Chiral Symmetry Breaking

Flavor Symmetry and Quark Condensate

Anomaly and t' Hooft Anomaly condition

Confinement implies chiral symmetry Breaking. for $\underset{\text{generalization}}{SU(3)}$ gauge group.

1°

Starting with massless fermions, the Lagrangian is

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \sum_{i=1}^{N_f} i\bar{\psi}_i \not{D}\psi_i$$

where $\not{D}\psi = \not{\partial}\psi - i\gamma^\mu A_\mu \psi$

If we use the chiral basis for Gamma matrices, $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$

$\sigma^\mu \equiv (1, \sigma^i)$ $\bar{\sigma}^\mu \equiv (1, -\sigma^i)$

$$\psi_i = \begin{pmatrix} \psi_{iL} \\ \psi_{iR} \end{pmatrix}, \quad \bar{\psi}\gamma^\mu\psi = (\psi_{iL}^\dagger, \psi_{iR}^\dagger)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\begin{pmatrix} \psi_{iL} \\ \psi_{iR} \end{pmatrix}$$

$$U(N_f)_L : \psi_{iL} \to L_{ij}\psi_{jL} \qquad U(N_f)_R : \psi_{iR} \to R_{ij}\psi_{jR}$$

Therefore the classical Lagrangian has the symmetry

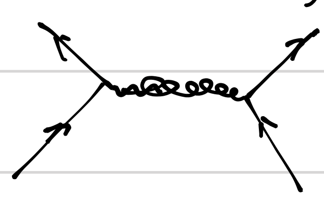
$$G_F = U(N_f)_L \times U(N_f)_R$$

However, the axial symmetry, where L, R transform differently, suffers anomaly.

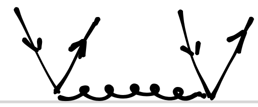
On Quantum level. the actual global symmetry is

$$G_F = SU(N_f)_L \times SU(N_f)_R \times U(1)_V$$

If we have a strong coupling theory.

$\Delta H_1 = g^2 \Big[$  $+ \cdots \Big]$

$\Rightarrow$ $q\bar{q}$ attract if they form color singlet.

$\delta H_2 = g^2 \Big[$  $+$  $+ \cdots \Big]$

provide matrix elements which mix the empty vacuum with $q\bar{q}$, which could possible lead to quark condensation

$$\langle \bar{\psi}_L \psi_R \rangle = -\sigma$$

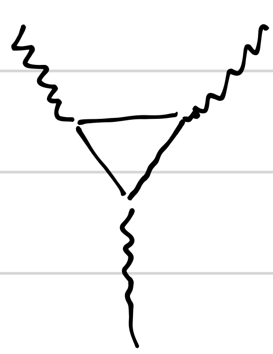
because the ground state is no longer invariant under flavor group, the symmetry is spontanneously breaking.

$$G_F = U(1)_V \times SU(N_f)_L \times SU(N_f)_R \to U(1)_V \times SU(N_f)_V$$

2°. t' Hooft Anomaly Condition.

It states that the calculation of any chiral anomaly for the flavor symmetry must not depend on what scale is chosen for the calculation if it is done by using the degrees of freedom of the theory at some energy scale.

One can prove that by introducing a gauge field which couples to the current related with this flavor symmetry.

 + other chiral fermions which only couple to this gauge field.

Gauge theory must be anomaly free to make any sense.

$$A_{IR} + A_S = A_{UV} + \underline{A_S} \text{ (scale independent).}$$

On the limit of zero coupling to this gauge field, we reproduce our theory with exactly the same mass spectrum.

$$A_{IR} = A_{UV}$$

$d^{abc} = \mathrm{Tr}[\lambda^a\{\lambda^b, \lambda^c\}]$ gives us anomaly.

$$\underbrace{\left(\sum_L - \sum_R\right)}_{\substack{\uparrow \\ \text{IR} \\ \text{degrees of freedom.}}} d^{abc}(r) = \underbrace{n_c}_{\text{color number.}} \big(d^{abc}(\underbrace{r_{0L}}_{\text{Fundamental representation of } G_F.}) - d^{abc}(r_{0R}) \big).$$

Q1: why is spin-$\frac{1}{2}$ particles so special.?

(Weinberg and Witten pointed out that one cannot form massless. bound states with $s \geq 1$)

Q2: why dont care about the caculation of Feynman intergral.

(Chiral anomaly just "care" massless parts, mass term mix them).

Q3: what do we mean by degrees of freedom in IR.

massless bounded fermions.

Reprensentation of Massless. Baryons

$$G_F = U(1)_V \times SU(N_f)_L \times SU(N_f)_R.$$

Both of these Weyl fermions have charge +1 under $U(1)_V$. For $SU(N_f)_L \times SU(N_f)_R$

Left-handed : $(N_f, 1)$.

Right-handed : $(1, N_f)$

Anomalies : $[SU(N_f)_L]^3$ and $[SU(N_f)_L]^2 \times U(1)_V$

what about $SU(N_f)_L \times [U(1)_V]^2$?

Rerepresentation for baryons:

L : $\boxed{L}\boxed{L}\boxed{L}$, $\begin{array}{c}\boxed{L}\\ \boxed{L}\\ \boxed{L}\end{array}$, $\boxed{L} \otimes \boxed{r}\boxed{r}$. $\begin{array}{ll}\boxed{L}\boxed{L}\\ \boxed{L}\end{array}$

R: $\boxed{R\,|\,R\,|\,R}$, $\begin{array}{|c|}\hline R\\ \hline R\\ \hline R\\ \hline\end{array}$, $\boxed{R} \otimes \boxed{L\,|\,L}$, $\begin{array}{|c|c|}\hline R & R\\ \hline R & \\ \cline{1-1}\end{array}$

column means antisymmetrization

row means symmetrization.

To be on the safe side, we use $|P_\alpha|$ to denote number of species of baryons.

$$\begin{cases} 27P_1 - 15P_3 = 3 \\ 15P_1 - 9P_3 + 6P_5 = 1 \end{cases}$$

No such solution could be found.

$N_f > 3$? Decoupling Massive Quarks.

when give a quark a mass. any baryon that contains this quark become massive (Vafa-Witten theorem)

$$U(1)_V \times SU(4)_L \times SU(4)_R \longrightarrow U(1)_V \times SU(3)_L \times SU(3)_R$$

explicitly broken $\qquad\qquad G_F'$

Any baryon can get a mass without breaking G_F' doesn't change t' Hooft Anomaly for G_F'.

G_F' should satisfy anomaly matching.

What about $N_f=2$?

$$[SU(2)]^2 \times U(1)_V$$

$$\Rightarrow 10 P_1 - 5P_3 + P_4 = 1$$

with many many solutions

We can't use t' Hooft anomaly to rule out massless baryon.

But lattice study shows that no such baryon could exist.