = 0.26$

$Z = \dfrac{x - m}{\sigma}$

$\therefore -0.26 = \dfrac{x - 1000}{200}$

$\therefore x = \cancel{1052}\ 948$



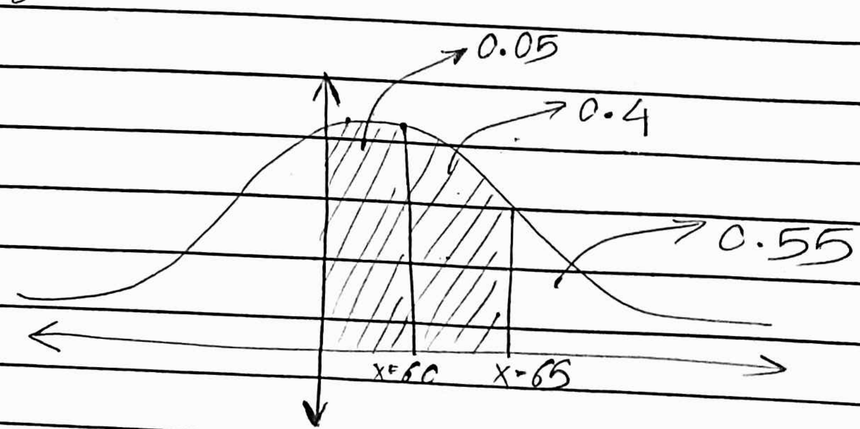
~~1052~~ 948 hours when ~~fi~~ 10% of the lamps have failed

Soln: 15) Given: $P(X<60) = 5\%$

$P(60<X<65) = 40\%$

To find: m, σ

Solution:



Area between $X=0$ and $X=60 = 0.05$

$\therefore Z_1 = 0.13$

Area between $X=0$ and $X=65 = 0.45$

$\therefore Z_2 = 1.66$

$\therefore\ 1.65 = \dfrac{65-m}{\sigma}$ and $0.13 = \dfrac{60-m}{\sigma}$

$\therefore\ m + 1.65\sigma = 65 \ \ldots(i)$

$\therefore\ m + 0.13\sigma = 60 \ \ldots(ii)$

$\therefore\ m = 59.57\ ,\ \sigma = 3.29$