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# A New Keynesian Model with Bounded Rational Agents and Asynchronous Updating

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## Abbreviation directory

| | |
|---|---|
| BR | Boundedly Rational |
| CB | Central Bank |
| HE | Heterogeneous Expectations |
| IRF | Impulse Response Function |
| NKM | New Keynesian Model |
| OSR | Optimal Simple Rule |
| RE | Rational Expectations |
| TR | Taylor Rule |

# Symbol directory

| | |
|---|---|
| $\pi$ | rate of inflation |
| $\pi^*$ | inflation target |
| $x$ | output gap |
| $i$ | nominal interest rate |
| $\tilde{E}_t$ | heterogenous expectation operator |
| $\mu$ | output-shock |
| $\kappa$ | interest rate shock |
| $\nu$ | cost-push shock |
| $\beta$ | coefficient of inflation expectations in inflation equation |
| $\gamma$ | coefficient of output in inflation equation |
| $\sigma$ | interest elasticity of output demand |
| $\delta_\pi$ | coefficient of inflation in Taylor equation |
| $\delta_x$ | coefficient of ouput in Taylor equation |
| $\alpha$ | degree of extrapolation in extrapolating heuristics |
| $\theta_{ada}$ | degree of adaption in adaptive heuristics |
| $\theta_{laa}$ | degree of anchoring in learning and anchoring heuristics |
| $\omega$ | fractions of agents using certain heuristics |
| $\zeta$ | memory parameter for the attractivity value |
| $\eta$ | asynchronous updating parameter |
| $\phi$ | intensity of choice |

# 1. Introduction

The New Keynesian Model (NKM) is a broadly used model for monetary policy analysis (Galí, 2015, p. 1). However, it is based on the assumptions of rational expectations (RE) of the representative household. This assumption implies that agents have a full understanding of the economy and all available information at hand when they form their expectations. The occurrence of RE in reality is questionable. Hayek (1945, p. 530) argues that every individual has its own set of available information and that assuming that all the knowledge about the economy is given to a single mind is disregarding reality. Moreover, recent empirical literature suggests that agents use simple heuristics for forecasting (e.g. Hommes et al., 2019). To account for the usage of simple forecast heuristics, there is an upcoming strang in the literature for including heterogeneous expectations (HE) into the NKM (e.g. Branch & McGough, 2009; De Grauwe, 2011).

This seminar paper also introduces HE in a NKM and analyses the implications for the central bank (CB). Hence the seminar paper is constructed in the following way. The second chapter gives an overview of the current status of research, by presenting recent theoretical and empirical findings. In the third Chapter, the HE model is derived and the response to an exogenous shock is presented. The last chapter deals with the implications for the monetary policy dealing with HE among agents. Therefore an optimal simple rule (OSR) is derived. Moreover, the stability properties of different model parameters are analyzed.

# 2. Discussion of Related Literature

The upcoming part gives an overview of the current status of research. In detail, the literature is mainly divided into two parts. The first part is the theoretical modeling HE in the NKM framework and the second part is the empirical investigation of the expectation formation process in a macroeconomic environment.

Giving contribution to the theoretical modeling of HE in NKM, one major finding is that the addition of two simple forecasting heuristics into the NKM can lead to positive autocorrelation which is one of the stylized facts about business cycle dynamics (De Grauwe, 2011). Furthermore, De Grauwe (2011) pointed out that HE have an impact on the stability of the NKM and therefore an implication for monetary policy. In detail, a CB can reach a higher degree of inflation stabilization if besides inflation targeting also output stabilization is practiced. Hommes et al. (2019), who set up a behavioral expectations model using four different heuristics, also investigated the implications for the CB in such an environment. Their results are in line with De Grauwe (2011). In addition to their theoretical setup, Hommes et al. (2019) conducted an experiment where the subjects had to forecast inflation and the output gap. The experimental findings coincide with those from the theoretical approach. In particular that the inflation rate is less volatile in the case of additional output stabilization by the monetary authority, compared to the scenario without output stabilization. These findings are consistent with those of Branch and Evans (2011) who have set up an NKM with HE as well. In contrast to the other approaches, they simulated a scenario of so-called "bad luck", which mimics the U.S. during the 1970s, where the economy was hit by a series of high price shocks. As a result, the occurrence of multiple equilibria is now possible even if the CB reacts more than one for one to changes in inflation, which is known as the Taylor Principle.

Anufriev et al. (2013) introduced an information gathering cost parameter for agents who use the heuristic of simply adopting the RE equilibrium, which is also used as a heuristic by De Grauwe (2011). By introducing the gathering cost parameter the heuristic gets less attractive and therefore is less used. They found evidence that for a low-cost parameter the CB is able to stabilize the economy by favoring an aggressive reaction to inflation variation. Furthermore, the Taylor Principle does not guarantee that the system reaches the stable RE equilibrium in the high and low-cost scenario. Moreover, the effectiveness of monetary policy in a boundedly rational (BR) setup depends crucially on the form of expectation formation. To find evidence for that Branch and McGough (2009) investigated in their micro-founded NKM with 2 agents, where only one period ahead forecast matters, the effect of agents extrapolating and dampening the past realization into the future. If the CB is faced with extrapolating agents, it has to react more aggressively to inflation variation. Instead, if they are confronted with dampening agents the opposite is the case. While Branch and McGough (2009) had to make assumptions about the expectation formation, to set up the micro-founded model, Massaro (2013) implemented a micro-founded model without restrictions. The results are in line with those of the other approaches.

The literature on expectation formation is either based on survey data or data from experiments. Becker et al. (2008) examined whether the RE hypothesis holds in a forecasting experiment in which the subjects have several sources of information and tried to forecast a stationary time series. They concluded that the average forecast can not be explained by the RE hypothesis when the forecasters only have a limited amount of information. Pfajfar and Zakelj (2011) designed a forecasting experiment in a macroeconomic environment similar to the NKM. The subjects had to forecast the inflation rate by using the time series of the past realizations of inflation, output gap, and the interest rate. They scrutinized the differences among forecast, considering different types and parameterizations of Taylor Rules (TR). They found evidence that on average across all treatments approximately one-third of the subjects are rational, this was also found by Bao et al. (2012). Moreover, the most used forecast model was the extrapolation of the trend. Pfajfar and Zakelj (2011) also observed that the subjects tend to switch the forecast model they use every fourth period. However, their approach is restricted as their subjects have access to all available information of the past, which in reality might cause some information gathering costs. This difference sets the model at risk to have explanatory power about stability after a shock (Anufriev et al., 2013). While in Pfajfar and Zakelj (2011) the approach was limited in the way that the subjects had to forecast inflation only, Hommes et al. (2019) examined an experiment, similar to Pfajfar and Zakelj (2011), where the major difference is that subjects had to forecast inflation and output gap. They observed a modest usage of trend extrapolating models. Instead, their subjects tended to use adaptive models, where the previous forecast is updated from period to period.

