Inverse Optimal Taxation in Closed Form

Jonathan Heathcote and Hitoshi Tsujiyama

February 15, 2017

1 Introduction

For any policy question with distributional implications, the planner's social welfare function will play an important role in shaping the optimal policy. In the context of an optimal taxation problem in which individuals differ ex ante with respect to labor productivity, the planner's social welfare function defines the relative Pareto weights placed on individuals with different productivities. In this paper, we show that if one assumes that the observed income tax schedule has been chosen optimally, then it is sometimes possible to derive a closed-form expression for the social welfare function. This social welfare function involves parameters defining the shape of the observed tax and transfer system, as well as parameters defining preference elasticities and the shape of the underlying cross-sectional productivity distribution.

There are two reasons why it is useful to have a closed-form expression for social welfare. First, one can readily use our expression to construct social welfare functions for countries or time periods that differ with respect to observed tax systems, or with respect to observed inequality. One could then attempt to develop political economic explanations for why the social welfare function differs across space or time.

The second reason we find our approach valuable is that tractability lends additional intuition to the standard optimal taxation question. The Mirrleesian approach takes as given a social welfare function (typically utilitarian) and then solves for the tax system that is optimal given that function, and the other details of the environment. A challenge is that this problem must typically be solved numerically, and it is difficult to gain intuition for the shape of the resulting optimal tax function. We show that the inverse optimum problem (finding a social welfare function that justifies a given tax function) is tractable in quantitatively relevant cases where the optimum problem (solving for the optimal tax function for a given social welfare function) is not. In particular, the inverse optimum problem is tractable given a standard utility specification with curvature over both consumption and labor effort. This tractability makes it easier to understand how optimal taxation works.

2 Deriving the Social Welfare Function

Our environment is a standard static Mirrleesian framework. Agents differ with respect to their labor productivity w, and the planner's social welfare function maps an individual's productivity into an individual-specific Pareto weight $W(w)$. The planner's goal is to design a tax and transfer system that maximizes social welfare, where taxes must be based on earnings rather than directly on productivity, because only earnings can be observed. We assume the distribution for productivity is continuous and let $F(w)$ denote the CDF. We let $c(w)$ denote consumption, and $y(w)$ denote labor earnings.

The analysis proceeds in four steps. In the first step we derive the familiar Diamond-Saez functional equation that ties together the marginal tax schedule $T'(y(w))$, the consumption function $c(w)$, and the social welfare function $W(w)$. The second step imposes parametric functional forms for the tax schedule $T(y)$ and the utility function $u(c) - v\left(\frac{y}{w}\right)$, which allow us to solve in closed form for $c(w)$ and $T'(y(w))$, and substitute out these functions from the Diamond Saez expression. The third step is to differentiate the Diamond-Saez functional equation with respect to w, which delivers a closed form expression for $W(w)$. The fourth and final step is to substitute in a functional form for $F(w)$.

2.1 Derivation of Diamond Saez Equation

Following Mirrlees (1971), suppose a planner seeks to solve the following problem:

$$\begin{cases} \max\limits_{c(w),y(w)} & \int W(w)\left[u\left(c(w)\right)-v\left(\frac{y(w)}{w}\right)\right]dF(w) \\ \text{s.t.} & u\left(c(w)\right)-v\left(\frac{y(w)}{w}\right)\geq u\left(c(w')\right)-v\left(\frac{y(w')}{w}\right) \quad \forall w,w' \\ & \int\left[c(w)-y(w)\right]dF(w)+G\leq 0. \end{cases}$$

The first line here is the planner's objective, which is to maximize aggregate utility where the utility for a worker with productivity w is weighted by her Pareto weight $W(w)$. The second line captures the familiar Mirrlees incentive compatibility (IC) constraints: given a productivity w, the worker must prefer the allocation $\{c(w),y(w)\}$ intended for his type to any alternative. The third line is the resource constraint, where G denotes an exogenous quantity of government purchases that must be financed.

