Exercise 5 (Optional)

Apply the Crank-Nicholson method to the time-dependent Schrödinger equation in 1D (assuming $\hbar = 1$ and $m = 1$):

$$i\frac{\partial \psi(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

for a free particle in an infinite potential well or for a quadratic potential (harmonic oscillator).

Note: You can consider that the problem is analogous to the diffusion equation but with an imaginary coefficient D and complex functions Ψ.