

Predict-and-Optimize Robust Unit Commitment with Statistical Guarantees via Weight Combination

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Abstract

The ongoing global transition toward greener power systems has significantly increased the penetration of renewable energy sources, introducing unprecedented levels of uncertainty into power grid operations. This evolution has transformed traditional deterministic power system scheduling into a complex stochastic optimization challenge. Among the critical operational problems, Unit Commitment (UC), the day-ahead decision process of determining which generators to activate, when to start them up or shut them down, and how much power to schedule, has become particularly challenging due to the intermittent and unpredictable nature of wind and solar generation. Unit Commitment fundamentally differs from Economic Dispatch (ED) in both temporal scope and decision complexity. While ED operates on a shorter time scale and assumes fixed generator commitment states to determine optimal power output levels, UC addresses a longer-term scheduling horizon and involves binary decisions regarding generator on and off states, startup and shutdown sequences, minimum up and down time constraints, and reserve allocation. Traditional approaches for uncertainty handling in UC have relied on Stochastic Programming (SP) and Robust Optimization (RO). SP depends on accurate probability distributions that are difficult to estimate and may lead to infeasible or suboptimal solutions when distributions are misspecified. RO addresses this limitation by considering worst-case realizations within predefined uncertainty sets; however, conventional RO methods often suffer from excessive conservatism or lack formal statistical guarantees when uncertainty sets are constructed heuristically. Distributionally Robust Optimization (DRO) provides theoretical guarantees but typically requires sample sizes that scale with the dimension of uncertainty, which becomes impractical for UC problems involving multiple time periods and spatially distributed net loads. This paper proposes a decision-centric predict-and-optimize framework that tightly integrates forecasting and optimization for robust unit commitment while providing rigorous finite-sample statistical guarantees on out-of-sample performance. Multiple heterogeneous forecasting models are treated as complementary information sources and combined through optimized weight parameters. Unlike conventional forecast aggregation methods that minimize prediction error, the proposed framework learns these weights directly from the downstream UC performance, aligning the forecasting objective with operational decision quality. The combined prediction and historical forecast residuals are used to construct a data-driven ellipsoidal uncertainty set that guarantees, with a prescribed confidence level, that the true uncertainty realization lies within the set, independent of uncertainty dimension. Building on this uncertainty representation, a two-stage robust UC model is formulated, where first-stage decisions determine generator commitment, dispatch, and reserve scheduling, and second-stage decisions model real-time redispatch actions under uncertainty. To reduce conservativeness, the initial ellipsoidal uncertainty set is reconstructed into a polyhedral form using problem structure and feasible solution information, excluding high-cost scenarios while preserving statistical guarantees. The resulting robust UC problem is solved efficiently using a column-and-constraint generation algorithm. To enable efficient optimization of forecast combination weights in daily operations, a neural network surrogate model is trained offline to approximate the mapping between forecast weights and evaluated UC cost. The trained network is equivalently reformulated as mixed-integer linear constraints using rectified linear unit activation functions, allowing the weight optimization problem to be solved as a mixed-integer linear program without differentiating through the UC solver. Case studies on modified IEEE 30-bus and 118-bus systems demonstrate that the proposed approach achieves superior robustness and lower operational costs compared to traditional stochastic, robust, and distributionally robust UC methods, while maintaining computational tractability suitable for practical day-ahead scheduling.

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Abstract—The growing uncertainty from renewable power and electricity demand brings significant challenges to unit commitment (UC). While various advanced forecasting and optimization methods have been developed to better predict and address this uncertainty, most previous studies treat forecasting and optimization as separate tasks. This separation can lead to suboptimal results due to misalignment between the objectives of the two tasks. To overcome this challenge, we propose a robust UC framework that integrates forecasting and optimization processes while ensuring statistical guarantees. In the forecasting stage, we combine multiple predictions derived from diverse data sources and methodologies for an improved prediction, aiming to optimize the UC performance. In the optimization stage, the combined prediction is used to construct an uncertainty set with statistical guarantees, based on which the robust UC model is formulated. The optimal robust UC solution provides feedback to refine the weight used for combining multiple predictions. To solve the proposed integrated forecasting-optimization framework efficiently and effectively, we develop a neural network-based surrogate model for acceleration and introduce a reshaping method for the uncertainty set based on the optimization result to reduce conservativeness. Case studies on modified IEEE 30-bus and 118-bus systems demonstrate the advantages of the proposed approach.

Index Terms—unit commitment, data-driven robust optimization, statistical guarantee, predict-and-optimize, surrogate model

NOMENCLATURE

A. Abbreviation

C&CG	Column-and-constraint generation.
DRO	Distributionally robust optimization.
i.i.d.	Independent and identically distributed.
MILP	Mixed-integer linear programming.
MLP	Multilayer perceptron.
PSO	Particle swarm optimization.
RO	Robust optimization.
SP	Stochastic programming.
UC	Unit commitment.

B. Indices and Sets

$i \in \mathcal{I}$	Set of buses.
$m \in \mathcal{M}$	Set of prediction methods.
$x \in \mathcal{X}$	Feasible set of the pre-dispatch variable.
$u \in \mathcal{U}$	Uncertainty set.
$y \in \mathcal{Y}(x, u)$	Feasible set of the re-dispatch variable.

$g \in \mathcal{G}$	Set of generators.
$t \in \mathcal{T}$	Set of periods in unit commitment.
$l \in \mathcal{L}$	Set of transmission lines.

C. Parameters

T	The number of periods in a day.
ε, δ	Probability tolerance parameters.
\hat{u}_{it}	Prediction of load i in period t .
o_g^+, o_g^-	Startup/shutdown cost of generator g .
ρ_g	Generation cost coefficient of generator g .
γ_g^+, γ_g^-	Upward/downward reserve cost coefficient of generator g .
S_l	Capacity of line l .
π_{gl}, π_{il}	Power transfer distribution factor from generator g /bus i to line l .
T_g^+, T_g^-	Minimum up/down time of generator g .
R_g^+, R_g^-	Maximum upward/downward reserve of generator g .
$\underline{P}_g, \bar{P}_g$	Minimum/maximum output of generator g .
K_g^+, K_g^-	Maximum upward/downward ramp of generator g in a period.
K_g^U, K_g^D	Maximum output increase/decrease when generator g startups/shutdowns in a period.
ρ_g^+, ρ_g^-	Upward/downward output adjustment cost coefficient of generator g .
\underline{U}, \bar{U}	Lower/upper bound of uncertain load.

D. Variables

u_{it}	Uncertain load at bus i in period t .
w	Weight vector of predictions.
$\theta_{gt}, \theta_{gt}^+, \theta_{gt}^-$	Indicator variable for the on/startup/shutdown state of generator g in period t .
p_{gt}	Day-ahead scheduled output of generator g in period t .
r_{gt}^+, r_{gt}^-	Upward/downward reserve of generator g in period t .
p_{gt}^+, p_{gt}^-	Upward/downward output adjustment of generator g in period t .

I. INTRODUCTION

THE ongoing transition towards greener power systems has significantly increased the deployment of renewable power generators, leading to higher volatility in power sources. This uncertainty, together with the randomness of electric loads, presents great challenges for power system operations. Unit commitment (UC) is one of such power system operation problems that require particular attention.

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If given the probability distribution of uncertainty, stochastic programming (SP) can be applied to UC problems to determine the optimal strategy [1]. However, an accurate probability distribution is difficult to obtain in reality and the inaccuracy of the distribution may jeopardize the feasibility and optimality of SP solutions. Robust optimization (RO) handles this problem by focusing on the worst-case scenario in a pre-defined uncertainty set. Fruitful research has been done in this area: A multi-stage robust UC model was proposed and solved by robust dual dynamic programming in [2]. Polyhedra were used to replace rectangular uncertainty sets to model the uncertainty correlations in [3]. However, RO can be overly conservative when the uncertainty set contains all the possible uncertainty realizations, because extreme scenarios rarely happen but can cause a steep cost increase. The method in [4] estimated quantiles from historical data to establish rectangular uncertainty sets, in which extreme scenarios were excluded. In [5], uncertainty budgets were used to restrict the uncertainty set. A decision-dependent uncertainty set was constructed in [6] to consider the impact of pricing strategies on demand. However, estimated quantiles and uncertainty budgets may still be inaccurate, and the resulting uncertainty set lacks a statistical guarantee. This makes the obtained robust UC strategy less reliable and trustworthy.

Distributionally robust optimization (DRO) is another way to deal with uncertainty, which considers the worst-case probability distribution in a pre-defined ambiguity set. Two-stage distributionally robust UC methods were developed in [7] and [8] using Wasserstein metric ambiguity sets, while unimodality skewness of wind power was utilized in [9]. The copula theory was combined with DRO in [10] to better capture the dependence between uncertainty. While data-driven DRO can offer statistical guarantees by constructing the ambiguity set based on the asymptotic properties of the empirical probability distribution [7], the theoretical number of required data points typically depends on the dimension of uncertainty [11], which is often much larger than what is practically available under multi-dimensional uncertainty, such as in the UC problem. Consequently, the size of the ambiguity set is typically adjusted through a trial-and-error process [12], without providing a satisfactory statistical guarantee.

Recently, a data-driven uncertainty set construction framework was proposed in [13] with a dimension-free statistical guarantee. Even for multi-dimensional uncertainty, it can provide a satisfactory statistical guarantee based on a moderate amount of data, enabling the decision-maker to effectively control the conservative degree and easily strike a balance between optimality and robustness. Therefore, this method is more suitable than DRO for the UC problem, where the dimension of uncertainty is proportional to the product of the number of periods and the number of uncertain net loads. This method was applied to economic dispatch [14], scheduling of thermostatically controlled loads [15], and UC [16]. However, the static RO models in [14]–[16] cannot account for the re-dispatch stage, resulting in overly conservative day-ahead strategies. Therefore, a two-stage adjustable RO method with statistical guarantees is needed for the robust UC problem.

In addition, the quality of the uncertainty set relies heavily

on the quality of the uncertainty forecasts. The predictions of different forecasting methods can be combined by a weight parameter to achieve a better performance than the individual methods [17]. Following this idea, an ensemble deep learning method was proposed in [18] for probabilistic wind power forecasting, where the weight parameter was determined according to a quantile loss index. The extreme prediction risk was modeled by the conditional value-at-risk in [19] to optimize the ensemble weight of renewable energy forecasts, and then the risk of renewable energy bidding strategy was evaluated. To enhance the performance in a changing environment, a deep deterministic policy gradient-based method was proposed in [20] to adjust the combination weight adaptively. The ensemble forecasting framework was integrated with flexible error compensation in [21], and the weight was optimized to minimize the worst-case forecast error. The above studies chose the weights to optimize the accuracy of the forecasts, without considering the impact of the forecasts on the subsequent decision-making.

