Comparative Analysis of Different Models of Checkpointing and Recovery

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Abstract—**Checkpointing and rollback-recovery is a common technique to keep the integrity of information and to enhance the reliability in database systems. Several models were studied in order to determine the optimum checkpointing policy which optimizes a certain performance measure. Among checkpointing strategies that have been considered are: 1) Poisson checkpointing, 2) a fixed time interval between checkpoints and 3) a specified number of completed transactions between checkpoints. Poisson checkpointing is often assumed because it is more tractable analytically. The second and third strategies are more realistic, but also more difficult to analyze. An important aspect of checkpointing and recovery models is the dependence of recovery periods on the checkpointing strategy. This dependence may be characterized as one of the following: 1) parametric dependence, 2) stochastic dependence and 3) deterministic dependence. Parametric dependence is the least realistic and simplest to model. Naturally, more realistic models are more complex to analyze. The purpose of this paper is to study and compare different models in order to select one which adequately represents a realistic system and yet tractable for analysis. The approach is analytical (whenever possible), otherwise numerical or by means of simulations. In particular, we consider the queueing analysis of a model with combined checkpointing strategies; the results can be specialized to any one of the combined strategies.**

Index Terms—**Checkpointing, database systems, Markovian models, performance analysis, rollback-recovery.**

I. Introduction

TRANSACTION oriented database systems have become very important in many critical and commercial applications. In most of these applications the demand is strong for high reliability and availability of the information stored in the system. A transaction is one or more tasks, requested by a single user, to be executed on the system. The response time of a transaction should be short in many of these applications. A common technique to keep the integrity of information in such systems is to save copies of the system status periodically. The system status include all files and information needed to restore the system to its state at the time of the copy. This information is saved in a reliable secondary storage device. The saving process is commonly known as a *checkpoint.* The transactions which modified the system files since the most recent checkpoint are stored in a file known as the *audit trail.* Upon the occurrence of a failure (due to hardware, software, or operator faults) a recovery action is initiated. It is assumed that failures are detected instantaneously. In a recovery action the cause of the failure is removed and the system state is restored to that at the time of the last checkpoint; this is known as a *rollback operation.* A recovery action is completed when all transactions stored in the audit trail are reprocessed and the system is brought back to its correct status just before the occurrence of the failure. Checkpoints and failure recoveries can be viewed as preemptive interruptions during which the system is unable to process new transactions. The processing of the preempted transaction is resumed after the interruption. Therefore, it is our objective to minimize the time spent by the system in checkpointing and recovery. Increasing the frequency of checkpointing decreases recovery time but increases checkpointing time. Decreasing the frequency of checkpointing decreases checkpointing time but increases recovery time. There is an optimum checkpointing strategy which maximizes the system availability (the fraction of time the system is available for processing new transactions). The optimum checkpointing strategy which minimizes the mean response time of a transaction is not necessarily the same as that which maximizes the system availability.

Several models of checkpointing and recovery have been proposed and analyzed in the literature. Most of these models were considered to determine the optimum checkpoint interval which maximizes the system availability [1], [4], [5], [7], [10], [11], [14], [16], [20], [21], [24]. These models did not include the queueing effects of checkpointing and recovery, and are often analytically tractable. Few authors have considered a queueing analysis in their models in order to determine the optimum checkpoint interval which minimizes the mean response time of a transaction [2], [3], [6], [15], [17]–[19]. These models are more complex and their analysis is often performed numerically or by means of simulation. A model in which checkpoints may be performed during recovery is considered in [15]; this is motivated by the fact that the probability of failure usually increases during recovery and therefore checkpoints are necessary to avoid repeating lengthy reprocessing times.

Among checkpointing strategies that have been considered in the literature are

C1) Poisson checkpointing,

C2) A fixed time interval between checkpoints,

C3) A specified number of completed transactions between checkpoints (load-dependent strategy).

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Poisson checkpointing (C1) is the least realistic, but it is the most tractable analytically. The deterministic checkpoint interval (C2) is proved to be optimal among all strategies with generally distributed time between checkpoints [11], but it may be difficult to implement. Checkpointing after the completion of a specified number of transactions (C3) is a load-dependent strategy in which the maximum number of transactions to be reprocessed during recovery cannot exceed a specified number. This is a realistic strategy which is considered in [16] and [18]. In practice, the above checkpointing strategies may be combined. For example, a load-dependent strategy (C3) may be combined with Poisson checkpointing strategy (C1); this prevents recovery periods from becoming too large by limiting the number of transactions to be reprocessed during any recovery. The analysis of this model is considered in this paper and the results are specialized to each of the combined strategies for the purpose of comparison.

The dependence of recovery periods on the checkpointing strategy is a basic feature in all checkpointing and recovery models. It is this dependence which defines the optimization problem noted earlier. Clearly, it is important to carefully choose a model for this dependence. Among possible recovery models are those characterized by

R1) Parametric dependence,

R2) Stochastic dependence,

R3) Deterministic dependence.

In model R1, recovery periods are assumed to be independent and identically distributed random variables. The parameters of the distribution depend on the parameters of the checkpointing strategy (e.g., the mean recovery period is proportional to the mean time between checkpoints). This model is often adopted due to its simplicity and analytical tractability. In model R2, recovery periods are independent but not identically distributed. A recovery period is drawn from a distribution whose parameters depend on the number of transactions processed since the last checkpoint. For example, an equal (or proportional) number of corresponding (but not identical) transactions is to be reprocessed in a recovery period. In fact the distributions of consecutive recovery periods in one checkpoint interval are not independent, since the number of transactions to be reprocessed increases in later recoveries. Note that recovery periods are stochastically dependent on the number of processed transactions since the last checkpoint. This model is reasonably accurate representation of a realistic recovery [23] and its analysis is considered in [18]. In model R3, recovery periods depend deterministically on the available busy (useful processing) time since the last checkpoint. For example, the recovery time is equal (or proportional) to the useful processing time since the last checkpoint. Clearly, consecutive recovery periods in one checkpoint interval are also dependent. This model is realistic [23] but its analysis is quite complex [11], therefore we resort to simulation to study this model.

TABLE I
DIFFERENT MODELS OF CHECKPOINGING AND RECOVERY AND THE METHODS OF ANALYSIS CONSIDERED IN THIS PAPER

Checkpointing \ Recovery	Parametric dependence (R1)	Stochastic dependence (R2)	Deterministic dependence (R3)
Poisson (C1)	A, $\bar{N}$ (analytic)	A, $\bar{N}$ (numeric)	A, $\bar{N}$ (simulation)
Deterministic (C2)	A (anal.), $\bar{N}$ (sim.)	A, $\bar{N}$ (simulation)	A, $\bar{N}$ (simulation)
Load-dependent (C3)	A (anal.), $\bar{N}$ (num.)	A, $\bar{N}$ (numeric)	A, $\bar{N}$ (simulation)

Different checkpointing strategies can be combined with different recovery models as shown in Table I. The level of accuracy of representation in these models increases from the upper-left to the lower-right of the table. Naturally, more realistic models are more complex to analyze (notice the different methods of analysis in Table I). The state-space analytic approach [17], [18] is pursued whenever possible. More complex models require numerical methods for their solution. When numerical techniques are not feasible we resort to simulation. For example, simulation is used to compare complex ("realistic") models with simpler models, and to check the accuracy of numerical solutions. The objective of this paper is to study different models in order to select an appropriate checkpointing strategy and an accurate recovery model which are tractable for analytical or numerical analysis.

Some of the models in Table I have been considered earlier by several authors. Model C1–R1 was first introduced in [10] and later extended and generalized in [17] and [19]. Model C2–R1 (with a deterministic checkpoint interval) has been treated in [2], [5], [6], [11], [20], [24]. Approximate analysis of model C3–R1 is considered in [16], the exact analysis of this model is a special case of model C3–R2 which is studied in [18]. In the present paper we use a state-space analytic or numeric approach as in [18], and consider a model with a combined checkpointing strategy (C1 and C3) and a stochastic recovery model (R2). The analysis of models with deterministic checkpointing (C2) or deterministic recovery model (R3) can be approached by numerical approximation, but we use simulation to avoid a large state-space and truncation errors.

