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A PRACTICAL METHOD FOR THE DIRECT ANALYSIS OF TRANSIENT STABILITY

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ABSTRACT

This paper describes the development and evaluation of an analytical method for the direct determination of transient stability. The method developed is based on the analysis of transient energy and accounts for the nature of the system disturbance as well as for the effects of transfer conductances on system behavior. It has been evaluated on a 10 generator 39 bus system and on a 20 generator 118 bus system. The method predicts critical clearing times for first swing transient stability which agree very closely with the results of simulations.

The main conclusion of the study is that the basic approach developed is practical, sufficiently accurate and could be applied to realistic problems in power system planning and operation.

INTRODUCTION

The size and complexity of modern power systems has placed increased emphasis on the development of improved analytical techniques. Direct methods for analyzing power system transient stability are particularly attractive as they have important applications in the area of power system planning, operation, and control. They can be used, for example, for predicting critical clearing times, for real-time security assessment and in developing strategies for emergency state control.

Mangnusson [1] and later Aylett [2] developed the original energy based methods for stability analysis. In recent years these have been considered a special case of the more general Lyapunov methods - the energy simply being one possible Lyapunov function. Considerable effort has been devoted to Lyapunov methods for power system stability analysis in the last decade. Due to space limitations, discussion of previous work is omitted here but may be found in [9]; see also the surveys [3,4] and the many references therein. A substantial part of the effort has involved searching for better Lyapunov functions, i.e., Lyapunov functions that either give larger regions of stability in state space or are valid for more complex system models. However, Ribbens-Pavella has shown [3] that, for the commonly used system models, the Lyapunov functions, derived using the correct state variables, are equivalent to the transient energy function developed by Aylett [2]. The transient energy remains useful for stability analysis and has the advantage that a physical interpretation of its properties is available.

The transient energy function contains both kinetic and potential terms. The system kinetic energy, associated with the relative motion of machine rotors, is formally independent of the network. The system potential energy, associated with the potential energy of network elements and machine rotors, is always defined for the post-fault system, whose stability is to be analyzed. The principal idea of the direct methods is that a system's transient stability can, for a given contingency, be determined directly by comparing the total system energy which is gained during the fault-on period, with a certain critical potential energy. For a two-machine system this critical energy is uniquely defined and the direct analysis is equivalent to the equal area criteria.

For a system with three or more machines the direct analysis becomes more difficult. In this case the critical energy is not uniquely defined and its determination becomes the key step in the analysis. It is in this step that the approach proposed here differs markedly and significantly from those adopted previously which are based on Lyapunov's second method. According to Lyapunov's theorem, the critical energy is chosen to be the potential energy at the unstable equilibrium point closest (in terms of energy) to the stable equilibrium point. This unstable equilibrium point is called the closest one [6] or the lowest saddle point [5]. This critical energy frequently yields results that are very conservative, especially for systems with more than 3 or 4 generators.

This conservativeness has severely limited the practical application of Lyapunov methods to the power system transient stability problem. Presently, the other major limitation of analytical methods is the requirement of simplified models, i.e., classically represented generators and constant impedance loads. The latter limitation is not as restrictive however, since much useful information can be obtained from studies with simplified models. An analytical method can potentially provide a broader perspective of the transient stability problem and valuably complement the step-by-step simulation of individual cases with detailed models.

In this paper the reasons for the conservativeness of direct methods are explored in relation to observed power system behavior and a new approach is proposed based on this understanding. Techniques which realize this modified transient energy method are discussed, including those which account for the effects of transfer conductances of power system transient behavior. The sum of these techniques constitute a complete algorithm that can be used to directly calculate critical clearing times without solving any differential equations, and results which illustrate the practical significance of this approach are presented.

MATHEMATICAL FORMULATION

For the system model being considered in this investigation the equations of motion are:

$$M_i \dot{\omega}_i = P_i - P_{ei} \qquad (1)$$

$$\dot{\delta}_i = \omega_i \qquad i = 1,\ldots n$$

where
$$P_{ei} = \sum_{\substack{j=1 \\ \neq i}}^{n} [C_{ij} \sin(\delta_i - \delta_j) + D_{ij} \cos(\delta_i - \delta_j)]$$

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$$P_i = P_{mi} - E_i^2 G_{ii}$$

$$C_{ij} = E_i E_j B_{ij},\ D_{ij} = E_i E_j G_{ij}$$

and, for unit i,

P_{mi} = mechanical power input

G_{ii} = driving point conductance

E_i = constant voltage behind the direct axis transient reactance

ω_i, δ_i = generator rotor speed and angle deviations, respectively

M_i = moment of inertia

$B_{ij}(G_{ij})$ = transfer susceptance (conductance) in the reduced bus admittance matrix.

Equations (1) are written with respect to an arbitrary synchronous reference frame. The transformation of these equations into the center of angle coordinates not only offers physical insight to the transient stability problem formulation in general, but in particular provides a concise framework for the analysis of systems with transfer conductances. Referring to (1) define:

$$\delta_0 \triangleq 1/M_T \sum_{i=1}^{n} M_i \delta_i,\ M_T \triangleq \sum_{i=1}^{n} M_i \tag{2}$$

then

$$M_T \dot{\omega}_0 = \sum_{i=1}^{n} P_i - P_{ei} = \sum_{i=1}^{n} P_i - 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} D_{ij} \cos \delta_{ij} \triangleq P_{COA} \tag{3}$$

$$\dot{\delta}_0 = \omega_0$$

The dynamics of the center of angle (or center of inertia) reference are governed by (3). By defining new angles and speeds relative to this reference, $\theta_i \triangleq \delta_i - \delta_0$ and $\tilde{\omega}_i \triangleq \omega_i - \omega_0$, the system equations of motion become

$$M_i \dot{\tilde{\omega}}_i = P_i - P_{ei} - M_i / M_T P_{COA} \tag{4}$$

$$\dot{\theta}_i = \tilde{\omega}_i. \qquad i=1,\ldots n$$

Notice that the center of angle variables satisfy the constraints

$$\sum_{i=1}^{n} M_i \theta_i = \sum_{i=1}^{n} M_i \tilde{\omega}_i = 0$$

The transient energy function V, which is always defined for the post fault system, can be derived in several ways. The expression given in (5) can be obtained as in [2] by first establishing from (1) the n(n-1)/2 relative acceleration equations, multiplying each of these by the corresponding relative velocity and integrating the sum of the resulting equations from a fixed lower limit of the stable equilibrium point (denoted by superscript "s") to a variable upper limit.

$$V = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \left[\frac{1}{2M_T} M_i M_j (\omega_i - \omega_j)^2 - \frac{1}{M_T}(P_i M_j - P_j M_i)(\delta_{ij} - \delta_{ij}^s) - C_{ij}(\cos \delta_{ij} - \cos \delta_{ij}^s) + \int_{\delta_i^s+\delta_j^s-2\delta_0^s}^{\delta_i+\delta_j-2\delta_0} D_{ij} \cos \delta_{ij}\, d(\delta_i + \delta_j - 2\delta_0) \right] \tag{5}$$

Equation (5) can be algebraically manipulated and written in a more convenient form using the center of angle variables. The resulting expression, given in (6), can alternatively be derived from (4) by applying the above steps to the n center of angle acceleration equations.

$$V = 1/2 \sum_{i=1}^{n} M_i \tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i (\theta_i - \theta_i^s) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \left[C_{ij}(\cos \theta_{ij} - \cos \theta_{ij}^s) - \int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij} \cos \theta_{ij}\, d(\theta_i + \theta_j) \right] \tag{6}$$

The terms of the transient energy function can be physically interpreted in the following way (all changes are with respect to the stable equilibrium point):

- $1/2 \sum_{i=1}^{n} M_i \tilde{\omega}_i^2 = 1/2 \sum_{i=1}^{n} M_i \omega_i^2 - 1/2 M_T \omega_0^2$

  Total change in rotor kinetic energy relative to COA = total change in rotor kinetic energy minus change in COA kinetic energy

- $\sum_{i=1}^{n} P_i \left(\theta_i - \theta_i^s\right) = \sum_{i=1}^{n} P_i \left(\delta_i - \delta_i^s\right) - \sum_{i=1}^{n} P_i \left(\delta_0 - \delta_0^s\right)$

  Change in rotor potential energy relative to COA = change in rotor potential energy minus change in COA potential energy

- $C_{ij} (\cos \theta_{ij} - \cos \theta_{ij}^s)$

  Change in magnetic stored energy of branch ij

- $\int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij} \cos \theta_{ij}\, d(\theta_i+\theta_j)$

  Change in dissipated energy of branch ij

The two expressions for the rotor kinetic and potential energies show that the change in energy associated with motion of the system center of angle is subtracted from the total system energy in order to obtain the transient energy function. This interpretation of transient energy corresponds with defining a stable system as one existing in a stable, synchronous equilibrium but not necessarily in frequency equilibrium [5].

