Demystifying Quantum Power Flow: Is It Fast?

Parikshit Pareek
Assistant Professor
Indian Institute of Technology Roorkee

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Joint work with — Abhijith Jayakumar, Carleton Coffrin and Sidhant Misra at LANL

LA-UR 24-28593

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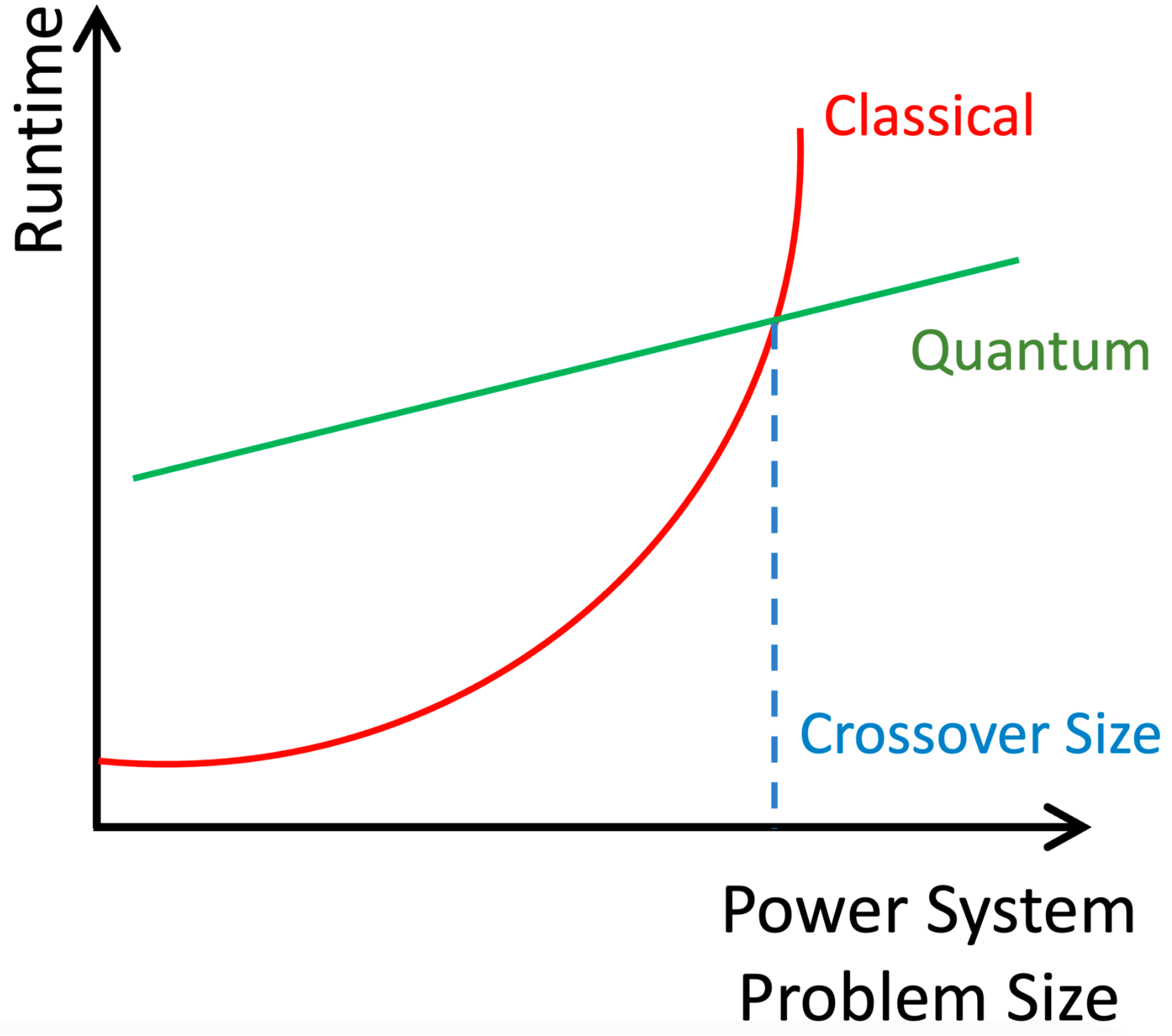

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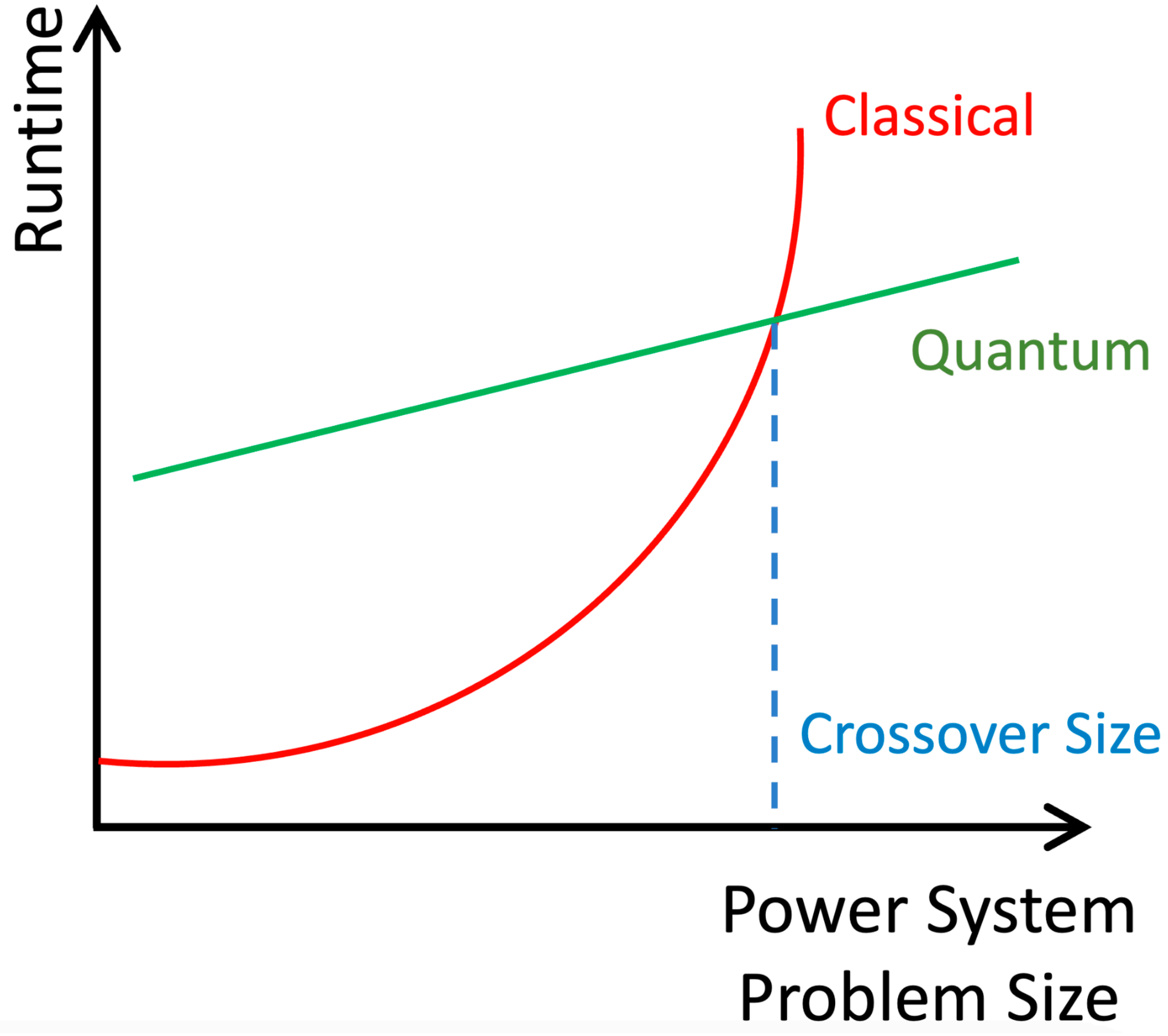

Potential Quantum Advantage

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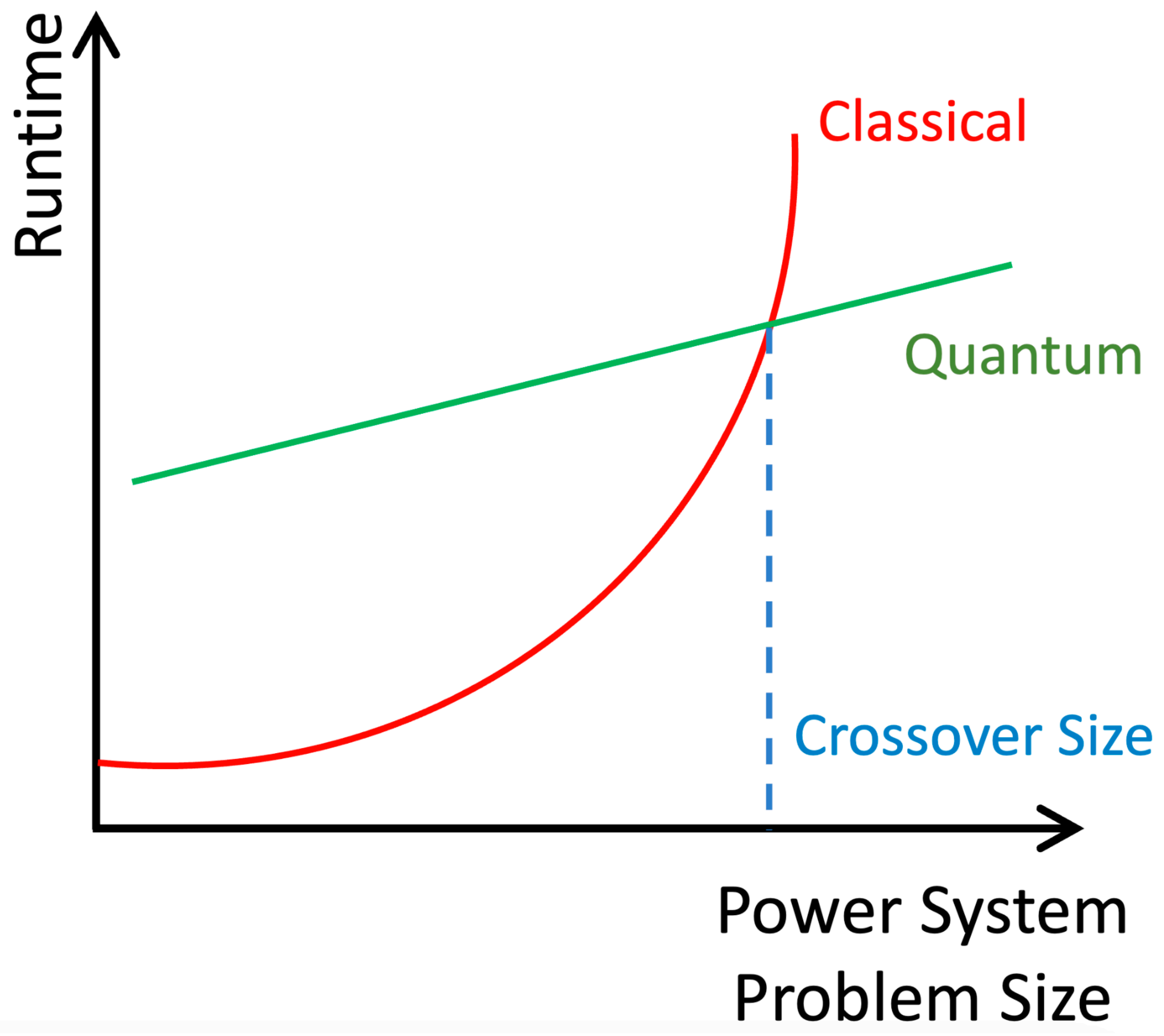

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Find Crossover Size (if exist) &

