Regge Theory Trivia
+ Some applications

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Outline

Regge model: theory of complex angular momentum

Analyticity and Crossing of the Regge amplitude

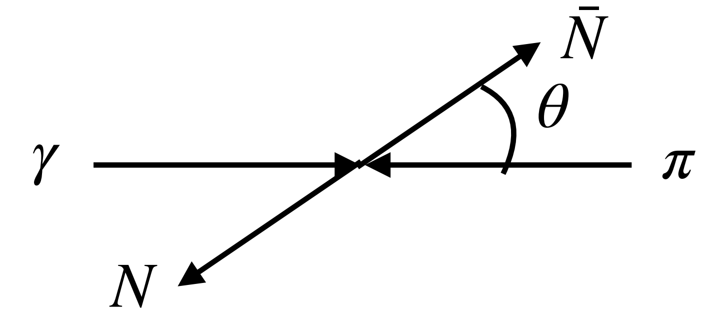
Regge + resonances: duality violation

Ways to cure (partly): saturated Regge

Phase rotations + dispersion relations

Regge model: Complex Angular Momentum

Consider the t-channel reaction



$$q_\gamma^\mu = ((t - m_\pi^2)/2\sqrt{t}, 0, 0, (t - m_\pi^2)/2\sqrt{t})$$
$$q_\pi^\mu = ((t + m_\pi^2)/2\sqrt{t}, 0, 0, (t - m_\pi^2)/2\sqrt{t})$$
$$p_{\bar{N}}^\mu = \sqrt{t}/2(1, \beta_N \hat{p}(\theta_t))$$
$$p_N^{'\mu} = \sqrt{t}/2(1, -\beta_N \hat{p}(\theta_t))$$

$$\beta_N = \sqrt{1 - 4M^2/t}$$

$$z = \cos\theta_t = \frac{s - u}{\beta_N (t - m_\pi^2)}$$

PW expansion

$$A_t(s,t) = \sum_\ell (2\ell + 1) f_\ell(t) P_\ell(z)$$

Definite parity in the t-channel:
either only even or only odd powers of z

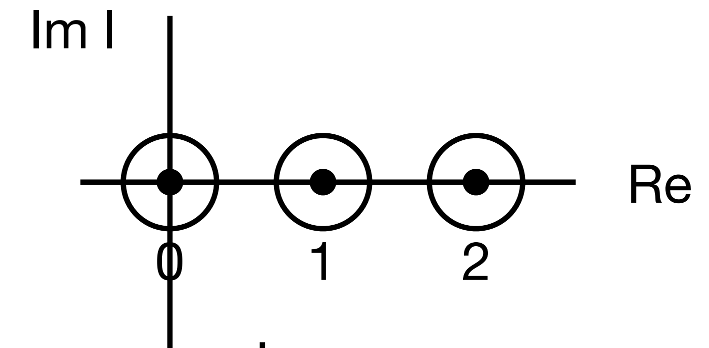
$$A_t^\pm(s,t) = \sum_\ell (2\ell + 1) f_\ell^\pm(t) [P_\ell(z) \pm P_\ell(-z)]$$

Large z asymptotics $\quad P_\ell(z \to \infty) \sim z^\ell \sim \nu^\ell$

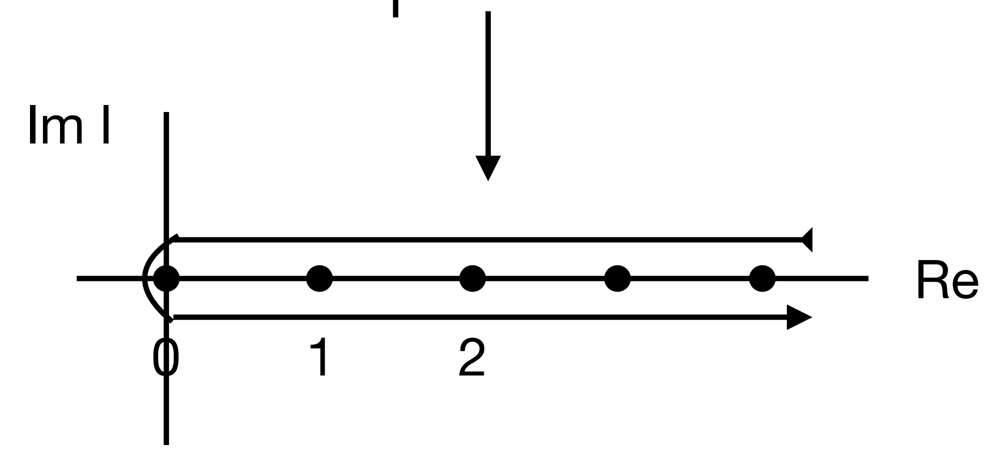
Which value of l governs the asymptotic behavior?

Analytical continuation of l from integer to complex plane

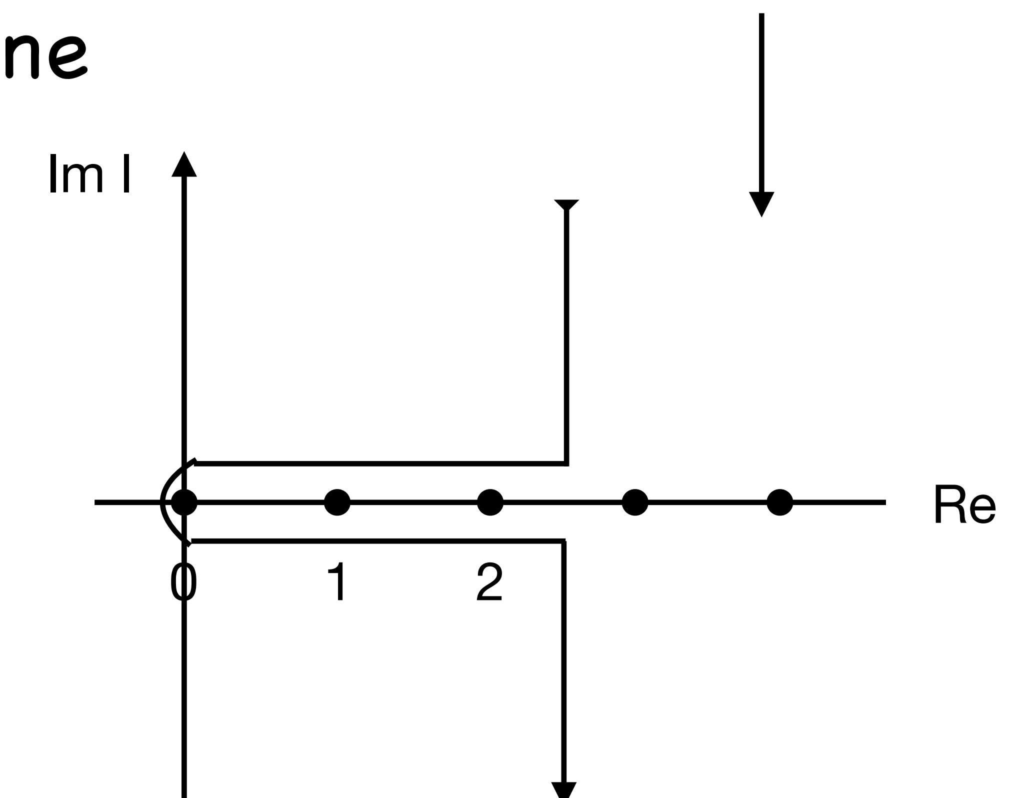
Sommerfeld-Watson representation



$$A(s,t) = \frac{1}{2i}\int_C \frac{d\ell}{\sin \pi\ell} f_\ell(t) P_\ell(-z)$$



Transform the contour in the complex plane



Im values of l only give oscillating behavior but do not add to asymptotics

Can do so until meet a singularity of $f_\ell(t)$ which lies at $\ell = \alpha(t)$

Why singularity?
Meson poles of definite spin at positive t

$$A_t^{\pm}(s,t) = \sum_{\ell} (2\ell+1) f_{\ell}^{\pm}(t)[P_{\ell}(z) \pm P_{\ell}(-z)]$$

The simplest model with singularity:

$$f_{\ell}^{\pm}(t) = \frac{r^{\pm}(t)}{\ell - \alpha^{\pm}(t)}$$

Residue at the pole:

$$A^{\pm}(s,t) = \frac{1}{2i}\int_C \frac{d\ell}{\sin \pi\ell} f_{\ell}(t)[P_{\ell}(-z) \pm P_{\ell}(z)] = -\frac{\pi r^{\pm}}{2}\frac{2\alpha+1}{\sin \pi\alpha}[P_{\alpha}(-z) \pm P_{\alpha}(z)]$$

