Preliminary results on single-energy PWA for γ,π

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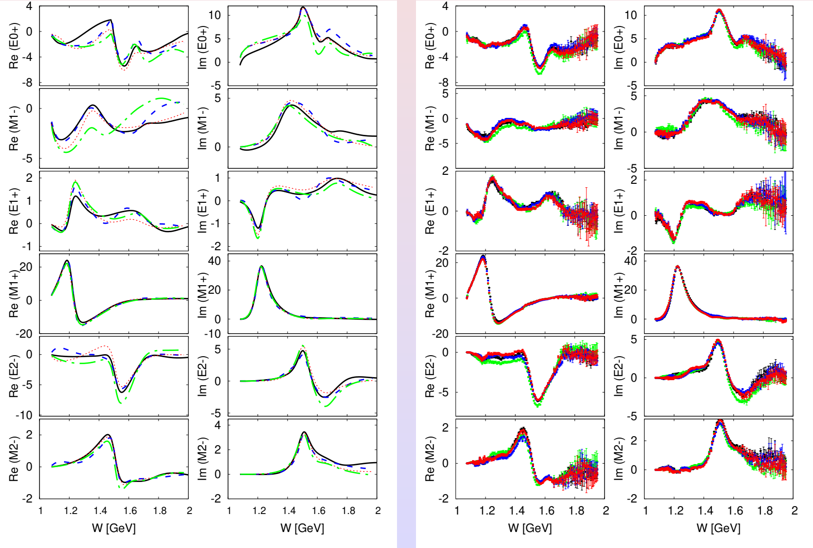
Outline

- Data prepare- s- and t-binning (Rifat's talk)
- Constraining PWA
- Preliminary results
- Conclusions

Results on SE PWA for $\pi^0 p$ (PWA10/ATHOS5 2018-Peking)

SE solutions obtained using ED solutions BnGa-black, JüBo-blue, MAID-green, SAID-red as a constraint.



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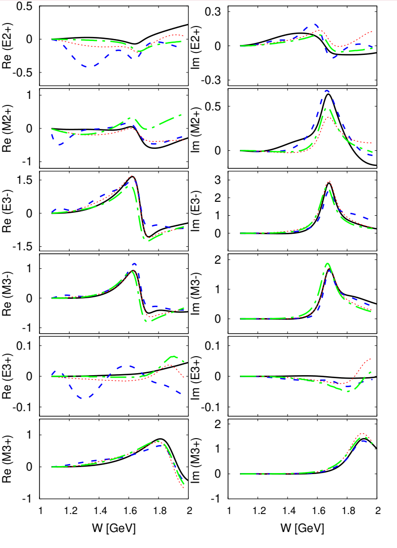

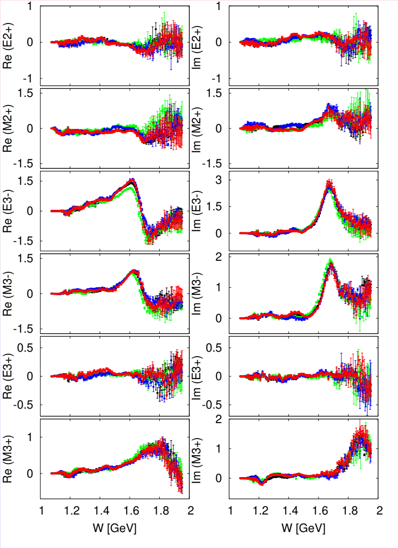

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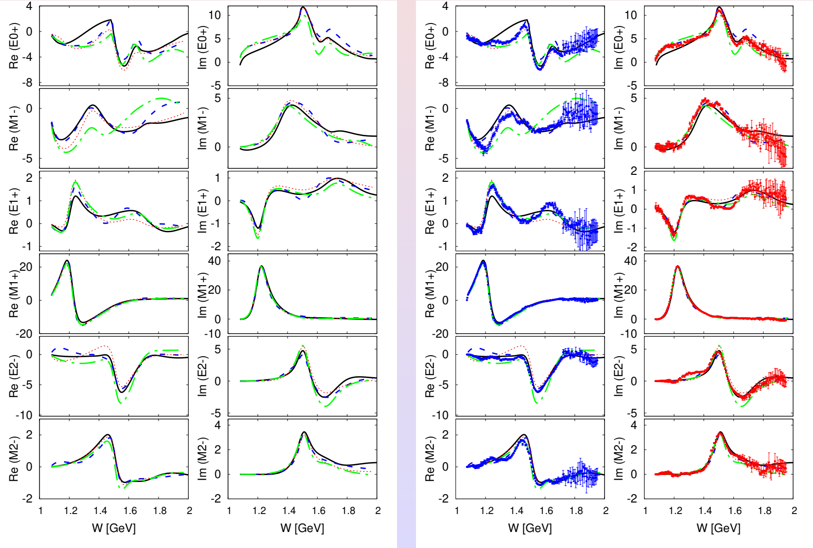
We calculated avarage value of SE solutions, and used them as constraint in our procedure.



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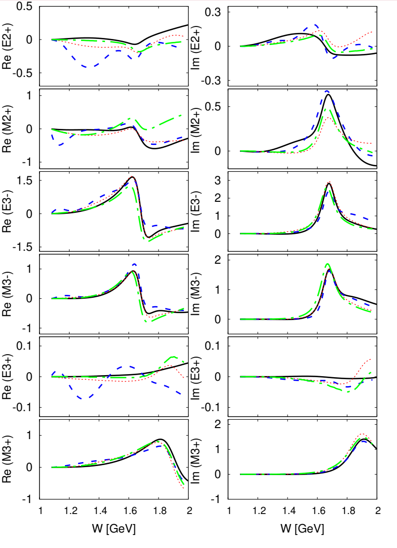
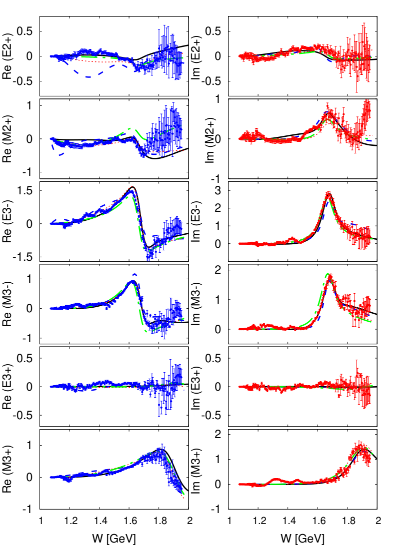