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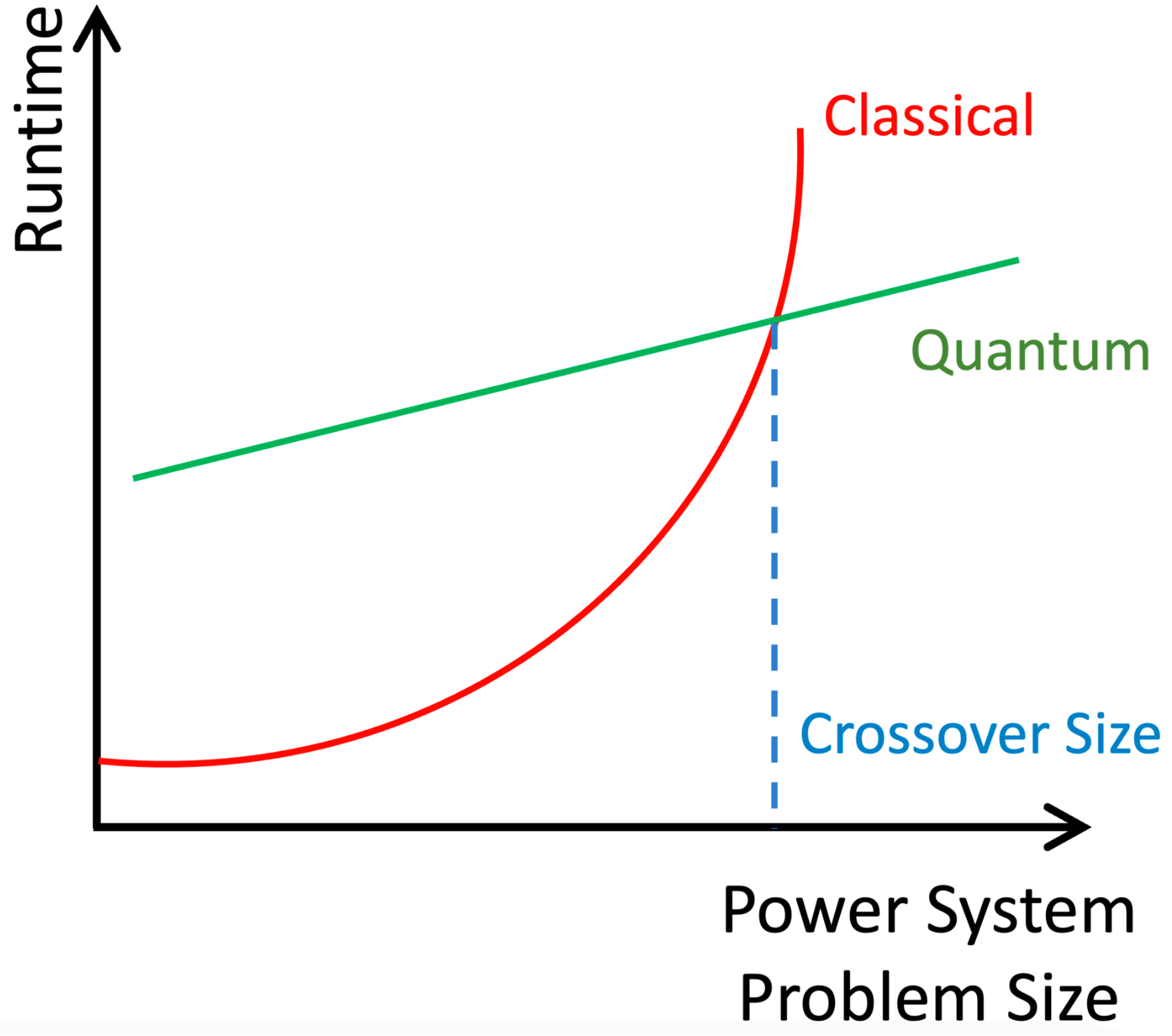

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This talk is **NOT** about proposing a 'New' Quantum algorithm & I am **NOT** a Quantum Guy

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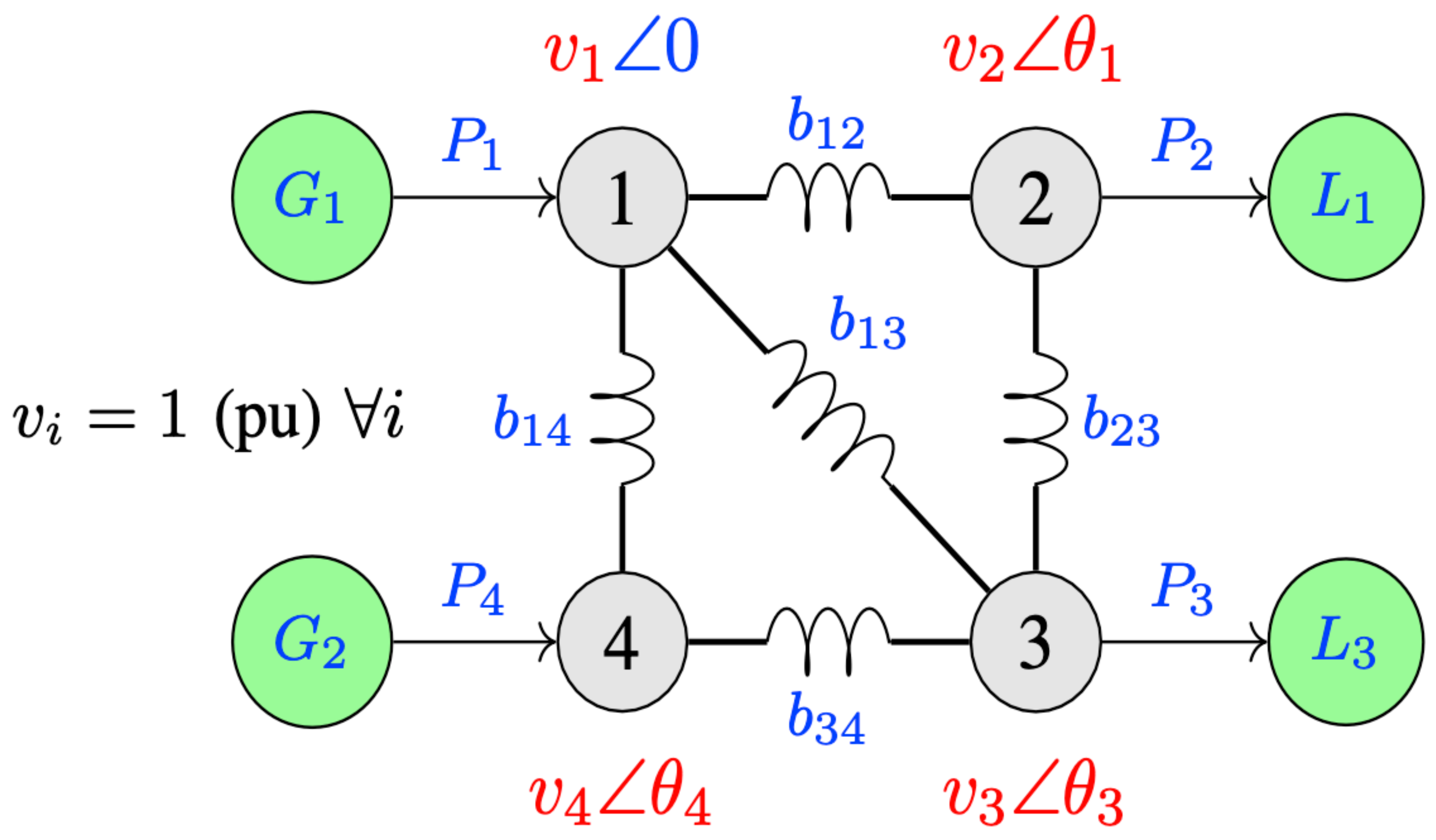

$$\begin{bmatrix} \overline{b_{2j}} & -b_{23} & -b_{24} \\ -b_{32} & \overline{b_{3j}} & -b_{34} \\ -b_{42} & -b_{43} & \overline{b_{4j}} \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

Angle Variable Vector

Injection Vector

Susceptance Matrix

$$A\,\mathbf{x} = \mathbf{b}$$

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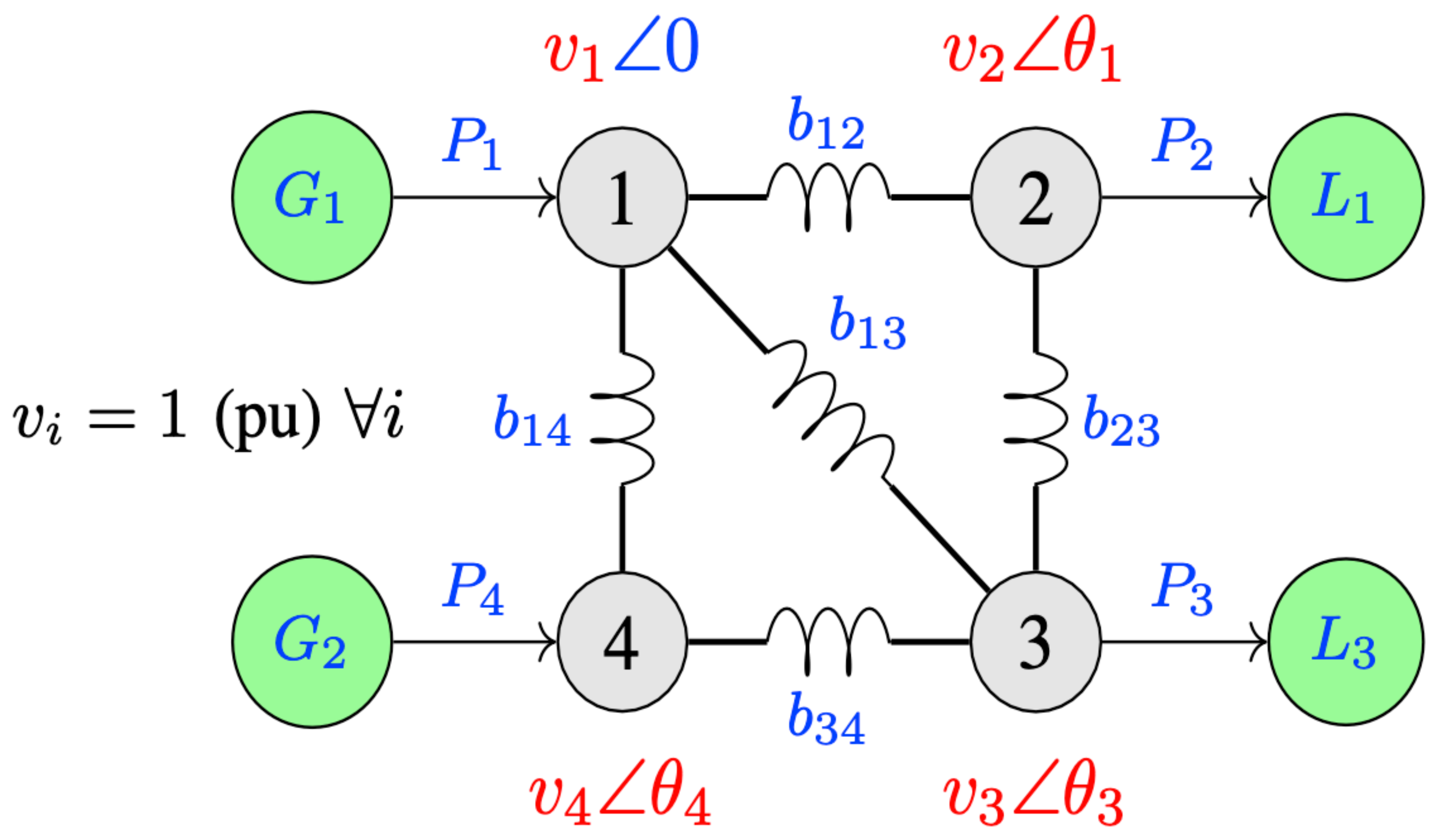

