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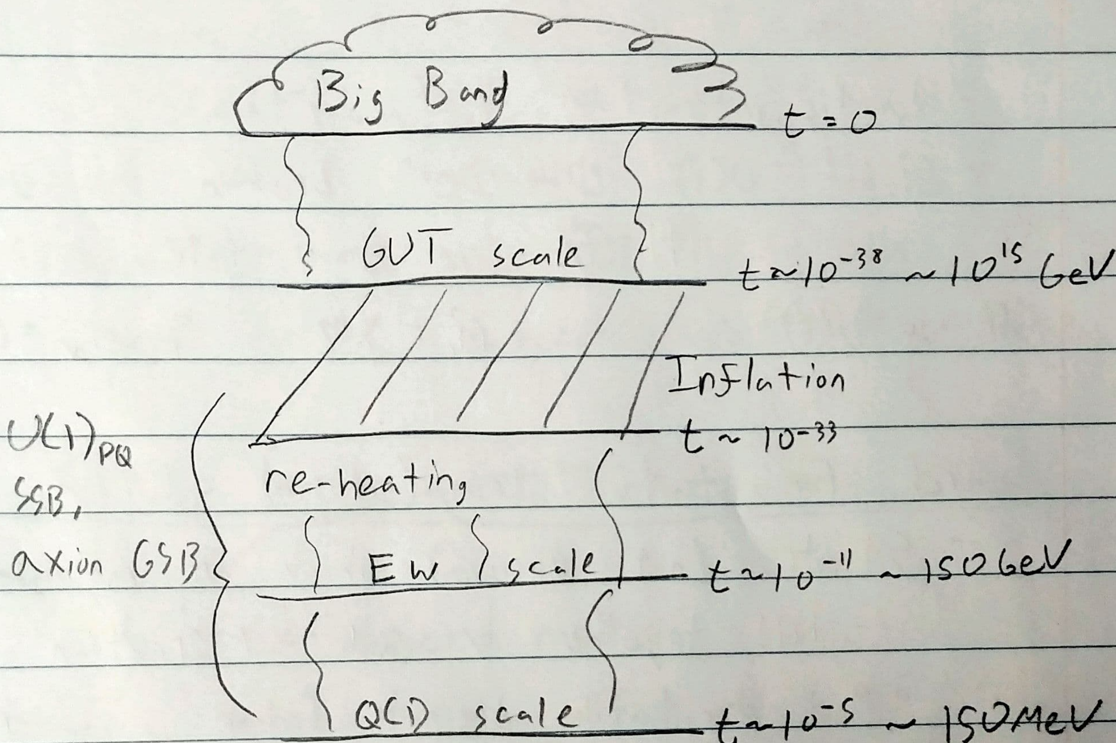
Cosmic String Formation & Evolution

- After the big bang the universe cooled and went through phase transitions



- Degenerate vacua at each phase transition
  - lead to DW, strings, monopoles $\pi_n(M) \neq 0$
  - "Regions of trapped excited phase"
  - Can be destroyed by further phase transitions
- Energy / Length of string ~ T_c^2
  - GUT string $1 r_\odot = 1 M_\odot$
  - EW string $1 r_\odot = 10$ mg
- high energy density causes matter to clump → galaxy/star formation
- 1994 Kibble used GUT string to explain CMB fluctuations ~ 10^{-5} (now ruled out)

Global $U(1)$ breaking & strings

- Simplest & relevent to axion, $\alpha(x)$

$$\mathcal{L}(\phi) = \partial_\nu \phi^* \partial^\nu \phi - V(\phi) \qquad V(\phi) = \frac{\lambda}{2}\left(|\phi|^2 - \eta^2/2\right)^2$$

- Quartic interaction & mass term
- Global $U(1)$ $\phi \to \phi e^{i\eta}$ broken by vacuum

