## Contents

# Study of Traffic Flow with Automaton

Pascal Sauerborn, Student ID: 4313894

*University of Nottingham, School of Physical Sciences*

**Abstract**

The flow of traffic can be effectively simulated using relatively simple models. The following study uses a Basic Automaton model to investigate how the flow of traffic is effected by several control parameters, such as road length, car density and driver profiles. By observing the flow around a single site of the Automaton, it is shown that flow rate is unchanged with varying road length, and, for low car density values, also increases with speed limit. Additionally, it is shown that an increase in the probability of random deceleration causes a global decrease in flow rate. By extending the model to include distributions of various driver profiles aimed to quantify the quality of a driver, it is shown that the speed gained by speeding drivers is not enough to compensate for their continuous acceleration and braking. Finally, it is shown that the flow rate is greatly increased by adding a second lane which allows drivers to switch lanes; this gives rise to a third flow regime. It is also shown that, at high densities, the overtaking rules become largely irrelevant

## 1. Introduction

Traffic flow is a particularly relevant subject since, for the majority of people, it presents a daily struggle; sitting in a traffic jam is an only too familiar experience for most, and becomes even more tiresome when one realizes there is seemingly no source for the jam. This raises the question as to what exactly causes the formation of a traffic jam.

In an ideal world, one might expect traffic to flow in a seemingly harmonious fashion and, provided there is no obstruction, traffic jams in general should be avoidable. This study aims to explore the nature of traffic jams and the natural flow of traffic by using a relatively simple Automaton model. Despite the simple nature of the model, the results obtained are strikingly reminiscent of data collected on real-world traffic flow. This gives the Automaton model some sense of relevance despite its simplicity, and one quickly arrives at the result that, for the simple case which assumes no road obstructions, traffic indeed flows fluidly if not for driver error.

One might naively come to the conclusion that bad drivers are the source of traffic jams (as indeed many do), but it then becomes interesting to ask how the dynamics of traffic can be controlled by varying external parameters that are independent of the drivers on the road (such as the speed limit, or the length of the road, for example).

Curiosity aside, there are many motivations for the investigation of the effects of such parameters on the reduction of traffic congestion. If the opportunity cost of passengers is not enough, traffic jams result in significant fuel wastage, which in turn leads to an increase in pollution and carbon dioxide emissions. The constant acceleration and deceleration results in significantly more wear on local infrastructure resulting in a higher need for repair and maintenance work. This, in turn, results in reduced flow and more congestion, and increased infrastructure cost, which is usually subsidized by an inflated tax cost.

Many studies also suggest that traffic congestion has a variety of negative effects on the physical and mental health of the general public; in fact, one study by Levy, Buoncore, von Stackelberg [2] from Harvard University suggest that, in just 83 urban areas in the United states, an estimated 3000 premature civilian deaths were at least in part attributed to release of fine particulate matter ($PM_{2.5}$), of which roughly 30% is emitted from road vehicles. The same study suggested that, in 2005, the total social cost due to congestion within the 83 studied areas reached an estimated $24 Billion.

## 2. Model

The model built for this study follows closely that constructed by Nagel and Schreckenberg [1], which is summarized below.

### *2.1. Automaton System for Modeling Traffic Flow*

An Automaton consists of a simple set of rules that are used to incrementally update the state of a system (in this case, the road). The road, in practice, consisted of a NumPy array object, in which each cell represented a road site. A car object was then defined and these car objects were stored in the array cells. Note that the road had periodic boundaries so that, if a car moves beyond the boundaries of the array it 'appears' on the opposite side. The periodic boundary conditions are implemented during the evolution of the system. The first five evolution's of an example run are schematically shown in Figure (1)

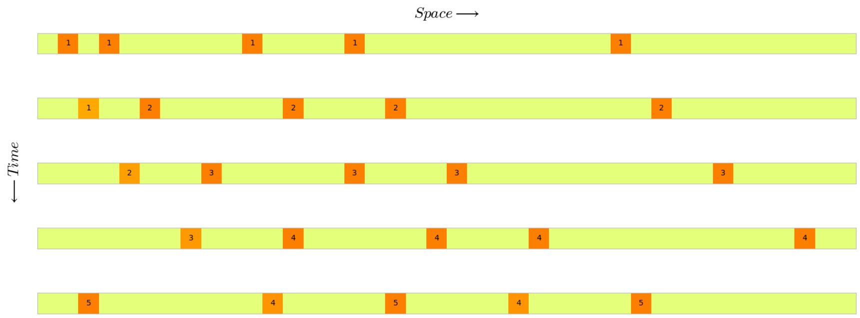


Figure 1: Example setup of road. Cars start with an initial velocity of 0 and evolve according to the rules defined below. Orange blocks are car objects and the number inside the block gives the velocity of the car

### *2.2. Automaton Rules*

The system is updated with each iteration according to the following rules. It should be noted that the below steps are performed in parallel for all cars.

