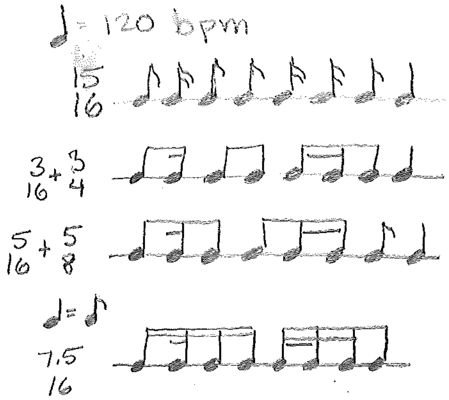


- Musical Notation: full of redundancies.

Ex.

- All technically expressing the same discretization of time

Music: temporal discretization on a lattice $\approx 8 \times 10^{-3}$ s

Attempting to express emotional journey

A purpose for each moment

Not easily translated into 2D sheet of paper

Comp. relies on performer to elaborate upon instructions

- to dynamically introduce features to the continuum.

and they influence the continuum interpretations by choosing among seemingly redundant notations. Diff. subliminal instructions.

Performers must extrapolate to continuum

Also occurs in our simulations of nature on a system with a finite number of degrees of freedom. Today we will be composers of Chiral Symmetry.

- Attempt to express the low-energy symmetries of QCD and see that the simulation also dynamically elaborates on our instructions.

outline of moods
$\begin{cases} 1{=}16 & \text{Determination} \\ 0 \to a & \\ \text{Lüscher} & \end{cases}$

Nature
boyish enthusiasm
No productive Discouragement
- Hope
- Clarity

As Any good composer, first Be clear on the Idea You want to Express Nature:

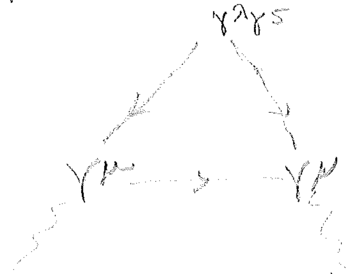
$$U(N_f)_L \times U(N_f)_R \to \underbrace{SU(N_f)_L \times SU(N_f)_R}_{} \times U(1)_V \times U(1)_A$$

$$\boxed{\{D, \gamma_5\} = 0}$$

Spontaneous Symm. Breaking
- goldstone Bosons
- No parity Partners N^* 1535 MeV

But our Story lives with $U(1)_A$

Wherever have global symmetries → Conserved Currents



$\langle 0 | J_5^\lambda(0) J^\mu(x_1) J^\nu(x_2) | 0 \rangle$

forced to make a decision

$\partial_\mu J^\mu = 0$

- conserves fermion #
- couple to photon
  $\frac{\xi k_\mu k_\nu}{k^2} J_\mu \to 0$
- 2 polarizations.

$\mathcal{L} = \bar{\psi} i \gamma^\mu (\partial_\mu - ieA_\mu) \psi$

$$\partial_\mu J_5^\mu \sim \varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu} F_{\lambda\sigma} \neq 0$$

due to instantons + carriers of topological charge

$$\partial_\mu J_5^\mu \sim Q_{top} = n_- - n_+$$

Q_{top}: Atiyah Singer

n_-: LH zero modes of Dirac operator; n_+: RH

$D|\psi\rangle = 0$

$D\gamma_5|\psi\rangle = 0$

$\Rightarrow \gamma_5|\psi_0\rangle = \pm|\psi_0\rangle$

chirality ✓

Obs. $\pi^0 \to \gamma + \gamma$, flavor Singlet (anomaly ~~SSB~~) $m_{\eta'} \gg m_\pi$

Start w/ Invariant Action

have a measure that is not!

