

## Econ 512

Fall 2019

### Homework 2 – Discrete Choice Demand Pricing Equilibrium

Due 10/07/2019

There are two single product firms in a market. There exist a unit mass of consumers. Consumer  $i$ 's' utility for product A is

$$u_{iA} = v_A - p_A + \varepsilon_{iA},$$

and her utility for product B is

$$u_{iB} = v_B - p_B + \varepsilon_{iB},$$

where  $p_A$  and  $p_B$  are the prices and  $v_A$  and  $v_B$  are the qualities of each product.

Each consumer chooses to consume a single unit of the product that gives her the highest utility, or the outside option with utility  $u_{i0} = \varepsilon_{i0}$ .

If  $\varepsilon \sim \text{iid Extreme Value}$  then consumer demand is the following:

$$\begin{aligned} D_A &= \frac{\exp(v_A - p_A)}{1 + \exp(v_A - p_A) + \exp(v_B - p_B)} \\ D_B &= \frac{\exp(v_B - p_B)}{1 + \exp(v_A - p_A) + \exp(v_B - p_B)} \\ D_0 &= \frac{1}{1 + \exp(v_A - p_A) + \exp(v_B - p_B)} \end{aligned}$$

Assume the firms have zero marginal costs and compete by simultaneously setting prices. The equilibrium concept is (Bertrand) Nash. Also, note that

$$\frac{\partial D_A}{\partial p_A} = -D_A(1 - D_A)$$

**1.** Consider the following parameterization:  $v_A = v_B = 2$ . What is the demand for each option if  $p_A = p_B = 1$ ?

**2.** Given the above parameterizations for product values, use Broyden's Method to solve for the Nash pricing equilibrium. (Hint: There is a unique equilibrium.) Report the starting value and convergence criteria (if it converges).

**3.** Now use a Gauss-Sidel method (using the secant method for each sub-iteration) to solve for the pricing equilibrium. Which method is faster? Why?

**3.** Lastly, use the following update rule to solve for equilibrium:

$$p^{t+1} = \frac{1}{1 - D(p^t)} \tag{1}$$

Does this converge? Is it faster or slower than the other two methods?

**5.** Solve the pricing equilibrium (using your preferred method) for  $v_A = 2$  and  $v_B = 0 : .2 : 3$ . On the same graph, plot equilibrium  $p_A$  and  $p_B$  as a function of the vector of  $v_B$ .