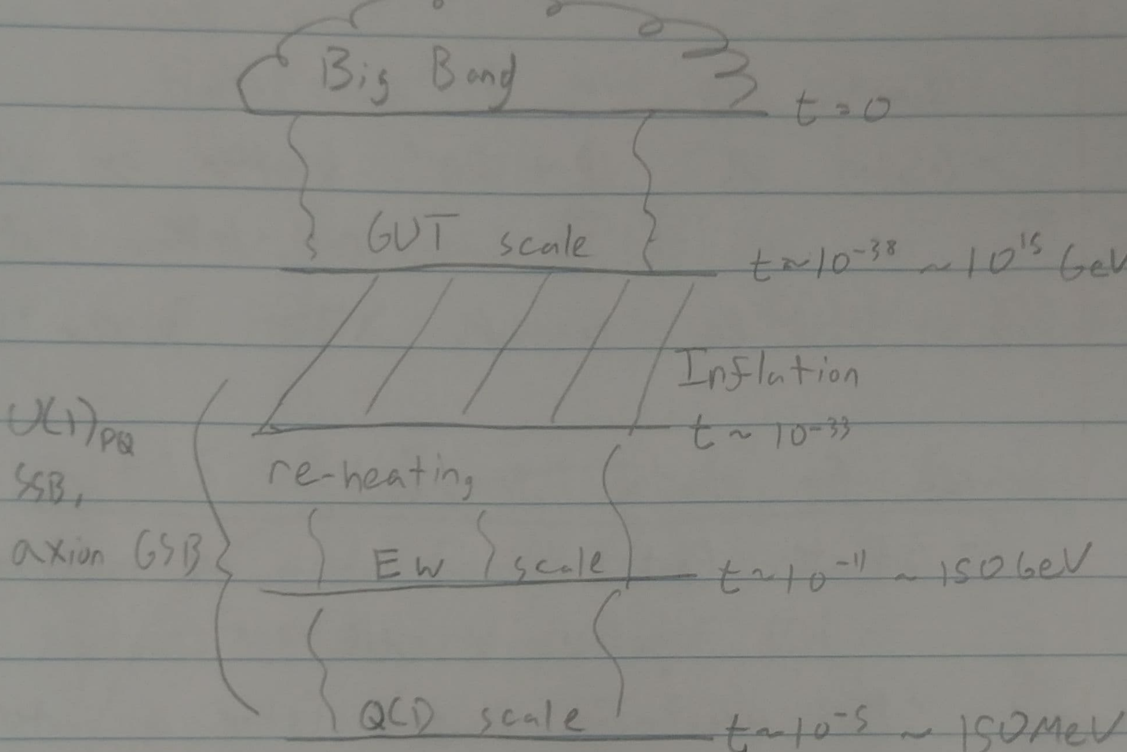
Cosmic String Formation & Evolution

- After the big bang the universe cooled and went through phase transitions



- Degenerate vacua at each phase transition
  - lead to DW, strings, monopoles $\pi_n(M) \neq 0$
  - "Regions of trapped excited phase"
  - Can be destroyed by further phase transitions
- Energy / Length of string ~ T_c^2
  - GUT string $1 r_\odot = 1 M_\odot$
  - EW string $1 r_\odot = 10 mg$
- high energy density causes matter to clump → galaxy / star formation
- ~~19~~ 1994 Kibble used GUT string to explain CMB fluctuations ~ 10^{-5} (now ruled out)

Global U(1) breaking & strings

- Simplest & relevent to axion, $\alpha(x)$

$$\mathcal{L}(\phi) = \partial_\nu \phi^* \partial^\nu \phi - V(\phi) \qquad V(\phi) = \frac{\lambda}{2}\left(|\phi|^2 - \eta^2/2\right)^2$$

- Quartic interaction & mass term
- Global U(1) $\phi \to \phi e^{i\eta}$ broken by vaccuum

$= \frac{1}{\sqrt{2}}(\eta + s(x))e^{i\alpha(x)/\eta}$ $\quad |\phi|^2 = \eta^2/2$ in ground state

SSB of U(1) leads to $m_s = \lambda \eta^2$ & $m_\alpha = 0$

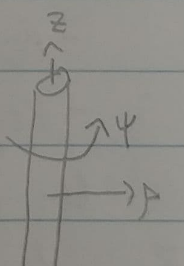
Search for static string (soliton) solutions