Jump in

parallel transport gauge

$$S = \int d^4x\, \bar{\psi}\, \gamma_\mu \partial_\mu \psi \longrightarrow a^d \sum_{x,\mu} \bar{\psi}_x \gamma_\mu \frac{U_\mu(x)\psi_{x+\hat{\mu}} - U_{-\mu}(x)\psi_{x-\hat{\mu}}}{2a}$$

$U^{\dagger}_{\mu}(x-\hat{\mu})$

prop $\sim \frac{1}{p^2}$ single pole LSZ $\longrightarrow$?

Bilinear Dirac op:

$$D_{x,y} = \sum_\mu \gamma_\mu \frac{1}{2a}\left(U_\mu(x)\delta_{y,x+\hat{\mu}} - U_{-\mu}(x)\delta_{y,x-\hat{\mu}}\right)$$

$$D^{-1} = \langle \bar{\psi}(-p)\psi(p)\rangle = \left[i \sum_\mu \gamma_\mu \frac{1}{a} \sin(p_\mu a)\right]^{-1}$$

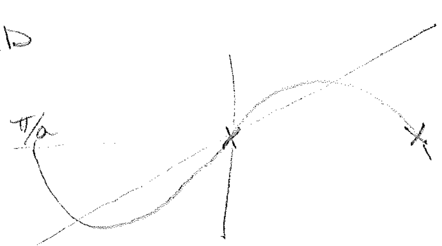
$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \sim \frac{-i\sum_\mu \gamma_\mu \sin(p_\mu a)\, a}{\sum_\mu \sin(p_\mu a)^2} \xrightarrow[a\to 0]{} \frac{-i\not{p}}{p_\mu^2}$$

High Momentum Modes? Regulated by lattice. Periodic Brillouin Zone $[-\pi/a, \pi/a]$

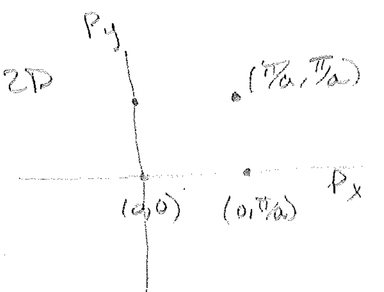
dangerous

mapping $p \rightarrow \frac{\sin(pa)}{a}$

1D



2D



Doubles for every dimension!

2^4 or 16 in D=4

$\therefore$ 16 = 1

But Not to Worry. Have a way to comment. EFT's. Higher Dim. op.

Exactly what Wilson did

back to Action

$$+a^d \sum_{x,\mu} \frac{1}{2a} \left[2\bar{\psi}_x \psi_x - \bar{\psi}_x \psi_{x+\hat{\mu}} - \bar{\psi}_x \psi_{x-\hat{\mu}} \right]$$

$$\sim \quad -\bar{\psi} \frac{ar}{2} \nabla^*_\mu \nabla_\mu \psi$$

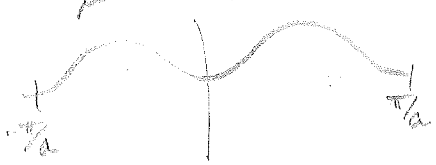
Irrelevant operator

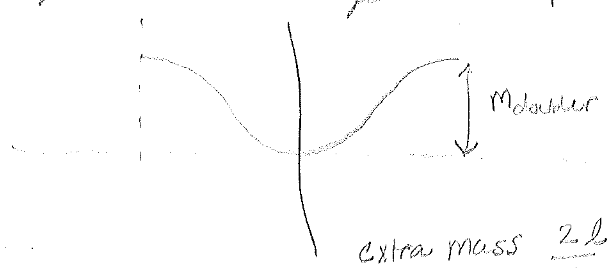
low momentum vanishes ; high mom. $\mathcal{O}$(cutoff) mass term

no γ_μ, transforms as a mass term (wave # dep)

Denom. of $\tilde{D}^{-1}$

$\sum_\mu \sin(p_\mu a)^2$ $\qquad$? $\qquad$ $\sum_\mu \sin(p_\mu a)^2 + \sum_\mu (1-\cos(p_\mu a))^2$





extra mass $\frac{2l}{a}$ $\quad l$ = # components $p_\mu = \pi/a$

Implications:

✓ $a \to 0$ decouple from theory.

