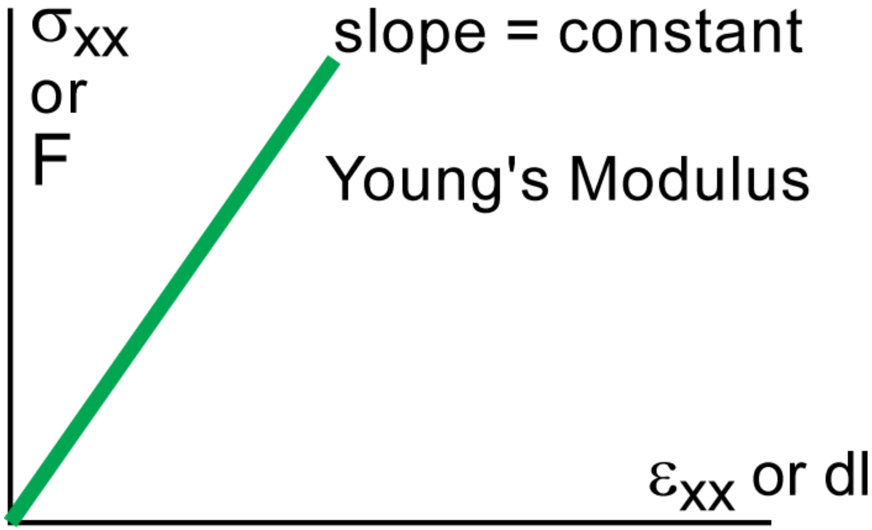
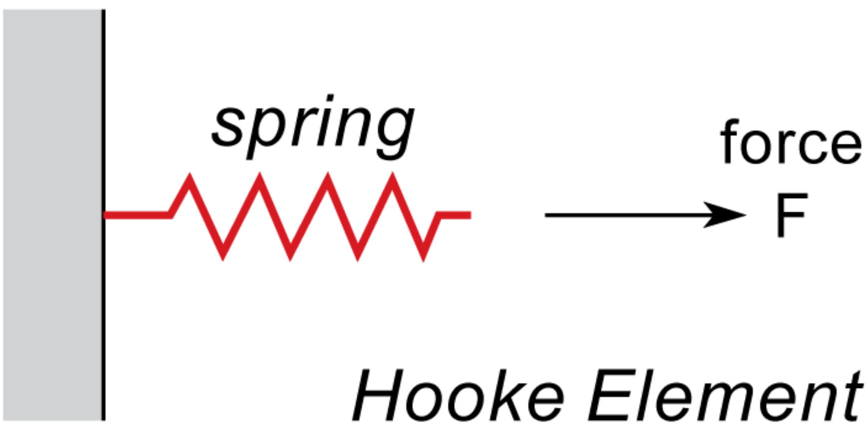
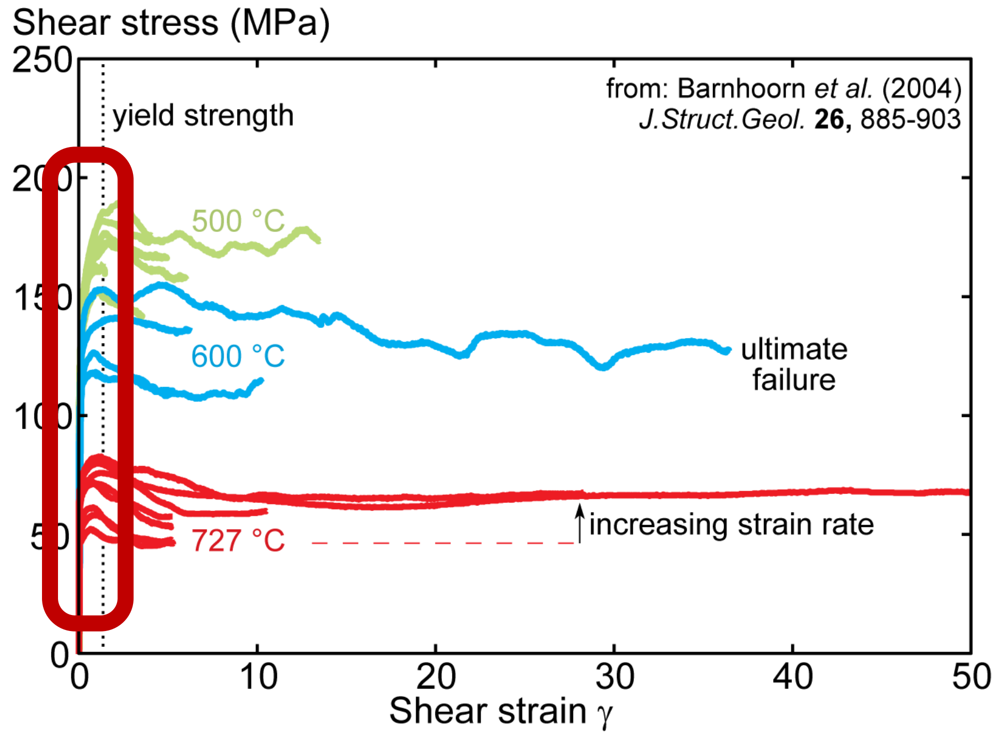
Elastic deformation