Conventionally, forecasting and optimization are performed separately, with forecasting focused on maximizing prediction accuracy and optimization aimed at minimizing costs. However, since the objectives of these two processes are distinct and may even conflict, conducting them separately can lead to suboptimal outcomes. To overcome this shortcoming, a predict-and-optimize framework was proposed in [22], which integrates the forecasting and optimization processes and aims at improving the performance of the final strategy.

The predict-and-optimize framework has been applied in various power system applications. For instance, in [23], the convergence of estimators derived from closed-loop prediction and optimization was examined in the context of linear programming. Solution methods utilizing MILP and heuristic algorithms were introduced and applied to reserve sizing in power systems. In [24], uncertainty prediction and network-constrained UC were integrated into a closed-loop predict-and-optimize framework, with the prediction model trained according to the induced UC cost. The predict-and-optimize framework was also employed in [25] for voltage regulation in distribution systems. In [26], stochastic UC was incorporated into the predict-and-optimize framework, where parameters in wind power forecasting models were adjusted based on risk costs using an iterative solution algorithm. Ref. [27] combined the predict-and-optimize framework with distributionally robust UC, adjusting the inertia prediction model according to UC cost feedback, while addressing price, demand, and renewable uncertainty through a scenario-based DRO technique. An end-to-end approach for multi-energy load data valuation was proposed in [28], where the model was trained through forward and backward propagation of the forecasting model and the optimization problem. Ref. [29] introduced a novel predict-and-optimize framework that considers the relative importance of uncertainty at different nodes, applying it to the optimal power flow problem in distribution networks.

However, there are still research gaps in the aforementioned predict-and-optimize framework: First, it directly adjusts the parameters of the forecasting model based on the induced cost from the optimized strategy, which may lead to biased

predictions with weak explainability and transparency. Second, the presence of numerous adjustable parameters in the forecasting model results in a significant computational burden during parameter optimization within the predict-and-optimize framework. Third, to the best of the authors' knowledge, the predict-and-optimize framework has not been combined with a statistical guarantee, which could theoretically ensure the out-of-sample performance of the optimized strategy.

To bridge the aforementioned research gaps, this paper proposes a novel UC framework that integrates the forecasting and optimization processes while ensuring statistical guarantees. The main contribution is two-fold:

1) An integrated forecasting and optimization framework is proposed to predict in a way that optimizes the performance of the robust strategy. Specifically, predictions from various sources are combined using weight parameters to generate an improved forecast for RO; and in turn, the weights are optimized based on the evaluated performance of the resulting strategy. To accelerate the weight optimization process, a neural network is trained as a surrogate model and equivalently transformed into mixed-integer linear constraints. By solving a mixed-integer linear programming (MILP) problem, the optimal weights can then be obtained. Notably, this integrated framework and solution methodology have not been previously reported in the literature.

2) A data-driven two-stage robust UC model with statistical guarantees is proposed. It employs the prediction and historical data to construct an ellipsoidal uncertainty set, which is then enhanced by reconstructing a polyhedral uncertainty set based on the identified feasible solution and the UC parameters. This method provides dimension-free statistical guarantees to ensure robustness at a specified confidence level, which extends the method in [13] for static RO to two-stage RO.

The rest of this paper is organized as follows: The integrated forecasting and optimization framework is proposed in Section II. A two-stage robust UC method with statistical guarantees is developed in Section III. Case studies are presented in Section IV and conclusions are drawn in Section V.

II. INTEGRATED FORECASTING AND OPTIMIZATION FRAMEWORK

In this section, we first propose an integrated forecasting and optimization framework for a two-stage decision-making problem under uncertainty. In fact, the proposed framework is generally applicable and not restricted to the UC problem. Then in Section III, we introduce the detailed UC model and explain how it fits within the proposed framework.

A. Forecasting

Consider the forecasting task for a multivariate stochastic process $\mathbf{u} := (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_t, \dots)$, where $\mathbf{u}_t = (u_{it}; i \in \mathcal{I})$ is the vector of uncertain variables in period t , and \mathcal{I} is the index set. A forecast is then a collection of future values over the horizon of interest, denoted by $\hat{\mathbf{U}} := (\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \dots, \hat{\mathbf{u}}_T)$. The hat notation indicates these are forecasts, and hence estimates of such future values. Additionally, the corresponding ground truth is denoted by \mathbf{U} .

Multiple forecasting methods can be used simultaneously. The prediction by method $m \in \mathcal{M}$ is denoted by $\hat{\mathbf{U}}^{(m)}$, and \mathcal{M} is the set of methods. The final prediction is a convex combination of these predictions, that is, $\hat{\mathbf{U}}(w) = \sum_{m \in \mathcal{M}} w^{(m)} \hat{\mathbf{U}}^{(m)}$, where $w = (w^{(m)}; m \in \mathcal{M})$ is the weight vector. It satisfies $w \in \mathcal{W} := \{w \mid w \geq 0, \sum_{m \in \mathcal{M}} w^{(m)} = 1\}$. Using the ground truth \mathbf{U} , the forecast error is calculated as $\mathbf{E}(w) = \mathbf{U} - \hat{\mathbf{U}}(w)$, which also depends on the weight w . We will denote the collection of N such forecast error groups as $\mathbf{E}^{(1:N)}(w)$. Forecasting methods are not the focus of this paper, and we apply multiple forecasting methods in case studies based on the related literature.

B. Two-Stage Optimization Under Uncertainty

The following is a two-stage optimization formulation under uncertainty:

$$O := \min_{x \in \mathcal{X}, \eta} f(x) + \eta \quad (1a)$$

$$\text{s.t. } \Pr[\min_{y \in \mathcal{Y}(x, u)} h(y) \leq \eta] \geq 1 - \varepsilon. \quad (1b)$$

In (1), the first-stage variables are collected in x and \mathcal{X} is its feasible region. The first-stage cost function is denoted by $f(x)$. The uncertainty is denoted by u , the second-stage variables are collected in y , $\mathcal{Y}(x, u)$ is the feasible region of y depending on x and u , and $h(y)$ is the second-stage cost function, which is minimized after the realization of u is observed. $\Pr[\cdot]$ denotes the probability. The second-stage cost value is denoted by η , and according to (1b), the probability of the second-stage cost being no larger than η is at least $1 - \varepsilon$, where ε is a specified probability threshold. Therefore, η represents a $(1 - \varepsilon)$ -quantile of the second-stage cost, and problem (1) minimizes the total cost.

Although problem (1) is a chance-constrained SP problem that has been studied and applied extensively in the literature, there is a key difficulty in its solution, i.e., the accurate probability distribution of uncertainty u is usually unknown. In the case considered, we need to extract the distribution information from the historical data. On the one hand, the number of historical data is limited and the error of the empirical distribution is inevitable. On the other hand, we want to ensure the robustness, or more specifically, guarantee the chance constraint (1b). For the sake of robustness, instead of directly using the empirical distribution in problem (1), we construct an uncertainty set \mathcal{U} subject to $\Pr[u \in \mathcal{U}] \geq 1 - \varepsilon$, and consider the following two-stage RO problem:

$$O_{\mathcal{U}} := \min_{x \in \mathcal{X}} \{f(x) + \max_{u \in \mathcal{U}} \min_{y \in \mathcal{Y}(x, u)} h(y)\}. \quad (2)$$

The effectiveness of problem (2) is revealed in Lemma 1.

Lemma 1: Suppose (x^*, η^*) and $x_{\mathcal{U}}^*$ are optimal solutions to problems (1) and (2) with optimal values O and $O_{\mathcal{U}}$, respectively. For any solution $x \in \mathcal{X}$, let

$$O_x := f(x) + \min \left\{ \eta \mid \Pr \left[\min_{y \in \mathcal{Y}(x, u)} h(y) \leq \eta \right] \geq 1 - \varepsilon \right\}.$$

Then $O = O_{x^*} \leq O_{x_{\mathcal{U}}^*} \leq O_{\mathcal{U}}$ whenever \mathcal{U} satisfies $\Pr[u \in \mathcal{U}] \geq 1 - \varepsilon$. Moreover, $O = O_{\mathcal{U}^*}$ and $\Pr[u \in \mathcal{U}^*] \geq 1 - \varepsilon$ for

$$\mathcal{U}^* := \{u \mid \min_{y \in \mathcal{Y}(x^*, u)} h(y) \leq \eta^*\}. \quad (3)$$

Lemma 1 shows that given $\Pr[u \in \mathcal{U}] \geq 1 - \varepsilon$, the RO problem (2) is a conservative approximation of problem (1), and it becomes exact when $\mathcal{U} = \mathcal{U}^*$. Moreover, the performance of the obtained solution $x_{\mathcal{U}}^*$ is represented by $O_{x_{\mathcal{U}}^*}$, which is bounded from above by the optimal value $O_{\mathcal{U}}$ of problem (2). The proof of Lemma 1 is in Appendix A.

The choice of the uncertainty set \mathcal{U} is the key to achieving good performance in problem (2), for which we will leverage the information we have: Given a fixed w , the uncertainty set $\mathcal{U}(w)$ will be constructed using the prediction $\hat{\mathbf{U}}(w)$ and its accuracy estimation deduced from the historical data of forecast error $\mathbf{E}^{(1:N)}(w)$ to approach $\Pr[u \in \mathcal{U}(w)] \geq 1 - \varepsilon$. Subsequently, RO in (2) is solved to find the optimal first-stage strategy $x_{\mathcal{U}}^*(w)$. The details of uncertainty set construction and solution method are in Section III.

C. Performance Evaluation and Integrated Framework

The proposed framework aims to solve the chance-constrained problem (1) to minimize costs while ensuring robustness. Problem (1) minimizes the summation of the first-stage cost and the $(1 - \varepsilon)$ -quantile of the second-stage cost. Thus, quantile estimation is required to evaluate a solution. In daily operational applications like UC, problem (1) is solved daily, but only one actual observed data point is available per day, which is insufficient for quantile estimation. To address this, we approximate the $(1 - \varepsilon)$ -quantile in cost evaluation using the $\lceil (1 - \varepsilon)N' \rceil$ -th smallest cost from the N' uncertainty data constructed based on the prediction and the historical data of forecast errors.

