Question 1 (15points)

Inductive proof (Hand in your proof as a pdf file q1.pdf).
Recall the mobile from HW 1 where we defined a mobile a follows.

```
type mobile = Object of int | Wire of mobile * mobile
```

Below we give two functions which both compute the overall size of a mobile by counting the wires and objects.

```
let rec size m = match m with
  | Obj w -> 1
  | Wire (m1, m2) -> 1 + size m1 + size m2

let rec size' m acc = match m with
  | Obj w -> acc + 1
  | Wire (m1, m2) -> size' m1 (size' m2 (1 + acc))
```

Prove that for all `m:mobile, size m = size' m 0`.

Base case:
Consider the mobile `m = Obj().` Applying the two functions for size, we get:

```
    size m = 1            |            size' m 0 = acc + 1 = 1
```

Assumptions:

```
              size m = size' m 0
          1 + size m = size' m 1
```

Inductive step:
Consider the mobile `m = Wire(m1,m2).` Computing for size, we get:

```
size m = 1 + size m1 + size m2

size' m 0    = size' m1 (size' m2 (1 + 0))
             = size' m1 (size' m2 1)
             = size' m1 (1 + size m2)
             = (size' m1 1) + size m2
             = 1 + size m1 + size m2

Therefore    size m = size' m 0
```

Prog Languages & Paradigms
Homework 2