

CSCE 633 - Homework 4 Solution

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Question 1: Convolution Operation

Input Matrix and Filter

The input matrix X and the filter F are as follows:

$$X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 5 & 2 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$F = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

1. Convolution without Max Pooling and with Stride = 1

For stride 1 without max pooling, we slide the filter over each possible position in the input matrix, compute element-wise multiplication, and sum the results.

Calculations for Each Position:

We show the calculations for some positions and others are calculated similarly to obtain the final output after using the convolution operation on the matrix using the 3x3 filter.

1. Position (1,1):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 0 + 0 + 0 - 2 + 0 + 0 - 1 = -3$$

2. Position (1,2):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4 \\ 3 & 0 & -1 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 0 + 0 + 0 - 4 + 3 + 0 - 1 = -2$$

3. Position (1,3):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 2 + 0 - 1 + 1 + 0 + 0 = 2$$

4. Position (1,4):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 4 + 0 + 0 + 1 + 0 - 1 = 4$$

5. Position (1,5):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 + 0 = 1$$

Continuing in this way for all positions, after applying the filter with stride = 1 and before taking the sum of values in each position, we get:

$$\left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -4 \\ 3 & 0 & -1 \\ 0 & 0 & -4 \\ 3 & 0 & -1 \\ 2 & 0 & -1 \\ 3 & 0 & -1 \\ 2 & 0 & -1 \\ 2 & 0 & -5 \\ 2 & 0 & -1 \\ 2 & 0 & -5 \\ 0 & 0 & -3 \\ 2 & 0 & -5 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \\ 4 & 0 & 0 \\ 1 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & -2 \\ 4 & 0 & 0 \\ 0 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 1 & 0 & -1 \\ 4 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \\ 5 & 0 & -2 \\ 1 & 0 & -1 \\ 5 & 0 & -2 \\ 3 & 0 & -1 \\ 5 & 0 & -2 \\ 3 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]$$

Taking some of values for matrices at each position, we get the output matrix:

$$\text{Output}_{\text{stride}=1} = \begin{bmatrix} -3 & -2 & 2 & 4 & 1 \\ -7 & -1 & 6 & 4 & 1 \\ -5 & 0 & 3 & 3 & 2 \\ -5 & -5 & 1 & 5 & 4 \\ -1 & -6 & -3 & 5 & 4 \end{bmatrix}$$

2. Convolution without Max Pooling and with Stride = 2

For stride 2, we slide the filter two elements at a time over the entire matrix, resulting in a 3x3 output matrix.

Calculations for Each Position:

We show the calculation for each position with stride = 2:

1. Position (1,1):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & 1 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 0 + 0 + 0 - 2 + 0 + 0 + 0 - 1 = -3$$

2. Position (1,2):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 0 + 2 + 0 - 1 + 1 + 0 + 0 = 2$$

3. Position (1,3):

$$\text{Region: } \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 + 0 = 1$$

4. Position (2,1):

$$\text{Region: } \begin{bmatrix} 0 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 2 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Sum: } 0 + 0 - 1 + 0 + 0 + 0 - 4 + 0 + 0 + 0 = -5$$

5. Position (2,2):

$$\text{Region: } \begin{bmatrix} 1 & 1 & 0 \\ 4 & 1 & 0 \\ 0 & 5 & 2 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\text{Element-wise Multiplication: } \begin{bmatrix} 1 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\text{Sum: } 1 + 0 + 0 + 4 + 0 + 0 + 0 + 0 - 2 = 3$$

6. Position (2,3):

$$\begin{aligned} \text{Region: } & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\ \text{Element-wise Multiplication: } & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \\ \text{Sum: } & 0 + 0 + 0 + 0 + 0 + 0 + 2 + 0 + 0 = 2 \end{aligned}$$

7. Position (3,1):

$$\begin{aligned} \text{Region: } & \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\ \text{Element-wise Multiplication: } & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \\ \text{Sum: } & 0 + 0 + 0 + 0 + 0 + 0 - 1 + 0 + 0 + 0 = -1 \end{aligned}$$

8. Position (3,2):

$$\begin{aligned} \text{Region: } & \begin{bmatrix} 0 & 5 & 2 \\ 1 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\ \text{Element-wise Multiplication: } & \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \\ \text{Sum: } & 0 + 0 - 2 + 1 + 0 - 2 + 0 + 0 + 0 = -3 \end{aligned}$$

9. Position (3,3):

$$\begin{aligned} \text{Region: } & \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{Filter: } \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \\ \text{Element-wise Multiplication: } & \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \text{Sum: } & 2 + 0 + 0 + 2 + 0 + 0 + 0 + 0 + 0 = 4 \end{aligned}$$

So, after applying the filter with stride = 2 and before taking the sum of values in each position, we get

$$\left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]$$

Taking sum of values for matrices at each position, we get the output matrix:

$$\mathbf{Output}_{\text{stride}=2} = \begin{bmatrix} -3 & 2 & 1 \\ -5 & 3 & 2 \\ -1 & -3 & 4 \end{bmatrix}$$

3. Convolution with Max Pooling and with Stride = 1

We have already calculated our output matrix after performing the convolution with stride = 1 in part (1) which is shown below:

$$\text{Output}_{\text{stride}=1} = \begin{bmatrix} -3 & -2 & 2 & 4 & 1 \\ -7 & -1 & 6 & 4 & 1 \\ -5 & 0 & 3 & 3 & 2 \\ -5 & -5 & 1 & 5 & 4 \\ -1 & -6 & -3 & 5 & 4 \end{bmatrix}$$

Now, performing max pooling with a pool size 3x3 and stride = 2 as given in the question:

$$\left[\begin{array}{cc} \begin{bmatrix} -3 & -2 & 2 \\ -7 & -1 & 6 \\ -5 & 0 & 3 \end{bmatrix} \rightarrow \text{Max: } 6 & \begin{bmatrix} 2 & 4 & 1 \\ 6 & 4 & 1 \\ 3 & 3 & 2 \end{bmatrix} \rightarrow \text{Max: } 6 \\ \begin{bmatrix} -5 & 0 & 3 \\ -5 & -5 & 1 \\ -1 & -6 & -3 \end{bmatrix} \rightarrow \text{Max: } 3 & \begin{bmatrix} 3 & 3 & 2 \\ 1 & 5 & 4 \\ -3 & 5 & 4 \end{bmatrix} \rightarrow \text{Max: } 5 \end{array} \right]$$

Thus, the final output matrix after max pooling is:

$$\text{Output}_{\text{Max Pooling}} = \begin{bmatrix} 6 & 6 \\ 3 & 5 \end{bmatrix}$$