Specifically, the performance of the optimized strategy $x_{\mathcal{U}}^*(w)$ is evaluated using the second training dataset, which is independent of the first training dataset used to construct \mathcal{U} . The evaluated cost will be utilized to adjust w in \mathcal{W} . Denote the historical data of forecast errors in the second training dataset by $\mathbf{E}^{(N+1:N+N')}(w)$. The uncertainty realization data $\tilde{\mathbf{U}}^{(1:N')}$ for cost evaluation is constructed by $\tilde{\mathbf{U}}^{(n)} = \hat{\mathbf{U}}(w) + \mathbf{E}^{(N+n)}(w)$, $n = 1, 2, \dots, N'$. For each $\tilde{\mathbf{U}}^{(n)}$, the total cost is

$$I^{(n)}(\hat{\mathbf{U}}, w) := f(x_{\mathcal{U}}^*(w)) + \min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*(w), \tilde{\mathbf{U}}^{(n)})} h(y).$$

Sort $I^{(1:N')}(\hat{\mathbf{U}}, w)$ in an ascending order. The evaluated cost is then determined as the $\lceil (1 - \varepsilon)N' \rceil$ -th cost (where $\lceil \cdot \rceil$ means rounding up to the nearest integer), denoted by $I(\hat{\mathbf{U}}, w)$.

The proposed integrated forecasting and optimization framework is illustrated and compared with other approaches in Fig. 1. The predictions from various forecasting methods are combined using the weight w . The combined prediction is then utilized in data-driven RO, where statistical guarantees are provided through the construction of uncertainty set. We propose to tune the weight w in \mathcal{W} to minimize the evaluated cost of the strategy. For applications like UC, optimizing w must be efficient. Therefore, we develop a surrogate model in the next subsection that captures the relationship between the inputs (predictions and weight) and the output (evaluated cost). This surrogate model is trained offline. For daily optimization, the surrogate model is utilized to establish an optimization

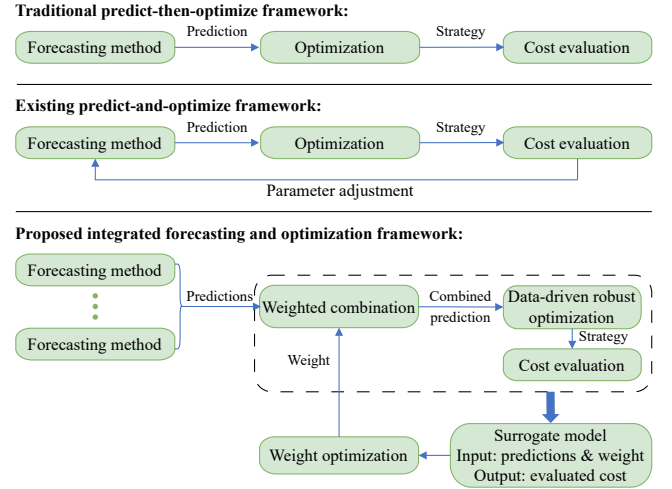


Fig. 1. Forecasting and optimization frameworks.

problem that optimizes the weight and minimizes the evaluated cost. This approach accelerates the weight optimization process. The optimized weight is then applied in the data-driven RO to derive the final strategy.

D. Surrogate Model to Speed up the Weight Optimization

We first establish the proposed surrogate model and then introduce an efficient method for optimizing the weight using this model.

1) *Multilayer Perceptron-Based Surrogate Model*: Fig. 2 illustrates the proposed surrogate model, which employs a multilayer perceptron (MLP) neural network to capture the mapping from predictions and weight to performance (i.e., the evaluated cost). The predictions ($\hat{\mathbf{U}}^{(m)}; m \in \mathcal{M}$) are high-dimensional, so we initially apply principal component analysis (PCA) [30] for dimensionality reduction. The resulting components are then incorporated as inputs to the neural network. Since the weight $w = (w^{(m)}; m \in \mathcal{M})$ satisfies $\sum_{m \in \mathcal{M}} w^{(m)} = 1$, we input the first $|\mathcal{M}| - 1$ weight components into the neural network, where $|\mathcal{M}|$ denotes the number of elements in \mathcal{M} . The inputs from the predictions and the weight are concatenated and scaled before being processed by the neural network. The rectified linear unit (ReLU) activation function is employed, which outputs $v := \max\{0, s\}$ for an input scalar s . The neural network's output represents the evaluated cost. The neural network is trained offline, and once trained, the surrogate model can forecast the evaluated cost based on the provided predictions and weight data.

2) *MILP-Based Weight Optimization*: We model the proposed surrogate model using mixed-integer linear constraints. These linear constraints represent PCA, scaling, and linear combinations. The ReLU activation function $v = \max\{0, s\}$ can be equivalently expressed through the following set of constraints [31]:

$$0 \leq v \leq Mz, \quad (4a)$$

$$s \leq v \leq s + M(1 - z), \quad (4b)$$

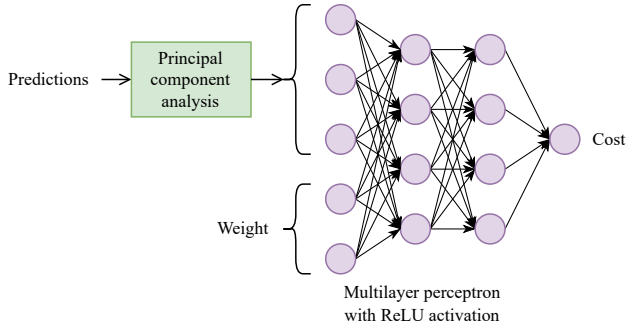


Fig. 2. The proposed surrogate model.

where z is an auxiliary binary variable and $M > 0$ is a sufficiently large constant. When $z = 0$, (4a) forces $v = 0$ and (4b) implies $v \geq s$; when $z = 1$, (4b) shows $v = s$ and by (4a) we have $v \geq 0$. Therefore, the ReLU activation function is equivalently modeled by the linear constraints in (4) with the binary variable z . The surrogate model is therefore expressed through these mixed-integer linear constraints:

$$\hat{\mathbf{U}} = \sum_{m \in \mathcal{M}} w^{(m)} \hat{\mathbf{U}}^{(m)}, \quad (5a)$$

$$v_{0j} = W_{0j} P_j \hat{\mathbf{U}} + B_{0j}, j = 1, 2, \dots, N_P, \quad (5b)$$

$$v_{0(m+N_P)} = W_{0(m+N_P)} w^{(m)} + B_{0(m+N_P)}, \\ m = 1, 2, \dots, |\mathcal{M}| - 1, \quad (5c)$$

$$s_{ij} = W_{ij} v_{i-1} + B_{ij}, i = 1, 2, \dots, N_L, j = 1, 2, \dots, N_{Li}, \quad (5d)$$

$$0 \leq v_{ij} \leq M z_{ij}, i = 1, 2, \dots, N_L, j = 1, 2, \dots, N_{Li}, \quad (5e)$$

$$s_{ij} \leq v_{ij} \leq s_{ij} + M(1 - z_{ij}), \\ i = 1, 2, \dots, N_L, j = 1, 2, \dots, N_{Li}, \quad (5f)$$

$$z_{ij} \in \{0, 1\}, i = 1, 2, \dots, N_L, j = 1, 2, \dots, N_{Li}, \quad (5g)$$

$$I = W_{N_L+1} v_{N_L} + B_{N_L+1}. \quad (5h)$$

The combined prediction is represented in (5a). The PCA process is modeled in (5b) with PCA parameters P_1, P_2, \dots, P_{N_P} , scaling parameters $W_{01}, W_{02}, \dots, W_{0N_P}$, and bias parameters $B_{01}, B_{02}, \dots, B_{0N_P}$. Similarly, the first $|\mathcal{M}| - 1$ components of the weight w are scaled in (5c). The values from (5b) and (5c) are concatenated to form the inputs of the neural network, denoted as $v_0 = (v_{01}, v_{02}, \dots, v_{0(N_P+|\mathcal{M}|-1)})$. Constraint (5d) models the relationship between the output v_{i-1} of the $(i-1)$ -th layer and the input s_i of i -th layer's ReLU function, where W_{ij} and B_{ij} are weight and bias parameters. The neural network consists of N_L layers, with the i -th layer containing N_{Li} units. Constraints (5e) and (5f) model the ReLU activation function, utilizing the binary variable z_{ij} defined in (5g). Constraint (5h) specifies the output I of the surrogate model.

To efficiently optimize the weight, we solve the MILP problem (6) to minimize the output of the neural network:

$$\min_{w, \hat{\mathbf{U}}, s, v, z, I} I \\ \text{s.t. (5), } w \geq 0, \sum_{m \in \mathcal{M}} w^{(m)} = 1. \quad (6)$$

Algorithm 1: Surrogate model training

Input: Uncertainty samples u_1, u_2, \dots, u_{N_U} ; weight samples w_1, w_2, \dots, w_{N_W} ; dimension N_P of the PCA output; number of layers N_L in the MLP; number of units N_{Li} in layer i , where $i = 1, 2, \dots, N_L$.

Output: PCA parameters P_1, P_2, \dots, P_{N_P} ; MLP weight parameter W_{ij} and bias parameter B_{ij} for $i = 0, 1, \dots, N_L$ and $j = 1, 2, \dots, N_{Li}$.

- 1 Compute PCA parameters P_1, P_2, \dots, P_{N_P} using uncertainty samples u_1, u_2, \dots, u_{N_U} .
 - 2 **for** $n_U = 1$ **to** N_U **do**
 - 3 Compute PCA outputs $d_{n_U 1} = P_1 u_{n_U}$,
 $d_{n_U 2} = P_2 u_{n_U}, \dots, d_{n_U N_P} = P_{N_P} u_{n_U}$.
 - 4 **for** $n_W = 1$ **to** N_W **do**
 - 5 Compute $\hat{\mathbf{U}}(w_{n_W}) = \sum_{m \in \mathcal{M}} w_{n_W}^{(m)} \hat{\mathbf{U}}^{(m)}$ and
 $\mathbf{E}^{(1:N)}(w_{n_W}) = \mathbf{U}^{(1:N)} - \hat{\mathbf{U}}^{(1:N)}(w_{n_W})$.
 - 6 Based on $\hat{\mathbf{U}}(w_{n_W})$ and $\mathbf{E}^{(1:N)}(w_{n_W})$, apply
Algorithm 2 to obtain the uncertainty set
 $\mathcal{U}(w_{n_W})$ and the optimal solution $x_{\mathcal{U}}^*(w_{n_W})$.
 - 7 Using $x_{\mathcal{U}}^*(w_{n_W})$, compute the evaluated cost
 $I(u_{n_U}, w_{n_W})$ as described in Section II-C.
 - 8 **end**
 - 9 **end**
 - 10 Use the input data $(d_{n_U 1}, d_{n_U 2}, \dots, d_{n_U N_P}, w_{n_W})$ and the output data $I(u_{n_U}, w_{n_W})$ for $n_U = 1, 2, \dots, N_U$ and $n_W = 1, 2, \dots, N_W$ to train the MLP with ReLU activation, and obtain the parameters W_{ij} and B_{ij} for $i = 1, 2, \dots, N_L$ and $j = 1, 2, \dots, N_{Li}$.
-

The optimal weight value is then utilized in data-driven RO to derive the final strategy.

