MATH 1830

UNIT 1 LIMITS

1.1 FINDING LIMITS GRAPHICALLY

Pre-Class:

- Take notes on the videos and readings (use the space below).
- Work and check problems #1-7 in the 1.1 NOTES section.
- Complete the 1.1 Pre-Class Quiz.

Introduction

1. Box Office Receipts

   The total worldwide box-office receipts for a long running indie film are approximated by the function

   $$T(x) = \frac{120x^2}{x^2+4}$$

   where T(x) is measured in millions of dollars and x is the number of months since the movie's release. What are the total box-office receipts after:

   a. The first month?

   b. The second month?

   c. The third month?

   d. The hundredth month?

   e. What will the movie gross in the long run? (When x is very large.)

2. Driving Costs

   A study of driving costs of 1992 model subcompact cars found that the average cost (car payments, gas, insurance, upkeep, and depreciation), measured in cents/mile, is approximated by

   $$C(x) = \frac{2010}{x^{2.2}} + 17.80$$

   where x denotes the number of miles (in thousands of miles) the car is driven in a year. What is the average cost of driving a subcompact car:

   a. 5,000 miles per year?

   b. 10,000 miles per year?

   c. 25,000 miles per year?

   d. 50,000 miles per year?

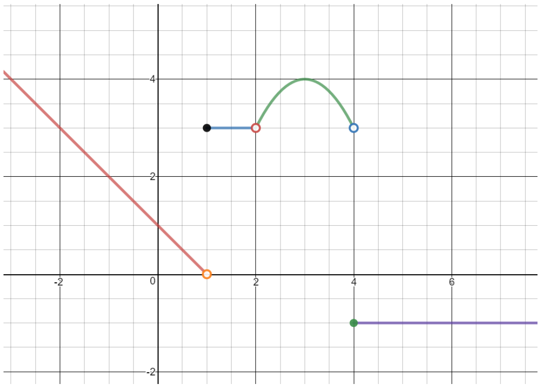
   e. What happens to the average cost as the number of miles driven increases without bound?

   f. Verify by evaluating the cost when the number of miles is 1,000,000 (or any large number).

Source https://domoremath.files.wordpress.com/2013/09/limits-word-prob-with-solns.pdf

Notes

Limits: A Graphical Approach



$$f(x) = \begin{cases} -x+1 & x < 1 \\ 3 & 1 \le x < 2 \\ -(x-3)^2+4 & 2 < x < 4 \\ -1 & x \ge 4 \end{cases}$$

Use the piecewise function to answer the questions below.

Evaluate the limits graphically. If the limit does not exist, explain why.

1. $\lim\limits_{x \to 0^-} f(x) =$

2. $\lim\limits_{x \to 0^+} f(x) =$

3. $\lim\limits_{x \to 0} f(x) =$

4. $f(0) =$

5. $\lim\limits_{x \to 1^-} f(x) =$

6. $\lim\limits_{x \to 1^+} f(x) =$

7. $\lim\limits_{x \to 1} f(x) =$

8. $f(1) =$

9. $\lim_{x \to 2^-} f(x) =$

10. $\lim_{x \to 2^+} f(x) =$

11. $\lim_{x \to 2} f(x) =$

12. $f(2) =$

13. Is it possible to define $f(1)$ so that $\lim_{x \to 1} f(x) = f(1)$? Explain.

14. Is it possible to redefine $f(2)$ so that $\lim_{x \to 2} f(x) = f(2)$? Explain.

15. $\lim_{x \to -1} f(x) =$

16. $\lim_{x \to 4} f(x) =$

17. $\lim_{x \to 6} f(x) =$

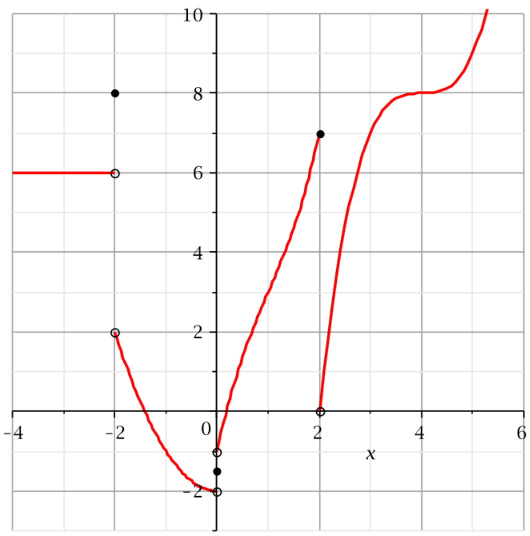
18. $\lim_{x \to 3} f(x) =$

19. $\lim_{x \to -2} f(x) =$

1.1 Finding Limits Graphically

Homework

Find the limits or evaluate the function. If the limit does not exist, explain why.



