

Proof Method & Propositional Resolution (Latihan 2)

1) Premis: $(\text{malas} \wedge \text{bolos}) \rightarrow \sim \text{lulus}$ Kesimpulan: $\sim \text{malas}$
 $\sim \text{lulus} \rightarrow \text{marah}$

$\text{bolos} \wedge \sim \text{marah}$

Buktikan $\{(\text{malas} \wedge \text{bolos}) \rightarrow \sim \text{lulus}, \sim \text{lulus} \rightarrow \text{marah}, \text{bolos} \wedge \sim \text{marah} \} \models \sim \text{malas}$

Jawab: gunakan inferensi

- | | | |
|----|--|--------------------------------|
| 1 | $(\text{malas} \wedge \text{bolos}) \rightarrow \sim \text{lulus}$ | (premis) |
| 2 | $\sim \text{lulus} \rightarrow \text{marah}$ | (premis) |
| 3 | $\text{bolos} \wedge \sim \text{marah}$ | (premis) |
| 4 | bolos | (simplification 3) |
| 5 | $\sim \text{marah}$ | (simplification 3) |
| 6 | $\text{lulus} \rightarrow \sim (\text{malas} \wedge \text{bolos})$ | (transposition 1) |
| 7 | $\sim \text{marah} \rightarrow \text{lulus}$ | (transposition 2) |
| 8 | lulus | (modus ponens 5 & 7) |
| 9 | $\sim (\text{malas} \wedge \text{bolos})$ | (modus ponens 6 & 8) |
| 10 | $\sim \text{malas} \vee \sim \text{bolos}$ | (de Morgan's Law 9) |
| 11 | $\sim \text{malas}$ | (disjunctive syllogism 10 & 4) |

2) Buktikan $\{p \rightarrow q, q \rightarrow r\} \models (q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$

a) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow ((q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p))$

p	q	r	$(p \rightarrow q)$	$(q \rightarrow r)$	$((p \rightarrow q) \wedge (q \rightarrow r))$	$((q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p))$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Karena valid, maka $\{p \rightarrow q, q \rightarrow r\} \models (q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$ ✓

b) $(p \rightarrow q) \wedge (q \rightarrow r) \wedge \neg ((q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p))$

p	q	r	$(p \rightarrow q)$	$(q \rightarrow r)$	$\neg ((q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p))$
T	T	T	T	T	F
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	F
F	T	F	T	F	F
F	F	T	T	T	F
F	F	F	T	T	F

(KIKY)

Karena unsatisfiable, maka $\{p \rightarrow q, q \rightarrow r\} \models (q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$ \square

c) premis $p \rightarrow q, q \rightarrow r$; konklusi $(q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$

(i) $p \rightarrow q$ premis (viii) $(p \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$ CR (vi)

(ii) $q \rightarrow r$ premis (ix) $(p \rightarrow \neg r) \rightarrow \neg p$ modus ponens (vi)(viii)

(iii) $(q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$ IS (ii) (x) $(q \rightarrow r) \rightarrow ((p \rightarrow \neg r) \rightarrow \neg p)$ IS (ix)

(iv) $p \rightarrow (q \rightarrow r)$ modus ponens (ii)(iii) \square

(v) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ ID (iv)

(vi) $(p \rightarrow q) \rightarrow (p \rightarrow r)$ modus ponens (iv)(v)

(vii) $p \rightarrow r$ modus ponens (i)(vi)

5) Buktilah set klausa berikut bersifat unsatisfiable!

a) $\{p, q\}, \{\neg p, r\}, \{\neg p, \neg r\}, \{p, \neg q\}$

(i) $\{p, q\}$ premis (v) $\{p\}$ (i), (iv)

(ii) $\{\neg p, r\}$ premis (vi) $\{\neg p\}$ (ii), (iii)

(iii) $\{\neg p, \neg r\}$ premis (vii) $\{\}$ (v)(vi) \square

(iv) $\{p, \neg q\}$ premis

b) $\{p, q, \neg r, s\}, \{\neg p, r, s\}, \{\neg q, \neg r\}, \{p, \neg s\}, \{\neg p, \neg r\}, \{r\}$

(i) $\{p, q, \neg r, s\}$ premis (vii) $\{q, s\}$ (i), (ii)

(ii) $\{\neg p, r, s\}$ premis (viii) $\{\neg q\}$ (iii), (vi)

(iii) $\{\neg q, \neg r\}$ premis (ix) $\{\neg r, \neg s\}$ (iv), (v)

(iv) $\{p, \neg s\}$ premis (x) $\{\neg s\}$ (vi), (ix)

(v) $\{\neg p, \neg r\}$ premis (xi) $\{s\}$ (vii), (viii)

(vi) $\{r\}$ premis (xii) $\{\}$ (x), (xi) \square

6) Premis: $p \rightarrow q, \sim p \rightarrow \sim t, \sim s \rightarrow \sim r, q \rightarrow r$; Konklusi: $t \rightarrow s$

Buktikan dengan propositional resolution!

