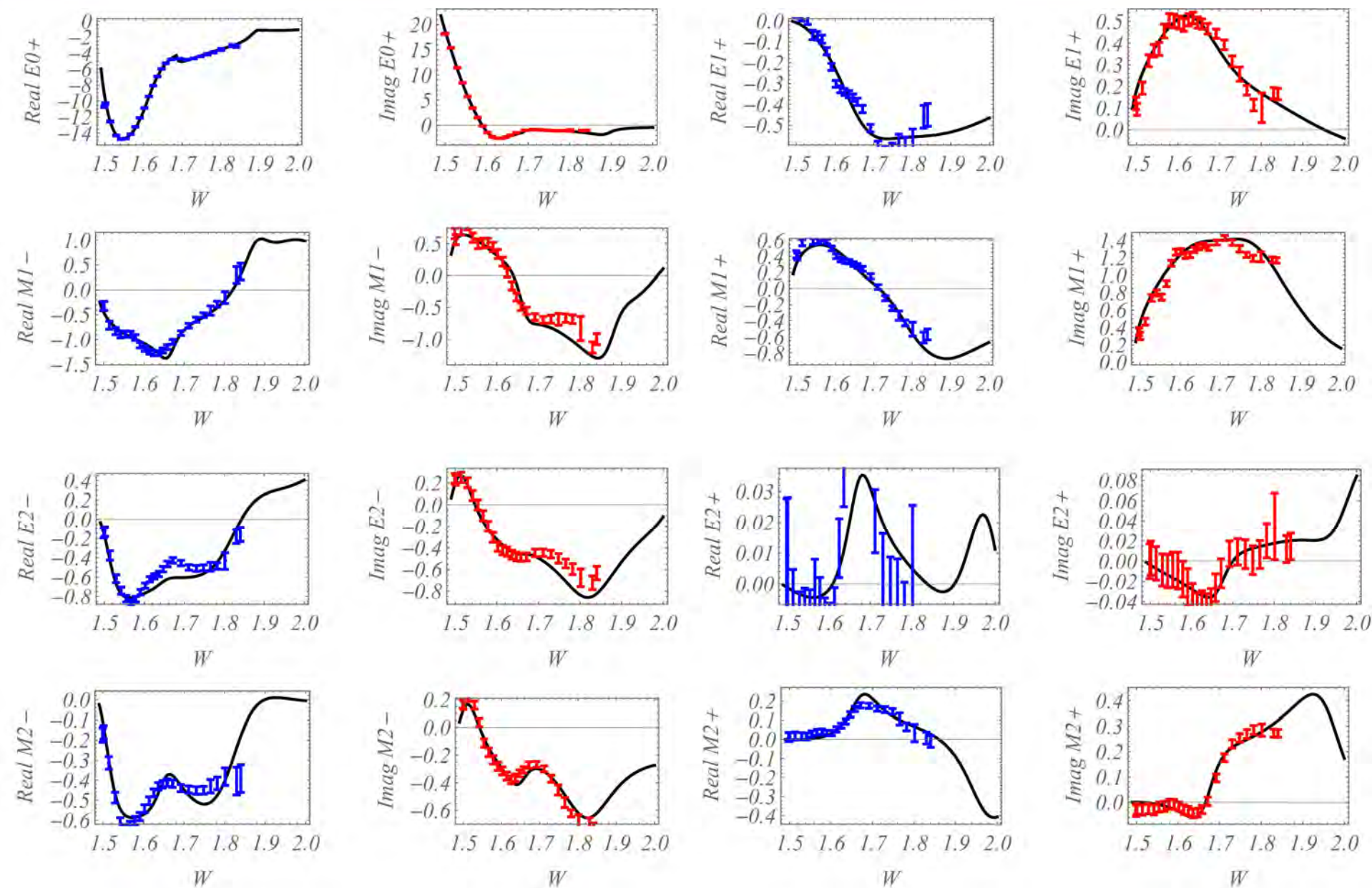
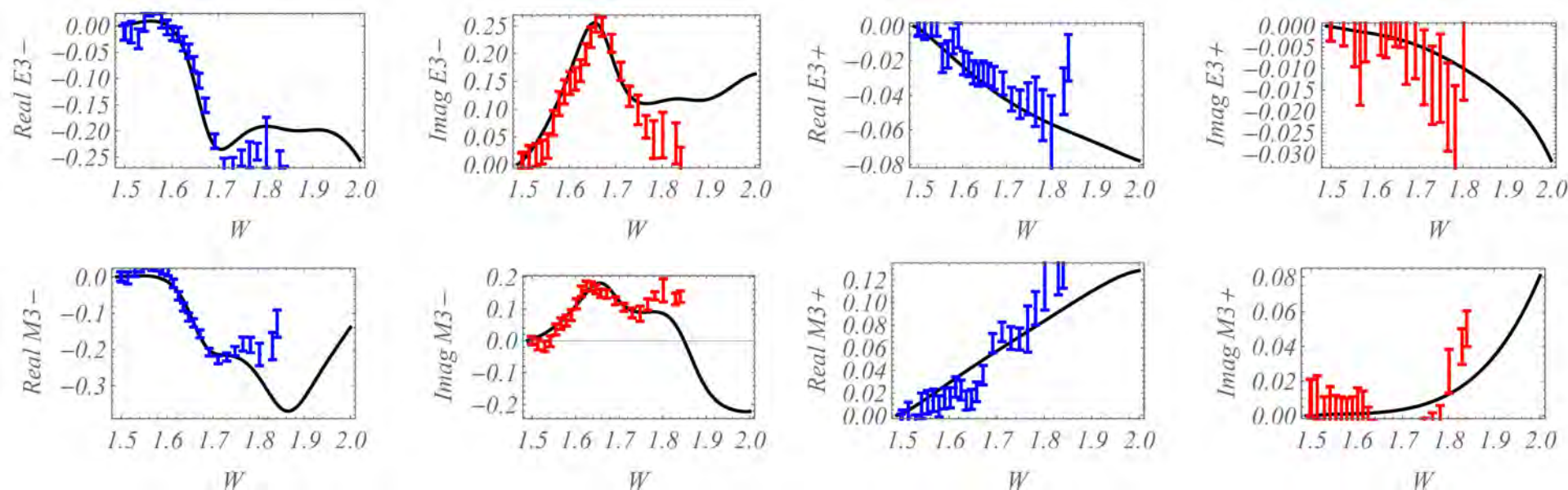


