(Kaliske, 1997)

- Initialize storage for previous time steps.

$$\mathrm{DEV}\,\underline{\underline{S}}_0^{\,n},\ \underbrace{\underline{\underline{H}}_1^{\,n},\ \underline{\underline{H}}_2^{\,n}}_{\text{internal stress variables}}$$

elastic stress

- Initialize material constants based on Abaqus per (Garimella, 2017).

$$g_1 = 0.65425,\quad g_1 = 0.0149$$

$$\tau_1 = 0.0066940,\quad \tau_2 = 0.15642$$

Note:

Set $K_1 = K_2 = 0$, so that Abaqus does not apply viscosity to hydrostatic stress.

- Convert material constants to Kaliske notation.

Kaliske

$$g_\infty + \sum_{i=1}^{N} g_i = 1 \;\rightarrow\; \underbrace{g_\infty = 1 - g_1 - g_2}_{\text{(Abaqus)}} \;\rightarrow\; \gamma_i = \frac{g_i}{\underbrace{g_\infty}_{\text{Abaqus}}}$$

- Calculate elastic PK2 stress at current time $\underline{\underline{S}}_0^{\,n+1}$.

- Calculate $\underline{\underline{C}} = \underline{\underline{F}}^T \underline{\underline{F}}$ for current time.

- Calculate the deviatoric component of $\underline{\underline{S}}_0^{\,n+1}$ w.r.t. the reference configuration.

$$\mathrm{DEV}\,\underline{\underline{S}}_0^{\,n+1} = I - \frac{1}{3}\left[\underbrace{\underline{\underline{C}} : \underline{\underline{S}}_0^{\,n+1}}\right]\underline{\underline{C}}^{-1}$$

$$\underline{\underline{C}} : \underline{\underline{S}}_0^{\,n+1} = \mathrm{tr}\left(\underline{\underline{C}}^T \underline{\underline{S}}_0^{\,n+1}\right) = \mathrm{tr}\left(\underline{\underline{C}}\,\underline{\underline{S}}_0^{\,n+1}\right)$$

- Calculate internal stress variables at current time $\underline{\underline{H}}_1^{n+1}$ and $\underline{\underline{H}}_2^{n+1}$.

current time step from n to n+1

$$\underline{\underline{H}}_j^{n+1} = \exp\left(-\frac{\Delta t}{\tau_j}\right) \underbrace{\underline{\underline{H}}_j^{n}}_{\text{Previous time step from storage.}} + \gamma_j \frac{1-\exp\left(-\frac{\Delta t}{\tau_j}\right)}{\frac{\Delta t}{\tau_j}} \left[\mathrm{DEV}\, \underline{\underline{S}}_0^{n+1} - \underbrace{\mathrm{DEV}\, \underline{\underline{S}}_0^{n}}_{\text{Previous time step from storage.}} \right]$$

- Calculate the modified deviatoric PK2 stress $\mathrm{DEV}\, \underline{\underline{S}}^{n+1}$.

$$\mathrm{DEV}\, \underline{\underline{S}}^{n+1} = \mathrm{DEV}\, \underline{\underline{S}}_0^{n+1} + \sum_{j=1}^{N} \underline{\underline{H}}_j^{n+1}$$

- Transform the modified deviatoric PK2 stress $\mathrm{DEV}\, \underline{\underline{S}}^{n+1}$ to the modified deviatoric kirchhoff stress $\mathrm{dev}\, \underline{\underline{\tau}}^{n+1}$.

$$\mathrm{dev}\, \underline{\underline{\tau}}^{n+1} = \underline{\underline{F}}^{n+1}\, \mathrm{DEV}\, \underline{\underline{S}}^{n+1} \left(\underline{\underline{F}}^{n+1}\right)^T$$

- Calculate the total kirchhoff stress $\underline{\underline{\tau}}^{n+1}$.

$$\underline{\underline{\tau}} = J^{n+1} U'^{\,n+1} \underline{\underline{I}} + \mathrm{dev}\, \underline{\underline{\tau}}^{n+1}$$

where

$$U' = \frac{\partial U}{\partial J} \qquad \hat{U}(J) \text{ volumetric strain energy function}$$

<u>Note:</u>

Mooney-Rivlin: $W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + \underbrace{\frac{1}{D_1}(J-1)^2}_{\hat{U}(J)}$

Ogden (N=1): $W = \frac{2\mu_1}{\alpha_1^2}\left[\bar{\lambda}_1^{\alpha_1} + \bar{\lambda}_2^{\alpha_1} + \bar{\lambda}_3^{\alpha_1} - 3\right] + \underbrace{\frac{1}{D_1}(J-1)^2}_{\hat{U}(J)}$

