Introduction

In the previous talks we've been discussing solutions and instantons in 2d Euclidean space to build up some background knowledge of instantons.

We've so for avoided discussing 4-d Euclidean space because of Perrick's Their we need to know how instantons work with gauge fields to go to 4-d. This is what we will do today

It will turn out that our SU(N) gange theories have important subtleties that give rise to a continuous range of degenerate vacuum states, which flies in the tace of conventional knowledge (of Coleman's time, at least).

to start heading down this direction, let us consider our gauge theory in a remiclosistical limit. We care about finite energy solutions here so since our action is $S = -\frac{1}{2} \int gg x \, Tr[ForFor]$ we need for to scale forster than is at infinity.

Let's say it has a rice taylor expansion here so that it is town O(1/2). This would rownly imply that Another On countries of but due to gauge invariance we are allowed to lieur Anifet (O(1/2)), where f is a member of sulw and a function only of analyse (not in).

This looks like it should be removable by a gauge transformation but as it turns out there are properties of I that are gauge invariant namely the winding number.

We've already seen some of this in 2-d space. Briefly turning to UIII in 2d for a moment, we can see that the functions figure are valid choices as long as vis an integer (because the the 2d angle and function).

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Winding number in SUPI in 4-8 Euclidean space.

New we will take a closer look at honotopy classes if SU(2) in 4-6.

In the previous example, we were essentially looking for mappings between the 5' of 2d space at fixed 5 to the 5' of the U(1) gauge theory. Here, we want to go from the spatial 5° at fixed 5 to the topology of SU(1) = 5° (because U=a+i++ with a+15!=))

The identity mapping here will be $f(x) = \frac{x_4 + i \cdot x_5}{c}$. Higher mappings are given by f(x) = f(x) just as with UU). The number v represents the number of times we raise around the hypersphere, and all shoices of f(x) can be determed into one of the above cases.

Let's define $\tilde{v} = \overline{24\pi^2}/88 \in Tr[43.5](53.5)(53.5)(53.5)$, which will strue as our formal definition of the winding number. Under an infinitesimal gauge transformation, we can show that by rearranging the derivatives using unitarity of f and integration by parts, the shift $\tilde{v} = 0$.

Finally consider the object 6 = 2 Engo Tr[Av & Aoig = Av Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 1 Engo tr[Av Frotig = Tv Ax Ao] = 1 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo tr[Av Frotig = Tv Ax Ao] = 2 Engo t

Thus for our minimal action solutions near infinity, we then have

[81x Tr[FF] = 185 Frencho Tr[An Frotig = Arabao] ~ [85 & jk Tr [= 30 (fort) (fort) (fort) +8 (fort)

= 160 = (12) [14x Tr[FF] = V)

Finally, there exists a throsen from Rapoul Both states that any continuous mapping from 53 to any simple lie group G can be continuously deformed into a mapping into an SU(2) subgroup of & Thus the work here tells you how to map to any Lie group namely all SU(1) theories.

Quantizing the Theory

We have looked at minimal action solytions asymptotically for a classical younge through, so now we must quantize the throng.

To have a well-defined path integral we need to choose a gauge to work in.
We will choose exical gauge Az-D. The reasons for this is that reasonables gauge contiguration can be put into axial gauge with non-singular transformations, there is no need for ghost fields or subsidiary conditions other games use Ao=O gauge, and it can be shifted into other gauges for specific calculations.

We will also work in a finite but large 3-volume U and a finite but large truckidean time T. Doing this may actually gain us into as theories with many vary may be made revident by showing that rectain properties always depend on boundary cond. Plus, the condition wed to eliminate to in the canonical quantization is made unique.

The surface term for a gauge field throng is given by 85 = \frac{1}{2} \land 80 \text{n} \text{First t} \text{...}

We need to shoose a boundary condition consistent with A=0 and our semiclassical calculation from before. This near that only field configurations of definite winding number are allowed in the box and this turns out to be the only aspect of finite volume that survives in the continuum limit.

Thus for large enough boxes we can forget about boundary conditions and integrate only over all configurations of definite winding number of this is denoted by the integral FUT N=N SCAR & Sun.