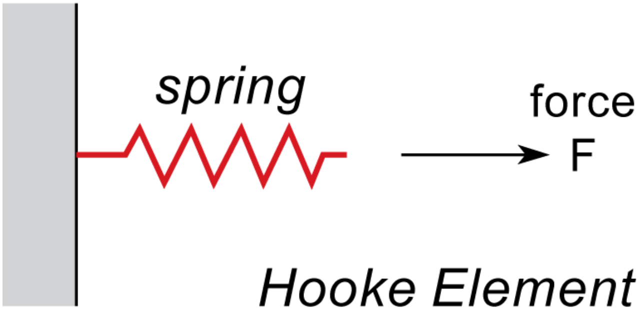




Rheological data from deformation experiments on Carrara marble under constant shear strain rate

Elastic deformation is active only for small strains in rock.

Elastic deformation



- no permanent deformation i.e. totally recoverable deformation

- linear relationship between stress and strain

$$\boxed{\sigma = \mathrm{E}\,\varepsilon = \mathrm{E}\left(\ell - \ell_0\right)/\ell_0}$$

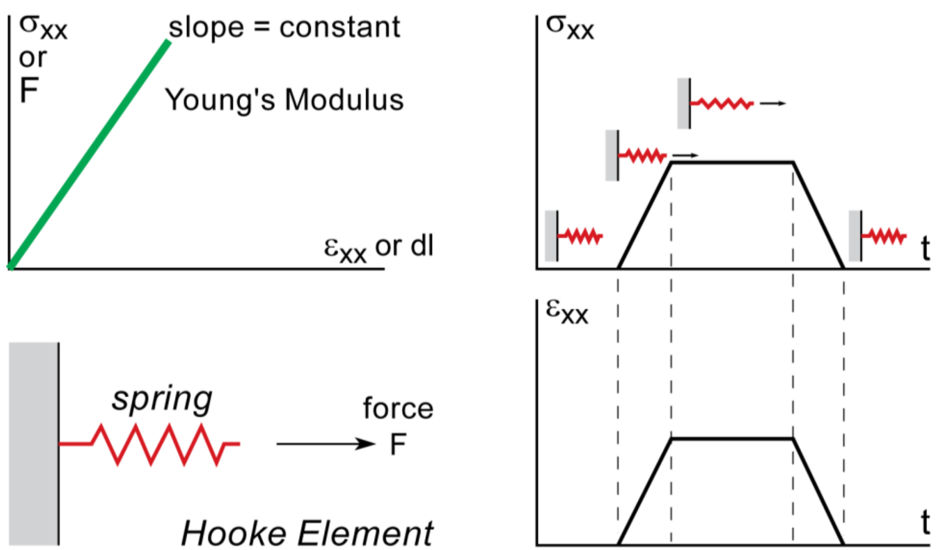
- **E = Young** or **Elasticity-Modul** (with the same dimension as stress [Pa])

$$\boxed{\sigma = E\,\varepsilon \quad \Leftrightarrow \quad [Pa] = [Pa][\;\;]}$$

