Exercise 6

When a frictional force, caused by the viscosity of the fluid in which a string is vibrating, is introduced, the wave equation takes the following form:

$$\frac{\partial^2 u}{\partial t^2} + \frac{2\kappa}{\rho}\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- Solve this problem using the Finite-Difference Time-Domain (FDTD) method (explicit method) for $0 \leq x \leq 1$, $0 < t \leq T = 40$, with $\rho = 0.01$ and $\kappa = 0.001$. Consider that the system starts from rest, with boundary conditions $u(0,t) = u(1,t) = 0$.

- Next, solve the same problem using an implicit method.

- Finally, if an additional source term is added, the resulting equation is known as the Telegraph Equation:

  $$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2u$$

  Modify the previous program to solve this equation and observe how the solution evolves with different initial conditions of the type $u(x,0) = \sin(n\pi x)$, where $n = 1, 2, \ldots$, which describe the normal modes.