Asymptotic behavior at large z $\sim \nu$

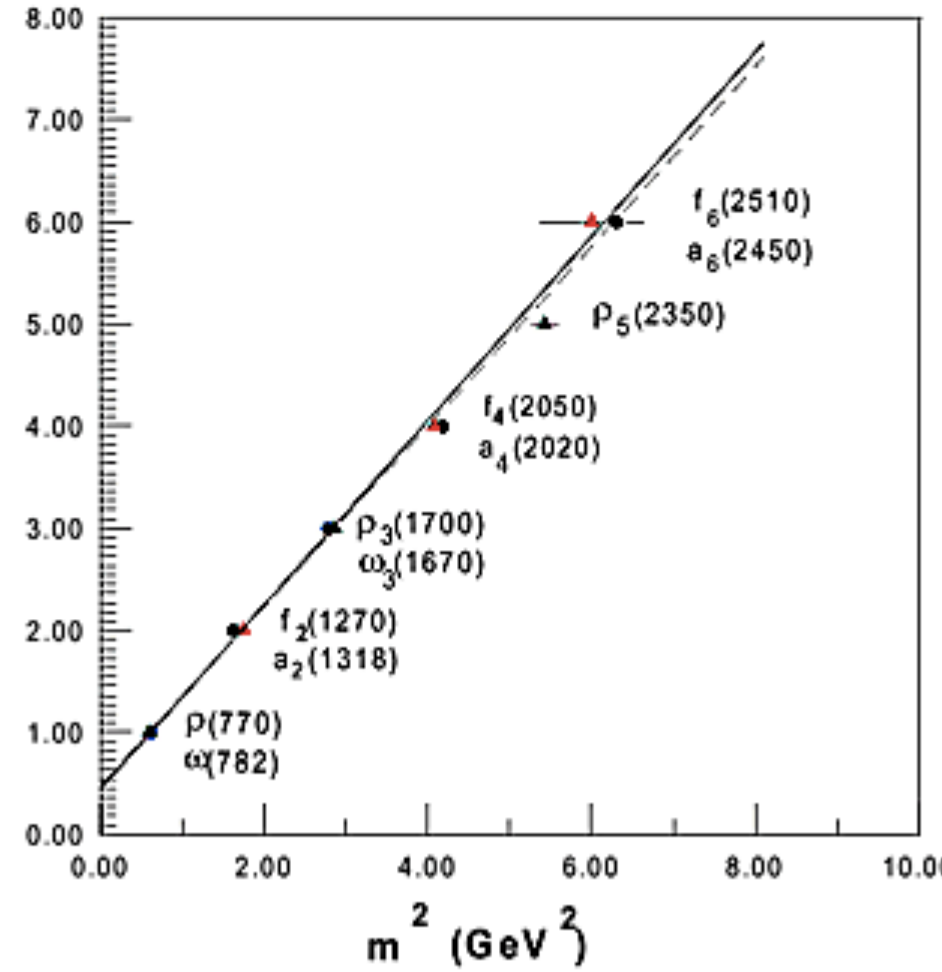
$$A^{\pm}(s \to \infty, t) \sim -\frac{\pi r^{\pm}(t)}{2}\frac{2\alpha(t)+1}{\sin \pi\alpha(t)}\nu^{\alpha(t)}[e^{-i\pi\alpha(t)} \pm 1]$$

This form of the Regge amplitude has all the crucial features:
analyticity in complex l and t; crossing symmetry in s, u;
and analyticity in z $\sim \nu$ (since $P_{\ell}(z)$ are analytical in z)

* used in derivation but not shown explicitly here

What is the form of the Regge trajectory?

The meson spectrum lies on linear trajectories with roughly the same slope



How to interpolate it to scattering t<0?

Minimal model: $\alpha(t) = \alpha_0 + \alpha' t$ for all t

For t<0 trajectory is unbound - there's a problem:
sin($\pi\alpha$) develops unphysical poles at t=0,-1,-2,...
Also: t-exchanges may have finite spin

$$A^{\text{Regge}}(\nu, t) = \beta(t)\pi\alpha' \frac{e^{-i\pi\alpha(t)} + (-1)^{J_0}}{2\sin\pi\alpha(t)\Gamma[\alpha(t) + J_0 - 1]} \left(\frac{\nu}{\nu_0}\right)^{\alpha(t)-J_0}$$

"JPAC model"

$$A^{\mathrm{Regge}}(\nu,t)=\beta(t)\pi\alpha'\frac{e^{-i\pi\alpha(t)}+(-1)^{J_0}}{2\sin\pi\alpha(t)\Gamma[\alpha(t)+J_0-1]}\left(\frac{\nu}{\nu_0}\right)^{\alpha(t)-J_0}$$

Obeys a dispersion relation:

$$\mathrm{Re}A^{\mathrm{Regge}}(\nu,t)=\frac{1}{\pi}\mathscr{P}\int_0^\infty d\nu'\left[\frac{1}{\nu'-\nu}\pm\frac{1}{\nu'+\nu}\right]\mathrm{Im}A^{\mathrm{Regge}}(\nu,t)$$

Master integral: $$\frac{2}{\pi}\mathscr{P}\int_0^\infty\frac{dx x^\alpha}{x^2-1}=\tan\left(\frac{\pi\alpha}{2}\right)$$

Another form used in the literature

$$A^{\mathrm{Regge}}(\nu,t)=\beta'(t)\pi\alpha'\frac{e^{-i\pi\alpha(t)}+(-1)^{J_0}}{2\sin\pi\alpha(t)\Gamma[\alpha(t)+J_0-1]}\left(\frac{s}{s_0}\right)^{\alpha(t)-J_0}$$

Asymptotically equivalent ($s \to 2M\nu$)

However, at sub-asymptotic energies crossing symmetry under $s \leftrightarrow u$ is violated

Since the DR above is explicitly crossing symmetric, also DR is not obeyed (but not a problem at high enough energy)

Analyticity violation of the s-Regge amplitude:

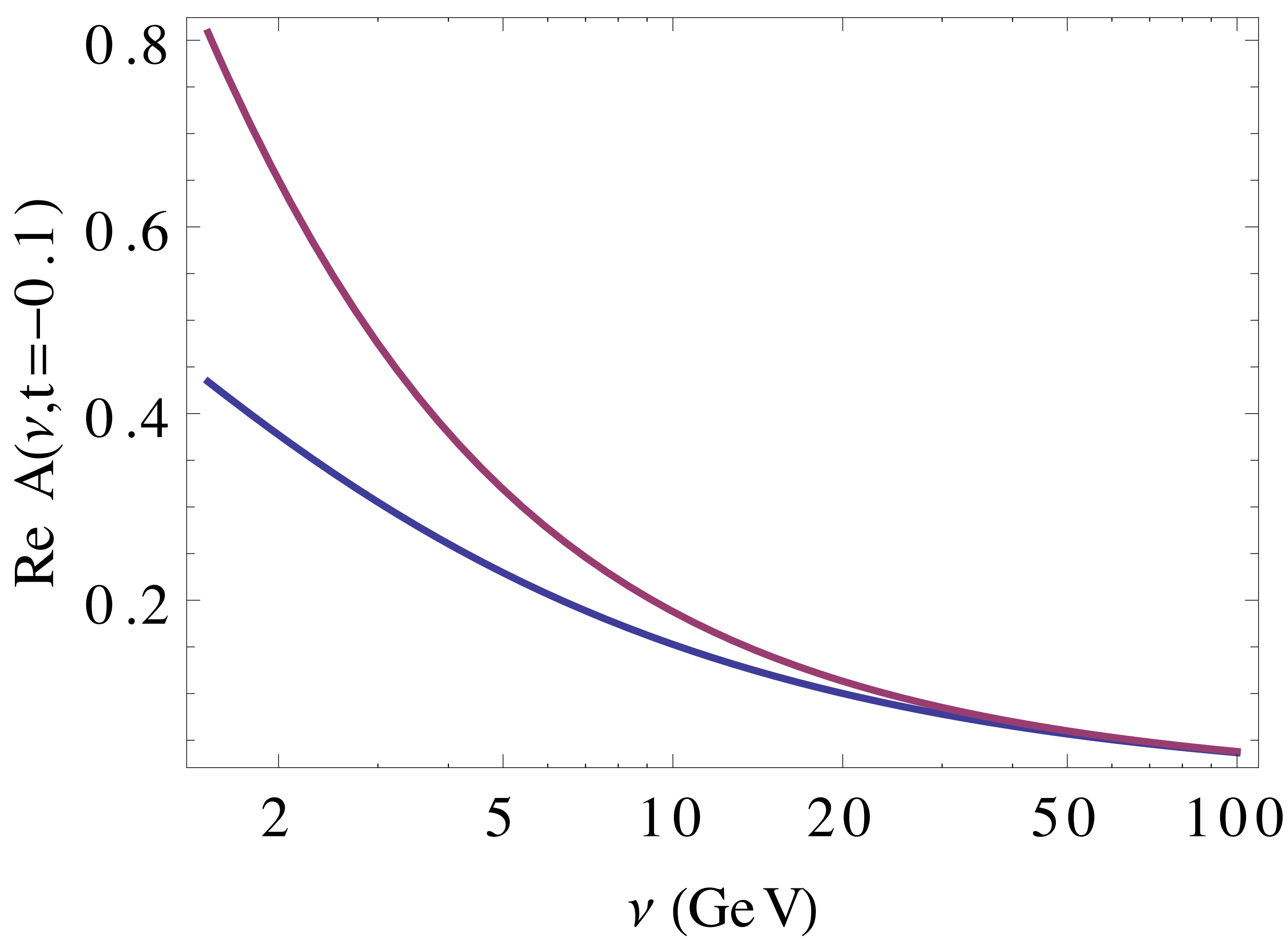
Regge model:

$$A_s^{\mathrm{Regge}} = \mathrm{Im} A_s^{\mathrm{Regge}} \times \left[i + \tan \frac{\pi\alpha(t)}{2} \right]$$

Compare

$$\mathrm{Re} A_s^{\mathrm{Regge}} = \tan \frac{\pi\alpha(t)}{2} \mathrm{Im} A_s^{\mathrm{Regge}}$$

$$\mathrm{Re} A_s^{\mathrm{Regge}}(\nu, t) = \frac{2}{\pi} \mathscr{P} \int_0^\infty \frac{d\nu' \nu'}{\nu'^2 - \nu^2} \mathrm{Im} A_s^{\mathrm{Regge}}(s', t)$$



Regge amplitude knows ~ all about t-channel singularities;
It knows something about the s-channel singularities
– the position of the unitarity cut
but nothing about the s-channel dynamics
(only that at asymptotic energies it does not matter)

On the other hand, Regge amplitude always has an absorptive part in the s-channel kinematics - which can only be due to on-shell (multi)-particle production in the s-channel

But if A is an analytical function of s, its value at high energy cannot be unrelated to its strength in the resonance region

Duality: a full theory knows all its states and their properties

Algebraic models (van Hove, Veneziano) - duality is trivial: spectra and couplings are exactly known

$$A(s,t,u) = \sum_{\mathrm{Res}_s}^{\infty} A^s(s,t,u) = \sum_{\mathrm{Res}_t}^{\infty} A^t(s,t,u)$$

Cannot directly test duality as written above.
But the strength of low-lying resonances and Regge related!
This correspondence is addressed by finite energy sum rules

FESR for π,η photoproduction

$$\gamma(k) + N(p) \to PS(q) + N'(p')$$

Mandelstam scalars

$$s = (p+k)^2, \;\; u = (p-q)^2, \;\; t = (k-q)^2$$

Crossing-odd variable

$$\nu = \frac{s-u}{4M}$$

CGLN decomposition:
invariant amplitudes

$$T_{fi} = \sum_i \bar{u}(p') M_i u(p) A_i(\nu, t)$$

$$M_1 = \frac{1}{2}\gamma_5 \gamma_\mu \gamma_\nu F^{\mu\nu} \,,$$

$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu} \,, \qquad P = (p+p')/2$$

$$M_3 = \gamma_5 \gamma_\mu q_\nu F^{\mu\nu} \,,$$

$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}$$

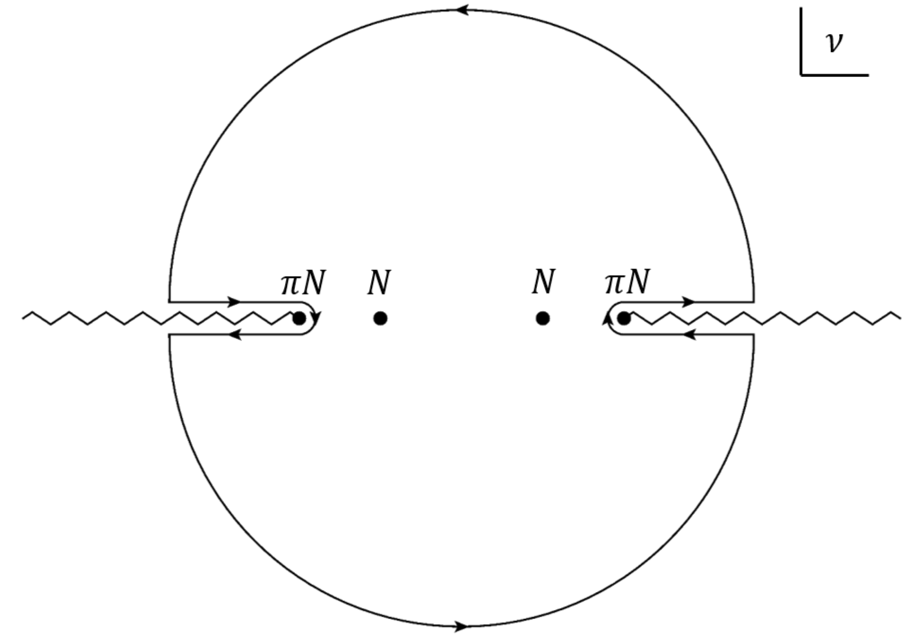
Crossing + isospin

$$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$$

σ = (+0,-) for pion, (s,v) for eta

$$\xi_1 \;=\; \xi_2 \;=\; -\xi_3 \;=\; \xi_4 \;=\; 1$$

Fixed-t dispersion relation



$$\mathrm{Re}\, A_i^I(\nu,t) = B_i^I \left[\frac{1}{\nu-\nu_N} + \xi_i^I \frac{1}{-\nu-\nu_N} \right] + \frac{\pi B_i^{(-)}}{t-m_\pi^2} + \frac{1}{\pi}\mathcal{P}\int_{\nu_\pi}^{\infty} d\nu' \left[\frac{1}{\nu'-\nu} + \xi_i^I \frac{1}{\nu'+\nu} \right] \mathrm{Im}\, A_i^I(\nu',t)$$

Above scale N: Regge dominates

Vector and axial vector: $$R_i^I(\nu,t) = \beta_i^I(t)\frac{\pi\alpha'}{2}\frac{e^{-i\pi\alpha_i^I(t)}-1}{\sin[\pi\alpha_i^I(t)]\Gamma[\alpha_i^I(t)]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Resembles V, A meson exchanges: $$R_i^I(\nu, t\to m_{V,A}^2) \to R_i^I(\nu,t)\big|_{\alpha_i^I=1} = \frac{\beta_i^I(m_{V,A}^2)}{t-m_{V,A}^2}$$

Regge amplitude obeys DR

$$\mathrm{Re}\, R_i^I(\nu,t) = \frac{1}{\pi}\mathcal{P}\int_0^{\infty} d\nu' \left[\frac{1}{\nu'-\nu} + \xi_i^I \frac{1}{\nu'+\nu} \right] \mathrm{Im}\, R_i^I(\nu',t)$$

Match full ampl. on Regge at ν > N

$$\mathrm{Re},\ \mathrm{Im}\, A_i^I(\nu,t) = \mathrm{Re},\ \mathrm{Im}\, R_i^I(\nu,t) \ \text{ for } \ \nu > N$$

Sidenote: A and R are analytical in slightly different regions of the nu-plane; mathematically still viable. But the two are only "identical" within finite experimental errors, not in exact sense.

$$\frac{\tilde{B}_i^I(t)}{N}\left(\frac{\nu_N}{N}\right)^k + \int_{\nu_\pi}^{N} \frac{d\nu'}{N}\left(\frac{\nu'}{N}\right)^k \mathrm{Im}\, A_i^I(\nu',t) = \frac{\beta_i^I(t)\pi\alpha'}{2(\alpha(t)+k)\Gamma[\alpha(t)]}\left(\frac{N}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Regge parameters: HE fit

Nys et al [JPAC] arXiv:1611.0465
Kashevarov, Ostrick, Tiator arXiv:1706.07376

LHS of FESR - fit to resonance data

FESR - a powerful tool for constraining resonance parameters by imposing duality, fixed-t analyticity, crossing etc.

