Partial wave analysis of eta meson photoproduction using fixed-t dispersion relations

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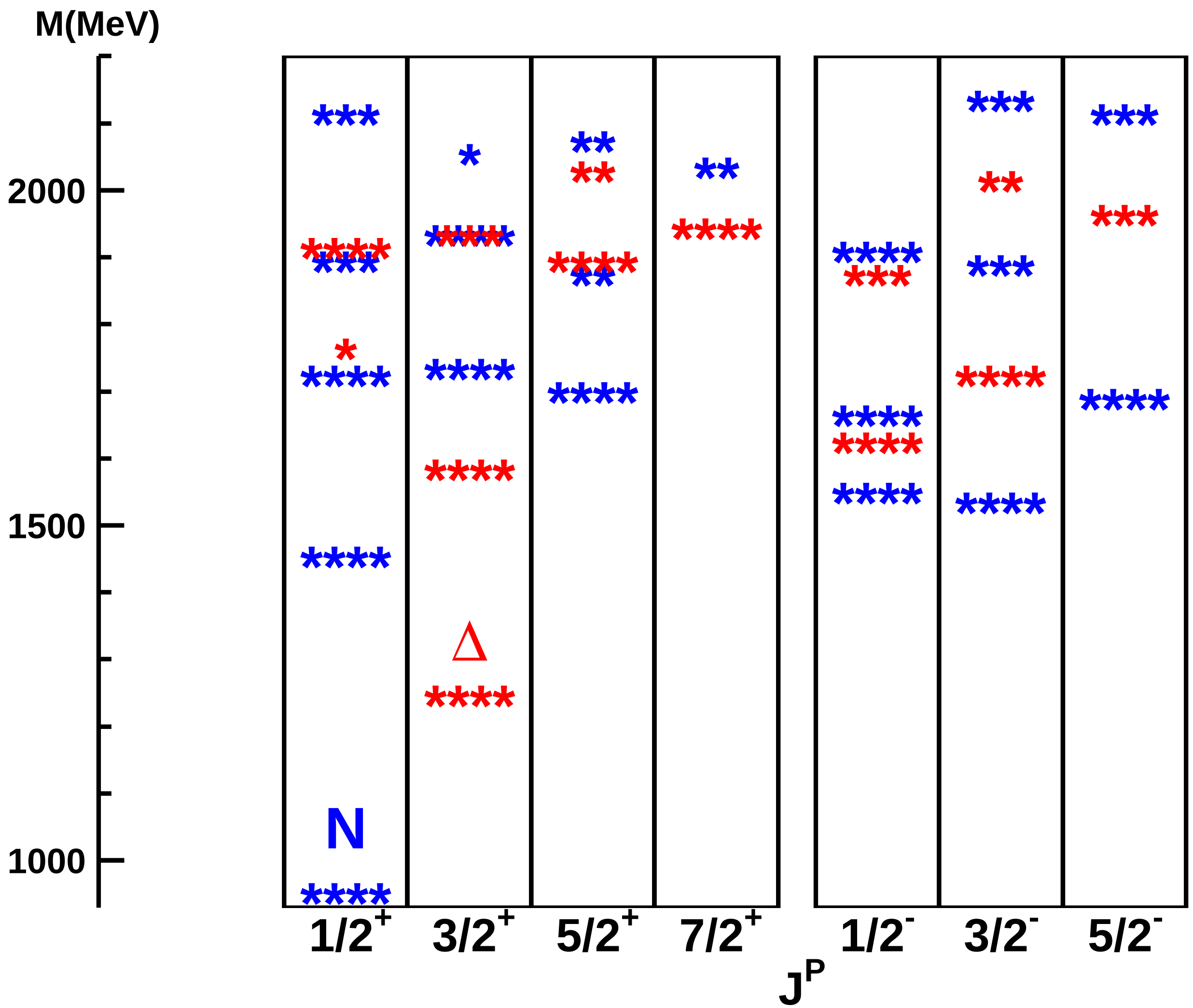
Important aspects

- Discussion of the PhD results
- Drawbacks of EtaMAID parametrization in DR procedure
- Behavior of the invariant amplitudes in the unphysical region
- Ways to improve
- Preliminary results

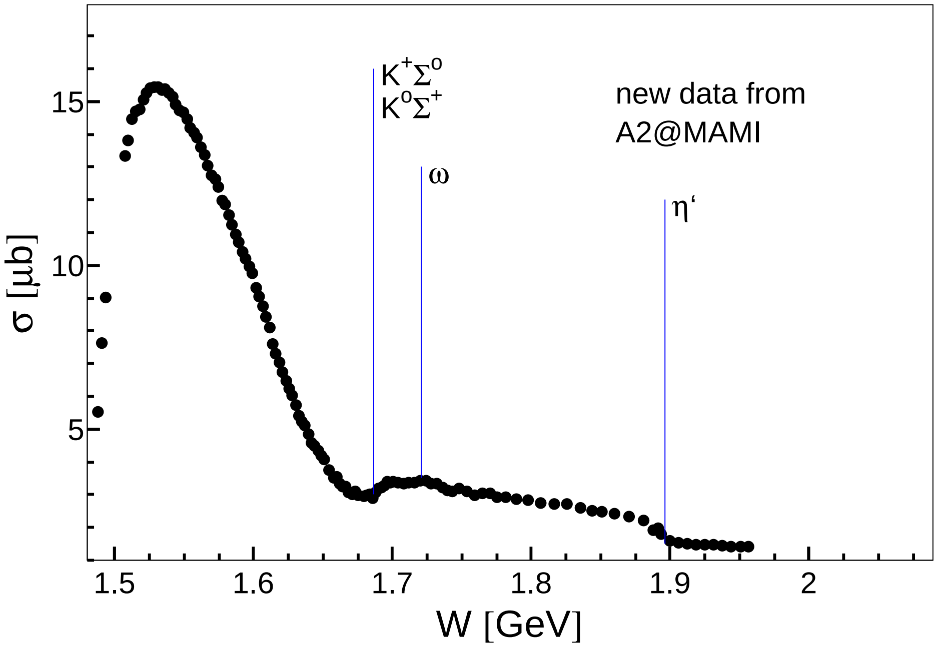
Spectrum of nucleon and delta resonances



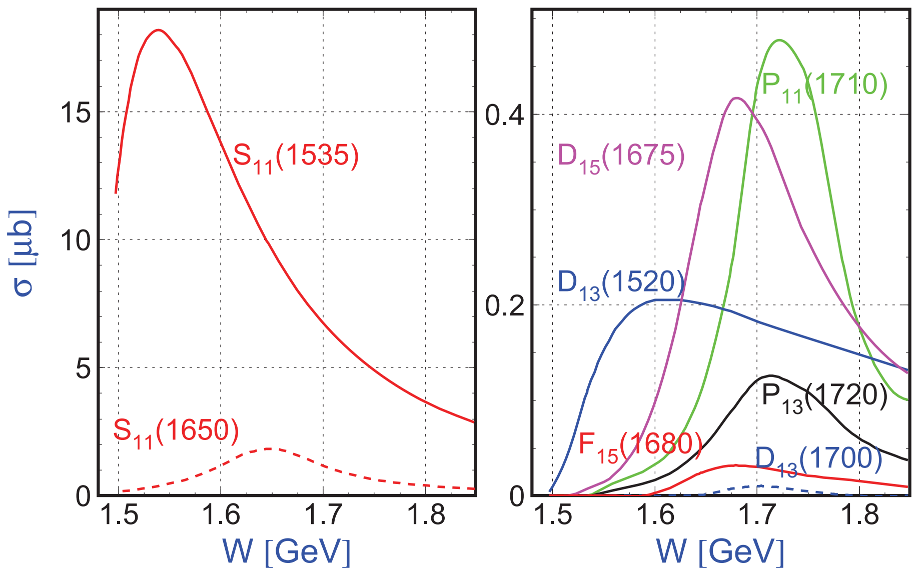
**** - PDG star rating (2018), $N: \; I = 1/2$ $\Delta: \; I = 3/2$

I-isospin, J-total spin, P-parity

Total cross section data from Mainz for $\gamma p \to \eta p$



Resonance contributions. Schematic picture



$$\overbrace{S_{11}(1535)}^{Old\ notation} = \overbrace{N(1535)\,1/2^-}^{New\ notation}, \ell = 0,\ I = 1/2,\ J = 1/2,\ P = -1,\ M = 1535 \text{ MeV}$$

$$D_{13}(1520) = N(1520)\,3/2^-, \ell = 2,\ I = 1/2,\ J = 3/2,\ P = -1,\ M = 1520 \text{ MeV}$$

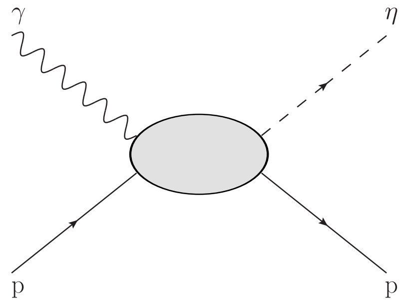
Angular distributions and polarization data play an important role,

where the structure comes from interferences of resonances

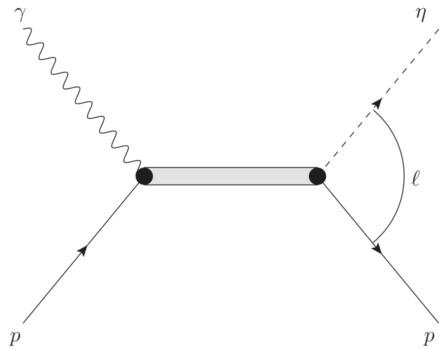
Introduction to Partial Wave Analysis

- Analyze the scattering amplitude to describe the scattering process
- $t_{\gamma,\eta}(W,\theta) \Rightarrow \sum_{\ell} f_{\ell}(W) \cdot \mathcal{P}_{\ell}(\cos\theta)$
- Main goal of the PWA: extraction of the resonances out of the scattering amplitude and determination of their parameters
- Different PWA approaches (models): MAID, SAID, BnGa, Jü-Bo, KSU

Scattering amplitude of $\gamma p \to \eta p$



Leading term of the "bubble"



Goals of PhD research

- Analyze the existing $\gamma p \to \eta p$ data using Mainz isobar model (EtaMAID)
- Improve the model
- Implement fixed-t dispersion relations for imposing analyticity and crossing symmetry

Why $\gamma p \to \eta p$?

- Isospin filter to remove $I = 3/2$ resonances
- For $\gamma p \to \eta p$ the background is small

Expected outcome

- More detailed information on nucleon resonances (mass, width, branching ratios, photon couplings)

Kinematics

Consider kinematical quantities independent of the reference frame

$$\gamma(k) \ + \ p(p_i) \to \eta(q) \ + \ p(p_f),$$

using 4-momenta we build variables that we operate with:

$$\begin{aligned} s &= (p_i + k)^2 = (q + p_f)^2, \\ t &= (q - k)^2 = (p_f - p_i)^2, \\ u &= (p_i - q)^2 = (p_f - k)^2, \end{aligned}$$

$$s + t + u = 2m_p^2 + m_\eta^2,$$

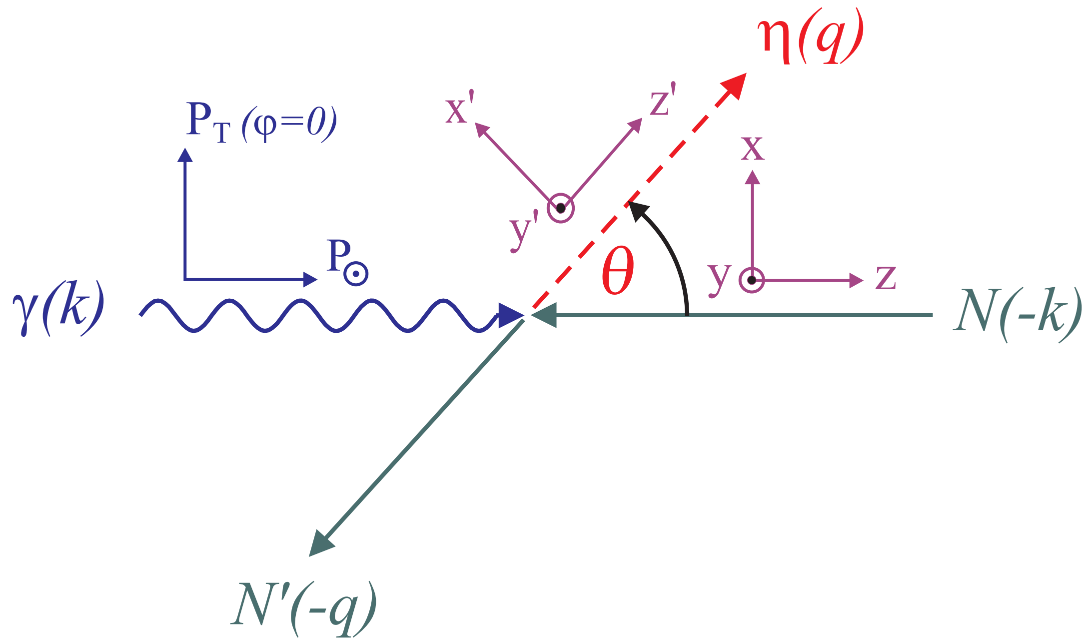
where t - momentum transfer squared from $photon$ to $meson$, $\sqrt{s} = W$ - total energy.

$$\nu = \frac{s-u}{4m_p}$$ crossing symmetrical variable.

Crossing EVEN $f(-\nu) = f^*(\nu)$,

crossing ODD $f(-\nu) = -f^*(\nu)$.

Polarization



Polarization of Beam, Target and Recoil give an opportunity to measure additional data.

Polarization in $\{x, y\}$ or $\{x', y'\}$ - transverse, polarization in z or z' - longitudinal.

In total 16 different observables can be measured.

$$\boxed{d\sigma/d\Omega,\ \Sigma,\ T},\ P,\ \boxed{E,\ F},\ G,\ H,\ O_x,\ O_z,\ C_x,\ C_z,\ T_x,\ T_z,\ L_x,\ L_z$$

Invariant and CGLN amplitudes

$\gamma p \to \eta p$ can be described by 4 amplitudes, the matrix element takes the form:

$$t_{\gamma,\eta} = \bar{u}(p_f) \sum_{i=1}^{4} A_i(\nu,t)\, \varepsilon_\mu M_i^\mu \, u(p_i) = -\frac{4\pi W}{m_p} \chi_f^\dagger \mathcal{F} \chi_i \,,$$

M_i^μ are dirac operators, $u(p)$ is the Dirac spinor, χ is the Pauli spinor, m_p is the proton mass.

$A_i(\nu,t)$ are crossing symmetric, Lorentz invariant, linear independent.

$$\mathcal{F} = i\,(\vec{\sigma}\cdot\hat{\epsilon})\,F_1 + (\vec{\sigma}\cdot\hat{q})\,(\vec{\sigma}\times\hat{k})\cdot\hat{\epsilon}\,F_2 + i\,(\hat{\epsilon}\cdot\hat{q})\,(\vec{\sigma}\cdot\hat{k})F_3 + i(\hat{\epsilon}\cdot\hat{q})(\vec{\sigma}\cdot\hat{q})\,F_4 \,,$$

where $\epsilon^\mu = (\epsilon_0, \vec{\epsilon})$ and $\vec{\epsilon}\cdot\vec{k} = 0$.

$F_i(W, \cos\theta)$ are amplitudes in the center of mass frame using Coulomb gauge.

Both types of amplitudes are used further.

Truncated partial wave expansion of CGLN amplitudes

CGLN amplitudes can be expanded in terms of the partial waves and Legendre polynomials.

$$
\begin{aligned}
F_1(W,x) &= \sum_{\ell=0}^{\ell_{max}} \left[\left(\ell M_{\ell+} + E_{\ell+}\right) P'_{\ell+1}(x) + \left((\ell+1)\, M_{\ell-} + E_{\ell-}\right) P'_{\ell-1}(x) \right], \\
F_2(W,x) &= \sum_{\ell=1}^{\ell_{max}} \left[(\ell+1)\, M_{\ell+} + \ell M_{\ell-} \right] P'_{\ell}(x), \\
F_3(W,x) &= \sum_{\ell=1}^{\ell_{max}} \left[\left(E_{\ell+} - M_{\ell+}\right) P''_{\ell+1}(x) + \left(E_{\ell-} + M_{\ell-}\right) P''_{\ell-1}(x) \right], \\
F_4(W,x) &= \sum_{\ell=2}^{\ell_{max}} \left[M_{\ell+} - E_{\ell+} - M_{\ell-} - E_{\ell-} \right] P''_{\ell}(x).
\end{aligned}
$$

ℓ is the orbital angular momentum of the ηN system, $\ell_{max} = 3$ was used in the analysis.

$x = \cos\theta$, where θ is the scattering angle in the center of mass frame.

$M_{\ell\pm}(W), E_{\ell\pm}(W)$ are multipoles (partial wave amplitudes of photoproduction), where $" \pm " \rightarrow J = \ell \pm 1/2$.

Resonances used in the analysis and related multipoles

Resonance	ℓ	J	P	Multipole
$N(1440)\,1/2^+$	1	1/2	$+$	M_{1-}
$N(1520)\,3/2^-$	2	3/2	$-$	E_{2-}, M_{2-}
$N(1535)\,1/2^-$	0	1/2	$-$	E_{0+}
$N(1650)\,1/2^-$	0	1/2	$-$	E_{0+}
$N(1675)\,5/2^-$	2	5/2	$-$	E_{2+}, M_{2+}
$N(1680)\,5/2^+$	3	5/2	$+$	E_{3-}, M_{3-}
$N(1700)\,3/2^-$	2	3/2	$-$	E_{2-}, M_{2-}

Resonance	ℓ	J	P	Multipole
$N(1710)\,1/2^+$	1	1/2	$+$	M_{1-}
$N(1720)\,3/2^+$	1	3/2	$+$	E_{1+}, M_{1+}
$N(1860)\,5/2^+$	3	5/2	$+$	E_{3-}, M_{3-}
$N(1875)\,3/2^-$	2	3/2	$-$	E_{2-}, M_{2-}
$N(1880)\,1/2^+$	1	1/2	$+$	M_{1-}
$N(1895)\,1/2^-$	0	1/2	$-$	E_{0+}
$N(1900)\,3/2^+$	1	3/2	$+$	E_{1+}, M_{1+}

In total 14 resonances up to $\ell_{max} = 3$ in the mass range up to 1900 MeV
were used in the analysis

MAID isobar model approach

Scattering amplitude for a given partial wave α

$$t^{\alpha}_{\gamma,\eta}(W) = t^{\alpha,bg}_{\gamma,\eta}(W) + t^{\alpha,Res}_{\gamma,\eta}(W),$$

is a sum of resonant and non-resonant parts.

Resonant part

$$t^{\alpha,Res}_{\gamma,\eta}(W) = \sum_{j=1}^{N_\alpha} t^{\alpha,BW,j}_{\gamma,\eta}(W) e^{i\Phi^{\alpha}_{j}},$$

is a sum of Breit-Wigner resonance functions with a unitarity phase Φ^{α}_{j} for each resonance, N_α number of resonances for each partial wave.

Non-resonant part

$$t^{\alpha,bg}_{\gamma,\eta}(W) = t^{\alpha,Born}_{\gamma,\eta}(W) + t^{\alpha,t-channel}_{\gamma,\eta}(W),$$

is a sum of Born terms and t-channel exchanges.

Parametrization of the resonances

The $t^{\alpha,BW,j}_{\gamma,\eta}$ projected to multipoles looks like a generalized Breit-Wigner form

$$\mathcal{M}_{\ell\pm}(W) = \bar{\mathcal{M}}_{\ell\pm}\, f_{\gamma N}(W)\, \frac{M_R \Gamma_{tot}(W)}{M_R^2 - W^2 - iM_R\Gamma_{\rm tot}(W)}\, f_{\eta N}(W) C_{\eta N}\,,$$

where:

$\bar{\mathcal{M}}_{\ell\pm}$ is related to the photodecay amplitudes $A_{1/2}$ and $A_{3/2}$ listed by PDG,

$\Gamma_{\rm tot}(W)$ is the energy-dependent width,

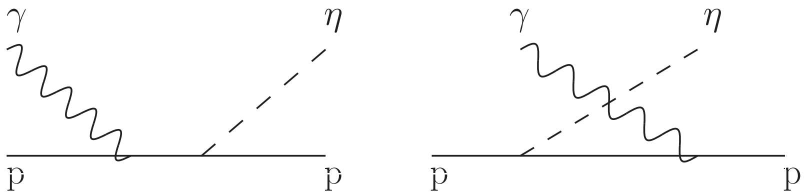
M_R is a mass of resonance

$f_{\gamma N}(W)$ and $f_{\eta N}(W)$ are vertex functions,

$C_{\eta N} = -1$ is an isospin factor.