1. **Update Velocity**
    - If the velocity is less than $v_{max}$ and the distance to the nearest car is greater than $v_{max}$, the velocity is increased by one
    - Else, the velocity is decreased to $j - 1$ where $j$ is the distance to the nearest car in front
2. **Random Slow Down**
    - If the velocity of the car is greater than 0, then the velocity is reduced by one with probability $p$
3. **Update Position**
    - The position of each car is advanced by $v$ sites, where $v$ is the velocity of the car

The random slow down in step 2) is essential if one wishes to obtain any non-trivial results from the simulation. In fact, if the probability of a vehicle randomly slowing down is set to 0, then the road system very quickly reaches a static equilibrium state; the random probability is therefore a way of simulating more realistic and dynamic traffic conditions.

One characteristic property of the Nagel-Schreckenberg model is the back-propagation of traffic that occurs when the system goes into a jammed state. This serves as an initial check for the model to ensure that the traffic flow rules are executed properly. Traffic density can be defined as

$$\rho = \frac{Number\ of\ Cars\ on\ Road}{Number\ of\ Sites\ on\ Road} \tag{1}$$

At low densities, as shown by Nagel and Schreckenberg [1], the flow of traffic is smooth and uniform, as shown in Figure (2). As the density of cars is increased, traffic jams begin to form. The jams themselves propagate backwards, as can be seen in Figure (3)

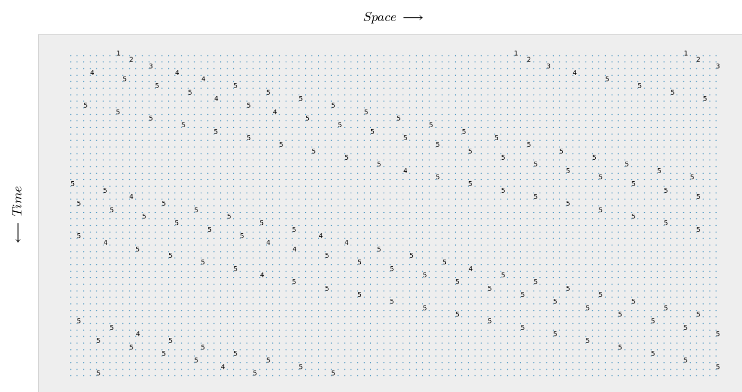


Figure 2: Smooth flow rate at low car density (0.03). Dots are used to represent empty spaces, while number show the speed of a car on the corresponding space

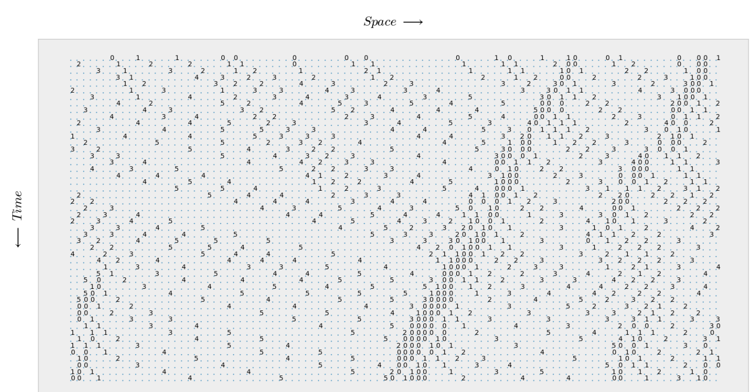


Figure 3: At higher densities (0.2 in this case), traffic jams begin to form. The jams propagate backwards, as found by Nagel-Schreckenberg

The results obtained are in agreement with the results obtained by Nagel and Schreckenberg in their study [1] and this indicates that the model was indeed working as expected.

### *2.3. Investigating Equilibrium Time*

An interesting initial point of study is the equilibrium time of the system. As mentioned previously, when the probability of random deceleration is set to 0, the system quickly reaches an equilibrium of constant average speed and constant maximum speed. The equilibrium time of the system can be measured as a function of road length and density. In order to measure the equilibrium time, the density was fixed at some particular value. The time taken for the system to reach equilibrium was then measured by generating road objects of various lengths and testing the number of iterations it took before a constant average speed was reached. The system was then defined to be in equilibrium if the average speed was left unchanged over the course of 10 iterations. The results are shown in figure (4)

The data shows that, at low densities and at very high densities, the equilibrium time is more or less constant, which makes sense since the cars are essentially either uncoupled at this stage and move independently or very strongly coupled due to the limited space on the road. However, at median density values, the equilibrium time changes noticeably with road length. More over, it was found that there is no clear relation between initial conditions. Rather, the equilibrium time varied greatly even when the simulations where run at the same densities. This suggests that the equilibrium time is very sensitive to the initial conditions of the system, which should be noted was random and different with each simulation.

