Exercise 3

Solve the differential equation for the damped harmonic oscillator:

$$\ddot{x} = -\omega^2 x - \alpha \dot{x}$$

using the following methods:

- An explicit method (Euler forward),
- An implicit method (Euler backward),
- A semi-implicit method (Crank-Nicholson).

Compare your results with the exact solution and study how the error in the numerical solution varies with the time step Δt and the stability of each method.

NOTE: Formulate the problem as a system of two first-order differential equations:

$$\dot{x} = v$$
$$\dot{v} = -\omega^2 x - \alpha v$$