- States (not vacuum) which are non-dissipative
  - held together by self interaction
- Most excitations dissipate (stone in water)
- topological arguments provide sufficient condition

Here we construct explicit solution

Cylindrical ansatz: $\phi = \frac{\eta}{\sqrt{2}} f(m_s \rho) e^{in\psi}$

- Winds around & cylindrically symmetric



EL eqn from $\mathcal{L}$:

$$\left[\partial^2 + \lambda\left(|\phi|^2 - \eta^2/2\right)\right]\phi = 0$$

Plug in ansatz & write $\partial^2 = -\partial_t^2 + \nabla_{cyl}^2$

$$f'' + \frac{1}{\bar{\rho}} f' - \frac{n^2}{\bar{\rho}^2} f - \frac{1}{2}(f^2 - 1)f = 0$$

$$\bar{\rho} \equiv m_s \rho \qquad f' = \frac{df}{d\bar{\rho}}$$

Boundary conditions: $f(0)=0$ so Φ continuous

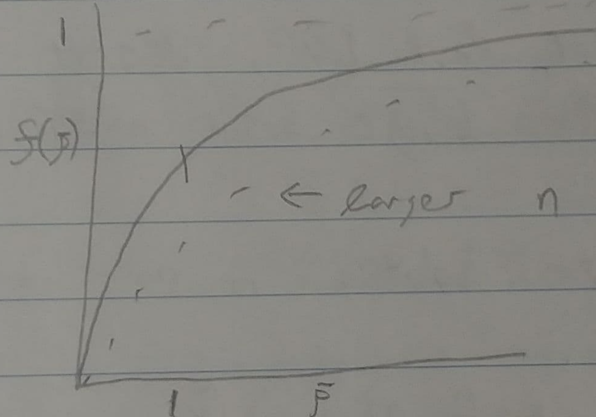
"local string" requires $f(\infty)=1$ (vacuum

by plugging in $f = 1 - \delta f$ as $\bar{\rho} \to \infty$ can show

$f(\bar{\rho}) \sim 1 - n^2/\bar{\rho}^2$

Mathematica can't solve ODE but when $f^2 = 0$

bessel function, numerically integrate outwards



Energy density: $\mathcal{E} \simeq |\dot{\Phi}|^2 + |\nabla\Phi|^2 + V(\Phi)$

$|\dot{\Phi}|^2 \to 0$, $\quad V(\Phi) \to 1/\rho^4$

asymptotically $\Phi \sim \frac{1}{\bar{\rho}^2}$ $\qquad \left|\frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho}\frac{\partial\Phi}{\partial\psi}\right|^2 \sim \left|\frac{1}{\rho^3} + \frac{1}{\rho}\left(1 - \frac{n^2}{\rho^2}\right)\right|$

$\hookrightarrow 1/\rho^2$

ψ gradient in energy is leading term, causes E/L to diverge

$E/L = \int \rho \, d\rho \, d\psi \, \mathcal{E} \sim \pi n^2 \eta^2 \ln(m_s R) \qquad R \gg m_s^{-1}$

- most of energy away from core
- Natural cutoff when hits another string $\sim H^{-1}$

Local (gauge) strings

- Abelian higgs (Nielsen Olsen Strings)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

$$D_\mu = \partial_\mu + ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Gauge transformation $\phi \to \phi e^{i\Lambda(x)}$ $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \Lambda(x)$
  - Can choose $\Lambda(x) = -\alpha(x)$ phase DOF not physical.
  - Find SSB with $m_s = \eta^2 \lambda$ $m_v = e^2 \eta^2$
    - Complex scalar lost DOF, vector gained spin DOF, gauge boson ate GSB

Search for string (soliton) solution in radial gauge $A_\rho = 0$

$$\phi = \frac{\eta}{\sqrt{2}} f(m_v \rho) e^{in\varphi} \qquad A = \frac{n}{e\rho} a(m_v \rho) \hat{\varphi}$$