The synchronous equilibrium points are the sets of system variables which satisfy (4) when the derivatives of speed and angle with respect to the center of angle are zero. These points correspond to extrema of the transient energy function and, because the speeds are zero, to extrema of the potential energy component as well. Within a periodic frame of rotor angles there exists at most one extremum which is a relative minimum of the potential energy, the stable equilibrium point. The rest of the extrema are unstable equilibria (at most $2^{n-1}-1$ for n even, $2^{n-1}+1/2\binom{n}{(n+1)/2}-1$ for n odd [5] which correspond to relative maxima and saddlepoints of the potential energy function.

The physical significance of the center of angle reference in the transient stability problem formulation can be further illustrated by considering in more detail the characterization of equilibria. For systems without transfer conductances, the equilibrium points can be obtained by solving, given appropriate initial angles, n-1 real power equations for an n machine system with one reference (or swing) machine as in [6]. This works satisfactorily for such systems because, irrespective of the operating point, there is no change in load.

For systems with transfer conductances however, the total load at an Unstable Equilibrium Point (UEP) will differ from that at the Stable Equilibrium Point (SEP) and, when the equations of motion are formulated with respect to a swing machine, the difference will be allocated to the swing bus resulting in a substantial mismatch. Thus the system is clearly not in equilibrium as the swing machine is accelerating relative to the rest of the system.

Equation (7), the re-written equilibrium characterization obtained from (4), shows that all absolute accelerations are equal or that the relative accelerations are zero.

$$(P_i - P_{ei}) / M_i = P_{COA} / M_T \quad i=1,\ldots,n-1 \tag{7}$$

The system is thus in synchronous equilibrium although it may not be in frequency equilibrium. The significance of defining the equilibrium condition in terms of (7) is that the relative accelerations are zero and the problem of a large mismatch at the reference bus is therefore eliminated.

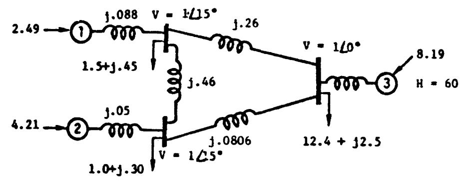
THE CONSERVATIVE NATURE OF LYAPUNOV METHODS

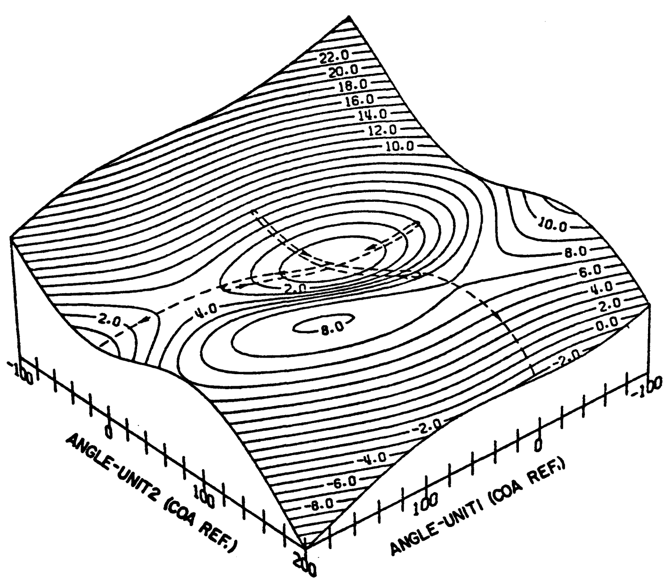
The reason for the conservative nature of the Lyapunov methods has until recently received little attention, probably because this has been widely accepted as an inherent characteristic of the method. For the particular case of the transient energy function, the conservativeness can be explained by separating the transient energy into kinetic energy and potential energy components and generalizing the well-known mechanical analogy to more than two machines. The multi-machine stability problem may be visualized as a ball rolling on the potential energy surface in multi-dimensional space. The coordinates at this space are the rotor angles and the post-fault steady state forms the minimum point of a multi-dimensional bowl. Depending upon the total kinetic plus potential energy of the ball at the time of fault clearing, the ball can either escape from the bowl over a saddle (i.e., an unstable case) or it can continue to oscillate within the bowl (i.e., a stable case). If the motion of the ball is undamped, it will continue to oscillate indefinitely and for a conservative estimate of stability it is assumed that the ball may at some time approach and escape over the lowest saddle point. (The often repeated argument that the system will always, given enough energy, escape over the lowest saddle point is suspect though, as this implies that the Lyapunov analysis yields necessary as well as sufficient conditions.) However, when this criteria is applied to power system stability the results are so conservative that they are of little practical value.

The reason for the conservative nature of the predicted stability region is that even though the system trajectory (i.e., the ball in the mechanical analogy) may ultimately approach the lowest saddle point, it may not do so in a reasonably finite time. A system that remains in synchronism during the first few swings is generally defined as transiently stable and critical clearing times are estimated on this basis. For those cases in which the system trajectory does not approach the lowest saddle point during the period of interest the Lyapunov method is conservative.

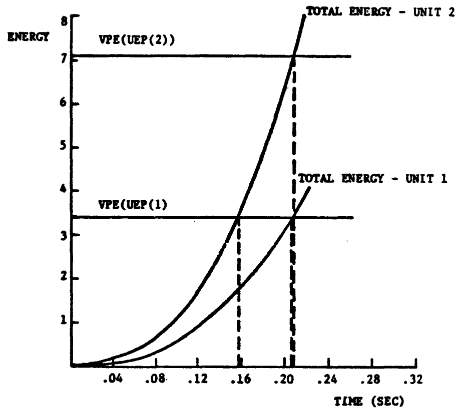
Further insight into the conservativeness problem can be obtained by realizing that the saddle points of the potential energy function can be uniquely associated with the different possible combinations of unstable generators [6], and that the consideration of the minimum saddle point, therefore, corresponds to assuming that the total transient fault energy contributes to causing the most weakly coupled group of generators to be unstable. This assumption is completely at odds with observable power system behavior since it is well known that the generators which tend to be unstable are influenced primarily by the fault location. In particular, in large systems it is possible to have very weakly coupled generators which remain stable because they are remote from the fault location. The existence of such cases explains why Lyapunov's results for large systems have been extremely conservative [7]. On the other hand, in small 3 or 4 machine studies, reasonable results are often obtained, because there is a much greater chance that the most weakly coupled generators will be unstable for any fault location.

These concepts can be reinforced by means of an illustrative example. Consider the three machine system shown in Figure 1. The potential energy surface for this system is shown in Figure 2 as are the actual stable and unstable system trajectories (determined from a step-by-step simulation) for two fault locations, buses 1 and 2. The stable cases were cleared without line switching at t=0.20 seconds and the unstable cases at t=0.22 seconds. In the mechanical analogy these trajectories correspond to the tracks of a ball rolling on the potential surface, where the effect of the fault is to give the ball (the system) an initial angle position (and corresponding potential energy) and an initial momentum vector (and corresponding kinetic energy). The stable equilibrium point, from which the trajectories in Figure 2 originate, is a local minimum of the potential energy surface where, by construction, the potential energy is zero.





