PS5

Shihong Pan
https://github.com/PSH-hub24/phys-ga2000

October 9, 2024

1 Q1

Fig 1 plots the numerical and analytic result of the derivative of $f(x)$ using a central difference. Fig 2 plots the numerical and analytic result of the derivative of $f(x)$ using the jax package which does work as advertised.

2 Q2

(a) Fig 3 plots the curves of the integrand with $a = 2, 3, 4$.

(b) Take the first and second derivative of the integrand:

$$D_1 = \frac{d}{dx}(x^{a-1}e^{-x}) = (a-1)x^{a-2}e^{-x} - x^{a-1}e^{-x} \tag{1}$$

$$D_2 = \frac{d^2}{dx}(x^{a-1}e^{-x}) = (a-1)(a-2)x^{a-3}e^{-x} - (a-1)x^{a-2}e^{-x} - D_1 \tag{2}$$

It is clear that substituting $x = a-1$ gives $D_1 = 0$ and $D_2 < 0$, i.e, the maximum falls at $x = a-1$.

(c) If $x = c$ then $z = 1/2$. Since the peak is at $x = a-1$, pick $c = a-1$.

(d) The integrand becomes

$$e^{(a-1)\ln(x)}e^{-x} = e^{(a-1)\ln(x)-x} \tag{3}$$

The new expression is better because we are now only evaluating one exponential, and the value of $(a-1)\ln(x) - x$ suffers less from large x since we are subtracting two large terms at large x.

(e) Fig 4 prints the values of $\Gamma(a)$ for $a = 3/2, 3, 6, 10$, the answers for both part (e) and (f).

(f) See above.

3 Q3

(a) Fig 5 plots the signal data.

(b) Fig 6 plots the best third-order polynomial fit in time to the signal, using the SVD technique.

(c) Fig 7 plots the residuals of the data wrt the model. This is not a good explanation of the data, because the RMSE value I calculated is around 2.526 which is a bit far away from the given standard deviation 2.0. Please see my codes for all the RMSE calculations.

(d) Fig 8 plots the model and data with the order of polynomial equals to 30. The conditional number of the corresponding A is around 2.74e12 which I think is still a reasonable value (and it does produce a reasonable curve). I also plot the residuals in Fig 9, and the corresponding RMSE value is 2.006 which is pretty close to the standard deviation of the signal data.

(e) Fig 10 plots the Lomb-Scarge model and the signal data, and Fig 11 plots the residuals. The RMSE value is 2.014 which is also close to the standard deviation of the data. The model looks like a reasonbale fit since the typical periodicity of the data is clearly shown by the orange curve.

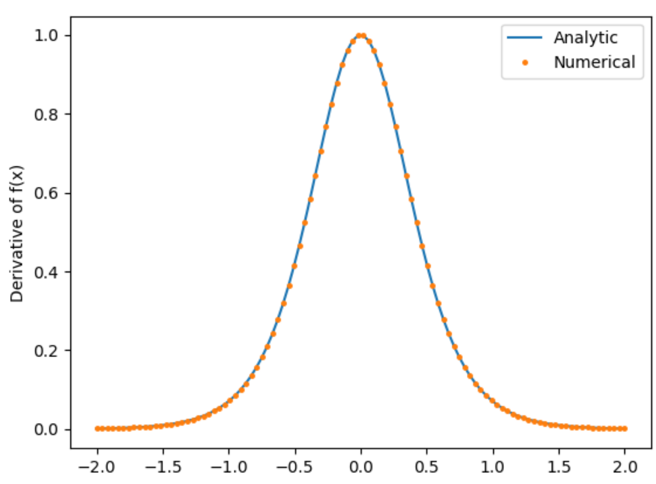

Figure 1: Q1: Derivative of f(x) using central difference.

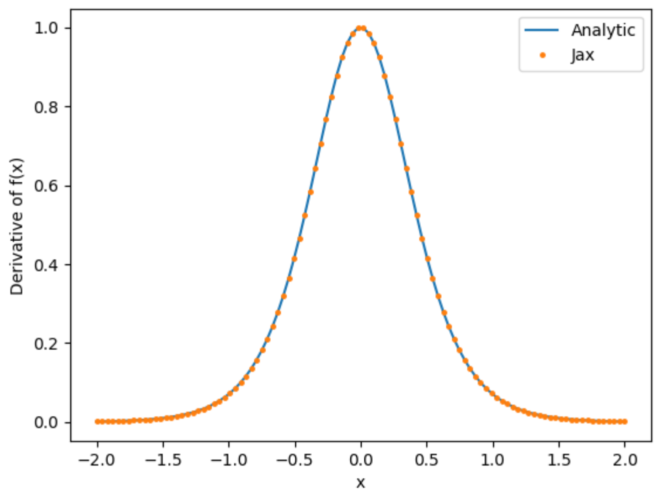

Figure 2: Q1: Derivative of f(x) using Jax.

