MATH 1830 NOTES

UNIT 3 APPLICATIONS OF DERIVATIVES

3.1 ANALYZING GRAPHS ALGEBRAICALLY

Pre-Class:

- Take notes on the videos and readings (use the space below).
- Complete the 3.1 Pre-Class Quiz.

PRELIMINARY ALGEBRAIC ANALYSIS OF THE FUNCTION

Use Algebra and Limits to Identify All Basic Components in the Graph of $f(x)$.

1. Identify the Domain of $f(x)$: Commonly, $f(x)$is undefined for at any x value where:
   a. The denominator equals zero.
   b. There is an even root of a negative number.
   c. There is a logarithm of a negative number or log of zero.
2. Identify x-intercepts and y-intercept of $f(x)$.
   a. x-intercepts: Set $y = 0$ and solve for x.
   b. y-intercept: Set $x = 0$ and solve for y.
3. Identify Vertical Asymptotes and Holes of $f(x)$.
   a. Vertical asymptote at a when $\lim\limits_{x \to a} f(x) = \frac{n}{0}$ if $n \neq 0$.
   b. Hole at a when $\lim\limits_{x \to a} f(x) = \frac{0}{0}$ and then, after factoring and reducing, $\lim\limits_{x \to a} f(x) = \frac{n}{c}$ if $n \neq 0$ and c is any real number.
4. Identify Horizontal Asymptote of $f(x)$: calculate $\lim\limits_{x \to \infty} f(x)$.

Notes

Use Algebra and Limits to Identify all the Basic Components of the Graph.

1. $f(x) = x^3 + 6x^2 + 9x$

   a. Domain:

   b. x int(s):

   c. y int:

   d. Asymptotes:

2. $f(x) = \frac{3x+4}{2x-5}$

a. Domain:

b. x int(s):

c. y int:

d. Asymptotes:

Vertical:

Horizontal:

3. **The annual first quarter change in revenue for Apple, Inc. is given in the table below.**

Year	% Revenue Growth
1998	-12.2
2000	27.1
2002	4.5
2004	29.4
2006	34.4
2008	42.7
2010	65.4
2012	58.9
2014	4.7
2016	-12.8

The regression model for this data is:

$$f(x) = -0.005x^4 + 0.113x^3 - 0.889x^2 + 7.946x - 5.346$$

where x is Years Since 1998.

Use Algebra and Limits to Identify All Basic Components in the Graph of $f(x)$:

a. Identify the domain of $f(x)$:

b. Identify all x-intercepts in the domain:

c. Identify the y-intercept, if $x = 0$ is in the domain:

d. Identify all Asymptotes:

4. Using data from the Federal Reserve, the Dow S&P 500 annual percent return on investments for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -1.64t^4 + 20.85t^3 - 86.05t^2 + 127.87t - 36.24$$

where t is in years since 2008 and A(t) is in percent.

Source: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

Analyze and interpret the characteristics of the function.

a. Identify the domain of $A(t)$.

b. Identify x-intercepts and y-intercepts of $A(t)$.

Evaluate and interpret any x-intercepts of $A(t)$.

Evaluate and interpret the y intercept of $A(t)$.

c. Identify any vertical asymptotes and holes in $A(t)$.

d. Identify any horizontal asymptotes in $A(t)$.

3.1 Analyzing Graphs Algebraically Homework

Use Algebra and Limits to Identify All Basic Components in the Graph of $f(x)$.

1. $f(x) = 3x^2 + 5x - 2$

    a. Domain:

    b. x int(s):

    c. y int:

    d. Asymptotes:

2. $f(x) = \frac{4x-7}{5x+1}$

a. Domain:

b. x int(s):

c. y int:

d. Asymptotes:

Vertical:

Horizontal:

3. Analyze the following function.

Using data from Statista, the total annual amount spent on the purchase of golf equipment in the United States for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -31.94t^3 + 301.16t^2 - 665.61t + 3454.63$$

where t is in years since 2008 and A(t) is in millions of dollars.

Source: http://www.statista.com/statistics/201038/purchases-of-golf-equipment-in-the-us-since-2007/

Analyze and interpret the characteristics of the function.

a. Identify the domain of $A(t)$.

b. Identify x-intercepts and y-intercepts of $A(t)$.

Evaluate and interpret any x-intercepts of $A(t)$.

Evaluate and interpret the y intercept of $A(t)$.

c. Identify any vertical asymptotes and holes in $A(t)$.

d. Identify any horizontal asymptotes in $A(t)$.

MATH 1830 NOTES

Unit 3 Applications of Derivatives

3.2 First Derivative Test

Pre-Class:

- Complete 3.1 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Complete the 3.2 Pre-Class Quiz.

Analyze $f'(x)$ to Identify Intervals of Increase/Decrease and Extrema on the Graph of $f(x)$.

1. Find $f'(x)$
2. Identify all critical numbers and partitions for the function.
   a. Values of x where $f'(x) = 0$ are critical numbers.
   b. Values of x where $f'(x)$ is undefined are partitions.
   c. Values of x where $f(x)$ is undefined are partitions.
3. Graph the critical numbers and partitions on a number line, separating the number line into intervals.
4. Determine the intervals on which $f(x)$ is increasing /decreasing
   a. Test one point contained in the interval (do not use the end points of the interval).
   b. $f'(x) < 0$ then the function $f(x)$ is DECREASING on the interval
   c. $f'(x) > 0$ then the function $f(x)$ is INCREASING on the interval
5. Identify local maxima and minima of $f(x)$ using the First Derivative Test.
   a. On the interval (a, c), a local maximum occurs at $f(b)$ when $f(x)$ is increasing for all x in the interval $(a, b]$ and $f(x)$ is decreasing for all x in the interval $[b, c)$.
   b. On the interval (a, c), a local minimum occurs at $f(b)$ when $f(x)$ is decreasing for all x in the interval $(a, b]$ and $f(x)$ is increasing for all x in the interval $[b, c)$.