For the unstable cases it can be seen that the ball escapes through a saddle point of the potential energy function. The ball escapes when it receives just enough energy from the fault condition to reach the saddle point that corresponds to the particular fault and pass through it. This is illustrated in Figure 3 where, for each of the two fault cases, the value of the total energy gained during the fault-on period is plotted as a function of time. The value of the potential energy at each of the saddle points (unstable equilibrium points) is also shown. For a particular fault case, the time at which the total energy becomes equal to the potential energy at the corresponding saddle point (the critical energy) is the critical clearing time. From Figure 3 the critical clearing time is, for each of the two fault cases, approximately 0.21 seconds. In the Lyapunov method however, the critical energy is taken to be the minimum value of unstable equilibrium point energies regardless of fault location. This corresponds in the example to UEP(1). From Figure 3 it can be seen that this results, for a fault-on Unit 2, in the very conservatively predicted critical clearing time of approximately 0.16 seconds.



The previous explanation of the conservativeness of the Lyapunov method naturally leads to the conclusion that a more realistic criterion for determining transient stability should be based upon considering the unstable equilibrium point corresponding to the generators which actually go unstable for a particular fault. That is, the critical energy should correspond to the actual boundary of separation rather than the weakest one.

The modified transient energy method proposed, which incorporates this key idea, consists of the following steps:

Step 1: Determine the actual boundary of separation for a particular fault location.

Step 2: Calculate the unstable equilibrium point corresponding to the boundary determined in Step 1 and calculate the potential energy at that point. This is the critical energy V_c.

Step 3: Calculate the system trajectory during the fault-on period and compute the total system energy as a function of time.

Step 4: If the total system energy at the time of fault clearing (from Step 3) is less than V_c the system is considered stable; if greater, the system is considered unstable. The time at which the total system energy becomes equal to V_c is the critical clearing time.

While the analytical approach developed for Step 1 as well as that for incorporating the effects of transfer conductances is discussed in the following two sections, several intermediate results are given here in order to illustrate the improved accuracy provided by this procedure. For these tests transfer conductances were neglected and the boundaries of separation for a particular fault location were determined by simulation studies. The 10 unit 39 bus system used is described later in the Evaluation section.

The critical clearing times for four fault locations (faults on generator terminals, cleared without line switching are shown in Table 1. In this table the entry "corresponding UEP" is the result predicted using the proposed procedure while the "closest UEP" result was obtained using the basic Lyapunov method. The critically stable and unstable clearing times determined by simulation are also shown. A comparison of these results shows that a substantial improvement in the prediction accuracy is obtained when the basic analytical method is modified to account for fault location.

Table 1
Critical Clearing Times for 10 Unit 39 Bus System

	Critical Clearing Times			
	Simulation		Transient Energy Method	
Fault Location	Stable	Unstable	Corresponding UEP	Closest UEP
Bus 31	.28	.30	.28	.23
Bus 32	.30	.32	.29	.22
Bus 35	.34	.36	.33	.29
Bus 38	.18	.20	.18	.18

ACCOUNTING FOR TRANSFER CONDUCTANCES

The major difficulty in the analysis of systems with transfer conductances is that a closed form expression for the total system energy cannot be obtained. Many previous researchers have neglected transfer conductances on the basis that these are small. However, the transfer conductances, i.e., the real part of the off-diagonal elements of the reduced bus admittance matrix, depend not only on the transmission line resistances but also on loads modeled as fixed impedances. Therefore, these terms can be quite large and in general cannot be neglected.

The integral terms in the transient energy function which arise when transfer conductances are not neglected are, from (6):

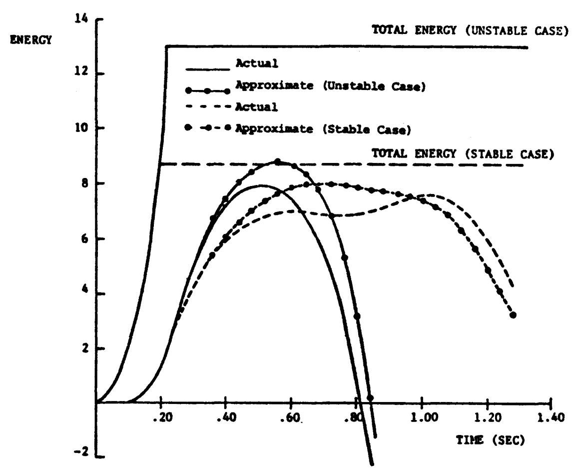
$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij} \cos \theta_{ij} \, d(\theta_i+\theta_j) \triangleq \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} I_{ij} \qquad (8)$$

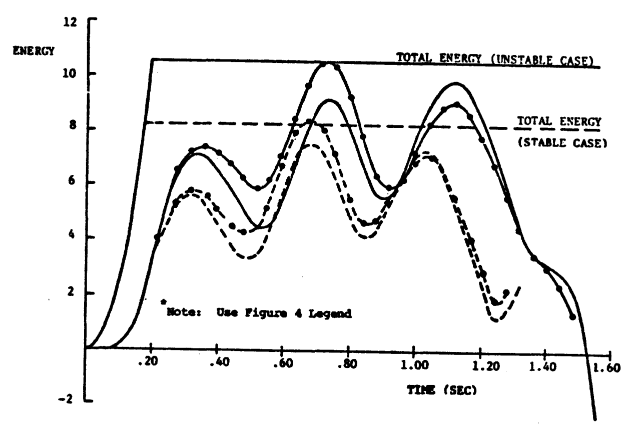
These terms represent energy dissipated in the transfer conductances and are naturally in the form of an integral which is path dependant. The correct path is that of the actual system trajectory and, when it is known, the terms of (8) can be very effectively evaluated numerically using the trapezoidal rule [9]. In the modified transient energy method the only step where an actual trajectory is (or needs to be) known is Step 3, however, where the fault trajectory is used to obtain the total energy which the system gains during the fault-on period.

In order to obtain the critical energy V_c, which is the energy at the UEP corresponding to the actual boundary of separation, a linear trajectory in the angle space is assumed. This allows the terms in (8) to be analytically evaluated between the limits θ^s and θ^u with the expression given in (9).

$$I_{ij} = D_{ij} \cdot \frac{\theta_i^{\,u}+\theta_j^{\,u}-\theta_i^{\,s}-\theta_j^{\,s}}{\theta_i^{\,u}-\theta_j^{\,u}-\theta_i^{\,s}+\theta_j^{\,s}}\left[\sin\theta_{ij}^{\,u} - \sin\theta_{ij}^{\,s}\right] \tag{9}$$

Figures 4 and 5 illustrate typical errors in the potential energy which this approximation introduces. In these figures the actual (as computed numerically in a simulation) and approximate potential energy variations as a function of time along the critical stable and unstable trajectories are shown for two different fault cases. In the former case a first swing instability occurs when the fault is cleared by line switching at t=0.22 seconds. In the latter case, although severe intermachine oscillations exist, the approximation is quite acceptable. Incidentally, because the total system energy is constant after fault clearing as shown, this figure dramatically illustrates the exchange of system potential and kinetic energy that fundamentally characterizes the multi-swing behavior.





DETERMINING BOUNDARIES OF SEPARATION

In addition to the generation parameters and prefault loading condition, which are independant of fault location, three factors determine the boundary of separation in a particular unstable case: the fault system trajectory; the post fault network; and the switching time. For the case of first swing instability, the effect of the fault system trajectory is dominant. Roughly speaking, in these cases the system begins to split up during the fault-on period and the synchronizing capability of the post fault network is such that the unit grouping pattern established during this period is never significantly altered. Generally, when this is the case, the switching time is not a significant factor either. Alternatively, when the post fault network does have a major impact on the ultimate boundary of separation, which is true in multi-swing cases, then the switching time is significant as well, so these two factors tend to go together.