This problem can be rewritten in a more convenient form. The incentive compability constraints state that

$$U(w)\equiv u\left(c(w)\right)-v\left(\frac{y(w)}{w}\right)=\max_{w'}u\left(c(w')\right)-v\left(\frac{y(w')}{w}\right).$$

The envelope condition (first-order condition with respect to w') is

$$0=u'\cdot c'-v'\cdot\frac{y'}{w}$$

Thus,

$$\begin{aligned} U' &= u'\cdot c'-v'\cdot\left[\frac{y'}{w}-\frac{y}{w^2}\right] \\ &= v'\frac{y}{w^2}. \end{aligned}$$

Thus, instead of thinking of the planner as choosing functions $\{c(w),y(w)\}$, we can think of it as choosing functions $\{U(w),y(w)\}$ and as solving

$$\begin{cases} \max\limits_{U(w),y(w)} & \int W(w)U(w)dF(w) \\ \text{s.t.} & U'=v'\frac{y}{w^2},\forall w \\ & \int\left[y(w)-c(w;U,y)\right]dF(w)-G\geq 0. \\ \text{where} & c(w;U,y)\text{ is determined by }U(w)=u\left(c(w)\right)-v\left(\frac{y(w)}{w}\right). \end{cases}$$

We now set up a Hamiltonian. Denoting by $\mu(w)$ and ζ the multipliers on the IC and resource constraints,

$$\boldsymbol{H}\equiv\left\{W(w)U(w)+\zeta\left[y(w)-c(w;U,y)-G\right]\right\}f(w)+\mu(w)v'\frac{y}{w^2},$$

where U is the state variable, and y is the control variable. From the dynamic optimization, the following equations hold

$$\begin{aligned} & \begin{cases} \partial \boldsymbol{H}/\partial y = 0 \\ \partial \boldsymbol{H}/\partial U = -\mu'(w), \\ \mu(0) = 0, \lim\limits_{w \nearrow \infty} \mu(w) = 0. \end{cases} \\ \Leftrightarrow & \begin{cases} 0 = \zeta \left[1 - \frac{v'}{wu'}\right] f(w) + \mu(w) \left(v' \frac{1}{w^2} + v'' \frac{y}{w^3}\right), \\ -\mu'(w) = \left\{W(w) - \frac{\zeta}{u'}\right\} f(w), \\ \mu(0) = \mu(\infty) = 0. \end{cases} \end{aligned} \tag{1}$$

Integrating the second equation over w and using $\mu(\infty) = 0$, we get

$$\mu(w) = \int_w^\infty \left\{W(s) - \frac{\zeta}{u'(c(s;U,y))}\right\} dF(s).$$

Using $\mu(0) = 0$, we get the expression for ζ:

$$0 = \int_0^\infty \left\{W(s) - \frac{\zeta}{u'(c(s;U,y))}\right\} dF(s),$$

and thus we have

$$\zeta = \frac{\int W(s) dF(s)}{\int \frac{1}{u'(c(s;U,y))} dF(s)} = \frac{1}{\int \frac{1}{u'(c(s;U,y))} dF(s)}.$$

We want to modify the first equation in (1), so that it involves the tax function. The first-order condition for labor supply, given a differentiable tax function $T(y)$ is

$$u' (1 - T') = \frac{v'}{w}.$$

So the first equation in (1) can be rewritten as

$$\begin{aligned} 0 &= \zeta T' f(w) + \mu(w) \left(u' (1 - T') \frac{1}{w} + v'' \frac{y}{w^3}\right) \\ 0 &= \zeta T' w f(w) + \mu(w) \left(u' (1 - T') + v'' \frac{y}{w^2}\right) \end{aligned}$$

Now, following Saez (2001), we want to express the last term in terms of labor supply elasticities. Standard algebra gives

$$0 = \zeta T' w f(w) + \mu(w) u' (1 - T') \left(\frac{1 + e^u}{e^c}\right)$$

where e^u and e^c are uncompensated and compensated labor supply elasticities. Substituting in the expression for $\mu(w)$

gives the following implicit expression for the optimal tax function, a la Saez:

$$\zeta T' w f(w) + u' (1 - T') \left(\frac{1+e^u}{e^c} \right) \int_w^\infty \left\{ W(s) - \frac{\zeta}{u'(c(s))} \right\} dF(s) = 0$$

$$\Leftrightarrow \qquad \frac{T'(y(w))}{1 - T'(y(w))} = A(w)B(w)C(w),$$

where

$$A(w) \equiv \frac{1+e^u}{e^c},$$

$$B(w) \equiv \frac{1-F(w)}{wf(w)},$$

$$C(w) \equiv \int_w^\infty \left[1 - \frac{W(s)u'(c(s))}{\zeta} \right] \frac{u'(c(w))}{u'(c(s))} \frac{dF(s)}{1-F(w)},$$

$$\zeta = \frac{1}{\int \frac{1}{u'(c(w;U,h))} dF(w)},$$

Note that the $C(w)$ term here contains both Pareto weights and endogenous allocations, via the consumption rule $c(w)$.