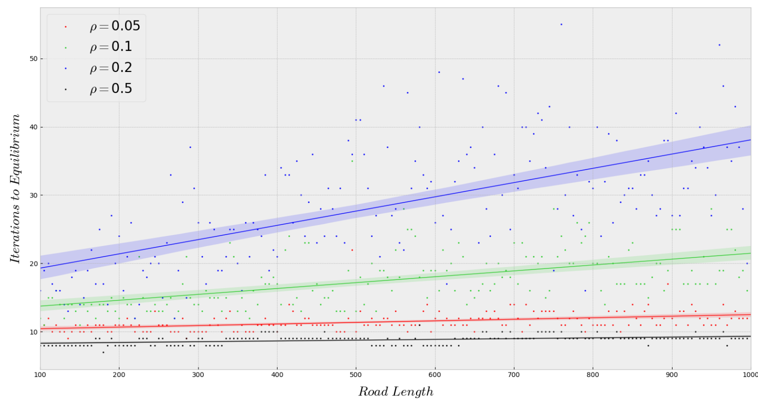


Figure 4: Total iterations taken for system to reach equilibrium. The equilibrium time was tested for car densities of 0.05, 0.1, 0.2 and 0.5. A linear regression fit line was then plotted along with the data

### *2.4. Investigating Flow Rate*

The flow rate is the fundamental point of interest of the study and was calculated by observing a single site on the road; the choice is arbitrary, but in order to make the code as simple as possible, the middle site on each road was chosen. The flow rate is measured by evaluating the number of cars that pass over the chosen site each iteration and then dividing by the total number of iterations to get the flow rate averaged over a period of time $T$. This is defined more quantitatively by

$$\bar{q}^T = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_{i,i+1}(t) \tag{2}$$

where

$$n_{i,i+1}(t) = \begin{cases} 1 \; if \; traffic \; is \; detected \; between \; sites \\ 0 \; otherwise \end{cases}$$

Figure (5) shows a typical flow rate vs density plot conducted with a road length of 500. The smooth curve was obtained from 10,000 iterations over each density point, while the black points show the flow rate over 100 iterations for each density point. Note that each flow rate plot has a point of maximum flow rate at some optimal density value; the flow rate increases more or less linearly up to this point, and, once the optimal density has been exceeded, the flow rate decays more or less linearly to 0. There are hence two distinct flow regimes;

at densities lower than the optimal density, the system is in a state of free flow analogous to Laminar flow in fluid dynamics. Once the critical density is reached, the system transitions into a regime of turbulent flow.

The flow rate plot produced is comparable to plots produced by Nagel-Schreckenberg [1] and to those produced by Larraga-Alvaro-Icaza [3] in a study of various models.

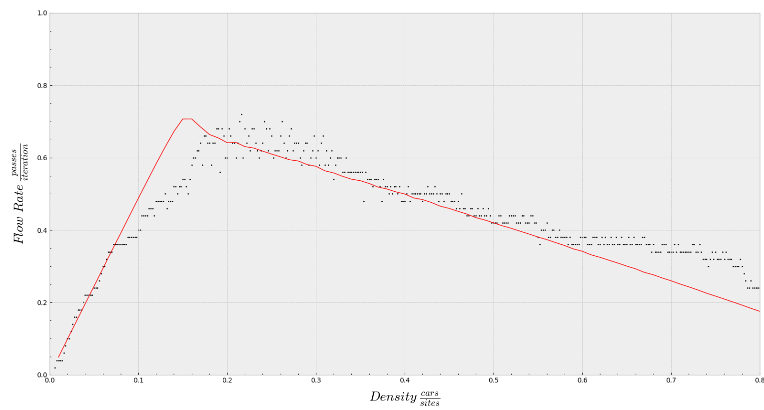


Figure 5: Average flow rate as a function of density for a road with 500 sites

By increasing the number of times the system is iterated for each density point, the statistical fluctuations that occur from random deceleration are gradually reduced, resulting in a smoother plot data bearing considerably less noise. Problematically, however, the density domain is not continuous, as would be required for a smooth curve; the number of cars on the road must be an integer number, and hence the density domain is restricted to the values that produce an whole number of cars when combined with the road length. Effectively, this means that the maximum number of data points that can be gathered is restricted to the length of the road itself; smaller road lengths lead to increasingly more discrete density domains, which consequently lead to worse data fits. Naturally, an ideal system is one with an infinitely long road.

Using larger road lengths introduced significant computational challenges; increasing the length of the road allows for a broader range of cars and hence more density points, but also results in significantly higher computation time since the system becomes larger and larger and needs to be evolved around more density values. All the data plots within this study (unless stated otherwise) were generated with a road length of 500, with 10,000 iterations for each density value; this yields results that have a good balance between data accuracy and computation time.

The effect on flow rate by the variation of road length, speed limit, and probability of deceleration can then be investigated, and the data shown in

Figures 6-8 illustrates the dependence on the respective parameters.

