Computational Finance

Handout 1: Return Measures

Discrete and Continuously Compounded Rate of Return

Related through the multiple

The discrete absolute return earned by an investment over a period of time is the multiple – 1:

$$\text{discrete absolute return} = \frac{P_2}{P_1} - 1$$

The continuously-compounded (aka CCR, log-return) absolute return is calculated by taking the natural log of the multiple:

$$\text{continuously compounded return} = \ln\left(\frac{P_2}{P_1}\right)$$

If, for example, you purchased a house for \$132k and sold it at \$250k):

$$\text{discrete return} = \frac{\$250\text{k}}{\$132\text{k}} - 1 = 1.89 - 1 = 0.894 = 89.4\%$$

$$\text{continuously compounded return} = \ln\left(\frac{\$250\text{k}}{\$132\text{k}}\right) = 0.639 = 63.9\%$$

“Absolute” vs. “Rate of Return”: Terminology

The qualifier “**absolute**” is taken to mean the overall return earned by an asset from one period to another. For example, if a house is bought for \$132,000 and sold for \$250,000 nine years later, then, using the continuously compounded return (CCR) as our metric, the **absolute return** would be:

Absolute Return = ln(\$250k / \$132k) = 0.639 = 63.9%

Note that there is no time unit in the absolute return – it simply gives an overall return from one point in time to another.

The **rate of return**, on the other hand, *does* have a time unit because it communicates the rate of return earned by an investment <u>per period of time</u>. The rate of return of the investment above might be expressed as follows:

Rate of Return = ln(\$250k / \$132k) / 9 years = 0.710 = 7.10% / year

Note that these metrics could just have just as easily been expressed in terms of any other measure such as the multiple, the raw return, or something else. For example, using the **raw return** as the metric, an equivalent way of expressing these returns would be as follows:

Absolute Return = (\$250k - \$132k) / \$132k = 0.894 = 89.4%
Rate of Return = (\$250k - \$132k) / \$132k / 9 years = 0.0993 = 9.93% / year

Arithmetic & Geometric Mean Rates of Return

Arithmetic Mean Rate of Return (AMRR) can be understood as the simple average of a time series of *n* individual returns (the AVERAGE() function in Excel).

Geometric Mean Rate of Return (GMRR) can be understood as the *n*th root of the product of the multiples of *n* returns.

For example, assume that changes in the price of an ounce of gold, measured for twelve (12) months at even intervals of 1 month, has the following time series:

returns = +25%, -20%, +25%, -20%, +25%, -20%, +25%, -20%, +25%, -20%, +25%, -20%

The **AMRR** of the returns, given by =AVERAGE(*returns*) in Excel, is 2.5%/month.

The **GMRR** is as follows:

GMRR = ((1.25)(.8)(1.25)(.8)(1.25)(.8) (1.25)(.8)(1.25)(.8)(1.25)(.8)))^(1/12) – 1

= (1)^(1/12) – 1

= 0% /month

Note that AMRR does not provide an accurate measure of the investment's performance.

An additional useful property of GMRR: A geometric mean return is a rate of return calculated over a discrete time interval, such as a month or a year. When that discrete time interval approaches 0 at the limit, the geometric mean return approaches the continuous log return.

Net Present Value (NPV) and Internal Rate of Return (IRR):

Cash in the future is considered to be worth less than cash now, but exactly ***how*** much less is a matter of interpretation. This concept is captured by the **discount rate**.

For example, I may be willing to loan you $100 now if you pay me back at least $110 in one month. If I make this loan, I'm implicitly stating that I'd rather have $110 a month from now than $100 today. The **discount rate** I am willing to accept is therefore a minimum of 10 % / month.

Equivalently, by making the loan, I'm stating that, at a discount rate of 9.99999…%/month, I don't care either way whether I have $100 now or $100 in a month. At any rate greater than that, I'll make the loan because I'd rather have money in the future. Less than that, and I'd rather keep the cash.

Observe that the discount rate I assign to the venture of giving you a loan actually encapsulates a lot of information about me, my beliefs about you, and the market as a whole:

1. If I anticipate the possibility needing cash in the next month (liquidity requirements), I'd only be willing to loan out at a higher discount rate.
2. If I believe you are very trustworthy/risky, I'll accept a lower/higher discount rate
3. If there were very few other investment opportunities available to me out in the wider economy, I might accept a lower discount rate.

