Theory and Application of the Hybrid Rocket Analysis Program (HRAP)

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This document outlines the theory behind and use of the Hybrid Rocket Analysis Program (HRAP) developed by Robert Nickel for use by the University of Tennessee Rocket Engineering Team. HRAP is a versatile tool utilizing a thermodynamic equilibrium model for simulation of self-pressurizing hybrid rocket motors, especially those powered with Nitrous Oxide stored as a saturated liquid-vapor mixture. In its current state this program can model an adiabatic oxidizer tank and combustion chamber and an isentropic nozzle, future iterations will account for subsonic flow and flow separation to better model thrust, add an option to export a .eng or .rse engine file for use in OpenRocket or RockSim, and will allow the use of other self-pressurizing oxidizers such as Nytrox.

Nomenclature

A	= area of injector orifice
A_e	= nozzle exit area
a_e	= speed of sound at nozzle exit plane
$A*$	= nozzle throat area
CEA	= NASA's Chemical Equilibrium Analysis program
C_d	= discharge coefficient
$C_{d_{nozzle}}$	= nozzle discharge coefficient
Cg	= center of gravity
Cp	= specific heat capacity of liquid oxidizer
C*	= characteristic velocity
dP	= injector pressure drop
dQ	= change in heat energy
dT	= change in oxidizer temperature
dV	= change in combustion chamber volume
F_{thrust}	= thrust force
G	= oxidizer mass flux
γ	= specific heat ratio
grainID	= propellant grain core diameter
grainLength	= propellant grain length
HRAP	= Hybrid Rocket Analysis Program
h_v	= specific enthalpy of vaporization
K	= injector loss coefficient
$\dot{m}_{fuel}$	= oxidizer mass flow rate
$\dot{m}_{gain}$	= combustion chamber mass gain rate
$\dot{m}_{gen}$	= combustion chamber mass generation rate
$\dot{m}_{nozzle}$	= mass flow rate out of nozzle
$\dot{m}_{ox}$	= oxidizer mass flow rate
m_{gas}	= combustion product mass
mliquid	= mass of liquid oxidizer at time, t

mLiquid_new	= mass of liquid oxidizer when tank is in equilibrium
mLiquid_old	= mass of liquid oxidizer prior to vaporization
$m_{oxidizer}$	= mass of oxidizer in tank at time, t
m_v	= mass of liquid vaporized
m_{vapor}	= mass of oxidizer vapor at time, t
Me	= exit Mach number
N	= number of injectors
n	= number of moles of substance
η_{comb}	= combustion or C* efficiency
η_{thrust}	= nozzle correction factor
O/F	= oxidizer to fuel ratio
$P_{ambient}$	= ambient pressure
P_c	= combustion chamber pressure
P_e	= exit pressure
R	= universal gas constant
$\dot{r}$	= fuel regression rate
ρ	= fluid density
ρ_{fuel}	= fuel density
ρ_{liquid}	= liquid oxidizer density
ρ_{vapor}	= oxidizer vapor density
RPA	= Rocket Propulsion Analysis program
T	= temperature
T_c	= combustion chamber temperature
T_e	= exit temperature
V	= combustion chamber volume
V_{tank}	= oxidizer tank volume
Ve	= exhaust velocity
Z	= compressibility factor

I. Introduction

This document discusses the theory behind the Hybrid Rocket Analysis Program, a versatile tool for modeling self-pressurizing hybrid rocket motors. The program utilizes a thermodynamic equilibrium model for simulation of the motor's oxidizer tank and in its current state, a look up table of thermodynamic properties of combustion products for use in an internal ballistics model, generated using the Rocket Propulsion Analysis program (RPA V. 1.2.9). This model can be applied to a wide variety of rocket motors, though relies on some empirical data such as experimentally determined injector discharge coefficients or propellant ballistic coefficients for determining fuel regression rates. The program can automatically generate plots and tables of important data such as time-varying thrust, chamber pressure and oxidizer tank pressures, propellant grain geometry throughout the burn, mass flow rates and shifting oxidizer to fuel ratios, specific impulse, and characteristic velocity, with the ability to add custom functionality with the use of additional scripting.

Self-pressurizing hybrid rocket motors are advantageous in that they do not require complex plumbing or valves to operate; however, modeling of such a system is non-trivial and requires numerical models to determine performance characteristics of a given system. For HRAP, simulation of the rocket motor is performed using six different numerical models which will be outlined in detail in later sections of this document. The six models used for this simulation, in simulation order, are:

1) Oxidizer Tank Model
2) Fuel Regression Model
3) Combustion Model
4) Chamber Pressure Model
5) Nozzle Model

6) Mass Properties Model

II. Oxidizer Tank Model

The oxidizer tank model used by HRAP assumes the tank is constantly in thermodynamic equilibrium, with the oxidizer remaining a saturated liquid-vapor mixture for the entirety of the motor's operation. To begin to model such a complex process requires an understanding of the thermophysical properties of the oxidizer being used, most often Nitrous Oxide.

A saturated liquid-vapor mixture is one in which both the liquid phase and vapor phase of a given substance can exist in thermodynamic equilibrium, meaning if left alone the amount of liquid or vapor in the container will not change with time. As a saturated mixture, knowledge of either the temperature or pressure of the oxidizer tank is enough to fully describe the physical properties of the saturated liquid or vapor. At any given temperature liquid will evaporate until enough vapor is produced to equal the substance's vapor pressure, at which point the mix is in thermodynamic equilibrium. If the substance is heated, more vapor will be produced and the vapor pressure will increase and conversely if the substance is cooled, vapor will condense and the vapor pressure will decrease. For simplicity heat transfer through the oxidizer tank wall is neglected; this is a fair assumption for most applications due to the relatively short burn times associated with self-pressurizing hybrid rocket motors. Plots of thermophysical properties of Nitrous Oxide are included in Appendix 1 for reference.

The idealized operation of this oxidizer tank is outlined in Figure 1. This complex process is broken into three simple states:

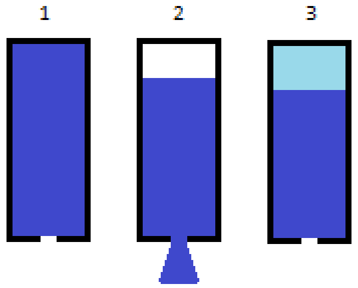


Figure 1 Oxidizer tank emptying process.

1) The initial temperature of the tank is used to determine the liquid and vapor density, specific enthalpy of vaporization, specific heat of liquid, and vapor pressure.
2) The vapor pressure and liquid density are used to determine the mass flow rate of oxidizer through the motor's injector and subsequently the mass of oxidizer discharged during each iteration.
3) The discharged oxidizer leaves a void which must be filled with vapor to re-pressurize the oxidizer tank. This process carries heat away from the liquid and causes a decrease in temperature which can be calculated using the specific enthalpy of vaporization and the specific heat capacity of the liquid oxidizer. This new temperature is used in step one of the next iteration.

Curve fits for Nitrous Oxide thermophysical properties were obtained from the Engineering Sciences Data Unit (ESDU) based out of the United Kingdom. These curve fits allow calculation of these parameters using only the temperature of the saturated mixture.

A. Determination of Initial Tank Properties

The first step in this oxidizer tank model is determination of oxidizer properties, which is done by evaluating curve fits at the tank's initial temperature in each time step. The oxidizer tank volume and oxidizer mass must also be known at the start of each iteration, as well as the mass of oxidizer which is in a liquid state.

B. Oxidizer Discharge

The second step in this oxidizer tank model is determination of the mass of oxidizer discharged during the timestep, to determine this, the injector's discharge coefficient and cross-sectional area must be known. For this application it is less computationally demanding to simply define a term, dLoss, as a constant which is specific to a given injector combination. The variable dLoss is a constant which accounts for the orifice size and discharge characteristics and simplifies calculation of mass flow rate, this value is calculated using Equation 1,

$$dLoss = \frac{K}{(NA)^2} \tag{1}$$

Where K is the orifice's loss coefficient, given as $K = \frac{1}{C_d^2}$, N is the number of injector orifices, and A is the cross-sectional area of each injector orifice. When using this approach, the mass flow rate of oxidizer out of the injector is calculated using equation 2,

$$\dot{m}_{ox} = \sqrt{\frac{2\rho\Delta P}{dLoss}} \tag{2}$$

where ρ is the liquid density at the current tank temperature, ΔP is the pressure drop between the oxidizer tank and combustion chamber (the difference between the oxidizer vapor pressure at the current tank temperature and the combustion chamber pressure at the current timestep). The mass flow rate is numerically integrated with respect to time to find the mass discharged during each timestep.

C. Re-pressurization of Oxidizer Tank

Once the mass of oxidizer discharged is determined, the mass of liquid in the tank is defined as the variable mLiquid_old. This value represents the mass of oxidizer in the case where no evaporation takes place, only a discharge of a finite mass of oxidizer. In this case both the total oxidizer mass and the mass of liquid oxidizer have decreased by $\dot{m}_{ox}dt$. Because there is a void left by this discharged mass of oxidizer the tank pressure decreases, and because there is initially no change in temperature and the oxidizer is stored as a saturated mixture, the liquid will begin to boil and heat will be carried away until the tank returns to equilibrium. To find the mass of liquid in this new equilibrium state, the current liquid and vapor density are used to determine the respective volume of liquid and vapor in the tank. Equations 3 and 4 state that the sum of the volume of liquid and vapor in the tank must always equal the volume of the oxidizer tank, and that the sum of the masses of liquid and vapor must equal the total mass of oxidizer remaining in the oxidizer tank.

$$V_{tank} = \frac{m_{liquid}}{\rho_{liquid}} + \frac{m_{vapor}}{\rho_{vapor}} \tag{3}$$

$$m_{oxidizer} = m_{liquid} + m_{vapor} \tag{4}$$

Equations 3 and 4 can be rearranged to get Equation 5, which gives the mass of liquid oxidizer remaining in the tank when the tank returns to thermodynamic equilibrium. This value is set as mLiquid_new in the simulation.

$$m_{liquid} = \frac{\left(V_{tank} - \frac{m_{oxidizer}}{\rho_{vapor}}\right)}{\left(\frac{1}{\rho_{liquid}} - \frac{1}{\rho_{vapor}}\right)} = mLiquid\_new \tag{5}$$

The difference between mLiquid_old and mLiquid_new represents the mass of oxidizer which must vaporize in order to re-pressurize the oxidizer tank.

Once the mass of oxidizer vaporized in the timestep is determined, the heat removed from the liquid can be determined using Equation 6:

$$dQ = h_v m_v \tag{6}$$

where dQ is the heat removed from the system, h_v is the specific heat of vaporization at the current temperature, and m_v is the mass of oxidizer vaporized during this timestep. The corresponding change in temperature is given by Equation 7:

$$dT = \frac{dQ}{mLiquid\_new \times C_p} \tag{7}$$

The change in temperature, dT, is numerically integrated to find the initial temperature for the next timestep and this process repeated.