X Explicitly broken Chiral Symmetry (w/o masses)

- No protection of M_π from Additive mass renormalization

What is keeping the masses so light? Hierarchy.

Rules — think we want - Symmetry of Action
Not Symmetry of Measure. continuum $\to \chi^{ral}$ Symm.

- Exact Symm. of regulated Action = exact Sym of Quantum theory
(No Anomalies)

If Anomalous in continuum $\longrightarrow$ broken on lattice

- $\partial_\mu J^\mu = 0$
Symm current have non-Zero DIV.

1.) original Action
2.) UV regulator
] explicitly violate Symmetry.

Nielson Ninomiya (1981)
So. Confusing and Unclear how to move forward
Telling Scientists: "You can't have what you want."

Changed Question: how do we express continuum sym. in a regulated theory
to: What of our continuum qualities must we give up?

1.) Bilinear $\quad S = \sum_{x,y} \bar{\psi}(x) M_{xy} \psi(y)$
2.) Real $\quad M = M^T$
3.) Translation Inv. $\quad M_{xy} = D(x-y)$
4.) Local $\quad \tilde{D}(p)$ - regular func. of p in B. zone
5.) Chirally Symmetric $\quad \{D, \gamma_5\} = 0$
6.) Doubler free. $\quad 1 = 1$

Performer cannot Do it

Of course, topology.

$\{\gamma_\mu, \gamma^5\} = 0$
keeps kinetic terms from mixing LH and RH

$\{D, \gamma^5\} = 0 \Rightarrow D = \sum_\mu \gamma_\mu \, d_\mu(p)$

↑ vector field on torus

$\Rightarrow$ Zero modes definite chirality

$\gamma_5 |\psi_0\rangle = \pm |\psi_0\rangle$

↑ handedness

Euler Characteristic: $\chi = 2 - 2H$ Holes: 1

$\chi = 0$

Hopf-Poincaré Index theorem

$\sum \text{Index}(d_\mu) = \chi = 0$

$\Rightarrow$ Zeros of d_μ come in pairs of opp index.

Doublers!

Lüscher, Ginsparg and Wilson

$$\psi \to \psi + i\varepsilon^a \tau^a \gamma_5 \psi \qquad \bar\psi \to \bar\psi + \bar\psi i \varepsilon^a \tau^a \gamma_5$$

generators of $SU(N_f)$ $a \in \{1, \dots N_f^2 - 1\}$

flavor singlet Axial (η') $a=0$ $\tau^a = \mathbb{I}$

1982 "A remnant of Chiral Symmetry on the Lattice"

Continuum Action and Spin blocking (Wilson) renormalization

$$\{D, \gamma^5\} = a D \gamma^5 D$$

Implications for quark prop (D^{-1} on both sides)

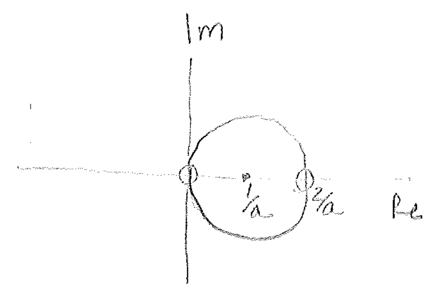
$$\{D^{-1}_{xy}, \gamma_5\} = a\gamma_5 \delta(x-y)$$

Added A contact term "Loop" correction

Dirac Op: Assume γ_5 hermiticity $\gamma_5 D \gamma_5 = D^\dagger$

$$\det\left[D - \lambda \mathbb{1}\right] = \det\left[D^\dagger - \lambda \mathbb{1}\right]$$

λ and λ^* are E. vals : <u>c.c. pairs</u>



Chirality: γ_5 M.E.