The approach to determining first swing boundaries is considered first. It relies primarily on a fault trajectory approximation and on several key properties of the potential energy function. The estimated system fault trajectory is obtained by explicitly integrating twice a simple approximation for the unit accelerations. Representing the center of angle unit accelerating powers of the faulted system by f_i (i.e., f_i = the right hand side of (4)), the form of the approximation is given in (10).

$$f_i \approx a_i + b_i \cos \eta t \tag{10}$$

This form was initially chosen based on careful observation of simulation results of the 10 and 20 unit test systems described later in the evaluation section. Computational details involved in determining the unknown constants a_i, b_i, i=1,2,...n and the frequency η are omitted here; basically, two power flow solutions are utilized [9]. The first, at the instant of fault application, fixes the vectors $\underline{a}$ and $\underline{b}$ for a given frequency η. The second, along an approximate trajectory shortly after the fault, is used to compute η. Angles obtained from this fault trajectory approximation are given in Table 2 for a particular case on the 10 unit system. Also shown for comparison are the actual angles and those obtained by assuming a constant acceleration during the fault. The cosine approximation errors illustrated in Table 2 are among the worst obtained for numerous fault cases.

Table 2
Comparison of Fault Trajectory Angles at t=0.4 Sec.
(Fault on Bus 15, 10-Unit System)

Unit	Actual	Cosine Approximation	Constant Acceleration Approximation
1	-38.0	-38.1	-40.8
2	55.7	55.4	64.1
3	63.2	63.2	73.1
4	98.5	98.8	114.7
5	91.2	91.0	85.7
6	95.0	95.7	98.7
7	100.7	101.2	111.3
8	43.9	42.4	55.6
9	71.7	71.5	75.3
10	8.2	9.4	1.7

The potential energy function is a first integral of the system differential equations written with respect to the center of angle. While the approximation introduced previously is used to evaluate the energy at a particular point in the angle space, the potential energy gradient is just the negative of the vector of accelerating powers calculated using the power flow equations (4). The power mismatch function F(θ) defined in (11) can therefore be interpreted as the euclidian norm squared of this gradient.

$$F(\theta) = \sum_{i=1}^{n} f_i^{\,2}(\theta) \tag{11}$$

The scalar function F(θ) is a measure of closeness to an equilibrium point. Indeed, this property is the basis of minimization techniques for calculating equilibrium points [8]; in this project a variation of the Davidon-

Fletcher-Powell (DFP) method has been used for this purpose.

Second order curvature information of the potential surface is contained in (the negative of) the Jacobian matrix. In the region containing the SEP the surface is convex and is referred to as the principle region; its closed boundary is the principle singular surface [5]. This is the boundary of the region of dynamic stability that corresponds in a two machine system to the peak of the power transfer curve. A load flow algorithm based on the minimization of F (θ) will, initialized inside the principle singular surface, converge to the SEP. Initialized outside, it will (almost always) converge to a UEP.

The basic procedure for determining first swing boundaries of separation, which utilizes these properties of the potential energy and the fault trajectory approximation, consists of the following steps:

Step 1: For the given fault, determine the fault trajectory approximation parameters a_i, b_i, $i = 1,2,...n$ and η.

Step 2: Calculate the post fault reduced admittance matrix and compute the post fault stable equilibrium point θ^{SEP2}.

Step 3: Using the approximation of Step 1, find the time at which the (post-fault) power mismatch function $F(\theta)$ is a maximum along the fault trajectory. The angles at this time are θ^{SS}: this is (very close to) the intersection point of the fault trajectory with the principle singular surface.

Step 4: Construct the vector $\theta^{SS} - \theta^{SEP2}$ and normalized it to form the direction vector h.

Step 5: Solve the one dimensional minimization problem

$$\min_{\chi > o} F(\theta(\chi)) \stackrel{\Delta}{=} F(\theta(\chi^*))$$

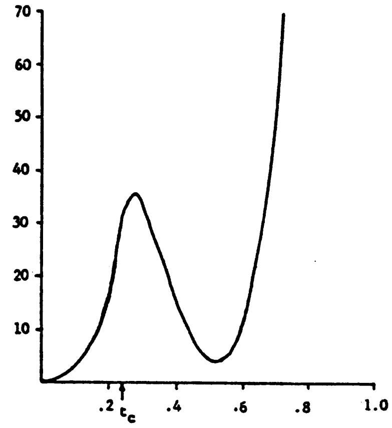
where $\theta(\chi) \stackrel{\Delta}{=} \theta^{SS} + \chi \cdot h$

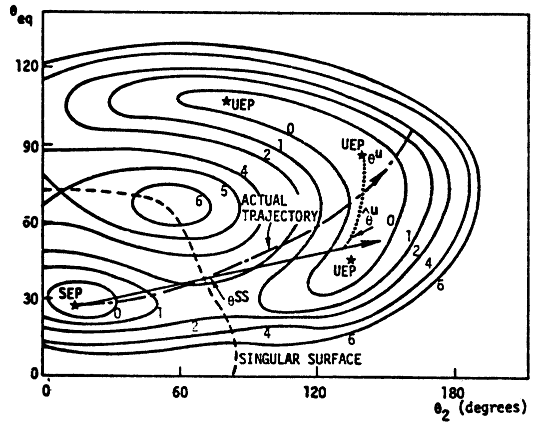
and $\theta(\chi^*) \stackrel{\Delta}{=} \hat{\theta}^u$

Step 6: With $\hat{\theta}^u$ as a starting point, use the DFP minimization load flow technique to obtain the unstable equilibrium point θ^u. θ^u characterizes the first swing boundary of separation and the potential energy at that point is the critical energy for the particular fault case.

Figures 6 and 7 illustrate this procedure. In Figure 6 $F(\theta)$ is plotted versus time along an actual (simulation) critically unstable trajectory for a case on the 10 unit system. The relative maximum shown corresponds to θ^{SS} in the procedure, while the relative minimum occurs with the trajectory passing by the UEP of interest. A contour plot, basically a slice out of the n-1 dimensional surface, of $F(\theta)$ is shown in Figure 7 for the post-fault network of this fault case. The actual unstable trajectory, cleared at θ^{SS} for illustrative purposes (slightly but insignificantly beyond the critical clearing point in this case), is also shown.

The fault trajectory approximation is quite accurate and its intersection with the principle singular surface θ^{SS}, determined in Step 3, is shown in Figure 7. The basic idea of this step, which typically requires several cubic interpolations and corresponding power flow solutions, is to let the fault trajectory run long enough for the effects of the faulted system to become





established. (A power flow solution is a single evaluation of the RHS of Equation (4) with a given set of angles.) The direction vector formed in Step 4 is also illustrated, as is the point of minimum $F(\theta)$ along it $\hat{\theta}^u$, which is determined in Step 5. Again, one or two cubic interpolations (two power flow solutions each) are the major computations here. Finally, although it appears that $\hat{\theta}^u$ is very near a UEP other than θ^u due to the slice effect, the UEP calculation of Step 6 does in fact converge to θ^u as represented by the dotted line in Figure 7.

Determining the boundary of separation, that is the set of unstable generators, is more difficult when the synchronizing intermachine oscillations of the post-fault system are significant which is the case for multiple swing instabilities. A simple and promising approach to this problem has been developed that is based on the predictor/corrector type of integration scheme.

