

## 1. Sets and Indices

- $I$ : Set of Carriers,  $I = \{1, 2, \dots, 8\}$ . (Index  $i$ )
- $J$ : Set of Shipping Lanes,  $J = \{1, 2, 3\}$ . (Index  $j$ )
- $A$ : Subset of carriers in Alliance A,  $A = \{1, 2, 3, 4\}$ .
- $B$ : Subset of carriers in Alliance B,  $B = \{5, 6, 7, 8\}$ .

## 2. Parameters

- $C_{ij}$ : Cost per unit of freight for carrier  $i$  on lane  $j$ .
- $D_j$ : Projected freight demand for lane  $j$ .
  - $D_1 = 650$
  - $D_2 = 775$
  - $D_3 = 880$
- $M_{min}$ : Minimum Quantity Commitment (MQC) a carrier must ship *in total* to participate.  $M_{min} = 1000$ .
- $P_{carrier}$ : Maximum percentage of a lane's demand a single carrier can handle.  $P_{carrier} = 0.60$ .
- $P_{alliance}$ : Maximum percentage of a lane's demand an alliance can handle.  $P_{alliance} = 0.90$ .
- $M_{max}$ : A "Big-M" constant (a valid upper bound on total shipment for one carrier), e.g.,  $M_{max} = \sum_{j \in J} D_j = 2305$ .

## 3. Decision Variables

- $x_{ij}$ : (Continuous) The amount of freight (in units) allocated to carrier  $i$  on lane  $j$ .
- $y_i$ : (Binary)  $y_i = 1$  if carrier  $i$  is selected to participate;  $y_i = 0$  otherwise.

## 4. Mathematical Model

### Objective Function

$$\min Z = \sum_{i \in I} \sum_{j \in J} C_{ij} \cdot x_{ij}$$

## Subject to Constraints

$$\begin{array}{ll} (1) & \sum_{i \in I} x_{ij} = D_j \quad \forall j \in J \\ (2) & x_{ij} \leq P_{carrier} \cdot D_j \quad \forall i \in I, j \in J \\ (3a) & \sum_{i \in A} x_{ij} \leq P_{alliance} \cdot D_j \quad \forall j \in J \\ (3b) & \sum_{i \in B} x_{ij} \leq P_{alliance} \cdot D_j \quad \forall j \in J \\ (4a) & \sum_{j \in J} x_{ij} \geq M_{min} \cdot y_i \quad \forall i \in I \\ (4b) & \sum_{j \in J} x_{ij} \leq M_{max} \cdot y_i \quad \forall i \in I \\ (5a) & x_{ij} \geq 0 \quad \forall i \in I, j \in J \\ (5b) & y_i \in \{0, 1\} \quad \forall i \in I \end{array}$$