D. Numerical Error at End of Liquid Phase

As the tank nears depletion, there is a point where the value mLiquid_new may exceed the value of mLiquid_old, indicating liquid is accumulating in the tank rather than continuing to vaporize. Because this is not physically accurate, the initial liquid phase model is ended and a secondary model using a constant value of dT is used. In this model, dT is set to the average value from the first phase, mLiquid_old is set to zero, and mLiquid_new is calculated using

Equation 5. Because dT is assumed constant for the rest of the liquid phase, there is no need to calculate dQ or the specific enthalpy of vaporization, and the model simply discharges oxidizer for the duration of the timestep, decreases the tank temperature by dT, and re-calculates the mass of liquid remaining. Once mLiquid_new is less than or equal to zero the liquid phase ends and transitions to the vapor phase.

E. Vapor Phase Model

Due to the extreme range of oxidizer tank pressures and temperatures over the course of operation, the oxidizer vapor cannot be approximated as an ideal gas. Instead, a real gas model must be utilized and compressibility factor must be calculated. Real gasses are modeled by Equation 8, where Z is the vapor's compressibility factor at a given temperature, T, and pressure, P. V represents the volume of the oxidizer tank, R is the universal gas constant, and n is the number of moles of vapor within the oxidizer tank. Figure 2 shows compressibility factor as a function of critical pressure and critical temperature for Nitrous Oxide.

$$PV = ZnRT \tag{8}$$

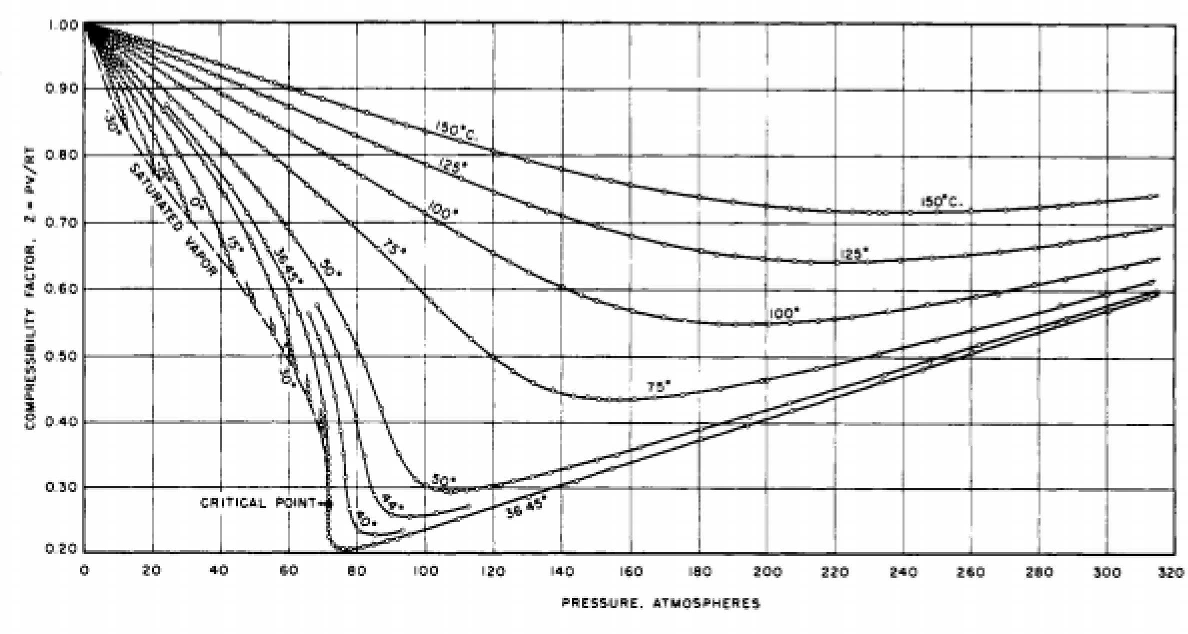


Figure 2 Compressibility of Nitrous Oxide as a function of pressure and temperature. Couch [1]

Because the oxidizer remains saturated for the duration of the burn, the compressibility factor of the vapor is simply a function of temperature and the curve of this compressibility factor equation is represented as a dashed line in Figure 2. The equation for compressibility factor along the saturation line as a function of temperature is derived using the ESDU curve fits for vapor pressure and vapor density as a function of temperature.

$$Z(T) = \frac{P(T)}{\rho(T)RT} \tag{8}$$

As with the liquid phase, the vapor phase model begins with discharging a finite mass of oxidizer. The remaining oxidizer mass and volume of the oxidizer tank are used to find the density of the oxidizer and the temperature is used to obtain an initial guess for compressibility factor using the curve fit, Z(T). The ratio of initial tank temperature to temperature after discharge of some of the vapor is given in Equation 9.

$$\frac{T_{i+1}}{T_i} = \left(\frac{Z_{i+1}m_{i+1}}{Z_i m_i}\right)^{\gamma-1} \tag{9}$$

In Equation 9, Z_{i+1} represents the current guess of compressibility factor, m_{i+1} represents the mass of oxidizer remaining after discharge, Z_i represents the initial compressibility factor at the tank's starting temperature, m_i is the mass of oxidizer prior to discharge, and γ is the ratio of specific heats for the oxidizer (1.3 for Nitrous Oxide). The temperature ratio is used to determine the tank temperature after discharge and this new temperature is used to re-calculate the compressibility factor. The average of the initial guess of compressibility factor and the value calculated using Equation 9 is used as the new guess, and the process is repeated until the error between the compressibility factor guess and calculated value is sufficiently small (on the order of 10^{-6}). With the compressibility factor and temperature determined for the oxidizer tank, the next iteration can be run. This is repeated until the simulation stop criteria is met (full consumption of the propellant grain, oxidizer tank depletion, combustion chamber or tank pressure is sufficiently low, or the maximum oxidizer run time is reached).

II. Fuel Regression Model

HRAP allows the use of one of two fuel regression models: either a constant O/F ratio model or a shifting O/F ratio model. The constant O/F regression model is useful in cases where ballistic coefficients for the propellant are unknown and provide a baseline for simulation of a hybrid rocket motor. Once sufficient testing is conducted to determine ballistic coefficients, the shifting O/F regression model can be used for higher fidelity simulation (particularly regarding the motor's mass properties).

A. Constant O/F Regression Model

For the constant O/F regression model the user specifies a given oxidizer to fuel ratio for the rocket motor, usually based on the ideal oxidizer to fuel ratio calculated in RPA or a similar chemical equilibrium tool. Based on the specified propellant grain geometry, the regression rate required at any given time to maintain a constant oxidizer to fuel ratio is calculated. The geometry of the propellant grain core is assumed to be cylindrical and it is assumed that regression on the forward and aft faces of the grain is negligible (propellant grain length is constant). Regression rate during each iteration is calculated using Equation 10.

$$\dot{r}(i) = \frac{\dot{m}_{ox}}{\pi \times grainID(i) \times grainLength \times \rho_{fuel} \times OF} \tag{10}$$

This regression rate value $\dot{r}$ is integrated with respect to time to determine the propellant grain inside diameter at the next iteration, and the mass flow rate of propellant is given as $\dot{m}_{fuel} = \frac{\dot{m}_{ox}}{OF}$. As with the regression rate and propellant grain inside diameter, the fuel mass flow rate is integrated to determine the mass of fuel remaining.

B. Shifting O/F Regression Model

For actual hybrid rocket motors, it is unlikely that a constant oxidizer to fuel ratio can be maintained without complicated propellant grain geometries or other novel features, so ballistic coefficients are utilized to determine the regression rate. As it turns out, fuel regression rates can often be adequately described as an exponential function of oxidizer mass flux, G, and propellant grain length, L. This equation is a variant of Saint Robert's law used for modeling burn rates in solid rocket motors.

$$\dot{r}(i) = aG^nL^m \tag{11}$$

In Equation 11, the values a, n and m are experimentally determined constants which depend on the propellant being used, and the oxidizer injection system. Injection systems which induce a vortex in the oxidizer flow substantially increase regression rates compared to simple axial injection as with impinging doublets or showerhead injectors. The mass flow rate of fuel is calculated using Equation 12, and the shifting oxidizer to fuel ratio in Equation 13.

$$\dot{m}_{fuel} = \pi \times grainID(i) \times grainLength \times \rho_{fuel} \times \dot{r}(i) \tag{12}$$

$$\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}} \tag{13}$$

Like the constant O/F regression model, the fuel mass flow rate and regression rate are integrated with respect to time to give the remaining fuel mass and fuel grain inside diameter.

III. Combustion Model

Due to the difficulty of calculating thermodynamic properties of combustion products in real-time, HRAP utilizes lookup tables of specific heat ratio, gas molecular mass, characteristic velocity, and adiabatic flame temperature as a function of O/F ratio and chamber pressure calculated using a chemical equilibrium tool such as RPA, CEA, or ProPep. As the simulation runs, these key values are determined using bi-linear interpolation in each of the look-up tables evaluated at the current chamber pressure and oxidizer to fuel ratio. At motor startup the chamber pressure is usually set to ambient pressure, though it's possible to model an initial starting pressure like what would be seen when using a large pre-heater grain. The specific heat ratio, gas molecular mass, characteristic velocity, and adiabatic flame temperature are used in calculation of chamber pressure and thrust for the next iteration. Due to imperfect combustion of fuel and oxidizer, a corrective term is added to characteristic velocity to simulate the effects of incomplete combustion. This corrective term is known as C^* efficiency, and typically ranges from 0.6 to 0.95. A typical U/C valve hybrid as is used by amateur rocketeers may have a C* efficiency between 67% and 75%, but more advanced injector designs and the addition of a post-combustion chamber can increase this value to 85% or more. Advanced mixing devices such as mixing plates can further increase combustion efficiency by forcing oxidizer rich gasses in the core of the propellant grain to mix with fuel rich gasses along the walls of the propellant grain, in some cases reaching C^* efficiency values on the order of 95%.

