Propositional Logic: Semantic Reasoning

Source: Computational Logic Lecture Notes
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Review

- Computational Logic
  - **Propositional Logic:**
    - Sintax ⇒ Simple sentence, Compound Sentence
    - Semantics ⇒ interpretation, evaluation, reverse evaluation, types of compund sentence
  - Relational Logic

Review Deduction

- In deduction, the conclusion is true whenever the premises are true.

  Premise: p

  Conclusion: $(p \vee q)$

  Premise: p

  Non-Conclusion: $(p \wedge q)$

  Premises: p, q

  Conclusion: $(p \wedge q)$

Logical Entailment

- Validity, satisfiability, unsatisfiability: property of single sentence
- Logical Reasoning: relationship between sentences
- Given sentences ⇒ other sentence is a conclusion or non-conclusion?

Example of Logical Entailment

- Price of pertamax is up only if $ is up
- $ is up only if Rp is down
- Price of pertamax is up
  - Conclusion…. Rp is down

⇒ logical entailment

Logical Entailment

- $\Delta \models \phi$
  - Set of premises Δ logically entails a conclusion ϕ iff every interpretation that satisfies the premises also satisfies the conclusion
- Propositional Logic: **Propositional entailment**
- Example:
  - $\{p\} \models (p \vee q)$
  - $\{p\} \not\models (p \wedge q)$
  - $\{p,q\} \models (p \wedge q)$

Logical Entailment Checking

- Semantic reasoning:
  - Truth table
  - Validity checking
  - Unsatisfiability checking
- Proof Method
  - Rules of Inference
  - Axiom schemata
  - Propositional Resolution

Truth Table

1. **Create two interpretation tables: premise, conclusion**
   Example : {p} |= (p ∨ q) ?

p	p	q	p ∨ q
T	T	T	T
T	T	F	T
F	F	T	T
F	F	F	F

2. **In table-1: eliminate all rows that do not satisfy the premises**

p	p	q	p ∨ q
T	T	T	T
T	T	F	T
F x	F	T	T
F x	F	F	F

Truth Table (2)

3. In table-2: eliminates all rows that do not satisfy the conclusion

p	p	q	p ∨ q
T	T	T	T
T	T	F	T
F	F	T	T
F x	**F**	**F**	**F x**

4. If the remaining rows in table-1 are subset of remaining rows of table-2, then the premises logically entail the conclusion

∴ {p} |= (p ∨ q)

{p} |= (p ∧ q) ?

- Truth Table

p	p	q	p ∧ q
T	T	T	T
T	T	F	F x
F ×	F	T	F x
F ×	F	F	F x

∴ {p} |# (p ∧ q)

Truth Table (3)

- Advantage : easy to understand
  - Direct implementation of the definition of logical entailments
- Drawbacks:
  - Managing two tables
  - Solution ⇒ one table method (Validity checking and Unsatisfiability checking)

One Table Approach

- $\{\phi_1,\phi_2,\ldots,\phi_n\} \models \phi$
- Validity checking:

  $\phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n \rightarrow \phi$: valid
- Unsatisfiability checking :
  - $\phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n \wedge \sim\phi$: unsatisfiable

{p} |= (p ∨ q) ?

- Validity checking: **p ⟶ (p ∨ q)** valid ?

p	q	p ∨ q	p ⟶ (p ∨ q)
T	T	T	**T**
T	F	T	**T**
F	T	T	**T**
F	F	F	**T**

∴ p⟶(p ∨ q) valid ⇒ {p} |= (p ∨ q)

{p} |= (p ∨ q) ?

- Unsatisfiability checking: p ∧ ~(p ∨ q) ?

p	q	p ∨ q	p ∧ ~(p ∨ q)
T	T	T	**F**
T	F	T	**F**
F	T	T	**F**
F	F	F	**F**

∴ p ∧ ~(p ∨ q) unsatisfiable ⇒ {p} |= (p ∨ q)

Example

- If Mary loves Pat, then Mary loves Quincy. If it is Monday, then Mary loves Pat or Quincy. Prove that if it is Monday, Mary loves Quincy.
- Premises: $p \rightarrow q$, $m \rightarrow p \vee q$
- Conclusion: $m \rightarrow q$
- $\{p \rightarrow q, m \rightarrow p \vee q\} \models m \rightarrow q$?