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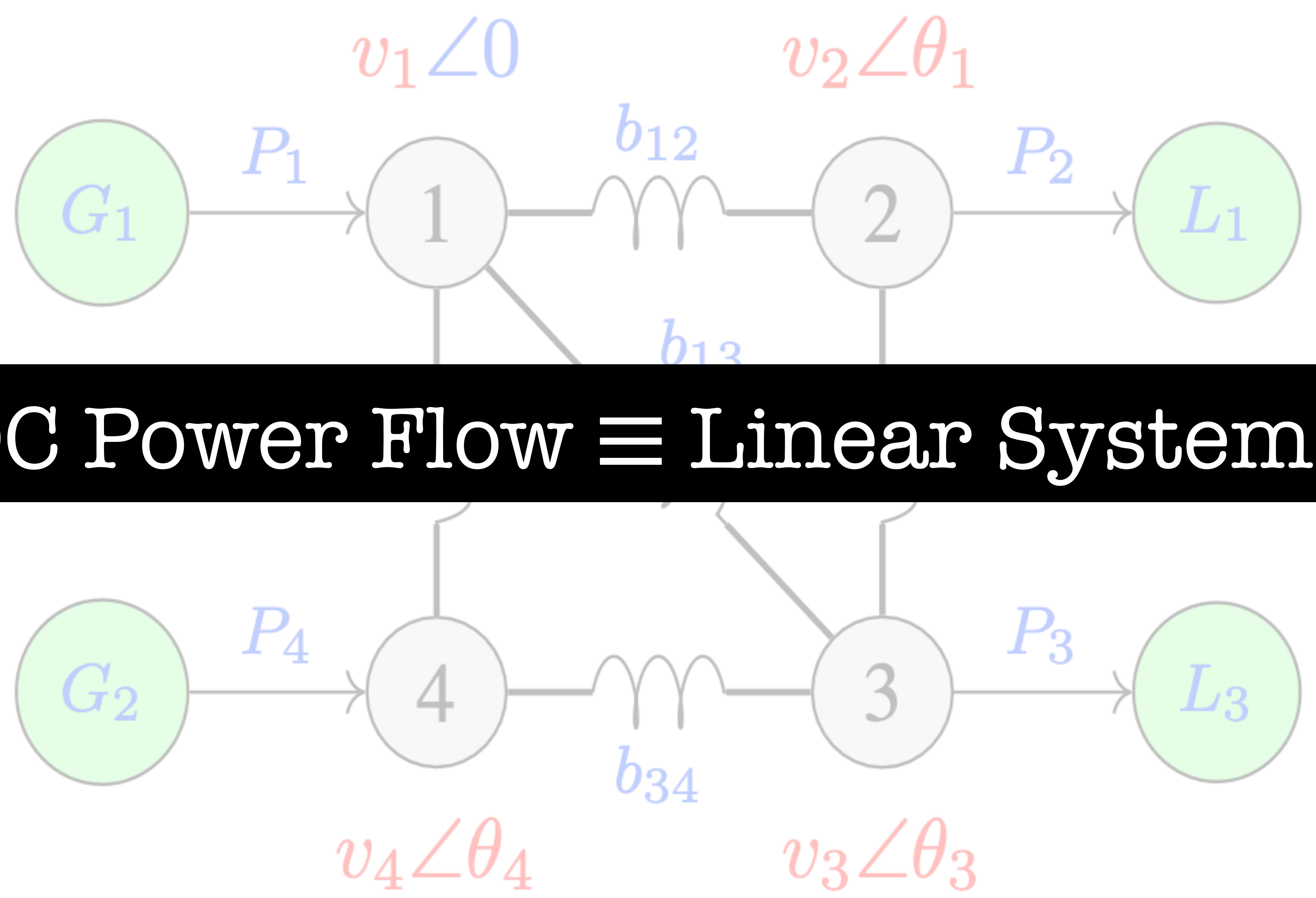

DC Power Flow ≡ Linear System Solve

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System Size

Sparsity

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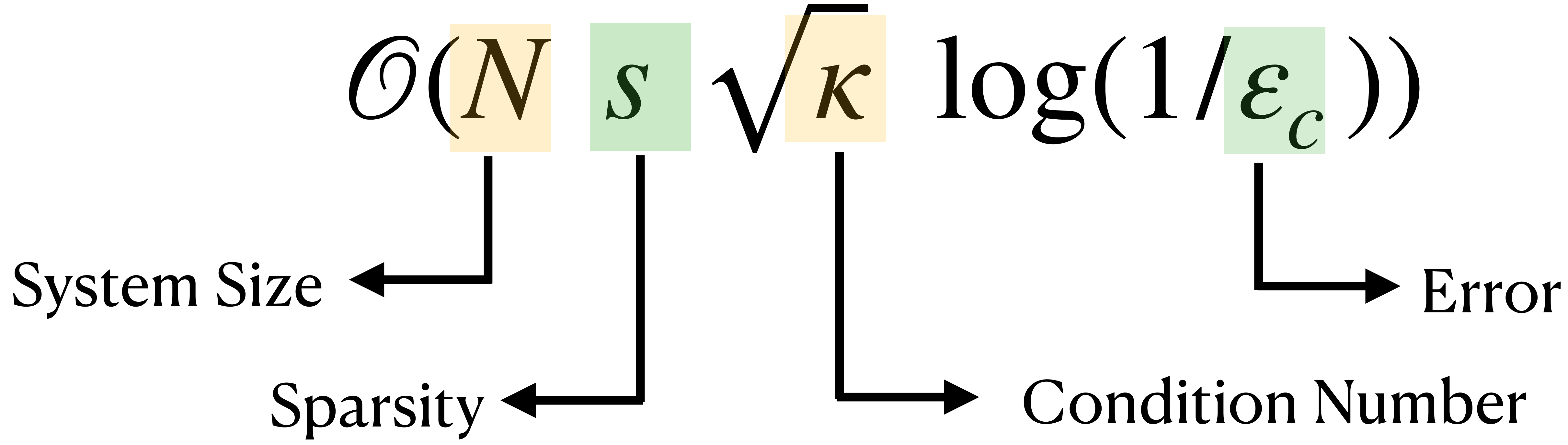

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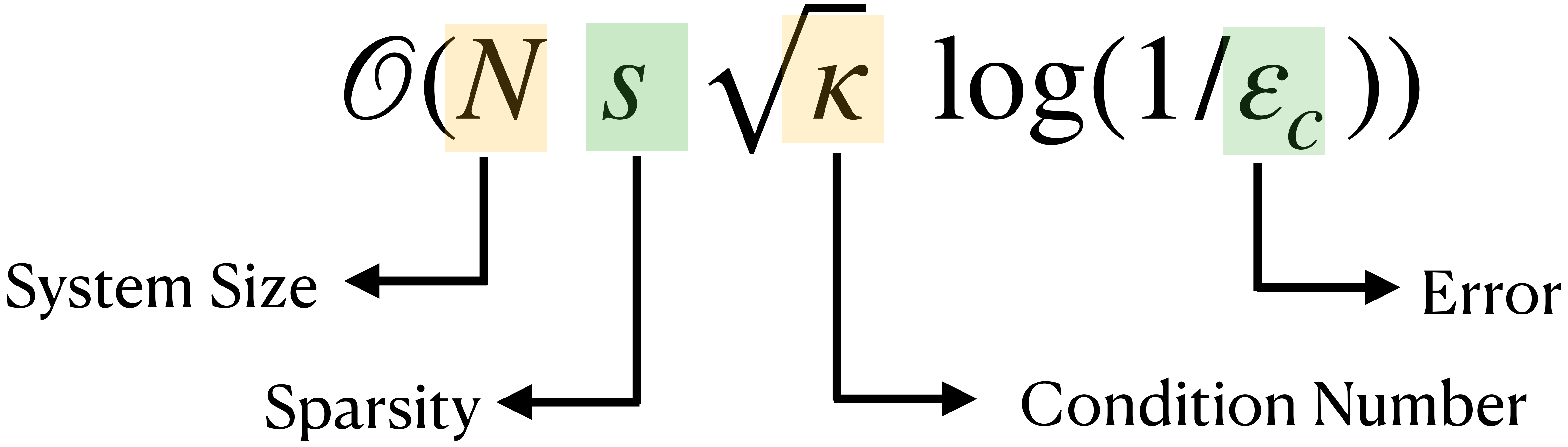

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Quantum Power Flow Claim!

Solving DCPF using Harrow-Hassidim-Lloyd (HHL) algorithm will lead to **Exponential** speed up

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HHL does scale better in terms of **System Size**, but scales worse in terms of **Condition Number** (& Sparsity)

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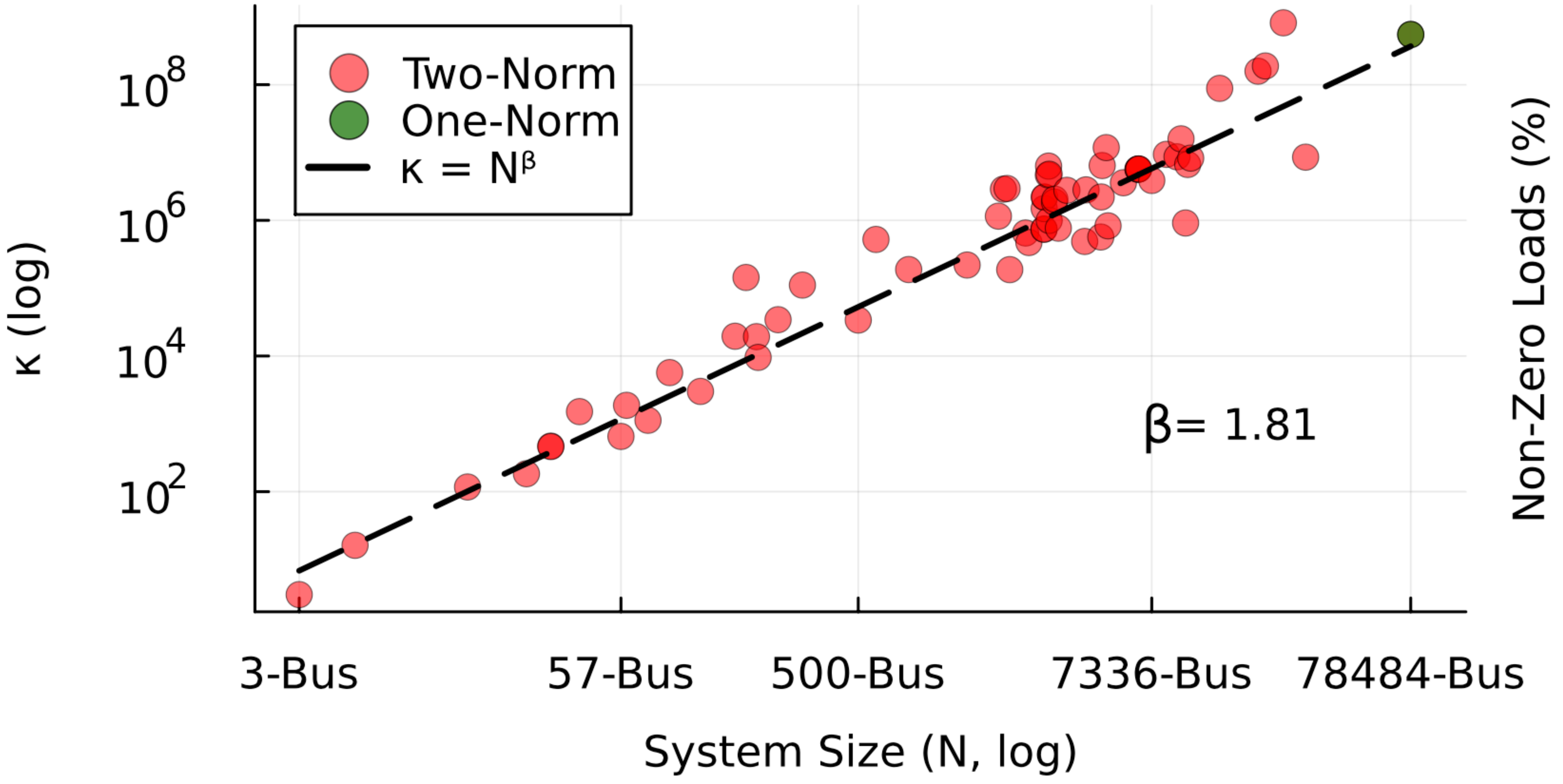
Scaling of condition number(k) as a function of buses (N) for the PGLib-OPF datasets



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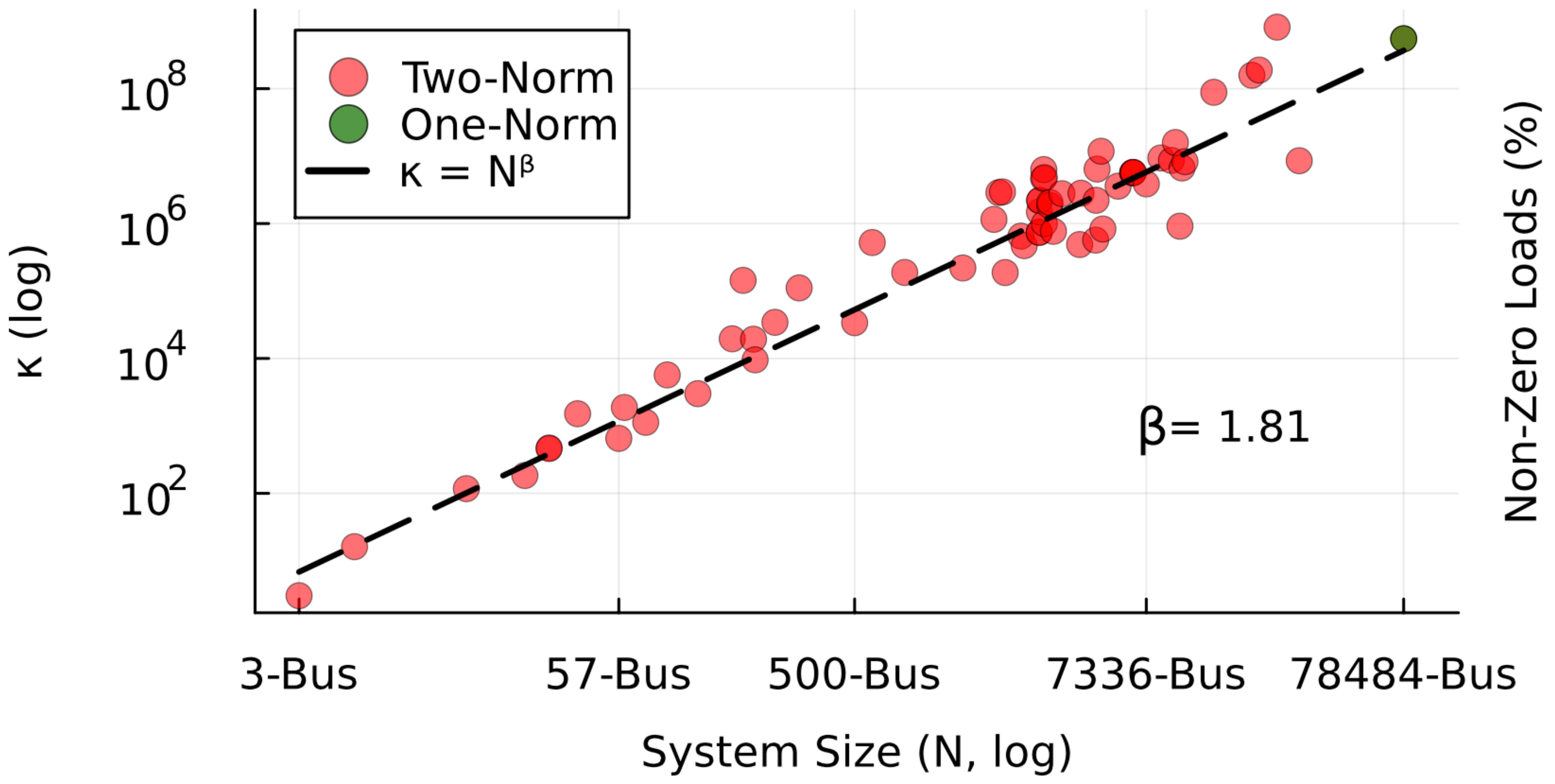
Condition number is scaling worse than N

Algorithms that manage condition number will be better for this application.



