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Poisson distribution:
for a random variable X, having mean m its Poisson distribution is given by:

$$P(X=x) = \frac{e^{-m} \cdot m^{x}}{x!} \quad \text{for } x = 1, 2, 3, 4, \ldots$$

1.) $n \to \infty$ (no. of trials) $p \to 0$ (success)
2.) For any trial, $p \to$ success, $q \to$ failure $p+q=1$
3.) Mean = variance = m
4.) For any given trial - $m = n \times p$. (mean)

Notation: $P(X=x) = p(x)$
$x=0$, $P(X=0) = p(0)$

Q.) Avg no. of customers who appear at a counter at a certain bank, per minute is 2. Find probability that during a given minute:
i.) No customer appears
ii.) 3 or more customers appear.

Binomial finite sample space coin toss

Poisson sample space ∞ death / no death

Example

→ ~~Poisson distribution~~

*1) $m = 2$

By Poisson Distribution:

$$P(X=x) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-2} \cdot 2^{x}}{x!} \quad x = 0, 1, 2, 3, \ldots$$

1.) no customer appears $\Rightarrow x=0$

$$P(X=0) = \frac{e^{-2} \cdot 2^{0}}{0!} = e^{-2} = 0.1353$$

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11.) 3 or more customers

$X \geq 3$

$= P(X \geq 3)$

$= 1 - P(X < 3)$

$= 1 - P(0) - P(1) - P(2)$

$= 1 - \frac{e^{-2} \cdot 2^0}{0!} - \frac{e^{-2} \cdot 2^1}{1!} - \frac{e^{-2} \cdot 2^2}{2!}$

$= 1 - e^{-2}(1 + 2 + 2)$

$= 1 - 5 \cdot e^{-2}$

$= \cancel{0.8646} = 0.3233$

Q.) If mean of poisson distribution is 2. find propability of $x = 1, 2, 3, 4$ using recurrence relation of probability

→ $m = 2$

$P(x) = \frac{e^{-m} \cdot m^x}{x!} = \frac{e^{-2} \cdot 2^x}{x!}$

$x = 0, 1, 2, 3, 4$

~~P(X)=/k/~~ recurrence relation of given formula

$\left\{ p(x+1) = \frac{m}{x+1} p(x) \right\}$

$P(0) = \frac{e^{-2} \cdot 2^0}{0!} = e^{-2} = \underline{0.1353}$

$P(1) = 2\, p(0+1) = \frac{2}{0+1} p(0) = 2 \cdot (0.1353)$

$p(1) = \underline{0.2706}$

$p(2) = p(1+1) = \frac{2}{1+1} p(1) = p(1) = \underline{0.2706}$

$p(3) = p(2+1) = \frac{2}{2+1} p(2) = \frac{2}{3} \cdot (0.2706) = \underline{0.1804}$

$p(4) = P(3+1) = \frac{2}{3+1} p(3) = \frac{1}{2} p(3) = \underline{0.0902}$

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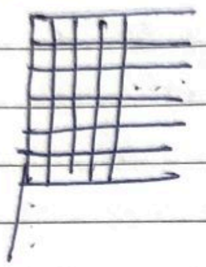
Q.) If a firm produces articles of which 0.1% are defective. It packs them in cases containing 500 articles. If a wholeseller purchases 100 such cases how many cases can be expected:

1.) To free from any defect
2.) To have 1 defect

→ 

500 articles $\frac{1 \text{ case}}{1 \text{ box}} \times 100$ cases (wholeseller)

P (defective articles) = 0.1%

$= \frac{0.1}{100} = 0.001$

P (~~number of~~ non defective) = 1 − 0.001

= 0.999

proceed by first finding no. of defective articles in 1 case (box)

1 box:

$m = n \times p$

$= 500 \times 0.001$

$= 0.5$

Poisson distribution given by

$P(X = x) = \frac{e^{-0.5}(0.5)^x}{x!}$

1.) No defect

$X = 0 \Rightarrow P(X = 0) = \frac{e^{-0.5}(0.5)^0}{0!} = 0.6065$

Expected ~~probability for~~ no. of cases to be free from any defect $= N \times P(X=0)$

$= 100 \times 0.6065$

$= 60.65 = 61$ cases

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11.) 1 defect $\Rightarrow X = 1$

$$P(X = \cancel{0}) = \frac{e^{-0.5}(0.5)^1}{1!} = 0.3032$$

expected no. of cases to ~~be~~ have 1 defect

$= N \times P(X=1)$

$= 100 \times 0.3032$

$= 30.32$

$= 30$ cases

$n \to$ individual case

$N \to$ whole

Q.16) A skilled typist on routine work kept record of mistakes per day during 300 working days.

