EE 678

Midsem

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I Question 1

§ Part A - Using the autocorrelation approach for the lowpass analysis filter, generate the Daubechies' analysis lowpass filters of as many lengths as you can

The autocorrelation function runs from – (L - 1) to (L - 1)

$$K_0(z) = \sum_{n=-(L-1)}^{L-1} k_0[n] \, . \, z^{-n}$$

Since, $K_O(z)$ is an even function, $K_O(n) = K_O(-n)$ ∴ $K_O(z)$ for length L must have L zeroes at z = -1 i.e it will have a factor of the form $(1+ z^{-1})^L$. Therefore we get L equations which are given as follows -

$$K_0(-1) = 0$$

$$\frac{dK_0(z)}{dz} = 0 \text{ (at z = -1)}$$

$$\frac{d^2K_0(z)}{dz^2} = 0 \text{ (at z = -1)}$$

$$\vdots$$

$$\frac{d^{L-1}K_0(z)}{dz^{L-1}} = 0 \text{ (at z = -1)}$$

From these L Homogeneous equations only $\frac{L}{2}$ equations are independent and we find using the function `rref(A)` in matlab.On solving these equations we get $k_0, k_1, \ldots k_{L-1}$.
Then
We find the zeros of $K_0(z)$ which are not equal to -1. zeros are present in the form of reciprocal pairs. So we include only those L/2 zeros which lie within the unit circle to make the $H_0(z)$. We

have now obtained the zeroes of the polynomial which we expand to get the filter coefficients α_i of $H_0(z)$

Last step is to find the normalizing coefficient of $H_0(z)$ which find using the following constraint

$$\sum_{i=0}^{L-1} \alpha_i^2 = 1$$

The fourier transform of scaling function and DTFT of $H_0(z)$ is related as follows

$$\widehat{\phi}(\omega) = [\prod_{m=1}^{\infty} 0.5\, H(e^{j\omega 2^{-n}})] . \widehat{\phi}(0)$$

Where,

$$H(e^{j\omega}) = \sum_{n=0}^{L-1} h[n]. e^{-j\omega n}$$

By analyzing the time domain of the above equation we get the following result,

$$\phi(t) = c_0 \times [\, h(2t) * h(4t) * h(8t) .\ .\ .\ .\,]$$

Therefore to replicate the above continuous time equation to the digital domain we use the following code -

```
for i=(1:17)
   x=conv(double(real(upsample(x,2))),double(real(poly)));
   x=x(1:end-1);
   t=(0:1/2^(i+1):(L-1)*(1-1/2^(i+1)))'
   if (i==1 || i==2 || i==3 || i==5 ||  i==10 || i==15 || i==17)
       plot(t,x*1.2/max(x));
   end
end
```

