SIT718 Real world Analytics
Assessment Task 3 2021 T3
Total Marks = 100, Weighting - 30%
Due date: 6 February 2022

Your final submission should cionsists of:

1. "name-report.pdf: A pdf file (created in any word processor) containing the solutions of the questions, appropriate graphs and tables. The file should be labelled with your name, to avoid confusion. **This file should consists of up to 8 pages, 9 pages with title page)**.

2. "name-code.R": Two codes combined in one R file, labelled with your name, with codes for Question 2 and Question 3.

Your assignment will not be assessed if we cannot reproduce your results with your R codes.

1. A brewery produces beer and ale. Beer sells for $ 5 per barrel, and ale for $ 2 per barrel. The production of a barrel of beer requires 5 pounds of corn and 2 pounds of hops. The production of a barrel of ale requires 2 pounds of corn and 1 pound of hops. 60 pounds of corn and 25 pounds of hops are available.

   a) Explain why a Linear Programming (LP) model would be suitable for this case study.

   [5 marks]

   b) Formulate a LP model to help the brewery management to maximise the revenue while satisfying all constraints.

   [5 marks]

   c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution(s) on the graph. Annotate all lines on your graph. What is the optimal daily profit for the factory?

   Note 1: it is possible to have multiple optimal points. **[10 marks]**

   Note 2: you can use graphical solvers available online but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.

   d) Find the range for the profit ($), of a barrel of beer (if any), that can be changed without affecting an optimal point of part (c)?

   [5 marks]

2. Vin Rouge owns two wineries: Winery A can produce up to 300 bottles per day and Winery B can produce up to 200 bottles a day. It supplies wine to three bottle shops: Fig, Bear and Sky. Assume that every 10 bottles are packed into a box. Fig needs at most 20 boxes a day; Bear needs at least 18 boxes a day; and Sky needs only 5 boxes a day. The costs, associated with the delivery of each box of wine from the wineries to the bottle shops, are shown in the table below. All costs are given in dollars.

Vin Rouge	Fig	Bear	Sky
Winery A	35	62	65
Winery B	28	36	32

(a) Why the scenario above can be modelled with a Linear Programming (LP) model?

[5 Marks]

(b) Formulate a linear programming (LP) model to determine a minimum cost delivery scheme that satisfies the need of the bottle shops and that does not exceed the production capacity of the wineries.

[10 Marks]

(c) Produce R code to solve the LP model in part (b).

[5 Marks]

(d) Solve the model in R/R Studio. Find the optimal cost and optimal values of the decision variables.

[5 Marks]

3. Two mining companies, Red and Blue, bid for the right to drill a field. The possible bids are $ 15 Million, $ 25 Million, $ 35 Million, $ 45 Million and $ 50 Million. The winner is the company with the higher bid.

   The two companies decide that in the case of a tie (equal bids), **Red** is the winner and will get the field.

   Company Red has ordered a geological survey and, based on the report from the survey, concludes that getting the field for more than $ 45 Million is as bad as not getting it (assume loss), except in case of a tie (assume win).

   (a) State reasons why/how this game can be described as a two-players-zero-sum game **[5 Marks]**

   (b) Considering all possible combinations of bids, formulate the payoff matrix for the game. **[5 Marks]**

   (c) Explain what is a saddle point. Verify: does the game have a saddle point? **[5 Marks]**

   (d) Construct a linear programming model for Company Red in this game. **[5 Marks]**

   (e) Produce an appropriate code to solve the linear programming model in part (d). **[5 Marks]**

   (f) Solve the game for Red using the linear programming model and the code you constructed in parts (d) and (e). Interpret your solution. **[5 Marks]**

4. Consider two companies, Company A and Company B, producing the same model of iPhones. The demand for the iPhones produced by Company A is D_A, and the demand for the iPhones produced by Company B is D_B. The demands are described by the following functions:

$$D_A = 200 - P_A - (P_A - \bar{P}) \tag{1}$$

$$D_B = 200 - P_B - (P_B - \bar{P}) \tag{2}$$

where P_A and P_B are the prices of iPhones for Factory A and Factory B respectively, and $\bar{P}$ is the average price over the prices P_A and P_B. For each company, the cost for producing one iPhone is $C = 20$. Suppose that each company can only choose one of the three prices $\{60, 70, 80\}$ for a sale.

(a) Compute the profits of each company under all sale price combinations and produce the payoff matrix for each company.

[Hint: the profit = the demand for the iPhones $\times$ the profit of one iPhone after sale.]

[10 Marks]

(b) Find the Nash equilibrium of this game. What are the profits at this equilibrium? Explain your reason clearly.

[5 Marks]

(C) If the cost C doubles to $C = 40$, would the Nash equilibrium from part (b) change? Give clear reasons.

[5 Marks]

This question is for students aiming for HD. Students who do not have such ambition do not need to attempt it.