DELFT UNIVERSITY OF TECHNOLOGY
FACULTY OF AEROSPACE ENGINEERING
Section Control & Simulation

AE4304P: Stochastic Aerospace Systems Practical

Name : **Pedro Tallon Marron** (5342120)	Date : **December 9, 2024**

Assignment Goal

The goal of this assignment is to gain practical hands-on experience with some of the concepts and methods covered in the AE4304 lecture series. In this assignment, you will familiarize yourself with the analysis of stochastic processes using time- and frequency-domain analysis techniques. Thereby, you will gain practical experience regarding the advantages, disadvantages, and limitations of these methods. Even though you will focus on the analysis of *simulated aircraft responses to atmospheric turbulence*, the applied methods and techniques are actually useful for the analysis of *any stochastic process or set of real, measured data!*

Assignment

You are requested to perform calculations on the asymmetrical aircraft responses for a rigid aircraft in asymmetrical atmospheric turbulence conditions. The Power Spectral Density function of atmospheric turbulence is according to the model of Dryden.

Perform the aircraft response's simulations to lateral turbulence, with

$$L_g = 150\, m \qquad \sigma_{v_g} = 2\, m/s.$$

Perform all calculations due to this turbulence for the following two aircraft models (see the lecture notes **[1,2]**):

1. the complete set of equations of motion,
2. the equations of motion that approximate the Dutch roll. Assume that the aircraft does not roll and only rotates along the vertical axis:
   $\varphi = \frac{pb}{2V} = 0, \quad \beta = -\psi.$

Use the aircraft configuration listed in Table 2. Use MATLAB or Python and carefully look at the examples in the lecture notes and the MATLAB files distributed on Brightspace.

Perform the following calculations:

1. **Stability Analysis**

   Before starting all calculations it is **essential** to first check whether the aircraft in the presented flight condition is stable, for both the full and the reduced set of equations of motion. If the aircraft is unstable you will have to design an autopilot (roll-damper) by using the control law:

$$\delta_a = K_\phi \phi + K_p p$$

Choose a value for K_ϕ, and if necessary for K_p, such that all eigenmodes are stable and well damped. *Note: this is not an exercise in controller design, so spend your effort accordingly!*

In your report give:

a) the state space system(s) you used in the simulations ,

b) a stability analysis of the aircraft models that evaluates the stability of all relevant eigenmodes (i.e., pole-zero map, eigenvalues, etc.) and, if the aircraft in the presented flight condition are unstable, the effect of the designed control law on aircraft stability ,

2. **Time-Domain Simulations**

To generate simulated stochastic measurement data, you will perform time-domain simulations of your aircraft model(s) in the specified turbulence condition(s). You will simulate the responses of the aircraft state variables, as well as the lateral acceleration a_y:

$$a_y = V(\dot{\psi} + \dot{\beta})$$

Note: the lateral acceleration does not necessarily have to be extracted from the time signals of V, $\dot{\beta}$ and $\dot{\psi}$, but can be directly calculated from the state-space system.

In your report give:

a) the time domain simulation results (time traces) of both the full and reduced aircraft models flying through turbulence. This includes all the aircraft states, including the lateral acceleration. It is not necessary to show the simulation results for the gust states.

3. **Spectral Analysis**

Calculate the analytical and experimental Power Spectral Density functions of the above mentioned variables (so not the gust states). Calculate the:

i) *analytical* Power Spectral Density functions using the state-space representation

ii) *experimental* Power Spectral Density functions (periodograms) estimated using the MATLAB routine `fft.m`

iii) *experimental* Power Spectral Density functions obtained by applying a smoothing filter,

$$\Phi_{estimate}[k] = 0.25 * \Phi[k-1] + 0.5 * \Phi[k] + 0.25 * \Phi[k+1]$$

for only one realization of the periodogram ($\Phi[k]$ is the periodogram).

In your report give:

a) the spectral analysis for the aircraft states and lateral acceleration only. Thus a spectral analysis regarding the gust states is not necessary

b) a discussion on all the results and how you obtained them. Clearly **explain** the differences between the responses to turbulence of the two sets of aircraft equations of motion

c) a discussion on any differences that were found between the calculation methods of the periodograms. This includes both a comparison between the methods and a possible explanation for any differences that were found

4. **Variances**

   Estimate the variances of all aircraft states and the lateral acceleration using:

   i) the analytical power spectra,
   ii) the unsmoothed and smoothed experimental power spectra,
   iii) by using the MATLAB routine `var.m` on the simulated state variable time traces.

