PS9

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https://github.com/PSH-hub24/phys-ga2000

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Q1

a) We have

$$\frac{dx}{dt} = v, \quad \frac{dv}{dt} = -\omega^2 x \tag{1}$$

Fig 1 plots the value of x as a function of time, using the given parameters and initial conditions.

b) Fig 2 plots the oscillations for the initial condition $x = 2$.

c) Fig 3 plots the oscillations for the anharmonic oscillator with $x = 1$ and $x = 2$. Indeed, the frequency of oscillation for $x = 2$ is twice of that for $x = 1$.

d) I modified the codes and used Fig 4 as an example of the output.

e) Fig 5 plots the phase space plot of the van der Pol oscillator with $\mu = 1, 2, 4$.

Q2

a) Let θ be the angle between the direction of movement and the level ground. The net forces in x and y directions are

$$F_x = m\frac{d^2x}{dt^2} = -\frac{1}{2}\pi R^2 \rho C v^2 \cos\theta \tag{2}$$

$$F_x = m\frac{d^2y}{dt^2} = -mg - \frac{1}{2}\pi R^2 \rho C v^2 \sin\theta. \tag{3}$$

Note that

$$v = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}, \quad \cos\theta = \frac{dx}{dt}\frac{1}{v}, \quad \sin\theta = \frac{dy}{dt}\frac{1}{v}. \tag{4}$$

Substituting eq. 4 into eq. 2 and 3 gives

$$\frac{d^2x}{dt^2} = -\frac{\pi R^2 \rho C}{2m} \frac{dx}{dt} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} \tag{5}$$

$$\frac{d^2y}{dt^2} = -g - \frac{\pi R^2 \rho C}{2m} \frac{dy}{dt} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}. \tag{6}$$

To obtain a unitless set of equations, define $T^2 \coloneqq R/g$ and consider the following parameters:

$$t' \coloneqq \frac{t}{T}, \quad x' \coloneqq \frac{x}{R}, \quad y' \coloneqq \frac{y}{R}. \tag{7}$$

Substituting eq. 7 into eq. 5 and 6 gives

$$\frac{d^2x'}{dt'^2} = -\frac{\pi}{2} A \frac{dx'}{dt'} \sqrt{(\frac{dx'}{dt'})^2 + (\frac{dy'}{dt'})^2} \tag{8}$$

$$\frac{d^2y'}{dt'^2} = -1 - \frac{\pi}{2} A \frac{dy'}{dt'} \sqrt{(\frac{dx'}{dt'})^2 + (\frac{dy'}{dt'})^2} \tag{9}$$

where $A \coloneqq R^2 \rho C g T^2 / m$. Eq. 8 and 9 are used in the following parts.

b) To obtain four first-order equations, the same trick as eq. 1 is applied, except now there are two directions $dx/dt = v_x$ and $dy/dt = v_y$. Fig 6 plots the trajectory of cannonball in the original coordinates x, y (the program gives unitless solutions, and I multiply them by R to get the x, y solutions). The plot assumes that the cannonball hits the level ground at the end.

c) Fig 7 plots the trajectories of the cannonball with $m = 1, 5, 10, 15$kg. It shows that as mass increases, the cannonball goes higher in y and travels further in x, which makes sense since the initial velocity is fixed and the de-accelerations in both directions are weaker as m increases. But note that the differences between the trajectories become smaller as m gets larger.

