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with(Physics) :
with(LinearAlgebra) :
Setup(mathematicalnotation = true) :
with(DynamicSystems) :
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$$xg(t) := x(t) + l \cdot \sin(\theta(t))$$

$$xg := t \mapsto x(t) + l \sin(\theta(t)) \tag{1}$$

$$Jw := mw \cdot r^2$$

$$Jw := mw\, r^2 \tag{2}$$

$$Jp := mp \cdot l^2$$

$$Jp := mp\, l^2 \tag{3}$$

$$x(t) := r \cdot \varphi(t)$$

$$x := t \mapsto r\, \varphi(t) \tag{4}$$

$$xg(t)$$

$$r\, \varphi(t) + l \sin(\theta(t)) \tag{5}$$

$$yg(t) := l \cdot \cos(\theta(t))$$

$$yg := t \mapsto l \cos(\theta(t)) \tag{6}$$

$$y(t) := 0$$

$$y := t \mapsto 0 \tag{7}$$

$$T(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)) := \frac{1}{2} \cdot (mp \cdot diff(yg(t), t)^2 + mp \cdot diff(xg(t), t)^2 + mw \cdot diff(x(t), t)^2 + mw \cdot diff(y(t), t)^2 + Jp \cdot diff(\theta(t), t)^2 + Jw \cdot diff(\varphi(t), t)^2)$$

$$T(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)) := \frac{mp\, l^2 \sin(\theta(t))^2 \dot{\theta}(t)^2}{2} + \frac{mp\, (r \dot{\varphi}(t) + l \cos(\theta(t))\, \dot{\theta}(t))^2}{2} + mw\, r^2 \dot{\varphi}(t)^2 + \frac{mp\, l^2 \dot{\theta}(t)^2}{2} \tag{8}$$

$$\Pi(t, \theta(t), \varphi(t)) := mp \cdot g \cdot l \cdot \cos(\theta(t))$$

$$\Pi(t, \theta(t), \varphi(t)) := mp\, g\, l \cos(\theta(t)) \tag{9}$$

$$L(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)) := T(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)) - \Pi(t, \theta(t), \varphi(t))$$

$$L(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)) := \frac{mp\, l^2 \sin(\theta(t))^2 \dot{\theta}(t)^2}{2} + \frac{mp\, (r \dot{\varphi}(t) + l \cos(\theta(t))\, \dot{\theta}(t))^2}{2} + mw\, r^2 \dot{\varphi}(t)^2 + \frac{mp\, l^2 \dot{\theta}(t)^2}{2} - mp\, g\, l \cos(\theta(t)) \tag{10}$$

$$dLd\theta := diff(L(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)), \theta(t))$$

$$dLd\theta := mp\, l^2 \sin(\theta(t))\, \dot{\theta}(t)^2 \cos(\theta(t)) - mp\, (r \dot{\varphi}(t) + l \cos(\theta(t))\, \dot{\theta}(t))\, l \sin(\theta(t))\, \dot{\theta}(t) + mp\, g\, l \sin(\theta(t)) \tag{11}$$

$$dLd\varphi := diff(L(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)), \varphi(t))$$

$$dLd\varphi := 0 \tag{12}$$

$dLdd\varphi := diff\left(L\left(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)\right), \dot{\varphi}(t)\right)$

$$dLdd\varphi := mp\left(r\,\dot{\grave{\varphi}}(t) + l\cos\left(\theta(t)\right)\dot{\theta}(t)\right) r + 2\,mw\,r^2\,\dot{\grave{\varphi}}(t) \tag{13}$$

$dLdd\theta := diff\left(L\left(t, \theta(t), \varphi(t), \dot{\varphi}(t), \dot{\theta}(t)\right), \dot{\theta}(t)\right)$

$$dLdd\theta := mp\,l^2\sin\left(\theta(t)\right)^2\dot{\theta}(t) + mp\left(r\,\dot{\grave{\varphi}}(t) + l\cos\left(\theta(t)\right)\dot{\theta}(t)\right) l\cos\left(\theta(t)\right) + mp\,l^2\,\dot{\theta}(t) \tag{14}$$