Truth Table:
$\{p \rightarrow q,\ m \rightarrow p \vee q\} \models m \rightarrow q$?

m	p	q	p→q	m→p∨q
T	T	T	T	T
T	T	F	**F**	T
T	F	T	T	T
T	F	F	T	**F**
F	T	T	T	T
F	T	F	**F**	T
F	F	T	T	T
F	F	F	T	T

m	p	q	m→q
T	T	T	T
T	T	F	**F**
T	F	T	T
T	F	F	**F**
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

One Table approach

- $\{p \rightarrow q, m \rightarrow p \vee q\} \models m \rightarrow q$
- Validity checking:
  $(p \rightarrow q) \wedge (m \rightarrow p \vee q) \rightarrow (m \rightarrow q)$ valid ?

$(p \rightarrow q) \wedge (m \rightarrow p \vee q) \rightarrow (m \rightarrow q)$

m	p	q	(p → q)	∧	(m	→	p ∨ q)	→	(m → q)
T	T	T	T	T		T	T	T	T
T	T	F	F	F		T	T	T	F
T	F	T	T	T		T	T	T	T
T	F	F	T	F		F	F	T	F
F	T	T	T	T		T	T	T	T
F	T	F	F	F		T	T	T	T
F	F	T	T	T		T	T	T	T
F	F	F	T	T		T	F	T	T

One Table Approach (2)

- $\{p \rightarrow q, m \rightarrow p \vee q\} \models m \rightarrow q$
- Unsatisfiability Checking:
  $(p \rightarrow q) \wedge (m \rightarrow p \vee q) \wedge \sim(m \rightarrow q)$ unsatisfiable ?

$(p \rightarrow q) \wedge (m \rightarrow p \vee q) \wedge \sim(m \rightarrow q)$

m	p	q	(p-->q)	∧	(m	-->	(p∨q))	∧	~	(m-->q)
T	T	T	T	T	T	T	T	F	F	T
T	T	F	F	F	T	T	T	F	T	F
T	F	T	T	T	T	T	T	F	F	T
T	F	F	T	F	T	F	F	F	T	F
F	T	T	T	T	F	T	T	F	F	T
F	T	F	F	F	F	T	T	F	F	T
F	F	T	T	T	F	T	T	F	F	T
F	F	F	T	T	F	T	F	F	F	T

Semantic Reasoning

- Advantage : easy to understand
- Drawbacks:
  - Number of interpretation: 2^n ⇒ Enumeration of all interpretation is time and space consuming
  - Some logical constants are irrelevant to the conclusion ⇒ time consuming and increase checking complexity

Exercise 1:

Terdapat tiga orang yang menjadi tersangka sebuah kasus yaitu Ang, Beng, dan Cing. Ang berkata," Beng bersalah dan Cing tidak bersalah." Beng berkata," Jika Ang bersalah, maka Cing juga." Cing berkata," Saya tidak bersalah, tapi setidaknya salah satu dari Ang atau Beng yang bersalah."

Dengan menggunakan proposisi:

a: Ang tidak bersalah;

b: Beng tidak bersalah;

c: Cing tidak bersalah;

jawablah pertanyaan berikut.

Exercise 1 (con't):

a) Tuliskan kembali tiga kalimat yang diucapkan di atas dalam representasi propositional logic. Setiap kalimat menjadi satu kalimat proposisi.

b) Jika Ang, Beng, dan Cing tidak bersalah, tentukan siapa yang berbohong dan siapa yang jujur. Gunakan tabel kebenaran untuk mendapatkan jawabannya.

c) Jika seseorang yang tidak bersalah berkata jujur, dan yang bersalah berkata bohong, siapakah yang tidak bersalah? Gunakan tabel kebenaran untuk mendapatkan jawabannya, dengan memanfaatkan operator biimplikasi ($\leftrightarrow$).

Exercise 2

Terdapat kumpulan premis (fakta) sebagai berikut. Jika anda bekerja keras, maka anda beruntung. Anda beruntung atau anda bekerja keras. Jika anda beruntung, maka anda bukan anak yang suka bermain atau anda bekerja keras (tapi tidak keduanya). Anda anak yang suka bermain. Buktikan apakah kesimpulan "Anda bekerja keras" dapat diturunkan dari kumpulan premis tersebut, dengan menggunakan ***unsatisfiability dan validity checking***. Gunakan proposisi sebagai berikut:

- p: Anda bekerja keras ; q: Anda beruntung ; r: Anda anak yang suka bermain.

THANK YOU