$\Delta \cup \{\sim p\}$ unsatisfiable

$$\Delta = \{p \rightarrow q, \sim p \rightarrow \sim t, \sim s \rightarrow \sim r, q \rightarrow r\} \quad \{\sim p\} = \{\sim (t \rightarrow s)\}$$

$$I \{ \sim p \vee q, p \vee \sim t, s \vee \sim r, \sim q \vee r \}$$

$$I \{ \sim (\sim t \vee s) \}$$

$$O \{ \{ \sim p, q \}, \{ p, \sim t \}, \{ s, \sim r \}, \{ \sim q, r \} \}$$

$$N \{ t \wedge \sim s \}$$

$$O \{ \{ t \}, \{ \sim s \} \}$$

$$\Delta \cup \{\sim p\} = \{ \{ \sim p, q \}, \{ p, \sim t \}, \{ s, \sim r \}, \{ \sim q, r \}, \{ t \}, \{ \sim s \} \}$$

$$(i) \{ \sim p, q \} \text{ premis} \quad (v) \{ t \} \text{ premis} \quad (ix) \{ q \} (i), (vii)$$

$$(ii) \{ p, \sim t \} \text{ premis} \quad (vi) \{ \sim s \} \text{ premis} \quad (x) \{ \sim q \} (iv), (viii)$$

$$(iii) \{ s, \sim r \} \text{ premis} \quad (vii) \{ p \} (ii), (v) \quad (xi) \{ \} (ix), (x)$$

$$(iv) \{ \sim q, r \} \text{ premis} \quad (viii) \{ \sim r \} (iii), (vi)$$

Karena $\Delta \cup \{\sim p\}$ unsatisfiable maka $\{p \rightarrow q, \sim p \rightarrow \sim t, \sim s \rightarrow \sim r, q \rightarrow r\} \models t \rightarrow s$

9) a: Ang tidak bersalah; b: Bery tidak bersalah; c: Cing tidak bersalah

$$a) \sim b \wedge c, \sim a \rightarrow \sim c, c \wedge (\sim a \vee \sim b)$$

b) Premis: a, b, c; Cari kalimat yang benar dan tidak benar pada pola a.

a	b	c	$\sim b \wedge c$	$\sim a \rightarrow \sim c$	$c \wedge (\sim a \vee \sim b)$
T	T	T	F	T	T
					F

Kalimat benar: $\sim a \rightarrow \sim c$: Bery jujur

Kalimat tidak benar: $\sim b \wedge c, c \wedge (\sim a \vee \sim b)$: Ang dan Cing berbohong

c) Premis: $a \leftrightarrow (\neg b \wedge c)$, $b \leftrightarrow (\neg a \rightarrow \neg c)$, $c \leftrightarrow (c \wedge (\neg a \vee \neg b))$

a	b	c	$a \leftrightarrow (\neg b \wedge c)$				$b \leftrightarrow (\neg a \rightarrow \neg c)$				$c \leftrightarrow (c \wedge (\neg a \vee \neg b))$								
T	T	T	T	<u>T</u>	F	F	T	<u>T</u>	F	T	F	T	<u>F</u>	T	F	F	F	F	
T	T	F	T	<u>F</u>	F	F	F	T	<u>T</u>	F	T	T	F	<u>T</u>	F	F	F	F	
T	F	T	T	<u>T</u>	T	T	T	F	<u>F</u>	F	T	F	T	<u>T</u>	T	T	F	T	T
T	F	F	T	<u>F</u>	T	F	F	F	<u>F</u>	F	T	T	F	<u>T</u>	F	F	F	T	T
F	T	T	F	<u>T</u>	F	F	T	T	<u>F</u>	T	F	F	T	<u>T</u>	T	T	T	T	F
F	T	F	F	<u>T</u>	F	F	F	T	<u>T</u>	T	T	T	F	<u>T</u>	F	F	T	T	F
F	F	T	F	<u>F</u>	T	T	T	F	<u>T</u>	T	F	F	T	<u>T</u>	T	T	T	T	T
F	F	F	F	<u>T</u>	T	F	F	F	<u>F</u>	T	T	T	F	<u>T</u>	F	F	T	T	T

Ketika seluruh premis benar, a bernilai salah, b bernilai benar, dan c bernilai salah.

Sehingga yang tidak benar adalah B dan C