Multiple regression

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Table of Contents

Background

Given the background and tools presented in linear regression, we will not extend the modeling approach to include additional variables, as well as relationships that are more complicated. This exercise provides the jumping off point for more automated modeling approaches, which will we see in the subsequent example(s).

Our assumption in this exercise is that multiple factors have explanatory value to explain the response variable of interest.

What does a model of this type look like? Some examples include:

1. Additive.

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon$

2. With interaction between the two terms.

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \epsilon$

Note: It is important to note that in modeling, when we add new explanatory variables that have merit (i.e., the sign makes sense in terms of the biological relation), the model will improve. This is not necessarily the same as being "biologically relevant". We should always consider the variable in the context of the question of interest.

```
library(tidyverse)
library(Hmisc)
library(corrplot)
library(readr)
```

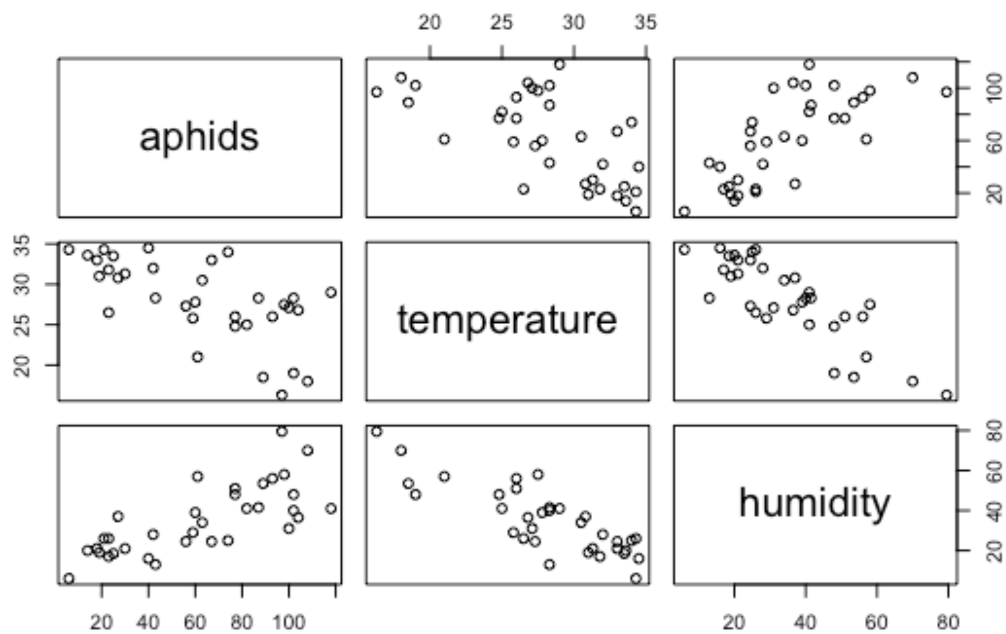
```r
library(HH)
library(car)
library(scatterplot3d)
library(olsrr)
```

Data and exploratory analysis

Our database comes from counts of the number of aphids in different lots, as well as measures of average temperature and relative humidity. We assume that there is a relationship between those two latter factors with the observed number of aphids, which means from a predictive value, we hope that by just measuring T and RH, we can estimate the number of expected aphids.

Where do we start? The main question is to determine if there is (are) a relationship between T and RH with the counts. We are also interested in trying to determine if there may be a complex relationship (i.e., that the predictive values have some degree of interpretable interaction).

```r
lot <- c(1:34)
aphids <- c(61, 77, 87, 93, 98, 100, 104, 118, 102, 74, 63, 43, 27, 19, 14,
23, 30, 25, 67, 40, 6, 21, 18, 23, 42, 56, 60, 59, 82, 89, 77, 102, 108, 97)
temperature <- c(21, 24.8, 28.3, 26, 27.5, 27.1, 26.8, 29, 28.3, 34, 30.5,
28.3, 30.8, 31, 33.6, 31.8, 31.3, 33.5, 33, 34.5, 34.3, 34.3, 33, 26.5, 32,
27.3, 27.8, 25.8, 25, 18.5, 26, 19, 18, 16.3)
humidity <- c(57,48, 41.5, 56, 58, 31, 36.5, 41, 40, 25, 34, 13, 37, 19, 20,
17, 21, 18.5, 24.5, 16, 6, 26, 21, 26, 28, 24.5, 39, 29, 41, 53.5, 51, 48,
70, 79.5)

aphids_data <- data.frame(lot, aphids, temperature, humidity)

# Quick exploratory analysis
summary(aphids_data)

##       lot            aphids        temperature       humidity
##  Min.   : 1.00   Min.   :  6.00   Min.   :16.30   Min.   : 6.00
##  1st Qu.: 9.25   1st Qu.: 27.75   1st Qu.:26.00   1st Qu.:21.88
##  Median :17.50   Median : 62.00   Median :28.30   Median :32.50
##  Mean   :17.50   Mean   : 61.91   Mean   :28.09   Mean   :35.19
##  3rd Qu.:25.75   3rd Qu.: 92.00   3rd Qu.:31.95   3rd Qu.:46.38
##  Max.   :34.00   Max.   :118.00   Max.   :34.50   Max.   :79.50

cor(aphids_data[,2:4])

##                 aphids temperature   humidity
## aphids       1.0000000  -0.6463022  0.7394570
## temperature -0.6463022   1.0000000 -0.8313696
## humidity     0.7394570  -0.8313696  1.0000000

plot(aphids_data[,2:4]) # Graphical matrix
pairs(aphids_data[,2:4]) # Gives us the same thing
```