Pion photoproduction

In terms of the three isospin amplitudes, the four physical amplitudes are obtained as

$$A(\gamma p \to \pi^+ n) = A_{\pi^+ n} = \sqrt{2}(A_p^{\frac{1}{2}} - \frac{1}{3}A^{\frac{3}{2}})$$

$$A(\gamma n \to \pi^- p) = A_{\pi^- p} = \sqrt{2}(A_n^{\frac{1}{2}} + \frac{1}{3}A^{\frac{3}{2}})$$

$$A(\gamma p \to \pi^0 p) = A_{\pi^0 p} = A_p^{\frac{1}{2}} + \frac{2}{3}A^{\frac{3}{2}}$$

$$A(\gamma n \to \pi^0 n) = A_{\pi^0 n} = -A_n^{\frac{1}{2}} + \frac{2}{3}A^{\frac{3}{2}}$$

Unconstrained single energy partial wave analysis

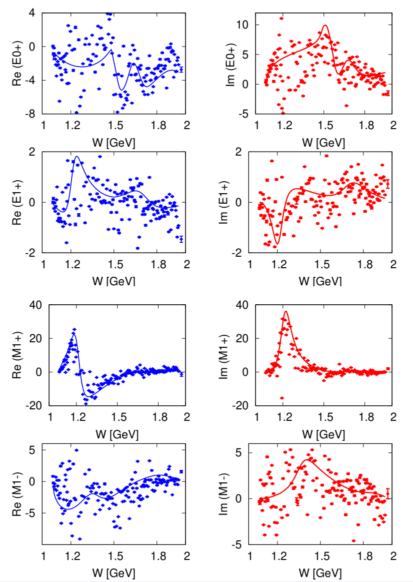
Problem setting:

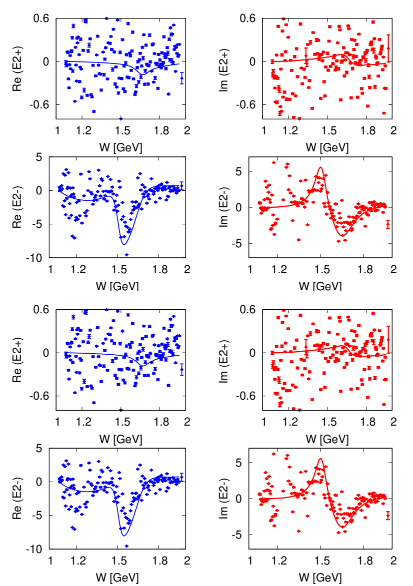
At a given energy W minimize a quadratic form:

$$\chi^2_{data} = \sum_D \sum_{k=1}^{N_D} \left(\frac{D_k^{exp}(\theta_k) - D_k^{fit}(\theta_k)}{\Delta_{D_k}} \right)^2$$

$D_k^{exp}(\theta_k)$– values of observable D measured at angles θ_k with errors Δ_{D_k}.
$D_k^{fit}(\theta_k)$ - predictions calculated from partial waves (multipoles) which are parameters in the fit.

Unconstrained SE PWA-Multipoles

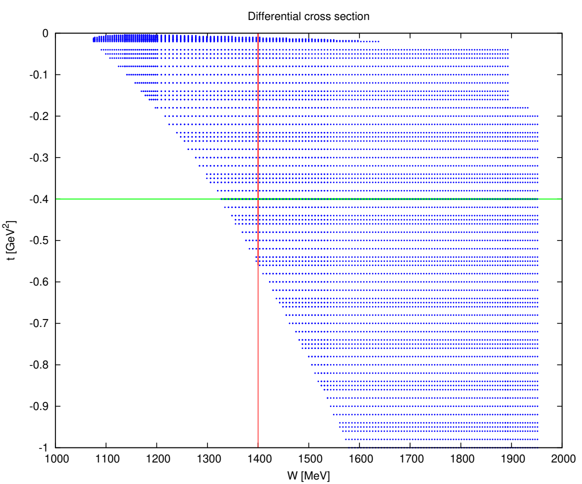




How to impose fixed-t analyticity in PWA of scattering data?

- The method consists of two separated analysis:-
  - Fixed-t amplitude analysis - determination of the invariant scattering amplitudes from exp. data at a given fixed-t value
  - Single energy partial wave analysis - SE PWA

- Fixed-t AA and SE PWA are coupled. Results from one analysis are used as constraint in another in an iterative procedure.

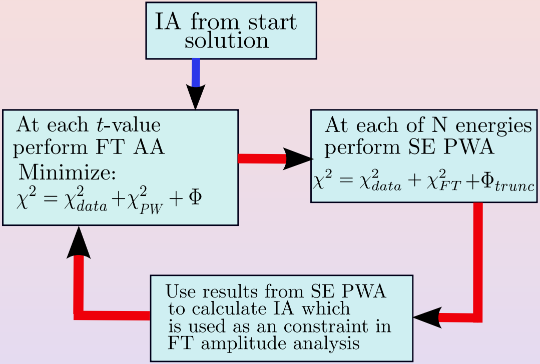


SE PWA is performed along red lines

Fixed-t AA is performed along green lines

Blue dots-experimental data in the physical region of pion

Coupled fixed-t AA and SE PWA



Connection between SE PWA and fixed-t AA

- Multipoles obtained from SE PWA at N^E energies are used to calculate helicity amplitudes which are used as constraint in the fixed-t amplitude analysis.
- The whole procedure has to be iterated until reaching reasonable agreement in two subsequent iterations

Pietarinen expansion of invariant amplitudes

We use Pietarinen's expansion with threshold πN and conformal variable

$$z_1 = \frac{\alpha - \sqrt{\nu_{th1}^2 - \nu^2 - i \cdot eps}}{\alpha + \sqrt{\nu_{th1}^2 - \nu^2 - i \cdot eps}}.$$

$\alpha = 0.3,\ Th(\pi N) = 1.07325\,GeV\ \ \nu = \frac{s-u}{4m}.$

$$\bar{B_1} = B_1 - B_{1N} = (1+z_1) \cdot \sum_i b_{1i} z_1^i$$

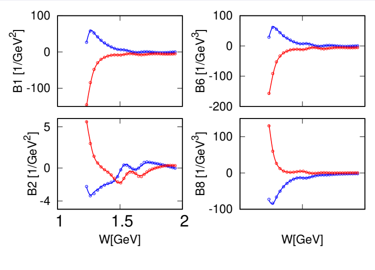
$$\bar{B_2} = B_2 - B_{2N} = (1+z_1) \cdot \sum_i b_{2i} z_1^i$$

$$\bar{B_6} = B_6 - B_{6N} = (1+z_1) \cdot \sum_i b_{6i} z_1^i$$

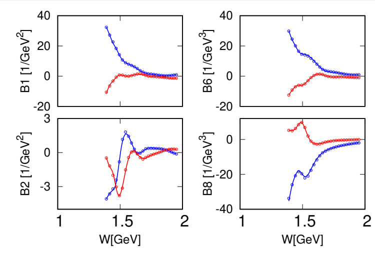
$$\bar{B_8} = \frac{B_8}{\nu} - \frac{B_{8N}}{\nu} = (1+z_1) \cdot \sum_i b_{8i} z_1^i$$

