

## Problem 1 – Calculate the Value of the Options

Assume you a call and a put option with the following

Current Stock Price \$165

Current Date 03/03/2023

Options Expiration Date 03/17/2023

Risk Free Rate of 4.25%

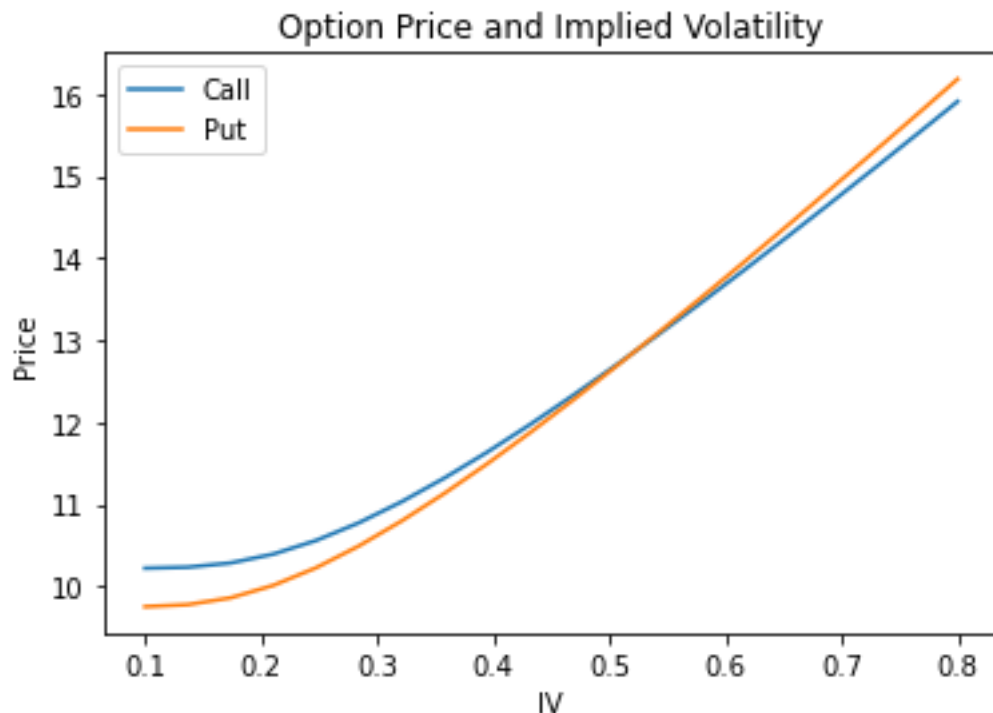
Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days).

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put.

Discuss these graphs. How does the supply and demand affect the implied volatility?

We use the Black Scholes formula to solve this problem. As there is no strike price in the question, I will assume the strike price of the call option is 155 and 175 for the put option. The result is shown as following:



From the graph we can know that:

1. The IV and the price of call and put option are positive related.
2. In the money options value more than out of money options (based on same structure of options).

Supply and demand affect price. Beside the basic data of the market like price, the supply and demand decide the price of the options from the perspective of economy. IV is the volatility calculated from the option price and other data. When then demand is larger than the supply, price will increase and the IV will increase with the price and vice versa.