1. $\lim_{x \to -2} f(x) =$
2. $\lim_{x \to -3} f(x) =$
3. $\lim_{x \to 0^-} f(x) =$
4. $\lim_{x \to 0^+} f(x) =$
5. $\lim_{x \to 0} f(x) =$
6. $f(0) =$
7. $\lim_{x \to 2^-} f(x) =$
8. $\lim_{x \to 2^+} f(x) =$
9. $\lim_{x \to 2} f(x) =$
10. $f(2) =$

Problems from https://www.whitman.edu/mathematics/california_calculus/calculus.pdf

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UNIT 1 LIMITS

1.2 FINDING LIMITS ALGEBRAICALLY/INFINTE LIMITS

Pre-Class:

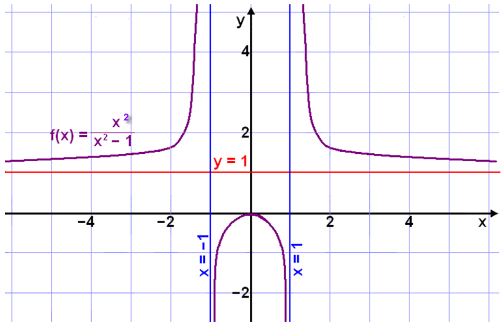
- Complete 1.1 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-#3b in the 1.2 NOTES section.
- Complete the 1.2 Pre-Class Quiz.

Introduction

Discuss this graph with your group.

Write down everything you observe.

Be prepared to share with the class.



Find each indicated quantity, if it exists.

1. $\lim\limits_{x \to 4} x^2 - 5x + 1 =$

2. $\lim\limits_{x \to -5} 2x^2 + 10x + 7 =$

3. $f(x) = \frac{3x^2 + 2x - 1}{x^2 + 3x + 2}$

    a. $\lim\limits_{x \to -3} f(x) =$

    b. $\lim\limits_{x \to -1} f(x) =$

    c. $\lim\limits_{x \to 2} f(x)$

    d. $\lim\limits_{x \to -2} f(x)$

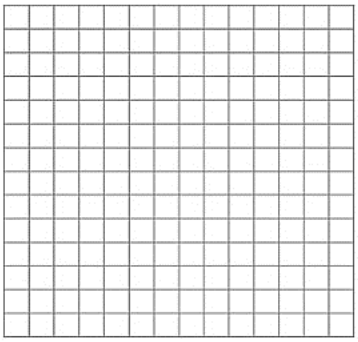
4. $\lim\limits_{x \to -5} \frac{x^2 + 7x + 10}{x^2 + 2x - 15}$

5. $\lim\limits_{x \to 4} \frac{x^2 - 16}{3x^2 - 13x + 4}$

6. $\lim\limits_{x \to 10} \frac{x^2 - 15x + 50}{(x - 10)^2}$

7. A taxi service charges \$3.00 per mile for the first 10 miles. If the trip is over 10 miles, they charge \$5.00 per mile for every mile. Write a piecewise definition of the charge G(x) for taxi fares of x miles.

   Graph G(x) for $0 < x \leq 25$.

   Find:

   $\lim_{x \to 10^-} G(x) =$

   $\lim_{x \to 10^+} G(x) =$

   $\lim_{x \to 10} G(x) =$

Identify the horizontal asymptotes of the following rational expression (if the horizontal asymptote exists)

8. $\lim_{x\to\infty} \frac{7x^3 - x^2 + 1}{5x^3 + 6x - 7} =$

9. $\lim_{x\to\infty} \frac{6x^4 - x^2 + 1}{2x^6 - 8x} =$

10. $\lim_{x\to\infty} \frac{4x^5 - 9x^3 - 1}{5x^3 + 3x^2 - 7} =$

Vertical and Horizontal Asymptotes: A summary

Find all vertical asymptotes, horizontal asymptotes, and holes of the function, showing all your work:

11. $f(x) = \frac{2x^2 - 32}{x^2 + 5x + 4}$

12. $f(x) = \frac{x^2 - 9}{x^2 - 4}$

Find all vertical asymptotes, horizontal asymptotes, and holes of the function by a quick analysis:

13. $f(x) = \frac{x + 2}{x^2 + 3}$

14. $f(x) = \frac{x^2 - 3x - 10}{x^2 - 4x - 5}$

15. $f(x) = \frac{x^2+5x-14}{x-2}$

1.2 Finding Limits Algebraically/Infinite Limits

Homework

Compute the limits. If the limit does not exist, explain why.

