CS208: Mathematical Foundations of CS **Fall 2018**

Lecture 13: September 11

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13.1 Function

Idea of basic function :
Function is just like a machine ,if you give some right input then you will get correct output.
or we can say

A function is a set of statements that take inputs, do some specific computation and produces output.

Definition : Function is a mapping such that no two of same element in domain can have different mapping.

Notation : f: A $\rightarrow$ B .

Function is defined from A to B where A is said to set of Domain and B is said to set of Co domain.

Lemma 1 13.1.1 Basic terms required to define a function :

- *Domain*
- *Co-domain*
- *Range*
- *Support of function*

Domain *- The set of all possible values for which the function may or may not be defined is called Domain.Set A is called domain of that function.*

Range *- Range is the set of all output of function after substituting domain values into it.*

Co-Domain *- Co-domain is the set of all values in which all the output of function is constrained to fall .Set B is called as co-domain of that function.*

Support of function *- The set of domain elements for which a function is defined is called the support of the function.*

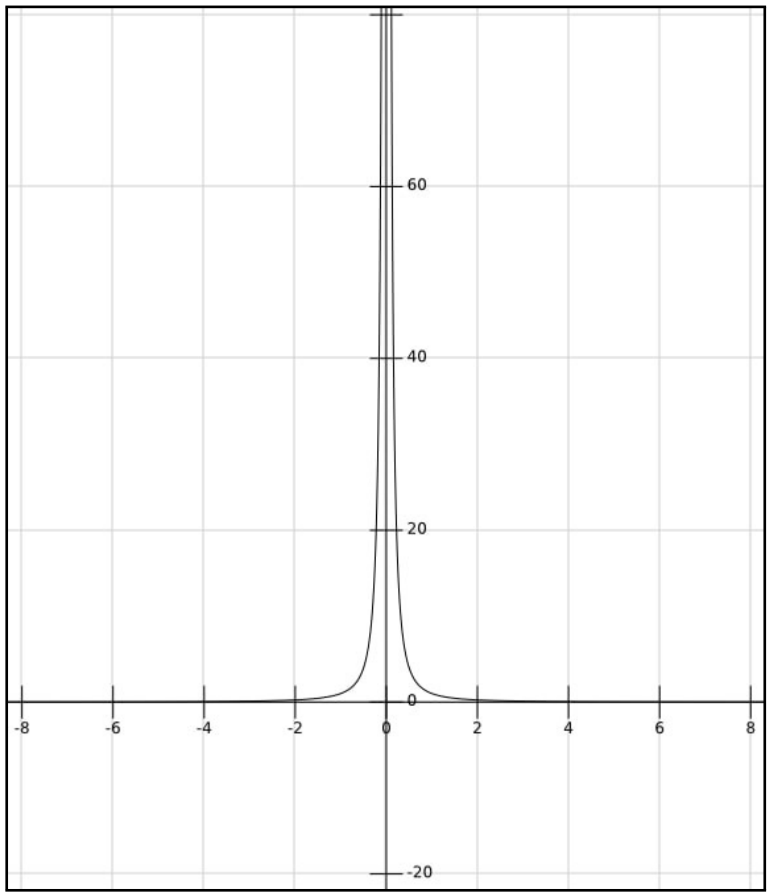
13.1.2 Types of function

- Partial Function
- Total Function

13.1.2.1 Partial Function : -

There may be domain elements for which function is not defined . Such functions are called as partial functions. **Examples** :-

1. F(x) = $\frac{1}{x^2}$



For x= 0 , this function does not return any value .
so , x=0 is the element in domain which does not contain any value under f.