Fixed-t - Invariant amplitudes-number of parameters 8x30

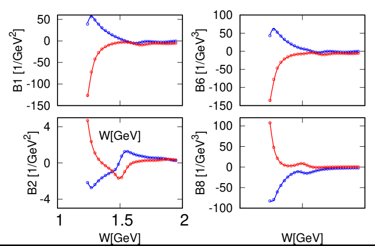
$t = -0.2\,GeV^2(\pi^0 p)$



$t = -0.5\,GeV^2(\pi^0 p)$



$t = -0.2\,GeV^2(\pi^0 n)$



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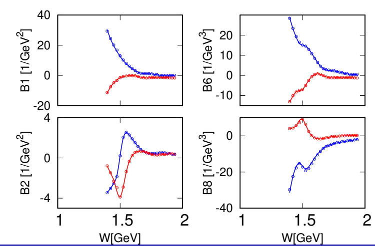


Fixed-t - Invariant amplitudes

IA from start solution

At each t-value perform FT AA

Minimize:

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi_{conv}$$

Parameters: Coefficients in Pietarinen's expansions

At each of energy perform SE PWA

Minimize:

$$\chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc}$$

Parameters: multipoles

Use results from SE PWA to calculate helicity amplitudes which are used as a constraint in FT amplitude analysis

Fixed-t amplitude analysis

Minimize:

$$\chi^2 = \chi^2_{data} + \chi^2_{PW} + \Phi$$

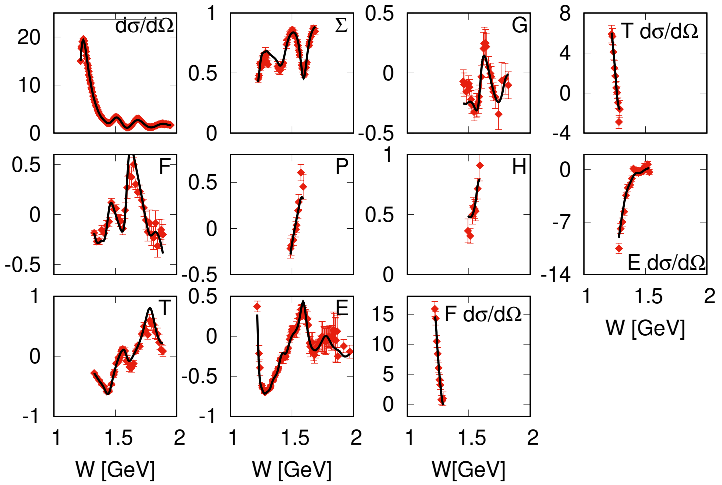
$$\chi^2_{PW} = q \sum_{k=1}^{4} \sum_{n=1}^{N_E} \left[\left(\frac{ReH_k(\omega, x_n)^{fit} - ReH_k(\omega, x_n)^{const}}{\varepsilon^{Re}_{k,n}} \right)^2 + \left(\frac{ImH_k(\omega, x_n)^{fit} - ImH_k(\omega, x_n)^{const}}{\varepsilon^{Im}_{k,n}} \right)^2 \right]$$

In 1st iteration H_k^{const} -helicity amplitudes from start solution (MAID07) $(-0.005\,GeV^2 > t > -1.00\,GeV^2)$.
Fitting parameters: Coefficients in Pietarinen's expansion.
q - adjustable weight factor.
$\varepsilon^{Re}_{k,n}$ and $\varepsilon^{Im}_{k,n}$ are errors.

Exp data- $(\pi^0 p + \pi^0 n)$

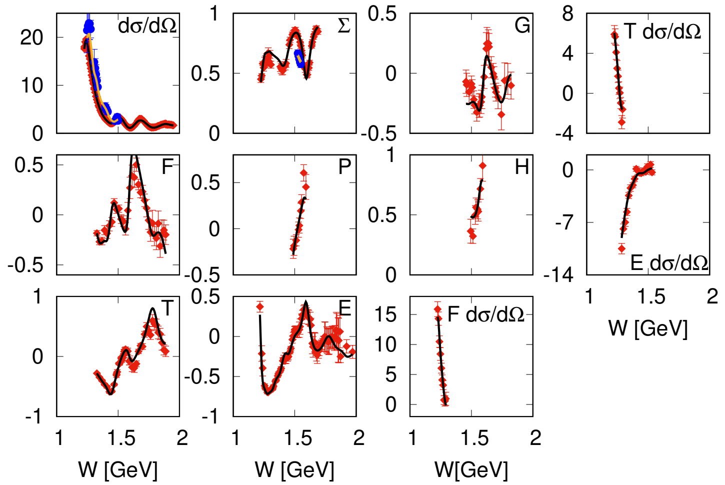
Fixed - t amplitude analysis - exp data- $(\pi^0 p)$

$t = -0.2 GeV^2$



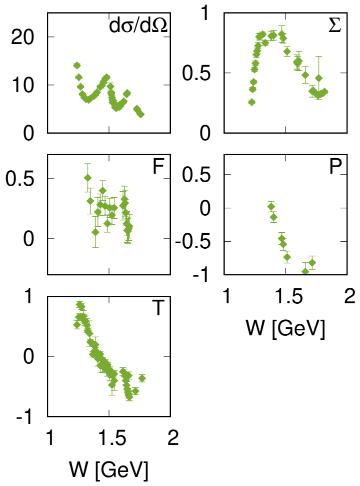
Fixed - t amplitude analysis - exp data-$(\pi^0 p + \pi^0 n)$

$t = -0.2 GeV^2$

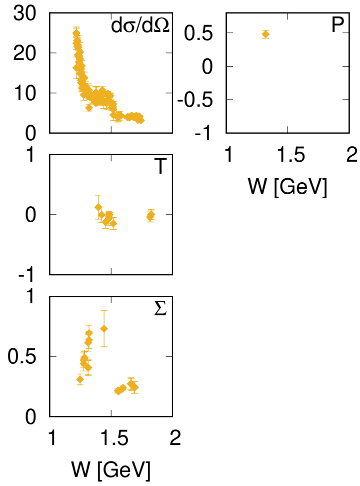


Fixed - t amplitude analysis - exp data (not included in the fit)