In Section II we introduce the basic model and the underlying assumptions, we also discuss the evaluation of performance measures. In Section III a Markovian queueing model with a combined (Poisson and load-dependent) checkpointing strategy is presented. Three different approaches to the analysis of this model are considered. In Section III-A we proceed analytically to determine the system availability and the average queue length in terms of the model parameters and a set of unknown boundary state probabilities. These unknown probabilities can be determined numerically, an efficient algorithm is described in Section III-B. The simulation approach is briefly discussed in Section III-C. Comparison and validation of different models with a discussion of the results are considered in Section IV. We conclude in Section V.

II. The Basic Model

The model considered here is basically an M/M/1/N queueing system subject to two sources of interruptions; namely, checkpointing and recovery. Transactions arrive according to a Poisson process at rate λ, they are serviced according to FCFS discipline when the system is available. The service time requirement of a transaction is exponentally distributed with a mean μ^{-1}. The queue size is limited to N (an arriving transaction is lost when the queue is full). Failures occur according to a Poisson process at different rates depending on the mode of the system operation. The failure rate is γ when the system is available or checkpointing, and is γ_r during recovery. Depending on the model under consideration a checkpointing strategy may be combined with any recovery model as shown in Table I. A constant checkpointing rate α is defined only for Poisson strategy (C1); otherwise, it should be interpreted as an effective rate for deterministic and load-dependent strategies (C2 and C3). The duration of a checkpoint is exponentially distributed with a mean β^{-1}. Therefore the distribution of the effective checkpoint duration in the presence of Poisson failures remains exponential with a mean β^{-1}. This assumption can be relaxed without adding to the complexity of the analysis. The characterization of recovery periods depends on the chosen recovery model as discussed in Section I. In the stochastic recovery model (R2) a recovery action starts with a rollback operation whose duration is exponentially distributed with a mean μ_0^{-1}. This is followed by reprocessing all processed transactions since the last checkpoint. The reprocessing time of a transaction is independent but of identical distribution to that of the original service time. (i.e., exponential with a mean μ^{-1}). A failure may occur during recovery (at rate γ_r), after which a corresponding (but not identical) recovery action is restarted.

We are primarily interested in evaluating performance measures such as the system availability and the mean response time of a transaction. In the following we discuss the evaluation of these performance measures in models of different complexity.

A. The System Availability

From renewal theoretic arguments [12], the system availability A is given by

$$A = \frac{E[a]}{E[a] + E[c] + E[r]}, \tag{2.1}$$

where

$E[a]$ is the expected time spent by the system in the available state between successive checkpoints,

$E[c]$ is the expected time spent by the system in the checkpointing state,

$E[r]$ is the expected time spent by the system in the recovery state between successive checkpoints.

Since $E(c)$ is known and $E(a)$ can be determined, an analytic expression for A can be obtained if we have an expression for $E[r]$. It follows that, in the absence of failures during recovery, an analytic expression for A can be obtained for all models with parametric recovery (R1), since then

$$E[r] = (\gamma E[a])(kE[a]) = k\gamma(E[a])^2. \tag{2.2}$$

Recall that, in the parametric recovery model, the expected recovery time following a failure is proportional to the mean available time between checkpoints. The constant of proportionality, k $(= \lambda/\mu A)$, is determined as the fraction of busy time in the available time [11], [17]. Note that for a load-dependent checkpointing strategy (C3) the distribution of the available time between checkpoints is difficult to obtain, but its mean is easily determined from the operational identity $(\lambda/A)E[a] = n$, where n is the number of completed transactions between checkpoints. In the presence of failures during recovery we need to know the distribution of recovery periods in order to determine $E[r]$. This distribution may be known (by assumption) only in the simple model R1. In realistic models such as R2 and R3 the distribution of recovery periods can not be obtained analytically. However, if there are no failures during recovery, then the mean can be determined if we know the distribution of the available time between checkpoints (as for C1 and C2). In this case, it is reasonable to assume that the mean recovery time is proportional to the mean available time interval between the failure occurrence and the last checkpoint. For Poisson failure process this is the mean residual life time $Y[a]$ of the available time interval between checkpoints, which is given by (see [12])

$$Y[a] = \frac{E[a^2]}{2E[a]},$$

where $E[a^2]$ is the second moment of the available time between checkpoints. Therefore

$$E[r] = (\gamma E[a])(kY[a]) = k\gamma\frac{E[a^2]}{2}. \tag{2.3}$$

For the load-dependent checkpointing strategy C3, the distribution of the available time between checkpoints is not known analytically, therefore no analytic expression can be obtained for A, except for the parametric recovery model R1 as noted earlier.

B. The Mean Response Time

The mean response time $\overline{R}$ is related to the steady-state average number of transactions in the system $\overline{N}$ by Little's formula $\overline{R} = \overline{N}/\overline{\lambda}$, where $\overline{\lambda}$ is the average arrival rate of transactions admitted to the system. For a system with a finite capacity of size N, $\overline{\lambda} = \lambda(1 - p(N))$, where $p(N)$

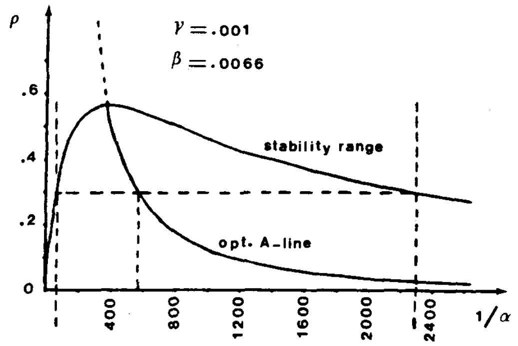

Fig. 1. The stability range and the optimal mean intercheckpoint interval as functions of ρ.

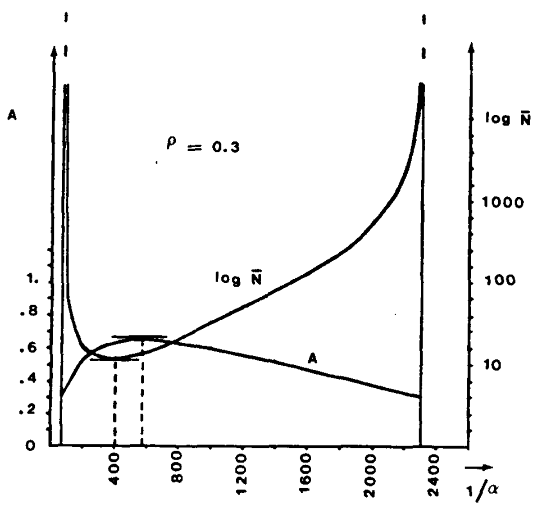

Fig. 2. The system availability and the average queue length as functions of α^{-1}.

is the probability that the queue is full. An analytic expression for $\bar{N}$ can not be obtained except for the simple model C1–R1 in Table I (with infinite capacity). It was first derived in [9] for an M/G/1 queue with a single type of Poisson interruptions and later generalized in [19] for mixed types of interruptions. A simple Markovian queueing model of checkpointing and recovery (in accordance with model C1–R1) was introduced in [10]. In this model the recovery periods are assumed to be exponentially distributed with a mean $\phi^{-1} = (\rho/A)\alpha^{-1}$, where α is the checkpointing rate and $\rho = \lambda/\mu$. The system availability (A) and the average queue length ($\bar{N}$) are given by [10]

$$A = \left(1 + \frac{\alpha}{\beta} + \frac{\gamma}{\phi}\right)^{-1}, \tag{2.4}$$

$$\bar{N} = \frac{1}{\left(1 - \frac{\rho}{A}\right)} \left[\frac{\rho}{A} + \lambda A\left(\frac{\alpha}{\beta^2} + \frac{\gamma}{\phi^2}\right)\right]. \tag{2.5}$$

The stability range and the optimum mean interval between checkpoints are shown in Fig. 1 as functions of ρ (vertical axis). Note that for a fixed value of ρ there are lower and upper limits on α beyond which the system is unstable, this stability range decreases as ρ increases. For fixed values of γ and ρ the system availability and the average queue length are plotted as functions of α in Fig. 2.

