MATH 1830 NOTES

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UNIT 3 APPLICATIONS OF DERIVATIVES

3.1 Analyzing and Interpreting Graphs

Introduction

Discuss this graph with your group. Be prepared to share your observations with the class.

Notes

Curve Sketching

Read It: DERIVATIVES AND THE SHAPE OF THE GRAPH

ANALYZE $F(X)$

1. Identify the Domain of $f(x)$: Commonly, $f(x)$is undefined for at any x value where:
    a. The denominator equals zero
    b. There is an even root of a negative number
    c. There is a logarithm of a negative number or log of zero
2. Identify x-intercepts and y-intercept of $f(x)$
    a. x-intercepts: Set $y = 0$ and solve for x
    b. y-intercepts: Set $x = 0$ and solve for y
3. Identify Vertical Asymptotes and Holes of $f(x)$
    a. Vertical asymptote at a when $\lim_{x \to a} f(x) = \frac{n}{0}$ if $n \neq 0$
    b. Hole at a when $\lim_{x \to a} f(x) = \frac{0}{0}$ and then, after factoring and reducing, $\lim_{x \to a} f(x) = \frac{n}{0}$ if $n \neq 0$
4. Identify Horizontal Asymptote of $f(x)$: calculate $\lim_{x \to \infty} f(x)$

ANALYZE $F'(X)$

1. Find $f'(x) = 0$
2. Find the critical numbers and partitions for the function.
    a. Values of x where $f'(x) = 0$ are critical numbers.
    b. Values of x where $f'(x)$ is undefined are partitions.
    c. Values of x where $f(x)$ is undefined are partitions.
3. Graph the critical numbers and partitions on a number line, separating the number line into intervals.
4. Determine the intervals on which $f(x)$ is increasing /decreasing
    a. Test one point contained in the interval (do not use the end points of the interval).
    b. $f'(x) < 0$ then the function $f(x)$ is DECREASING on the interval
    c. $f'(x) > 0$ then the function $f(x)$ is INCREASING on the interval
5. Identify local maxima and minima of $f(x)$ using the First Derivative Test.
    a. On the interval (a, c), a local maximum occurs at $f(b)$ when $f(x)$ is increasing for all x in the interval $(a, b]$ and $f(x)$ is decreasing for all x in the interval $[b, c)$.
    b. On the interval (a, c), a local minimum occurs at $f(b)$ when $f(x)$ is decreasing for all x in the interval $(a, b]$ and $f(x)$ is increasing for all x in the interval $[b, c)$.

ANALYZE $F''(X) = 0$

1. Find $f''(x)$
2. Find the critical numbers for the function.
   a. Values of x where $f''(x) = 0$ are critical numbers.
   b. Values of x where $f''(x)$ is undefined are partitions.
   c. Values of x where $f(x)$ is undefined are partitions.
3. Graph the critical numbers and partitions on a number line, separating the number line into intervals.
4. Determine the intervals on which $f(x)$ is concave up or concave down.
   a. Test one point contained in the interval (do not use the end points of the interval).
   b. $f(x)$ is concave down on the interval if $f''(x) < 0$.
   c. $f(x)$ is concave up on the interval if $f''(x) > 0$.
5. Identify inflection points of $f(x)$. A point of inflection occurs at $x = a$ when $f''(a) = 0$ and $f''(x)$ changes concavity across a.

GRAPH THE FUNCTION $F(X)$

1. Determine the interval and scale for the x- and y-axes.
2. Graph the asymptotes and holes.
3. Graph the x- and y-intercepts.
4. Graph the maxima, minima, and point(s) of inflection.
5. Sketch the graph using the intervals where the function is increasing, decreasing, concave up and concave down.

The annual first quarter change in revenue for Apple, Inc. is given in the table below.

Year	% Revenue Growth
1998	-12.2
2000	27.1
2002	4.5
2004	29.4
2006	34.4
2008	42.7
2010	65.4
2012	58.9
2014	4.7
2016	-12.8

The regression model for this data is:

$$f(x) = -0.005x^4 + 0.113x^3 - 0.889x^2 + 7.946x - 5.346$$

where x is Years Since 1998.

Analyze and interpret the characteristics of the function identified below. Graph the function clearly marking each characteristic.

(Use graphing paper and colored pencils. Each graph should cover an entire piece of graph paper.)

1. Analyze $f(x)$
    a. Domain

    b. x-intercepts:

    y-intercept:

    c. Asymptotes:

2. Analyze $f'(x)$

Increasing and Decreasing

a. Values of x where $f'(x) = 0$

Values of x where $f'(x)$ is undefined

Values of x where $f(x)$ is undefined

Increasing:

Decreasing:

b. Local maximum

Local minimum

3. Analyze $f''(x)$

Concave Up and Concave Down

a. Values of x where $f''(x) = 0$

Values of x where $f''(x)$ is undefined

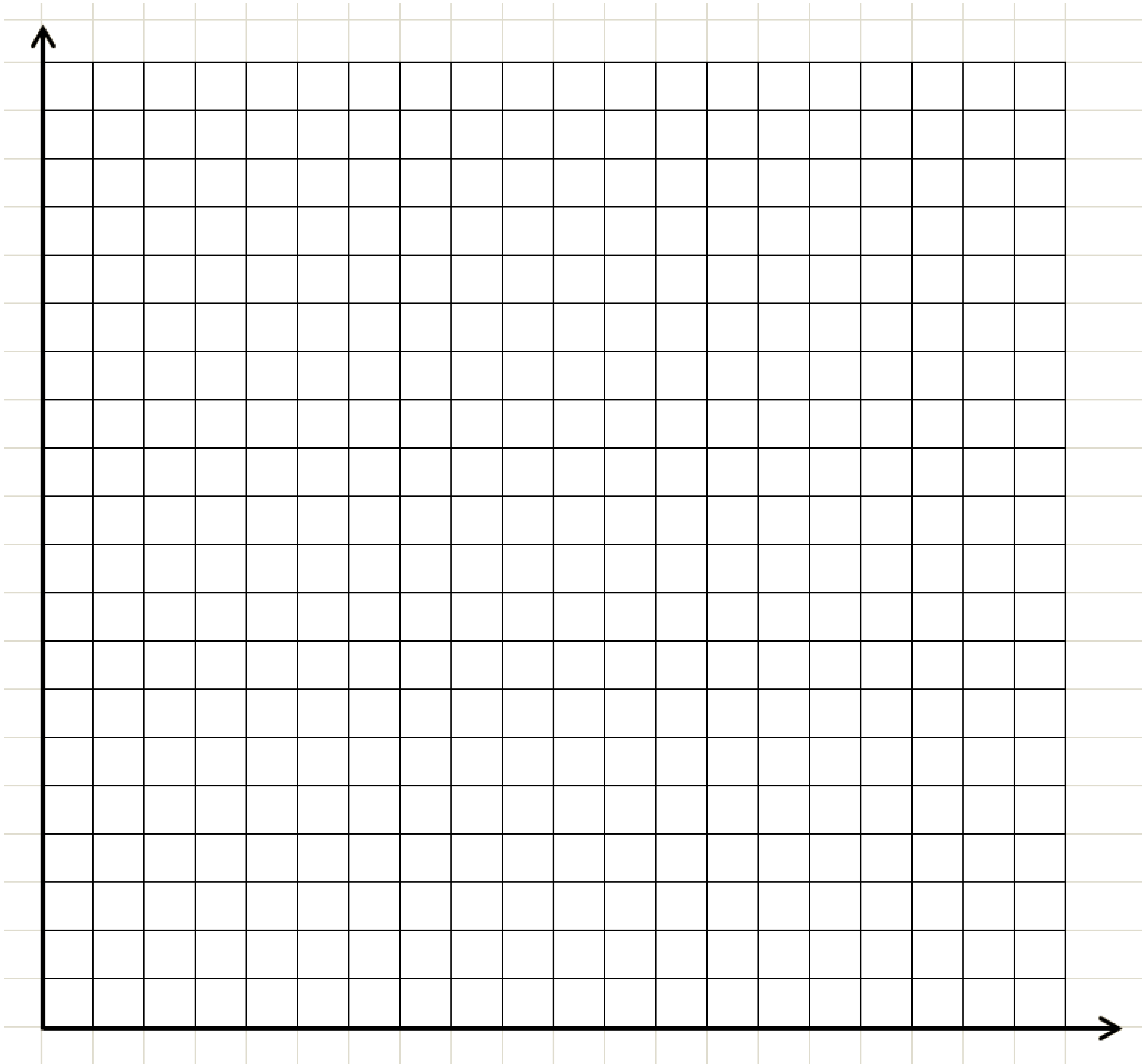
Values of x where $f(x)$ is undefined

Concave Up:

Concave down:

b. Inflection points:

4. Graph $f(x)$



3.2 Analyzing and Interpreting Graphs Part 2

See Homework Assignment for 3.2

Read It: DERIVATIVES AND THE SHAPE OF THE GRAPH

3.3 Curve Sketching

Read It: DERIVATIVES AND THE SHAPE OF THE GRAPH

Analyze and interpret the characteristics of the function identified below. Graph the function clearly marking each characteristic. (Use graphing paper and colored pencils. Each graph should cover an entire piece of graph paper.)

1. $f(x) = \frac{3x+4}{2x-5}$

   Domain:

   x int(s):

   y int:

Asymptotes:

Vertical:

Horizontal:

Increasing and Decreasing

Increasing:

Decreasing:

Local Maxima:

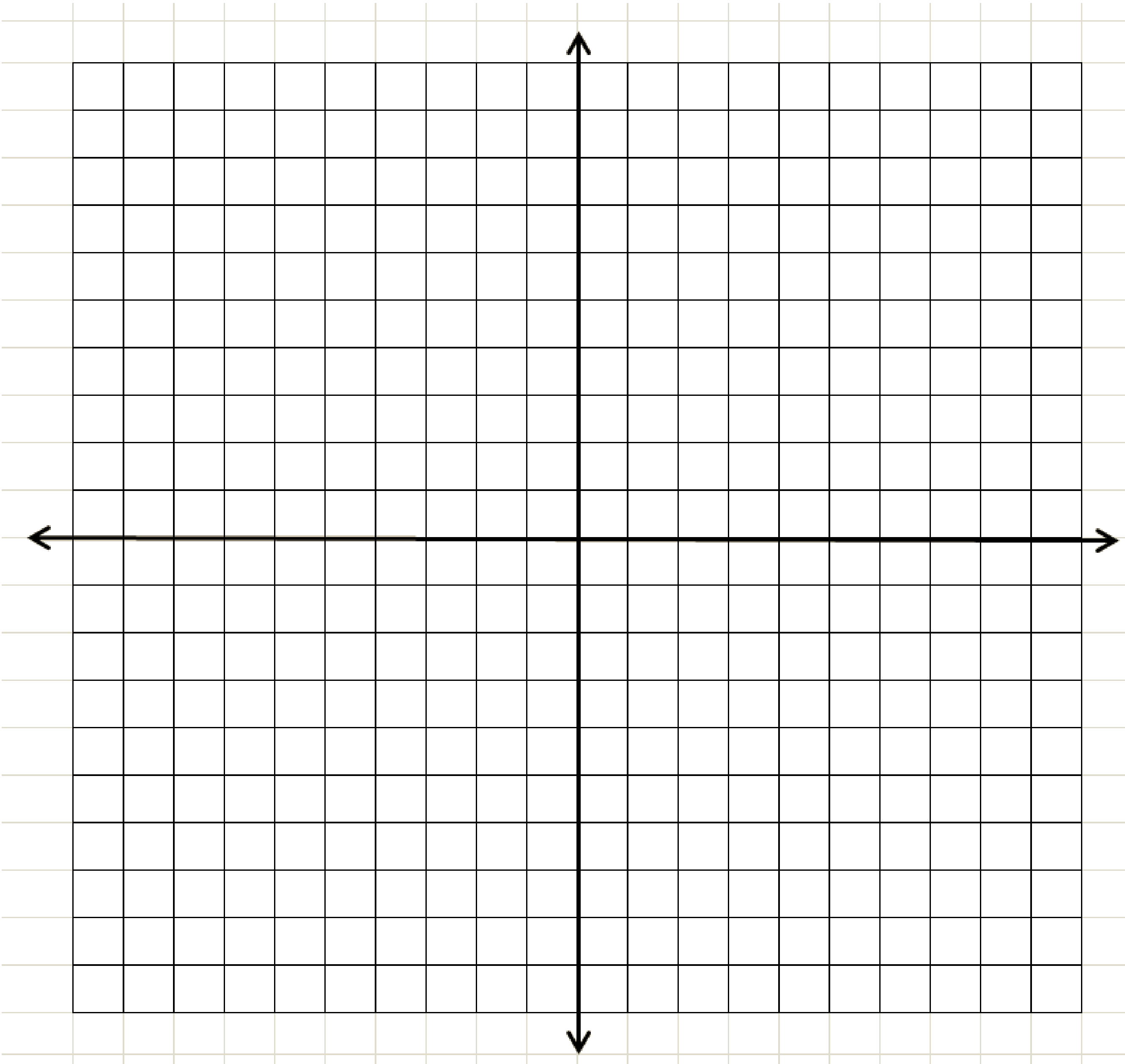
Local Minima:

Concave Up and Concave Down

Concave up:

Concave down:

Inflection Points:



2. $f(x) = e^x(5x - 7)$

Domain:

x int(s):

y int:

Asymptotes:

Vertical:

Horizontal:

Increasing and Decreasing

Increasing:

Decreasing:

Local Maxima:

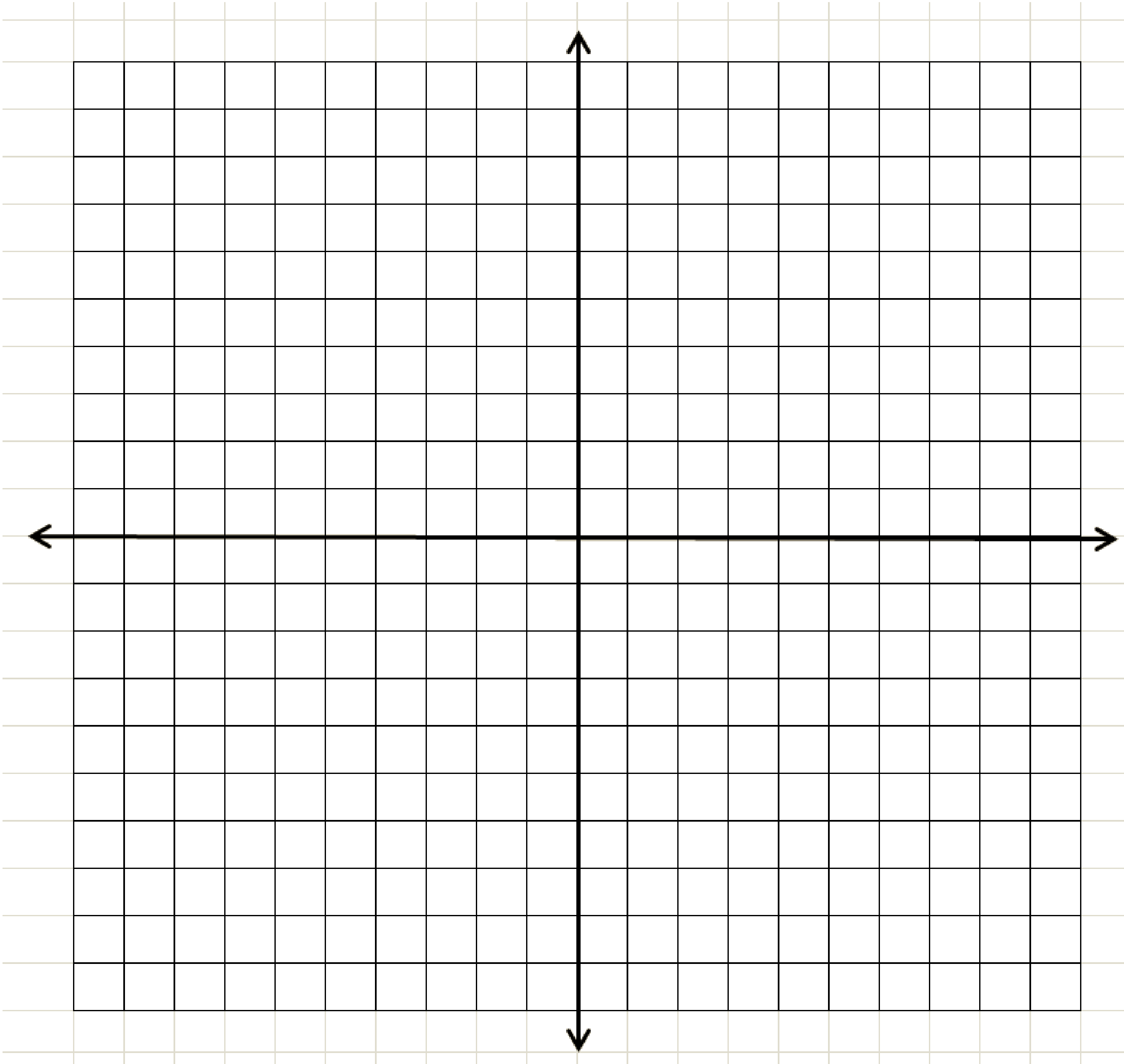
Local Minima:

Concave Up and Concave Down

Concave up:

Concave down:

Inflection Points:



3. $f(x) = 3x^2 + 5x - 2$

Domain:

x int(s):

y int:

Asymptotes:

Increasing and Decreasing

Increasing:

Decreasing:

local max:

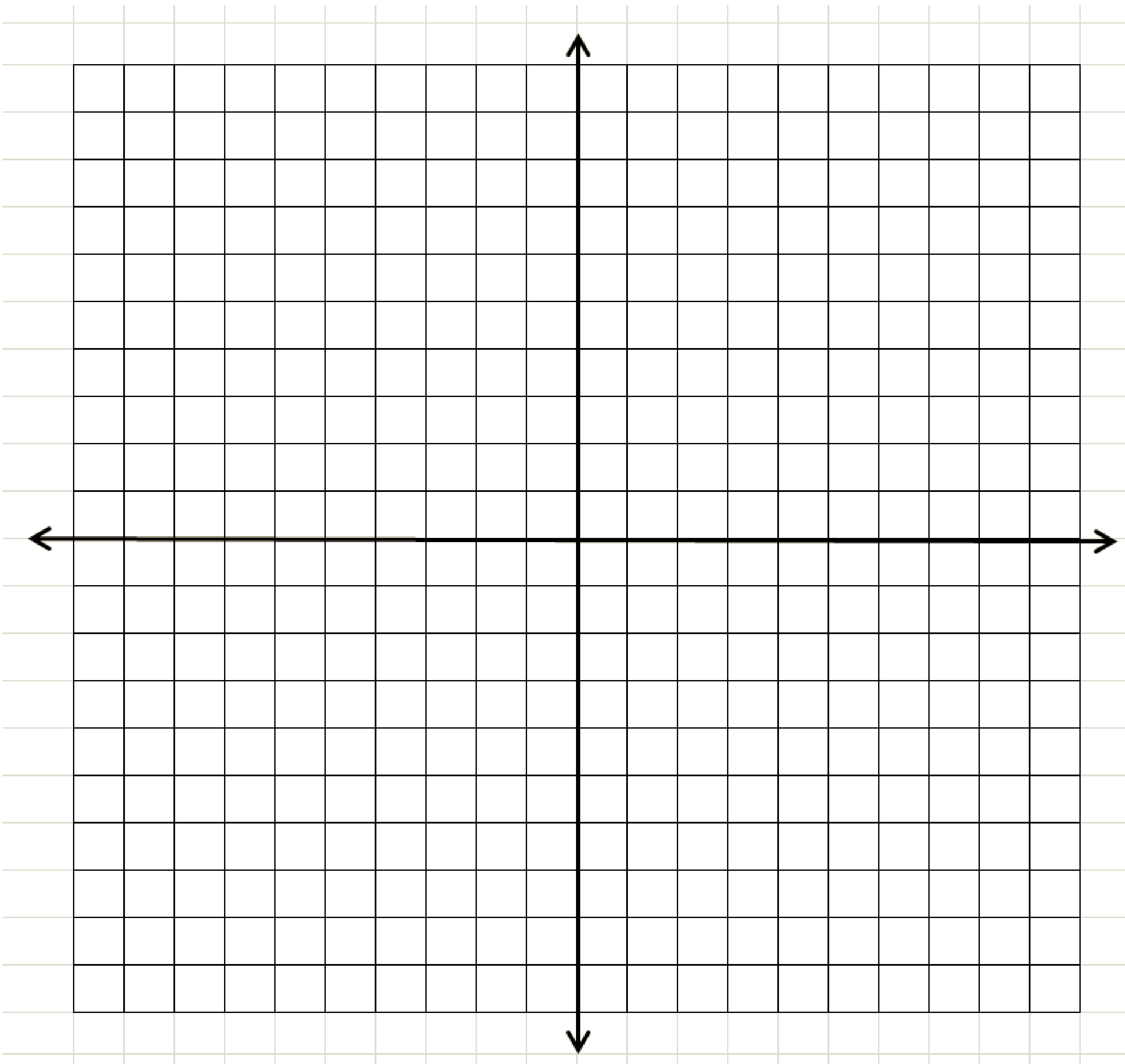
local min:

Concave Up and Concave Down

Concave up:

Concave down:

Inflection Points:



4. $f(x) = x^3 + 6x^2 + 9x$

Domain:

x int(s):

y int:

Asymptotes:

Increasing and Decreasing

Increasing:

Decreasing:

Local Maxima:

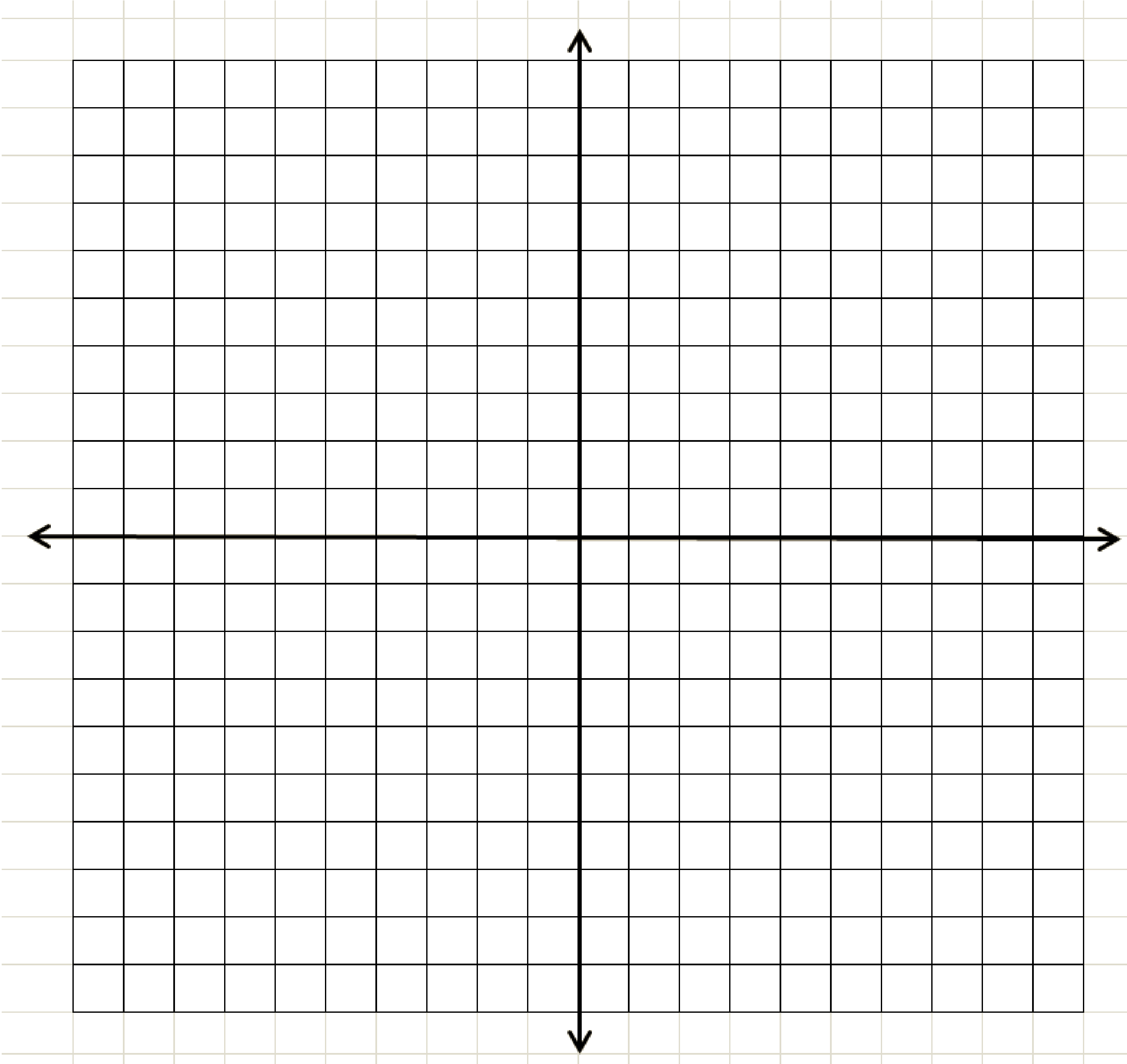
Local Minima:

Concave Up and Concave Down

Concave up:

Concave down:

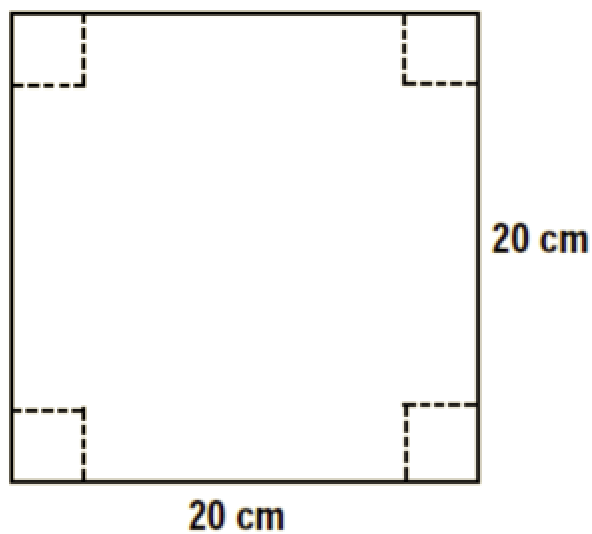
Inflection Points:



3.4 Introduction to Optimization

Introduction

1. Construct a 20 cm by 20 cm square on the white piece of paper.
2. Draw four congruent squares in each corner of your original square (see diagram below), the size of the four squares you draw will be assigned for your group.
3. Using the scissors and tape, cut out your square and its corners to create an open-topped box.

