MATH 1830

Unit 4: Integration

4.1 Area by Sums

Pre-Class:

- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 4.1 NOTES section.
- Complete the 4.1 Pre-Class Quiz.

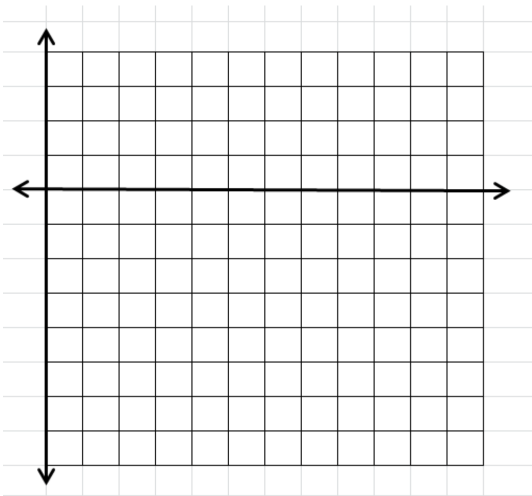
Introduction

1. The rate of change of the population (in thousands of people per year) of North Dakota between 1985 and 1996 can be modeled by

$$p(t) = \begin{cases} -7.35 & 0 \le t \le 6 \\ 2.5 & 6 < t \le 11 \end{cases}$$ where t represents the number of years since 1985.

(Source: Statistical Abstract, 1998)

   a. Sketch a graph of the rate of change function.



b. Find the area of the region between the graph of p and the horizontal axis from 0 to 6. Interpret your answer.

c. Find the area of the region between the graph of p and the horizontal axis from 6 to 11. Interpret your answer.

d. Was the population of North Dakota in 1996 greater or less than the population in 1985? By how much did the population change between 1985 and 1996?

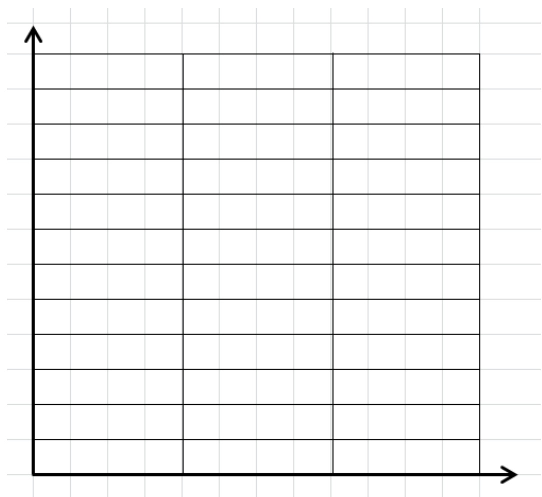
e. What information would you need to determine the population of North Dakota in 1996?

f. What is the relationship between the area of the regions and the population of North Dakota?

2. An office worker assembles advertising portfolios. As fatigue sets in, the number of portfolios he can assemble per hour decreases. Using regression, it is determined that he can assemble $f(t) = 20 - t^2$ portfolios per hour t hours after he begins work.

   a. How many portfolios can he assemble in the third hour?

   b. Graph the equation on the interval [0,3] on the graph below and approximate the area under the curve using three left rectangles.



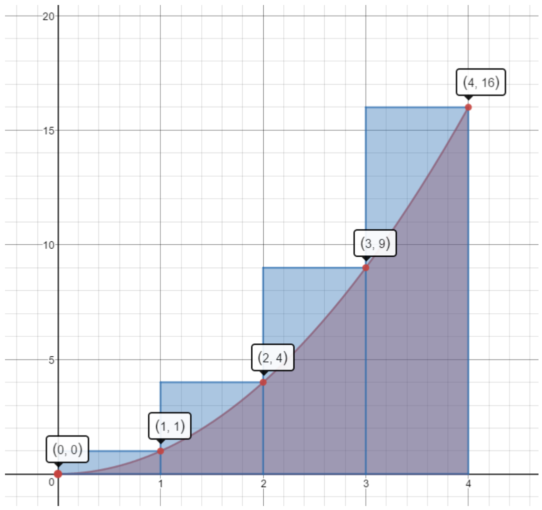
c. Find the area under the curve using 6 left rectangles and then 9 left rectangles.

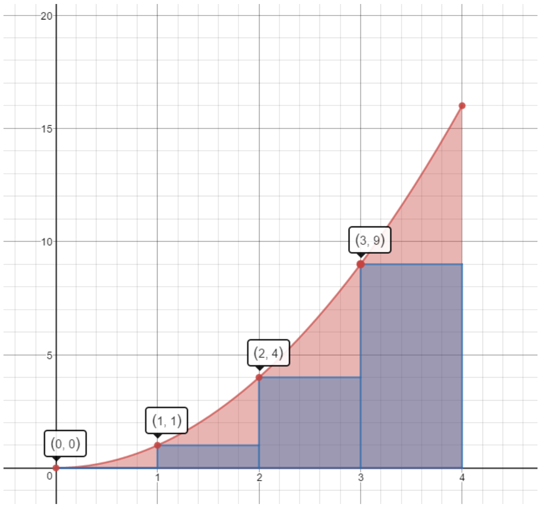
d. Do you think using 3 or 6 or 9 rectangles is a more accurate measure of the area? What could you do to get an even better measure?

e. What does the area under the curve represent?

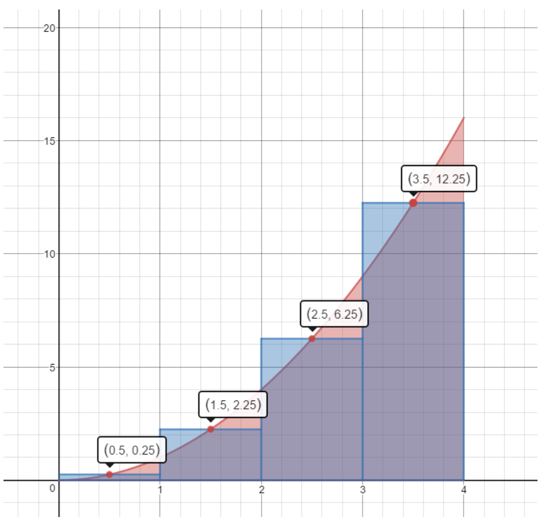
Notes

1. Estimate the area under the curve $f(x) = x^2$ on the domain [0,4] by summing the areas of the four Right Hand Rectangles. Is your estimate greater than the actual area or less than the actual area?