EL eqns give asymptotic:

$$f \sim \begin{cases} \bar{\rho}^{|n|} \\ 1 - \exp\left(-\frac{m_s}{m_v} \bar{\rho}\right) \end{cases} \qquad a \sim \begin{cases} \bar{\rho}^{2} & \bar{\rho} \to 0 \\ 1 - \exp(-\bar{\rho}) & \bar{\rho} \to \infty \end{cases}$$

$\bar{\rho} \equiv m_v \rho$

- Exponentially localized
- A in $\hat{\varphi}$, axial magnetic field concentrated to $\bar{\rho} < 1$
- Energy density exponentially localized

° Magnetic Flux is quantized in string

EL $\partial^\nu F_{\mu\nu} = j_\mu = \left(\frac{-1}{e}\partial_\mu \frac{\alpha(x)}{\eta} + A_\mu\right)\frac{e^2}{2}\left(\eta+s(x)\right)^2$

vanishing current → $A_\mu = \frac{1}{e}\partial_\mu \frac{\alpha(x)}{\eta} = \underbrace{\frac{1}{e\rho}\partial_\varphi \frac{\alpha(x)}{\eta}}$

Flux $\Phi = \int \rho\, d\rho\, d\varphi\, \underset{\nabla\times A}{B}\cdot\hat{z} = \oint_{\rho\to\infty} \rho\, d\varphi$

$= \frac{2\pi n}{e}$ quantized!

Alternatively finite energy requires

$|D\phi|^2 = 0$ at ∞

$= \left|\frac{1}{\rho}\frac{\partial}{\partial\varphi}\phi + ieA_\varphi\right|^2 = 0$

$A_\varphi = \frac{i}{\rho e}\frac{\partial\phi}{\partial\varphi}$

String stability: $m_s < m_v$ stable Type I

$m_s > m_v$ $|n|>1$ strings decay into $|n|=1$ strings Type II

~~Imagine~~ Consider $n=2$ decay into 2 $n=1$

i.e. compare energy 2 $n=1$ on top of each other vs. slight displacement

Scalar field interaction is attractive
(minimize volume w/ $V(\phi) \neq 0$)
Gauge field is repulsive

range of interaction $\sim \frac{1}{m_s}$ or $\frac{1}{m_v}$ so
lighter particle dominates

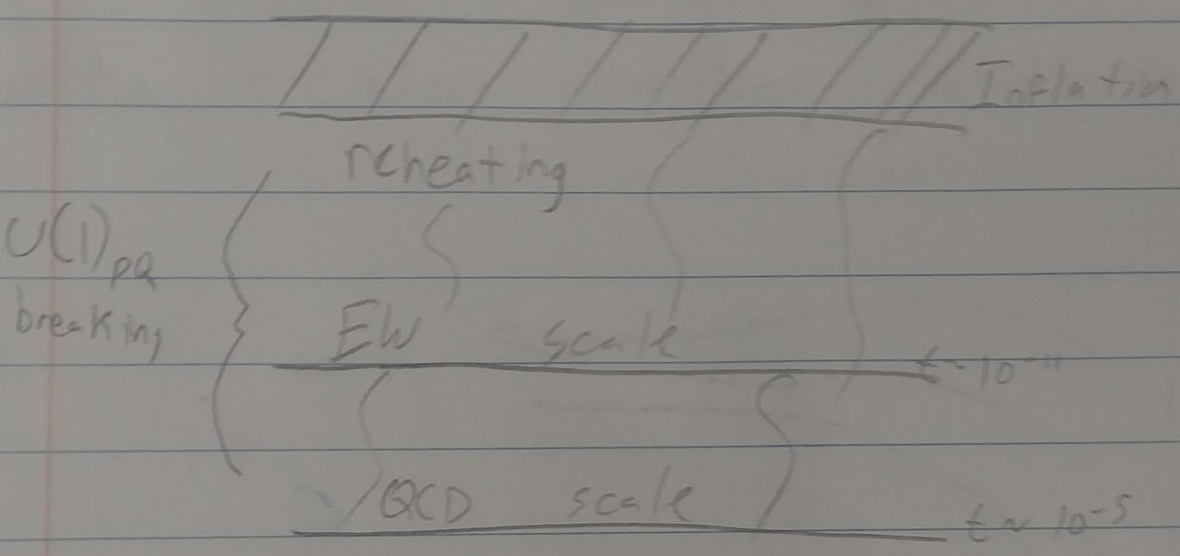
$E_B = \int d^2x |B|^2 : \quad \int d^2x \; |2B_1(x)|^2 \quad$ vs
$\int d^2x \; |B_1(x) + B_1(x+\epsilon)|^2$