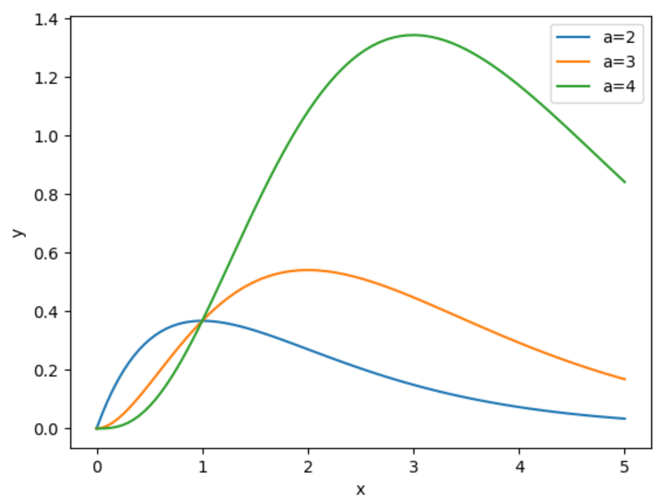

Figure 3: Q2(a): Three separate curves for $a = 2, 3, 4$.

```
The value for part e is 0.886226961308722.
gamma(3) = 2.0000000000000018, 2! = 2.
gamma(6) = 120.00000000000009, 5! = 120.
gamma(10) = 362880.00000000023, 9! = 362880.
[Finished in 2.0s]
```

Figure 4: Q2(e),(f): The values of $\Gamma(a)$ for $a = 3/2, 3, 6, 10$.

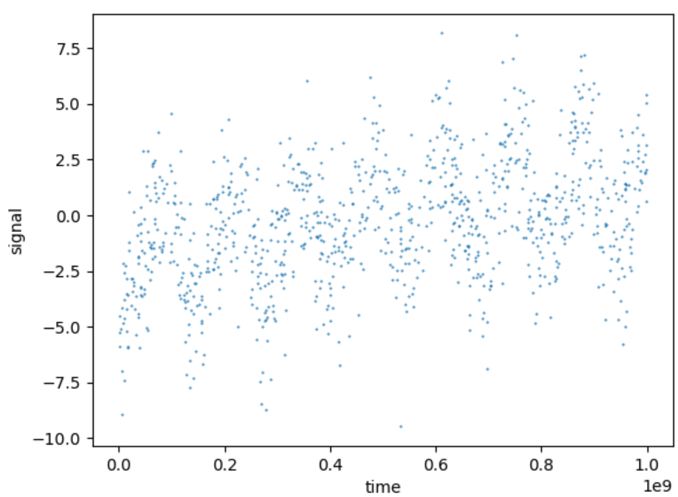

Figure 5: Q3(a): The scatter plot of raw signal data.

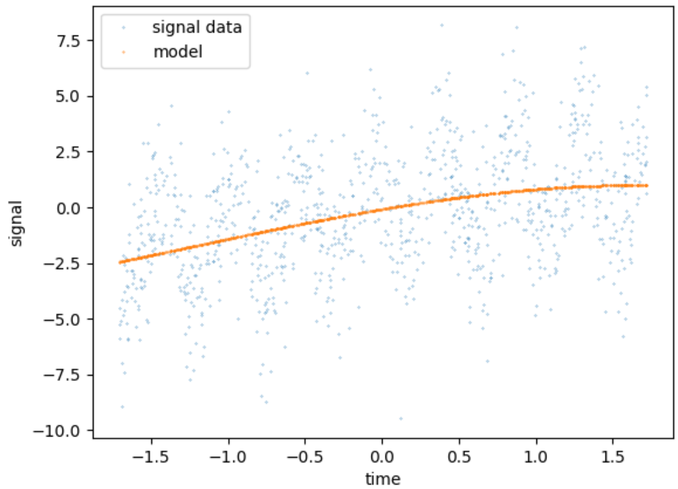

Figure 6: Q3(b): The best third-order polynomial fit in time to the signal, using the SVD technique.

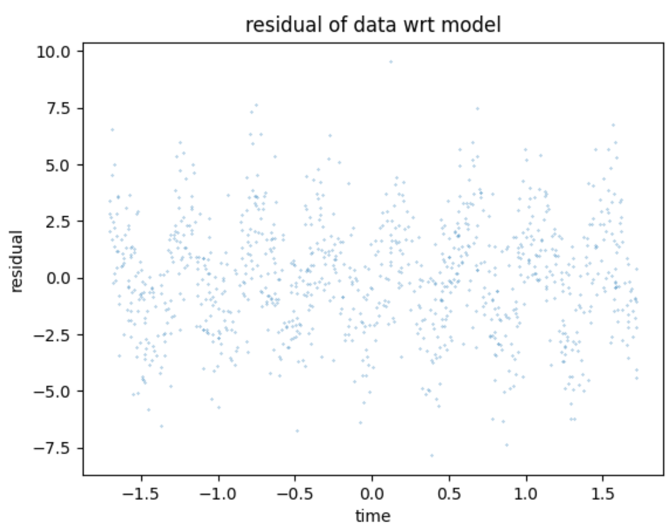

Figure 7: Q3(c): The residuals of the data wrt the model in part (b).