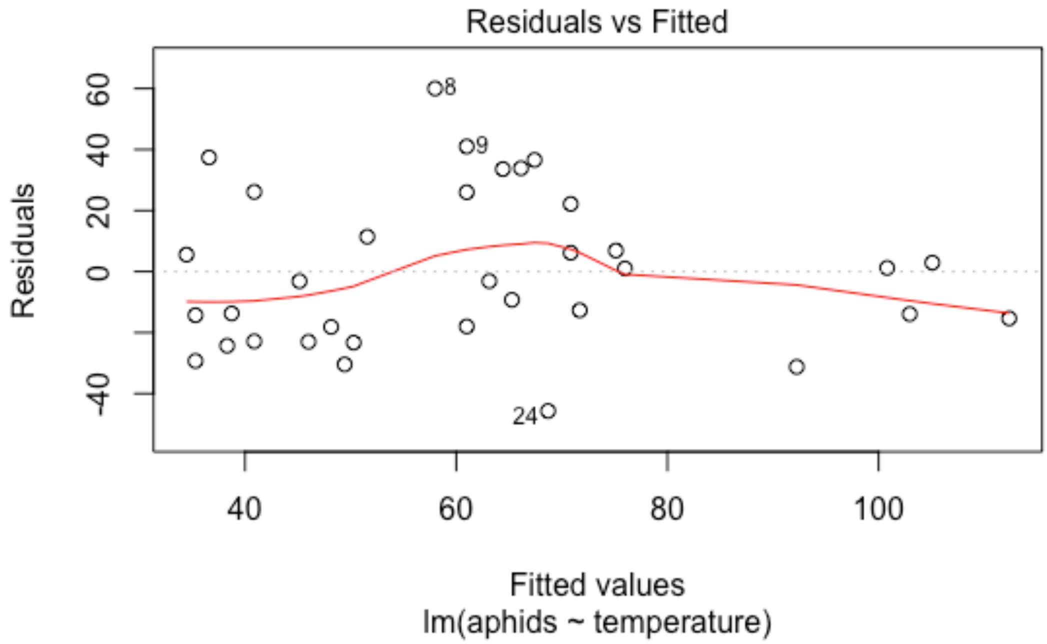


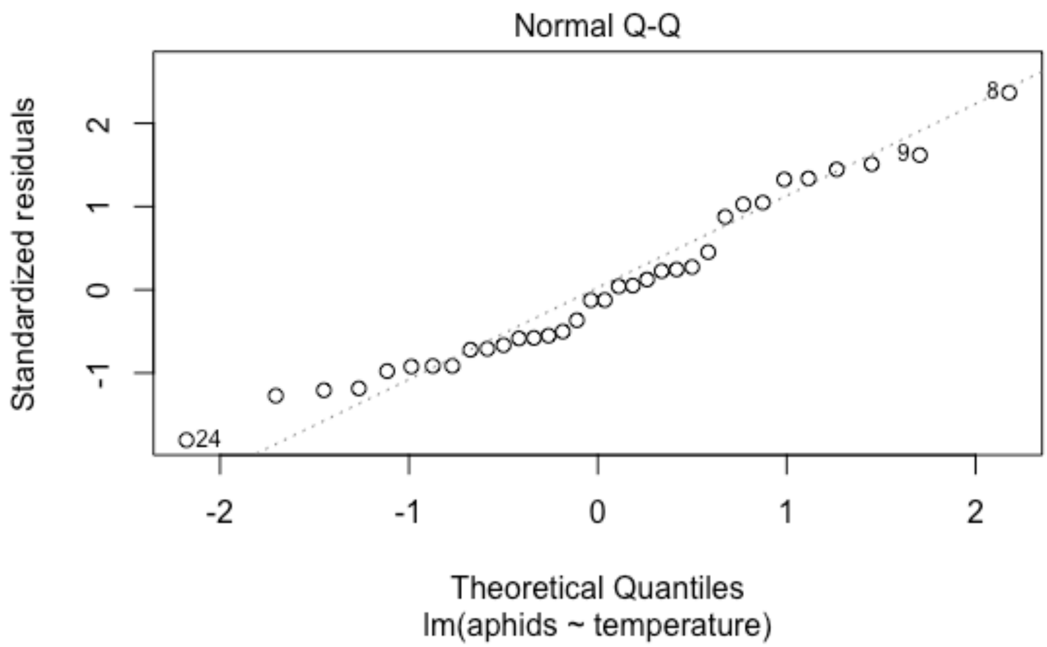
Linear regression

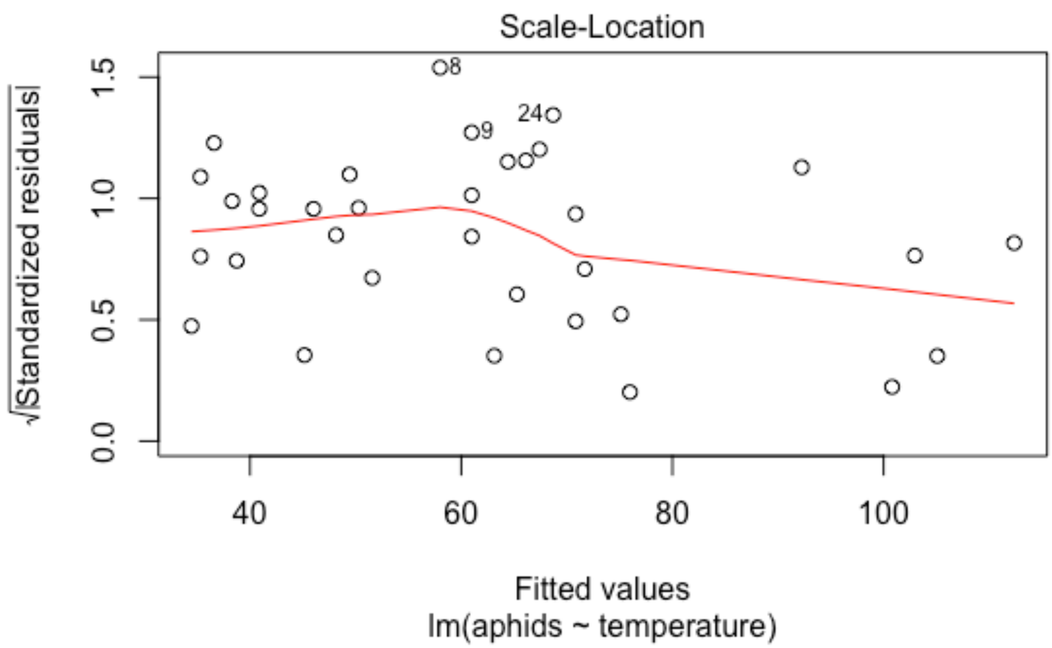
```
# Factor = temperature (X)

model1<-with(aphids_data, lm(aphids~temperature))
anova(model1)

## Analysis of Variance Table
##
## Response: aphids
##             Df Sum Sq Mean Sq F value    Pr(>F)
## temperature  1  15195 15194.8  22.955 3.643e-05 ***
## Residuals   32  21182   661.9
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

summary(model1)

##
## Call:
## lm(formula = aphids ~ temperature)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -45.698 -18.111  -3.143  19.477  60.004
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

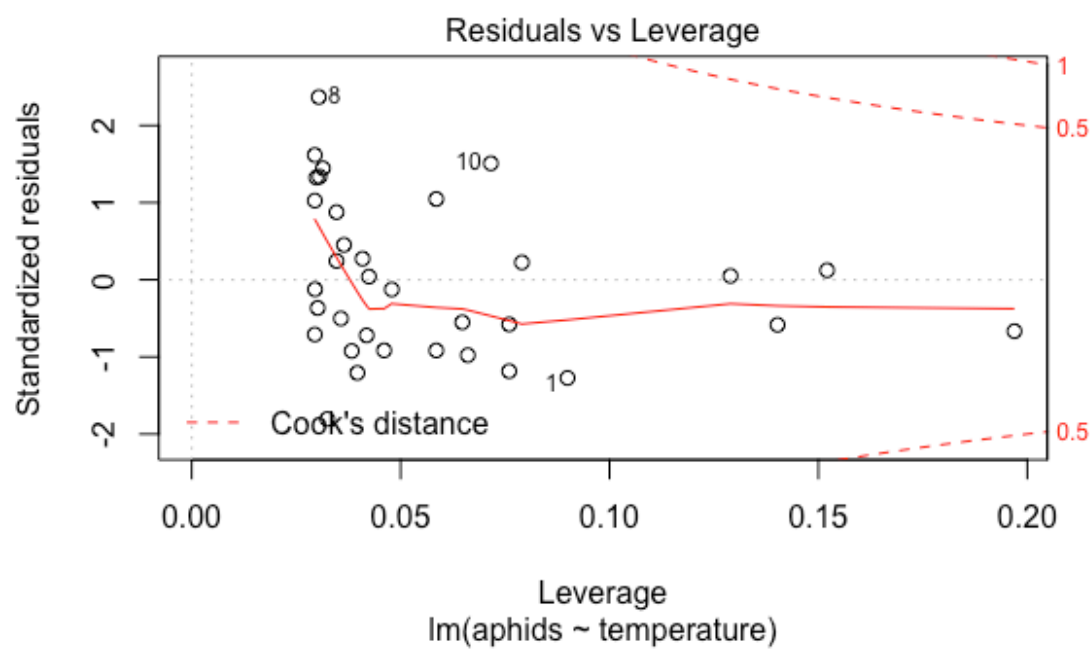
```
## (Intercept) 182.1386    25.4785   7.149 4.10e-08 ***
## temperature  -4.2808     0.8935  -4.791 3.64e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 25.73 on 32 degrees of freedom
## Multiple R-squared:  0.4177, Adjusted R-squared:  0.3995
## F-statistic: 22.96 on 1 and 32 DF,  p-value: 3.643e-05
```

```
plot(model1)
```


Residuals vs Fitted
Residuals
Fitted values
lm(aphids ~ temperature)
60
40
20
0
-40
40
60
80
100
8
9
24


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(aphids ~ temperature)
2
1
0
-1
-2
8
9
24






```
# Assumptions = values, model1

rstudent(model1)

##             1            2            3            4            5            6
7            8            9           10           11           12           13
```

```
14          15          16          17          18          19          20
21          22          23          24          25          26          27
28          29          30          31          32          33          34
## -1.28585895  0.04005818  1.02696012  0.87344711  1.34166917  1.35439948
1.47093718  2.56708773  1.66182645  1.54106791  0.44669460 -0.70426322 -
0.92092001 -1.21610745 -0.97682293 -0.91330799 -0.71519631 -0.54584549
1.04821427  0.22134400 -1.19286546 -0.57242992 -0.91388994 -1.87539948 -
0.12367881 -0.36099265 -0.12169500 -0.49651581  0.26909704 -0.57840180
0.24013268  0.04903167  0.12114783 -0.66038629
```

```
dfbetas(model1)
```

```
##     (Intercept)  temperature
## 1  -0.366682839  0.331661207
## 2   0.005815635 -0.004670407
## 3   0.023305013  0.007772618
## 4   0.089808162 -0.064378045
## 5   0.067723562 -0.027686588
## 6   0.087212625 -0.047068694
## 7   0.110092705 -0.066711426
## 8  -0.004133934  0.082814320
## 9   0.037712163  0.012577648
## 10 -0.276055170  0.328520764
## 11 -0.024068323  0.038160258
## 12 -0.015981987 -0.005330265
## 13  0.059303647 -0.088531881
## 14  0.086856812 -0.125611236
## 15  0.160634492 -0.193579923
## 16  0.091034920 -0.120629792
## 17  0.058637009 -0.081569976
## 18  0.087768241 -0.106135446
## 19 -0.149512336  0.184383492
## 20 -0.043753984  0.051380499
## 21  0.226920851 -0.267823576
## 22  0.108894329 -0.128522648
## 23  0.130352947 -0.160755509
## 24 -0.160003296  0.104963953
## 25  0.013206774 -0.017231640
## 26 -0.020732461  0.009996651
## 27 -0.004874275  0.001223897
## 28 -0.054538615  0.040127888
## 29  0.037156513 -0.029440595
## 30 -0.223031160  0.207642177
## 31  0.024690533 -0.017699151
## 32  0.017885539 -0.016575675
## 33  0.049290028 -0.046078814
## 34 -0.318930693  0.301601424
```

```
dffits(model1)
```

```
##           1           2           3           4           5
6           7           8           9          10          11
12          13          14          15          16          17
18          19          20          21          22          23
24          25          26          27          28          29
30          31          32          33          34
## -0.404272806  0.008432063  0.178944815  0.165495030  0.235239322
0.240562702  0.264859612  0.454709984  0.289568427  0.427975171  0.086872782
-0.122715819 -0.183780341 -0.247127470 -0.259852603 -0.200671906 -0.149517430
-0.143648162  0.261386769  0.064842854 -0.342084958 -0.164159053 -0.227891133
-0.343410519 -0.027739070 -0.063654708 -0.021220776 -0.095548871  0.055563960
-0.233580039  0.045498765  0.018866121  0.051306429 -0.327009697
```

```
covratio(model1)
```

```
##         1         2         3         4         5         6         7
8         9        10        11        12        13        14        15
16        17        18        19        20        21        22        23
24        25        26        27        28        29        30        31
32        33        34
## 1.0553084 1.1126545 1.0268519 1.0514226 0.9810703 0.9797855 0.9612415
0.7474350 0.9256398 0.9902076 1.0917586 1.0636026 1.0497679 1.0108127
1.0738384 1.0592292 1.0763152 1.1177641 1.0556567 1.1533544 1.0541908
1.1291908 1.0732083 0.8882885 1.1180537 1.0895089 1.0969090 1.0876492
1.1058148 1.2130097 1.0997154 1.2231239 1.2554800 1.2902753
```

```
cooks.distance(model1)
```

```
##            1            2            3            4            5
6            7            8            9           10           11
12           13           14           15           16           17
18           19           20           21           22           23
24           25           26           27           28           29
30           31           32           33           34
## 8.008297e-02 3.669472e-05 1.598333e-02 1.379652e-02 2.699386e-02
2.819990e-02 3.384457e-02 8.800702e-02 3.973731e-02 8.780865e-02 3.870253e-03
7.650078e-03 1.696816e-02 3.008573e-02 3.381010e-02 2.023952e-02 1.135101e-02
1.054883e-02 3.405642e-02 2.166690e-03 5.774783e-02 1.376327e-02 2.610161e-02
5.466541e-02 3.969427e-04 2.082560e-03 2.323129e-04 4.674868e-03 1.589759e-03
2.785916e-02 1.066474e-03 1.836918e-04 1.357989e-03 5.442676e-02
```

```
# Factor = humedad (X)

model2<-with(aphids_data, lm(aphids~humidity))
anova(model2)
```

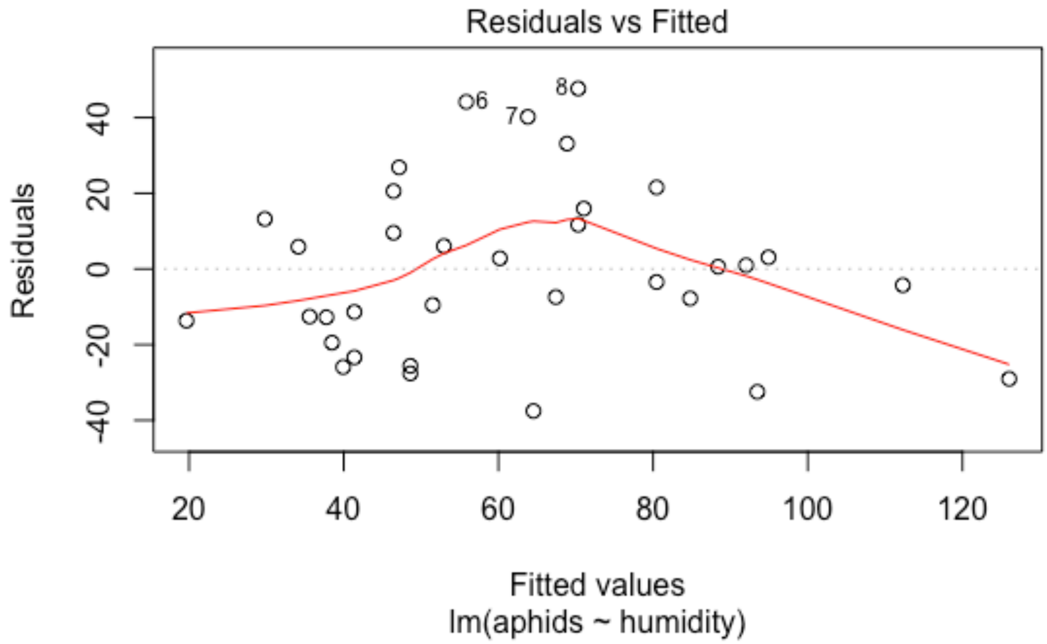
```
## Analysis of Variance Table
##
## Response: aphids
##           Df Sum Sq Mean Sq F value    Pr(>F)
## humidity   1  19891 19890.7  38.608 5.857e-07 ***
```

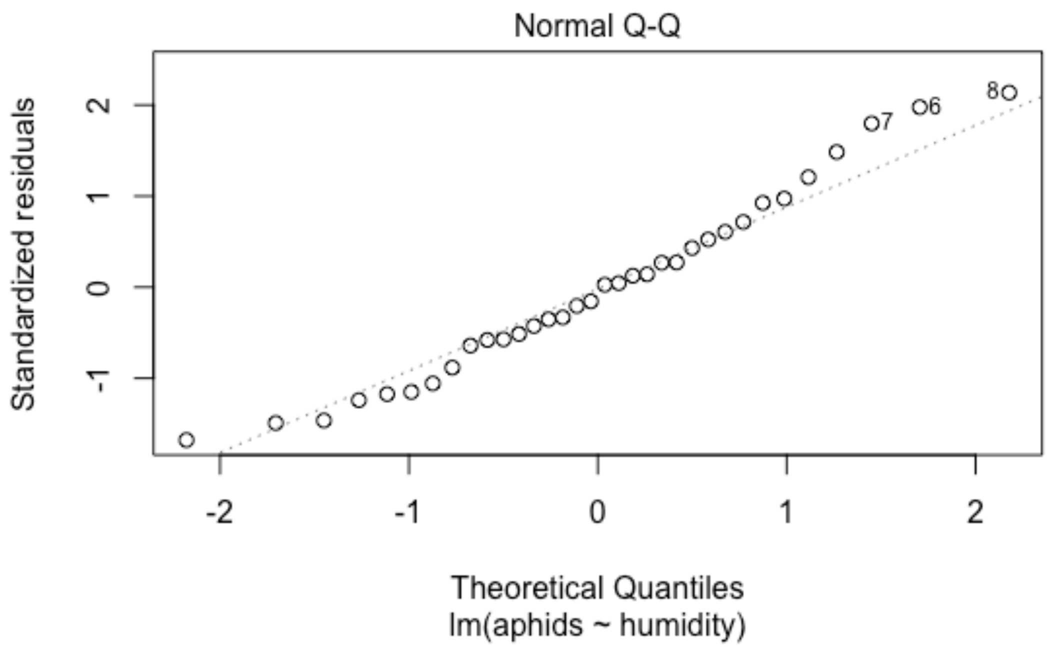
```
## Residuals 32  16486   515.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

