Exercise 4

Solve the diffusion equation

$$\frac{\partial T(x,t)}{\partial t} = D\frac{\partial^2 T(x,t)}{\partial x^2}$$

for the following cases:

1. Initial condition $T = 100$ (constant), and Dirichlet boundary conditions $T = 0$ at both ends of a domain of unit length. Use an explicit method with $D = 10^{-2}$.

2. Repeat the exercise, changing the boundary condition at only one of the ends to $T = 50$ and use the Crank-Nicolson method.

3. Now, decrease the coefficient to $D = 10^{-3}$ between $x = [0.4, 0.6]$ (this represents, for example, a zone within the conductor with a material with a higher specific heat) and repeat the previous case.

Show the results with several curves of $T(x)$ for different values of t or in an animation. Discuss and interpret the results, both from a numerical and physical perspective.