

1. Sets and Indices

- I : Set of Carriers, $I = \{1, 2, \dots, 8\}$. (Index i)
- J : Set of Shipping Lanes, $J = \{1, 2, 3\}$. (Index j)
- A : Subset of carriers in Alliance A, $A = \{1, 2, 3, 4\}$.
- B : Subset of carriers in Alliance B, $B = \{5, 6, 7, 8\}$.

2. Parameters

- C_{ij} : Cost per unit of freight for carrier i on lane j .
- D_j : Projected freight demand for lane j .
 - $D_1 = 650$
 - $D_2 = 775$
 - $D_3 = 880$
- M_{min} : Minimum Quantity Commitment (MQC) a carrier must ship *in total* to participate. $M_{min} = 1000$.
- $P_{carrier}$: Maximum percentage of a lane's demand a single carrier can handle. $P_{carrier} = 0.60$.
- $P_{alliance}$: Maximum percentage of a lane's demand an alliance can handle. $P_{alliance} = 0.90$.
- M_{max} : A "Big-M" constant (a valid upper bound on total shipment for one carrier), e.g., $M_{max} = \sum_{j \in J} D_j = 2305$.

3. Decision Variables

- x_{ij} : (Continuous) The amount of freight (in units) allocated to carrier i on lane j .
- y_i : (Binary) $y_i = 1$ if carrier i is selected to participate; $y_i = 0$ otherwise.

4. Mathematical Model

Objective Function

$$\min Z = \sum_{i \in I} \sum_{j \in J} C_{ij} \cdot x_{ij}$$

Subject to Constraints

- $$(1) \quad \sum_{i \in I} x_{ij} = D_j \quad \forall j \in J$$
- $$(2) \quad x_{ij} \leq P_{carrier} \cdot D_j \quad \forall i \in I, j \in J$$
- $$(3a) \quad \sum_{i \in A} x_{ij} \leq P_{alliance} \cdot D_j \quad \forall j \in J$$
- $$(3b) \quad \sum_{i \in B} x_{ij} \leq P_{alliance} \cdot D_j \quad \forall j \in J$$
- $$(4a) \quad \sum_{j \in J} x_{ij} \geq M_{min} \cdot y_i \quad \forall i \in I$$
- $$(4b) \quad \sum_{j \in J} x_{ij} \leq M_{max} \cdot y_i \quad \forall i \in I$$
- $$(5a) \quad x_{ij} \geq 0 \quad \forall i \in I, j \in J$$
- $$(5b) \quad y_i \in \{0, 1\} \quad \forall i \in I$$