2.2 Plug in Functional Forms

To start, we assume the following utility function

$$u(c) - v\left(\frac{y}{w}\right) = \log c - \frac{\left(\frac{y}{w}\right)^{1+\sigma}}{1+\sigma}$$

where the curvature parameter σ defines the Frisch elasticity as σ^{-1}.

Given this utility function,

$$A(w) = \frac{1+e^u}{e^c} = 1 + \frac{v'' h}{v'} = 1 + \sigma$$

and

$$\zeta = \frac{1}{\int \frac{1}{u'(c(w;U,h))} dF(w)} = \frac{1}{C}$$

where C is aggregate consumption.

Plugging all this into the DS equation gives

$$\frac{T'(y(w))}{1 - T'(y(w))} = (1+\sigma) \frac{1}{wf(w)} \frac{1}{c(w)} \int_w^\infty \left[c(s) - W(s)C \right] f(s) ds$$

which can be rearranged to give

$$\int_w^\infty W(s) f(s) ds = \frac{1}{C} \left[\int_w^\infty c(s) f(s) ds - \frac{T'(y(w))}{1 - T'(y(w))} \frac{wf(w)c(w)}{1+\sigma} \right]$$

We assume the following functional form for taxes, following Heathcote, Storesletten and Violante:

$$T(y) = y - \lambda y^{1-\tau}$$

The marginal tax function is then

$$T'(y) = 1 - \lambda(1-\tau)y^{-\tau}$$

Given this utility function and tax function it is straightfoward to solve for allocations in closed form:

$$c(w) = \lambda\left((1-\tau)^{\frac{1}{1+\sigma}} w\right)^{1-\tau},$$

and

$$y(w) = (1-\tau)^{\frac{1}{1+\sigma}} w$$

Substituting the latter into the marginal tax function gives

$$T'(y(w)) = 1 - \lambda(1-\tau)^{\frac{1+\sigma-\tau}{1+\sigma}} w^{-\tau}$$

The government budget constraint can be used to solve for λ :

$$\lambda = \frac{(1-\tau)^{\frac{\tau}{1+\sigma}}(1-g)}{\int w^{1-\tau} dFw}$$

where g is G/Y, i.e., government consumption as a share of output.

Thus, consumption and the marginal tax rate function can be simplified further to give

$$c(w) = \frac{(1-\tau)^{\frac{1}{1+\sigma}}(1-g)}{\int w^{1-\tau} dFw} w^{1-\tau}$$

$$T'(y(w)) = 1 - \frac{(1-g)(1-\tau)}{\int w^{1-\tau} dFw} w^{-\tau}$$

Substituting the expressions for $c(w)$ and $T'(y(w))$ into th expression defining the inverse optimum weights gives

$$\int_w^\infty W(s)f(s)ds = \frac{1}{\int_0^\infty s^{1-\tau} f(s)ds}\left[\int_w^\infty s^{1-\tau} f(s)ds - \left(\frac{\int s^{1-\tau} f(s)ds}{(1-g)(1-\tau)} w^\tau - 1\right)\frac{wf(w)w^{1-\tau}}{1+\sigma}\right]$$

which can be rewritten as

$$\int_0^w W(s)f(s)ds = 1 - \frac{\int_w^\infty s^{1-\tau} f(s)ds}{\int_0^\infty s^{1-\tau} f(s)ds} + \frac{w^2 f(w)}{(1-g)(1-\tau)(1+\sigma)} - \frac{1}{1+\sigma}\frac{w^{2-\tau} f(w)}{\int_0^\infty s^{1-\tau} f(s)ds}$$