NPV: Future cash payments – or a SERIES of them – can be discounted and summed.
You hear that a famous painting can be purchased in one year for $1.5 million. You believe you can sell the painting in three years for $2.5 million. Taking into account your liquidity requirements, your assessment of the risk of actually being able to sell the painting in 3 years, and your other economic opportunities, you assign a discount rate *r* = 20%, discrete, to this venture.

The question is: **how much is this entire venture worth to you right now**, taking into account the cash flows of buying the painting in a year, selling it in three years, as well as the fact that cash later is worth less than cash now?

Net Present Value (NPV) is the name given to the parameter that measures this quantity. It is defined simply as **the sum of all future cash flows of an asset, discounted to the present**.

For the painting example:

$$NPV = \frac{-\$1.5\text{MM}}{1.2^1} + \frac{\$2.5\text{MM}}{1.2^3} = -\$1{,}250{,}000 + \$1{,}446{,}759 = \$196{,}759$$

In other words, as a venture, overall, flipping the painting is worth $196,759 to you. You need to know that number because there are many other things you could do with that money – the NPV can be used to help you compare present values and decide your most profitable allocation of resources.

IRR: The Internal Rate of Return (IRR) is a unique discount rate that sets the NPV to zero.
Let's say that – all things considered – someone pitched you the idea of buying & selling the painting as laid out in the previous section. You heard the pitch, believed it, and went out feeling <u>completely indifferent</u> about the whole thing – you could take it or leave it.

If you're indifferent towards an investment, it means that, to you, that investment has a net present value of zero. This observation can be useful because it can be used to calculate the IRR – the discount rate that you're implicitly assuming for a venture. IRRs can also be used to compareopportunities.

In the above example, the IRR is the *r* that satisfies *NPV* = 0, and can be found in **Solver**:

$$NPV = 0 = \frac{-\$1.5\text{MM}}{(1+r)^1} + \frac{\$2.5\text{MM}}{(1+r)^3} \rightarrow r = 29.1\%$$

It can also be found using Excel's NPV function, which you are welcome to use, **<u>BUT BEWARE</u>**: that function excludes the initial cash outlay, so you must always remember to account for it!!!

Nominal vs. Real (Inflation Adjusted) Returns

The purchasing power of a currency may vary over time. Reduction in currency value relative to typical goods and services is known as "inflation". Increase in the currency value is known as "deflation".

Investment returns expressed in nominal currency prices, without taking inflation or deflation into account, are known as **nominal returns**. During periods of high inflation or deflation, the difference between nominal and **real** (adjusted for inflation) returns can be large.

Inflation or deflation over a given time interval can be calculated by taking the ratio of the Consumer Price Index ("CPI") which represents the dollar cost of a basket of typical goods and services for the beginning and end date of the period.

$P_1 \frac{CPI_2}{CPI_1} = P_{1:2} = P_1$ Converted to time 2 (inflation-adjusted) dollars

$P_2 \frac{CPI_1}{CPI_2} = P_{2:1} = P_2$ Converted to time 1 (inflation-adjusted) dollars

$$\text{Inflation-adjusted returns} = \ln\left(\frac{P_{2:1}}{P_1}\right) = \ln\left(\frac{P_2}{P_{1:2}}\right)$$

Example:

The S&P 500 Index adjusted closing price was **$76.98** on **02 Jan 1975** and **$179.63** on **02 Jan 1985**.

The Consumer Price Index (CPI) was **51.9** on **31 Dec 1974** and **105.3** on **31 Dec 1984**.

The annual rate of inflation:

$$= \ln\left(\frac{CPI_2}{CPI_1}\right) / 10 \text{ years}$$

$$= \ln\left(\frac{105.3}{51.9}\right) / \text{ 10 years}$$

= 0.0707 / yr

= 7.07% / yr

The annualized log real return

$$= \ln\left(\frac{P_2}{P_{1:2}}\right)$$

$$= \ln\left(\frac{179.63}{76.98 \frac{105.3}{51.9}}\right) / 10 \text{ years}$$

= 0.014 / year

= 1.4% / year

Note: When using log returns, Nominal return – Inflation Rate = Real Return

In the example above, 8.47% - 7.07% = 1.4%.

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