Use the First Derivative Test to analyze the function. Identify intervals of increase/decrease and extrema on the graph of the function.

1. $f(x) = x^3 + 6x^2 + 9x$

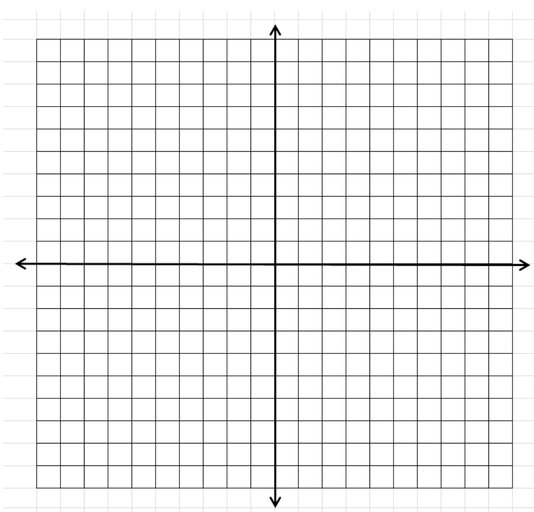
    a. Increasing and Decreasing

Increasing:

Decreasing:

b. Local Maxima:

Local Minima:



2. $f(x) = \frac{3x+4}{2x-5}$

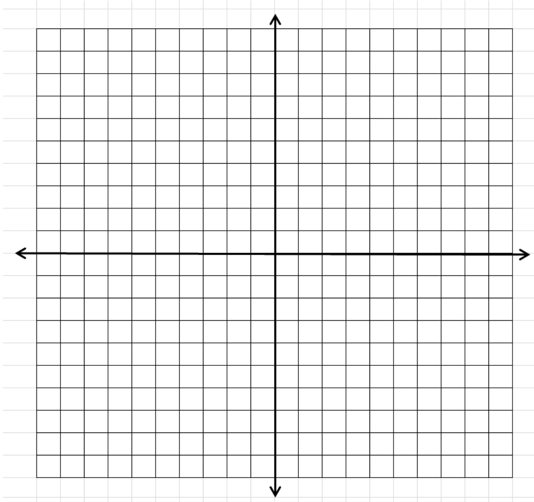
a. Increasing and Decreasing

Increasing:

Decreasing:

b. Local Maxima:

Local Minima:



3. The annual first quarter change in revenue for Apple, Inc. is given by the regression model:

$$f(x) = -0.005x^4 + 0.113x^3 - 0.889x^2 + 7.946x - 5.346$$

where x is Years Since 1998.

Use the First Derivative Test to analyze $f'(x)$ and identify intervals of increase/decrease and extrema on the graph of $f(x)$.

a. Create the sign chart for $f'(x)$:

Identify the values of x where $f'(x) = 0$:

Values of x where $f'(x)$ is undefined:

Values of x where $f(x)$ is undefined:

Intervals of Increase and Intervals of Decrease

Increasing:

Decreasing:

b. Extrema

Local maximum:

Local minimum:

4. Using data from the Federal Reserve, the Dow S&P 500 annual percent return on investments for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -1.64t^4 + 20.85t^3 - 86.05t^2 + 127.87t - 36.24$$

where t is in years since 2008 and A(t) is in percent.

Source: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

<u>Use the First Derivative Test to analyze $A'(t)$ and identify intervals of increase/decrease and extrema on the graph of $A(t)$.</u>

a. Graph the critical numbers on a number line and determine the sign for each interval.

   Values of t where $A'(t) = 0$:

   Values of t where $A'(t)$ is undefined:

   Values of t where $A(t)$ is undefined:

   Determine the intervals on which $A(t)$ is increasing/decreasing.

b. Identify local maxima and minima for $A(t)$.

3.2 First Derivative Test Homework

Use the First Derivative Test to analyze the function. Identify intervals of increase/decrease and extrema on the graph of the function.

1. $f(x) = 3x^2 + 5x - 2$

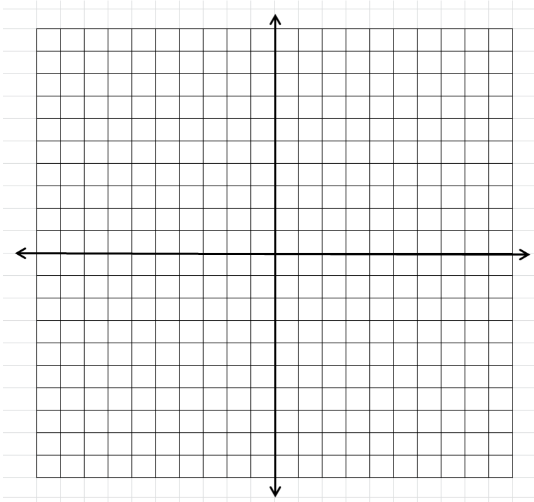
   a. Increasing and Decreasing

   Increasing:

   Decreasing:

   b. local max:

   local min:



2. $f(x) = \frac{4x-7}{5x+1}$

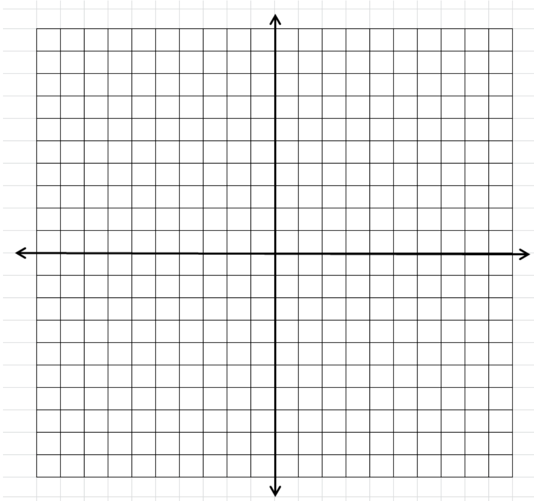
a. Increasing and Decreasing

Increasing:

Decreasing:

b. Local Maxima:

Local Minima:



3. Using data from Statista, the total annual amount spent on the purchase of golf equipment in the United States for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -31.94t^3 + 301.16t^2 - 665.61t + 3454.63$$

where t is in years since 2008 and A(t) is in millions of dollars.

Source: href="https://www.statista.com/statistics/201038/purchases-of-golf-equipment-in-the-us-since-2007/">http://www.statista.com/statistics/201038/purchases-of-golf-equipment-in-the-us-since-2007/

<u>Use the First Derivative Test to analyze the function. Identify intervals of increase/decrease and extrema on the graph of the function.</u>

a. Graph the critical numbers on a number line and determine the sign for each interval.

Values of t where $A'(t) = 0$:

Values of t where $A'(t)$ is undefined:

Values of t where $A(t)$ is undefined:

Determine the intervals on which $A(t)$ is increasing/decreasing.

b. Identify local maxima and minima for $A(t)$.

MATH 1830 NOTES

UNIT 3 APPLICATIONS OF DERIVATIVES

3.3 SECOND DERIVATIVE TEST

Pre-Class:

- Complete 3.2 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Complete the 3.3 Pre-Class Quiz.

SECOND DERIVATIVE TEST

Analyze $f''(x)$ to Identify Intervals of Concavity and Points of Inflection on the Graph of $f(x)$.

1. Find $f''(x)$.
2. Find the critical numbers for the function.
   a. Values of x where $f''(x) = 0$ are critical numbers.
   b. Values of x where $f''(x)$ is undefined are partitions.
   c. Values of x where $f(x)$ is undefined are partitions.
3. Graph the critical numbers and partitions on a number line, separating the number line into intervals.
4. Determine the intervals on which $f(x)$ is concave up or concave down.
   a. Test one point contained in the interval (do not use the end points of the interval).
   b. $f(x)$ is concave down on the interval if $f''(x) < 0$.
   c. $f(x)$ is concave up on the interval if $f''(x) > 0$.
5. Identify inflection points of $f(x)$. A point of inflection occurs at $x = a$ when $f''(a) = 0$ and $f''(x)$ changes concavity across a.

Use the Second Derivative Test to analyze the function. Identify intervals of concavity and points of inflection on the graph of the function.

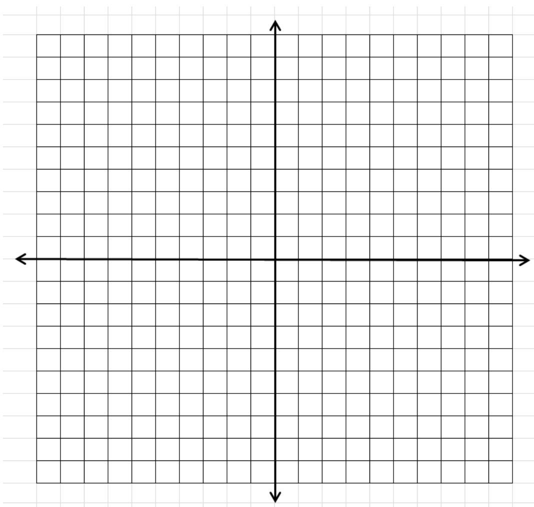
1. $f(x) = x^3 + 6x^2 + 9x$

    a. Concave Up and Concave Down

Concave up:

Concave down:

b. Inflection Points:



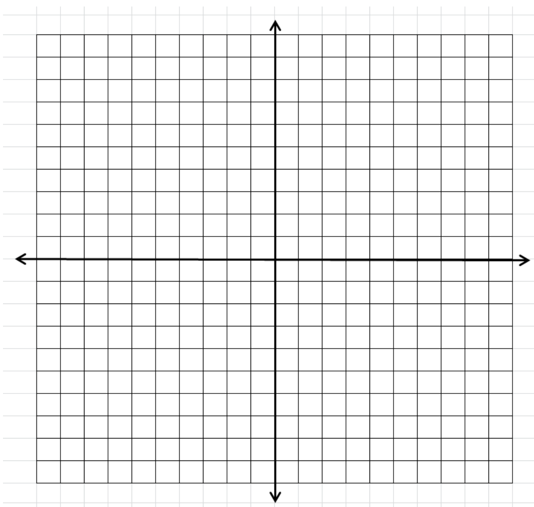
2. $f(x) = \frac{3x+4}{2x-5}$

a. Concave Up and Concave Down

Concave up:

Concave down:

b. Inflection Points:



3. The annual first quarter change in revenue for Apple, Inc. is given by the regression model:

$$f(x) = -0.005x^4 + 0.113x^3 - 0.889x^2 + 7.946x - 5.346$$

where x is Years Since 1998.

