ECON 3201
Econometrics for Economics and Finance

Assignment 2
Due Friday October 3, 2025

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Instructions

Answer all questions. Please complete your assignment using Quarto. Submit both your Quarto file and pdf output file.

1. The Normal Distribution

Write your solutions using LaTeX. Show your work. You may use R to compute the answers.

1.1. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$ be a uniformly distributed variable with probability distribution function $f_X(x) = 1/12$.

(a) Check that $f_X(x)$ is a true probability distribution. # We need to satisfy the following 2 conditions: # Condition 1: $f_X(x) = 1/12 \geq 0$ for all x # Condition 2: $\sum_{x=1}^{12} \frac{1}{12} = 12 \times \frac{1}{12} = 1$ # Conditions met, hence verified.

(b) Compute the expected value of X. #Ans (b) $E[X] = \sum_{x=1}^{12} x \cdot \frac{1}{12} = \frac{1}{12} \times 78 = 6.5$

I did this one manually since the use of R was optional for this one.

(c) Compute the variance of X. $E[X^2] = \sum_{x=1}^{12} x^2 \cdot \frac{1}{12} = \frac{1}{12} \times 650 = 54.1667$ $Var(X) = 54.1667 - (6.5)^2 = 11.9167$

1.2. Let $X \sim N(4, 5)$. Find the probability of each event:

(a) $Pr(X \leq 6)$

`Pr(X   6) = 0.8145`

(b) $Pr(X > 4)$

`Pr(X > 4) = 0.5`

(c) $Pr(-1 \leq X < 1)$

`Pr(-1   X < 1) = 0.0772`

1.3. Let $Z \sim N(0, 1)$.

(a) Compute $Pr(Z \leq 0.83)$

`Pr(Z   0.83) = 0.7967`

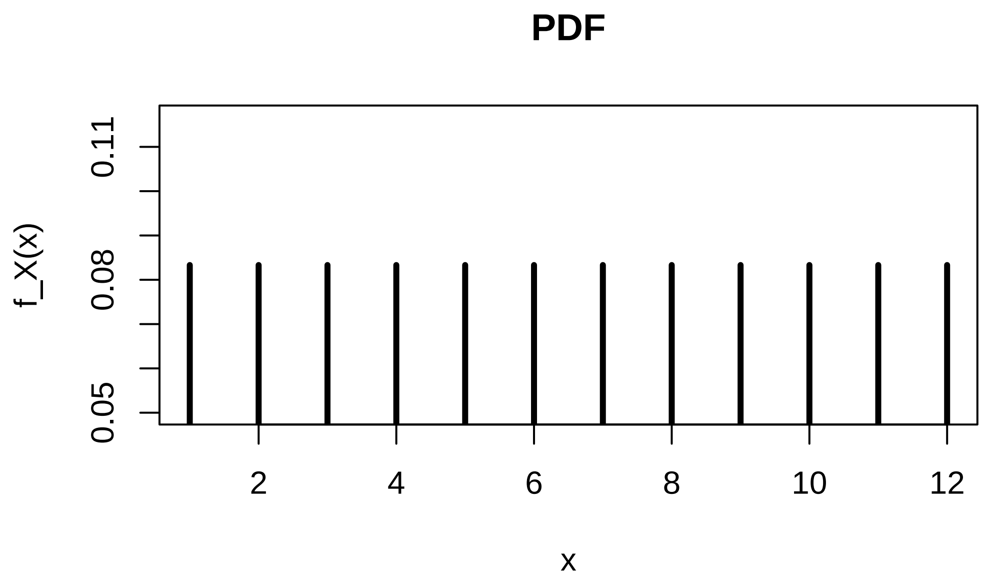
(b) Compute $Pr(-0.83 \leq Z \leq 0.83)$

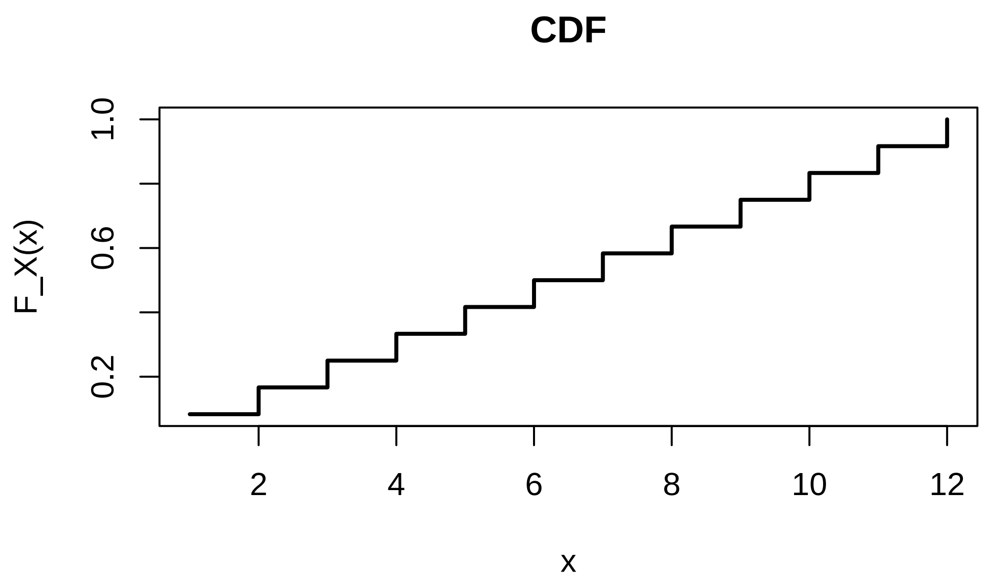
`Pr(-0.83   Z   0.83) = 0.5935`

2. R Excercises

2.1. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$ be a uniformly distributed variable with probability distribution function $f_X(x) = 1/12$.

