

Practical 3: Recursion

What am I doing today?

Today's practical focuses on 3 things:

***pre-requisite: Github classroom set up and commits.**

1. Quick questions about Recursion
2. Comparing an iterative fibonacci algorithm to a recursive one
3. Help the monks solve the Towers of Hanoi

Instructions

Try all the questions. Ask for help from the demonstrators if you get stuck.

Solutions will be posted afterward.

*You can re-use the helper classes from previous weeks.

*****Grading: Remember to commit your work to your repository and push to the main branch on the origin.**

Warm-up questions

1. What are the two principal characteristics of a recursive algorithm?

A recursive algorithm always has a base case.

A recursive algorithm always calls itself recursively, i.e., with a smaller instance of the input argument.

2. Recursion is..

Answer	
FALSE	theoretically interesting but rarely used in actual programs
FALSE	theoretically uninteresting and rarely used in programs
TRUE	theoretically powerful and often used in algorithms that could benefit from recursive methods

3. True or false: All recursive functions can be implemented iteratively. **True**

4. True or false: if a recursive algorithm does NOT have a base case, the compiler will detect this and throw a compile error? **False, it would just be an infinite loop**

5. True or false: a recursive function must have a void return type.
False

6. True or False: Recursive calls are usually contained within a loop.
False

7. True or False: Infinite recursion can occur when a recursive algorithm does not contain a base case. True

8. **Which of these statements is true about the following code?**

```
int mystery(int n)
{
    if (n>0) return n + mystery(n-1);
    return 0;
}
```

Your answer	
False	The base case for this recursive method is an argument with any value which is greater than zero.
True	The base case for this recursive function is an argument with the value zero.
False	There is no base case.

9. List common bugs associated with recursion.

	Lack of base case, in turn creating an infinite loop
	Incorrect recursive call, i.e., the recursive call doesn't break the problem into smaller problems with the recursive call, i.e., never reaching the base case
	Not returning the recursive call
	Excessive memory requirements/re-computation

10. What method can be used to address recursive algorithms that excessively recompute?

Attempt to use another method of recursion i.e., binary recursion, or convert the recursive algorithm into an iterative algorithm

Fibonacci

The Fibonacci numbers are a sequence of integers in which the first two elements are 0 and 1, and each following element is the sum of the two preceding elements:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, and so on...

The Nth Fibonacci number is output with the following function:

$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2) \rightarrow \text{for } n > 1 \text{ fib}(n) = 1 \rightarrow \text{for } n = 0, 1$ <p>The first two terms of the series are 0, 1.</p> <p>For example: $\text{fib}(0) = 0$, $\text{fib}(1) = 1$, $\text{fib}(2) = 1$</p>

Exercises

1. Below is an iterative algorithm that computes Fibonacci numbers. Write a recursive function to do the same.
2. Test both algorithms with various sizes of N s. What do you find?
3. What is the time complexity of both functions?