III. A Model with Combined Checkpointing Strategies

In this section we consider a model with a combined, Poisson and load-dependent, checkpointing strategy. In other words, checkpoints are performed according to a Poisson process and a checkpoint is forced if the number of completed transactions since the last checkpoint reaches a specified number, say n. As discussed earlier in Section I, such a strategy limits the number of transactions to be reprocessed during any recovery to a maximum equal to n. Furthermore, the results of the same analysis can be specialized to each of the combined strategies. In the analysis of this model we consider a stochastic recovery model (R2) in which all processed transactions since the last checkpoint should be reprocessed in a recovery action following a failure. As mentioned in Section I, this model is an adequate representation of a realistic environment [23]. The analysis with a parametric recovery model (R1) is a special case of the present analysis. For a deterministic recovery model (R3) we use simulation to compare it with other models. The queueing model is represented by a continuous-time Markov chain, the state-transition diagram is shown in Fig. 3(a) and (b), for a system with a finite capacity of size N. This model reduces to a model with Poisson checkpointing as $n \to \infty$, and reduces to a model with n completed transactions between checkpoints as $\alpha \to 0$. The obtained results will be specialized to these cases as we proceed throughout this section. The three methods of analysis, analytical, numerical and simulation are considered in Sections III-A, B, and C, respectively.

The following notation is associated with the Markovian model in Fig. 3(a) and (b); it will be used throughout this section.

Index "m"	indicates the mode of system operation, $m = a$ for available, c for checkpointing, or r for recovery.
Index "i"	indicates the number of transactions in the system, queued and in processing, $0 \leq i \leq N$ (N is the capacity of the system).
Index "j"	indicates the number of transactions processed since the most recent checkpoint, including the transaction being processed, $1 \leq j \leq n$ (n is the number of completed transactions between successive checkpoints).
Index "k"	indicates the number of reprocessed transactions in a recovery opera-

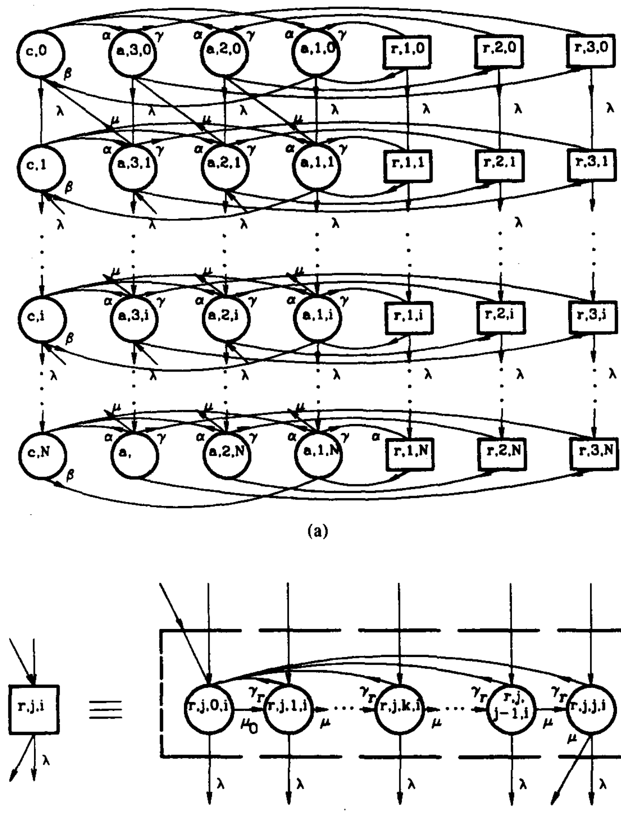

Fig. 3. (a) State-transition diagram representing a queueing model with combined Poisson and load-dependent checkpointing strategies ($n = 3$ and a queue of capacity N). (b) A detailed representation of the aggregate state "r, j, i".

tion, $0 \le k \le j$ (j is the total number of transactions to be reprocessed in the recovery operation). $k = 0$ corresponds to the rollback operation.

State "c, i" corresponds to the checkpointing mode of operation, with i transactions in the system. $p(c, i)$ is the associated probability.

State "a, j, i" corresponds to the available mode of operation during the processing of the jth transaction after the most recent checkpoint, with i transactions in the system. $p(a, j, i)$ is the associated probability.

State "r, j, k, i" corresponds to the recovery mode of operation during the processing of the kth transaction, with i transactions in the system. j is the total number of transactions to be reprocessed during recovery. $p(r, j, k, i)$ is the associated probability.

We also define the following aggregate states and the associated probabilities.

State "c" corresponds to all the states "c, i", $i = 0, 1, \cdots, N$. $A(c)$ is the associated probability.

State "a, j" corresponds to all the states "a, j, i", $i = 0, 1, \cdots, N$. $A(a, j)$ is the associated probability, $1 \le j \le n$.

State "a" corresponds to all the states "a, j", $j = 1, 2, \cdots, n$. $A(a)$ is the associated probability.

State "r, j, k" corresponds to all the states "r, j, k, i", $i = 0, 1, \cdots, N$. $A(r, j, k)$ is the associated probability, $0 \le k \le j$, $1 \le j \le n$.

State "r, j" corresponds to all the states "r, j, k", $k = 0, 1, \cdots, j$. $A(r, j)$ is the associated probability, $1 \le j \le n$.

State "r" corresponds to all the states "r, j", $j = 1, 2, \cdots, n$. $A(r)$ is the associated probability.

State "i" corresponds to all the states "a, j, i",

"r, j, k, i", $k = 0, 1, \cdots, j, j = 1, 2, \cdots, n$, and the state "$c, i$". $p(i)$ is the associated probability, $0 \le i \le N$.

A. Analytical Approach

In this section we carry out a state-space analysis of the queueing model in Fig. 3.1, with infinite capacity ($N \to \infty$), in order to derive performance measures such as the system availability and the mean response time of a transaction. The method of analysis is very similar to that in [18] and therefore many details are omitted. The derived expressions for the performance measures are semianalytic, i.e., they are not exclusively expressed in terms of the model parameters but also in terms of a set of boundary state probabilities which can be determined numerically. First, let us define the following probabilities

$$B(c) = \sum_{i=1}^{\infty} p(c, i),$$

$$B(a, j) = \sum_{i=1}^{\infty} p(a, j, i), \qquad 1 \le j \le n,$$

$$B(r, j, k) = \sum_{i=1}^{\infty} p(r, j, k, i), \qquad 0 \le k \le j,\ 1 \le j \le n.$$

It follows that

$$A(c) = p(c, 0) + B(c), \tag{3.1}$$

$$A(a, j) = p(a, j, 0) + B(a, j), \qquad 1 \le j \le n, \tag{3.2}$$

$$A(a) = \sum_{j=1}^{n} A(a, j), \tag{3.3}$$

$$A(r, j, k) = p(r, j, k, 0) + B(r, j, k), \qquad 0 \le k \le j,\ 1 \le j \le n, \tag{3.4}$$

$$A(r, j) = \sum_{k=0}^{j} A(r, j, k), \qquad 1 \le j \le n, \tag{3.5}$$

$$A(r) = \sum_{j=1}^{n} A(r, j). \tag{3.6}$$

Using transaction balance equations and following similar steps as in [18] we get

$$A(r, j) = \frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1\right)A(a, j), \qquad 1 \le j \le n, \tag{3.7}$$

with

$$P_k = \left(\frac{\mu_0}{\gamma_r + \mu_0}\right)\left(\frac{\mu}{\gamma_r + \mu}\right)^k. \tag{3.8}$$