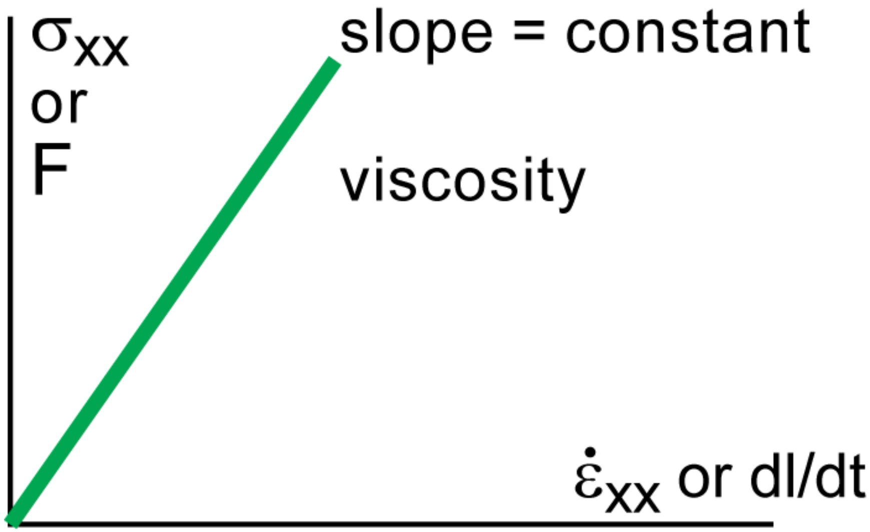
Elastic

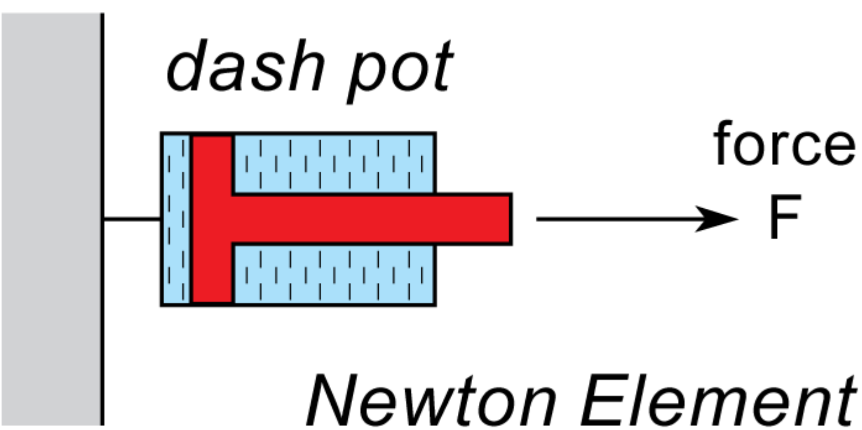
Elastic

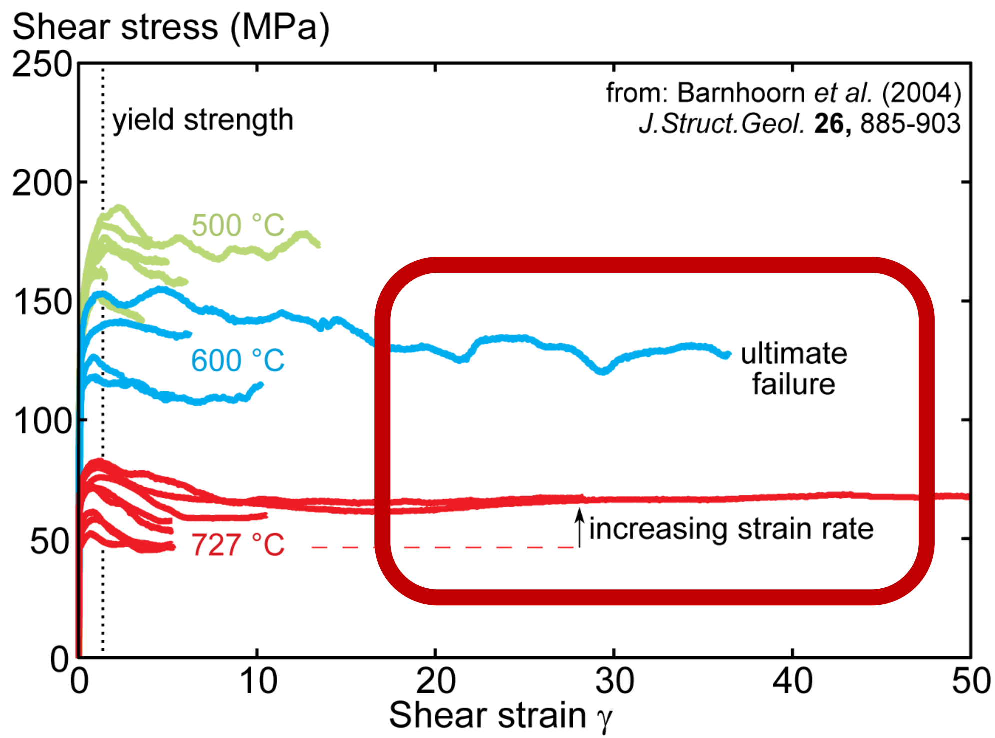
- Time independent
- Energy is stored
- Recoverable deformation
- Stress - strain



Viscous deformation







Rheological data from deformation experiments on Carrara marble under constant shear strain rate

Viscous deformation