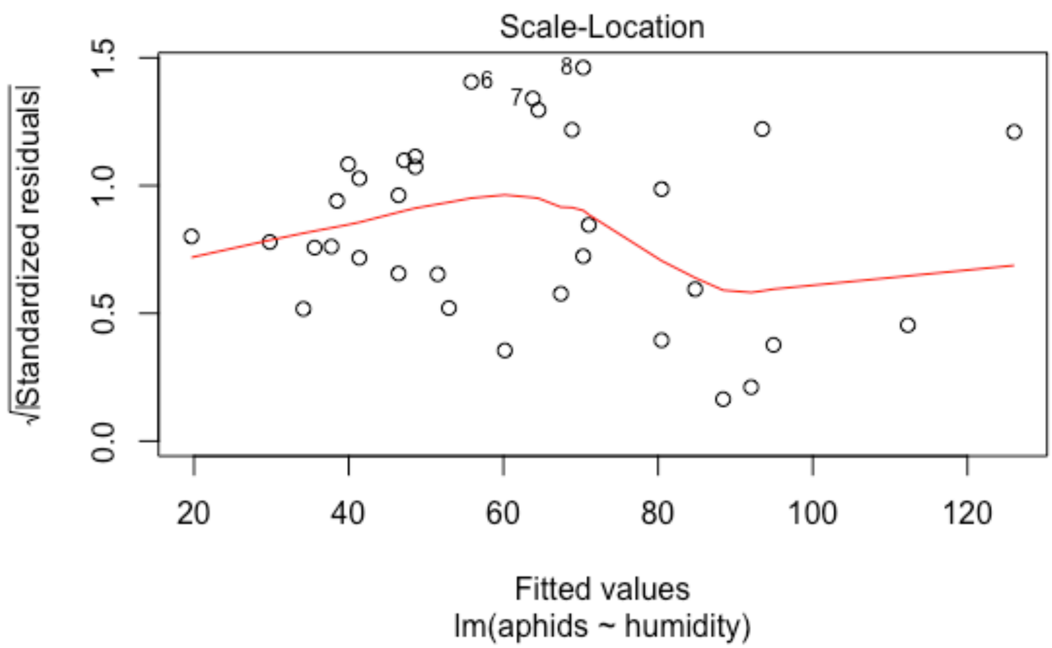
```r
summary(model2)
```

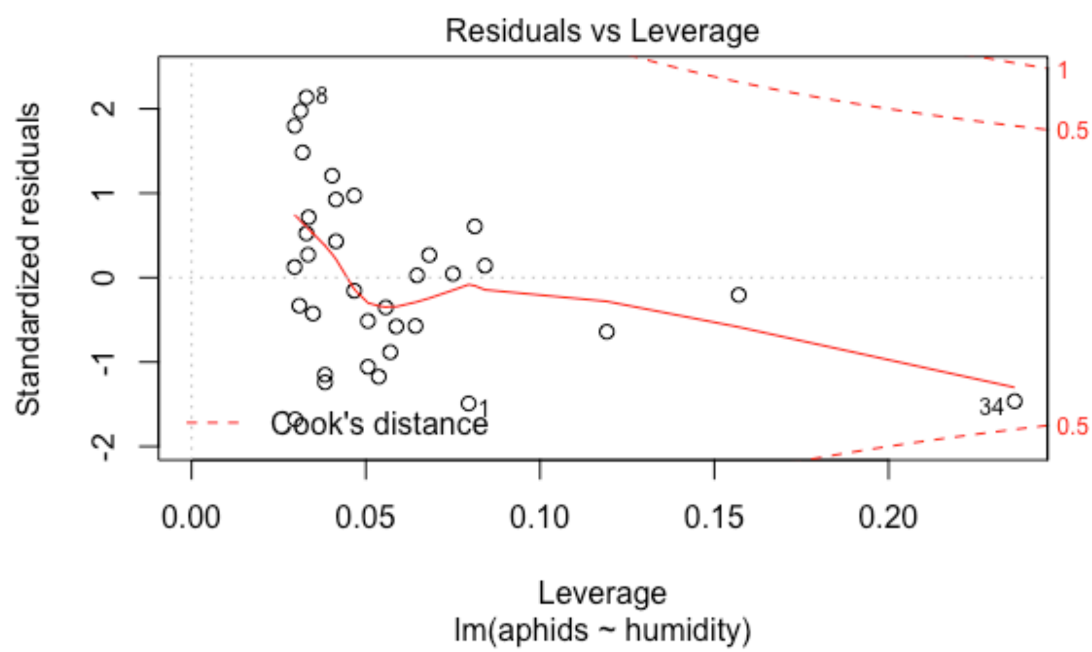
```
##
## Call:
## lm(formula = aphids ~ humidity)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -37.53 -13.44  -1.43  12.82  47.68
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.9787     9.0744   1.210    0.235
## humidity      1.4473     0.2329   6.214 5.86e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.7 on 32 degrees of freedom
## Multiple R-squared:  0.5468, Adjusted R-squared:  0.5326
## F-statistic: 38.61 on 1 and 32 DF,  p-value: 5.857e-07
```

```r
plot(model2)
```


Residuals vs Fitted
Residuals
Fitted values
lm(aphids ~ humidity)


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(aphids ~ humidity)






```
# Assumptions = values, model2

rstudent(model2)

##             1            2            3            4            5            6
7            8            9           10           11           12           13
```

```
14          15          16          17          18          19          20
21          22          23          24          25          26          27
28          29          30          31          32          33          34
## -1.52166116 -0.15329366  0.70958337  0.04378704  0.13944816  2.07616739
1.86604477  2.27067980  1.51293060  1.21600986  0.12382267  0.60092052 -
1.73010695 -0.88059890 -1.18139920 -0.56699356 -0.50823171 -0.57307681
0.92313524  0.26372262 -0.63535605 -1.25128763 -1.05882161 -1.15657354 -
0.42068829  0.42473282 -0.32760978  0.26710592  0.51730586  0.02642984 -
0.34840648  0.97153856 -0.20281479 -1.49172395
```

```r
dfbetas(model2)
```

```
##     (Intercept)     humidity
## 1   0.203962802 -0.354960067
## 2   0.007091915 -0.020637509
## 3   0.010888321  0.046732070
## 4  -0.005432893  0.009722237
## 5  -0.020090271  0.034108001
## 6   0.237139151 -0.090726882
## 7   0.116375070  0.025442918
## 8   0.045533868  0.137646115
## 9   0.044574968  0.075879906
## 10  0.208590411 -0.129821254
## 11  0.010635007 -0.001536502
## 12  0.175091541 -0.142772652
## 13 -0.099765368 -0.032603908
## 14 -0.202822717  0.150676741
## 15 -0.260370374  0.189329385
## 16 -0.141963444  0.109421496
## 17 -0.106991971  0.075962840
## 18 -0.134852336  0.101178572
## 19  0.162812727 -0.103448492
## 20  0.068702630 -0.053805692
## 21 -0.232992130  0.202795860
## 22 -0.202585666  0.120351428
## 23 -0.222901106  0.158256745
## 24 -0.187251287  0.111241631
## 25 -0.060049112  0.031601350
## 26  0.074909835 -0.047596460
## 27 -0.012732795 -0.013008080
## 28  0.035580371 -0.017261761
## 29  0.010373518  0.031358513
## 30 -0.002627836  0.005134802
## 31  0.026166587 -0.058167303
## 32 -0.044946864  0.130795599
## 33  0.055027998 -0.078907786
## 34  0.575503750 -0.776106220
```

```r
dffits(model2)
```

```
##           1           2           3           4           5 
6           7           8           9          10          11 
12          13          14          15          16          17 
18          19          20          21          22          23 
24          25          26          27          28          29 
30          31          32          33          34 
## -0.447191192 -0.033924951  0.132317421  0.012469416  0.042283274 
0.372962648  0.325861688  0.419240517  0.274398674  0.249344653  0.021611110 
0.178729735 -0.302985776 -0.216541182 -0.281470975 -0.148585846 -0.117356163 
-0.143175941  0.191962176  0.071346658 -0.233677243 -0.249738808 -0.244493286 
-0.230835253 -0.079949336  0.088321443 -0.058538076  0.049688881  0.095511299 
0.006952189 -0.084642540  0.215008230 -0.087530558 -0.829472811
```

```
covratio(model2)
```

```
##         1         2         3         4         5         6         7 
8         9        10        11        12        13        14        15 
16        17        18        19        20        21        22        23 
24        25        26        27        28        29        30        31 
32        33        34 
## 1.0022713 1.1160516 1.0676446 1.1518269 1.1620673 0.8477864 0.8874781 
0.8100232 0.9544553 1.0115565 1.0969300 1.1332631 0.9133416 1.0755087 
1.0311057 1.1154780 1.1038994 1.1084567 1.0529470 1.1384310 1.1787935 
1.0040208 1.0453923 1.0182323 1.0915424 1.0988072 1.0920026 1.0973744 
1.0831002 1.1392331 1.1196610 1.0526653 1.2606798 1.2144142
```

```
cooks.distance(model2)
```

```
##            1            2            3            4            5 
6            7            8            9           10           11 
12           13           14           15           16           17 
18           19           20           21           22           23 
24           25           26           27           28           29 
30           31           32           33           34 
## 9.604190e-02 5.935641e-04 8.891911e-03 8.024605e-05 9.221958e-04 
6.302998e-02 4.927114e-02 7.777970e-02 3.618960e-02 3.062822e-02 2.409338e-04 
1.629755e-02 4.320873e-02 2.361072e-02 3.912909e-02 1.127801e-02 7.049632e-03 
1.046940e-02 1.851025e-02 2.621394e-03 2.782097e-02 3.064301e-02 2.977580e-02 
2.636426e-02 3.280316e-03 4.002862e-03 1.762520e-03 1.271389e-03 4.668043e-03 
2.494547e-05 3.683311e-03 2.315487e-02 3.949133e-03 3.313265e-01
```