Numerical implementation can be tricky: strong cancellation

Historically, models used in the resonance region and at high energy are completely different (VM exchanges <—> Regge exchanges)

Matching is plagued by a potentially significant systematic uncertainty; May still depend on the matching point N

Use Regge amplitude modified in the resonance region joining smoothly onto Regge

Exploit duality for extracting few low-lying resonances

$$A(s,t,u) = \sum_{\mathrm{Res}_s}^{\infty} A^s(s,t,u) = \sum_{\mathrm{Res}_t}^{\infty} A^t(s,t,u)$$

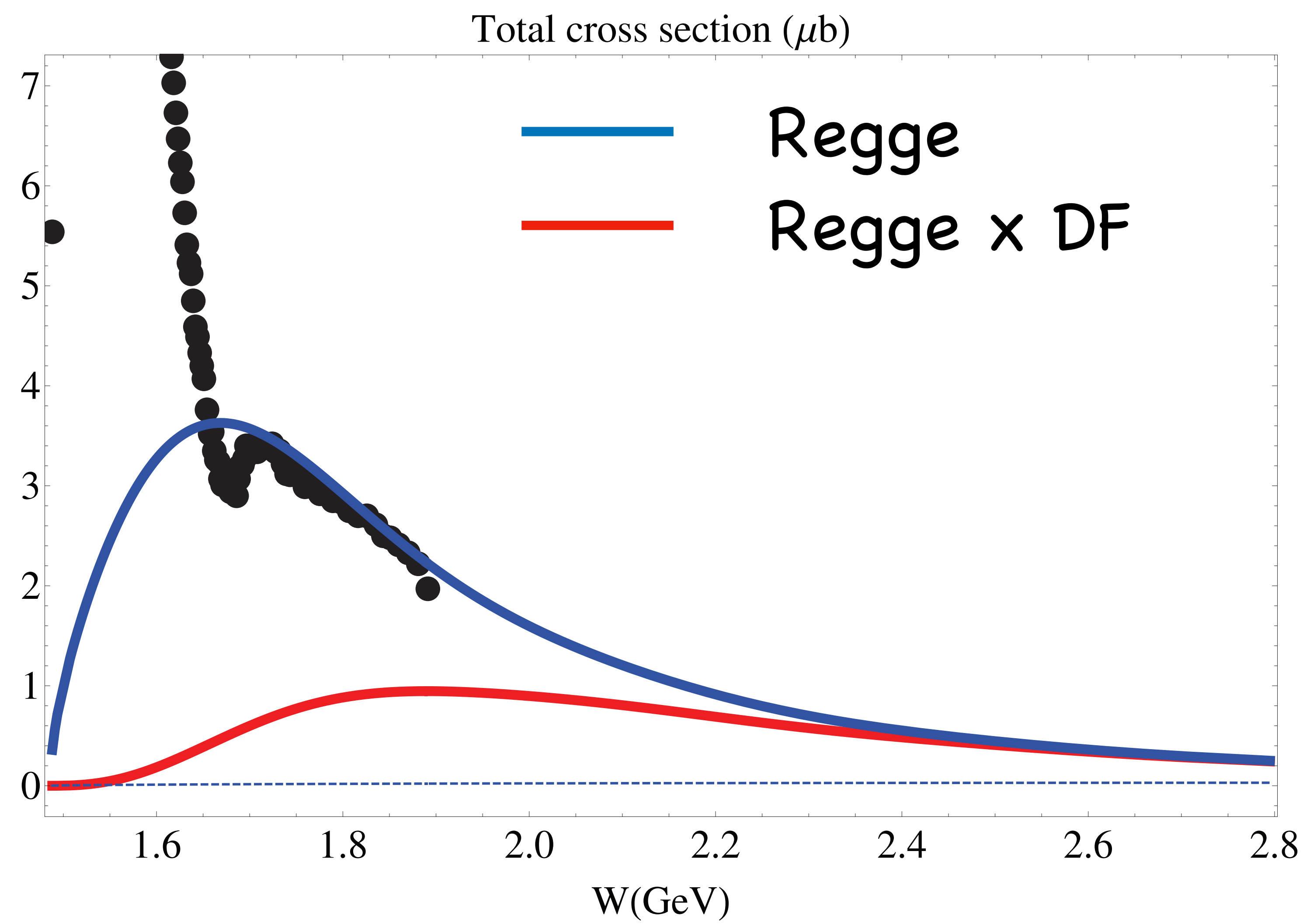
Remove part of the strength of Regge in the resonance region to leave space for resonances

$$A(s,t,u) = \sum_{\mathrm{Res}_s=1}^{N} A^{\mathrm{Res}}(s,t,u) + \sum_{\mathrm{Res}_t}^{\infty} A^t(s,t,u) - \sum_{\mathrm{Res}_s=1}^{N} A^{\mathrm{Res}}(s,t,u)$$
$$\approx \sum_{\mathrm{Res}_s=1}^{N} A^{\mathrm{Res}}(s,t,u) + DF(W) \times A^{\mathrm{Regge}}(s,t,u)$$

Damping factor removes double counting:
DF(W) -> 0 at threshold;
DF(W) -> 1 at high energy

The form of DF is unknown; we chose $DF(W) = 1 - e^{-\frac{W^2 - W_{thr}^2}{\Lambda^2}}$
Λ - fit parameter (of order 1 GeV)

Effect of the damping factor (Λ ~ 1.5 GeV)



The phase of Regge amplitude is a unique prediction.
Go one step beyond FESR: match multipoles (with phases) at the boundary Resonance-Regge regions

Application to pion and eta production - work in progress

Multipole decomposition is needed for the full amplitude, not only resonance part

Follow MAID approach: $t_{\gamma\pi}(W,\theta) = t_{\gamma\pi}^{Bg}(W,\theta) + t_{\gamma\pi}^{Res}(W,\theta)$

Resonance contributions - $BW_{\ell,\,\alpha}^{Res}(W) \to BW_{\ell,\,\alpha}^{Res}(W)e^{i\phi_\alpha^{Res}(W)}$

Background = Born + Regge x DF

Born $t_{\gamma\pi}^{B}(W,\theta) = \sum_\ell \mathcal{M}_{\ell,\,\alpha}^{B}(W)\,\{P_\ell(\cos\theta)\}$

Unitarization (K-m.) $\mathcal{M}_{\ell,\,\alpha}^{B}(W) \to \mathcal{M}_{\ell,\,\alpha}^{B}(W)[1 + it_{\pi N}^{\alpha}]$ $\quad t_{\pi N}^{\alpha} = \dfrac{\eta_\alpha e^{2i\delta_\alpha^{\pi N}} - 1}{2i}$

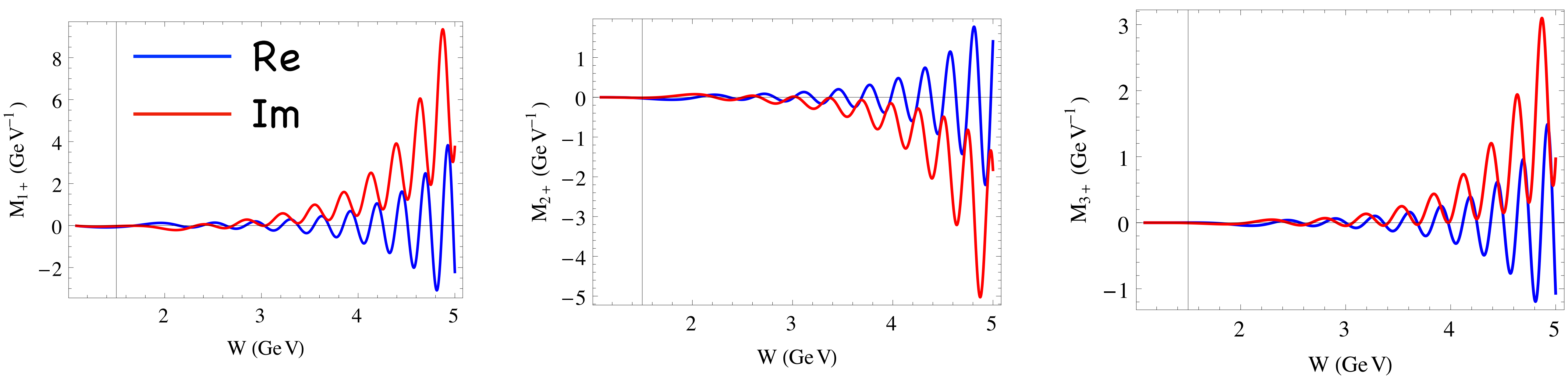
Pi-N phases and inelasticities - e.g. from GWU analysis, W < 2 GeV