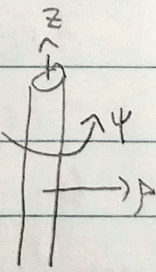
$= \frac{1}{\sqrt{2}}(\eta + s(x))e^{i\alpha(x)/\eta}$ $\qquad |\phi|^2 = \eta^2/2$ in ground state

SSB of $U(1)$ leads to $m_s = \lambda \eta^2$ & $m_\alpha = 0$

Search for static string (soliton) solutions

- States (not vacuum) which are non-dissipative
  - held together by self interaction
- Most excitations dissipate (stone in water)
- topological arguments provide sufficient condition

Here we construct explicit solution



Cylindrical ansatz: $\phi = \frac{\eta}{\sqrt{2}} f(m_s \rho) e^{in\psi}$

- Winds around & cylindrically symmetric

EL eqn from $\mathcal{L}$:

$$\left[\partial^2 + \lambda\left(|\phi|^2 - \eta^2/2\right)\right]\phi = 0$$

Plug in ansatz & write $\partial^2 = -\partial_t^2 + \nabla_{cyl}^2$

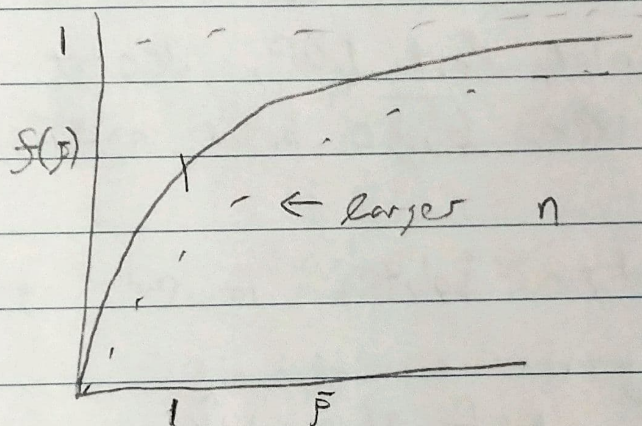
$$f'' + \frac{1}{\tilde{\rho}} f' - \frac{n^2}{\tilde{\rho}^2} f - \frac{1}{2}(f^2 - 1) f = 0$$

$\tilde{\rho} \equiv m_s \rho \qquad f' = \frac{df}{d\tilde{\rho}}$

Boundary conditions: $f(0)=0$ so Φ continuous
"local string" requires $f(\infty)=1$ (vacuum

by plugging in $f=1-\delta f$ as $\bar{\rho}\to\infty$ can show "
$f(\bar{\rho}) \sim 1 - n^2/\bar{\rho}^2$ "

Mathematica can't solve ODE but when $f^2=0$
bessel function, numerically integrate outwards



Energy density: $\mathcal{E} \simeq |\dot{\Phi}|^2 + |\nabla\Phi|^2 + V(\Phi)$

$|\dot{\Phi}|^2 \to 0$, $V(\Phi) \to 1/\rho^4$

asymptotically $\Phi \sim \frac{1}{\bar{\rho}^2}$ $\quad \left|\frac{\partial\Phi}{\partial\bar{\rho}} + \frac{1}{\bar{\rho}}\frac{\partial\Phi}{\partial\psi}\right|^2 \sim \left|\frac{1}{\bar{\rho}^3} + \frac{1}{\bar{\rho}}\left(1-\frac{n^2}{\bar{\rho}^2}\right)\right|$

$\to 1/\rho^2$

ψ gradient in energy is leading term, causes
E/L to diverge

$E/L = \int \rho\, d\rho\, d\psi\, \mathcal{E} \sim \pi n^2 \eta^2 \ln(M_s R) \qquad R \gg M_s^{-1}$

- most of energy away from core
- Natural cutoff when hits another string $\sim H^{-1}$

Local (gauge) strings

- Abelian higgs (Nielsen Olsen Strings)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

$$D_\mu = \partial_\mu + ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Gauge transformation $\phi \to \phi e^{i\Lambda(x)}$ $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \Lambda(x)$
  - Can choose $\Lambda(x) = -\alpha(x)$ phase DOF not physical.
  - Find SSB with $m_s = \eta^2 \lambda$ $m_v = e^2 \eta^2$
    - Complex scalar lost DOF, vector gained spin DOF, gauge boson ate GSB