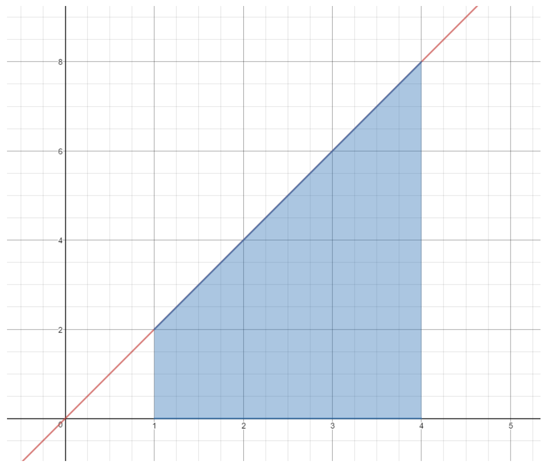


Left Hand Rectangles



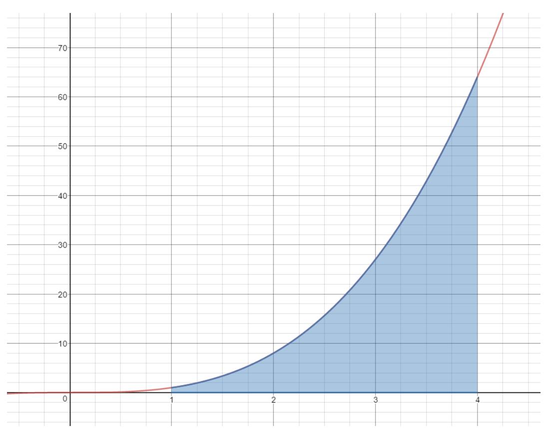
Mid-Point Rectangles

2. Use 3 Right Hand Rectangles to estimate the area under the curve $f(x) = 2x$ on the domain [1,4].



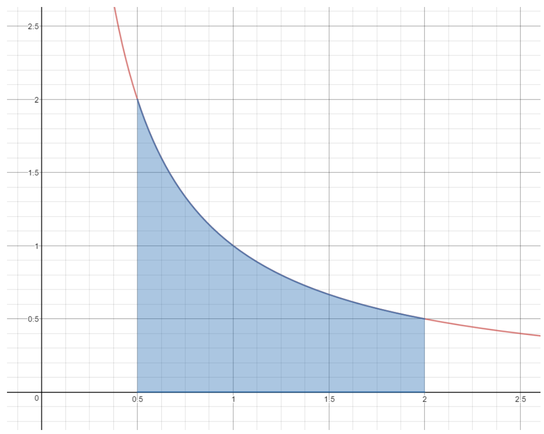
Is this estimate less than the actual area or greater than the actual area?

3. Use 3 Mid-Point Rectangles to estimate the area under the curve $f(x) = x^3$ on the domain [1,4].



Is this estimate less than the actual area or greater than the actual area?

4. Use 3 Left Hand Rectangles to estimate the area under the curve $f(x) = \frac{1}{x}$ on the domain [0.5, 2.0].



Is this estimate less than the actual area or greater than the actual area?

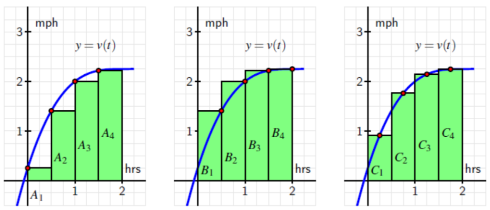
4.1 Area by Sums

Homework

1. A person walking along a straight path has her velocity in miles per hour at time t given by the function

$$v(t) = 0.25t^3 - 1.5t^2 + 3t + 0.25,$$

for times in the interval $0 \le t \le 2$. The graph of this function is also given in each of the three diagrams below.



a. Estimate the area under each of the curves by summing the areas of the rectangles.

Left Rectangles: $A_1 + A_2 + A_3 + A_4$

Right Rectangles: $B_1 + B_2 + B_3 + B_4$

Midpoint Rectangles: $C_1 + C_2 + C_3 + C_4$

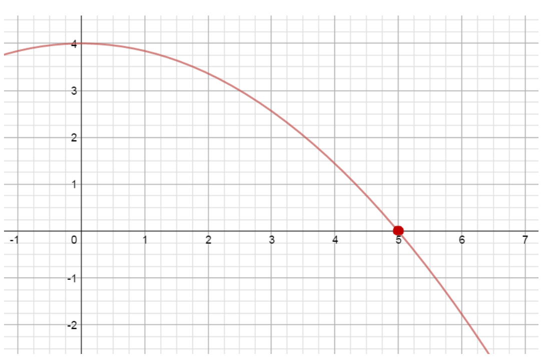
b. Why are the three answers different?

c. Of the three estimates from part a, which do you think is the best approximation of the area under the curve?

d. How could you get a better approximation for the area under the curve?

e. What does the area under the curve represent?

2. Given the function $f(x) = 4 - 0.16x^2$,

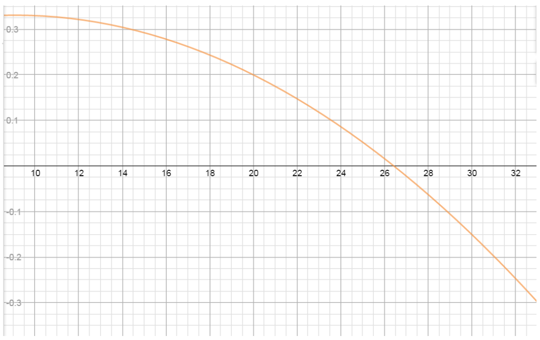


approximate the area under the curve on the interval [0, 6] using 6 right rectangles.

3. The rate of change of per capita consumption of a certain type of cheese in the United States from 1982 through 2002 can be modeled as

$$C'(x) = -0.0011x^2 + 0.02x + 0.2399$$

pounds per person per year where x is the number of years since 1970.



a. According to the graph of $C'(x)$, when was the per capita consumption of this cheese growing and when was it declining?
b. Find the point of the graph of $C'(x)$ that corresponds to the time when the per capita consumption of this cheese, $C(x)$, was the greatest. Explain.
c. Estimate using 7 right rectangles the area lying above the x-axis and below the graph of $C'(x)$ over the interval in part a. Interpret your answer.
d. Estimate using 3 right rectangles the area lying below the x-axis and above the graph of $C'(x)$ over the interval in part a. Interpret your answer.
e. By how much did the per capita consumption of this cheese change between 1982 and 2002?
f. What information do we need to determine the per capita consumption of this cheese in 2002?