Additive multiple regression

Model form:

$$aphids = intercept + temperature + humidity + error$$

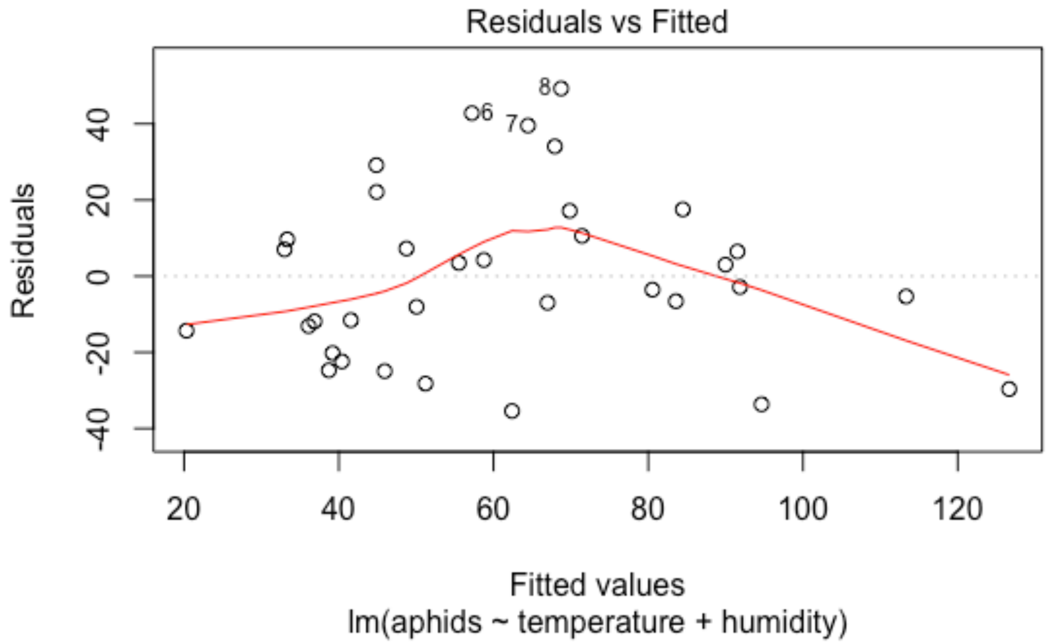
```
model3<-with(aphids_data, lm(aphids~temperature+humidity))
anova(model3)
```

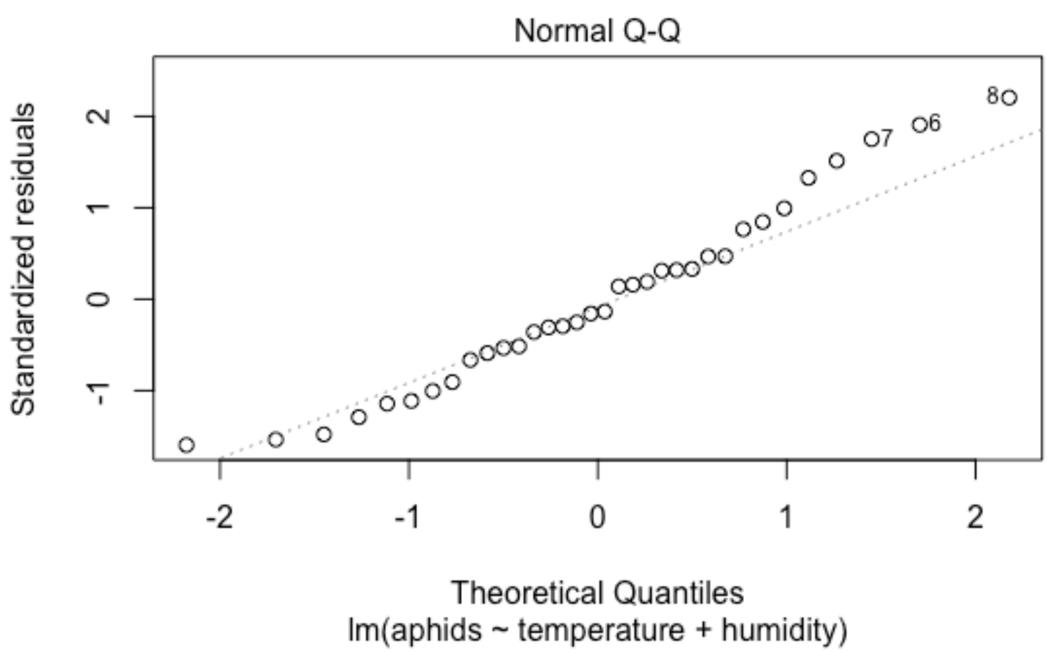
```
## Analysis of Variance Table
##
## Response: aphids
##             Df  Sum Sq Mean Sq F value    Pr(>F)
## temperature  1 15194.8 15194.8 28.7765 7.554e-06 ***
## humidity     1  4813.1  4813.1  9.1151  0.005038 **
## Residuals   31 16368.9   528.0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

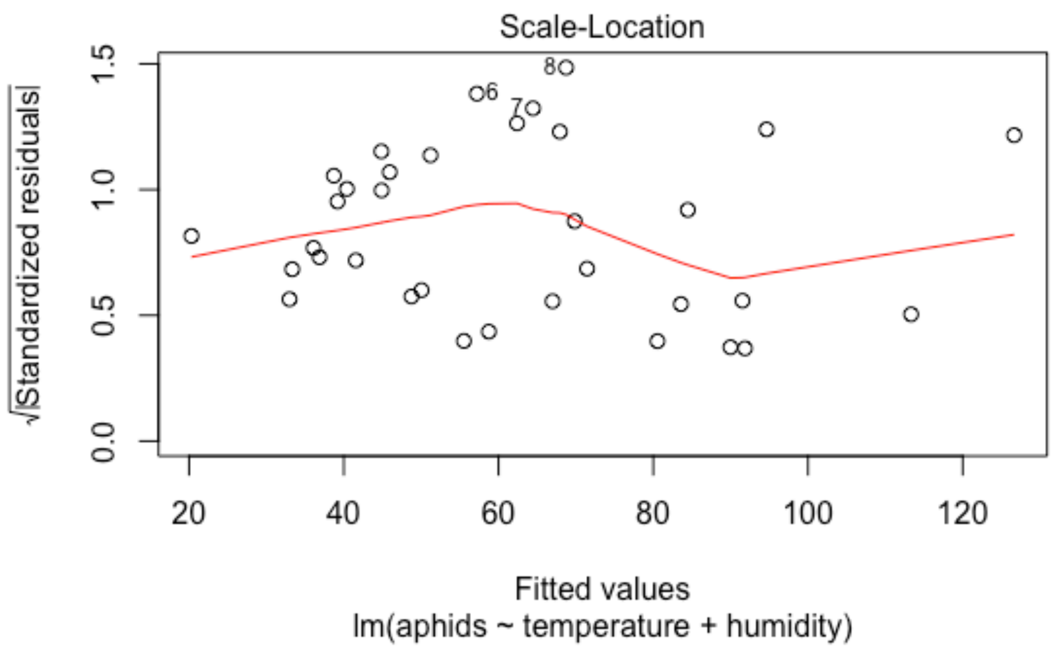
```r
summary(model3)
```

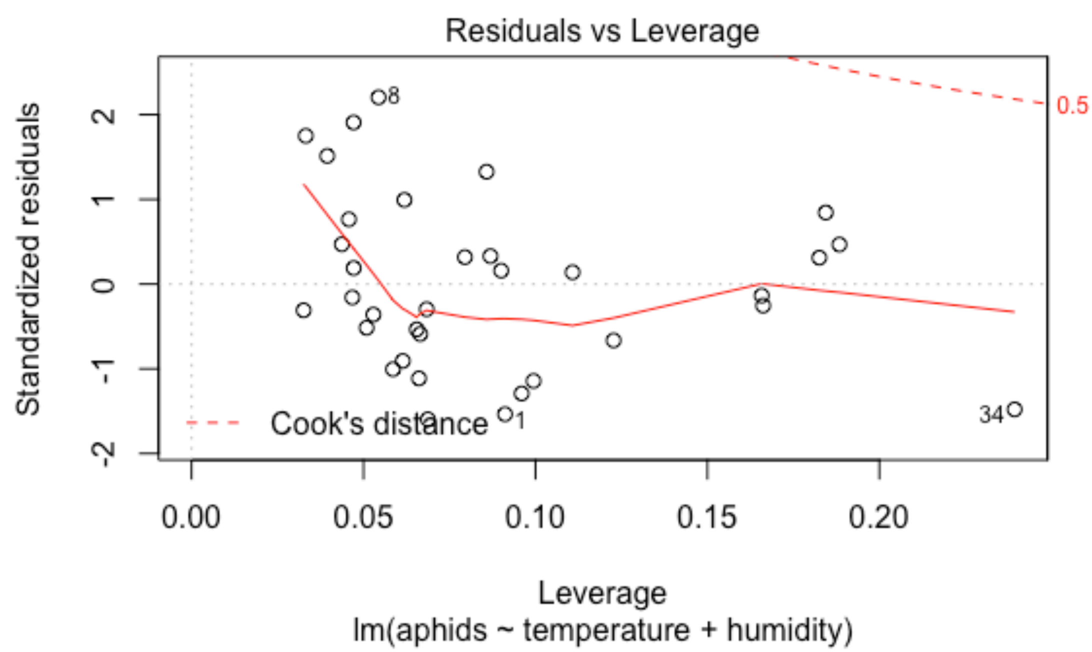
```
##
## Call:
## lm(formula = aphids ~ temperature + humidity)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -35.393 -14.006  -3.198  10.335  49.265
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  35.8255    53.5388   0.669  0.50835
## temperature  -0.6765     1.4360  -0.471  0.64089
## humidity      1.2811     0.4243   3.019  0.00504 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.98 on 31 degrees of freedom
## Multiple R-squared:   0.55,  Adjusted R-squared:  0.521
## F-statistic: 18.95 on 2 and 31 DF,  p-value: 4.212e-06
```

```r
plot(model3)
```


Residuals vs Fitted
Residuals
Fitted values
lm(aphids ~ temperature + humidity)
8
6
7
40
20
0
-20
-40
20
40
60
80
100
120


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(aphids ~ temperature + humidity)
8
6
7
2
1
0
-1
-2






```
vif(lm(aphids~temperature+humidity, data=aphids_data))

## temperature    humidity
##    3.238084    3.238084
```

```r
# Assumptions = values, model3
rstudent(model3)

##          1          2          3          4          5          6 
7          8          9         10         11         12         13 
14         15         16         17         18         19         20 
21         22         23         24         25         26         27 
28         29         30         31         32         33         34 
## -1.5718383 -0.1554587  0.7588243  0.1370895  0.3068861  1.9975726 
1.8135772  2.3619288  1.5465227  1.3436594  0.1864002  0.4608253 -1.6388797 -
0.9045838 -1.1176402 -0.5834426 -0.5100256 -0.5278939  0.9931907  0.3134810 -
0.6587793 -1.1492651 -1.0051359 -1.3057363 -0.3548909  0.3255590 -0.3044695 
0.1559741  0.4639182 -0.1337120 -0.2920664  0.8408516 -0.2500954 -1.5096341

dfbetas(model3)

