STRONG CP PROBLEM AND AXION SOLUTION

• OUTLINE

1) $N_f = 1$ QCD $\rightarrow$ ANOMALOUS $U(1)_A$ $\rightarrow$ CP VIOLATION IN QCD IS <u>NATURAL</u>

2) $N_f = 2$ QCD $\rightarrow$ EFT of $\pi^{\pm,0}$ $\rightarrow$ CP IS <u>NOT</u> VIOLATED IN QCD!

EXP. DATA $\rightarrow$ CP IS NOT VIOLATED IN QCD!

3) AXION SOLUTION

$N_f = 1$ QCD

ADJ. REP of SU(3)

$$\not{D} = \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu + A_\mu)$$

$$\bullet\ \mathcal{L} \sim \bar{\psi}\not{D}\psi + F^2 + \theta \tilde{F}F$$

FUND. REP of SU(3)

$$F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} + [A_\mu, A_\nu]$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

• OBS: $\theta\tilde{F}F$ is CP VIOLATING!

$$\bullet\ \bar{\psi}\not{D}\psi \sim \psi_L^\dagger \bar{\sigma}^\mu D_\mu \psi_L + \psi_R^\dagger \sigma^\mu D_\mu \psi_R \quad ; \quad \sigma^\mu = (1, \vec{\sigma}), \quad \bar{\sigma}^\mu = (1, -\vec{\sigma})$$

CHIRAL DEC.

- GLOBAL SYMMETRIES $\quad G = U(1)_V \times U(1)_A$

$$\psi \to e^{i\alpha}\psi \iff \psi_{L,R} \to e^{i\alpha}\psi_{L,R}$$

$$\psi \to e^{i\alpha\gamma^5}\psi \iff \psi_{L,R} \to e^{\mp i\alpha}\psi_{L,R}, \quad \gamma^5\psi_{L,R} = \mp\psi_{L,R}$$

- MASS TERM OF QUARKS

$$\mathcal{L} \supset m\bar{\psi} e^{i\theta_q\gamma^5}\psi = m e^{i\theta_q}\psi_L^\dagger\psi_R + m e^{-i\theta_q}\psi_R^\dagger\psi_L$$

CP VIOLATING
COMPLEX PHASE!
(ELECTROWEAK)

PRESERVES $U(1)_V$
BREAKS $U(1)_A$ (RESTORED $m \to 0$)

- ANOMALOUS $U(1)_A$

$$\psi \to e^{i\alpha\gamma^5}\psi \Rightarrow D\bar{\psi}\,D\psi \to D\bar{\psi}\,D\psi \exp\left(i\int\alpha\tilde{F}F\right)$$

$$\partial_\mu\langle J^\mu\rangle \sim \alpha\tilde{F}F \quad \left(\delta\mathcal{L} \sim \alpha\tilde{F}F\right)$$

- OBS$_1$: $\mathcal{L} \supset m\bar{\psi}e^{i\theta_q}\psi + \theta\tilde{F}F \xrightarrow[U(1)_A]{} m\bar{\psi}e^{i(2\alpha+\theta_q)}\psi + (\theta-2\alpha)\tilde{F}F$

$$\theta_q \to \theta_q - 2\alpha, \quad \theta \to \theta + 2\alpha \Rightarrow \text{EXACT SYMMETRY!} \Rightarrow \text{OBSERVABLES DEPEND ON } \bar{\theta} = \theta_q + \theta$$

CAN GET RID of θ_q:

$$\alpha = \frac{\theta_q}{2} \Rightarrow \theta \to \bar{\theta}$$

- OBS$_2$: $\bar{\theta} = \underbrace{\theta_q}_{\text{ELECTROWEAK}} + \underbrace{\theta}_{\text{STRONG}}$ $\Rightarrow$ CP VIOLATION IN QCD is <u>NATURAL</u>

$\boxed{N_f = 2 \text{ QCD}}$

- $$\mathcal{L} \sim \sum_{\psi = u,d} \left(\bar{\psi} \not{D} \psi + m_\psi \bar{\psi} e^{i\theta_\psi \gamma^5} \psi \right) + F^2 + \theta \tilde{F} F$$

$$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} ; \quad \psi_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\mathcal{L} \supset \psi_L^\dagger \bar{\sigma}^\mu D_\mu \psi_L + \psi_R^\dagger \sigma^\mu D_\mu \psi_R + \left(\psi_L^\dagger M \psi_R + h.c \right)$$

$$M = \begin{pmatrix} m_u e^{i\theta_u} & 0 \\ 0 & m_d e^{i\theta_d} \end{pmatrix}$$

- GLOBAL SYMMETRIES

i) $\psi_{L,R} \to e^{i\alpha} \psi_{L,R}$ $\quad U(1)_B$

ii) $\psi_{L,R} \to e^{\mp i\alpha} \psi_{L,R}$ $\quad U(1)_A \to$ APPROXIMATE ANOMALOUS