MATH 1830

UNIT 4: INTEGRATION

4.2 FUNDAMENTAL THEOREM OF CALCULUS

Pre-Class 4.2A:

- Complete 4.1 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-2 in the 4.2 NOTES section.
- Complete the 4.2A Pre-Class Quiz.

Pre-Class 4.2B:

- Complete 4.2A Homework assignment: check and correct.
- Complete the 4.2B Pre-Class Quiz.

Introduction

From 4.1 homework problem #2:

Given the function $f(x) = 4 - 0.16x^2$,

approximate the area under the curve on the interval [0, 6] using 6 right rectangles.

Riemann Sum Exploration: http://webspace.ship.edu/msrenault/GeoGebraCalculus/integration_riemann_sum.html

Notes

ANTIDERIVATIVES

General Antiderivative Formulas

$$\int u^n \; du \;\; = \;\; \frac{u^{n+1}}{n+1} + C \;\; n \neq -1$$

$$\int e^u \; du \;\; = \;\; e^u + C$$

$$\int \frac{1}{u} \; du \;\; = \;\; \ln|u| + C$$

$$\int E'\left[I(x)\right] I'(x) \; dx \; = E\left[I(x)\right] + C$$

$$\int \left[f(x)\right]^n f'(x) \; dx \;\; = \;\; \frac{\left[f(x)\right]^{n+1}}{n+1} + C \quad n \neq -1$$

$$\int e^{f(x)} \cdot f'(x) \; dx \; = e^{f(x)} + C$$

$$\int \frac{1}{f\left(x\right)} \; f'\left(x\right) \; dx \;\; = \;\; \ln[f\left(x\right)] + C$$

The Fundamental Theorem of Calculus

Let f be continuous on [a, b]. If F is any antiderivative for f on [a,b], then

$$\int_a^b f\left(x\right) \mathrm{d}\, x \;\; = F(b) - F(a).$$

Evaluate the Definite Integral: Compare your answer to the area you calculated in 4.1 notes.

1. $S(x) = \displaystyle\int_0^4 x^2 dx$

2. $S(x) = \int_{1}^{4} 2x dx$

3. $S(x) = \int_{1}^{4} x^3 dx$

4. $S(x) = \int_{0.5}^{2} \frac{1}{x}\, dx$

5. $\int_{0}^{7} 8x\ dx$

6. $\displaystyle\int_{2}^{3} 4x^3 \, dx$

7. $\displaystyle\int_{0}^{3} 5e^x \, dx$

8. $\displaystyle\int_{1}^{4} \frac{7}{x} \, dx$

9. $\int_{3}^{7} \left(2 - 4x^2\right)\, dx$

10. $\int_{4}^{25} \frac{5}{\sqrt{x}}\, dx$

11. Cost: A company manufactures mountaineering 75-liter backpacks. The research department produced the marginal cost function

$$B'(x) = 400 - \frac{x}{5} \qquad 0 \le x \le 1000$$

where $B'(x)$ is in dollars and x is the number of backpacks produced per week. Compute the increase in cost when production level increases from 0 backpacks per week to 600 backpacks per week. Set up a definite integral and evaluate it.

12. Costs of Upkeep of a Marina: Maintenance costs for a marina generally increase as the structures at the marina age. The rate of increase in maintenance costs (in dollars per year) for a particular marina is given approximately by

$$M'(x) = 30x^2 + 2000$$

where x is the age of marina, in years, and M(x) is the total accumulated costs of maintenance for x years. Write a definite integral that gives the total maintenance costs from the third through the seventh year, and evaluate the integral.

4.2A Fundamental Theorem of Calculus

Homework

Use the Fundamental Theorem of Calculus to evaluate each of the following integrals.

1. $\int_{-1}^{5} (1-2x)dx$
2. $\int_{0}^{3} (x^3+2x^2-e^x)dx$
3. $\int_{1}^{3} \frac{1}{x}dx$
4. $\int_{0}^{1} \left(\sqrt{x}+\sqrt[3]{x^2}\right) \mathrm{d}\, x$
5. $\int_{0}^{9} (3\sqrt{x}+2x+1)dx$
6. $\int_{1}^{e} \frac{2}{x}dx$
7. $\int_{1}^{3} \left(\frac{4x^3-8x^2+2x-3}{x}\right) \mathrm{d}\, x$
8. $\int_{2}^{8} \left(5x^{1/5}-2x^{2/3}-2x^{-2}+\frac{7}{x}-6\right) \mathrm{d}\, x$
9. The rate of change of annual U. S. factory sales of electronics from 1990 through 1996 can be modeled by the equation

$$s\left(t\right)=-0.23t^3+2.257t^2-1.51t+42.8$$

in billions of dollars per year where t is the number of years since 1990. Evaluate $\int_{0}^{6} s\left(t\right)$ and interpret your answer.

10. The rate of change of the length of the average hospital stay between 1980 and 1996 can be modeled by the equations below where t is the number of years since 1980. Determine the value of the following definite integrals and interpret your answers. Note: $s(t)$ is in days per year.

$$s(t) = \begin{cases} 0.028t - 0.23 & 0 \le t \le 10 \\ -0.0408t + 0.30883 & 10 < t \le 16 \end{cases}$$

a. $S(t) = \int_0^{10} s(t)\ dt$

b. $\int_{10}^{16} s(t)\ dt$

c. $\int_0^{16} s\left(t\right)\ dt$

MATH 1830

UNIT 4: INTEGRATION

4.3 INDEFINITE INTEGRALS

Pre-Class:

- Complete 4.2B Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problems #1-4 in the 4.3 NOTES section.
- Complete the 4.3 Pre-Class Quiz.

Introduction

The marginal cost of producing x units of a commodity is given by

$$C'\left(x\right) = 3x^2 + 2x.$$

1. Find the cost function $C\left(x\right)$ for this commodity.

2. If the fixed costs are $2000, find the cost of producing 20 units.

Notes

General Integral Formulas

$$\int u^n \; du \;=\; \tfrac{u^{n+1}}{n+1} + C \;\; n \neq -1$$

$$\int e^u \; du \;=\; e^u + C$$

$$\int \frac{1}{u} \; du \;=\; \ln|u| + C$$

$$\int E'\left[I(x)\right] I'(x) \; dx \;=\; E\left[I(x)\right] + C$$

$$\int \left[f(x)\right]^n f'(x) \; dx \;=\; \tfrac{\left[f(x)\right]^{n+1}}{n+1} + C \quad n \neq -1$$

$$\int e^{f(x)} \cdot f'(x) \; dx \;=\; e^{f(x)} + C$$

$$\int \frac{1}{f(x)} \; f'(x) \; dx \;=\; \ln[f(x)] + C$$

Find the indefinite integral. Check by differentiating.

