

1. (3 points) Convert the following units:

Unit 1		Unit 2	
1	Kg	1000	mg
20	nm	200	Å
0.5	kN	500	N
2	MPa	$2 \times 10^6$	Pa
5	m <sup>2</sup>	50000	cm <sup>2</sup>
3000	cm <sup>3</sup>	$3 \times 10^{-3}$	m <sup>3</sup>

2. (3 points) Complete the following table on materials classifications

	Materials Class Name	2 Characteristic Properties	Two Examples of Each Class
1	Ceramics	Brittle, high melting point	Quartz, glass
2	Metals	Ductile, conductive	6000 series aluminum, brass
3	Polymers	Strong in tension, non-conductive	Polyethylene, Teflon

3. (6 points) Define Hooke's Law, engineering stress and engineering strain with words and then define the equations that define each as function of force, length and/or cross-sectional area.

**Solution:** Hooke's law states that the change in length of certain materials is linearly proportional to the force applied to the material. Such materials are called *elastic* materials.

$$F = -k\Delta x$$

where  $F$  is the force applied,  $k$  is a material dependent proportionality constant, and  $\Delta x$  is the material's change in length. The relationship is negative since the restoring force is opposite to the direction of the force applied.

Engineering stress is the ratio between an applied force and the starting cross-sectional area of a material.

$$\sigma = \frac{F}{A_0}$$

where  $F$  is the applied force and  $A_0$  is the original cross sectional area of a material.

Engineering strain is the ratio of between the change in length of a material and its original length when a force is applied.

$$\varepsilon = \frac{\Delta L}{L}$$

where  $\Delta L$  is the material's change in length, and  $L$  is its original length.

4. You work at an independent lab that conducts materials testing. To determine the tensile properties of a new alloy you are provided a tensile coupon with the following dimensions in the reduced section: 2mm x 8mm x 50mm (assuming the longest dimension is the length).

- (a) (2 points) If the coupon elongated by 0.1mm when loaded axially with 1kN, what is the Young's Modulus of the alloy?

**Solution:**

$$\begin{aligned}
 E &= \frac{\sigma}{\epsilon} \\
 &= \frac{1000 / (2 \cdot 8 \cdot 10^{-6})}{0.1/50} \\
 E &= 3.125 \times 10^{10} \text{ Pa} \\
 \therefore E &= \boxed{3 \times 10^{10} \text{ Pa (1 s.f.)}}
 \end{aligned}$$

- (b) (2 points) If the component has a density of 6 g/cm<sup>3</sup>, is the Performance Index for a light stiff beam fabricated from this material going to be better or worse than steel?

**Solution:**

$$\begin{aligned}
 \text{MPI mystery material} &= \frac{\sqrt{E}}{\rho} & \text{MPI steel} &= \frac{\sqrt{E}}{\rho} \\
 &= \frac{\sqrt{3.125 \cdot 10^{10}}}{6 \cdot 10^{-3} / 10^{-6}} & &= \frac{\sqrt{200 \cdot 10^9}}{7850} \\
 &\approx 29.46 & &\approx 56.96 \\
 &= \boxed{30 \text{ m}^{2.5} \text{s}^{-1} \text{kg}^{-0.5} \text{ (1 s.f.)}} & &= \boxed{60 \text{ m}^{2.5} \text{s}^{-1} \text{kg}^{-0.5} \text{ (1 s.f.)}}
 \end{aligned}$$

*I Googled the density to be 7850 kg/m<sup>3</sup> and used  $E_{\text{steel}} = 200 \text{ GPa}$ . The MPI for steel is higher, so the mystery material does not perform as well as steel.*

5. For an unknown alloy, the stress at which plastic deformation begins is 345 MPa, and the modulus of elasticity is 103 GPa. You are given 0.73m long hollow brass cylinder with an internal external diameter of 1.9cm and an internal diameter of 1.75cm, which will be required to support a large chandelier.

- (a) (3 points) What is the maximum load that can be supported without plastic deformation?

**Solution:**

$$\begin{aligned}
 \sigma_{\text{cr}} &= \frac{F}{A} \\
 F &= \sigma_{\text{cr}} A \\
 &= 345 \cdot 10^6 \cdot (\pi(1.9^2 - 1.75^2) \cdot 0.25 \cdot 10^{-4}) \\
 &= 14835 \text{ N} \\
 &= \boxed{15 \text{ kN (2 s.f.)}}
 \end{aligned}$$

- (b) (2 points) What is the maximum length to which it can be stretched without causing plastic deformation?

**Solution:**

$$\begin{aligned}\Delta L &= L \cdot \frac{\sigma_{cr}}{E} \\ &= 0.73 \cdot \frac{345 \cdot 10^6}{103 \cdot 10^9} \\ &= 2.5 \text{ mm (2 s.f.)}\end{aligned}$$

The maximum length it can be stretched is  $0.73 \text{ m} + 2.5 \text{ mm} = \boxed{733 \text{ mm 3 s.f.}}$

6. A cylinder ( $\varnothing = 20\text{cm}$ ,  $l = 0.5\text{m}$ ) is to be manufactured and will be required support up to  $3 \times 10^6 \text{ N}$  in tension. The current material chosen has been shown to elongate  $0.5\text{mm}$  under these loading conditions. Due to the tolerances of fit, the final component must not decrease in diameter more than  $55\mu\text{m}$ . From the provided list, choose an appropriate material for this component.

	Poisson's Ratio	E	Shear Modulus
<b>Aluminum, 6061-T6</b>	0.35	69 GPa	26 GPa
<b>Aluminum, 2024-T4</b>	0.32	73 GPa	28 GPa
<b>Beryllium Copper</b>	0.285	125 GPa	50 GPa
<b>Brass, 70-30</b>	0.331	110 GPa	37 GPa
<b>Bronze</b>	0.34	70 GPa	25 GPa
<b>Copper</b>	0.355	110 GPa	48 GPa
<b>Cast Iron</b>	0.211	120 GPa	39 GPa
<b>Lead</b>	0.431	13 GPa	4.9 GPa
<b>Magnesium Alloy</b>	0.281	42 GPa	60 GPa

**Solution:** Since the rod has strains radially and longitudinally, we can use the relation  $G = \frac{E}{2(1+\nu)}$ . We have two parameters we can solve for with given info:  $E$  and  $\nu$ . Once we solve this, we can find the minimum shear modulus that the material should possess in order to be selected.

Calculating  $E$

$$\begin{aligned}E &= \frac{\sigma_{cr}}{\epsilon} \\ &= \frac{3 \cdot 10^6 / \pi (10^2) \cdot 10^{-4}}{0.5/500} \\ &= 95 \text{ GPa (2 s.f.)}\end{aligned}$$

Calculating  $\nu$

$$\begin{aligned}\nu &= \frac{\epsilon_{radial}}{\epsilon_{axial}} \\ &= \frac{55 \cdot 10^{-6} / 20 \cdot 10^{-2}}{0.5 \cdot 10^{-3} / 0.5} \\ &= 0.28 \text{ (2 s.f.)}\end{aligned}$$

This suggests that  $G = \frac{95 \cdot 10^9}{2(1+0.28)} = 38 \text{ GPa}$ .

Cast iron is a suitable material since  $\nu < 0.28$ ,  $E > 95 \text{ GPa}$ ,  $G > 38 \text{ MPa}$ .