Discussions
on
Single-Energy Solutions
and
 $L + P$

single-energy (SE-3) vs energy-dependent (ED) PWA for $p(\gamma,\eta)p$

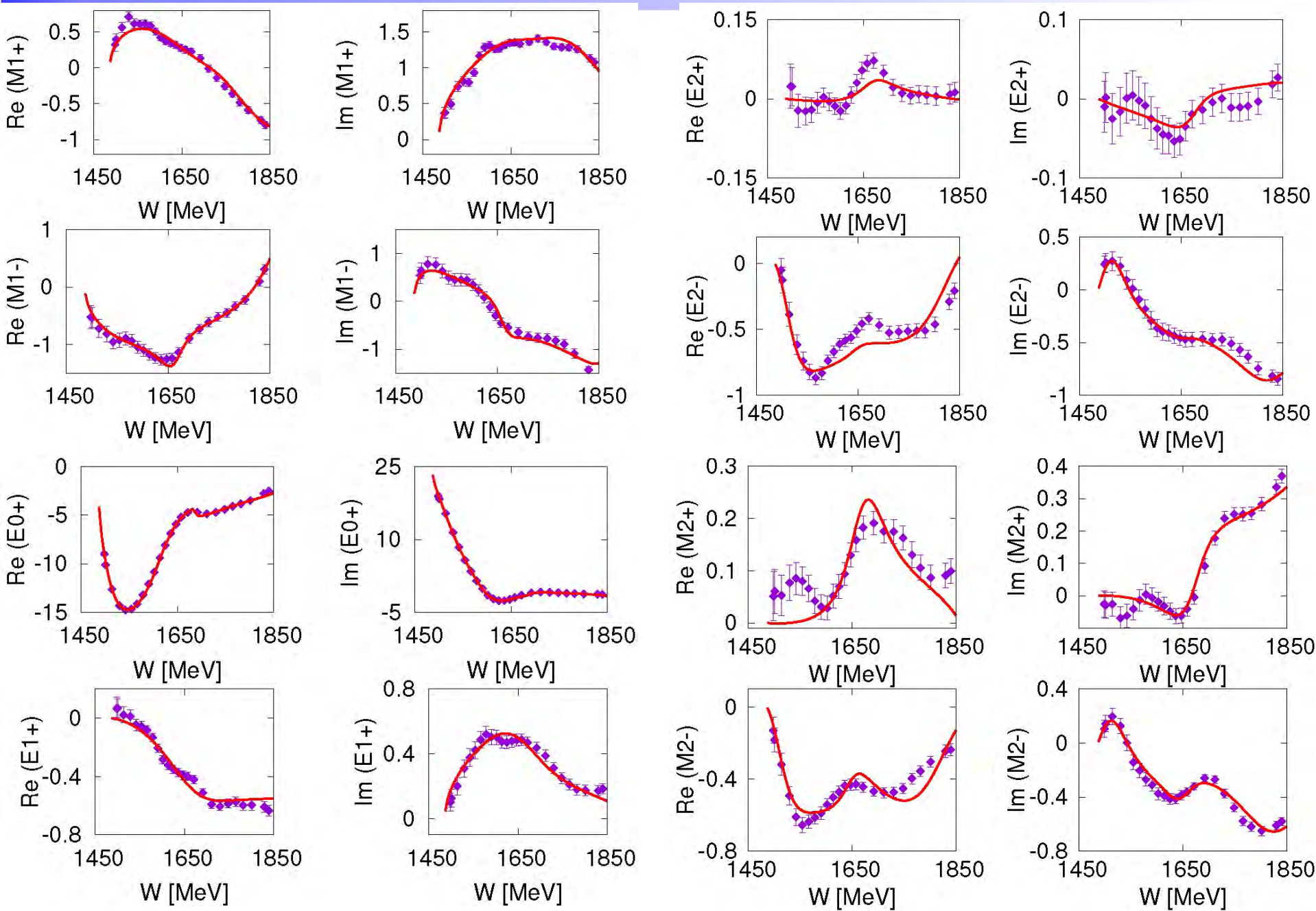


single-energy (SE-3) vs energy-dependent (ED) PWA for $p(\gamma,\eta)p$

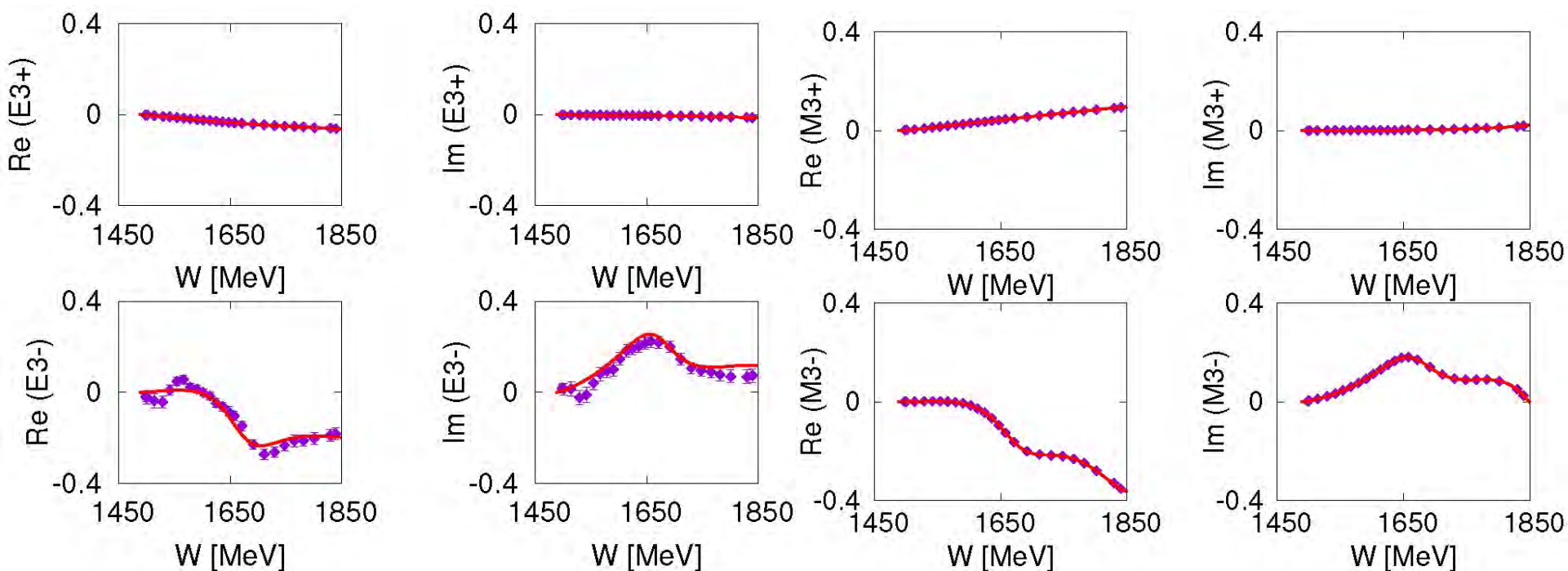


in SE-3 all multipoles up to $L=5$ have been freely fitted

single-energy (SE-4) vs energy-dependent (ED) PWA for $p(\gamma,\eta)p$

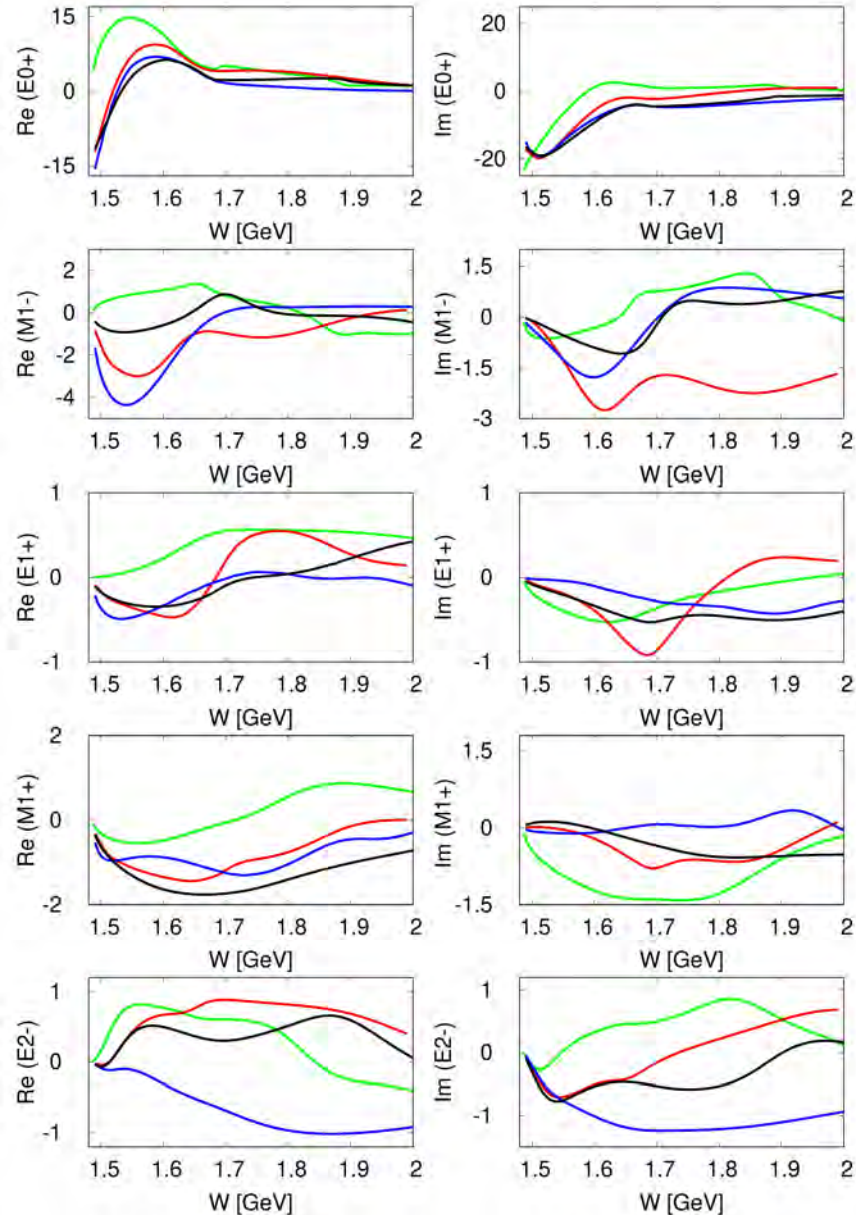


single-energy (SE-4) vs energy-dependent (ED) PWA for $p(\gamma,\eta)p$