$ddLdd\varphi dt := diff\left(dLdd\varphi, t\right)$

$$ddLdd\varphi dt := mp\left(r\,\ddot{\grave{\varphi}}(t) - l\sin\left(\theta(t)\right)\dot{\theta}(t)^2 + l\cos\left(\theta(t)\right)\ddot{\theta}(t)\right) r + 2\,mw\,r^2\,\ddot{\grave{\varphi}}(t) \tag{15}$$

$ddLdd\theta dt := diff\left(dLdd\theta, t\right)$

$$ddLdd\theta dt := 2\,mp\,l^2\sin\left(\theta(t)\right)\dot{\theta}(t)^2\cos\left(\theta(t)\right) + mp\,l^2\sin\left(\theta(t)\right)^2\ddot{\theta}(t) + mp\left(r\,\ddot{\grave{\varphi}}(t) - l\sin\left(\theta(t)\right)\dot{\theta}(t)^2 + l\cos\left(\theta(t)\right)\ddot{\theta}(t)\right) l\cos\left(\theta(t)\right) - mp\left(r\,\dot{\grave{\varphi}}(t) + l\cos\left(\theta(t)\right)\dot{\theta}(t)\right) l\sin\left(\theta(t)\right)\dot{\theta}(t) + mp\,l^2\ddot{\theta}(t) \tag{16}$$

$$Q(t) := \begin{bmatrix} \varphi(t) \\ \theta(t) \end{bmatrix}$$

$$Q := t \mapsto Vector_{column}\left(1..2, \{1 = \varphi(t), 2 = \theta(t)\}, datatype = anything, storage = rectangular, order = Fortran\_order, subtype = Vector_{column}\right)$$

$dQ(t) := diff\left(Q(t), t\right)$

$$dQ := t \mapsto \dot{Q}(t) \tag{18}$$

$ddQ(t) := diff\left(dQ(t), t\right)$

$$ddQ := t \mapsto d\dot{Q}(t) \tag{19}$$

$dQ(t)$

$$\begin{bmatrix} \dot{\grave{\varphi}}(t) \\ \dot{\theta}(t) \end{bmatrix} \tag{20}$$

$ddQ(t)$

$$\begin{bmatrix} \ddot{\grave{\varphi}}(t) \\ \ddot{\theta}(t) \end{bmatrix} \tag{21}$$

$Momentum(t) := Mk(t)$

$$Momentum := t \mapsto Mk(t) \tag{22}$$

$$\tau := \begin{bmatrix} Mk(t) \\ 0 \end{bmatrix}$$

$$\tau := \begin{bmatrix} Mk(t) \\ 0 \end{bmatrix} \tag{23}$$

$LeqLSide\varphi := collect\left(collect\left(collect\left(simplify(ddLdd\varphi dt - dLd\varphi), \ddot{\grave{\varphi}}(t)\right), \ddot{\theta}(t)\right), \dot{\theta}(t)\right)$

$$LeqLSide\varphi := r\cos\left(\theta(t)\right) l\,mp\,\ddot{\theta}(t) + r\left(mp\,r + 2\,mw\,r\right)\ddot{\grave{\varphi}}(t) - r\sin\left(\theta(t)\right)\dot{\theta}(t)^2\,l\,mp \tag{24}$$