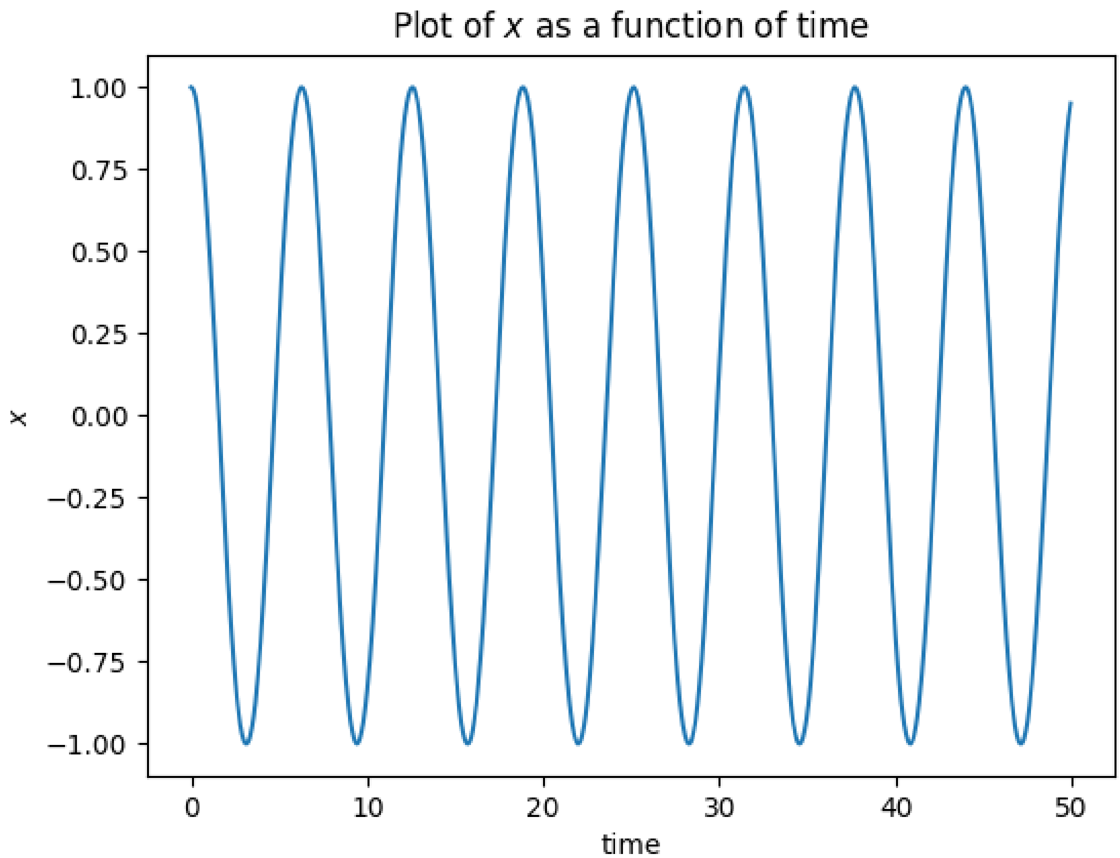

Figure 1: The value of x of SHO as a function of time.

