SIT718 Real world Analytics

Assessment Task 3 2021 T3

Q1.

a): This case study is a suitable example for an optimization problem, where the end goal is to optimize (minimize or maximize) the decision variable. In this case study, we are given a brewery which has two products: beer and ale, production of these two products is subject to cost constraints which directly drive the amount of revenue that the brewery can achieve. Linear Programming offers the ability to formulate a model which can give insights into maximizing the revenue that can be achieved under the given constraints. This makes Linear Programming a suitable technique for this case study, which would allow the brewery company to get maximum revenue based on the information derived from the LP model (discussed in following questions).

b): To formulate the linear programming model for the given case study let us first tabulate the information related to the production costs and constraints:

	Corn required (in lbs)	Hop required (in lbs)	Revenue per barrel
Beer	5	2	\$5
Ale	2	1	\$2
Limits on raw material	60	25	

Based on the above information we can formalize our decision variables, objective function, and constraints:

Decision variables:

let x be the barrels of beer and y be the barrels of ale.

Objective function:

Since we want the revenue to be maximized our objective function would be:

Max $z = 5x + 2y$

Constraints:

Total available corn and hop are 60 and 25 pounds respectively. From the table above we can say that:

$5x + 2y \le 60$ (Constraint for corn)

$2x + y \le 25$ (Constraint for hop)

Non-negativity constraints (since we cannot have a negative number of beer and ale).

$x, y \ge 0$

Combining all the equations together, our LP model will be

Max $z = 5x + 2y$

s.t

5x +2y <= 60 (Constraint for corn)

2x + y <= 25 (Constraint for hop)

x, y >= 0

c): To solve the above problem graphically we treat the above equations as equality. The points of interest obtained after solving the equations are:

P1: (0,30), P2: (0,25), **P3: (10, 5), P4: (12, 0),** P5: (12.5, 0)

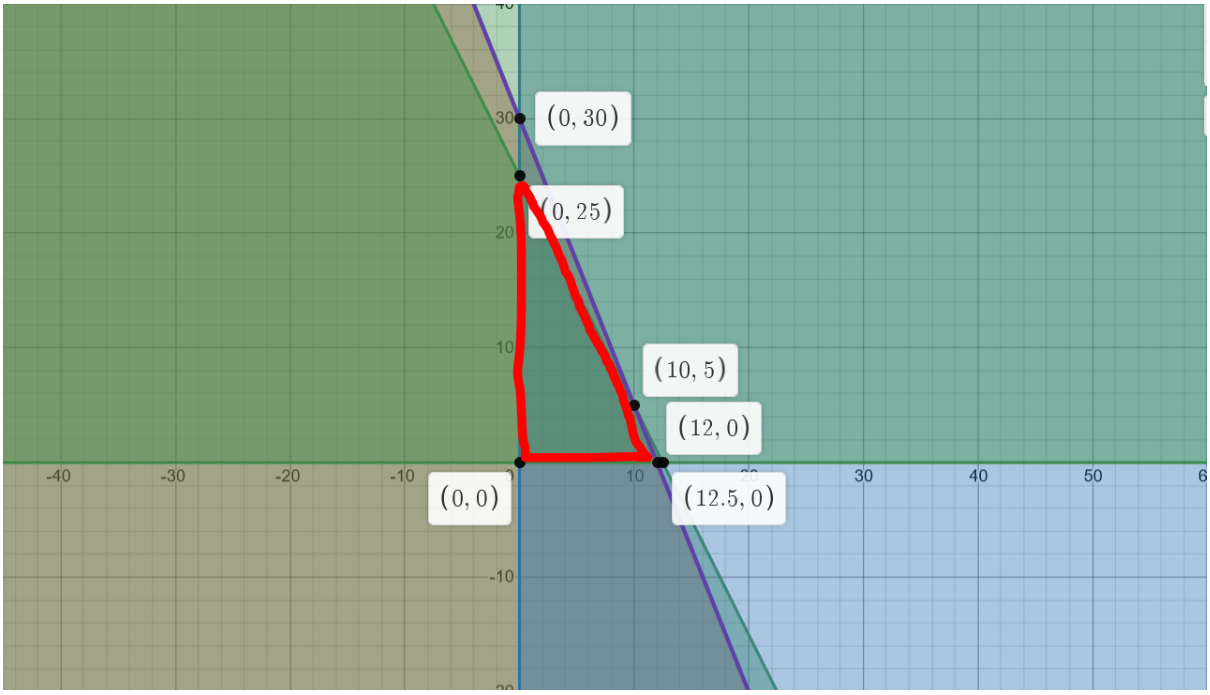

Fig: *The area enclosed in the red outline denotes the feasible region. All the vertices have been annotated.*