2.3 Differentiating to derive $W(s)$

Differentiating each term with respect to w gives

$$W(w)f(w) = \frac{w^{1-\tau} f(w)}{\int_0^\infty s^{1-\tau} f(s)ds} + \frac{2wf(w) + w^2 f'(w)}{(1-g)(1-\tau)(1+\sigma)} - \frac{1}{1+\sigma}\frac{\left((2-\tau)w^{1-\tau} f(w) + w^{2-\tau} f'(w)\right)}{\int_0^\infty s^{1-\tau} f(s)ds}$$

which can alternatively be written as

$$W(w) = \frac{w^{1-\tau}}{(1+\sigma)\int_0^\infty s^{1-\tau} f(s)ds}\left((1+\sigma+\tau) + \left(\frac{\int_0^\infty s^{1-\tau} f(s)ds}{(1-g)(1-\tau)} w^\tau - 1\right)\left(2 + w\frac{f'(w)}{f(w)}\right)\right)$$

We can also define a marginal social welfare weight as

$$M(w) = u'(c(w))W(w)$$

This denotes the value to the planner of giving an extra unit of consumption (dollar) to an agent with a wage w.

$$
\begin{aligned}
M(w) &= \frac{W(w)}{c(w)} \\
&= \frac{1}{(1-g)(1+\sigma)(1-\tau)^{\frac{1}{1+\sigma}}}\left((1+\sigma+\tau)+\left(\frac{\int_0^\infty s^{1-\tau}f(s)ds}{(1-g)(1-\tau)}w^\tau-1\right)\left(2+w\frac{f'(w)}{f(w)}\right)\right)
\end{aligned}
$$

2.4 Special Cases without Making Assumptions on $F(w)$

Suppose labor supply is exogenous, so that $\sigma \to \infty$. Then

$$
W(w) = \frac{w^{1-\tau}}{\int_0^\infty s^{1-\tau}f(s)ds}
$$

Suppose we observe $\tau = 0$.Then

$$
\begin{aligned}
W(w) &= \frac{w}{(1+\sigma)}\left((1+\sigma)+\left(\frac{1}{(1-g)}-1\right)\left(2+w\frac{f'(w)}{f(w)}\right)\right) \\
&= w+\frac{1}{1+\sigma}\frac{g}{(1-g)}\left(2+w\frac{f'(w)}{f(w)}\right)w
\end{aligned}
$$

Now if, in addition, $g = 0$, then

$$
W(w) = w
$$

Also if $g = 0$ and $\tau = 0$

$$
M(w) = 1
$$

2.5 Specific Wage Distributions

Suppose we assume that $\log(w)\tilde{}EMG(\mu_\alpha, v_\alpha, \lambda_\alpha)$, or equivalently that productivity follows a Pareto log-normal distribution. Heathcote and Tsujiyama (2016) show that this is a close approximation to the actual wage distribution in the United States.

We will eventually set

$$
\mu_\alpha = \log\left(\frac{\lambda_\alpha - 1}{\lambda_\alpha}\right) - \frac{1}{2}v_\alpha
$$

which ensures that the average wage is equal to one.

The wage density is given by

$$
f(w) = \lambda_\alpha w^{-\lambda_\alpha - 1}\exp\left(\lambda_\alpha\mu_\alpha + \frac{\lambda_\alpha^2 v_\alpha}{2}\right)\Phi\left(\frac{\log w - \mu_\alpha - \lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)
$$

and the derivative of the density is

$$
f'(w) = \frac{-(\lambda_\alpha+1)}{w}f(w) + \frac{f(w)}{w\sqrt{v_\alpha}}\frac{\Phi'\left(\frac{\log w - \mu_\alpha - \lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)}{\Phi\left(\frac{\log w - \mu_\alpha - \lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)}
$$

where

$$
\begin{aligned}
\Phi(x) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^x \exp\left(-\frac{t^2}{2}\right)dt \\
\Phi'(x) &= \phi(x) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-x^2}{2}\right)
\end{aligned}
$$

Thus,

$$W(w)=\frac{w^{1-\tau}}{(1+\sigma)\int_0^\infty s^{1-\tau}f(s)ds}\left((1+\sigma+\tau)+\left(\frac{\int_0^\infty s^{1-\tau}f(s)ds}{(1-g)(1-\tau)}w^\tau-1\right)\left(1-\lambda_\alpha+\frac{1}{\sqrt{v_\alpha}}\frac{\phi\left(\frac{\log w-\mu_\alpha-\lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)}{\Phi\left(\frac{\log w-\mu_\alpha-\lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)}\right)\right)$$