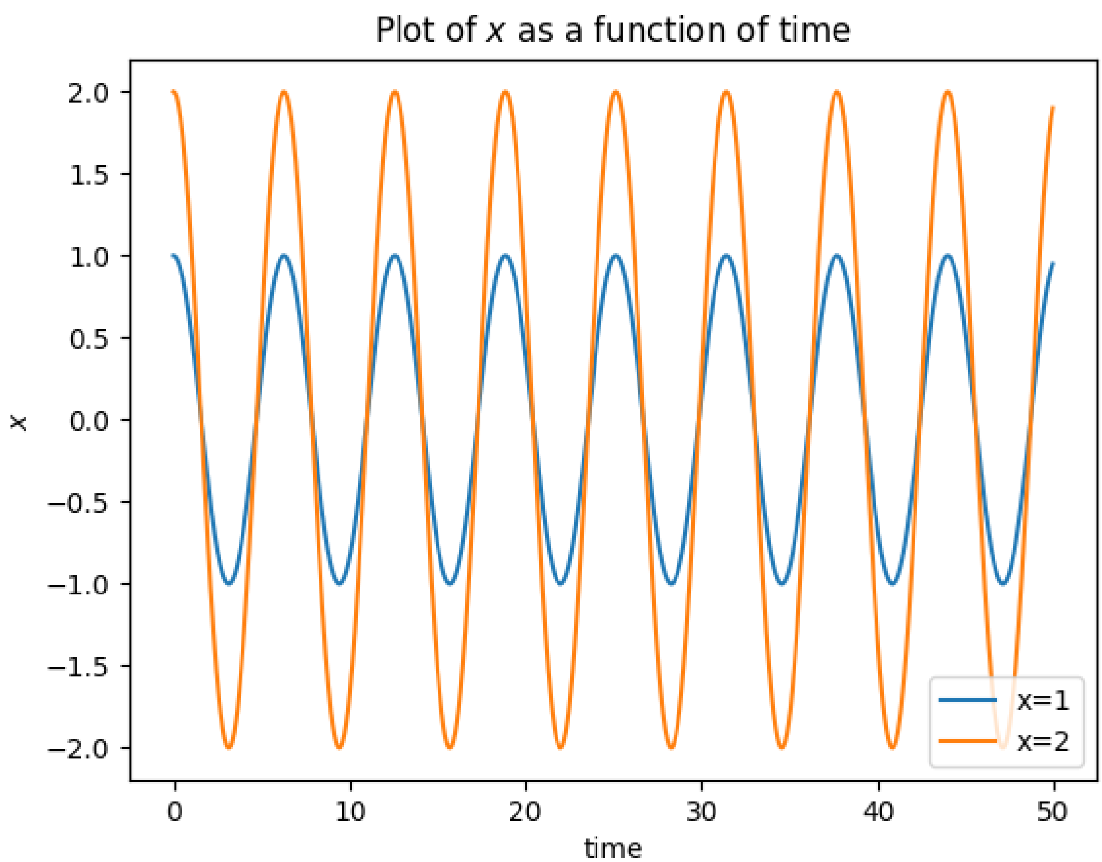

Figure 2: The amplitude is doubled ($x = 2$).

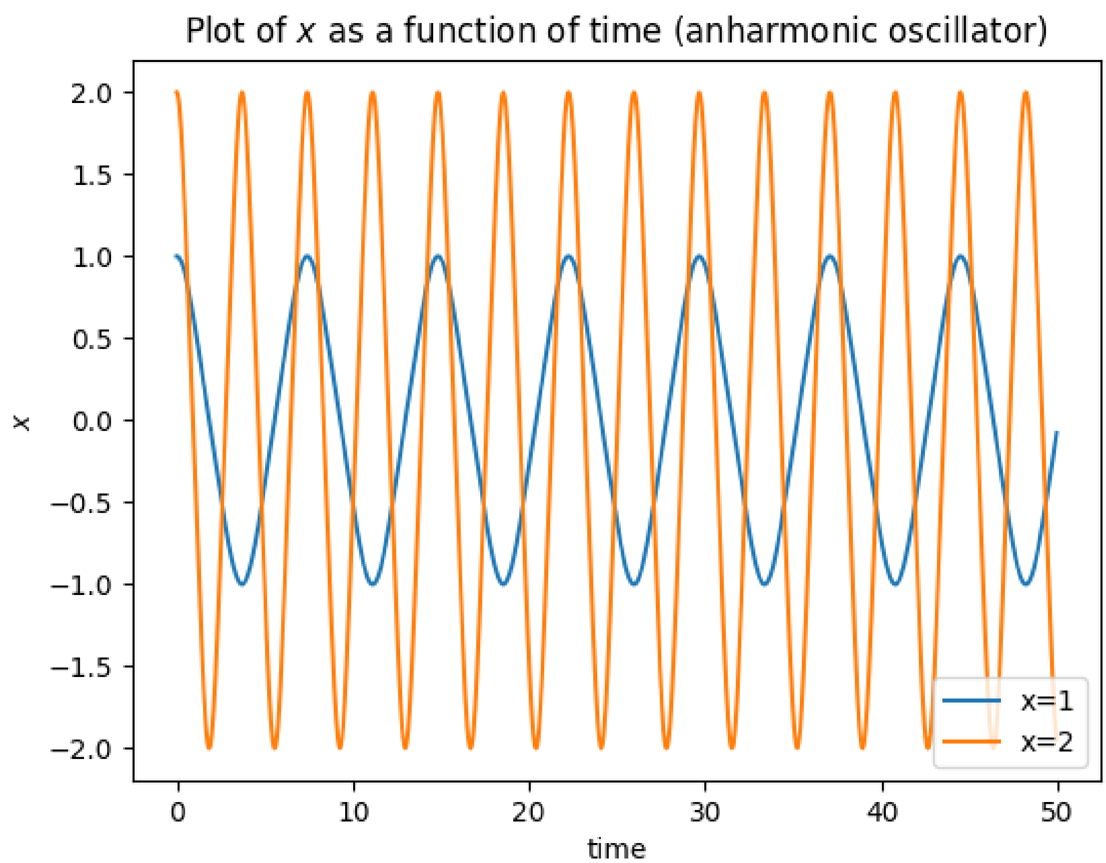

Figure 3: Anharmonic oscillator with different amplitudes.