$$\lambda(\phi_\lambda, \gamma_5 \phi_\lambda) = \lambda^*(\phi_\lambda, \gamma_5 \phi_\lambda)$$

$$\mathrm{Im}\,\lambda(\phi_\lambda, \gamma_5\phi_\lambda) = 0$$

only eigenvecs w/ real eigenvalues can have non-vanishing chirality

G + W could not find solution

- Looking for Local
  nearest neighbor

Spoiling it, too restrictive. Next week, solutions

locality: Couplings in Action decay w/ distance faster than any signal e.g. $e^{-\kappa r/a}$

κ $O(1)$ is "local"

So: Currently have (w/ continuum transformations)

Backwards!

[Local actions + No Doubling $\Rightarrow$ not a sym of Action

Fermion Measure : Invariant

Lüscher 1998 : Modify transformation

$$\psi \to \psi + i\varepsilon^a T^a \gamma_5 \left(1 - \frac{a}{2} D\right)$$

Gauge field Dependent!

continuum: χrality is a local concept.

$$P_L = \frac{1 - \hat{\gamma}_5}{2} \quad \hat{\gamma}_5 = \gamma_5 (\mathbb{1} - aD)$$

Is it a symmetry?

$$\bar{\psi} D \psi \to \bar{\psi} \left(1 + i\varepsilon^a T^a \left(1 - \frac{a}{2} D\right)\gamma_5\right) D \left(1 + i\varepsilon^a T^a \gamma_5 \left(1 - \frac{a}{2} D\right)\right)\psi$$

$$\to \bar{\psi} D \psi + \bar{\psi}\, i\varepsilon^a T^a \Big[\underbrace{\gamma_5 D + D\gamma_5 - aD\gamma_5 D}_{\text{vanishes if sol. to GW!}}\Big]\psi + O(\varepsilon^2)$$

What About the Measure?

Grassman Int → det of trans. Matrix

$$\mathrm{Det}\left[\mathbb{1} + i\varepsilon^a T^a \gamma_5 \left(\mathbb{1} - \tfrac{a}{2} D\right)\right]^2$$

$\mathrm{Det}\, X = e^{\overbrace{\mathrm{tr} \log X}^{\text{CFDS}}}$

$$= 1 + 2i\varepsilon^a \, \mathrm{tr}_{FDc}\left[T^a \gamma_5 \left(\mathbb{1} - \tfrac{a}{2} D\right)\right] + \mathcal{O}(\varepsilon^2)$$

$\log(1+x) = x + \ldots$

$$= 1 - \frac{2i\varepsilon^a a}{2} \underbrace{\left(\mathrm{tr}_F T^a\right)}_{N_f} \underbrace{\mathrm{Tr}_{cD}\left(\gamma_5 D\right)}_{\sim Q_{top}} + \mathcal{O}(\varepsilon^2) \neq 1$$

Invariant Action And Anomalous Measure! Not Just Any Anomaly...

$$\frac{a}{2} \mathrm{tr}_{cD} \gamma_5 D = -\frac{1}{2} \mathrm{tr}\left(\gamma_5 (2 - aD)\right)$$

tr: Sum of evals

$$= -\frac{1}{2} \sum_\lambda \left(\phi_\lambda, \gamma_5 (2 - aD) \phi_\lambda\right)$$

$$= -\frac{1}{2} \sum_\lambda (2 - a\lambda) \underbrace{\left(\phi_\lambda, \gamma_5 \phi_\lambda\right)}_{\langle\gamma_5\rangle}$$

vanishes unless λ real

$\lambda = 0$, $\cancel{\lambda = 2}$

Σ over Zero Mode Chiralities

$$= n_- - n_+ = Q_{top}$$

Just like in Continuum, topologically-related Axial Anomaly

Message: No Go thm bypassed IF modify χ^{real} sym.

Next: Sils to GW: overlap and Domain Walls