Search for string (soliton) solution in radial gauge $A_\rho = 0$

$$\phi = \frac{\eta}{\sqrt{2}} f(m_v \rho) e^{in\varphi} \qquad A = \frac{n}{e\rho} a(m_v \rho) \hat{\varphi}$$

EL eqns give asymptotic:

$$f \sim \begin{cases} \bar{\rho}^{|n|} \\ 1 - \exp\left(-\frac{m_s}{m_v} \bar{\rho}\right) \end{cases} \qquad a \sim \begin{cases} \bar{\rho}^2 & \bar{\rho} \to 0 \\ 1 - \exp(-\bar{\rho}) & \bar{\rho} \to \infty \end{cases}$$

$\bar{\rho} \equiv m_v \rho$

- Exponentially localized
- A. in $\hat{\varphi}$, axial magnetic field concentrated to $\bar{\rho} < 1$
- Energy density exponentially localized

° Magnetic Flux is quantized in string

EL $\partial^\nu F_{\mu\nu} = j_\mu = \left(\frac{-1}{e}\partial_\mu \frac{\alpha(x)}{\eta} + A_\mu\right)\frac{e^2}{2}(\eta + s(x))^2$

Vanishing current → $A_\mu = \frac{1}{e}\partial_\mu \frac{\alpha(x)}{\eta} = \underbrace{\frac{1}{e\rho}\partial_\varphi \frac{\alpha(x)}{\eta}}$

Flux $\Phi = \int \rho\, d\rho\, d\varphi \underbrace{B}_{\nabla\times A}\cdot\hat{z} = \oint_{\rho\to\infty} \rho\, d\varphi$

$= \frac{2\pi n}{e}$ quantized!

Alternatively finite energy requires

$|D\phi|^2 = 0$ at ∞

$= \left|\frac{1}{\rho}\frac{\partial}{\partial\varphi}\phi + ieA_\varphi\right|^2 = 0$

$A_\varphi = \frac{i}{\rho e}\frac{\partial\phi}{\partial\varphi}$

String stability: $m_s < m_v$ stable Type I

$m_s > m_v$ $|n|>1$ strings decay into $|n|=1$ strings Type II

~~Imagine~~ Consider $n=2$ decay into 2 $n=1$

i.e. compare energy 2 $n=1$ on top of each other vs. slight displacement

Scalar field interaction is attractive
(minimize volume w/ $v(\phi) \neq 0$)
Gauge field is repulsive

range of interaction $\sim \frac{1}{m_s}$ or $\frac{1}{m_v}$ so
lighter particle dominates

$$E_B = \int d^2x |B|^2 : \quad \sim \int d^2x \; |2B_1(x)|^2 \quad \text{vs}$$
$$\int d^2x \; |B_1(x) + B_1(x+\epsilon)|^2$$

$$\int B(x)^2 \overset{?}{>} \int B(x) B(x+\epsilon)$$

yes! if $B(x)$ square integrable then this is like
in QM $\int \psi(x) \psi^*(x) \geq \int \psi(x) \phi(x)^*$