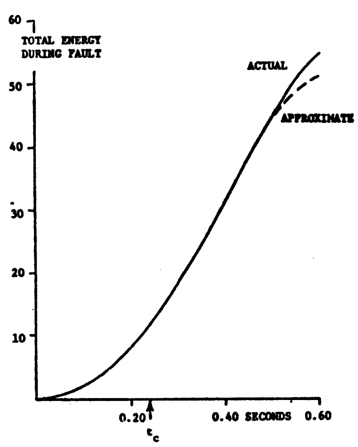
Consider the situation at the completion of Step 5 where the UEP estimate $\hat{\theta}^u$ has been determined. The assumption in searching along a straight line from the principle singular surface is that intermachine synchronizing torques due to the post fault network will not significantly affect the grouping pattern which the fault trajectory has established. The basic idea now is to treat the angles $\hat{\theta}^u$ as if they were the result of the prediction step in an integration step and to compute a correction step based on the post-fault power flow equations. Assuming that the fault is cleared at the principle singular surface, the initial unit accelerating powers and their time derivatives can be obtained from a power flow solution at that point and the speeds as obtained from the fault trajectory approximation. Using the accelerating powers computed by a power flow at $\hat{\theta}^u$ and an estimate for the time step obtained by assuming a linear variation of the norm of machine angles over a short time interval, a cubic polynomial in time can be fitted for the accelerating powers on each unit between the points θ^{SS} and θ. This can then be analytically integrated in order to complete the correction step [9].

Based on this correction step, a new direction vector is formed and Step 5 of the basic procedure is repeated. While the step taken can be quite large, in which case the predicted angles may be in absolute error with respect to time, the procedure effectively detects relative motion, i.e., the relative amount of synchronizing energy which each machine exchanges with the network. In the case of a first swing instability, the predictor/corrector step makes very little change in the initial direction vector assumed and the effect is to confirm the first UEP estimate.

DETERMINING SYSTEM ENERGY DURING FAULT

Direct methods of stability analysis are limited to stationary systems. For the power system transient stability problem they are applicable only to the post-fault network and it is still necessary to obtain the total energy which the system gains during the fault-on period. This has previously been accomplished by performing a step-by-step integration of the faulted system equations and simultaneously calculating the appropriate energy or Lyapunov functions.

While initially developed to aid boundary of separation determination, the fault trajectory approximation introduced in the previous section can be used to obtain this total energy very effectively. This is illustrated in Figure 8 where the actual (from a simulation) and the approximate energy gained during the fault is plotted for a typical case on the 10 unit system. The approximation is valid well beyond the critical clearing time. As a result the modified transient energy method reported here is truly a direct method in the sense that the stability assessment which it provides can be obtained without solving any differential equations.



EVALUATION

The basic approach to evaluation which has been adopted is to perform a reasonably comprehensive series of classical stability analyses using the accepted method of simulation and to compare the results with those obtained from the modified transient energy method. While these two techniques differ markedly, both can be used to calculate critical clearing times, so this familiar transient stability performance measure provides an explicit basis of comparison.

Two test systems have been used. The first is a 10 unit 39 bus system which is representative of the New England system and was studied in [7]. A line diagram, generator parameters and initial conditions are provided in the Appendix. The second is a 20 unit 118 bus IEEE test system which was also studied in [7]. Generator parameters and initial conditions for this system are in the Appendix. Complete data for both systems is in [9].

A range of faults were tested including faults on both generator and load buses, cleared with and without line switching. Considerable insight has been gained on the different characteristics of these systems, and many of the faults were chosen so as to create situations that critically test the method. Under these faults, the systems exhibited a wide range of stability behavior including single and multi-unit as well as first swing and multiswing instability.

Table 3 shows critical clearing times calculated directly along with those obtained via simulation for the 10 unit system. The first direct result corresponds to using the first swing boundary determination procedure in Step 1 of the proposed method while the second result corresponds to using, in addition, the predictor/corrector step. Table 4, similarly arranged, shows critical clearing times calculated for the 20 unit system.

Table 3
Direct Calculation of Critical Clearing Times for 10 Unit System

Line Tripped * - faulted bus	SIMULATION		DIRECT CALCULATIONS	
	stable	unstable	First Swing	Predictor/ Corrector
2*-3	0.24	0.26	0.235	0.225
4*-14	0.22	0.24	0.215	0.215
5*-11	0.20	0.22	0.21	0.21
15*-16	0.22	0.24	0.23	0.23
23-24*	0.18	0.20	0.175	0.175
25*-26	0.18	0.20	0.19	0.22
28-29*	0.04	0.06	0.06	0.06
31*	0.22	0.24	0.22	0.22
35*	0.24	0.26	0.25	0.25
37*	0.22	0.24	0.23	0.23

Table 4
Direct Calculation of Critical Clearing Times for 20 Unit System

Line Tripped * - faulted bus	SIMULATION		DIRECT CALCULATIONS	
	stable	unstable	First Swing	Predictor/ Corrector
2	0.18	0.20	0.18	0.18
3	0.46	0.48	0.50	0.39
4	0.32	0.34	0.31	0.33
5	0.38	0.40	0.36	0.36
9	0.46	0.48	0.46	0.00
13	0.34	0.36	0.33	0.35
18	0.32	0.34	0.33	0.35

Details of individual cases are provided in [9]. The general conclusion drawn from these results is that, for an instability that is essentially first swing, the basic procedure yields results of practical significance. The predictor/corrector is effective in detecting the presence of major post fault synchronizing oscillations and determining the relative unit motions that therefore occur. In some cases this additional step is sufficient for obtaining the correct multiswing boundary whereas in other cases it is not.

CONCLUSION

This paper has contributed to the clear understanding and effective removal of several major difficulties that have seriously impeded the application of direct methods to the power system transient stability problem for many years. By accounting for the effects of fault location and transfer conductances on the system's transient behavior, the approach to transient energy stability analysis developed has provided results which have practical meaning and importance. The method developed is able to efficiently and directly calculate critical clearing times associated with first swing transient instability without explicitly solving any differential equations. The results obtained are practical in the sense that they are sufficiently accurate and they are derived using power system models which, although simplified, are still widely used by the industry.

As a result of this progress further research is planned in three major areas. Firstly, continued development of techniques for analyzing multiple swing instability and for unstable equilibrium point classification is planned that will increase the reliability of the basic approach. Secondly it is intended that the algorithms be adapted to exploit network sparsity in order to further increase their computational efficiency. Thirdly, an investigation will be made of the implications and effects of modeling assumptions on transient energy stability analysis including those of variations in field and internal machine voltages, damping, and non-linear loads.

With further development, the transient energy method is likely to yield new and powerful computational tools for stability analysis that have potentially valuable applications in system planning and operation. For system planning it could be employed as a screening tool which would be useful for preliminary and/or long range studies by allowing a large number of alternative plans to be studied in a fast and approximate manner. For system operation, methods based on transient energy can be used for on-line assessment and enhancement of transient stability.

ACKNOWLEDGEMENT

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REFERENCES

[1] P.C. Magnusson, "The Transient - Energy Method of Calculating Stability", AIEE Transactions Vol. 66, 1947, pp. 747-755.

[2] P. D. Aylott, "The Energy-Integral Criterion of Transient Stability Limits of Power Systems", Proceedings IEE (London), Vol. 105(C), July 1958, pp. 527-536.

[3] M. Ribbens-Pavella, "Critical Survey of Transient Stability Studies of Multimachine Power Systems by Liapunov's Direct Method", Proceedings of the 9th Annual Allerton Conference on Circuits and Systems Theory, October 1971, pp. 751-767.

[4] A. A. Fouad, "Stability Theory-Criteria for Transient Stability", Engineering Foundation Conference Systems Engineering for Power, Henniker, New Hampshire, 1975.

[5] C. J. Tavora and O. J. M. Smith, "Stability Analysis of Power Systems", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, May/June 1972, pp. 1138-1145.

[6] F. S. Prabhakara and A. H. El-Abiad, "A Simplified Determination of Transient Stability Regions for Liapunov Methods", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-94, March/April 1975, pp. 672-689.

[7] G. W. Bills, et.al., "On-Line Stability Analysis Study" RP90-1 Report for the Edison Electric Institute, October 12, 1970.