In both cases

$$U' = \frac{2}{D_1}(J-1)$$

Note: (Gurtin)

$\underline{\underline{T}}$: Cauchy stress (i.e. $\underline{\underline{\sigma}}$) $\qquad \underline{\underline{\sigma}} = \frac{1}{J}\underline{\underline{F}}\,\underline{\underline{S}}\,\underline{\underline{F}}^T$ (25.10)

$\underline{\underline{T}}_R$: Piola Stress (i.e. $\underline{\underline{P}}$) $\qquad \underline{\underline{P}} = J\,\underline{\underline{\sigma}}\,\underline{\underline{F}}^{-T}$ (24.1)

$\underline{\underline{T}}_{RR}$: second Piola Stress (i.e. $\underline{\underline{S}}$) $\qquad \underline{\underline{S}} = \underline{\underline{F}}^{-1}\underline{\underline{P}}$ (25.2)

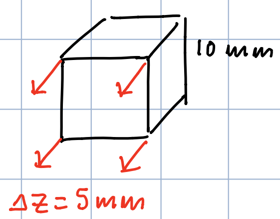
$\underline{\underline{T}}_K$: Kirchhoff stress (i.e. $\underline{\underline{\tau}}$) $\qquad \underline{\underline{\tau}} = J\,\underline{\underline{\sigma}}$ pg. 329

$$\boxed{\underline{\underline{\sigma}} = \frac{1}{J}\underline{\underline{\tau}}}$$

$$\underline{\underline{S}} = \underline{\underline{F}}^{-1}\underline{\underline{P}} = \underline{\underline{F}}^{-1}\left(J\,\underline{\underline{\sigma}}\,\underline{\underline{F}}^{-T}\right) = \underline{\underline{F}}^{-1}J\left(\frac{1}{J}\underline{\underline{\tau}}\right)\underline{\underline{F}}^{-T}$$

$$\boxed{\underline{\underline{S}} = \underline{\underline{F}}^{-1}\underline{\underline{\tau}}\,\underline{\underline{F}}^{-T}}$$

Test Problem – tension



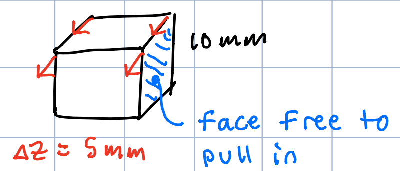
- Mooney-Rivlin.
- double precision for Abaqus

Δt	σ_{xx} EEMA	σ_{xx} ABAQUS	σ_{zz} EEMA	σ_{zz} ABAQUS	R_{tot} EEMA	R_{tot} ABAQUS
0.001	-2,776.26	-2,892.54	34,651.2	34,535	2.28537	2.302516
0.010	-27.838	-28.9317	24,207.3	24,206.2	1.61367	1.613756
0.100	-0.285623	-0.29577	14241.7	14,241.7	0.949451	0.949452
1.000	-0.003628	-0.00364687	13,299.5	13,299.5	0.886638	0.886636
10.000	-0.000114	-0.000105816	13,207.7	13,207.7	0.880518	0.880516
20.000	-0.000050	-0.0000462	13,203.2	13,203.2	0.880213	0.880212

$$\varepsilon = \frac{\delta}{L} = \frac{5\text{ mm}}{10\text{ mm}} \rightarrow \varepsilon = 0.5$$

$$\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t}$$

Test Problem - shear



- Mooney-Rivlin.
- double precision for Abaqus

viscoelasticity = OFF

	τ_{YZ}		σ_{YY}		σ_{ZZ}	
Δt	EEMA	ABAQUS	EEMA	ABAQUS	EEMA	ABAQUS
0.010	6769.0006	6769	-3901.3408	-3901.4	-516.8405	-516.9
0.100	6769.0006	6769	-3901.4686	-3901.48	-516.9683	-516.974

viscoelasticity = ON

	τ_{YZ}		σ_{YY}		σ_{ZZ}	
Δt	EEMA	ABAQUS	EEMA	ABAQUS	EEMA	ABAQUS
0.001	19,534.8183	19,534.8	-11,417.3209	-11,418.9	-1,813.3466	-1,814.97
0.010	14,185.0155	14,185	-9,040.1743	-9,040.25	-2,840.1971	-2,840.27
0.100	8,014.5600	8,014.56	-5,008.7737	-5,008.78	-1,456.1534	-1,456.16