1. $\int 10 \; dx$

2. $\int 15x^2 \;\; dx$

3. $\int x^{-4} \;\; dx$

4. $\int 8\ x^{1/3} \quad dx$

5. $\int x^2\ \left(1+x^3\right)\ dx$

6. $\int \left(4x^3+\frac{2}{x^3}\right)\ dx$

7. $\int \frac{1-xe^x}{x} dx$

Find the particular antiderivative of each derivative that satisfies the given condition.

8. $R'(x) = 600 - 0.6x$ when $R(0) = 0$

9. $\frac{dQ}{dt} = \frac{100}{t^2}$ when $Q(1) = 400$

10. $\frac{dy}{dt} = 5e^t - 4$ when $y(0) = -1$

11. Renewable Energy: According to the Energy Research Institute, in 2012, US consumption of renewable energy was 8.45 quadrillion Btu (or 8.45×10^{15} Btu). Since the 1960's, consumption has been growing at a rate (in quadrillion Btu per year) given by

$$f'(t) = 0.004t + 0.062$$

where t is in years after 1960. Find $f(t)$ and estimate US consumption of renewable energy in 2024.

12. Sales Analysis: The rate of change of the monthly sales of a newly released video game is given by

$$S'(t) = 400t^{1/3}$$

$$S(0) = 0$$

where t is the number of months since the game was released and $S(t)$ is the number of games sold each month (in thousands). Find $S(t)$. When will monthly sales reach 20,000,000 games?

13. Efficiency of a Machine Operator: The rate at which a machine operator's efficiency Q (in percent) changes with respect to time on the floor without a break is modeled by the function

$$\frac{dQ}{dt} = 0.3t - 7 \qquad 0 \le t \le 16\ hrs$$

where t is the number of house the operator has been working. Find $Q(t)$ given that the operator's efficiency after working 2 hours is 82%.

Find the operators efficiency after 4 hours. After 8 hours.

4.3 Indefinite Integrals

Homework

Integrate the following.

1. $\int (2x^4 - 5x^3 - x^2 + 5x - 8)\ dx$
2. $\int (3x^{-2} - 4x^{-3} + 8)\ dx$
3. $\int (4x^{-1} + 3x - 2e^x - 1)\ dx$
4. $\int \left(2\sqrt[3]{x^2} - 4\sqrt{x} + \frac{3}{x^2} - \frac{5}{x} + 2\right)\ dx$
5. $\int \left(e^x + \frac{2}{x} - \frac{3}{x^2} - \frac{4}{\sqrt{x}} + 5\right)\ dx$
6. $\int \frac{2x^2 + 6x - 7}{x}\ dx$
7. $\int \frac{4x^4 - 5x^3 + 6x^2 - 7x}{x^2}\ dx$
8. The MORF dress in one dress with no buttons, zippers, Velcro, front or back that can be worn 24 different ways. After completing a successful Kickstarter campaign, the company found that the marginal cost for making the dress can be modeled by the function $C'(x) = x^2 - 16x + 70$ dollars per dress where x is the number of dresses produced.

   a. Find the cost function for the MORF dress.
   b. If $C(0) = 500$ find the total cost for producing 20 dresses.
   c. What is the fixed cost and what is the total variable cost of 20 dresses?

9. Suppose that when it is t years old, a particular industrial machine generates revenue at the rate $R'(t) = 5000 - 20t^2$ dollars per year.

   The operating and servicing costs related to the machine accumulate at the rate of $C'(t) = 2000 + 10t^2$ dollars per year.

   Find the profit function given that when the machine is first purchased no profit has been generated from it.

10. The rate of change for the lower limit for the top 5% of earners for all households in the United States from 2000 to 2014 can be modeled by

$$I'(t) = 38.58t^2 - 523.56t + 5416.05$$

   where t is the number of years after 2000.

   a. Find a general antiderivative for the lower limit for the top 5% of earners.
   b. Find a specific antiderivative given that in 2000 the lower limit for the top 5% of earners was $145,220.
   c. What will the lower limit for the top 5% of earners be in 2015 using this model?

MATH 1830

Unit 4: Integration

4.4 Integration by Substitution

Pre-Class 4.4A:

- Complete 4.3 Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Complete the 4.4A Pre-Class Quiz.

Pre-Class 4.4B:

- Complete 4.4A Homework assignment: check and correct.
- Complete the 4.4B Pre-Class Quiz.

Introduction

Chain Rule Review.

Find the derivative.

1. $f(x) = \left(3x^5 + 5x\right)^4$

2. $f(x) = \sqrt[5]{(3x^2 + 7)^4}$

3. $y = \ln\left(3x^2 + 5x\right)$

4. $f(x) = e^{5x^3 - 6x + 1}$

Notes

General Indefinite Integral Formulas

General Antiderivative Formulas

$$\int u^n \, du \;=\; \tfrac{u^{n+1}}{n+1} + C \quad n \neq -1$$

$$\int e^u \, du \;=\; e^u + C$$

$$\int \frac{1}{u} \, du \;=\; \ln|u| + C$$

$$\int E'\left[I(x)\right] I'(x) \, dx \;= E\left[I(x)\right] + C$$

$$\int \left[f(x)\right]^n f'(x) \, dx \;= \tfrac{\left[f(x)\right]^{n+1}}{n+1} + C \quad n \neq -1$$

$$\int e^{f(x)} \cdot f'(x) \, dx \;= e^{f(x)} + C$$

$$\int \frac{1}{f\left(x\right)} \, f'\left(x\right) \, dx \;=\; \ln[f\left(x\right)] + C$$

Find each indefinite integral and check the result by differentiating.

1. $\displaystyle\int (x^9 + 1)^4 \, (9x^8) \, dx$

2. $\displaystyle\int (6x^3 - 7)^{-6} \, (18x^2) \, dx$

3. $\displaystyle\int \frac{2}{9x - 4} \, (9) \, dx$

4. $\displaystyle\int (3t+5)^4 \, dt$

5. $\displaystyle\int e^{-2x} \, dx$

6. $\displaystyle\int \frac{x}{5+x^2} \, dx$

7. $\int \frac{t^2}{(t^3 - 1)^4} \, dt$

8. $\int \frac{x - 1}{x^2 - 2x + 5} \, dx$

9. $\displaystyle\int \frac{1}{x \ln x} \; dx$

10. $\displaystyle\int \frac{x}{\sqrt{x+3}} \, dx$

11. $\int x(x+2)^5 \, dx$

12. Continuous Money Flow: Suppose money is flowing continuously into a savings account at a rate of $1000 per year at interest rate of 2%, compounded continuously. The amount that is paid over time, dt, is

$$A'(t) = 1000e^{0.02t}.$$

What is the accumulation of the amount over the first 5 years?