Note that P_k is the probability of no failure during the rollback operation and the reprocessing of the first k transactions in a recovery action. We also have

$$B(a, j) = \frac{\lambda}{\mu}\left(\frac{\alpha}{\alpha + \mu}\right)\left(\frac{\mu}{\alpha + \mu}\right)^{j-1}\left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)^{-1}, \qquad 1 \le j \le n, \tag{3.9}$$

where we made use of the relation $\lambda = \mu \sum_{j=1}^{n} B(a, j)$,

$$A(a) = \frac{\lambda}{\mu} + \sum_{j=1}^{n} p(a, j, 0), \tag{3.10}$$

and

$$A(c) = \frac{\alpha}{\beta}\left[\frac{\lambda}{\mu}\left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)^{-1} + \sum_{j=1}^{n} p(a, j, 0)\right]. \tag{3.11}$$

Now using the definition (3.2) and the normalizing condition $A(a) + A(r) + A(c) = 1$ yields

$$\frac{\alpha}{\beta}\sum_{j=1}^{n} p(a, j, 0) + \sum_{j=1}^{n} \frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1 + \frac{\gamma_r}{\gamma}\right)p(a, j, 0)$$
$$= 1 - \frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)^{-1} \cdot \left(\frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha - \gamma_r}\right)\left(1 - \left(\frac{\gamma_r + \mu}{\alpha + \mu}\right)^n\right)\frac{1}{P_1}\right)\right]. \tag{3.12}$$

A necessary and sufficient condition for ergodicity follows from the fact that for a stable system $p(a, j, 0) > 0$, for $j = 1, 2, \cdots, n$. This yields the stability condition

$$\frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)^{-1}\left(\frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha - \gamma_r}\right) \cdot \left(1 - \left(\frac{\gamma_r + \mu}{\alpha + \mu}\right)^n\right)\frac{1}{P_1}\right)\right] < 1. \tag{3.13}$$

Note that the system availability A given by (3.10) is expressed in terms of the sum of the boundary state probabilities, $p(a, j, 0)$, $1 \le j \le n$, which can not be determined from the normalizing condition in (3.12). Numerical techniques are used to determine these unknown probabilities as will be shown in Section III-B.

A saturated system corresponds to a system operating in a batch environment where transactions are processed in sequence. For such a system it is of interest to determine the system availability A_s which can be derived analytically. Let λ_s be the threshold arrival rate at which the system becomes saturated, then $\lambda_s = \mu A_s$. Now substituting in the normalizing condition (3.12) and making use of the fact that at saturation $p(a, j, 0) = 0$, $1 \le j \le n$, we obtain

$$A_s = \left[1 - \frac{\gamma}{\gamma_r} + \left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)^{-1}\left(\frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r} \cdot \left(\frac{\alpha}{\alpha - \gamma_r}\right)\left(1 - \left(\frac{\gamma_r + \mu}{\alpha + \mu}\right)^n\right)\frac{1}{P_1}\right)\right]^{-1}. \tag{3.14}$$

For Poisson checkpointing, $n \to \infty$ and A_s reduces to

$$A_s = \left[1 - \frac{\gamma}{\gamma_r} + \frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha - \gamma_r}\right)\frac{1}{P_1}\right]^{-1}, \tag{3.15}$$

where we have made use of the inequality $\gamma_r < \alpha$; this is a necessary condition for a finite recovery period with probability one. For checkpointing after completing n transactions (load-dependent strategy), $\alpha \to 0$ and A_s reduces to

$$A_s = \left[1 - \frac{\gamma}{\gamma_r} + \frac{\mu}{n\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\mu}{n\gamma_r}\right) \cdot \left(\left(\frac{\gamma_r + \mu}{\mu}\right)^n - 1\right)\frac{1}{P_1}\right]^{-1}. \tag{3.16}$$

Before we proceed to derive an expression for the average number of transactions in the system $\overline{N}$, we introduce the following definitions.

$$N(c) = \sum_{i=1}^{\infty} ip(c, i),$$

$$N(a, j) = \sum_{i=1}^{\infty} ip(a, j, i), \qquad 1 \le j \le n,$$

$$N(a) = \sum_{j=1}^{n} N(a, j),$$

$$N(r, j, k) = \sum_{i=1}^{\infty} ip(r, j, k, i), \qquad 0 \le k \le j,\ 1 \le j \le n,$$

$$N(r, j) = \sum_{k=0}^{j} N(r, j, k), \qquad 1 \le j \le n,$$

$$N(r) = \sum_{j=1}^{n} N(r, j).$$

The probability that there are i transactions in the system is given by

$$p(i) = p(c, i) + \sum_{j=1}^{n}\left[p(a, j, i) + \sum_{k=0}^{j} p(r, j, k, i)\right].$$

The average number of transactions in the system $\overline{N}$ is therefore

$$\overline{N} = \sum_{i=1}^{\infty} ip(i) = N(c) + N(a) + N(r). \tag{3.17}$$

In the following we relate the quantities defined above in order to obtain an expression for $\overline{N}$. Using transition balance equations and after some algebraic manipulations, it can be shown that

$$N(c) = \left(\frac{\alpha + \mu}{\beta}\right)N(a, 1) - \frac{\lambda}{\beta}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_1} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, 1), \tag{3.18}$$

$$N(a, j) = \left(\frac{\mu}{\alpha + \mu}\right)^{j-1} N(a, 1) + \left(\frac{\lambda}{\alpha + \mu}\right)\sum_{k=2}^{j} \left(\frac{\mu}{\alpha + \mu}\right)^{j-k}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right) \cdot A(a, k) - \frac{\mu}{\lambda}B(a, k-1)\right],$$
$$2 \le j \le n. \tag{3.19}$$

Thus we have the following for $N(a)$

$$N(a) = \left(\frac{\alpha + \mu}{\alpha}\right)\left(1 - \left(\frac{\mu}{\alpha + \mu}\right)^n\right)N(a, 1) + \left(\frac{\lambda}{\lambda + \mu}\right)\sum_{j=2}^{n}\sum_{k=2}^{j} \left(\frac{\mu}{\alpha + \mu}\right)^{j-k}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right) \cdot A(a, k) - \frac{\mu}{\lambda}B(a, k-1)\right]. \tag{3.20}$$

After some lenghty manipulations it can also be shown that

$$N(r, j) = \frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1\right)N(a, j) + \lambda\sum_{k=0}^{j} A(r, j, k)\,\bar{t}(r, j, k), \tag{3.21}$$

with

$$\bar{t}(r, j, 0) = \frac{1}{\gamma_r}\left(\frac{1}{P_j} - 1\right)$$

and

$$\bar{t}(r, j, k) = \frac{1}{\gamma_r}\left(\frac{1}{P_j} - \frac{1}{P_{k-1}}\right), \qquad 1 \le k \le j.$$

Note that $\bar{t}(r, j, k)$ can be interpreted as the expected time remaining in recovery with "r, j, k" as an initial state.