$LeqLSide\theta := collect\left(collect\left(collect\left(simplify(ddLdd\theta dt - dLd\theta), \ddot{\grave{\varphi}}(t)\right), \ddot{\theta}(t)\right), \dot{\theta}(t)\right)$

$$LeqLSide\theta := \ddot{\grave{\varphi}}(t)\cos\left(\theta(t)\right) l\,mp\,r - mp\,g\,l\sin\left(\theta(t)\right) + 2\,mp\,l^2\,\ddot{\theta}(t) \tag{25}$$

$$LeqLSide := Vector(2, \{1 = LeqLSide\varphi, 2 = LeqLSide\theta\})$$

$$LeqLSide := \begin{bmatrix} r\cos(\theta(t))\ l\ mp\ \ddot{\theta}(t) + r\ (mp\ r + 2\ mw\ r)\ \ddot{\varphi}(t) - r\sin(\theta(t))\ \dot{\theta}(t)^2\ l\ mp \\ \ddot{\varphi}(t)\ \cos(\theta(t))\ l\ mp\ r - mp\ g\ l\sin(\theta(t)) + 2\ mp\ l^2\ \ddot{\theta}(t) \end{bmatrix} \quad (26)$$

$$M := [coeff(LeqLSide, \ddot{\varphi}(t)), coeff(LeqLSide, \ddot{\theta}(t))]$$

$$M := \left[\begin{bmatrix} r\ (mp\ r + 2\ mw\ r) \\ r\cos(\theta(t))\ l\ mp \end{bmatrix}, \begin{bmatrix} r\cos(\theta(t))\ l\ mp \\ 2\ mp\ l^2 \end{bmatrix}\right] \quad (27)$$

$$M := convert(M, Matrix)$$

$$M := \begin{bmatrix} r\ (mp\ r + 2\ mw\ r) & r\cos(\theta(t))\ l\ mp \\ r\cos(\theta(t))\ l\ mp & 2\ mp\ l^2 \end{bmatrix} \quad (28)$$

$$ddQ(t)$$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix}$$

$$MatrixVectorMultiply(M, ddQ(t))$$

$$\begin{bmatrix} r\cos(\theta(t))\ l\ mp\ \ddot{\theta}(t) + r\ (mp\ r + 2\ mw\ r)\ \ddot{\varphi}(t) \\ \ddot{\varphi}(t)\ \cos(\theta(t))\ l\ mp\ r + 2\ mp\ l^2\ \ddot{\theta}(t) \end{bmatrix} \quad (30)$$

$$tmp := LeqLSide - MatrixVectorMultiply(M, ddQ(t))$$

$$tmp := \begin{bmatrix} -r\sin(\theta(t))\ \dot{\theta}(t)^2\ l\ mp \\ -mp\ g\ l\sin(\theta(t)) \end{bmatrix}$$

$$C := \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, coeff(tmp, \dot{\theta}^2) \cdot \dot{\theta}(t)\right]$$

$$C := \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\dot{\theta}(t)\ r\sin(\theta(t))\ l\ mp \\ 0 \end{bmatrix}\right] \quad (33)$$

$$C := convert(C, Matrix)$$

$$C := \begin{bmatrix} 0 & -\dot{\theta}(t)\ r\sin(\theta(t))\ l\ mp \\ 0 & 0 \end{bmatrix} \quad (34)$$

$$C$$

$$\begin{bmatrix} 0 & -\dot{\theta}(t)\ r\sin(\theta(t))\ l\ mp \\ 0 & 0 \end{bmatrix} \quad (35)$$

$dQ(t)$

$$\begin{bmatrix} \dot{\grave{\varphi}}(t) \\ \dot{\theta}(t) \end{bmatrix} \tag{36}$$

$MatrixVectorMultiply(C, dQ(t))$

$$\begin{bmatrix} -r\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp \\ 0 \end{bmatrix} \tag{37}$$

$G := LeqLSide - MatrixVectorMultiply(M, ddQ(t)) - MatrixVectorMultiply(C, dQ(t))$

$$G := \begin{bmatrix} 0 \\ -mp\,g\,l\sin(\theta(t)) \end{bmatrix} \tag{38}$$

$M\,ddQ(t) + C\,dQ(t) + G = \tau$

$$\begin{bmatrix} r\cos(\theta(t))\,l\,mp\,\ddot{\theta}(t) + r\,(mp\,r + 2\,mw\,r)\,\ddot{\grave{\varphi}}(t) - r\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp \\ \ddot{\grave{\varphi}}(t)\cos(\theta(t))\,l\,mp\,r - mp\,g\,l\sin(\theta(t)) + 2\,mp\,l^2\,\ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} Mk(t) \\ 0 \end{bmatrix} \tag{39}$$