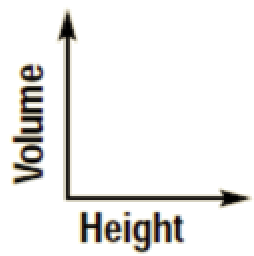


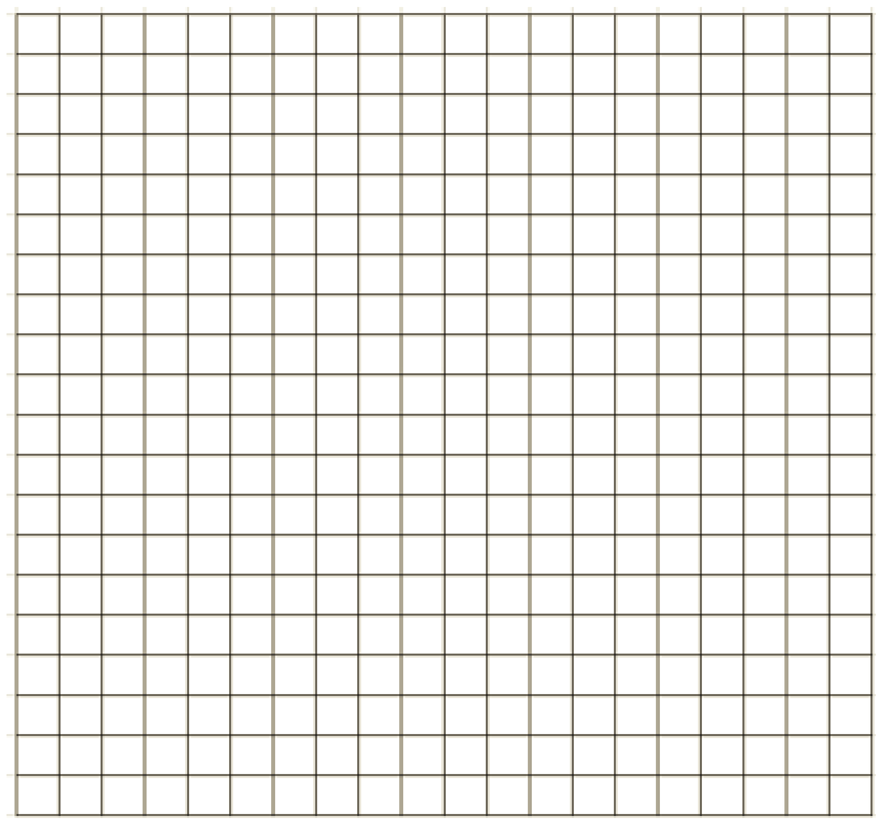
4. Complete the following questions:
    a. The width of our box is:
    b. The length of our box is:
    c. The height of our box is:
    d. Calculate the volume of your box.

    A summary of the data collected from the class is on the board. Copy this data into the chart below.

Height (cm)	**Volume (cubic cm)**
0	0
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	0

5. Using the graph paper, construct a graph of height vs volume by plotting the above ordered pairs. Join the points with a smooth curve. Answer the following questions based on your graph.





6. What is the maximum volume? (According to your graph.)
7. What size of square cut out of the corner would result in the maximum volume?
8. What type of function models your graph?
9. Could we write a mathematical function representing the graph?
10. How could our knowledge of derivatives be used to find the maximum volume?

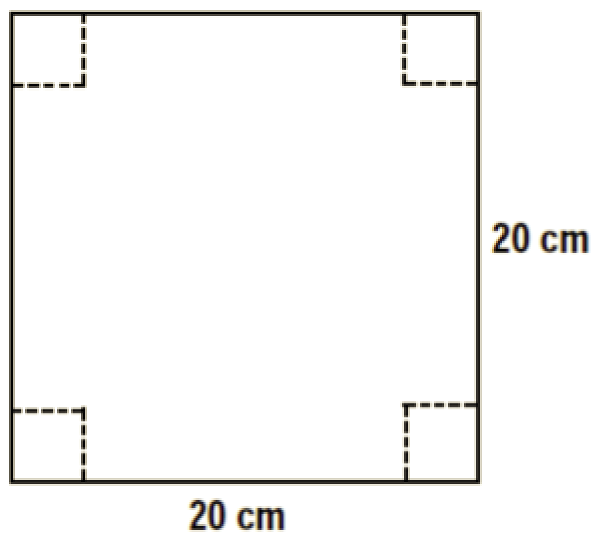
3.5 Modeling Optimization

Notes

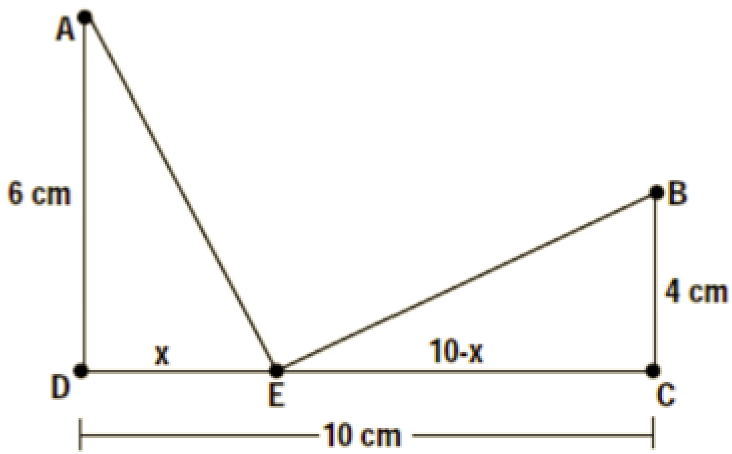
Write a mathematical function for each of the following.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. Find the formula for volume of an open-topped box created from a 20 cm square sheet of paper.

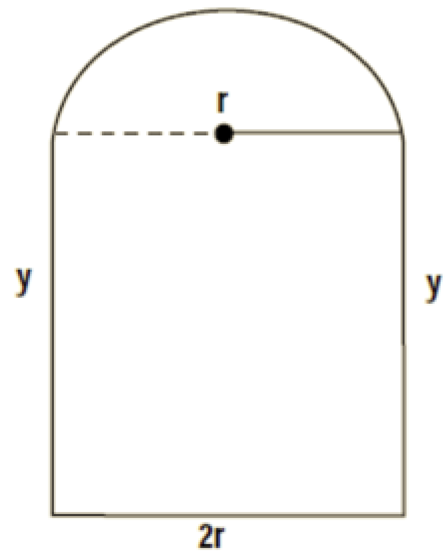


2. Determine the formula for the distance from A to B through E.



3. When a theater owner charges \$3 for admission, there is an average attendance of 100 people. For every \$0.10 increase in admission, there is a loss of 1 customer from the average number. Find the revenue formula.

4. A Norman window is a rectangle with a semi-circle on top. If the perimeter is 24 feet, express the area as a function of the radius (r).



Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?

