CS208: Mathematical Foundations of CS **Fall 2018**

Lecture 15: September 13

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15.1 Range of numbers in nature

15.1.1 Smallest Number

Plack constant

$$6.626176 * 10^{-34} Js$$

15.1.2 Largest Number

Mass of the observable universe

$$1.5 * 10^{53} kg$$

15.2 How google got it's name from googol

Their search engine was intended to provide information in large quantity. So they The name of the google is originated from misspelling of ”googol”, the number is equiavalent to

$$10^{100}$$

Which signify that the search engine is intended to provide large information.

15.3 Finite Set

A set which is either empty or have finite number of member.
Eg:-set of vowels.
A $\rightarrow$ finite set
$\left|A\right| \rightarrow$ size/cardinality
f-A $\rightarrow$ B
f is a function,

15.3.1 Surjective

A surjective B iff there is a surjectve function f from A $\rightarrow$ B
Suppose f is surjective funciton
bϵB, atleast an arrwow going in
$\Big|A\Big| \geqslant \Big|B\Big|$

15.3.2 Bijective

A bijective B iff there is a bijectve function f from A $\rightarrow$ B
Suppose f is bijective funciton
$\Big|A\Big| = \Big|B\Big|$

15.3.3 Injective

A injective B iff there is a injectve total realtion f from A $\rightarrow$ B
Suppose f is injective total relation
$\Big|A\Big| \leqslant \Big|B\Big|$

Lemma 1 *Let suppose , A and B are finite set*

- *A surjective B* $\Longrightarrow \Big|A\Big| \geqslant \Big|B\Big|$
- *A injective B* $\Longrightarrow \Big|A\Big| \leqslant \Big|B\Big|$
- *A bijective B* $\Longrightarrow \Big|A\Big| = \Big|B\Big|$

Proof: if $\Big|A\Big| \geqslant \Big|B\Big|$,then A surjective B ,where A and B are finite set,
$A = \{a_0, a_1, a_2, a_3, a_4, a_5\}$
$B = \{b_0, b_1, b_2, b_3\}$
there is a relation f between A and B,such that
$f(A, B) = \{(a_0, b_0), (a_1, b_0), (a_2, b_1), (a_3, b_2), (a_4, b_3), (a_5, b_3)\}$
So, it is surjective and total
if f-surjective funciton
[$\leqslant$1 out,$\geqslant$in property]
That means R^{-1} satisfies
[$\geqslant$1 out,$\leqslant$in property]
is injetive total

■

15.4 Redefined Lemma

Lemma 2 *if A and B are finite set*

- *A surjective B* $\Longleftrightarrow \Big|A\Big| \geqslant \Big|B\Big|$
- *A injective B* $\Longleftrightarrow \Big|A\Big| \leqslant \Big|B\Big|$
- *A bijective B* $\Longleftrightarrow \Big|A\Big| = \Big|B\Big|$

15.5 Power set

$\Big|A\Big| = n$
$\Big|pow(A)\Big| = 2^n$

Proof: $A = \{a_0, a_1, a_2, a_3, a_4, a_5, \ldots, a_{n-1}\}$
we can define a n bit sequences a n bit sequences, each bit correspond to the position in A
So, ith bit is 1 $\Longleftrightarrow a_i \epsilon$ subset
i.e $b_i = 1 \Longleftrightarrow a_i \epsilon$ s
eg $\rightarrow$ $s = \{a_3, a_4, a_5\}$

0	0	0	1	1	1
0	1	2	3	4	5

So, if we show n element, then there are 2^n combination of bits.
So, we proved
A bijective B so $\Big|A\Big| = \Big|B\Big|$ thus the number of subset of B is same. ■

References

[] GLYNN WINSKEL , "Discrete Mathematics II" *: Set Theory for Computer Science*,