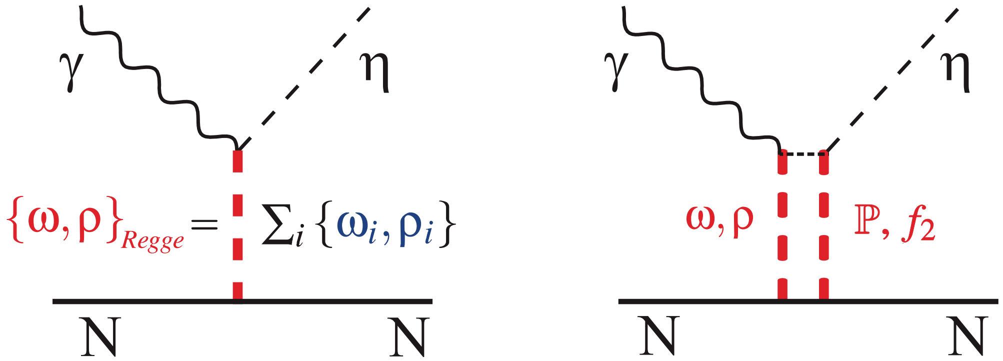
Non-resonant background

s- and u- channel Born terms



Born terms for $\gamma p \to \eta p$ are small but can be significant in interferences.

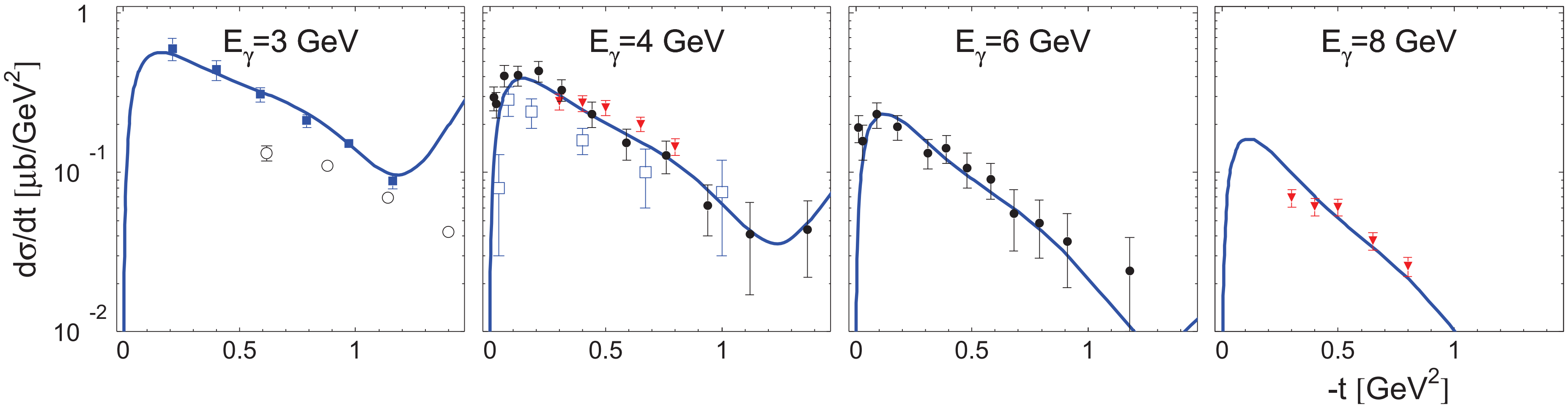
t-channel exchanges



- Regge trajectories: $\rho,\ \omega$
- Regge and Regge cuts trajectories: $\rho,\ \omega,\ \rho f_2, \omega f_2, \rho\mathbb{P}, \omega\mathbb{P}$

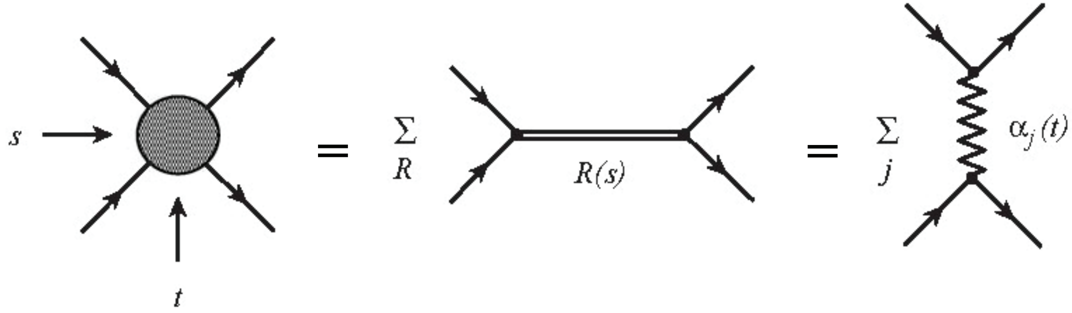
t-channel Regge exchanges used in the analysis

Solution obtained with Regge poles (formalism written in terms of ν)



Solution has been taken from: *Kashevarov, Tiator, Ostrick: Phys.Rev.C96, 035207(2017)*

Quark-hadron duality



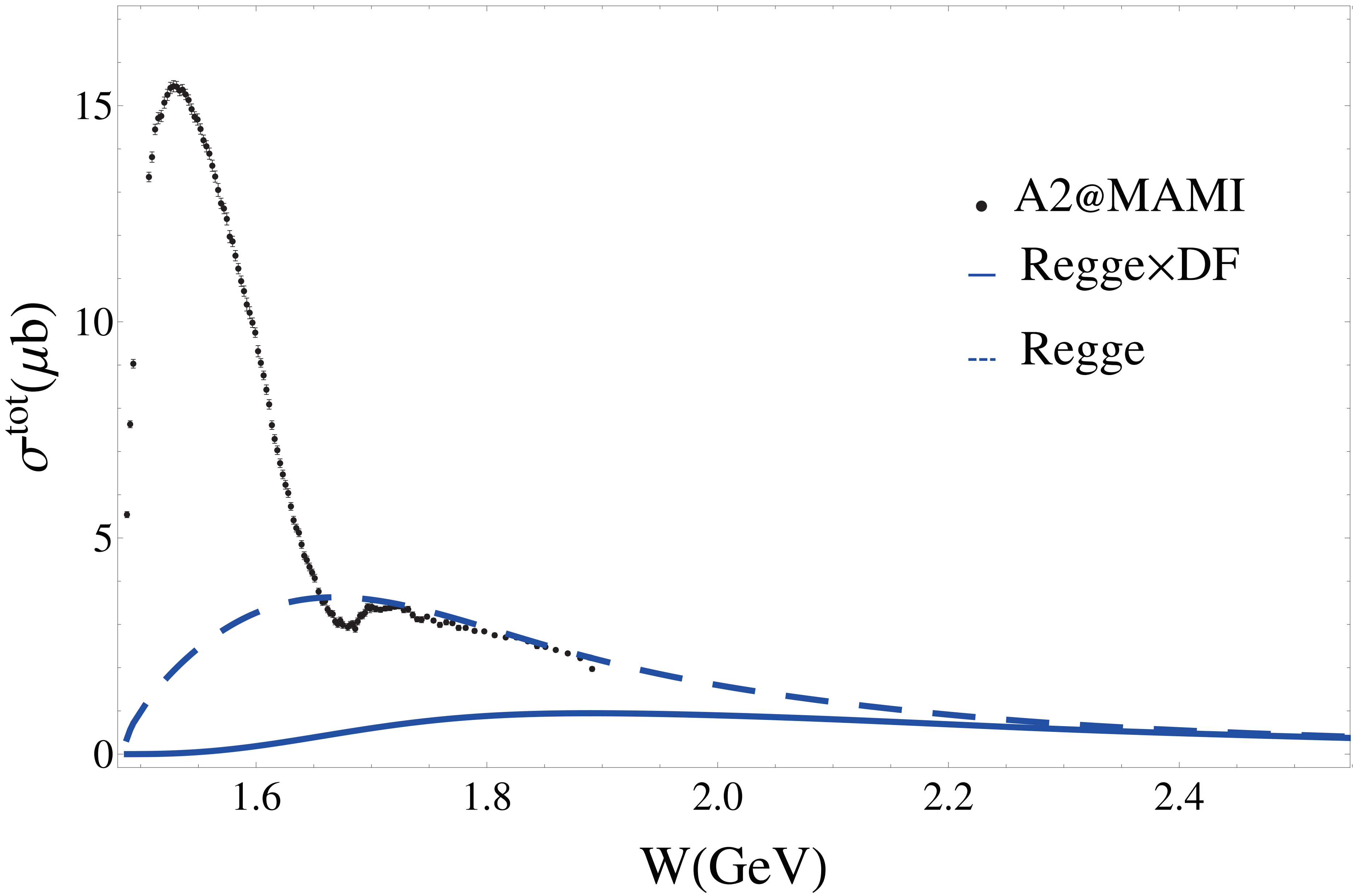
Sum of all s-channel resonances is equivalent to sum over all t-channel resonances, therefore keeping both will lead to a double counting

$$
\begin{aligned}
M &= \sum_{i=1}^{\infty} M_s^{Res_i} = \sum_{i=1}^{\infty} M_t^{Res_i} = \sum_{i=1}^{N} M_s^{Res_i} + \left[\sum_{i=1}^{\infty} M_t^{Res_i} - \sum_{i=1}^{N} M_s^{Res_i} \right] \\
&\approx \sum_{i=1}^{N} M_s^{Res_i} + M^{Regge} \times DF \qquad \text{: our approach}
\end{aligned}
$$

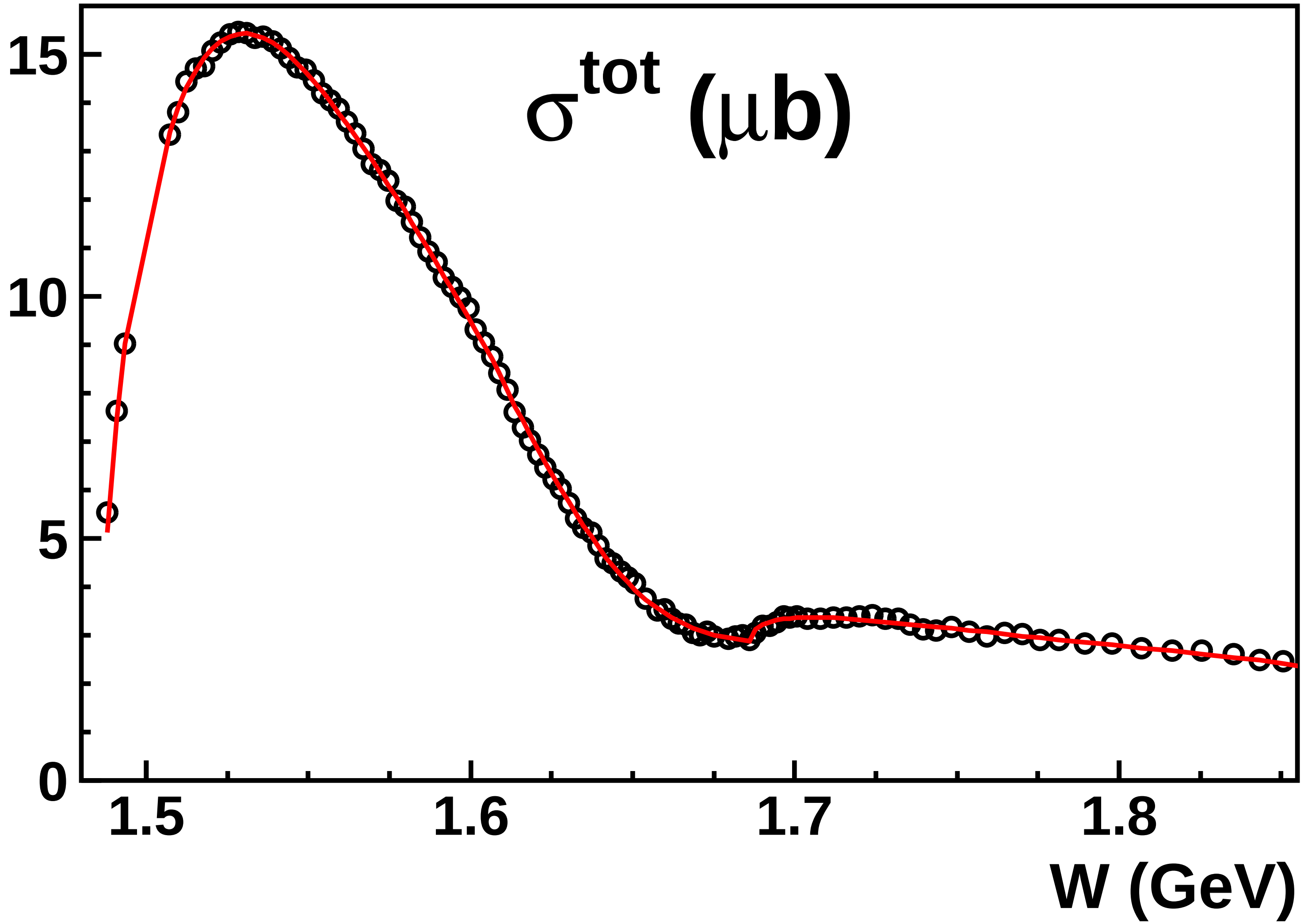
Low energy behavior

In order to use Regge as a background one has to deal with contributions at low energies

Regge contributions to the total cross section



Fit result for the total cross section with isobar model EtaMAID



Fixed-t dispersion relations

Important: We want more than describing the data!

$\Downarrow$

Use dispersion relations: Invariant amplitudes are analytic functions of complex variables, one can derive dispersion relations at a fixed value of t.

Crossing even:

$$\text{Re}A_i(\nu,t) \quad = \quad A_i^{pole}(\nu,t) + \frac{2}{\pi}\,\mathcal{P}\!\int_{\nu_{thr}(t)}^{\infty} d\nu' \, \frac{\nu' \; \text{Im}A_i(\nu',t)}{\nu'^2 - \nu^2}\,, \quad \text{for } i = 1,2,4.$$

Crossing odd:

$$\text{Re}A_i(\nu,t) \quad = \quad A_i^{pole}(\nu,t) + \frac{2\nu}{\pi}\,\mathcal{P}\!\int_{\nu_{thr}(t)}^{\infty} d\nu' \, \frac{\text{Im}A_i(\nu',t)}{\nu'^2 - \nu^2}\,, \quad \text{for } i = 3,$$

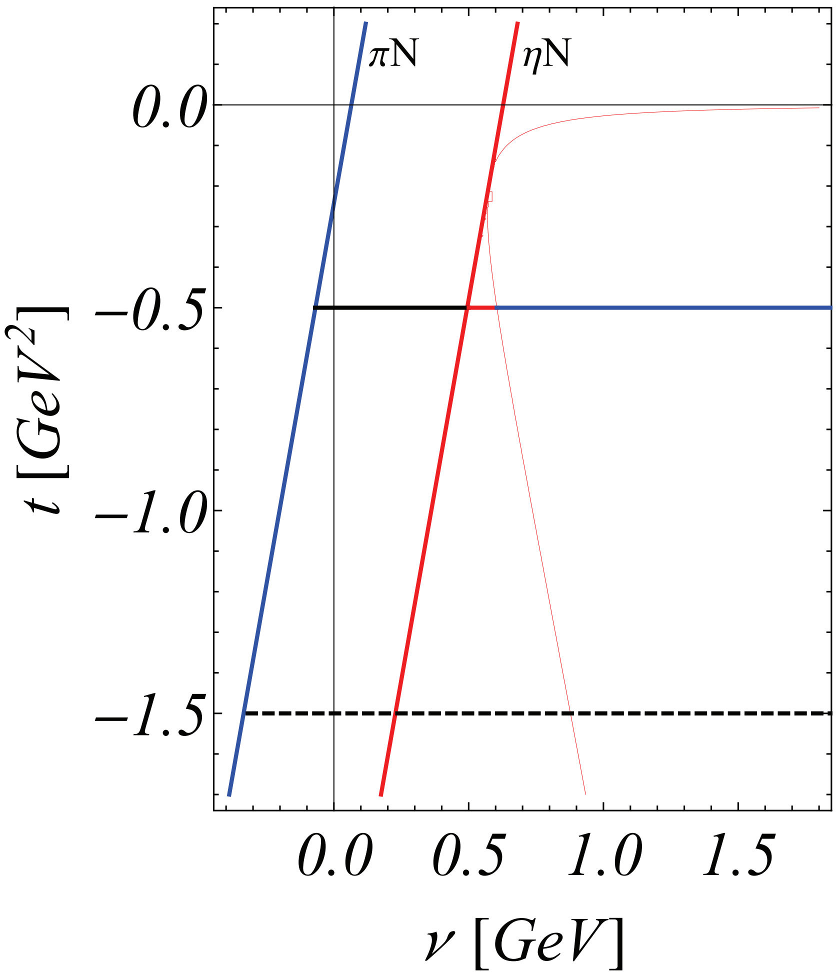
where $\nu_{thr}(t)$ corresponds to the πN threshold, $A_i^{pole}(\nu,t)$ are pole terms.

Real part is calculated via dispersive integral out of the Imaginary part

$\Downarrow$

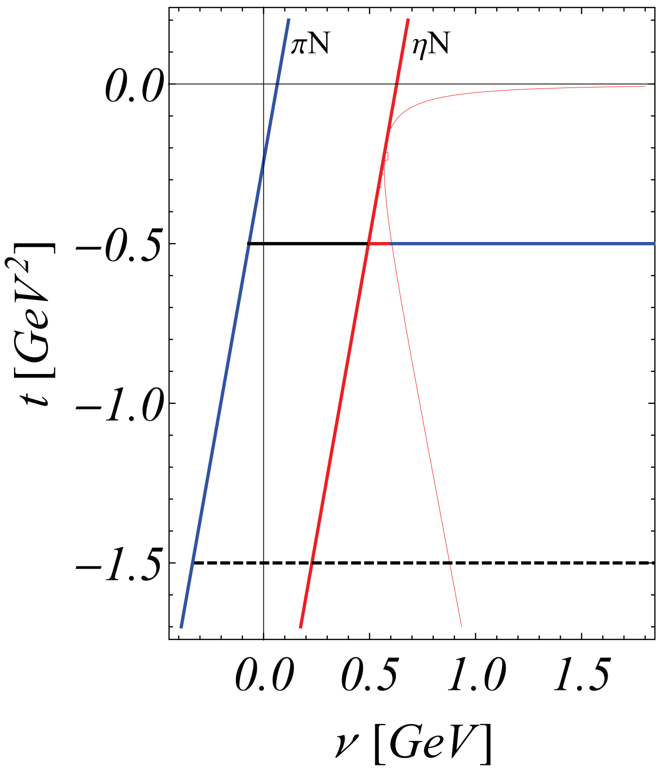
Real and Imaginary parts are not independent anymore

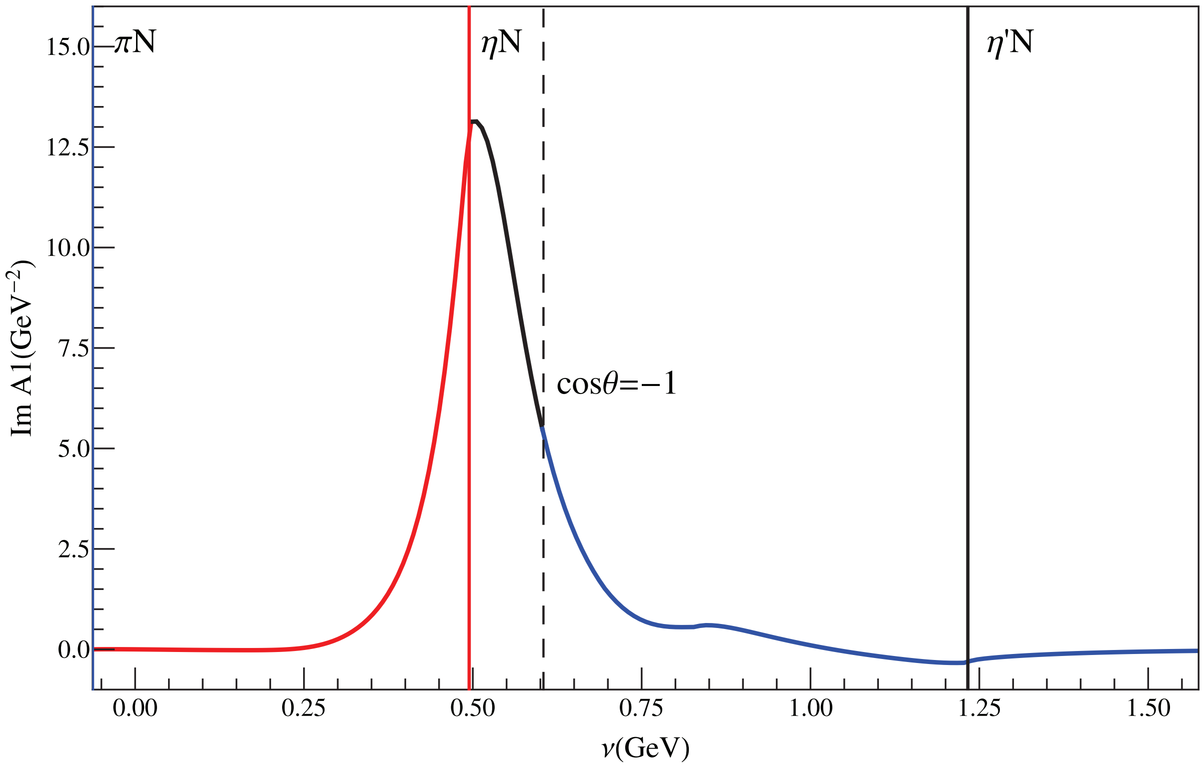
Mandelstam plane for $\gamma p \to \eta p$



$$\cos\theta = \frac{(s-m_p^2)^2 - m_\eta^2(s+m_p^2) + 2\,s\,t}{2q\sqrt{s}(s-m_p^2)}$$

Integration regions





However, Im. parts of invariant amplitudes not always look like this, I will discuss this later

Fitted data

$\gamma p \to \eta p$	Observable	Energy range W (MeV)	$\cos(\theta)$
A2@MAMI	$d\sigma/d\Omega$	1488-1851	[-0.958,0.958]
A2@MAMI	T	1495-1850	[-0.916,0.916]
A2@MAMI	F	1495-1850	[-0.916,0.916]
GRAAL	Σ	1490-1863	[-0.95,0.84]
CLAS	E	1525-1825	Different angular binning for each energy

T - unpolarized beam and transverse polarized target

F - circularly polarized beam and transverse polarized target

Σ - linearly polarized beam and unpolarized target

E - circularly polarized beam and longitudinally polarized target

Fit results

In the thesis, 7 important scenarios are investigated and reported.