4.4A Integration by Substitution

Homework

Integrate the following.

1. $\int (5x+4)^2 \, dx$

2. $\int (4x-5)^{1/2} \, dx$

3. $\int \left(x^3+x^2\right)^{1/2}(3x^2+2x) \, dx$

4. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2} \, dt$

5. $\int \frac{2x+2}{x^2+2x+1} \, dx$

6. $\int xe^{x^2} \, dx$

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

8. $\int \frac{e^x}{4-e^x} \, dx$

9. $\int \frac{1}{2x \ln x} \, dx$

10. $\int \frac{(\ln x)^5}{x} \, dx$

11. The population growth rate of a certain bacteria can be modeled by

$$\frac{dP}{dt} = 1000e^{t/3}$$

where t is measured in hours. What is the population after 3 hours?

12. The annual marginal revenue for the sale of x iPhones can be modeled by

$$R'(x) = 20x + 700 + \frac{200}{x+5} \qquad R(0) = 0,$$

where $R(x)$ is revenue in dollars and x is number of iPhones in thousands.

a. Find the revenue function.

b. Find the revenue from the sale of 5 million iPhones.

13. A city's population is expected to grow at the rate

$$P'(t) = \frac{126e^{16t}}{1+e^{16t}},$$

where t is the number of months from now when the population is 30,000.

a. Find the function for the city's population.
b. Find the population 12 months from now.
c. Approximate the increase in the population from month 6 to month 12.

4.4B Integration by Substitution

Homework

Integrate the following.

1. $\int 3t^2 \left(t^3+4\right)^5 \, dt$
2. $\int x^2 \left(x^3+4\right)^{-1/2} \, dx$
3. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}} \, dx$
4. $\int \frac{x+1}{\left(x^2+2x+2\right)^3} \, dx$
5. $\int x^2 e^{-4x^3} \, dx$
6. $\int \frac{1+e^{3x}}{e^{3x}+3x} \, dx$
7. $\int e^{-x} \left(2e^{-x}+3\right)^9 \, dx$
8. $\int \frac{2\ln x}{x} \, dx$
9. $\int \frac{\left(5+\ln x\right)^5}{x} \, dx$
10. The proportion of disposable income which individuals spend on consumption is known as the marginal propensity to consume (MPC). MPC is the proportion of additional income that an individual consumes. For example, if a household earns one extra dollar of disposable income, and the marginal propensity to consume is 0.65, then of that dollar, the household will spend 65 cents and save 35 cents. For a family of four in 2012, the MPC can be modeled by

$$\frac{dQ}{dx} = \frac{0.98}{(x-23,049)^{0.02}}$$

where x is the family income and Q is the income consumed. Given that the poverty level, in 2012, for a family of four was \$23,050 and all \$23,050 were consumed, use the model to estimate the amount consumed by a family of four whose 2012 income was \$35,000.

11. Target's market research department has determined that the marginal price for a particular brand of deodorant can be modeled by the function

$$p'(x) = -0.016e^{-0.02x},$$

where x is the number deodorant sticks sold per week in thousands.

   a. Find the price-demand equation if the weekly demand is 40,000 deodorant sticks when the price is \$3.99.
   b. Find the demand when the price of the deodorant is \$3.79 per stick.

4.4c In-class Homework

Evaluate each indefinite integral.

1. $\displaystyle\int -12x^2\left(-4x^3+2\right)^{-3}\,dx$

2. $\displaystyle\int \left(e^{4x}-4\right)^{1/3}\cdot 8e^{4x}\,dx$

3. $\displaystyle\int 5\sqrt{2x+3}\,dx$

4. $\displaystyle\int \frac{12x^2}{x^3+2}\,dx$

5. $\displaystyle\int \frac{5e^{-3+\ln 3x}}{x}\,dx$

MATH 1830

Unit 4: Integration

4.5 Area between Curves

Pre-Class 4.5A:

- Complete 4.4B Homework assignment: check and correct.
- Take notes on the videos and readings (use the space below).
- Work and check problem #1 in the 4.5 NOTES section.
- Complete the 4.5A Pre-Class Quiz.

Pre-Class 4.5B:

- Complete 4.5A Homework assignment: check and correct.
- Work and check problems #9 in the 4.5 NOTES section.
- Complete the 4.5B Pre-Class Quiz.

Introduction

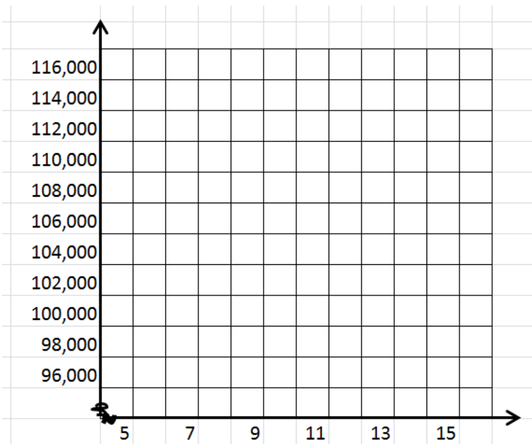
Before 1995, the U. S. Census Bureau used the model below to project the number (in thousands) of households in the United States, where t is the number of years after 1990.

$$N_1 = 1.35t^2 + 1078.4t + 92,323$$

For the years 1995-2005, the actual number of households N in the United States can be modeled by

$$N_2 = 18.32t^2 + 1178.3t + 92,099$$

1. Graph N_1 on the interval $5 \le t \le 15$ and shade the area under the curve. What does this shaded area represent?



2. Now graph N_2 on the interval $5 \leq t \leq 15$ on the same graph and shade the area under the curve. What does this shaded area represent?