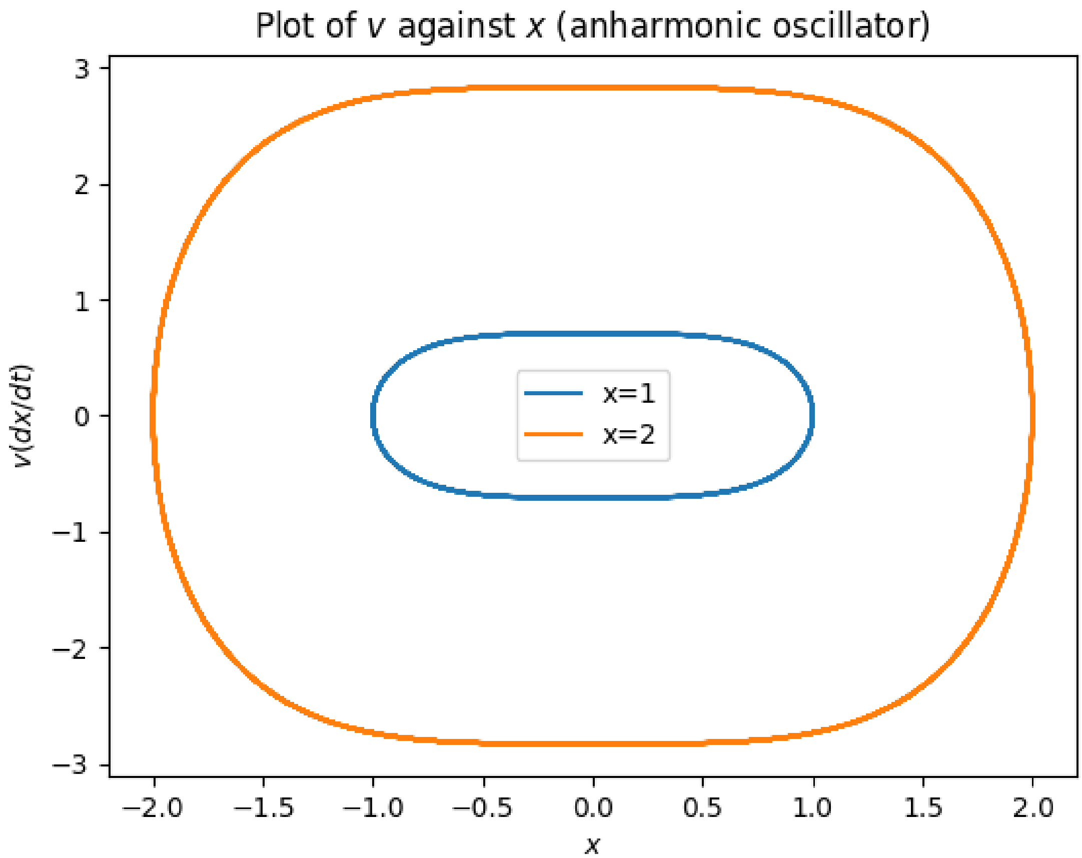

Figure 4: Example of phase space plot.

