from ① & ②

$m = 42.276 \sim 42$ marks

$\sigma = 13.736$

Sampling Theory:

Testing of hypothesis

Q.3) The mean weekly sales of a chocolate bar in a candy store were 146.3 bars per store. After an advertising campaign, the mean weekly sales in 22 stores for a typical week increased to 153.7 & showed standard deviation of 17.2. Was the advertising campaign successful

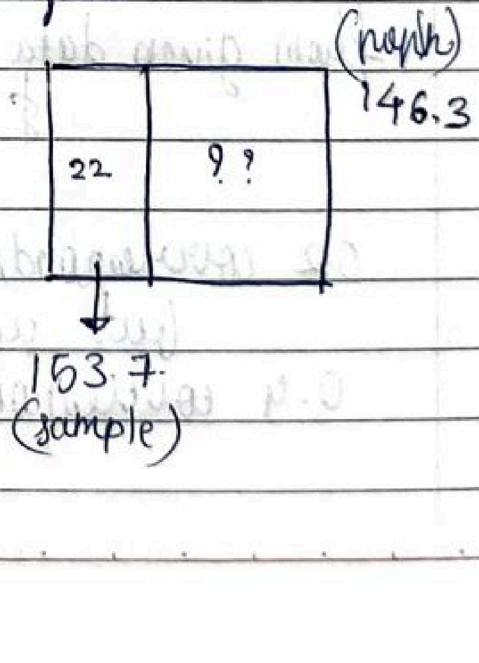
→

$\mu = 146.3$ (popln mean)

$\bar{X} = 153.7$ (sample mean)

$n = 22$

$s = 17.2$



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Null hypothesis - H_0

$\mu = 146.3$

Alternate hypothesis - H_a

$\mu \neq 146.3$

Test statistic:

As $n < 30$,

we proceed by t distribution

$$|t| = \left| \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \right| = \left| \frac{-146.3 + 153.7}{17.2/\sqrt{22-1}} \right|$$

$$|t| = 1.9715$$

level of significance: (los)

At 5% level of significance

$\alpha = 0.05$

degree of freedom (dof)

dof $= n - 1$

$= 22 - 1$

$= 21$

dof:

$x_1 + x_2 = 0$ — 2 var., 1 eqn, $2-1 = 1$ LIS, dof $= 1$

$x_1 + x_2 + x_3 = 0$ — 3 var., 1 eqn, $3-1 = 2$ LIS, dof $= 2$

22 var, 1 eqn, $22-1 = 21$ LIS, dof 21

Critical value of t_α at 5% los & 21 dof is equal to

<from t-distribution log table>

$= 2.080$

Decision:

As the calculated value of $t = 1.9715$ is less than critical value, $t_\alpha = ~~0.~~ 2.080$

$t < t_\alpha$

$\therefore$ Null hypothesis is accepted

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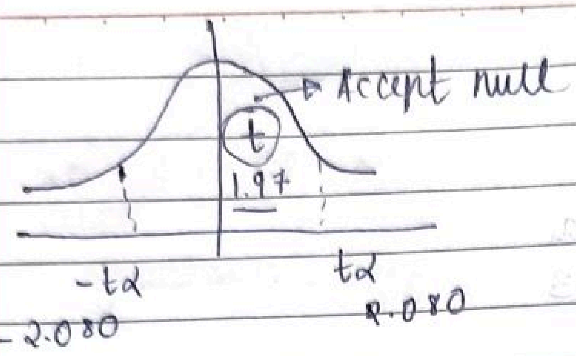


Conclusion:
mean weekly sales of population is $\mu = 146.3$ only
$\therefore$ advertising campaign was not successful.

