PS 9

1. a) $\Diamond$ false

~~b) $\Diamond(p \wedge \bullet(\neg q \, U p))$~~

b) $\neg p \, U [p \wedge [\bigcirc((\neg q \wedge \neg p) \, U [p \wedge \neg \bigcirc \Diamond p]]$

c) $(\Diamond \Box \neg r) \rightarrow (\Diamond \Box p)$

~~d) $r \, U [\bigcirc p \wedge q \wedge \bigcirc($~~

e) ~~$\Diamond \Box (p \wedge \bigcirc p)$~~ $\Box \Diamond (p \wedge \bigcirc p)$

~~f) $\Box$~~

d) $r \, U (p \wedge q \wedge \bigcirc(\Box((q \vee r) \wedge (q \rightarrow \bigcirc r)$
$\wedge (r \rightarrow \bigcirc q))))$

[q r q r . . .

$r \, U (p \wedge q \wedge \bigcirc(\Diamond(q \vee r) \wedge \Box((q \rightarrow \bigcirc r)$
$\wedge (r \rightarrow \Diamond q))))$

e) $\Box \Diamond (p \wedge \bigcirc p)$

f) Not expressible
LTL $\equiv$ FO logic
$\therefore$ prefix of even length cannot be counted

i) Traces(TS) = Traces(TS') $\rightarrow$
TS and TS' satisfy the same set of LT properties

Take any property $P \subseteq (2^{AP})^{\omega}$

~~Traces~~(TS) $\models P \rightarrow$ Traces(TS) $\subseteq P$ ~~$\rightarrow$ Traces(TS~~
$\rightarrow$ Traces(TS') $\subseteq P$
~~$\rightarrow$ Traces(TS') $\subseteq P$~~
$\rightarrow$ TS' $\models P$

Similarly TS' $\models P \rightarrow$ TS $\models P$

$\therefore$ TS $\models P \leftrightarrow$ TS' $\models P$

ii) TS and TS' satisfy same set of LT properties
$\rightarrow$ Traces(TS) = Traces(TS')

Consider P = Traces(TS) [Valid property as Traces(TS) $\subseteq (2^{AP})^{\omega}$]

clearly TS $\models P$
$\rightarrow$ TS' $\models P$
$\rightarrow$ Traces(TS') $\subseteq$ Traces(TS)

Similarly Traces(TS) $\subseteq$ Traces(TS')

$\therefore$ Traces(TS) = Traces(TS')

- **LTL can express $\mathcal{X}$**:
  $\phi \Delta \psi \equiv \bigcirc(\phi \cup \psi)$
- **$\mathcal{X}$ can express LTL**:
  $\phi \cup \psi \equiv \phi \wedge (\bigcirc(\phi \cup \psi)) \equiv \phi \wedge (\phi \Delta \psi)$
  $\bigcirc\phi = \bigcirc(\bot \cup \phi) \equiv (\bot \Delta \phi)$ (and $\bot$ can be written as $p \wedge \neg p$)

5.1

4 1 a) $\{s_1, s_2, s_3, s_4\}$

b) $\{s_1, s_2, s_3, s_4\}$

c) $\{\}$

d) $\{s_1, s_2, s_3, s_4\}$

e) $\{s_1, s_2, s_3, s_4\}$

f) $\{s_1, s_2, s_3, s_4\}$

5.2 i) $TS \not\models \varphi_1$

Take $\pi = s_2 s_4 s_2 s_4 s_2 \ldots$

~~TS~~ ii) $TS \models \varphi_2$

Any path will visit one of s_2, s_3, s_5 infinite times, as s_1 has no incoming edge, and s_4 has no loop

iii) $TS \models \varphi_3$

LHS is satisfied only when second state of path is s_4, and then third state must be one of s_2, s_3, s_5 each of which have c in their labels

iv) $TS \not\models \varphi_4$

$\pi = s_2 s_4 s_2 s_4 \ldots,$

v) $TS \models \varphi_5$

~~If in~~ From state s_2, $\Box(b \vee c)$ holds, from s_1, ~~we~~ first a holds, then we have $\Box(b \vee c)$

vi) ~~If~~ $TS \not\models \varphi_6$

$\pi = s_1 s_4 s_2 \ldots$

5.5 Elevator has stopped at floor i : x_i

Door is open at floor i : z_i

Elevator has been called at floor i : y_i

a) $\Box(z_i \to x_i)$ for every i

b) $\Box(y_i \to \Diamond(x_i \wedge z_i))$ for every i

c) $\Box\Diamond x_0$

d) $\Box(y_3 \to \bigcirc(x_3 \wedge z_3))$

5.7 a) $(\neg\Diamond\psi) \vee [\neg\psi \, U(\psi \wedge \varphi)]$

b) $\varphi \, U \, \neg\psi$

c) $\Diamond\psi \to (\neg\psi \, U \, \varphi)$