IV. Chamber Pressure Model

The combustion chamber of a rocket can be thought of as a sort of gas capacitor; as oxidizer enters the combustion chamber, fuel is burned and produces hot gasses which accumulate in the combustion chamber before exiting the rocket's nozzle. If the rate of gas production inside the combustion chamber exceeds the rate at which gas exits the nozzle, the chamber pressure will rise; conversely, if the rate of gas production is lower than the mass flow rate out of the nozzle, the chamber pressure will fall. In steady-state operation the mass flow rate out of the nozzle is dependent on the characteristic velocity, C*, of the exhaust gasses, where C* is defined by the following equations:

$$C^* = \frac{P_c A^*}{\dot{m}_{nozzle}} = \eta_{comb} \sqrt{\frac{RT_c}{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \qquad (14)$$

To account for transients, a more complicated differential equation must be derived. First a mass balance is performed with Equation 15:

$$\dot{m}_{gen} - \dot{m}_{nozzle} = \dot{m}_{gain} \qquad (15)$$

The mass generation rate $\dot{m}_{gen}$ is equal to the sum of the oxidizer mass flow rate and fuel mass flow rate. The mass flow rate out of the nozzle is determined by solving Equation 14 in terms of $\dot{m}_{nozzle}$ as is shown in Equation 16.

$$\dot{m}_{nozzle} = \frac{P_c A^*}{C^*} \qquad (16)$$

This value for mass flow rate out of the nozzle is a best-case scenario, not taking into account phenomena such as vena contracta which result in reduced flow rates from a given orifice, thus a discharge coefficient must also be defined for the nozzle. Typical values range from 0.9 for a nozzle with a conical convergent and divergent section, to 0.95 with a radiused convergent section and conical divergent, all the way up to 0.99 for a proper bell nozzle. The nozzle mass flow rate equation accounting for this discharge coefficient is given in Equation 17.

$$\dot{m}_{nozzle} = C_{d_{nozzle}} \frac{P_c A^*}{C^*} \qquad (16)$$

The mass of combustion products in the combustion chamber is found by integrating $\dot{m}_{gain}$ with respect to time, using the initial chamber volume and ambient air density to provide the initial mass of combustion gasses. Because the propellant grain takes up a large portion of the combustion chamber volume, the chamber volume is time-dependent and must be re-calculated after determining new propellant grain dimensions. The differential equation describing chamber pressure as a function of time is given by Equation of 18, where V and $\dot{V}$ are the instantaneous volume and rate of change of volume of the combustion chamber, m_{gas} and $\dot{m}_{gain}$ are the current mass and rate of change of mas of combustion products, and P(t) is the current chamber pressure.

$$\frac{\Delta P}{\Delta t} = P(t)\left(\frac{\dot{m}_{gain}}{m_{gas}} - \frac{\dot{V}}{V}\right) \qquad (18)$$

This equation is numerically integrated to find the chamber pressure at a time t + dt.

V. Nozzle Model

To simplify the analysis, the current nozzle model used assumes the nozzle is choked at all times, and that there is no flow separation in the nozzle. Future iterations will attempt to more accurately model these phenomena, but they are only relevant at motor startup and shut-down. The area ratio of a nozzle is defined as the nozzle exit area divided by the nozzle throat area, and is defined by Equation 19.

$$\frac{A_e}{A^*} = \left(\frac{\gamma + 1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e} \tag{19}$$

Because the area ratio is defined by nozzle geometry, it is set as a constant. The value of γ calculated in the combustion model for each iteration is used, and a root finding algorithm is used to find the exit Mach number, M_e. Once the exit Mach number is determined, the exhaust velocity is calculated. Mach number is defined as the ratio of velocity to the local speed of sound. The local speed of sound at the nozzle exit can be found with Equation 20, where R is the gas molecular mass calculated in the combustion model and T_e is the temperature of exhaust products at the nozzle exit.

$$a_e = \sqrt{\gamma R T_e} \tag{20}$$

The temperature at the nozzle exit is not equal to the temperature of exhaust gasses inside of the combustion chamber, but can be found by assuming isentropic flow. Using the isentropic relation in Equation 21, the exhaust temperature, T_e, can be calculated.

$$T_e = T_c\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1} \tag{21}$$

Finally the exhaust velocity is calculated as $V_e = M_e a_e$.

Thrust is comprised of both a momentum component due to the mass flow rate and velocity of the exhaust and also a pressure component due to the residual pressure at the nozzle exit plane. The generalized thrust equation used by HRAP considers both this momentum thrust and corresponding pressure thrust and is shown in Equation 22.

$$F_{thrust} = \dot{m}_{nozzle} V_e + A_e(P_e - P_{ambient}) \tag{22}$$

The nozzle exit pressure must also be calculated using isentropic flow relations, and is given by Equation 23.

$$P_e = P_c\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{23}$$

Finally, a thrust correction factor must be added to Equation 22 to account for friction and divergence losses. This is given by Equation 24.

$$F_{thrust_{actual}} = F_{thrust_{ideal}} \eta_{thrust} \tag{24}$$

VI. Mass Properties Model

The last model created for HRAP was the mass properties model, which determines the total mass of the assembled rocket motor and the center of mass as measured from a reference point at the forward end of the rocket motor. This mass properties model is entirely optional as it has no real benefit when simulating a motor designed only for static test fires, but is absolutely essential for accurate simulation of a flight vehicle. The oxidizer usually accounts for a large portion of the motor's total mass, and as such, dominates the center of mass of the motor. As the motor burns, the oxidizer mass drops far faster than the fuel mass, causing the center of mass of the motor to shift towards the aft end of the rocket motor. There are three primary components used in the calculation of a flight motor's mass properties: the rigid or structural component which includes everything that is not consumed, the oxidizer, and the fuel.

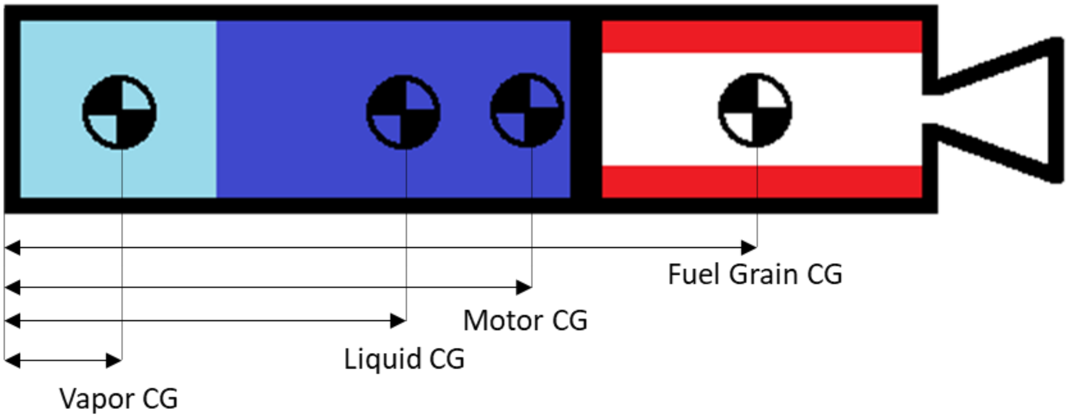


Figure 3 Diagram of individual mass components for mass property calculations.

A. Structural Mass Component

The structural mass component consists of the rocket motor casing, oxidizer tank, closures, nozzle, motor insulator, injectors, valves, and fittings required for operation of the motor. When mass properties are to be calculated, the user will define the total structure mass and the center of mass position for the empty rocket motor.

B. Oxidizer Mass Component

The oxidizer mass component considers the mass of both liquid and vapor in the oxidizer tank. The oxidizer tank geometry is approximated as a cylinder with a diameter equal to the average inside diameter of the oxidizer tank. The height of the liquid column is found by dividing the liquid mass, mLiquid_new(i), by the current liquid density and the cross sectional area of the oxidizer tank. The liquid column starting location is defined by the user and is measured from the motor's structural reference point at the forward end of the motor. Above this liquid column is a column of vapor with a height similarly determined by the current mass of vapor, the cross-sectional area of the tank, and the current density of the vapor.

C. Fuel Mass Component

The fuel mass component relies on the mass of propellant calculated during the fuel regression model, and the position of the propellant grain base as measured from the motor's structural reference.

D. Mass Property Calculation

The total mass of the rocket motor is assumed to be equal to the sum of the structural mass component, the oxidizer mass component and the fuel mass component. The center of mass of the vehicle is then calculated using a moment balance about the structural reference point, as is shown in Equation 25.

$$X_{cg} = \frac{M_{structure} \times CG_{structure} + M_{liquid} \times \left(X_{Tank\_Base} - \frac{Liquid\_Column\_Height}{2}\right) + M_{vapor} \times \left(X_{Tank\_Base} - Liquid\_Column\_Height - \frac{Vapor\_Column\_Height}{2}\right) + M_{fuel} \times \left(X_{Fuel\_Base} - \frac{Grain\_Length}{2}\right))}{M_{structure} + M_{liquid} + M_{vapor} + M_{fuel}} \quad (25)$$

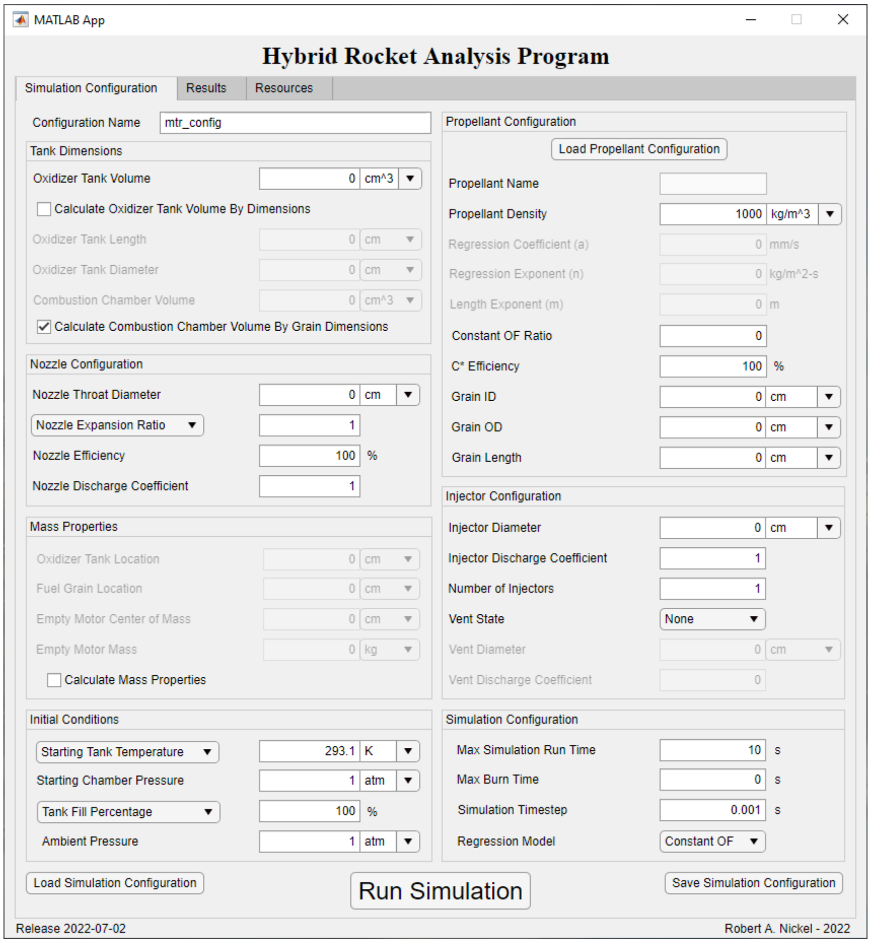
This gives the value X_{cg} in terms of distance aft of the structural reference point.

VII. Using HRAP

HRAP is built into a standalone MATLAB application, out of convenience primarily as I did not possess the ability to program a full user interface in Python or Java. The user interface has three tabs at the top: one for configuring your simulation, one for displaying and processing results, and one with a quick link to this document. There are many built in features that are particularly useful for certain circumstances, though not all are required to run a simulation. The following sections will cover in detail how to use HRAP to its full potential.

A. Getting Started

When first opening HRAP, you are presented with the Simulation Configuration tab. Until a simulation is run, you will not be able to do anything in the Results tab. You'll notice for various properties, mass properties in particular, that the fields are not able to be interacted with. This is intentional, as you only need to fill out the editable fields in order to run a basic simulation. Once all fields have been filled out and a propellant configuration imported, you can save the motor configuration. When you re-open HRAP, you can load and even modify the motor configuration if desired.