M^{-1}

$$\begin{bmatrix} -\dfrac{2}{r^2\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)} & \dfrac{\cos(\theta(t))}{r\,l\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)} \\ \dfrac{\cos(\theta(t))}{r\,l\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)} & -\dfrac{mp + 2\,mw}{l^2\,mp\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)} \end{bmatrix} \tag{40}$$

$(\tau - G - C\,dQ(t))$

$$\begin{bmatrix} Mk(t) + r\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp \\ mp\,g\,l\sin(\theta(t)) \end{bmatrix} \tag{41}$$

$ResultDDQ(t) := factor(simplify(MatrixVectorMultiply(M^{-1}, (\tau - G - MatrixVectorMultiply(C, dQ(t))))))$

$$ResultDDQ := t \mapsto factor\left(simplify\left(LinearAlgebra:-MatrixVectorMultiply\left(\frac{1}{M}, \tau - G - LinearAlgebra:-MatrixVectorMultiply(C, dQ(t))\right)\right)\right) \tag{42}$$

$ddQ(t) = ResultDDQ(t)$

$$\begin{bmatrix} \ddot{\grave{\varphi}}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \left[\left[\frac{-2\,r\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp + \cos(\theta(t))\,mp\,g\sin(\theta(t))\,r - 2\,Mk(t)}{r^2\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)}\right], \left[\frac{1}{r\,l\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)}\left(\cos(\theta(t))\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp\,r - g\sin(\theta(t))\,r\,mp - 2\,g\sin(\theta(t))\,r\,mw + \cos(\theta(t))\,Mk(t)\right)\right]\right] \tag{43}$$

$$sys1 := [ddQ(t)[1] = ResultDDQ(t)[1], ddQ(t)[2] = ResultDDQ(t)[2]]$$

$$sys1 := \left[\ddot{\varphi}(t) = \frac{-2\, r \sin(\theta(t))\, \dot{\theta}(t)^2\, l\, mp + \cos(\theta(t))\, mp\, g \sin(\theta(t))\, r - 2\, Mk(t)}{r^2 \left(\cos(\theta(t))^2\, mp - 2\, mp - 4\, mw\right)}, \ddot{\theta}(t) \right. \tag{44}$$

$$= \frac{1}{r\, l \left(\cos(\theta(t))^2\, mp - 2\, mp - 4\, mw\right)} \left(\cos(\theta(t))\, \sin(\theta(t))\, \dot{\theta}(t)^2\, l\, mp\, r \right.$$

$$\left. - g \sin(\theta(t))\, r\, mp - 2\, g \sin(\theta(t))\, r\, mw + \cos(\theta(t))\, Mk(t) \right) \Bigg]$$

$$subs(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQ(t)[2])$$

$$\frac{\cos(x)\, \sin(x)\, q^2\, l\, mp\, r - g \sin(x)\, r\, mp - 2\, g \sin(x)\, r\, mw + \cos(x)\, Mk(t)}{r\, l \left(\cos(x)^2\, mp - 2\, mp - 4\, mw\right)} \tag{45}$$

$$ResultDDQLineal(t)$$

$$:= \begin{bmatrix} subs(q = \dot{\theta}(t), x = \theta(t), taylor(subs(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQ(t)[1]), x, 3)) \\ subs(q = \dot{\theta}(t), x = \theta(t), taylor(subs(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQ(t)[2]), x, 3)) \end{bmatrix}$$

$$ResultDDQLineal := t \mapsto Vector_{column}(1..2, \{1 = subs(q = \dot{\theta}(t), x = \theta(t), taylor(subs(\dot{\theta}(t) = q, \tag{46}$$