Problem 2 – Calculate the Implied Volatility of the Options for AAPL

Use the options found in AAPL\_Options.csv

Current AAPL price is 151.03

Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

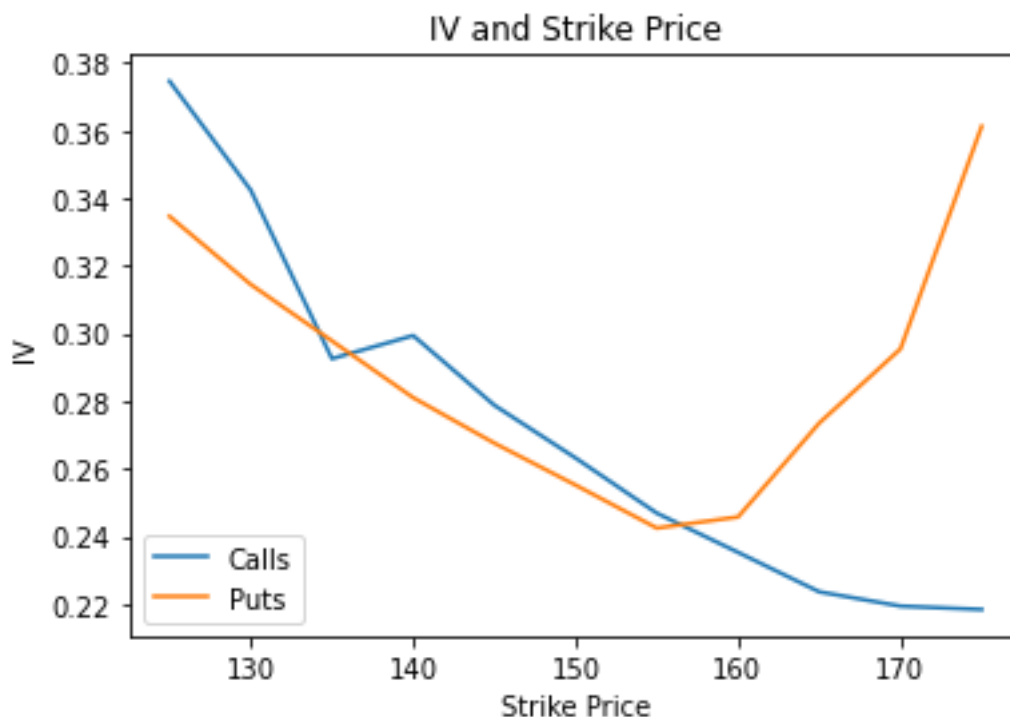
Calculate the implied volatility for each option.

Plot the implied volatility vs the strike price for Puts and Calls.

Discuss the shape of these graphs. What market dynamics could make these graphs?

There are bonus points available on this question based on your discussion. Take some time to research if needed.

The IV is calculated from price and all the other data given. And the plot is shown as following:



What strange in the graph is that volatility smile is not shown for the call options.

Volatility smile is used to describe the phenomenon of implied volatility of options with the same expiration but different strike prices, forming a U-shaped curve. It is called a "smile" because of its visual appearance.

In a normal market, the implied volatility of options with higher strike prices tends to be lower than options with lower strike prices, assuming they have the same expiration date. However, in some cases, the implied volatility for the at-the-money options may be lower than for the out-of-the-money options or in-the-money options. This creates the U-shaped curve that is known as a volatility smile.

The volatility smile indicates that the market expects greater variability in the underlying asset's price in the near future, leading to higher implied volatility for some option contracts.

It is possible for a stock to have a volatility smile for puts options but not for call options. This can occur when there are differences in market participants' expectations of future price movements for the underlying stock for different strike prices.

One possible explanation is that market participants are more concerned about downside risks, which leads them to demand more protection for out-of-the-money puts than for out-of-the-money calls. This results in higher implied volatility for put options with lower strike prices, leading to a volatility smile.

On the other hand, the implied volatility for call options may not show the same pattern because market participants may have different expectations for the stock's potential upside. They may be less concerned about upside risks and therefore do not demand as much protection for out-of-the-money calls as they do for out-of-the-money puts.

Another possible explanation is that there may be differences in the supply and demand dynamics of the options market for calls and puts, leading to different shapes of the implied volatility curve. This can occur when there is an imbalance in the number of market participants trading calls versus puts or when there are differences in the trading strategies employed by market participants for calls and puts.

Problem 3 – Calculate the Value of the Options for AAPL  
Use the portfolios found in problem3.csv

Current AAPL price is 151.03

Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above).

Calculate Mean, VaR and ES. Discuss.