We use the proposed surrogate model based on MLP with ReLU activation to describe how the predictions and weight influence the performance of the optimized strategy. MLP with a non-polynomial activation function can approximate any continuous function to any desired degree of accuracy [32]. Additionally, ref. [33] examined the width and depth needed for MLP with ReLU activation to serve as universal approximators. These theoretical results demonstrate the underlying rationale for the proposed MLP-based surrogate model.

E. Overview of the Proposed Framework

The steps of the proposed integrated forecasting and optimization framework are summarized as follows:

Step 1 (offline training): Train the surrogate model using historical data on uncertainty. Details are provided in Algorithm 1.

Step 2 (daily optimization for UC): Solve the MILP problem (6) to obtain the optimal weight w and the combined prediction $\hat{\mathbf{U}}$. Solve the data-driven RO problem (2) using Algorithm 2 and obtain the final strategy.

Compared to existing predict-and-optimize frameworks in power system applications, the proposed method has several advantages: First, instead of adjusting the parameters of forecasting models, the proposed framework optimizes the weight

$w \in \mathcal{W}$ assigned to different forecasting methods. Thus, the prediction used for optimization must be a convex combination of the original predictions, which prevents severe deviations and enhances the reliability of the induced strategy. Second, we introduce a MLP-based surrogate model to represent the relationship between the weight and evaluated cost. This allows for rapid weight optimization via MILP, which significantly reduces computational burdens compared to existing frameworks that rely on forward and backward propagation or other iterative algorithms. Third, the proposed method incorporates data-driven RO in the optimization process, which provides statistical guarantees within the integrated forecasting and optimization framework. This approach ensures the out-of-sample performance of the optimized strategy from a theoretical perspective.

III. DATA-DRIVEN TWO-STAGE ROBUST UNIT COMMITMENT WITH STATISTICAL GUARANTEES

In this section, we first establish the robust UC problem and then construct the uncertainty set, which is later reconstructed using the problem information. The solution algorithm is introduced in the end.

A. Robust Unit Commitment Formulation

As introduced in Section II-B, the robust UC problem has the compact form in (2). Now we specify the components of this problem. The pre-dispatch variable $x = (\theta_{gt}, \theta_{gt}^{\pm}, p_{gt}, r_{gt}^{\pm}; g \in \mathcal{G}, t \in \mathcal{T})$, where \mathcal{G} and \mathcal{T} are the index sets of generators and periods, respectively. For generator g in period t , the on, startup, and shutdown states are denoted by binary variables θ_{gt} , θ_{gt}^+ , and θ_{gt}^- , respectively; the day-ahead scheduled power output is denoted by p_{gt} ; the upward and downward reserve power values are denoted by r_{gt}^+ and r_{gt}^- , respectively. The pre-dispatch cost is given by

$$f(x) = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (o_g^+ \theta_{gt}^+ + o_g^- \theta_{gt}^- + \rho_g p_{gt} + \gamma_g^+ r_{gt}^+ + \gamma_g^- r_{gt}^-), \quad (7)$$

where o_g^{\pm} , ρ_g , and γ_g^{\pm} are cost coefficients. The pre-dispatch feasible region is defined as follows:

$$\mathcal{X} = \{x = (\theta_{gt}, \theta_{gt}^{\pm}, p_{gt}, r_{gt}^{\pm}; g \in \mathcal{G}, t \in \mathcal{T}) \mid \sum_{g \in \mathcal{G}} p_{gt} = \sum_{i \in \mathcal{I}} \hat{u}_{it}, \forall t \in \mathcal{T}, \quad (8a)$$

$$-S_l \leq \sum_{g \in \mathcal{G}} \pi_{gl} p_{gt} - \sum_{i \in \mathcal{I}} \pi_{il} \hat{u}_{it} \leq S_l, \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \quad (8b)$$

$$\theta_{gt}, \theta_{gt}^+, \theta_{gt}^- \in \{0, 1\}, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \quad (8c)$$

$$\sum_{\tau=t}^{t+T_g^+-1} \theta_{g\tau} \geq T_g^+ \theta_{gt}^+, 1 \leq t \leq T - T_g^+ + 1, \forall g \in \mathcal{G}, \quad (8d)$$

$$\sum_{\tau=t}^T (\theta_{g\tau} - \theta_{gt}^+) \geq 0, T - T_g^+ + 2 \leq t \leq T, \forall g \in \mathcal{G}, \quad (8e)$$

$$\sum_{\tau=t}^{t+T_g^--1} (1 - \theta_{g\tau}) \geq T_g^- \theta_{gt}^-, 1 \leq t \leq T - T_g^- + 1, \forall g \in \mathcal{G}, \quad (8f)$$

$$\sum_{\tau=t}^T (1 - \theta_{g\tau} - \theta_{gt}^-) \geq 0, T - T_g^- + 2 \leq t \leq T, \forall g \in \mathcal{G}, \quad (8g)$$

$$\theta_{gt} - \theta_{g(t-1)} = \theta_{gt}^+ - \theta_{gt}^-, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \quad (8h)$$

$$\theta_{gt}^+ + \theta_{gt}^- \leq 1, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \quad (8i)$$

$$0 \leq r_{gt}^+ \leq R_g^+ \theta_{gt}, 0 \leq r_{gt}^- \leq R_g^- \theta_{gt}, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \quad (8j)$$

$$\underline{P}_g \theta_{gt} + r_{gt}^- \leq p_{gt} \leq \bar{P}_g \theta_{gt} - r_{gt}^+, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \quad (8k)$$

$$(p_{gt} + r_{gt}^+) - (p_{g(t-1)} - r_{g(t-1)}^-) \leq K_g^+ \theta_{g(t-1)} + K_g^U \theta_{gt}^+, \forall g \in \mathcal{G}, 2 \leq t \leq T, \quad (8l)$$

$$-(p_{gt} - r_{gt}^-) + (p_{g(t-1)} + r_{g(t-1)}^+) \leq K_g^- \theta_{gt} + K_g^D \theta_{gt}^-, \forall g \in \mathcal{G}, 2 \leq t \leq T \}. \quad (8m)$$

The power balance of the transmission network is stipulated in (8a), where \hat{u}_{it} is the day-ahead prediction of the load power. In (8b), \mathcal{L} is the index set of transmission lines; S_l is the power capacity of line l ; parameters π_{gl} and π_{il} are the power transfer distribution factors. Thus, (8b) bounds the line flow in the DC power flow model. Binary variables are set in (8c). Constraints (8d)-(8g) are for the minimum up time T_g^+ and minimum down time T_g^- of generators [34]. The generator state change is modeled in (8h) and the simultaneous startup and shutdown of a generator is prohibited in (8i). Constraint (8j) contains bounds R_g^{\pm} for the upward and downward reserve power. The minimum and maximum outputs of generators, i.e., \underline{P}_g and \bar{P}_g for generator g , are stipulated in (8k). For generator g , K_g^+ and K_g^- are the maximum upward and downward ramp values in a period; K_g^U and K_g^D are the maximum increase and decrease of power if the generator startups or shutdowns in a period. Thus, constraints (8l) and (8m) are the bounds for the change in generator power over a period.

The construction of the uncertainty set \mathcal{U} is deferred and will be addressed in Sections III-B and III-C. In the re-dispatch stage, the variable is $y = (p_{gt}^{\pm}; g \in \mathcal{G}, t \in \mathcal{T})$, where p_{gt}^+ and p_{gt}^- are the upward and downward power adjustments of generator g in period t , respectively. The re-dispatch cost function is the total power adjustment cost, i.e.,

$$h(y) = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (\rho_g^+ p_{gt}^+ + \rho_g^- p_{gt}^-), \quad (9)$$

where ρ_g^+ and ρ_g^- are cost coefficients. The re-dispatch feasible region $\mathcal{Y}(x, u)$ depends on the pre-dispatch decision x and the realization $u = (u_{it}; i \in \mathcal{I}, t \in \mathcal{T})$ of uncertain load:

$$\mathcal{Y}(x, u) = \{y = (p_{gt}^{\pm}; g \in \mathcal{G}, t \in \mathcal{T}) \mid \sum_{g \in \mathcal{G}} (p_{gt} + p_{gt}^+ - p_{gt}^-) = \sum_{i \in \mathcal{I}} u_{it}, \forall t \in \mathcal{T}, \quad (10a)$$

$$-S_l \leq \sum_{g \in \mathcal{G}} \pi_{gl} (p_{gt} + p_{gt}^+ - p_{gt}^-) - \sum_{i \in \mathcal{I}} \pi_{il} u_{it} \leq S_l, \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \quad (10b)$$

$$0 \leq p_{gt}^+ \leq r_{gt}^+, 0 \leq p_{gt}^- \leq r_{gt}^-, \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \}. \quad (10c)$$

In (10), constraint (10a) is for the power balance of the network. The transmission line flow constraints are in (10b). The power adjustments of the generators are bounded by the reserve power in (10c).

It is crucial to consider ramp rate constraints to ensure the feasibility of the UC strategy [35]. Lemma 2 shows that

the re-dispatch feasible region $\mathcal{Y}(x, u)$ adheres the ramp rate constraints, provided that $x \in \mathcal{X}$. The proof of Lemma 2 is in Appendix B.

Lemma 2: Assume $x = (\theta_{gt}, \theta_{gt}^\pm, p_{gt}, r_{gt}^\pm; g \in \mathcal{G}, t \in \mathcal{T}) \in \mathcal{X}$ and $y = (p_{gt}^\pm; g \in \mathcal{G}, t \in \mathcal{T}) \in \mathcal{Y}(x, u)$. Then the following ramp rate constraints hold for the re-dispatch solution:

$$(p_{gt} + p_{gt}^+ - p_{gt}^-) - (p_{g(t-1)} + p_{g(t-1)}^+ - p_{g(t-1)}^-) \leq K_g^+ \theta_{g(t-1)} + K_g^U \theta_{gt}^+, \forall g \in \mathcal{G}, 2 \leq t \leq T, \quad (11a)$$

$$-(p_{gt} + p_{gt}^+ - p_{gt}^-) + (p_{g(t-1)} + p_{g(t-1)}^+ - p_{g(t-1)}^-) \leq K_g^- \theta_{gt} + K_g^D \theta_{gt}^-, \forall g \in \mathcal{G}, 2 \leq t \leq T. \quad (11b)$$

According to the formulations of $f(x)$ and $h(y)$ in (7) and (9), the two functions are linear. Equation (10) shows that the constraints that define $\mathcal{Y}(x, u)$ are linear in x and u . Therefore, the compact form of the robust UC problem can be further written as follows:

$$\min_{x \in \mathcal{X}} \{C^\top x + \max_{u \in \mathcal{U}} \min_{y: Ay \geq Bx + Du + E} F^\top y\}, \quad (12)$$

where A , B , C , D , E , and F are coefficient matrices and vectors.