Here I present most extended and the best one.

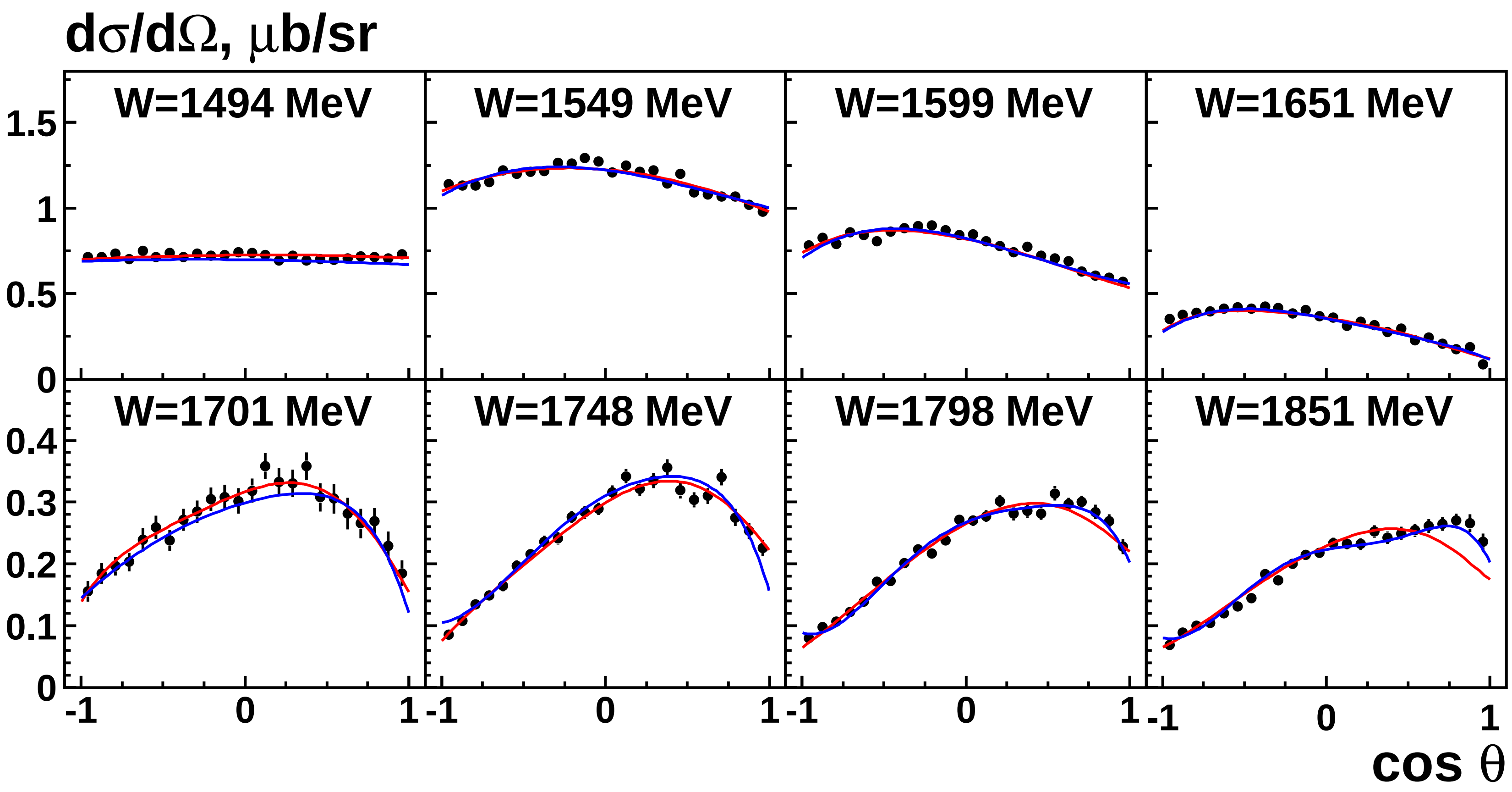
Resonances and Regge $\times$ DF

$$\chi^2_{IB}/N_{dof} = 1.61 \quad \chi^2_{DR}/N_{dof} = 1.61$$

$\gamma p \to \eta p$	Observable	χ^2_{IB}	χ^2_{DR}	Number of points
A2@MAMI	$d\sigma/d\Omega$	3448	3388	2544
A2@MAMI	T	456	423	144
A2@MAMI	F	318	426	144
GRAAL	Σ	323	353	130
CLAS	E	38	31	42
DESY,WLS,Daresbury,CEA	$d\sigma/dt$	11	13	52
Daresbury	Σ	7	13	12
Daresbury	T	1	2	3

Differential cross section $d\sigma/d\Omega$

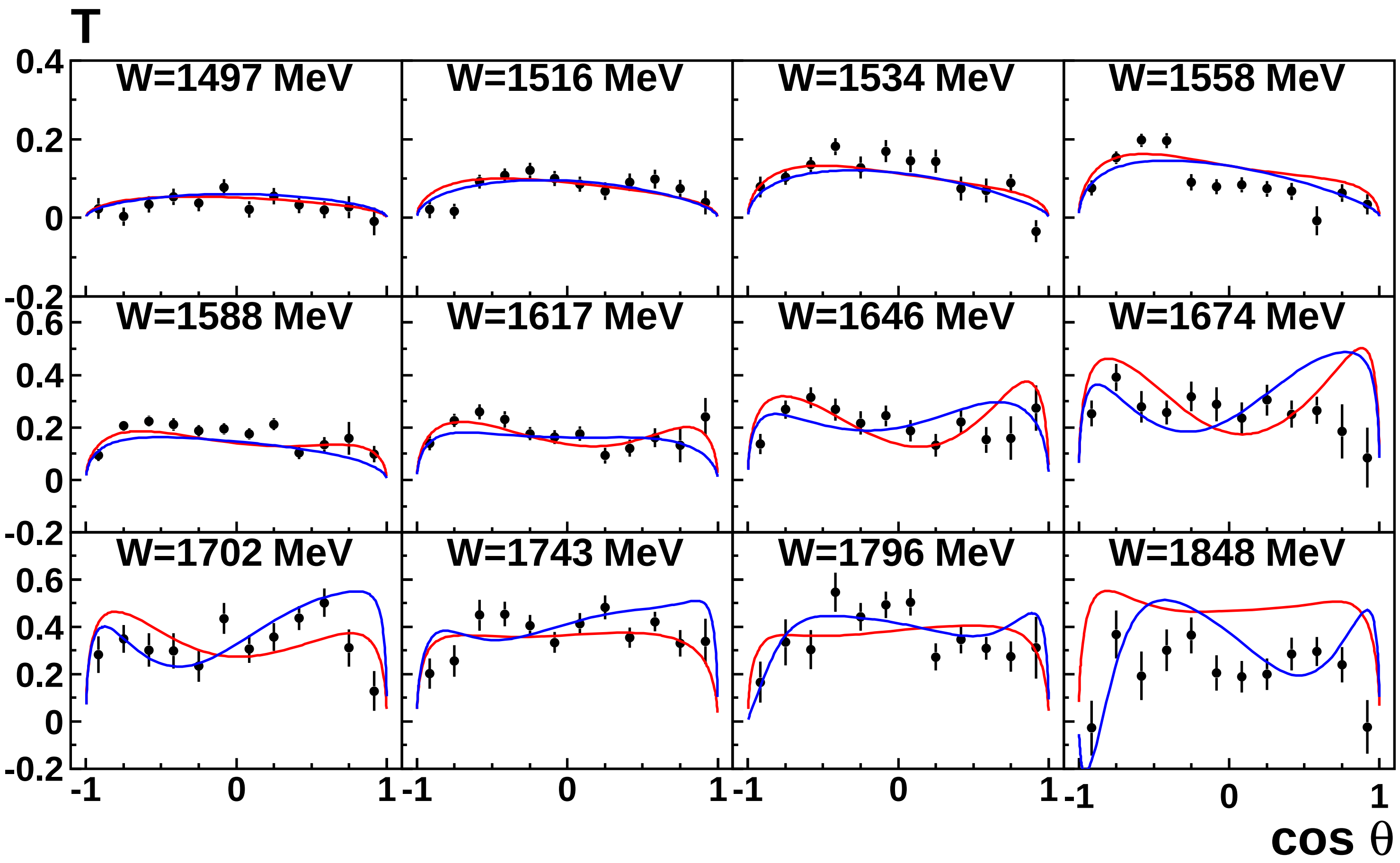
$$\chi^2_{IB} = 3448/2544 \qquad \chi^2_{DR} = 3388/2544$$