OBS:

$$\theta_u \to \theta_u - 2\alpha$$
$$\theta_d \to \theta_d - 2\alpha \quad \Rightarrow \text{EXACT SYMMETRY} \Rightarrow \bar{\theta} = \theta + \theta_u + \theta_d \text{ is OBSERVABLE}$$
$$\theta \to \theta + 4\alpha$$

iii) $\psi_L \to L\psi_L$
$\psi_R \to R\psi_R$ $\qquad SU(2)_L \times SU(2)_R \to$ APPROXIMATE

$L = R$ is BROKEN UNLESS $m_u = m_d$

$L \neq R$ is BROKEN UNLESS $m_u = m_d = 0$

$\Rightarrow SU(2)_L \times SU(2)_R = SU(2)_V \times SU(2)_A$

$SU(2)_V \to$ ISOSPIN

• EFT of $\pi^{\pm,0}$

EXP. OBSERVATION: QCD CONFINES $\Rightarrow \langle (\psi_L^\dagger)_a (\psi_R)_b \rangle = -\Lambda^3 \delta_{ab}$

$\langle (\psi_L^\dagger)_a (\psi_R)_b \rangle$ → QUARK CONDENSATE; $a = u, d$; $\Lambda \sim 100\,\mathrm{MeV}$

$$\langle (\psi_L)^\dagger_a (\psi_R)_b \rangle \xrightarrow{SU(2)_L \times SU(2)_R} (L^\dagger)^c{}_a R_b{}^d \langle (\psi_L^\dagger)_c (\psi_R)_d \rangle$$

$$= -\Lambda^3 (RL^\dagger)_{ba} =: -\Lambda^3 \Sigma_{ab} \; ; \quad \Sigma \in SU(2)$$

$R = L \Rightarrow SU(2)_V$ INVARIANT

$R \neq L \Rightarrow SU(2)_A$ SB

3PGB

$$(\psi_L^\dagger)_a (\psi_R)_b = -\Lambda^3 \Sigma'_{ab}(x) + (\ldots)$$

HEAVY MODES

$$\Sigma' = e^{i \frac{\vec{\pi}(x)}{f_\pi} \cdot \vec{\sigma}}$$

$$f_\pi \simeq 130\, \mathrm{MeV}$$

KINETIC TERM:

$$\mathcal{L} \supset \mathrm{Tr}\left(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma\right)$$

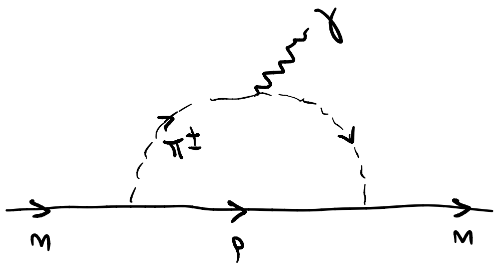
$$= \frac{1}{2}(\partial\pi)^2 + \frac{1}{f_\pi^2}(\partial\pi)^2 \pi^2 + \mathcal{O}\left(\frac{1}{f_\pi^3}\right)$$

QUARK MASS:

$$\mathcal{L} \supset \Lambda^3 \left[\mathrm{Tr}(M\Sigma) + h.c\right] \Rightarrow \begin{cases} m_{\pi^\pm}^2 \sim \frac{\Lambda^3}{f_\pi^2}(m_u + m_d) \\ m_{\pi^0}^2 \sim \frac{\Lambda^3}{f_\pi^2}\sqrt{(m_u+m_d)^2 + 2 m_u m_d(\cos\bar{\theta} - 1)} \end{cases}$$

EXP. DATA: $m_{\pi^\pm}^2 \simeq m_{\pi^0}^2 \Rightarrow \bar{\theta} \simeq 0$

BEST CONSTRAINT: NEUTRON EDM



$\Rightarrow$ $d_n \sim 10^{-16}\,\bar\theta$ e cm

EXP. DATA: $|d_n| \lesssim 10^{-26}$ e cm

$\Rightarrow |\bar\theta| \lesssim 10^{-10}$

$\Rightarrow$ CP is <u>NOT</u> violated in QCD!

<u>IDEA</u>: $\bar\theta \to \bar\theta(x)$ with $\langle\bar\theta\rangle = 0$ E.F

$U(1)_{PQ}$

$\downarrow$

ANOMALOUS AXIAL

$(\psi, \Phi, \ldots)$

$E \lesssim f_a \Rightarrow$ GB $a(x)$ (AXION FIELD)

$U(1)_{PQ}$ SSB

$a \to a + \alpha f_a$

$U(1)_{PQ}$ is ANOMALOUS $\xRightarrow{Q.C} \mathcal{L}_{eff} \supset \left(\frac{a}{f_a} + \theta\right)\tilde{F}F$

$\Rightarrow \theta \to \theta + \frac{a}{f} \left(\bar\theta \to \bar\theta + \frac{a}{f}\right) \Rightarrow$

$\langle a \rangle = -\bar\theta f$

$d_n \approx 0$

$m^2_{\pi^\pm} \approx m^2_{\pi^0}$

- $m_a \sim \frac{1}{f_a}$; $g_{a\gamma} \sim m_a \sim \frac{1}{f_a} \Rightarrow$ AVENUE of DIRECT DETECTION
- ALP's
- AXIONS ARE DM CANDIDATES