1. $\lim\limits_{x \to 3} \frac{x^2+x-12}{x-3} =$
2. $\lim\limits_{x \to 1} \frac{x^2+x-12}{x-3} =$
3. $\lim\limits_{x \to 10} 10 =$
4. $\lim\limits_{x \to 4} 3x^2 - 5x =$
5. $\lim\limits_{x \to 0} \frac{4x-5x^2}{x-1} =$
6. $\lim\limits_{x \to 1} \frac{x^2-1}{x-1} =$
7. $\lim\limits_{x \to 0^+} \frac{\sqrt{2-x^2}}{x} =$
8. $\lim\limits_{x \to 0^+} \frac{\sqrt{2-x^2}}{x+1} =$
9. $\lim\limits_{x \to 2} (x^2+4)^3 =$

Problems 1-9 from http://www.whitman.edu/mathematics/california_calculus/calculus.pdf

Find the vertical and horizontal asymptotes and holes, if they exist.

10. $f(x) = \frac{2x-7}{x-4}$
11. $f(x) = \frac{x^2-5}{x^3+x^2+1}$
12. $f(x) = \frac{x^3+6x^2+8x}{x^2-16}$
13. $f(x) = \frac{x-5}{\sqrt{4x^2+8}}$
14. $f(x) = \frac{x^2-1}{x^2-x-2}$

15. DIRECTV offers the following packages:

   - Select Package: 145 channels (0-145) for \$19.99 per month
   - Choice Package: Add 30 additional channels (146-175) for \$29.99 per month

   a. Write a piecewise definition of the charge G(x) for service with x channels.

   a. Graph G(x) for $0 < x \ \leq 175.$

   b. Find: $\lim\limits_{x \to 145^-} G(x)$ =

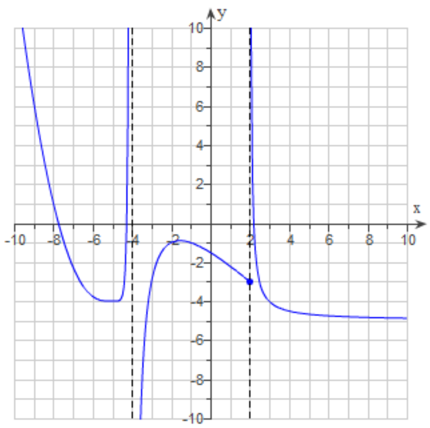
   c. Find: $\lim\limits_{x \to 145^+} G(x)$ =

   d. Find: $\lim\limits_{x \to 145} G(x)$

Source: https://www.directv.com/DTVAPP/pepod/configure.jsp?CMP=&keninvocaid=#package-section

EXTRA PRACTICE FOR LIMITS AND ASYMPTOTES

Find the limits.



1. $\lim\limits_{x \to -4^-} f(x) =$

2. $\lim\limits_{x \to -4^+} f(x) =$

3. $\lim\limits_{x \to -4} f(x) =$

4. $f(-4) =$

5. $\lim\limits_{x \to 2^-} f(x) =$

6. $\lim\limits_{x \to 2^+} f(x) =$

7. $\lim\limits_{x \to 2} f(x) =$

8. $f(2) =$

9. $\lim\limits_{x \to -5^-} f(x) =$

10. $\lim\limits_{x \to -5^+} f(x) =$

11. $\lim\limits_{x \to -5} f(x) =$

12. $f(-5) =$

<u>Find the limit.</u>

13. $\lim\limits_{x \to 1} \frac{10x}{x-1} =$

14. $\lim\limits_{x \to -12} \frac{x^2+11x-12}{x+12} =$

<u>Find the vertical and horizontal asymptotes and holes, if they exist.</u>

15. $f(x) = \frac{9x}{x+7}$

16. $f(x) = 9x^8 + 7x^6 + 21$

17. $f(x) = \frac{10x^2+49}{x^2-49}$

18. $f(x) = \frac{6x+7}{7x^2+4}$

19. $f(x) = \frac{2x^2-7x-15}{x^2+3x-40}$

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UNIT 1 LIMITS

1.3 CONTINUITY

Pre-Class:

- Complete 1.2 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 1.3 NOTES section.
- Complete the 1.3 Pre-Class Quiz.

Introduction

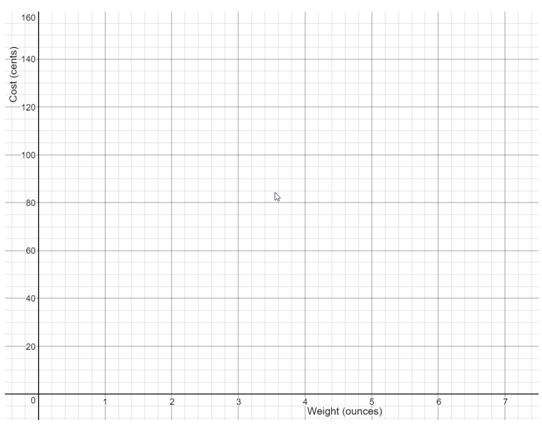
1. The table below shows the cost of mailing a letter that weighs x ounces.

