CS 228 : Logic in Computer Science

S. Krishna

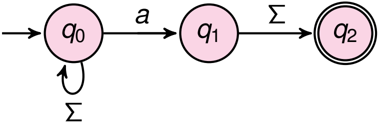
Recap

- Discussed determinism of DFAs : every word has a unique path in the DFA, starting from any state
- In particular, every word has a unique path in the DFA starting from the start state
- If this path leads to a good state, the word is accepted, else it is rejected.
- Looked at closure properties : complementation, intersection, union.
- Looked at proof techniques for correctness of a constructed DFA.

Moving on to Non-determinism

- We looked at DFA
- Showed closure under union, intersection and complementation
- Now we look at a more relaxed model, which is as good as a DFA

The Comfort of Non-determinism



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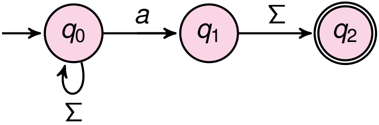

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  - $\delta : Q \times \Sigma \to 2^Q$
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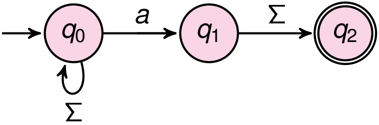

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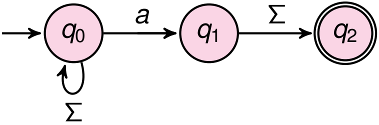

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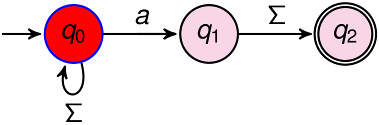

One run of *aabb*

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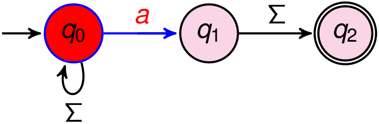

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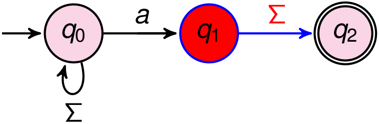

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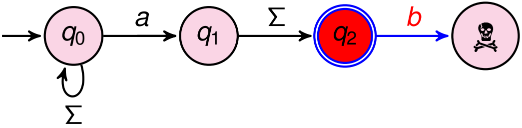
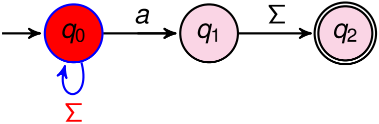


- A non-accepting run for *aabb*

Another run of *aaab*

Is *aaab* accepted?



Another run of *aaab*

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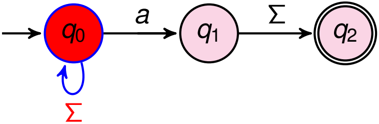

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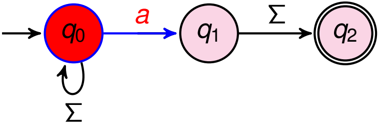

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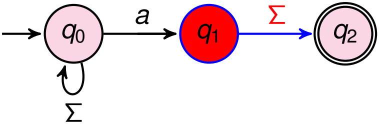
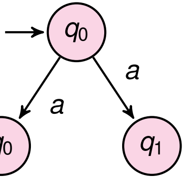


- An accepting run for *aaab*

Nondeterministic Finite Automata(NFA)

- $N = (Q, \Sigma, \delta, Q_0, F)$
  - Q is a finite set of states
  - $Q_0 \subseteq Q$ is the set of initial states
  - $\delta : Q \times \Sigma \to 2^Q$ is the transition function
  - $F \subseteq Q$ is the set of final states
- Acceptance condition : A word w is accepted iff it has atleast one accepting path

Run Tree of *aaab*



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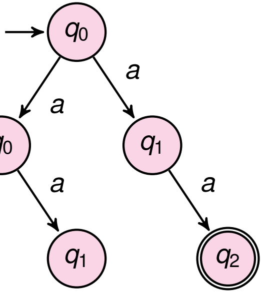

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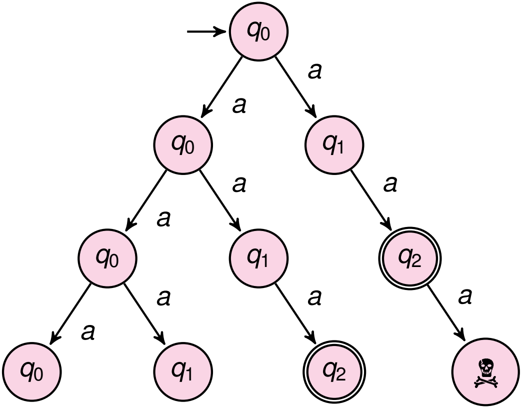

