(Kaliske, 1997)

- Initialize storage for previous time steps.

$$\mathrm{DEV}\,\underline{\underline{S}}_0^{\,n}\,,\ \underbrace{\underline{\underline{H}}_1^{\,n}\,,\ \underline{\underline{H}}_2^{\,n}}_{\text{internal stress variables}}$$

elastic stress

- Initialize material constants based on Abaqus per (Garimella, 2017).

$$g_1 = 0.65425\,,\quad g_1 = 0.0149$$

$$\tau_1 = 0.0066940\,,\quad \tau_2 = 0.15642$$

Note:
Set $K_1 = K_2 = 0$, so that Abaqus does not apply viscosity to hydrostatic stress.

- Convert material constants to Kaliske notation.

$$g_\infty + \sum_{i=1}^{N} g_i = 1 \;\rightarrow\; \underbrace{g_\infty = 1 - g_1 - g_2}_{\text{(Abaqus)}} \;\rightarrow\; \gamma_i = \frac{g_i}{\underbrace{g_\infty}_{\text{Abaqus}}}$$

Kaliske

- Calculate elastic PK2 stress at current time $\underline{\underline{S}}_0^{\,n+1}$.

- Calculate $\underline{\underline{C}} = \underline{\underline{F}}^T \underline{\underline{F}}$ for current time.

- Split $\underline{\underline{S}}_0^{\,n+1}$ into deviatoric and spherical components w.r.t. the reference configuration.

$$\mathrm{SPH}\,\underline{\underline{S}}_0^{\,n+1} = \frac{1}{3}\left[\underbrace{\underline{\underline{C}} : \underline{\underline{S}}_0^{\,n+1}}\right]\underline{\underline{C}}^{-1}$$

$$\underline{\underline{C}} : \underline{\underline{S}}_0^{\,n+1} = \mathrm{tr}\left(\underline{\underline{C}}^T \underline{\underline{S}}_0^{\,n+1}\right) = \mathrm{tr}\left(\underline{\underline{C}}\,\underline{\underline{S}}_0^{\,n+1}\right)$$

$$\text{DEV}\,\underline{\underline{S}}_0^{n+1} = 1 - \text{SPH}\,\underline{\underline{S}}_0^{n+1}$$

- Calculate internal stress variables at current time $\underline{\underline{H}}_1^{n+1}$ and $\underline{\underline{H}}_2^{n+1}$.

$$\underline{\underline{H}}_j^{n+1} = \exp\left(-\frac{\overbrace{\Delta t}^{\text{current time step from } n \text{ to } n+1}}{\tau_j}\right)\underbrace{\underline{\underline{H}}_j^n}_{\substack{\text{Previous time}\\\text{step from storage.}}} + \gamma_j \frac{1-\exp\left(-\frac{\Delta t}{\tau_j}\right)}{\frac{\Delta t}{\tau_j}}\left[\text{DEV}\,\underline{\underline{S}}_0^{n+1} - \underbrace{\text{DEV}\,\underline{\underline{S}}_0^n}_{\substack{\text{Previous time}\\\text{step from storage.}}}\right]$$

- Calculate the modified deviatoric PK2 stress $\text{DEV}\,\underline{\underline{S}}^{n+1}$.

$$\text{DEV}\,\underline{\underline{S}}^{n+1} = \text{DEV}\,\underline{\underline{S}}_0^{n+1} + \sum_{j=1}^{N}\underline{\underline{H}}_j^{n+1}$$

- Calculate the modified PK2 stress.

$$\boxed{\underline{\underline{S}}^{n+1} = \text{SPH}\,\underline{\underline{S}}_0^{n+1} + \text{DEV}\,\underline{\underline{S}}^{n+1}}$$

Final version used in code and comparison table below.

Note:

In (Kaliske, 1997), they do the last steps a little different.

$$\text{dev}\,\underline{\underline{\tau}}^{n+1} = \underline{\underline{F}}^{n+1}\,\text{DEV}\,\underline{\underline{S}}^{n+1}\left(\underline{\underline{F}}^{n+1}\right)^T$$

$$\underline{\underline{\tau}} = J^{n+1}U'^{\,n+1}\underline{\underline{I}} + \text{dev}\,\underline{\underline{\tau}}^{n+1}$$

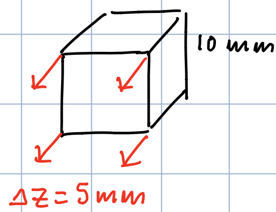
where

$$U' = \frac{\partial U}{\partial J}$$

$\hat{U}(J)$ volumetric strain energy function

When I tried this, the stress values were coming out too high. I do not know for sure, but I suspect this is because our D1 is very high.

Test Problem



• Mooney-Rivlin.

	σ_{xx}		σ_{zz}		R_{tot}	
Δt	EEMA	ABAQUS	EEMA	ABAQUS	EEMA	ABAQUS
0.001	-2,776.26	-2,280.48	34,651.3	35,147.1	2.28537	2.341204
0.001		-2,892.54		34,535	single prec. double prec.	2.302516
0.010	-27.838	-14.337	24,207.3	24,222.2	1.61367	1.614392
0.010		-28.9317		24,206.2		1.613756
0.100	-0.285623	134.526	14241.7	14,384.8	0.949451	0.957314
0.100		-0.29577		14,241.7		0.949452
1.000	-0.0036	-28.8369	13,299.5	13,334.6	0.886638	0.889314
1.000		-0.00364687		13,299.5		0.886636
10.000	-0.000114	-22.9668	13,207.7	14,355.6	0.880518	0.939071
10.000		-0.000105816		13,207.7		0.880516
20.000	-0.000050	-55.6562	13,203.2	17,371.1	0.880213	1.034663
20.000		-0.0000462		13,203.2		0.880212

$$\varepsilon = \frac{\delta}{L} = \frac{5\text{ mm}}{10\text{ mm}} \rightarrow \varepsilon = 0.5$$

$$\dot{\varepsilon} = \frac{\Delta \varepsilon}{\Delta t}$$