$t = -0.2 GeV^2 (\pi^+ n)$

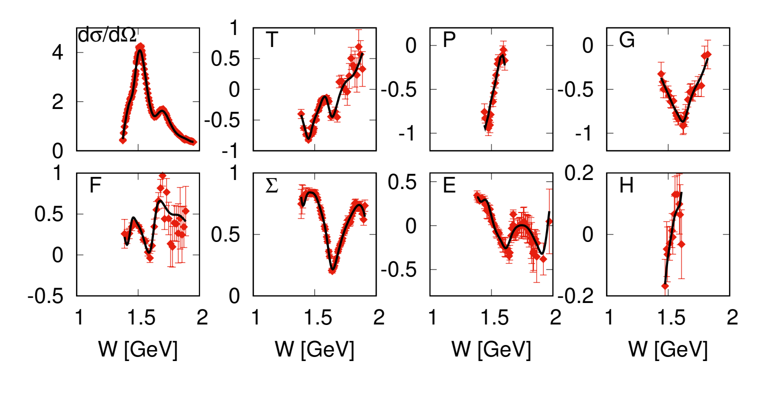


$t = -0.2 GeV^2\ (\pi^- p)$



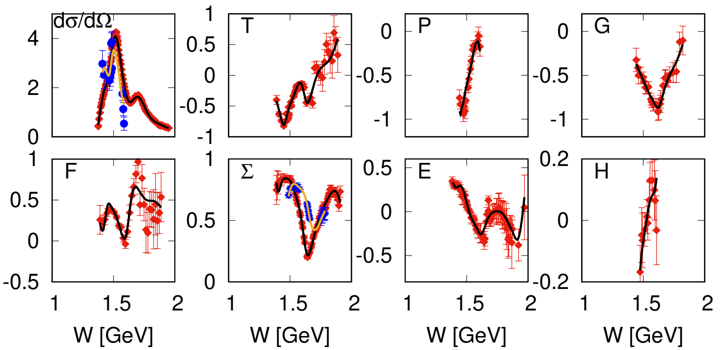
Fixed - t amplitude analysis - exp data- $(\pi^0 p)$

$t = -0.5\,GeV^2$



Fixed - t amplitude analysis - exp data-($\pi^0 p + \pi^0 n$)

$t = -0.5\, GeV^2$



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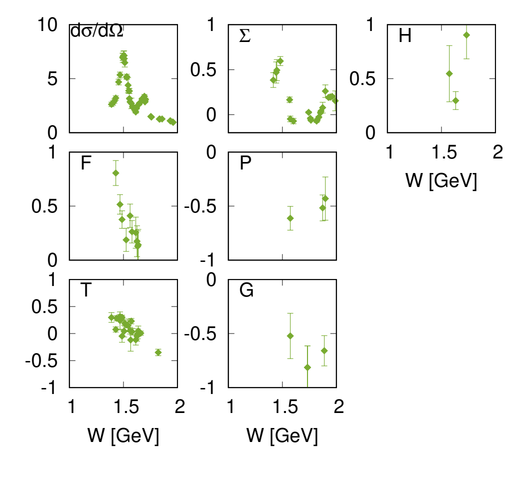

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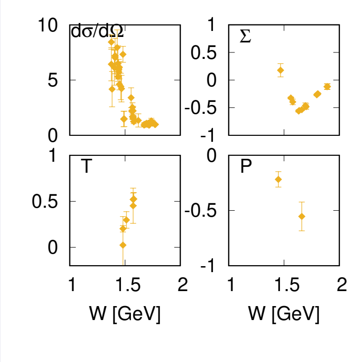
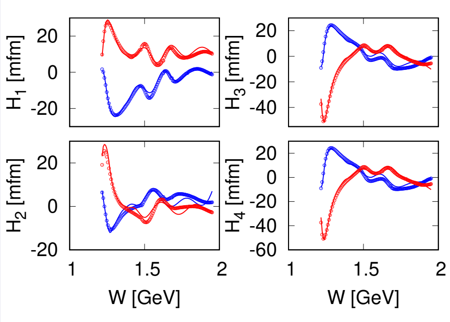
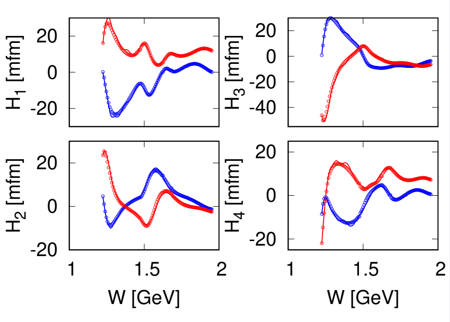


Fixed - t amplitude analysis - helicity amplitudes

$t = -0.2 GeV^2 (\pi^0 p)$



$t = -0.2 GeV^2 \; (\pi^0 n)$



Fixed - t amplitude analysis - helicity amplitudes

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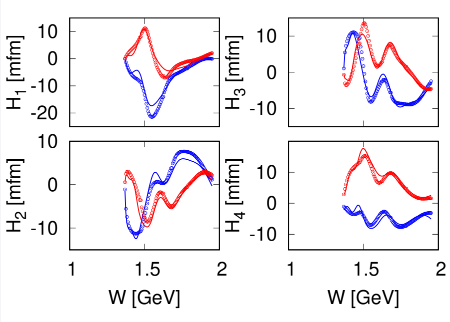
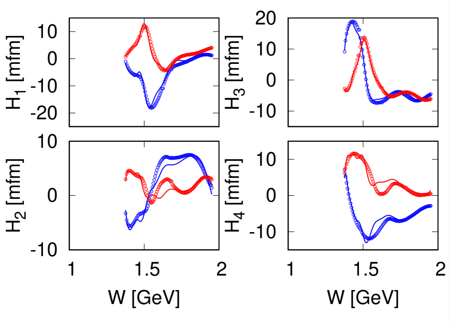


