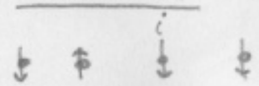
Dualities

- 1+1d bosonization
- other dualities (2+1d)
  - ↳ "gauging symmetry"
  - ↳ bosonization

1+1d bosonization

Bosonic

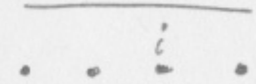


At site i: σ^z_i, σ^x_i

$$\sigma^z_i \sigma^x_i = -\sigma^x_i \sigma^z_i$$

$$i \neq j \quad \sigma^z_i \sigma^x_j = \sigma^x_j \sigma^z_i$$

Fermionic



At site i: $a^\dagger_i, a_i$

$$a^\dagger_i a_i = \mathbb{1} - a_i a^\dagger_i$$

$$i \neq j, \quad a_i a_j = -a_j a_i$$

$$a^\dagger_i a_j = -a_j a^\dagger_i$$

$$a^\dagger_i a^\dagger_j = -a^\dagger_j a^\dagger_i$$

Bosonic ctd	Fermionic ctd
$\sigma^z_i \longrightarrow$	$\mathbb{1} - 2a^\dagger_i a_i$
$\sigma^x_i \longrightarrow$	$(a^\dagger_i + a_i) \prod_{j>i} (\mathbb{1} - 2a^\dagger_j a_j)$
$\sigma^x_i \, \sigma^x_{i+1} \longrightarrow$	$(a^\dagger_i + a_i)(\mathbb{1} - 2a^\dagger_{i+1} a_{i+1})(\mathbb{1} - 2a^\dagger_{i+2} a_{i+2}) \cdots$ $(a^\dagger_{i+1} + a_{i+1})(\mathbb{1} - 2a^\dagger_{i+2} a_{i+2}) \cdots$

Bosonic ctd ctd		Fermionic ctd ctd
$\sigma_i^- \prod_{j>i} \sigma_j^z$	$\longleftarrow$	$a_i^\dagger$
$\sigma_i^+ \prod_{j>i} \sigma_j^z$	$\longleftarrow$	a_i

Quantum Ntd Ising chain

$$H = -K \sum_{i=1}^{N} \sigma_i^z - J \sum_{i=1}^{N-1} \sigma_i^x \sigma_{i+1}^x$$

$$U_{\mathbb{Z}_2} = \prod_i \sigma_i^z , \quad U_{\mathbb{Z}_2}^2 = \mathbb{1}$$

$$[H, U_{\mathbb{Z}_2}] = 0$$

$K \gg J$)

groundstate $\approx |\uparrow\uparrow\uparrow \cdots \rangle$

$J \gg K$)

groundstate $\approx$

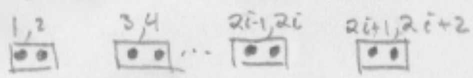
$|\rightarrow\rightarrow\rightarrow\cdots\rangle + |\leftarrow\leftarrow\leftarrow\cdots\rangle$

$|\rightarrow\rightarrow\rightarrow\cdots\rangle - |\leftarrow\leftarrow\leftarrow\cdots\rangle$

physical gs:

$|\rightarrow\rightarrow\rightarrow\cdots\rangle$ or $|\leftarrow\leftarrow\leftarrow\cdots\rangle$

Kitaev chain



$$\gamma_{2i-1} = a_i^\dagger + a_i$$

$$\gamma_{2i} = i(a_i^\dagger - a_i)$$

$$\gamma_k^\dagger = \gamma_k$$

$$\gamma_k^2 = 1$$

$$k \neq l \quad \{\gamma_k, \gamma_l\} = 0$$

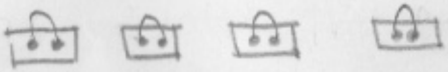
$$\sigma_i^z \longleftrightarrow (-i\gamma_{2i-1}\gamma_{2i})$$

$$\sigma_i^x \sigma_{i+1}^x \longleftrightarrow (-i\gamma_{2i}\gamma_{2i+1})$$

$$H = -K\sum_{i=1}^{N}(-i\gamma_{2i-1}\gamma_{2i}) - J\sum_{i=1}^{N-1}(-i\gamma_{2i}\gamma_{2i+1})$$

$$P_f = \prod_i(-i\gamma_{2i-1}\gamma_{2i}),\quad P_f^2 = 1,\quad [H, P_f] = 0$$

$\boxed{K >> J}$ $H \approx -K\sum_{i=1}^{N}(-i\gamma_{2i-1}\gamma_{2i})$



groundstate $\approx$ no fermions

$\boxed{J >> K}$ $H \approx -J\sum_{i=1}^{N-1}(-i\gamma_{2i}\gamma_{2i+1})$

γ_1 γ_{2N}

$$\tilde{a}^{+} = \frac{1}{2}(\gamma_1 - i\gamma_{2N})$$

$$\hat{a} = \frac{1}{2}(\gamma_1 + i\gamma_{2N})$$

groundstates:

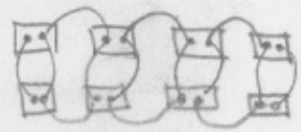
MZM occupied/unoccupied

$$P_f = \prod_i(-i\gamma_{2i-1}\gamma_{2i})$$

on gs; $P_f = -i\gamma_1\gamma_{2N}$

$P_L = -i\gamma_1,\ P_R = \gamma_{2N}$

$P_L P_R = -P_R P_L$



trivial