in SE-4 only 9 multipoles have been freely fitted,
M3- was kept constant as all other higher multipoles

comparison of partial waves for $p(\gamma, \eta)p$



— EtaMAID — BnGa
— JüBo — KSU

comparison of S and P waves

between new (2018) PWA

from:

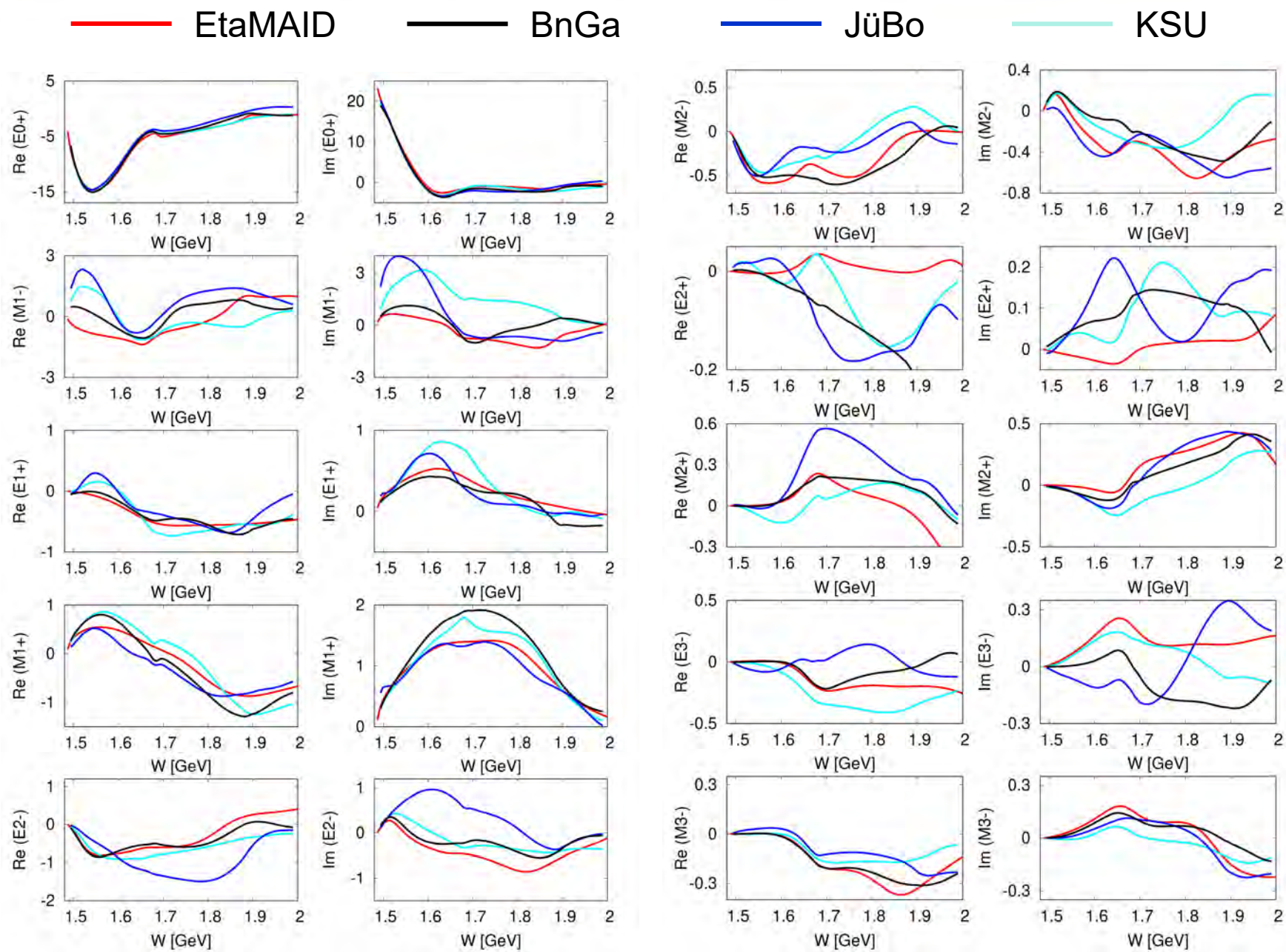
our MAID solution

Bonn-Gatchina

Jülich-Bonn

Kent-State

comparison of partial waves after phase rotation



comparison of partial waves after phase rotation for $p(\gamma,\eta)p$

comparison of S and P waves

between new (2018) PWA

from:

our MAID solution

Bonn-Gatchina

Jülich-Bonn

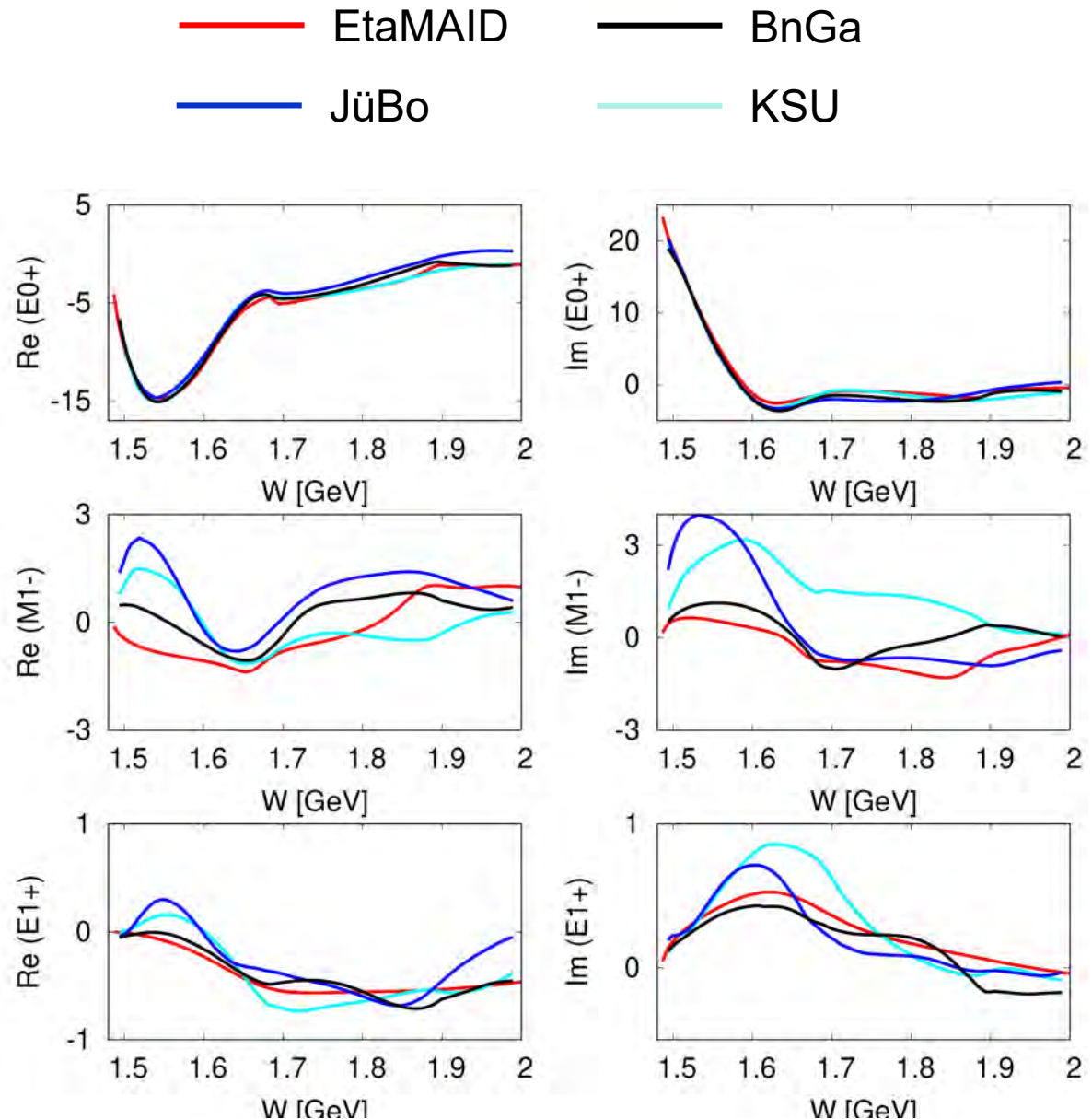
Kent-State

S waves are almost identical

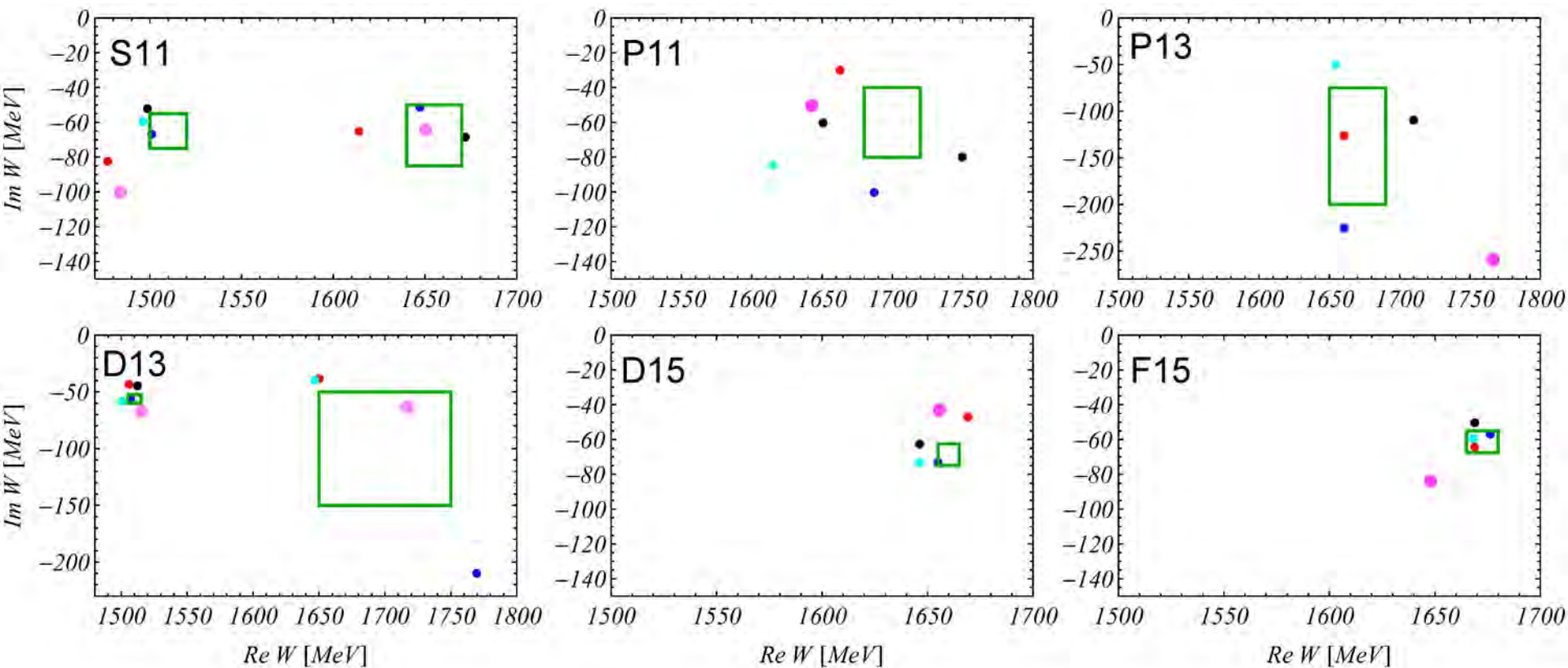
some higher pw are close

other pw differ a lot,

due to incomplete experiments!