Now, the term $\int s^{1-\tau}dFs$ can also be simplified:

$$\int s^{1-\tau}dFs=\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)^{1-\tau}\exp\left(-\frac{v_\alpha}{2}\tau(1-\tau)\right)\frac{\lambda_\alpha}{\lambda_\alpha-(1-\tau)}$$

So

$$\begin{aligned}W(w)\quad&=\quad\frac{w^{1-\tau}}{(1+\sigma)\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)^{1-\tau}\exp\left(-\frac{v_\alpha}{2}\tau(1-\tau)\right)\frac{\lambda_\alpha}{\lambda_\alpha-(1-\tau)}}\times\\&\quad\left(1+\sigma+\tau+\left(\frac{\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)^{1-\tau}\exp\left(-\frac{v_\alpha}{2}\tau(1-\tau)\right)\frac{\lambda_\alpha}{\lambda_\alpha-(1-\tau)}}{(1-g)(1-\tau)}w^\tau-1\right)X\right)\end{aligned}$$

where

$$X=\left(1-\lambda_\alpha+\frac{1}{\sqrt{v_\alpha}}\frac{\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-\left(\frac{\log w-\left(\log\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)-\frac{1}{2}v_\alpha\right)-\lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)^2}{2}\right)}{\text{NormalDist}\left(\frac{\log w-\left(\log\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)-\frac{1}{2}v_\alpha\right)-\lambda_\alpha v_\alpha}{\sqrt{v_\alpha}}\right)}\right)$$

The marginal social welfare weights are

$$\begin{aligned}M(w)\quad&=\quad\frac{W(w)}{c(w)}\\&=\quad\frac{1}{(1-g)(1-\tau)^{\frac{1}{1+\sigma}}(1+\sigma)}\left(1+\sigma+\tau+\left(\frac{\left(\frac{\lambda_\alpha-1}{\lambda_\alpha}\right)^{1-\tau}\exp\left(-\frac{v_\alpha}{2}\tau(1-\tau)\right)\frac{\lambda_\alpha}{\lambda_\alpha-(1-\tau)}}{(1-g)(1-\tau)}w^\tau-1\right)X\right)\end{aligned}$$

2.6 Special Case: Lognormal Distribution

Suppose instead we had the lognormal distribution

$$\begin{aligned}f(w)\quad&=\quad\frac{1}{w}\frac{1}{\sqrt{2\pi v_\alpha}}\exp\left(-\frac{(\log w-\mu_\alpha)^2}{2v_\alpha}\right)\\f'(w)\quad&=\quad-w^{-2}\frac{1}{\sqrt{2\pi v_\alpha}}\exp\left(-\frac{(\log w-\mu_\alpha)^2}{2v_\alpha}\right)+\frac{1}{w}\frac{1}{\sqrt{2\pi v_\alpha}}\exp\left(-\frac{(\log w-\mu_\alpha)^2}{2v_\alpha}\right)\frac{-2(\log w-\mu_\alpha)}{2v_\alpha}\frac{1}{w}\\&=\quad\frac{1}{w}\left(-f(w)+f(w)\frac{-2(\log w-\mu_\alpha)}{2v_\alpha}\right)\end{aligned}$$

So

$$w\frac{f'(w)}{f(w)}=-1-\frac{(\log w-\mu_\alpha)}{v_\alpha}$$

So in that case we would have

$$W(w)=\frac{w^{1-\tau}}{(1+\sigma)\exp\left(-\frac{1}{2}\tau(1-\tau)v_\alpha\right)}\left((1+\sigma+\tau)+\left(\frac{\exp\left(-\frac{1}{2}\tau(1-\tau)v_\alpha\right)}{(1-g)(1-\tau)}w^\tau-1\right)\left(1-\frac{\log w+\frac{v_\alpha}{2}}{v_\alpha}\right)\right)$$

2.7 Special Case: Pareto Distribution

In this case, the average wage is not necessarily 1. Re-deriving the weights,

$$W(w) = \frac{w^{1-\tau}}{(1+\sigma)\int_0^\infty s^{1-\tau} f(s)ds}\left(1+\sigma+\tau+\left(\frac{\int_0^\infty s^{1-\tau}f(s)ds}{(1-g)(1-\tau)\int sdFs}w^\tau - 1\right)\left(2+w\frac{f'(w)}{f(w)}\right)\right)$$