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5. A lifeguard has 200 m of rope and some buoys with which she intends to enclose a rectangular area at a lake for swimming. The beach will form one side of the rectangle, with the rope forming the other three sides. Find the formula for the area of the enclosure.

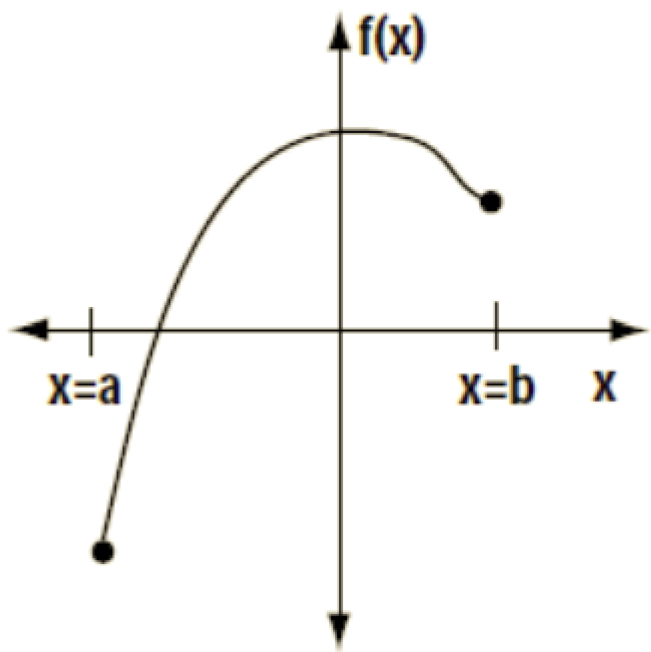
3.6 Absolute Maximum and Minimum

Introduction

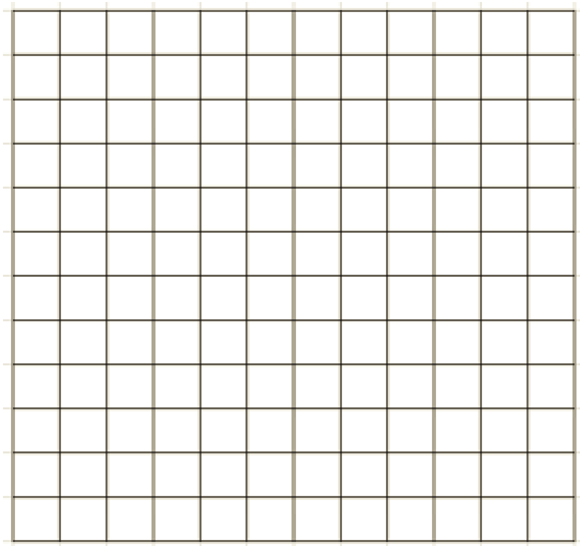
On a sheet of paper, each member of the group is to draw 3 functions f(x) over an interval a < x < b.

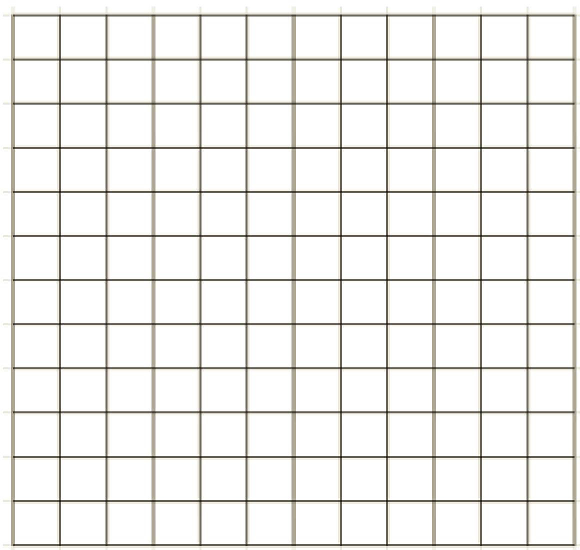
Be creative! Try to draw as many different possibilities as you can.

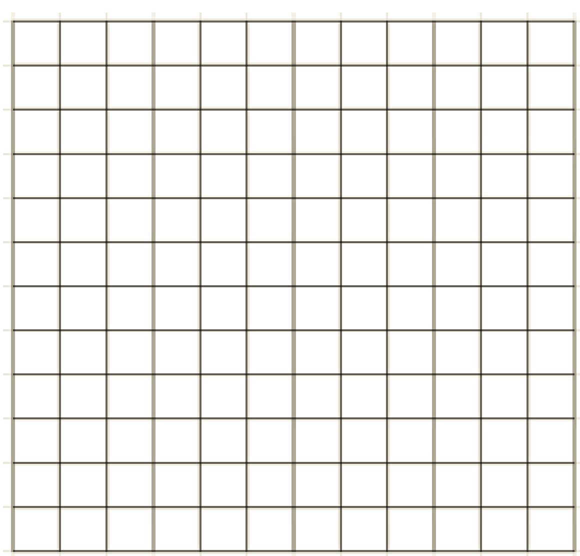
I have drawn one for you as an example.



Label each of your graphs (ex: graph #1, graph #2, etc)







For each of your graphs, answer the following questions:

1. Where does f (x) have its maximum value? That is, where on your graph does y have the largest value?

2. Where does f (x) have its minimum value?

   Clearly indicate the answers to these two questions on each graph.

3. Based on your answers above, can your group arrive at a conclusion?

   a. Could you make a general statement about how to determine the absolute maximum or minimum values of a function over a given interval?
   b. Can you think of any exceptions?
   c. How can our knowledge of derivatives assist us?
   d. Summarize your responses on this sheet. Be prepared to share your results to the rest of the class.

Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?

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Notes

ALGORITHM FOR DETERMINING EXTREME VALUES

Suppose that $f(x)$ is a continuous function over a closed interval [a, b].