$t = -0.5\,GeV^2\ (\pi^0 n)$



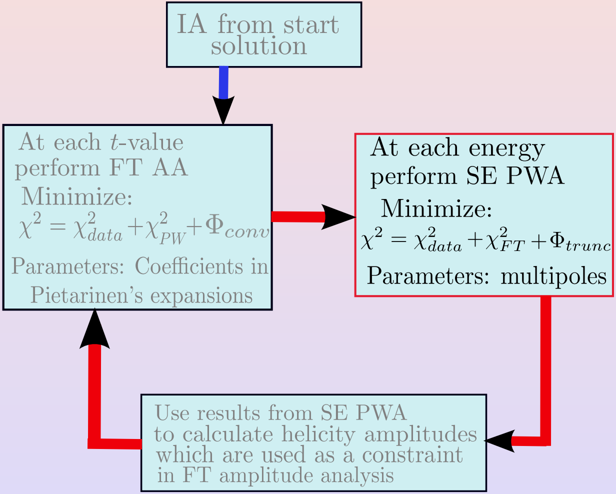
Conection between fixed-t AA and SE PWA

After performing fixed-t amplitude analysis at predetermined t-values, helicity amplitudes may be calculated at any energy W at N_c values of scattering angle

$$cos\theta_i = \frac{t_i - m_\eta^2 + 2k\omega}{2kq} \qquad |cos\theta_i| \leq 1, \quad t_i \in [t_{min}, t_{max}]$$

These values of helicity amplitudes are used as constraint in SE PWA.

Constrained SE PWA



Constrained SE PWA

Minimize:

$$\chi^2 = \chi^2_{data} + \chi^2_{FT} + \Phi_{trunc}$$

$$\chi^2_{FT} = q \sum_{k=1}^{4} \sum_{i=1}^{N^C} \left(\frac{Re\, H_k(\theta_i)^{const} - Re\, H_k(\theta_i)^{fit}}{(\varepsilon_R)_{ki}} \right)^2 + q \sum_{k=1}^{4} \sum_{i=1}^{N^C} \left(\frac{Im\, H_k(\theta_i)^{const} - Im\, H_k(\theta_i)^{fit}}{(\varepsilon_I)_{ki}} \right)^2$$

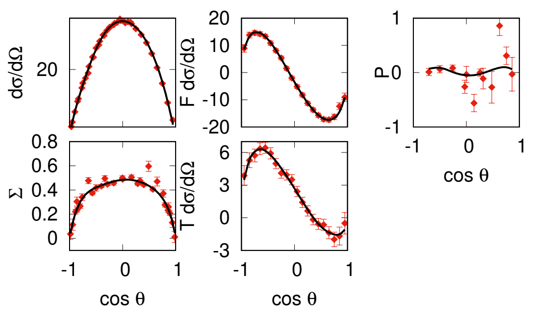
H^{const} constraining helicity amplitudes from fixed-t AA.
Fitting parameters: electric ($E_{\ell\pm}$) and magnetic ($M_{\ell\pm}$) multipoles.

Connection between SE PWA and fixed-t AA

Multipoles obtained from SE PWA at N^E energies are used to calculate helicity amplitudes which are used as constraint in the fixed-t amplitude analysis.

Constrained SE PWA - fit to the data
$W = 1.201\,GeV$;E=0.3GeV

$(\pi^0 p)$

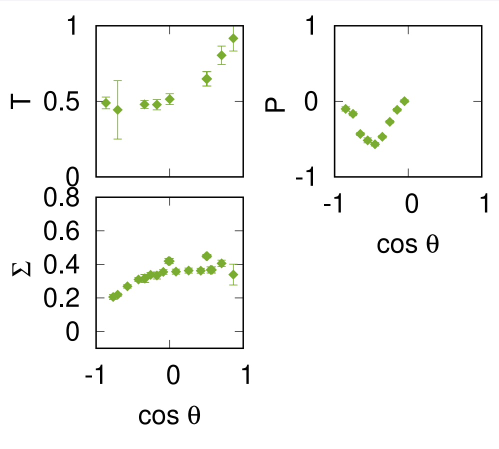


$(\pi^0 n)$

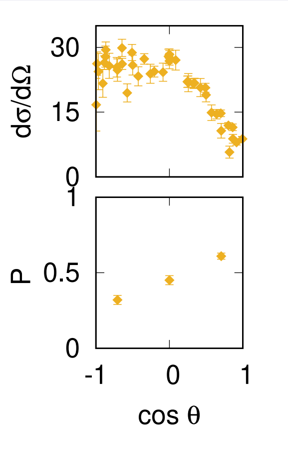
Constrained SE PWA - (not included in the fit)