With a Pareto distributionn $\log(w) \sim \exp(\lambda_\alpha)$

$$\begin{aligned} f(w) &= \frac{\lambda_\alpha}{w^{1+\lambda_\alpha}} \\ f'(w) &= -\frac{f(w)}{w}(1+\lambda_\alpha) \end{aligned}$$

So

$$w\frac{f'(w)}{f(w)} = -(1+\lambda_\alpha)$$

So in that case we would have

$$W(w) = \frac{\sigma+\tau+\lambda_\alpha}{1+\sigma}\frac{\lambda_\alpha - 1+\tau}{\lambda_\alpha}w^{1-\tau} - \frac{1}{(1+\sigma)(1-g)(1-\tau)}\frac{(\lambda_\alpha-1)^2}{\lambda_\alpha}w$$

If $\tau = 0$

$$W(w) = \frac{1}{1+\sigma}\frac{\lambda_\alpha - 1}{\lambda_\alpha}\left[\sigma+\lambda_\alpha - \frac{\lambda_\alpha - 1}{1-g}\right]w$$

which is a linear function.

If in addition $g = 0$,

$$W(w) = \frac{\lambda_\alpha - 1}{\lambda_\alpha}w$$

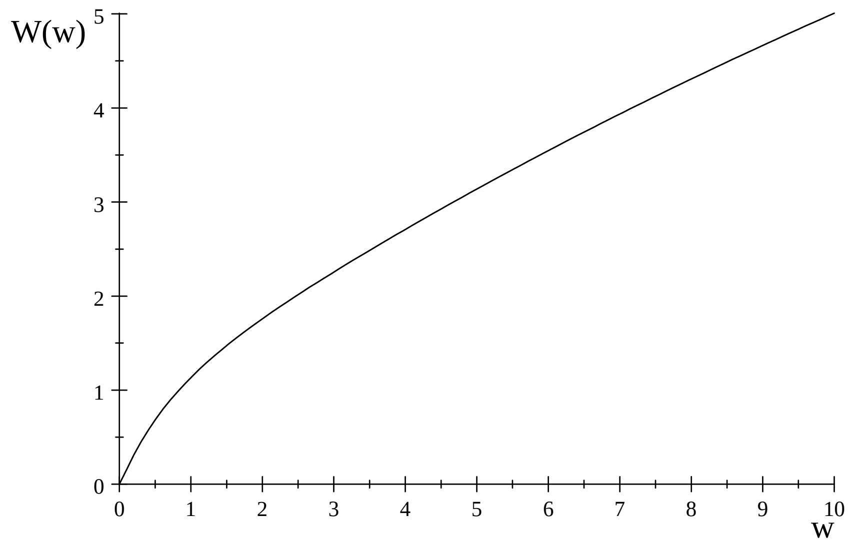
3 Numerical Example

We set $\sigma = 2$, a standard value.

We set $\tau = 0.181$, which is the estimate of Heathcote, Storesletten and Violante (2017) for the United States. We set $g = 0.188$, which is government purchases share of output in the United States.

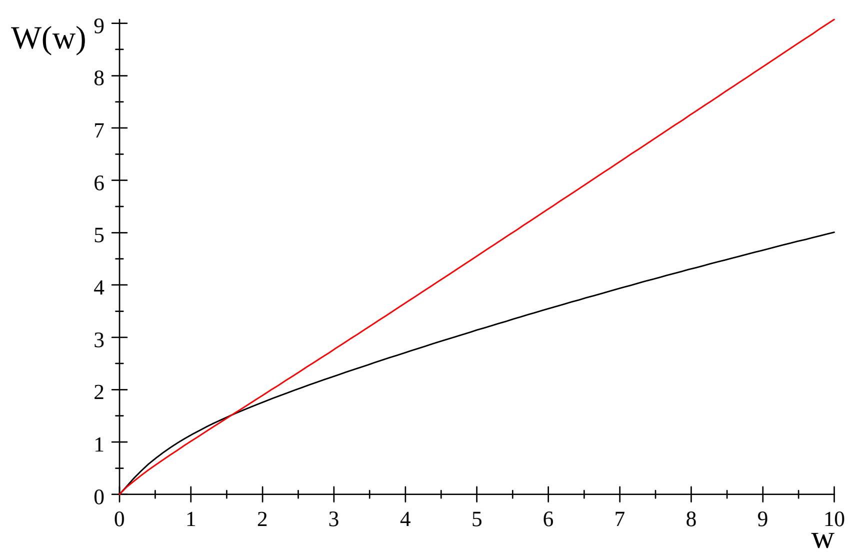
Using data from the 2007 Survey of Consumer Finances, Heathcote and Tsujiyama (2016) fit an EMG distribution to the observed distribution for labor earnings. They estimate an exponential coefficient for the Pareto right tail of 2.2, and a variance for the normal component v_a of 0.412.