MATLAB App
Hybrid Rocket Analysis Program
Simulation Configuration
Results
Resources
Load Propellant Configuration
Load Simulation Configuration
Run Simulation
Save Simulation Configuration
Release 2022-07-02
Robert A. Nickel - 2022

Configuration Name mtr_config

Tank Dimensions
Oxidizer Tank Volume 0 cm^3
□ Calculate Oxidizer Tank Volume By Dimensions
Oxidizer Tank Length 0 cm
Oxidizer Tank Diameter 0 cm
Combustion Chamber Volume 0 cm^3
☑ Calculate Combustion Chamber Volume By Grain Dimensions

Nozzle Configuration
Nozzle Throat Diameter 0 cm
Nozzle Expansion Ratio 1
Nozzle Efficiency 100 %
Nozzle Discharge Coefficient 1

Mass Properties
Oxidizer Tank Location 0 cm
Fuel Grain Location 0 cm
Empty Motor Center of Mass 0 cm
Empty Motor Mass 0 kg
□ Calculate Mass Properties

Initial Conditions
Starting Tank Temperature 293.1 K
Starting Chamber Pressure 1 atm
Tank Fill Percentage 100 %
Ambient Pressure 1 atm

Propellant Configuration
Propellant Name
Propellant Density 1000 kg/m^3
Regression Coefficient (a) 0 mm/s
Regression Exponent (n) 0 kg/m^2-s
Length Exponent (m) 0 m
Constant OF Ratio 0
C* Efficiency 100 %
Grain ID 0 cm
Grain OD 0 cm
Grain Length 0 cm

Injector Configuration
Injector Diameter 0 cm
Injector Discharge Coefficient 1
Number of Injectors 1
Vent State None
Vent Diameter 0 cm
Vent Discharge Coefficient 0

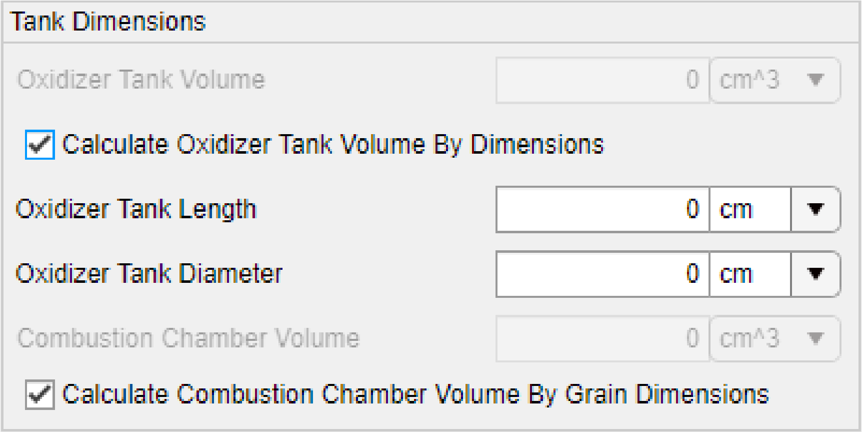
Simulation Configuration
Max Simulation Run Time 10 s
Max Burn Time 0 s
Simulation Timestep 0.001 s
Regression Model Constant OF

Figure 4 HRAP Simulation Configuration Tab

Starting off, you'll need to name your motor configuration so that it can be saved. Configuration names can be anything you want so long as there are no spaces, no slashes, and the name does not start with a number (think MATLAB filename rules).

B. Defining the Oxidizer Tank

Oxidizer tank definition can be completely satisfied by just inputting the volume of your oxidizer tank. If you are using a cylindrical tank and prefer not to directly measure the volume you can instead select "Calculate Oxidizer Tank Volume By Dimensions". Doing so will disable the Oxidizer Tank Volume input and instead use volume calculated by the length and diameter of the tank.


Tank Dimensions

Oxidizer Tank Volume 0 cm^3

☑ Calculate Oxidizer Tank Volume By Dimensions

Oxidizer Tank Length 0 cm

Oxidizer Tank Diameter 0 cm

Combustion Chamber Volume 0 cm^3

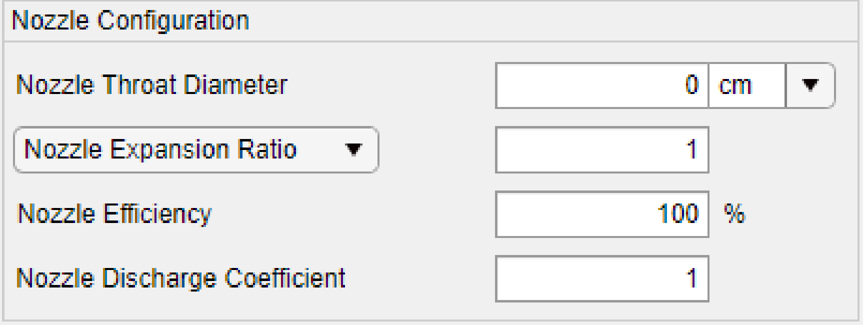
☑ Calculate Combustion Chamber Volume By Grain Dimensions

Figure 5 Tank Dimension Definition

In either case, be sure to select the appropriate unit for each. Any simulation is limited by the accuracy of inputs so between accurate measurements and using the correct units, you can avoid unnecessary error in predictions. You'll notice in addition to tank volume and dimensions, you have the option to manually override combustion chamber volume. If the "Calculate Combustion Chamber Volume By Grain Dimensions" checkbox is left checked, HRAP will assume the chamber volume is equal to 1.25x the volume of the fuel grain, including the core. If one wishes to enable this function, define chamber volume as the total volume of water it would take to fill the combustion chamber from the outlet of the injectors to the nozzle throat.

C. Defining the Nozzle

The rocket nozzle is defined using four parameters: throat diameter, efficiency, discharge coefficient, and either expansion ratio or exit diameter. Throat diameter is defined as the minimum diameter within the nozzle, exit diameter is the diameter at the nozzle exit, discharge coefficient is the ratio of actual mass flow rate to theoretical mass flow rate and can be determined either by water flow testing or CFD, and nozzle efficiency is the ratio between the actual thrust and theoretical thrust at a given pressure. The throat diameter must always be lower than the exit diameter (i.e. expansion ratio must always be greater than 1). Discharge coefficient is driven primarily by the profile of the nozzle convergent section and typically ranges from 0.9 to 0.99. Nozzle efficiency is driven primarily by the exit cone angle, which diverts a portion of the exhaust radially and reduces the effective thrust, and is normally between 90-99%.


Nozzle Configuration

Nozzle Throat Diameter 0 cm

Nozzle Expansion Ratio 1

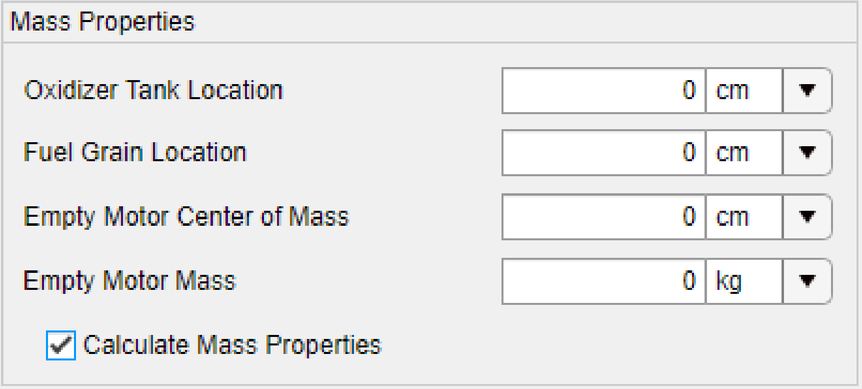
Nozzle Efficiency 100 %

Nozzle Discharge Coefficient 1

Figure 6 Nozzle Definition

D. Defining Mass Properties

Mass properties are entirely optional for a generic simulation in HRAP, and so these do not have to be defined. In order to export a .rse file for flight simulations, mass properties must be turned on. If you wish to use mass properties, check the "Calculate Mass Properties" box and fill out each of the fields. The location of the oxidizer tank is defined as the distance from the bottom of the oxidizer tank to the forward-most point on the motor; likewise, the fuel grain location is defined as the distance from the bottom of the fuel grain to the forward-most point on the motor. The empty motor center of mass can be determined by balancing the motor in full flight configuration minus fuel and oxidizer and measuring the distance to the cg; the empty motor center of mass is the fully assembled mass of the motor without fuel or oxidizer. When mass properties are turned on, the oxidizer tank diameter must be specified.



Mass Properties

Oxidizer Tank Location 0 cm ▼

Fuel Grain Location 0 cm ▼

Empty Motor Center of Mass 0 cm ▼

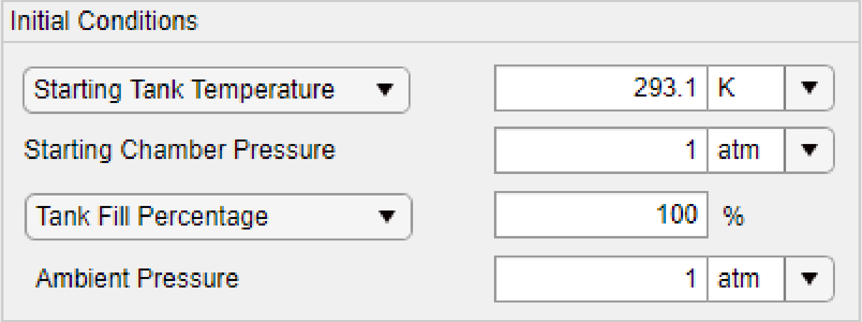
Empty Motor Mass 0 kg ▼

☑ Calculate Mass Properties

Figure 7 Mass Properties Definition

E. Defining Initial Conditions

The initial conditions are more of an advanced feature than some of the other parameters that need to be defined. For the most part you should be fine to leave this section alone, but if you know the starting tank temperature or alternatively the starting tank pressure, or the starting fill percentage or fill mass, you can override them. Starting chamber pressure mainly impacts the initial startup transient in the burn, and ambient pressure affects the pressure thrust from the nozzle.