comparison of pole positions



PDG ranges

● EtaMAID-SE4

● EtaMAID

● BnGa

● JüBo

● KSU

comparison of pole positions and residues

EtaMAID and JüBo: analytical continuation, SE-4: L+P

	EtaMAID 2018	SE-4	JüBo 2017
S11(1535) E0+	1477 – 165/2 i 1971, 21°	1484 – 201/2 i 3037, 26°	1495 – 112/2 i 736, 149°
S11(1650) E0+	1614 – 131/2 i 351, –176°	1650 – 129/2 i 325, –96°	1674 – 130/2 i 102, 57°
D13(1520) E2 – M2 –	1506 – 88/2 i 38.6, –13° 25.4, –13°	1515 – 135/2 i 71.9, 12° 70.7, 19°	1509 – 98/2 i 13.4, 123° 10.4, 108°
D15(1675) E2+ M2+	1669 – 94/2 i 38.6, –13° 25.4, –13°	1656 – 86/2 i 3.3, –121° 8.4, –158°	1647 – 135/2 i 3.7, 59° 22.6, –31°
F15(1680) E3 – M3 –	1669 – 128/2 i 15.1, 36° 11.8, 36°	1648 – 168/2 i 23.3, –18° 20.0, 14°	1666 – 80/2 i 2.9, 126° 1.7, 125°

comparison of pole positions and residues

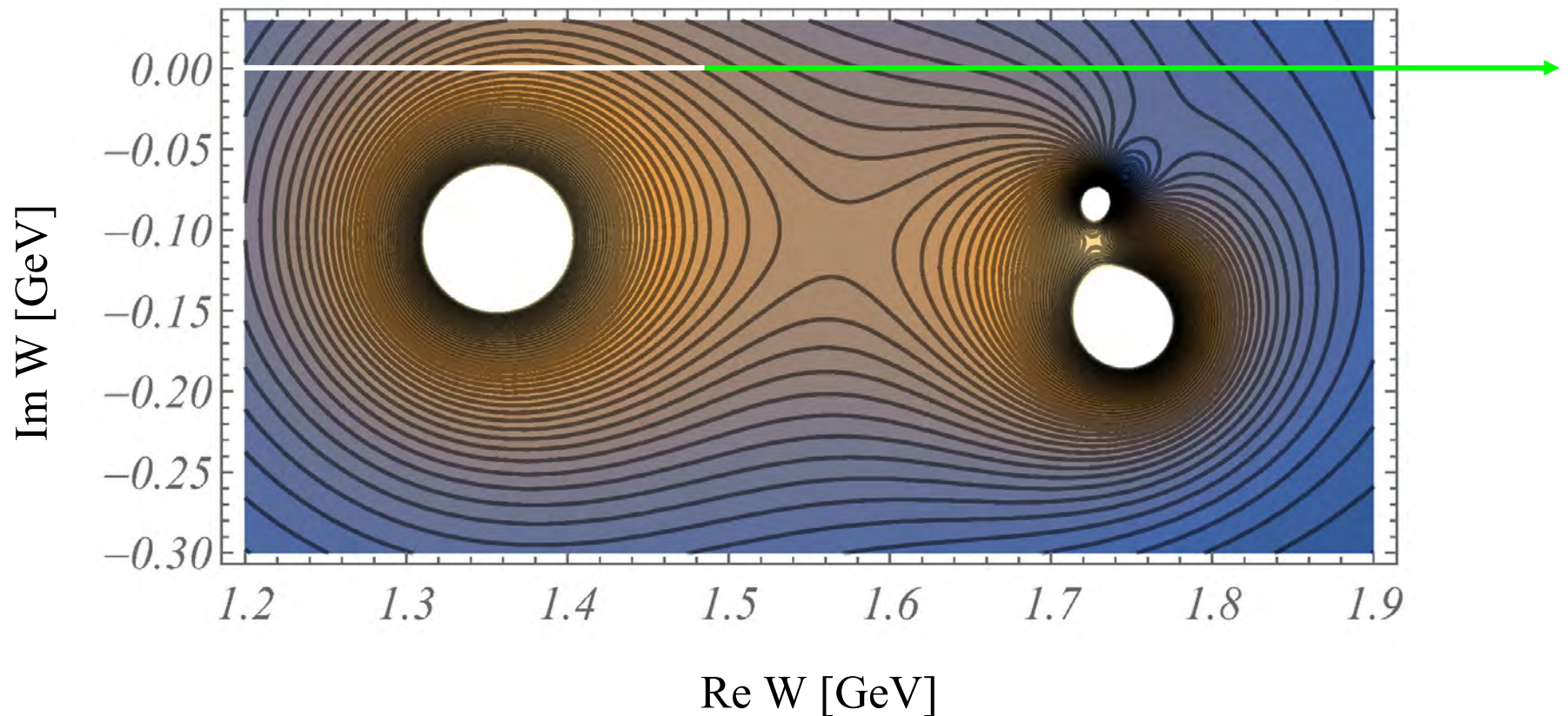
	EtaMAID 2018	SE-4	JüBo 2017
D13(1700) E2 – M2 –	1650 – 76/2 i 3.3, –137° 8.6, –137°	1717 – 128/2 i 20.4, 3° 23.3, 45°	
P11(1710) M1 –	1663 – 60/2 i 16.9, 120°	1643 – 101/2 i 30.9, 95°	1731 – 158/2 i 30.9, 95°
P13(1720) E1+ M1+	1660 – 251/2 i 68.4, 56° 81.2, 56°	1766 – 517/2 i 428, 123° 291, 82°	1689 – 190/2 i 3.7, –165° 3.3, –90°

JüBo has 2 poles in P11 partial wave in the region $1650 \text{ MeV} < W < 1750 \text{ MeV}$,
the pole masses are too close for a separation with data on the real axis

1750 – 316/2 i
57.0, 161°

simplified P11 map of JüBo model

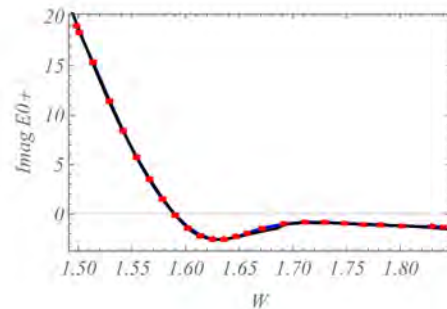
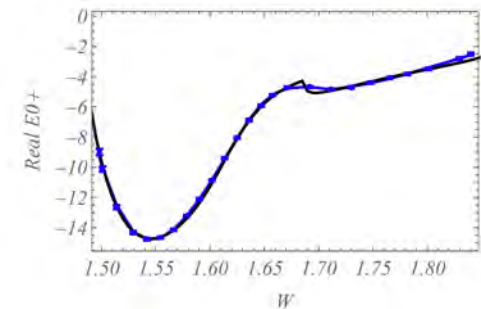
no bg is added



such a situation is too difficult for L+P

Details on L+P
with
Single-Energy Solution SE-4 (2019)

L+P result for S11 partial wave of SE-4 solution



the fit is perfect

the 2 pole positions are close to PDG range

$$M_{p1} = 1484 \text{ MeV}, \Gamma_{p1} = 201 \text{ MeV}$$

$$M_{p2} = 1650 \text{ MeV}, \Gamma_{p2} = 129 \text{ MeV}$$

the first residue is a bit too large

$$r_{E0+,1} = 3\,037 \text{ mfm MeV}$$

$$r_{E0+,2} = 325 \text{ mfm MeV}$$

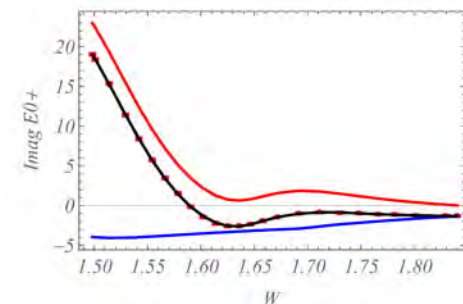
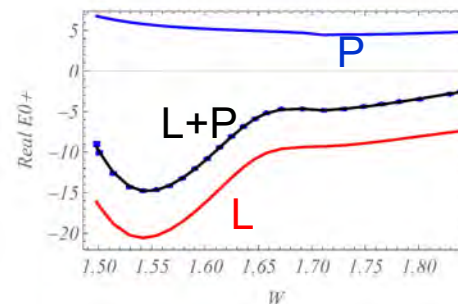
S11(1535) is difficult,