Given these values, we now plot the social welfare function $W(w)$.

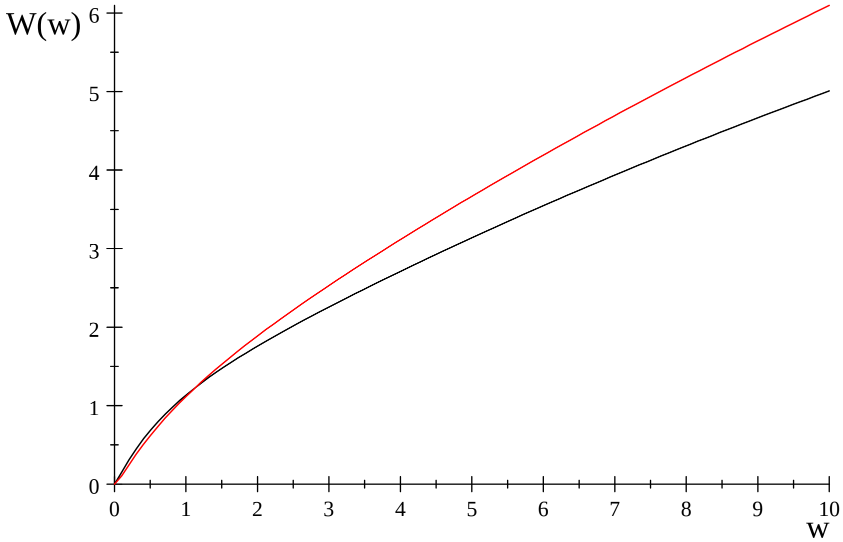


Note that these weights are increasing in productivity, indicating that the US social planner puts relatively more weight on the more productive agents. For very low productivity values ($w < 0.01$) and very high values ($w > 600$) Pareto weights are actually negative. At the very top, this reflects the fact that the HSV tax function implies marginal tax rates that eventually reach 1, and such high marginal rates imply that the planner is beyond the peak of the Laffer curve. Punishing the super rich in this fashion is only optimal if the planner puts negative weight on their utility.

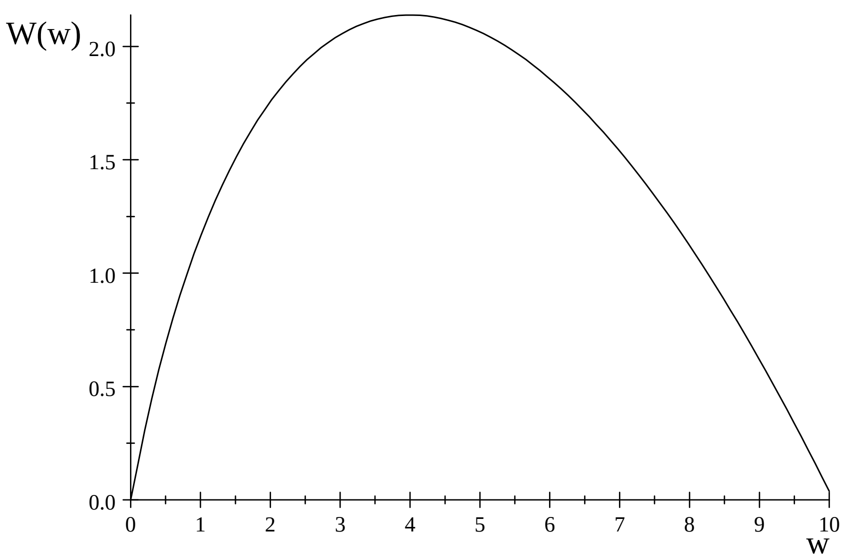
The next two plots show sensitivity with respect to τ and g. In the first plot, the black line is the SWF given $\tau = 0.181$, while the red line is what would be inferred given $\tau = 0$ (a flat tax). Clearly, if we observe a less progressive tax system, we can infer that the planner puts more weight on richer agents. In the even simpler case in which both $\tau = 0$ and $g = 0$, the social welfare function collapses to $W(w) = w$.