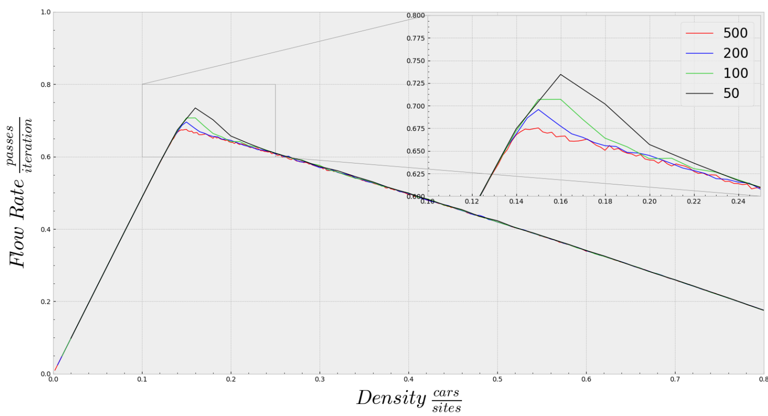


Figure 6: Average flow rate computed from 10,000 iterations of road lengths 500, 200, 100 and 50 with a maximum velocity $v_{max} = 5$.

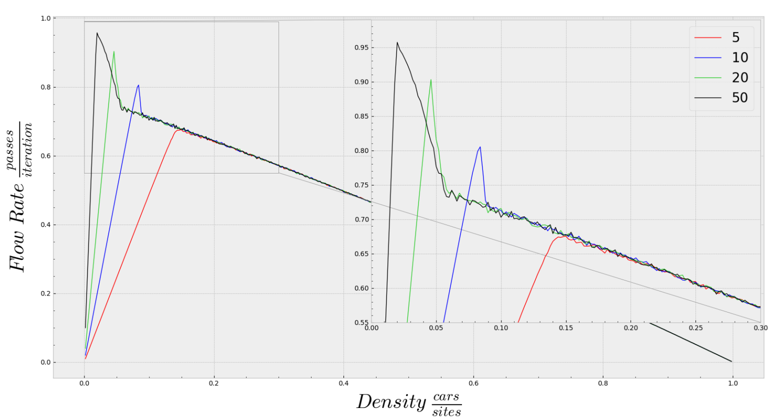


Figure 7: Average flow rate computed from 10,000 iterations of road lengths 500 with a maximum velocities $v_{max} = 50, 20, 10, 5$. There is a notable shift in the maximum flow rate to higher flow rate values, and the optimal density shifts to lower densities.

The data in Figure (6) shows that the flow rate is affected by the length of the road only within a very limited density regime around the optimal density value. Outside of this region, the flow rate is identical to for all road lengths when slight statistical variations are ignored. Within this region, however, the optimal density is shifted to lower density values as the length of the road

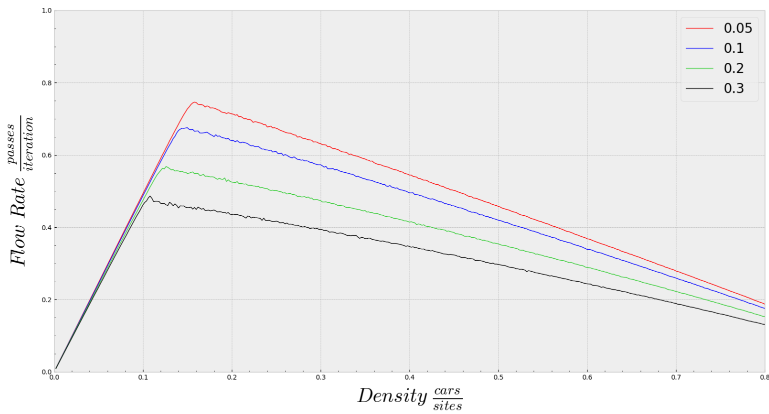


Figure 8: Average flow rate computed from 10,000 iterations of road lengths 500 with probability of random deceleration $p_{slow} = 0.05$, 0.1, 0.2, 0.3. Note that increasing the probability of random deceleration globally reduces the flow rate

increases; similarly, the maximum flow rate corresponding to the optimal density shifts to lower values with increasing road length.

This result is somewhat contradicting; the road length should not have any effect on the flow rate, and more importantly, shorter roads should not increase the flow rate. It then becomes reasonable to suspect that the deviation that occurs in critical regime is an error in the model as opposed to an insight into the rate of flow. One obvious candidate for the deviation is the discrete of the density domain, as discussed above. In order to get accurate results, the density domain must be as continuous as possible. However, as mentioned above, the number of cars on the road must be an integer, and this restricts how continuous the density domain can be. The deviation in the error becomes notably smaller as longer road lengths are used, which supports the conclusion that the discrete nature of the model is to blame for the error in the results.

