

# Relational Logic: Syntax & Semantic

Source: Computational Logic Lecture Notes  
Stanford University

IF1221 Computational Logic  
Semester II - 2024/2025

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# Review

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- Reasoning: information  $\Rightarrow$  conclusion
- Computational Logic
  - **Propositional Logic:**
    - Syntax  $\Rightarrow$  Simple sentence, Compound Sentence
    - **[something] is [anything]: [something]  $\Rightarrow$  [anything]**
    - Semantics  $\Rightarrow$  interpretation, evaluation, reverse evaluation, types of compound sentence
    - Logical Entailment :
      - Semantic Reasoning  $\Rightarrow$  Two tables, Validity Checking, Unsatisfiability Checking
      - Proof Method  $\Rightarrow$  Rules of Inference, Axiom Schemata, Propositional Resolution
  - **Relational Logic  $\Rightarrow$  Today**

# Propositional Logic vs Relational Logic

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- Constants refer to atomic propositions/ logical constants.

*raining    snowing    wet*

- Compound sentences capture relationships among propositions

*raining ∨ snowing ⇒ wet*

- How to represent general concepts??

In Propositional Logic:

- If Ali knows Budi, then Budi knows Ali ( $a \Rightarrow b$ )
- Ali knows Budi ( $a$ )
- Conclusion by Modus Ponens : Budi knows Ali ( $b$ )

What if we want to say something more general, such as If person I knows person II, then person II knows person I

In Relational Logic:  $\forall x,y (\text{knows}(x,y) \Rightarrow \text{knows}(y,x))$

# Relational Logic Syntax

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- Two new vocabularies:
  - Variables : begin with letters from the end of the alphabet
  - Constants:
    - *begin with either alphabetic letters (other than u,v, w, x, y, z),*
    - *mathematical characters (+, -, etc.),*
    - *or digits*
  - Example:
    - $u, v, w, x, y, z \Rightarrow ???$
    - $a, b, c, arthur, betty, cathy, 1, 2, \dots \Rightarrow ???$

# Relational Logic Syntax (2): Constants

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- Object constants refer to objects in the universe of discourse.
- Function constants denote functions.
  - father, mother, age, plus, times
- Relation constants refer to relations.
  - person, happy, parent, loves
- There is no syntactic distinction between object constants, function constants, and relation constants. The type of each such word is determined from context

# Relational Logic Syntax (3): Arity

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- The arity of a function constant or a relation constant is the number of arguments it takes.
  - Unary Function constants: *father1*, *mother*
  - Binary Function constants: *plus2*, *times2*
  - Ternary Function constants: *price3*
  - Unary Relation constants: *person1*, *happy1*
  - Binary Relation constants: *parent2*, *loves2*
  - Ternary Relation constants: *between3*
- The arity of a function constant or a relation constant is optionally notated as a subscript on the constant

# Relational Logic Syntax (4): Term

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- A term is either a variable, an object constant, or a functional term.
- Terms refer to items in the universe of discourse.
- Terms are analogous to noun phrases in natural language.

# Functional Terms

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- A functional term is an expression formed from an n-ary function constant and n terms enclosed in parentheses and separated by commas.
  - *fatherI(joe)*
  - *ageI(joe)*
  - *plus2(x,2)*
- Functional terms are terms and, as such, can be nested.
  - *plus2(ageI(fatherI(joe)),ageI(motherI(joe)))*

# Relational Logic Syntax (5): Sentences

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- There are three types of sentences:
  - Relational sentences - analogous to the simple sentences in Propositional Logic
  - Logical sentences - analogous to the compound sentences in Propositional Logic
  - Quantified sentences - sentences that express the significance of variables

# Relational Sentences

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- A relational sentence is an expression formed from an n-ary relation constant and n terms enclosed in parentheses and separated by commas.
  - $\text{happy}_1(\text{art})$
  - $\text{loves}_2(\text{art}, \text{cathy})$
- Relational sentences are not terms and cannot be nested in terms or relational sentences.