Multipole decomposition of Regge amplitude

$$R_i^I(\nu,t) = \beta_i^I(t)\frac{\pi\alpha'}{2}\frac{e^{-i\pi\alpha_i^I(t)}-1}{\sin[\pi\alpha_i^I(t)]\Gamma[\alpha_i^I(t)]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Several possibilities:
-> unitarize few lowest partial waves, match them back to Regge at HE;
-> match the one's favorite low-energy multipoles onto Regge multipoles above resonance region

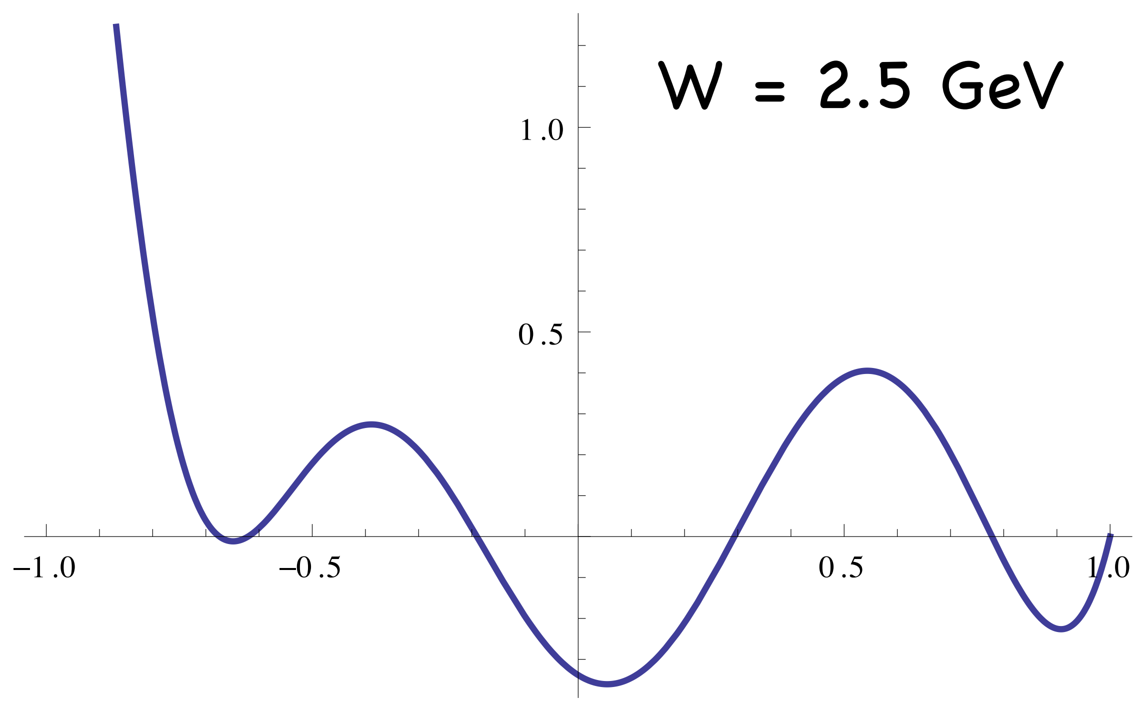
Example: ρ-exchange in $\pi^+ n$ channel
Vector coupling to the nucleon; M_{l+} multipoles



Oscillations observed: make matching impossible!

What's the reason for these oscillations?
Consider the integrand of R -> M_{1+} conversion
Strong backward peak plus oscillations between
But one expects t-channel Regge exchanges
to dominate forward angles

Combination of two factors: ν decreases for -t>>
Oscillations - $1/\Gamma[\alpha(t)]$ for large negative t
Gamma-fn. removes unphysical poles at t = -1, -3, ...

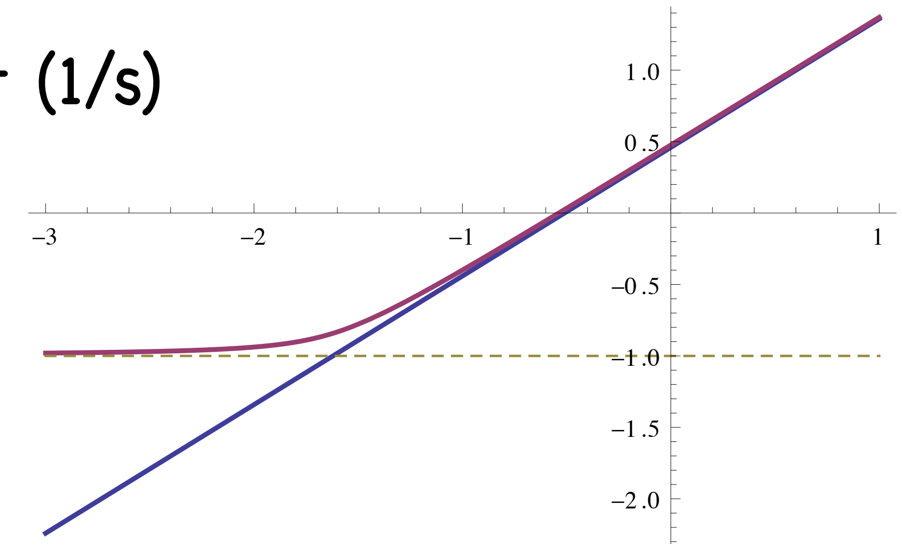


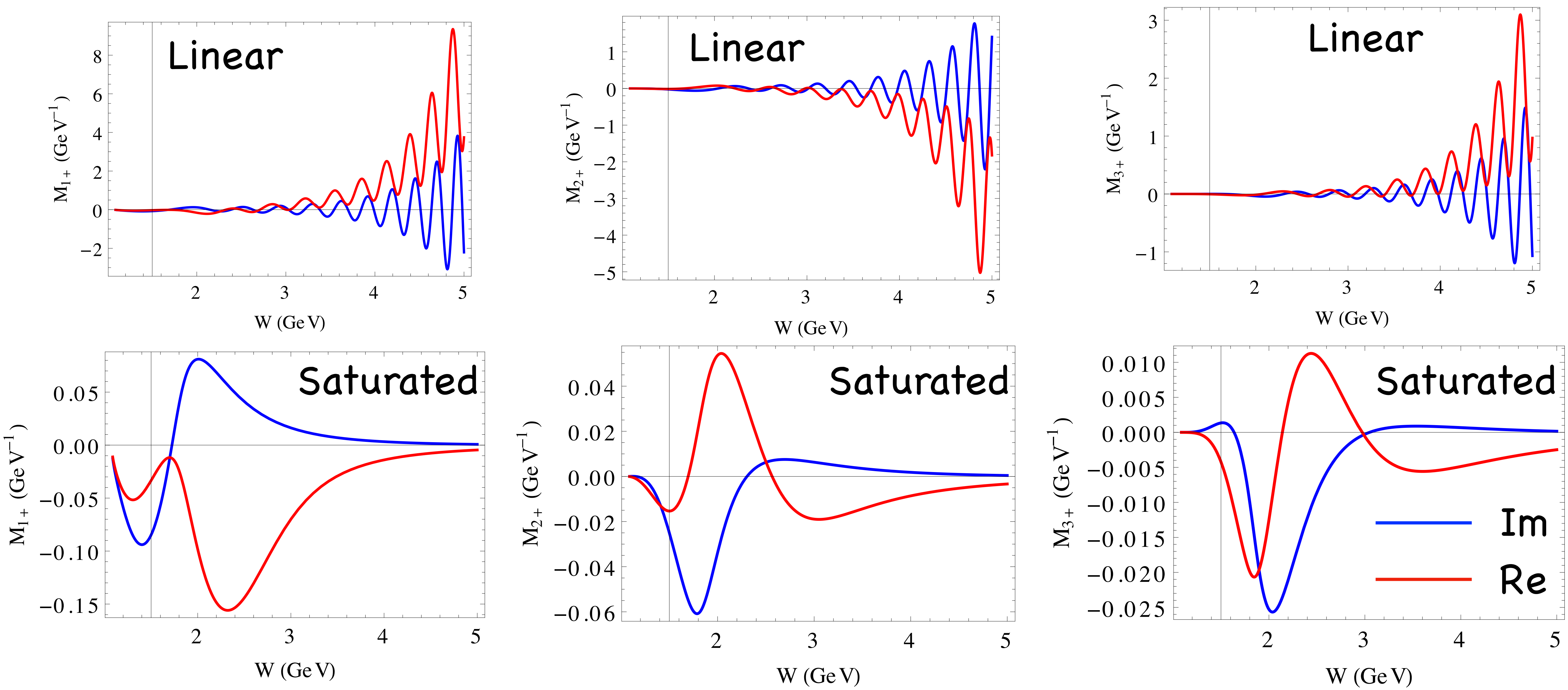
Saturated Regge trajectory

$\alpha(t)$ - linear at positive t (Frautschi plot, meson poles)
- at large $|t|{\sim}s$: pQCD quark exchange - expect $1/t$ ($1/s$)