While the previous authors have analyzed the expectation formation based on data from experiments Pfajfar and Santoro (2010) considered survey data, in particular the Michigan Survey Household's Expectation distribution. They investigated how frequently agents update their expectations and conclude that on average the frequency is about seven months and depends on the volatility. Furthermore, their research "highlights the presence of a marked degree of heterogeneity in the process of expectation formation." (Pfajfar & Santoro, 2010, p. 440).

# 3. A New Keynesian Model with behavioral expectations

In this chapter, the structure of the underlying model is described. The model is based on the baseline NKM, presented in the first subsection. Yet instead of applying RE, the model templates heterogeneous beliefs among the expectation formation procedure. These heterogeneous beliefs are developed in the second subsection and the resulting distribution of the usage of the heuristics is shown. Finally, the behavior of the model after an exogenous one-off shock is presented.

## 3.1. The baseline New Keynesian Model

The observed model is a standard NKM, which is in line with the basic NKM presented in Galí (2015, pp. 63–65). Therefore the model consists of a purely forward-looking aggregate demand equation (IS-equation) and a purely forward-looking aggregate supply equation (Phillips Curve). Additionally, an ad hoc given TR is part of the model. Following Branch and McGough (2009) one can replace the RE operator with a convey sum of adaptive expectation operators to introduce HE in the model and still consider a microfounded NKM.

The aggregate demand equation is specified in the following way,

$$x_t = \widetilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \tilde{E}_t[\pi_{t+1}]\right) + \mu_t \tag{1}$$

where $\sigma > 0$ stands for the inverse of the intertemporal elasticity of substitution between the present and future consumption. $x_t$ is the output gap in period t, $i_t$ is the nominal interest rate in period t and $\pi_t$ is the inflation rate in period t. Since I consider BR agents in my model, $\widetilde{E}$ is the expectation operator which indicates the usage of BR agents. A white noise output shock is expressed by $\mu_t$. Often the aggregate-demand equation additionally consists of a lagged output. Since De Grauwe (2011) pointed out that this is not necessary to create inertia, this term is neglected here.

The aggregate supply equation is stated as follows,

$$\pi_t = \beta\tilde{E}_t[\pi_{t+1}] + \gamma x_t + \nu_t \tag{2}$$

where $0 \leq \beta \geq 1$ and $\gamma > 0$. Similar to equation (1) white noise cost-shock $\nu_t$ added.

Finally, the ad hoc given TR, which is the monetary policy rule, is described as,

$$i_t = \delta_\pi(\pi_t - \pi^*) + \delta_x x_t + \kappa_t \tag{3}$$

where $\delta_\pi, \delta_x > 0$ measures how strong the CB reacts to deviations of the inflation rate and the output gap from their corresponding target values. In this approach, the target value of $\pi$ is set equal to zero. In section four, where the implications for the monetary authority are discussed, different types of TRs will be considered as well.

## 3.2. Households expectation formation

In the baseline NKM agents form their expectations fully rational. This means that agents have a deep understanding of the model so that predictions are the same as those of the underlying model (Muth, 1961). In this seminar paper, the assumption of rational agents is eased. For that reason, agents build up their forecast about the future development of the state variables on the usage of different types of heuristics. This is in line with the discussed findings in section 1 of this seminar paper.

The set of used heuristics is shown in table 1. The heuristics, in particular, "Targeter" and "Extrapolator" are similar to the one used in (De Grauwe, 2011). Furthermore, the "LAA" rule and the "ADA" rule are used by (Hommes et al., 2019). They additionally observed that the "LAA" heuristics has the lowest mean squared forecast error if one compares the forecast performance considering all heuristics in their setup separately. This is the reason for including it into the set of heuristics. In the following $y \in \{\pi, x\}$.

| | | | |
|---|---|---|---|
| TAR | Targeter | $\tilde{E}_t^{tar}[y_{t+1}] = y^*$ | (4) |
| EXT | Extrapolator | $\tilde{E}_t^{ext}[y_{t+1}] = y_t + \alpha_y[y_t - y_{t-1}]$ | (5) |
| ADA | Adaptive Rule | $\tilde{E}_t^{ada}[y_{t+1}] = \theta_{ada} y_t + (1 - \theta_{ada})\tilde{E}_{t-1}^{ada}[y_t]$ | (6) |
| LAA | Anchoring and Adjustment Rule | $\tilde{E}_t^{laa}[y_{t+1}] = \theta_{laa}(y_{t-1}^{av} + y_t) + y_t - y_{t-1}$ | (7) |

Table 1: Set of heuristics. Source: own illustration, $y_{t-1}^{av}$ is the avergage of all observations up to t-1.

One can categorize the heuristics into two groups. The first group including "Targeters" and "Extrapolators" display a rather naïve way of forecasting, in which the target of the CB is used as a forecast or the observed trend is followed. The second group of heuristics including the "Adaptive Rule" and the "Anchoring and Adjustment Rule" gives the agents the possibility to update the forecast rule and therefore account for their past errors.

The overall market forecast is calculated by a weighted average of these four forecasts,

$$\tilde{E}_t[y_{t+1}] = \omega_t^{y,tar} \tilde{E}_t^{tar}[y_{t+1}] + \omega_t^{y,ext} \tilde{E}_t^{ext}[y_{t+1}] + \omega_t^{y,ada} \tilde{E}_t^{ada}[y_{t+1}] + \omega_t^{y,laa} \tilde{E}_t^{laa}[y_{t+1}] \quad (8)$$

with $\omega_t^{y,j}$ $j \in \{tar, ext, ada, laa\}$ being the fractions of agents using certain heuristics.

Assuming that agents favor parsimony, the weights corresponding to the different forecast techniques are updated in every period via an "attractivity value". In line with this, the "attractivity" of each forecast heuristic is measured by the past squared forecast error:

$$A_t^{y,j} = -\left(y_{t-1} - \tilde{E}_{t-2}^{y}[y_{t-1}]\right)^2 + \zeta A_{t-1}^{y,j} \quad (9)$$

where $\zeta \geq 0$ is a memory parameter, indicating how valuable the past forecast errors are for the agents.