Initial Conditions

Starting Tank Temperature ▼ 293.1 K ▼

Starting Chamber Pressure 1 atm ▼

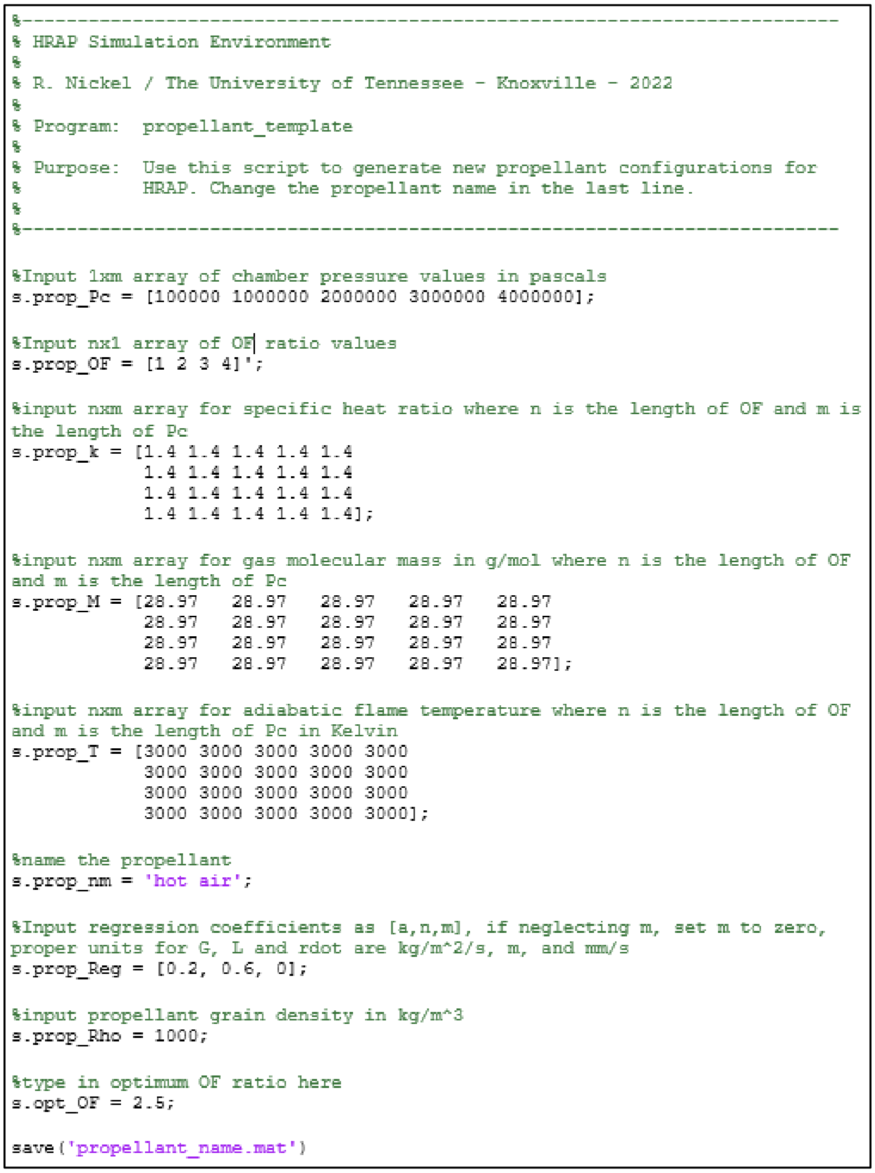
Tank Fill Percentage ▼ 100 %

Ambient Pressure 1 atm ▼

Figure 8 Defining Initial Conditions

F. Defining Propellant Configurations

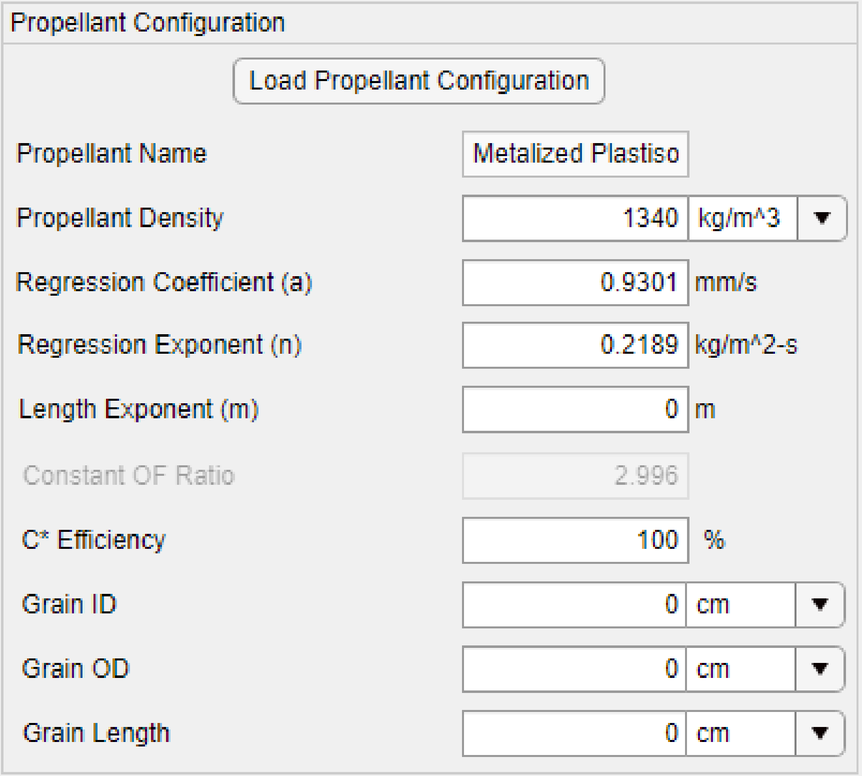
This section will likely be the most difficult to navigate, as very few propellants are pre-defined for HRAP. A script is included in the main HRAP directory in the propellant_configs folder that can be used for generating new propellant configurations either in MATLAB or its open source alternative, Octave. The propellant configuration file generates a .mat structure with 9 fields: one is an nx1 array of O/F ratio values, one is a 1xm array of chamber pressure values, there are separate nxm arrays for specific heat ratio (k), gas molecular mass (M), and adiabatic flame temperature (T) which represents each of these properties at a given combination of O/F ratio and chamber pressure (i.e. index (3,5) in s.prop_M would return the gas molecular mass at the third value of O/F ratio and fifth value of chamber pressure specified in their respective arrays). Any chemical equilibrium tool such as NASA's CEA, ProPep, or RPA can be used to obtain these values. In addition to gas properties, you must also define a propellant name, the density of your propellant, the optimum OF ratio, and the coefficients for the hybrid rocket regression law (see Eq 11). For the regression law, the array of coefficients is arranged as [a,n,m], and typically m can be assumed 0. With a new propellant configuration saved, you can open it in HRAP by clicking "Load Propellant Configuration".



```
%-------------------------------------------------------------------------
% HRAP Simulation Environment
%
% R. Nickel / The University of Tennessee - Knoxville - 2022
%
% Program:  propellant_template
%
% Purpose:  Use this script to generate new propellant configurations for
%           HRAP. Change the propellant name in the last line.
%
%-------------------------------------------------------------------------

%Input 1xm array of chamber pressure values in pascals
s.prop_Pc = [100000 1000000 2000000 3000000 4000000];

%Input nx1 array of OF ratio values
s.prop_OF = [1 2 3 4]';

%input nxm array for specific heat ratio where n is the length of OF and m is
the length of Pc
s.prop_k = [1.4 1.4 1.4 1.4 1.4
            1.4 1.4 1.4 1.4 1.4
            1.4 1.4 1.4 1.4 1.4
            1.4 1.4 1.4 1.4 1.4];

%input nxm array for gas molecular mass in g/mol where n is the length of OF
and m is the length of Pc
s.prop_M = [28.97   28.97   28.97   28.97   28.97
            28.97   28.97   28.97   28.97   28.97
            28.97   28.97   28.97   28.97   28.97
            28.97   28.97   28.97   28.97   28.97];

%input nxm array for adiabatic flame temperature where n is the length of OF
and m is the length of Pc in Kelvin
s.prop_T = [3000 3000 3000 3000 3000
            3000 3000 3000 3000 3000
            3000 3000 3000 3000 3000
            3000 3000 3000 3000 3000];

%name the propellant
s.prop_nm = 'hot air';

%Input regression coefficients as [a,n,m], if neglecting m, set m to zero,
proper units for G, L and rdot are kg/m^2/s, m, and mm/s
s.prop_Reg = [0.2, 0.6, 0];

%input propellant grain density in kg/m^3
s.prop_Rho = 1000;

%type in optimum OF ratio here
s.opt_OF = 2.5;

save('propellant_name.mat')
```

Figure 9 Propellant Configuration Template

With a propellant configuration loaded, you'll see you can still modify both regression coefficients and density. This is because these values may change with both scale and injector configuration, and because density may vary from the ideal theoretical density. If static test fires have been performed with chamber pressure measurements, HRAP results can be improved by altering C* efficiency. C* efficiency is defined as the ratio of measured characteristic velocity to ideal characteristic velocity, and significantly affects calculated chamber pressure. For the time being HRAP only models cylindrical fuel grains, but in the event a non-cylindrical port geometry is to be used you can input the hydraulic diameter of the port instead of the grain ID (hydraulic diameter being the diameter of a circle with area equal to a given port geometry).

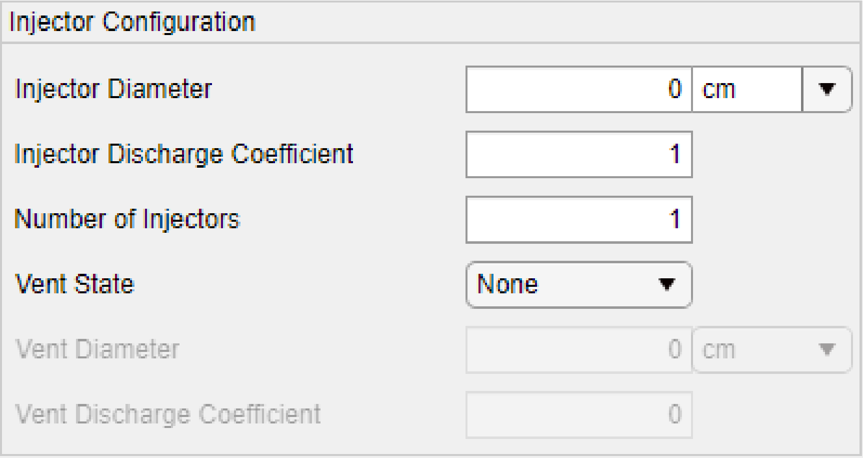


Propellant Name	Metalized Plastiso	
Propellant Density	1340	kg/m^3 ▼
Regression Coefficient (a)	0.9301	mm/s
Regression Exponent (n)	0.2189	kg/m^2-s
Length Exponent (m)	0	m
Constant OF Ratio	2.996	
C* Efficiency	100	%
Grain ID	0	cm ▼
Grain OD	0	cm ▼
Grain Length	0	cm ▼

Figure 10 Propellant Configuration Definition

G. Defining Injector Configuration

Injector configuration requires a great deal of care when setting up. For typical injector orifices, the inlet geometry and any internal geometry can greatly impact the discharge coefficient, and by extension flow rates. Injector diameter is somewhat arbitrary, as HRAP uses CdA when calculating flow rates. This means that you do not have to know the actual injector diameter, so long as the product of discharge coefficient and injector area is equal to what you measure either through CFD or cold flow tests. Typically injector diameter is defined as the minimum diameter within the injector. Small errors in discharge coefficient can have a disproportionate impact on the accuracy of simulations, so it is best to physically test your injectors either in a cold flow test or a water flow test. Currently, the injector model used assumes Single-Phase-Incompressible flow, though future releases of HRAP will attempt to model the choked two phase flow behavior seen at high pressure drops using the Homogeneous-Equilibrium-Model. By default no nitrous oxide vent is modeled in HRAP, but if one is used either an internal or external vent model can be added. The external vent simply empties nitrous oxide vapor from the top of the tank, while the internal vent is essentially an additional injector that supplies vapor rather than liquid (internal vents are common in motors produced by Contrail Rockets). As with injectors, it is beneficial to directly measure the diameter and discharge coefficient of the vent if one is to be used.



