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## RSA Public-Key Encryption and Signature Lab (Lab3)

### RSA Encryption and Decryption:

## RSA Algorithm

### Key Generation

Select $p, q$	$p$ and $q$ both prime; $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p-1)(q-1)$	
Select integer $e$	$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate $d$	$de \bmod \phi(n) = 1$
Public key	$KU = \{e, n\}$
Private key	$KR = \{d, n\}$

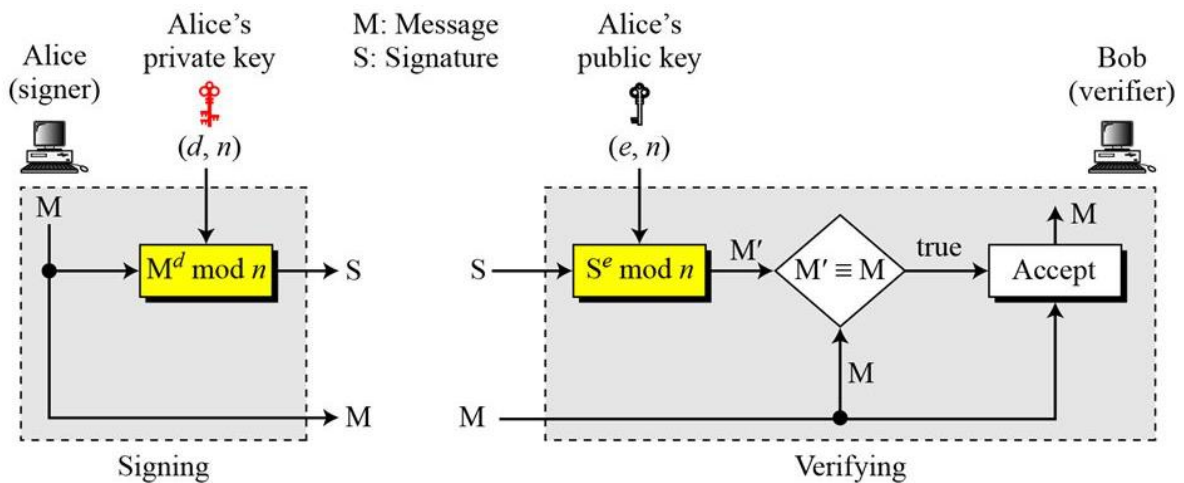
### Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

### Decryption

Plaintext:	$C$
Ciphertext:	$M = C^d \bmod n$

### RSA Signing and Verifying:



$$M' \equiv M \pmod{n} \rightarrow S^e \equiv M \pmod{n} \rightarrow M^{d \times e} \equiv M \pmod{n}$$

## Task 1: Deriving the Private Key

```
Task1.c
~/Desktop

1#include <stdio.h>
2#include <openssl/bn.h>
3
4#define NBITS 128
5
6void printBN(char *msg, BIGNUM * a)
7{
8    /* Use BN_bn2hex(a) for hex string
9     * Use BN_bn2dec(a) for decimal string */
10   char * number_str = BN_bn2hex(a);
11   printf("%s %s\n", msg, number_str);
12   OPENSSL_free(number_str);
13 }
14
```

C Tab Width: 8 Ln 1, Col 1 INS

```
Task1.c
~/Desktop

14
15int main ()
16{
17    BN_CTX *ctx = BN_CTX_new();
18
19    BIGNUM *p = BN_new();
20    BIGNUM *q = BN_new();
21    BIGNUM *e = BN_new();
22    BIGNUM *p_1 = BN_new();
23    BIGNUM *q_1 = BN_new();
24    BIGNUM *n = BN_new();
25    BIGNUM *phi_n = BN_new();
26    BIGNUM *d = BN_new();
27
28    BN_hex2bn(&p,
29    "F7E75FDC469067FFDC4E847C51F452DF");
30    BN_hex2bn(&q,
31    "E85CED54AF57E53E092113E62F436F4F");
32    BN_hex2bn(&e, "0D88C3");
33
34    BN_sub(p_1, p, BN_value_one());
35    BN_sub(q_1, q, BN_value_one());
36    BN_mul(n, p, q, ctx);
37    BN_mul(phi_n, p_1, q_1, ctx);
38
39    BN_mod_inverse(d, e, phi_n, ctx);
40
41    printBN("n = ", n);
42    printBN("d = ", d);
43
44    return 0;
45 }
```

C Tab Width: 8 Ln 1, Col 1 INS

```
seed@VM: ~/Desktop
[04/18/24] seed@VM:~/Desktop$ gcc Task1.c -lcrypto -o Task1
[04/18/24] seed@VM:~/Desktop$ ./Task1
n = E103ABD94892E3E74AFD724BF28E78366D9676BCCC70118BD0AA1968DBB143D1
d = 3587A24598E5F2A21DB007D89D18CC50ABA5075BA19A33890FE7C28A9B496AEB
[04/18/24] seed@VM:~/Desktop$
```

