1. i) $L(MSO_0) \subseteq L(MSO)$

Every atomic formula of MSO_0 can be expressed in MSO:

$Sing(X)$: $\exists x\, [X(x) \wedge \forall y\, (X(y) \rightarrow y = x)]$

$X \subseteq Y$: $\forall x\, [X(x) \rightarrow Y(x)]$

~~$X < Y$: $[Sing(X)$~~

$X < Y$: $\exists x \exists y\, [X(x) \wedge Y(y) \wedge x < y$
$\wedge \forall z\, [(X(z) \rightarrow x = z) \wedge (Y(z) \rightarrow y = z)]]$

$S(X, Y)$: Replace $x < y$ with $S(x, y)$ above

$Q_a(X)$: $\forall x\, [X(x) \rightarrow Q_a(x)]$

Every connective of MSO_0 is a connective of MSO

ii) $L(MSO) \subseteq L(MSO_0)$

Atomic Formulae

$x = y$: $\exists X \exists Y [Sing(X) \wedge Sing(Y) \wedge X(x) \wedge Y(y) \wedge X \subseteq Y]$

ii) $L(MSO) \subseteq L(MSO_0)$

Atomic Formulas

$x = y$: $Sing(X) \wedge Sing(Y) \wedge X \subseteq Y$

$x < y$: $X < Y$

$S(x, y)$: $S(X, Y)$

$X(x)$: $Sing(X)$

$Q_a(x)$: $Sing(X) \wedge Q_a(X)$

$\forall x \, \varphi$: $\forall X [Sing(X) \rightarrow \varphi]$

Other Quantifiers are common to both MSO_0 and MSO

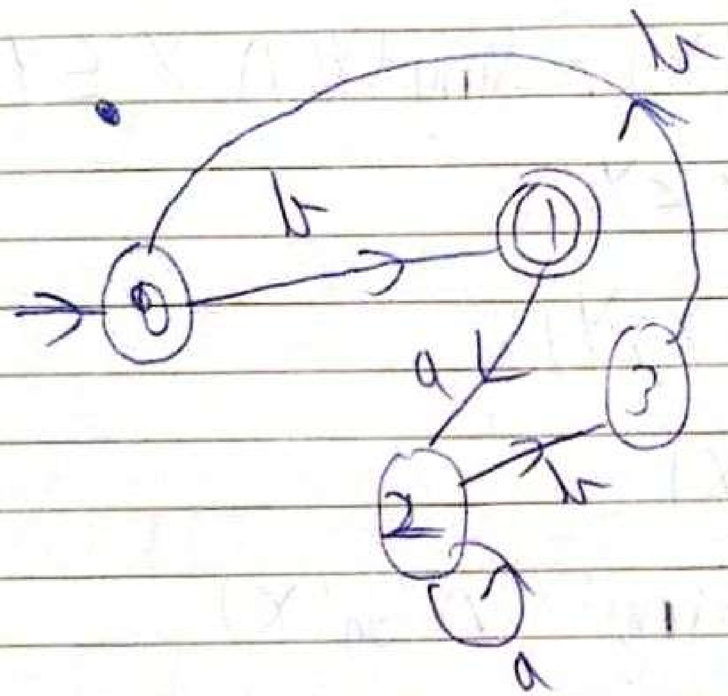
From i) and ii)

$L(MSO_0) = L(MSO)$

1. $\exists x \forall y (x < y \to \theta_a(y))$

$\exists X \forall Y [Sing(X) \land (Sing(Y) \to (X < Y \to \theta_a(Y)))]$

3.



$b(a^+b^3)^*$

X	Y	State
0	0	0
0	1	1
1	0	2
1	1	3

$\exists X, Y \{ [\exists x (First(x) \land \neg X(x) \land \neg Y(x))]$

$\land \exists x (Last(x) \land \neg X(x) \land \neg Y(x) \land \theta_b(x))$

$\land \forall x \forall y [S(x,y) \to [\neg X(x) \land \neg Y(x) \land \theta_b(x) \land \neg X(y) \land Y(y)]$

$\lor [\neg X(x) \land Y(x) \land \theta_a(x) \land X(y) \land \neg Y(y)]$

$\lor [X(x) \land \neg Y(x) \land \theta_a(x) \land X(y) \land \neg Y(y)]$

Date

$\vee\ [X(x) \wedge \neg Y(x) \wedge \theta_{\mu}(x) \wedge X(y) \wedge Y(y)]$

$\vee\ [X(x) \wedge Y(x) \wedge \theta_{\mu}(x) \wedge \neg X(y) \wedge \neg Y(y)]\ ]]$

$\exists z$

$\exists z\ [x + z = y \wedge \neg (z + z = z)]$

$x \leq y$ $\quad$ z is not 0

$t \leq 1$

$x \leq y$

$s \neq 0 \rightarrow t \leq s$

$\exists t\, [\, x + t = y \wedge \forall s\, [\neg(s + s = s) \rightarrow \exists z\, (t + z = s)]$

$\wedge \neg (t + t = t)\,]$

$t \neq 0$