[8] A. M. Sasson, C. Trevino and F. Aboytes, "Improved Newton's Load Flow Through a Minimization Technique", IEEE Transactions, Vol. PAS-90, 5, pp 1974-1981, September-October, 1971.

[9] T. Athay, R. Podmore and S. Virmani, "Transient Energy Stability Analysis". Engineering Foundation Conference-System Engineering for Power, Henniker, New Hampshire, August 21-26, 1977.

APPENDIX

Table 5
Generator Parameters and Initial Conditions for 10 Unit 39 Bus System (100 MW Base)

Unit No.	H pu	X'_d pu	E pu	Angle (radians)
1	500.	0.006	1.0368	-0.1344
2	30.30	0.0697	1.1966	0.3407
3	35.80	0.0531	1.1491	0.3417
4	38.60	0.0436	1.0808	0.2985
5	26.00	0.132	1.3971	0.5088
6	34.80	0.05	1.1910	0.3376
7	26.40	0.049	1.1394	0.3499
8	24.30	0.057	1.0709	0.3070
9	34.50	0.057	1.1368	0.5335
10	42.0	0.031	1.0929	-0.0087

Table 6
Generator Parameters and Initial Conditions for 20 Unit, 118 Bus System (100 MW Base)

Unit (Bus) No.	H pu	X'_d pu	E pu	Angle (radians)
1	8.0	0.0875	0.9875	-0.2495
2	22.0	0.0636	1.0955	0.3693
3	8.0	0.1750	1.1808	-0.1688
4	14.0	0.1000	1.1280	0.1660
5	26.0	0.0538	1.0525	0.1697
6	8.0	0.0875	0.9778	-0.2492
7	8.0	0.0875	1.0005	-0.2805
8	8.0	0.0875	1.0025	-0.4251
9	8.0	0.0875	1.0283	-0.4139
10	12.0	0.1167	1.2072	0.0463
11	10.0	0.1400	1.1351	0.0205
12	12.0	0.1167	0.9793	0.1006
13	20.0	0.0700	1.1491	0.2091
14	20.0	0.0700	1.0852	0.2108
15	30.0	0.0467	1.0342	0.2367
16	28.0	0.0500	1.1266	0.1967
17	32.0	0.0438	1.0423	0.4337
18	8.0	0.0875	1.0425	-0.0069
19	16.0	0.0875	1.1511	0.1641
20	15.0	0.0467	0.9957	-0.2730

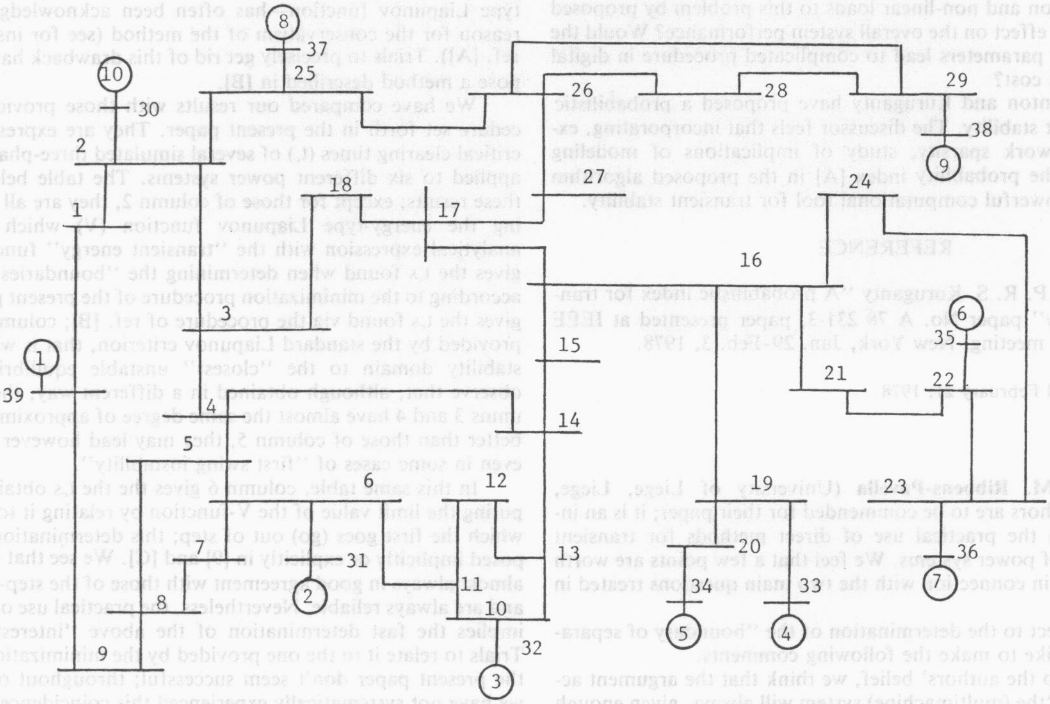
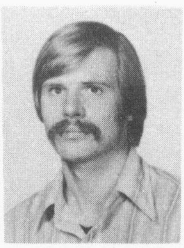


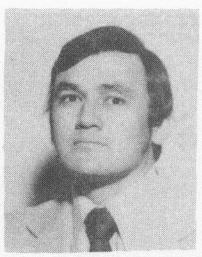
Figure 9 10 Unit 39 Bus New England Test System



Thomas M. Athay has obtained three degrees in electrical engineering; the B.S. in 1974 from Michigan Technological University, and the M.S. and Engineer's degrees both in 1976 from Massachusetts Institute of Technology.

While at MIT he served as both a Teaching Assistant and a Research Assistant, the latter assignment being with the Electronics Systems Laboratory helping in the development of decentralized strategies for the control of interconnected power systems.

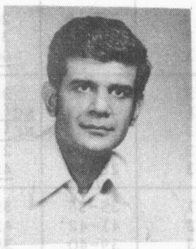
Since joining SCI in 1976, Mr. Athay has been working with the Power Systems Analysis Division. He has been working primarily on the development of direct methods for transient stability analysis and on the development of advanced automatic generation control strategies and algorithms. These two projects are being conducted for ERDA.



Robin Podmore was born in New Zealand on July 20, 1947. He received a Bachelor of Engineering degree with first class honors, from the University of Canterbury, Christchurch, New Zealand, in 1968 and a Ph.D. from the same university in 1973.

From 1973 to 1974 he was a Post-Doctoral Fellow with the Power System Research Group at the University of Saskatchewan, Canada. In May 1974 he joined Systems Control, Inc. He is presently Manager of the Power Systems Analysis Division and is responsible for supervision and management of R&D activities in the areas of computer applications for power system planning, operation and control. He has managed and worked as principle investigator on several major research and development projects sponsored by Electric Power Research Institute and U.S. Department of Energy. His principle project assignments have included development of dynamic equivalents and analytical techniques for use in transient stability studies, development of new techniques for on-line monitoring and rating of underground cables, and development and implementation of new methods for automatic generation control.

Dr. Podmore is a member of the PES System Dynamic Performance Subcommittee and is a Registered Professional Engineer in the State of California. He has authored or co-authored approximately 20 publications in the areas of network analysis, transient stability, economic dispatch, automatic generation control and security assessment.



Sudhir Virmani obtained his B. Tech (Hons.) degree from the Indian Institute of Technology and the M.S. and Ph.D. degrees from the University of Wisconsin.

He is currently Senior Project Manager at Systems Control, Inc. where he is working the areas of power system dynamics and control. Prior to joining Systems Control, Inc., Dr. Virmani worked at American Electric Power Service Corporation and Stagg Systems, Inc. in New York.

Discussion

R. P. Sood (Department of Electrical Engineering, Regional Engineering College, Kurukshetra, India): The authors are to be highly commended for their efforts to improve upon existing methods of evaluation of transient stability of large multimachine power systems using direct methods.

