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Newmark β-Equations:

$$d^{n+1} = \tilde{d}^{n+1} + \beta \Delta t^2 a^{n+1} \quad \text{where} \quad \tilde{d}^{n+1} = d^n + \Delta t v^n + \frac{\Delta t^2}{2}(1-2\beta) a^n$$

$$v^{n+1} = \tilde{v}^{n+1} + \gamma \Delta t a^{n+1} \quad \text{where} \quad \tilde{v}^{n+1} = v^n + (1-\gamma)\Delta t a^n$$

where

$$\Delta t = t^{n+1} - t^n$$

explicit central difference method: $\beta = 0$, $\gamma = \frac{1}{2}$

$$d^{n+1} = \tilde{d}^{n+1} + \beta \Delta t^2 a^{n+1} \quad \text{where} \quad \tilde{d}^{n+1} = d^n + \Delta t v^n + \frac{\Delta t^2}{2}(1-2\beta) a^n$$

$$\rightarrow \boxed{d^{n+1} = d^n + \Delta t\, v^n + \frac{\Delta t^2}{2} a^n}$$

Reference (Box 6.1):

$$d^{n+1} = d^n + \Delta t^{n+1/2} v^{n+1/2}$$

where

$$v^{n+1/2} = v^n + (t^{n+1/2} - t^n) a^n$$

$$t^{n+1/2} = \frac{1}{2}(t^{n+1} + t^n)$$

$$\Delta t^{n+1/2} = t^{n+1} - t^n$$

$$\rightarrow v^{n+1/2} = v^n + \left[\frac{t^{n+1}}{2} + \frac{t^n}{2} - t^n\right] a^n$$

$$\rightarrow v^{n+1/2} = v^n + \frac{1}{2}(t^{n+1} - t^n) a^n$$

$$\rightarrow v^{n+1/2} = v^n + \frac{1}{2} \Delta t^{n+1/2} a^n$$

$$\rightarrow \boxed{d^{n+1} = d^n + \Delta t^{n+1/2} v^n + \frac{1}{2}\left(\Delta t^{n+1/2}\right)^2 a_n} \quad \text{(proven)}$$

$$v^{n+1} = \tilde{v}^{n+1} + \gamma \Delta t a^{n+1} \quad \text{where} \quad \tilde{v}^{n+1} = v^n + (1-\gamma)\Delta t a^n$$

($\gamma \rightarrow 1/2$; $(1-\gamma) \rightarrow 1/2$)

$$\rightarrow v^{n+1} = v^n + \frac{\Delta t a^n}{2} + \frac{\Delta t a^{n+1}}{2}$$

$$\rightarrow \boxed{v^{n+1} = v^n + \frac{\Delta t}{2}\left(a^n + a^{n+1}\right)}$$

Reference (Box 6.1):

$$v^{n+1} = v^{n+1/2} + \left(t^{n+1} - t^{n+1/2}\right) a^{n+1}$$

$$\rightarrow v^{n+1} = \left[v^n + \frac{1}{2}\Delta t^{n+1/2} a^n\right] + \left[t^{n+1} - \frac{1}{2}\left(t^{n+1} + t^n\right)\right] a^{n+1}$$

$$\rightarrow v^{n+1} = v^n + \frac{\Delta t^{n+1/2} a^n}{2} + \frac{\Delta t^{n+1/2} a^{n+1}}{2}$$

$$\rightarrow \boxed{v^{n+1} = v^n + \frac{\Delta t^{n+1/2}}{2}\left(a^n + a^{n+1}\right)} \quad \text{(proven)}$$