Econ 512

Fall 2019

Homework 3 – Nonlinear Optimization
Due 10/21/2019

In 1969, the popular magazine *Psychology Today* published a 101-question survey on affairs. Professor Ray Fair (1978) extracted a sample of 601 observations on men and women who are currently married for the first time and analyzed their responses to a question about extramarital affairs. He used the tobit model as his estimation framework for this study. The dependent variable is a count of the number of affairs which suggests that a standard Poisson model may be a better choice. Download the data set hw3.mat, and estimate the parameters by the methods of nonlinear least squares and maximum likelihood using different algorithms.

Data description:

y - count data: number of affairs in the past year

$\mathbf{x}$ - constant term=1, age, number of years married, religiousness (scale $1-5$), occupation (scale $1-7$), self-rating of marriage (scale $1-5$)

The data generating assumptions for the Poisson model, where j= number of affairs, are:

$$\begin{aligned} \Pr[y_i = j] &= \frac{e^{-\lambda_i}\lambda_i^j}{j!} \\ \log \lambda_i &= \mathbf{x}_i'\beta \\ E(y_i \,|x_i\,) &= e^{\mathbf{x}_i\prime\beta} \end{aligned}$$

for some $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)'$.

The log-likelihood function is:

$$\begin{aligned} \ln L &= \sum_{i=1}^{n} \ln f(y_i \,|x_i\,, \beta) \\ &= \sum_{i=1}^{n} \ln \frac{e^{-\lambda_i}\lambda_i^j}{j!} \\ &= \sum_{i=1}^{n} [-\lambda_i + y_i \ln \lambda_i - \ln j!] \\ &= \sum_{i=1}^{n} \left[-e^{\mathbf{x}_i\prime\beta} + y_i x_i'\beta - \ln y_i!\right] \end{aligned}$$

The residual sum of squares is:

$$S(\beta) = \sum_{i=1}^{n} \left(y_i - e^{\beta' x_i} \right)^2$$

1. Estimate the parameter vector β using the maximum likelihood estimator computed via the Nelder-Mead simplex method.

2. Estimate the parameter vector β using the maximum likelihood estimator computed via a quasi-Newton optimization method, report which method you choose.

3. Estimate the parameter vector β using nonlinear least squares estimator computed using the command `lsqnonlin`. What computation method are you using?

4. Estimate the parameter vector β using the nonlinear least squares estimator computed using the Nelder-Mead simplex method.

5. Test all four approaches with regard to the choice of initial values. Roughly rank them in order of robustness and time to convergence. Submit a short writeup summarizing your results.