$W = 1.201\,GeV$;E=0.3GeV

$(\pi^+ n)$



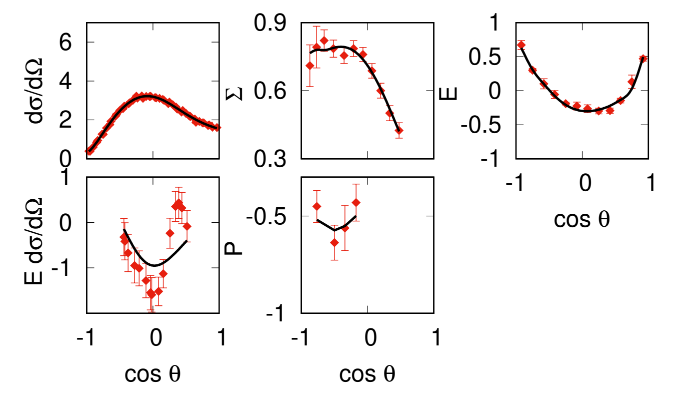
$(\pi^- p)$



Constrained SE PWA - fit to the data

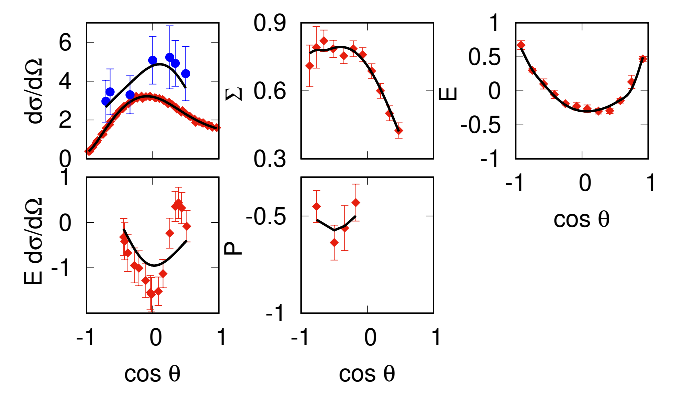
$W = 1.403 GeV$;E=0.58GeV

$(\pi^0 p)$



Constrained SE PWA - fit to the data
$W = 1.403 GeV$;E=0.58GeV

$(\pi^0 p + \pi^0 n)$



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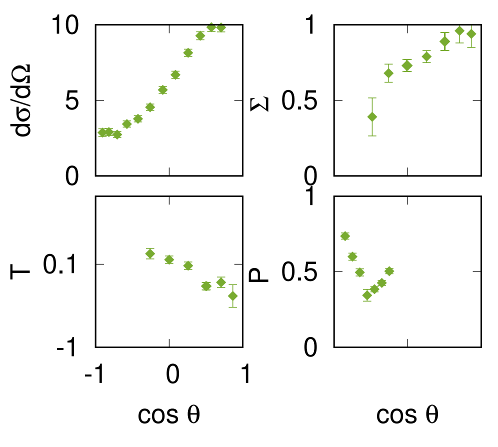
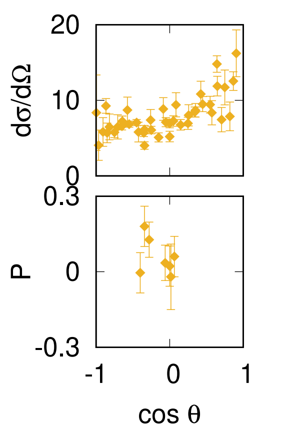
$(\pi^+ n)$



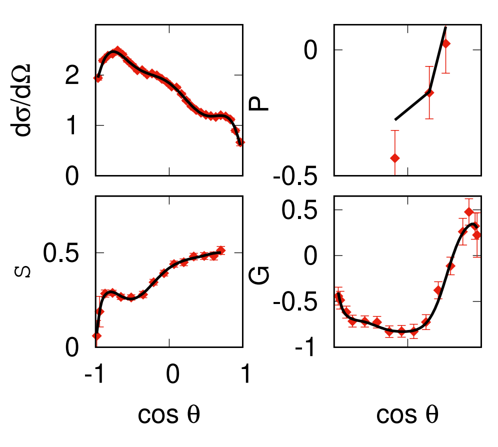
$(\pi^- p)$



Constrained SE PWA - fit to the data

$W = 1.602\,GeV$;E=0.9GeV

$(\pi^0 p)$



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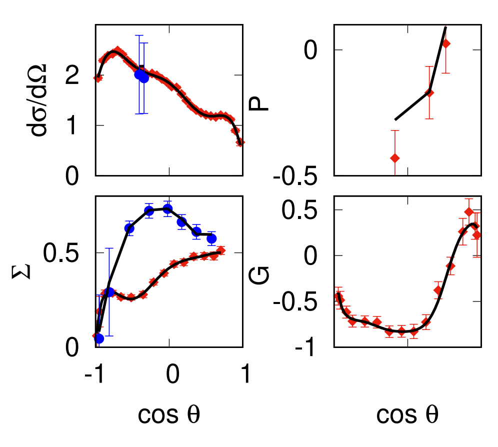
$(\pi^0 p + \pi^0 n)$



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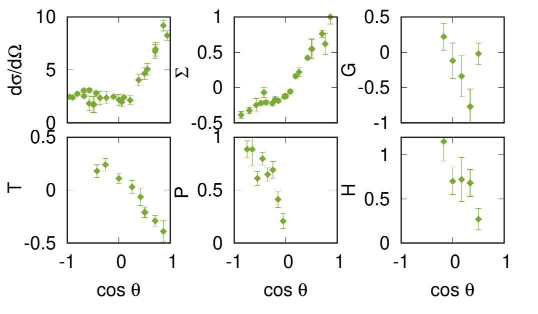
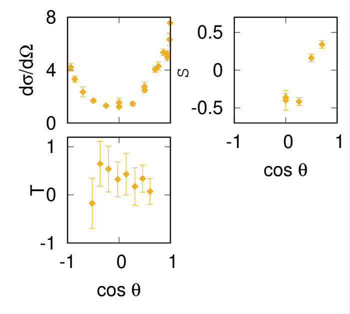


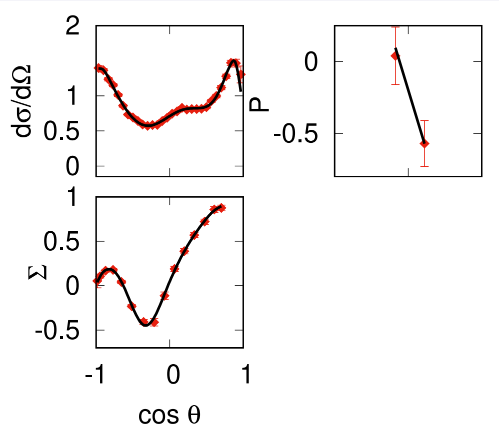
$(\pi^- p)$



Constrained SE PWA - fit to the data

$W = 1.801 GeV$;E=1.26GeV

$(\pi^0 p)$



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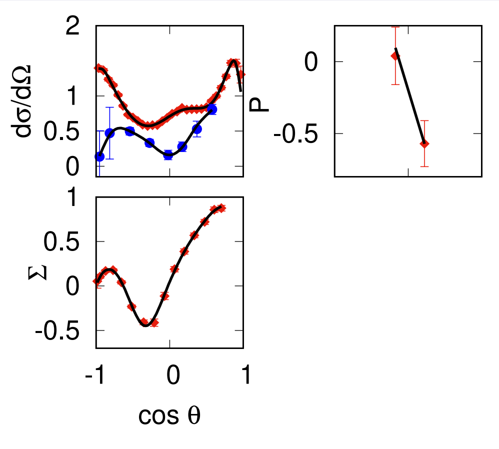
$(\pi^0 p + \pi^0 n)$



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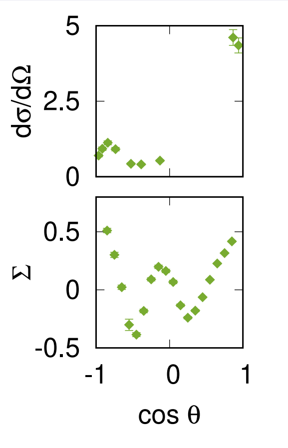
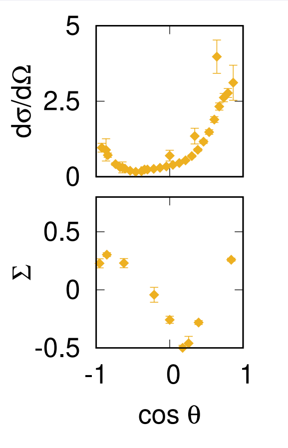