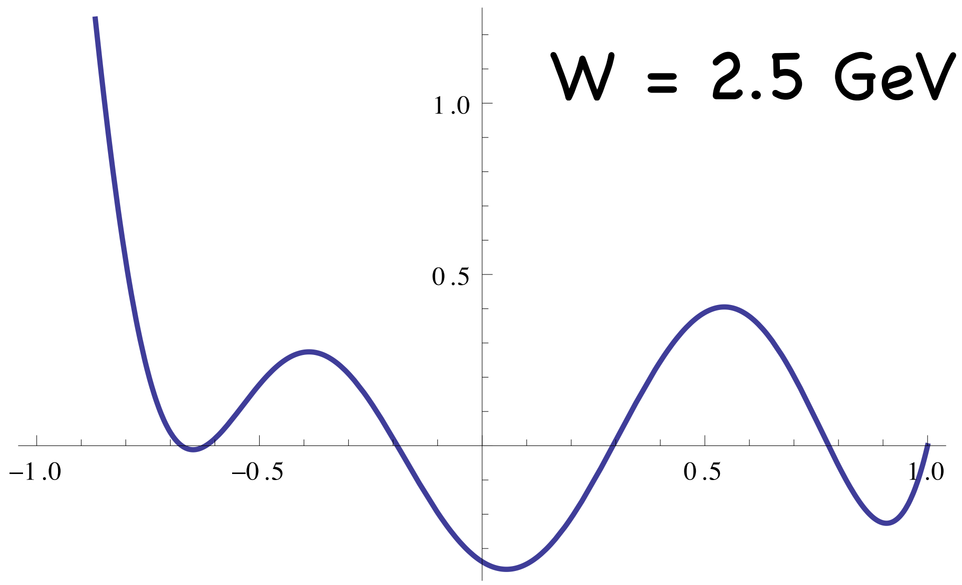
$$\tilde{\alpha}(t) = \frac{\alpha(t)-1}{2} + \frac{1}{2}\sqrt{(\alpha(t)+1)^2 + 4\lambda^2}$$

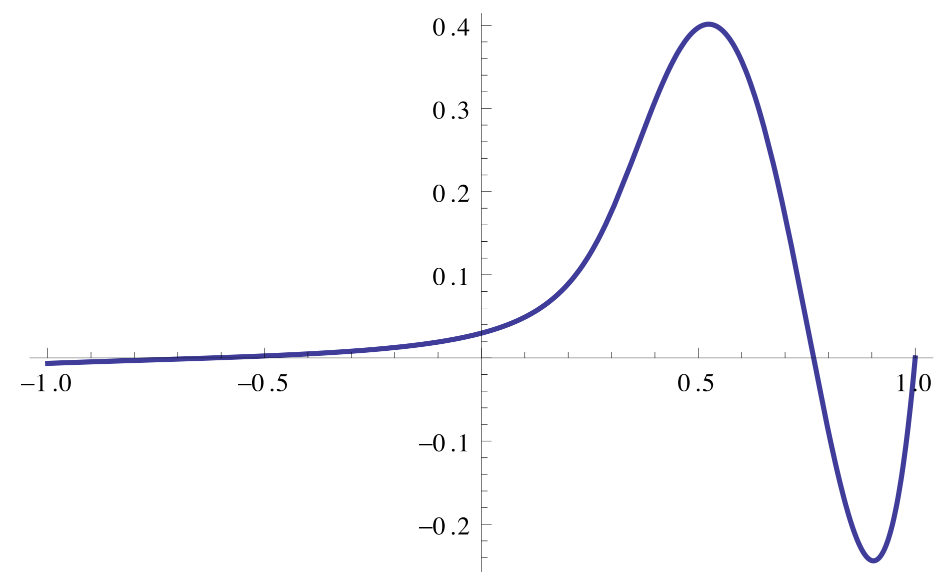
Transition: λ



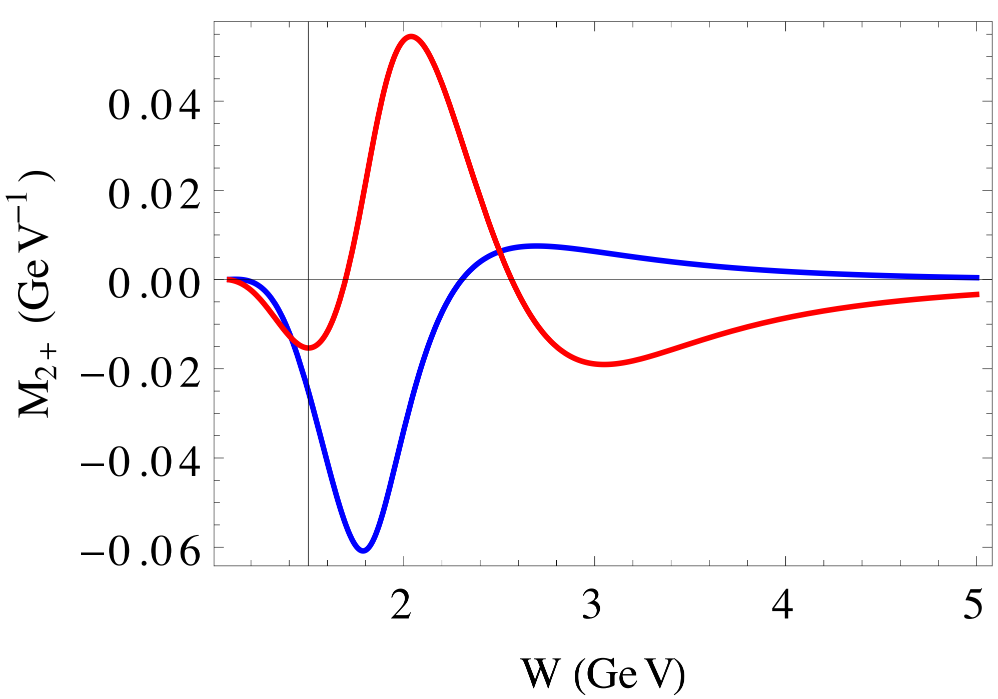


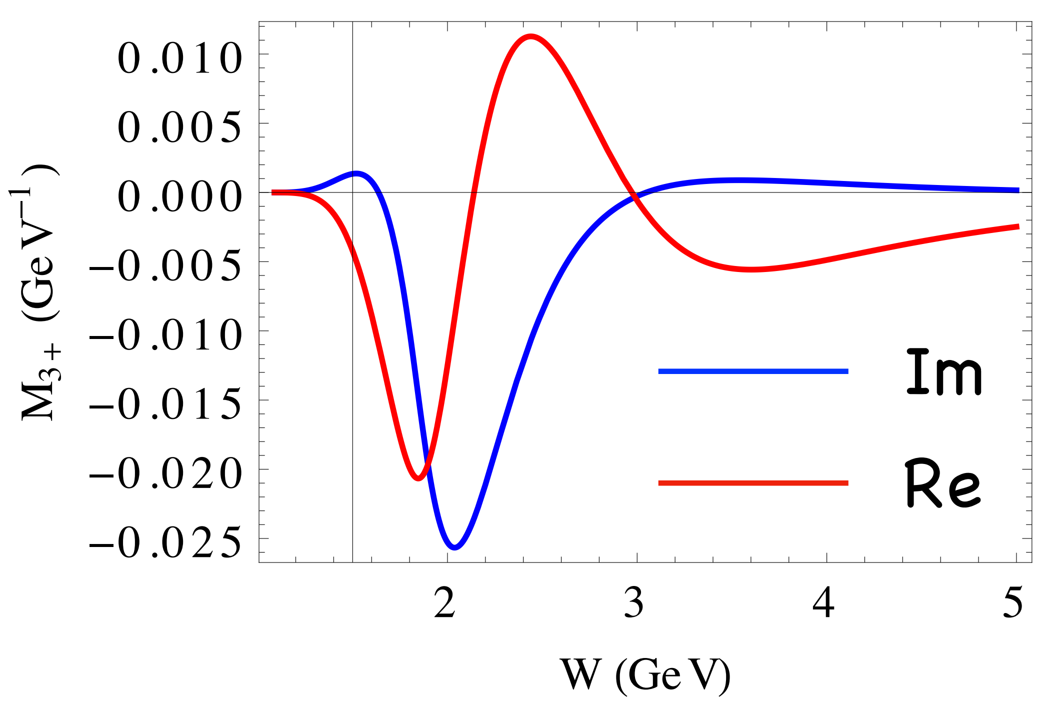
Saturated Regge trajectory
Eliminates backward structure and unphysical oscillations!