3. Did the projection model over-project or under-project the number of households?

4. How would you determine the difference in the number of households given by each model?

5. Find the difference in the number of households for the two models.

Notes

Area Between Curves

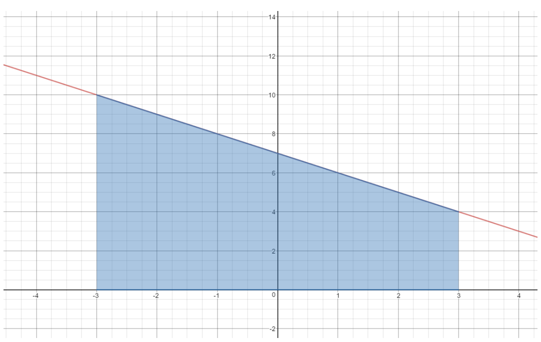
If f and g are continuous and $f(x) > g(x)$ over the interval $[a, b]$, then the area bounded by $y = f(x)$ and $y = g(x)$ for $a \leq x \leq b$ is given exactly by

$$A = \int_a^b [f(x) - g(x)]\, dx\,.$$

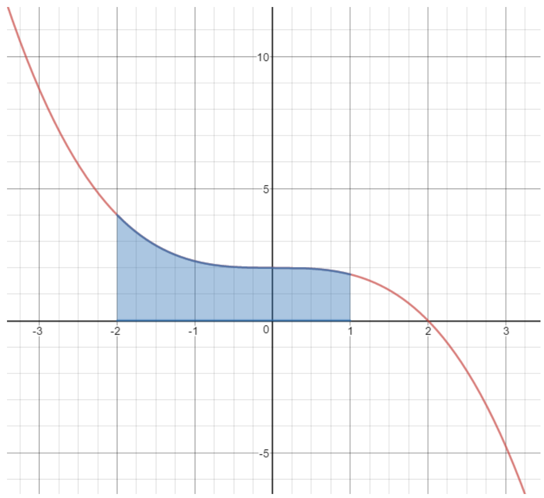
Area Bounded by an Interval

Find the area bounded by the graphs of the indicated equations over the given interval. Compute answers to three decimal places.

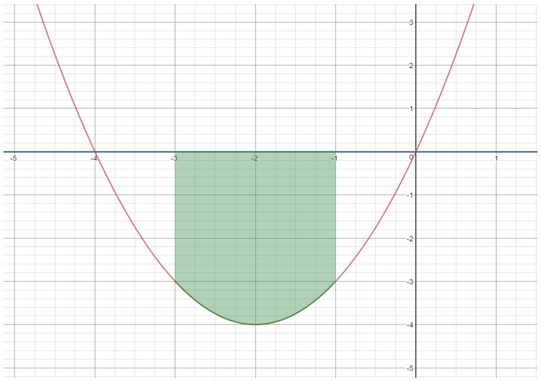
1. $y_1 = -x + 7$ and $y_2 = 0$ on the interval $-3 \leq x \leq 3$



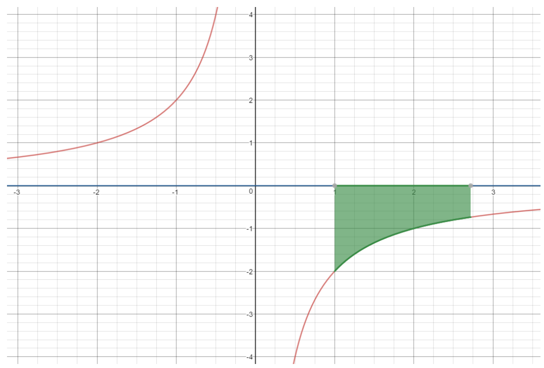
2. $y_1 = -\frac{1}{4}x^3 + 2$ and $y_2 = 0$ on the interval $-2 \leq x \leq 1$



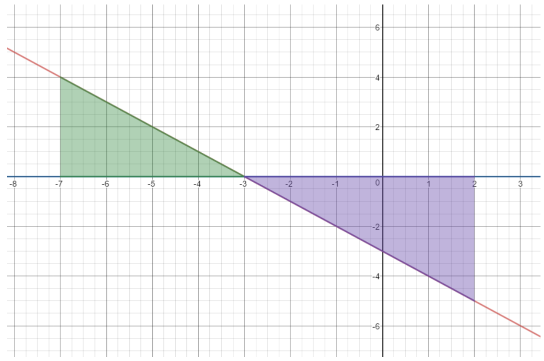
3. $y_1 = x(4 + x)$ and $y_2 = 0$ on the interval $-3 \le x \le -1$



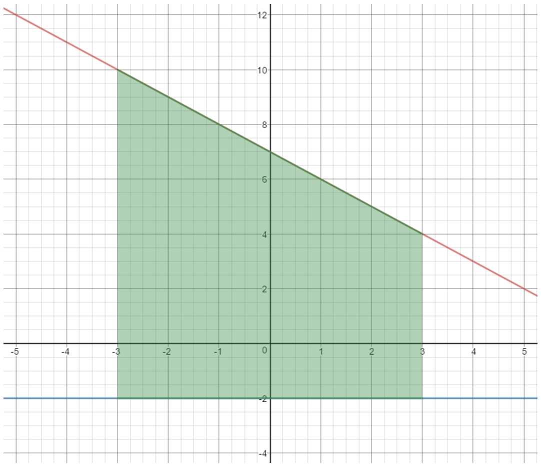
4. $y_1 = -\frac{2}{x}$ and $y_2 = 0$ on the interval $1 \le x \le e$



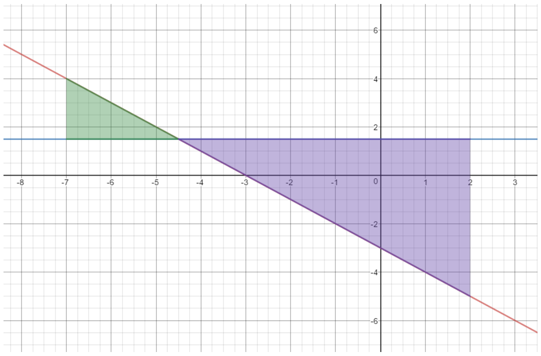
5. $y_1 = \; -x-3$ and $y_2 = 0$ on the interval $-7 \le x \le 2$



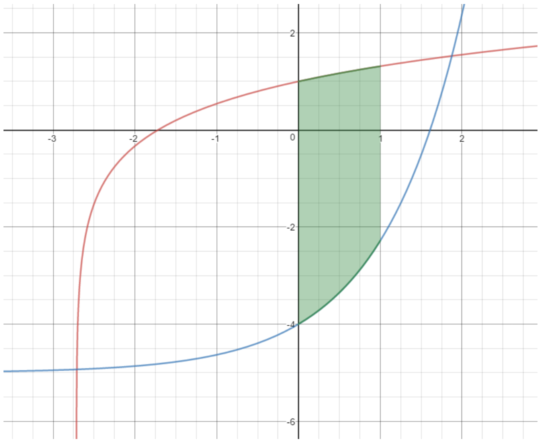
6. $y_1 = -x + 7$ and $y_2 = -2$ on the interval $-3 \le x \le 3$



