

To implement the function  $f(x) = \sqrt{x^2 + 1} - 1$  we will need to address two issues:

1. Overflow:  $\sqrt{x^2 + 1} \approx |x|$  when  $|x|$  is large, but the intermediate value  $x^2$  may overflow. To prevent this, we can express  $f(x)$  as

$$f(x) = |x|\sqrt{1 + x^{-2}} - 1.$$

2. Cancellation:  $\sqrt{x^2 + 1} \approx 1$  when  $|x|$  is close to zero, and this can lead to catastrophic cancellation. To prevent this, we can express  $f(x)$  as

$$f(x) = (\sqrt{x^2 + 1} - 1) \frac{\sqrt{x^2 + 1} + 1}{\sqrt{x^2 + 1} + 1} = \frac{x^2}{\sqrt{x^2 + 1} + 1}.$$

We will use this expression when  $x$  is small.

Combining the above observations, we arrive at the following implementation:

```
double feval(double x) {
    double t = fabs(x);
    if (t > 2.0)
        // avoid overflow
        return t*sqrt(1.0+1.0/(t*t))-1.0;
    else {
        // avoid cancellation
        t = x*x;
        return t/(sqrt(t+1.0)+1.0);
    }
}
```