We define β_i's as follows, $i \in [0, L - 1]$

$$\beta_i = (-1)^i \alpha_{L-1-i}.$$

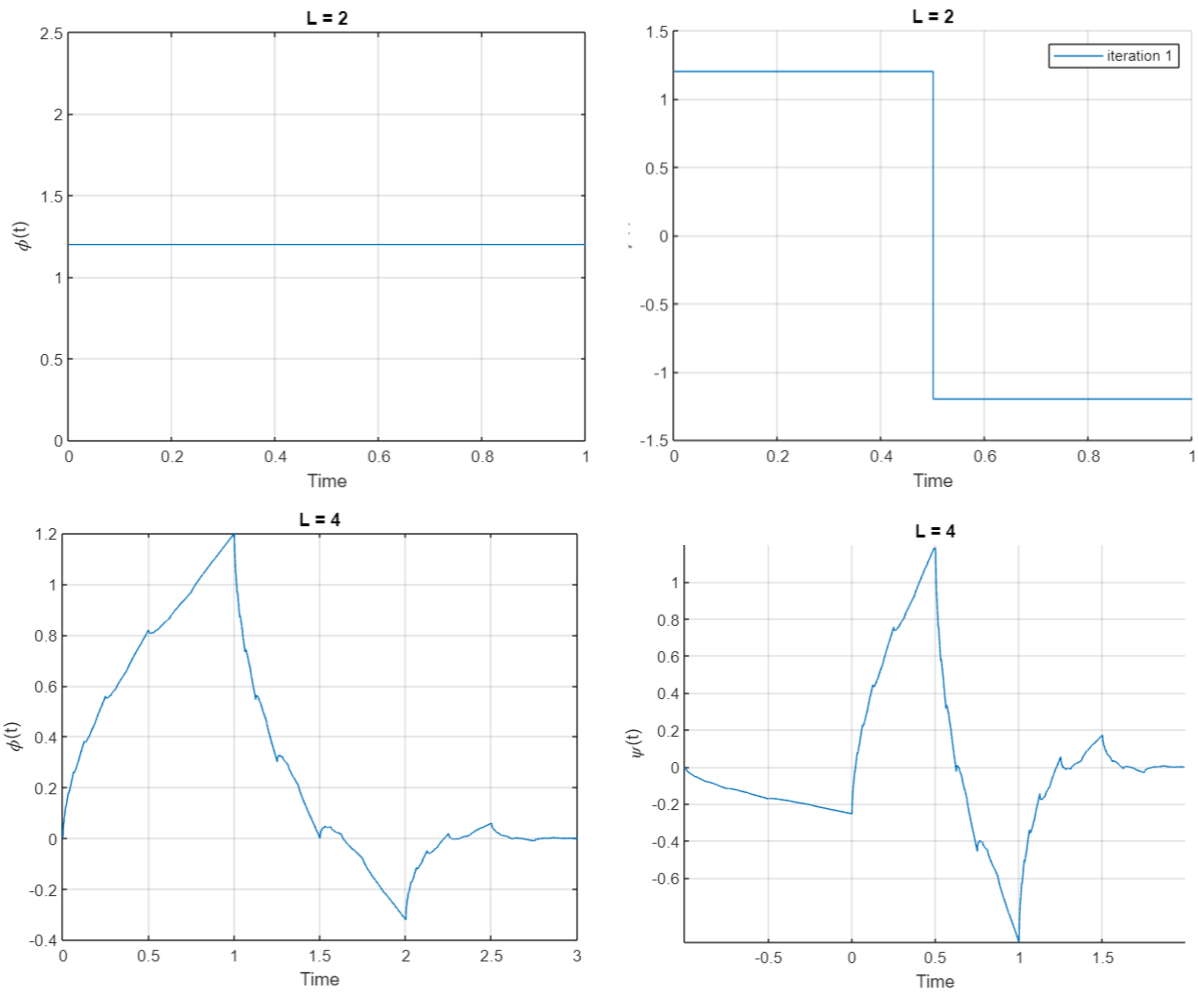
Now we construct the wavelet function as follows

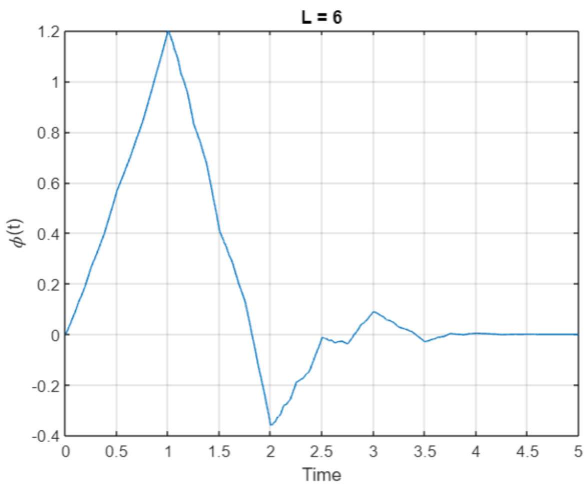
$$\psi(t) = \sum_{i=0}^{L-1} \beta_i \,.\, \phi(2t - i)$$