as it appears close or even below threshold

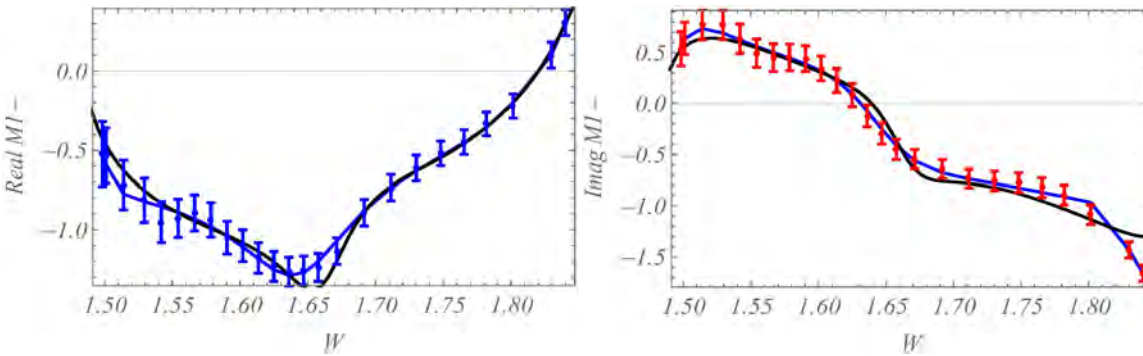
S11(1650) is easy

```

-----
Pole 1
-----
ReRes= 2.7334( 0.02151)
ImRes= 1.3230( 0.02216)
Modul= 3.0367( 0.02163)
Phase= 25.8277( 0.00726)
Mass= 1.4840( 0.00062) GeV
-2Im= 0.2009( 0.00056) GeV
*****
Pole 2
-----
ReRes= -0.0361( 0.00910)
ImRes= -0.3227( 0.00844)
Modul= 0.3247( 0.00845)
Phase= -96.3896( 0.02800)
Mass= 1.6500( 0.00112) GeV
-2Im= 0.1291( 0.00106) GeV
*****
TH1= -9.4380-i0.0000
TH2= 1.4860-i0.0000
TH3= 1.7072-i0.0000
del1= 1.8518
del2= 0.6808
del3= 0.5644
*****
chi squared total + penalty = 16.74+ 1.37= 18.11
50 29 21
chi squared total (reduced) = 0.80 0.86
    
```



L+P result for P11 partial wave of SE-4 solution



the fit is perfect

the pole position is near PDG range

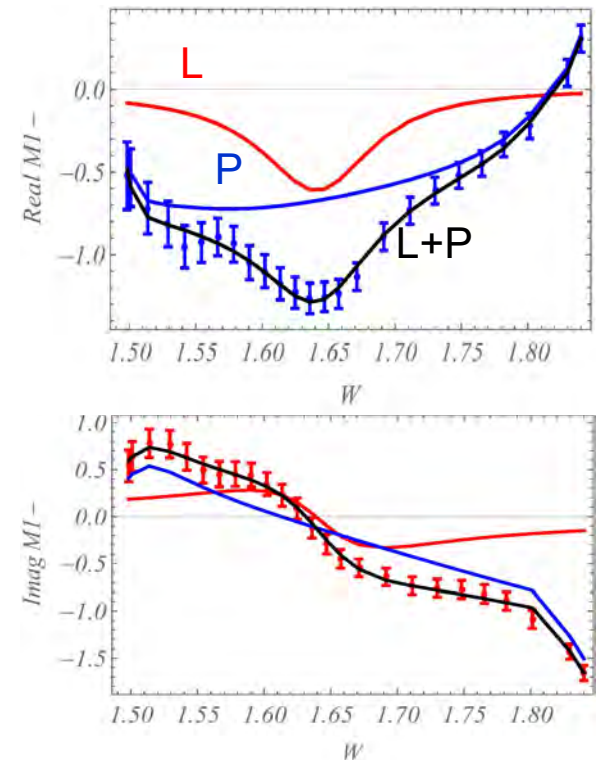
$$M_p = 1643 \text{ MeV}, \Gamma_p = 101 \text{ MeV}$$

$$r_{M1^-} = 30.9 \text{ mfm MeV}$$

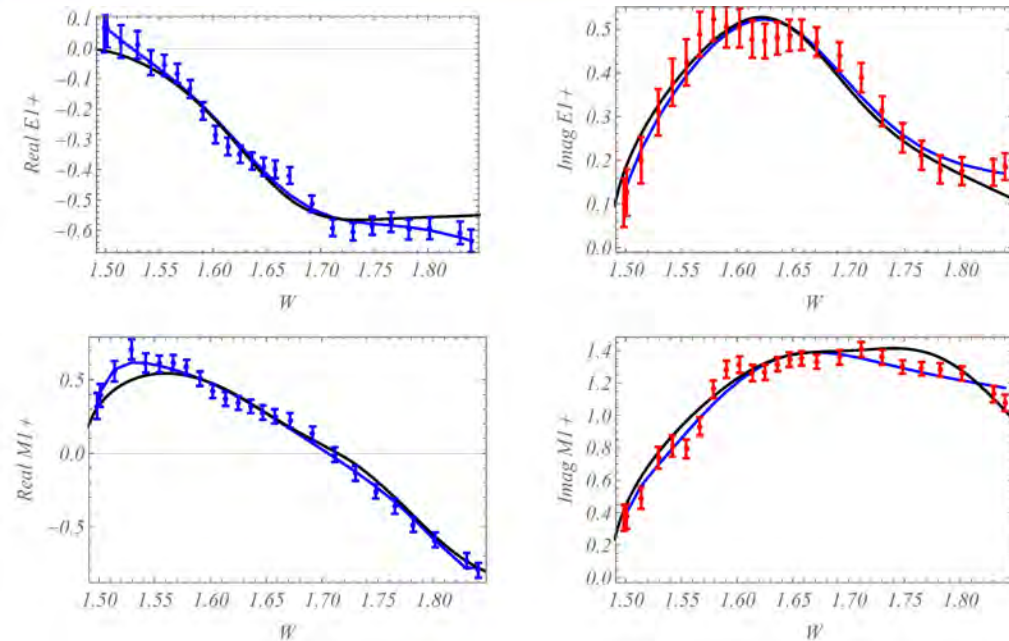
no need for a second pole

```

-----
Pole 1
-----
ReRes=  -0.0027( 0.00594)
ImRes=   0.0308( 0.00635)
Modul=   0.0309( 0.00635)
Phase=  95.0009( 0.19255)
Mass=    1.6432( 0.00804) GeV
-2Im=    0.1006( 0.00818) GeV
*****
TH1=     1.3349-i0.0000
TH2=     1.4860-i0.0000
TH3=     1.8247-i0.0000
del1=    0.7613
del2=    0.4044
del3=    0.3152
*****
chi squared total + penalty =      6.35+      2.24=      8.60
   50  25  25
chi squared total (reduced) =      0.25      0.34
    
```



