

## Learning Theory

- Using learning theory, we can make formal statements or give guarantees on:
  - Expected performance of a learning algorithm.
  - Number of examples required to attain a certain level of test accuracy.
  - Hardness of learning problems in general.

## Bias, Variance and Model Complexity

### Def. Bias

The tendency to consistently learn the same wrong thing.

- The bias is error from erroneous assumptions in algorithm.
- High bias causes an algorithm to miss the relevant relations between features and outputs.

### Def. Variance

The tendency to learn random things irrespective of the real signal.

- The variance is error from sensitivity to small fluctuations in the training set.
- High variance causes an algorithm to model the random noise rather than the outputs.

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- Loss function for measuring errors between  $Y$  and  $\hat{f}(X)$

$$L(Y, \hat{f}(X)) = \begin{cases} (Y - \hat{f}(X))^2, & \text{squared error} \\ |Y - \hat{f}(X)|, & \text{absolute error} \end{cases}$$

- Test / generalized error  $\text{Err}_{\mathcal{D}} = \mathbb{E}[L(Y, \hat{f}(X)) \mid \mathcal{D}]$ , where  $\mathcal{D}$  denotes the training set.
- Expected prediction / test error  $\text{Err} = \mathbb{E}[L(Y, \hat{f}(X))] = \mathbb{E}[\text{Err}_{\mathcal{D}}]$ .
- Training error  $\overline{\text{err}} = \frac{1}{m} \sum_{i=1}^m L(y_i, \hat{f}(x_i))$

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- Simple model have high bias and small variance.
  - Complex models have small bias and high variance.
  - The bad performance (low accuracy on test data) could be due to either high bias (underfitting) or high variance (overfitting).

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- High bias: Both training and test error are large.
  - High variance: Small training error, large test error.