Selected bins are shown for convenience

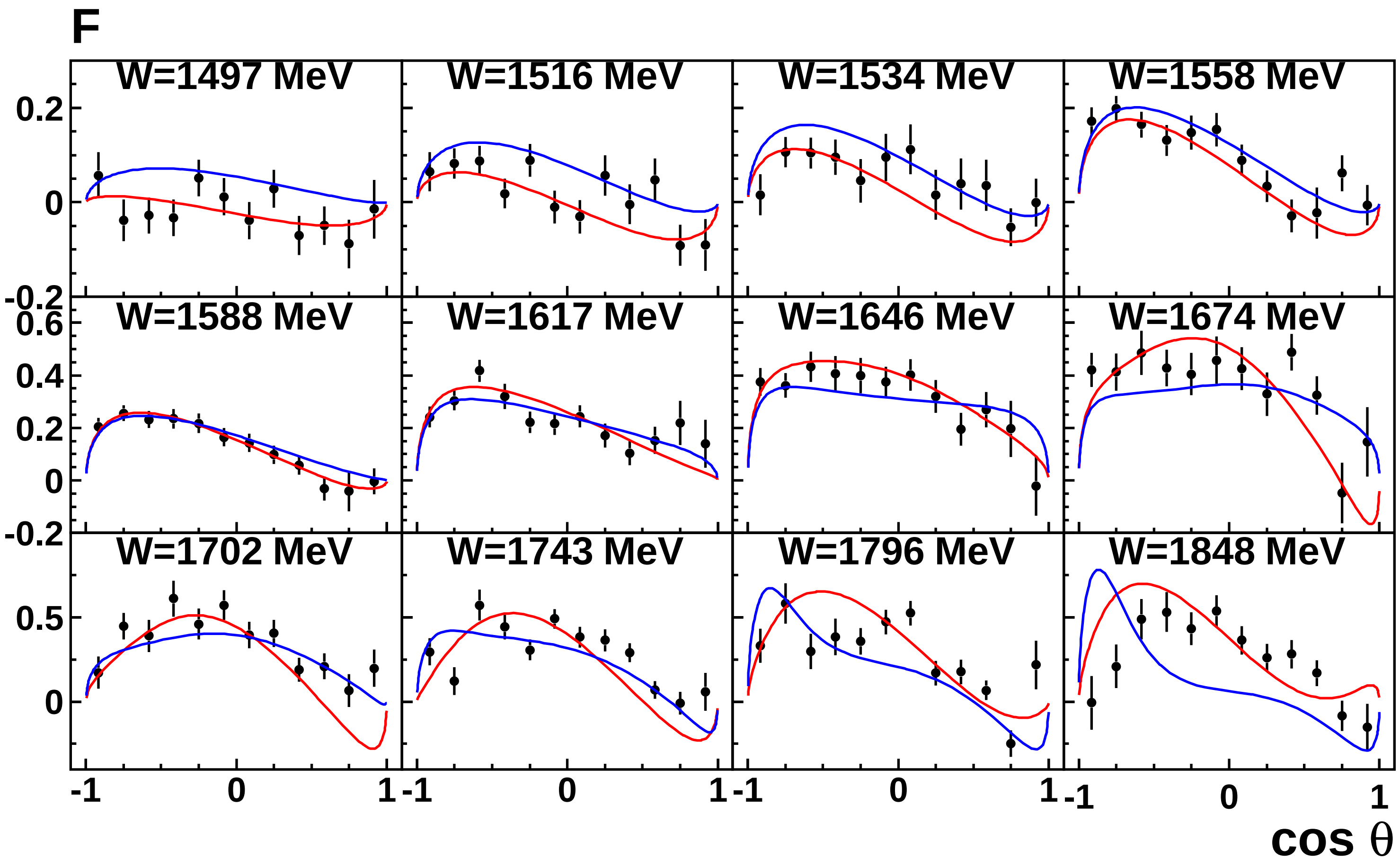
Target asymmetry T

$$\chi^2_{IB} = 456/144 \qquad \chi^2_{DR} = 423/144$$



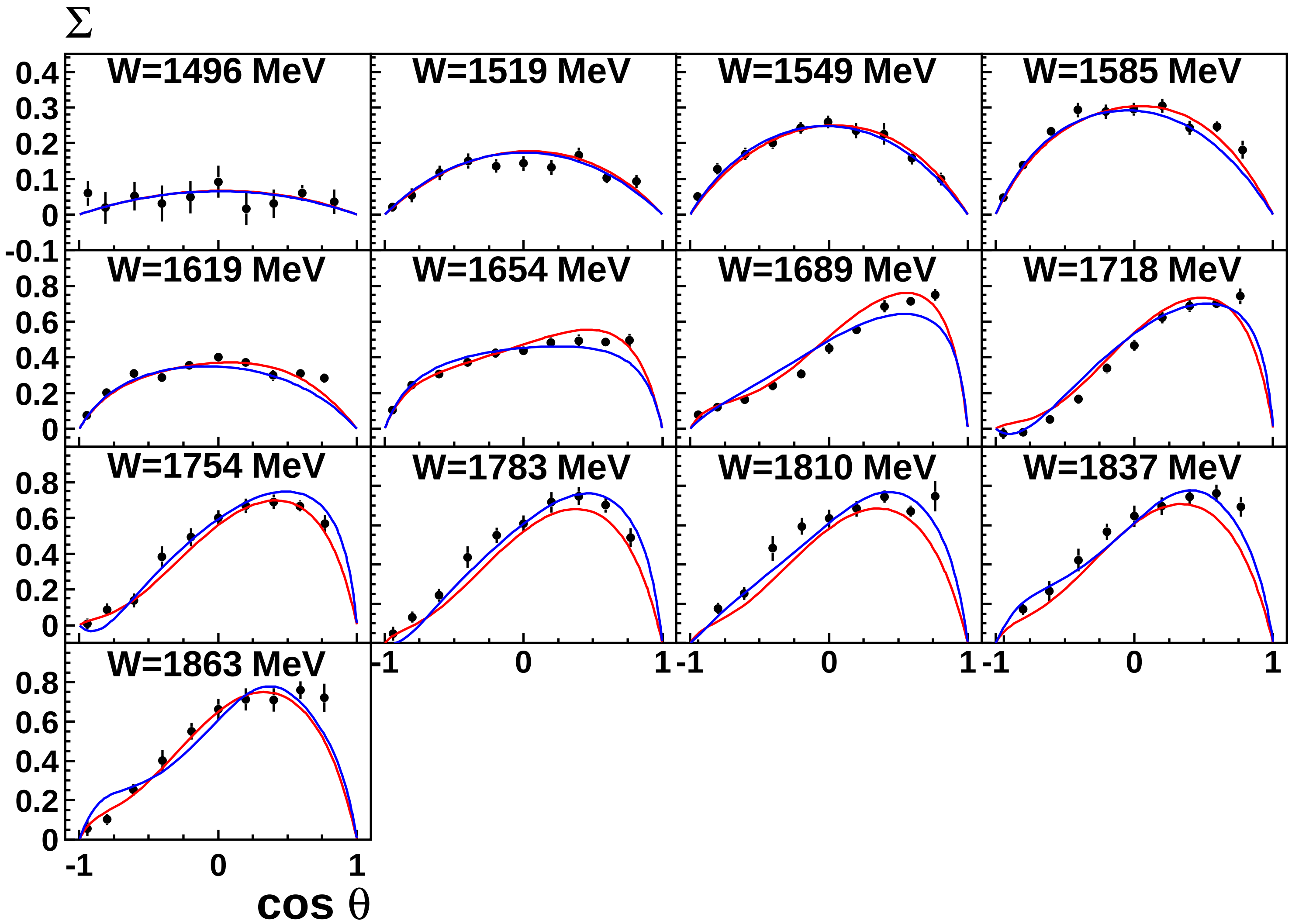
Beam-target asymmetry F

$\chi^2_{IB} = 318/144$ $\chi^2_{DR} = 426/144$



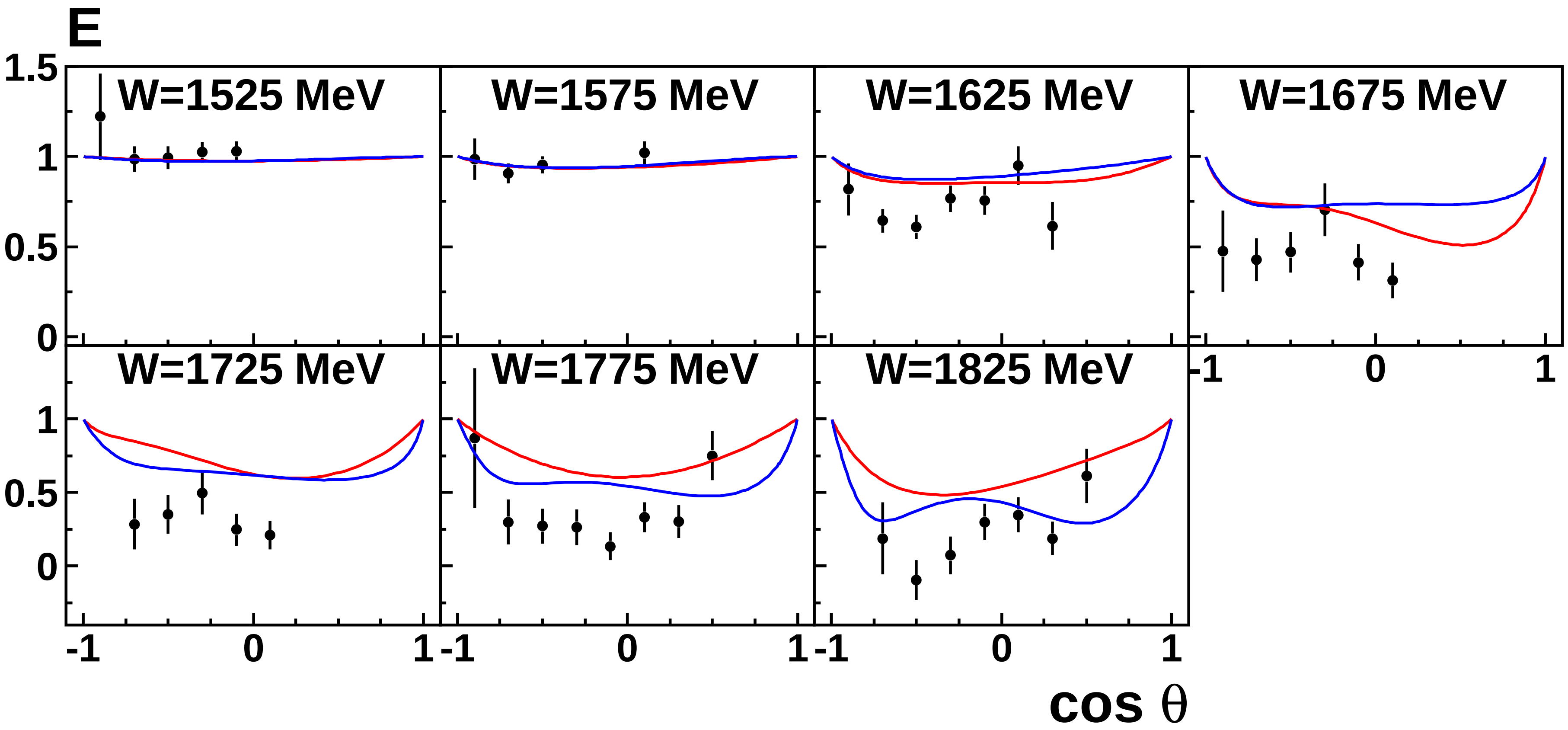
Beam asymmetry Σ

$\chi^2_{IB} = 323/130$ $\quad$ $\chi^2_{DR} = 353/130$



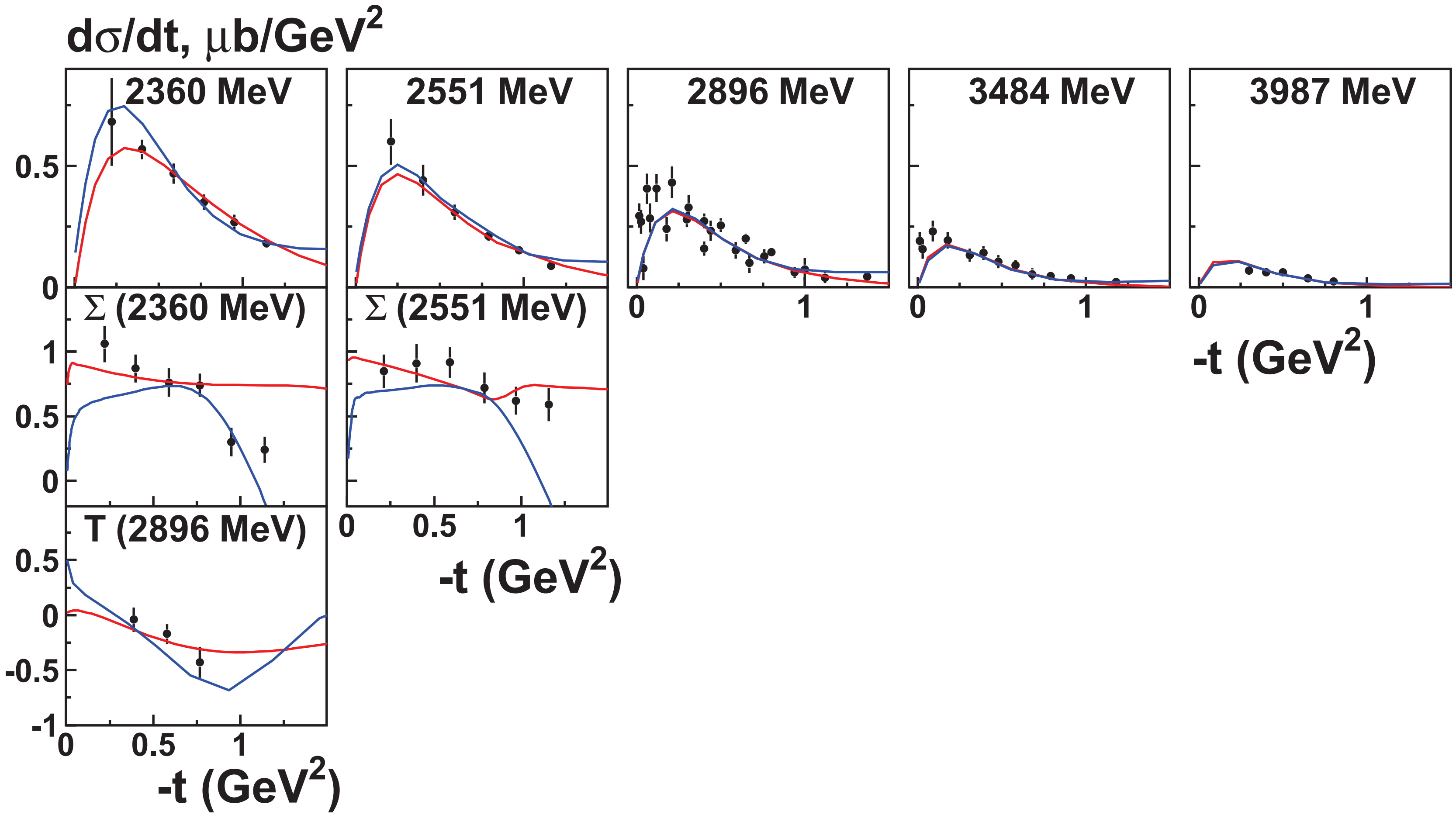
Beam-target asymmetry E

$$\chi^2_{IB} = 38/42 \qquad \chi^2_{DR} = 31/42$$



High energy data, $d\sigma/dt$, Σ, T

$\chi^2_{IB} = 11/52,\ \ 7/12,\ \ 1/3$ $\qquad$ $\chi^2_{DR} = 13/52,\ \ 13/12,\ \ 2/3$



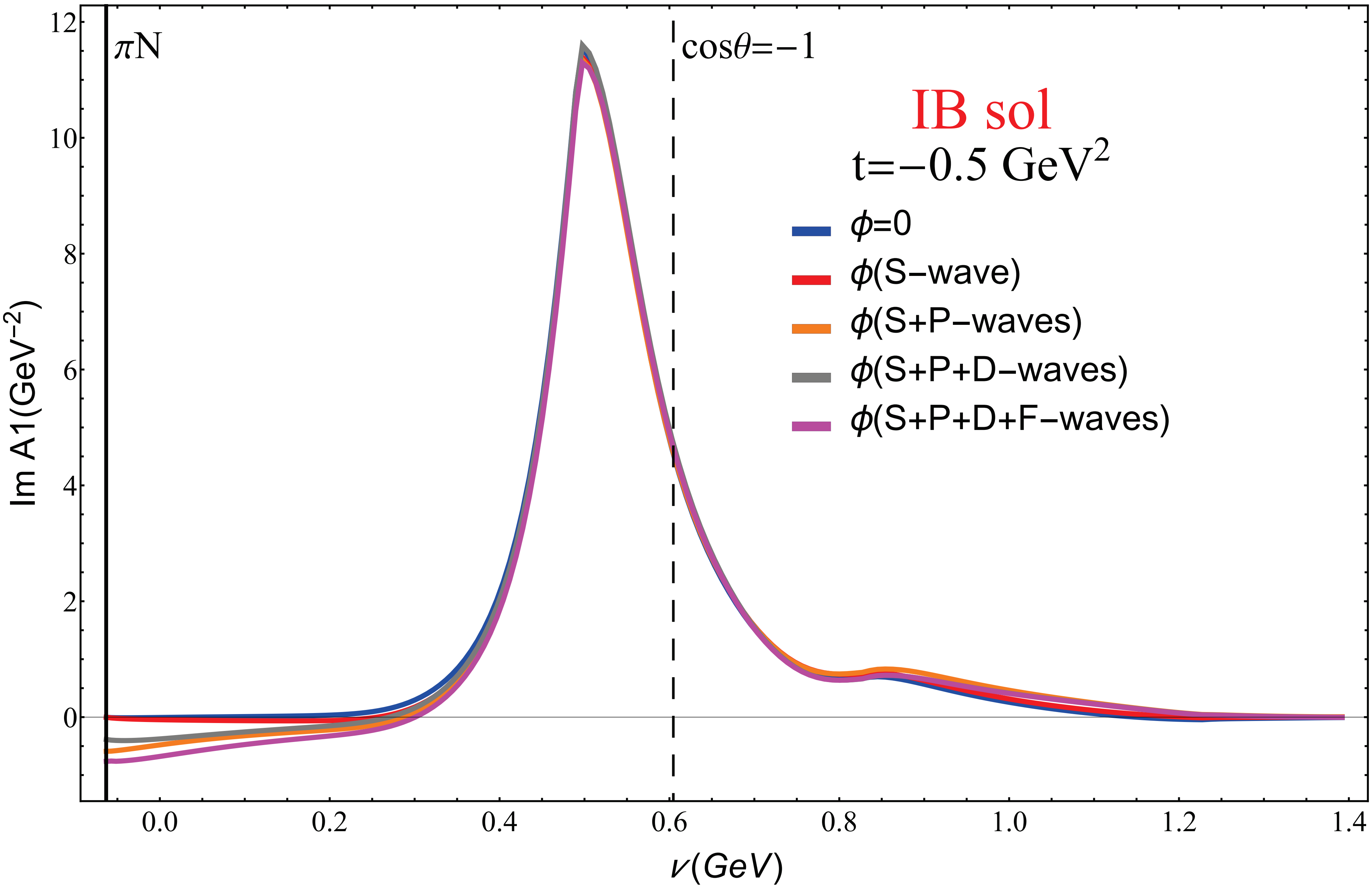
Intermediate conclusion

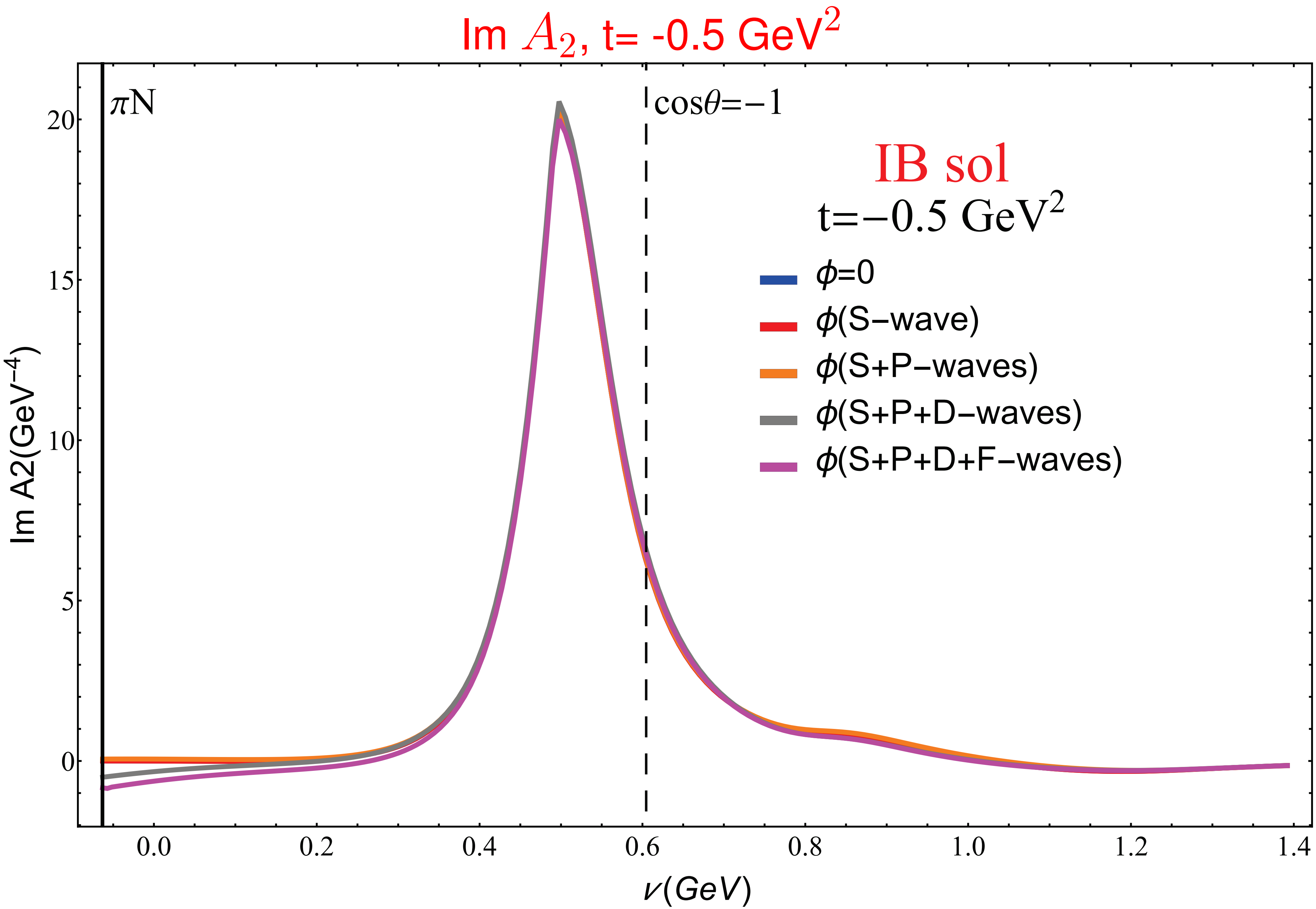
- Fit results are satisfactory, χ^2/N_{dof} is good
- BUT: Imaginary parts of invariant amplitudes in the unphysical region were not studied
- Resonance phase $\Phi_j =$ const $\Rightarrow Im\ A_i \neq 0$ at πN threshold

To DO

- Look at the imaginary part for both IB and DR solutions
- Use $t = -0.5$ and $t = -1.5\ GeV^2$ as a reference values
- Track which wave mostly affects the structure
- Possible ways of improvement

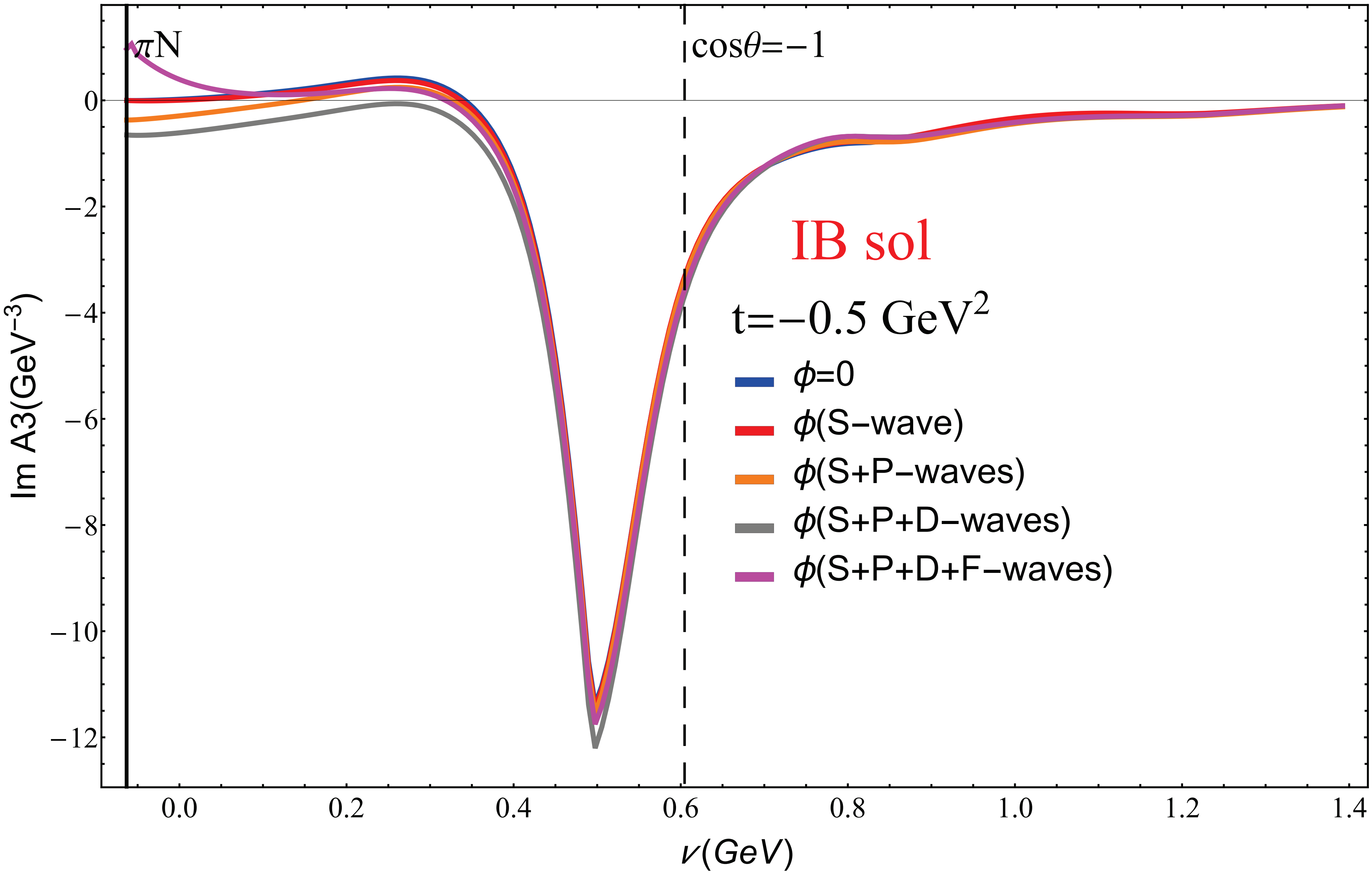
Im A_1, t= -0.5 GeV^2



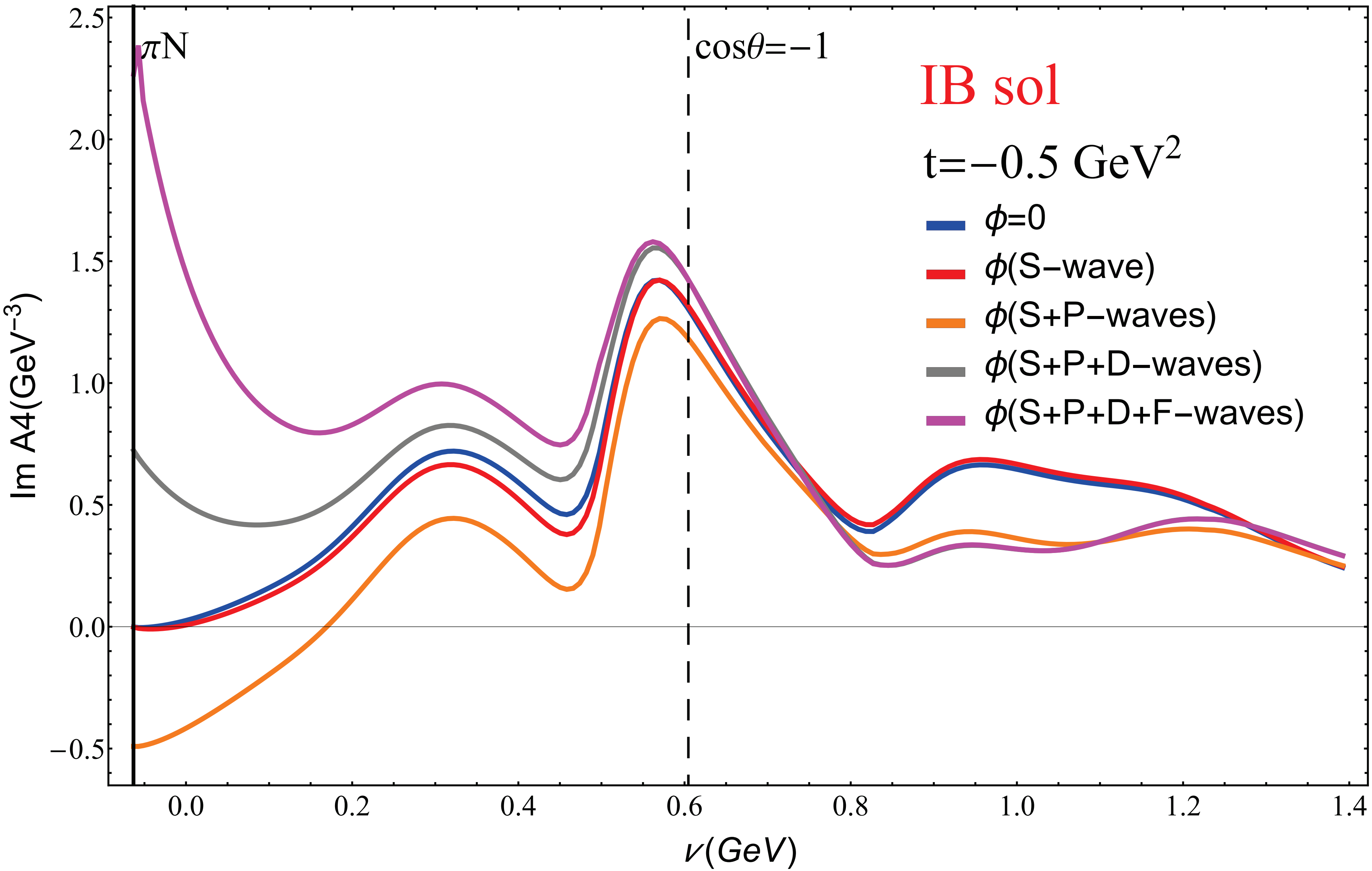


Im A_2, t= -0.5 GeV^2
πN
cosθ=−1
IB sol
t=−0.5 GeV^2
ϕ=0
ϕ(S−wave)
ϕ(S+P−waves)
ϕ(S+P+D−waves)
ϕ(S+P+D+F−waves)
Im A2(GeV^{-4})
ν(GeV)
0
5
10
15
20
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