Regge amplitude generates resonance-like structures - that's why need to take double counting seriously

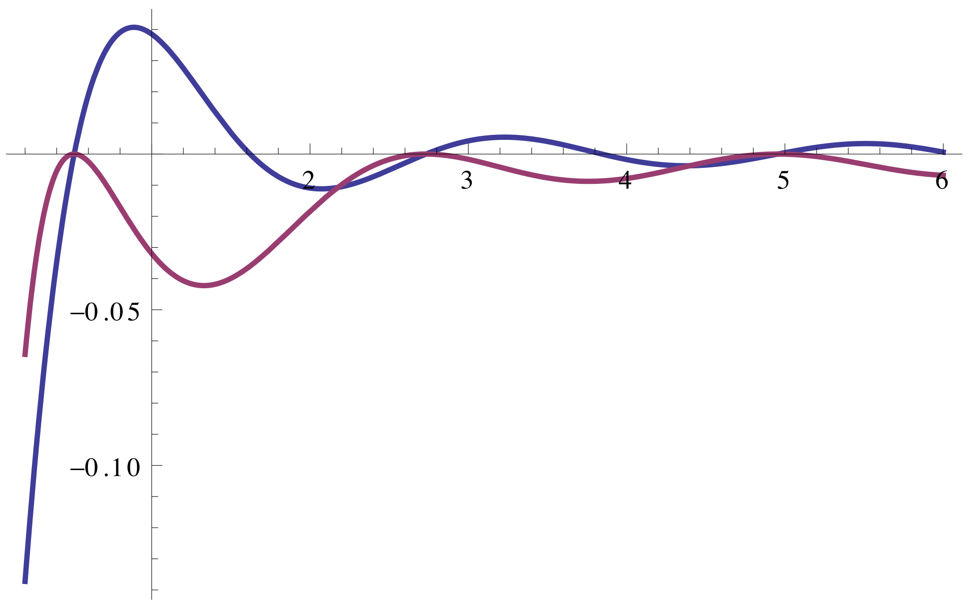


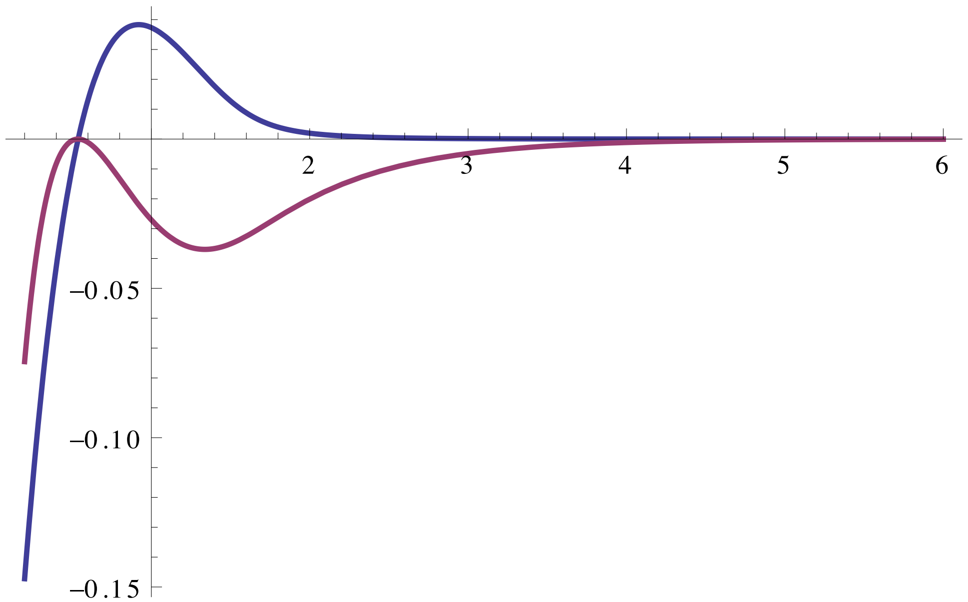


Saturated Regge trajectory

Oscillations seen not only in multipoles but in cross sections

Invariant amplitude: linear vs. saturated (ν = 5 GeV, function of t)





Unitarize the lowest PW of Regge background for W < 2 GeV

For W > 2.5 GeV - pure Regge $$\mathcal{M}_\alpha^R = |\mathcal{M}_\alpha^R| e^{i\phi_\alpha^R}$$

For W < 2 GeV - pi-N phase $$t_{\gamma\pi}^{R,\alpha} = \mathcal{M}_\alpha^R e^{-i\phi_\alpha^R} \left[1 + i t_{\pi N}^\alpha\right]$$

Smooth interpolation in between

Or use more conventional MAID parametrization in the resonance region and match low multipoles onto Regge multipoles

WORK IN PROGRESS

t-channel Regge exchanges: correct physics input at forward angles;
Saturated Regge removes artifacts from the backward angles;

To provide a more complete description
– desirable to include baryon Regge exchanges, as well
not trivial: formal problems - parity doubling; fixed-t DR don't work at large negative t;

Ideally, a combination of fixed-t DR at forward angles
and interior DR at backward angles

