牛顿一次迭代求Logistic解析解

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathsf{log}(1 + e^{-y^{(i)}\theta^T x^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} \mathsf{log}(h_\theta(y^{(i)}x^{(i)}))$$

- Solution:

$$\begin{aligned}
&\nabla J(\theta) = -\tfrac{1}{m}\textstyle\sum_{i=1}^{m} y^{(i)}x^{(i)}(1 - h_\theta(y^{(i)}x^{(i)})) \\
&H = \frac{\partial J}{\partial\theta\partial\theta^T} = \frac{\partial J}{\partial\theta}(\frac{1}{m}\sum_{i=1}^{m} y^{(i)}x^{(i)}h_\theta(y^{(i)}x^{(i)}))^T \\
&\qquad\qquad\quad = \frac{1}{m}\sum_{i=1}^{m} {y^{(i)}}^2 h_\theta(y^{(i)}x^{(i)})(1 - h_\theta(y^{(i)}x^{(i)}))x^{(i)}{x^{(i)}}^T \\
&\qquad\qquad\quad = \frac{1}{m}\sum_{i=1}^{m} h_\theta(y^{(i)}x^{(i)})(1 - h_\theta(y^{(i)}x^{(i)}))x^{(i)}{x^{(i)}}^T
\end{aligned}$$

- Iteration:

$$x := x - \frac{\nabla J(\theta)}{H} = x + \frac{\sum_{i=1}^{m} y^{(i)}x^{(i)}(1 - h_\theta(y^{(i)}x^{(i)}))}{\sum_{i=1}^{m} h_\theta(y^{(i)}x^{(i)})(1 - h_\theta(y^{(i)}x^{(i)}))x^{(i)}{x^{(i)}}^T}$$