   In your report give:

   a) a table with the variances calculated using the different methods ,
   b) an explanation for differences in variance between the two aircraft models ,
   c) an explanation for the differences between the different calculation methods .

Report Deadline & Assessment

The deadline for submitting the report is **April 7, 2024, 17:00**. If you miss the deadline, your next chance for handing in an AE4304P report will be next year! Please note that you get separate grades for the AE4304P practical (report) and for the AE4304 course (exam). Both grades must be $\geq$ 6. The scoring distribution for this AE4304P assignment is given in Table 1.

Table 1: The score distribution for AE4304P. The final grade is 1 + your score.

Assignment (1 pt)	pt
Correct implementation of turbulence condition and/or model reduction	0.5
Correct aircraft stability analysis	0.5
Spectral Analysis (3 pt)	
Spectra calculated for all required variables and methods	0.5
Spectra calculated correctly	1
Spectra plotted clearly (plot size, x/y-axis labels, units)	0.5
Discussion of results for different conditions/models	0.5
Discussion of results for different methods	0.5
Variances (3 pt)	
Variances calculated for required variables and methods	0.5
Variances calculated correctly	1
Variances presented clearly (table, significant digits, units)	0.5
Discussion of results for different conditions/models	0.5
Discussion of results for different methods	0.5
Report (2 pt)	
Quality of report Introduction	0.5
Quality of report Conclusion	0.5
Overall report quality	1.0

Getting help

For questions regarding your assignment, please send an e-mail to dr.ir. E. J. J. Smeur (ae4304-project@tudelft.nl). Always first make an appointment by email before coming to my office.

References

1. Mulder, J. A. & Van Staveren, W.H.J.J. & Van der Vaart, J.C. & De Weerdt, E., *Flight Dynamics*. Lecture Notes AE3302.

2. Mulder, J. A. & Van der Vaart, J. C. & Van Staveren, W.H.J.J. & Chu, Q.P. & Mulder, M., *Aircraft Responses to Atmospheric Turbulence*. Lecture Notes AE4304.

Table 2: Stability and control derivatives of the Cessna Citation 500, cruise (4).

Aircraft : Cessna Ce500 Citation I

Configuration : cruise

$x_{c.g.}$	=	0.30	c̄							
W	=	53361	N	V	=	181.9	m/sec	μ_b	=	32
m	=	5445	kg	h	=	9144	m	$\mathrm{K_X^2}$	=	0.013
S	=	24.2	$\mathrm{m^2}$	ρ	=	0.4587	$\mathrm{kg/m^3}$	$\mathrm{K_Z^2}$	=	0.037
$\bar{c}$	=	2.022	m	μ_c	=	209		$\mathrm{K_{XZ}}$	=	0.002
b	=	13.36	m	l_h	=	5.5	m	$\mathrm{K_Y^2}$	=	0.950

C_{X_0}	=	0	C_{Z_0}	=	-0.2510			
C_{X_u}	=	-0.0642	C_{Z_u}	=	-0.5020	C_{m_u}	=	0
C_{X_α}	=	0.0844	C_{Z_α}	=	-5.8000	C_{m_α}	=	-0.5350
			$C_{Z_{\dot{\alpha}}}$	=	-1.8300	$C_{m_{\dot{\alpha}}}$	=	-4.7500
C_{X_q}	=	0	C_{Z_q}	=	-4.3800	C_{m_q}	=	-8.0000
C_{X_δ}	=	0	C_{Z_δ}	=	-0.5798	C_{m_δ}	=	-1.4440

C_{Y_β}	=	-1.4900	C_{ℓ_β}	=	-0.1240	C_{n_β}	=	0.1865
C_{Y_p}	=	-0.1450	C_{ℓ_p}	=	-0.4344	C_{n_p}	=	0.0135
C_{Y_r}	=	0.4300	C_{ℓ_r}	=	0.1550	C_{n_r}	=	-0.1930
$C_{Y_{\delta_a}}$	=	0	$C_{\ell_{\delta_a}}$	=	-0.2108	$C_{n_{\delta_a}}$	=	0.0031
$C_{Y_{\delta_r}}$	=	0.3037	$C_{\ell_{\delta_r}}$	=	0.0469	$C_{n_{\delta_r}}$	=	-0.1261