- **Newton** or **ideal viscous** materials are able to undergo large permanent deformation, whose amount is a function of time.

$$\boxed{\sigma = \eta \dot{\varepsilon} = \eta \frac{dv_x}{dx}}$$

- Stress proportional to strain rate

- Proportionality constant η = **Viscosity**

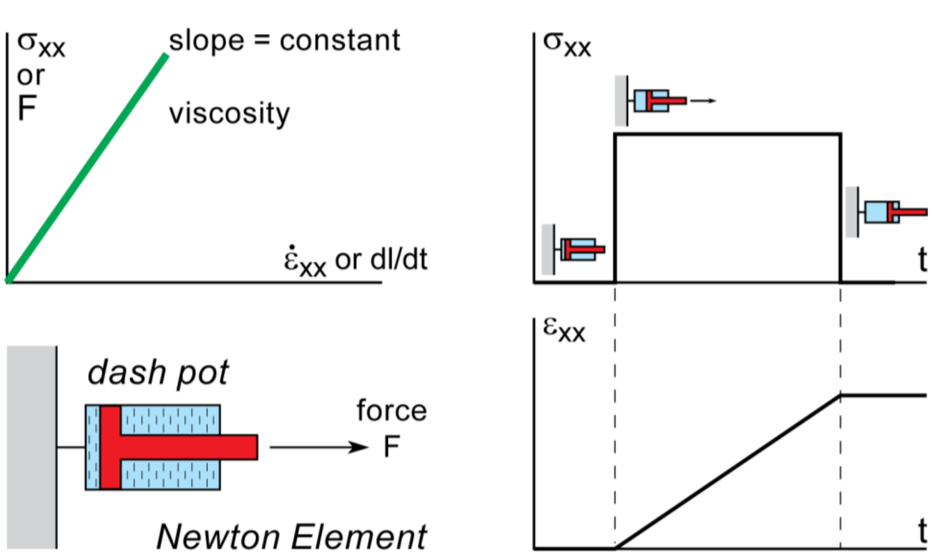
Unit : [Pa·s] = Dimension of stress multiplied by time

$$\boxed{\sigma = \eta \dot{\varepsilon} \quad \Leftrightarrow \quad [Pa] = [Pa \cdot s]\left[\frac{1}{s}\right]}$$