Injector Configuration

Injector Diameter 0 cm

Injector Discharge Coefficient 1

Number of Injectors 1

Vent State None

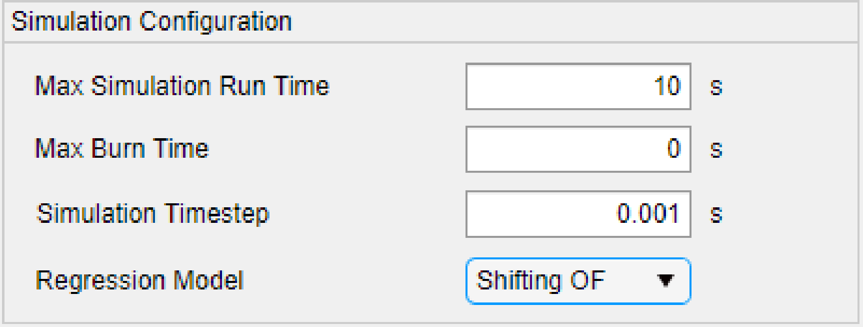
Vent Diameter 0 cm

Vent Discharge Coefficient 0

Figure 11 Injector Configuration Definition

H. Defining Simulation Configuration

The final step before running a simulation is to define simulation controls. The max simulation run time is the time at which the simulation is ended, provided no other simulation terminations are met first (fuel grain depletion, oxidizer depletion, or completion of the burn). The max burn time is primarily used for motors with actuated valves and essentially just stops flow through the injectors at a predetermined time, after which the simulation will continue to run until the shut down transient is finished or the max simulation time is reached. The simulation timestep is critical for stability of the simulation, if its value is too high error between timesteps will accumulate and the simulation could just fail all-together. One millisecond is usually sufficient, but re-running the simulation varying timestep can show if the simulation is converging to a fixed answer. The final option in simulation configuration is the regression model, which can be either a shifting O/F ratio model or a constant O/F ratio model. The constant O/F ratio model can be used as a "best-first-guess" for an un-characterized propellant, though the shifting O/F model will provide better accuracy. By default the constant O/F ratio used is the optimum O/F ratio specified in the propellant configuration file, but it can be overridden to a custom value.



Simulation Configuration

Max Simulation Run Time 10 s

Max Burn Time 0 s

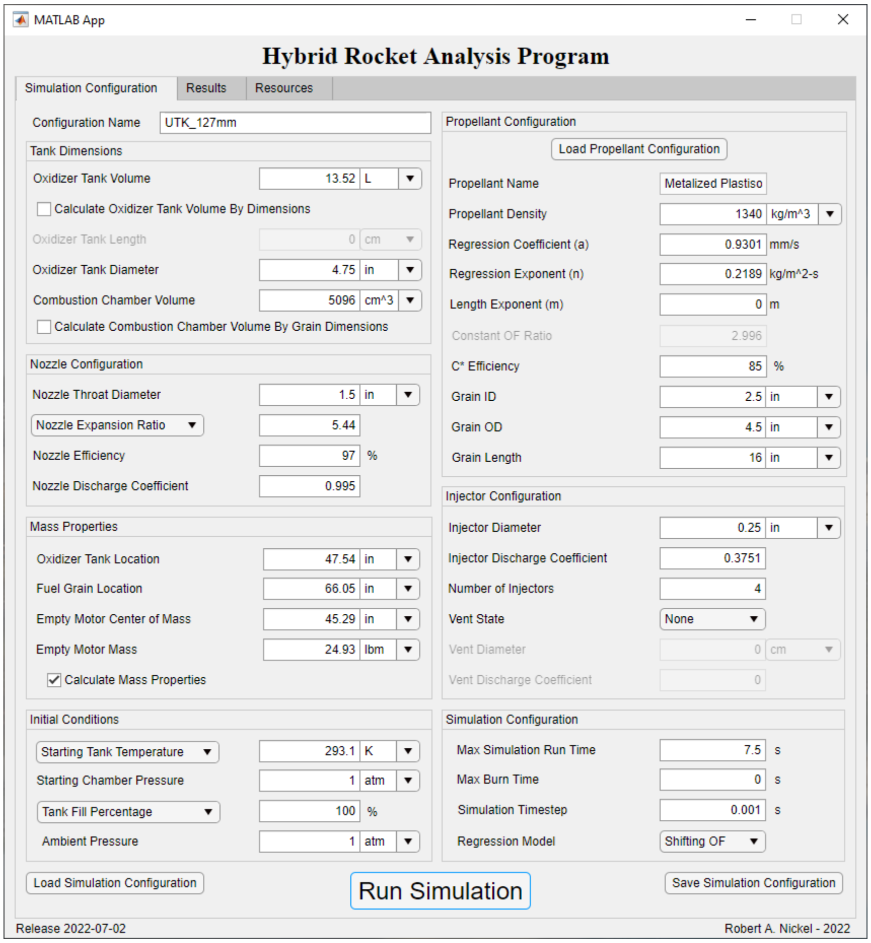
Simulation Timestep 0.001 s

Regression Model Shifting OF

Figure 12 Simulation Configuration Definition

I. Results Tab

With all required parameters defined, you can now run a simulation, usually the simulation will take only a few seconds to run depending on burn time and your chosen timestep. Upon completion of the simulation, you will notice the Performance Summary is displayed on the bottom left corner of the Results tab. The last entry in the Performance Summary is the Simulation Termination condition and specifies why the simulation ended when it did. The performance summary can be useful when seeing how different parameters impact the performance of the motor, but plotting and export options are arguably far more useful.



MATLAB App

Hybrid Rocket Analysis Program

Simulation Configuration | Results | Resources

Configuration Name: UTK_127mm

Tank Dimensions

- Oxidizer Tank Volume: 13.52 L
- □ Calculate Oxidizer Tank Volume By Dimensions
- Oxidizer Tank Length: 0 cm
- Oxidizer Tank Diameter: 4.75 in
- Combustion Chamber Volume: 5096 cm^3
- □ Calculate Combustion Chamber Volume By Grain Dimensions

Nozzle Configuration

- Nozzle Throat Diameter: 1.5 in
- Nozzle Expansion Ratio: 5.44
- Nozzle Efficiency: 97 %
- Nozzle Discharge Coefficient: 0.995

Mass Properties

- Oxidizer Tank Location: 47.54 in
- Fuel Grain Location: 66.05 in
- Empty Motor Center of Mass: 45.29 in
- Empty Motor Mass: 24.93 lbm
- ☑ Calculate Mass Properties

Initial Conditions

- Starting Tank Temperature: 293.1 K
- Starting Chamber Pressure: 1 atm
- Tank Fill Percentage: 100 %
- Ambient Pressure: 1 atm

Propellant Configuration

Load Propellant Configuration

- Propellant Name: Metalized Plastiso
- Propellant Density: 1340 kg/m^3
- Regression Coefficient (a): 0.9301 mm/s
- Regression Exponent (n): 0.2189 kg/m^2-s
- Length Exponent (m): 0 m
- Constant OF Ratio: 2.996
- C* Efficiency: 85 %
- Grain ID: 2.5 in
- Grain OD: 4.5 in
- Grain Length: 16 in

Injector Configuration

- Injector Diameter: 0.25 in
- Injector Discharge Coefficient: 0.3751
- Number of Injectors: 4
- Vent State: None
- Vent Diameter: 0 cm
- Vent Discharge Coefficient: 0

Simulation Configuration

- Max Simulation Run Time: 7.5 s
- Max Burn Time: 0 s
- Simulation Timestep: 0.001 s
- Regression Model: Shifting OF

Load Simulation Configuration | Run Simulation | Save Simulation Configuration

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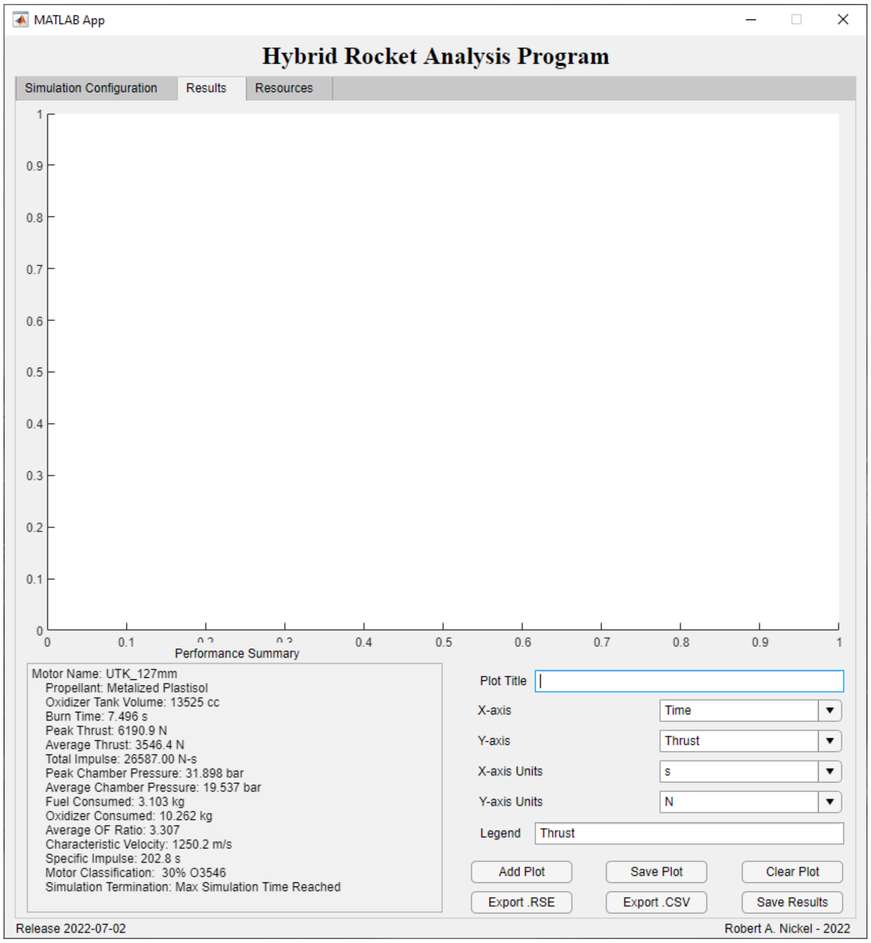
Figure 13 Fully defined Simulation Configuration Tab