Also, the allocation of the values corresponding to the fractions is done via discrete decision making following Brock and Hommes (1997). This approach is extended by an asynchronous updating which is based on (Diks & van der Weide, 2005). Therefore,

$$\omega_t^{y,j} = \eta\omega_{t-1}^{y,j} + (1-\eta)\frac{\exp\left\{\phi A_t^{y,j}\right\}}{\exp\left\{\phi A_t^{y,tar}\right\}+\exp\left\{\phi A_t^{y,ext}\right\}+\exp\left\{\phi A_t^{y,ada}\right\}+\text{ex}\ \left\{\phi A_t^{y,laa}\right\}} \quad (10)$$

where $\phi$ measures "the intensity of choice", sometimes called the "degree of rationality" since it parameterizes how important the "attractivity value" is, for the determination of the fractions. If $\phi = \infty$ agents select the best performing heuristic for each period in time. The parameter $0 \leq \eta \leq 1$ accounts for the asynchronous updating of the agents which do not update every period their used forecast heuristic (Pfajfar & Zakelj, 2011).

In order to simulate the model, the model has to be solved. This is done by inserting (3) into (1) and replacing the HE operator with (8). The solution to the model is then given by (compare Appendix A1):

$$\begin{pmatrix}\pi_t\\x_t\end{pmatrix} = A_t^{-1}B_t\begin{pmatrix}\pi_{t-1}\\x_{t-1}\end{pmatrix} + A_t^{-1}C_t\begin{pmatrix}\tilde{E}_{t-1}^{ada}[\pi_t]\\\tilde{E}_{t-1}^{ada}[x_t]\end{pmatrix} + A_t^{-1}D_t\begin{pmatrix}\pi^{av}\\x^{av}\end{pmatrix} + A_t^{-1}\begin{pmatrix}\sigma\mu_t-\kappa_t\\nu_t\end{pmatrix} \quad (11)$$

with time-dependent matrices:

$$A_t = \begin{pmatrix}\delta_\pi - a_\pi & \sigma(1-a_x)+\delta_x\\1-\beta a_\pi & -\gamma\end{pmatrix}$$

$$B_t = \begin{pmatrix}-b_\pi & -\sigma b_x\\-\beta b_\pi & 0\end{pmatrix}$$

$$C_t = \begin{pmatrix}c_\pi & \sigma c_x\\\beta c_\pi 0 & 0\end{pmatrix}$$

$$D_t = \begin{pmatrix}d_\pi & \sigma d_x\\\beta d_\pi & 0\end{pmatrix}$$

and: $y \in \{\pi, x\}$

$$a_y = \omega_t^{y,ext}\left[1+\alpha_y\right) + \omega_t^{y,ada}\theta_{ada} + \omega_t^{y,laa}(1+\theta_{laa})$$

$$b_y = \omega_t^{y,ext}\alpha_y + \omega_t^{y,laa}$$

$$c_y = \omega_t^{y,ada}(1-\theta_{ada})$$

$$d_y = \omega_t^{y,laa}\theta_{laa}$$

A detailed parameterization of the model used for all the following simulations is given in Appendix (A2)[1].

The distribution of the heuristics to forecast inflation which is used by the agents and the corresponding time path, for one simulation run, is presented in Figure 1. Figure 2 shows the development of the used heuristics to forecast the output gap and the corresponding time path. The evolution of the weights is almost similar for output and inflation. This is in line with experimental results confirming that agents tend to change the way of forming expectations

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[1] compare e.g. Lengnick and Wohltmann (2016)

in similar ways, regardless if they forecast inflation or output gap (Hommes et al., 2019).

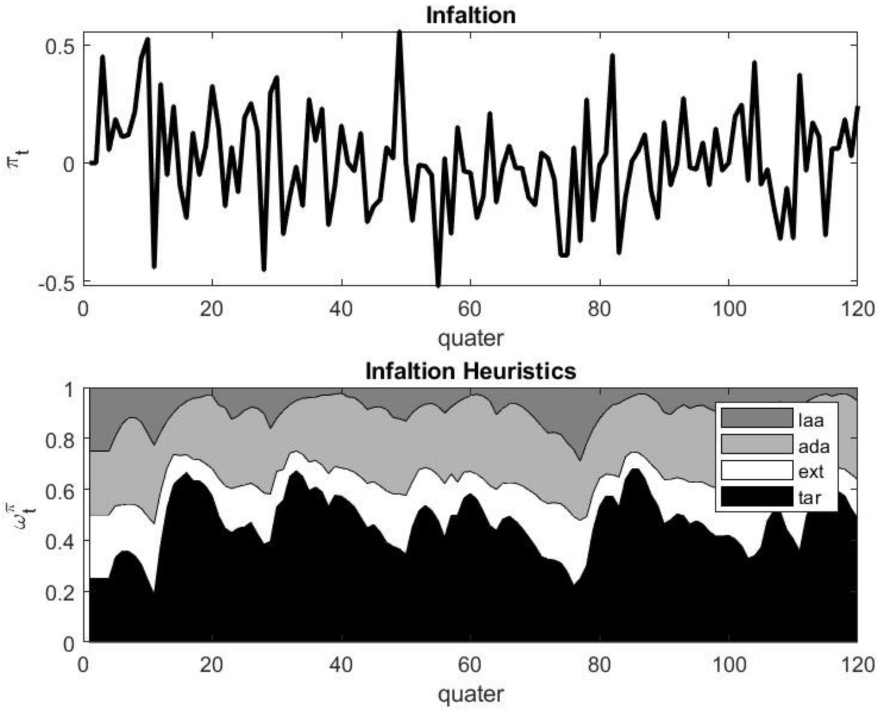


Figure 1: Inflation and inflation heuristics time path. Source: own illustration

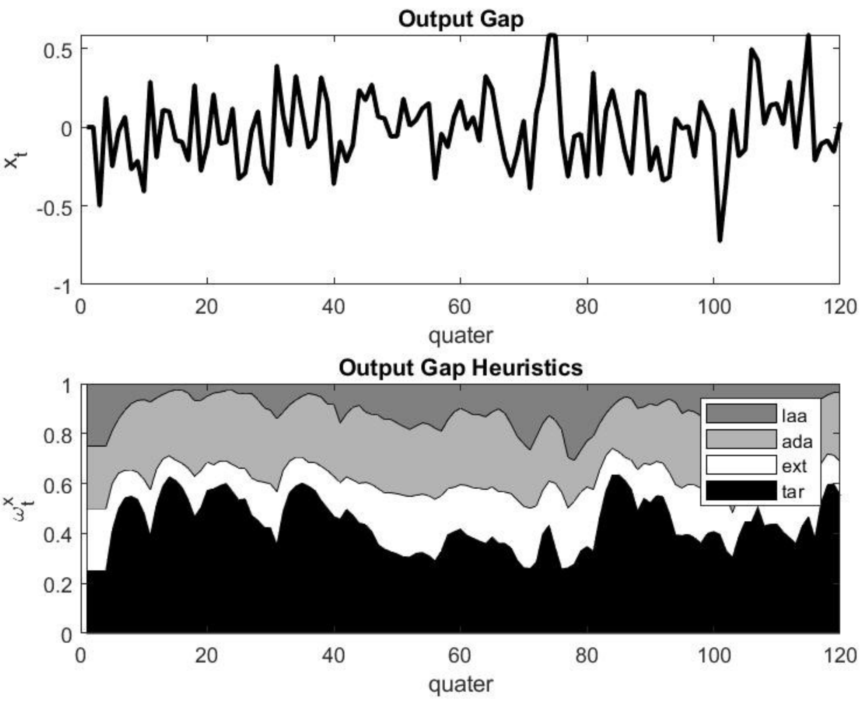


Figure 2: Output gap and output gap heuristics time path. Source: own illustration