Weight	Cost
$0 < x \le 1$	49¢
$1 < x \le 2$	70¢
$2 < x \le 3$	91¢
$3 < x \le 4$	112¢
$4 < x \le 5$	133¢

a. Complete the table of letters with the following weights.

Weight	Cost
.98	
1.26	
2.55	
3.01	
4.29	

b. Graph the function.

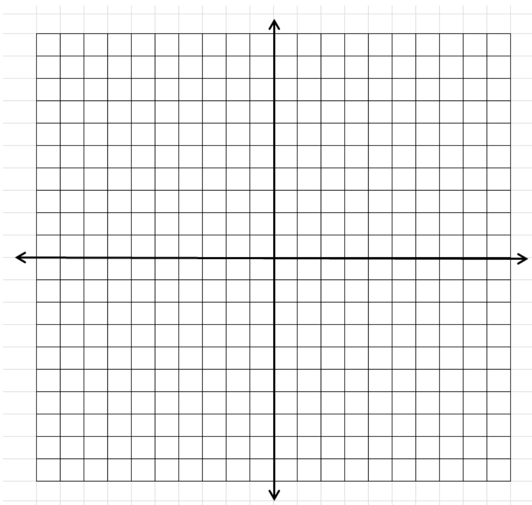


Source http://www.stamps.com/usps/postage-rate-increase/

2. Given: $f(x) = \frac{3x^2-12x-15}{x^2-3x-10}$

a. From looking at the given function (and NOT graphing), where would you expect to see vertical asymptote(s)?

b. Graph the function. Where are the vertical asymptote(s)?



c. Do you get the same answer for a & b? Why or why not?

Notes

Informal Definition: Continuity

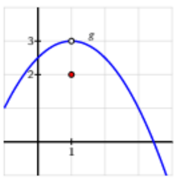
A function is continuous over an interval if the graph over the interval can be drawn without removing the pencil from the paper.

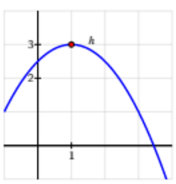
Formal Definition: Continuity

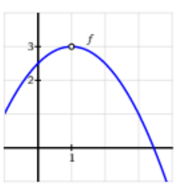
A function, f(x), is continuous at the point $x = c$ if all three of the following requirements are met:

1. $\lim_{x \to c} f(x)$ exists
2. $f(c)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Examples of Continuous and Discontinuous Functions:

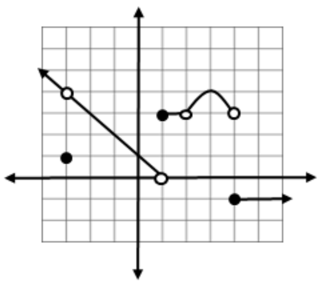






Active Calculus https://open.umn.edu/opentextbooks/

In Groups: Use the formal definition of continuity to discuss the continuity of the function whose graph is shown below.



1. Continuity at $x = -3$

2. Continuity at $x = -2$

3. Continuity at $x = 1$

4. Continuity at $x = 2$

5. Continuity at $x = 3$

6. Continuity at $x = 4$

Rules for Continuity

- Constant functions $f(x) = k$ are continuous for all x.
  Example: $f(x) = -2$

- Power functions $f(x) = x^n$ are continuous for all x, where n is a positive integer.
  Example: $f(x) = x^5$

- Polynomial Functions are continuous for all x.
  Example: $f(x) = 2x^3 - 5x + 1$

- Rational Function s are continuous for all x except where the denominator = 0.
  Example: $f(x) = \frac{x^2+5}{x-3}$, where numerator and denominator are polynomials.

- $\sqrt[n]{f(x)}$ functions are continuous for all x where n is an odd positive integer > 1.
  Example: $\sqrt[3]{x}$

- $\sqrt[n]{f(x)}$ functions are continuous for all x where n is an even positive integer and f(x) is positive.
  Example: $\sqrt[4]{x}$

Assessing Continuity

Are the functions continuous? Use the Rules for Continuity to explain your answers.

7. $h\left(x\right) = 5 - 3x$

8. $n\left(x\right) = \frac{x-3}{x^2 + 2x-15}$

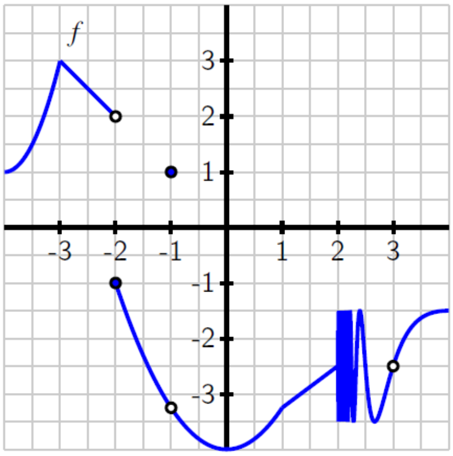
9. $f\left(x\right) = \sqrt{25 - x^2}$

10. $g\left(x\right) = \sqrt[3]{x^2 - 4}$

1.3 Continuity

Homework

Below is a graph of $y = f(x)$. Use the graph to answer the following questions.