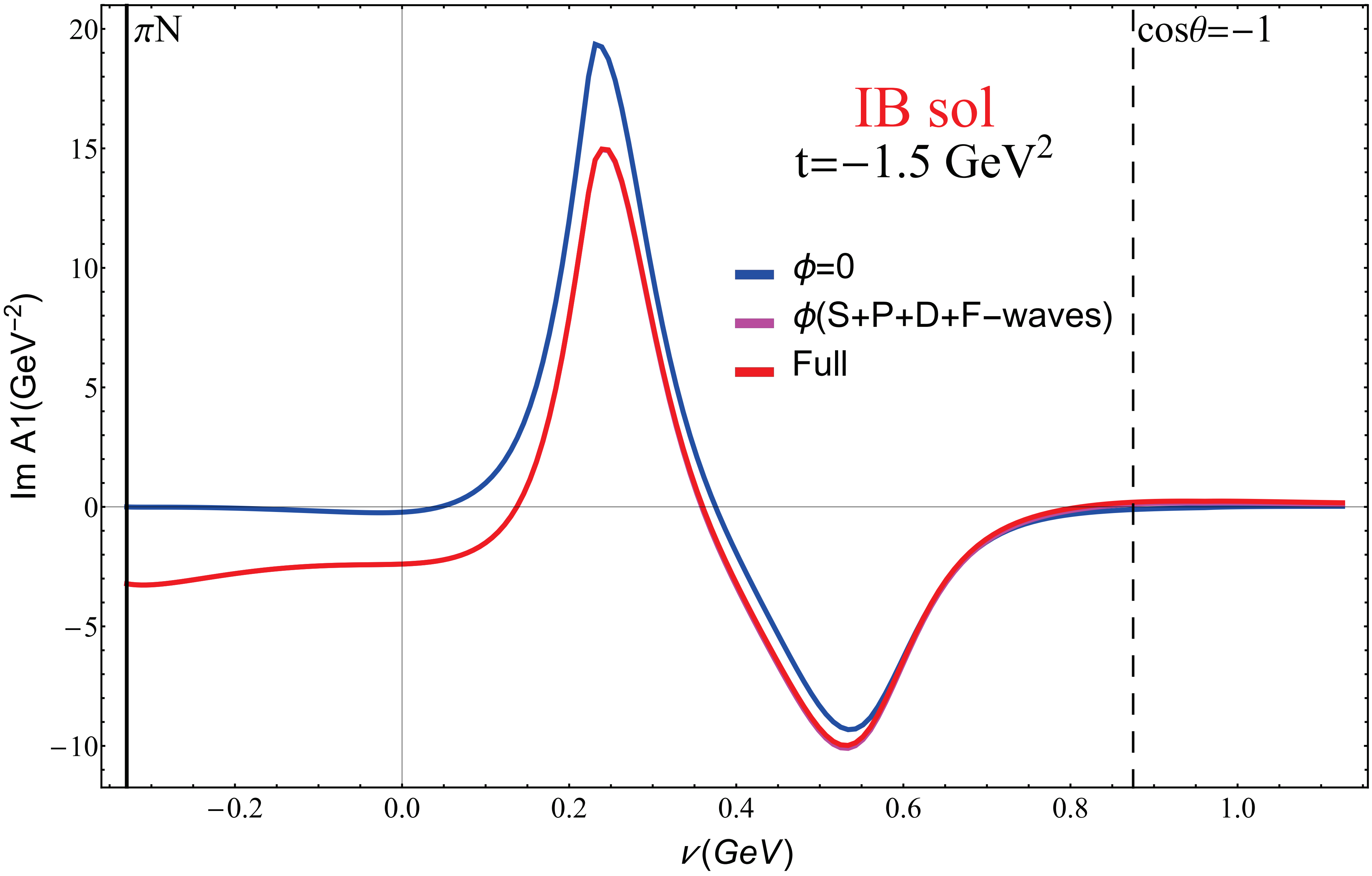
Im A_3, t= -0.5 GeV^2

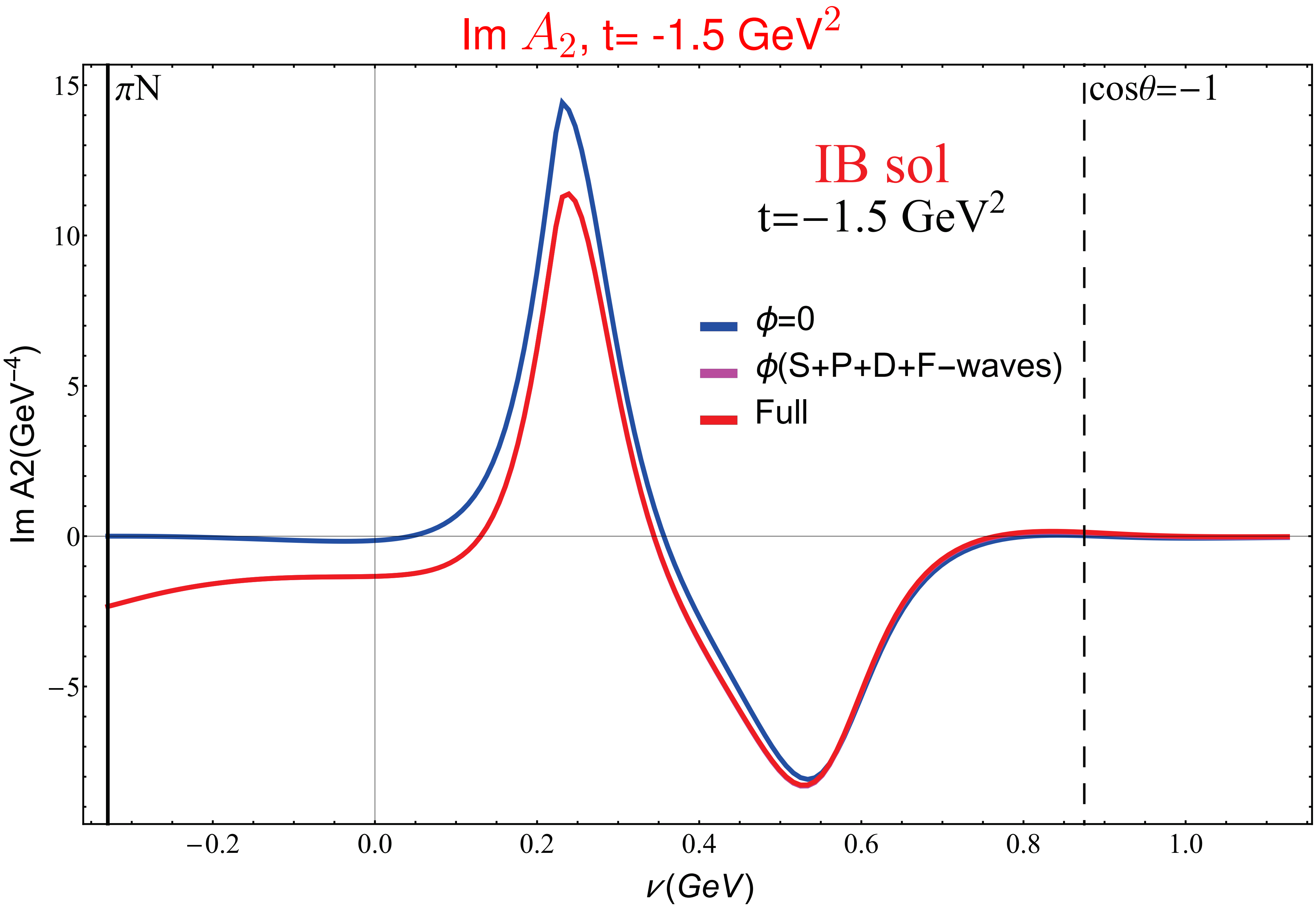


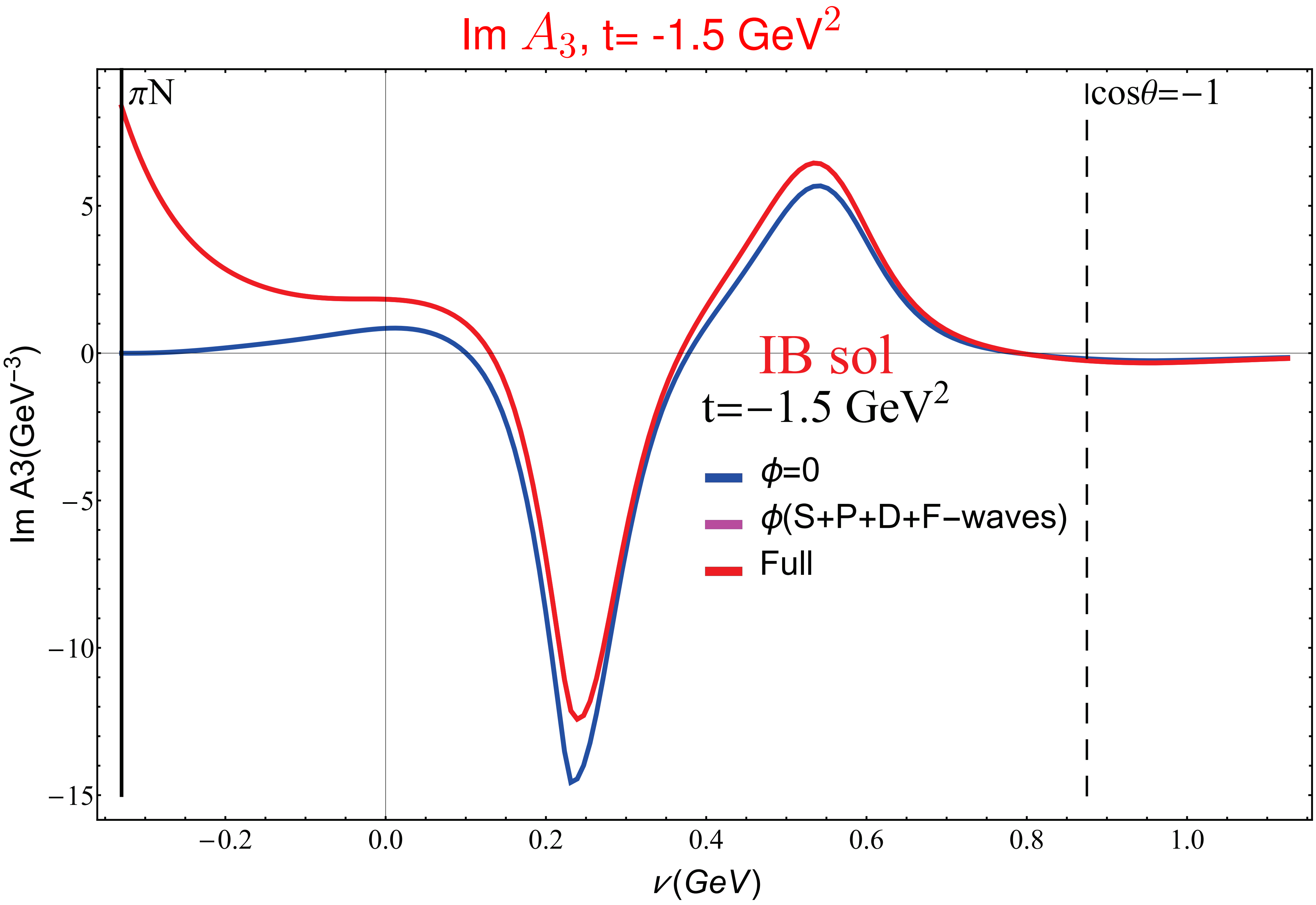
Im A_4, t= -0.5 GeV^2



Im A_1, t= -1.5 GeV^2

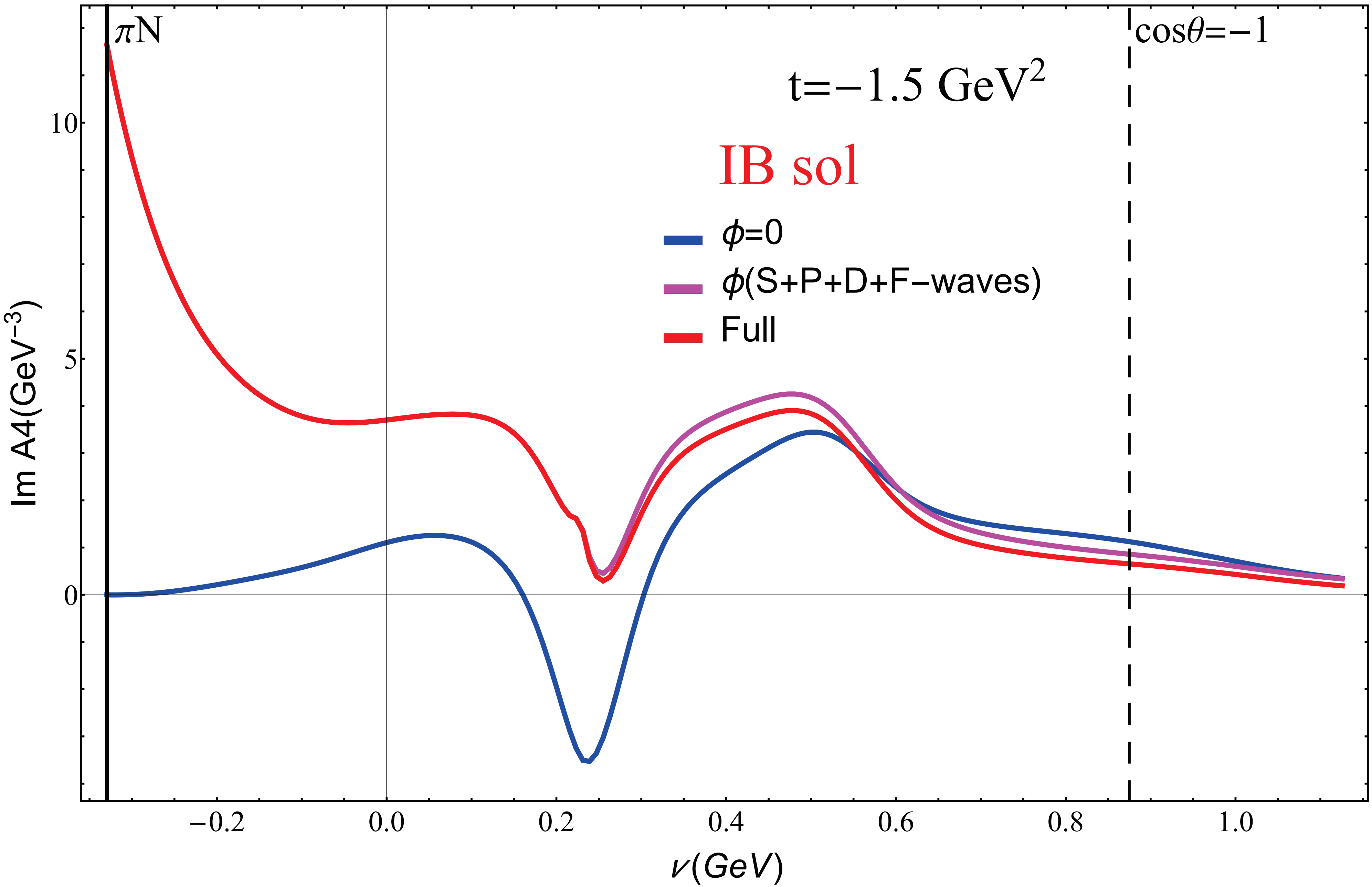


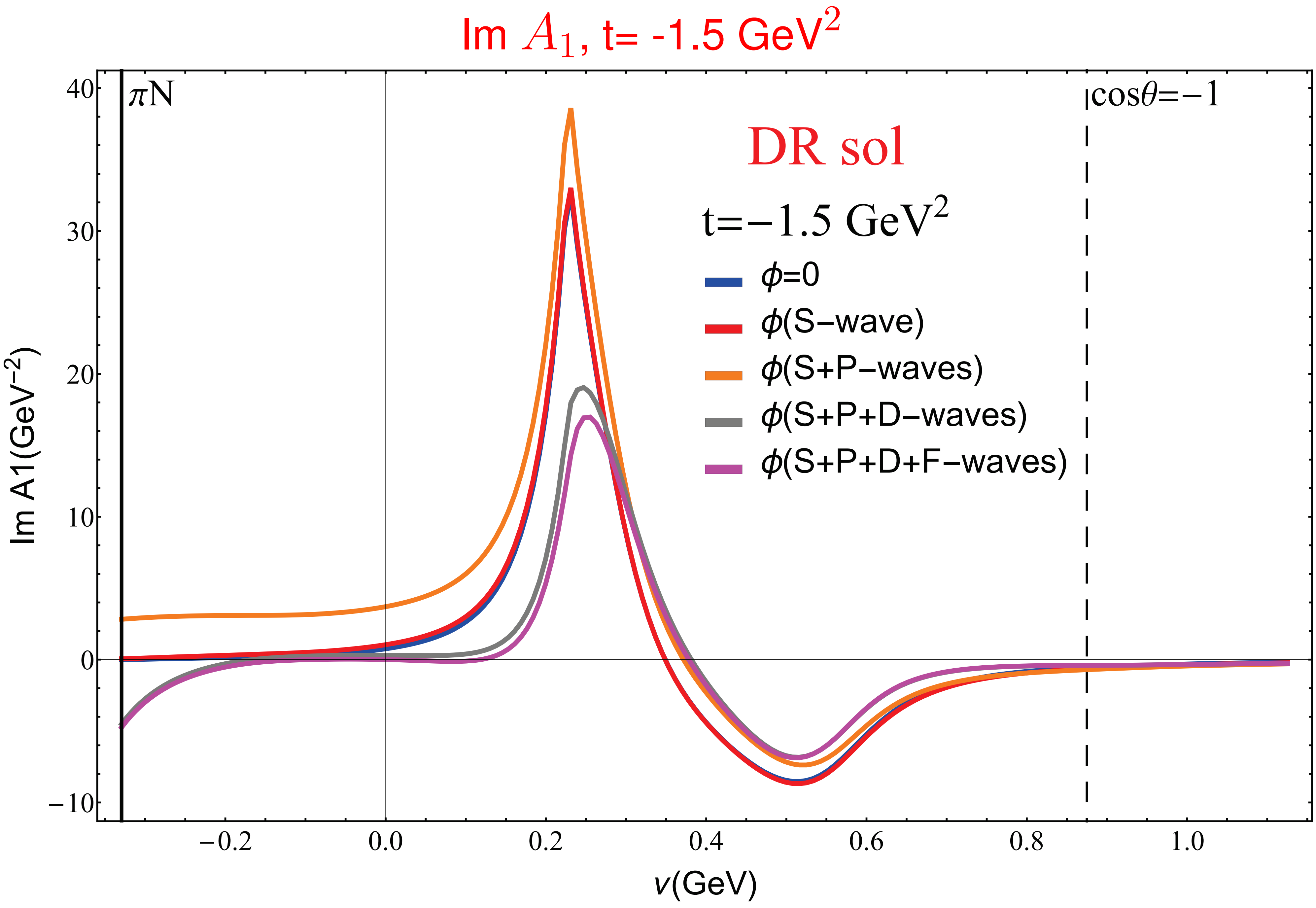

Im A_2, t= -1.5 GeV^2
πN
$|\cos\theta = -1$
IB sol
t=−1.5 GeV^2
ϕ=0
ϕ(S+P+D+F−waves)
Full
Im A2(GeV^{-4})
ν(GeV)
15
10
5
0
−5
−0.2
0.0
0.2
0.4
0.6
0.8
1.0