L+P result for P13 partial wave of SE-4 solution



the fit is good

the pole position is far outside PDG range

$$M_p = 1766 \text{ MeV}, \Gamma_p = 517 \text{ MeV}$$

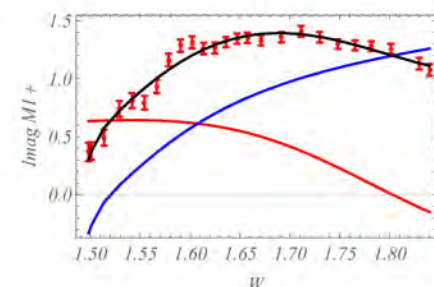
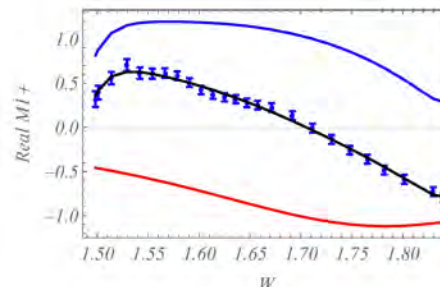
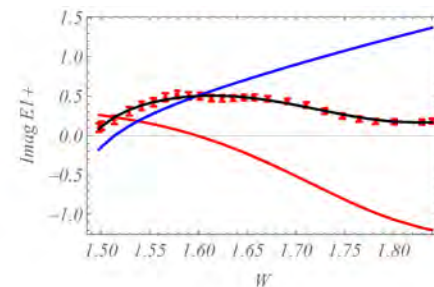
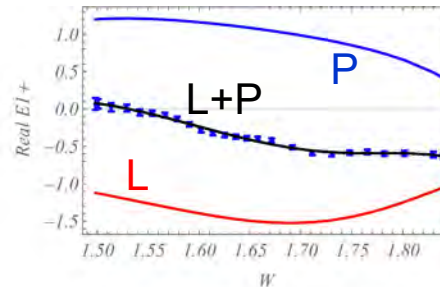
the residues are very much too large

$$r_{E1+} = 428 \text{ mfm MeV}$$

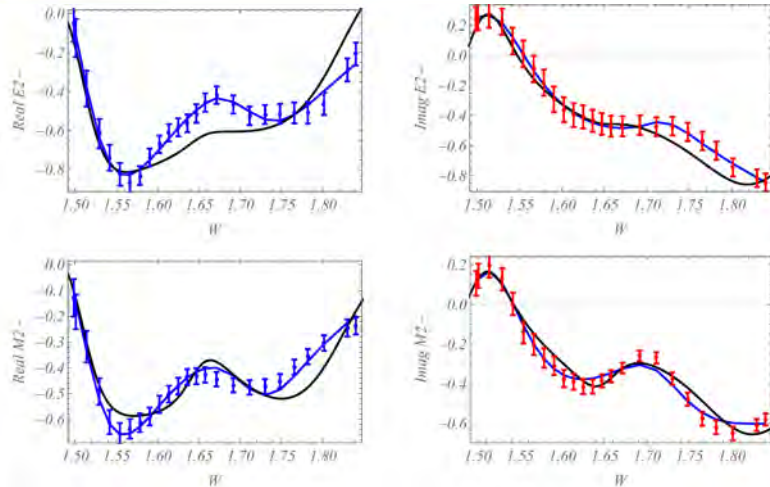
$$r_{M1+} = 291 \text{ mfm MeV}$$

P13 is the most difficult partial wave

```
-----
Pole 1
-----
ReRes=  -0.2320( 0.00715)    0.0416( -0.00910)
ImRes=   0.3595( 0.00656)    0.2881( 0.01018)
Modul=   0.4279( 0.00674)    0.2911( 0.01015)
Phase=  122.8278( 0.01632)   81.7907( 0.03133)
Mass=    1.7657( 0.00335) GeV
-2Im=    0.5168( 0.00309) GeV
*****
TH1=    1.2003-i0.0000    1.3000-i0.0000
TH2=    1.4860-i0.0000    1.4860-i0.0000
TH3=    1.8876-i0.0000    1.8500-i0.0000
del1=    1.0713          1.2679
del2=    0.8917          0.5500
del3=    0.6592          0.5805
*****
chi squared total + penalty =      60.58+      4.75=      65.33
100 48 52
chi squared total (reduced) =      1.16      1.26
```



L+P result for D13 partial wave of SE-4 solution



```

Pole 1
-----
ReRes= 0.0703( 0.02872) 0.0669( 0.02360)
ImRes= 0.0148( 0.01836) 0.0228( 0.01717)
Modul= 0.0719( 0.02836) 0.0707( 0.02302)
Phase= 11.9186( 0.26331) 18.8335( 0.25381)
Mass= 1.5147( 0.01020) GeV
-2Im= 0.1346( 0.01076) GeV
*****

Pole 2
-----
ReRes= 0.0204( 0.00710) 0.0163( 0.00745)
ImRes= 0.0010( 0.00647) 0.0166( 0.00679)
Modul= 0.0204( 0.00710) 0.0233( 0.00712)
Phase= 2.8681( 0.31777) 45.4624( 0.30664)
Mass= 1.7173( 0.01089) GeV
-2Im= 0.1283( 0.01072) GeV
*****

TH1= 1.3419-i0.0000 -0.0147-i0.0000
TH2= 1.4860-i0.0000 1.4860-i0.0000
TH3= 1.6885-i0.0000 1.6800-i0.0000
del1= 1.1439 1.5116
del2= 0.4832 1.5708
del3= 0.8091 1.0000
*****
chi squared total + penalty = 38.71+ 0.95= 39.66
100 54 46
chi squared total (reduced) = 0.84 0.86
    
```

the fit is good

the 2 pole positions are close to PDG ranges

$M_{p1} = 1515 \text{ MeV}$, $\Gamma_{p1} = 135 \text{ MeV}$

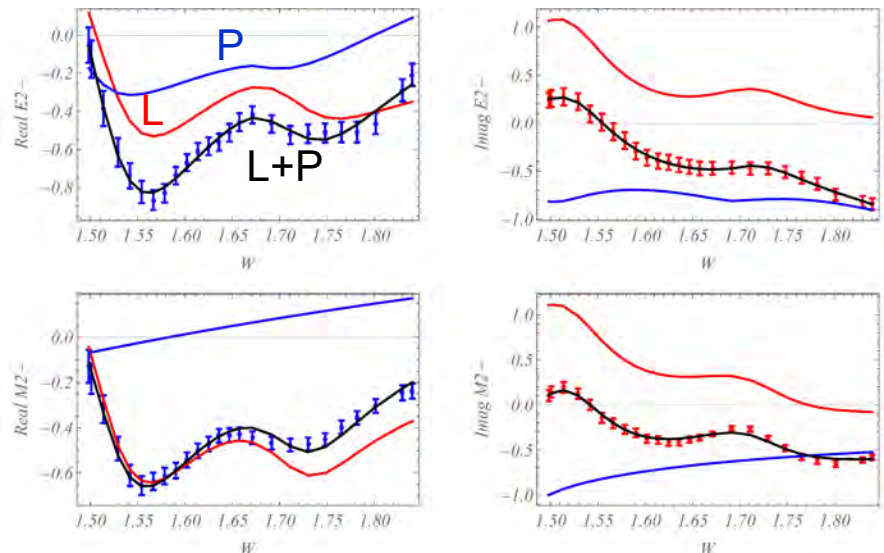
$M_{p2} = 1717 \text{ MeV}$, $\Gamma_{p2} = 128 \text{ MeV}$