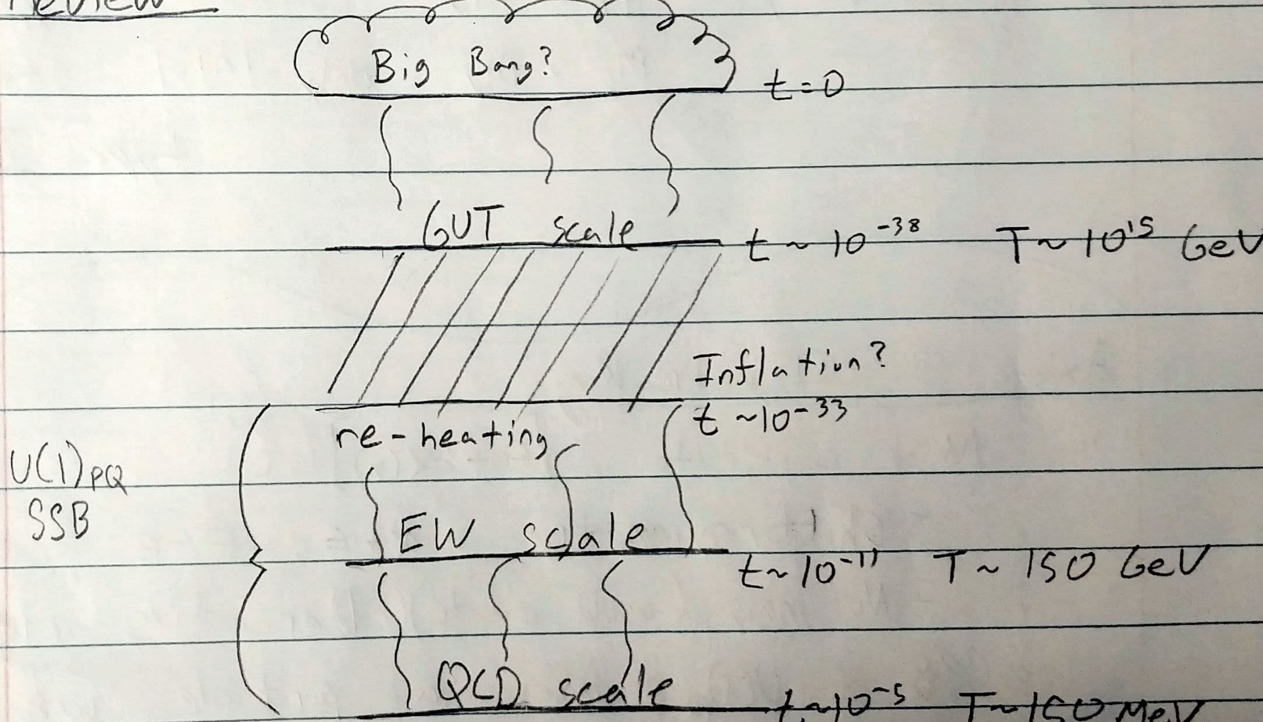
Talk 2

Outline:

- Review
- Axions, strong CP & $U(1)_{PQ}$ strings
- Attractive solution for cosmic string evolution

→ Superconducting axion strings

Review:



- Topology of degenerate vacua can lead to "strings" of excited phase (topologically protected)

– Global $U(1)$ strings
  - "localized" objects, approach vacuum as $1/\rho^2$
  - Enenergy "non-local" stored away from core

  → diverges logarithmically $E/L \sim \log(m_s R)$

  cutoff by distance to next string $\sim H^{-1}$

- Local (gauged) $U(1)$ strings
  - Complex scalar coupled to $U(1)$ gauge field
    - Abelian Higgs
  - Strings are exponentially localized $e^{-m_s \rho}$
  - Magnetic flux in string is quantized (classical!)

String stability: $m_s < m_v$ stable Type I

$m_s > m_v$ only $|n| = 1$ stable

type II

Axions & $U(1)_{PQ}$ breaking:

- Axion coupled to $[\theta + \alpha(x)] G\tilde{G}$
  - Shift symmetry $\alpha + \epsilon$, $\theta - \epsilon$
  - No mass (would violate). Derivative interactions
- SSB of $U(1)_{PQ}$ produces a particle which fits
- At QCD scale, instanton effects generate $V(\alpha) \propto \cos(\alpha)$,
  - mass $\sim \frac{\Lambda_{QCD}^2}{T_{PQ}}$ small

- axion DM abundance
- string implication for current obs.

° Model independent axion string dynamics!