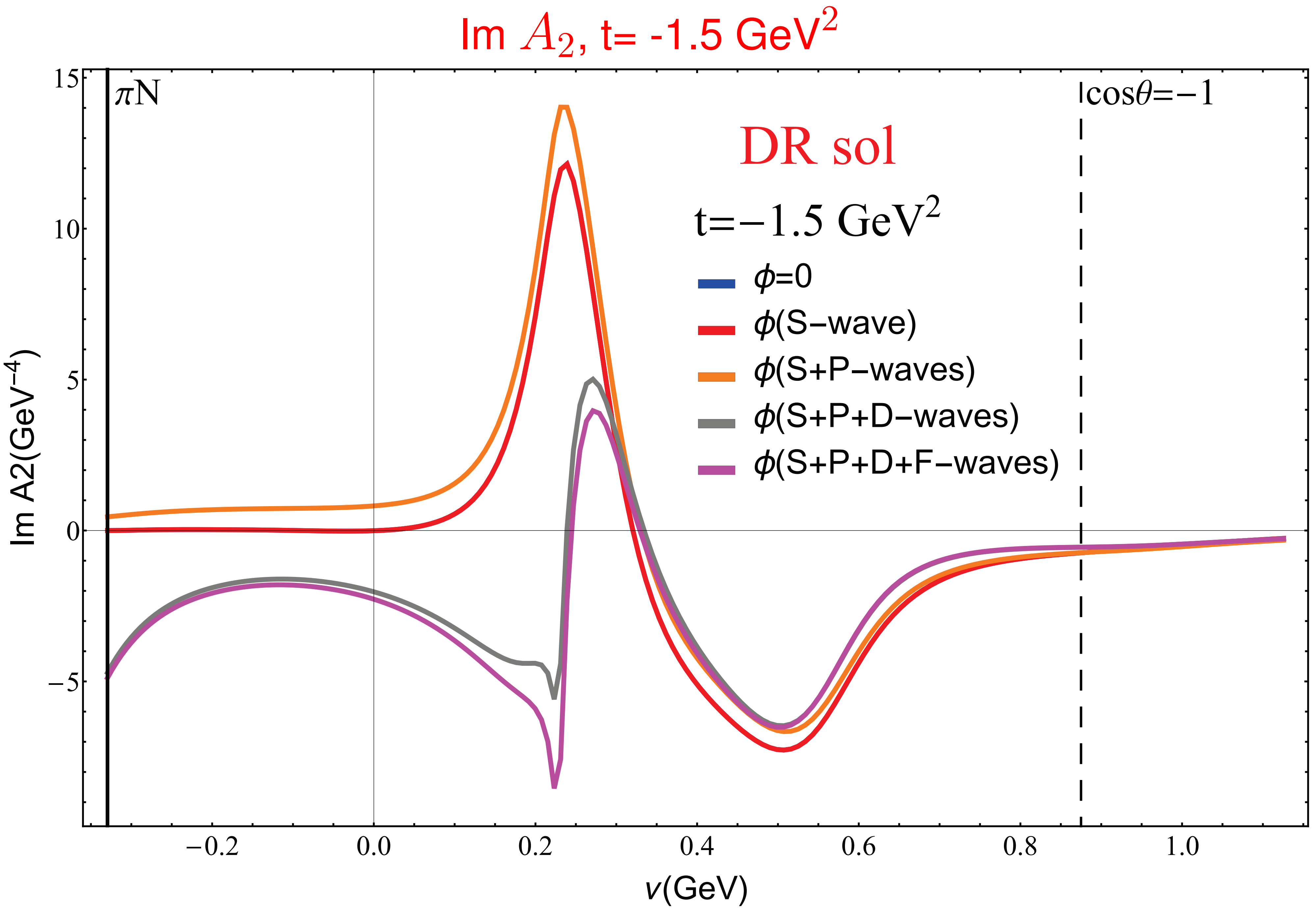
Im A_3, t= -1.5 GeV^2
πN
cosθ=−1
IB sol
t=−1.5 GeV^2
ϕ=0
ϕ(S+P+D+F−waves)
Full
Im A3(GeV^{-3})
5
0
−5
−10
−15
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ν(GeV)

Im A_4, t= -1.5 GeV^2

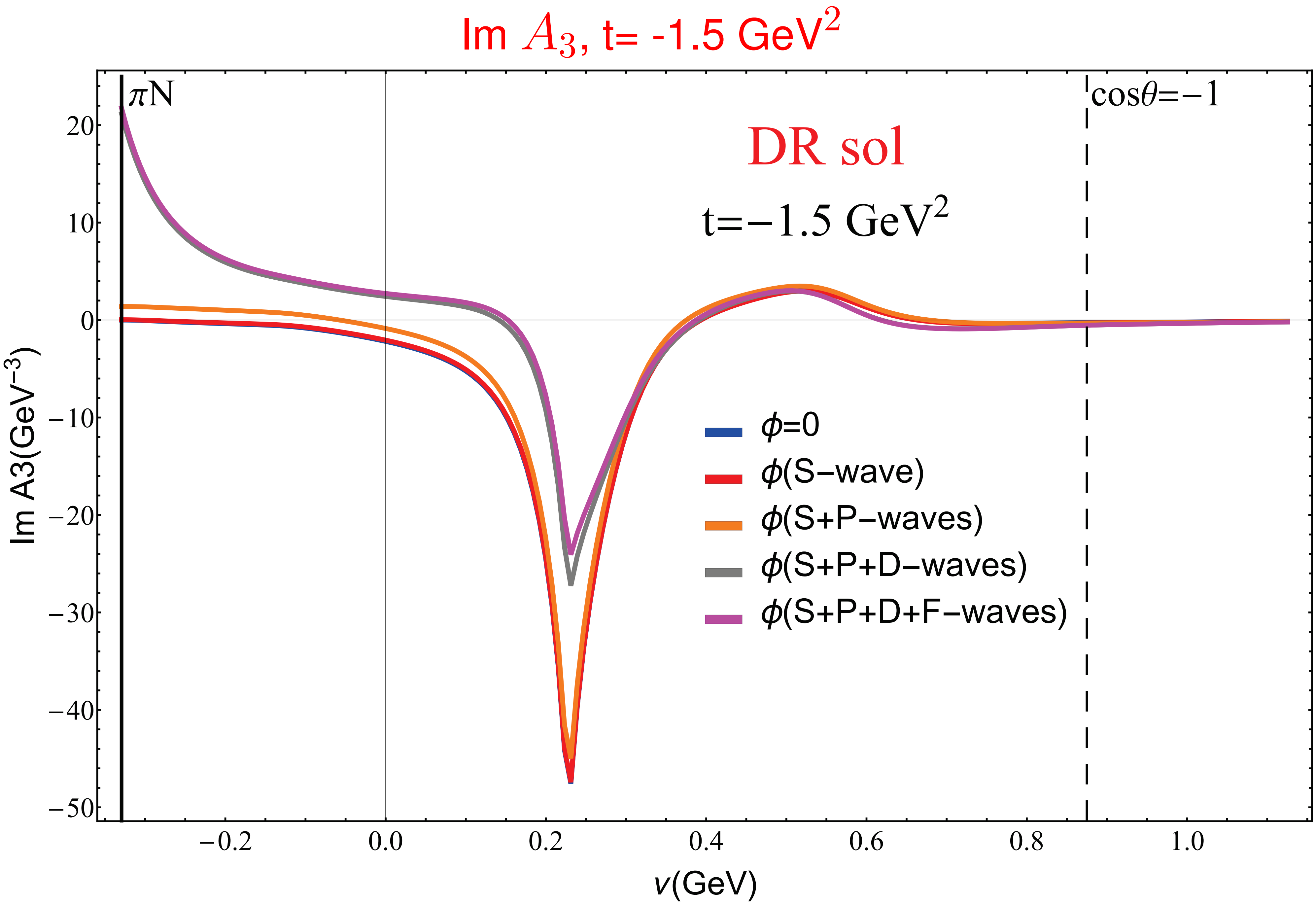




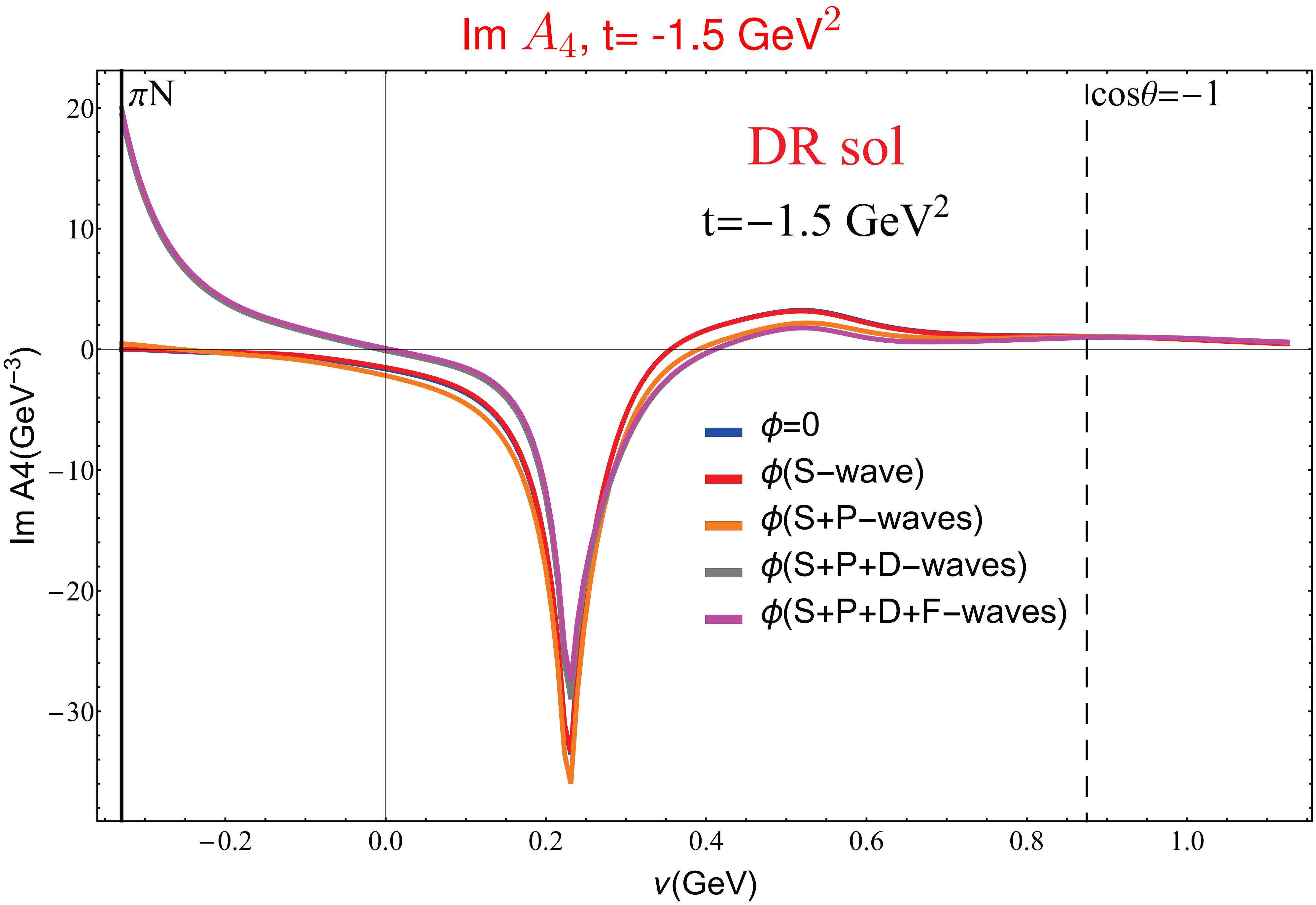
Im A_1, t= -1.5 GeV^2
πN
cosθ=−1
DR sol
t=−1.5 GeV^2
ϕ=0
ϕ(S−wave)
ϕ(S+P−waves)
ϕ(S+P+D−waves)
ϕ(S+P+D+F−waves)
Im A1(GeV^{-2})
ν(GeV)
40
30
20
10
0
−10
−0.2
0.0
0.2
0.4
0.6
0.8
1.0



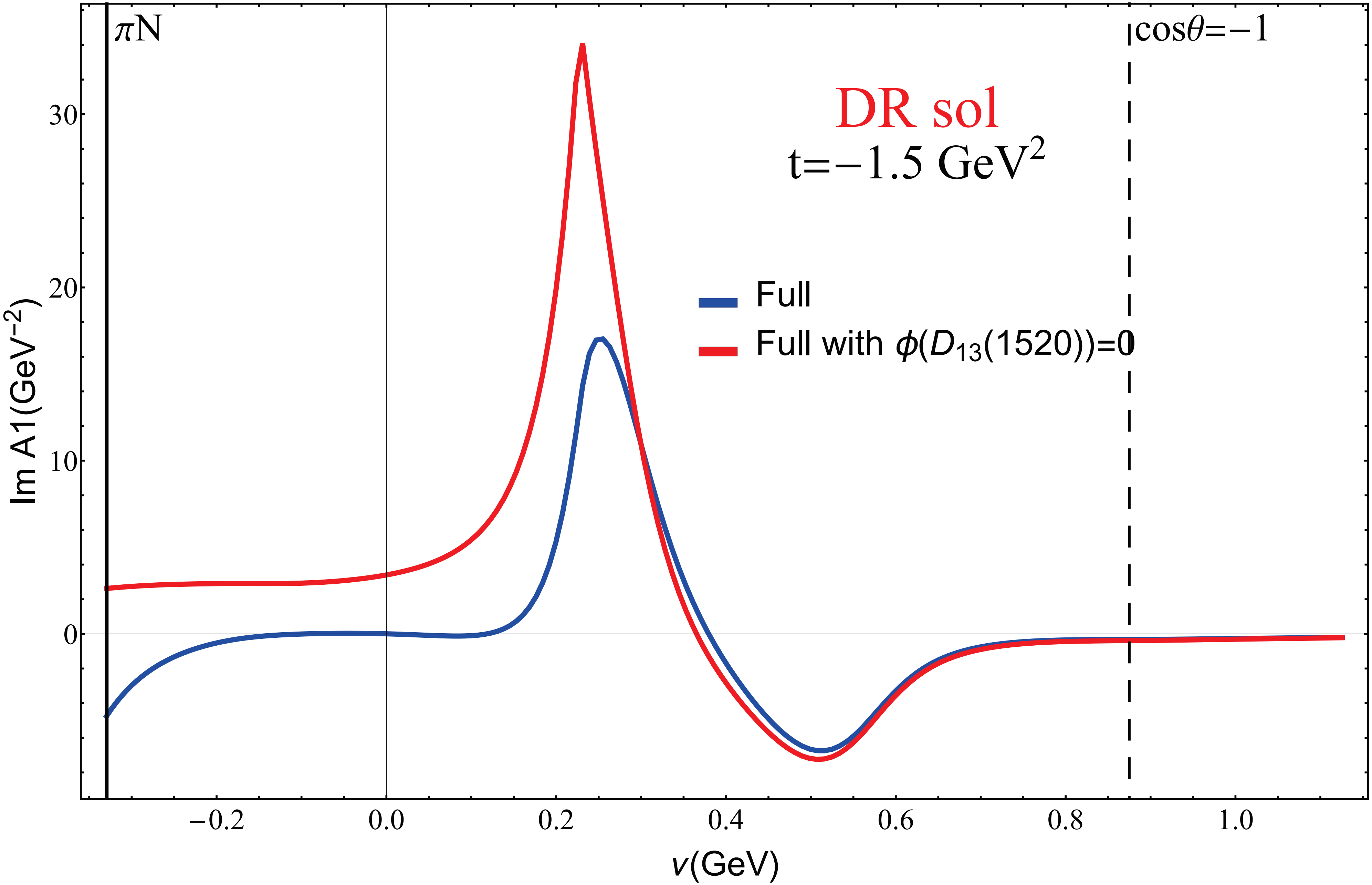
Im A_2, t= -1.5 GeV^2
πN
cosθ=−1
DR sol
t=−1.5 GeV^2
ϕ=0
ϕ(S−wave)
ϕ(S+P−waves)
ϕ(S+P+D−waves)
ϕ(S+P+D+F−waves)
Im A2(GeV^{-4})
15
10
5
0
−5
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ν(GeV)



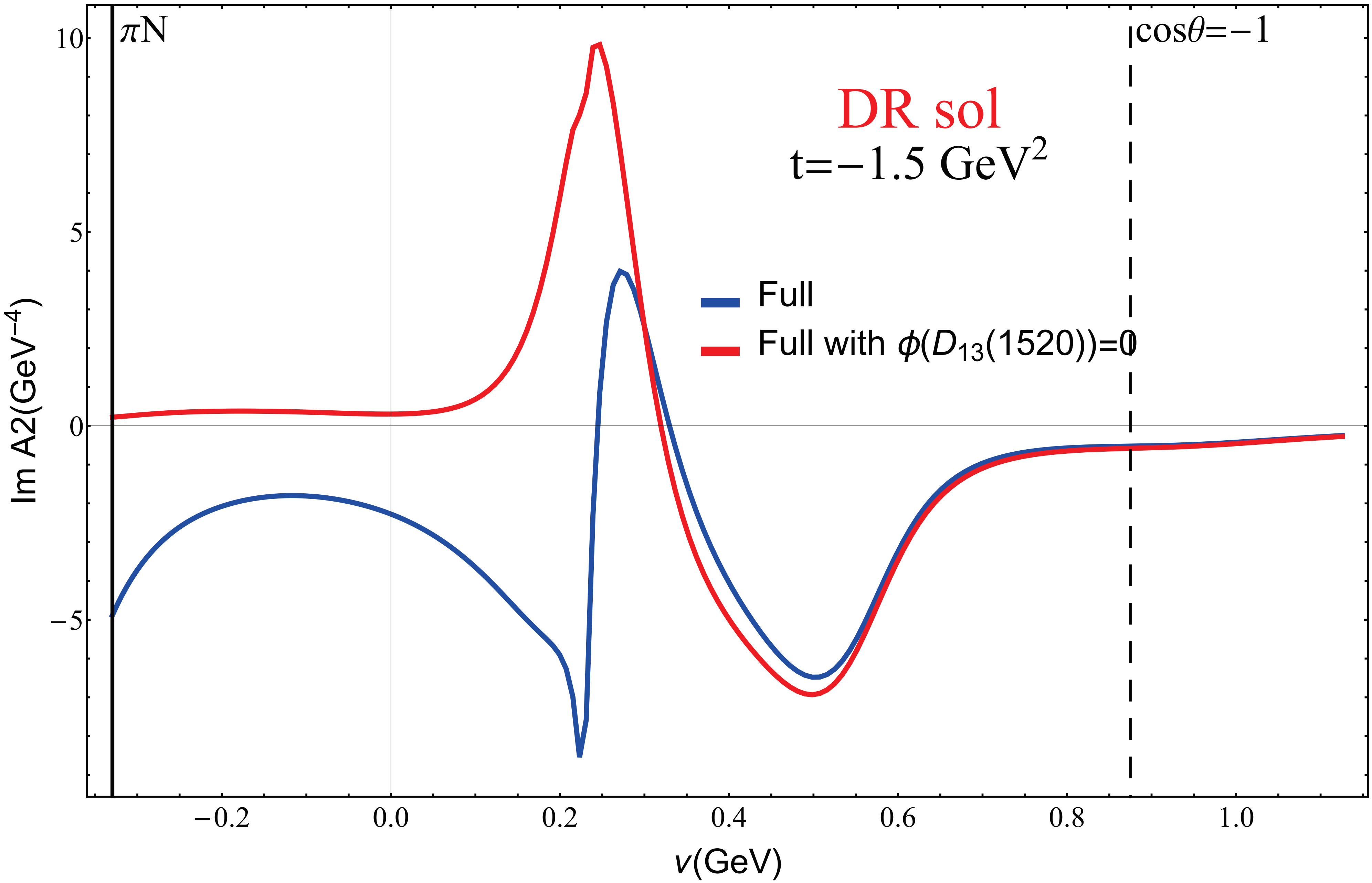
Im A_3, t= -1.5 GeV^2
πN
cosθ=−1
DR sol
t=−1.5 GeV^2
20
10
0
−10
−20
−30
−40
−50
Im A3(GeV^{-3})
ϕ=0
ϕ(S−wave)
ϕ(S+P−waves)
ϕ(S+P+D−waves)
ϕ(S+P+D+F−waves)
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ν(GeV)


Im A_4, t= -1.5 GeV^2
πN
cosθ=−1
DR sol
t=−1.5 GeV^2
20
10
0
−10
−20
−30
Im A4(GeV^{-3})
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ν(GeV)
ϕ=0
ϕ(S−wave)
ϕ(S+P−waves)
ϕ(S+P+D−waves)
ϕ(S+P+D+F−waves)