## Task 2: Encrypting a Message

```
Task2.c
~/Desktop
Open Save
15 int main ()
16 {
17     BN_CTX *ctx = BN_CTX_new();
18
19     BIGNUM *n = BN_new();
20     BIGNUM *e = BN_new();
21     BIGNUM *M = BN_new();
22     BIGNUM *d = BN_new();
23     BIGNUM *C = BN_new();
24     BIGNUM *res = BN_new();
25
26     BN_hex2bn(&n,
27 "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
28 BN_hex2bn(&e, "010001");
29 BN_hex2bn(&M, "4120746f702073656372657421");
30 BN_hex2bn(&d,
31 "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");
32
33 BN_mod_exp(C, M, e, n, ctx);
34 BN_mod_exp(res, C, d, n, ctx);
35
36 printBN("M = ", M);
37 printBN("C = ", C);
38 printBN("res = ", res);
39
40 return 0;
41 }
```

C Tab Width: 8 Ln 1, Col 1 INS

```
seed@VM: ~/Desktop
[04/18/24] seed@VM:~/Desktop$ python3 -c 'print("A top secret!".encode("utf-8").hex())'
4120746f702073656372657421
[04/18/24] seed@VM:~/Desktop$ gcc Task2.c -lcrypto -o Task2
[04/18/24] seed@VM:~/Desktop$ ./Task2
M = 4120746f702073656372657421
C = 6fb078da550b2650832661e14f4f8d2cfaef475a0df3a75cacdc5de5cfc5fadc
res = 4120746f702073656372657421
[04/18/24] seed@VM:~/Desktop$
```

M is plaintext before encryption while res is plaintext after encryption (decrypt the ciphertext). We verified the encryption result by comparing M and res (both same values).

### Task 3: Decrypting a Message

```
Task3.c
~/Desktop
Open Save
15 int main ()
16 {
17     BN_CTX *ctx = BN_CTX_new();
18
19     BIGNUM *n = BN_new();
20     BIGNUM *e = BN_new();
21     BIGNUM *M = BN_new();
22     BIGNUM *d = BN_new();
23     BIGNUM *C = BN_new();
24
25     BN_hex2bn(&n,
26 "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
27     BN_hex2bn(&e, "010001");
28     BN_hex2bn(&C,
29 "8C0F971DF2F3672B28811407E2DABBE1DA0FEBBDFC7DCB67396567EA1E2493F");
30     BN_hex2bn(&d,
31 "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");
32
33     BN_mod_exp(M, C, d, n, ctx);
34
35     printBN("C = ", C);
36     printBN("M = ", M);
37
38     return 0;
39 }
```

```
seed@VM: ~/Desktop
[04/18/24] seed@VM:~/Desktop$ gcc Task3.c -lcrypto -o Task3
[04/18/24] seed@VM:~/Desktop$ ./Task3
C = 8C0F971DF2F3672B28811407E2DABBE1DA0FEBBDFC7DCB67396567EA1E2493F
M = 50617373776F72642069732064656573
[04/18/24] seed@VM:~/Desktop$ python3 -c 'print(bytes.fromhex("50617373776F72642069732064656573").decode("utf-8"))'
Password is dees
[04/18/24] seed@VM:~/Desktop$
```

#### Task 4: Signing a Message

```
Task4.c
~/Desktop
Open Save
15 int main ()
16 {
17     BN_CTX *ctx = BN_CTX_new();
18
19     BIGNUM *n = BN_new();
20     BIGNUM *e = BN_new();
21     //python3 -c 'print("I owe you $2000.".encode("utf-8").hex())'
22     BIGNUM *M1 = BN_new(); //2000
23     BIGNUM *M2 = BN_new(); //3000
24     BIGNUM *d = BN_new();
25     BIGNUM *S1 = BN_new();
26     BIGNUM *S2 = BN_new();
27
28     BN_hex2bn(&n,
29 "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
30     BN_hex2bn(&e, "010001");
31     BN_hex2bn(&M1, "49206f776520796f752024323030302e");
32     BN_hex2bn(&M2, "49206f776520796f752024333030302e");
33     BN_hex2bn(&d,
34 "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");
35
36     BN_mod_exp(S1, M1, d, n, ctx);
37     BN_mod_exp(S2, M2, d, n, ctx);
38     printf("M = I owe you $2000.\n");
39     printBN("Sign = ", S1);
40     printf("M = I owe you $3000.\n");
41     printBN("Sign = ", S2);
42
43     return 0;
44 }
```

```
seed@VM: ~/Desktop
[04/18/24]seed@VM:~/Desktop$ python3 -c 'print("I owe you $2000.".encode("utf-8").hex())'
49206f776520796f752024323030302e
[04/18/24]seed@VM:~/Desktop$ python3 -c 'print("I owe you $3000.".encode("utf-8").hex())'
49206f776520796f752024333030302e
[04/18/24]seed@VM:~/Desktop$ gcc Task4.c -lcrypto -o Task4
[04/18/24]seed@VM:~/Desktop$ ./Task4
M = I owe you $2000.
Sign = 55A4E7F17F04CCFE2766E1EB32ADDBA890BBE92A6FBE2D785ED6E73CCB35E4CB
M = I owe you $3000.
Sign = BCC20FB7568E5D48E434C387C06A6025E90D29D848AF9C3EBAC0135D99305822
[04/18/24]seed@VM:~/Desktop$
```

