

Mathematical Foundations of CS (CS 208)

Assignment 2

Due: September 11, 2018

Note: Each question will be graded as per the following criteria:

- (a) 5 - Thorough understanding of the topic and conceptual clarity
 - (b) 4 - Adequate understanding of the topic and conceptual clarity
 - (c) 3 - Some understanding of the topic and conceptual clarity
 - (d) 2 or 1 - Poor understanding of the topic and messed up concepts
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1. From the following, choose correct negation of the statement: The river is shallow or polluted.
 - (a) It is false that the river is shallow or polluted.
 - (b) The river is neither shallow nor polluted.
 - (c) The river is deep and unpolluted.
 - (d) The river is not shallow or not polluted.
2. Rewrite each of the following statements in the form “If A, then B.”
 - (a) Healthy plant growth follows from sufficient water.
 - (b) Increased availability of information is a necessary condition for further technological advances.
 - (c) Errors were introduced only if there was a modification of the program.
 - (d) Fuel savings implies good insulation or storm windows throughout.
3. Four machines, A, B, C, and D, are connected on a computer network. It is feared that a computer virus may have infected the network. Your security team makes the following statements:
 - (a) If D is infected, then so is C.
 - (b) If C is infected, then so is A.
 - (c) If D is clean, then B is clean but C is infected.
 - (d) If A is infected, then either B is infected or C is clean.

Assuming that these statements are all true, what can you conclude? Explain your reasoning.

4. Using propositional logic, prove that the statement

$$A \wedge (B \rightarrow C) \wedge [(A \wedge B) \rightarrow (D \vee \overline{C})] \wedge B \rightarrow D$$

is valid.

5. Use a truth table to verify that $A \rightarrow (B \rightarrow C) \leftrightarrow (A \wedge B) \rightarrow C$ is valid.
6. Which of the following statements expresses the negation of, “Everybody loves somebody sometime”? Justify your answer by first writing the predicate formula for the original statement and then negating it.
 - (a) Everybody hates somebody sometime.

- (b) Somebody loves everybody all the time.
(c) Everybody hates everybody all the time.
(d) Somebody hates everybody all the time.
7. Consider the predicate formula $(\forall x)[P(x) \vee Q(x)] \rightarrow (\forall x)P(x) \vee (\forall x)Q(x)$. Is it valid. (*Hint:* To prove that a formula is invalid you only need to give one counterexample when it does not hold.)
8. Prove the following properties of Boolean algebra. Justify each step.
- (a) $x + y' = x + (x' \cdot y + x \cdot y)'$
(b) $(x + y)' \cdot z + x' \cdot y \cdot z = x' \cdot z$
(c) $(x \cdot y') + (y \cdot z') + (z \cdot x') = (x' \cdot y) + (y' \cdot z) + (z' \cdot x)$