Use the Second Derivative Test to analyze the function. Identify intervals of concavity and points of inflection on the graph of the function.

a. Identify Critical Values and Partitions for the Sign Chart.
   Values of x where $f''(x) = 0$:

   Values of x where $f''(x)$ is undefined:

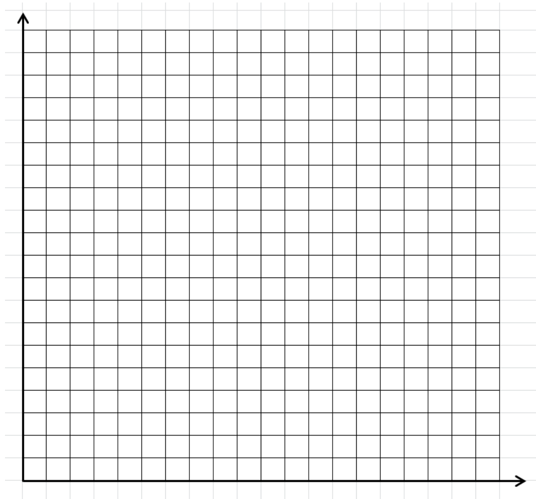
   Values of x where $f(x)$ is undefined:

   Intervals of Concavity
   Concave Up:

   Concave down:

b. Inflection points:

c. Graph $f(x)$.



4. Using data from the Federal Reserve, the Dow S&P 500 annual percent return on investments for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -1.64t^4 + 20.85t^3 - 86.05t^2 + 127.87t - 36.24$$

where t is in years since 2008 and A(t) is in percent.

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<u>Use the Second Derivative Test to analyze the function. Identify intervals of concavity and points of inflection on the graph of the function.</u>

a. Graph the critical numbers on a number line and determine the sign for each interval.

Values of t where $A''(t) = 0$.

Values of t where $A''(t)$ is undefined.

Values of t where $A(t)$ is undefined.

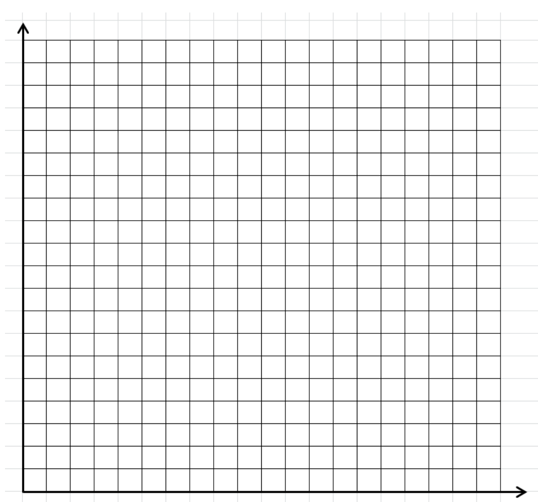
b. Determine the intervals on which $A(t)$ is concave up / concave down.

On what intervals is A(t) concave up? Interpret these results.

On what intervals is $A(t)$ concave down? Interpret these results.

c. Identify points of inflection for $A(t)$. Interpret these results.

d. Graph $A(t)$.



3.3 Second Derivative Test Homework

Use the Second Derivative Test to analyze the function. Identify intervals of concavity and points of inflection on the graph of the function.

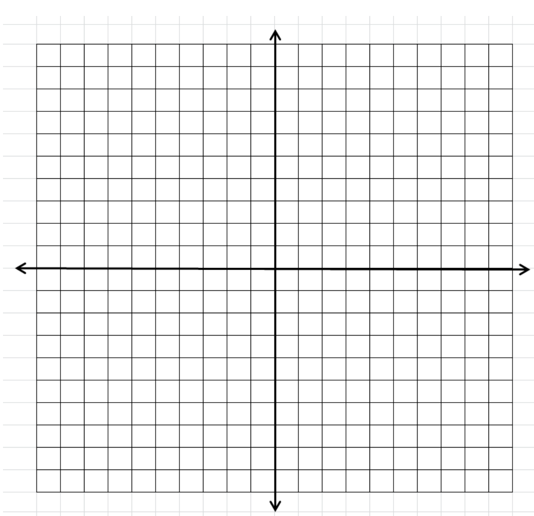
1. $f(x) = 3x^2 + 5x - 2$

a. Concave Up and Concave Down

Concave up:

Concave down:

b. Inflection Points:



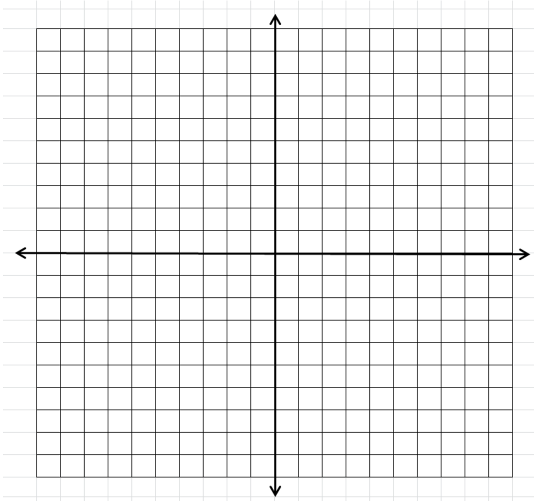
2. $f(x) = \frac{4x-7}{5x+1}$

a. Concave Up and Concave Down

Concave up:

Concave down:

b. Inflection Points:



3. Using data from Statista, the total annual amount spent on the purchase of golf equipment in the United States for the years 2008-2014 can be modeled by the following equation:

$$A(t) = -31.94t^3 + 301.16t^2 - 665.61t + 3454.63$$

where t is in years since 2008 and A(t) is in millions of dollars.

Source: href="https://www.statista.com/statistics/201038/purchases-of-golf-equipment-in-the-us-since-2007/"

Use the Second Derivative Test to analyze the function. Identify intervals of concavity and points of inflection on the graph of the function.

Analyze $A''\left(t\right)$

a. Graph the critical numbers on a number line and determine the sign for each interval.