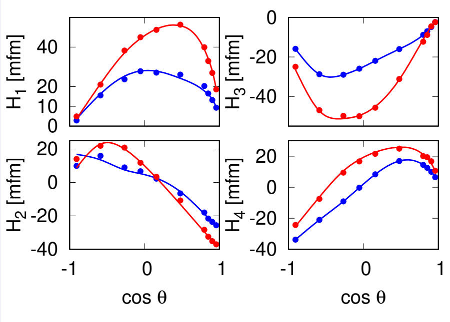
$(\pi^- p)$



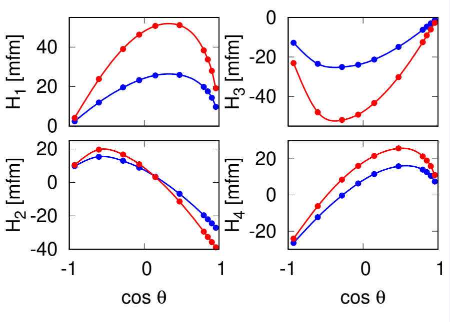
Constrained SE PWA-helicity amplitudes

$W = 1.201\,GeV$;E=0.3GeV

$(\pi^0 p)$



$(\pi^0 n)$

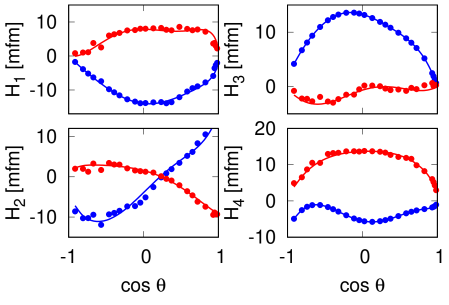


Real and imaginary parts of helicity amplitudes (blue and red dots) are obtained from independent fixed-t AA at different t-values. (1st iteration)

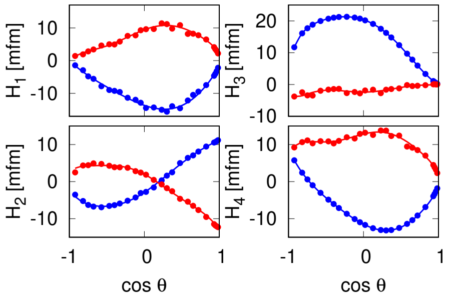
Constrained SE PWA-helicity amplitudes

$W = 1.403 GeV$;E=0.58GeV

$(\pi^0 p)$



$(\pi^0 n)$



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Constrained SE PWA-helicity amplitudes

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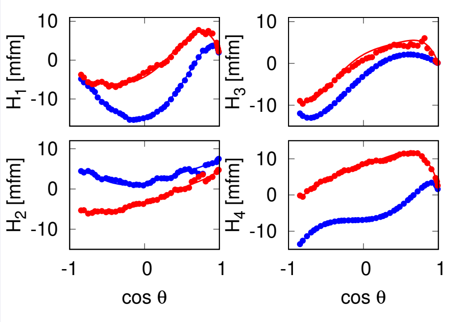
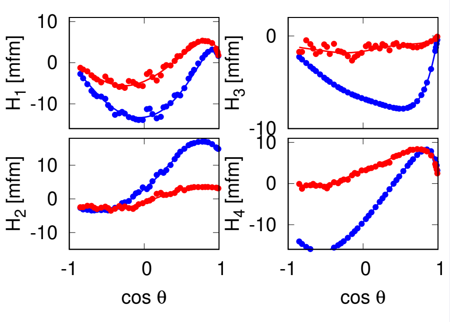
$(\pi^0 p)$



$(\pi^0 n)$

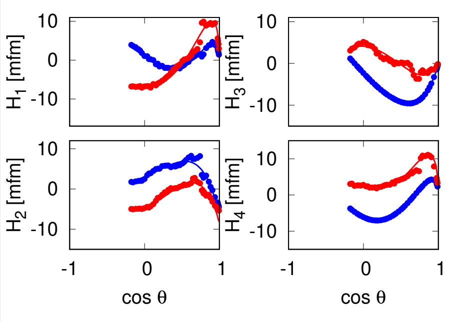


Real and imaginary parts of helicity amplitudes (blue and red dots) are obtained from independent fixed-t AA at different t-values. (1st iteration)

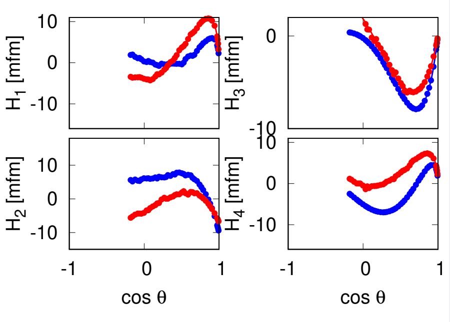
Constrained SE PWA-helicity amplitudes

$W = 1.801\,GeV$;E=1.26GeV

$(\pi^0 p)$



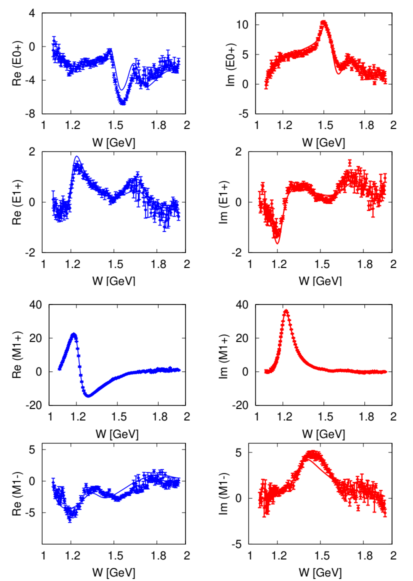
$(\pi^0 n)$



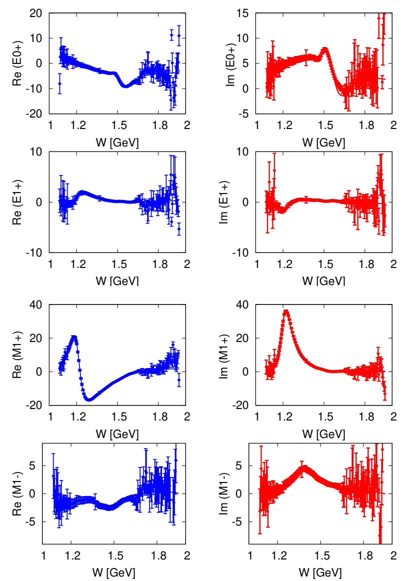
Real and imaginary parts of helicity amplitudes (blue and red dots) are obtained from independent fixed-t AA at different t-values. (1st iteration)