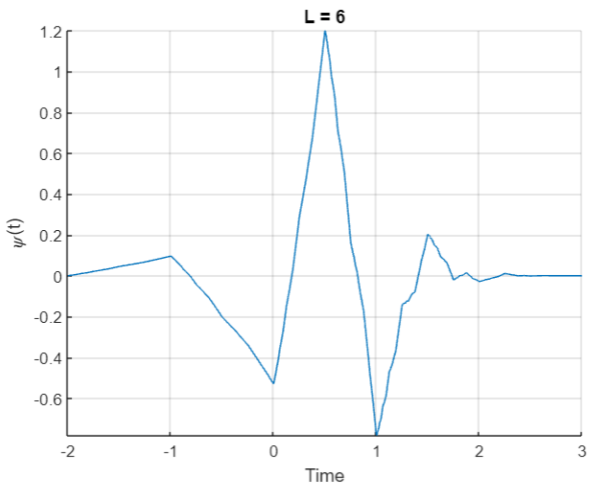
The values of filter coefficients for various L are as follows

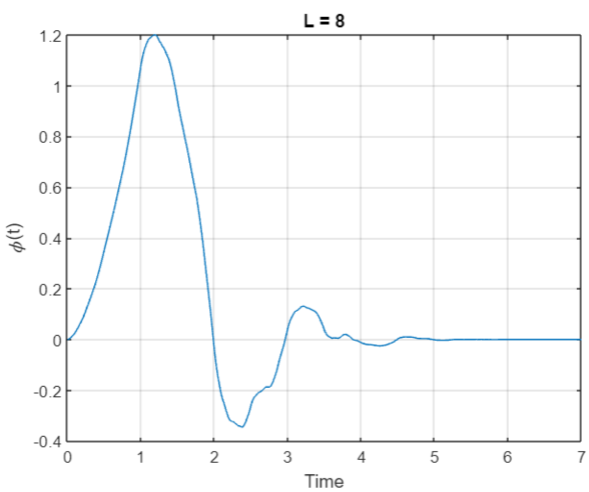
Filter Length (L)	Filter coefficients
2	0.7071 0.7071
4	0.4830 0.8365 0.2241 -0.1294
6	0.3327 0.8069 0.4599 -0.1350 -0.0854 0.0352
8	0.2304 0.7148 0.6309 -0.0280 -0.1870 0.0308 0.0329 -0.0106

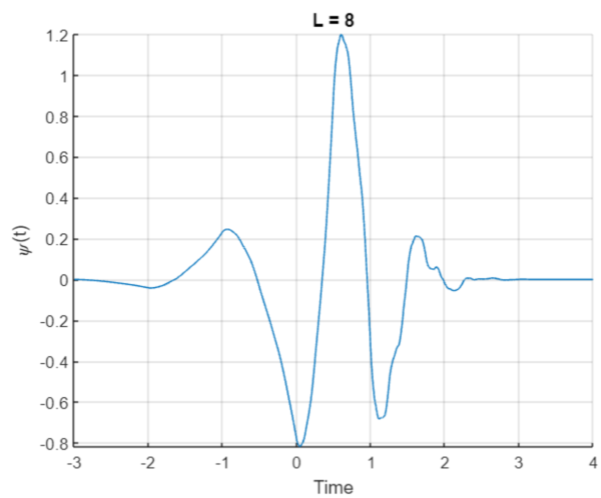
The scaling and wavelet function for L = 2, 4, 6, 8 obtained are as follows -