## 3.3. Impulse Response function

To examine how the model reacts to exogenous shocks the mean impulse response function (IRF) is computed. Therefore two scenarios are simulated. One with an extra cost-shock (without persistence) in period forty and one without the additional shock. The shock size is two times the standard deviation of the regular cost-shock. Since the exogenous shocks are white noise, the state of the economy is different for each simulation. Therefore the average response over all simulations is obtained. To compare both scenarios the economy is faced with the same shocks (excluded period forty) for every simulation run. The difference of both situations, which is the IRF, for 1000 simulations of 15 quarter after the shock, is given in Figure 2[2].

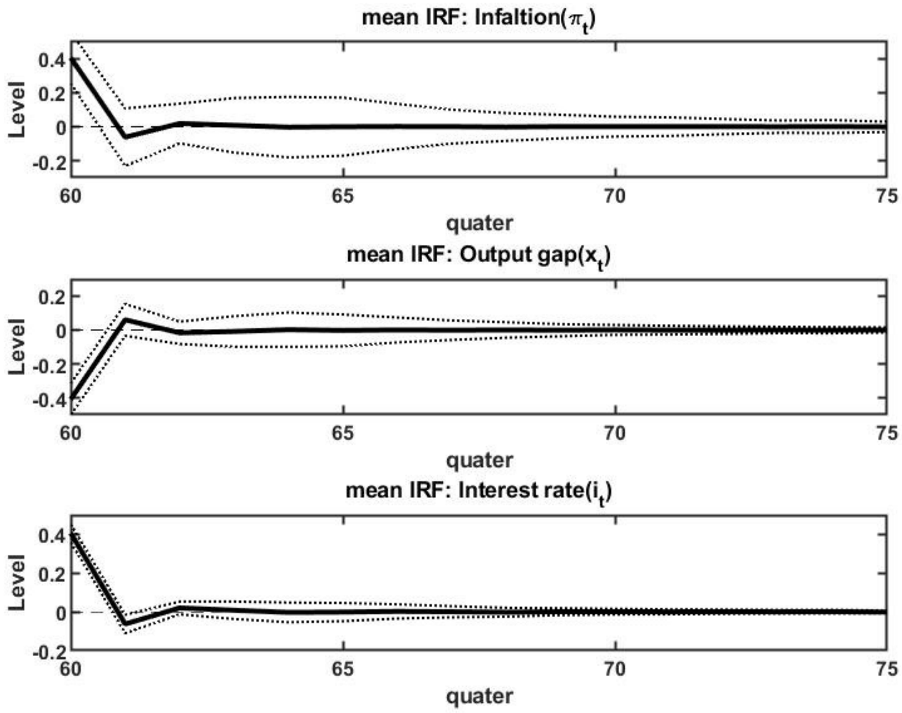


Figure 3: Mean IRF for inflation, output gap, and interest rate. Source: own illustration

The two dotted lines are the mean value plus/minus two times the standard deviation, and so approximately represent the 95% confidence interval (CI).

[2] To ensure that the system has converged to its ergodic distribution, the model was simulated with 20 initital periods (compare De Grauwe (2011) In additon to guarantee stability the paramter ϕ was set to 4.

The dashed line represents the equilibrium at the level of zero. Figure 3 displays humped shaped behavior of the IRFs. Therefore the system does not converge immediately to its equilibrium value. This stands in contrast to the baseline NKM where the state variables immediately jump to their equilibrium level after a one-off shock. Moreover, the exact course is uncertain, as indicated by the CI. Therefore in some scenarios, the economy can even be considered with a high degree of deflation, considering a positive cost-push shock. To examine the optimal response of the CB in such an environment, in chapter 4 implications are discussed, which are deduced from the shown outcomes.

# 4. Monetary Policy Implications

In this chapter, the question is answered if and how the CB can stabilize the economy in a HE framework. Therefore, an optimal simple rule is derived. Besides the optimal parameterization of different types of TR are presented and checked whether the CB can stabilize the economy.

## 4.1. Optimal Simple Rule

In order to derive an OSR for the CB a loss function for the CB which is given as the unconditional variance of inflation and output[3] is set up:

$$L = \mathrm{var}(\pi) + \frac{1}{2}\mathrm{var}(x) \tag{12}$$

In line with (8), the OSR is given by the parameter combination of $(\delta_\pi^*, \delta_x^*)$ that minimizes the loss function. Since the model is driven by the stochastic terms in (1-3), the average loss is taken. This is done to account for the different time patterns of the economy the CB is faced with.

In this context, the question arises if the OSR changes if different scenarios are considered.

In the first scenario the monetary policy depends on the contemporaneous values (compare Eq. (3)):

[3] Compare e.g. Svensson (2003) and Lengnick and Wohltmann (2016)

$i_t = \delta_\pi \pi_t + \delta_x x_t + \kappa_t$

In the second scenario it depends on past values:

$$i_t = \delta_\pi \pi_{t-1} + \delta_x x_{t-1} + \kappa_t \quad (13)$$

In the last scenario it depends on the expectations:

$$i_t = \delta_\pi \tilde{E}[\pi_{t+1}] + \tilde{E}[\delta_x x_{t+1}] + \kappa_t \quad (14)$$

Table 2 therefore presents the optimal parameter combinations for all three scenarios[4]. In addition, the corresponding loss and the percentage change in loss compared to the baseline scenario is given[5].

| | $[\pi_t, x_t]$ | $[\pi_{t-1}, x_{t-1}]$ | $\left[\tilde{E}_t[\pi_{t+1}], \tilde{E}_t[x_{t+1}]\right]$ |
|---|---|---|---|
| $(\delta_\pi^*, \delta_x^*)$ | (5.5,4.5) | (0,0) | (5.8,4.9) |
| $L$ | 0.0568 | $\rightarrow \infty$ | 0.0593 |
| $\Delta L$ | | $\rightarrow \infty$ | 0.0025 |

Table 2: Optimal Simple Rule for different Taylor Rules. Source: own illustration.