Constrained SE PWA-Multipoles

$(\pi^0 p)$

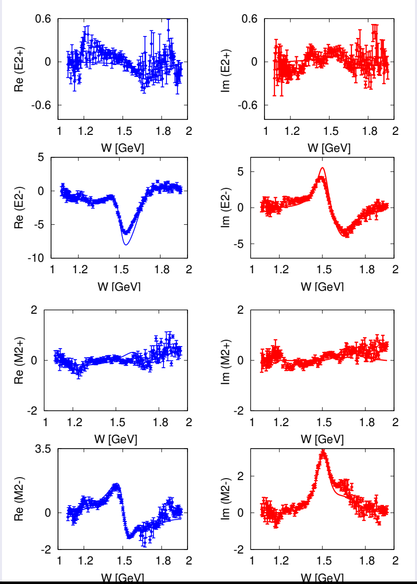


$(\pi^0 n)$

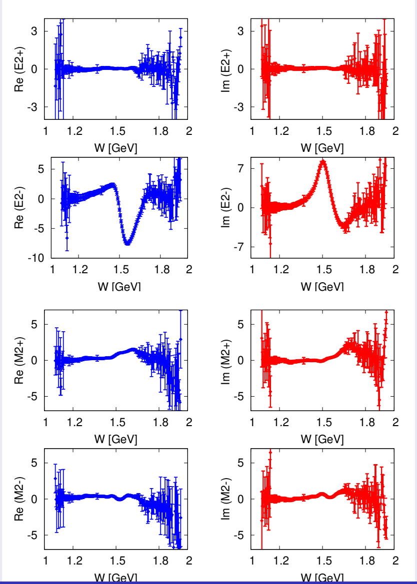


Constrained SE PWA-Multipoles

$(\pi^0 p)$

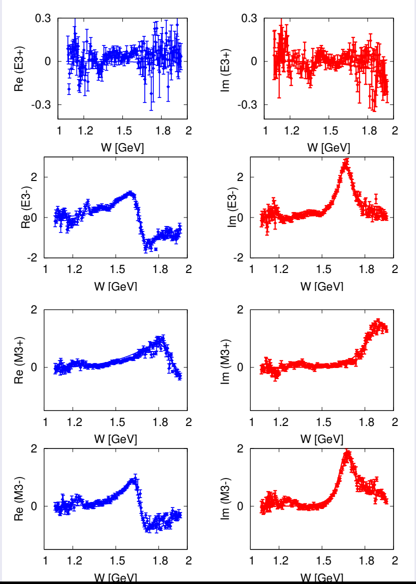


$(\pi^0 n)$

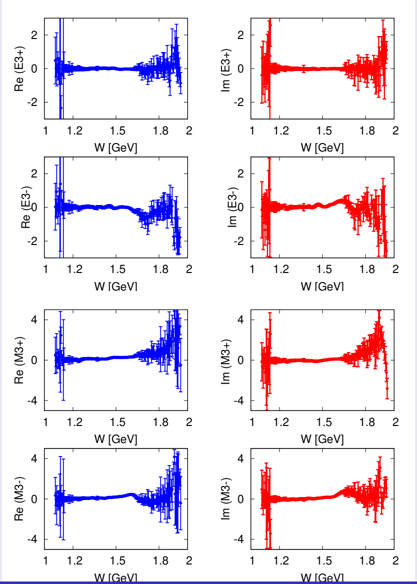


Constrained SE PWA-Multipoles

$(\pi^0 p)$

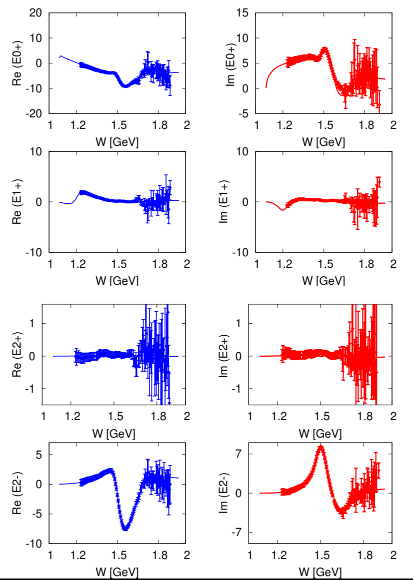


$(\pi^0 n)$

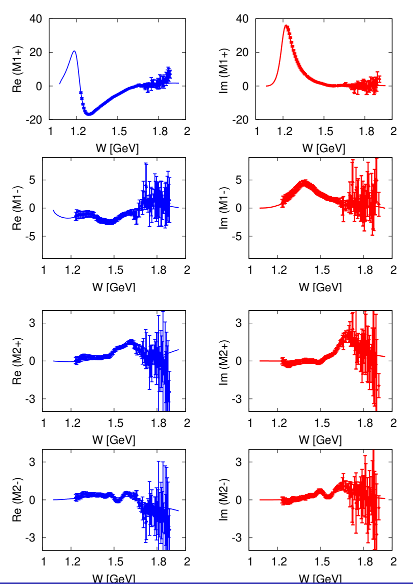


Constrained SE PWA-Multipoles

$(\pi^0 n)$

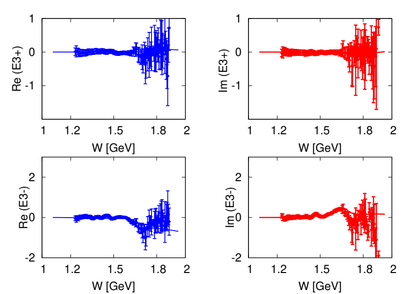

Re (E0+)
Im (E0+)
Re (E1+)
Im (E1+)
Re (E2+)
Im (E2+)
Re (E2-)
Im (E2-)
W [GeV]

$(\pi^0 n)$

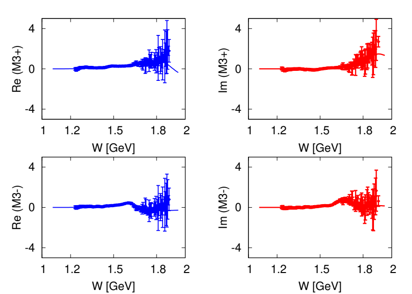

Re (M1+)
Im (M1+)
Re (M1-)
Im (M1-)
Re (M2+)
Im (M2+)
Re (M2-)
Im (M2-)
W [GeV]

Constrained SE PWA-Multipoles

$(\pi^0 n)$



$(\pi^0 n)$



Fixed $-$ t -SE PWA

Pion photoproduction

Work in progres ...