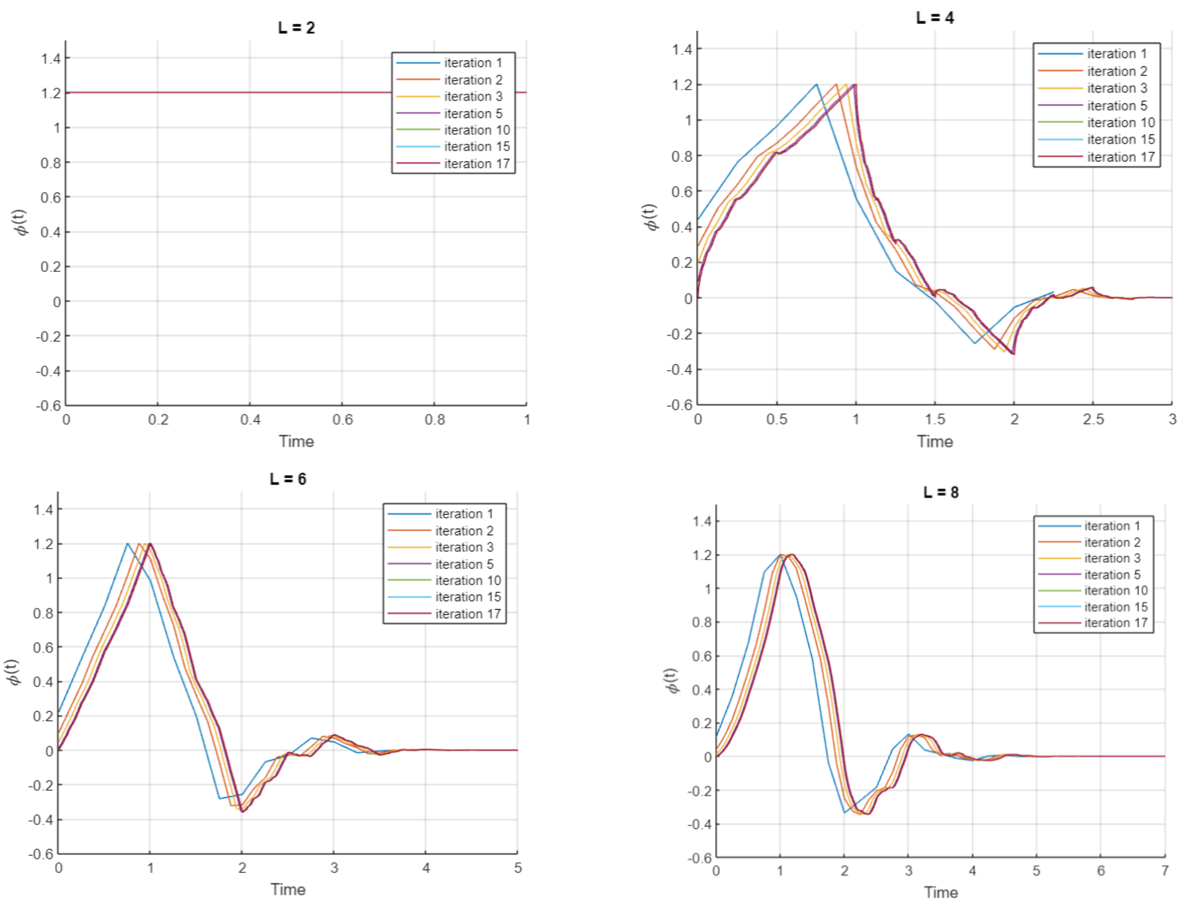

L = 6
ϕ(t)
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


L = 6
ψ(t)
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-2
-1
0
1
2
3

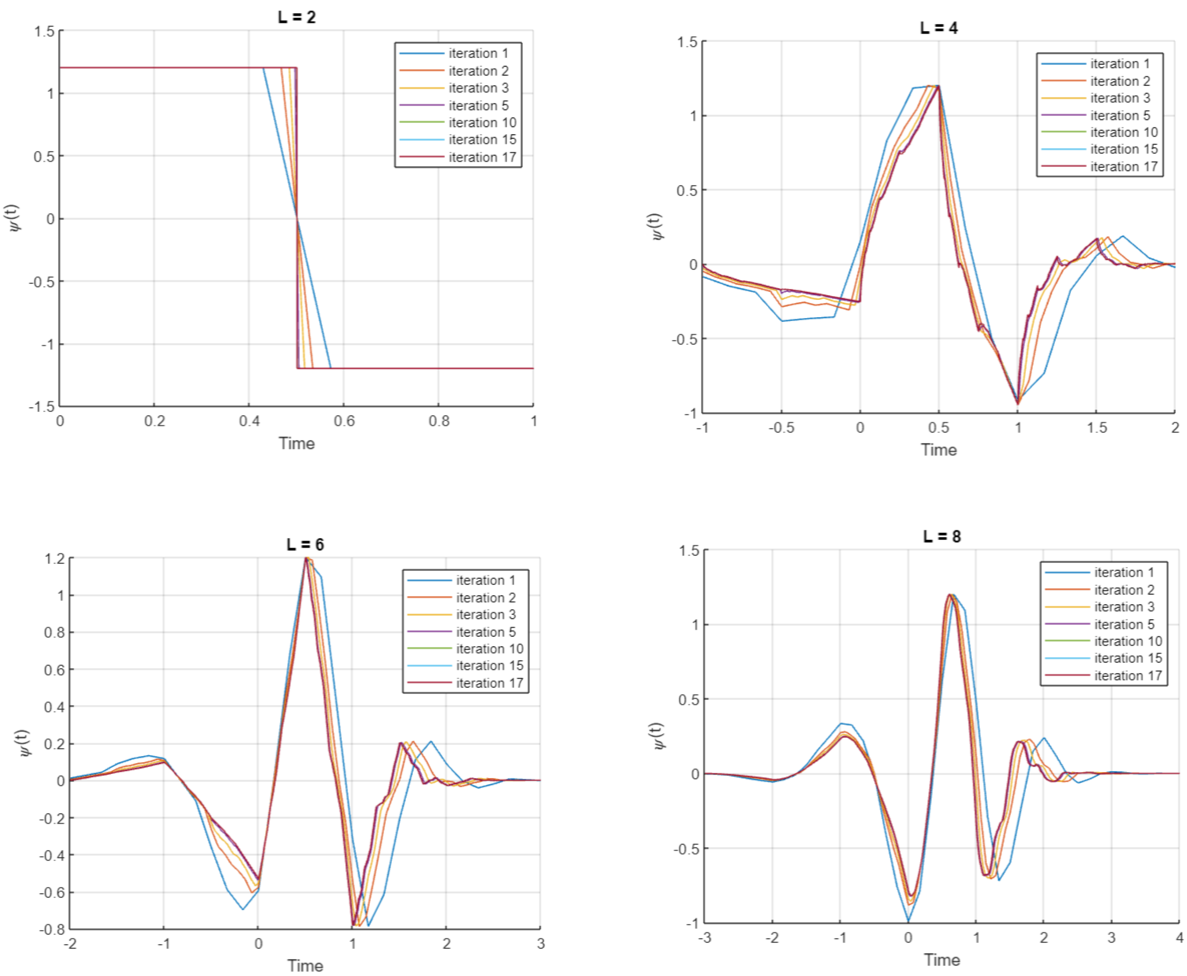

L = 8
ϕ(t)
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
1
2
3
4
5
6
7


L = 8
ψ(t)
Time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3
-2
-1
0
1
2
3
4

§ Part B - Show a few iterations of convolution of the filter impulse responses, successively contracted, to generate the underlying continuous scaling function and then the underlying wavelet



Scaling functions for L = 2, 4, 6, 8



Wavelet functions for L = 2, 4, 6, 8

II Insights

The main distinction between wavelets and Fourier basis functions is that the former's sine function, which ranges from negative to positive infinity, is nonlocal and consequently less effective for approximating sharp, localized anomalies in data. As a result, if the function to be approximated is largely smooth and periodic, Fourier analysis produces good results. The basis functions used by wavelets, on the other hand, are localized in finite domains (compact support), which makes them ideal for identifying both sharp abnormalities and rounded oscillations in data.

There is one vanishing moment in D2, two in D4, etc. The wavelet's capacity to capture polynomial activity or information in a signal is constrained by a vanishing moment. For instance, D2, which has a single vanishing moment, can readily encode constant signal components or polynomials with a single coefficient. Constant and linear signal components are represented by polynomials with two coefficients in D4 while constant, linear, and quadratic signal components are represented by polynomials with three coefficients in D6. Despite being able to encrypt signals, this capability is nevertheless hampered by the scale leaking and lack of shift-invariance phenomena that result from the discrete shifting operation (below) during the application of the transform.

From studying the graphs

While generating multiple daubechies wavelets for length 2 to 4 and ahead we observe that the wavelet function(which represents high pass nature of filter) becomes smoother.. The filter impulse responses take about 15-20 iterations to converge, which seems to be constant regardless of the length of the filter. Same follows for the scaling function.

The code for the exam is available here