Attractive solution:

° $U(1)_{PQ}$ broken in regime considered
- axion is massless
- After inflation (not predictive)

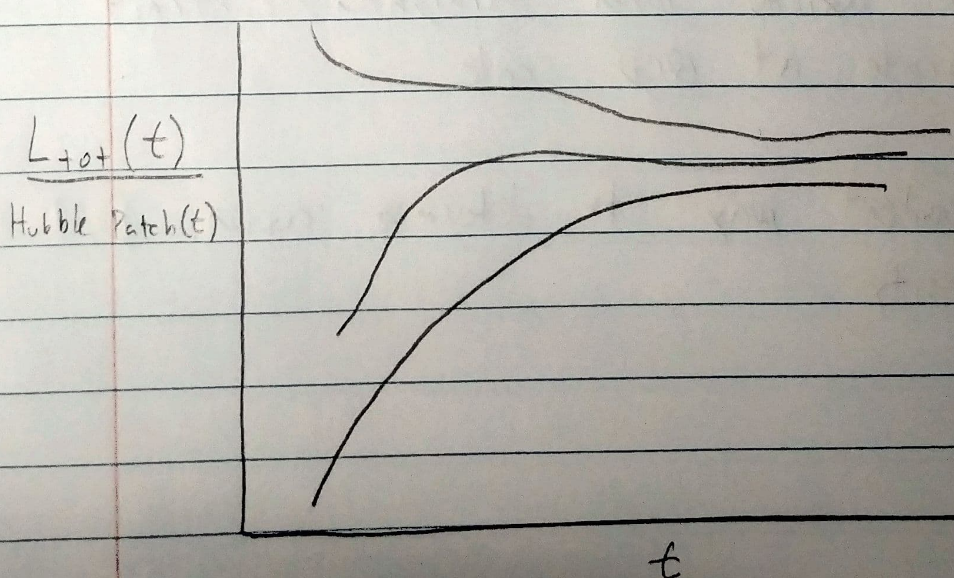
° Numerical simulations done by starting in broken phase w/ random field values & evolving w/ EL eqns.

$$V(\phi) = \frac{\lambda}{2}\left(|\phi|^2 - \frac{\eta^2}{2}\right)^2$$

$$\left[\partial^2 + 3H\partial_t + \lambda\left(|\phi|^2 - \eta^2/2\right)\right]\phi = 0$$

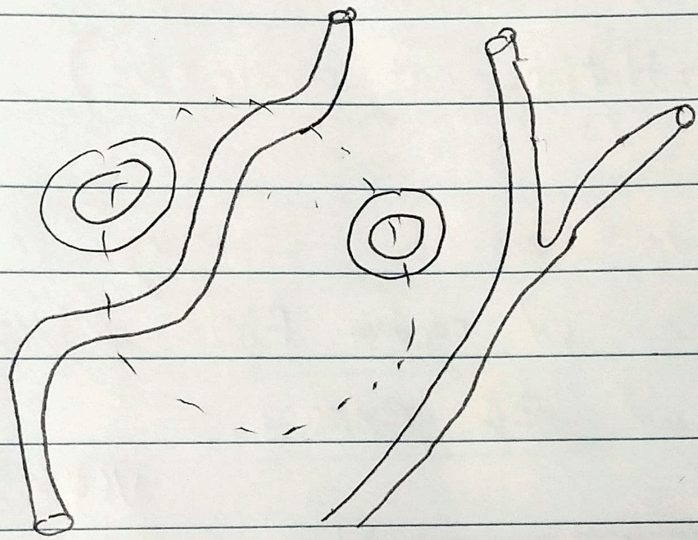
- extra term due to $g_{\mu\nu}(t)$

Many initial conditions evolve to attractive solution:



L_{tot} is total length of string

Physical motivation:



recombine

- As univere expands, hubble patch grows
- Also strings straighten out & loops contract
  - Net effect is they balance out
    - converge on universal behavior
    - radiate axions when straighten / contract
    - disintegrate at QCD scale

- Model independent way to determine axion DM density