To find the maximum revenue, we evaluate our objective function at these points:

z = 5 + 2 * 25 = 50

z = 5 * 12 = 60 (max) [Here all the corn is used but 1 barrel of hop is not consumed]

z = 5 * 10 + 2 * 5 = 60 (max)

This shows that the maximum revenue is $60 and to achieve this, the brewery should **produce 10 barrels of beer and 5 barrels of ale.**

d): If we keep the optimal point of part c) same, that is (10, 5), then the range of beer can be change to 4, i.e., range will be [4,5].

If we consider (12,0) as the optimal point, then because we are considering only beer then the optimum point will stay the same in all case when range is >= 5.

Q2:

a): The given scenario is an example of a supply-demand scenario (Transportation Model), where the end goal is to minimize the cost incurred while meeting supply requirements from every demand centre. Here the demand centres are the three bottle shops each with their minimum demand constraints, Linear Programming can help knowing the cost required to meet these demand constraints while minimizing the cost required to ship the bottles to the required shops. This shows why Linear Programming can be effectively used to solve such problems [Solution below].

b): To formulate the linear programming model for the given case study let us first tabulate the information related to the shipping costs and demand constraints:

It is given that,

1 box = 10 bottles

Supply constraints:

Winery A can produce at max 300 bottles per day => 30 boxes

Winery B can produce at max 200 bottles per day => 20 boxes

Demand constraints:

Fig needs at least 20 boxes a day => 200 bottles

Bear needs at least 18 boxes a day => 180 bottles

Sky needs at least 5 boxes a day => 5 bottles

Using the above conditions with the given shipping costs, we get the following table **(All costs are in dollars and all quantities in boxes):**

	Fig	**Bear**	**Sky**	**Supply Constraints**
Winery A	35	62	65	30
Winery B	28	36	32	20
Demand Constraints	20	18	5	

A general LP formulation of such scenarios can be represented as:

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \sum_{j=1}^{n} x_{ij} \leq s_i \quad \forall i = 1, \dots, m \text{ (Supply Constraints)}$$

$$\sum_{i=1}^{m} x_{ij} \geq d_i \quad \forall i = 1, \dots, m \text{ (Demand Constraints)}$$

$$x_{ij} \geq 0 \quad \forall i = 1, \dots, m; j = 1, \dots, m$$

Using this, we get the following LP model:

Min $z = 35x_{11} + 62x_{12} + 65x_{13} + 12x_{21} + 36x_{22} + 32x_{23}$

s.t:

$x_{11} + x_{12} + x_{13} <= 30$

$x_{21} + x_{22} + x_{23} <=20$

Supply Constraints

$x_{11} + x_{21} >= 20$

$x_{12} + x_{22} >= 18$

$x_{13} + x_{23} >= 5$

Demand Constraints

$x_{ij} >=0; i = 1, 2; j = 1, 2, 3$ [Non-Negativity Constraints]

d):

After calculation from R model (shown in fig)

```
Model name:
              C1    C2    C3    C4    C5    C6
Minimize      35    62    65    28    36    32
R1             1     1     1     0     0     0  <=  30
R2             0     0     0     1     1     1  <=  20
R3             1     0     0     1     0     0  >=  20
R4             0     1     0     0     1     0  >=  18
R5             0     0     1     0     0     1  >=   5
Kind         Std   Std   Std   Std   Std   Std
Type        Real  Real  Real  Real  Real  Real
Upper        Inf   Inf   Inf   Inf   Inf   Inf
Lower          0     0     0     0     0     0
```

The optimal cost is $**1586**

for the decision variables:

$x_{11} = 20, x_{12} = 3, x_{13} = 0, x_{21} = 0, x_{22} = 15, x_{23} = 5$

Q3):

a): A zero sum game is a game where one player's gain is equal to another player's loss. In this scenario there are two players, the Red company and the Blue company, who are competing for the right to drill a field. In this scenario the players are going to gain what the other loses hence making this game a two-players-zero-sum game.