$\text{happy}_1(\text{person}_1(\text{joe})) \Rightarrow \text{WRONG}$

- Should be written:

$\text{happy}_1(\text{joe})$

$\text{person}_1(\text{joe})$

# Logical Sentences

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- Logical sentences in Relational Logic are analogous to those in Propositional Logic (Compound Sentences).
  - $\neg \text{loves}(\text{art}, \text{cathy})$
  - $(\text{loves}(\text{art}, \text{betty}) \wedge \text{loves}(\text{betty}, \text{art}))$
  - $(\text{loves}(\text{art}, \text{betty}) \vee \text{loves}(\text{art}, \text{cathy}))$
  - $(\text{loves}(x, y) \Rightarrow \text{loves}(y, x))$
  - $(\text{loves}(x, y) \Leftarrow \text{loves}(y, x))$
  - $(\text{loves}(x, y) \Leftrightarrow \text{loves}(y, x))$
- Parenthesization rules are the same as for Propositional Logic

# Quantified Sentences

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- Quantified sentences can be nested within other sentences.

$$\forall x.(apple(x) \vee \exists x.pear(x))$$
$$\forall x. \forall y.loves(x,y)$$

- Universally quantified sentences:
  - is used to assert that all objects have a certain property
  - $\forall x.p(x)$
- Existentially quantified sentences:
  - is used to assert that some object has a certain property
  - $\exists x.p(x)$

# Other Sentences

- A sentence is ground if and only if it contains no variables
  - Example:
  - Ground Sentence: *human(joe)*
  - Not Ground Sentence:  $\forall x.\text{human}(x)$
- A sentence is open if and only if it has free variables (iff it is not in the scope of a quantifier of that variable)
  - Example:
  - $P(y) \Rightarrow \exists x.q(x, y)$  - - > Open sentence since *y* is a free variable
  - $\forall y.(P(y) \Rightarrow \exists x.q(x, y))$  - - > Close sentence since *x* & *y* are bound

# Reminder

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- Variables
- Constants: Object, [Functional, Relational]  $\Rightarrow$  has arguments (n-ary)
- Term: variables, object constants, functional terms  $\Rightarrow$  noun phrase in natural language
- Sentence: relational, logical, quantified
- Functional Term vs Relational Sentence:
  - Functional terms *may be used within other functional terms.*
  - Functional terms *may be used within relational sentences.*
  - Relational sentences may *not be used in functional terms.*
  - Relational sentences may *not be used in relational sentences.*

# Natural Language Representation

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- $\forall x.P(x)$   
For all  $x$ ,  $x$  is  $P$
- $\sim \forall x.P(x)$   
Not all  $x$  are  $P$ .  
Some  $x$  are  $P$ .
- $\forall x.\sim P(x)$   
All  $x$  are not  $P$ .

# Natural Language Representation(2)

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- $\exists x.P(x)$   
Some  $x$  is  $P$
- $\exists x.\sim P(x)$   
Some  $x$  is not  $P$
- $\forall x. \exists y.P(x,y)$   
For all  $x$ , there is  $y$  such that  $P$

# Natural Language Representation(3)

- $\forall x.(P(x) \Rightarrow Q(x))$   
For all  $x$ , all  $P$  are  $Q$   
All  $P$  is  $Q$
- $\forall x.(P(x) \Rightarrow \sim Q(x))$   
For all  $x$ , no  $P$  are  $Q$   
No  $P$  is  $Q$
- $\exists x.(P(x) \wedge Q(x))$   
For some  $x$ ,  $x$  are  $P$  and  $Q$   
Some  $P$  is  $Q$
- $\exists x.(P(x) \wedge \sim Q(x))$   
For some  $x$ ,  $x$  are  $P$  and not  $Q$   
Some  $P$  is not  $Q$

# Examples

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- All human is mortal.
  - If a thing is human, then it is mortal.
$$\forall x.(\text{human}(x) \Rightarrow \text{mortal}(x))$$
- Purple mushrooms are poisonous.
  - If a thing is a purple mushroom, then it is poisonous.
  - If a thing is mushroom and it is purple, then it is poisonous.
$$\forall x.(\text{mushroom}(x) \wedge \text{purple}(x) \Rightarrow \text{poisonous}(x))$$

# Exercise 1

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- Given:
  - Object Constants: art, betty, cathy
  - Functional Constants: father<sub>1</sub>, mother<sub>1</sub>, age<sub>1</sub>, plus<sub>2</sub>, times<sub>2</sub>
  - Relational Constants: person<sub>1</sub>, sad<sub>1</sub>, reflexive<sub>1</sub>, parent<sub>2</sub>, loves<sub>2</sub>, friends<sub>2</sub>,
- Syntax Test:
  1. friends(father(art), betty)
  2. friends(mother(art), friends(father(art), betty))
  3. sad(person(cathy))
  4. loves(x,y)  $\Rightarrow$  loves(y,x)
  5. reflexive(z)  $\Rightarrow$  z(x,x)

