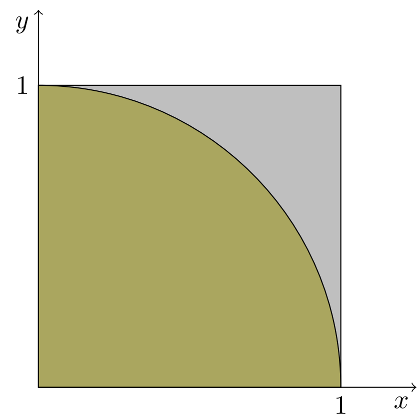
Econ 512
Fall 2018

Homework 4 – Numerical Integration
Due 10/10/2018

There are multiple ways to compute the mathematical constant π via numerical integration. Probably the simplest is the "dart-throwing" method, which is a special case of an *accept-reject* simulator. A somewhat more elegant approach comes from an integral representation. This homework will consider both. Consider the quarter-circle arc of radius 1, centered at the origin inscribed within the unit square:



Clearly, the area of the quarter-circle (green shaded area) is $\frac{\pi}{4}$ whereas the area of the unit square is 1. For any point inside the unit square, we can tell whether it lies within the quarter-circle using the Pythagorean formula. That is, (x, y) lies within the shaded are if $x^2 + y^2 \leq 1$. The dart-throwing method simply draws points uniformly from within the unit square and accepts them when they lie within the circle and rejects them otherwise. The ratio of acceptances to total draws converges to the area of the quarter circle. In other words, we can calculate π using the double-integral,

$$\pi = 4 \int_0^1 \int_0^1 \mathbf{1}[x^2 + y^2 \leq 1] dy dx.$$

1. Write an algorithm that uses the dart-throwing method to approximate π using a quasi-Monte Carlo approach to computing integrals.
2. Write an algorithm that uses the dart-throwing method to approximate π using a Newton-Cotes approach to computing integrals.

However, all we are doing is calculating the area under the curve. From the same figure and again using Pythagorean formula, one can solve for y as a function of x, $y = \sqrt{1 - x^2}$. Thus, we have a definition for π based on a single dimensional integral,

$$\pi = 4 \int_0^1 \sqrt{1 - x^2} dx.$$

3. Approximate π based on the above integral using a quasi-Monte Carlo approach.
4. Approximate π based on the above integral using a Newton-Coates approach.
5. Prepare a table which shows the mean squared error of 200 simulations of each method using 1000, 10,000 and 1,000,000 draws. Compare this to the squared error of the Newton-Coates methods for the same number of quadrature nodes.