

## Logarithm cheat sheet (M J Rhoades)

Notes: When  $\log_b$  is written, it is for any log base even base "e"

When "ln" is written it means base "e" which is log base 2.718283

If just log is written, with no subscript, it implies base 10

When ln is written it means base "e"

### Rules

#### Algebra

$$\log_b(x) = N \quad \text{means} \quad b^N = x$$

$\log_b(0)$  is undefined

$$\log_b b = 1$$

$$\log_b 1 = 0$$

$$\log_b b^x = x \quad \text{or} \quad \ln(e^x) = x$$

$$\log_{\log_b x} x = x$$

$$\log_b(xy) = \log_b(x) + \log_b y \text{ and vice versa}$$

$$\log_b(x^r) = r \log_b x$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y) \text{ and vice versa}$$

Inverse function of logs: example  $f(x) = b^x$  and  $g(x) = \log_b x$

Inverse exponent functions of ln

$$f(f^{-1}(x)) = e^{\ln(x)} = x$$

$$f^{-1}(f(x)) = \ln(e^x) = x$$

Special logs  $x = \log_e x$  and  $\log x = \log_{10} x$  natural and common logs

Change of base formula  $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

### Trig

$$\log_b \tan(x) = \log_b \sin(x) - \log_b \cos(x)$$

$$\log_b \cot(x) = \frac{1}{\log_b \sin(x) - \log_b \cos(x)}$$

### Calculus

#### Limits

$$\lim_{x \rightarrow 0} \ln x = -\infty$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

#### Derivatives:

$$f'(a^x) = a^x \ln(a)$$

$$f'(e^x) = e^x$$

$$f'(\ln(x)) = \frac{1}{x}, \quad x > 0$$

$$f'(\log_a(x)) = \frac{1}{x \ln a}, \quad x > 0$$

$$f'(\log_a u) = \frac{1}{u} \frac{du}{dx} (\log_a e)$$

#### Integrals:

$$\int f(x) dx = \int \ln(x) dx = (x)(\ln(x) - 1) + c$$

$$\int \frac{dx}{x} = \ln(x) + c$$

$$\int (x)e^x dx = (x - 1) e^x + c$$

$$\int \frac{dx}{(x) \ln(x)} = \ln |\ln (x)| + c$$

$$\int x^n \ln (x) \, dx = x^{n+1} \left( \frac{\ln(x)}{n+1} - \frac{1}{(n+1)^2} \right) + c \quad n \neq -1$$

$$\int x^n (\ln (x))^m \, dx = \frac{x^{n+1}}{n+1} (\ln x)^m - \frac{m}{n+1} \int x^n (\ln x)^{m-1} \, dx \quad n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln x + c$$

$$\int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} \, dx \quad \text{when } g(x) = \tan(x)$$

$$\int \tan(x) \, dx = \int \frac{-\frac{d}{dx} \cos(x)}{\cos(x)} \, dx$$