CS208: Mathematical Foundations of CS **Fall 2018**

Lecture 12: September 6

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12.1 Sets

- A set is a collection of distinct objects.
- Elements of a set can be anything.
- In a set order of elements is not significant.
- Have distinct elements.

Examples: A = {a, b}, B = {circle, mango, 1, truck}

12.1.1 Representation of Set

Following three methods are commonly used for representing sets:

1. Statement form
2. Tabular or Roster form
3. Set builder form

12.1.1.1 Statement form

A well defined description of the set is given.
Example: A set of numbers greater than 10 and less than 16

12.1.1.2 Tabular or Roster form

All elements of set are written inside a pair of braces {} and separated by commas.
Example: A = {11, 12, 13, 14, 15}

12.1.1.3 Set builder form

The set is defined using a rule or formula. All elements of the set must possess a single property to be a part of that set.
Example: A = $\{x : x \in N, 10 < x < 16\}$

12.1.2 Power Set

The set of all the subsets, denoted by power set(A).
Number of elements in a power set(A) = $2^{|A|}$, where $|A|$ is cardinality of set A.
Examples: A = $\{1, 2\}$, powerset(A) = $\{ \phi, \{1\}, \{2\}, \{1, 2\} \}$
B = $\{\phi, \{\phi\}\}$, powerset(B) = $\{ \phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\} \}$

12.1.3 Equality of sets

Let A and B be two sets. They are equal if they have the same elements.
So, A = B if following condition is satisfied, x $\in$ A iff x $\in$ B.

Exercise:

1. Prove equality: A $\cap$ (B $\cup$ C) = (A $\cap$ B) $\cup$ (A $\cap$ C)
   Proof:
   Let x $\in$ A $\cap$ (B $\cup$ C)
   x $\in$ A AND x $\in$ (B $\cup$ C)
   $\implies$ (x $\in$ A AND x $\in$ B) OR (x $\in$ A AND x $\in$ C)
2. Prove equality: $\overline{A \cap B}$ = $\overline{A} \cup \overline{B}$
   Proof:
   Let x $\in \overline{A \cap B}$
   x $\notin$ (A $\cap$ B)
   x $\notin$ A OR x $\notin$ B
   NOT(x $\in$ A) OR NOT(x $\in$ B)

12.2 Sequence

- A sequence is also a collection of objects.
- Elements can be anything.
- Order of elements matters.
- Repetition is allowed.

Example: (a, b) $\neq$ (b, a)

12.2.1 How sets and sequences are related

Let S_1, S_2, ..., S_n be n sets.
$S_1 \times S_2 \times ... \times S_n = \{a_1, a_2, ..., a_n \mid a1 \in S_1, a2 \in S_2, ..., a_n \in S_n\}$