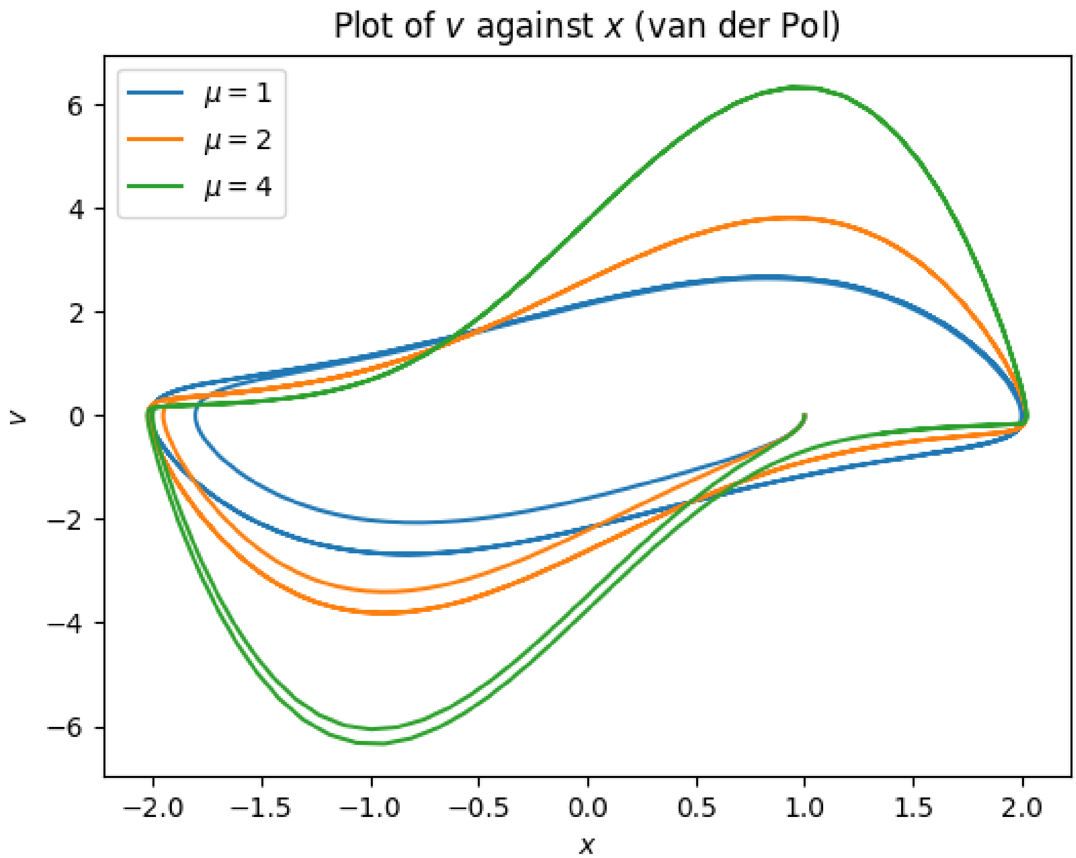

Figure 5: Phase space plot of the van der Pol oscillator with different values of μ.