$$\theta(t) = x, ResultDDQ(t)_1), x, 3)), 2 = subs(q = \dot{\theta}(t), x = \theta(t), taylor(subs(\dot{\theta}(t) = q, \theta(t)$$

$$= x, ResultDDQ(t)_2), x, 3))\}, datatype = anything, storage = rectangular, order$$

$$= Fortran\_order, subtype = Vector_{column})$$

$$ddQ(t) = ResultDDQLineal(t)$$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \left[\left[-\frac{2\, Mk(t)}{r^2\, (-mp - 4\, mw)} + \frac{\left(-2\, r\, \dot{\theta}(t)^2\, l\, mp + mp\, g\, r\right) \theta(t)}{r^2\, (-mp - 4\, mw)} \right. \right. \tag{47}$$

$$\left. + \frac{2\, Mk(t)\, mp\, \theta(t)^2}{r^2\, (mp + 4\, mw)\, (-mp - 4\, mw)} + O(\theta(t)^3) \right],$$

$$\left[\frac{Mk(t)}{r\, l\, (-mp - 4\, mw)} + \frac{\left(r\, \dot{\theta}(t)^2\, l\, mp - mp\, g\, r - 2\, g\, r\, mw\right) \theta(t)}{r\, l\, (-mp - 4\, mw)} \right.$$

$$\left. \left. + \frac{\left(-\frac{Mk(t)}{2\, r\, l} - \frac{Mk(t)\, mp}{r\, l\, (mp + 4\, mw)}\right) \theta(t)^2}{-mp - 4\, mw} + O(\theta(t)^3) \right] \right]$$

$$ResultDDQLineal(t) := subs[inplace]\left(\cos(\theta(t))^2 = 1, \cos(\theta(t)) = 1, \sin(\theta(t))^2 = (\theta(t))^2, \right.$$

$$\left. \sin(\theta(t)) = \theta(t), \dot{\theta}(t)^2 = 0, ResultDDQ(t)\right)$$

$$ResultDDQLineal := t \mapsto subs_{inplace}\left(\cos(\theta(t))^2 = 1, \cos(\theta(t)) = 1, \sin(\theta(t))^2 = \theta(t)^2, \right. \tag{48}$$

$$\left. \sin(\theta(t)) = \theta(t), \dot{\theta}(t)^2 = 0, ResultDDQ(t)\right)$$

$$ddQ(t) = factor(simplify(expand(ResultDDQLineal(t))))$$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{g\,\theta(t)\,r\,mp - 2\,Mk(t)}{r^2\,(mp + 4\,mw)} \\ \dfrac{g\,\theta(t)\,r\,mp + 2\,g\,\theta(t)\,r\,mw - Mk(t)}{r\,l\,(mp + 4\,mw)} \end{bmatrix} \tag{49}$$

$$InputV(t) := \begin{bmatrix} \varphi(t) \\ diff(\varphi(t), t) \\ \theta(t) \\ diff(\theta(t), t) \end{bmatrix}$$

$$InputV := t \mapsto Vector_{column}\left(1..4,\ \{1 = \varphi(t), 2 = \dot{\varphi}(t), 3 = \theta(t), 4 = \dot{\theta}(t)\},\ datatype = anything,\ storage = rectangular,\ order = Fortran\_order,\ subtype = Vector_{column}\right) \tag{50}$$

$$InputV(t)$$

$$linealsys1 := mtaylor(sys1, [InputV])$$

$$linealsys1 := \left[\ddot{\varphi}(t) = \frac{-2\,r\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp + \cos(\theta(t))\,mp\,g\sin(\theta(t))\,r - 2\,Mk(t)}{r^2\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)},\right. \tag{51}$$

$$\ddot{\theta}(t) = \frac{1}{r\,l\left(\cos(\theta(t))^2\,mp - 2\,mp - 4\,mw\right)}\left(\cos(\theta(t))\sin(\theta(t))\,\dot{\theta}(t)^2\,l\,mp\,r\right.$$