Performance Summary

```
Motor Name: UTK_127mm
   Propellant: Metalized Plastisol
   Oxidizer Tank Volume: 13525 cc
   Burn Time: 7.496 s
   Peak Thrust: 6190.9 N
   Average Thrust: 3546.4 N
   Total Impulse: 26587.00 N-s
   Peak Chamber Pressure: 31.898 bar
   Average Chamber Pressure: 19.537 bar
   Fuel Consumed: 3.103 kg
   Oxidizer Consumed: 10.262 kg
   Average OF Ratio: 3.307
   Characteristic Velocity: 1250.2 m/s
   Specific Impulse: 202.8 s
   Motor Classification:  30% O3546
   Simulation Termination: Max Simulation Time Reached
```

Plot Title

X-axis Time

Y-axis Thrust

X-axis Units s

Y-axis Units N

Legend Thrust

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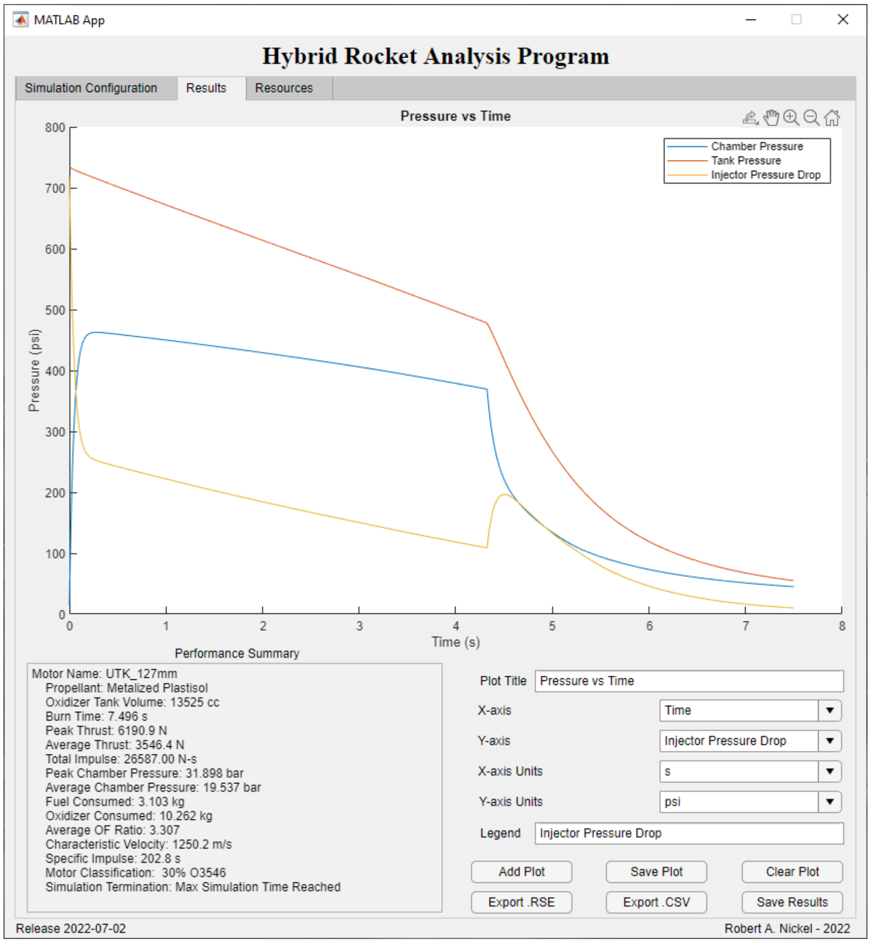
Figure 14 Results Tab

HRAP’s built in plotter has the ability to plot multiple different types of data on a single plot, even from different simulations. To use the plot, give the plot a title, specify the x and y axis variables and their corresponding units, and click “Add Plot”. Once the plot is added you can add other data to the same plot, though one should try and stick to data with consistent units (i.e. a plot of tank pressure, chamber pressure, and pressure drop, or a plot of oxidizer mass, fuel mass, total propellant mass, and total motor mass). For each variable, the legend is automatically updated to specify what the data is. Alternatively you can re-name the data set using the “Legend” field, which is especially useful when comparing different motor simulations.

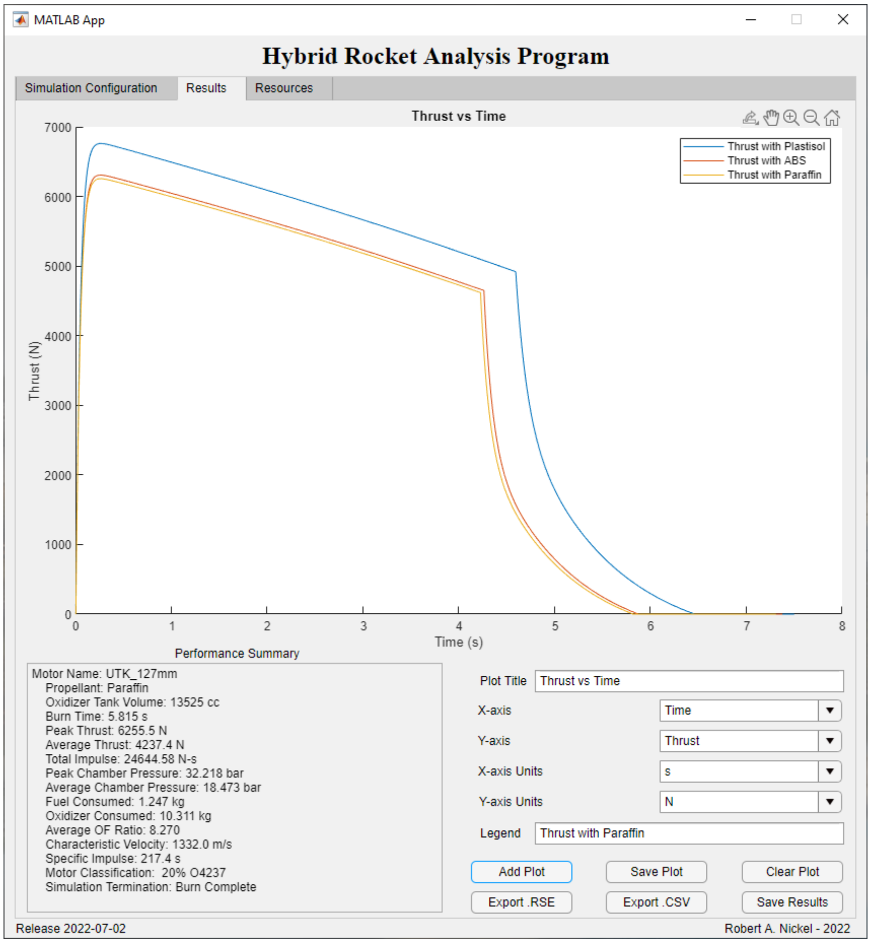


Performance Summary

Motor Name: UTK_127mm
Propellant: Metalized Plastisol
Oxidizer Tank Volume: 13525 cc
Burn Time: 7.496 s
Peak Thrust: 6190.9 N
Average Thrust: 3546.4 N
Total Impulse: 26587.00 N-s
Peak Chamber Pressure: 31.898 bar
Average Chamber Pressure: 19.537 bar
Fuel Consumed: 3.103 kg
Oxidizer Consumed: 10.262 kg
Average OF Ratio: 3.307
Characteristic Velocity: 1250.2 m/s
Specific Impulse: 202.8 s
Motor Classification: 30% O3546
Simulation Termination: Max Simulation Time Reached

Plot Title Pressure vs Time
X-axis Time
Y-axis Injector Pressure Drop
X-axis Units s
Y-axis Units psi
Legend Injector Pressure Drop

Figure 15 Example plot in HRAP



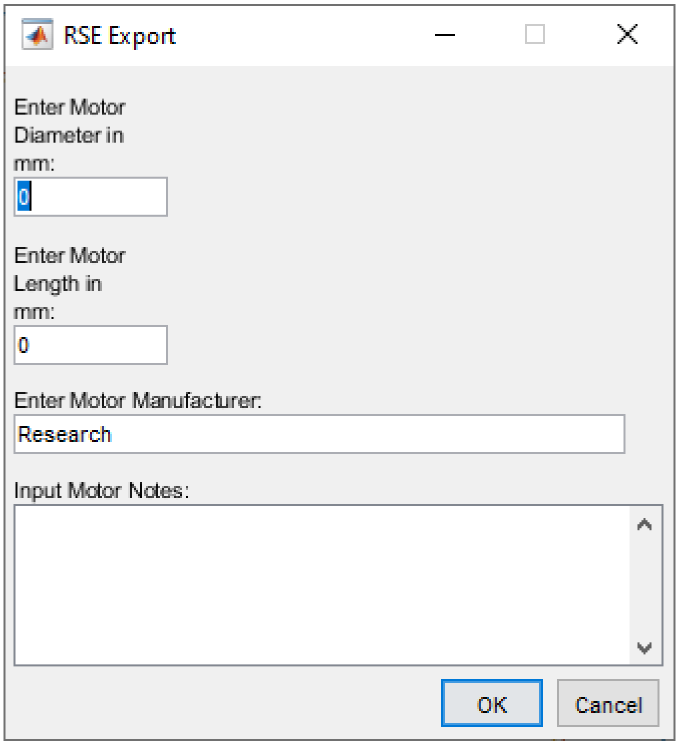
Motor Name: UTK_127mm
Propellant: Paraffin
Oxidizer Tank Volume: 13525 cc
Burn Time: 5.815 s
Peak Thrust: 6255.5 N
Average Thrust: 4237.4 N
Total Impulse: 24644.58 N-s
Peak Chamber Pressure: 32.218 bar
Average Chamber Pressure: 18.473 bar
Fuel Consumed: 1.247 kg
Oxidizer Consumed: 10.311 kg
Average OF Ratio: 8.270
Characteristic Velocity: 1332.0 m/s
Specific Impulse: 217.4 s
Motor Classification: 20% O4237
Simulation Termination: Burn Complete

Plot Title Thrust vs Time
X-axis Time
Y-axis Thrust
X-axis Units s
Y-axis Units N
Legend Thrust with Paraffin

Figure 16 Example of plotting from multiple simulations

With plots generated, clicking “Save Plot” will export the plot to the HRAP output folder as a .png. If you wish to reset the plot, simply click “Clear Plot”.

The other export options available are “Export .RSE”, “Export.CSV”, and “Save Results”. The “Export .RSE” option will open a window asking for additional information on the motor necessary for the .RSE file. The “Export .CSV” option will save all data output by HRAP to a .csv file. The “Save Results” option will save the text from the Performance Summary as a .txt file. All HRAP outputs by default are sent to C:\Program Files\HRAP\application\output.


RSE Export

Enter Motor Diameter in mm: 0

Enter Motor Length in mm: 0

Enter Motor Manufacturer: Research

Input Motor Notes:

Figure 17 Prompt that appears when exporting a .RSE file

J. Final Notes About Usage

Because HRAP was compiled using MATLAB, there are some somewhat annoying nuances in getting it to work. The two biggest issues are that you must use the default installation directory, and that you must run HRAP as an administrator. In the future I plan to place the default install directory in the AppData folder so that it does not require running as an administrator. Finally, while HRAP itself is free to use, it has taken years of education, programming, and hands on research to deliver this program to the general public. If HRAP is to be used for any work that will be published please cite it, and if you have any comments, questions, concerns, suggestions, or features you want implemented, please email me.