Point #1:

During speedup analysis consider runtime complexity with respect to ALL Parameters

Our Favorite Problem might not have so Favorable Parameters

Overheads: Data Loading

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$$\textsc{Classical Input} \xRightarrow{\textsc{Classical Algorithm}} \textsc{Classical Output}$$

$$\textsc{Classical Input} \xRightarrow{\textsc{State Prep.}} \left|\textsc{Input}\right\rangle \xRightarrow{\textsc{Quantum Algorithm}} \left|\textsc{Output}\right\rangle \xRightarrow{\textsc{Readout}} \textsc{Classical Output}$$

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$$\begin{tikzcd}
\mathcal{O}(1) & & \\
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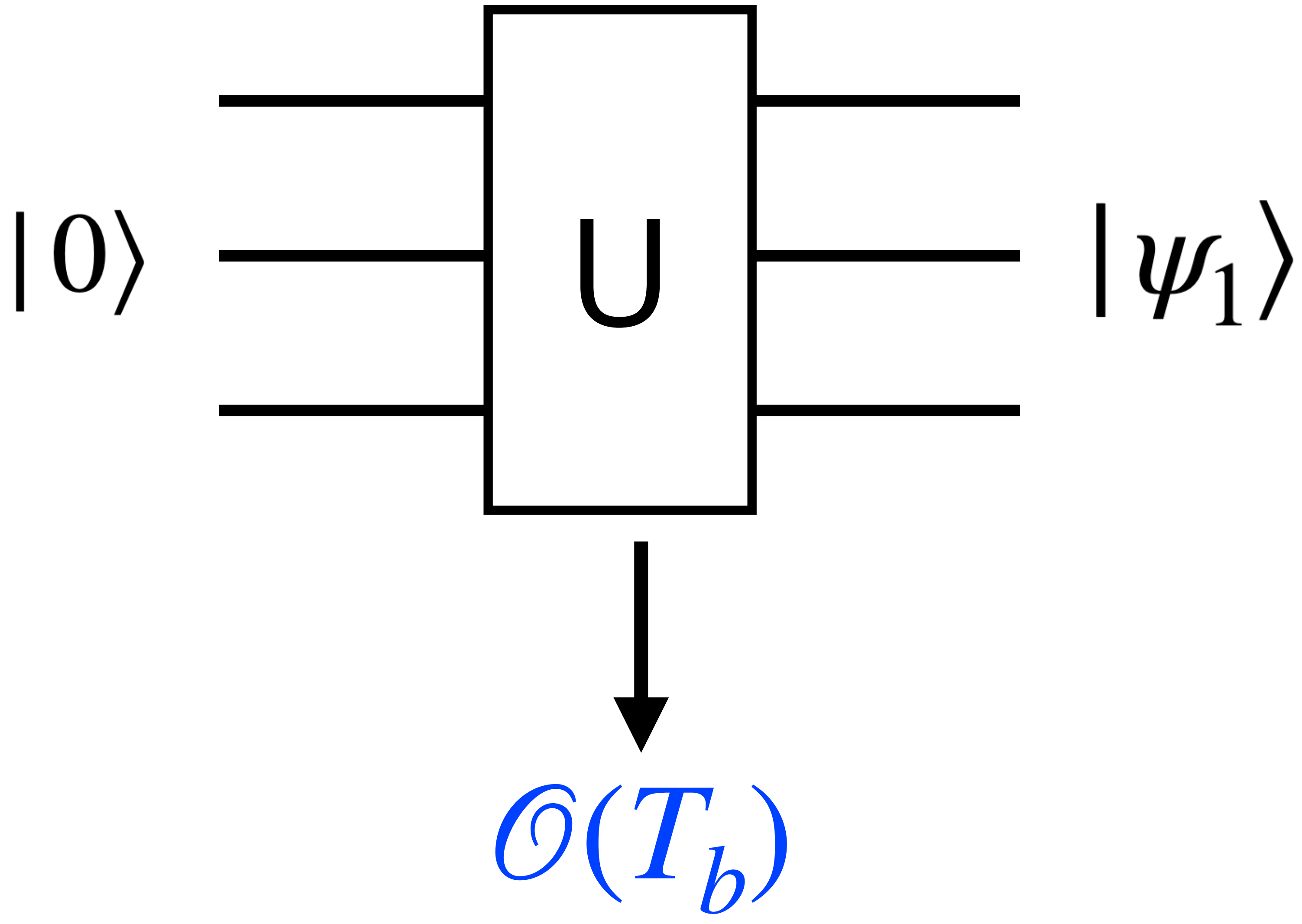

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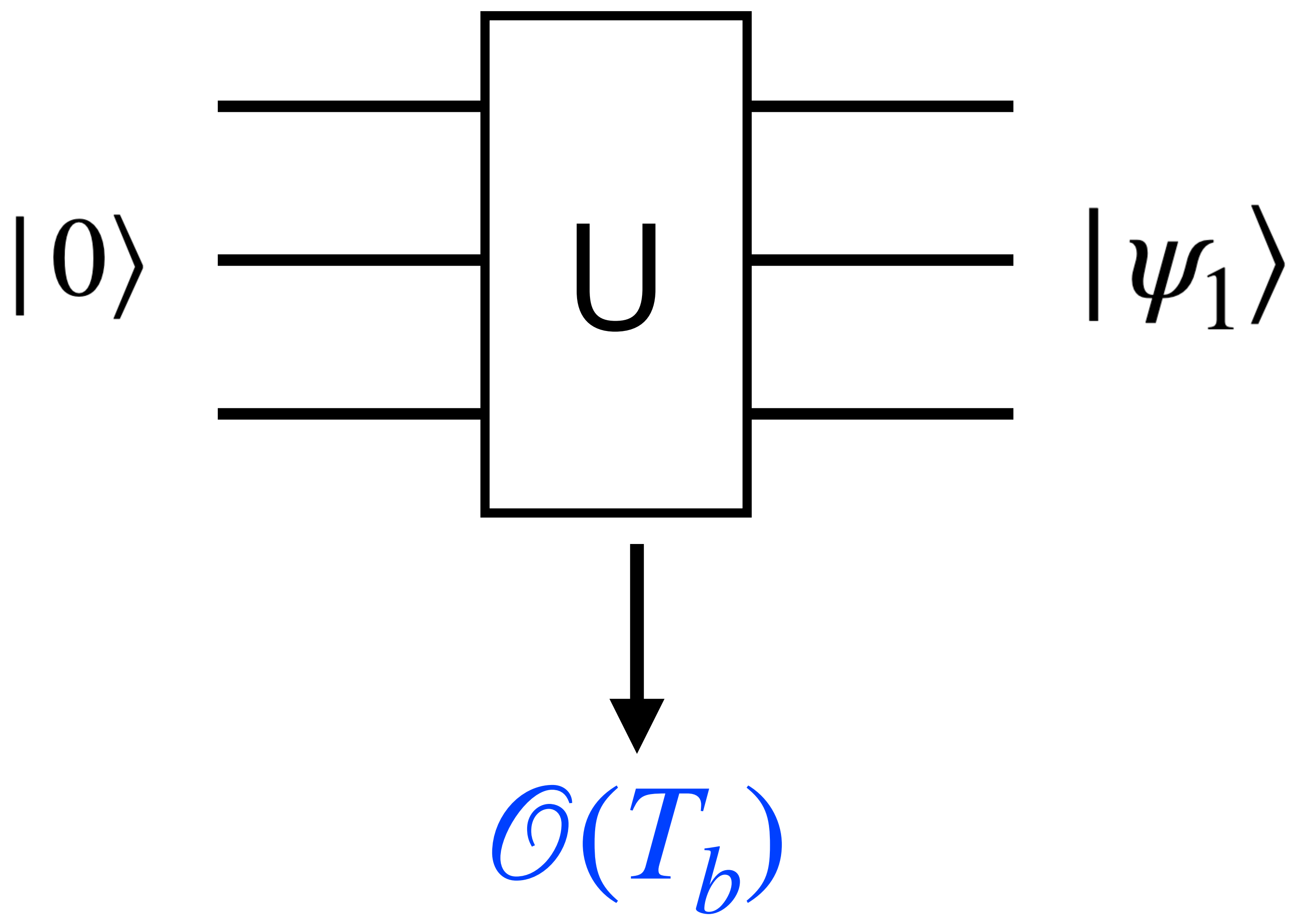

To Load $N-$Dimensional b vector

Amplitude Encoding → $\mathcal{O}(N)$

With Quantum RAM → $\mathcal{O}(\log N)$

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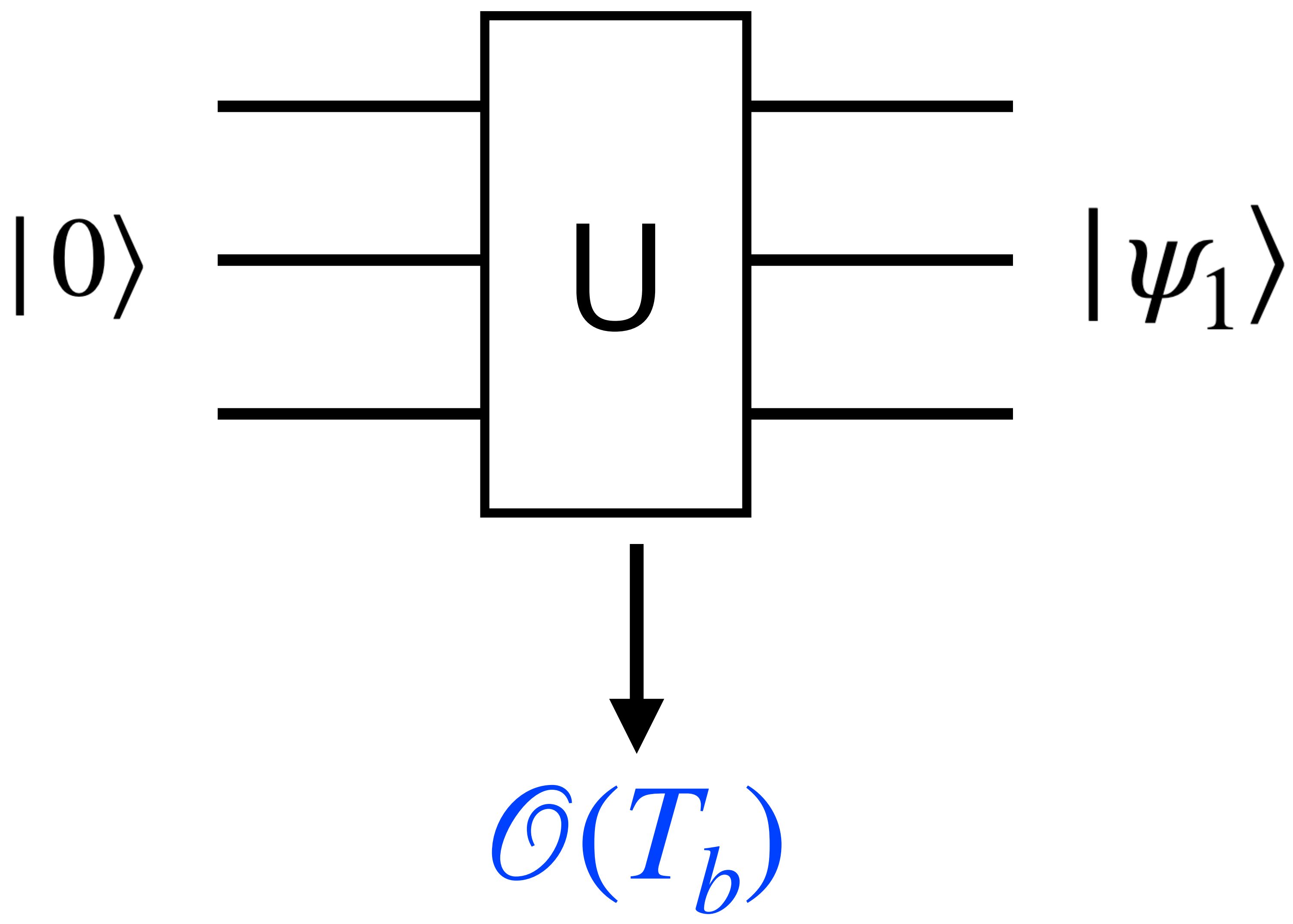

