1. We have two major inputs to fe_newtonRhapson. First, we have the 3×1 vector nat_coord_nodes; which are the actual coordinates of the fiber element node in Ω_0. In fe_newtonRhapson, this becomes nat_coord. Second, we have the 8×1 vectors xcoord, ycoord, zcoord; which are the coordinates of the host element nodes in Ω_0.

2. We start by assembling the 24×1 vector coord; which contains the coordinates of the host element nodes in Ω_0. We then use fe_shapes_8 to populate the 8×1 vector shapes; which is initialized at 0,0,0 in $\square$. Next, we use fe_shapeMatrix to populate the 3×24 matrix shape_mat.

$$\text{shape\_mat} = \underbrace{\begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \cdots & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \cdots & N_8 \end{bmatrix}}_{3 \times 24}$$

$$\text{coord} = \underbrace{\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ X_2 \\ Y_2 \\ Z_2 \\ \vdots \\ Z_8 \end{bmatrix}}_{24 \times 1}$$

Each shape function is evaluated at a specific point ξ, η, ζ in $\square$.

$$\text{shapes} = \underbrace{\begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_8 \end{bmatrix}}_{8 \times 1}$$

3. Overall, we are trying to find the 3×1 vector iso_coord; which are the <u>unknown</u> coordinates of the fiber element node in $\square$. We use a while loop to solve for iso_coord.

Pg. 159

Newton - Raphson Method

$f(x) = 0$ ← This is what we want to solve.

$$\underbrace{f(x + \Delta x)}_{\text{improved solution}} = f(x) + J(x)\,\Delta x$$ ← Talyor series expansion about x.

$$J_{ij} = \frac{\partial f_i}{\partial x_j}$$ (Jacobian Matrix)

$$J(x)\,\Delta x = -f(x)$$ ← set of linear equations for Δx.

In our case, we are trying to find the root of the following:

$$\underset{3\times1}{\overbrace{f(\xi)}^{fx}} = \underset{3\times1}{\overbrace{X_{actual}}^{nat\_coord}} - \underset{3\times1}{\overbrace{X_{estimate}}^{[shape\_mat][coord]}}$$

$$\underbrace{\xi_{n+1}}_{iso\_coord\_new} = \overbrace{\xi_n}^{iso\_coord} - J(\xi_n)^{-1}\,\overbrace{f(\xi)}^{fx}$$

At each pass through the while loop, we calculate the error eps as follows:

$$eps = norm\big(\underset{3\times1}{f(\xi)}\big)$$

The while loop stops if eps < 1e-6 OR if iterations > 100.

4. In the end, fe_newtonRhapson returns the 3×1 vector iso_coord; which are the estimated coordinates of the fiber element node in $\square$.