$$\left.\left. - g\sin(\theta(t))\,r\,mp - 2\,g\sin(\theta(t))\,r\,mw + \cos(\theta(t))\,Mk(t)\right)\right]$$

$$ddQ(t) = simplify\left(subs\left(r = \frac{50}{1000}, l = 1, g = 9.8, mp = 1, mw = 0.1, ResultDDQLineal(t)\right)\right)$$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -140.0000000\,\theta(t) + 571.4285716\,Mk(t) \\ 8.400000000\,\theta(t) - 14.28571429\,Mk(t) \end{bmatrix} \tag{52}$$

$$\frac{1}{-14.28571429}$$

$$-0.06999999998 \tag{53}$$

$$\frac{\left(\dfrac{8.400000000 \cdot 3.14}{4} - 1\right)}{14.28571429}$$

$$0.3915799999 \tag{54}$$

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{g\cdot r\cdot mp}{r^2\cdot(mp+4\,mw)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{g\cdot r\cdot(mp+2\,mw)}{r\cdot l\cdot(mp+4\,mw)} & 0 \end{bmatrix}$$

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{mp\,g}{r\,(mp+4\,mw)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{g\,(mp+2\,mw)}{l\,(mp+4\,mw)} & 0 \end{bmatrix} \tag{55}$$

$$B := \begin{bmatrix} 0 \\ \dfrac{2}{r^2\cdot(mp+4\,mw)} \\ 0 \\ -\dfrac{1}{r\cdot l\cdot(mp+4\,mw)} \end{bmatrix}$$

$$B := \begin{bmatrix} 0 \\ \dfrac{2}{r^2\,(mp+4\,mw)} \\ 0 \\ -\dfrac{1}{r\,l\,(mp+4\,mw)} \end{bmatrix} \tag{56}$$

$InputV(t) = A\,InputV(t) + B\cdot Mk(t)$

$$\begin{bmatrix} \varphi(t) \\ \dot{\varphi}(t) \\ \theta(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \dot{\varphi}(t) \\ -\dfrac{mp\,g\,\theta(t)}{r\,(mp+4\,mw)} + \dfrac{2\,Mk(t)}{r^2\,(mp+4\,mw)} \\ \dot{\theta}(t) \\ \dfrac{g\,(mp+2\,mw)\,\theta(t)}{l\,(mp+4\,mw)} - \dfrac{Mk(t)}{r\,l\,(mp+4\,mw)} \end{bmatrix} \tag{57}$$

$subs(g = 9.8,\ mp = 1,\ mw = 0.01,\ l = 1,\ r = 0.01,\ A)$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -942.3076923 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 9.611538462 & 0 \end{bmatrix} \tag{58}$$

$subs(g = 9.8, mp = 1, mw = 0.01, l = 1, r = 0.01, B)$

$$\begin{bmatrix} 0 \\ 19230.76923 \\ 0 \\ -96.15384615 \end{bmatrix} \tag{59}$$

$MatrixVectorMultiply(A, InputV(t)) + B \cdot Mk(t)$

$$\begin{bmatrix} \dot{\varphi}`(t) \\ -\frac{mp\, g\, \theta(t)}{r\,(mp + 4\, mw)} + \frac{2\, Mk(t)}{r^2\,(mp + 4\, mw)} \\ \dot{\theta}(t) \\ \frac{g\,(mp + 2\, mw)\, \theta(t)}{l\,(mp + 4\, mw)} - \frac{Mk(t)}{r\, l\,(mp + 4\, mw)} \end{bmatrix} \tag{60}$$