We observed that a slight change to the message M can result to totally different signatures.

## Task 5: Verifying a Signature

```
Task5.c
~/Desktop
Open Save
15 int main ()
16 {
17     BN_CTX *ctx = BN_CTX_new();
18
19     BIGNUM *n = BN_new();
20     BIGNUM *e = BN_new();
21     BIGNUM *M = BN_new();
22     BIGNUM *d = BN_new();
23     BIGNUM *S1 = BN_new();
24     BIGNUM *S2 = BN_new();
25     BIGNUM *res1 = BN_new();
26     BIGNUM *res2 = BN_new();
27
28     BN_hex2bn(&n,
29 "AE1CD4DC432798D933779FBD46C6E1247F0CF1233595113AA51B450F18116115");
30     BN_hex2bn(&e, "010001");
31     BN_hex2bn(&S1,
32 "643D6F34902D9C7EC90CB0B2BCA36C47FA37165C0005CAB026C0542CBDB6802F");
33     BN_hex2bn(&S2,
34 "643D6F34902D9C7EC90CB0B2BCA36C47FA37165C0005CAB026C0542CBDB6803F");
35     BN_hex2bn(&M, "4c61756e63682061206d697373696c652e");
36
37     BN_mod_exp(res1, S1, e, n, ctx);
38     BN_mod_exp(res2, S2, e, n, ctx);
39
40     printBN("Message = ", M);
41     printBN("Verified message 1 = ", res1);
42     printBN("Verified message 2 = ", res2);
43
44     return 0;
45 }
```

```
Task5.c
~/Desktop
Open Save
39 printf("Verified message 2 = ", res2),
40 if (BN_cmp(res1, M) == 0)
41 {
42     printf("Signature 1 is Alice's!\n");
43 }
44 else
45 {
46     printf("Signature 1 is not Alice's!\n");
47 }
48 if (BN_cmp(res2, M) == 0)
49 {
50     printf("Signature 2 is Alice's!\n");
51 }
52 else
53 {
54     printf("Signature 2 is not Alice's!\n");
55 }
56
57 return 0;
58 }
```

```
seed@VM: ~/Desktop
[04/18/24]seed@VM:~/Desktop$ python3 -c 'print("Launch a missile.".encode("utf-8").hex())'
4c61756e63682061206d697373696c652e
[04/18/24]seed@VM:~/Desktop$ gcc Task5.c -lcrypto -o Task5
[04/18/24]seed@VM:~/Desktop$ ./Task5
Message = 4C61756E63682061206D697373696C652E
Verified message 1 = 4C61756E63682061206D697373696C652E
Verified message 2 = 91471927C80DF1E42C154FB4638CE8BC726D3D66C83A4EB6B7BE0203B41AC294
Signature 1 is Alice's!
Signature 2 is not Alice's!
[04/18/24]seed@VM:~/Desktop$
```

Signature 1's last byte is 2F which generate Verified message 1; Signature 2's last byte is 3F which generate Verified message 2. Compare both Verified message 1 and 2 with Message, we observed that Verified message 1 is similar with Message while Verified message 2 not, hence we say that Signature 1 is Alice's while Signature 2 is not.