The smallest loss value occurs if the CB responds to the contemporaneous values. The loss value increases by about 4.4% if expectations are used instead of contemporaneous values. Since the expectations depend on the contemporaneous and past values (compare Tabel 1), one can conclude that using past values is destabilizing. This can be confirmed by the third column of Table 2: The Loss value gets explosive, which implies that a backward-looking TR is destabilizing the economy. This result is in line with the macroeconomics literature, namely that optimal monetary policy and the structure of the model should have an inverse relationship. Therefore, if the model is backward-looking, which is the case for the underlying model, monetary policy should use the most recent information available (e.g. Carlstrom & Fuerst, 2000; Svensson & Woodford, 2004). Moreover, the monetary authority has to react very strongly to inflation and output gap variation to reach a low loss level.

[4] A derivation of the state space model for scenario 2 and 3 can be found in the Appendix (A3)

[5] To obtain the values in table 2, the model was simmulated for 250 times each time for 260 quaters and the avergae Loss over the 250 simmulation runs was taken.

In addition, Figure 4 displays the loss values for parameter combinations of $\delta_\pi$ and $\delta_x$ if the CB uses contemporaneous values in the TR. The white area indicates explosive/divergent regions and darker areas indicate a lower loss. Therefore, even if the CB reacts more than one for one to inflation a guaranteed stabilization of the economy does not occur. Moreover, the CB can reach a lower loss value if, in addition to inflation stabilization, output stabilization is practiced. These findings coincide with those presented in Chapter 2[6].

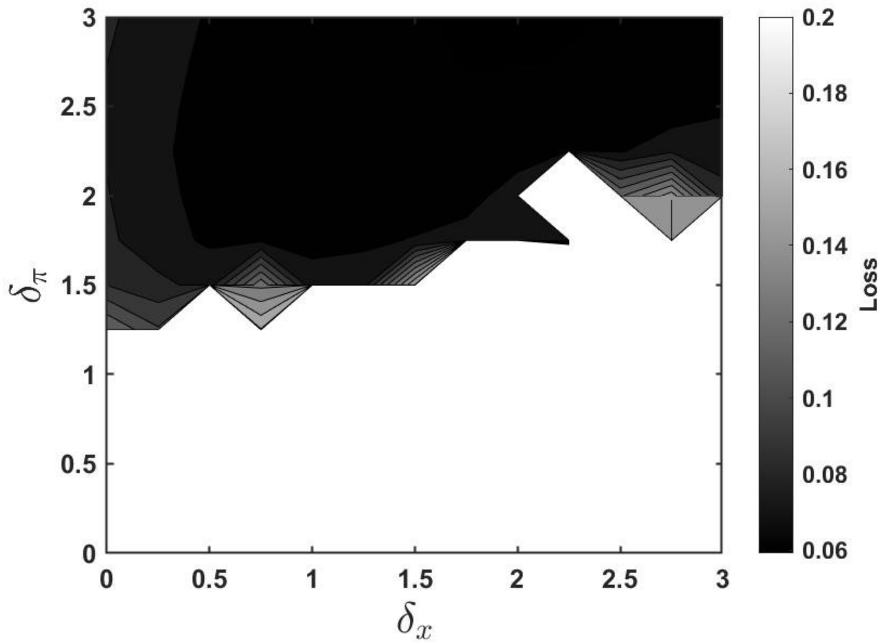


Figure 4: Stability of the model with contemporaneous TR for different parameterization. Source: own illustration

## 4.2. Stability Analysis

This chapter deals with the question if the asynchronous updating parameter is stabilizing or destabilizing the economy if in addition also the "degree of rationality" is varying. The analysis is done for the baseline model with HE. To contribute to this question the model, given the optimal values from Chapter

[6] e.g. De Grauwe (2011); Hommes et al. (2019)

4.1[7], is simulated[8], for different parameter combinations, and obtained the loss value for this parametrization.

The results can be summed up as follows: The asynchronous updating parameter has a stabilizing effect for the CB. The lowest loss is reached if the CB is faced with a high intensity of choice of the agents and also the agents update their used heuristics on average between every fourth and tenth period. This frequency of updating is in line with the findings from macroeconomic experiments in the lab[9]. Moreover, the system tends to explode if the agents behave more rational and update their used forecast heuristics more frequently (white area). Whereas $\phi \leq 4$ has a stabilizing impact. In the case of $\phi = 0$ and $\eta = 1$, the distribution of the heuristics is time-invariant. In this scenario, the CB is faced with a higher loss but can stabilize the economy. Similar results hold if one considers different parametrization in the TR.

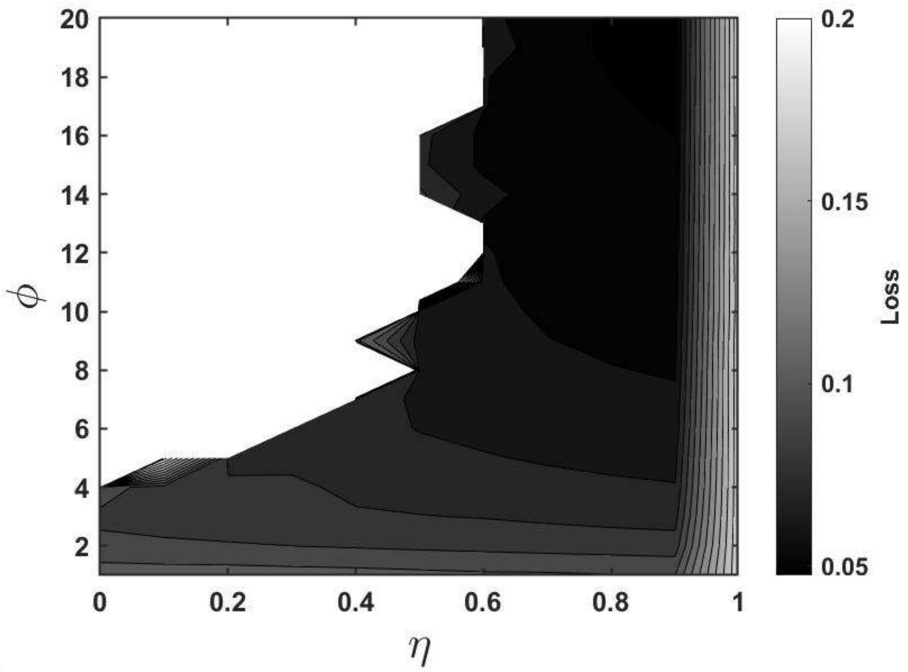


Figure 5: Impact of intensity of choice and asynchronous updating on the stability. The white areas indicate explosive/divergent behavior of the system. Furthermore darker areas indicate lower loss values.