$\int B(x)^2 \overset{?}{>} \int B(x) B(x+\epsilon)$

yes! if $B(x)$ square integrable then this is like
in QM $\int \psi(x) \psi^*(x) \geq \int \psi(x) \phi(x)^*$

Scaling (attractive) solution & axions

- Numerical evidence shows features of string evolution in early universe independent of initial conditions
  - For $U(1)_{PQ}$ strings (axions) allow prediction of DM abundance
  - Understand string implications for current observations



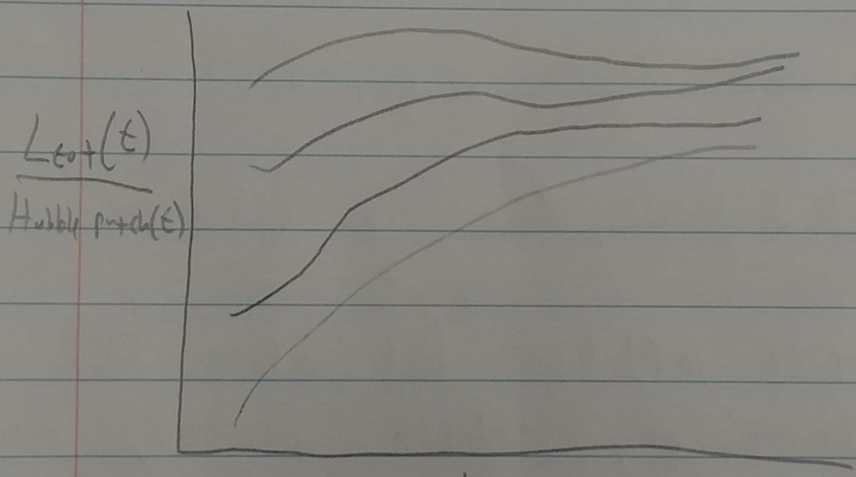
- $U(1)$ PQ after inflation but before QCD scale
  - if $U(1)_{PQ}$ broken during inflation no strings formed
- Want axion to be massless so has shift symmetry $\alpha \to \alpha + 2\pi$ since appears in $(\theta + \alpha) G\tilde{G}$
  - gains mass at QCD scale due to instanton effects $V(\alpha) \propto \cos(\alpha)$
  - in regime of consideration α massless

Numerical simulations done by starting in broken phase w/ random field values & evolve w/ EL:

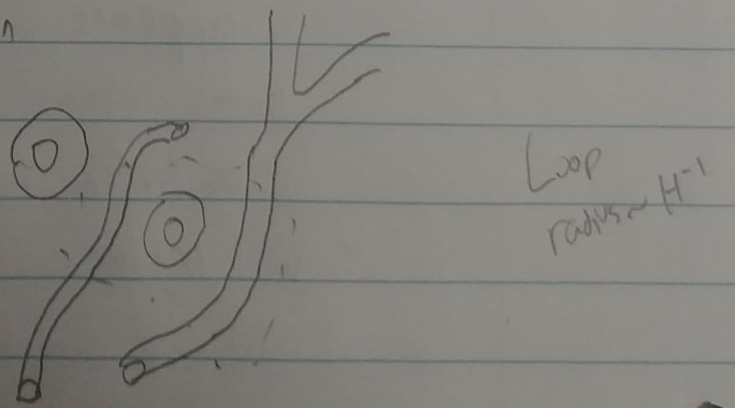
$$\left[\partial^2 + 3H\partial_t + \frac{m_s^2}{\eta^2}\left(|\phi|^2 - \frac{\eta^2}{2}\right)\right]\phi = 0$$

- extra term due to $g_{\mu\nu}(t)$

- Many initial conditions evolve to attractive solution



~~Motivat~~ physical motivation



Two competing forces:
$H^{-1}(t)$ grows so more strings but strings decay (loops contract) & strings straighten. These converge on universal behavior
- radiate axions