1. State all values of x for which f is not continuous at $x = a$.

   (In other words, at what x values is the graph not continuous?)

2. At which values of a does $\lim\limits_{x \to a} f(x)$ not exist?

   (In other words, where on this graph does the limit not exist?)

3. At which values of a does f have a limit, but $\lim\limits_{x \to a} f(x) \neq f(a)$?

   (In other words, where on the graph are the limit and the point not the same?)

4. Which condition is stronger, and hence implies the other: **f has a limit at** $x = a$ or **f is continuous at** $x = a$? Explain.

   Based on your answer, choose the correct statement below :

   a. If f is continuous at $x = a$, then f has a limit at $x = a$
   b. If f has a limit at $x = a$, then f is continuous at $x = a$

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Unit 1 Limits

1.4 Definition of the Derivative

Pre-Class:

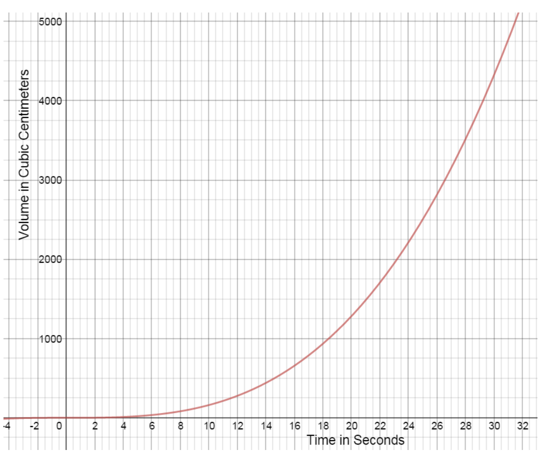
- Complete 1.3 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 1.4 NOTES section.
- Complete the 1.4 Pre-Class Quiz.

Introduction

A decorative birthday balloon is being filled with helium. The table shows the volume of the helium in the balloon at 3 second intervals for 30 seconds.

t(seconds)	V (cubic centimeters)
0	0
3	4.2
6	33.5
9	113.0
12	267.9
15	523.3
18	904.3
21	1436.0
24	2143.6
27	3052.1
30	4186.7

This function can be approximated by the equation $f(x) = 0.16x^3 + 0.0003x^2 - 0.007x + 0.0161$ (graphed below).



1. What are the dependent and independent variables for this problem? In what units is the rate of changed expressed?

2. A secant line is a line that intersects two points on a curve. Draw a secant line on the graph for each of the following. Calculate the slope of the secant line for each of the following intervals.

   a. 21 s to 30 s

   b. 21 s to 27 s

   c. 21 s to 24 s

3. What does the slope of the secant line represent?

4. A tangent line is a line that intersects a curve at only one point. Draw a tangent line at the point on the graph corresponding to 21 s and estimate the slope of this line.

5. What does the slope of the tangent line represent?

6. Compare the secant slopes to the slope of the tangent line. What do you notice?

Source http://mysite.science.uottawa.ca/iabde083/ch01.pdf

Notes

Limit Definition of the Derivative of a Function: 4 Step Process

Given $f(x)$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Step 1: Find $f\left(x + h\right)$

Step 2: Find $f\left(x + h\right) - f\left(x\right)$

Step 3: Find $\frac{f(x+h) - f(x)}{h}$

Step 4: Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

1. Use the limit definition of the derivative to find $f'\left(x\right)$, given $f\left(x\right) = x^2 - 3x - 2.$

2. Use the limit definition of the derivative to find $f'(x)$, given $f(x) = 5x^2 + 2x - 8$.

Write the equation of the tangent line at x=2.

3. Use the limit definition of the derivative to find $f'(x)$, given $f(x) = -4x^2 + x + 2.$

Write the equation of the tangent line at x = 1.

4. Use the limit definition of the derivative to find $f'(x)$, given $f(x) = \frac{6}{x} - 2$.

Find the equation of the tangent line at $x = 3$.

5. Friedman Academy is a charter school in South Texas. The school is not at capacity, and enrollment increases monthly. The charter school's enrollment is modeled by

$$f(x) = x^2 + x + 100$$

where y is in students and x is in years since 2008.

a. Estimate the average change in enrollment (the average rate of change of y with respect to x) from the year 2010 to the year 2012.

b. Find the function for the instantaneous rate of change in enrollment using the limit definition of the derivative.

c. Use the equation to calculate the instantaneous change in enrollment in 2010, 2011, and 2012.