$ddQ(t) = Expand(subs(g = 9.8, mp = 1, mw = 0.1, l = 0.3, r = 0.1, ResultDDQ(t)))$

$$\begin{bmatrix} \ddot{\varphi}`(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{\cos(\theta(t))^2 - 2.400000000} \left(-6.000000000 \sin(\theta(t))\, \dot{\theta}(t)^2 \right. \\ \left. + 98.00000000 \cos(\theta(t))\, \sin(\theta(t)) - 200.0000000\, Mk(t) \right), \\ \frac{1}{\cos(\theta(t))^2 - 2.400000000} \left(0.9999999999 \cos(\theta(t))\, \sin(\theta(t))\, \dot{\theta}(t)^2 \right. \\ \left. - 39.20000000 \sin(\theta(t)) + 33.33333333 \cos(\theta(t))\, Mk(t) \right) \end{bmatrix} \tag{61}$$

$ResultDDQLineal2(t)$

$$:= \begin{bmatrix} subs\left(q = \dot{\theta}(t), x = \theta(t), taylor\left(subs\left(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQLineal(t)[1]\right), q, 1\right)\right) \\ subs\left(q = \dot{\theta}(t), x = \theta(t), taylor\left(subs\left(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQLineal(t)[2]\right), q, 1\right)\right) \end{bmatrix}$$

$$ResultDDQLineal2 := t \mapsto Vector_{column}\left(1..2, \left\{1 = subs\left(q = \dot{\theta}(t), x = \theta(t), taylor\left(subs\left(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQLineal(t)_1\right), q, 1\right)\right), 2 = subs\left(q = \dot{\theta}(t), x = \theta(t), taylor\left(subs\left(\dot{\theta}(t) = q, \theta(t) = x, ResultDDQLineal(t)_2\right), q, 1\right)\right)\right\}, datatype = anything, storage = rectangular, order = Fortran\_order, subtype = Vector_{column}\right) \tag{62}$$

$ResultDDQLineal2(t)$

$$\begin{bmatrix} \dfrac{g\,\theta(t)\,r\,mp - 2\,Mk(t)}{r^2\,(-mp - 4\,mw)} \\ \dfrac{-g\,\theta(t)\,r\,mp - 2\,g\,\theta(t)\,r\,mw + Mk(t)}{r\,l\,(-mp - 4\,mw)} \end{bmatrix} \tag{63}$$

$ddQ(t) = Expand(subs(g = 9.8, mp = 1, mw = 0.04, l = 0.5, r = 0.09, ResultDDQ(t)))$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \left[\dfrac{1}{\cos(\theta(t))^2 - 2.160000000}\left(-11.11111111\,\sin(\theta(t))\,\dot{\theta}(t)^2 + 108.8888889\,\cos(\theta(t))\,\sin(\theta(t)) - 246.9135802\,Mk(t)\right)\right], \\ \left[\dfrac{1}{\cos(\theta(t))^2 - 2.160000000}\left(0.9999999999\,\cos(\theta(t))\,\sin(\theta(t))\,\dot{\theta}(t)^2 - 21.16800000\,\sin(\theta(t)) + 22.22222222\,\cos(\theta(t))\,Mk(t)\right)\right] \end{bmatrix} \tag{64}$$

$subs(g = 9.8, mp = 1, mw = 0.04, l = 0.5, r = 0.09, A)$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -93.86973179 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 18.24827586 & 0 \end{bmatrix} \tag{65}$$

$subs(g = 9.8, mp = 1, mw = 0.04, l = 0.5, r = 0.09, B)$

$$\begin{bmatrix} 0 \\ 212.8565347 \\ 0 \\ -19.15708812 \end{bmatrix} \tag{66}$$

$ddQ(t) = Expand(subs(g = 9.8, mp = 1, mw = 0.04, l = 0.5, r = 0.09, ResultDDQLineal2(t)))$

$$\begin{bmatrix} \ddot{\varphi}(t) \\ \ddot{\theta}(t) \end{bmatrix} = \begin{bmatrix} -93.86973181\,\theta(t) + 212.8565347\,Mk(t) \\ 18.24827586\,\theta(t) - 19.15708812\,Mk(t) \end{bmatrix} \tag{67}$$