2. A function that take binary strings as input.

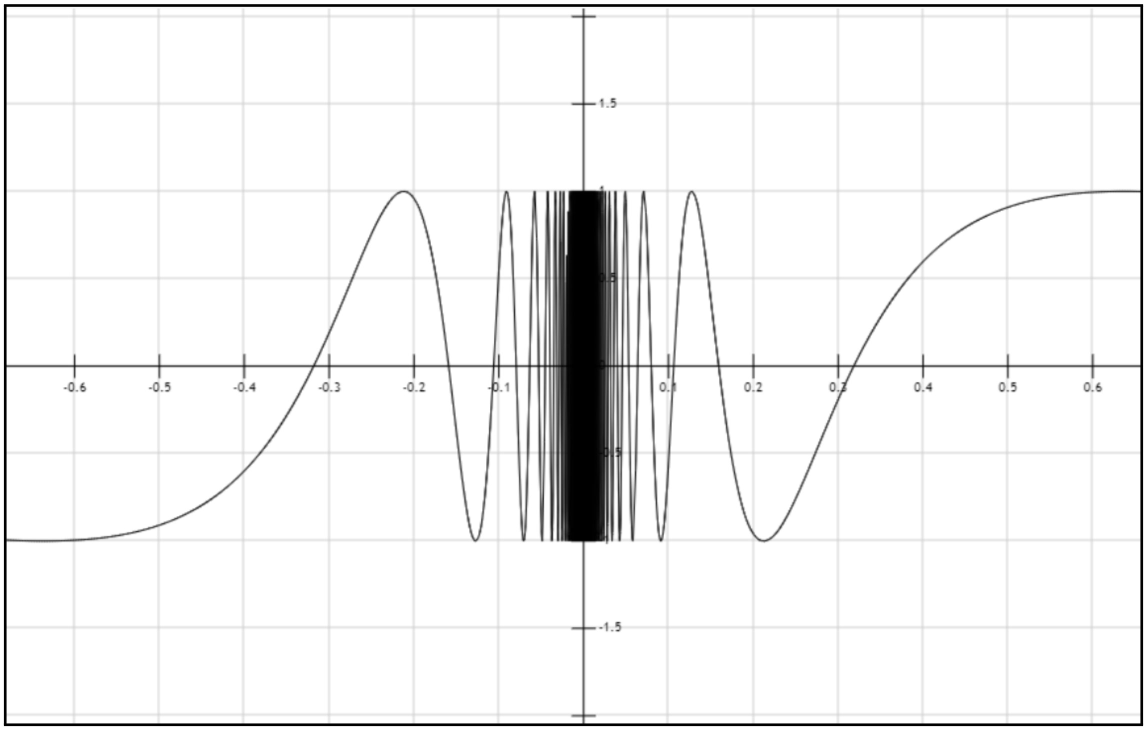
f : length of binary bits from left to right of string y until a 1 appears.

f(0001001) = 4 f(1000) = 1
f(0000) = undefined .
so , this function is also partial function as it does not assign any value to 0000.

13.1.2.2 Total Function : -

If function is defined for each and every element of domain then such functions are called as Total Functions.
Examples :-

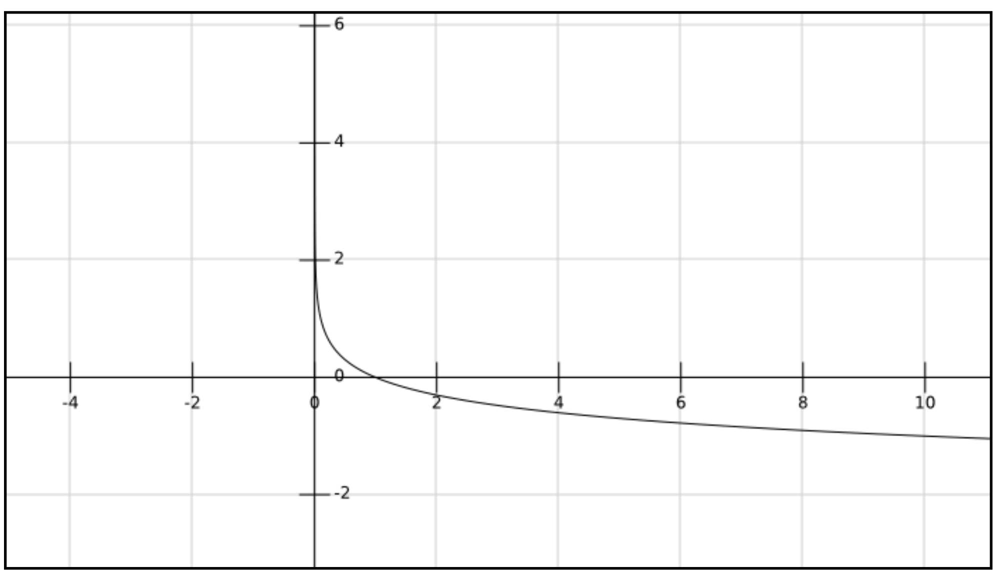
1. $\sin(\frac{1}{x})$



$\sin(\frac{1}{x})$ wobbles as go to the x=0 from infinite(or - infinite) its oscillates around the -1,0 and 1 infinite times.

There are an infinite number of these values, and all are between 0 and 1. We can conclude that as x $\rightarrow$0 from the right, the function $\sin(\frac{1}{x})$ does not settle down on any value , and so the limit as x approaches 0 from the right does not exist.

2. $\log(\frac{1}{x})$



As x $\rightarrow$ 0 from the right the limit of the function goes to the infinity .