To find the absolute maximum and minimum values of the function $f(x)$ on [a, b]:

1. Find $f'(x)$
2. Determine the critical points of $f'(x)$ in [a, b]
   (that is, find all x values for which $f'(x) = 0$, $f'(x)$ does not exist, or $f(x)$ does not exist).
3. List the critical points of $f(x)$ and the endpoints of the interval [a & b]
4. Find the function values of critical points and the endpoints
5. The largest of these values is the absolute maximum of on the interval [a, b].
6. The smallest of these is the absolute minimum of on the interval [a, b].

Find the absolute minimum and maximum values of the function, if they exist, over the given interval.

1. $f(x) = x^2 - 6x - 3 \qquad [-1, 5]$

2. $f(x) = 2x^3 - 3x^2 - 36x + 62 \qquad [-3, 4]$

3. $f(x) = x + \frac{1}{x} \qquad [1, 20]$

4. $f(x) = \frac{x^2}{x^2+1} \qquad [-2, 2]$

5. $f(x) = \frac{x}{(x+9)^2} \qquad [-1, 8]$

6. $f(x) = -3 \qquad [-2, 2]$

7. An employee's monthly production M, in number of units produced, is found to be a function of the number of year of service, t. For a certain product, a productivity function is being given by: $M(t) = -2t^2 + 100t + 180, \quad 0 \leq t \leq 40$

   Find the maximum productivity and the year in which it is achieved.

8. A firm determines that its total profit in dollars from the production and sale of x thousand units of a product is given by:

$$P(x) = \frac{1500}{x^2 - 6x + 10} \qquad x \geq 0$$

Find the number of units x for which the total profit is a maximum.

Norris, Ken. (1999). Optimization Problems. Retrieved from https://www.stf.sk.ca/portal.jsp?

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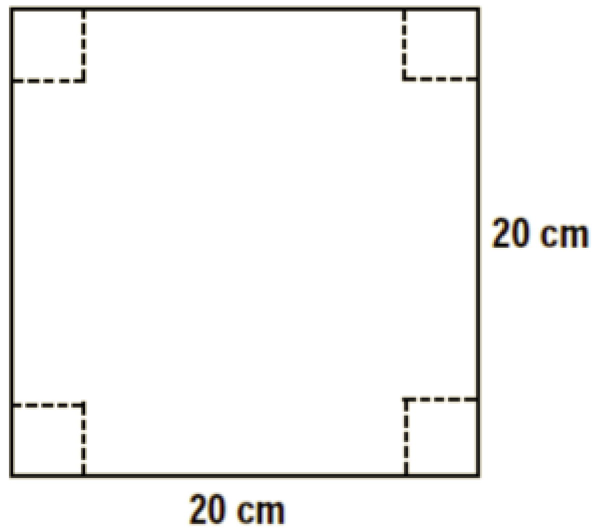
3.7 Applications of Optimization

Notes

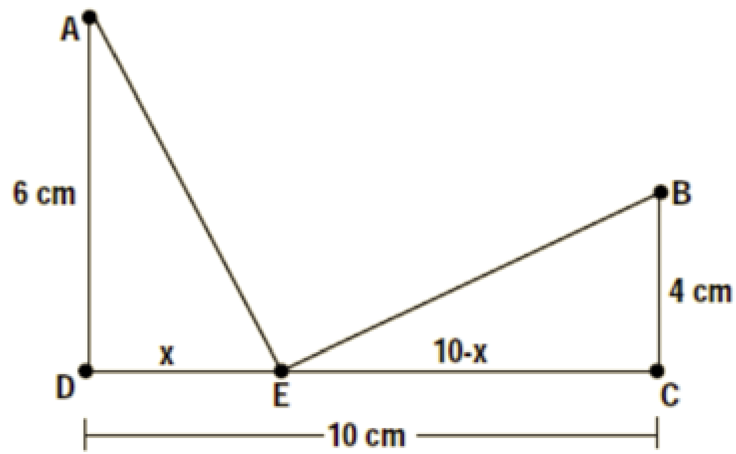
Write a mathematical function for each of the following and then find the requested information.

Be sure to include a properly labeled diagram (if applicable) and variable statements. State the restrictions on the independent variable.

1. Find the maximum volume of an open-topped box created from a 20 cm square sheet of paper.



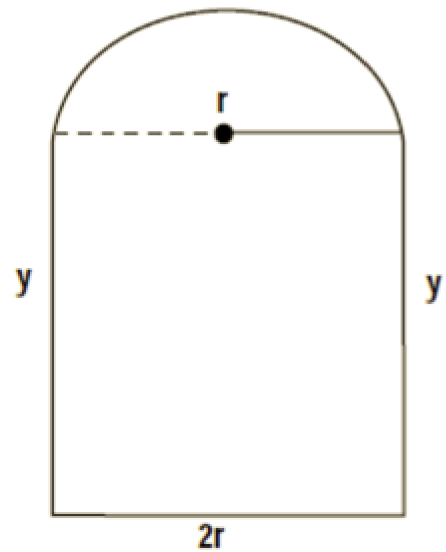
2. Determine the shortest possible distance from A to B through E.



x	f(x)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

3. When a theater owner charges \$3 for admission, there is an average attendance of 100 people. For every \$0.10 increase in admission, there is a loss of 1 customer from the average number. How much should the theater owner charge to maximize revenue and what is the maximum revenue?

4. A Norman window is a rectangle with a semi-circle on top. Find the maximum area if the perimeter is 24 feet, express the area as a function of the radius (r).



5. A lifeguard has 200 m of rope and some buoys with which she intends to enclose a rectangular area at a lake for swimming. The beach will form one side of the rectangle, with the rope forming the other three sides. Find the dimensions that will produce the maximum area.