##      (Intercept)  temperature     humidity
## 1  -0.1389605773  0.177983609 -0.057094981
## 2  -0.0001234515  0.001378033 -0.010485457
## 3  -0.0823800284  0.085661206  0.099164620
## 4  -0.0300681549  0.027499212  0.040114310
## 5  -0.1128940158  0.106442807  0.132644719
## 6   0.2933256395 -0.257672477 -0.263133078
## 7   0.1288811603 -0.111084942 -0.078585304
## 8  -0.3419564458  0.355447009  0.375970378
## 9  -0.1277781057  0.137669370  0.157728821
## 10 -0.2545839017  0.299547009  0.167359803
## 11 -0.0221748178  0.025322754  0.019755369
## 12  0.1894356505 -0.167405618 -0.203910662
## 13  0.3137416001 -0.335266154 -0.296248797
## 14 -0.0969427761  0.062028386  0.137785229
## 15  0.0843787510 -0.128835461 -0.006914942
## 16 -0.0531420869  0.028468182  0.086313818
## 17 -0.0271836261  0.008889032  0.049759567
## 18  0.0227872585 -0.044844187  0.014698315
## 19 -0.1140684195  0.146626191  0.059379069
## 20 -0.0200956838  0.034709112 -0.006903602
## 21 -0.0829693398  0.042055524  0.152053987
## 22  0.2620078051 -0.299442541 -0.185468011
## 23  0.0546306003 -0.092463785  0.006968177
## 24 -0.3623565160  0.329834636  0.346198714
## 25  0.0394488839 -0.048949485 -0.025740050
## 26  0.0816581713 -0.072640776 -0.081164117
## 27  0.0103635254 -0.012582384 -0.017184593
## 28  0.0419895172 -0.038892078 -0.038106780
## 29  0.0500352973 -0.049159176 -0.025153876
## 30 -0.0434351938  0.046540937  0.023406939
## 31  0.0372940235 -0.034009119 -0.055554804
## 32  0.3331825303 -0.345523841 -0.219245482
```

```
## 33 -0.0141526880  0.026249474 -0.032547183
## 34  0.0042654848  0.097322196 -0.356471323
```

```
dffits(model3)
```

```
##           1           2           3           4           5           6 
7           8           9          10          11          12          13 
14          15          16          17          18          19          20 
21          22          23          24          25          26          27 
28          29          30          31          32          33          34 
## -0.49779546 -0.03443304  0.16617790  0.04839027  0.14501981  0.44419213 
0.33617659  0.56641172  0.31345118  0.41159699  0.04146285  0.22201280 -
0.44522486 -0.23142890 -0.29739864 -0.15570300 -0.11812321 -0.13975284 
0.25511469  0.09211577 -0.24640080 -0.38190475 -0.25074744 -0.42548793 -
0.08385349  0.10043884 -0.05588462  0.04905946  0.09917484 -0.05960708 -
0.07911715  0.39982748 -0.11165807 -0.84678558
```

```
covratio(model3)
```

```
##         1         2         3         4         5         6         7 
8         9        10        11        12        13        14        15 
16        17        18        19        20        21        22        23 
24        25        26        27        28        29        30        31 
32        33        34 
## 0.9574605 1.1547079 1.0921820 1.2385180 1.3371269 0.7961237 0.8353191 
0.6995172 0.9125707 1.0128230 1.1539505 1.3310018 0.9160657 1.0844138 
1.0454007 1.1426135 1.1328307 1.1483997 1.0673803 1.1869404 1.2046850 
1.0766525 1.0611755 1.0340267 1.1504192 1.1956712 1.1300346 1.2095848 
1.1293150 1.3202770 1.1742902 1.2615329 1.3150607 1.1644730
```

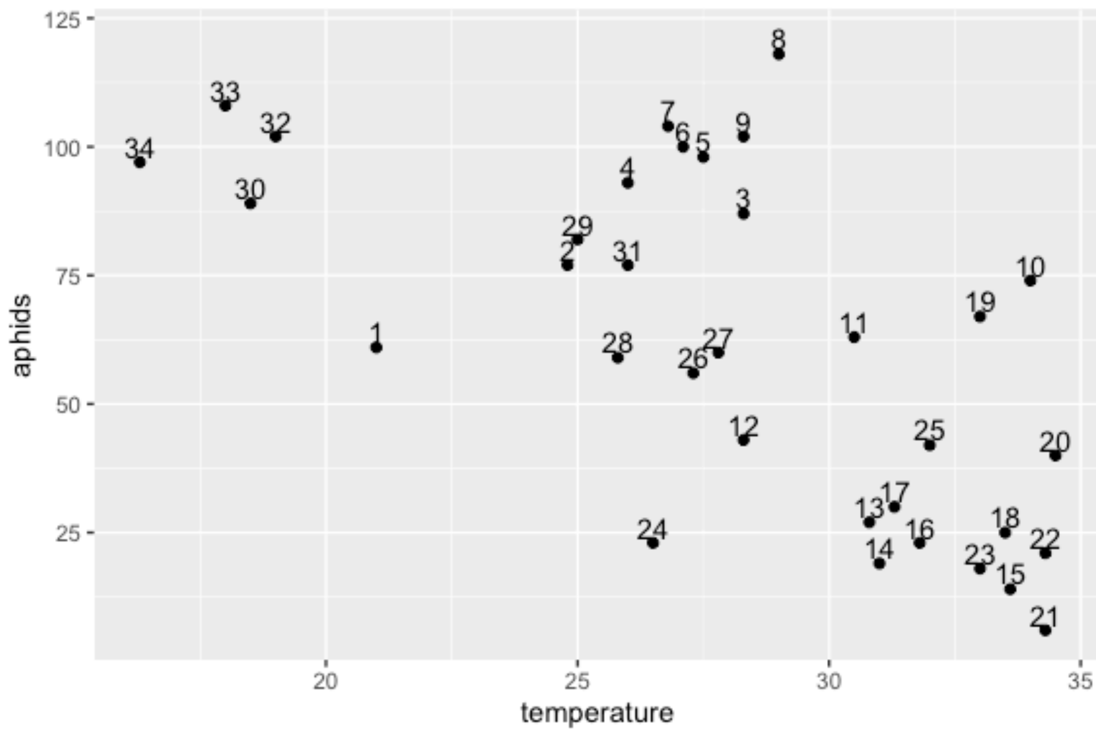
```
cooks.distance(model3)
```

```
##            1            2            3            4            5 
6            7            8            9           10           11 
12           13           14           15           16           17 
18           19           20           21           22           23 
24           25           26           27           28           29 
30           31           32           33           34 
## 0.0788589481 0.0004080564 0.0093327348 0.0008060524 0.0072212533 
0.0599828648 0.0350811487 0.0931782892 0.0313433979 0.0550406641 0.0005914727 
0.0168582238 0.0626669338 0.0179583880 0.0292469522 0.0082568244 0.0047647510 
0.0066653796 0.0217040047 0.0029131770 0.0206141656 0.0481191084 0.0209511332 
0.0590048734 0.0024118042 0.0034625093 0.0010724176 0.0008283478 0.0033637037 
0.0012230834 0.0021499446 0.0537957394 0.0042854350 0.2295447363
```

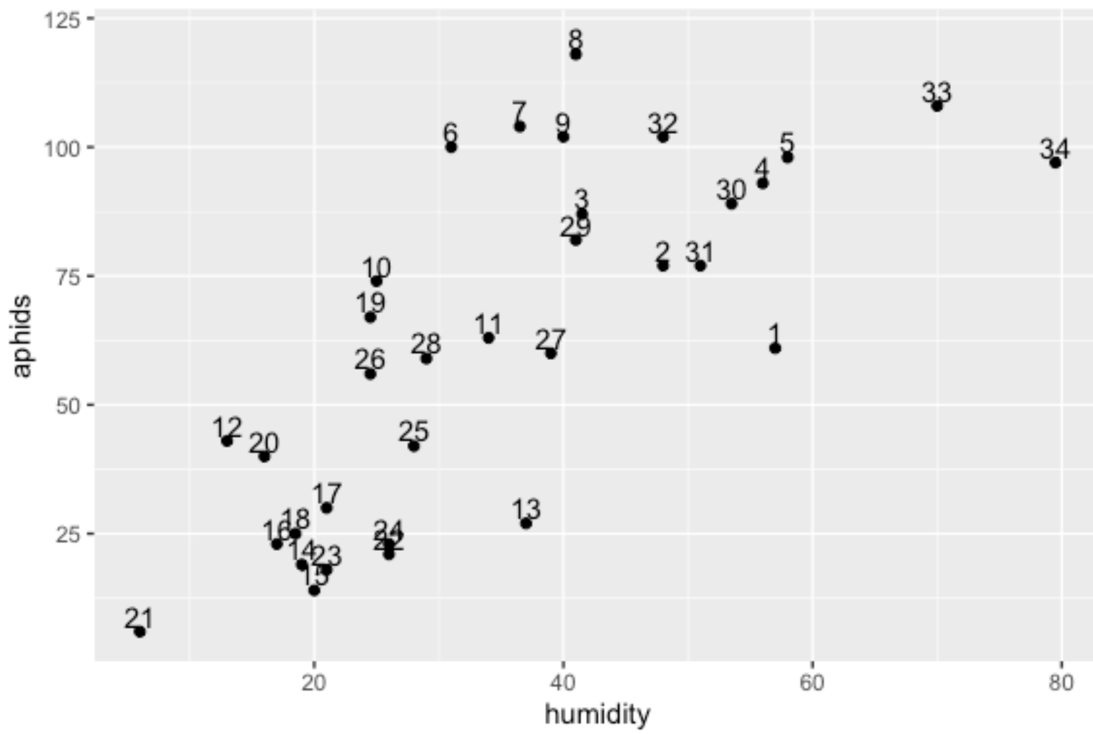
Visualizing more complicated relationships

So far, we cannot say with certainty that the additive model is the best fitting model. Before we commit to another analysis, it is important to take a step back and think about the visualization of the data to be better informed about what has occurred. Another reason for doing this is to be able to better interpret the observed results about the model assumptions (i.e., influential observations, some unhidden spatial structure in the data collection process).

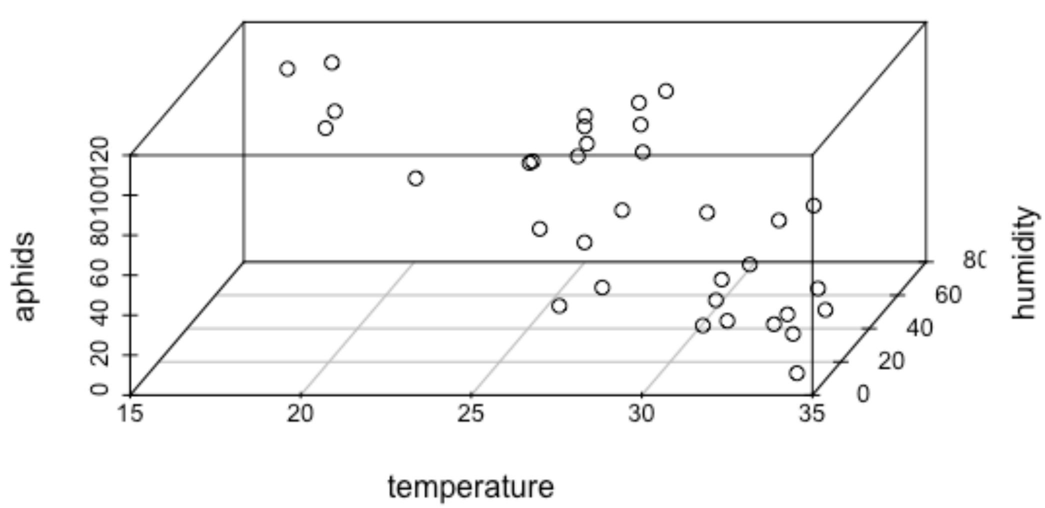
```
# Start with temperature, let's add to the graph infomration about the lots
temp <- ggplot(aphids_data, aes(x=temperature, y=aphids, label=lot))
temp + geom_point() + geom_text(hjust=0.5, nudge_y=3) #Have a look a few of
the observations like 30, 32, 33, 34 and also 8 (maybe 9)
```