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[7] I set the parameter $\delta_\pi = 5.5$ and $\delta_x = 4.5$.
[8] The number of simulations was equal to 1000 and the time span was 500 periods.
[9] e.g. Hommes et al. (2019)

# 5. Conclusion

This seminar work investigates the impact of HE in a NKM with four types of agents who forecast the state variables of the model. The model is a combination of the model used by De Grauwe (2011) and Hommes et al. (2019). Therefore the model consists of an aggregate demand curve, an aggregate supply curve, and a Taylor-type interest rate rule.

In this environment, I analyze the ability of the CB to react to an exogenous shock via IRFs. The CB can stabilize the economy, but the exact course of the state variables is uncertain. The uncertainty exists due to the dependence on the state of the economy at the time of the shock. Besides that, the IRFs display humped shape behavior of the state variables which is in clear contrast to the baseline NKM with RE.

Moreover, I derive an OSR for different types of TRs and compare the resulting loss values. The CB reaches the lowest loss value if the contemporaneous values of the state variables are included in the TR. Furthermore, a backward-looking TR has no stabilizing power and is even destabilizing the economy. Also, I showed that the Taylor Principle does not guarantee a stable economy and that if in addition to inflation stabilization also output stabilization is practiced the CB can reach a lower loss value.

Finally, I investigated the impact of the asynchronous updating parameter in combination with varying intensity of choice. A low intensity of choice is stabilizing the economy where the opposite is destabilizing. Also, the less frequent the agents update their forecast model the more the CB can stabilize the economy even for a high rationality level.

This seminar paper still leaves further questions open. In reality, using more advanced forecast heuristics might come at a higher cost than simple heuristics. This is not taken into account in the model. Moreover, in reality, the CB cannot set the interest rate lower than zero. Future research should analyze whether the CB has to change the optimal monetary policy in an environment with a zero lower bound and agents who have costs for gathering information.

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# Appendix

## A.1. Derivation of the State Space Model in Matrix Notation

The baseline Model:

$x_t = \tilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \tilde{E}_t[\pi_{t+1}]\right) + \mu_t$ compare (1)

$\pi_t = \beta\tilde{E}_t[\pi_{t+1}] + \gamma x_t + \nu_t$ compare (2)

$i_t = \delta_\pi(\pi_t - \pi^*) + \delta_x x_t + \kappa_t$ compare (3)

Market expectations:

1. Market expectation for the inflation rate

   Heuristics:

   $$\tilde{E}_t^{tar}[\pi_{t+1}] = \pi^* \tag{15}$$

   $$\tilde{E}_t^{ext}[\pi_{t+1}] = \pi_t + \alpha_y[\pi_t - \pi_{t-1}] \tag{16}$$

   $$\tilde{E}_t^{ada}[\pi_{t+1}] = \theta_{ada}\pi_t + (1-\theta_{ada})\tilde{E}_{t-1}^{ada}[\pi_t] \tag{17}$$

   $$\tilde{E}_t^{laa}[\pi_{t+1}] = \theta_{laa}(\pi_{t-1}^{av} + \pi_t) + \pi_t - \pi_{t-1} \tag{18}$$

   Market expectation:

   $$\tilde{E}_t[\pi_{t+1}] = \omega_t^{\pi,tar}\tilde{E}_t^{tar}[\pi_{t+1}] + \omega_t^{\pi,ext}\tilde{E}_t^{ext}[\pi_{t+1}] + \omega_t^{\pi,ada}\tilde{E}_t^{ada}[\pi_{t+1}] + \omega_t^{\pi,laa}\tilde{E}_t^{laa}[\pi_{t+1}] \tag{19}$$

   Inserting (15)-(18) into (19) and assuming $\pi^* = 0$ yields market expectation for the inflation rate:

   $$\tilde{E}_t[\pi_{t+1}] = a_\pi\pi_t - b_\pi\pi_{t-1} + c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + d_\pi\pi^{av} \tag{20}$$

   with:

   $a_\pi = \omega_t^{\pi,ext}[1+\alpha_\pi) + \omega_t^{\pi,ada}\theta_{ada} + \omega_t^{\pi,laa}(1+\theta_{laa})$

   $b_\pi = \omega_t^{\pi,ext}\alpha_\pi + \omega_t^{\pi,laa}$

   $c_\pi = \omega_t^{\pi,ada}(1-\theta_{ada})$

   $d_\pi = \omega_t^{\pi,laa}\theta_{laa}$

2. Market expectations for the output gap

   Heuristics:

   $$\tilde{E}_t^{tar}[x_{t+1}] = x^* \tag{21}$$

   $$\tilde{E}_t^{ext}[x_{t+1}] = x_t + \alpha_y[x_t - x_{t-1}] \tag{22}$$

   $$\tilde{E}_t^{ada}[x_{t+1}] = \theta_{ada}x_t + (1-\theta_{ada})\tilde{E}_{t-1}^{ada}[x_t] \tag{23}$$

$$\tilde{E}_t^{laa}[x_{t+1}] = \theta_{laa}(x_{t-1}^{av} + x_t) + x_t - x_{t-1} \quad (24)$$

Market expectation:

$$\tilde{E}_t[x_{t+1}] = \omega_t^{x,tar}\tilde{E}_t^{tar}[x_{t+1}] + \omega_t^{x,ext}\tilde{E}_t^{ext}[x_{t+1}] +$$
$$\omega_t^{x,ada}\tilde{E}_t^{ada}[x_{t+1}] + \omega_t^{x,laa}\tilde{E}_t^{laa}[x_{t+1}] \quad (25)$$

Inserting (21)-(24) into (25) and assuming $x^* = 0$ yields <u>market expectation for the output gap:</u>

$$\tilde{E}_t[x_{t+1}] = a_x x_t - b_x x_{t-1} + c_x \tilde{E}_{t-1}^{ada}[x_t] + d_x x^{av} \quad (26)$$

with:

$$a_x = \omega_t^{x,ext}[1 + \alpha_x) + \omega_t^{x,ada}\theta_{ada} + \omega_t^{x,laa}(1 + \theta_{laa})$$
$$b_x = \omega_t^{x,ext}\alpha_x + \omega_t^{x,laa}$$
$$c_x = \omega_t^{x,ada}(1 - \theta_{ada})$$
$$d_x = \omega_t^{x,laa}\theta_{laa}$$