the residues are reasonable

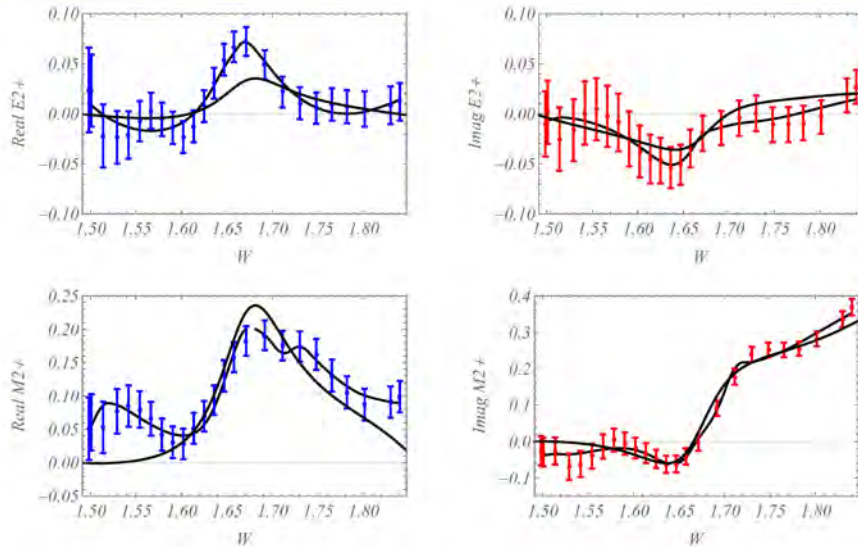
$r_{E2-} = 71.9$ and 20.4 mfm MeV

$r_{M2-} = 70.7$ and 23.3 mfm MeV

D13 is an easy partial wave



L+P result for D15 partial wave of SE-4 solution



the fit is good

the pole position is far outside PDG range

$$M_p = 1656 \text{ MeV}, \Gamma_p = 86 \text{ MeV}$$

the residues look very reasonable

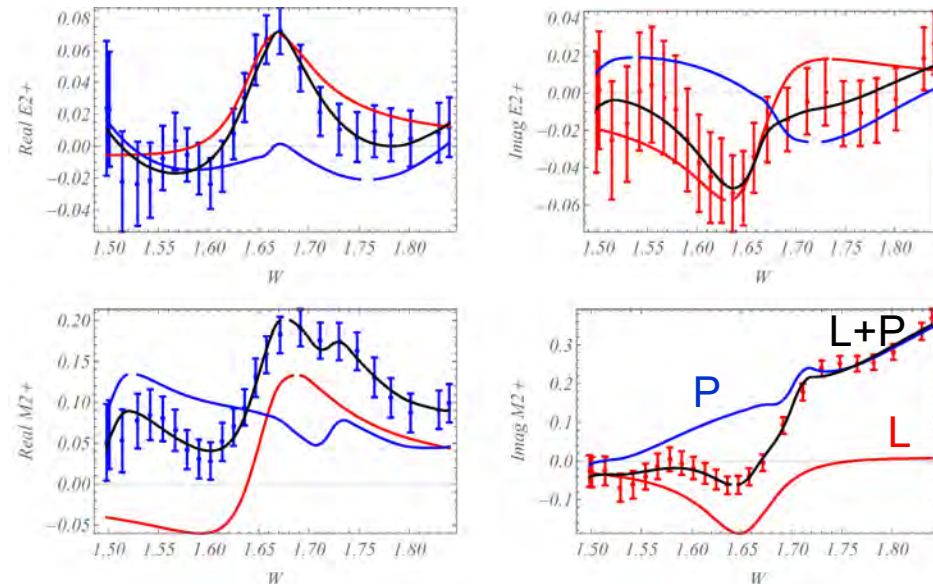
$$r_{E2+} = 3.3 \text{ mfm MeV}$$

$$r_{M2+} = 8.4 \text{ mfm MeV}$$

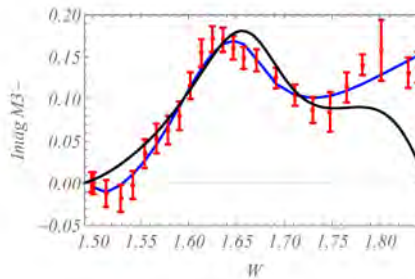
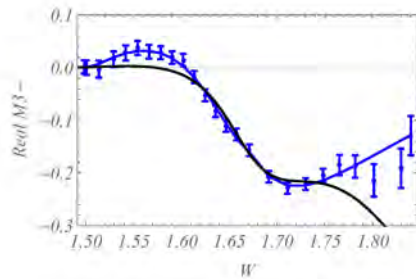
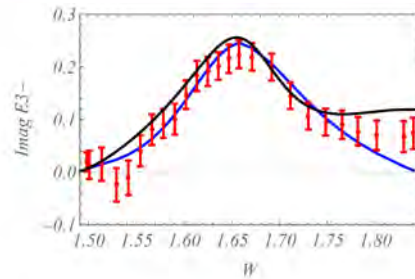
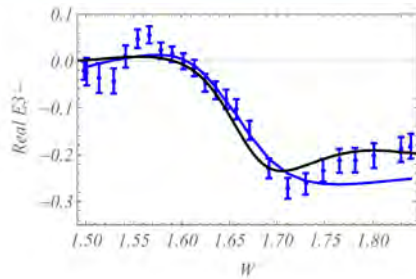
D15 is an easy partial wave

```

-----
Pole 1
-----
ReRes=  -0.0017( 0.00051)   -0.0078( 0.00096)
ImRes=  -0.0028( 0.00056)   -0.0032( 0.00114)
Modul=   0.0033( 0.00054)    0.0084( 0.00099)
Phase= -121.3284( 0.16075)  -157.6509( 0.13285)
Mass=    1.6562( 0.00490) GeV
-2Im=    0.0863( 0.00444) GeV
*****
TH1=    1.2454-i0.0000      1.2860-i0.0000
TH2=    1.4860-i0.0000      1.4860-i0.0000
TH3=    1.6713-i0.0000      1.7016-i0.0000
del1=    0.6510             0.9840
del2=    0.9775             0.5500
del3=    0.7570             0.5805
*****
chi squared total + penalty =      21.99+      0.77=      22.76
100  48  52
chi squared total (reduced) =       0.42       0.44
    
```



L+P result for F15 partial wave of SE-4 solution



the fit is good

the pole position is far outside PDG range

$$M_p = 1648 \text{ MeV}, \Gamma_p = 168 \text{ MeV}$$

the residues look reasonable

$$r_{E3-} = 23.3 \text{ mfm MeV}$$

$$r_{M3-} = 20.0 \text{ mfm MeV}$$

SE solution is questionable for $W > 1.7 \text{ GeV}$

```

-----
Pole 1
-----
ReRes=   0.0222( 0.00139)   0.0195( 0.00112)
ImRes=  -0.0073( 0.00168)   0.0047( 0.00134)
Modul=   0.0233( 0.00142)   0.0200( 0.00113)
Phase=  -18.2888( 0.07065)  13.6600( 0.06647)
Mass=    1.6475( 0.00353) GeV
-2Im=    0.1683( 0.00329) GeV
*****
TH1=    1.1127-i0.0000    1.3000-i0.0000
TH2=    1.4860-i0.0000    1.4860-i0.0000
TH3=    1.6576-i0.0000    1.6698-i0.0000
del1=    0.6498          0.4685
del2=    0.7155          0.5500
del3=    1.1898          0.5805
*****
chi squared total + penalty =    56.41+    0.83=    57.24
100 48 52
chi squared total (reduced) =    1.08    1.10
    
```