$\lim_{x\to\infty} f(x)$

$\lim_{x\to\infty} -log(x) \to -\infty$

13.1.3 Piece wise Application

If f : is a Function from set A to set B , S is subset of A.

f(s) = { b ϵ B , f(s) = b ,∀ s∈S }

This is called image of S under f.

f : A → B

Domain = A

f : S → B , S ⊆ A then ,

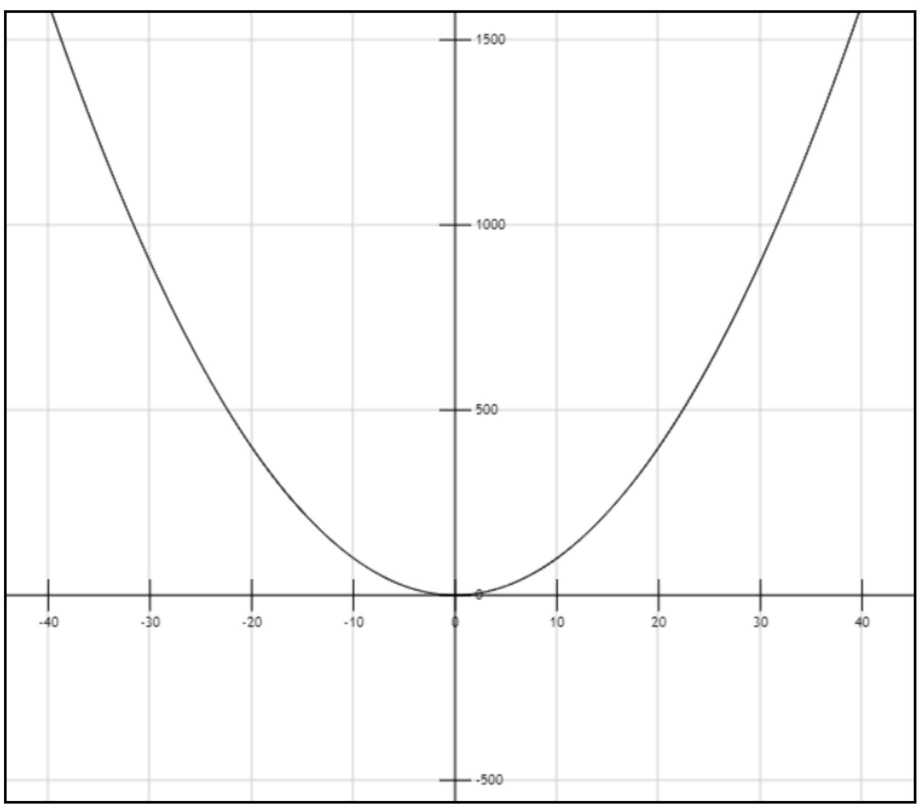
Domain = pow(A)

⇒ Range(f) = f ⊏ domain(f) ⊐

⇒ Range(f) ⊆ Co-domain

Exampless :-

1. F(x) = x^2 , R → R



note : square of any number is positive.

Range = $\{$x $\geq$ 0 , x $\in$ R $\}$

$\Rightarrow$ Range $\subseteq$ R and co-domain=R .

$\Rightarrow$ Range $\subseteq$ co-domain

2. F(x) = $\sqrt{x}$, $R^+ \rightarrow$ R

   Graph of this function is

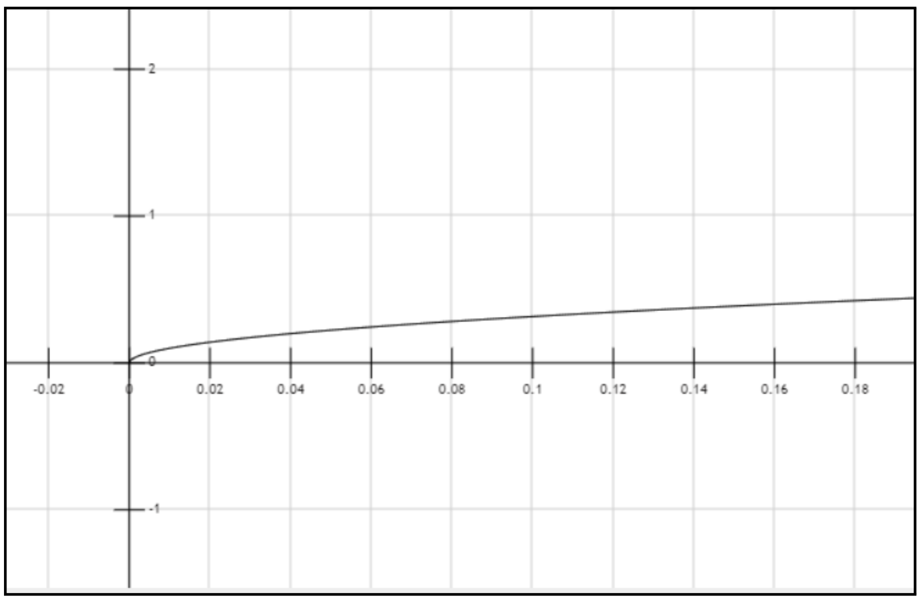
   Domain $\geq$ 0

   Range = R^+

   co-domain = R

   $\Rightarrow$ Range $\subseteq$ co-domain .