7. $y_1 = -x - 3$ and $y_2 = 1.5$ for the interval $-7 \leq x \leq 2$



8. $y_1 = \ln(x + e)$ and $y_2 = e^x - 5$ on the interval $0 \le x \le 1$

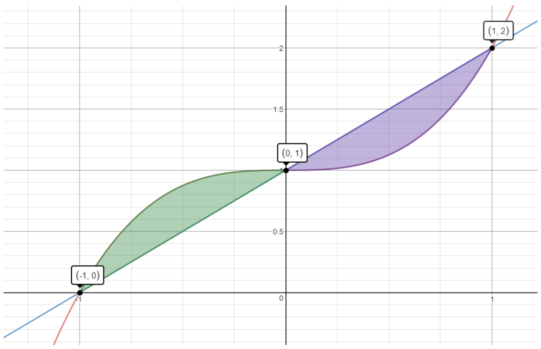


Define the total area bounded by the two functions. How would you determine the interval, if you were asked to calculate the total area bounded between the two functions?

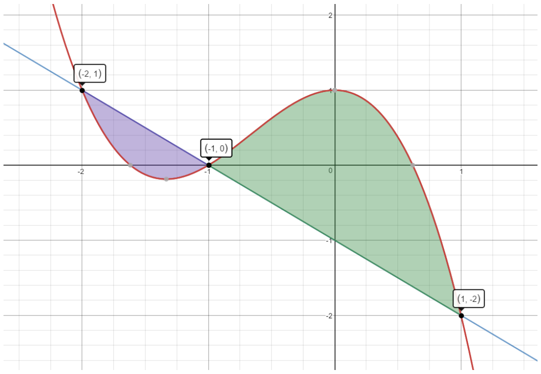
Area Not Bounded by an Interval

Find the area bounded by the graphs of the indicated equations.

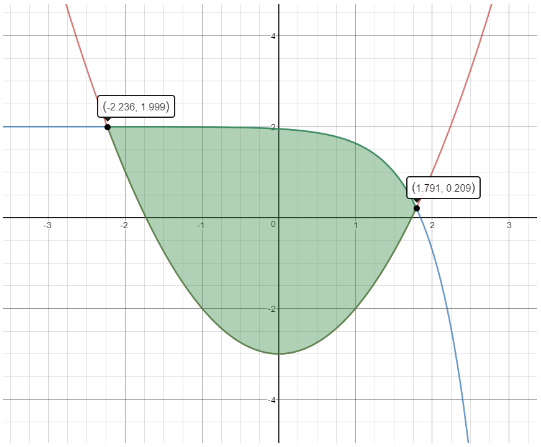
9. $y_1 = x^3 + 1$ and $y_2 = x + 1$



10. $y_1 = -x^3 - 2x^2 + 1$ and $y_2 = -x - 1$



11. $y_1 = -3 + x^2$ and $y_2 = -e^{2x-3} + 2$



12. The useful life of a piece of rental equipment is the duration for which the equipment will be profitable to the rental business, and not how long the equipment will actually last. Many factors affect a piece of equipment's useful life, including the frequency of use, the age when acquired and the repair policy and certain environmental conditions. The change in revenue for renting the equipment, over time, is modeled by

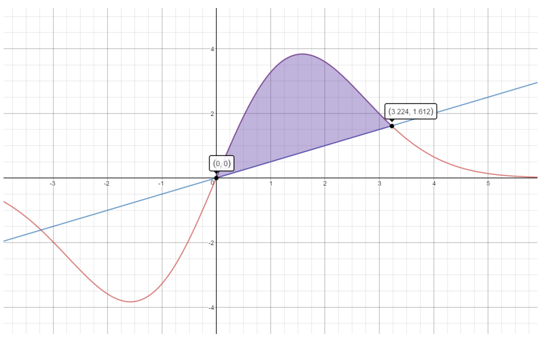
$$R'(t) = 4te^{-0.2t^2},$$

in thousands of dollars per year. The change in cost for routine maintenance of the equipment is modeled by

$$C'(t) = .5t,$$

also in thousands of dollars per year. Find the area between the graphs of C' and R' over the interval from the time the equipment is purchased until the equipment reaches the end of its useful life. Interpret the results.

Note: The end of the useful life is t where $C'(t) = R'(t)$.

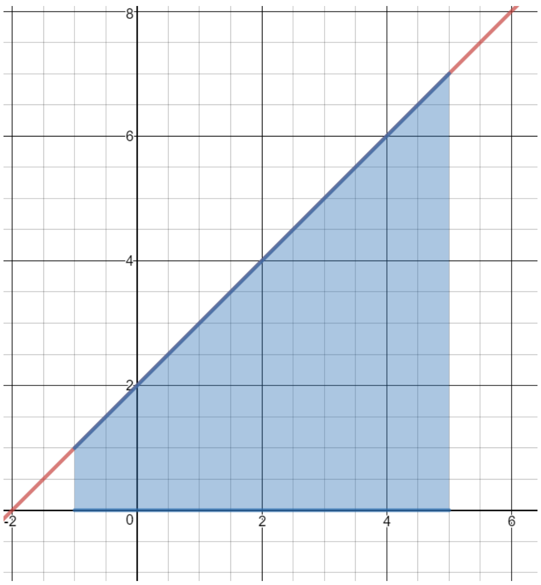


4.5A Area Between Curves

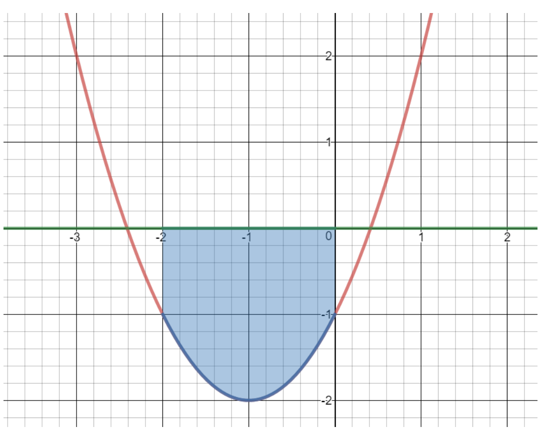
Homework

Find the area bounded by the graphs of the given equations over the given interval. Compute answers to 3 decimal places.

