Econ 512

Fall 2019

Homework 2 – Discrete Choice Demand Pricing Equilibrium
Due 10/07/2019

There are two single product firms in a market. There exist a unit mass of consumers. Consumer i's' utility for product A is

$$u_{iA} = v_A - p_A + \varepsilon_{iA},$$

and her utility for product B is

$$u_{iB} = v_B - p_B + \varepsilon_{iB},$$

where p_A and P_B are the prices and v_A and v_B are the qualities of each product.

Each consumer chooses to consume a single unit of the product that hives her the highest utility, or the outside option with utility $u_{i0} = \varepsilon_{i0}$.

If $\varepsilon \sim$ iid Extreme Value then consumer demand is the following:

$$D_A = \frac{exp(q_A - p_A)}{1 + exp(q_A - p_A) + exp(q_B - p_B)}$$
$$D_B = \frac{exp(q_B - p_B)}{1 + exp(q_A - p_A) + exp(q_B - p_B)}$$
$$D_0 = \frac{1}{1 + exp(q_A - p_A) + exp(q_B - p_B)}$$

Assume the firms have zero marginal costs and compete by simultaneously setting prices. The equilibrium concept is (Bertrand) Nash. Also, note that

$$\frac{\partial D_A}{\partial p_A} = -D_A(1 - D_A)$$

1. Consider the following parameterization: $v_A = v_B = 2$. What is the demand for each option if $p_A = p_B = 1$?

2. Given the above parameterizations for product values, use Broyden's Method to solve for the Nash pricing equilibrium. (Hint: There is a unique equilibrium.) Report the starting value and convergence criteria (if it converges).

3. Now use a Guass-Sidel method (using the secant method for each sub-iteration) to solve for the pricing equilibirum. Which method is faster? Why?

3. Lastly, use the following update rule to solve for equilibrium:

$$p^{t+1} = \frac{1}{1 - D(p^t)} \tag{1}$$

Does this converge? Is it faster or slower than the other two methods?

5. Solve the pricing equilibrium (using your preferred method) for $v_A = 2$ and $v_B = 0 : .2 : 3$. On the same graph, plot equilibrium p_A and p_B as a function of the vector of v_B.