FOR large times Typ, this should be FLV, T. T. J. E FLV, T. M. FUTZ, M. as the sub-boxes should be by enough that their boundary terms dent matter either.

thomover we want to find a solution with definite energy, which would have a purely multiplicative composition rule. Luckily we can make this happen with a Fourier transform $F(V,T,\theta) = \sum_{i=1}^{n} F(V,T,\eta) = V(\xi dA) e^{\frac{\pi}{2}} e^{iv\theta}$ $\longrightarrow F(V,T,\theta) = F(V,T,\theta) + F(V,T,\theta)$

Thus we identify F(V,T,E) as proportional to (O) = N' ([dn] = eint where
the & state represents a distinct vacuum state. So we have shown that there
is an infinite continuum of vacua, each with nearly identical action some
for an FF term.

The instanton rolutions

Now we can finally proceed with constructing the v=1 solutions for our gauge theory. Just as in previous talks, we can take this state and time v=1 antimortantans and get approximate solutions for other winding numbers out of v=n=1 (anti) instantons.

We will approximate $F(V,T,\theta)$ by summing over these configurations to get $(\theta | e^{HT}(\theta) | x \leq (Ke^{50})^{HT} (VT)^{HT} e^{i(n-7)\theta} | = \exp(2KVTe^{50}\cos\theta)$. Thus the energy density can be read off as $\frac{E(\theta)}{V} = -2K\cos\theta e^{50}$.

We can also calculate <0/77/F(x) F(x) [(b) = 1/7 (8x (0) Tr(FF) (b) by translational invariance = 167 (5CAA) v es eive = 167 (167 (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (167) (1

Note that this is independent of Vand T, and is imaginary a that the Michauski space quantity is quaranteed to be real.

Also, note the dependence on the differentiating the vacua.

From the Schwartz inequality, $\int d^4x \operatorname{Tr}[FF] = \int \int d^4x \operatorname{Tr}[FF] \int \int d^4x \operatorname{Tr}[FF] \geq \int d^4x \operatorname{Tr}[FF]$ Thus for any winding number we have $5 \geq \frac{8}{2} \operatorname{TV}$, where the equality holds when $F = \pm F$. If solutions to this exist, then they must be wining for the artion, $\pm = \operatorname{sign}(u)$, and are the only solutions we need to easilier here.

for v=1, we know that the field can be transformed to be $A_n = \frac{1}{9}f'' \partial_n f^{ort} + \mathcal{O}(\frac{1}{7})$.

For $f''' = \frac{xy + i \times v}{r}$. This is rotationally invariant in the sense that it may be reverted back with a gauge transformation, since $SO(4) \simeq SU(3) \times SU(3)$. Thus our solution should have the same property.

. We make the ansatz An= Ren't for aft with ROJED. Doing the algebra gives Ron= 10 for arbitrary po

We can then got now relations by applying symmetries of the Lagrangian to this one. The symmetries we have are scale transformations, rotalisms, spatial transformations, and goings transformations.

Scale transformations only change of the sizes of the instantion.

Rotalisms to gauge transformations are equivalent, give more general ties spatial transformations will give as now idulting the center of the instantion (center of this over to a).

Tortismal turns out to also be equivalent to gauge to translatival.

Tixed gauge still counts because we can still make global transformations.

Total of 8 parameters.

Approximate solutions for higher 12 have 81 parameters.

Evaluation of $K = \left(\frac{S_0}{24\pi}\right)^2 \left|\frac{\det\left(-\partial_t^2 + w^4\right)}{\det\left(-\partial_t^2 + w^2\right)^2}\right|^2$ for dA

tradly one last thing we might want to accomplish is evaluating K for this instanton. In this case, we know that So= \$2. There are 8 parameters -> 8 eyes modes -> factor of fixed. Instanton location integral is taken bear of and the integral over gaings transformations gives a constant factor. Thus only an integral over scale remains

thus Ele) = - cost. & # 18 5 de X(pM) where X is an unknown function and M is no notifical was needed to detine a renormalization condition. The \$5 is required by dimensional analysis.

Renormalization group analysis says that the combination of B. In M + Ely) is observable. Bits tron the 250 factor this times X to be AP 8+3, In pm = Apm 3. A is a difficult contact to obtain, so our final answer will be Ele = A cost = 84 g8 (ode (p. M 3 [1+8]))

getting large for large of meaning our small of expansion fails in the large of limit. It is tearlike that if we had exact intentor solutions that worked in high of regimes that this integral (and therefore K) would be firste.

ter 5003 the precedure for infantor follows very similarly. Cur 50(2) instanton can be used as a starting point by doing the 5 other gauge transformation. Then we would have not 3 but 7 parameters for gauge (to computer with this) the gives replaced with a given and grant of the province with the gives all instanton following for 5003.

Conclusion