Let's say that there are two numbers, a and b. Let's say GCD(a,b)=x.

Surely one number must the greater than the another. Let's say that a is greater than b.

If we divide a by b then, we get q as quotient and r as remainder.

Now,

$a=bq+r$

GCD(a,b)=x must divide both a and b.

a must be divisible by x, leaving remainder as 0.

By the equation, $(bq+r)$ must be divisible by x, since $a=bq+r$.

By definition, x should divide b. So, x also divides bq, since q is just another integer.

Now, for $(bq+r)$ to be divisible by x, it's intuitive and clear that r should be divisible by x.

Now the theorem follows, GCD(a,b)=GCD(b,r)=GCD(a,r)

Usually, GCD(b,r) is taken, since b is smaller than a, thereby making life easy.

Now again, GCD(b,r) is treated as like the mirror GCD(a,b) and this process continues until the remainder r becomes zero.

The last non-zero remainder r is the value of the GCD of the original numbers, GCD(a,b).