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So Optimistically $\mathcal{O}(T_b) \equiv \mathcal{O}(\log N)$

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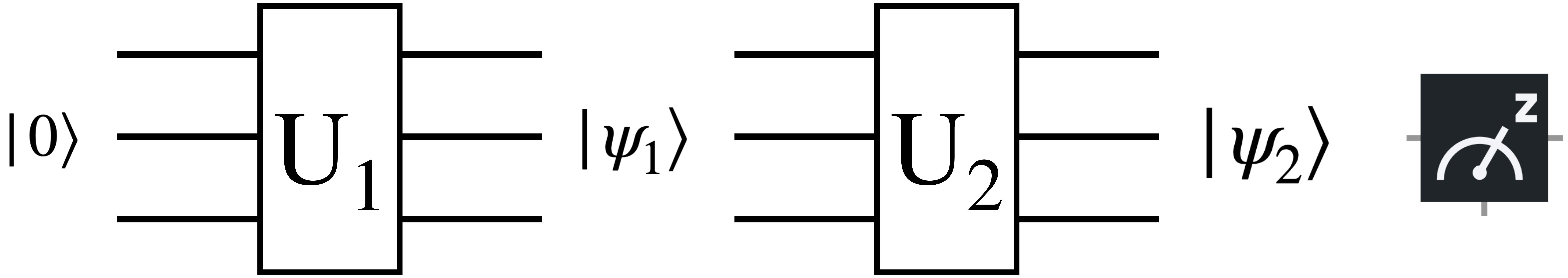

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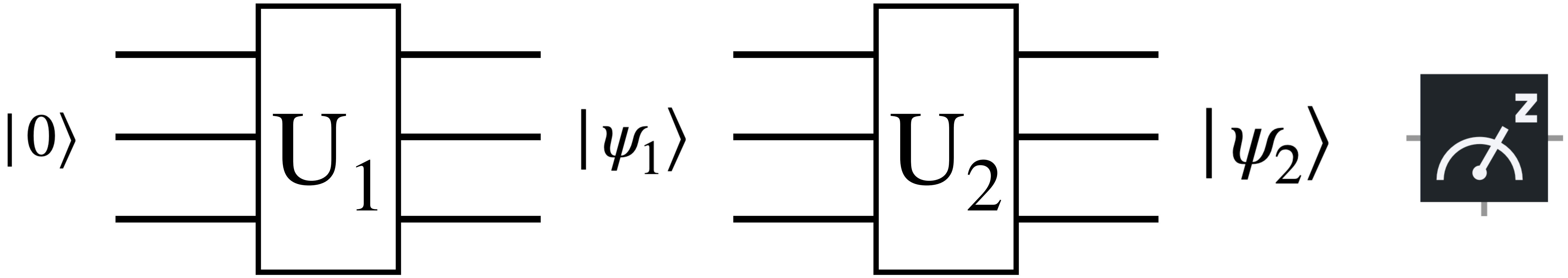

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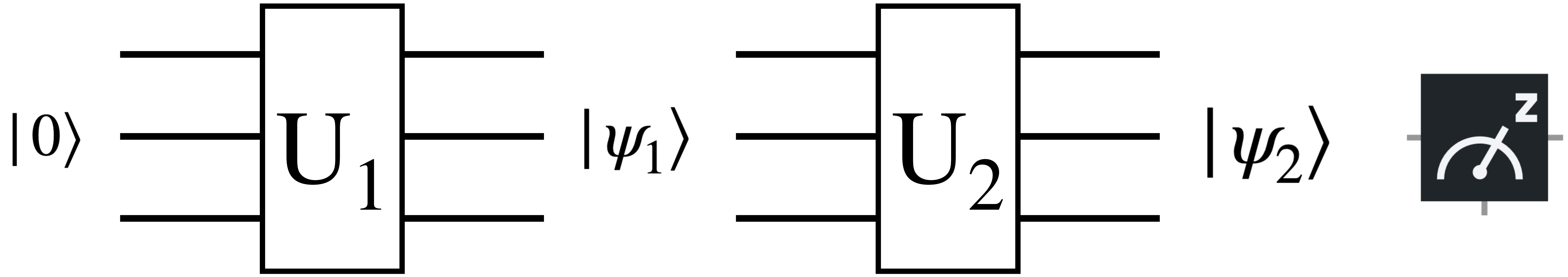

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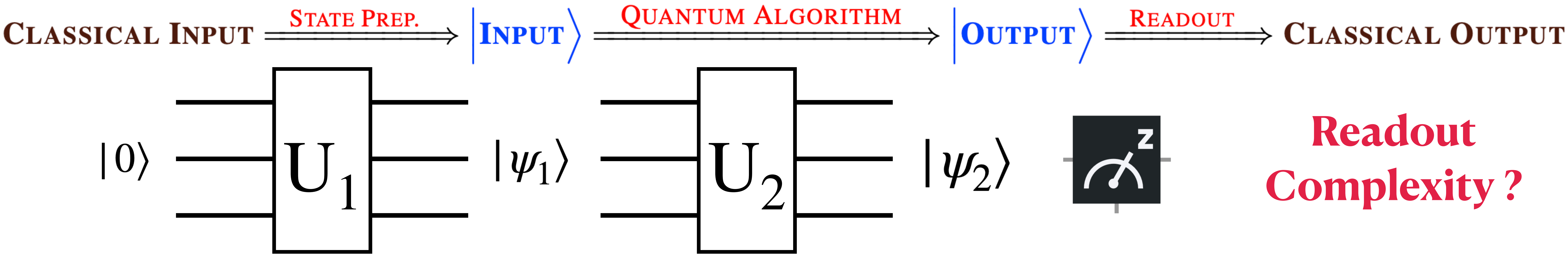

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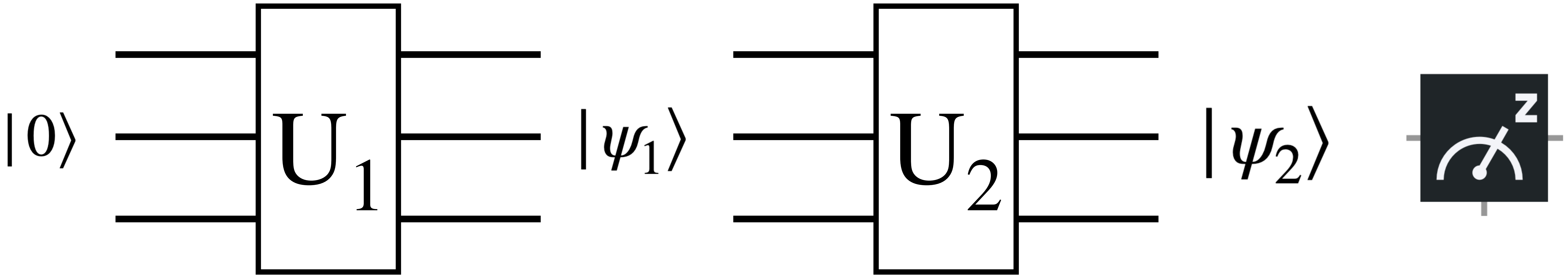

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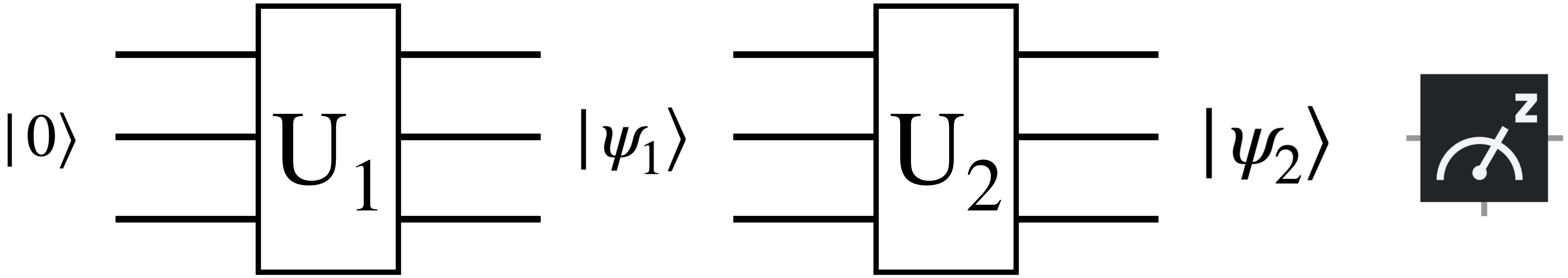

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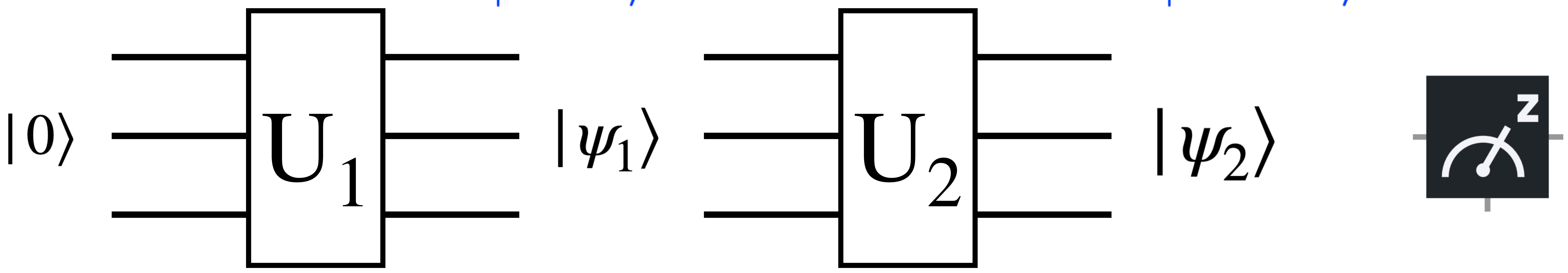

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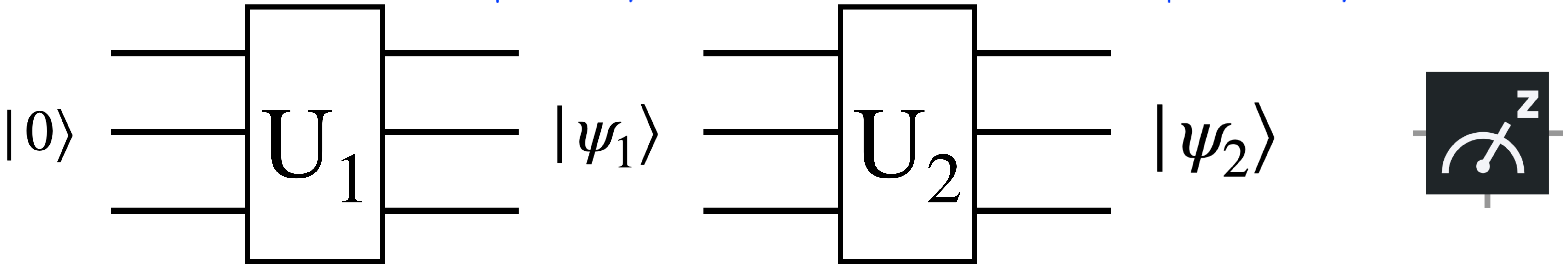