Mistakes/day	0	1	2	3	4	5	6
day	143	90	42	12	9	3	1

fit a poisson distribution to the above data & hence calculate theoretical frequencies

$\to$ To fit a poisson curve means to find expected frequency when $x = 0, 1, 2, 3, 4, 5, 6$

$N = 300$

formula for poisson distribution is $\frac{e^{-m} m^x}{x!} = P(X = x)$

$x = 0, 1, 2, \ldots, 6$

$$m = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0 \times 143 + 1 \times 90 + 2 \times 42 + 3 \times 12 + 4 \times 9 + 5 \times 3 + 6 \times 1}{143 + 90 + 42 + 12 + 9 + 3 + 1}$$

$m = 0.89$

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$$P(X=x) = \frac{e^{-0.89} \times (0.89)^x}{x!}$$

Expected frequency when $x=0$;

$= N \times P(X=0)$

$= 300 \times \frac{e^{-0.89}(0.89)^0}{0!}$

$= 123.196$

≈ 123.

Expected frequency when $x=1$

$= N \times P(x=1)$

$= \frac{300 \times e^{-0.89} \times (0.89)^1}{1!}$

$= 109.645$

$\approx$ ~~10 110 days~~ ~~109 day~~ 110 days

Expected frequency when $x=2$

$= N \times P(x=2)$

$= \frac{300 \times e^{-0.89} \times (0.89)^2}{2!}$

$= 48.79$

$=$ 49 days

expected frequency when $x=3$

$= N \times P(x=2)$

$= \frac{300 \times e^{-0.89} \times (0.89)^3}{3!}$

$=$ 14 days

expected frequency when $x = 4$

$$= N \times P(x=4)$$

$$= \frac{300 \times e^{-0.89} \times (0.89)^4}{4!}$$

$$= 3 \text{ days}$$

expected frequency when $x = 5$

$$= N \times P(x=5)$$

$$= \frac{300 \times (e^{-0.89}) \times (0.89)^5}{5!}$$

$$= 1 \text{ day}$$

Normal distribution

Let x be a random variable with parameter m & variance σ^2, then normal distribution given by

$$f(x) = \frac{1}{(\sqrt{2\pi})\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \qquad -\infty < x, m < \infty$$

$$\sigma^2 \geq 0$$

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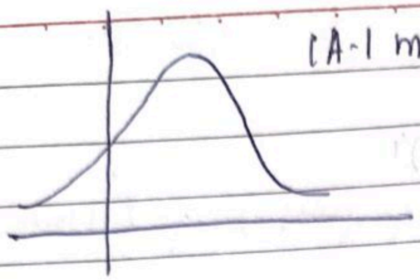
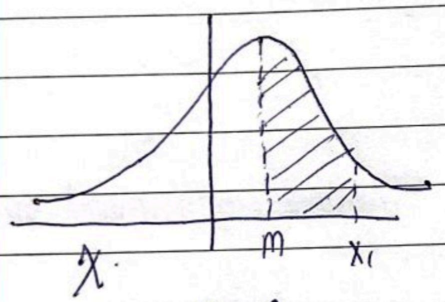


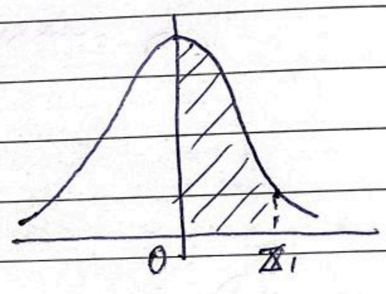
Note-1 If normal distribution is a standard normal distribution, then $Z = \frac{X - m}{\sigma}$

m: mean σ: standard deviation

Note-2 Real data → Z data → worked upon → Real data



Real.



Z

$$f(x) = \frac{1}{(\sqrt{2\pi})\sigma} \int_{m}^{x_1} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} \qquad \frac{1}{(\sqrt{2\pi})\sigma} \int_{m}^{z_1} e^{-\frac{1}{2}z^2}$$

Q.8) i) $z = 0$ & $z = 1.2$

ii) $z = -0.68$ & $z = 0$

iii) $z = -0.48$ to $z = 2.21$

iv) $z = 0.81$ to $z = 1.94$

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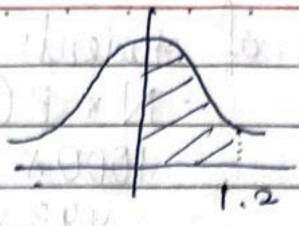
1) $z = 0$ & $z = 1.2$

$P(0 < z < 1.2)$

$= 0.3849$

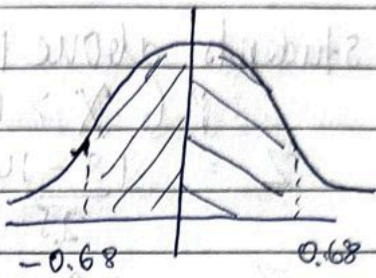


ii) $P(-0.68 < z < 0)$

By Symmetry

$= P(0 < z < 0.68)$

$= 0.2517$



Q.6) In a sample of 1000 cases, the mean of a certain test is 14 & SD is 2.5. Assuming the distribution to be normal, find