Viscous versus elastic

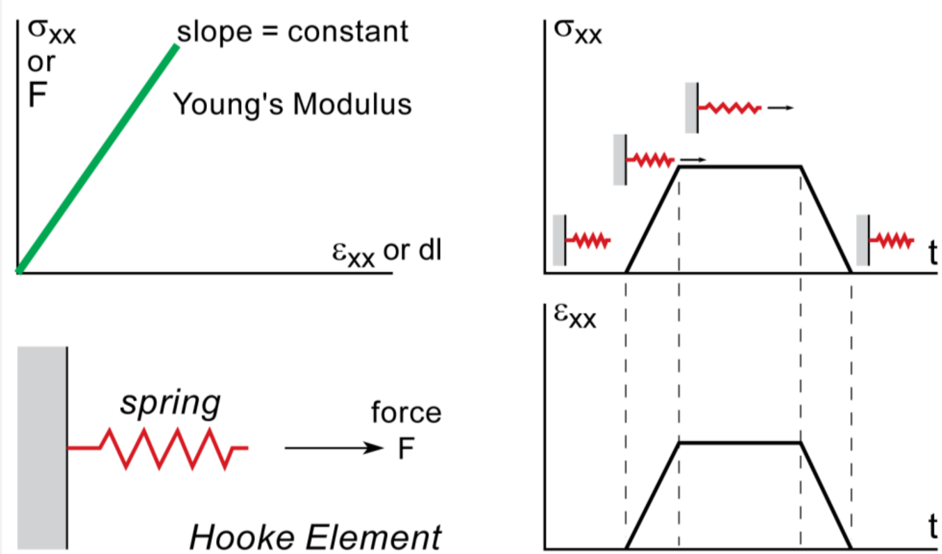
Viscous

- Time dependent
- Energy is dissipated
- Unrecoverable deformation
- Stress – strain rate



Elastic

- Time independent
- Energy is stored
- Recoverable deformation
- Stress - strain

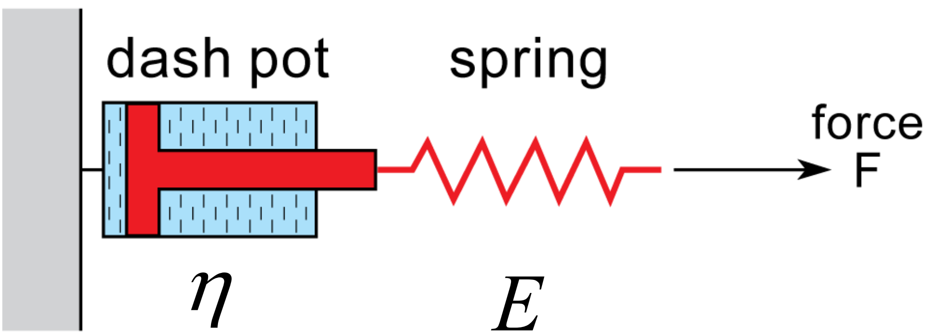


Viscoelastic deformation

- Most rocks are viscoelastic, i.e. they can deform like solids (elastic) and like fluids (viscous). Seismic waves through the mantle are elastic deformations but mantle convection is viscous deformation.
- Important is the time scale on which the deformation occurs.