From (3.19) and (3.21) we get

$$N(a) + N(r)$$
$$= N(a, 1) \sum_{j=1}^{n} \frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1 + \frac{\gamma_r}{\gamma}\right)\left(\frac{\mu}{\alpha+\mu}\right)^{j-1} + \left(\frac{\lambda}{\alpha+\mu}\right)\sum_{j=2}^{n}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1 + \frac{\gamma_r}{\gamma}\right) \cdot \sum_{k=2}^{j}\left(\frac{\mu}{\alpha+\mu}\right)^{j-k} \cdot \left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, k) - \frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1}\right] + \frac{\lambda}{\gamma_r}\sum_{j=1}^{n}\frac{A(a, j)}{P_j}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1\right) - \left(\frac{j\gamma}{\gamma_r+\mu} + \frac{\gamma}{\gamma_r+\mu_0}\right)\right]. \tag{3.22}$$

From the transition balance equations

$$\lambda p(i-1) = \mu \sum_{j=1}^{n} p(a, j, i), \qquad i \geq 1,$$

we have

$$N(a) = \frac{\lambda}{\mu}(\overline{N} + 1). \tag{3.23}$$

From (3.20) and (3.23) we obtain the following for $N(a, 1)$

$$N(a, 1) = \left(\frac{\alpha}{\alpha+\mu}\right)\left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1} \cdot \left[\frac{\lambda}{\mu}(\overline{N}+1) - \left(\frac{\lambda}{\alpha+\mu}\right)\sum_{j=2}^{n}\sum_{k=2}^{j}\left(\frac{\mu}{\alpha+\mu}\right)^{j-k}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, k) - \frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1}\right]\right]. \tag{3.24}$$

Substituting from (3.18), (3.22), and (3.24) into (3.17) we finally get the following for $\overline{N}$

$$\overline{N} = \left[1 - \frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1} \cdot \left(\frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha-\gamma_r}\right)\left(1 - \left(\frac{\gamma_r+\mu}{\alpha+\mu}\right)^n\right)\frac{1}{P_1}\right)\right]\right]^{-1} \cdot \left\{\frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1} \cdot \left(\frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha-\gamma_r}\right)\left(1 - \left(\frac{\gamma_r+\mu}{\alpha+\mu}\right)^n\right)\frac{1}{P_1}\right)\right] \cdot \left[1 - \left(\frac{\mu}{\alpha+\mu}\right)\sum_{j=2}^{n}\sum_{k=2}^{j}\left(\frac{\mu}{\alpha+\mu}\right)^{j-k} \cdot \left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, k) - \frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1}\right]\right] - \frac{\lambda}{\beta}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_1} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, 1) + \left(\frac{\lambda}{\alpha+\mu}\right)\sum_{j=2}^{n}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1 + \frac{\gamma_r}{\gamma}\right)\sum_{k=2}^{j}\left(\frac{\mu}{\alpha+\mu}\right)^{j-k}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, k) - \frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\left(1 - \left(\frac{\mu}{\alpha+\mu}\right)^n\right)^{-1}\right] + \frac{\lambda}{\gamma_r}\sum_{j=1}^{n}\frac{A(a, j)}{P_j}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j} - 1\right) - \left(\frac{j\gamma}{\gamma_r+\mu} + \frac{\gamma}{\gamma_r+\mu_0}\right)\right]\right\}, \tag{3.25}$$

with $A(a, j)$, $1 \leq j \leq n$, as given by (3.2). Equation (3.25) expresses $\overline{N}$ in terms of the model parameters and the boundary state probabilities $p(a, j, 0)$, $1 \leq j \leq n$. If estimates of the values of the boundary state probabilities are provided (for example, from measurements or by numerical means) then an estimate of $\overline{N}$ follows from (3.25). Note that the denominator in the expression for $\overline{N}$ should be greater than zero for a stable system. This yields the same stability condition as that obtained earlier in (3.13).

For Poisson checkpointing ($n \to \infty$), and making use of the condition $\alpha > \gamma_r$, $\overline{N}$ reduces to

$$\overline{N} = \left[1 - \frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha-\gamma_r}\right)\frac{1}{P_1}\right]\right]^{-1} \cdot \left\{\frac{\lambda}{\mu}\left[1 - \frac{\gamma}{\gamma_r} + \frac{\alpha}{\beta} + \frac{\gamma}{\gamma_r}\left(\frac{\alpha}{\alpha-\gamma_r}\right)\frac{1}{P_1}\right] \cdot \left[1 - \left(\frac{\mu}{\alpha+\mu}\right)\sum_{j=2}^{\infty}\sum_{k=2}^{j}\left(\frac{\mu}{\alpha+\mu}\right)^{j-k} \cdot \left(\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, k) - \frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\right)\right] - \frac{\lambda}{\beta}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_1} - 1 + \frac{\gamma_r}{\gamma}\right)A(a, 1)$$

$$+\left(\frac{\lambda}{\alpha+\mu}\right)\sum_{j=2}^{\infty}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j}-1+\frac{\gamma_r}{\gamma}\right)\sum_{k=2}^{j}$$
$$\left(\frac{\mu}{\alpha+\mu}\right)^{j-k}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k}-1+\frac{\gamma_r}{\gamma}\right)A(a,k)\right.$$
$$\left.-\frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{k-1}\right]$$
$$+\frac{\lambda}{\gamma_r}\sum_{j=1}^{\infty}\frac{A(a,j)}{P_j}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j}-1\right)\right.$$
$$\left.\left.-\left(\frac{j\gamma}{\gamma_r+\mu}+\frac{\gamma}{\gamma_r+\mu_0}\right)\right]\right\}, \tag{3.26}$$

with

$$A(a,j)=P(a,j,0)+\frac{\lambda}{\mu}\frac{\alpha}{\mu}\left(\frac{\mu}{\alpha+\mu}\right)^{j}, \qquad 1\le j\le n.$$

For load-dependent checkpointing, i.e., checkpointing after completing n transactions ($\alpha \to 0$), $\overline{N}$ reduces to

$$\overline{N}=\left[1-\frac{\lambda}{\mu}\left[1-\frac{\gamma}{\gamma_r}+\frac{\mu}{n\beta}+\frac{\gamma}{\gamma_r}\left(\frac{\mu}{n\gamma_r}\right)\right.\right.$$
$$\left.\left.\cdot\left(\left(\frac{\gamma_r+\mu}{\mu}\right)^n-1\right)\frac{1}{P_1}\right]\right]^{-1}$$
$$\cdot\left\{\frac{\lambda}{\mu}\left[1-\frac{\gamma}{\gamma_r}+\frac{\mu}{n\beta}+\frac{\gamma}{\gamma_r}\left(\frac{\mu}{n\gamma_r}\right)\right.\right.$$
$$\left.\cdot\left(\left(\frac{\gamma_r+\mu}{\mu}\right)^n-1\right)\frac{1}{P_1}\right]$$
$$\cdot\left[1-\sum_{k=2}^{n}(n-k+1)\left(\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k}-1+\frac{\gamma_r}{\gamma}\right)\right.\right.$$
$$\left.\left.\cdot A(a,k)-\frac{1}{n}\right)\right]$$
$$-\frac{\lambda}{\beta}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_1}-1+\frac{\gamma_r}{\gamma}\right)A(a,1)$$
$$+\frac{\lambda}{\mu}\sum_{j=2}^{n}\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j}-1+\frac{\gamma_r}{\gamma}\right)\sum_{k=2}^{j}$$
$$\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_k}-1+\frac{\gamma_r}{\gamma}\right)A(a,k)-\frac{1}{n}\right]$$
$$+\frac{\lambda}{\gamma_r}\sum_{j=1}^{n}\frac{A(a,j)}{P_j}\left[\frac{\gamma}{\gamma_r}\left(\frac{1}{P_j}-1\right)\right.$$
$$\left.\left.-\left(\frac{j\gamma}{\gamma_r+\mu}+\frac{\gamma}{\gamma_r+\mu_0}\right)\right]\right\}, \tag{3.27}$$

with

$$A(a,j)=P(a,j,0)+\frac{\lambda}{n\mu}, \qquad 1\le j\le n.$$

If no failures occur during recovery ($\gamma_r = 0$) then the above expression for $\overline{N}$ reduces to that obtained in [18].