Values of t where $A''(t) = 0$

Values of t where $A''\left(t\right)$ is undefined.

Values of t where $A\left(t\right)$ is undefined.

b. Determine the intervals on which $A(t)$ is concave up / concave down

On what intervals is $A(t)$ concave up? Interpret these results.

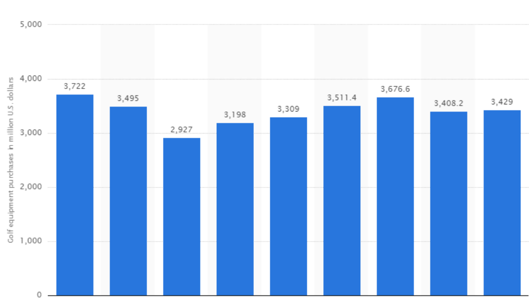
On what intervals is $A(t)$ concave down? Interpret these results.

Identify points of inflection for A(t). Interpret these results.

c. Graph $A(t)$

Clearly marking the characteristics identified above. Use graphing paper and colored pencils. Your graph should cover an entire piece of graph paper.

d. In order to accurately use the model, t values should be interpreted discretely, since the data used was given in discrete values (2008, 2009, etc) Using only discrete values for years, identify the relative maximums and minimums from the function:



MATH 1830

UNIT 3 APPLICATIONS OF DERIVATIVES

3.4 A SUMMARY OF CURVE SKETCHING

Pre-Class:

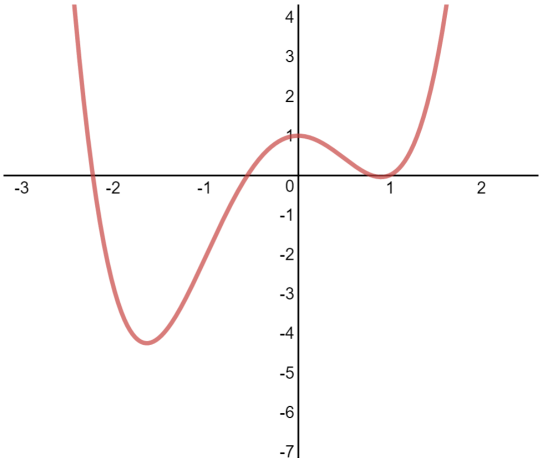
- Complete 3.3 Homework assignment: check and correct.
- Print Handouts for 3.4 In-Class Activity

NearPod: Derivative Rules

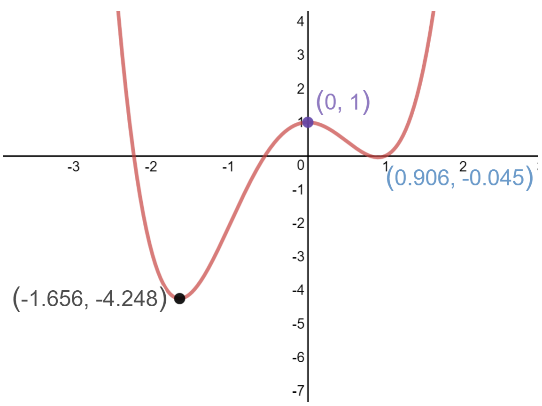
1. Which rule would you use to find the rate of change for the function $f(x) = x^3\left(2x^2 + 3x - 7\right)$?
    - Power Rule
    - e Rule
    - ln Rule
    - Product Rule
    - Quotient Rule
    - Chain Rule
2. Which rule would you use to find the derivative of the function $f(x) = (2x + 7)^2$?
    - Power Rule
    - e Rule
    - ln Rule
    - Product Rule
    - Quotient Rule
    - Chain Rule
3. Which rule would you use to find the rate of change for the function $f(x) = 4e^x$?
    - Power Rule
    - e Rule
    - ln Rule
    - Product Rule
    - Quotient Rule
    - Chain Rule

4. Which rule would you use to find the derivative of the function $y = \frac{x^2+3x-7}{x}$?
   - Power Rule
   - e Rule
   - ln Rule
   - Product Rule
   - Quotient Rule
   - Chain Rule
5. Which rule would you use to find the rate of change for the function $g(x) = \frac{x^3}{(x+1)(x-3)}$?
   - Power Rule
   - e Rule
   - ln Rule
   - Product Rule
   - Quotient Rule
   - Chain Rule
6. Which rule would you use to find the derivative of the function $y = e^x\sqrt{x}$?
   - Power Rule
   - e Rule
   - ln Rule
   - Product Rule
   - Quotient Rule
   - Chain Rule

NEARPOD ACTIVITY: FIRST DERIVATIVE TEST

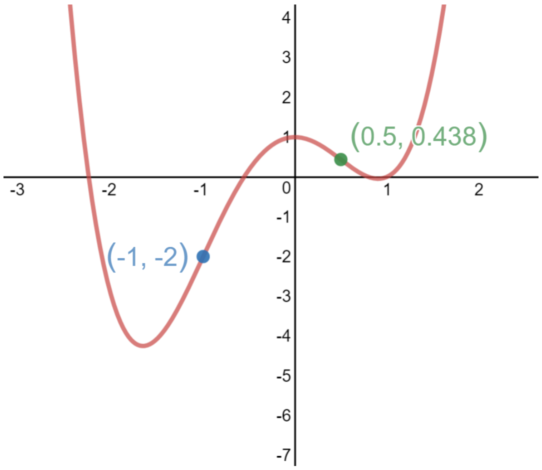


1. Draw and label 3 lines tangent to the function.
    a. a tangent line with a positive slope
    b. tangent line with a negative slope
    c. a tangent line with a slope of zero

