CSCE 633 - Homework 1

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Problem 1: Gradient Calculation

Part 1:

Given the function:

$$f(x, y) = x^2 + \ln(y) + xy + y^3$$

We need to find the gradient of this function at the point $(x, y) = (10, -10)$.

The gradient vector is given by:

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

1. Compute $\frac{\partial f}{\partial x}$:

$$\frac{\partial f}{\partial x} = 2x + y$$

Substitute $x = 10$ and $y = -10$:

$$\frac{\partial f}{\partial x} = 2(10) + (-10) = 20 - 10 = 10$$

2. Compute $\frac{\partial f}{\partial y}$:

$$\frac{\partial f}{\partial y} = \frac{1}{y} + x + 3y^2$$

Substitute $x = 10$ and $y = -10$:

$$\frac{\partial f}{\partial y} = \frac{1}{-10} + 10 + 3(-10)^2 = -0.1 + 10 + 300 = 309.9$$

Thus, the gradient at the point $(10, -10)$ is:

$$\nabla f(10, -10) = (10, 309.9)$$

Part 2:

Given the function:

$$f(x, y, z) = \tanh(x^3y^3) + \sin(z^2)$$

We need to find the gradient of this function at the point $(x, y, z) = (-1, 0, \frac{\pi}{2})$.

The gradient vector is given by:

$$\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

1. Compute $\frac{\partial f}{\partial x}$:

$$\frac{\partial f}{\partial x} = 3x^2y^3\text{sech}^2(x^3y^3)$$

At $(x, y) = (-1, 0)$, this simplifies to:

$$\frac{\partial f}{\partial x} = 0$$

2. Compute $\frac{\partial f}{\partial y}$:

$$\frac{\partial f}{\partial y} = 3x^3y^2\text{sech}^2(x^3y^3)$$

At $(x, y) = (-1, 0)$, this simplifies to:

$$\frac{\partial f}{\partial y} = 0$$

3. Compute $\frac{\partial f}{\partial z}$:

$$\frac{\partial f}{\partial z} = 2z\cos(z^2)$$

Substitute $z = \frac{\pi}{2}$:

$$\frac{\partial f}{\partial z} = 2\left(\frac{\pi}{2}\right)\cos\left(\left(\frac{\pi}{2}\right)^2\right) = \pi \cdot \cos\left(\frac{\pi^2}{4}\right)$$

Thus, the gradient at the point $(-1, 0, \frac{\pi}{2})$ is:

$$\nabla f(-1, 0, \frac{\pi}{2}) = (0, 0, \pi\cos(\frac{\pi^2}{4}))$$

Problem 2: Matrix Multiplication

Part 1:

Given the matrices:

$$\mathbf{A} = \begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

Perform the matrix multiplication $\mathbf{A} \times \mathbf{B}$:

$$\mathbf{A} \times \mathbf{B} = \begin{bmatrix} 10 \cdot 0 & 10 \cdot 3 & 10 \cdot 0 & 10 \cdot 1 \\ -5 \cdot 0 & -5 \cdot 3 & -5 \cdot 0 & -5 \cdot 1 \\ 2 \cdot 0 & 2 \cdot 3 & 2 \cdot 0 & 2 \cdot 1 \\ 8 \cdot 0 & 8 \cdot 3 & 8 \cdot 0 & 8 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

Part 2:

Given the matrices:

$$\mathbf{C} = \begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

Perform the matrix multiplication $\mathbf{C} \times \mathbf{D}$:
Step-by-step calculation:

$$\mathbf{C} \times \mathbf{D} = \begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

First row of $\mathbf{C} \times \mathbf{D}$:

$$1 \cdot 6 + (-1) \cdot 0 + 6 \cdot (-3) + 7 \cdot 3 = 6 + 0 - 18 + 21 = 9$$

$$1 \cdot 2 + (-1) \cdot (-1) + 6 \cdot 0 + 7 \cdot 4 = 2 + 1 + 0 + 28 = 31$$

$$1 \cdot 0 + (-1) \cdot 1 + 6 \cdot 4 + 7 \cdot 7 = 0 - 1 + 24 + 49 = 72$$

First row: $[9, 31, 72]$
Second row of $\mathbf{C} \times \mathbf{D}$:

$$9 \cdot 6 + 0 \cdot 0 + 8 \cdot (-3) + 1 \cdot 3 = 54 + 0 - 24 + 3 = 33$$

$$9 \cdot 2 + 0 \cdot (-1) + 8 \cdot 0 + 1 \cdot 4 = 18 + 0 + 0 + 4 = 22$$

$$9 \cdot 0 + 0 \cdot 1 + 8 \cdot 4 + 1 \cdot 7 = 0 + 0 + 32 + 7 = 39$$

Second row: $[33, 22, 39]$
Third row of $\mathbf{C} \times \mathbf{D}$:

$$-8 \cdot 6 + 1 \cdot 0 + 2 \cdot (-3) + 3 \cdot 3 = -48 + 0 - 6 + 9 = -45$$

$$-8 \cdot 2 + 1 \cdot (-1) + 2 \cdot 0 + 3 \cdot 4 = -16 - 1 + 0 + 12 = -5$$

$$-8 \cdot 0 + 1 \cdot 1 + 2 \cdot 4 + 3 \cdot 7 = 0 + 1 + 8 + 21 = 30$$

Third row: $[-45, -5, 30]$

Fourth row of $\mathbf{C} \times \mathbf{D}$:

$$10 \cdot 6 + 4 \cdot 0 + 0 \cdot (-3) + 1 \cdot 3 = 60 + 0 + 0 + 3 = 63$$

$$10 \cdot 2 + 4 \cdot (-1) + 0 \cdot 0 + 1 \cdot 4 = 20 - 4 + 0 + 4 = 20$$

$$10 \cdot 0 + 4 \cdot 1 + 0 \cdot 4 + 1 \cdot 7 = 0 + 4 + 0 + 7 = 11$$

Fourth row: $[63, 20, 11]$

Thus, the result of $\mathbf{C} \times \mathbf{D}$ is:

$$\mathbf{C} \times \mathbf{D} = \begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$