Viscoelastic deformation

- One of the simplest models of a viscoelastic medium is the ***Maxwell model*** which is an viscous and elastic element connected in series.
- The stress in both elements must be the same but the strain or strain rate can be different.



$$\sigma = \eta \dot{\varepsilon}_v$$

$$\sigma = E\varepsilon_e$$

$$\dot{\sigma} = E\dot{\varepsilon}_e$$

$$\varepsilon = \varepsilon_e + \varepsilon_v$$

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_v$$

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

Viscoelastic deformation

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \quad \Leftrightarrow \quad \dot{\sigma} = -\frac{E}{\eta}\sigma + E\dot{\varepsilon}$$

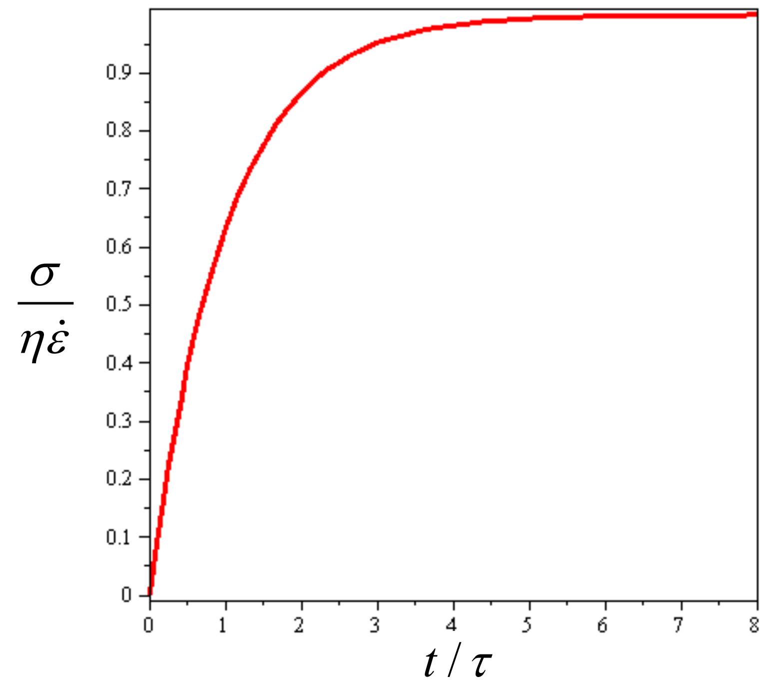
$$\frac{dy}{dt} = ay + b$$

- Deformation under constant strain rate.
- Solve rheological equation for stress (σ(t=0)=0):