## Exercise 2

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- Apakah ekspresi logika relasional di bawah ini legal atau tidak ? Jika tidak, jelaskan dimana letak kesalahannya dan mengapa, dengan ketentuan variabel dan konstanta sebagai berikut:
- *variables: x, y, z*
- *object constants: patrick, joe, kevinKW, PR, cemilan*
- *function constants: mother, anak, plus*
- *relational constants dengan aritas satu: hantu, biru, ramah, senang, sepupu*
- *relational constants dengan aritas dua: takutpada, suka, mengerjakan, teman*

## Exercise 2 (2)

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- (a)  $\neg \text{suka}(\text{joe}, \text{PR}) \vee \text{mengerjakan}(\text{joe}, \text{PR})$
- (b)  $\text{plus}(\text{joe}, z) \Rightarrow \text{suka}(\text{mother}(\text{kevinKW}), \text{joe})$
- (c)  $\text{hantu}(x) \wedge \text{ramah}(x) \Rightarrow \text{patrick}(x)$
- (d)  $\neg \text{senang}(z) \wedge \text{takutpada}(\text{kevinKW}, \text{anak}(\text{sepupu}(\text{patrick})))$
- (e)  $\text{biru}(\text{mother}(\text{anak}(\text{patrick}))) \vee \text{ramah}(\text{anak}(\text{mother}(\text{kevinKW})))$   
 $\Leftrightarrow \text{senang}(\text{joe})$
- (f)  $\text{suka}(\text{joe}, \text{cemilan}) \wedge \text{suka}(\text{kevinKW}, \text{cemilan}) \Rightarrow \text{teman}(\text{kevinKW})$
- (g)  $\text{takutpada}(\text{joe}, \text{sepupu}(\text{patrick}))$
- (h)  $\text{teman}(\text{patrick}, \text{joe}) \wedge \neg \text{takutpada}(\text{kevinKW}, \text{hantu}(\text{patrick}))$
- (i)  $\text{suka}(\text{kevinKW}, \text{mother}(x)) \wedge \text{teman}(\text{kevinKW}, x) \Rightarrow \text{hantu}(x) \wedge \text{biru}(y)$
- (j)  $\text{plus}(\text{mother}(\text{anak}(z)), \text{anak}(\text{anak}(\text{anak}(\text{cemilan})))) \wedge \text{plus}(\text{joe}, \text{kevinKW})$

# Relational Logic Semantics

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- **Review Propositional Logic Semantic:**

A Propositional logic *interpretation* is an association between the propositional constants in a propositional language and the truth values T or F

- The big question: what is a relational logic interpretation?
- There are no propositional constants, just object constants, function constants, and relation constants.
- To what do they refer?

# Introduction to Relational Logic Semantic

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- **Modeling the World**
  - Objects, Functions, Relations
  - Data
  - Models
- **Semantics of Relational Logic**
  - Atomic Sentences
  - Logical Sentences
  - Quantified Sentences

# Exercise 3: Translate into FOL

Relation Constants:

- person( $x$ )
  - femur( $x$ )
  - leg( $x$ )
  - eye( $x$ )
  - has( $x, y$ )
  - heart( $x$ )
  - part\_of( $x, y$ ):  $x$  part of  $y$
  - sinus\_rhythm( $x$ )
  - seeing( $x$ )
  - living( $x$ )
  - rhythm( $x$ )
  - regular( $x$ )
  - differ( $x, y$ ):  $x$  and  $y$  are different
- 
- a) **All femurs are part of some legs**
  - b) **All living hearts have a rhythm**
  - c) **Not all eyes are seeing**
  - d) **All people have two eyes**

# Exercise 4: Translate into FOL

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Relation Constants:

- $\text{person}(x)$
- $\text{child}(x)$
- $\text{parent}(x,y)$ :  $x$  is the parent of  $y$
- $\text{male}(x)$
- $\text{female}(x)$
- $\text{ancestor}(x,y)$ :  $x$  is the ancestor of  $y$
- $\text{sibling}(x,y)$
- $\text{differ}(x,y)$ :  $x$  and  $y$  are different

- a) **All people have two parents**
- b) **No person is both male and female**
- c) **All people have one male parent and one female parent**
- d) **One child is a sibling of another if they both have the same two parents**



**THANK YOU**