1.) how many students between 12 & 15

2.) how many score above 18

3.) how many score below 18

$N = 1000$, $m = 14$, $\sigma = 2.5$

As distribution is normal

$$\therefore Z = \frac{X - m}{\sigma} = \frac{X - 14}{2.5}$$

1.) no. of students between 12 & 15

$P(12 < X < 15) =$

when $X = 12$, $Z = -0.8$

$X = 15$, $Z = 0.4$

$$P(12 < X < 15) = P(-0.8 < Z < 0.4)$$
$$= P(-0.8 < Z < 0) + P(0 < Z < 0.4)$$
$$= P(0 < z < 0.8) + P(0 < z < 0.4)$$
$$= 0.2881 + 0.1554$$
$$= 0.4435.$$

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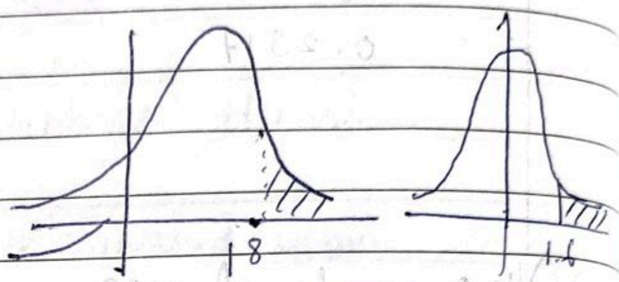
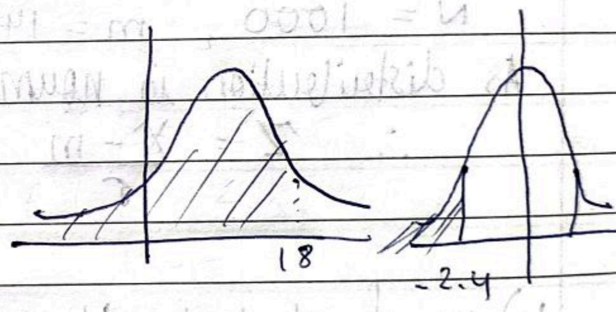
no. of students using b/w 12 & 15
$= N \times P(12 < X < 15)$
$= 1000 \times 0.4435$
$= 443.5$
≈ 443 students.

2.) students above 18
$P(X > 18)$
$Z = \frac{18-14}{2.5}$
$Z = 1.6$



$P(X > 18) = P(Z > 1.6)$
$= 0.5 - P(0 < Z < 1.6)$
$= 0.5 - 0.4452$
$= 0.0548$

no. students b > 18
$= N \times P(X > 18)$
$= 1000 \times 0.0548$
$= 54.8$
≈ 55 students



3.) Students below 8 ≈ 8 students ans
$P(X < 8)$
$Z = \frac{8-14}{2.5} = -2.4$

$P(0 < X < 8) = P(0 < Z < 1.8)$
$= P(-2.4 < Z < 0)$
$= P(0 > Z > 2.4)$
$= 0.4918$

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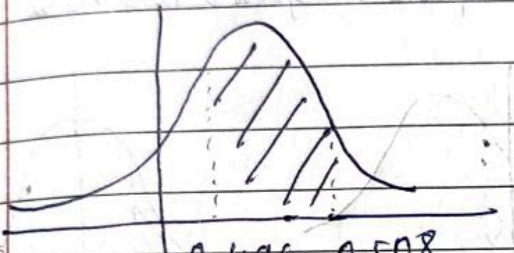
Q.7) Mean inside diameter of a sample of 200 washers produced by a machine is 0.502 cm & SD 0.005. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 cm, otherwise the washers are considered defective. Determine the %age of defective washers produced by the machine, assuming the diameters are normally distributed.

→ $N = 200$, $m = 0.502$ cm, $\sigma = 0.005$.



0.496 0.508

As distribution is normal

$$Z = \frac{X - m}{\sigma} = \frac{X - 0.502}{0.005}$$

no. of washers between 0.496 & 0.508 is

$P(0.496 > X > 0.508)$

$$Z = \frac{0.496 - 0.502}{0.005} = -1.2 \qquad Z = \frac{0.508 - 0.502}{0.005} = 1.2$$

$$\begin{aligned} P(0.496 > X > 0.508) &= P(-1.2 > Z > 1.2) \\ &= P(-1.2 > Z > 0) + P(0 < Z < 1.2) \\ &= 2 \times P(0 < Z < 1.2) \\ &= 2 \times 0.3849 \\ &= 0.7698 \end{aligned}$$

no. of nondefective washers ~~btw~~ $= N \times P(0.496 > X > 0.508)$

$= 200 \times 0.7698 = 153.96 \approx 154$

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no. of non defective = 154

no. of dective = 200 − 154

$= \underline{46}$.

