1. $A_1 = (Q_{A_1}, \Sigma, \delta_{A_1}, Q_{0A_1}, F_{A_1})$

$A_2 = (Q_{A_2}, \Sigma, \delta_{A_2}, Q_{0A_2}, F_{A_2})$

$A_3 = (Q, \Sigma, \delta, Q_0, F)$ such that $L(A_3) = L(A_1) \cap L(A_2)$

$Q = Q_{A_1} \times Q_{A_2} \times \{1, 2\}$

$\delta = \delta_1 \cup \delta_2$

$\delta_1 = \{((q_{A_1}, q_{A_2}, 1), a, (q'_{A_1}, q'_{A_2}, i)) \mid$
$(q_{A_1}, a, q'_{A_1}) \in \delta_{A_1}$ and
$(q_{A_2}, a, q'_{A_2}) \in \delta_{A_2}$ and
if $q_A \in F_A$ then $i = 2$ else $i = 1\}$

$\delta_2 = \{((q_{A_1}, q_{A_2}, 2), a, (q'_{A_1}, q'_{A_2}, i)) \mid$
$(q_{A_1}, a, q'_{A_1}) \in \delta_{A_1}$ and
$(q_{A_2}, a, q'_{A_2}) \in \delta_{A_2}$ and
if $q_B \in F_B$ then $i = 1$ else $i = 2\}$

$Q_0 = Q_{A_1} \times Q_{A_2} \times \{1\}$

$F' = \{(q_{A_1}, q_{A_2}, 2) \mid q_{A_1} \in Q_{A_1}, q_{A_2} \in F_{A_2}\}$

Consider an ω-word α accepted by A_3

There is a state $(q_{A_1}, q_{A_2}, 2)$ ~~such~~ that is visited infinitely many times.

Thus,

i) $q_{A_2} \in F_{A_2}$ is visited infinitely times,

$\therefore \alpha \in L(A_2)$

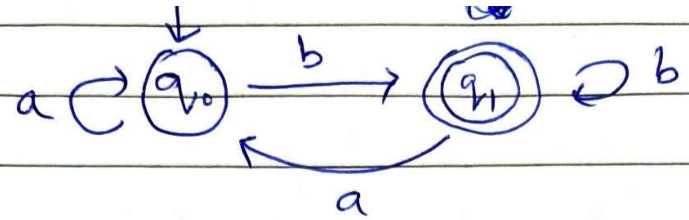
ii) Everytime $(q_{A_1}, q_{A_2}, 2)$ is visited the run goes to a 1 state.

To visit $(q_{A_1}, q_{A_2}, 2)$ infinitely many times, there have to be infinite transitions from a 1-state to a 2-state i.e. an accepting state of A_1 is also visited infinitely many times.

$\therefore \alpha \in L(A_3) \rightarrow \alpha \in L(A_1) \cap L(A_2)$

Similarly we can argue the converse as well.

2 (a)



$A = (Q, \Sigma, \delta, q_0, Acc)$ where

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$\delta(q_0, a) = q_0$, $\delta(q_0, b) = q_1$, $\delta(q_1, a) = q_0$

$\delta(q_1, b) = q_1$

with $\mathcal{G} = \{\{q_0\}\} \subseteq 2^Q$

(b) Let G be the set of good states for the NBA corresponding to some ω-regular language L.

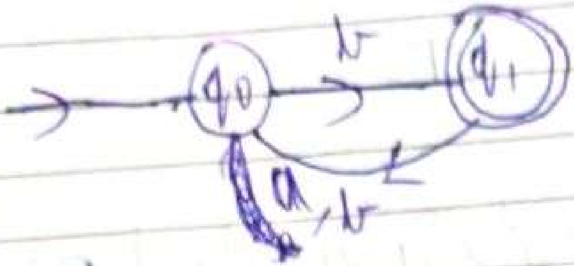
$\mathcal{G} = 2^Q \setminus 2^{\underbrace{Q \setminus G}_{\text{Bad states}}}$

(c) $\mathcal{G}' = 2^Q \setminus \mathcal{G}$

4. $L(A) = ((a+b)b)^\omega$

(There is b in all the even positions)

$\Sigma = \{a, b\}$



Cannot be expressed in LTL as there is no way to enable the even positions.

3 A finite set of infinite words is ω-regular

Consider $\Sigma = \{0, 1 \ldots 9\}$

Take the word π (the irrational number)

As π does not end with any infinitely repeating sequence, it cannot be written as EF^ω for any regular expressions E and F.

$\therefore$ Statement is false.