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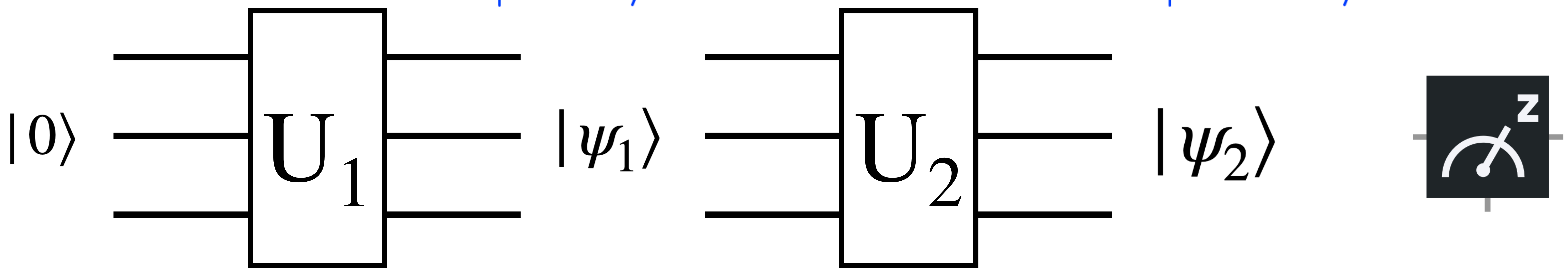

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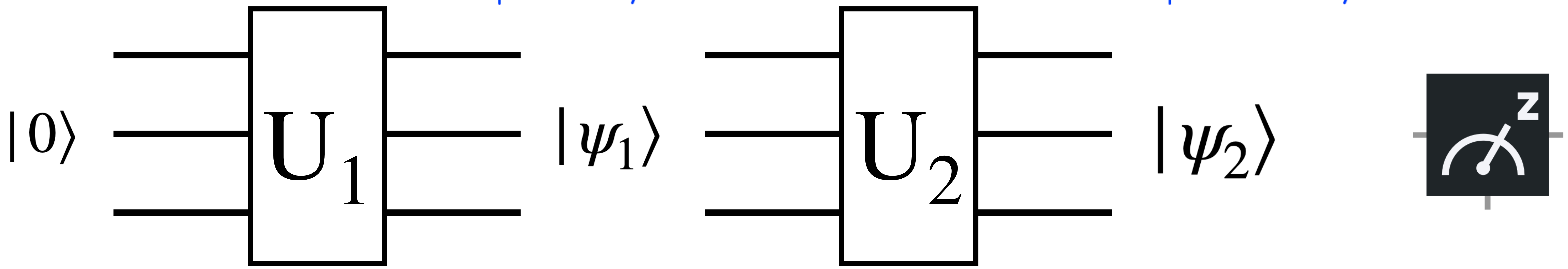

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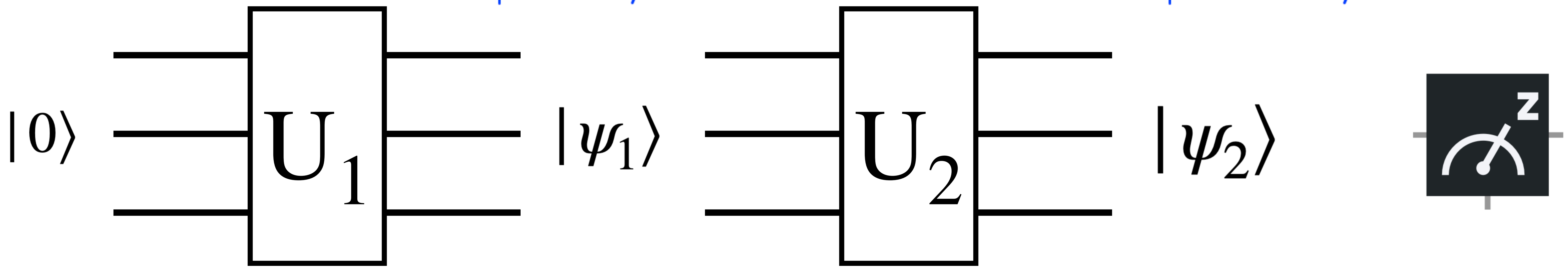

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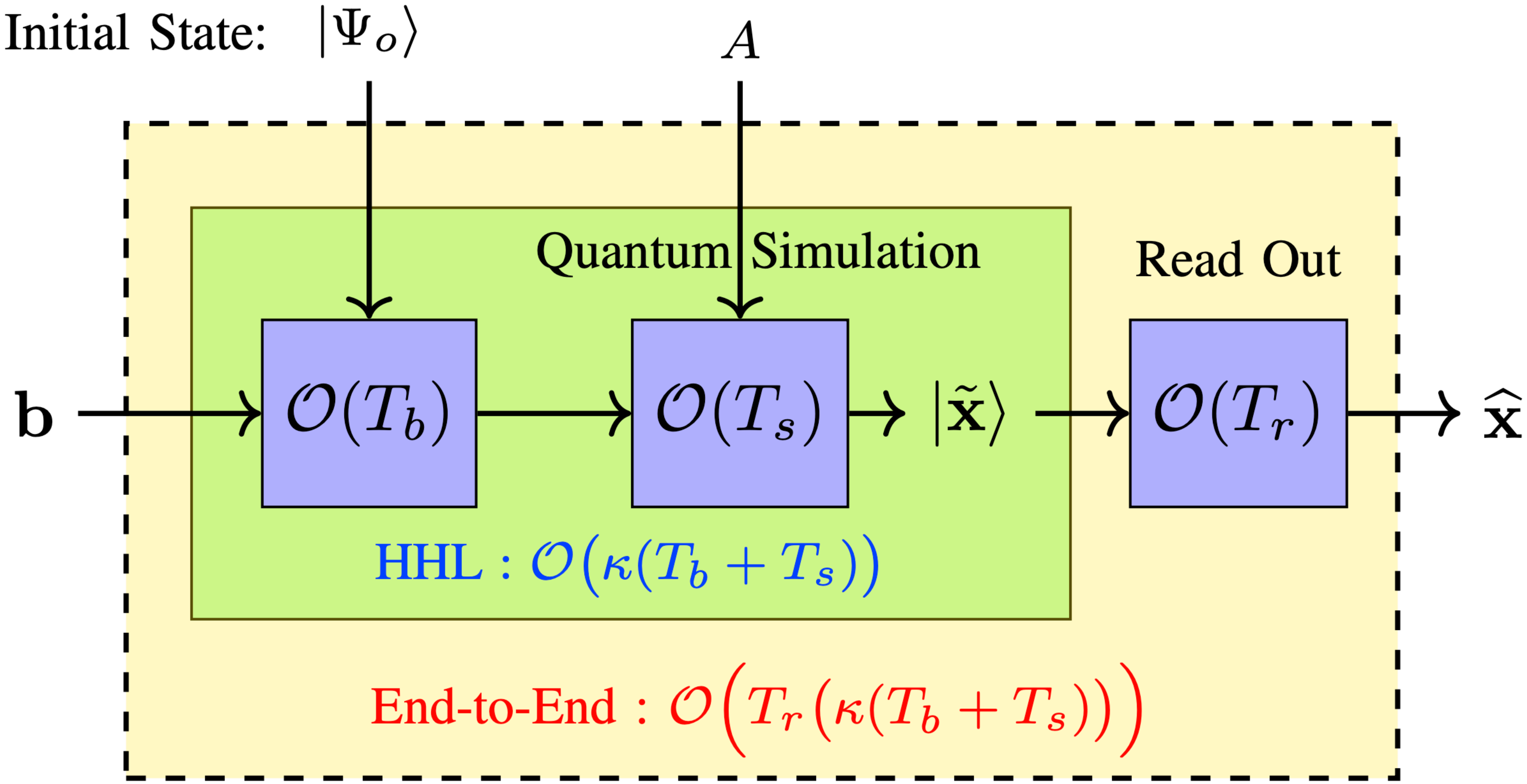
How many copies ? $\longrightarrow$ $\Theta(poly(N)/\varepsilon)$

J. van Apeldoorn, A. Cornelissen, A. Gilye´n, and G. Nannicini, "Quantum tomography using state-preparation unitaries," in *Proceedings of the 2023 Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*. SIAM, 2023, pp. 1265–1318.

Complete Quantum Picture: End-to-End


Initial State: $|\Psi_o\rangle$
A
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HHL : $\mathcal{O}\big(\kappa(T_b + T_s)\big)$
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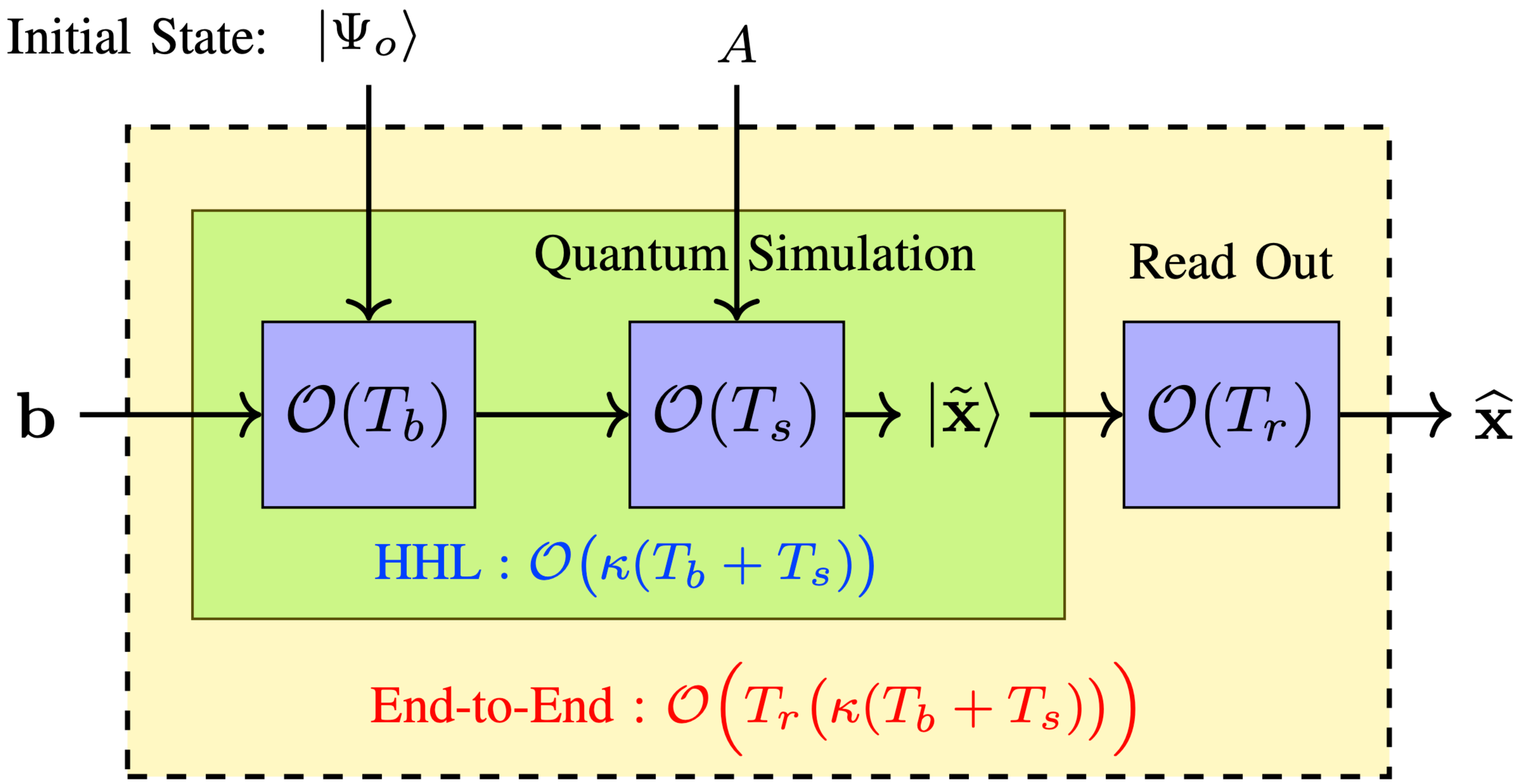
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Evaluating End-to-End Complexity of Solving Linear Power Flows using Quantum Linear System Solving Algorithms (HHL Family)

Point #2:

During speedup analysis consider End-to-End runtime complexity

Readout alone is enough to Kill any advantage in general Power Flow setting

Comparative Picture

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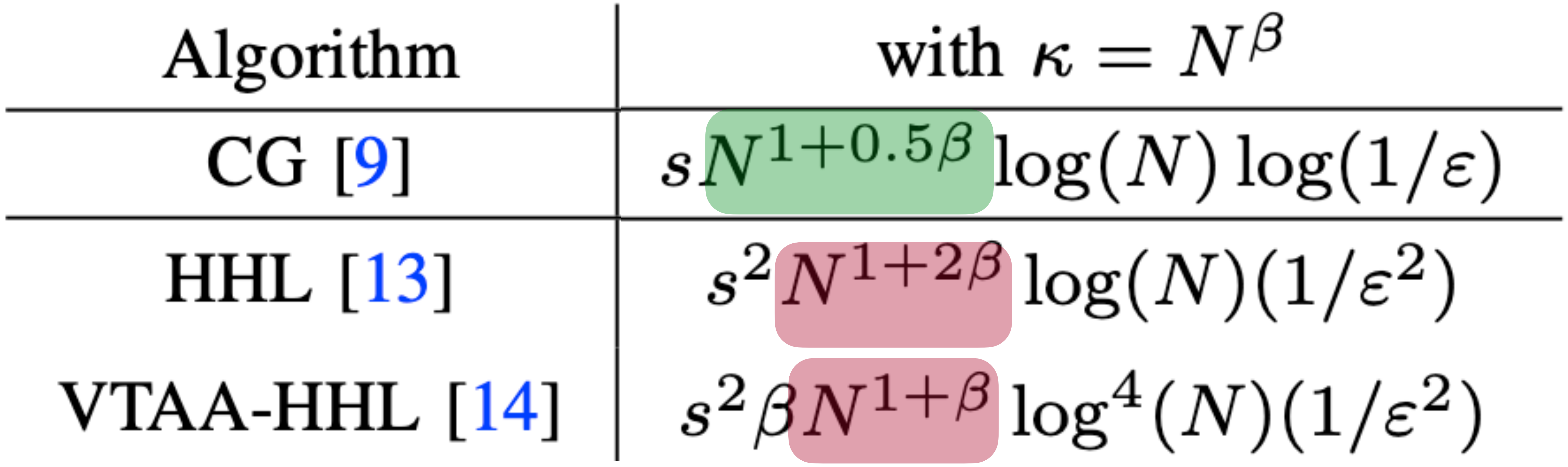

Algorithm	with $\kappa = N^{\beta}$
CG [9]	$sN^{1+0.5\beta}\log(N)\log(1/\varepsilon)$
HHL [13]	$s^2N^{1+2\beta}\log(N)(1/\varepsilon^2)$
VTAA-HHL [14]	$s^2\beta N^{1+\beta}\log^4(N)(1/\varepsilon^2)$

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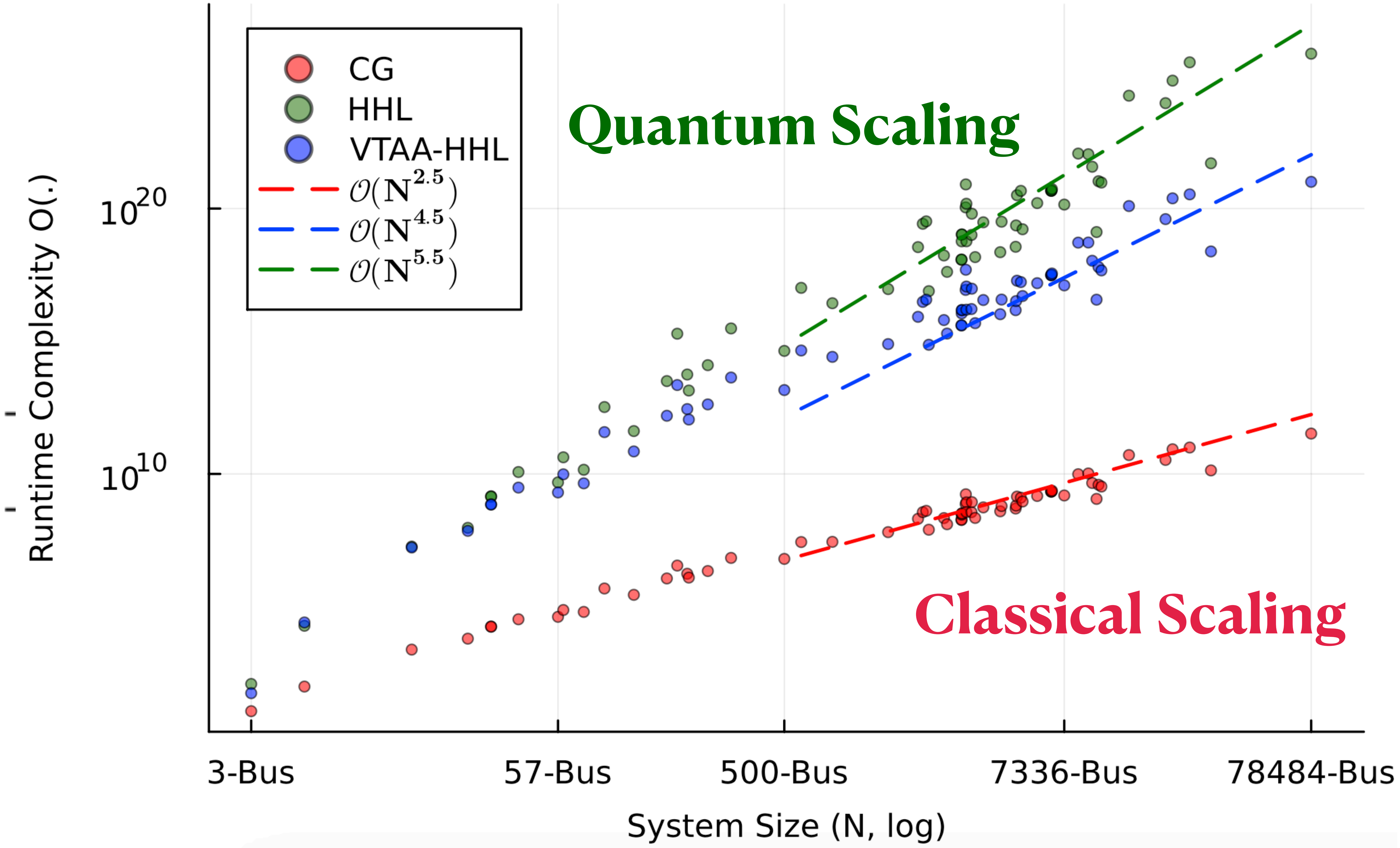

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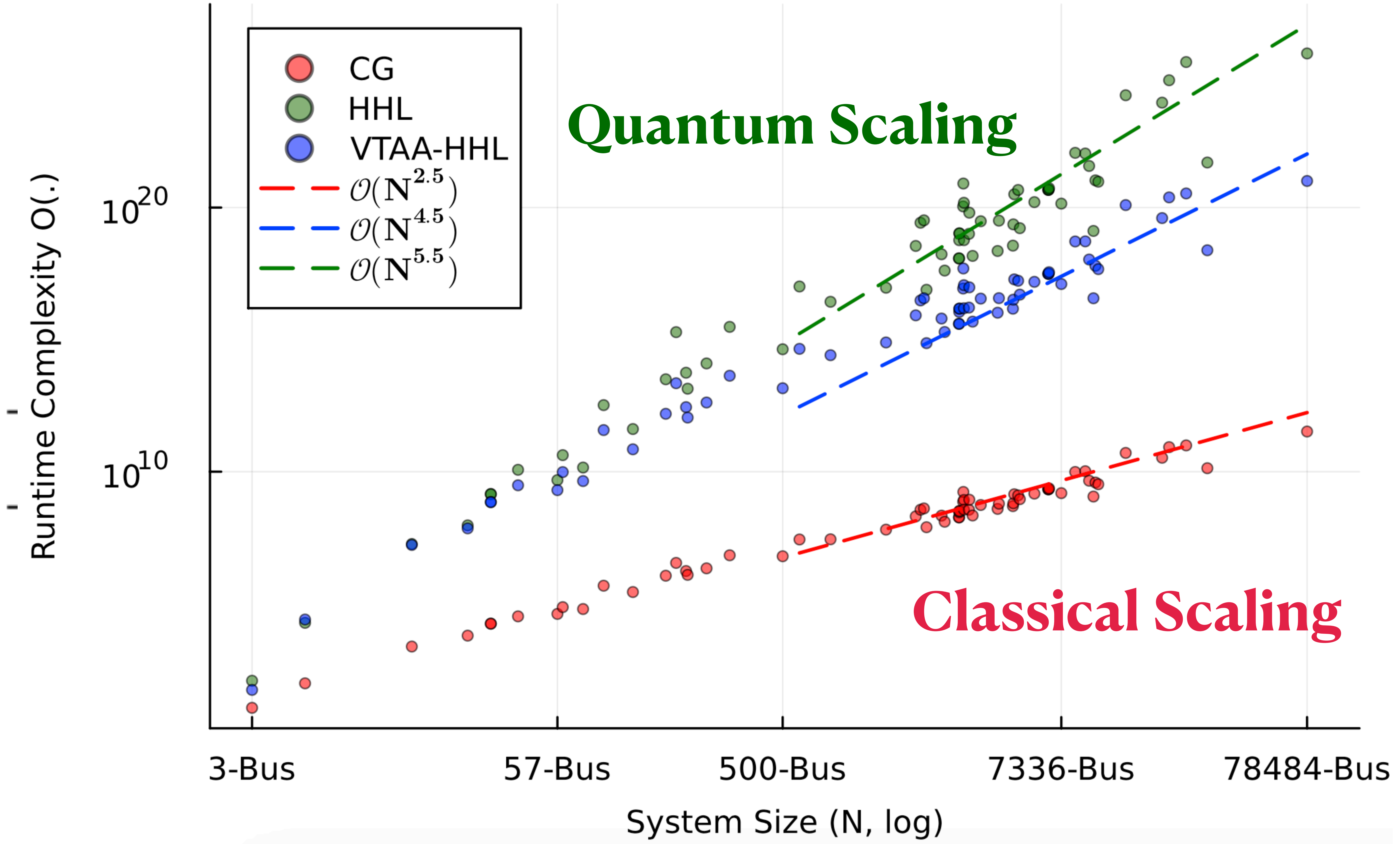

Current Quantum Linear Solving Algorithms offer **No Advantage** in solving Power Flow in Standard Formulations

What is needed for ‘Potential’ Speedup?

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$$s^2\,\kappa^2\,N\log(N)(1/\varepsilon^2)$$

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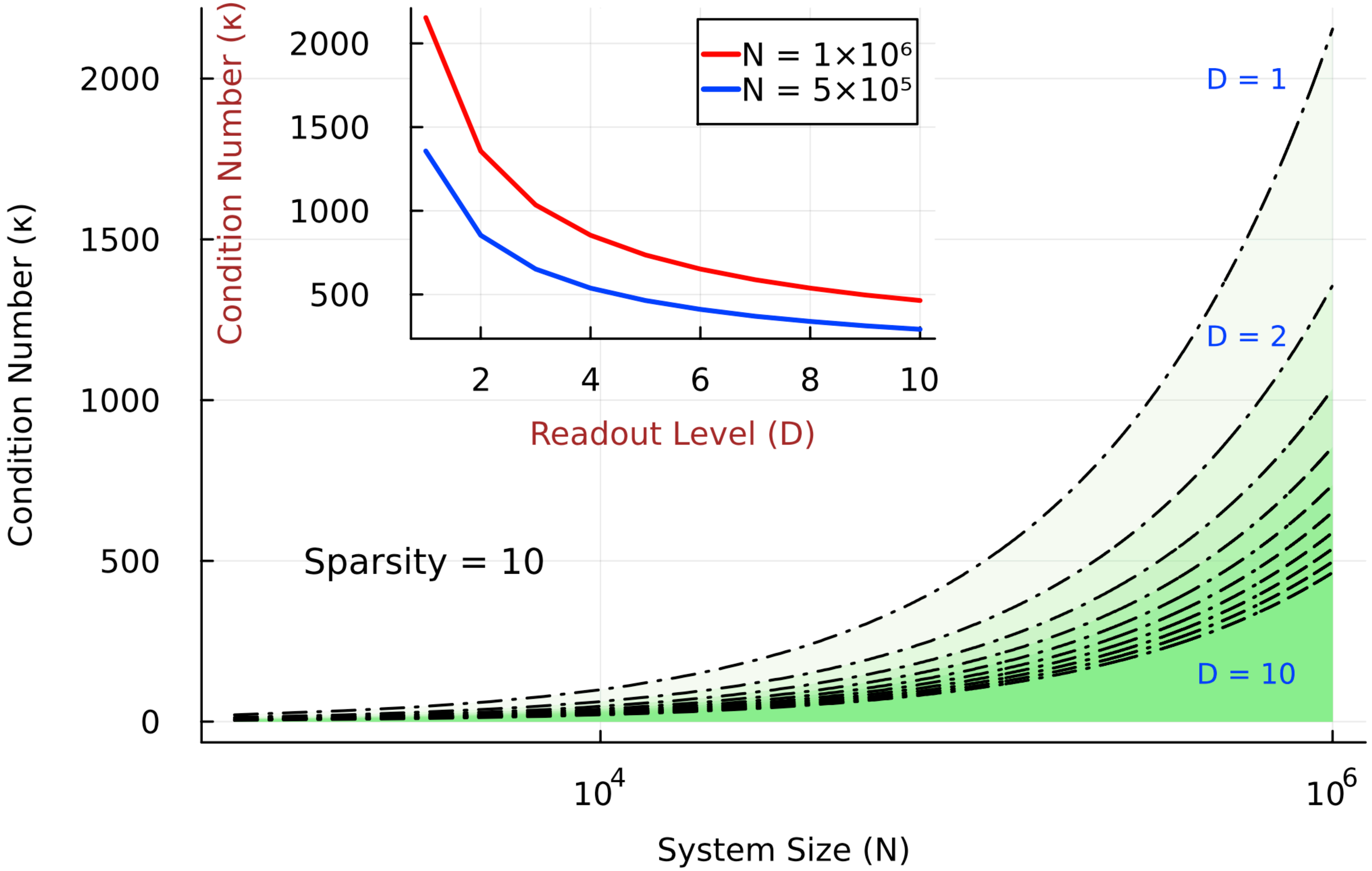

