Overview Graphtheory

1 Basics

- Definitions: hypercube Q_n, Euler Tour, connected component, tree, forest, d-degenerate, edge contracting, girth, diameter, radius, minimum maximum average degree, subgraph, induced subgraph, bipartite, independent set, walk, closed walk, A-B-path, independent path, proper edge colouring (chromatic index $\chi'(G)$), proper vertex coloring $\chi(G)$
- Handshake Lemma
- Proposition: Bipartite iff no odd cycle
- Eulerian Tour Condition
- Tree Equivalences

2 Matchings

- Definitions: matching, vertexcover, k-factor, 1-factor (perfect matching), f-factor H-factor
- Halls's Marriage Theorem
- Kőnigs Theorem
- Tutte's theorem

3 Connectivity

- Definitions: k-connected, $\kappa(G)$, k-linked, cut set, cut vertex, cut edge, bridge, l-edge-connected, $\kappa'(G)$, block, block-cut-vertex graph
- $\kappa(G) \leq \kappa'(G) \leq \delta(G)$
- Menger's Theorem
- Global Version of Menger's Theorem
- Line graph, Beineke Theorem: Forbidden induced subgraphs of a line graph
- ear-decomposition
- Tutte: 3-connected graph from K_4
- Mader theorem: average degree $\geq 4k \Rightarrow k$-connected subgraph

4 Plane Graphs

- Definitions: faces, outer face, inner face, boundary of face, maximally plane, plane triangulation, planar embedding, planar graph, outerplanar, Minor, subdivision, topological minor
- Plane triangulation Theorem
- Euler's Formula
- upperbound for edges for plane graphs $(3n - 6)$
- upperbound edges triangle free plane graph $(2n - 4)$
- Kuratowski's Theorem
- 5-Color Theorem
- 5-List-Color Theorem

5 Coloring

- Definitions: clique number $\omega(G)$, co-clique number $\alpha(G)$, perfect graph (for each induced subgraph H: $\chi(G) = \omega(G)$)
- Greedy estimate for the chromatic number
- Brook's Theorem
- simple coloring results
- Mycielski's Construction
- Tutte's Construction
- (Kőnig 1916) G bipartite with maximum degree Δ then $\chi'(G) = \Delta$
- Vizing's Theorem

6 Extremal Graphtheory

- Definitions: extremal number ex(n,H), EX(n,H), Turan graph, complete r-partite graph, density $d(X, Y)$, ϵ-regular pair, ϵ-regular partition
- Mantel's theorem
- Turan's theorem
- (Erdős-Stone-Simonovits)
- Zarankiewicz function
- Kővari-Sos-Turan Theorem
- Szemeredis's Regularity Lemma
- Erdős-Stone Theorem

7 Ramsey theory

- Definitions: Ramsey number, asymmetric Ramsey, graph Ramsey, hypergraph Ramsey, induced Ramsey, anti-Ramsey
- Ramsey Theorem
- Applications of Ramsey theory
- Cvatal-Rödl-Szemeredi-Trotter

8 Random Graphs

- Definitions: Erdős-Renyi model, property, threshold function, probabilistic method,
- expected number of cycled of length k
- Erdős: $R(k,k) \geq 2^{k/2}$
- Erdős-Hajnal: For any integer $k \geq 3$ there is a graph with girth greater than k and chromatic number greater than k.

9 Hamiltonian Cycle

- Definitions: Hamilton cycle
- Necessary condition for existence
- Dirac Theorem

10 Flows

- Definitions: network, source, sink, capacity, network flow, cut, capacity of cut
- Ford-Fulkerson Theorem