B. Data-Driven Uncertainty Set and Statistical Guarantees

The possible realization values of the uncertain load u constitute a set $\mathcal{U}_0 = \{u \mid \underline{U} \leq u \leq \overline{U}\}$, where \underline{U} and \overline{U} are the lower and upper bounds for u . \mathcal{U}_0 contains all the possible values of u , so $\Pr[u \in \mathcal{U}_0] = 1$. \mathcal{U}_0 can be constructed using the capacity parameters of the facilities. For example, if bus i connects a wind farm with a power capacity of P_i^W and no loads, then the net load power at bus i satisfies $-P_i^W \leq u_{it} \leq 0$ for any $t \in \mathcal{T}$. The box defined by these bounds for all $i \in \mathcal{I}$ and $t \in \mathcal{T}$ establishes a choice of \mathcal{U}_0 such that $\Pr[u \in \mathcal{U}_0] = 1$. If the accurate \mathcal{U}_0 is hard to obtain, we may just let \mathcal{U}_0 to be the entire Euclidean space, and the proposed method will still function effectively.

According to the analysis in Section II-B, we need to construct an uncertainty set $\mathcal{U}_1 \subset \mathcal{U}_0$ such that $\Pr[u \in \mathcal{U}_1] \geq 1 - \varepsilon$, where the information we have include prediction \hat{u} and historical forecast error $e_{1:N} := (e_1, e_2, \dots, e_N)$ in the training dataset (e_n is the column vector reshaping $\mathbf{E}^{(n)}(w)$). We assume that the daily load forecast errors are i.i.d. continuous random variables. Our plan is first to construct a set \mathcal{E} for the uncertain forecast error $e := u - \hat{u}$ so that $\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon$, followed by letting

$$\mathcal{U}_1 := \{u \in \mathcal{U}_0 \mid u - \hat{u} \in \mathcal{E}\}. \quad (13)$$

Clearly, such \mathcal{U}_1 satisfies $\mathcal{U}_1 \subset \mathcal{U}_0$ and $\Pr[u \in \mathcal{U}_1] \geq 1 - \varepsilon$, so the uncertainty set construction problem comes down to finding a set \mathcal{E} such that $\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon$, based on the historical data $e_{1:N}$.

Since $e_{1:N}$ are random variables, the set \mathcal{E} constructed using $e_{1:N}$ is also random, and so is the event $\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon$. Therefore, instead of directly attempting $\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon$, we consider the following statistical guarantee:

$$\mathbb{P}^N[\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon] \geq 1 - \delta, \quad (14)$$

where \mathbb{P} is the underlying distribution of $e_n, n = 1, 2, \dots, N$; \mathbb{P}^N denotes the N times distribution product of \mathbb{P} , which models the uncertainty of historical data and \mathcal{E} ; δ is a probability tolerance parameter. Thus, equation (14) means that the probability of the random event $\Pr[e \in \mathcal{E}] \geq 1 - \varepsilon$ is at least $1 - \delta$. The parameters ε and δ jointly control the conservative degree of the robust UC problem.

To achieve the statistical guarantee (14), we adopt the data-driven uncertainty set proposed in [13] to establish \mathcal{E} , whose procedure and statistical guarantee are summarized in Theorem 1. The idea is to pull two disjoint groups from the dataset. The first group determines the shape of \mathcal{E} , where an ellipsoid is established based on the sample mean and covariance to consider the correlation. The second group is for the size of \mathcal{E} , where the differences of the points from the ellipsoid center are measured and sorted to provide a threshold, and the independence of the two data groups forms the foundation of the statistical guarantees.

Theorem 1 (Uncertainty set construction and statistical guarantee): Select two disjoint groups from $e_{1:N}$. Denote their realizations by $e_{1:N_1}^{(1)}$ and $e_{1:N_2}^{(2)}$, where $N_1 + N_2 \leq N$ and $N_2 \geq \log_{1-\varepsilon} \delta$. Let

$$\mu := \frac{1}{N_1} \sum_{n=1}^{N_1} e_n^{(1)}, \quad (15a)$$

$$\Sigma := \frac{1}{N_1 - 1} \sum_{n=1}^{N_1} (e_n^{(1)} - \mu) (e_n^{(1)} - \mu)^\top. \quad (15b)$$

In other words, μ and Σ are the sample mean and the sample covariance matrix in the first dataset $e_{1:N_1}^{(1)}$. Assume Σ is invertible. Define function $a(e) := (e - \mu)^\top \Sigma^{-1} (e - \mu)$. Let

$$n_N^* := \min \left\{ n \in \mathbb{N} \mid \sum_{m=0}^{n-1} C_N^m (1 - \varepsilon)^m \varepsilon^{N-m} \geq 1 - \delta \right\}, \quad (16)$$

where C_N^m denotes the binomial coefficient of N choose m . Let α be the n_N^* -th smallest value in $a(e_1^{(2)}), a(e_2^{(2)}), \dots, a(e_{N_2}^{(2)})$. Define the set \mathcal{E} as

$$\mathcal{E} := \{e \mid (e - \mu)^\top \Sigma^{-1} (e - \mu) \leq \alpha\}. \quad (17)$$

Then the statistical guarantee (14) holds. Moreover, if $\mathcal{U} = \mathcal{U}_1$ defined in (13) is used in the robust UC problem (2) and $x_0 := x_{\mathcal{U}_1}^*$ is an optimal solution, then

$$\mathbb{P}^N[O \leq O_{x_0} \leq O_{\mathcal{U}_1}] \geq 1 - \delta. \quad (18)$$

Theorem 1 gives a method to generate the uncertainty set \mathcal{U}_1 and approximate problem (1) with confidence $1 - \delta$. The proof of Theorem 1 can be found in Appendix C.

Net loads often exhibit strong spatial and temporal correlations, making uncertainty correlation modeling essential for decision-making under multi-dimensional uncertainty [36]. The ellipsoidal uncertainty set \mathcal{U}_1 of $u = (u_{it}; i \in \mathcal{I}, t \in \mathcal{T})$ is able to consider the correlation between u_{it} and u_{js} for any $i, j \in \mathcal{I}$ and $t, s \in \mathcal{T}$ through the sample covariance matrix Σ of the forecast errors [37].

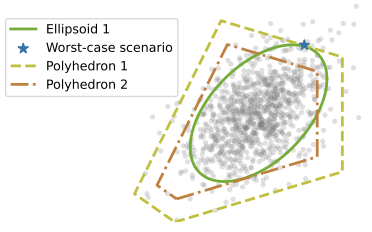


Fig. 3. The basic idea of uncertainty set reconstruction.

C. Uncertainty Set Reconstruction

The uncertainty set \mathcal{U}_1 obtained in Theorem 1 guarantees robustness with confidence $1 - \delta$, but it can be conservative. Our goal in this subsection is to mitigate the conservativeness by reconstructing the uncertainty set. The construction of \mathcal{U}_1 uses only the prediction and historical forecast error but does not involve any information on the UC problem. In the following, we integrate the problem information into the reconstruction.

We illustrate the basic idea in Fig. 3. Ellipsoid 1 represents \mathcal{U}_1 . The worst-case scenario in ellipsoid 1 is marked as a blue star, which reaches the highest cost, with nearby scenarios also incurring high costs. Considering the range where costs do not exceed that of the blue star, we obtain polyhedron 1 in Fig. 3. This range forms a polyhedron due to the linear structure in the UC problem. Polyhedron 1 includes ellipsoid 1, and the blue star remains the worst-case scenario within polyhedron 1. Furthermore, polyhedron 1 may contain more data points than ellipsoid 1, which can lead to conservativeness. This suggests that polyhedron 1 can be shrunk into polyhedron 2 according to the desired probability guarantee thresholds. In contrast to ellipsoid 1, polyhedron 2 excludes the high-cost scenarios near the blue star, while including additional low-cost scenarios to maintain the statistical guarantees. Consequently, the worst-case scenario in polyhedron 2 will not exceed the blue star, resulting in a less conservative strategy.

Now we explain the reconstruction formally. Recall from Lemma 1 that when (x^*, η^*) is optimal for the chance-constrained problem (1), the uncertainty set \mathcal{U}^* defined in (3) can equivalently transform problem (1) into (2). Ideally, \mathcal{U}^* is used as the uncertainty set to eliminate conservativeness. However, it is impractical because (x^*, η^*) is unknown. To this end, \mathcal{U}^* is approximated using the data we have. Suppose x_0 is a feasible solution, then we can estimate its performance O_{x_0} in a historical dataset and construct an uncertainty set using $(x, \eta) = (x_0, O_{x_0} - f(x_0))$. This idea is refined to maintain the statistical guarantees in Theorem 2.

Theorem 2 (Uncertainty set reconstruction): Suppose $e_{1:N_3}^{(3)}$ is a subgroup of $e_{1:N}$ and $N_3 \geq \log_{1-\epsilon} \delta$. Assume that the solution x_0 is independent of $e_{1:N_3}^{(3)}$. Define function $b(u) := \min_{y \in \mathcal{Y}(x_0, u)} h(y)$. Define $n_{N_3}^*$ according to (16). Let β be the $n_{N_3}^*$ -th smallest value in $b(\hat{u} + e_1^{(3)}), b(\hat{u} + e_2^{(3)}), \dots, b(\hat{u} + e_{N_3}^{(3)})$. Define the uncertainty set

$$\mathcal{U}_2 := \{u \in \mathcal{U}_0 \mid \min_{y \in \mathcal{Y}(x_0, u)} h(y) \leq \beta\}. \quad (19)$$

Let $x_1 := x_{\mathcal{U}_2}^*$ be optimal in problem (2) with $\mathcal{U} = \mathcal{U}_2$. Then the following statistical guarantees hold:

$$\mathbb{P}^N[\Pr[u \in \mathcal{U}_2] \geq 1 - \epsilon] \geq 1 - \delta, \quad (20a)$$

$$\mathbb{P}^N[O \leq O_{x_1} \leq O_{\mathcal{U}_2} \leq f(x_0) + \beta] \geq 1 - \delta. \quad (20b)$$

In Theorem 2, the shape of the new uncertainty set is formed based on both the old solution and the UC problem, whereas the size β is determined by the evaluations on an independent dataset so that the statistical guarantees remain valid. The conclusion indicates that the reconstruction of the uncertainty set will lead to a new solution probably better than the old one, represented by $O_{x_1} \leq f(x_0) + \beta$. The proof of Theorem 2 can be found in Appendix D.

