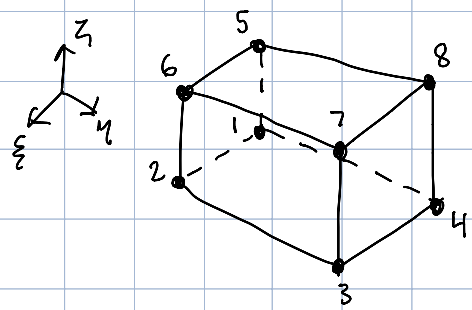
1. We start in fe_mainExplicit. We need to calculate the system-level vector m_system, which is sdof x 1. Therefore, there is a mass associated with each system-level dof. We use the function fe_calculateMass and provide type = direct_lumped.

2. The function fe_calculateMass goes one of two ways, depending on if embedded_constraint = true or not. For now, assume embedded_constraint ≠ true. In this case, we use fe_calculateMassDirectLumped and assume mesh_id = 0.

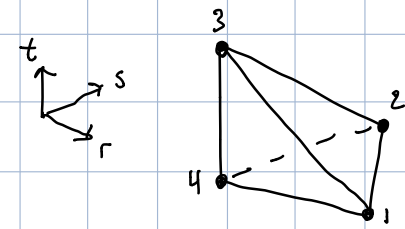
3. We extract node and element info for mesh[0]. Then we loop over each element and use fe_massLumped to calculate element-level vector m_element, which is 24x1 for a hex element. We then use fe_scatter_pbr to add the element-level m_element to the system-level m_system. Note that multiple elements contribute to the total mass at a single node.

4. In fe_massLumped we first extract the x,y,z coordinates for each node in the element under consideration. We then use fe_get_mats, with obj_interest = 0 to extract the material density rho. Next, we use fe_calVolume to calculate volume_element.

5. In fe_calVolume we use the nodal x,y,z coordinates of the element to calculate the volume. This is done by splitting up the hexahedron into five tetrahedrons.



Hexahedral Element



Tetrahedral Element

Pg. 172

The volume of a tetrahedron is

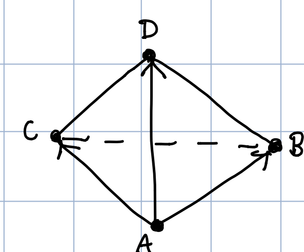
$$V = \frac{1}{6} \det \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & \underbrace{x_4 & y_4 & z_4}_{\text{coordinates of four nodes}} \end{bmatrix}$$

Pg. 599

Alternatively, the volume of a tetrahedron can be calculated as follows

$$V = \frac{1}{6} \left\| \underbrace{\vec{AB} \times \vec{AC} \cdot \vec{AD}}_{\text{scalar triple product}} \right\|$$

norm, meaning absolute value in this context



where

$\vec{AB} = (AB_1, AB_2, AB_3)$

$\vec{AC} = (AC_1, AC_2, AC_3)$

$\vec{AD} = (AD_1, AD_2, AD_3)$

or

determinant

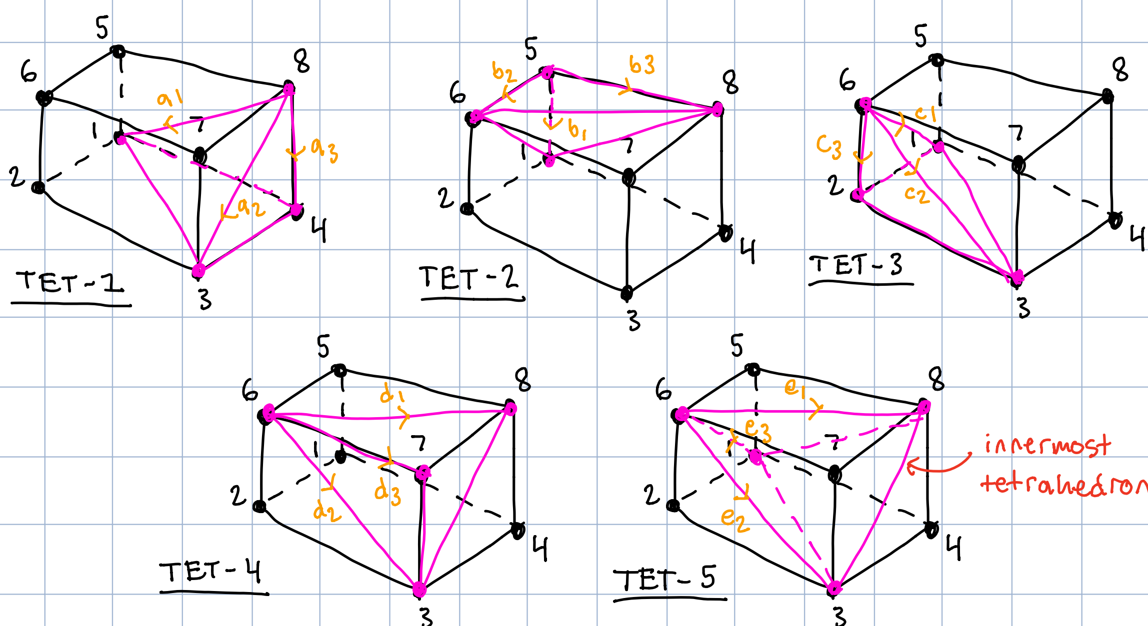
$$\text{Signed Volume} = \frac{1}{6}\begin{vmatrix} AB_1 & AB_2 & AB_3 \\ AC_1 & AC_2 & AC_3 \\ \underbrace{AD_1 \quad AD_2 \quad AD_3}_{\text{components of three vectors}} \end{vmatrix}$$

This means we need to take the absolute value.

Pg. 541

A hexahedron can be divided into five tetrahedrons.

Per EEMA,



6. In fe_massLumped we then calculate mass = rho × volume_element. Finally, we take the mass of the element (mass) and divide it by the number of nodes per element (nnel) and assign this value to each component of the vector m_element, which is 24x1.

Pg. 212

$$M_{IJ} = I\tilde{M}_{IJ} = I\int_{\Omega_0} \rho_0 N_I N_J \, d\Omega_0$$ consistent mass matrix

I and J represent the number of nodes in the mesh.

Pg. 39

$$M^D_{II} = \sum_J M^C_{IJ}$$ row-sum technique

consistent mass matrix

diagonal mass matrix or lumped mass matrix