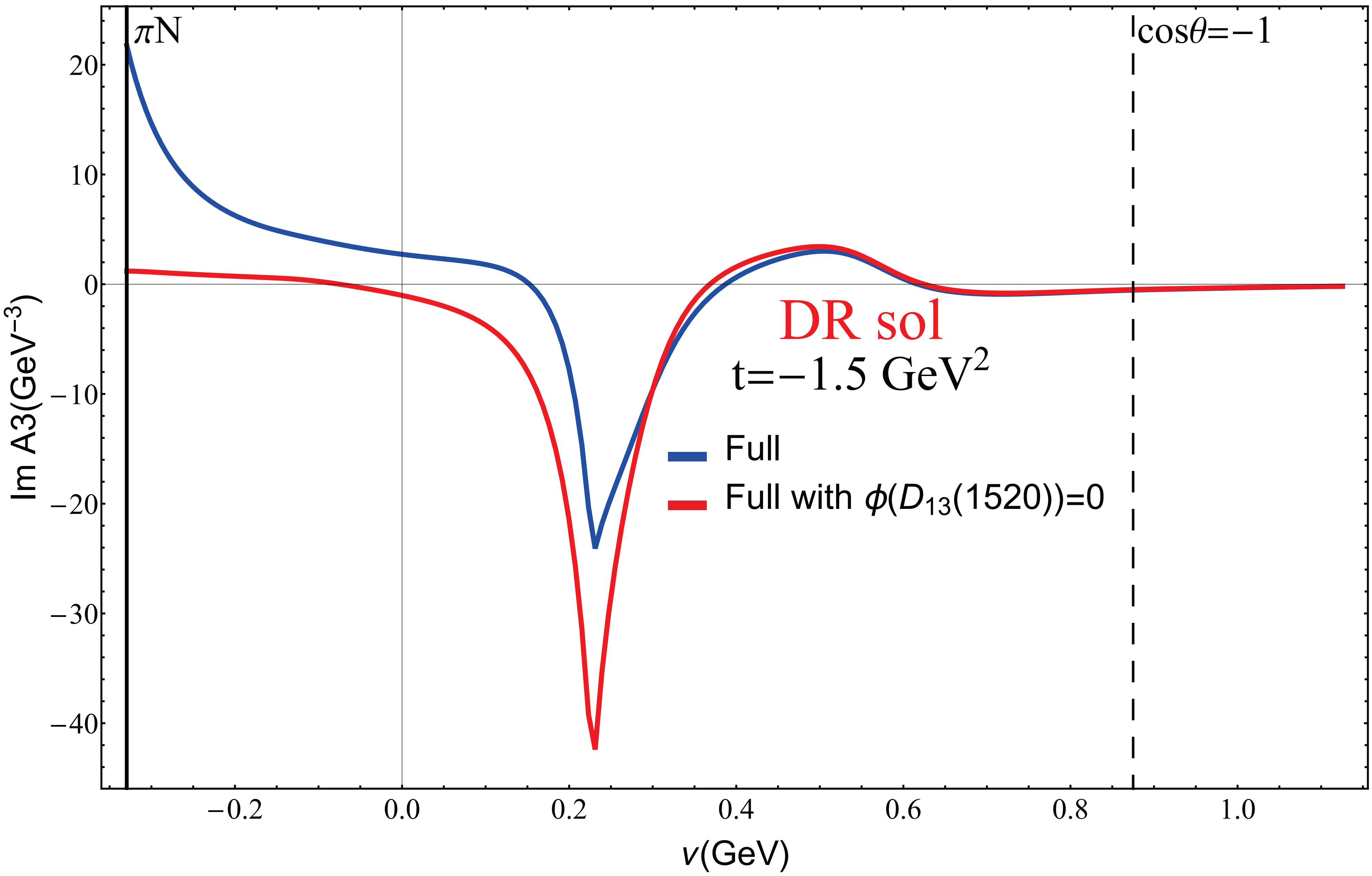
Influence of D13(1520) on Im A_1, t= -1.5 GeV^2



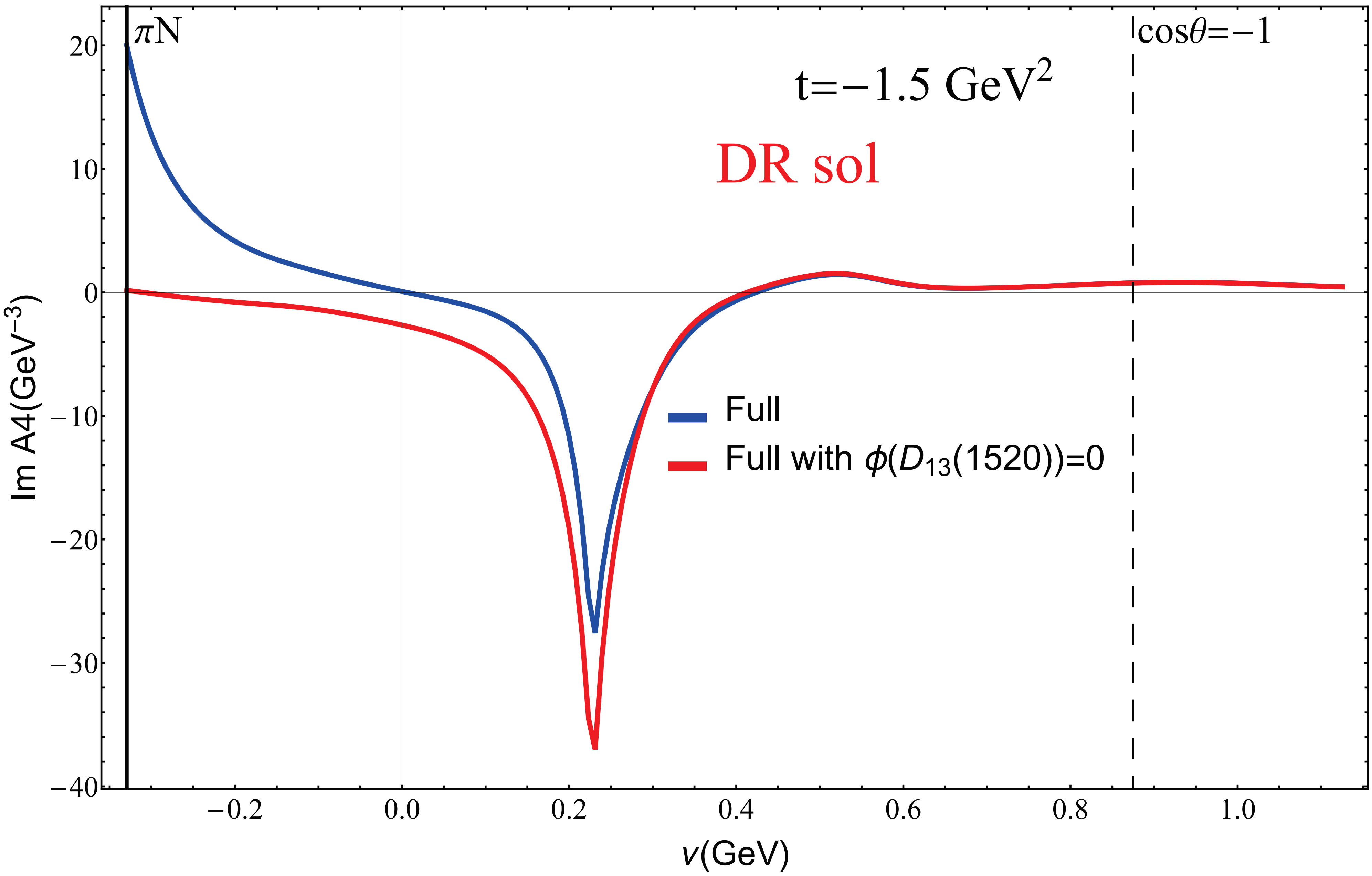
Influence of D13(1520) on Im A_2, t= -1.5 GeV^2



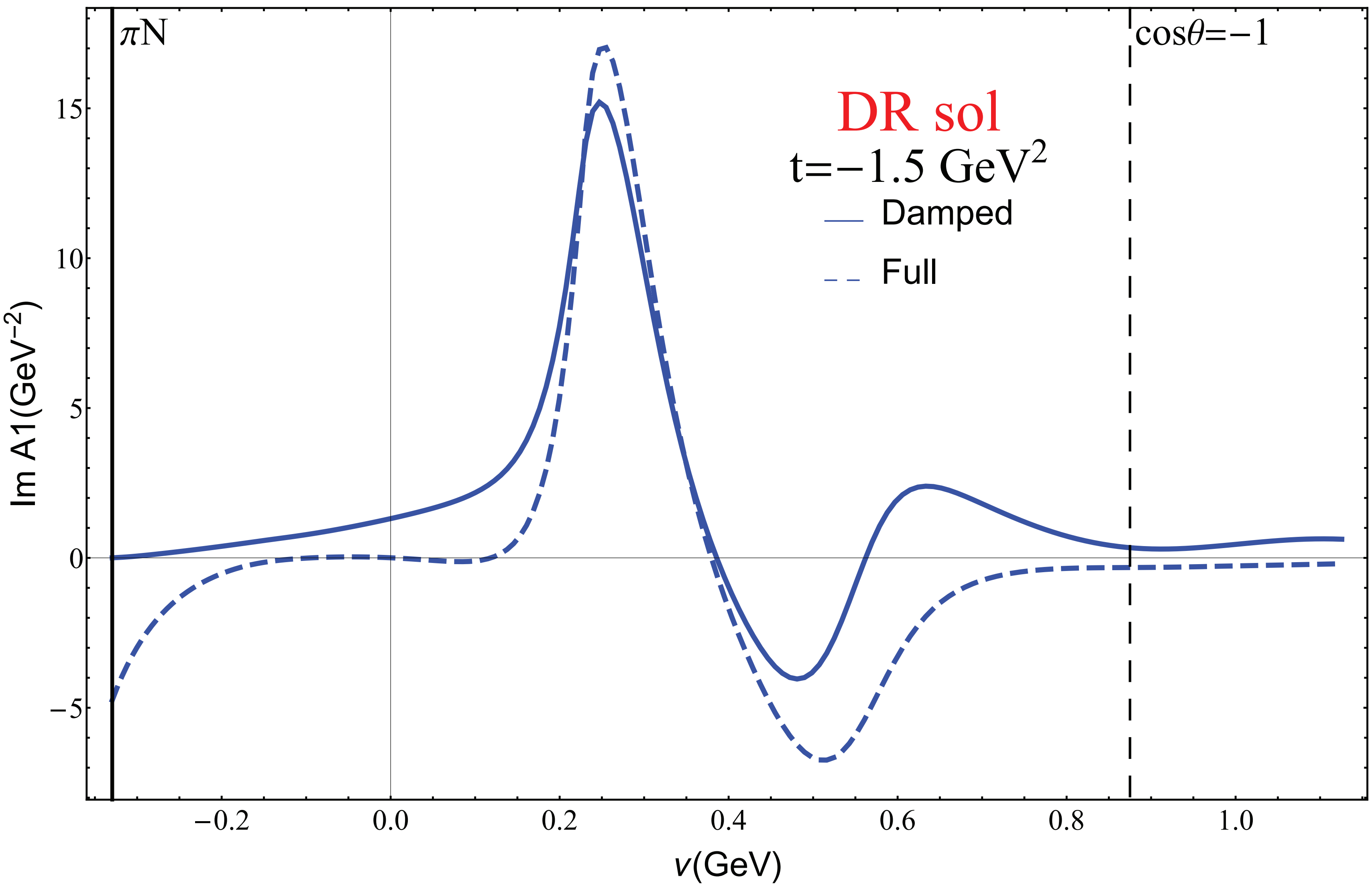
Influence of D13(1520) on Im A_3, t= -1.5 GeV^2



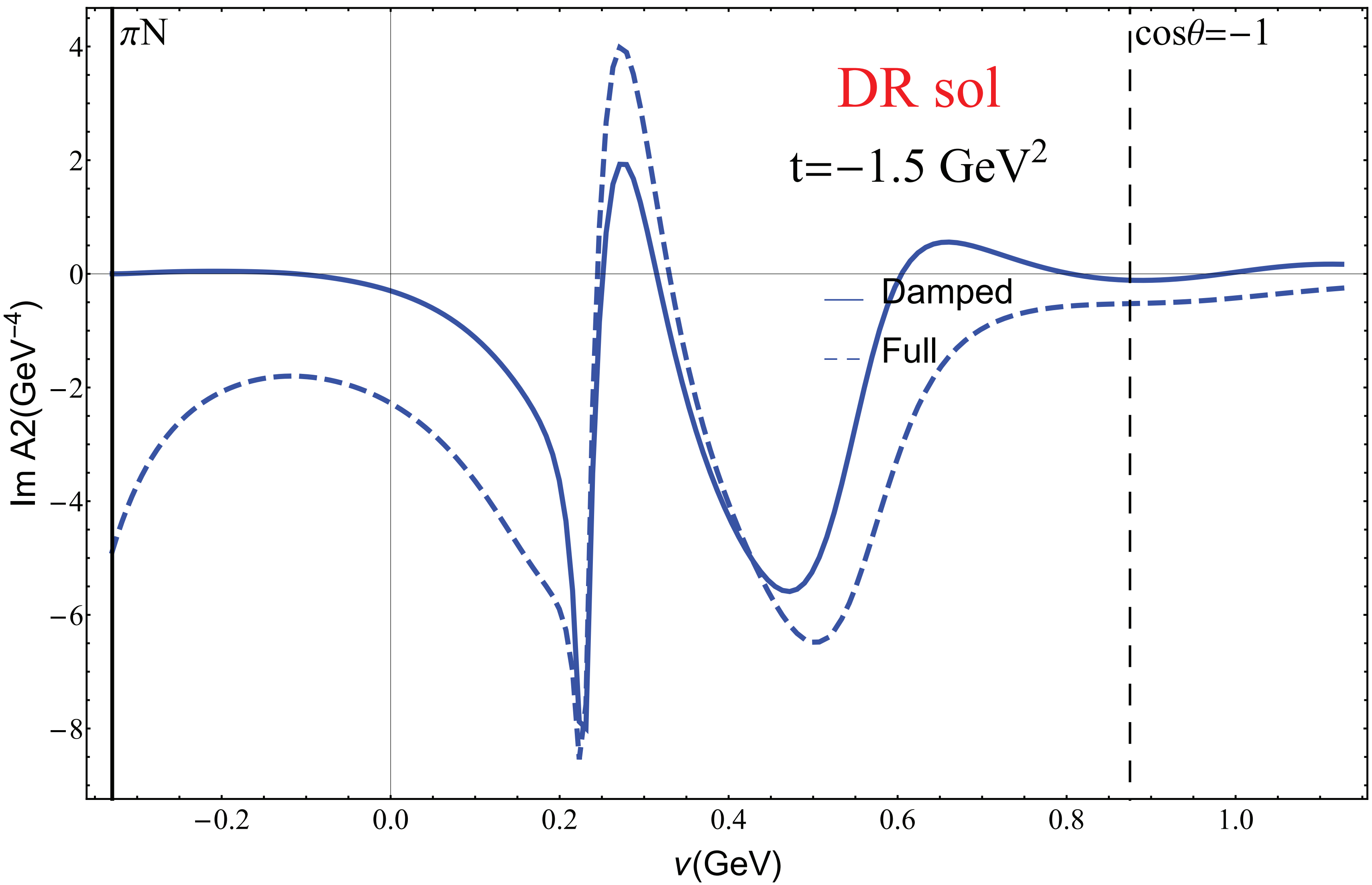
Influence of D13(1520) on Im A_4, t= -1.5 GeV^2



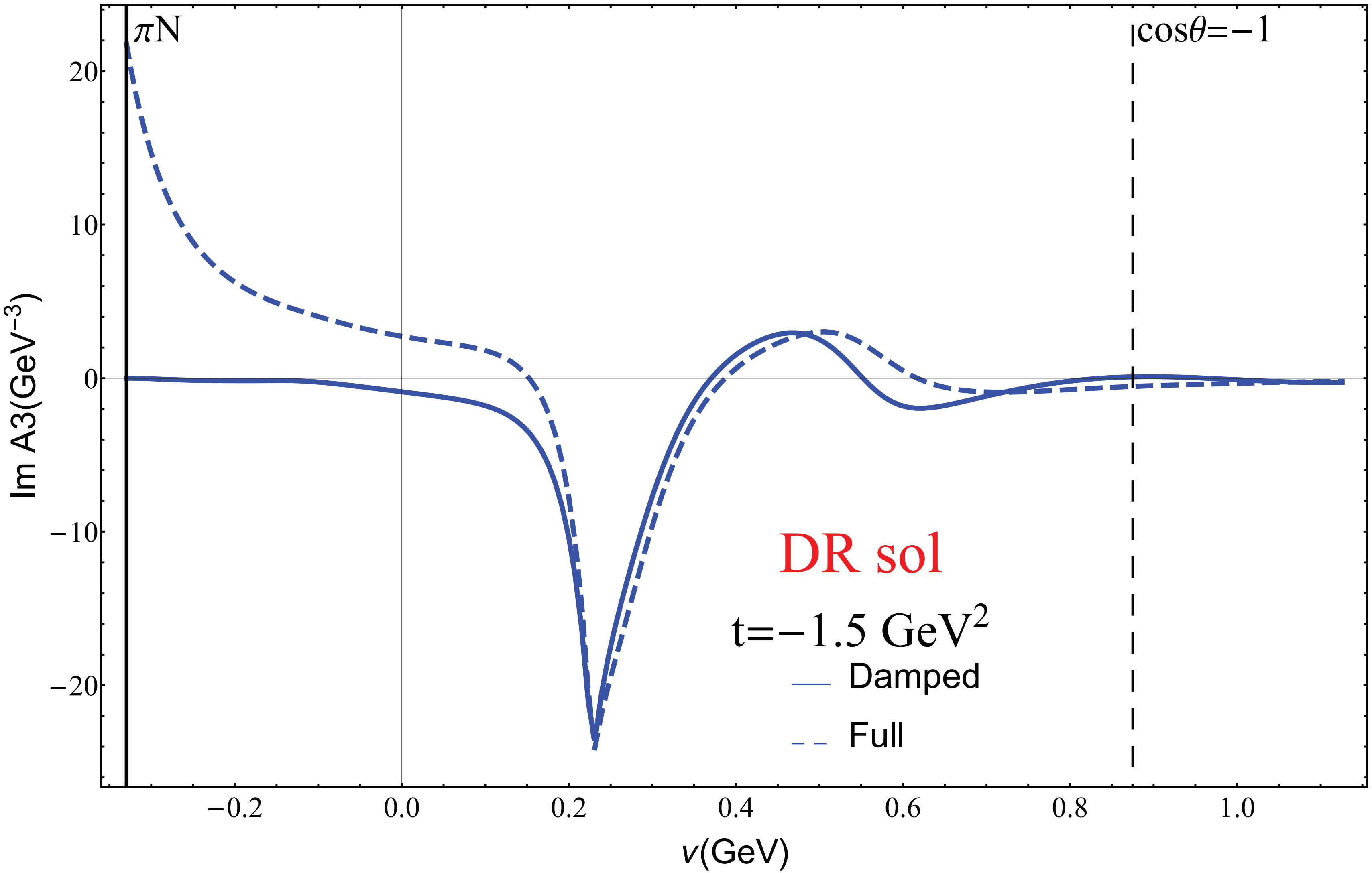
Possible solution: $\Phi_j \to \Phi_j \times \left(\frac{q_\pi(\nu)}{q_{\pi,\eta N}}\right)^{2\ell+1}$



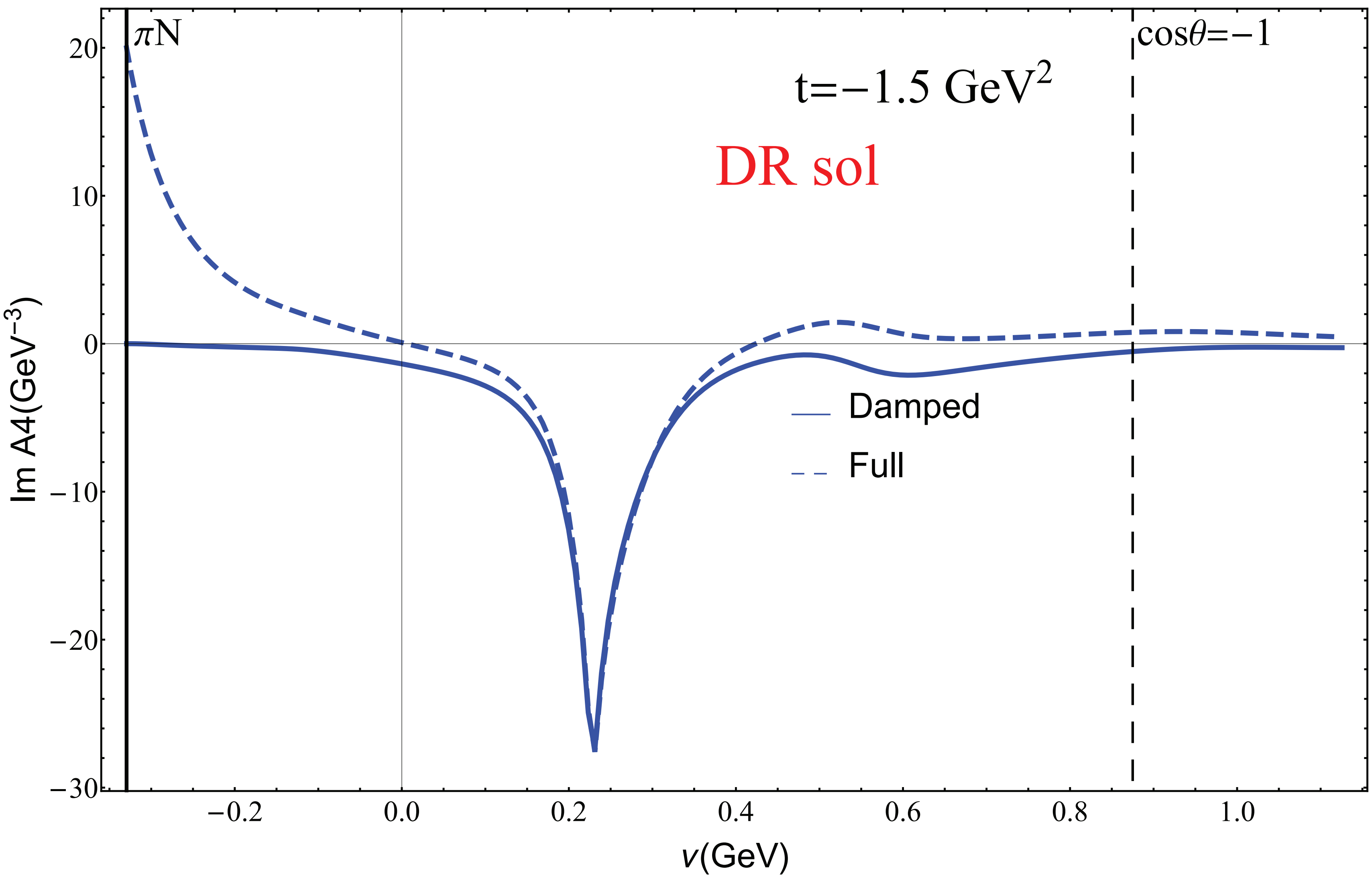
Possible solution: $\Phi_j \to \Phi_j \times \left(\frac{q_\pi(\nu)}{q_{\pi,\eta N}}\right)^{2\ell+1}$