The polyhedral uncertainty set \mathcal{U}_2 also takes into account the spatial and temporal correlations of net loads. However, unlike the ellipsoidal uncertainty set \mathcal{U}_1 , which uses the sample covariance matrix of historical data, \mathcal{U}_2 describes correlations in a problem-driven way, through linear constraints derived from the UC problem. As will be elaborated in Fig. 5 in Section IV, \mathcal{U}_2 is not box-shaped, demonstrating that it incorporates correlations across different dimensions.

D. Solution Algorithm

\mathcal{U}_0 is polyhedral. According to (13) and (17),

$$\mathcal{U}_1 = \{u \in \mathcal{U}_0 \mid (u - \hat{u} - \mu)^\top \Sigma^{-1}(u - \hat{u} - \mu) \leq \alpha\},$$

is the intersection of an ellipsoid and a polyhedron. Using the compact form of the robust UC problem in (12), \mathcal{U}_2 in (19) can be further written as follows:

$$\begin{aligned} \mathcal{U}_2 &= \{u \in \mathcal{U}_0 \mid \exists y \in \mathcal{Y}(x_0, u), \text{ s.t. } h(y) \leq \beta\} \\ &= \{u \in \mathcal{U}_0 \mid \exists y, \text{ s.t. } Ay \geq Bx_0 + Du + E, F^\top y \leq \beta\}, \end{aligned}$$

which shows that \mathcal{U}_2 is polyhedral. Problem (12) with these uncertainty sets can be effectively solved by the column-and-constraint generation (C&CG) algorithm [38], which is omitted here for the sake of conciseness.

The solution procedure for the robust UC problem considering uncertainty set reconstruction is summarized in Algorithm 2. The historical dataset is divided into two groups. The first group forms the first ellipsoidal uncertainty set and leads to a solution x_0 . Then the reconstruction procedure in Theorem 2 is performed to obtain an improved uncertainty set, resulting in the final solution x_1 . Since the two datasets are independent, the statistical guarantees in (20) are maintained.

Remark: The proposed method does not require explicit consideration of the robust feasibility requirement and can effectively address the infeasible re-dispatch cases. For a fixed first-stage variable $x' \in \mathcal{X}$, if the re-dispatch problem is infeasible for a given uncertainty realization $u' \in \mathcal{U}$, then $\mathcal{Y}(x', u') = \emptyset$ and the second-stage cost

$$\max_{u \in \mathcal{U}} \min_{y \in \mathcal{Y}(x', u)} h(y) \geq \min_{y \in \mathcal{Y}(x', u')} h(y) = +\infty.$$

Since the two-stage RO problem (2) minimizes the sum of the first- and second-stage costs, the optimal solution x^* must be robust feasible, i.e., $\mathcal{Y}(x^*, u) \neq \emptyset$ for all $u \in \mathcal{U}$, provided that the optimal value is finite. Therefore, the robust feasibility requirement naturally holds for the optimal solution, effectively excluding scenarios with infeasible re-dispatch.

Algorithm 2: Solution of robust unit commitment**Input:** Parameters of (12); ε ; δ ; \mathcal{U}_0 ; \hat{u} ; $e_{1:N}$; N_2 .**Output:** UC strategy x_1 .

- 1 Let $N_1 \leftarrow N - N_2$.
- 2 Divide $e_{1:N}$ into $e_{1:N_1}^{(1)}$ and $e_{1:N_2}^{(2)}$.
- 3 Calculate μ and Σ according to (15).
- 4 Let $\alpha \leftarrow \max\{(e_n^{(1)} - \mu)^\top \Sigma^{-1} (e_n^{(1)} - \mu) \mid n = 1, 2, \dots, N_1\}$.
- 5 Let $\mathcal{U}'_1 \leftarrow \{u \in \mathcal{U}_0 \mid (u - \hat{u} - \mu)^\top \Sigma^{-1} (u - \hat{u} - \mu) \leq \alpha\}$.
- 6 Solve problem (12) with $\mathcal{U} = \mathcal{U}'_1$ using the C&CG algorithm and obtain the optimal solution x_0 .
- 7 Let $b_n \leftarrow \min_{y: Ay \geq Bx_0 + D(\hat{u} + e_n^{(2)}) + E} F^\top y$, for $n = 1, 2, \dots, N_2$.
- 8 Arrange $b_n, n = 1, 2, \dots, N_2$ from small to large and get $b'_n, n = 1, 2, \dots, N_2$.
- 9 Let $n^* \leftarrow \min\{n \mid \sum_{m=0}^{n-1} C_{N_2}^m (1 - \varepsilon)^m \varepsilon^{N_2-m} \geq 1 - \delta\}$.
- 10 Let $\beta \leftarrow b'_{n^*}$.
- 11 Let $\mathcal{U}_2 \leftarrow \{u \in \mathcal{U}_0 \mid \exists y, \text{ s.t. } Ay \geq Bx_0 + Du + E, F^\top y \leq \beta\}$.
- 12 Solve problem (12) with $\mathcal{U} = \mathcal{U}_2$ using the C&CG algorithm and return the optimal solution x_1 .

IV. CASE STUDIES

This section examines the proposed robust UC method using modified IEEE 30-bus and 118-bus systems. All experiments are carried out on a laptop with an Intel i7-12700H processor and 16 GB RAM. The neural network is established and trained by PyTorch 2.1.2. The MILP problems in the C&CG algorithm are solved by Gurobi 11.0.2. In the following, we first introduce the prediction data. The performance of the proposed method is then investigated in the modified IEEE 30-bus system, where different methods are compared and sensitivity analysis is performed. Finally, the modified IEEE 118-bus system is used to demonstrate the scalability of the proposed method.

A. Prediction Data

The hourly load data in one and a half years are extracted from the dataset in [39], based on real data in Ireland [40]. The hourly wind power data are generated according to historical weather data [41]. The load and wind data are aligned according to the date information. The dataset is divided into samples for training, validation, and testing.

We predict uncertainty using the following forecasting methods:

- M1: Forecast nodal power using local data and bidirectional long short-term memory (BiLSTM) neural network [42].
- M2: Apply federated learning between nodes and forecast using the adapted global BiLSTM model [43].
- M3: Forecast nodal power using BiLSTM networks and nodal power subprofiles [17].
- C1: Combine the predictions of M1, M2, and M3 by a weight w to minimize the mean square error (MSE).

TABLE I
AVERAGE FORECAST ERRORS OF DIFFERENT METHODS

Method	RMSE	MAE
M1	84.39	54.64
M2	80.93	52.37
M3	80.44	55.23
C1	76.14	51.26
C2 (30-bus)	76.95	52.29
C2 (118-bus)	78.72	53.80

- C2: Combine the predictions of M1, M2, and M3 by a weight w according to the method proposed in Section II.

The loss function used in the training is MSE. We adopt root mean square error (RMSE) and mean absolute error (MAE) to measure the test forecast errors.

The average forecast errors under different methods are shown in TABLE I. By combining three forecasting methods and optimizing the weight to minimize the MSE, C1 has the lowest forecast errors, where its RMSE is 11.1%, 7.3%, and 6.8% lower than M1, M2, and M3, respectively. This verifies that the combination of predictions is effective in improving forecast accuracy by leveraging the potential of different data sources and forecasting methods. The forecast errors of C2 are larger than those of C1 because instead of minimizing the MSE, the weight in C2 is chosen to minimize the UC cost.

B. Modified IEEE 30-Bus System

Based on the prediction data mentioned above, the integrated framework in Fig. 1 and the proposed data-driven RO method in Algorithm 2 are used for the robust UC problem. In the modified IEEE 30-bus system, there are six controllable generators and four wind farms, where $\underline{P}_g = (100, 40, 0, 0, 0, 0)$ MW, $\overline{P}_g = (360, 140, 100, 100, 100, 100)$ MW, and $T_g^+ = T_g^- = 6$ h. Other parameters are in [44].

1) *Benchmark:* In the benchmark case, the probability tolerance parameters are set as $\varepsilon = \delta = 5\%$. We divide the training dataset into two subsets with N_1 and N_2 data, respectively. According to Theorem 1 and Theorem 2, N_2 should be at least 59. We set $N_1 = 212$ and $N_2 = 124$.

Based on the performance on the validation dataset, we use the first three components of PCA in the surrogate model for weight optimization. The MLP neural network has two hidden layers, and each layer has 16 units. The neural network is trained using the Adam algorithm and the learning rate is 0.001. To help prevent overfitting, we adopt the L2 regularization technique. After about 1000 epochs, the neural network is trained.

The test day for UC is from the test dataset. The MILP problem for weight optimization is solved in 0.02 s and the result is $w = (0.28, 0.23, 0.49)$. The weight for M3 is the highest, reflecting that the sub-profiles are valuable in power forecasting and robust UC, which is consistent with the results in TABLE I and [17]. With the optimized weight, the UC results are obtained after 121 s, and the optimal value is \$89725. The power outputs of some controllable generators are depicted in Fig. 4, including the pre-dispatch power, the

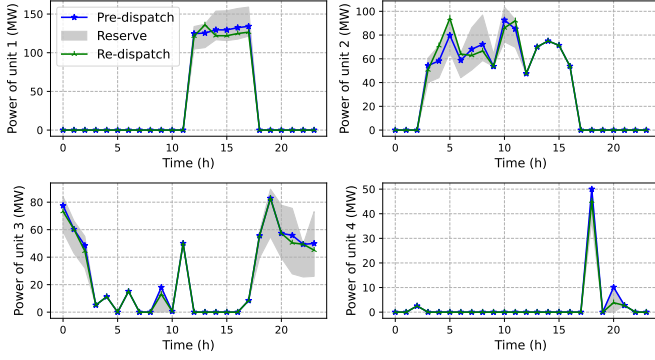


Fig. 4. Power outputs of controllable generators.

reserve region, and the re-dispatch power in the worst-case scenario. The re-dispatch power is always within the reserve region.

2) *Comparison:* We use out-of-sample tests to examine the performance of the proposed method under uncertainty. Meanwhile, we compare the proposed method with the following alternative methods:

- SP: The traditional chance-constrained SP method using the estimated distribution based on historical data.
- RO1: The traditional data-driven RO method with an ellipsoidal uncertainty set that includes all data.
- RO2: The same method as RO1 except that the ellipsoidal uncertainty set includes $1 - \varepsilon$ proportion of the data.
- P1: The data-driven RO method using the ellipsoidal uncertainty set with statistical guarantees.
- P2: The data-driven RO method using Algorithm 2, where the weight w is optimized to minimize the MSE.
- PSO: Similar to the proposed method, but the weight is optimized using the particle swarm optimization (PSO) algorithm instead of the surrogate model. The number of particles is 10, the inertia weight is 0.5, the cognitive constant is 1.0, the social constant is 1.5, and the computation time limit is 10000 s.
- EXACT: An adapted version of the MILP-based exact method for the predict-and-optimize approach in [23], whose details will be introduced later.