Figure (7) illustrates the effect of a variation in speed limit on the flow rate and shows a clear positive correlation between flow rate and maximum speed. As the maximum speed is increased, the maximum flow rate also increases notably. Additionally, the optimal density shifts to lower densities values. It can also be observed that the gradient of the flow rate at low densities becomes steeper with increasing maximum speed. This is to be expected since, at low densities, the vehicles are essentially independent and uncoupled and free to move without restriction. In fact, the gradient of the flow rate is proportional to the maximum speed of the cars in the laminar flow regime. A study done by Alejandro Salcido [4] in which the flow rate of the Nagel-Schreckenberg model was investigated with several maximum speed values in a similar fashion showed qualitatively comparable results.

One interesting aspect of the data shown in Figure (7) is that the flow rate only differs in the regions below the optimal density i.e in the laminar flow regime; once the maximum possible flow rate is achieved, all curves converge onto the same trajectory. This indicates that the benefits of increasing the speed limit on roads is very much dependent on the expected car density; if a road experiences a traffic density greater than its optimal value, then increasing the speed has no effect on the flow rate of traffic. A similar observation again can be seen in the study by Salcido [4].

The final parameter to vary is the probability of random deceleration. The naive expectation is that the flow rate decreases with increased probability of deceleration since on average, more cars will decelerate resulting in an overall lower average seed and consequently a lower mean flow rate; as it turns out, the naive guess is correct. The relationship between flow rate and probability is shown in Figure (8).

As $p_{slow}$ is increased, the maximum flow rate decreases and the optimal road density shifts to lower densities. In contrasts to the other parameters tested, the flow rate does not converge unto some limiting curve. Rather, all four probabilities have an initially equal flow rate and gradients until the peak flow rate is reached, after which each setup more or less linearly decays to 0. This also concurs with the findings of Salcido [4].

## 3. Model Extensions

### *3.1. Investigating the Effect of Driver Profiles*

The model was then altered slightly to accommodate the possibility of various driver profiles. The driver profiles are intended to quantify the quality of a driver and the quality of a driver, in this case, was defined by the probability of random slow down and by an additive to the maximum speed of the road (i.e. $v_{max} = v_{max} + \alpha$). The better the driver, the lower the probability of slowdown and the lower the additive to the maximum speed. The four profiles used are shown in figure (9).

| Profile | $p_{slow}$ | $\alpha$ |
|---|---|---|
| Perfect | 0.0 | 0% |
| Excellent | 0.1 | 10% |
| Good | 0.2 | 25% |
| Bad | 0.35 | 100% |

Figure 9: Driver profiles used in simulations. Note that alpha refers to a constant value added unto the speed limit of the road

The effect that the variation in profiles has is not entirely obvious or intuitive; it has already been shown that there is a negative correlation with flow rate and probability of slowdown, and hence one might expect that the increase in slowdown probability would reduce the flow rate. However, the flow rate has also been shown to have a positive correlation with maximum speed and therefore it is not unreasonable to expect the flow rate to increase with the effective 'speeding' of the drivers. The question therefore becomes whether or not the increased speed compensates for the constant acceleration and braking accompanied with bad driving.

In order to compare the effects of the variation in profiles, the flow rate was computed for a road with length of 500 and a probability of random slowdown of 0 (i.e. a road with perfect drivers). A series of driver distributions where then simulated and then compared to the perfect driver setup. The driver profile distributions used are given in figure (10). Note that the table shows the percentage of drivers that had the corresponding profile i.e profile 3 consisted of 10% perfect drivers, 30% excellent driver etc. The data obtained is given by figure (11).

The data clearly shows that the presence of bad drivers results in a significantly lower flow rate even at low densities where the cars are essentially uncoupled and one might naively assume that the increase in speed results in a higher flow rate. The road with only perfect drivers has a higher flow rate for all densities, with the exception of a few spikes at lower densities. The data gathered from this model clearly implies that the speed advantage gained does

| Profile | Perfect (%) | Excellent (%) | Good (%) | Bad (%) |
|---|---|---|---|---|
| 1 | 100 | 0 | 0 | 0 |
| 2 | 10 | 50 | 20 | 20 |
| 3 | 10 | 30 | 30 | 30 |
| 4 | 10 | 20 | 20 | 50 |

Figure 10: The four Driver profiles used in simulations

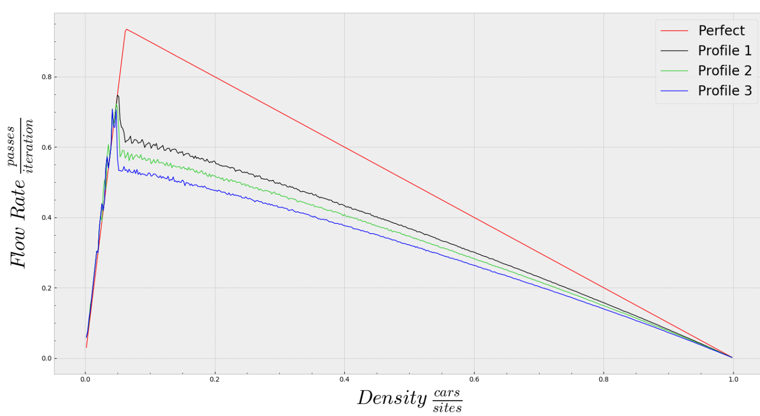


Figure 11: Data comparing the flow rate with the three drivers profiles of increasingly bad driver ratios. The red line gives the run with nothing but perfect drivers

not produce a higher flow rate. Rather, the constant braking and accelerating disturbs the system as a whole and results in a significant decrease in average traffic flow.