Q.) 10 individuals are chosen at random from a population & their heights are found in inches

63 63 64 65 66 69 69 70 70 71

In the light of the foll. data, discuss the suggestion that mean height in population is 66 inches

null hypothesis (H_0)
$\mu = 66$

Alternate hypothesis (H_a) $\mu \neq 66$

Test statistic
as $n = 10 < 30$
$\therefore$ we proceed by t distribution

$$|t| = \left| \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \right|$$

to find s & $\bar{x}$

Sr.	Height	d_i $X-67$	d_i^2
1	63	-4	16
2	63	-4	16
3	64	-3	9
4	65	-2	4
5	66	-1	1
6	69	2	4
7	69	2	4
8	70	3	9
9	70	3	9
10	71	4	16
$N=10$	670	$\Sigma d_i = 0$	$\Sigma d_i^2 = 88$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{670}{10} = 67.$$

$$s^2 = \frac{\Sigma (X_i - \bar{X})^2}{n}$$ { sum of square of deviation from mean)

$$s^2 = \Sigma d_i^2 - \frac{(\Sigma d_i)^2}{N}$$

$$= 88 - \frac{(0)^2}{10}$$

df	los (0.05)
(9)	

$$s^2 = 88$$

$$s = 9.380\cancel{4}8$$

$$|t| = \left| \frac{\bar{X} - \mu}{s/\sqrt{n-1}} \right| = \left| \frac{67 - 66}{9.3808/\sqrt{10-1}} \right|$$

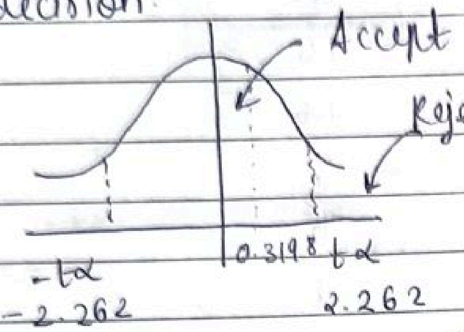
$$= 0.3198.$$

let 5% los $\Rightarrow \alpha = 0.05.$

dof $= n-1 = 10-1 = 9.$

Critical value of t at 5% los & 9 dof is = 2.262

decision



As the calculated value of t is less than critical value

$t_{cal} < t_{critical}$

$0.3198 < 2.262$

Null hypothesis is accepted

∴ mean height of population is 66 inches

* t distribution for 2 samples:

1.) Non-independent sample.

there is exactly one sample & two tests are being conducted.

Ex: IA1 & IA2 marks of Comp-B

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n-1}}$$

$\mu = 0$ { since population same for both sample }

2.) Independent Sample

Ex: IA1 marks of Comp-A & Comp-B

there is exactly one test & two sets are conducted tests upon

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S.E}$$

$$SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

when standard deviation.

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

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when unbiased standard deviation is given

$$S_p = \sqrt{\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}}$$

when sum of squares of deviation from mean given

$$S_p = \sqrt{\frac{\Sigma (x_{1i} - \bar{x}_1)^2 + \Sigma (x_{2i} - \bar{x})^2}{n_1 + n_2 - 2}}$$

Q.) Samples of tubes of two companies were tested for length of their lifes

	Company A	Company B.
~~A~~ No. of sample	8	7
~~B~~ mean life	1210	1314
Standard deviation	36	42

test as 5% level of significance whether diff. btween samples is significant.

<THIS IS AN EX OF INDEPENDENT SAMPLES SO WE PROCEED AS FOLLOWS>.

Null hypothesis (H_0): $\mu_1 = \mu_2$

Alternate hypothesis (H_a): $\mu_1 \neq \mu_2$

Test Statistic:

$$t = \left| \frac{\bar{x}_1 - \bar{x}_2}{SE} \right| \quad SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{8(36)^2 + 7(42)^2}{8 + 7 - 2}}$$

$$= 41.8017$$

$$SE = 41.8017 \sqrt{\frac{1}{8} + \frac{1}{7}} = 21.6344$$

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$$t = \left| \frac{\bar{x}_1 - \bar{x}_2}{SE} \right|$$