6. Suppose an object moves along the y axis so that its location is $f\left(x\right)=2x^2\ +3x$ at time x. $f(x)$ is in meters and x is in seconds.

   a. Find the average velocity (the average rate of change of y with respect to x) for x changing from 2 to 4 sec.

   b. Use the limit definition of the derivative to find the instantaneous velocity.

   c. The instantaneous velocity at x = 2 seconds, 3 seconds, and 4 seconds.

1.4A Definition of the Derivative

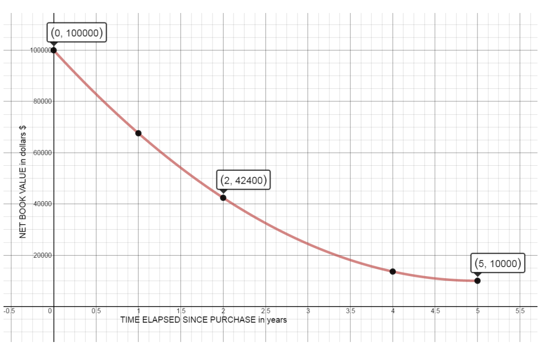
Homework

ABC Company purchases a machine for $100,000. It has an estimated salvage value of $10,000 and has a useful life of five years. The double declining balance depreciation calculation is:

Year	Net book value, beginning of year	Net book value, end of year
0	$100000	$100000
1	$100000	$67600
2	$67600	$42400
3	$42400	$24400
4	$24400	$13600
5	$13600	$10000

Source http://www.accountingtools.com/double-declining-balance-depre

This depreciation can be modeled by the function $y = 3600x^2 - 36000x + 100000$, which is graphed below.



1. Draw the secant line that intersects the two points on the curve and calculate the slope of the secant line for each of the following intervals.
   a. 2 years to 3 years
   b. 2 years to 2.5 years
   c. 2 years to 2.1 years
2. What does the slope of each of these secant lines represent?
3. Draw the tangent line on the graph that goes through the given points (2, 42400) and (4, 400). Calculate the slope of this line that goes through (2, 42400) and (4, 400).

4. What does the slope of the tangent line represent?

5. Compare the secant slopes to the slope of the tangent line. What do you notice?

1.4B Definition of the Derivative

Homework

Functions of the form $f(x) = x^n$, where n = 1,2,3, . . ., are often called power functions.

1. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2$.
2. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^3$.
3. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^4$.

   (Hint: $(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$)

4. Complete the table.

$f(x)$	$f'(x)$
x^2	
x^3	
x^4	

5. Based on your work in (a), (b), and (c), what do you conjecture is the derivative of $f(x) = x^5$? of $f(x) = x^{13}$?
6. Conjecture a formula for the derivative of $f(x) = x^n$ that holds for any positive integer n. That is, given $f(x) = x^n$ where n is a positive integer, what do you think the formula for $f'(x)$ is?

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Unit 1 Limits

1.5 Derivatives: The Power Rule

Pre-Class 1.5A:

- Complete 1.4 homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-4 in the 1.5 NOTES section.
- Complete the 1.5A Pre-Class Quiz

Pre-Class 1.5B:

- Complete 1.5A Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- 1.5B Work and check problems #9-10 in the 1.5 NOTES section.
- Complete 1.5B Pre-Class Quiz

Notes

Basic Differentiation Properties

Three Equivalent Terms:

If $y \;=\; f(x)$, you can use any of these to represent the derivative $y' = f'(x) = \frac{dy}{dx}$

THE POWER RULE: If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Using the Power Rule, find the indicated derivative:

1. $g(x) = x^4$

2. $y = 2x^3$

3. $\frac{d}{dx}(5)$

4. $y = \frac{1}{x^7} = x^{-7}$

5. $y = \frac{x^4}{16}$

6. $y = 8 + 3t - 5t^3$

7. $g(x) = 6x^{-5} - 5x^{-4}$

8. $\frac{d}{dx}\left(\frac{4x^3}{10} - \frac{2}{3x^4}\right)$

9. $H'(w)$ if $H(w) = \frac{5}{w^6} - 7\sqrt{w}$

10. $\frac{d}{du}\left(7u^{2/3} + 4u^{-3/5}\right)$

11. Find and approximate the value(s) of x where the graph of f has a horizontal tangent line. Use a graphing calculator to verify. $f(x) = 2x^2 - 5x$

12. A company's total sales (in millions of dollars) t months from now are given by $S(t) = 0.032t^4 + 0.5t^3 + 2.8t^2 + 9t - 4.$

    a. Find $S'(t)$.

    b. Find $S(4)$ and $S'(4)$. Write a brief verbal interpretation of these results.

    c. Find $S(8)$ and $S'(8)$. Write a brief verbal interpretation of these results.

13. A company decides to develop a cost equation based on the quantity of the product produced in a day. They collected the following data:

Quantity	20	35	50	65	80	95	110
Cost	642.35	766.48	858.82	928.83	1005.32	1078.82	1140.79

    a. Enter the data in a graphing calculator and find a cubic regression equation for the data.
    Let x represent the quantity produced in a day.