One possible explanation for this arises when one considers the average speed of the cars on the road; looking at the data, it becomes reasonable to expect that, despite the fact that the speeding drivers have a significantly higher potential speed, they are hindered by the cars that are traveling at the designated speed limit. In order to investigate this, the average velocity was recorded at several density values for profile 3 and then compared to the average velocity for the road with perfect drivers. Given the data in figure (11), the expectation is that the average speeds are comparable, up to the optimal density, after which the average velocity is expected to drop significantly for profiles with a greater proportion of bad drivers.

The results of this investigation are shown in figures (12-14)

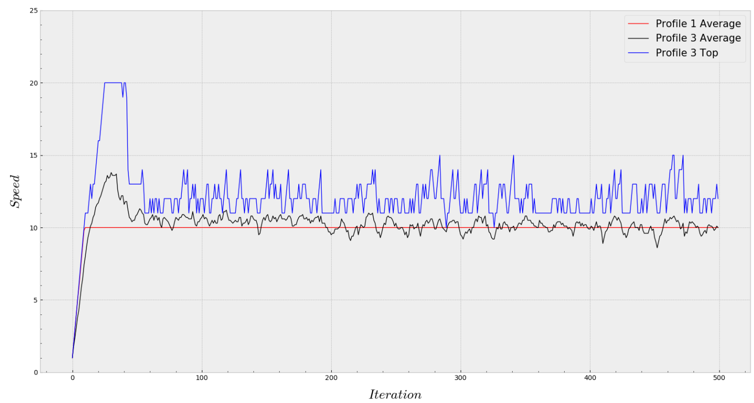


Figure 12: Data comparing the average speed (black line) and top speed (blue line) of profile 3 with that of the perfect driver profile (red line) at density of 0.02. Note that the average speed of the perfect driver profile and that of profile 3 are comparable and, on average, equal

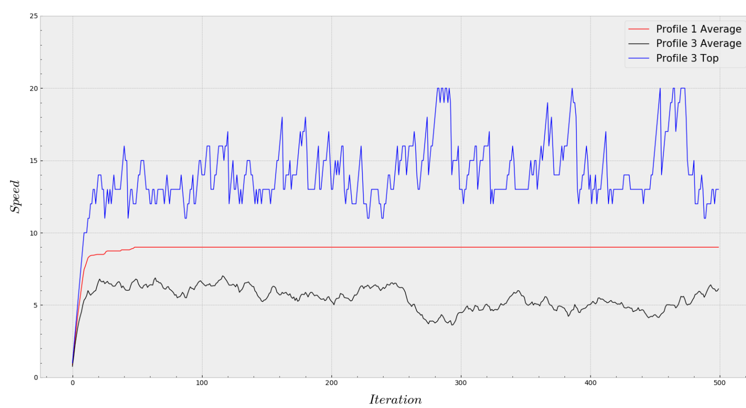


Figure 13: Data comparing the average speed (black line) and top speed (blue line) of profile 3 with that of the perfect driver profile (red line)at density of 0.1. At higher densities, the average speed of profile 3 is significantly lower than that of the perfect profile

The results are as what would be expected from figure (11); at low densities, despite the fact that the maximum speed is significantly higher than that of the perfect profile, the average speeds are comparable, as shown in figure (12). However, once density exceeds the optimum value, the average velocity of the perfect profile begins to overtake that of the other profiles, resulting in a higher flow rate, as shown in figure (13). As the density is then increased far beyond the optimal value, the average speeds of the profiles begin to converge unto that

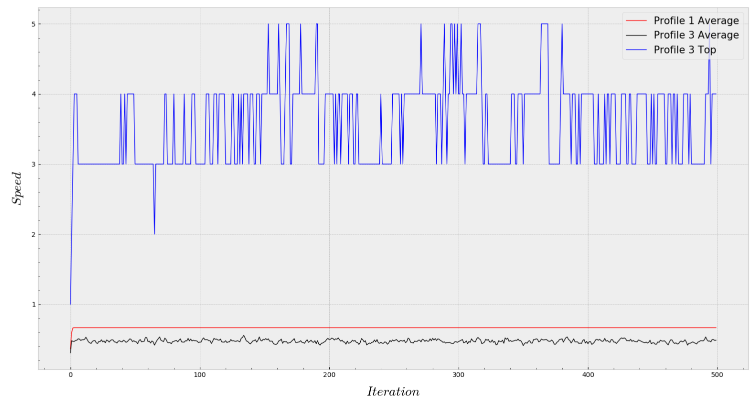


Figure 14: Data comparing the average speed (black line) and top speed (blue line) of profile 3 with that of profile 1 at density of 0.6

of the perfect profile, which can be seen in figure (14)