<u>The state-space model:</u>

<u>First state equation:</u>

Inserting (3) into (1) and again assuming $\pi^* = 0$ yields:

$$x_t = \tilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\Big((\delta_\pi \pi_t + \delta_x x_t + \kappa_t) - \tilde{E}_t[\pi_{t+1}]\Big) + \mu_t \quad (27)$$

Inserting (26) and (20) into (27) and rearranging terms yields the <u>first state equation</u>

$$(\delta_\pi - a_\pi)\pi_t + (\sigma(1 - a_x) + \delta_x) = -b_\pi \pi_{t-1} - \sigma b_x x_{t-1} + c_\pi \tilde{E}_{t-1}^{ada}[\pi_t] +$$
$$\sigma c_x \tilde{E}_{t-1}^{ada}[x_t] + d_\pi \pi^{av} + \sigma d_x x^{av} + \sigma\mu_t - \kappa_t \quad (28)$$

<u>Second state equation:</u>

Inserting (15) into (2) yields the <u>second state equation:</u>

$$(1 - \beta a_\pi)\pi_t - \gamma x_t = -\beta\pi_{t-1} + \beta c_\pi \tilde{E}_{t-1}^{ada}[\pi_t] + \beta d_\pi \pi^{av} + \nu_t \quad (29)$$

Reformulating (28) and (29) into matrix notation yields:

$$A_t \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = B_t \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + C_t \begin{pmatrix} \tilde{E}_{t-1}^{ada}[\pi_t] \\ \tilde{E}_{t-1}^{ada}[x_t] \end{pmatrix} + D_t \begin{pmatrix} \pi^{av} \\ x^{av} \end{pmatrix} + \begin{pmatrix} \sigma\mu_t - \kappa_t \\ nu_t \end{pmatrix} \quad (30)$$

By solving (19) for $(\pi_t \quad x_t)'$ the explicit state dynamics is obtained

$$\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} = A_t^{-1}B_t\begin{pmatrix}\pi_{t-1}\\ x_{t-1}\end{pmatrix} + A_t^{-1}C_t\begin{pmatrix}\tilde{E}_{t-1}^{ada}[\pi_t]\\ \tilde{E}_{t-1}^{ada}[x_t]\end{pmatrix} + A_t^{-1}D_t\begin{pmatrix}\pi^{av}\\ x^{av}\end{pmatrix} +$$

$$A_t^{-1}\begin{pmatrix}\sigma\mu_t - \kappa_t\\ nu_t\end{pmatrix} \qquad \text{compare (11)}$$

with time-dependent matrices

$$A_t = \begin{pmatrix}\delta_\pi - a_\pi & \sigma(1-a_x)+\delta_x\\ 1-\beta a_\pi & -\gamma\end{pmatrix}$$

$$B_t = \begin{pmatrix}-b_\pi & -\sigma b_x\\ -\beta b_\pi & 0\end{pmatrix}$$

$$C_t = \begin{pmatrix}c_\pi & \sigma c_x\\ \beta c_\pi 0 & 0\end{pmatrix}$$

$$D_t = \begin{pmatrix}d_\pi & \sigma d_x\\ \beta d_\pi & 0\end{pmatrix}$$

## A.2. Parameterization of the model

| Structure: | Learning: | Noise: |
|---|---|---|
| $\sigma = 1.0$ | $\zeta = 0.5$ | $\sigma_\mu = 0.15$ |
| $\beta = 0.99$ | $\phi = 10$ | $\sigma_\kappa = 0.15$ |
| $\gamma = 0.33$ | $\alpha_x = \alpha_\pi = 0.2$ | $\sigma_\nu = 0.15$ |
| $\delta_\pi = 1.5$ | $\theta_{ada} = 0.65$[10] | |
| $\delta_x = 0.5$ | $\theta_{\mathrm{laa}} = 0.5$[11] | |
| | $\eta = 0.75$[12] | |

[10] compare Hommes et al. (2019)
[11]compare Hommes et al. (2019)
[12] I set $\eta = 0.75$ in order to let the agents fully update their weights every fourth period on anverage (compare with the emprical findings of Pfajfar and Zakelj (2011)).

## A.3. Derivation of the state dynamics for a backward- and forward-looking Taylor Rule

### A.3.1. The baseline Model with backward-looking Taylor Rule:

$x_t = \widetilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \tilde{E}_t[\pi_{t+1}]\right) + \mu_t$ compare (1)

$\pi_t = \beta\widetilde{E}_t[\pi_{t+1}] + \gamma x_t + \nu_t$ compare (2)

$i_t = \delta_\pi(\pi_{t-1} - \pi^*) + \delta_x x_{t-1} + \kappa_t$ compare (13)

Market expectations compare with equations (15) –(26):

$\widetilde{E}_t[\pi_{t+1}] = a_\pi\pi_t - b_\pi\pi_{t-1} + c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + d_\pi\pi^{av}$ compare (20)

$\widetilde{E}_t[x_{t+1}] = a_x x_t - b_x x_{t-1} + c_x\tilde{E}_{t-1}^{ada}[x_t] + d_x x^{av}$ compare (26)

The state-space model:

First state equation:

Inserting (13) into (1) and again assuming $\pi^* = 0$ yields:

$$x_t = \widetilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\Big((\delta_\pi\pi_{t-1} + \delta_x x_{t-1} + \kappa_t) - \tilde{E}_t[\pi_{t+1}]\Big) + \mu_t \qquad (31)$$

Inserting (26) and (20) into (27) and rearranging terms yield the first state equation

$$-a_\pi\pi_t + \sigma(1 - a_x)x_t = -(b_\pi + \delta_\pi)\pi_{t-1} - (\sigma b_x + \delta_x)x_{t-1} + c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + \sigma c_x\tilde{E}_{t-1}^{ada}[x_t] + d_\pi\pi^{av} + \sigma d_x x^{av} + \sigma\mu_t - \kappa_t \qquad (37)$$

Second state equation :

$(1 - \beta a_\pi)\pi_t - \gamma x_t = -\beta\pi_{t-1} + \beta c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + \beta d_\pi\pi^{av} + \nu_t$ compare (29)

Reformulating (37) and (29) into matrix notation yields:

$$A_t\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} = B_t\begin{pmatrix}\pi_{t-1}\\ x_{t-1}\end{pmatrix} + C_t\begin{pmatrix}\tilde{E}_{t-1}^{ada}[\pi_t]\\ \tilde{E}_{t-1}^{ada}[x_t]\end{pmatrix} + D_t\begin{pmatrix}\pi^{av}\\ x^{av}\end{pmatrix} + \begin{pmatrix}\sigma\mu_t - \kappa_t\\ nu_t\end{pmatrix} \qquad (38)$$