% of defective $= \frac{46}{200} \times 100 = 23\%$.

3) ii) $N = 2000$, $m = 1000$, $\sigma = 200$

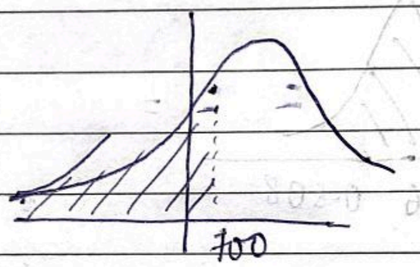
→ i) $Z = \frac{X - m}{\sigma}$ (∵ Distribution is normal)

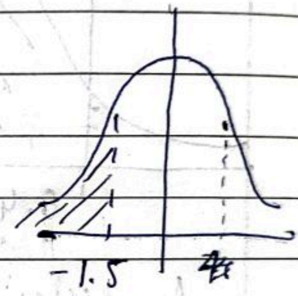
P no. of lamps expected to fail in first 700 hrs

$P(X < 700)$

$Z = \frac{700 - 1000}{200}$

$Z = -1.5$





$P(X < 700) = P(Z < -1.5)$

$P(Z < -1.5) = 0.5 - P(-1.5 < Z < 0)$

$= 0.5 - P(0 > Z > 1.5)$

$= 0.5 - 0.4332$

$= 0.0668$

no. of lamps expected to fail in first 700 hrs

$N \times P(X < 700)$

$= 2000 \times 0.0668$

$= 133.6$

≈ 134 lamps

ii)

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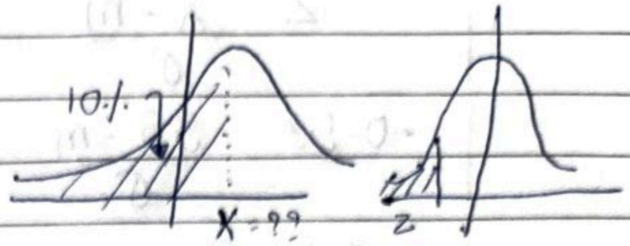
11.) $Z = \frac{X - m}{\sigma}$

~~10% of 2000 = $\frac{10}{100} \times 2000$~~

~~$p = 200$~~



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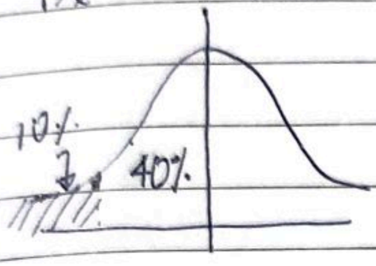


40% = 0.4

~~0.1~~ 40% of area, 0.4 area.
corresponds to $Z = -1.28$.

$$Z = \frac{X - m}{\sigma}$$

$$X = Z\sigma + m$$

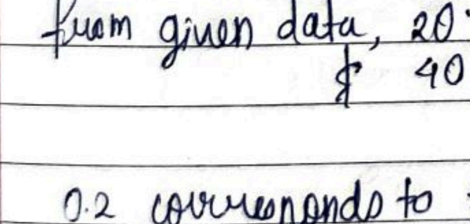
$X = 744$ (10% lamps failed)

~~$X = 1000 - 744$ (10% lamps failed)~~

after 744 hrs 10% lamps failed.

8.) Marks obtained by students in an examination follow normal distribution. If 30% of students got < 35 marks & 10% of students > 60 marks find m & σ.

→



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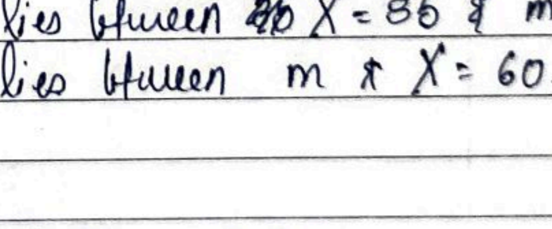

from given data, 20% of area lies between X = 35 & m
& 40% of area lies between m & X = 60.

0.2 corresponds to $z = 0.52$
but ordinate is -ve $\therefore z = -0.52$

0.4 corresponds to $Z = 1.28$

$$Z = \frac{X - m}{\sigma} \quad (\because \text{ curve is normal}).$$

$$-0.53 = \frac{35 - m}{\sigma}$$

$$0.53\sigma - m + 35 = 0 \quad — (1)$$

$$1.28 = \cancel{X/1+}\ \frac{60 - m}{\sigma}$$

$$1.28\sigma + m - 60 = 0 \quad — (2)$$

from (1) & (2)

$m = 42.276 \sim 42$ marks

$\sigma = 13.736$

Sampling Theory: