**CS208: Mathematical Foundations of CS** **Fall 2018**

# Lecture 16: September 14

*Lecturer: Samar* *Scribes: Vinay (B17068),Deepak(B17039)*

**Disclaimer**: *These notes have not been subjected to the usual scrutiny reserved for formal publications. They may be distributed outside this class only with the permission of the Instructor.*

## 16.1 Infinite Sets

**Lemma 1** *A strict B iff $|A| < |B|$*
*Here A strict B is defined as NOT(A surjective B).*

*Proof:* A strict B iff NOT(A surj B).
A strict B iff NOT ( $|A| \geq |B|$ ).
A strict B iff $|A| < |B|$.

■

### 16.1.1 A few more lemma

For set A,B and C

- if A surjective B iff B injective A.
- if A surjective B and B surjective C then A surjective C.
- if A bijective B and B bijective C then A bijective C.
- if A bijective B then B bijective A.

### 16.1.2 Lemma and its proof

Lemma 1.
A strict B and B strict C then A strict C.

*Proof:* PROOF BY CONTRADICTION
assume that NOT(A strict C) is true.
$\implies$ A surjective C.
$\implies$ B strict C
C surjective B
From above we get

A surjective C and C surjective B.
$\implies$ A surjective B.
but A strict B implies NOT(A surjective B)
Here it contradicts hence lemma is true. ■

Lemma 2.
Let A be a set and $b \notin A$. Then A is infinite iff A bij $A \cup \{b\}$.

*Proof:* Since A is not the same size as $A \cup \{b\}$ when A is finite,
we only have to show that $A \cup \{b\}$ is the same size as A when A is infinite.
$A = \{a_0, a_1, a_2, .......a_n\}$
Lets define bijection $e : A \cup \{b\} \implies A$
$e(b) ::= a_0,$
$e(b_n) ::= a_{n+1}$
$e(a) ::= a$
for $a \in A - \{b, a_0, a_1, a_2, .......a_n\}$

■

## References

[CW87] D. Coppersmith and S. Winograd, "Matrix multiplication via arithmetic progressions," *Proceedings of the 19th ACM Symposium on Theory of Computing*, 1987, pp. 1–6.