    Let y represent the daily cost for production.

    b. If $F(x)$ denotes the regression equation found in part A, find $F(70)$ and $F'(70)$.

    c. Write a brief verbal interpretation of these results.

1.5A DERIVATIVES: THE POWER RULE

Homework

Determine the derivative of each of the following functions. State your answer using full and proper notation, labeling the derivative with its name. For example, if you are given a function $h(z)$, you should write $h'(z)$ or $\frac{dh}{dz}$ as part of your response.

1. $f(x) = x^7$
2. $h(z) = \pi$
3. $y = \frac{1}{x^9}$
4. $\frac{d}{dx}\left(\frac{x^6}{36}\right)$
5. $p(a) = 3a^4 - 2a^3 + 7a^2 - a + 12$
6. $\frac{d}{dx}\left(-5x^4 - 6x^2 - \frac{2}{x^3}\right)$
7. $q(x) = \frac{x^3 - x + 2}{x}$
8. $f(x) = \frac{2x^5 - 3x^3 + x}{x^2}$
9. Find the slope of the tangent line to the curve $p(x) = 3x^4 - 2x^3 + 7x^2 - x + 12$ at the point where $x = -1$.
10. Find the equation of the tangent line at $x = -1$ for the function $p(x)$ in problem 9.

1.5B Derivatives: The Power Rule

Homework

Determine the derivative of each of the following functions. State your answer using full and proper notation, labeling the derivative with its name. For example, if you are given a function $h(z)$, you should write $h'(z)$ or $\frac{dh}{dz}$ as part of your response.

1. $f(x) = \sqrt[7]{x^4}$
2. $h(z) = \frac{5}{\sqrt{x}}$
3. $y = \frac{x^6}{12} - 3\sqrt[3]{x}$
4. $\frac{d}{dx}\left(-\frac{1}{x^4} + 9x^5\right)$
5. $p(a) = -3a^{-5} + a^{-3} + 7a - \sqrt{a} + 12$
6. $\frac{d}{dx}\left(-5\sqrt[4]{x^3} - 6\sqrt[3]{x^2} - \frac{2}{x^3}\right)$
7. $q(x) = \frac{-8x^3+5x-6}{x^4}$
8. Find the equation of the tangent line to the graph of $g(x) = -\frac{5}{\sqrt{x}}$ at the point $x = 4$.
9. The population of a bacteria colony is modeled by the function $p(t) = 200 + 20t - t^2$, where t is time in hours, $t \geq 0$, and p is the number of bacteria, in thousands.

   a. Determine the growth rate of the bacteria population at each of the following times.
      i. 3 hours
      ii. 8 hours
      iii. 13 hours
      iv. 18 hours
   b. What are the implications of the growth rates in part a?
   c. Determine the equation of the tangent to p(t) at the point t=8.
   a. When does the bacteria population stop growing? What is the population at this time?

MATH 1830

UNIT 1 LIMITS

1.6 MARGINAL ANALYSIS IN BUSINESS AND ECONOMICS

Pre-Class:

- Complete 1.5B Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-7, 11-16 in the 1.6 NOTES section.
- Complete the 1.6 Pre-Class Quiz.

Introduction

The cost, in dollars, of producing x frozen fruit yogurt bars per day can be modeled by the function

$$C(x) = 3450 + 1.5x - 0.0001x^2 \quad when \quad 0 \leq \ x \ \leq 5000.$$

The revenue from selling x yogurt bars is

$$R(x) = 3.25x.$$

Cost, Revenue, & Profit

1. What does 3450 represent in the cost function?

2. What does 3.25 represent in the revenue function?

3. What is the cost of producing 0 yogurt bars? What is the revenue generated from selling this many bars? What is the profit for selling 0 bars?

4. What is the cost of producing 1000 yogurt bars? What is the revenue generated from selling this many bars? What is the profit for selling 1000 bars?

5. What is the cost of producing 5000 yogurt bars? What is the revenue generated from selling this many bars? What is the profit for selling 5000 bars?

6. Find the model for the profit function, $P\left(x\right)$.

7. Answer the following questions about the model:

   When is the profit function positive?

   When is it negative?

   What important information does this provide the business owner?

Average Cost, Revenue, & Profit

8. It is clear from 3, 4, & 5 that the cost, revenue, and profit change based on the number of yogurt bars produced/sold. How would you determine the AVERAGE cost, revenue, and profit?

   Use the following equations that were identified earlier in this introduction:

   $C(x) = 3450 + 1.5x - 0.0001x^2 \quad when \quad 0 \leq \ x \ \leq 5000$

   $R(x) = 3.25x$

   $P(x) = 0.0001x^2 + 1.75x - 3450$

   a.

   b.

   c.