B. Numerical Approach

In the last section we have derived semianalytic expressions for the system availability and the steady-state average number of transactions in the system. These expressions are in terms of a set of unknown boundary state probabilities. In this section we discuss the numerical evaluation of these unknown probabilities. We consider the same model with a finite capacity, say N. (For models with infinite capacity, N is chosen sufficiently large.) The computational algorithm presented here is partially recursive and requires the solution of a reduced system of linear equations in the boundary state probabilities. The algorithm is briefly outlined in the following. Consider the Markov chain representing the queueing model in Fig. 3. This Markov chain contains D ($D = (n^2 + 5n + 2)(N + 1)/2$) states. The state probabilities can be determined by making use of $(D - 1)$ independent transition balance equations at $(D - 1)$ different states, together with the normalizing condition (all state probabilities sum up to one). This forms a system of linear equations in the D unknown state probabilities. It is evident that D can be large for small values of n and N (for example, if $N = 9$ and $n = 50$, $D = 13760$). Significant reduction in the size of the system can be achieved by making use of the model structure. The system can be solved partially in a recursive manner. For this we use $(D - n)$ transition balance equations at $(D - n)$ different states. The remaining $(n - 1)$ independent transition balance equations, together with the normalizing condition, form a reduced system of linear equations in the n unknown boundary state probabilities $p(a, j, 0)$, $j = 1, 2, \cdots, n$. This system of n linear equations can be solved simultaneously to determine the values of the unknown boundary state probabilities, which then can be used in the expressions of the other state probabilities (or the performance variables) to determine their actual values. A simple numerical algorithm for the recursive computation of the state probabilities is outlined as follows [13]. Any state probability (or performance measure) p in the Markov chain of Fig. 3 can be written as a linear sum of the n boundary state probabilities $p(a, j, 0)$, $j = 1, 2, \cdots, n$, so that $p = \Sigma_{j=1}^{n} g_j p(a, j, 0)$, where g_j is the coefficient of $p(a, j, 0)$ in the linear sum. It is then possible to determine the values of all the coefficients g_j for all state probabilities in the Markov chain by letting $p(a, j, 0)$ be equal to one and all other boundary state probabilities set equal to zero. The same recursive procedure to determine the state probabilities can now be used to obtain the values of their corresponding coefficients g_j. Note that the $(n - 1)$ independent balance equations at the end of the Markov chain are not needed to compute the coefficients; they are used after evaluating the coefficients together with the normalizing condition to form a reduced system of linear equations in the unknown boundary state probabilities. If we are interested only in evaluating certain per-

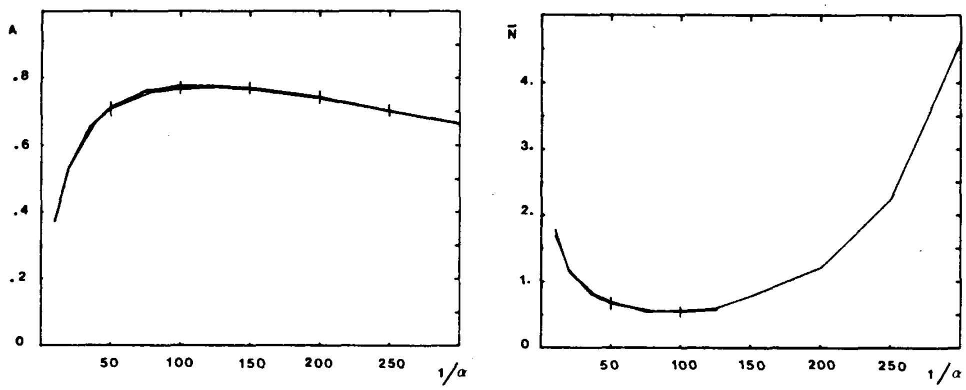

Fig. 4. Numerical versus simulation results with Poisson checkpointing ($\mu = 1$, $\lambda = 0.1$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [numerical (--------------), simulation (--+----+----+--)].

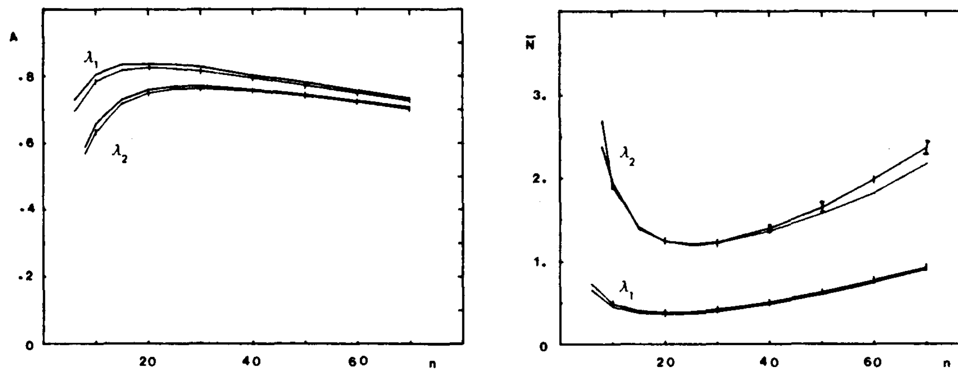

Fig. 5. Numerical versus simulation results with load-dependent checkpointing ($\mu = 1$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [numerical (--------------), simulation (--+----+----+--)].

formance measure then we need only to update and store the coefficients g_j, $j = 1, 2, \cdots, n$, for that measure as we compute recursively the coefficients for the state probabilities in the Markov chain.

The algorithm described above is used for models with a finite capacity (represented by a finite-state Markov chain). For models with infinite capacity, the solution is approached with an arbitrary accuracy by successively using the above algorithm with increasing system capacity (N). When further increase gives the same solution within the specified accuracy then we have an approximate solution for the model with infinite capacity. One possible stopping criterion is $| p_N(a, j, 0) - p_{N-1}(a, j, 0)| < \epsilon p(a, 0)$, for $j = 1, 2, \cdots, n$, with ϵ arbitrarily small.

For models with Poisson checkpointing ($n \rightarrow \infty$), the Markov chain of Fig. 3 is extended infinitely to the left (i.e., $j = 1, 2, \cdots$). In order to obtain a numerical solution for this model we set n sufficiently large to approximate a model with pure Poisson checkpointing. One way to do this is to successively solve the model for increasing n until further increase gives the same solution within a specified accuracy. Note that for smaller values of α (the checkpointing rate), n (and hence the number of boundary state probabilities) should be larger to obtain the same accuracy. As a result, the size of the system of linear equations grows rapidly as α decreases, this could lead to storage problems and numerical instability. For a model with Poisson checkpointing (C1) and a stochastic recovery model (R2) the numerical results are compared with the simulation results in Fig. 4. Similar comparison for a model with load-dependent checkpointing (C3) is shown in Fig. 5. For both checkpointing strategies the numerical results are in good agreement with the simulation results, particularly in the stable operating range. We conclude that numerical solutions with acceptable accuracy can be obtained for these models of checkpointing and recovery; this is useful since numerical solutions are much less expensive than simulations.

C. Simulation Approach

Simulation is a very useful solution technique, particularly when analytical and numerical solutions are not feasible. In this paper we use simulations to study complex ("realistic") models in order to validate other simpler models. Once we select a model the numerical solu-

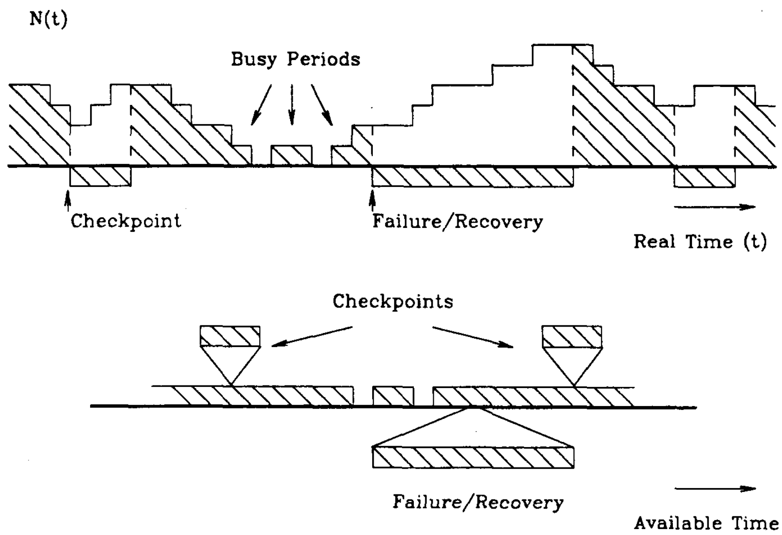

Fig. 6. A sample sequence of events taking place in the real time and in the available time ($N(t)$ is the number of transactions in the system at real time t).

tion of this model is also validated by comparison with simulation results. The main disadvantage of simulation is the high cost due to the large number of simulation runs which is often required to obtain results with acceptable accuracy.