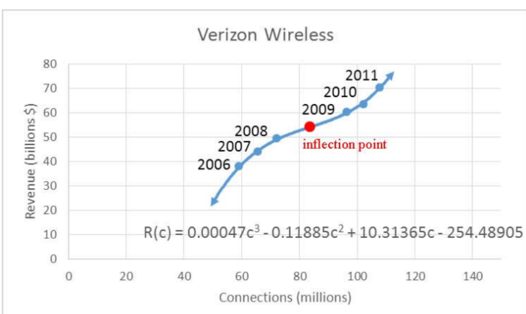


2. Use different colors to mark the intervals where the function is increasing and decreasing. Include a color legend.
3. State the intervals where the function is:
    a. Increasing
    b. Decreasing

Nearpod Activity: Second Derivative Test

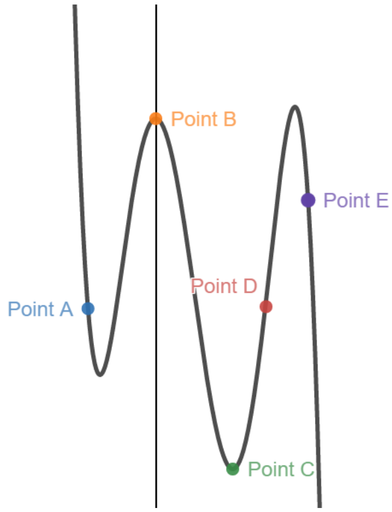


1. Use different colors to mark the intervals where the function is concave up and concave down. Include a color legend.
2. State the intervals where the function is:
    a. Concave Up
    b. Concave Down



3. a. Describe what is happening to the revenue from 2006 until mid-2008.
    b. Describe what is happening to the revenue from mid-2008 until 2011.

Nearpod Activity: Derivative Analysis

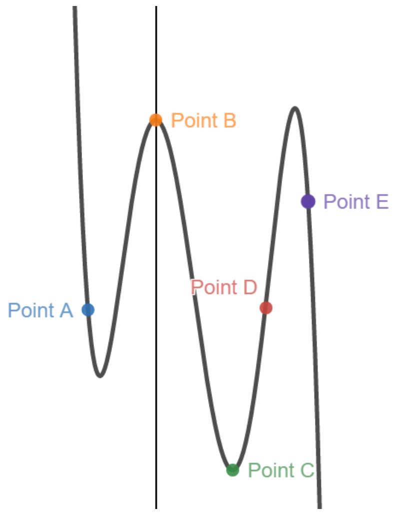


FIRST DERIVATIVE TEST:

In the first blank of each statement, choose one: **positive**, **negative**, **equal to zero.**

In the second blank of each statement, choose one: **increasing**, **decreasing**, **has a horizontal tangent.**

1. At Point A, $f'(x)$ is ________, therefore the graph (is) ________.
2. At Point B, $f'(x)$ is ________, therefore the graph (is) ________.
3. At Point C, $f'(x)$ is ________, therefore the graph (is) ________.
4. At Point D, $f'(x)$ is ________, therefore the graph (is) ________.
5. At Point E, $f'(x)$ is ________, therefore the graph (is) ________.



SECOND DERIVATIVE TEST:

In the first blank of each statement, choose one: **positive**, **negative**, **equal to zero.**

In the second blank of each statement, choose one: **concave up**, **concave down**, **has a possible point of inflection.**

1. At Point A, $f''(x)$ is , therefore the graph (is) .
2. At Point B, $f''(x)$ is , therefore the graph (is) .
3. At Point C, $f''(x)$ is , therefore the graph (is) .
4. At Point D, $f''(x)$ is , therefore the graph (is) .
5. At Point E, $f''(x)$ is , therefore the graph (is) .

3.4 Curve Sketching Homework

1. $f(x) = e^x(5x - 7)$

   Domain:

   x int(s):

   y int:

   Asymptotes:

   Vertical:

   Horizontal:

Increasing and Decreasing

Increasing:

Decreasing:

Local Maxima:

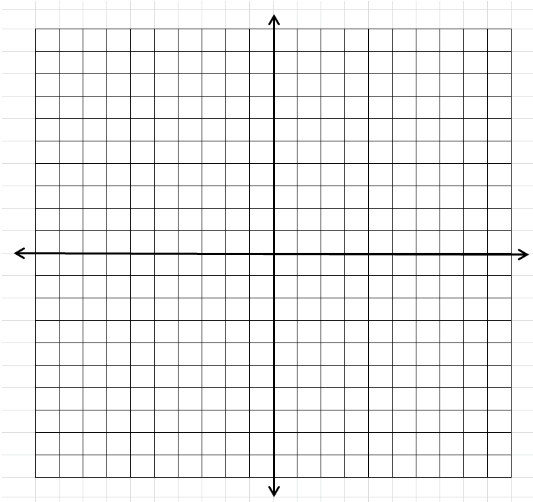
Local Minima:

Concave Up and Concave Down

Concave up:

Concave down:

Inflection Points:



MATH 1830 NOTES

Unit 3 Applications of Derivatives

3.5 Absolute Extrema

Pre-Class:

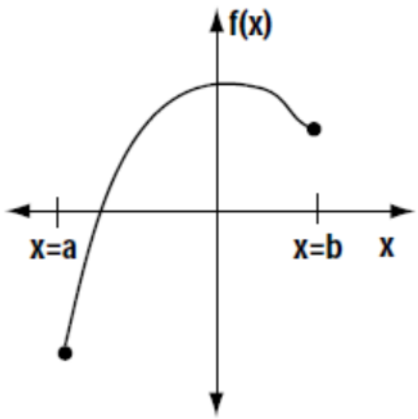
- Complete 3.4 NearPod Activity, if you did not complete it in class.
- Take notes on the videos and readings (use the space below).
- Complete the 3.5 Pre-Class Quiz.

