CS208: Mathematical Foundations of CS **Fall 2018**

Lecture 23: October 4

Lecturer: Samar *Scribes: Yash Khanna(B17070), Saurbh(B17060)*

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23.1 Perturbation Methods

Perturbation theory comprises mathematical methods for finding an approximate solution to a problem, by starting from the exact solution of a related, simpler problem. A critical feature of the technique is a middle step that breaks the problem into "solvable" and "perturbation" parts.

$$S = 1 + x + x^2 + ...x^n = \frac{1-x^{n+1}}{1-x}$$
$$Sx = x + x^2 + ...x^{n+1}$$

Subtracting:-

$$S = \frac{1-x^{n+1}}{1-x}$$

23.1.1 A few items of note

$\sum_1^n i = \frac{n(n+1)}{2}$

Proof:-

$$S = 1 + 2 + 3 + ... + n - 1 + n$$

$$S = n + n - 1 + n - 2 + ... + 2 + 1$$

$$2S = n + 1 + ...(nterms)$$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

$\sum_1^n i^2$
$S = 1^2 + 2^2 + ...n^2$

There are n terms so,roughly polynomial of n^3.

$\sum_1^n i^2 = ax^3 + bx^2 + cx + d$
For general summation

$\sum_1^n i^k = a_0 n^{k+1} + a_1 n^k + ... + a_k$
$f : R^+ \to R^+$

-Weakly increasing

Here is an equation:

$$S = \sum_1^n f(i)$$

$$I = \int_1^n f(x)dx$$

$$\text{then } I + f(1) \leq S \leq I + f(n)$$

Here is an example:

Example:
Find the upper and lower bound of $\sum_1^n \sqrt{i}$

$$\frac{2x^{\frac{3}{2}}}{3} + \frac{1}{3} \leq \sum_1^n \sqrt{i} \leq \frac{2x^{\frac{3}{2}}}{3} + \sqrt{n} - n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

23.2 Products and Factorials

$$p = \prod_1^n f(i)$$

$$\ln p = \sum_1^n \ln f(i)$$

$$\ln n! = \sum_1^n \ln i$$

$$n \ln n - n + 1 \leq \sum_1^n lni \leq n \ln n - n + 1 + \ln n$$

23.3 Striling's approximation

Comparison of Stirling's approximation with the factorial In mathematics, Stirling's approximation (or Stirling's formula) is an approximation for factorials. It is a good approximation, leading to accurate results even for small values of n

$$\ln n! = n \ln n - n + O(\ln n)$$

Specifying the constant in the O(ln n) error term gives $\frac{1}{2}$ln(2n), yielding the more precise formula:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n,$$

Simple bounds valid for all positive integers n

$$\sqrt{2\pi}\, n^{n+\frac{1}{2}} e^{-n} \leq n! \leq e\, n^{n+\frac{1}{2}} e^{-n}$$

References

1. https://en.wikipedia.org/wiki/Perturbation-theory

2. https://en.wikipedia.org/wiki/Stirling