   Graph of this function is



3. F(x) = $\sin x$, R $\rightarrow$ $\sqsubset$ -2 , 2 $\sqsupset$.

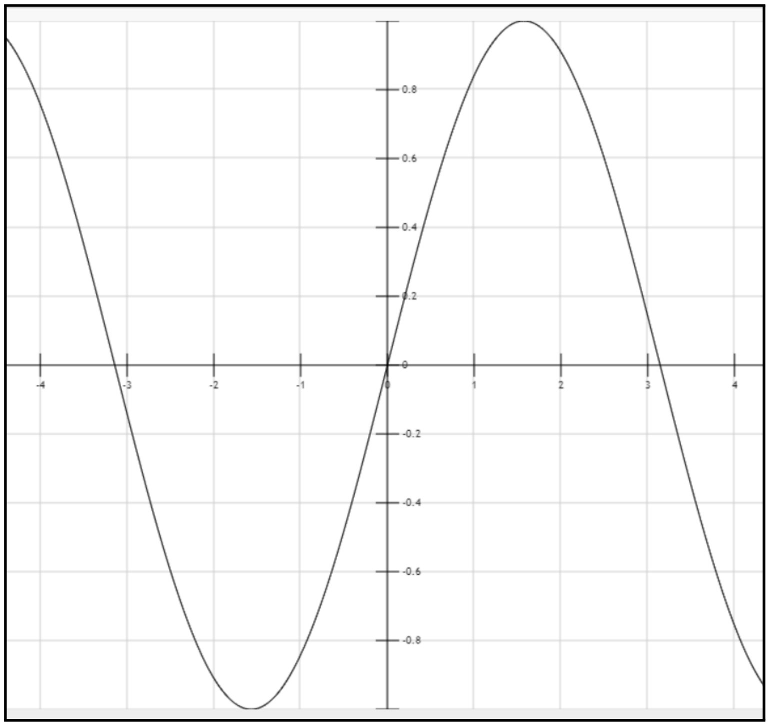
   From Graph ,

   Domain = R

   co-domain = $\sqsubset$ -2 , 2 $\sqsupset$

   Range = $\sqsubset$ -1 , 1 $\sqsupset$

   $\Rightarrow$ Range $\subseteq$ co-domain .



13.1.4 Function Composition

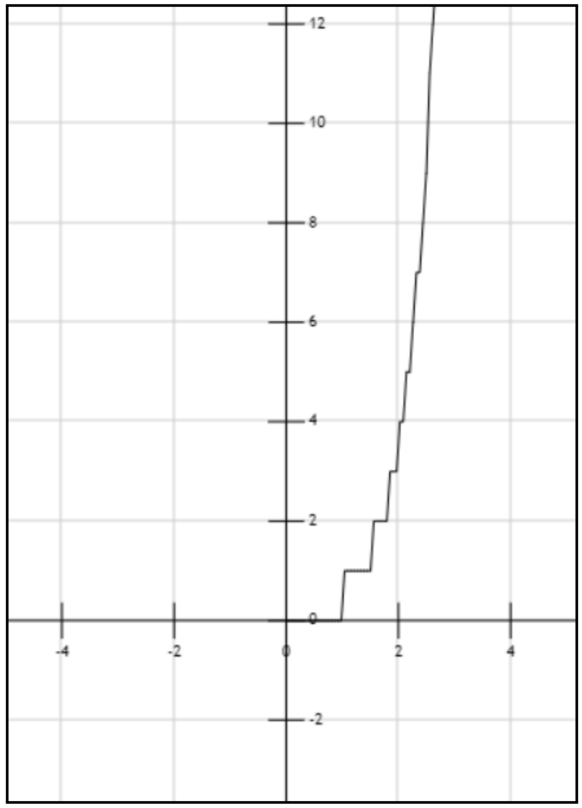
Taking a step amounts to applying a function, and going step by step corresponds to applying functions one after the other.

Composition the function f and g means first calculate f on argument x to produce f(x) and then calculate g on argument f(x) to produce g $\sqsubset$ f(x) $\sqsupset$.

Definition :- If we have two function f : A $\rightarrow$ B and g : B $\rightarrow$ C then , composition g o f , of g and f is defined to be the function from A to C defined by :

$$g \text{ o } f(x) = g(f(x))$$

Examples :
f(x) = x^x
g(x) = floor(x)
then , g o f(x) is defined as
g o f = floor(x^x) Domain = (0 ,∞)
Co-domain = $\sqsubset$ 0,∞)
Range = $\sqsubset$ $e^{\frac{-1}{e}}$,∞)



References

[4.3.1] Mathematics for Computer Science by Eric Lehman.
[4.3.2] Mathematics for Computer Science by Eric Lehman.
FOOPLOT for ploting the graphs.