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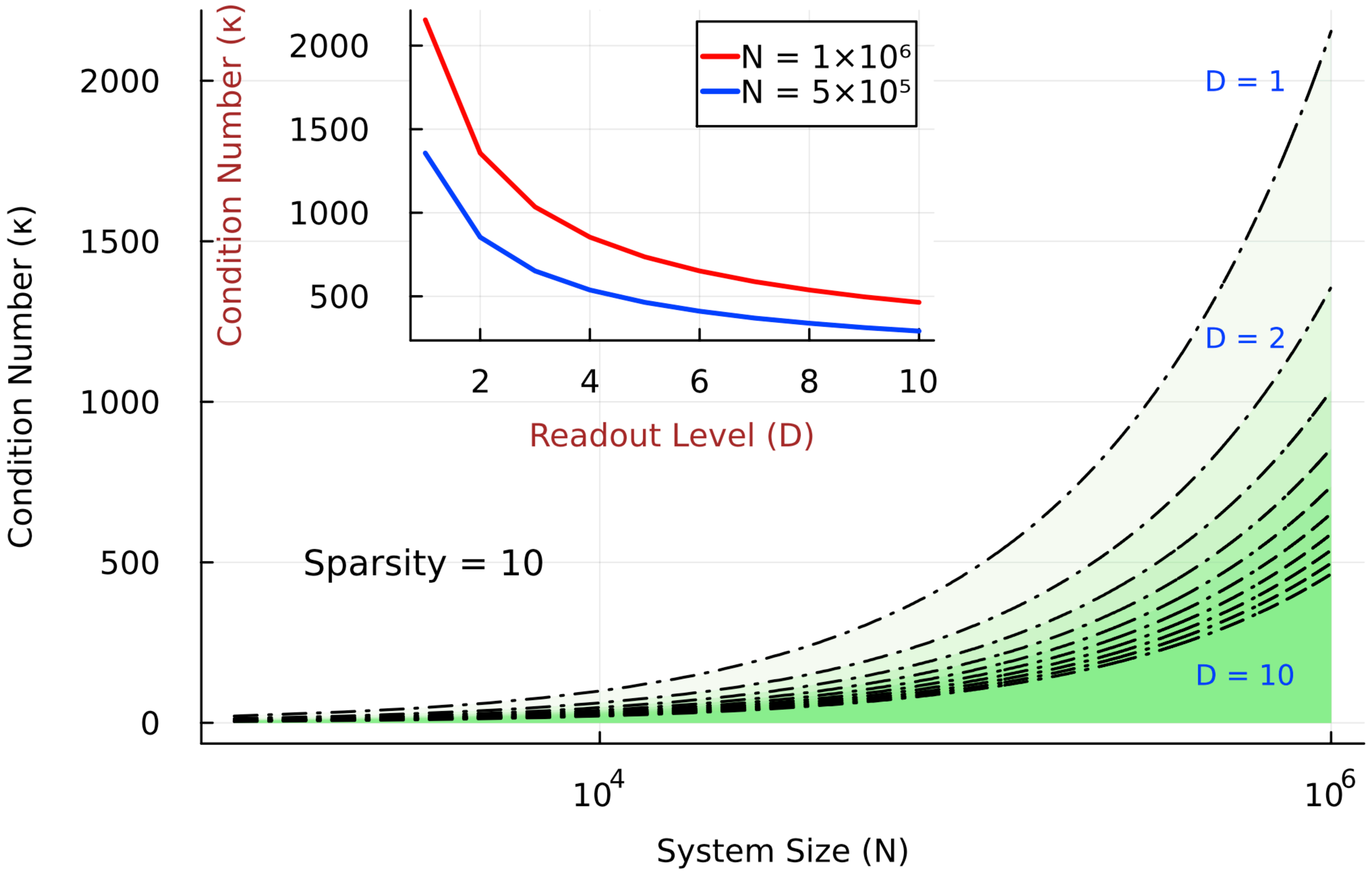

Pre-Condition & Read Less

Ok! Forget DC, What About AC Power Flow?

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Newton Raphson Load Flow

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Newton Raphson Load Flow → Solving Linear Systems of Equations with Jacobian Matrix Multiple Times

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As long as K is same for Quantum & Classical, We have Less Hopes![†]

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[†] Note that exact Quantum Complexity will depends on how the proposed algorithm handles error propagation within Quantum.

Another Issue

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κ = N^β
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10^4
10^6
10^8
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1803-Bus
78484-Bus
System Size (N, log)
10^6
10^7
10^8
κ (log)
2869-bus
4601-bus
9241-bus
1
2
3
4
5
6
7
#Iterations
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24464-bus
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Tamas Terlaky and his group from Lehigh University have some work on it, in the context of Interior Point Methods

Overall— What will it take to have Hope?

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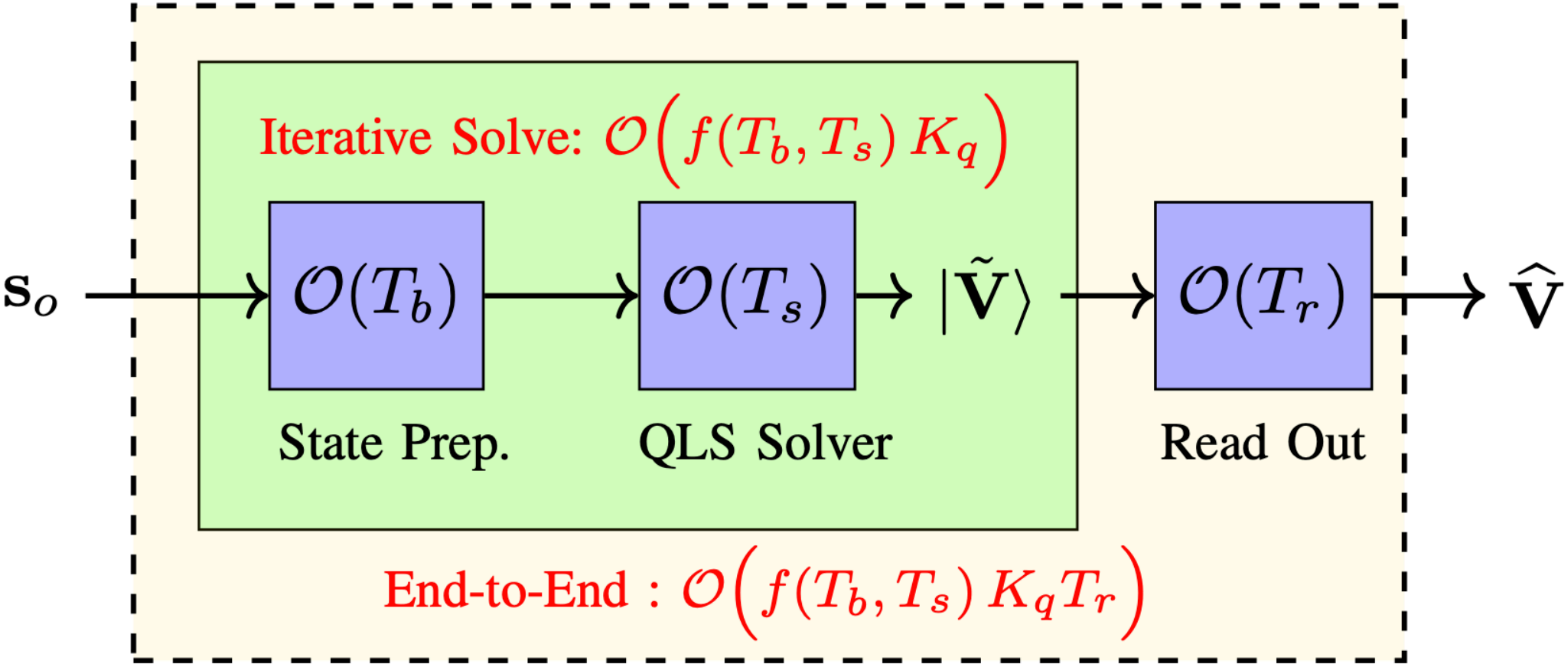
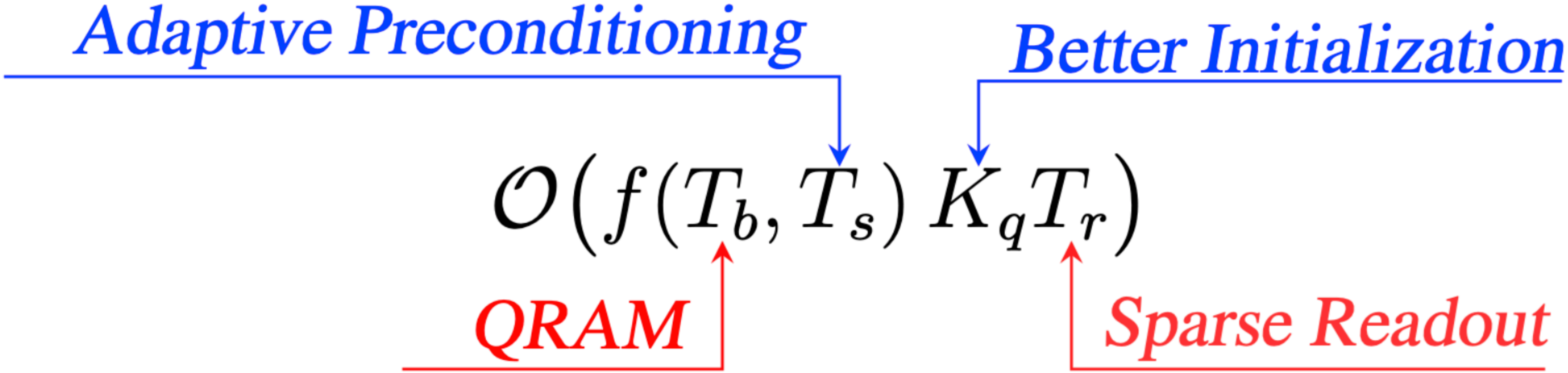

Iterative Solve: $\mathcal{O}\Big(f(T_b, T_s)\, K_q\Big)$
$\mathbf{s}_o$
$\mathcal{O}(T_b)$
$\mathcal{O}(T_s)$
$|\tilde{\mathbf{V}}\rangle$
$\mathcal{O}(T_r)$
$\widehat{\mathbf{V}}$
State Prep.
QLS Solver
Read Out
End-to-End : $\mathcal{O}\Big(f(T_b, T_s)\, K_q T_r\Big)$

Overall— What will it take to have Hope?



Adaptive Preconditioning

Better Initialization

$$\mathcal{O}\left(f(T_b, T_s)\, K_q T_r\right)$$

QRAM

Sparse Readout

By the way! What best can be achieved by doing all of this?

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State Prepration

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State Prepration

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Readout

Reading the Solution

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So……

Is All of This Worth it?

Or

Is it Watt We are Looking for?

Conclusion

End-to-End Complexity based Potential Quantum Speedup Analysis must be done for Your Favorite Problem....

..... Before starting to solve it.

Demystifying Quantum Power Flow:
Unveiling the Limits of Practical Quantum Advantage

Parikshit Pareek, Abhijith Jayakumar, Carleton Coffrin, and Sidhant Misra
Los Alamos National Laboratory, NM, USA.
pareek@lanl.gov; abhijithj@lanl.gov; cjc@lanl.gov, sidhant@lanl.gov.

arXiv:2402.08617

pareek@ee.iitr.ac.in

https://psquare-lab.github.io/

Indian Institute of Technology Roorkee

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If Need someone who asks Very Stupid Questions in your research group meetings related to an interesting problem on

ML + Power or **Quantum + Power**

Let me know at: *pareek@ee.iitr.ac.in*