$$= \left| \frac{1210 - 1314}{21.6344} \right|$$

$$t = 4.8071$$

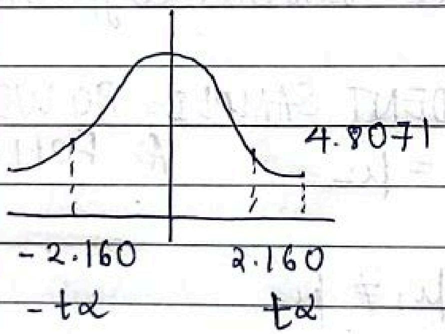
level of significance = 5%

$\therefore \alpha = 0.05$

$$d.of = n_1 + n_2 - 2$$
$$= 8 + 7 - 2$$
$$= 13$$

critical value of t at ~~to~~ dof 13 & $\alpha = 0.05$

$$= 2.160$$



As calculated value of t greater than critical value

$\Rightarrow 4.8071 > 2.160$

$\therefore$ Null hypothesis is rejected

Conclusion:

mean production of company A & B isn't the same & diff. btween sample is significant table dika , 3rd formula

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1(d.) 2 independent samples of sizes 8 & 7 contained the following values

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16.	

$n_1 = 8$, $n_2 = 7$

As sample is independent, we proceed as follows.

Null hypothesis (H_0) : $\mu_1 = \mu_2$

Alternate hypothesis (H_a): $\mu \neq \mu_2$.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{SE} \qquad SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p = \sqrt{\frac{\Sigma(x_{1i} - \bar{x}_1)^2 + (\Sigma x_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

Sr.	X_1	d_i $\bar{X}_1 - 17$	d_i^2	X_2	d_2 $\bar{X}_2 - 16$		d_2^2
1	19	2	4	15	-1	4	1
2	17	0	0	14	-2	4	4
3	15	-2	4	15	-1	4	1
4	21	4	16	19	3	4	9
5	16	-1	1	15	-1	4	1
6	18	1	1	18	2		4
7	16	-1	1	16	0		0
8	14	-3	9				
	136	0	36	112	0		20

$$\bar{X}_1 = \frac{\Sigma X_1}{n} = \frac{136}{8} = 17 \qquad \bar{X}_2 = \frac{\Sigma X_2}{n} = \frac{112}{7} = 16$$

$$\Sigma (X_{1i} - \bar{X}_1)^2 = \Sigma d_1^2 - \frac{(\Sigma d_1)^2}{N_1}$$

$$= 36.$$

$$\Sigma (x_{2i} - \bar{x}_2) = \Sigma d_2^2 - \frac{(\Sigma d_2)^2}{N_2}$$

$$= 20$$

$$s_p = \sqrt{\frac{\Sigma (X_{1i} - X_1)^2 + \Sigma (X_{2i} - \bar{X}_2)^2}{n_1 + n_2 - 2}} = 2.075$$

$$SE = sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.0739$$

$$t = 0.9311$$

Sr.	x_i	d_1: $x_1 - 17$	d_i^2	x_2	d_2: $x_2 - 16$	d_2^2
1	19	2	4	15	-1	1
2	17	0	0	14	-2	4
3	15	-2	4	15	-1	1
4	21	4	16	19	3	9
5	16	-1	1	15	-1	1
6	18	1	1	18	2	4
7	16	-1	1	16	0	0
8	14	-3	9			
	136	0	36	112	0	20

$\bar{x_1} = \frac{\Sigma x_1}{n} = \frac{136}{8} = 17$ $\quad \bar{x_2} = \frac{\Sigma x_2}{n} = \frac{112}{7} = 16$

$\Sigma (x_{1i} - \bar{x})^2 = \Sigma d_1^2 - \frac{(\Sigma d_1)^2}{N_1} = 36$

$\Sigma (x_{2i} - \bar{x_2})^2 = \Sigma d_2^2 - \frac{(\Sigma d_2)^2}{N_2} = 20$

$S_p = \sqrt{\frac{\Sigma (x_{1i} - \bar{x_1})^2 + \Sigma (x_{2i} - \bar{x})^2}{n_1 + n_2 - 2}} = \sqrt{\frac{36 + 20}{8 + 7 - 2}} = 2.0754$