```
# Now let's consider RH and do the same thing
temp2 <- ggplot(aphids_data, aes(x=humidity, y=aphids, label=lot))
temp2 + geom_point() + geom_text(hjust=0.5, nudge_y=3) #Maybe a bit different
grouping" 6-9, 33 and 34
```



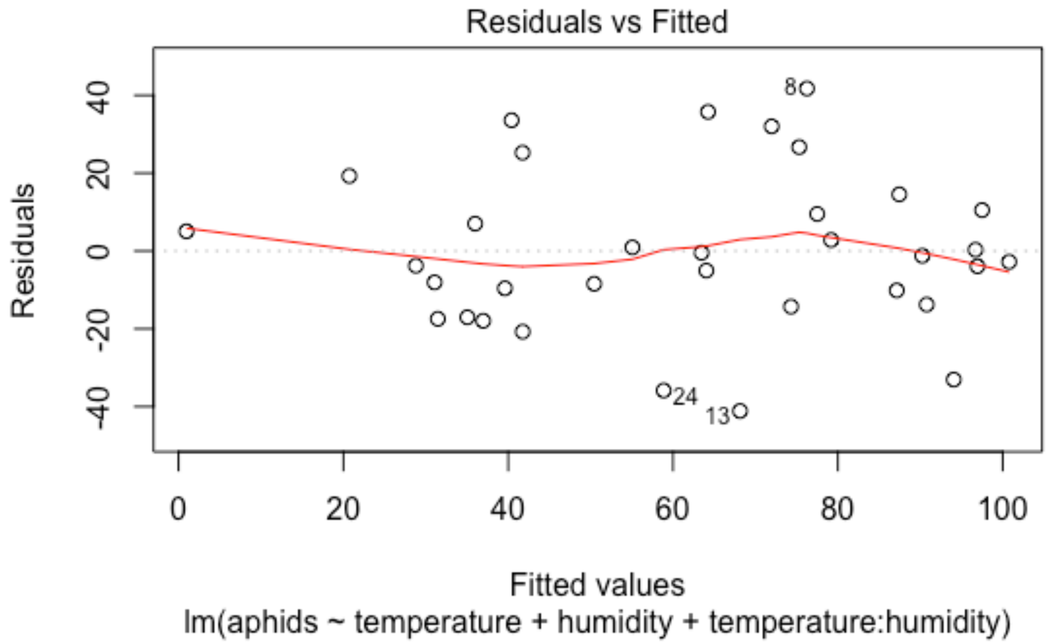

```r
# In 3-dimensiones? This example comes from the package *scatterplot3d*

with(aphids_data, scatterplot3d(temperature, humidity, aphids, angle=75))
```

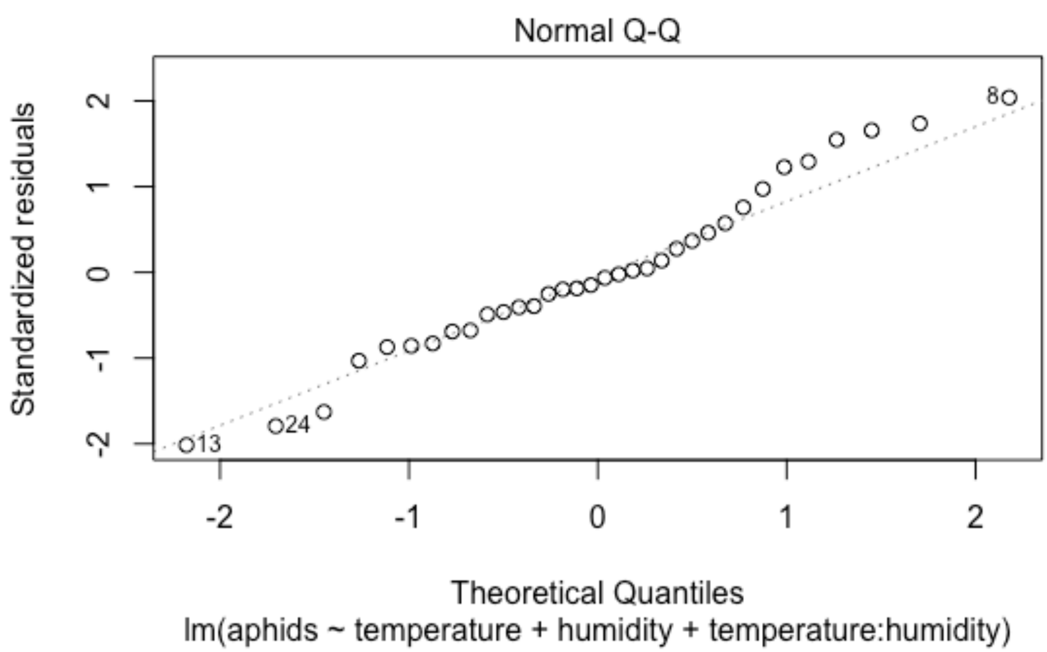


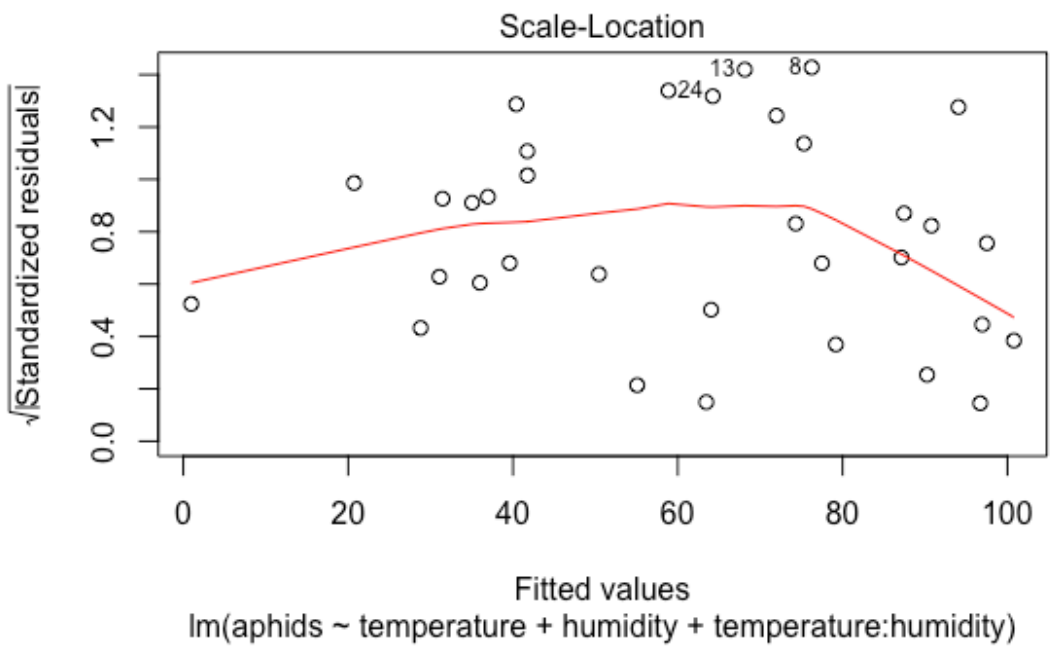
Multiple regression with interactions

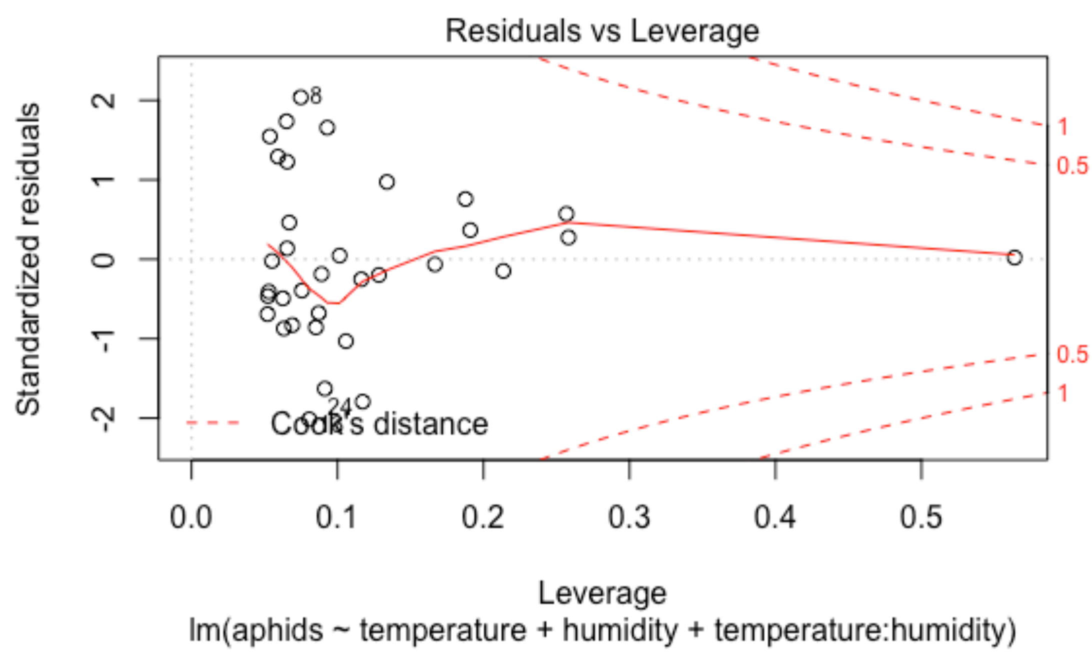
Given that individually, we see different relationships between the number of aphids with temperature or relative humidity, we might want to consider if there is an interaction between those two factors that helps to explain the overall relationship.