VIII. Simulation Validation

In December of 2019, Equatorial Space Industries tested their 750 Newton thrust subscale demonstration motor using their proprietary HRF1Al propellant. Because this is a medium scale rocket motor, heat transfer into the oxidizer tank can be more or less neglected and friction losses normally associated with smaller motors are negligible. The motor used a 2500 cubic centimeter oxidizer tank filled with 1.78 kg of Nitrous Oxide and had a target chamber pressure of 3 MPa. Data from this test fire was provided for validation of the HRAP simulation environment and closely matched predicted performance. Plots of key performance parameters are shown below in Figures 7 through 10. An image of this particular motor test is shown in Figure 6 below.

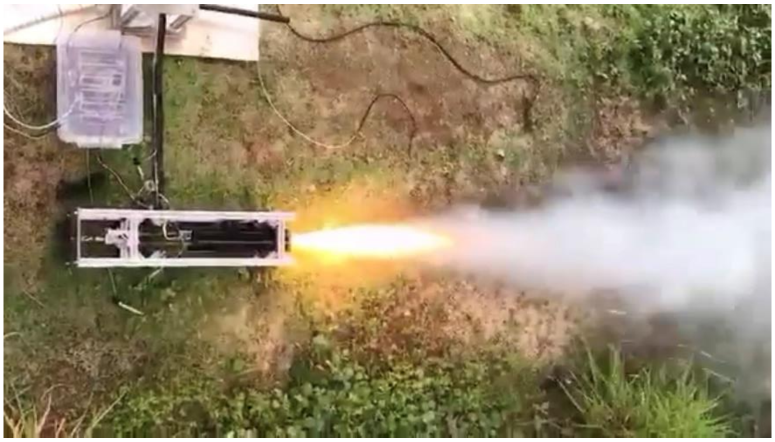


Figure 18 Equatorial Space Industries 750 Newton subscale motor.

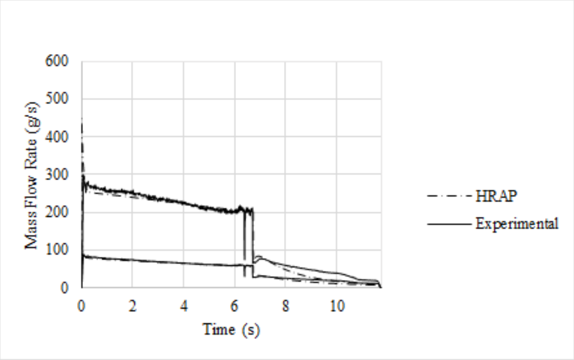


Figure 7 Fuel and oxidizer mass flow rate vs time.

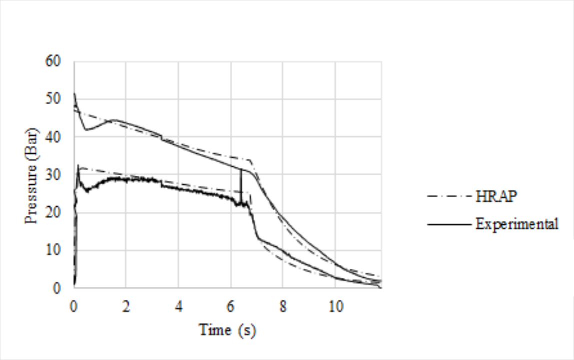


Figure 8 Oxidizer tank and combustion chamber pressure vs time.

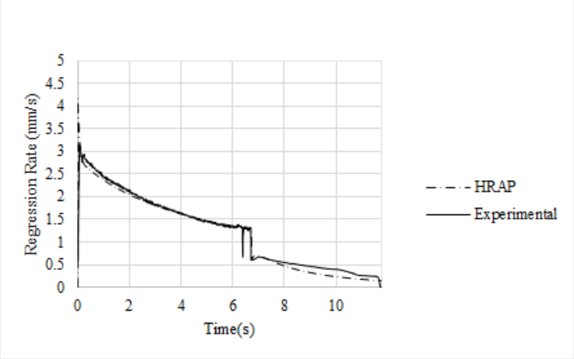


Figure 9 Fuel regression rate vs time.

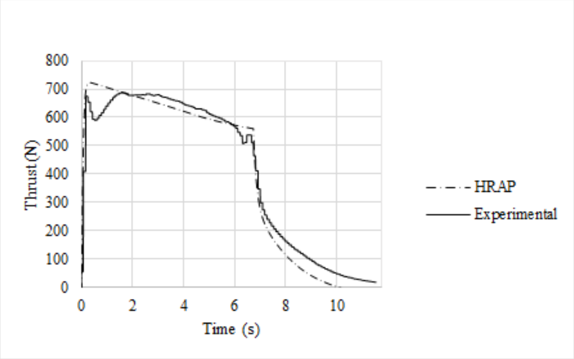


Figure 10 Thrust vs time.

The simulation's predictions are close to measured motor performance, there is some noticeable error at motor startup and towards the end of the motor burn, though much of the error between simulated and experimental data can be readily explained.

A. Error at Motor Startup

HRAP assumes the oxidizer tank remains in equilibrium for the duration of the burn and that all valves open instantaneously; however, this is not entirely accurate. Nitrous oxide vaporizes at a finite rate and with some delay between initial de-pressurization and the liquid boiling off to re-pressurize. In addition to non-equilibrium. Depending on the scale of the motor, non-equilibrium effects may have no meaningful impact on the simulation or they could induce a dip in tank pressure as is seen in the above static test fire. The oxidizer valve also opens at a finite rate, usually on the order of milliseconds, which can lead to substantial error in the motor's startup transient.

B. Error at Motor Shutdown

Towards the end of the motor burn, the simulation and experimental values diverge. This is likely due to the assumption that the compressibility factor of the Nitrous Oxide remains along the saturation line for the entirety of the vapor phase and the fact the vapor discharge is modeled with a Single-Phase-Incompressible (SPI) model. In addition to error from this assumption, the nozzle model does not account for flow separation or shifting equilibrium. As the motor transitions to the vapor phase, there is a sudden decrease in chamber pressure. Once the nozzle exit pressure drops sufficiently (usually around 40% of ambient pressure), the flow will separate and the effective expansion ratio of the nozzle will go down as the flow retracts into the nozzle. Because this is not modeled, the rocket motor reaches a point at which the pressure component of thrust is negative and the vacuum produced is pulling the motor in the aft direction. This causes calculated thrust under these conditions to be lower than what is actually observed.

Acknowledgments

I would like to thank Dr Richard Newlands with Aspire Space for his extensive help in the development of HRAP. His hybrid rocket model laid the foundation for HRAP, and I could not have gotten this far without him. I would like to thank James Anderson with Equatorial Space Industries in Singapore for providing test data for model verification. Lastly, I would like to thank Dr. Evans Lyne, Dr. Kuvanc Ekici, Dr. Mark Barker, Dr. Matthew Mench, Dr. Robert Jacobsen, and James Hudspeth for their academic and/or financial support of this endeavor.

References

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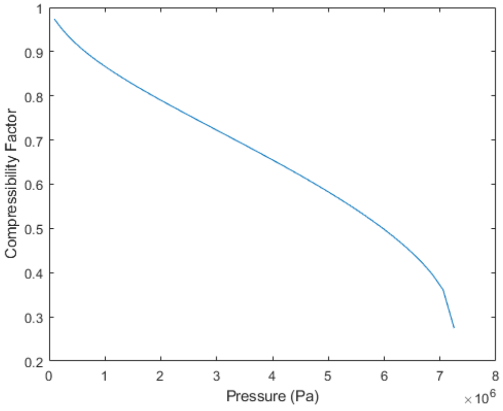
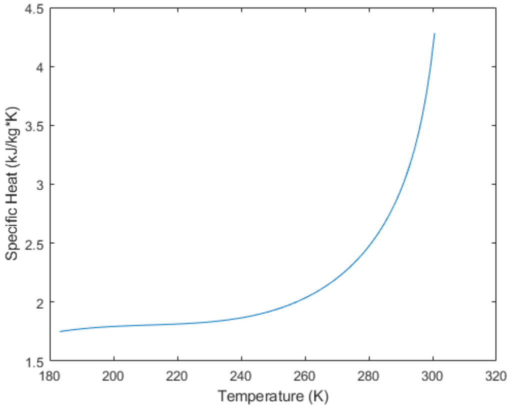
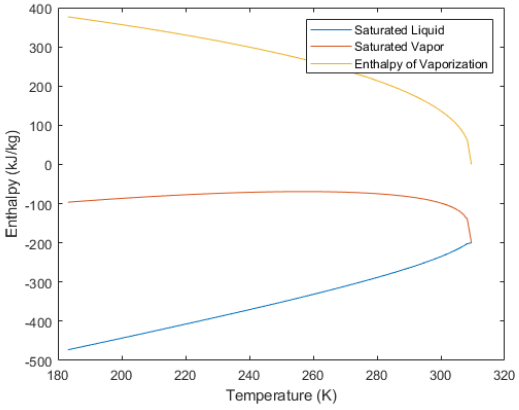
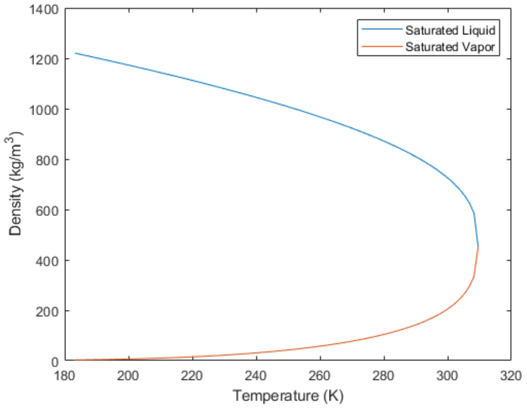
[9] Sutton, J. P. and Biblarz O., “Rocket Propulsion Elements – Ninth Edition,” Wiley, 2010.

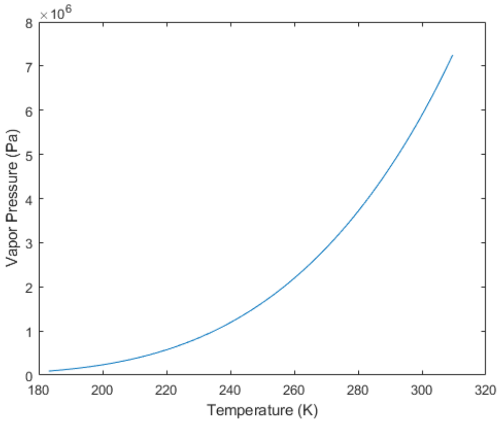
Appendix A. Thermophysical Properties of Saturated Nitrous Oxide


Compressibility Factor
Pressure (Pa)
× 10^6


Specific Heat (kJ/kg*K)
Temperature (K)


Enthalpy (kJ/kg)
Temperature (K)
Saturated Liquid
Saturated Vapor
Enthalpy of Vaporization


Density (kg/m^3)
Temperature (K)
Saturated Liquid
Saturated Vapor


× 10^6
Vapor Pressure (Pa)
Temperature (K)