Despite the fact that the maximum speed of any individual car is greater as a result of the speeding drivers, the average speed never exceeds that of the road with perfect drivers. This justifies the conclusion that the majority of cars that have the potential to reach higher speeds are simply stuck behind those traveling at the speed limit. As a result, the potential gain in speed has no effect on the overall flow. The negative effect of the increased deceleration probability is therefore the only thing that effectively acts on the flow rate, and, because of this, the road full of perfect drivers is the optimal configuration of the model. Occasional spikes of increased and decreased average speed do occur, as both figure (11) and figure (12) show; However, they occur on both sides of the flow rate relatively equally, and hence the affect averages out over enough iterations.

### *3.2. Flow rate of Multi-lane System*

The model was then altered in order to include a second lane of drivers, who could switch between lanes if conditions were preferable. However, in order to integrate smooth transitions between lanes, a pre-step needed to be added to the Automaton rules defined. Before the speeds are updated, the conditions for every car were checked in order to see if a lane switch is favorable. If it was, the car was moved over into the other lane. Once all eligible cars had switched lanes, the Automaton proceeded with the speed/position updates as before.

One immediate consequence of this is the significant increase in computational time when running simulations; firstly, since the number of lanes was doubled, the number of cars doubled for each density value. In the 1D model, a road of length 100 at maximum density had 99 cars on it. The corresponding

2D system, however, contains 198 cars that need to be updated. Additionally, each car needs to be checked every iteration to see if conditions are preferable for a lane switch. Switching to a compiled language such as C or Fortran becomes a necessary step if extended simulations are to be run. However, in this case, a relatively small amount of data was gathered, and as a result the cost of computational time was more than compensated by the significant decrease in development and analysis time accompanied with Python or similar scripting languages. Additionally, when one considers that the model at its core is identical to the 1D model previously developed, the choice to use Python becomes a valid one.

Figure (15) shows the flow rate of the two lane system in contrast to that of the single lane system. Both had a road length of 500 and the simulation was carried out with 10,000 iterations per density value, as before. It should be noted however, that, in this case, the flow rate is defined as the average number of passes over a site i on both lanes i.e. instead of monitoring a single site on one lane, the passes over two parallel sites (one on each lane) is measured.

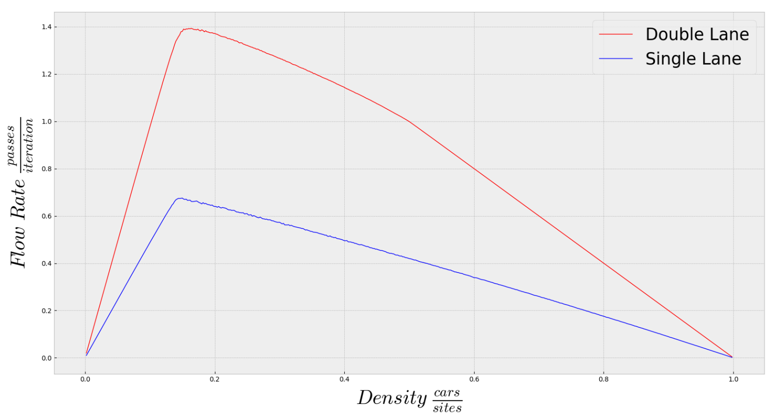


Figure 15: Flow Rate of 2 lane road with overtaking (red curve) allowed vs single lane road (blue curve

The data shows that the two lane system has a similar profile to the one lane system; both the laminar flow and the turbulent flow as present, along with the critical density. Interestingly, a third flow regime appears in the 2D model. Note that the flow rate is essentially double that of the single lane at low, which is as expected since, at low densities, the system is essentially two single roads. The cars are assumed to be independent at low densities, and hence overtakes are rare.

An equally interesting observation is made when the probability of random deceleration is varied in the 2D system, the results of which are shown in figure (16). The presence of a third flow regime becomes increasingly evident and the nature of the regime changes. Additionally, the second critical point always occurs at a density value of precisely 0.5, the maximum occupancy of one of the lanes. Moreover, the corresponding flow rate is roughly 1 car per iteration.

