PS3

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https://github.com/PSH-hub24/phys-ga2000

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1 Q1

Fig 1 shows the computation time scaled with N, using different methods. It is obvious that the scaling of the explicit function matches that of the N^3 curve.

2 Q2

Fig 2 shows the simulation results.

3 Q3

Fig 3 shows the simulation results.

4 Q4

The expectation and variance of the given exponential distribution are

$$\mu_x = 1, \sigma_x^2 = 1 \tag{1}$$

Then those of y are

$$\begin{aligned} \mu_y &= \frac{1}{N}\sum_{i=1}^{N}\mu_x = \mu_x = 1 \\ \sigma_y^2 &= \frac{1}{N^2}\sum_{i=1}^{N}\sigma_x^2 = \frac{1}{N} \end{aligned} \tag{2}$$

Fig 4 shows the change in the distribution of y as N increases. Fig 5 plots the mean, variance, skewness, and kurtosis of the distribution of y as function of N. Fig 6 shows an estimation of N where the skewness and kurtosis have reached about 1% of their value for $N = 1$.

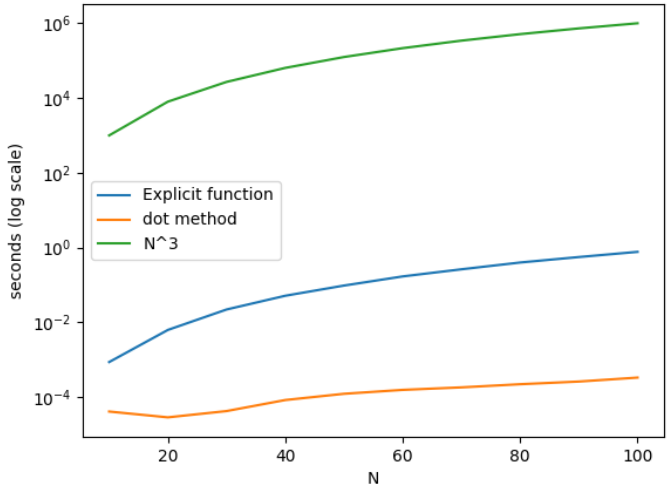

Figure 1: The computation time (in log scale) of computing $N \times N$ matrix multiplication, using an user-defined explicit function and the dot() method, compared to the N^3 scaling.

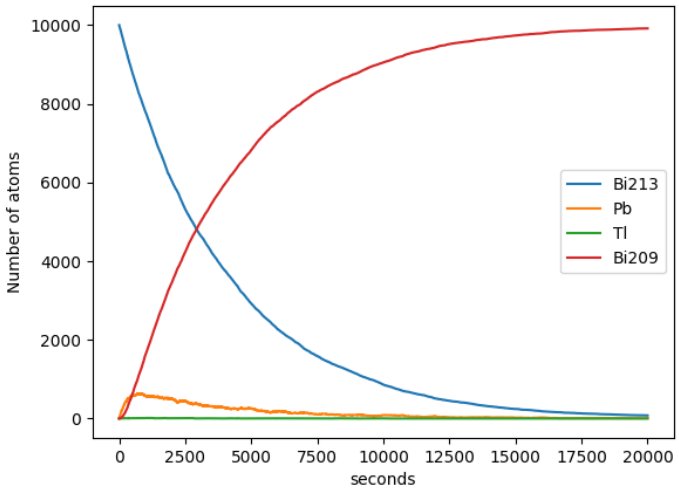

Figure 2: Decay simulation. The figure plots the number of Bi213, Pb209, Tl209, and Bi209 atoms over 20000 seconds.

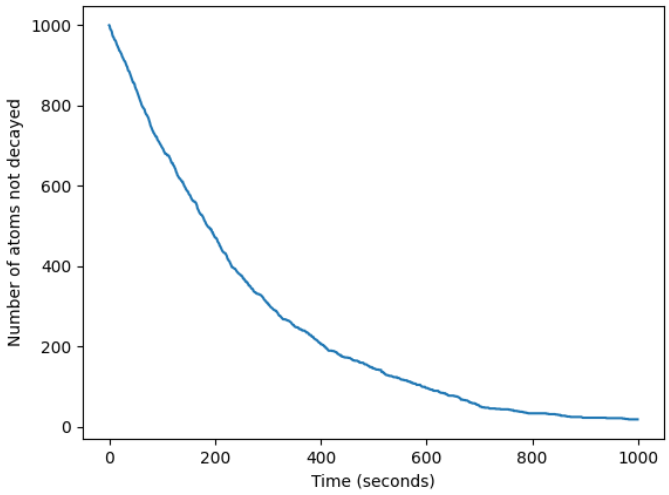

Figure 3: Decay simulation. The figure plots the number of Tl208 atoms over 1000 seconds, using the transformation method to speed up the computation time.

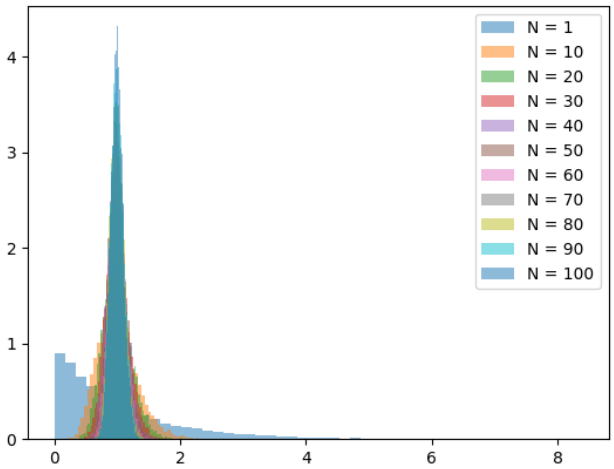

Figure 4: The distribution of y for large N.

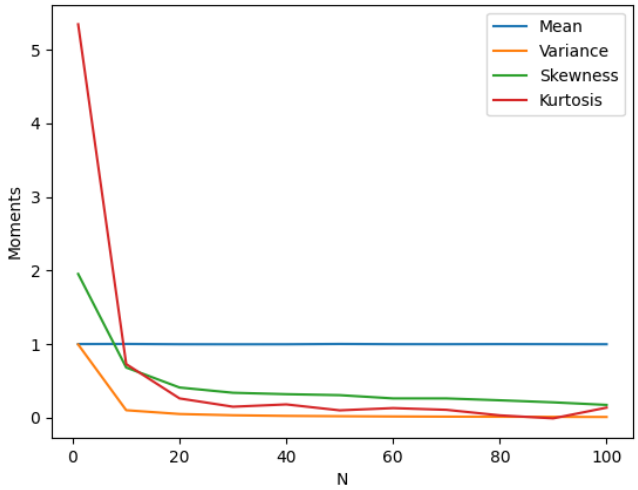

Figure 5: The mean, variance, skewness, and kurtosis of the distribution of y as function of N.

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The skewness has reached about 1% of its value for N = 1 at N = 10
The kurtosis has reached about 1% of its value for N = 1 at N = 10
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Figure 6: Estimation of the value of N where the skewness and kurtosis have reached about 1% of their value for $N = 1$.