Unitarize the Regge background for W < 2 GeV

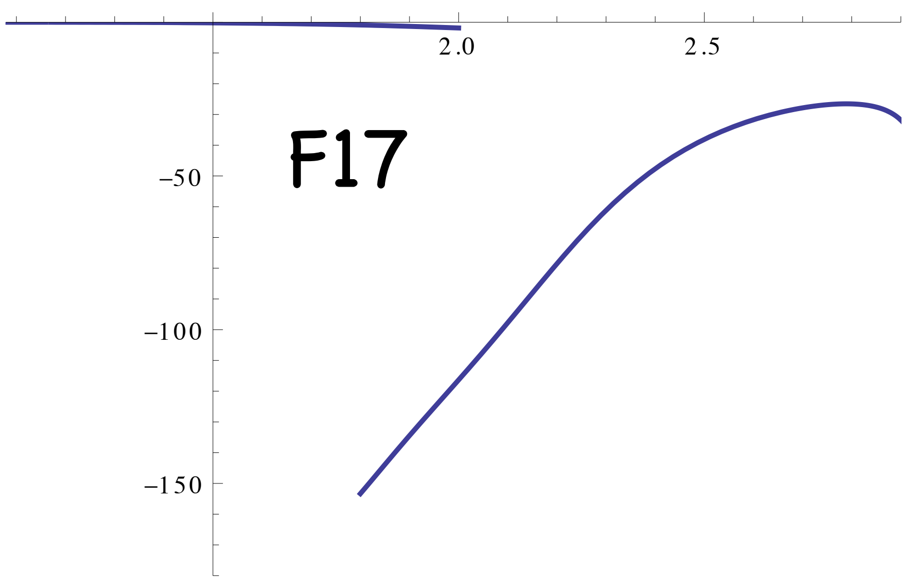
For W > 2.5 GeV - pure Regge

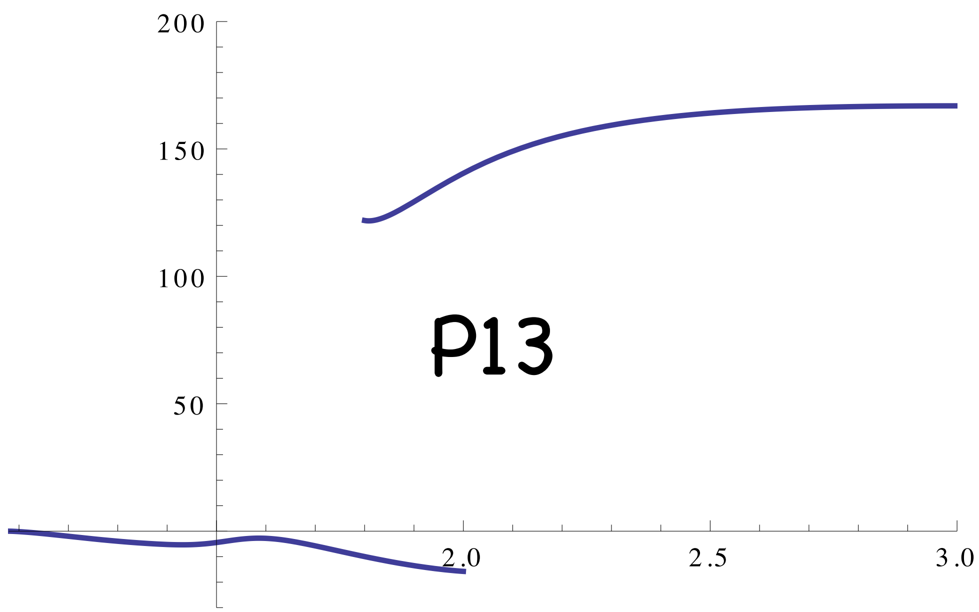
$$\mathcal{M}^R_\alpha = |\mathcal{M}^R_\alpha| e^{i\phi^R_\alpha}$$

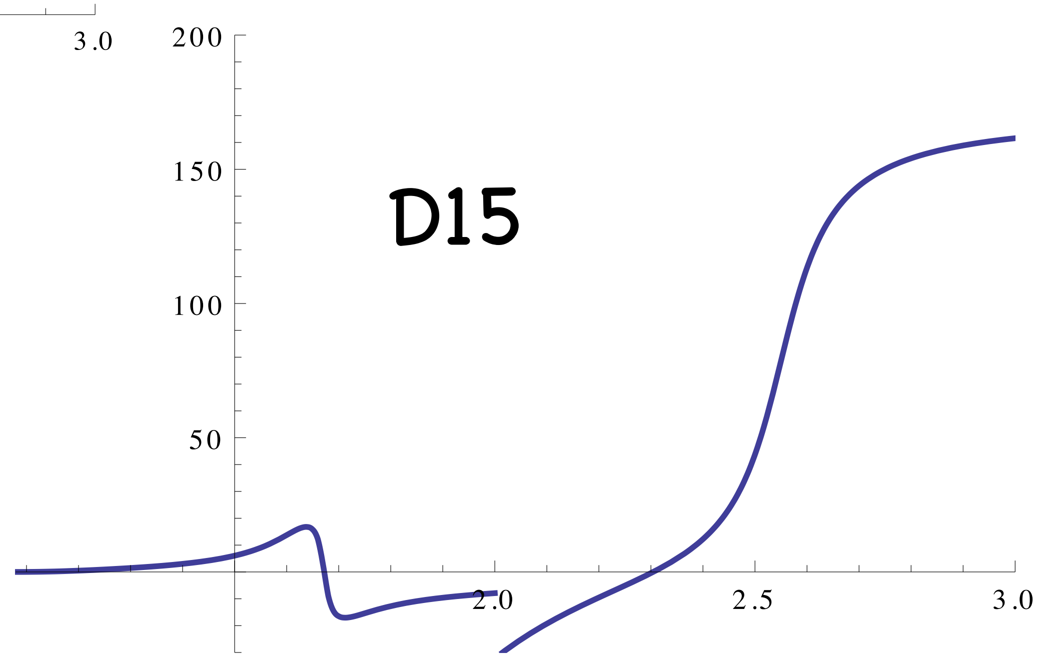
For W < 2 GeV - pi-N phase

$$t^{R,\alpha}_{\gamma\pi} = \mathcal{M}^R_\alpha e^{-i\phi^R_\alpha}\left[1 + i t^\alpha_{\pi N}\right]$$

Smooth interpolation in between







Phase rotations + Dispersion relations

BW resonance with E-dependent width (S-wave)

$$BW_R(s+i\epsilon) \sim \frac{1}{s - M_R^2 + M_R\Gamma_R\sqrt{\frac{s_{thr}-s-i\epsilon}{M_R^2-s_{thr}}}}$$

Im part: $$\mathrm{Im}BW_R(s+i\epsilon) \sim \frac{M_R\Gamma_R BR(R\to\gamma N)BR(R\to\pi N)}{(s-M_R^2)^2+M_R^2\Gamma^2(s)}$$

Re part may be reconstructed from a DR (more or less)

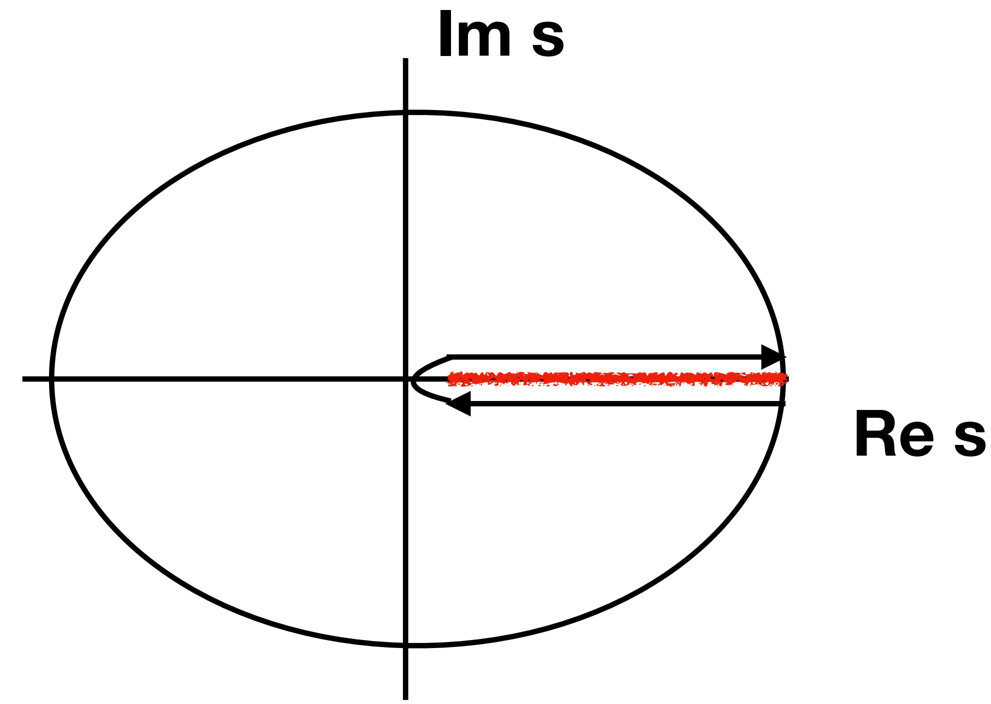
What happens if we introduce a phase $e^{i\phi}$?

$$\mathrm{Im}BW_R(s+i\epsilon) \sim \frac{M_R\Gamma_R BR(R\to\gamma N)BR(R\to\pi N)}{(s-M_R^2)^2+M_R^2\Gamma^2(s)}\cos\phi + \frac{s-M_R^2}{(s-M_R^2)^2+M_R^2\Gamma^2(s)}\sin\phi$$

Choosing an appropriate phase one can suppress a resonance at its position (e.g. phase = 90 deg)

But the overall phase should not matter (Alfred's talk)

What matters is not the Im part but rather the absorptive part (discontinuity along the real axis)



$$BW_R(s \in C) = \frac{1}{2\pi i}\oint_C \frac{ds'}{s'-s} BW_R(s')$$

$$BW_R(s+i\epsilon) = \frac{1}{\pi}\frac{ds'}{s'-s-i\epsilon}\frac{BW_R(s'+i\epsilon)-BW_R(s'-i\epsilon)}{2i} + \frac{1}{2\pi i}\oint_{C_\infty}\frac{ds'}{s'-s}BW_R(s')$$

$$\mathrm{Abs}BW_R(s) = BW_R(s+i\epsilon) - BW_R(s-i\epsilon) = 2i\mathrm{Im}BW_R(s+i\epsilon)$$

A mere redefinition of Im part: it is now a complex function

$$\mathrm{Abs}BW_R(s)\times e^{i\phi} = BW_R(s+i\epsilon)\times e^{i\phi} - BW_R(s-i\epsilon)\times e^{i\phi} = 2i\mathrm{Im}BW_R(s+i\epsilon)\times e^{i\phi}$$

The only assumption: phase has no discontinuity on the real s axis