b): For formulating the payoff matrix we need to consider all the bids, also as the question mentions that for any bid for more than 45 million dollars red loses (except in case of tie):

This can be represented as

Payoff for Red (50, 50) = 0, in all the other cases Red loses so the payoff = 45 – 50 = -10 [Shown in the last column]

For other bids payoff for both players can be written as:

45 – bid amount

Using this, we get the following payoff matrix [All amounts in million dollars]:

Red (across), Blue (Down)	15	25	35	45	50	Lower Value
15	30	20	10	0	-10	-10
25	0	20	10	0	-10	-10
35	0	0	10	0	-10	-10
45	0	0	0	0	-10	-10
50	0	0	0	0	0	0
Upper Value	30	20	10	0	0	

Table: Denotes the payoff amount for both the players.

c): A saddle point is a state where both the players have no incentive to change their strategy (tend to stay in the equilibrium state). This is the situation when we say the two-player-zero-sum game is in equilibrium. This means that we can choose a pure strategy for each player involved in the game, with no player having any incentive to divert from pure strategy.

To verify whether the given scenario has a saddle point, we examine the lower and upper values of the game. From the table we can not find any point which satisfies this condition; therefore, this game does not have a saddle point.

d): Linear programming model for company Red is:

max z = v

s.t. constrains:

$v - 30x_1 <= 0$
$v - (20x_1 + 20x_2) <= 0$
$v - (10x_1 + 10x_2 + 10_{x3}) <= 0$
$v - (-10x_1 - 10x_2 - 10_{x3} - 10_{x4}) <= 0$
x1 + x2 + x3 + x4 + x5 = 1
$x_i >= 0$; i: 1, 2, 3, 4, 5

e): The optimum value achieved from this LP model is 0. This suggests that company Red should go with the strategy 4 (bid $45 million). This means that for any other value company Blue can have a strategy which would result in larger payoff, whereas for this strategy, Blue has only one better strategy ($50 million bid), hence this is the best strategy for Red to minimize the payoff and maximize the chance to win. [We can also solve this problem as a minimization problem which would also result in the same optimum point, in that scenario we would again be minimizing loss due to payoff].

Q4):

For computing the payoff for all sale price combinations, we need to examine each scenario and calculate profits.

Profit can be calculated by:

the demand for the iPhones × the profit of one iPhone after sale

- ⇨ Demand * (Selling Price – Cost)
- ⇨ Where demand is:
    - $D_A = 200 - P_A - (P_A - P')$
    - $D_B = 200 - P_B - (P_B - P')$
    - $P' = (P_A + P_B)/2$

Using this we calculate profits for all cases:

When A and B both choose 60:

Profit for A = 5600

Profit for B = 5600

When A chooses 60 and B chooses 70:

Profit for A = 5800

Profit for B = 6250

When A chooses 60 and B chooses 80:

Profit for A = 6000

Profit for B = 6600

When A chooses 70 and B chooses 60:

Profit for A = 6250

Profit for B = 5800

When A chooses 70 and B chooses 70:

Profit for A = 6500

Profit for B = 6500

When A chooses 70 and B chooses 80:

Profit for A = 6750

Profit for B = 6900

When A chooses 80 and B chooses 60:

Profit for A = 6600

Profit for B = 6000

When A chooses 80 and B chooses 70:

Profit for A = 6900

Profit for B = 6750

When A chooses 80 and B chooses 80:

Profit for A = 8400

Profit for B = 8400

Using the above profits, the payoff matrix is:

A (across), B (Down)	**60**	**70**	**80**
60	5600, 5600	5800, 6260	6000, 6600
70	6250, 5800	6500, 6500	6750, 6900
80	6600, 6000	6900, 6750	8400, 8400

b): This payoff matrix can be used to observe that that Nash equilibrium is obtained at (80,80) and the profit is maximum for A and B both i.e., 8400.

Any diversion from this the strategy which yields Nash Equilibrium might end up in less-than-optimal profits for both the companies. This provides incentives to both the companies to choose this strategy. This is also the saddle point in this scenario.

c) No, the Nash equilibrium would stay the same even if the cost doubles to 40 as the cost for producing one iPhone is same for both the companies, which makes it a shared variable. This will affect both the companies equally. Although the accrued profits would change for both the companies.

So, if the cost doubles to 40 then the new profits at equilibrium (80, 80) would be = 4800.