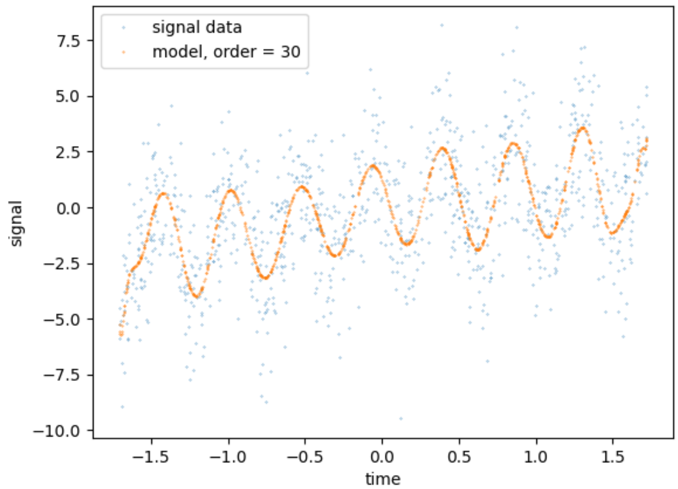

Figure 8: Q3(d): The signal data and the model with the order of polynomial = 30.

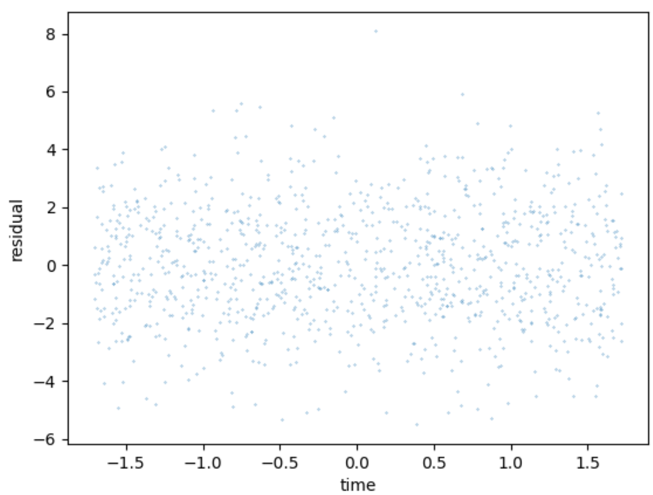

Figure 9: Q3(d): The residuals of the data wrt the model with the order of polynomial = 30.

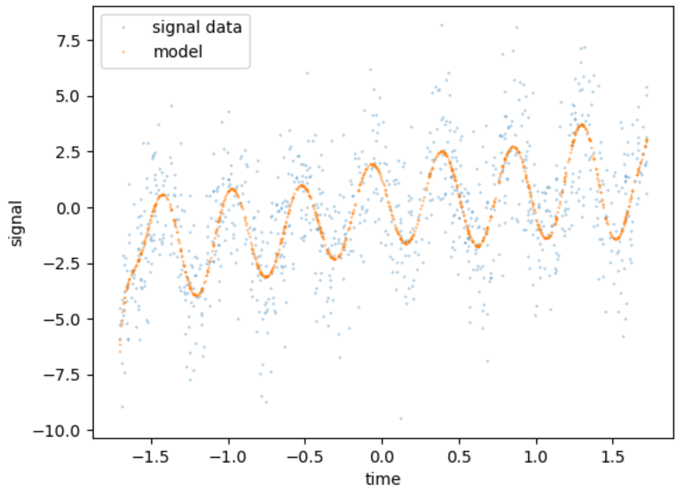

Figure 10: Q3(e): Use the Lomb-Scargle model to fit the data.

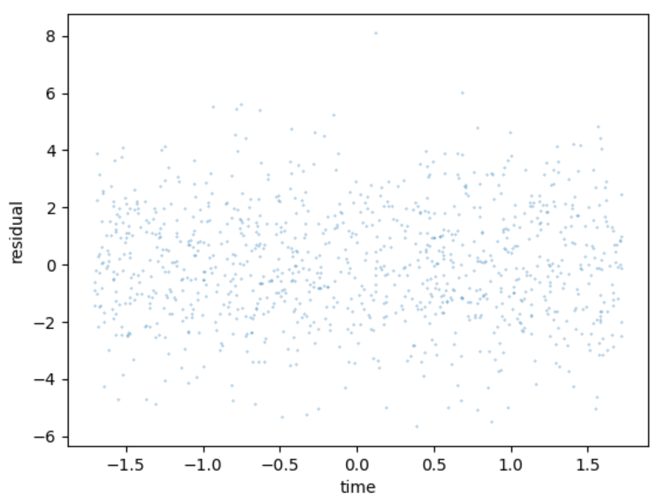

Figure 11: Q3(e): The residuals of the data wrt the Lomb-Scargle model.