```
model4 <- with(aphids_data, lm(aphids ~ temperature + humidity +
temperature:humidity))

anova(model4)

## Analysis of Variance Table
##
## Response: aphids
##                      Df  Sum Sq Mean Sq F value    Pr(>F)
## temperature           1 15194.8 15194.8  33.506 2.522e-06 ***
## humidity              1  4813.1  4813.1  10.613  0.002789 **
## temperature:humidity  1  2764.0  2764.0   6.095  0.019474 *
## Residuals            30 13604.8   453.5
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

summary(model4)

##
## Call:
## lm(formula = aphids ~ temperature + humidity + temperature:humidity)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -41.13 -12.87  -2.02  10.25  41.75
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)          150.70989   68.02395   2.216   0.0345 *
## temperature           -4.72276    2.11121  -2.237   0.0329 *
## humidity              -1.29670    1.11576  -1.162   0.2543
## temperature:humidity   0.09728    0.03940   2.469   0.0195 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 21.3 on 30 degrees of freedom
## Multiple R-squared:  0.626,  Adjusted R-squared:  0.5886
## F-statistic: 16.74 on 3 and 30 DF,  p-value: 1.414e-06

plot(model4)
```


Residuals vs Fitted
Residuals
Fitted values
lm(aphids ~ temperature + humidity + temperature:humidity)
8
24
13
40
20
0
-20
-40
0
20
40
60
80
100


Normal Q-Q
Standardized residuals
Theoretical Quantiles
lm(aphids ~ temperature + humidity + temperature:humidity)
8
24
13
2
1
0
-1
-2
-2
-1
0
1
2






```
# Assumptions = values, model4
rstudent(model4)

##              1              2              3              4              5              6
7              8              9             10             11             12             13
14             15             16             17             18             19             20
```

```
21          22          23          24          25          26          27
28          29          30          31          32          33          34
## -1.67738460 -0.48588149  0.45590218 -0.19523576 -0.14531052  1.79972032
1.58449094  2.15886689  1.30727332  1.70840183 -0.02181518  0.36068272 -
2.12994645 -0.86808331 -0.85354897 -0.38866846 -0.45693592 -0.18387477
1.23775270  0.97153371  0.27052672 -1.03205411 -0.82508336 -1.86536030 -
0.40145315  0.04498291 -0.68446574 -0.24802524  0.13423194 -0.06323885 -
0.67115336  0.75218833  0.56505021  0.02067808
```

```
dfbetas(model4)
```

```
##      (Intercept)  temperature     humidity temperature:humidity
## 1  -0.0949334782  0.104709362 -0.039584196          0.019350004
## 2  -0.0434083991  0.051676363  0.047346765         -0.063039334
## 3   0.0104511179 -0.020483667 -0.043004153          0.068644149
## 4   0.0125973436 -0.003428996  0.005585795         -0.027700059
## 5   0.0200060874 -0.009982149  0.004443134         -0.028864368
## 6   0.3651795277 -0.341307478 -0.317703892          0.249350618
## 7   0.2422524883 -0.242541747 -0.242297450          0.232776103
## 8  -0.0109075193 -0.042140260 -0.177929199          0.320976539
## 9   0.0498830523 -0.072826504 -0.129655838          0.189283832
## 10 -0.3408072725  0.358785369  0.217261311         -0.151702669
## 11  0.0005263790 -0.000316099  0.001062068         -0.002009342
## 12  0.1221549093 -0.098424293 -0.075269257          0.020228157
## 13  0.1333989598 -0.088122021  0.090421220         -0.242562866
## 14 -0.0418864697  0.008011790  0.011031748          0.038058296
## 15  0.1317295973 -0.158260224 -0.117120422          0.123141864
## 16  0.0004605123 -0.017956896 -0.015762200          0.038604856
## 17 -0.0058300098 -0.008533870 -0.000619253          0.017463648
## 18  0.0260840050 -0.032918740 -0.025833607          0.029557943
## 19 -0.1563373872  0.174924757  0.097882894         -0.076671310
## 20 -0.2133463221  0.258866254  0.220017614         -0.243411245
## 21 -0.0520591385  0.077908687  0.084254567         -0.115605524
## 22  0.2314600667 -0.237322197 -0.139947666          0.086596590
## 23  0.0925507109 -0.115811898 -0.079585871          0.087209322
## 24 -0.5799971247  0.525151313  0.447129697         -0.289295996
## 25  0.0304683047 -0.032541539 -0.007414919         -0.003043043
## 26  0.0122044840 -0.010813140 -0.009331422          0.005713728
## 27 -0.0499259797  0.058126678  0.078027579         -0.098076089
## 28 -0.0786659661  0.072746679  0.061679949         -0.042751118
## 29  0.0246830038 -0.024958336 -0.021748326          0.020466850
## 30 -0.0135483304  0.012244012  0.001928902          0.002110430
## 31 -0.0027225130  0.024986567  0.044656549         -0.096290927
## 32  0.2502942582 -0.232060720 -0.113674644          0.047457190
## 33 -0.1102356148  0.113536109  0.212046646         -0.197252978
## 34 -0.0122551182  0.012733935  0.018960381         -0.017832210
```

```
dffits(model4)
```

```
##            1            2            3            4            5
6            7            8            9           10           11
```

```
12          13          14          15          16          17
18          19          20          21          22          23
24          25          26          27          28          29
30          31          32          33          34
## -0.531609206 -0.125502391  0.122090070 -0.074914358 -0.075725395
0.474770847  0.377244053  0.613987196  0.327877853  0.546849311 -0.005271111
0.175211526 -0.630865880 -0.225538194 -0.260429456 -0.111153445 -0.107334987
-0.057484681  0.327640435  0.381923652  0.159601123 -0.354887689 -0.224599512
-0.679748019 -0.094906727  0.015111134 -0.160394451 -0.089969303  0.035517257
-0.028285597 -0.207379175  0.361507505  0.332123809  0.023506774
```

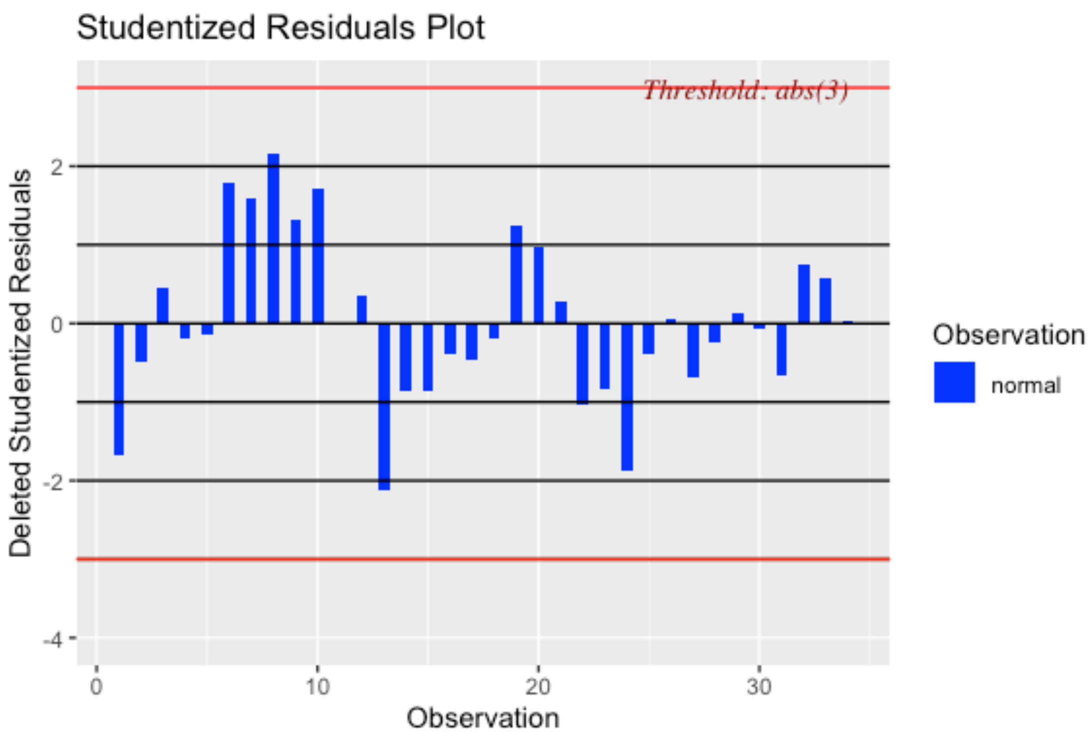
```r
covratio(model4)
```

```
##         1         2         3         4         5         6         7
8         9        10        11        12        13        14        15
16        17        18        19        20        21        22        23
24        25        26        27        28        29        30        31
32        33        34
## 0.8701623 1.1826549 1.1927973 1.3069641 1.4520128 0.8020052 0.8681666
0.6819798 0.9680977 0.8603410 1.2120133 1.3903653 0.6965068 1.1033099
1.1335691 1.2134151 1.1742397 1.2513168 0.9974108 1.1632187 1.5283511
1.1085842 1.1210624 0.8245063 1.1827248 1.2741137 1.1331044 1.2849849
1.2223690 1.3735892 1.1795590 1.3049070 1.4748564 2.6250656
```

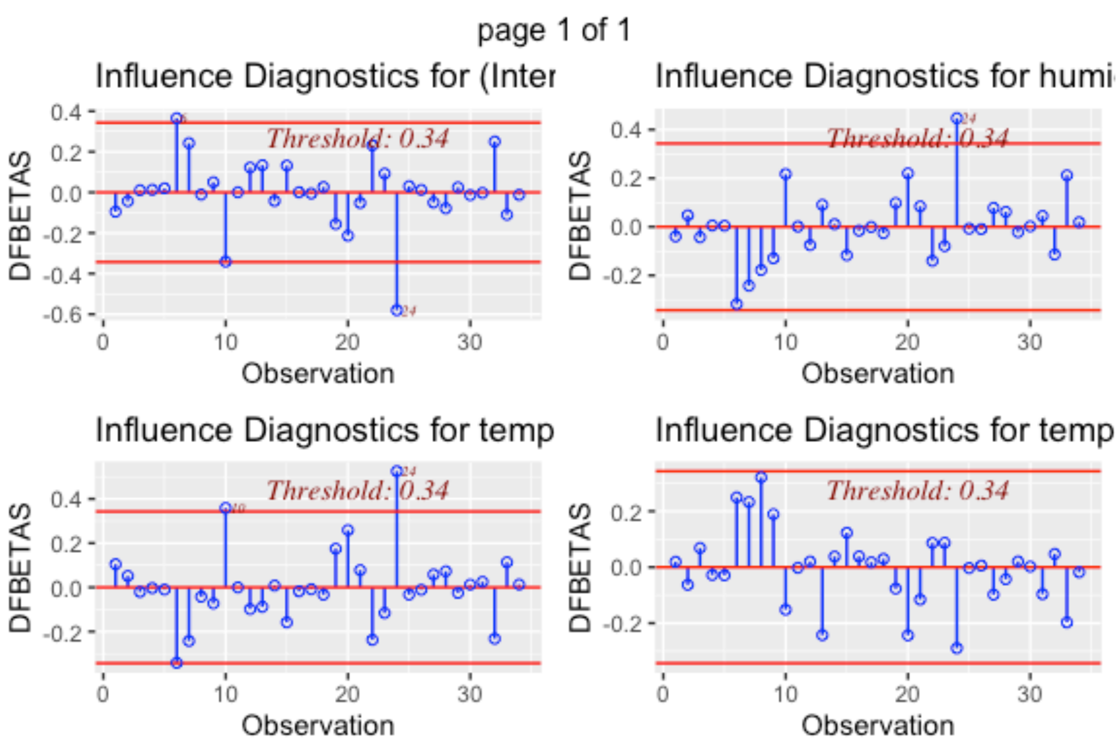
```r
cooks.distance(model4)
```