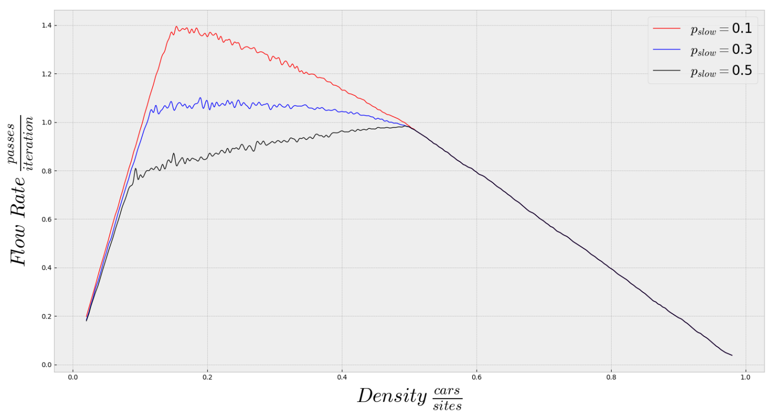


Figure 16: Flow Rate of 2 lane road at various probabilities of random deceleration. Note that the addition of a third regime becomes increasingly evident

The data gathered by the model suggests that the system has a native potential maximum flow rate of at least 1 car per iteration that it can reach, even at very high deceleration probability and relatively high density. This shows that the 2D model is significantly more robust to the effects of the formation of traffic jams. This can be explained by the ability of the cars to overtake; if a jam begins to form at low car densities, the car can simply jump into the next lane and overtake. By measuring the average number of lane switches that occur at various density values (as shown in figure (17)), it can be seen that at low probabilities of deceleration, the system quickly reaches a quasi static state after which virtually no overtakes occur. At higher probabilities of random deceleration, the number of lane switches remains fairly constant until higher densities are reached. Hence, the ability of the cars to overtake only becomes relevant at high probabilities where jams begin to form. At higher densities, the road is too crowded to allow cars to overtake and the ability to switch lanes becomes more or less irrelevant for all system configurations.

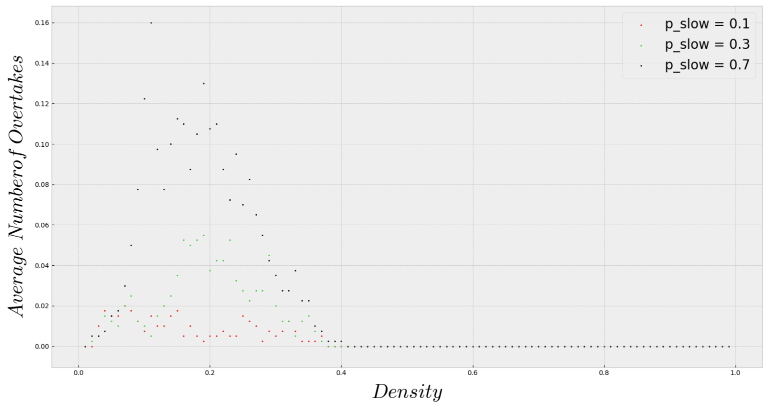


Figure 17: Average number of lane switches as a function of density. Note that the lane switches occur more frequently at higher probabilities of random deceleration, and, at high densities, overtaking becomes practically irrelevant

## 4. Conclusion

While the flow of traffic is inherently a very complex problem consisting of large number of unique, individual components, a relatively large amount of information can be inferred from a very simple model. The Automaton model analyzed in this study might appear to be a gross over-simplification at first glance, but it can be used to quantitatively show how the flow rate of traffic is affected by various parameters such as road length and maximum speed. Moreover, as shown by Nagel-Schreckenberg [1] the general trends shown in the data math those of actual traffic data gathered.

The model constructed for this study showed that the flow rate of traffic had no dependence on the length of the road used; furthermore, it was shown that, by increasing the maximum speed of the road, a significant flow rate gain is achieved at low densities. However, once the optimal road density was reached, the flow rate of all speed limits converged unto some limiting curve. This suggest that increasing the speed limit of a road is only beneficial if the expected road density is lower than or near its optimal value. Moreover, it was shown that the optimal road density shifts to lower densities as the speed limit is increased. It was then shown that, when the probability of spontaneous slow-down is increased, the flow rate is reduced significantly at densities higher than the optimal road density.

Finally, the model was extended to include a range of various driver profiles ranging from bad drivers to perfect drivers. This was then used to show that, at least in this basic model, the increase of speed accompanied by bad drivers was not enough to compensate for their continuous braking/acceleration and the flow rate of the system suffered as a whole; even at low densities where one might otherwise, the average speed does not exceed that of a system with perfect drivers. Additionally, the model was extended to include a second lane, which was used to show that the flow rate is increased significantly with the addition of a second lane, while the optimal density occurred at significantly higher densities than for the single lane road; it was also shown that the ability of cars to overtake only became relevant at high probabilities of random deceleration, at which a third flow regime appeared. However, for low probabilities of deceleration, the system enters a quasi-static state very quickly, and overtaking becomes a rare occurrence.

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