Iterative Fibonacci

```
static int fibonacciIterative(int n){
    if (n<=1)        return 1;
        int fib = 1;
    int prevFib = 1;

    for (int i = 2; i < n; i++)
    {
        int temp = fib;    fib =
fib + prevFib;    prevFib =
temp;
    }
    return fib;
}

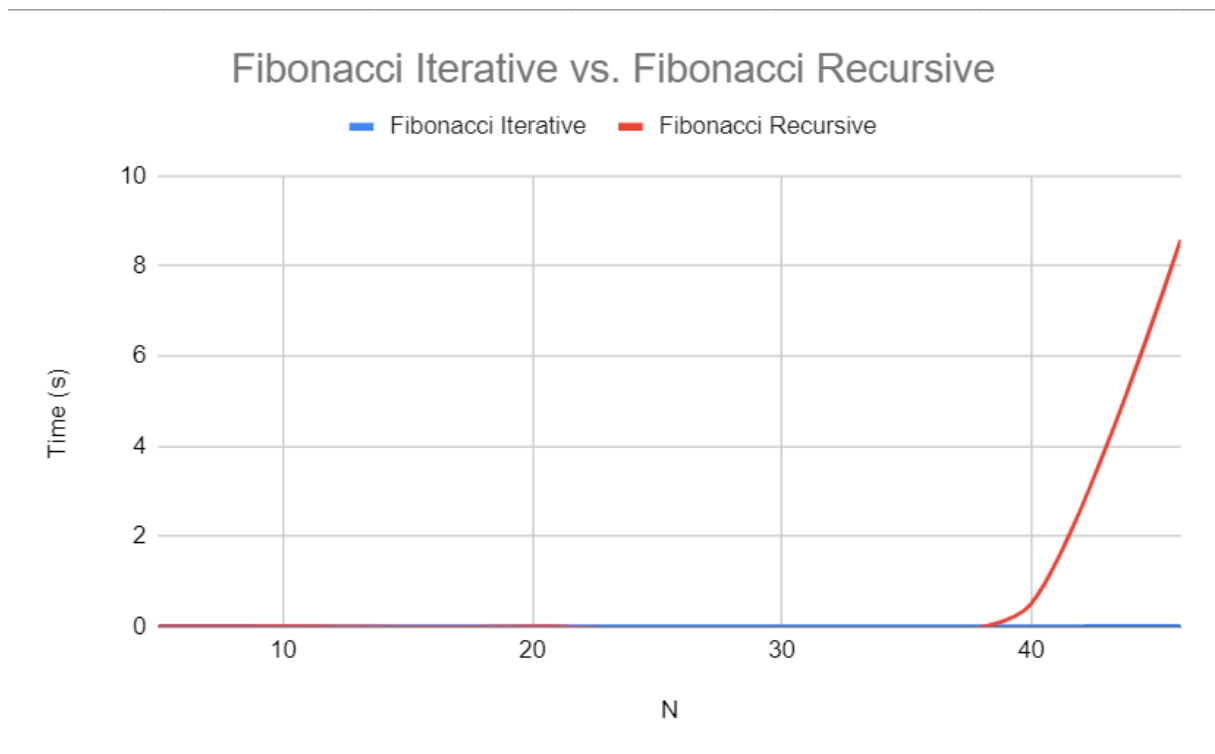
public static void main (String
args[])
{
    int n = 9;
    System.out.println(fibonacciIterative(n));
}
```

Q1 – fibonacciRecursive.java

Q2 –

N	Elapsed Time (Iterative)	Elapsed Time (Recursive)
5	0.0	0.0
10	0.0	0.0
20	0.0	0.0
40	0.0	0.495
46	0.0	8.58

46 is the upper bound, i.e., the 46th term of the sequence gets the closest to the maximum integer value in Java before giving minus values



Q3 – Time complexity of fibonacciIterative – $O(N)$, time complexity of fibonacciRecursive – $O(2^n)$

3.

Hanoi - The Monks need your help!



Convert the pseudo-code into java and add your own output instructions so junior monks can learn how to perform the legal moves in the Tower of Hanoi so they can end the world.

There are two rules:

- Move only one disc at a time.
- Never place a larger disc on a smaller one.

Tasks:

1. Implement Hanoi in java
2. Test with various size disks
3. Output the moves for the monks as step-by-step instructions so the monks can end the world

Pseudocode for Hanoi

```
towersOfHanoi(disk, source, dest, auxiliary):  
IF n == 0, THEN:  
    move disk from source to dest
```

```
ELSE:  
    towersOfHanoi(disk - 1, source, auxiliary, dest)  
    towersOfHanoi(disk - 1, auxiliary, dest, source)  
END IF
```

*alternative Palindrome or Factorial (iterative and recursive solution)

Number of moves to move N disks from A to C is $= 2^n - 1$

Q1 – towersOfHanoi.java

Q3 – n = 2 – Disk 1 from A to B
 Disk 2 from A to C
 Disk 1 from B to C

n = 3 – Disk 1 from A to C
 Disk 2 from A to B
 Disk 1 from C to B
 Disk 3 from A to C
 Disk 1 from B to A
 Disk 2 from B to C
 Disk 1 from A to C

n = 4 – Disk 1 from A to B
 Disk 2 from A to C
 Disk 1 from B to C
 Disk 3 from A to B
 Disk 1 from C to A
 Disk 2 from C to B
 Disk 1 from A to B
 Disk 4 from A to C
 Disk 1 from B to C
 Disk 2 from B to A
 Disk 1 from C to A
 Disk 3 from B to C
 Disk 1 from A to B
 Disk 2 from A to C
 Disk 1 from B to C