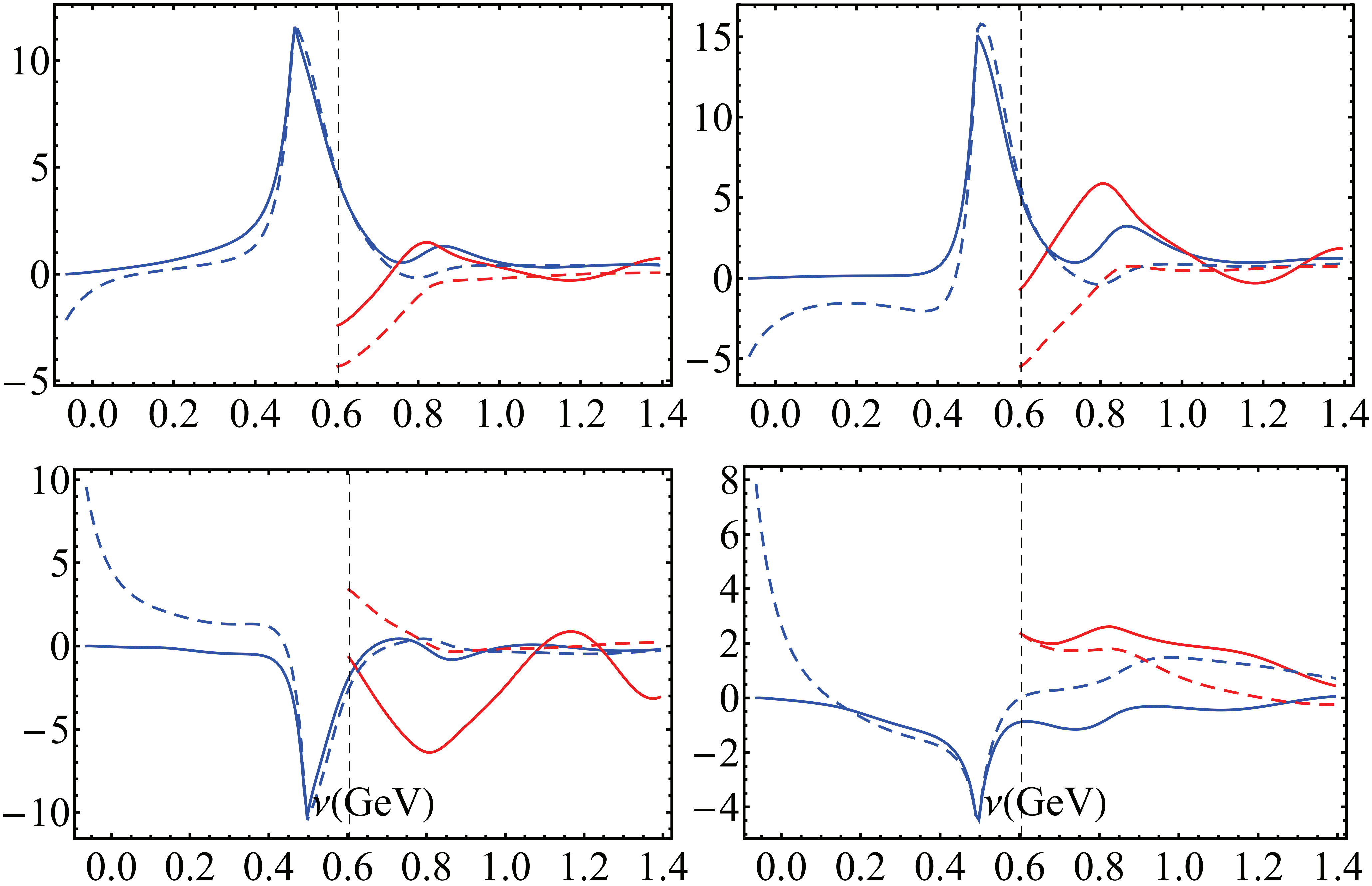
Possible solution: $\Phi_j \to \Phi_j \times \left(\frac{q_\pi(\nu)}{q_{\pi,\eta N}}\right)^{2\ell+1}$



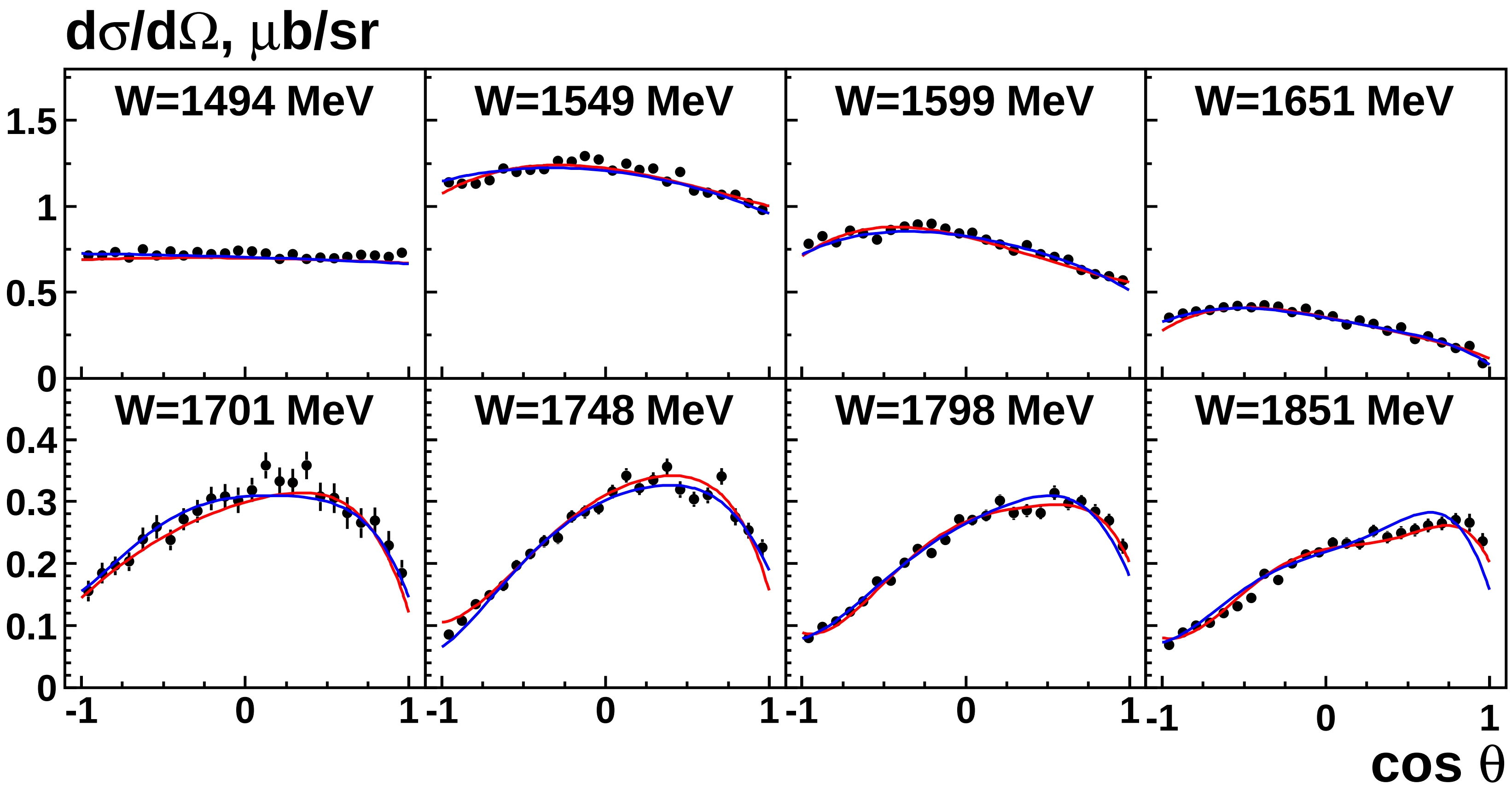
Possible solution: $\Phi_j \to \Phi_j \times \left(\frac{q_\pi(\nu)}{q_{\pi,\eta N}}\right)^{2\ell+1}$



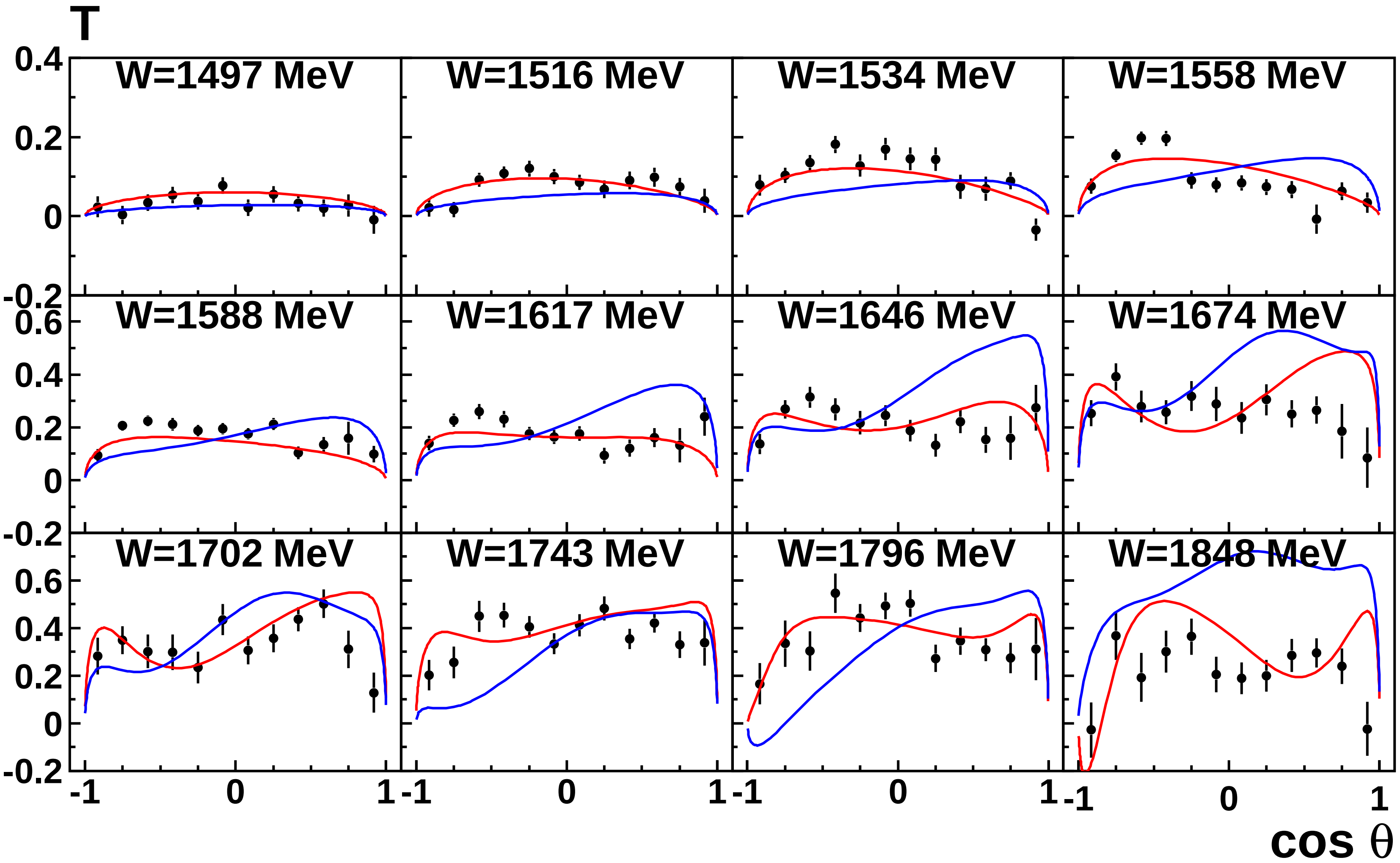
Comparison of Re and Im parts of invariant amplitudes



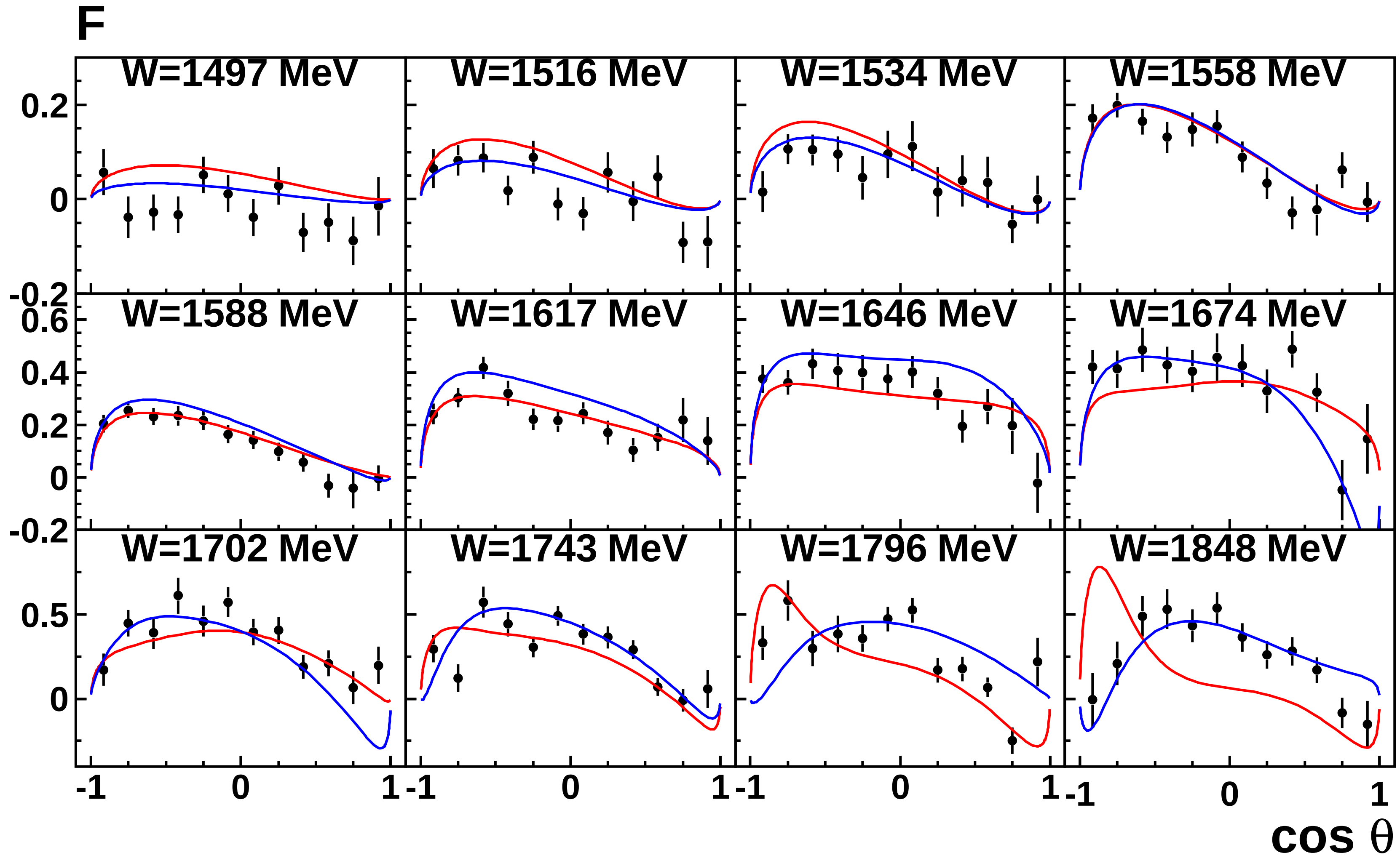
Preliminary results , - old DR, - new DR



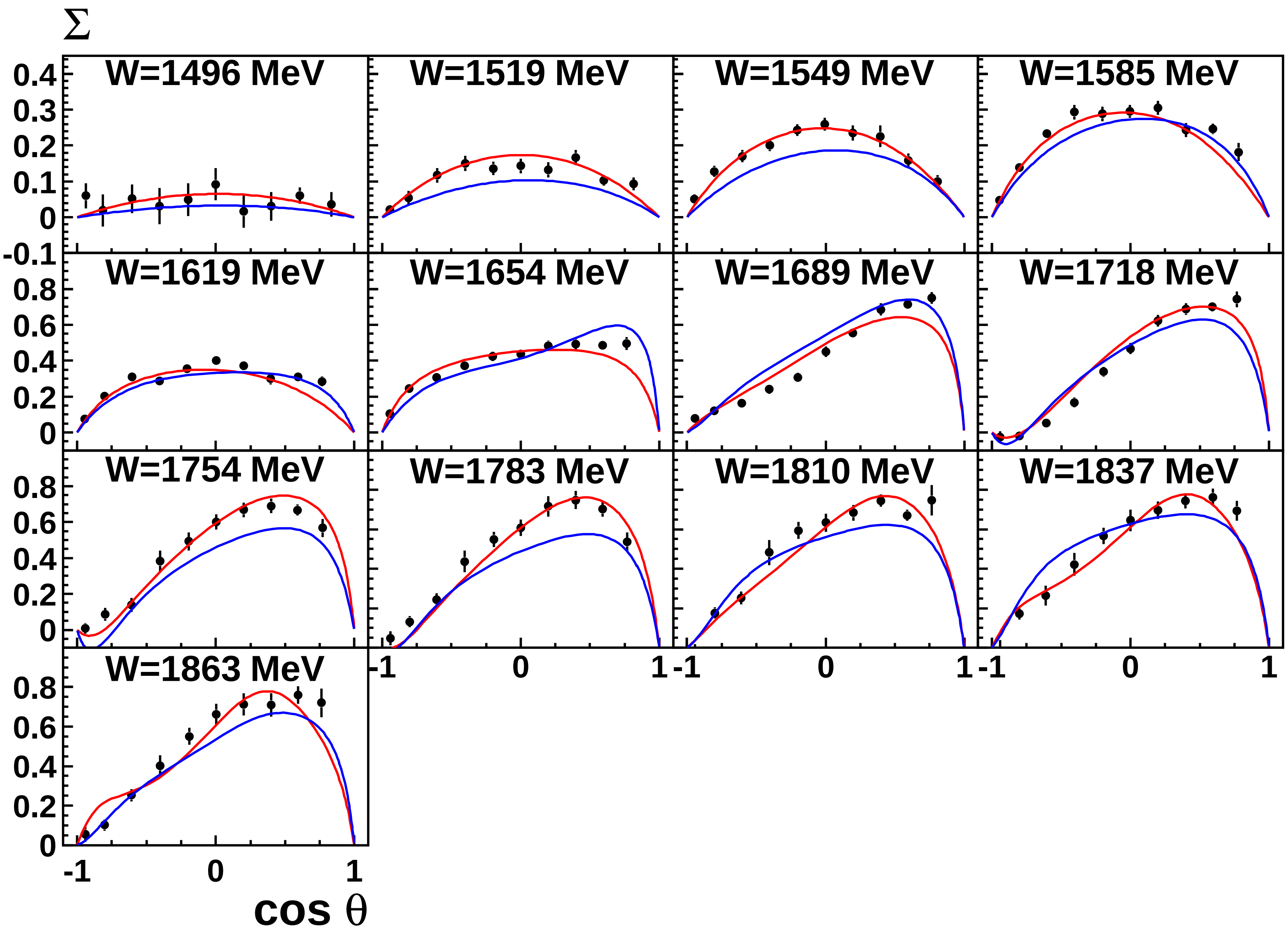
Preliminary results , - old DR, - new DR



Preliminary results , - old DR, - new DR



Preliminary results , - old DR, - new DR



Comparison of previous and new solutions