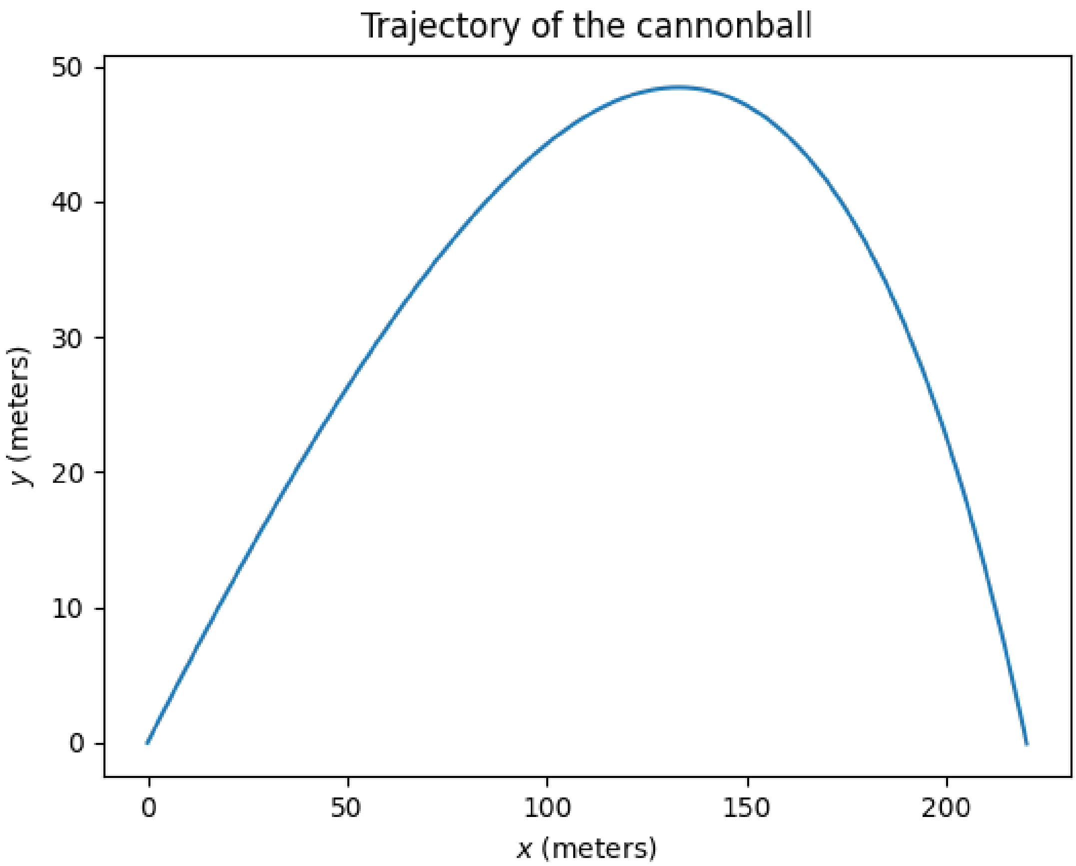

Figure 6: Trajectory of the cannonball.

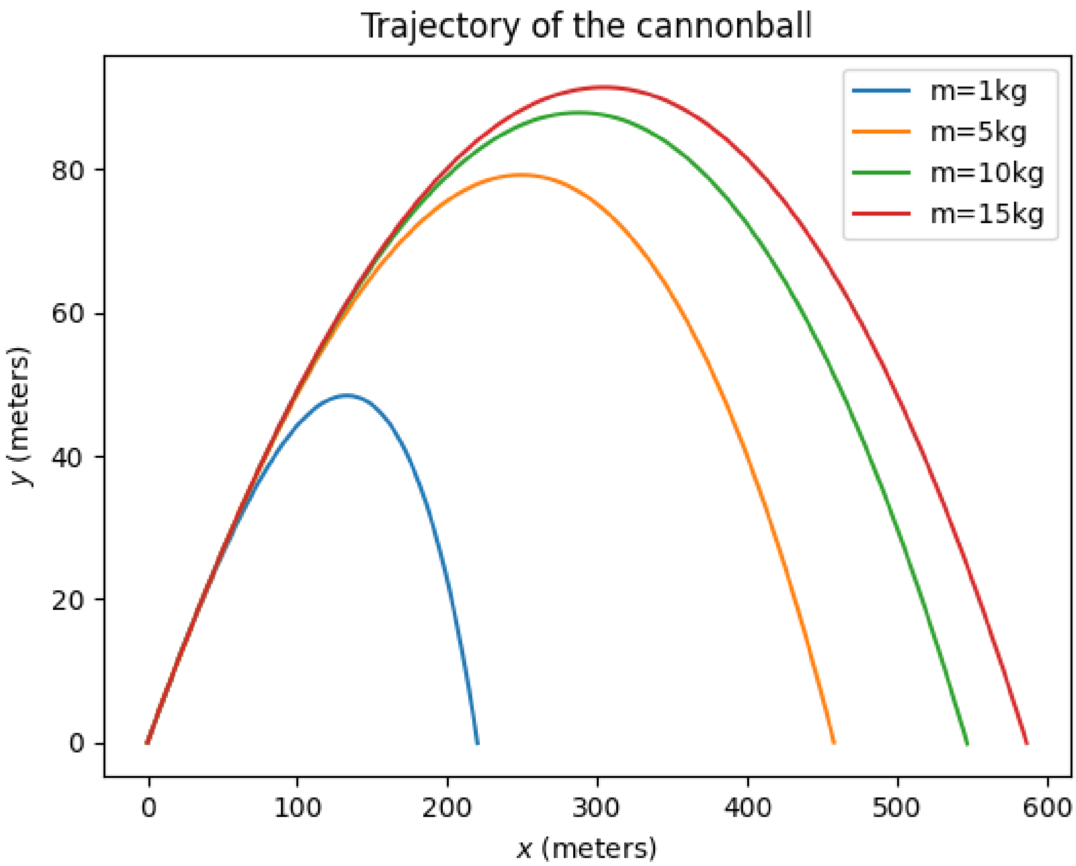

Figure 7: Trajectories of the cannonball with different masses.