For clarity, the settings of these methods are compared in TABLE II, with results listed in TABLE III. The feasible rate is calculated using the 100 forecast error samples on the test dataset. The total cost is calculated using the actual observed data of uncertainty on the test days.

As TABLE III shows, SP has the lowest objective value and test total cost. However, its test feasible rate is 88%, much lower than the desired threshold 95%, which shows that SP lacks robustness. The other five methods are RO-based and their test feasible rates all meet the requirement. However, RO1 is rather conservative and has the highest objective value and test total cost. RO2 is a traditional data-driven RO method with no statistical guarantees, which shows its theoretical limitation. P1, P2, and the proposed method have statistical guarantees, and the proposed method achieves the lowest optimal objective and test total cost among them. Using uncertainty set reconstruction, the objective decreases

TABLE II
SETTINGS OF UNIT COMMITMENT METHODS FOR COMPARISON

Method	Statistical guarantee	Integrated forecasting and optimization	Uncertainty set reconstruction
SP	×	×	×
RO1	×	×	×
RO2	×	×	×
P1	✓	✓	×
P2	✓	×	✓
PSO	✓	✓	✓
EXACT	×	✓	×
Proposed	✓	✓	✓

TABLE III
UNIT COMMITMENT RESULTS OF DIFFERENT METHODS IN THE MODIFIED IEEE 30-BUS SYSTEM

Method	Objective (\$)	Feasible rate	Test total cost (\$)	Time (s)
SP	84832	88%	82985	218
RO1	106810	100%	92652	143
RO2	97350	97%	90468	94
P1	97848	98%	89149	124
P2	90122	98%	88318	147
PSO	89111	98%	88154	10000
Proposed	89725	98%	88243	121

8.30%. The integrated forecasting and optimization framework also contributes to improving the objective.

To validate the effectiveness of the surrogate model, we compare the proposed method to a method that directly optimizes the weight. Since the impact of the weight on the evaluated cost of the UC strategy cannot be expressed as a compact function, we choose the PSO algorithm, which is derivative-free and highly flexible [45]. As shown in TABLE III, the PSO algorithm uses a much longer time (10000 s) compared to the proposed method (121 s), achieving a reduction in the objective value by 0.7% and a decrease in the test total cost by 0.1%. Thus, the proposed method substantially reduces the computation time to an acceptable duration for UC, with only a mild sacrifice in performance. This underscores the effectiveness of the proposed surrogate model in balancing computational efficiency and performance.

The EXACT method [23] solves problem (21):

$$\min_{w, \hat{\mathbf{U}}(w), x^*(w)} \frac{1}{N} \sum_{n=1}^N \left(f(x_n^*(w)) + \min_{y \in \mathcal{Y}(x_n^*(w), \hat{\mathbf{U}}_n)} h(y) \right) \quad (21a)$$

$$\text{s.t. } \hat{\mathbf{U}}_n(w) = \sum_{m \in \mathcal{M}} w^{(m)} \hat{\mathbf{U}}_n^{(m)}, \forall n, \quad (21b)$$

$$x_n^*(w) \in \arg \min_{x \in \mathcal{X}_n} \left\{ f(x) + \min_{y \in \mathcal{Y}(x, \hat{\mathbf{U}}_n(w))} h(y) \right\}, \forall n. \quad (21c)$$

The predictions are combined using the weight $w = (w^{(m)}; m \in \mathcal{M})$ in (21b). Based on the combined predictions, UC strategies are optimized in (21c). The objective (21a) minimizes the average cost of UC strategies under observed uncertainty realizations.

The EXACT method uses the KKT conditions of the optimization problem in (21c) to equivalently reformulate problem (21) into a single-level MILP. However, for the UC problem considered in this paper, there are binary variables in x , i.e., the indicator variables for the on/startup/shutdown states of generators, so the KKT conditions do not imply optimality. If we take a step back and relax the binary variables to continuous values in $[0, 1]$, the equivalent MILP for problem (21) can be formulated. However, in the benchmark case with $N = 336$ training samples, the equivalent MILP deduced by KKT conditions contains over one million binary variables, and the Gurobi solver runs out of memory before finding a feasible solution. Even in a reduced dataset with $N = 40$ training data, the solver fails to converge within 10000 s. Therefore, the EXACT method is not suitable for UC problems due to its computational complexity and limitations in handling binary variables in the original optimization problem.

Therefore, the above method comparison verifies that the proposed method is effective. It outperforms other methods considering the comprehensive performance of robustness, optimality, and computational efficiency. Moreover, the proposed weight optimization and uncertainty set reconstruction processes help improve the performance.

To visualize different uncertainty sets, we examine a specific case involving two random loads. The uncertainty sets are projected onto a two-dimensional plane. The projections of the uncertainty sets are drawn in Fig. 5. To emphasize the impact of uncertainty set construction and reconstruction, the weight w is fixed to that of C1. The black polygon is the projection of \mathcal{U}_0 . All other uncertainty sets are the intersections of \mathcal{U}_0 and ellipsoids or polyhedrons. The projected uncertainty set of RO1 is framed by an ellipse that includes all the data points. RO2's ellipse has the same shape and center but only contains 95% data points. RO2 does not have a statistical guarantee for its out-of-sample performance. P1 has a larger uncertainty set than RO2 to maintain the statistical guarantees, but P1 does not reconstruct the uncertainty set to decrease the conservativeness. Proposed_1 and Proposed_2 are the projections of the first and second uncertainty sets of the proposed method (\mathcal{U}'_1 and \mathcal{U}_2 in Algorithm 2), respectively. Proposed_1 is framed by an ellipse similar to that of P1. After the reconstruction, Proposed_2 excludes the right part of the ellipse, thereby omitting some high-cost scenarios in the UC problem. At the same time, Proposed_2 includes additional regions with relatively low costs, ensuring that the data points within it are enough for the statistical guarantees. This approach effectively reduces conservativeness while maintaining statistical guarantees.

3) *Sensitivity Analysis:* We investigate the impacts of weight w , the number of data points N_2 , and the probability tolerance parameters ε and δ for sensitivity analysis. The total costs under different values of weight w are depicted in Fig. 6. As Fig. 6 shows, there is a valley in the middle of the weight's feasible region, which means that the combination of the three kinds of predictions has the best performance. The optimal weight computed by the MILP problem of the surrogate model lies in the center region of the valley and achieves a low total cost, showing the effectiveness of the surrogate model.

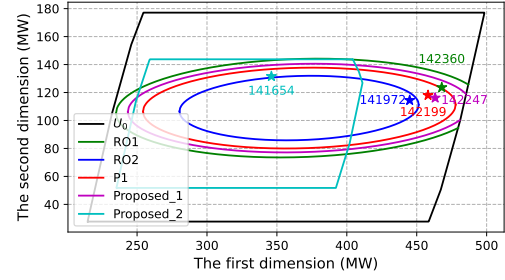


Fig. 5. Projections of uncertainty sets onto a two-dimensional plane in the case of two random loads, where the projections of worst-case scenarios within these uncertainty sets are marked by stars, with the costs (\$) labeled.

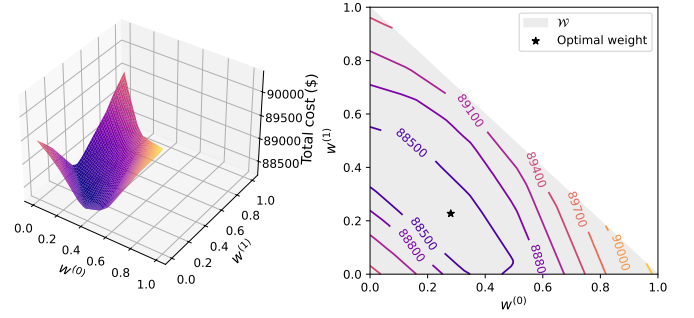


Fig. 6. Total costs under different weights. (Because $w^{(2)} = 1 - w^{(0)} - w^{(1)}$, we only draw the relationship between the total cost, $w^{(0)}$, and $w^{(1)}$, where the weight satisfies $w^{(0)} \geq 0$, $w^{(1)} \geq 0$, and $w^{(0)} + w^{(1)} \leq 1$.)

To illustrate the relationship between statistical guarantees and the parameters ε and δ , we examine the number and proportion of the N_2 data points that fall outside the uncertainty set as N_2 , ε , and δ vary. In the benchmark case, $N_2 = 124$ and $\varepsilon = \delta = 5\%$. In Fig. 7, as N_2 increases, the number of points outside steps up because it must be an integer. The proportion of points outside becomes closer to $\varepsilon = 5\%$ under a larger N_2 , indicating that statistical guarantees are more easily maintained with a larger dataset. When ε is increased while keeping N_2 and δ fixed, the proportion of points outside the uncertainty set rises but never exceeds ε . This suggests that more data points should be included in the uncertainty set than the specified proportion to achieve a statistical guarantee of out-of-sample performance. As δ increases, the proportion of points outside increases.

Fig. 8 shows the results of the optimal value and the test feasible rate under different values of ε . As ε increases, the robustness decreases, leading to a decline in both the objective value and the test feasible rate. When $\varepsilon \in [0.05, 0.17]$, the test feasible rate is larger than $1 - \varepsilon$, demonstrating the effectiveness of the probability guarantee. The test feasible rate decreases approximately linearly as ε increases, when $\varepsilon \geq 0.05$.

The effects of the probability threshold δ are illustrated in Fig. 9. As δ increases, the confidence in the chance constraint $\Pr[u \in \mathcal{U}] \geq 1 - \varepsilon$ decreases, resulting in less conservative outcomes and decreases in both the objective value and the test feasible rate. When $\delta \leq 0.13$, the test feasible rate remains above $1 - \varepsilon = 95\%$. However, if δ becomes too large, the desired probability $1 - \varepsilon$ is no longer met in the test dataset.