Hints:

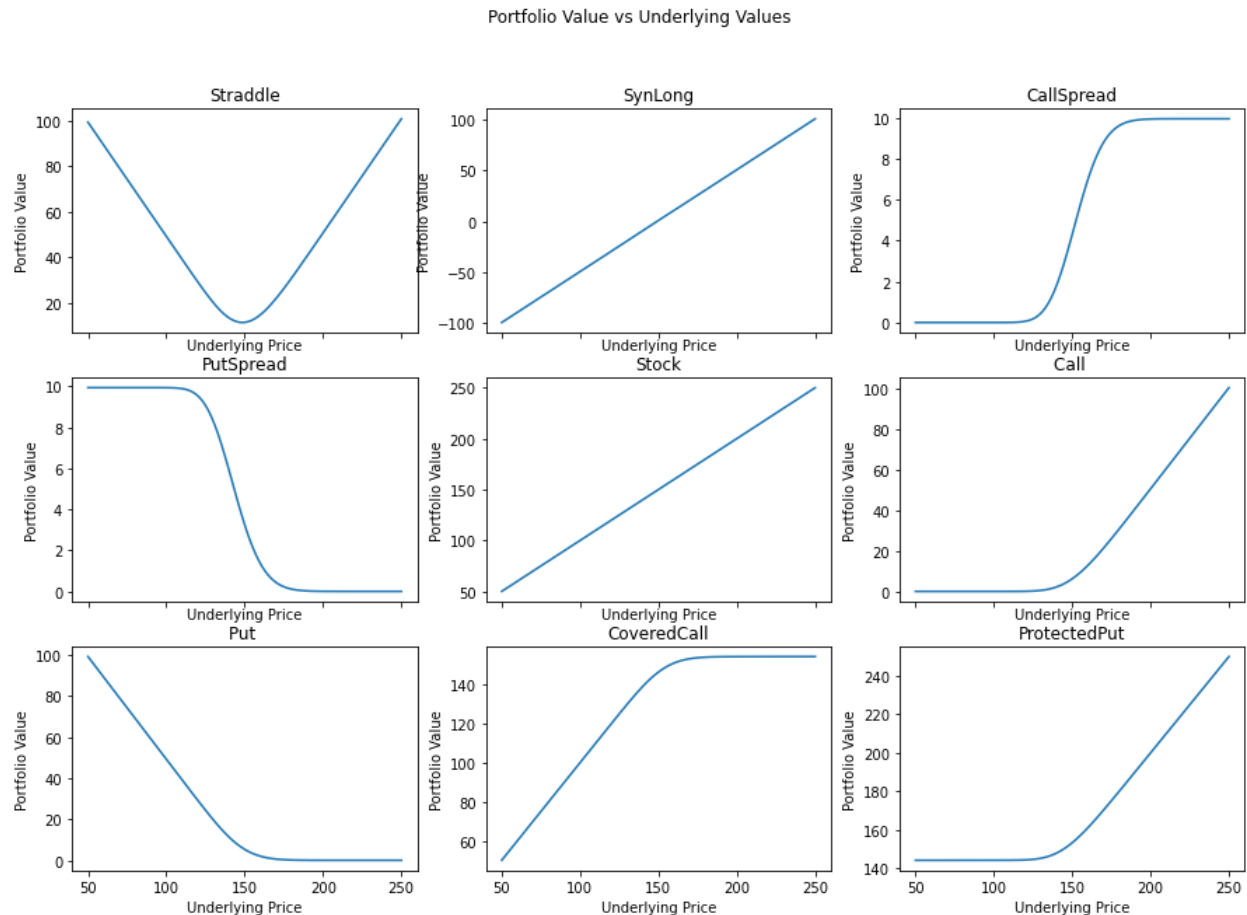
you will need to calculate the implied volatility – might not be the same as #2

you need to take into account the change in dates for option valuations.