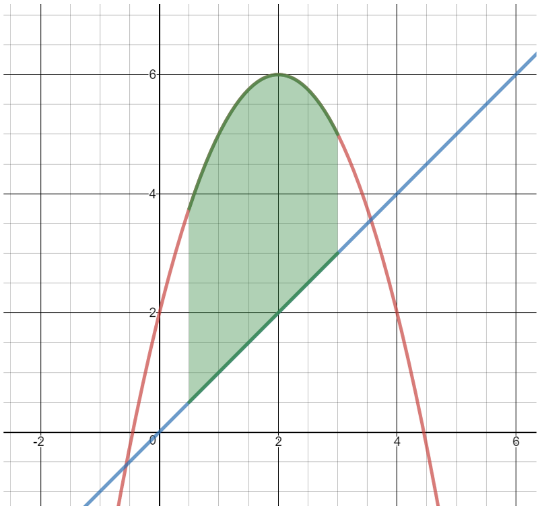
1. $y_1 = x + 2$ and $y_2 = 0$ on the interval $-1 \le x \le 5$



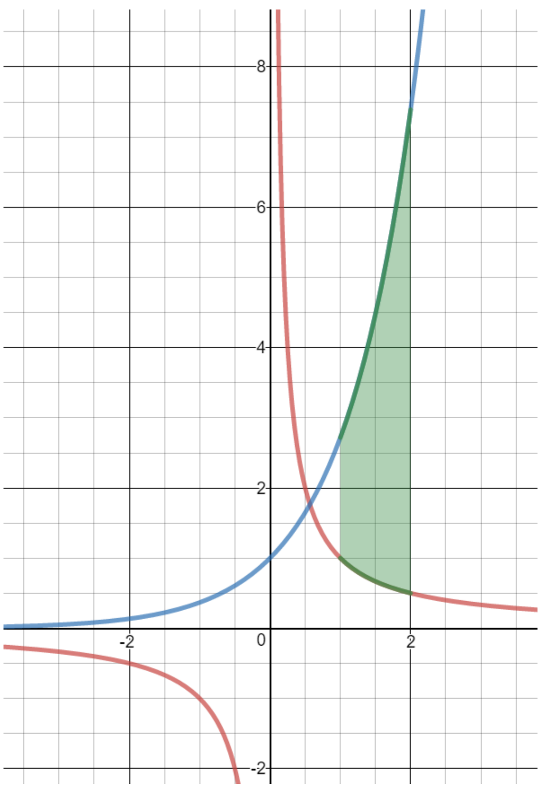
2. $y_1 = x^2 + 2x - 1$ and $y_2 = 0$ on the interval $-2 \le x \le 0$



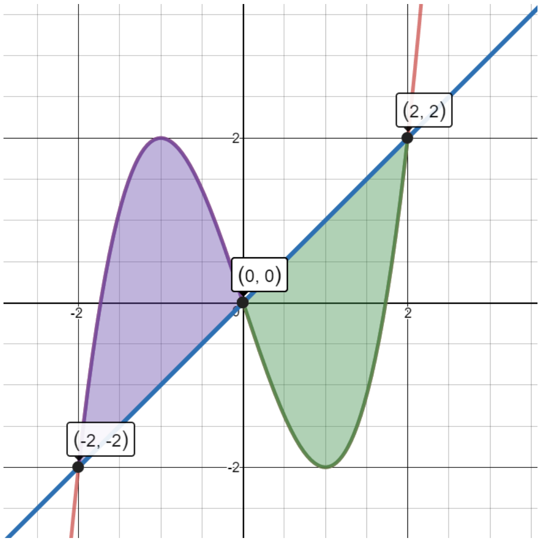
3. $y_1 = -x^2 + 4x + 2$ and $y_2 = x$ on the interval $\frac{1}{2} \leq x \leq 3$



4. $y_1 = \frac{1}{x}$ and $y_2 = e^x$ on the interval $1 \leq x \leq 2$

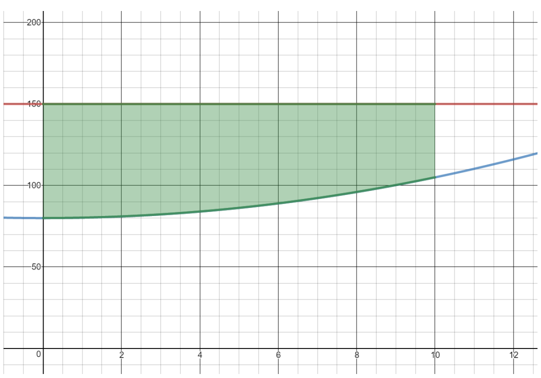


5. $y_1 = x^3 - 3x$ and $y_2 = x$



6. The revenue from a manufacturing process is projected to follow the model $R'(x) = 150$ million per year for 10 years. The cost is projected to follow the model $C'(x) = 80 + 0.25x^2$ millions of dollars per year over the same period of time, where x is time in years.

Approximate the profit over the 10-year period.

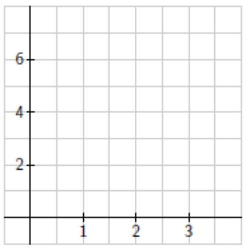


4.5B Area Between Curves

Homework

1. Consider the functions given by $f(x) = -x^2 + 2x + 4$ and $g(x) = 4 - x$.

   a. Use algebra to find the points where the graphs of f and g intersect.
   b. Sketch an accurate graph of f and g on the axes provided, labeling the curves by name and the intersection points with ordered pairs.
   c. What is the exact area between f and g between their intersection points?



2. Find the area of the region bounded by $y_1 = \sqrt{x}$ and $y_2 = \frac{1}{4}x$.

3. Find the area of the region bounded by $y_1 = 12 - 2x^2$ and $y_2 = x^2 - 8$.

4. Find the area of the region bounded by the curves $y_1 = x^3 - x$ and $y_2 = x^2$.

5. The rate of change of the value of goods exported from the United States between 2000 and 2014 can be modeled as

$$E'(t) = -2688.96x^2 + 43524x - 35103.48$$

billion dollars per year t years after the end of 2000.The rate of change of the value of goods imported into the United States between 2000 and 2014 can be modeled as

$$I'(t) = 4485.04x + 144,264.16$$

billion dollars per year t years after the end of 2000.

Source: U.S. Census Bureau, Economic Indicator Division

Find the difference in the accumulated value of imports and exports from 2000-2014.