9. Find the average cost and explain the result:

   a. $\bar{C}(0) \ =$

   b. $\bar{C}(1000) \ =$

   c. $\bar{C}(5000)$ =

10. Is there a difference in the average cost when the number of bars produced changes? Why or why not?

Marginal Cost, Revenue, & Profit

Use these equations generated earlier:

$$C(x) = 3450 + 1.5x - 0.0001x^2 \quad when \quad 0 \le \ x \ \le 5000$$

$$R(x) = 3.25x$$

$$P(x) = 0.0001x^2 + 1.75x - 3450$$

11. Determine $C'\left(x\right)$. What does this represent?

12. Determine $C'\left(1000\right)$. Interpret this result.

13. When is $C'\left(x\right) = 0$? Explain your answer.

14. Determine $R'\left(x\right)$. What does this represent?

15. Determine $P'\left(x\right)$. What does this represent?

16. How do marginal cost, revenue, & profit relate to our discussions about average rate of change and instantaneous rate of change?

Notes

Cost and Marginal Cost

Marginal Cost, Revenue and Profit

1. A company's market research department recommends the manufacture and marketing of a new 3 meter lightening-to-USB power cord. After suitable test marketing, the research department presents the following price demand equation:

$$p = 12 - 0.001x$$

where x is demand at price p. The financial department provides the cost function that includes a fixed cost of $5600 (tooling and overhead) and variable costs of $1.80 per power cord (materials, labor, marketing, transportation, storage):

$$C(x) = 5600 + 1.80x$$

   a. **Marginal Cost Function: Find and interpret the Marginal Cost Function**

   b. **Revenue Function: Find the Revenue Function as a function of x.**

   c. **Marginal Revenue Function: Find the Marginal Revenue function and find the marginal revenue at x = 2000, x = 5000, and x = 7000. Interpret the results.**

   d. **Cost and Revenue Functions: Graph the cost function and the revenue function in the same coordinate system. Find the intersection points of these two graphs and interpret the results.**

e. **Profit Function: Find the profit function. Sketch the graph.**

f. **Marginal Profit Function: Find the marginal profit function and evaluate the marginal profit at x = 1000, x = 4000, and x = 6000. Interpret the results.**

Marginal and Average Cost, Revenue, and Profit:

2. A shop manufactures performance bikes. The manager estimates that the total cost (in dollars) of producing b bikes is:

$$C(b) = 1200 + 25b - 0.14b^2$$

a. **Average Cost: Find the average cost function, $\bar{C}(b)$. Calculate $\bar{C}(7)$ and interpret the results.**

b. **Marginal Cost: Find the Marginal Cost Function,$C'(b)$. Calculate $C'(7)$ and interpret the results.**

c. **Explain the difference between parts a and b.**

1.6 Marginal Analysis

Homework

1. You are the manager of Sassy Surf Creations, a new trend-setting clothing manufacturer. The cost function for your very exclusive Tai Kwon Do Dragon T-shirts is

$$C\left(x\right)=0.02x^2+7.5x+600$$

in dollars, and you sell the shirts for $\$20.00$ each. Determine the following.

   a. The revenue function
   b. The marginal revenue function
   c. The profit function
   d. The marginal profit function
   e. The marginal revenue and marginal profit for the sale of 300 T-shirts
   f. Explain what marginal revenue and profit mean, in general.
   g. Explain what the marginal revenue and marginal profit mean for the sale of 300 T-shirts.

2. The total cost, in dollars, of operating a factory that produces gourmet gas ranges is

$$C(x)=0.5x^2+40x+8000,$$

where x is the number of gas ranges produced.

   a. Determine the marginal cost of producing 5000 gas ranges and compare this with the actual cost of producing the 5001st gas range.
   b. Determine the average cost of producing 5000 gas ranges. Compare this value to those in part a. What do you notice?

3. The cost, in dollars, of producing x hot tubs is modeled by $C(x)=3450x-1.02x^2$, when the company produces up to 1500 hot tubs.

   a. Determine the marginal cost when 750 hot tubs are made. What does this mean to the manufacturer?
   b. Find the cost of producing the 751st hot tub.
   c. Compare and comment on the values in parts a and b.
   d. Each hot tub is sold for $9200. Write a revenue function for the sale of x hot tubs.
   e. Determine the rate of change of the profit for the sale of 750 hot tubs.

4. The National Honor Society at a local high school sells T-shirts for its yearly fundraiser. The cost of producing the x shirts is

$$C(x) = -0.0005x^2 + 7.5x + 200,$$

and each shirt sells for $\$15.00$.

a. Determine the revenue function.
b. Determine the marginal cost function.
c. Determine the marginal revenue function.
d. Determine the marginal cost AND the marginal revenue when 1500 shirts are sold.
e. Determine the ACTUAL cost of producing the 1501st shirt, according to the model.
f. Determine the profit and the marginal profit from the sale of 1500 shirts.