PS2

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1 Q1

Fig 1 is the output for Q1.

2 Q2

Fig 2 is the output for Q2. I found those exponents just by trial and error.

3 Q3

See the output of the two versions in Fig 3. The no-for-loop version runs faster. The value of L used is 100.

4 Q4

The Mandelbrot set is plotted in Fig 4. I used 100 iterations for each c and $N = 2000$. The running time was between 30 and 60 seconds which is within the acceptable range as described in the exercise.

5 Q5

5.1 part a

Fig 5 shows the solutions of the given quadratic equation. It also contains the solutions for part b.

5.2 part b

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \times \frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} = \frac{b^2 - (b^2 - 4ac)}{-2ab \mp 2a\sqrt{b^2 - 4ac}} = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}} \tag{1}$$

The solutions obtained using the new form of x are shown in Fig 5. I see that there are discrepancies between the solutions and their order of magnitude are quite different (the difference between the

first ones is on the order of 1e-7, while that between the second ones is on the order of 1e1). This happens because the value of b is relatively large compared to the values of a and c, such that $\sqrt{b^2 - 4ac} \approx b$. Thus in part a, the first solution has small number divided by small number, the second solution has large (in terms of magnitude) number divided by small number. In part b, the first solution has small number divided by large number, the second solution has small number divided by small number.

5.3 part c

My quadratic module passed the test. The output is in Fig 6

```
The binary representation of 100.98763 in np.float32:
01000010110010011111100110101011
The 32-bit number is:
100.98763275146484
The difference between the actual number from its binary representation:
2.7514648479609605e-06
[Finished in 340ms]
```

Figure 1: Q1 answer

```
Smallest increment to 1 in np.float32 is approx. 2^(-23), and the result is:
1.0000001
Smallest increment to 1 in np.float64 is approx. 2^(-52), and the result is:
1.0000000000000002
Minimum of np.float32 is approx. -2^(127), which reads:
-1.7014118e+38
Maximum of np.float32 is approx. 2^(127), which reads::
1.7014118e+38
Minimum of np.float64 is approx. -2^(1023), which reads::
-8.98846567431158e+307
Maximum of np.float64 is approx. 2^(1023), which reads::
8.98846567431158e+307
[Finished in 351ms]
```

Figure 2: Q2 answer

```
The for-loop version calculates M = 1.7418198158396654
26.209017
The no-for-loop version calculates M = 1.7418198158362388
0.4042652000000011
[Finished in 26.9s]
```

Figure 3: The value of M is the value of the Madelung constant of NaCl. The two numbers 26.209017 and 0.404265 are the running time of the for-loop version and the no-for-loop version respectively.

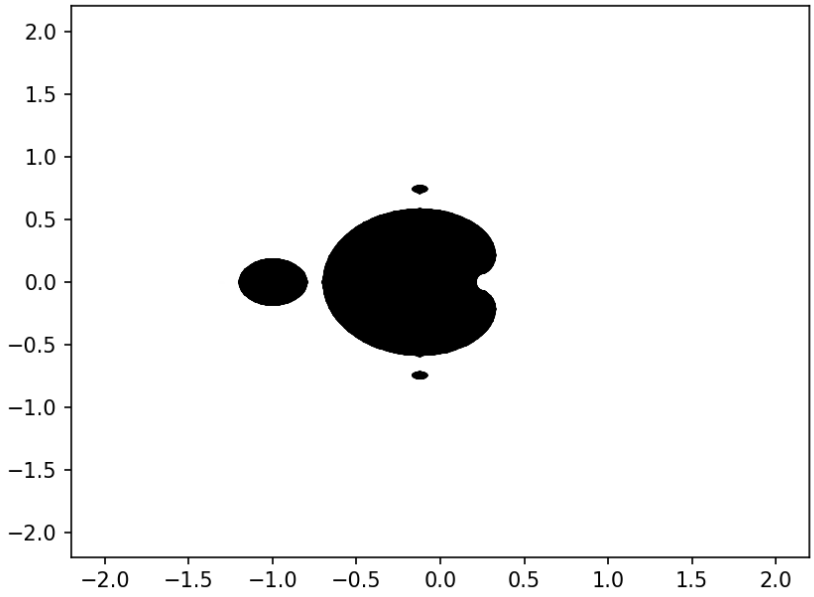

Figure 4: The plot for Q4.

```
The solutions for part a:
(-9.999894245993346e-07, -999999.999999)
The solutions for part b:
(-1.000000000001e-06, -1000010.5755125057)
[Finished in 1.3s]
```

Figure 5: The solutions of the given quadratic equation, for both part a and part b.

```
(base) D:\PSH\PhD\Courses\Computational Physics\Github workspace\phys-ga2000\ps-2>pytest test_quadratic.py
============================= test session starts =============================
platform win32 -- Python 3.8.8, pytest-6.2.3, py-1.10.0, pluggy-0.13.1
rootdir: D:\PSH\PhD\Courses\Computational Physics\Github workspace\phys-ga2000\ps-2
plugins: anyio-2.2.0
collected 1 item

test_quadratic.py .                                                      [100%]

============================= warnings summary ================================
C:\Users\peter\anaconda3\lib\site-packages\pyreadline\py3k_compat.py:8
  C:\Users\peter\anaconda3\lib\site-packages\pyreadline\py3k_compat.py:8: DeprecationWarning: Using or importing the ABCs from 'collections' instead of from 'collections.abc' is deprecated since Python 3.3, and in 3.9 it will stop
 working
    return isinstance(x, collections.Callable)

-- Docs: https://docs.pytest.org/en/stable/warnings.html
======================== 1 passed, 1 warning in 0.21s =========================

(base) D:\PSH\PhD\Courses\Computational Physics\Github workspace\phys-ga2000\ps-2>
```

Figure 6: The output of pytest test_quadratic.py.