Problem 1 - Price Return Calculation

Calculate and compare the expected value and standard deviation of price at time t (Pt) ,given each of the 3 types of price returns, assuming rt ~ N(0,σ^2).

Simulate each return equation using rt ~ N(0,σ^2) and show the mean and standard deviation match your expectations.

Problem 1.1

Calculate and compare the expected value and standard deviation of price at time t (Pt) ,given each of the 3 types of price returns, assuming rt ~ N(0,σ^2).

The expected value and standard deviation of three types of price return used is:

	Distribution	Mean	Std
Classical Brownian	Pt+1 ~ N(Pt, σ^2)	P0	σ
Arithmetic Return	Pt+1 ~ N(Pt, Pt^2 * σ^2)	P0	σ * P0
Log return	Pt+1 ~ LN(LN(Pt), σ^2)	exp(np.log(P0) + 0.5 * σ ** 2)	sqrt(exp(σ ** 2) - 1) * exp(np.log(P0) + 0.5 * σ ** 2)

For the Geometric Brownian Motion, we have

$$P_{t+t} \sim LN\left(\mu + ln\left(P_t\right), \sigma^2\right)$$

The initial value P0 = 100, sigma = 0.1, mu = 0.
The result is shown as:

	Expected Mean		Expected Std	
Classical Brownian	100.00		0.1	
Arithmetic Return	100.00		10.00	
Log return	100.50		10.07	

The Expected Mean of the three methods are similar. But the Expected standard deviation of Classical Brownian Motion is much smaller as it is not infected by Pt. But this method is the least used of the 3, as it can not assure prices greater than 0. The Expected value and expected Std of log return is the largest.

Problem 1.2

Simulate each return equation using rt ~ N(0,σ^2) and show the mean and standard deviation match your expectations.
The three types of price return used is:

1. Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$

2. Arithmetic Return System

$$P_t = P_{t-1}\left(1 + r_t\right)$$

3. Log Return or Geometric Brownian Motion

$$P_t = P_{t-1} e^{r_t}$$

The initial value P0 = 100, sigma = 0.1, mu = 0.
The result is shown as:

	Expected Mean	Actual Mean	Expected Std	Actual Std
Classical Brownian	100.00	100.00	0.10	0.10
Arithmetic Return	100.00	100.11	10.00	9.93
Log return	100.50	100.44	10.07	10.02

The result shows that the mean and standard deviation match expectations. But as the sigma become bigger, the deviation of actual and simulated becomes bigger.

Problem 2 - VaR Calculation

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

Remove the mean from the series so that the mean(META)=0

Calculate VaR

Using a normal distribution.

Using a normal distribution with an Exponentially Weighted variance (λ = 0. 94)

Using a MLE fitted T distribution.

Using a fitted AR(1) model.

Using a Historic Simulation.

Compare the 5 values.

Problem 2.1 & 2.2 & 2.3

Implement a function similar to the “return_calculate()” in this week’s code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

Remove the mean from the series so that the mean(META)=0

A: Implement the function as the one in Julia.

Problem 2.4

Calculate VaR

Using a normal distribution. The normal distribution package is used here. With actual mu and sigma.

Using a normal distribution with an Exponentially Weighted variance (λ = 0. 94). The normal distribution package is used here. With actual mu and sigma is transformed by Exponentially Weighted variance written in previous project.

Using a MLE fitted T distribution. MLE_t function written in previous project is used in this one.

Using a fitted AR(1) model. AR(1) model is implemented by ARIMA in python, which is a more general model that extends the ARMA model to non-stationary time series data.

Using a Historic Simulation. Calculate by use the actual data.

Compare the 5 values. The result is shown as:

	VaR
normal distribution	0.06683
normal distribution with an Exponentially Weighted variance	0.09221
MLE fitted T distribution	0.05647
fitted AR(1) model	0.06533
Historic Simulation	0.05462

1. VaR calculated by normal distribution with an Exponentially Weighted variance is larger than all of the other method. Exponentially Weighted variance gives more weight to the

recent data. This implies that META's recent stock price is volatile which leads to higher standard deviation and higher VaR.

2. VaR calculated by normal distribution and fitted AR(1) model is similar. Which means the simulation result of those two functions are similar.
3. MLE fitted T distribution best fit the historic result. As the VaR calculated by those two methods are quite similar.

Problem 3 - Portfolio VaR Calculation

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

Discuss your methods and your results.

Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

Problem 3.1 - Portfolio VaR Calculation

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

The method used here is delta normal VaR. The procedure used is in the example:

Example

Assume we have

- 3 assets $A = [A_1, A_2, A_3]$ with current values, $[1, 8, 20]$
- 2 prices $P = [P_1, P_2]$ with current prices, $[7, 10]$
- We hold Assets 1-3 in quantities $Q = [10, 20, 5]$
- Total portfolio value is then $A'Q = 1 * 10 + 8 * 20 + 5 * 20 = 270$
- $\frac{dA_1}{dP_1} = 0.5$
- $\frac{dA_2}{dP_1} = 1$
- $\frac{dA_3}{dP_2} = 1$
- all other partial derivatives are 0
- $\Sigma =$
  0.01 0.0075
  0.0075 0.0225
- $\frac{dR}{dr_1} = \frac{7}{270}(10 * 0.5 + 20 * 1) = 0.6481$
- $\frac{dR}{dr_2} = \frac{10}{310}(5 * 1) = 0.1852$
- $\sqrt{\nabla R^T \Sigma \nabla R} = 0.0823$
- Assuming a normal distribution and $\alpha = 0.05$, $F^{-1}(\alpha) = 1.645$
- $VaR =- 270 * 1.645 * 0.0823 = \$36.55$
- If we remove the PV from the equation, we get VaR as a %
- $VaR =- 1.645 * 0.0823 = 13.54\%$

As the VaR is not sub additive We must use other method like delta normal VaR to calculate the portfolio VaR. And the result is shown as:

	A	B	C	All
Delta Normal ($)	5760.2	4494.6	3786.6	13577.1

This method has two assumptions: 1. Normality returns. 2. linear assumption of portfolio value with returns.

Problem 3.2

Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

The second method is Normal Monte Carlo VaR. Returns are assumed to be multivariate normal in this method. But we want to break both assumptions of delta normal VaR. So, the Historical VaR method is choose. This method using observed past returns mimic the distribution. Historical VaR is a non-parametric statistic – we make no distributional assumptions on returns. And the result is shown as:

	A	B	C	All
Delta Normal ($)	5760.2	4494.6	3786.6	13577.1
Historical VaR ($)	9005.1	7273.8	5773.5	21103.4

We can see that the Historical VaR method has larger VaR, as this method has no assumptions and use the real historical data, there exist more “extreme” data as the return is usually fat-tailed which indicate larger VaR. As the data we used has relative long time, the historical method is preferred.