In the following we discuss briefly the simulation of a transactional system supported by checkpointing and recovery strategies (as described in Section I). A sample sequence of events taking place in the system in real time is shown in Fig. 6. It is more convenient to view the sequence of events in the available time, this also is shown in Fig. 6. Checkpoints and failure recovery take place in real time during which the system is not available, therefore, the corresponding durations are not included in the available time; they are indicated above and below the available time axis. The system is idle if it is available and there are no transactions to be processed in the system.

The following are the main variables in the simulation program which we discuss briefly.

NIT (Next Interruption Time) is the time at which the next interruption occurs; it is the minimum of *NCT* (Next Checkpointing Time) and *NFT* (Next Failure Time).

NTT (Next Transaction Time) is the time of the next transaction arrival to the system.

NDT (Next Departure Time) is the time of the next transaction departure out of the system.

IND (Interruption Duration) is the duration of performing a checkpoint or a recovery after a failure.

The program consists of four main segments

Segment-1: While $NTT < NIT$ transactions keep arriving and departing the system after being processed. New arrivals and departures times are determined.

Segment-2: Now $NTT > NIT$, while the queue is not empty and $NDT < NIT$ transactions depart the system.

Segment-3: Now $NTT > NIT$ and $NDT > NIT$, the next interruption is determined (a checkpoint or a failure) with its duration. During the interruption new transactions arrive and placed in the queue.

Segment-4: Depending on the type of event the appropriate variables are updated (e.g., the total checkpointing or recovery time during a simulation run, queue statistics, etc.).

Each simulation run contains a fixed number of checkpointing cycles, at the end of which we obtain the mean values for the system availability and the queue length. The final results are obtained from a number of simulation runs; these include means, variances, and confidence intervals for performance measures of interest.

IV. Results and Discussion

In this section we compare different models in order to select one which is a good representation of a realistic system and which is tractable for analysis. It is also interesting to present a performance comparison between different checkpointing strategies. We use simulation to compare and validate different models; recovery models and checkpointing strategies are considered in Sections IV-A and B, respectively.

A. Recovery Models

For a simulation model with Poisson checkpointing we compare different recovery models, this is shown in Fig. 7. It is seen that the three recovery models agree on the system availability. For the average queue length we observe that, for lower checkpointing frequencies, the parametric recovery model differs from the other two models.

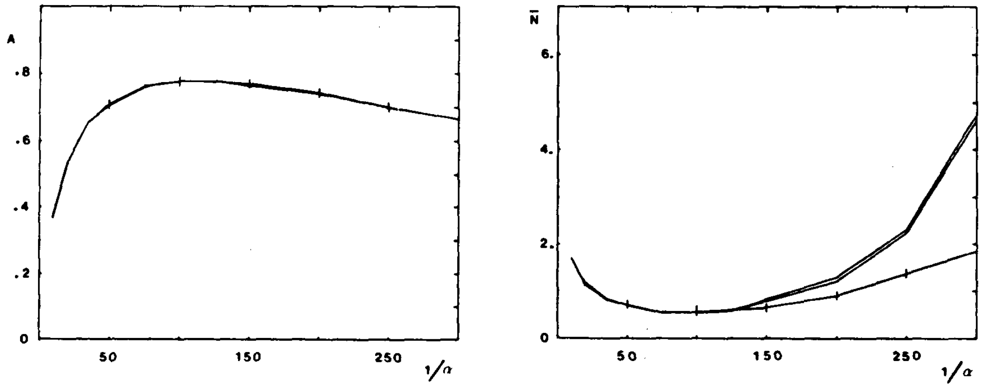

Fig. 7. Comparison of different recovery models with Poisson checkpointing ($\mu = 1$, $\lambda = 0.1$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [parametric (--+----+----+--), stochastic and deterministic (---------------)].

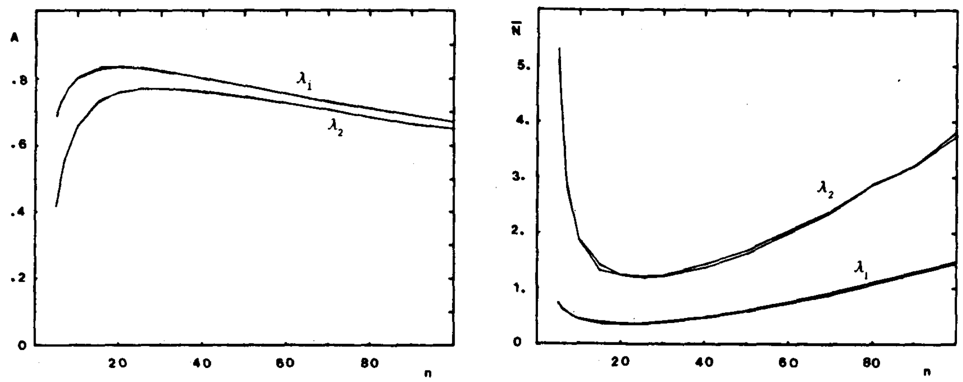

Fig. 8. Comparison of stochastic and deterministic recovery models with load-dependent checkpointing ($\mu = 1$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$).

This disagreement is caused mainly because of the increasing probability of two or more failures between two successive checkpoints. In the deterministic recovery model a recovery period is at least as long as the preceding one within the same intercheckpoint interval. This (deterministic) dependency between recovery periods increases for lower checkpointing frequencies, and is responsible for the increase in the average queue length. Similar (stochastic) dependency exists in the stochastic recovery model (R2). In the parametric recovery model (R1) recovery periods are independent even when they occur in the same intercheckpoint interval; this explains the lower average queue length. It was mentioned in Section I that the deterministic recovery model (R3) is the most realistic. We conclude that the parametric recovery model is the least accurate representation of a realistic recovery. The deterministic and stochastic recovery models are compared in Fig. 8 for a model with load-dependent checkpointing, and in Fig. 9 for a model with deterministic checkpointing. Clearly, the stochastic recovery model is in good agreement with the deterministic recovery model and therefore can be selected as an accurate recovery model which is also tractable for numerical analysis. This model is used in the following comparisons and discussions.

B. Checkpointing Models

In this section simulation is used to compare between different checkpointing strategies. Poisson checkpointing (C1) is often assumed in simple models for analytical tractability. Both deterministic (C2) and load-dependent (C3) checkpointing are close to real system implementations [11], [16], [18], [23]. For a fixed number of completed transactions between checkpoints, say n, the mean available time interval between checkpoints is given by the relation $E[a] = nA/\lambda$; this transformation is needed in order to compare load-dependent checkpointing with other strategies.

For a stochastic recovery model, Fig. 10 compares the system availability and the average queue length for Poisson and load-dependent checkpointing strategies. It is seen that the performance of Poisson checkpointing is inferior; the system availability is lower and the average queue

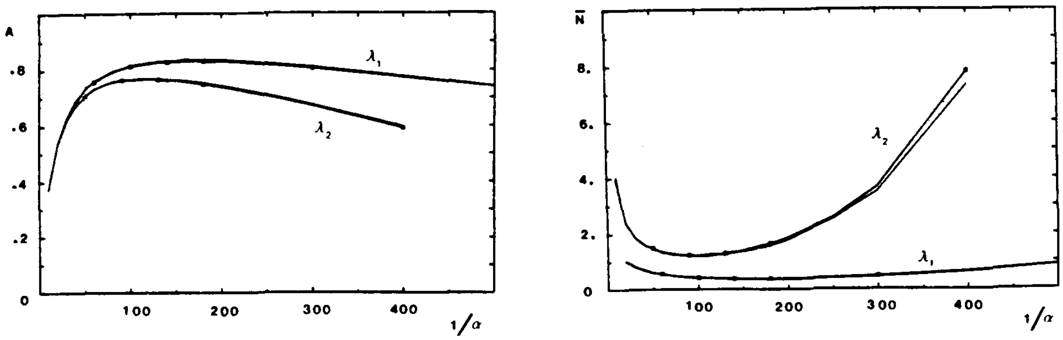

Fig. 9. Comparison of stochastic and deterministic recovery models with deterministic checkpointing ($\mu = 1$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$).