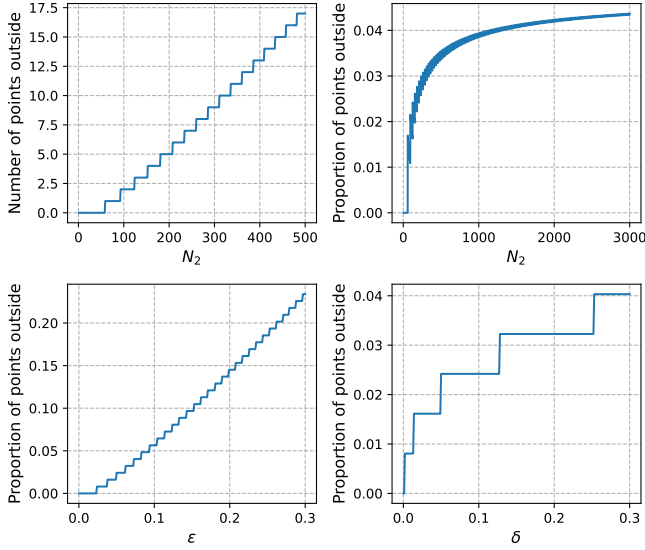


Fig. 7. Proportion of points outside the proposed uncertainty set under different settings.

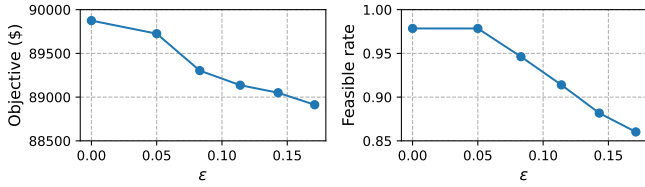


Fig. 8. Results under different ε .

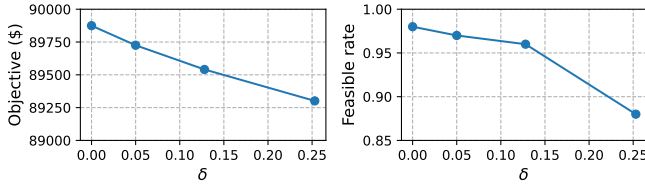


Fig. 9. Results under different δ .

Therefore, the parameters ε and δ should be chosen carefully based on the decision makers' risk preference. Smaller values of ε and δ result in more conservative strategies. We recommend using a small value for δ to maintain good out-of-sample performance, such as $\varepsilon = \delta = 5\%$.

C. Modified IEEE 118-Bus System

We use a modified IEEE 118-bus system with 54 controllable generators and four wind farms to show the scalability. The parameter settings are $\varepsilon = \delta = 5\%$, $N_1 = 212$, and $N_2 = 124$. More details can be found in [44]. The UC results under different methods are shown in TABLE IV. The method relationships are similar to those in the modified IEEE 30-bus system. In addition, the computation time increases but is still acceptable for day-ahead UC. We also test the computational efficiency under different numbers of random loads in the modified IEEE 118-bus system, as shown in Table V. These results verify the scalability of the proposed method.

TABLE IV
UNIT COMMITMENT RESULTS OF DIFFERENT METHODS IN THE MODIFIED IEEE 118-BUS SYSTEM

Method	Objective (\$)	Feasible rate	Test total cost (\$)	Time (s)
SP	2061915	84%	2055123	1638
RO1	2096428	100%	2069461	256
RO2	2086813	100%	2064276	634
P1	2092717	100%	2068213	1304
P2	2066737	100%	2058488	1076
Proposed	2065591	100%	2056996	763

TABLE V
COMPUTATIONAL EFFICIENCY IN THE MODIFIED IEEE 118-BUS SYSTEM

Number of random loads	25	21	17	13
Number of iterations	23	21	21	18
Computation time (s)	763	600	551	391

V. CONCLUSION

To enhance out-of-sample performance and ensure robustness, this paper develops a new data-driven two-stage robust UC method. The proposed integrated forecasting and optimization framework combines different predictions using weights optimized based on performance outcomes. A surrogate model is established to accelerate the weight optimization process. In the two-stage robust UC, the uncertainty set is constructed from data to have statistical guarantees, and it is then reconstructed using the information from the optimization problem to reduce conservativeness. Comparative analysis on modified IEEE 30-bus and 118-bus systems demonstrates that the proposed method surpasses traditional SP and RO methods in balancing robustness with out-of-sample performance. The case studies also show that the computational complexity of the proposed method is comparable to that of traditional RO methods, making it scalable. Future work could explore multi-stage robust UC with statistical guarantees.

APPENDIX A PROOF OF LEMMA 1

1) Prove that $O = O_{x^*} \leq O_{x_{\mathcal{U}}} \leq O_{\mathcal{U}}$ when $\Pr[u \in \mathcal{U}] \geq 1 - \varepsilon$:

Because (x^*, η^*) is optimal in (1), we have $O = O_{x^*}$ and

$$\begin{aligned}
 O &= f(x^*) + \min \left\{ \eta \mid \Pr \left[\min_{y \in \mathcal{Y}(x^*, u)} h(y) \leq \eta \right] \geq 1 - \varepsilon \right\} \\
 &\leq f(x_{\mathcal{U}}^*) + \min \left\{ \eta \mid \Pr \left[\min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*, u)} h(y) \leq \eta \right] \geq 1 - \varepsilon \right\} \\
 &= O_{x_{\mathcal{U}}^*}.
 \end{aligned}$$

Let

$$\eta_{\mathcal{U}}^* = \max_{u \in \mathcal{U}} \min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*, u)} h(y).$$

Then

$$\Pr \left[\min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*, u)} h(y) \leq \eta_{\mathcal{U}}^* \right] \geq \Pr[u \in \mathcal{U}] \geq 1 - \varepsilon.$$

Therefore,

$$\begin{aligned} O_{x_{\mathcal{U}}} &= f(x_{\mathcal{U}}^*) + \min \left\{ \eta \mid \Pr \left[\min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*, u)} h(y) \leq \eta \right] \geq 1 - \varepsilon \right\} \\ &\leq f(x_{\mathcal{U}}^*) + \eta_{\mathcal{U}}^* = f(x_{\mathcal{U}}^*) + \max_{u \in \mathcal{U}} \min_{y \in \mathcal{Y}(x_{\mathcal{U}}^*, u)} h(y) = O_{\mathcal{U}}, \end{aligned}$$

where the last equation follows from the optimality of $x_{\mathcal{U}}^*$ in (2). Hence, $O = O_{x^*} \leq O_{x_{\mathcal{U}}^*} \leq O_{\mathcal{U}}$.

2) Prove that $\Pr[u \in \mathcal{U}^*] \geq 1 - \varepsilon$ and $O = O_{\mathcal{U}^*}$:

By the feasibility of (x^*, η^*) in (1), we have

$$\Pr[u \in \mathcal{U}^*] = \Pr[\min_{y \in \mathcal{Y}(x^*, u)} h(y) \leq \eta^*] \geq 1 - \varepsilon,$$

and then $O \leq O_{\mathcal{U}^*}$ by the previous conclusion. According to the definition of \mathcal{U}^* in (3), $\min_{y \in \mathcal{Y}(x^*, u)} h(y) \leq \eta^*$ for any $u \in \mathcal{U}^*$. Therefore,

$$\begin{aligned} O_{\mathcal{U}^*} &= \min_{x \in \mathcal{X}} \{f(x) + \max_{u \in \mathcal{U}^*} \min_{y \in \mathcal{Y}(x, u)} h(y)\} \\ &\leq f(x^*) + \max_{u \in \mathcal{U}^*} \min_{y \in \mathcal{Y}(x^*, u)} h(y) \\ &\leq f(x^*) + \eta^* = O. \end{aligned}$$

This completes the proof.

APPENDIX B PROOF OF LEMMA 2

According to the definitions of \mathcal{X} and $\mathcal{Y}(x, u)$, we have (8l), (8m), and (10c). Constraint (10c) implies that

$$\begin{aligned} p_{gt} + p_{gt}^+ - p_{gt}^- &\leq p_{gt} + p_{gt}^+ \leq p_{gt} + r_{gt}^+, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \\ p_{gt} + p_{gt}^+ - p_{gt}^- &\geq p_{gt} - p_{gt}^- \geq p_{gt} - r_{gt}^-, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}. \end{aligned}$$

Thus, by (8l),

$$\begin{aligned} &(p_{gt} + p_{gt}^+ - p_{gt}^-) - (p_{g(t-1)} + p_{g(t-1)}^+ - p_{g(t-1)}^-) \\ &\leq (p_{gt} + r_{gt}^+) - (p_{g(t-1)} - r_{g(t-1)}^-) \\ &\leq K_g^+ \theta_{g(t-1)} + K_g^U \theta_{gt}^+, \forall g \in \mathcal{G}, 2 \leq t \leq T, \end{aligned}$$

which shows (11a). Constraint (11b) follows similarly.

APPENDIX C PROOF OF THEOREM 1

The function $a(e)$ is a continuous random variable, where e follows the distribution \mathbb{P} . Therefore, the assumptions of Theorem 1 in [13] are satisfied, and then its conclusion holds, i.e., \mathcal{E} satisfies the statistical guarantee (14).

Because $u \in \mathcal{U}_0$ must hold, we have $u - \hat{u} = e \in \mathcal{E} \iff u \in \mathcal{U}_1$. Combine it with Lemma 1, then

$$\begin{aligned} \Pr[e \in \mathcal{E}] &\geq 1 - \varepsilon \iff \Pr[u \in \mathcal{U}_1] \geq 1 - \varepsilon \\ &\implies O \leq O_{x_0} \leq O_{\mathcal{U}_1}. \end{aligned}$$

Hence, (18) follows from (14), which completes the proof.

APPENDIX D PROOF OF THEOREM 2

1) Prove (20a):

According to linear programming theory [46], the function $b(\hat{u} + e)$ in e is continuous on the closed set

$$\begin{aligned} &\{e \mid \min_{y \in \mathcal{Y}(x_0, \hat{u} + e)} h(y) < +\infty\} \\ &= \{e \mid \mathcal{Y}(x_0, \hat{u} + e) \neq \emptyset\} \\ &= \{e \mid \exists y, \text{ s.t. } Ay \geq Bx_0 + D(\hat{u} + e) + E\}. \end{aligned}$$

When $\beta = +\infty$, $\mathcal{U}_2 = \mathcal{U}_0$ and (20a) holds. For the finite case, Lemma 3 and Theorem 1 in [13] can be applied to $b(\hat{u} + e)$ to obtain (20a).

2) Prove (20b):

Similar to the proof of Theorem 1, the statistical guarantee (20a) implies

$$\mathbb{P}^N[O \leq O_{x_1} \leq O_{\mathcal{U}_2}] \geq 1 - \delta.$$

In addition,

$$\begin{aligned} O_{\mathcal{U}_2} &= \min_{x \in \mathcal{X}} \{f(x) + \max_{u \in \mathcal{U}_2} \min_{y \in \mathcal{Y}(x, u)} h(y)\} \\ &\leq f(x_0) + \max_{u \in \mathcal{U}_2} \min_{y \in \mathcal{Y}(x_0, u)} h(y) \leq f(x_0) + \beta, \end{aligned}$$

where the last inequality follows from (19), so (20b) holds.

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