In the next plot, we consider the baseline alongside a version in which $g = 0$. Thus, holding fixed τ, reducing g implies higher Pareto weights on rich agents. The logic is that reducing g pushes the planner towards a more progressive tax system (see Heathcote et al., 2017). To rationalize the same tax system requires putting more weight on relatively productive agents.

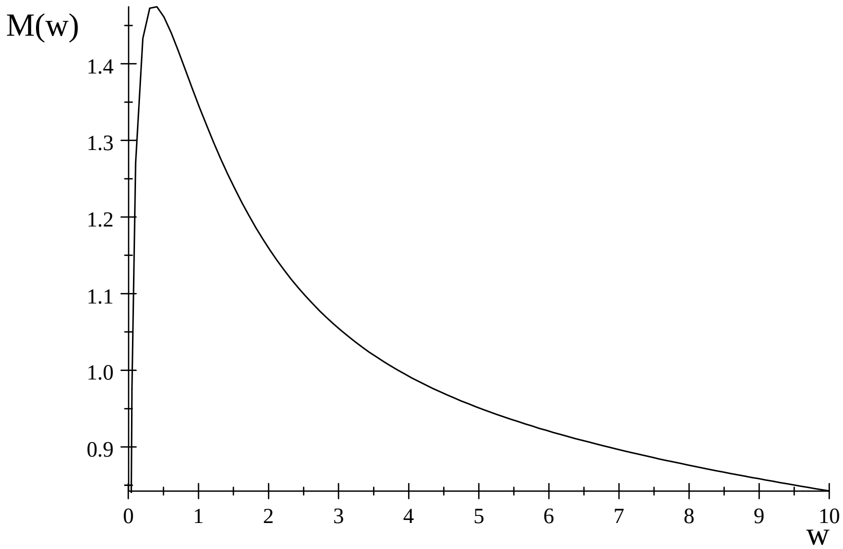


Suppose now, that we were to observe the same tax system, but a lognormal wage distribution with variance $v_l = 0.412 + \frac{1}{2.2^2} = 0.619$ (so that the total variance of log wages is the same as the baseline case). In this case, the social welfare function looks quite different:



Now the social welfare function is hump-shaped, and Pareto weights turn negative at wages around 10 times the mean. The logic is that, given a lognormal productivity distribution, optimal marginal tax rates at high productivity levels are lower – holding fixed the social welfare function – than when the productivity distribution has a Pareto tail. Thus lower Pareto weights at high productivity levels are required to rationalize the same marginal rates in the economy with a lognormal distribution.

We can also plot marginal social welfare weights. For the baseline EMG case, they look as follows:



Note that these weights are mostly declining in w, but not at the very bottom. The downward sloping portion is easy to understand: an extra dollar for a middle class household is worth more to the planner than an extra dollar for a very rich household, but the planner doesn't make the tax system more redistributive because the associated distortions would be too large. The logic for the upward sloping portion is that with the HSV tax function, the planner gives small transfers to the very poor, and larger transfers to the only moderately poor. Thus, we infer that the planner puts much more weight on the moderately poor than the very poor, so that the planner wants to redistribute upwards in this range. It achieves this goal with negative marginal tax rates. Doing even more upward redistribution (which other things equal the planner would like given an upward sloping marginal social welfare weight function) would require even more negative marginal tax rates, and that would be more distortionary (it is best to have marginal tax rates close to zero).

4 To Do

Explore GHH preferences of the form

$$u(c, y) = \log\left(c - \frac{\left(\frac{y}{w}\right)^{1+\sigma}}{1+\sigma}\right)$$

Explore affine tax systems of the form

$$T(y) = \lambda + \tau y$$