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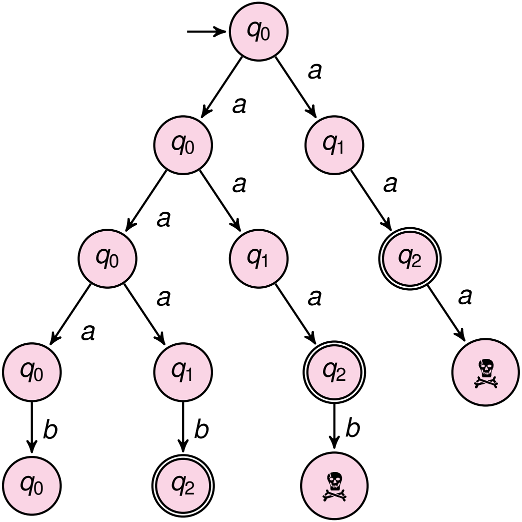

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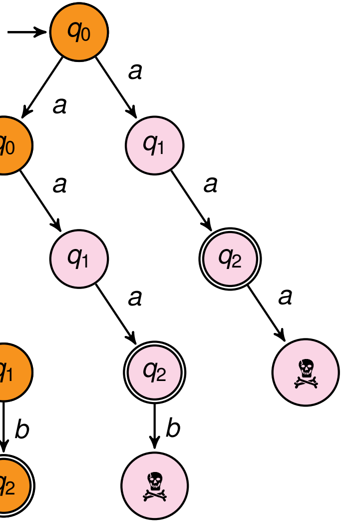

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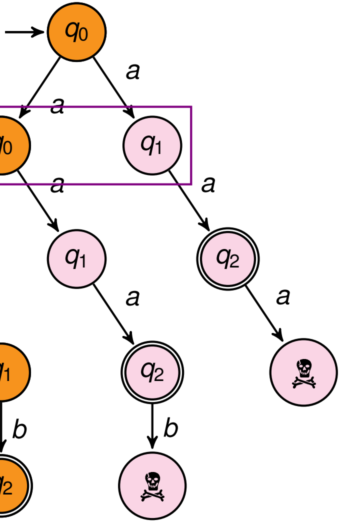

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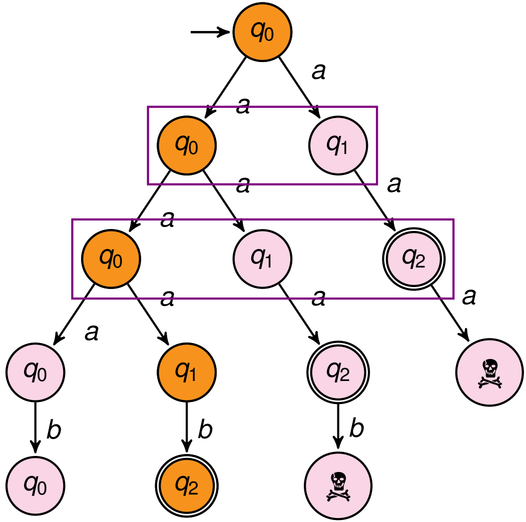

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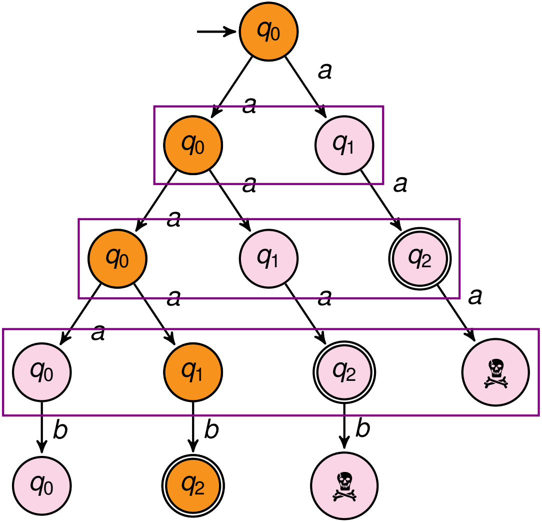

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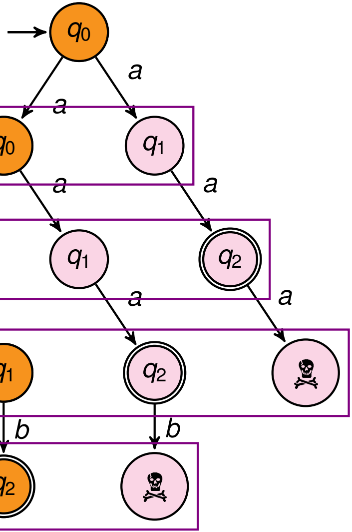
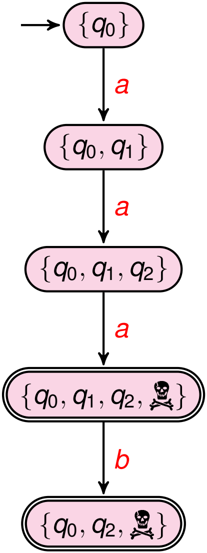


The Single Run



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  - Accept if the obtained set of states contains a final state

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NFA = DFA

$$x \in L(D) \leftrightarrow \hat{\Delta}(Q_0, x) \in F'$$

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$$\leftrightarrow$$

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$$\leftrightarrow$$

$$x \in L(N)$$

Regularity

A language L is regular iff there exists an NFA A such that $L = L(A)$