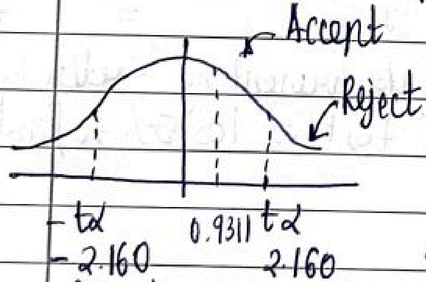
$SE = S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2.075 \sqrt{\frac{1}{8} + \frac{1}{7}} = 1.0741$

$t = \frac{\bar{x_1} - \bar{x_2}}{SE} = \frac{17 - 16}{1.0741} = 0.9311$

level of significance $= \alpha = 5\% = 0.05$

dof $= n_1 + n_2 - 2 = 7 + 8 - 2 = 13$

critical value of t at dof 13 & α 0.05 is 2.160



As calculated value of t less than critical value

$\Rightarrow 0.9311 < 2.160$

$\therefore$ Null hypothesis is accepted

Conclusion: Mean value of sample A & B is same

14.) Q.) 10 students were given intensive coaching & 4 tests were conducted in a month

Marks Test 1 : 50 42 51 42 60 41 70 55 62 38

_/_/_

Marks Test 2 : 62 40 61 52 68 51 64 63 72 50

Do the scores from test1 & test 4 show improve significantly test at los 5%

< Non independent example > null hypothesis $(H_0): \mu = 0$ $\quad H_a: \mu \neq 0$

Test Statistic: $|t| = \left|\frac{\bar{x}-\mu}{s/\sqrt{n-1}}\right|$ $\quad \bar{x}+7 \quad \bar{x}-a$ $= \left|\frac{-7.2-0}{5.8103/\sqrt{10-1}}\right| = 3.7175$

Sr. No.	X_1	X_2	$X (X_1 - X_2)$	d_i	d_i^2
1	50	62	-12	-5 -~~19~~	25
2	42	40	2	9 ~~8~~	81
3	51	61	-10	-3 -~~17~~	9
4	42	52	-10	-3 -~~17~~	9
5	60	68	-8	-1 -~~15~~	1
6	41	51	-10	-3 -~~17~~	9
7	70	64	6	13 -~~1~~	169
8	55	63	-8	-1 -~~15~~	1
9	62	72	-10	-3 -~~17~~	9
10	38	50	-12	-5 -~~19~~	25
N=10	511	583	-72	-2	338

$\Sigma X = -72$

$\bar{X} = \Sigma X / N = -7.2$

let $a = -7$.

$\bar{x} = a + \frac{\Sigma d_i}{N} = -7 - \frac{2}{10} = -7.2$

$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$

$\Sigma (x_i - \bar{x})^2 = \Sigma d_i^2 - (\Sigma d_i)^2 / N$

$= 338 - (-2)^2/10$

$s^2 = 337.6/10 = 33.76$

$s = 5.8103$

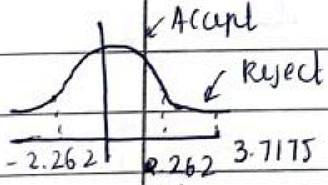
LOS = 5% $\Rightarrow \alpha = 0.05$ $\quad$ dof = $n-1 = 10-1 = 9$

critical value for dof 9 & $\alpha = 0.05$ is 2.262

$\therefore$ calculated value of t greater than critical value.

$\Rightarrow 3.7175 > 2.262$ $\therefore$ null hypothesis rejected

$\therefore \mu \neq 0$ after coaching marks show improvement



condition of child	Condition of Home	
	Clean	Dirty
clean	70	50
Fair	80	20
dirty	35	45

χ^2 test: (Chi square)

Contingency table:

Q.) Calculate expected frequency of foll. data presenting 2 attributes condition of home & child is independent. Use χ^2 test at los 5% to find if 2 attributes are independent.