Introduction

On the following page, each member of the group is to draw 3 functions f(x) over an interval $a < x < b$.

Be creative! Try to draw as many different possibilities as you can.

I have drawn one for you as an example.



Label each of your graphs (ex: graph #1, graph #2, etc).

1.

2.

3.

For each of your graphs, answer the following questions:

1. Where does f (x) have its maximum value? That is, where on your graph does y have the largest value?
2. Where does f (x) have its minimum value?

   Clearly indicate the answers to these two questions on each graph.

3. Based on your answers above, can your group arrive at a conclusion?
   a. Could you make a general statement about how to determine the absolute maximum or minimum values of a function over a given interval?
   b. Can you think of any exceptions?
   c. How can our knowledge of derivatives assist us?
   d. Summarize your responses on this sheet. Be prepared to share your results with the rest of the class.

Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?Sy3uQUnbK9L2RmSZs02CjV/Lfyjbyjsxsd+sU7CJwaIY=F

Notes

ALGORITHM FOR DETERMINING EXTREME VALUES

Suppose that $f(x)$ is a continuous function over a closed interval [a, b].

To find the absolute maximum and minimum values of the function $f(x)$ on [a, b]:

1. Find $f'(x)$.
2. Determine the additional points to test in the interval [a, b].
   (that is, find all x values for which $f'(x) = 0$ (critical points), $f'(x)$ does not exist (possible cusps, etc.).
3. Create a table for (x, y) coordinates and list all x values you plan to check: the endpoints a, b, and all critical values, possible cusps, etc.
4. Find the function values of all the x values in the table.
   a. The largest of these values is the absolute maximum of on the interval [a, b].
   b. The smallest of these is the absolute minimum of on the interval [a, b].

Find the absolute minimum and maximum values of the function, if they exist, over the given interval.

1. $f(x) = x^2 - 6x - 3 \qquad [-1, 5]$

2. $f(x) = 2x^3 - 3x^2 - 36x + 62 \qquad [-3, 4]$

3. $f(x) = x + \frac{1}{x}$ $\qquad [1, 20]$

4. $f(x) = \frac{x^2}{x^2+1} \qquad [-2, 2]$

5. $f(x) = \frac{x}{(x+9)^2}$ $\qquad [-1, 8]$

6. $f(x) = -3 \qquad [-2, 2]$

7. An employee's monthly production M, in number of units produced, is found to be a function of the number of year of service, t. For a certain product, a productivity function is given by:
$M(t) = -2t^2 + 100t + 180, \quad 0 \leq t \leq 40$

   Find the maximum productivity and the year in which it is achieved.

8. A firm determines that its total profit in dollars from the production and sale of x thousand units of a product is given by:

$$P(x) = \frac{1500}{x^2 - 6x + 10} \qquad x \geq 0$$

Find the number of units x for which the total profit is a maximum.

Note: this function is continuous on the interval $(0, \infty)$.

Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?Sy3uQUnbK9L2RmSZs02CjV/Lfyjbyjsxsd+sU7CJwaIY=F

3.5 Absolute Maximum and Absolute Minimum

Homework

Find the absolute minimum and maximum values of the function, if they exist, over the given interval.

1. $f(x) = x^3 - 3x + 6 \qquad [-2, 2]$
2. $y = x^4 - 4x^3 \qquad [-4, 6]$
3. $y = \frac{x^2}{3x-6} \qquad [3, 6]$
4. $y = (x+2)^{2/3} \qquad [-4, -2]$
5. Find the absolute minimum and maximum values of the function, if they exist, over the given interval.
   a. $g(x) = \frac{1}{3}x^3 - 2x - 2 \qquad [-2, 3]$
   b. Would your answers in part a change if we considered [-2,2] instead?
   c. What if we changed the interval to $-2 \leq x \leq 1$?
6. The temperature, T, of person during an illness is given by:

$$T(t) = -0.1t^2 + 1.2t + 98.6 \quad 0 \leq t \leq 12$$

   where T = temperature (°F) at time t, in hours since noon. Find the maximum value of the temperature and when it occurs.

7. Technicians working for the Ministry of Natural Resources have found that the amount of a pollutant in a certain river can be represented by $P(t)$ where t is the time (in years) since a clean-up campaign started. At what time was the pollution at its lowest level?

$$P(t) = 2t + \frac{1}{162t+1} \qquad 0 \leq t \leq 1$$

Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?Sy3uQUnbK9L2RmSZs02CjV/Lfyjbyjsxsd+sU7CJwaIY=F

MATH 1830 NOTES

UNIT 3 APPLICATIONS OF DERIVATIVES

3.6 INTRODUCTION TO OPTIMIZATION

Pre-Class:

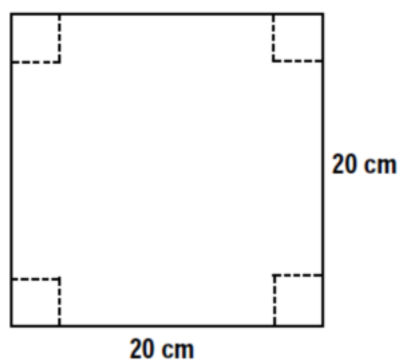
- Complete 3.5 Homework assignment: check and correct.
- Take notes on the readings (use the space below).

Introduction

1. Construct a 20 cm by 20 cm square on the white piece of paper.

2. Draw four congruent squares in each corner of your original square (see diagram below), the size of the four squares you draw will be assigned for your group.