```
##            1            2            3            4            5
6            7            8            9           10           11
12           13           14           15           16           17
18           19           20           21           22           23
24           25           26           27           28           29
30           31           32           33           34
## 6.662438e-02 4.040602e-03 3.827564e-03 1.449516e-03 1.481939e-03
5.243821e-02 3.387265e-02 8.399563e-02 2.625550e-02 7.026714e-02 7.185558e-06
7.903960e-03 8.900520e-02 1.282220e-02 1.711070e-02 3.178723e-03 2.958219e-03
8.536139e-04 2.636942e-02 3.653477e-02 6.571137e-03 3.141810e-02 1.274688e-02
1.066957e-01 2.316596e-03 5.905097e-05 6.547598e-03 2.088968e-03 3.260411e-04
2.068874e-04 1.095216e-02 3.315175e-02 2.821681e-02 1.429035e-04
```

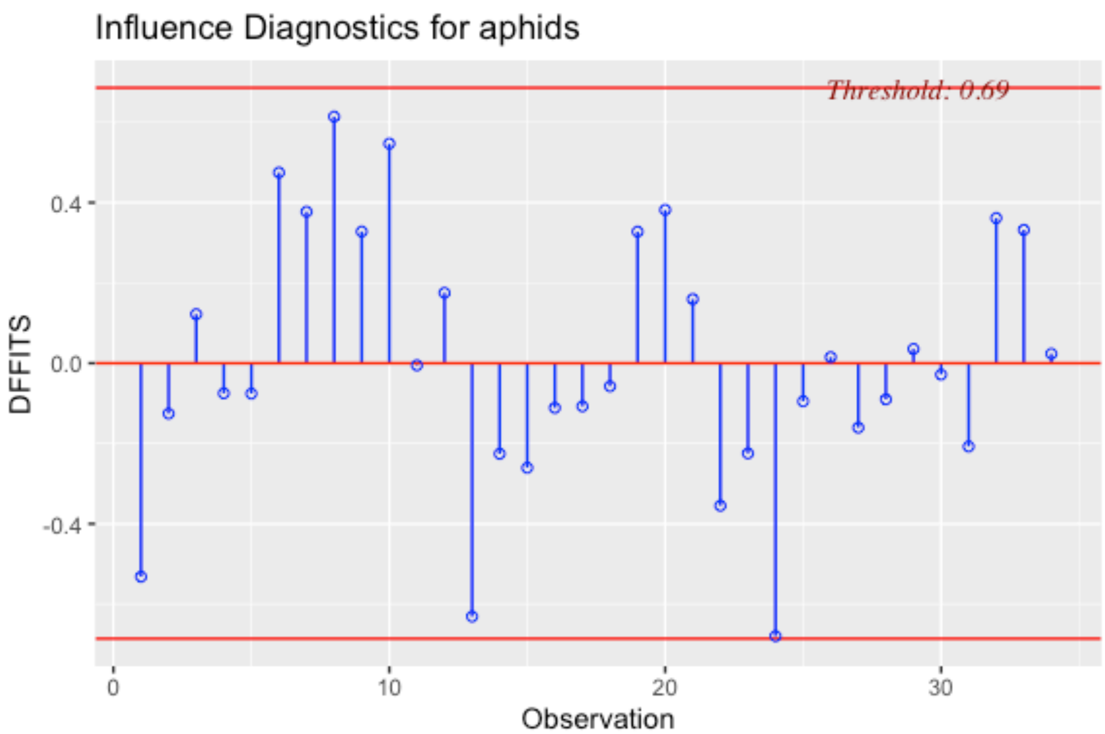
```r
# Graphically from olsrr package
ols_plot_resid_stud(model4)
```



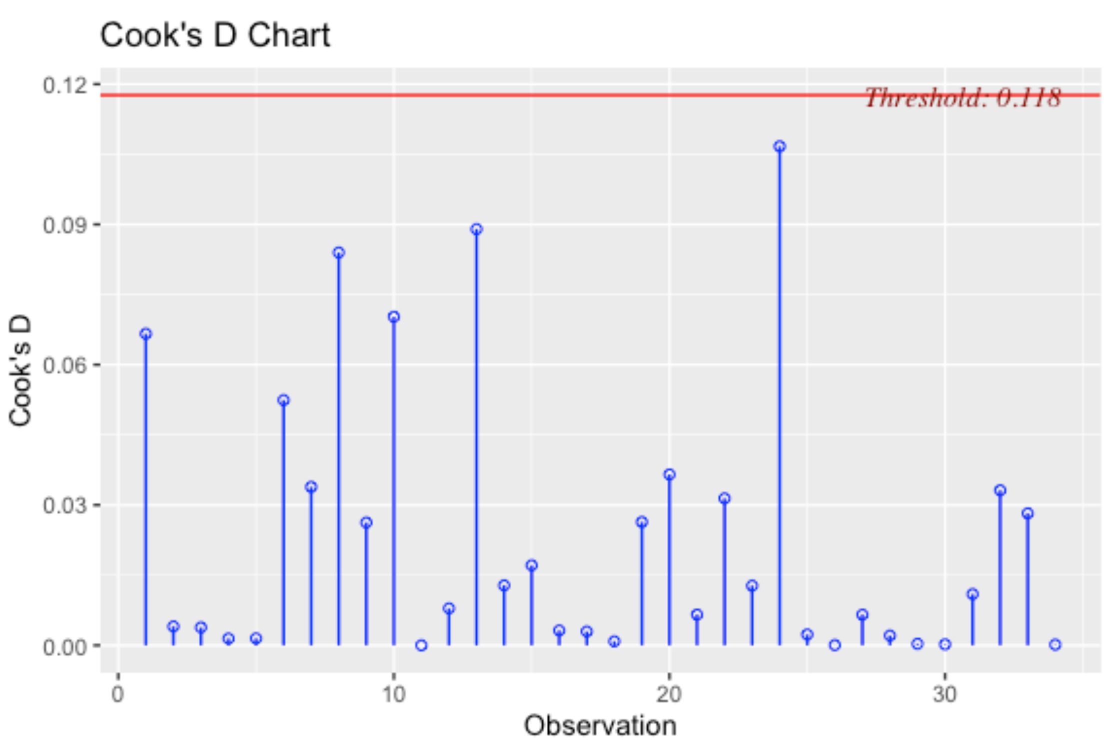

```r
ols_plot_dfbetas(model4)
```




```r
ols_plot_dffits(model4)
```




```r
ols_plot_cooksd_chart(model4)
```




```r
# Compare the different models
anova(model1, model3) # model 3 better
```

```
## Analysis of Variance Table
##
## Model 1: aphids ~ temperature
## Model 2: aphids ~ temperature + humidity
##   Res.Df   RSS Df Sum of Sq      F   Pr(>F)
## 1     32 21182
## 2     31 16369  1    4813.1 9.1151 0.005038 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(model2, model3) # model 2 mejor (only RH)
```

```
## Analysis of Variance Table
##
## Model 1: aphids ~ humidity
## Model 2: aphids ~ temperature + humidity
##   Res.Df   RSS Df Sum of Sq      F Pr(>F)
## 1     32 16486
## 2     31 16369  1    117.18 0.2219 0.6409
```

```r
anova(model2, model4) # the interaction improved the model?
```

```
## Analysis of Variance Table
##
## Model 1: aphids ~ humidity
## Model 2: aphids ~ temperature + humidity + temperature:humidity
##   Res.Df   RSS Df Sum of Sq      F  Pr(>F)
## 1     32 16486
## 2     30 13605  2    2881.2 3.1767 0.05606 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
anova(model3, model4) # the interaction improved the model
```

```
## Analysis of Variance Table
##
## Model 1: aphids ~ temperature + humidity
## Model 2: aphids ~ temperature + humidity + temperature:humidity
##   Res.Df   RSS Df Sum of Sq     F  Pr(>F)
## 1     31 16369
## 2     30 13605  1      2764 6.095 0.01947 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Remember that once we have a model selected, we should examine the
assumptions in greater detail
```

Predictions

To close our discussion, let's again look at the function *predict* using model 4.

```r
# Let's start by considering the average values for temperature and relative
humidity
mean(temperature)

## [1] 28.08529

mean(humidity)

## [1] 35.19118

observation <- data.frame(temperature=mean(temperature),
humidity=mean(humidity))

predict(object=model4, newdata=observation, interval="confidence")

##        fit      lwr      upr
## 1 68.58645 59.30643 77.86646

predict(object=model4, newdata=observation, interval="predict")

##        fit      lwr      upr
## 1 68.58645 24.11635 113.0565

# Looking at all observations in the database
intervals<-predict(model4, interval="confidence")
intervals

##            fit        lwr       upr
## 1   94.0665692  80.927158 107.20598
## 2   87.1482874  76.271599  98.02498
## 3   77.4955105  66.245093  88.74593
## 4   96.9454425  81.365016 112.52587
## 5  100.7900752  80.691122 120.88903
## 6   64.2519748  53.158443  75.34551
## 7   71.9715847  61.898562  82.04461
## 8   76.2533767  64.356209  88.15054
## 9   75.3109441  64.730640  85.89125
## 10  40.4082060  27.149656  53.66676
## 11  63.4592842  53.244672  73.67390
## 12  35.9887485  16.985340  54.99216
## 13  68.1334763  55.782306  80.48465
## 14  36.9661205  26.029726  47.90252
## 15  31.4646380  18.772561  44.15671
## 16  31.0728691  19.114485  43.03125
## 17  39.6002445  29.654831  49.54566
## 18  28.7989865  15.821797  41.77618
## 19  41.7421198  30.613084  52.87116
## 20  20.7271297   4.815514  36.63875
## 21   0.9596999 -21.139235  23.05864
## 22  41.7610595  27.618756  55.90336
## 23  35.0445138  23.621299  46.46773
## 24  58.8698369  43.979310  73.76036
```

```
## 25  50.4385954  40.432771  60.44442
## 26  55.0764466  41.227039  68.92585
## 27  74.3189517  64.396268  84.24164
## 28  64.0447598  49.214277  78.87524
## 29  79.1902050  68.065480  90.31493
## 30  90.2502594  72.492852 108.00767
## 31  90.7822943  77.942971 103.62162
## 32  87.4570502  68.617765 106.29634
## 33  97.5065531  75.468470 119.54464
## 34  96.7041862  64.049441 129.35893
```

```r
predictions<-predict(model4, interval="predict")
```

```
## Warning in predict.lm(model4, interval = "predict"): predictions on
current data refer to _future_ responses
```

```r
predictions
```

```
##             fit        lwr       upr
## 1   94.0665692  48.634034 139.49910
## 2   87.1482874  42.317791 131.97878
## 3   77.4955105  32.572877 122.41814
## 4   96.9454425  50.747815 143.14307
## 5  100.7900752  52.879335 148.70082
## 6   64.2519748  19.368375 109.13557
## 7   71.9715847  27.329263 116.61391
## 8   76.2533767  31.164423 121.34233
## 9   75.3109441  30.551432 120.07046
## 10  40.4082060  -5.058928  85.87534
## 11  63.4592842  18.784802 108.13377
## 12  35.9887485 -11.472822  83.45032
## 13  68.1334763  22.922609 113.34434
## 14  36.9661205  -7.878900  81.81114
## 15  31.4646380 -13.840548  76.76982
## 16  31.0728691 -14.032275  76.17801
## 17  39.6002445  -5.013457  84.21395
## 18  28.7989865 -16.586898  74.18487
## 19  41.7421198  -3.150269  86.63451
## 20  20.7271297 -25.583243  67.03750
## 21   0.9596999 -47.823843  49.74324
## 22  41.7610595  -3.971597  87.49372
## 23  35.0445138  -9.921706  80.01073
## 24  58.8698369  12.900294 104.83938
## 25  50.4385954   5.811388  95.06580
## 26  55.0764466   9.433515 100.71938
## 27  74.3189517  29.710312 118.92759
## 28  64.0447598  18.094631 109.99489
## 29  79.1902050  34.298885 124.08152
## 30  90.2502594  43.273705 137.22681
## 31  90.7822943  45.435637 136.12895
## 32  87.4570502  40.060956 134.85314
```

```
## 33  97.5065531  48.750546 146.26256
## 34  96.7041862  42.318494 151.08988
```

Summary

The objective in this exercise was to introduce the concept of using multiple regression to build a model. This provides a base also as you move forward in your modeling work to think about things like *hidden interactions*, which is often very common in complex datasets and can often drive aspects of things like machine learning.