The authors have experimentally demonstrated a better accuracy in results by accounting for the effects of fault location and transfer conductances on the system's transient behaviour for evaluating the critical clearing time associated with the first swing transient stability. Would the authors please express their opinion for assessing the influence of inclusion of other machine parameters such as pole saliency, damping or fast governor action and non-linear loads to this problem by proposed algorithm and the effect on the overall system performance? Would the inclusion of these parameters lead to complicated procedure in digital computations and cost?

Recently Billinton and Kuruganty have proposed a probabilistic index for transient stability. The discussor feels that incorporating, exploitation of network sparsity, study of implications of modeling assumption and the probability index [A] in the proposed algorithm could lead to a powerful computational tool for transient stability.

REFERENCE

[A] R. Billinton, P. R. S. Kuruganty "A probabilistic index for transient stability" paper No. A 78 231-3, paper presented at IEEE PES, Winter meeting, New York, Jan. 29–Feb. 3, 1978.

Manuscript received February 21, 1978.

J. Sabatel and **M. Ribbens-Pavella** (University of Liege, Liege, Belgium): The authors are to be commended for their paper; it is an interesting work on the practical use of direct methods for transient stability analysis of power systems. We feel that a few points are worth to be brought out in connection with the two main questions treated in the paper.

A. With respect to the determination of the "boundary of separation", we would like to make the following comments.

1. Contrary to the authors' belief, we think that the argument according to which "the (multimachine) system will always, given enough energy, escape over the lowest saddle point", has seldom been used; it is well known, instead, that the consideration of this saddle point-which provides a theoretical estimate of the stability domain for the energy-type Liapunov function—has often been acknowledged as the main reason for the conservatism of the method (see for instance p. 176 of ref. [A]). Trials to precisely get rid of this drawback have led us to propose a method described in [B].

We have compared our results with those provided by the procedure set forth in the present paper. They are expressed in terms of critical clearing times (t_c) of several simulated three-phase short circuits applied to six different power systems. The table below contains all these results; except for those of column 2, they are all obtained by using the energy-type Liapunov function (V) which has the same analytical expression with the "transient energy" function: column 3 gives the t_cs found when determining the "boundaries of separation" according to the minimization procedure of the present paper; column 4 gives the t_cs found via the procedure of ref. [B]; column 5 gives the t_cs provided by the standard Liapunov criterion, that is when relating the stability domain to the "closest" unstable equilibrium point. We observe that, although obtained in a different way, the results of columns 3 and 4 have almost the same degree of approximation: generally better than those of column 5, they may lead however to large errors, even in some cases of "first swing instability".

In this same table, column 6 gives the the t_cs obtained when computing the limit value of the V-function by relating it to the machine(s) which the first goes (go) out of step; this determination has been proposed implicitly or explicitly in [9] and [C]. We see that these results are almost always in good agreement with those of the step-by-step method and are always reliable. Nevertheless, the practical use of this procedure implies the fast determination of the above "interesting" machine. Trials to relate it to the one provided by the minimization procedure of the present paper don't seem successful; throughout our simulations, we have not systematically experienced this coincidence. We would appreciate the author's comments on this point.

TABLE

Network	Location of the disturbance at bus-bar	1 Machine which first goes out of step	2 t_c according to the step-by-step procedure (10^{-2}s)	3 t_c according to the " Transient Energy" procedure (10^{-2}s)	4 t_c according to reference [B] (10^{-2}s)	5 t_c according to the standard Liapunov criterion (10^{-2}s)	6 t_c provided via the machine of column 1 (10^{-2}s)	7 t_c as in column 6 and accounting for transfer conductances (10^{-2}s)
3-machine system	3	3	18-19	20	17	17	21	20
	2*	2	32-33	34	34	31	33	31
	1*	1	37-38	>60	39	37	36	38
5-machine system	11A	3	41-42	38	38	29	38	33
	1A	1	42-43	NC(1) 50	43	27	34	40
	1B*	3	38-39	NC(1) 50	43	28	34	34
	5B	3	18-19	21	18	13	18	15
	3A*	3	28-29	45	45	21	26	20
7-machine system	1	1	35-36	35	35	26	35	34
	2	2	41-42	42	42	34	36	36
	3	3	39-40	39	39	28	39	38
	4*	5	50-51	52	50	30	47	42
	5	5	35-36	36	36	26	36	36
	6	6	52-53	53	53	39	50	50
9-machine system	3	3	26-27	27	27	21	27	27
	11*	1	46-47	NC(1) 56	39	39	39	40
	24	7	73-74	75	75	53	67	71
15-machine system	Q1*	Q1	41-42	45	38	28	38	37
	C1	C1	41-42	46	46	38	38	43
	M1	M1	36-37	38	38	33	38	34
40-machine system	COO1	COO1	32-33	31	18	15	24	>43
	AWIR3	AWIR3	24-25	>35	12	10	24	>30
	MERC1	DOEL1A	25-26	NC(1)	10	8	20	>37

(1) Non convergence of the algorithm
* Multiple swings instability case

2. A subsidiary question related to the procedure followed in the present paper is the amount of computing time needed for the minimization via the Davidon–Fletcher–Powell method. Could the authors give an estimate of the increase of the computing time due to this additional step? (Let us note that the procedure of ref. [B] is quite simple and needs an almost negligible computing time).

3. Expression (10) of the paper implies the consideration of the same frequency η for all the systems's machines: this approximation does not seem to us reliable enough; moreover, it could be easily replaced by a more reliable one, without significant computational effort (see, for instance, the Taylor's expansion series as used in ref [B]). This is a minor remark, nevertheless.

B. The second important question concerns a trial to taking into account transfer conductances in an approximate way, as their strict analytical consideration seems quite hard—at least in the "energy-type" Liapunov function case.

The procedure proposed consists in (1) evaluating numerically terms (8) of the paper using the trapezoidal rule in order to obtain the value of the total energy during the fault-on period, and (2) linearizing the system's trajectory in the angle space between the limits θ^s and θ^u.

Conceptually, unlike to (1), the approximation of the (2) does not seem so reliable, as the trajectory followed by the system in the angle space could be quite non-linear, especially in the neighbourhood of θ^u, and in "multiple swing instability" cases.

We have performed computations of the t_cs provided when accounting for the transfer conductances and when adopting the procedure of column 6 for the determination of the limit value of the Liapunov function. The results are given in column 7. Their comparison with those of column 6 shows that they are slightly less good in all cases except for the 40-machine system where they are too optimic and therefore unreliable.

In conclusion, the ideas proposed in this paper are quite attractive; undoubtedly, they deserve further work with good chances of succeeding to relaxe stability conditions of direct methods.

REFERENCES

[A] M. Ribbens-Pavella and J. L. Horward, "Practical considerations of the transient stability power system studies by Liapunov's direct method", *Revue E,* Vol. VII, No. 7. 1973, pp. 167–179.

[B] M. Ribbens-Pavella et al., "Transient stability analysis by scalar Liapunov functions: recent improvements and practical results", *Coll. des Public. de la Fac. des Sc. Appl., Univ. of Liege,* No. 67, 1977, pp. 75–100.

[C] O. Saito et al., "Security Monitoring Systems Including Fast Transient Stability Study Using Pattern Recognition", *Proceedings of the 5th PSCC,* Cambridge, Sept. 1975, paper 3.2/2.

Manuscript received February 23, 1978.

T. Athay, R. Podmore, and **S. Virmani:** We would like to thank the discussors for their interest in our paper. In response to R. P. Sood, the extension of direct methods to more detailed system representations appears to be a formidable task. Our intent to investigate this area was indicated in the Conclusion, but we cannot at this time answer your questions on performance effects or computational complexity.