You are simulating forward in time and options valuations are a function of time

Calculate the PL from the current portfolio value using Current Date

The graph is shown as following:



By Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price, we have:

	Mean	VaR	ES
Straddle	1.326614	1.380597	1.387930
SynLong	-0.424802	16.414656	20.073559
CallSpread	-0.187994	3.910205	4.199130
PutSpread	0.359855	2.553987	2.748505
Stock	-0.223259	16.170964	19.807763
Call	0.450906	6.060526	6.382465
Put	0.875708	4.265698	4.520131
CoveredCall	-0.844624	12.346409	15.870225
ProtectedPut	0.495880	8.112426	8.718021

The strategies here can be divided into four kinds:

1. Stock and SynLong: "Synthetic Long" is a trading strategy that involves creating a position that behaves similarly to owning a long call option on an underlying asset, but with a different combination of other options and/or the underlying asset.  
To create a synthetic long position, an investor would typically combine a long position in a stock or futures contract with a long position in a put option on the same underlying asset. This combination effectively creates a position that mimics the behavior of owning a long call option on the underlying asset, because the put option provides downside protection (similar to the long call) while the stock or futures position provides upside potential.  
By constructing a synthetic long position, an investor may be able to achieve the same exposure and potential profit as owning a long call option, while potentially reducing the cost of the position or adjusting the risk profile. However, it is important to note that synthetic long positions may also have different risks and costs compared to owning the actual long call option, depending on the specific options and underlying assets used in the strategy.  
These two have the two largest VaR and ES.
2. Call, Put, CoveredCall, ProtectPut. These four are quite similar as call and put options have put call parity. A covered call is a strategy that involves holding a long position in an asset and writing (selling) call options on that same asset. A protective put, also known as a married put, is a strategy that involves buying a put option on an asset while also holding a long position in that same asset. These two are not naked as an asset is held. CoveredCall and ProtectPut typically have larger Value-at-Risk (VaR) than buying a Call or Put option because they involve holding a long position in the underlying asset in addition to the option position.

In a CoveredCall strategy, the investor holds a long position in the underlying asset and sells call options on that same asset. This means that if the price of the underlying asset falls, the value of the long position will also decrease, which can lead to larger losses than just holding the call option. The VaR of a CoveredCall strategy will therefore be higher than that of a call option alone.

Similarly, in a ProtectPut strategy, the investor holds a long position in the underlying asset and buys a put option on that same asset. While the put option can protect against losses if the price of the underlying asset falls, the cost of the put option (the premium) will be an additional expense that adds to the overall risk of the strategy. This means that the VaR of a ProtectPut strategy will also be higher than that of a put option alone.

In contrast, buying a Call or Put option alone involves only the premium paid for the option, which is the maximum loss that can be incurred. This means that the VaR of a simple option position will typically be lower than that of a strategy that involves holding a long position in the underlying asset.

3. Callspread and putspread: A call spread involves buying a call option on an asset with a certain strike price, while also simultaneously selling a call option on the same asset with a higher strike price. This strategy is used when the investor expects the underlying asset to increase in price, but wants to limit the potential downside risk. The maximum profit that can be earned with this strategy is the difference between the two strike prices, minus the cost of the options, while the maximum loss is limited to the premium paid for the options. A put spread involves buying a put option on an asset with a certain strike price, while also simultaneously selling a put option on the same asset with a lower strike price. This strategy is used when the investor expects the underlying asset to decrease in price, but wants to limit the potential downside risk. The maximum profit that can be earned with this strategy is the difference between the two strike prices, minus the cost of the options, while the maximum loss is limited to the premium paid for the options. So these two strategies have small VaR and ES.
4. Straddle: Straddle is an options trading strategy that involves buying both a call option and a put option on the same underlying asset, with the same strike price and expiration date. This strategy has the smallest VaR and ES.

The goal of a straddle is to profit from significant price movements in the underlying asset, regardless of whether the price goes up or down. If the asset price increases significantly, the call option will be profitable, while the put option will expire worthless. On the other hand, if the asset price decreases significantly, the put option will be profitable, while the call option will expire worthless.