## Task 6: Manually Verifying an X.509 Certificate

```
Open Task6.c Save
1#include <stdio.h>
2#include <openssl/bn.h>
3
4#define NBITS 256
5
6void printBN(char *msg, BIGNUM * a)
7{
8    /* Use BN_bn2hex(a) for hex string
9     * Use BN_bn2dec(a) for decimal string */
10   char * number_str = BN_bn2hex(a);
11   printf("%s %s\n", msg, number_str);
12   OPENSSL_free(number_str);
13}
14
15int main ()
16{
17   BN_CTX *ctx = BN_CTX_new();
18
19   BIGNUM *e = BN_new();
20   BIGNUM *n = BN_new();
21   BIGNUM *S = BN_new();
22   BIGNUM *res = BN_new();
23
24   BN_hex2bn(&n,
25   "CCF710624FA68B636FED905256C56D277B7A12568AF1F4F9D6E7E18FB095ABF260411570DB1200FA270AB557385B7DB2519371950E6A41945B3518FA7BFABBC5BE2430FE56EFC4F37D97E314F5144D03E8A1390C090C32C1AF16574C9447427CA2C89C7DE6D44D54AF4299A8C104C2779C0648E4CE11E02AB099F04370CF3F766B014C49AB245EC20D82FD46A8AB6C93C6252427592F89AF45E8B2B061E51F1FB97F0998E830FA837F4769A1");
26   BN_hex2bn(&S,
27   "04e16e023e0de32346f4e396350593522020b845de27386d4744ffc1b27af3ecaadc3ce46d6fa0fe271f90d1a9a13b7d50848bd5058b35e20638629ca3ecccc7826e1598f5");
28
29   BN_mod_exp(res, S, e, n, ctx);
30   printBN("Verify signature:", res);
31
32   return 0;
33}
34
```

```
seed@VM: ~/Desktop
[04/18/24]seed@VM:~/Desktop$ openssl x509 -in c1.pem -noout -modulus
Modulus=CCF710624FA68B636FED905256C56D277B7A12568AF1F4F9D6E7E18FB095ABF260411570DB1200FA270AB557385B7DB2519371950E6A41945B3518FA7BFABBC5BE2430FE56EFC4F37D97E314F5144D03E8A1390C090C32C1AF16574C9447427CA2C89C7DE6D44D54AF4299A8C104C2779C0648E4CE11E02AB099F04370CF3F766B014C49AB245EC20D82FD46A8AB6C93C6252427592F89AF45E8B2B061E51F1FB97F0998E830FA837F4769A1
[04/18/24]seed@VM:~/Desktop$ openssl x509 -in c1.pem -text -noout
Certificate:
    Data:
        Version: 3 (0x2)
        Serial Number:
            0c:f5:bd:06:2b:56:02:f4:7a:b8:50:2c:23:cc:f0:66
        Signature Algorithm: sha256WithRSAEncryption
        Issuer: C = US, O = DigiCert Inc, OU = www.digicert.com, CN = DigiCert Global Root G2
        Validity
            Not Before: Mar 30 00:00:00 2021 GMT
            Not After : Mar 29 23:59:59 2031 GMT
        Subject: C = US, O = DigiCert Inc, CN = DigiCert Global G2 TLS RSA SHA256 2020 CA1
        Subject Public Key Info:
            Public Key Algorithm: rsaEncryption
            RSA Public-Key: (2048 bit)
            Modulus:
                00:cc:f7:10:62:4f:a6:bb:63:6f:ed:90:52:56:c5:
                6d:27:7b:7a:12:56:8a:f1:f4:f9:d6:e7:e1:8f:bd:
                95:ab:f2:60:41:15:70:db:12:00:fa:27:0a:b5:57:
                38:5b:7d:b2:51:93:71:95:0e:6a:41:94:5b:35:1b:
                fa:7b:fa:bb:c5:be:24:30:fe:56:ef:c4:f3:7d:97:
                e3:14:f5:14:4d:cb:a7:10:f2:16:ea:ab:22:f0:31:
                22:11:61:69:90:26:ba:78:d9:97:1f:e3:7d:66:ab:
                75:44:95:73:c8:ac:ff:ef:5d:0a:8a:59:43:e1:ab:
                b2:3a:0f:f3:48:fc:d7:6b:37:c1:63:dc:de:46:d6:
                db:45:fe:7d:23:fd:90:e8:51:07:1e:51:a3:5f:ed:
                49:46:54:7f:2c:88:c5:f4:13:9c:97:15:3c:03:e8:
                a1:39:dc:69:0c:32:c1:af:16:57:4c:94:47:42:7c:
                a2:c8:9c:7d:e6:d4:4d:54:af:42:99:a8:c1:04:c2:
                77:9c:d6:48:e4:ce:11:e0:2a:80:99:f0:43:70:cf:
                3f:76:6b:d1:4c:49:ab:24:5e:c2:0d:82:fd:46:a8:
                ab:6c:93:cc:62:52:42:75:92:f8:9a:fa:5e:5e:b2:
                b0:61:e5:1f:1f:b9:7f:09:98:e8:3d:fa:83:7f:47:
                69:a1
            Exponent: 65537 (0x10001)
        X509v3 extensions:
            X509v3 Basic Constraints: critical
                CA:TRUE, pathlen:0
            X509v3 Subject Key Identifier:
                74:85:80:C0:66:C7:DF:37:DE:CF:BD:29:37:AA:03:1D:BE:ED:CD:17
```



