

EEC 351 Concentration Inequalities Homework

Suggested Completion: 20th August

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Objective

Simulate repeated coin flips to empirically estimate probabilities and compare your results with theoretical bounds given by Markov, Chebyshev, and Hoeffding inequalities.

Tasks

1. Simulation:

- For a fair coin (probability of heads $p = 0.5$), simulate flipping the coin N times for various values of N (e.g., $N = 10, 50, 100, 500, 1000, 5000, 10000$).
- Repeat each simulation M times ($M = 10,000$). For each repetition, calculate the sample mean ν .

2. Probability Calculation:

- For each N , and for selected values of ϵ (e.g., $\epsilon = 0.01, 0.05, 0.1$), estimate the probability

$$P(|\nu - \mu| > \epsilon),$$

where $\mu = 0.5$.

- Compare this simulated probability to the bounds provided by:
 - Markov's Inequality
 - Chebyshev's Inequality
 - Hoeffding's Inequality

3. Visualization:

- Plot the simulated probability (scatter points) and each bound (lines) as functions of N , for each ϵ .
- Use a log-log scale for clarity.

4. Analysis:

- For each bound and ϵ , discuss:
 - How tight or loose each bound is compared to the actual simulated probabilities.
 - How the bounds behave as N increases.
 - In which regimes the bounds are useful (or not).

(Optional) Try with a biased coin ($p \neq 0.5$) and observe what changes.