(a) Plot the probability distribution function $f_X(x)$ in R.
(b) Plot the cumulative probability distribution function $f_X(x) inR$.
(c) Compute the expected value of X in R.
(d) Compute the variance of X in R.






```
Expected value = 6.5

Variance = 11.9167
```

2.2 The standard normal distribution function is given by

$$f(x) = \frac{e^{-(x^2/2)}}{\sqrt{2\pi}\sigma}$$

(a) Define this function in R. (Note: π is coded as `pi` and e is coded as `exp`.
(b) Integrate this function over the limit $[-\infty, \infty]$ using the `integrate()` command in R. Infinity in R is written as `Inf`. To evaluate the integral, use `integrate()$value`
(c) Compute the expected value of the standard normal distribution in R and confirm it is equal to 0.
(d) Compute the variance of the standard normal distribution in R and confirm it is equal to 1.
(e) Compute $Pr(Z \leq 0.83)$
(f) Compute $Pr(-0.83 \leq Z \leq 0.83)$
(g) Compute $Pr(-0.83 \leq Z \leq 0.83)$ using the `pnorm()` command and compare your results to (f).

```
Integral = 1

Expected value = 0

Variance = 1

Pr(Z   0.83) = 0.7967

Pr(-0.83   Z   0.83) = 0.5935

Using pnorm = 0.5935
```

2.3 Let X denote the prison sentence, in years, for people convicted of auto theft in a particular province in Canada. Suppose the PDF of X is given by:

$$f(X) = \frac{1}{9}x^2,\ 0 < x < 3.$$

(a) Define $f(x)$ in R.
(b) Integrate $f(x)$ over the interval [0,3].
(c) Compute the expected prison sentence.

```
Integral = 1

Expected sentence = 2.25 years
```

2.4 Let X follow a Student t distribution with M degrees of freedom. Then

$$X = \frac{Z}{\sqrt{W/M}} \sim t_M,$$

where Z is a standard normal variable and W is a chi-squared variable with M degrees of freedom.

(a) On the same figure plot the following:

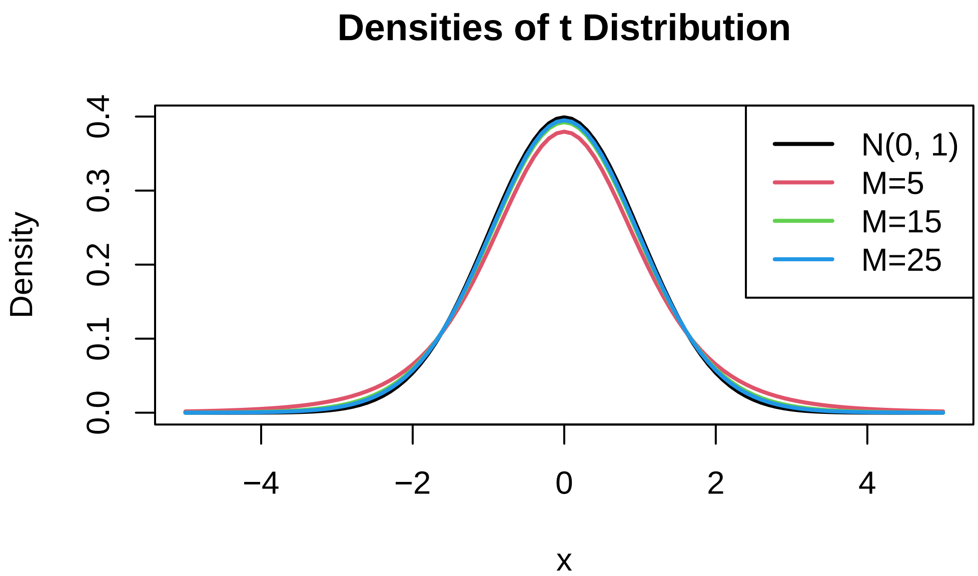
(i) Plot the standard normal probability distribution using `curve()`. Specify the range as [-5,5] using `xlim`. Name the `ylab` "Density". The main title should be "Densities of t Distribution".

(ii) Plot the Student t distribution with 5 degrees of freedom using `curve()` and `dt()`. Specify the range as [-5,5] using `xlim`. Choose a different colour, i.e. `col=2`. To learn more about `dt`, type `?dt` in your console.

(iii) Plot the Student t distribution with 15 degrees of freedom using `curve()` and `dt()`. Specify the range as [-5,5] using `xlim`. Choose a different colour, i.e. `col=3`. To learn more about `dt`, type `?dt` in your console.

(iv) Plot the Student t distribution with 25 degrees of freedom using `curve()` and `dt()`. Specify the range as [-5,5] using `xlim`. Choose a different colour, i.e. `col=4`. To learn more about `dt`, type `?dt` in your console.

(v) Include the following legend in you plot:
`legend("topright", c("N(0, 1)", "M=2", "M=4", "M=25"), col = 1:4, lty = c(1, 1, 1, 1))`



(b) What do you observe as the degrees of freedom increase? ## As degrees of freedom increase, the t-distribution becomes more similar to the standard normal distribution i.e., its curve flattens out more and more. I think we also touched upon this briefly in one of our recent classes.