$$\sigma = \eta\dot{\varepsilon}\left(1 - \mathrm{e}^{-\frac{E}{\eta}t}\right)$$

$$\frac{\sigma}{\eta\dot{\varepsilon}} = 1 - \mathrm{e}^{-\frac{t}{\tau}}$$

$$\boxed{\tau = \frac{\eta}{E}}$$

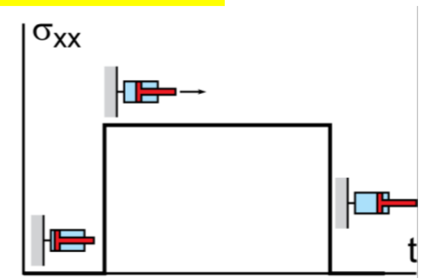


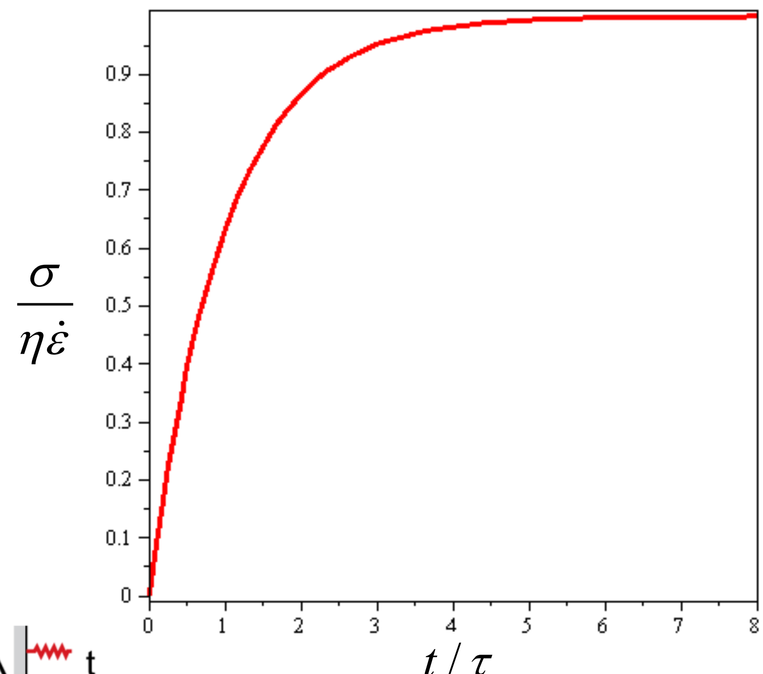
Viscoelastic deformation

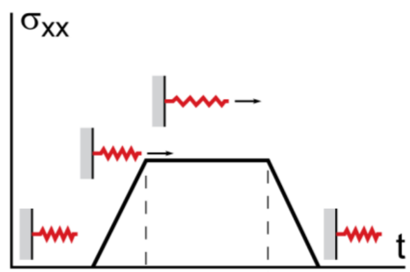
- Maxwell relaxation time: $$\tau = \frac{\eta}{E} \Leftrightarrow [s] = \frac{[Pa \cdot s]}{[Pa]}$$
- Deborah number: $$De = \frac{\tau}{t} = \tau\dot{\varepsilon}$$

„The mountains flowed before the Lord“
Judges, 5:5

- $De >> 1 \Rightarrow$ elastic
- $De << 1 \Rightarrow$ viscous







Viscoelastic deformation

- Maxwell relaxation time: $\tau = \dfrac{\eta}{E}$

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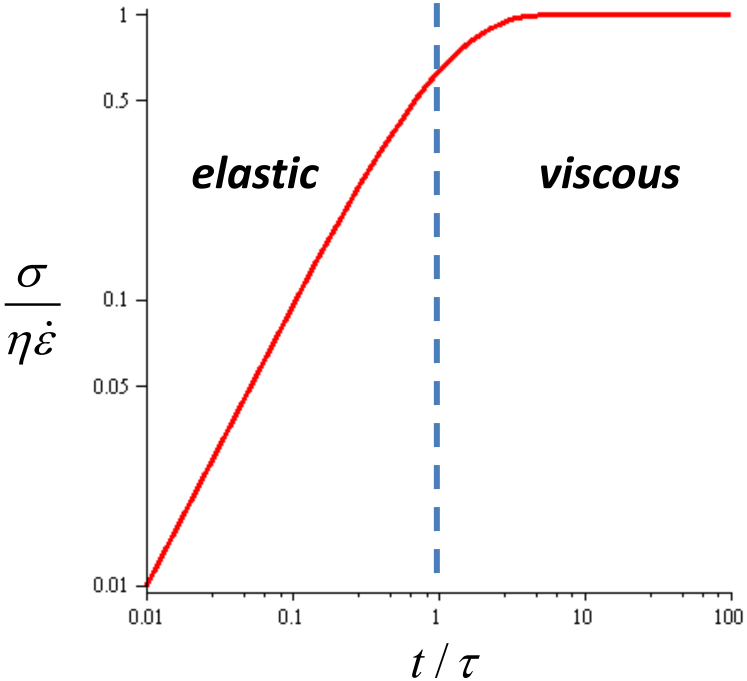

Viscoelastic deformation under constant bulk strain rate