Getting Started with Abaqus, 9.5.1 Bulk Viscosity

Abaqus Theory Guide, 2.4.5 Explicit Dynamic Analysis

$$p_1 = b_1 \, \rho \, c_d \, L^e \, \dot{\varepsilon}_{vol}$$

- $\dot{\varepsilon}_{vol}$: volumetric strain rate
- L^e: element characteristic length
- c_d: current dilatational wave speed
- ρ: current density
- b_1: damping coefficient (default value 0.06)
- p_1: bulk viscosity pressure

Abaqus Theory Guide, 1.2.1-8

strain rate

$$\dot{\varepsilon} = \frac{1}{2}\left(\frac{\partial \dot{u}}{\partial x} + \left[\frac{\partial \dot{u}}{\partial x}\right]^T\right)$$

where $\dot{u}$ is the velocity of the material currenty flowing thru point x in space

Abaqus Theory Guide, 1.4.3 Rate of Deformation and Strain Increment

strain rate or rate of deformation tensor

$$\dot{\varepsilon} = \frac{1}{2}\left(L + L^T\right)$$

$$\dot{\varepsilon} = \frac{1}{2}\left(\left[\frac{\partial v}{\partial x}\right] + \left[\frac{\partial v}{\partial x}\right]^T\right)$$

$\dot{\varepsilon}$: Commonly denoted D (symmetric matrix)

Abaqus Analysis User's Guide, vol 3, 22.7.2 Frequency Domain Viscoelasticity

rate of volumetric strain

$\dot{\varepsilon}_{vol} = \text{trace}\,(\dot{\varepsilon})$

LS-DYNA Theory Manual, 24.2 Bulk Viscosity

$\dot{\varepsilon}_{kk}$ is the trace of the strain rate tensor

ℓ is the characteristic length ($\sqrt{A}$ in 2D and $\sqrt[3]{\forall}$ in 3D)

LS-DYNA Theory Manual, 4.3 Hourglass Control

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\sum_{k=1}^{8} \frac{\partial \phi_k}{\partial x_i} \dot{x}_j^{\,k} + \frac{\partial \phi_k}{\partial x_j} \dot{x}_i^{\,k} \right) \qquad \text{(strain rate)}$$

for 8-node solid element

LS-DYNA Theory Manual, 16.3 Element Formulations

$\dot{\varepsilon}$ is the strain rate (or rate of deformation)

Dowling, pg. 210

$$\varepsilon_v = \frac{d\forall}{\forall} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

* Small-displacement definition per Boresi, pg. 93.