H. Sabatel and M. Ribbon-Pavella from the University of Liege have prepared a detailed and interesting discussion. They comment that the Lyapunov stability boundary which is defined by the potential energy at the lowest UEP has often been acknowledged as the main reason for the conservatism of direct methods that utilize this particular boundary. In a general sense we agree, i.e., the fundamental reason that stability assessments obtained from the second method of Lyapunov are conservative is simple that this theory provides sufficient but not necessary conditions for stability. In our paper however this basic fact is explored in terms of observed power system behavior and in this context reasons for the conservativeness of direct methods are identified which are more specific and which provide a cogent understanding of the problem that has not been widely shared. We feel that this understanding is an important result of our research and a significant contribution to the field.

The discussors compare the basic method proposed in the paper for first swing stability analysis with a new version of their own direct method by providing a table of critical clearing times for numerous fault cases. (We would like to thank them for responding to our request for an explanation of how these results were obtained as well as for providing the systems data used so that we could repeat some of these studies.) The main difference between the discussor's method and the one described in the paper is that the former neglects the effects of transfer conductances and obtains the critical energy in a different manner. A brief summary of the discussor's approach will be useful in replying to their discussion.

In order to determine the critical energy, the discussors evaluate the potential energy (VPE) at the following n "interesting point" [B]:

$$\delta_j = [\delta^s_{1n}\ \delta^s_{2n} \ldots s_j \cdot \pi - \delta^s_{jn} \ldots \delta^s_{jn} \ldots \delta^s_{n-1,\,n}]^T, \quad j = 1,2,\ldots,n-1,$$

and

$$\delta n = [-s_n\,\pi - \delta^s] = [-s_n\pi - \delta^s_{1n} \ldots -s_n\pi - \delta^s_{n-1,\,n}]$$

where

$$s_j = \text{sign}\,(P_j - M_j/M_T P_T), \quad j = 1, 2, \ldots, n.$$

In the discussors' previous method the interesting point with the lowest potential energy was used as a starting point in a load flow calculation to obtain the critical unstable equilibrium point. The new version used by the discussors to obtain the results in column 4 applies an additional step in which the Euclidian state-space (normed) distances between a certain point on the fault trajectory $[\delta, \omega]$ and the n interesting points $[\delta_j, 0]$ defined above are calculated. The smallest state-space distance determines the starting point used in the equilibrium point calculation. Although this modification indicates that the discussors recognize the importance of accounting for fault location in a direct method of stability analysis, we believe that their procedure is unlikely to be satisfactory for the reasons discussed below.

The most significant difference between the discussors' approach and the approach proposed in the paper is the procedure used for selecting the critical UEP. It is important to recognize that by considering a particular UEP we are inferring that the system generators will lose synchronism according to a particular grouping pattern. This grouping pattern may be defined by a cut set or a "boundary of separation" between the generating units. By restricting only one of the angles of the UEP to be in the neighborhood of $\pm\,\pi$ radians, the discussors implicitly assume that system will split into only two groups—a single machine and the rest of the system. Each of the "interesting points" are approximations to the UEPs associated with one of the n machines losing synchronism with the other n-1 machines. We have found many actual boundaries of separation that include more than one machine in an unstable group as well as multiple groups. For example, in the 6*-11 case of the 10 unit system cited in the paper, the UEP associated with the actual boundary of separation is known in Table 1 along with the UEP obtained using the discussor's procedure. Notice that the UEP which the critically unstable trajectory passes closest to corresponds to the three groups into which the system actually separates as shown by swing curves in Figure 1. (For the unstable case, the swing curves are not shown are essentially coherent with unit 6.) On the other hand, the UEP obtained using the discussor's method is associated with only two groups. This is an important point of distinction between the two methods.

In summary, our method attempts to obtain the UEP which is closest to the critically unstable trajectory and the critical energy is the potential energy at this UEP. In its current stage of development, our procedure for finding this critical UEP, i.e., for determining the actual boundary of separation, is almost always successful for first swing cases but less successful for multiple swing cases. More work is needed in this area, and we are currently investigating several refinements and extensions of the approach described in the paper.

The convergence difficulties which the discussors experienced with our approach can generally be avoided by a simple extension of the Fletcher–Powell method. This minimization procedure may converge to a singular surface surrounding a UEP which is of higher order than type one, so that the gradient of $F(\theta)$ is zero although $F(\theta)$ is non-zero. By simply moving the point away from the singular surface when this occurs, the minimum $F(\theta) = 0$ can be obtained.

In developing the set of programs used for the research reported herein, computational efficiency was not stressed. The reduced bus admittance matrix is computed and the basic calculations, including the calculation of a stable and unstable equilibrium point by minimizing $F(\theta)$ and potential energy evaluation(s), are performed using this full matrix. We are now developing a more powerful and efficient set of programs that exploit network sparsity and greatly reduce the number of trigonometric function evaluations. This development is not yet at a stage, however, where explicit time and storage requirements can be provided.

Regarding the cosine fault trajectory approximation, considerable work was done before this choice was made. Taylor series expansions with up to six even terms (odd terms are zero) were evaluated for

TABLE 1

GENERATOR	ANGLE (COA coordinates, degrees)	
	Critical UEP VPE = 9.31 ($\hat{t}_c$ = .21 sec)	Closest Interesting UEP VPE = 5.93 ($\hat{t}_v$ = .16 sec)
1	-44.7	-25.7
2	138.8	135.4
3	89.2	47.0
4	75.3	35.6
5	86.7	46.8
6	77.2	37.6
7	78.3	38.5
8	60.1	27.9
9	84.2	44.9
10	34.2	7.2

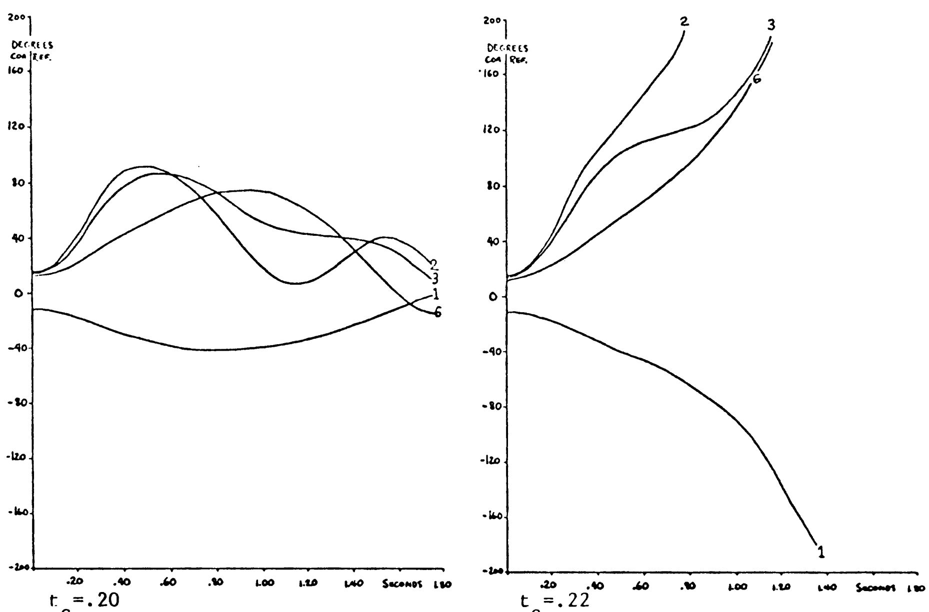


Figure 1

numerous cases, and the cosine approximation was found to be simpler and more accurate.

The discussors question the linear trajectory approximation used in accounting for transfer conductances. This is a simple approximation, although the errors introduced have been tolerable for very nonlinear trajectories, e.g., see Figure 5. The transfer conductances have a significant affect on system behavior, and that approximation is much more satisfactory than ignoring them completely. They also have a strong influence on the location and nature of equilibrium points, which is a separate issue from the evaluation of the potential energy at the UEP. For example, the approximation used by the discussors for determining interesting UEPs can be quite inaccurate when the transfer conductances are included.

In closing we would like to again thank the discussors for their efforts in preparing their comments.

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