3. Using the scissors and tape, cut out your square and its corners to create an open-topped box.



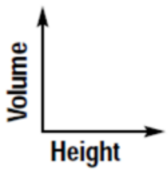
4. Complete the following questions:

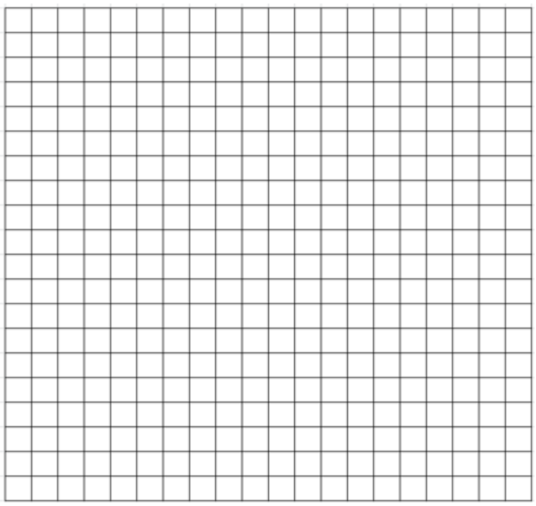
   a. The width of our box is:
   b. The length of our box is:
   c. The height of our box is:
   d. Calculate the volume of your box.

A summary of the data collected from the class is on the board. Copy this data into the chart below.

Height (cm)	**Volume (cubic cm)**
0	0
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	0

5. Using the graph paper, construct a graph of height vs volume by plotting the above ordered pairs. Join the points with a smooth curve. Answer the following questions based on your graph.





6. What is the maximum volume? (According to your graph.)
7. What size of square cut out of the corner would result in the maximum volume?
8. What type of function models your graph?
9. Could we write a mathematical function representing the graph?
10. How could our knowledge of derivatives be used to find the maximum volume?

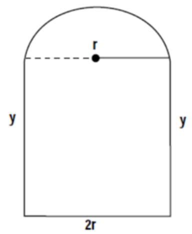
3.6 Optimization Homework

Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. When a theater owner charges \$3 for admission, there is an average attendance of 100 people. For every \$0.10 increase in admission, there is a loss of 1 customer from the average number. How much should the theater owner charge to maximize revenue and what is the maximum revenue?

2. A Norman window is a rectangle with a semi-circle on top. Find the maximum area if the perimeter is 24 feet, express the area as a function of the radius (r).



3. A lifeguard has 200 m of rope and some buoys with which she intends to enclose a rectangular area at a lake for swimming. The beach will form one side of the rectangle, with the rope forming the other three sides. Find the dimensions that will produce the maximum area.

4. A rancher wants to enclose two rectangular areas near a river, one for sheep and one for cattle. There is 240 meters of fencing available. If the area of the pens is to be maximized, what are their dimensions?

5. An apple farm yields an average of 30 bushels of apples per tree when 20 trees are planted on an acre of ground. Each time 1 more tree is planted per acre, the yield decreases 1 bushel per tree due to the extra congestion. How many trees should be planted to maximize the yield?

6. A rectangular pen is to be built with 1200 meters of fencing. The pen is to be divided into three parts using parallel partitions. Find the dimensions that will maximize the enclosed area.

7. There are 50 apple trees in an orchard, and each tree produces an average of 200 apples each year. For each additional tree planted within the orchard, the average number of apples produced drops by 5. What is the optimal number of trees to plant in the orchard?

8. For an outdoor concert, a ticket price of $30 typically attracts 5000 people. For each $1 increase in the ticket price, 100 fewer people will attend.

    a. Find the ticket price that maximizes revenue.

    b. Will your answer change if the concert area holds a maximum of 1200 people? Explain.

MATH 1830

Unit 3 Applications of Derivatives

3.7 Optimization in Packaging: Cans and Other Right Cylinders

Pre-Class:

- Complete 3.6 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Complete the 3.7 Pre-Class Quiz.

Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. A cylindrical soup can has a volume of 355 cm^3. Find the dimensions (radius and height) that minimize the surface area of the can.

2. You have 320 cm^2 of aluminum to make a cylinder shaped coke can. Find the dimensions that would produce a maximum volume.

3. What are the dimensions of the lightest open-top right cylindrical container that can hold a volume of 1000 cm^3?

3.7 Optimization in Packaging: Cans and Other Right Cylinders Homework

Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. A cylinderical container is to be made with 3000 cm^2 of sheet metal. What dimensions would result in the maximum volume? Assume a lid.
2. A cylindrical can has a volume of 900 cm^3. The metal costs \$15.50/cm^2. What dimensions produce a can with minimized cost? What is the cost of making the can?

MATH 1830

UNIT 3 APPLICATIONS OF DERIVATIVES

3.8 OPTIMIZATION IN PACKAGING: BOXES AND OTHER RIGHT RECTANGULAR PRISMS

Pre-Class:

- Complete 3.7 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Complete the 3.8 Pre-Class Quiz.

Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. A rectangular package to be sent by a postal service can have a maximum combined length and girth (distance around the package) of 108 inches. Find the dimensions of the package that contains a maximum volume. Assume that the package has square ends.

2. Your iron works company has been contracted to build a $500 ft^3$ holding tank for a paper company. The square-based, open-top tank is to be made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible. What dimensions do you tell the shop to use?

3. A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produces a box with a maximum volume?

3.8 Optimization in Packaging: Boxes and Other Right Rectangular Prisms Homework

Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. A container company is designing an open-top, square-based, rectangular box that will have a volume of 62.5 cm^3. Find the dimensions of the box that will minimize the surface area.
2. Find the maximum volume of a box with a square base that has a surface area of 96 cm^2.