By solving (38) for $(\pi_t \quad x_t)'$ the explicit state dynamics is obtained

$$\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} = A_t^{-1}B_t\begin{pmatrix}\pi_{t-1}\\ x_{t-1}\end{pmatrix} + A_t^{-1}C_t\begin{pmatrix}\tilde{E}_{t-1}^{ada}[\pi_t]\\ \tilde{E}_{t-1}^{ada}[x_t]\end{pmatrix} + A_t^{-1}D_t\begin{pmatrix}\pi^{av}\\ x^{av}\end{pmatrix} +$$

$$A_t^{-1}\begin{pmatrix}\sigma\mu_t - \kappa_t\\ nu_t\end{pmatrix} \quad (39)$$

with time-dependent matrices

$$A_t = \begin{pmatrix}-a_\pi & \sigma(1-a_x)\\ 1-\beta a_\pi & -\gamma\end{pmatrix}$$

$$B_t = \begin{pmatrix}-(b_\pi+\delta_\pi) & -(\sigma b_x+\delta_x)\\ -\beta b_\pi & 0\end{pmatrix}$$

$$C_t = \begin{pmatrix}c_\pi & \sigma c_x\\ \beta c_\pi 0 & 0\end{pmatrix}$$

$$D_t = \begin{pmatrix}d_\pi & \sigma d_x\\ \beta d_\pi & 0\end{pmatrix}$$

### A.3.2. The baseline Model with forward-looking Taylor Rule:

$$x_t = \tilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - \tilde{E}_t[\pi_{t+1}]\right) + \mu_t \quad \text{compare (1)}$$

$$\pi_t = \beta\tilde{E}_t[\pi_{t+1}] + \gamma x_t + \nu_t \quad \text{compare (2)}$$

$$i_t = \delta_\pi\left(\tilde{E}_t[\pi_{t+1}] - \pi^*\right) + \delta_x\tilde{E}_t[x_{t+1}] + \kappa_t \quad \text{compare (14)}$$

<u>Market expectations compare with equations (15) –(26):</u>

$$\tilde{E}_t[\pi_{t+1}] = a_\pi\pi_t - b_\pi\pi_{t-1} + c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + d_\pi\pi^{av} \quad \text{compare (20)}$$

$$\tilde{E}_t[x_{t+1}] = a_x x_t - b_x x_{t-1} + c_x\tilde{E}_{t-1}^{ada}[x_t] + d_x x^{av} \quad \text{compare (26)}$$

<u>The state-space model:</u>

<u>First state equation:</u>

Inserting (14) into (1) and again assuming $\pi^* = 0$ yields:

$$x_t = \tilde{E}_t[x_{t+1}] - \frac{1}{\sigma}\Big((\delta_\pi\pi_{t-1} + \delta_x x_{t-1} + \kappa_t) - \tilde{E}_t[\pi_{t+1}]\Big) + \mu_t \quad (40)$$

Inserting (26) and (20) into (40) and rearranging terms yields the <u>first state equation</u>

$$(\delta_\pi - 1)a_\pi\pi_t + (\sigma + (\delta_x - \sigma)a_x)x_t = (\delta_\pi - 1)b_\pi\pi_{t-1} + (\delta_x - \sigma)b_x x_{t-1} +$$
$$(1-\delta_\pi)c_\pi\tilde{E}_{t-1}^{ada}[\pi_t] + (\sigma - \delta_x)c_x\tilde{E}_{t-1}^{ada}[x_t] + (1-\delta_\pi)d_\pi\pi^{av} + (\sigma -$$
$$\delta_x)d_x x^{av} + \sigma\mu_t - \kappa_t \quad (41)$$

Second state equation

$(1-\beta a_\pi)\pi_t - \gamma x_t = -\beta\pi_{t-1} + \beta c_\pi \tilde{E}^{ada}_{t-1}[\pi_t] + \beta d_\pi \pi^{av} + \nu_t$ compare (29)

Reformulating (17) and (18) into matrix notation yields:

$$A_t \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = B_t \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + C_t \begin{pmatrix} \tilde{E}^{ada}_{t-1}[\pi_t] \\ \tilde{E}^{ada}_{t-1}[x_t] \end{pmatrix} + D_t \begin{pmatrix} \pi^{av} \\ x^{av} \end{pmatrix} + \begin{pmatrix} \sigma\mu_t - \kappa_t \\ nu_t \end{pmatrix} \tag{42}$$

By solving (42) for $(\pi_t \quad x_t)'$ the explicit state dynamics is obtained

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = A_t^{-1} B_t \begin{pmatrix} \pi_{t-1} \\ x_{t-1} \end{pmatrix} + A_t^{-1} C_t \begin{pmatrix} \tilde{E}^{ada}_{t-1}[\pi_t] \\ \tilde{E}^{ada}_{t-1}[x_t] \end{pmatrix} + A_t^{-1} D_t \begin{pmatrix} \pi^{av} \\ x^{av} \end{pmatrix} +$$

$$A_t^{-1} \begin{pmatrix} \sigma\mu_t - \kappa_t \\ nu_t \end{pmatrix} \tag{43}$$

with time-dependent matrices

$$A_t = \begin{pmatrix} (\delta_\pi - 1)a_\pi & \sigma + (\delta_x - \sigma)a_x \\ 1 - \beta a_\pi & -\gamma \end{pmatrix}$$

$$B_t = \begin{pmatrix} (\delta_\pi - 1)\mathrm{b}_\pi & -(\delta_\mathrm{x} - \sigma)b_x \\ -\beta b_\pi & 0 \end{pmatrix}$$

$$C_t = \begin{pmatrix} (1 - \delta_\pi)c_\pi & (\sigma - \delta_\mathrm{x})c_x \\ \beta c_\pi 0 & 0 \end{pmatrix}$$

$$D_t = \begin{pmatrix} (1 - \delta_\pi)d_\pi & (\sigma - \delta_\mathrm{x})d_x \\ \beta d_\pi & 0 \end{pmatrix}$$

## Affirmation

I hereby declare that I have composed my seminar paper “A New Keynesian Model with Bounded Rational Agents and Asynchronous Updating” independently using only those resources mentioned, and that I have as such identified all passages which I have taken from publications verbatim or in substance. I agree that my thesis may be checked for plagiarism using testing software. Neither this thesis, nor any extract of it, has been previously submitted to an examining authority, in this or a similar form.

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Date Signature