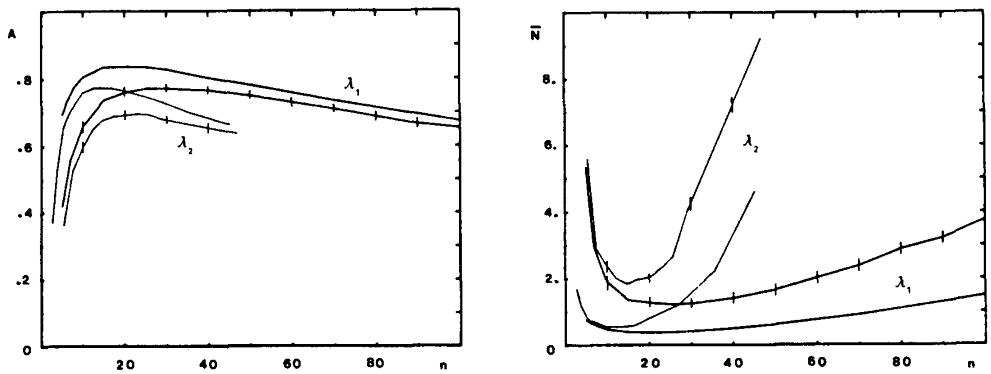

Fig. 10. Comparison of Poisson and load-dependent checkpointing ($\mu = 1$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [Poisson (--+----+----+--), load-dependent (---------------)].

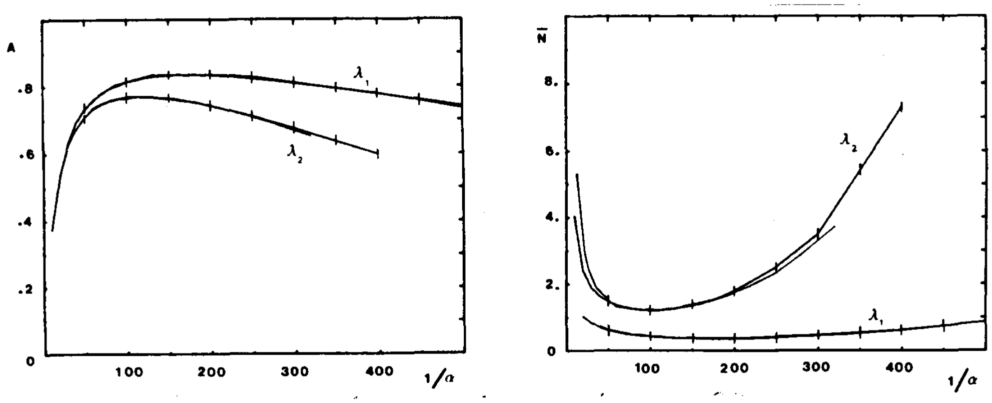

Fig. 11. Comparison of deterministic and load-dependent checkpointing ($\mu = 1$, $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [deterministic (--+----+----+--), load-dependent (---------------)].

length is higher. Note also that for Poisson checkpointing, performance measures are more sensitive to variations in the checkpointing frequency and the optimal frequency is higher. The low performance of Poisson checkpointing is due to a larger mean and variance of recovery periods. It is therefore obvious that Poisson checkpointing is not a good strategy for real systems. Fig. 11 compares the system availability and the average queue length for deterministic and load-dependent strategies. There is a good correspondence between the two checkpointing strategies over a wide range of checkpointing frequencies. In addition to being a good and realistic strategy, load-

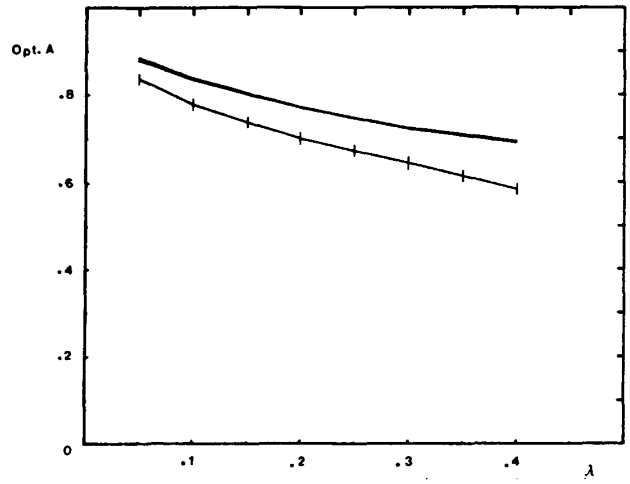

Fig. 12. The optimal system availability as a function of the system load ($\mu = 1$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [Poisson (--+----+----+--), deterministic and load-dependent (---------------)].

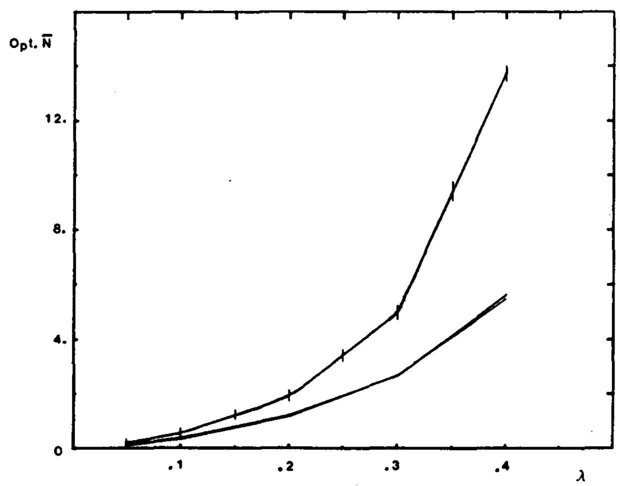

Fig. 13. The optimal average queue length as a function of the system load ($\mu = 1$, $\beta = 0.06$, $\gamma = 0.01$, $\gamma_r = 0$), [Poisson (--+----+----+--), deterministic and load-dependent (---------------)].

dependent checkpointing is also tractable for numerical analysis as shown in Section III.

Finally, we compare the optimal system availability and the optimal average queue length as functions of the system load (λ), for different checkpointing strategies, this is shown in Figs. 12 and 13. Generally, as the system load increases the optimal system performance decreases. Again the superiority and agreement of the deterministic and the load-dependent checkpointing strategies is observed.

V. Conclusions

Different checkpointing strategies are combined with recovery models of different levels of refinement. The complexity of the resulting model of checkpointing and recovery increases with its accuracy in representing a realistic system. Three different analysis approaches; namely, analytic, numerical, and simulation are used depending on the complexity of the model. A Markovian queueing model is developed, for a combined Poisson and load-dependent checkpointing strategy with stochastic recovery. In this model, failures may occur during any mode of system's operation. A state-space analysis approach is used to derive semianalytic expressions for the performance variables in terms of a set of unknown boundary state probabilities. An efficient numerical algorithm for the evaluation of these unknown probabilities is outlined. The validity of the numerical solution is checked against simulation results and shown to be of acceptable accuracy, particularly in the stable operating range. Simulations have shown that the realistic load-dependent checkpointing (a specified number of transactions between checkpoints) results in a performance close to the optimal deterministic checkpointing. Furthermore, the stochastic recovery model (reprocessing all transactions processed between the failure and the last checkpoint) is an accurate representation of a realistic recovery. It remains of much interest to develop more general queueing models (other than Markovian) to include other characteristics of existing systems, such as deterministic dependencies and queue-dependent behavior.

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