9.3 Dimensionality Reduction

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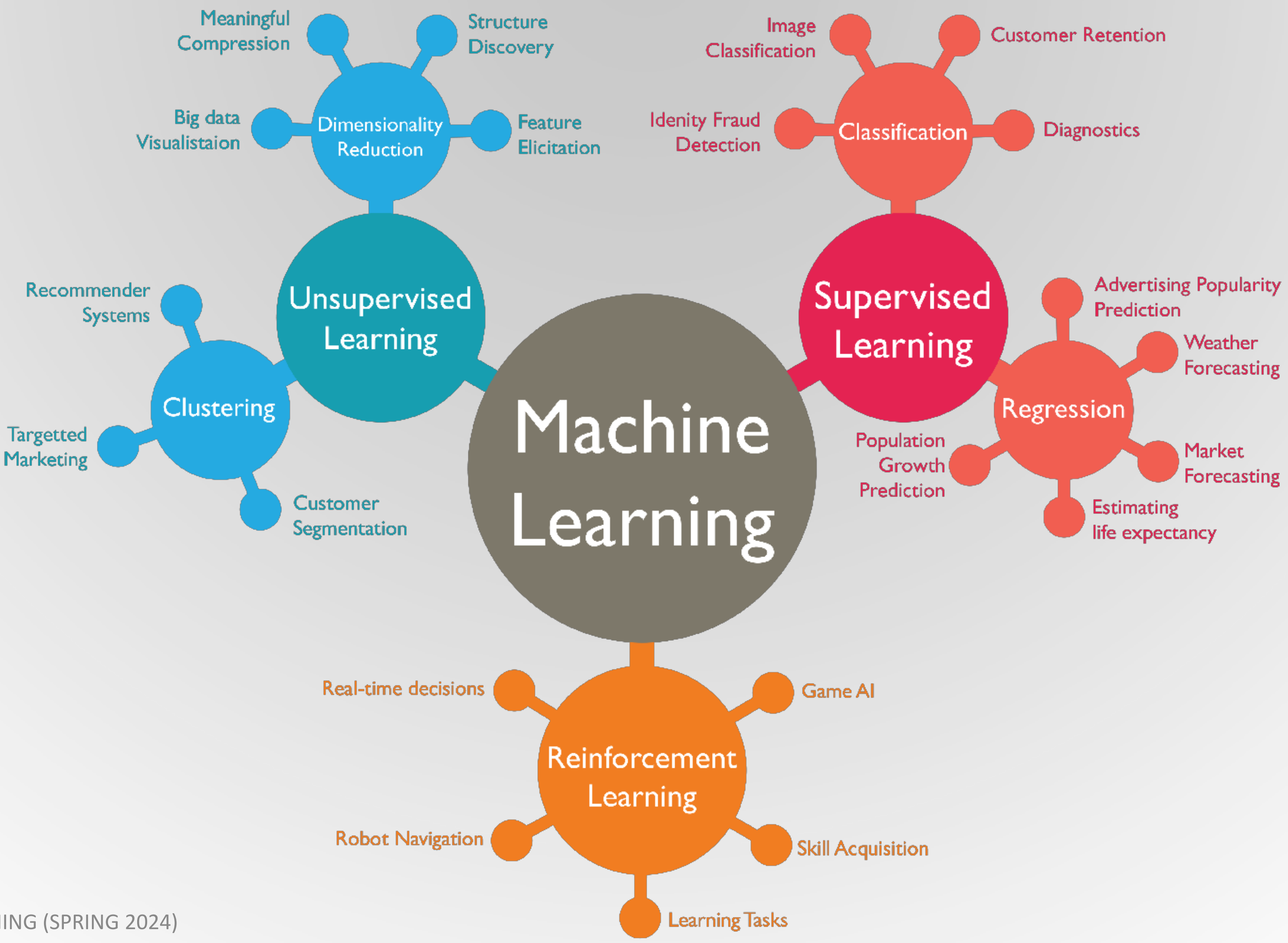
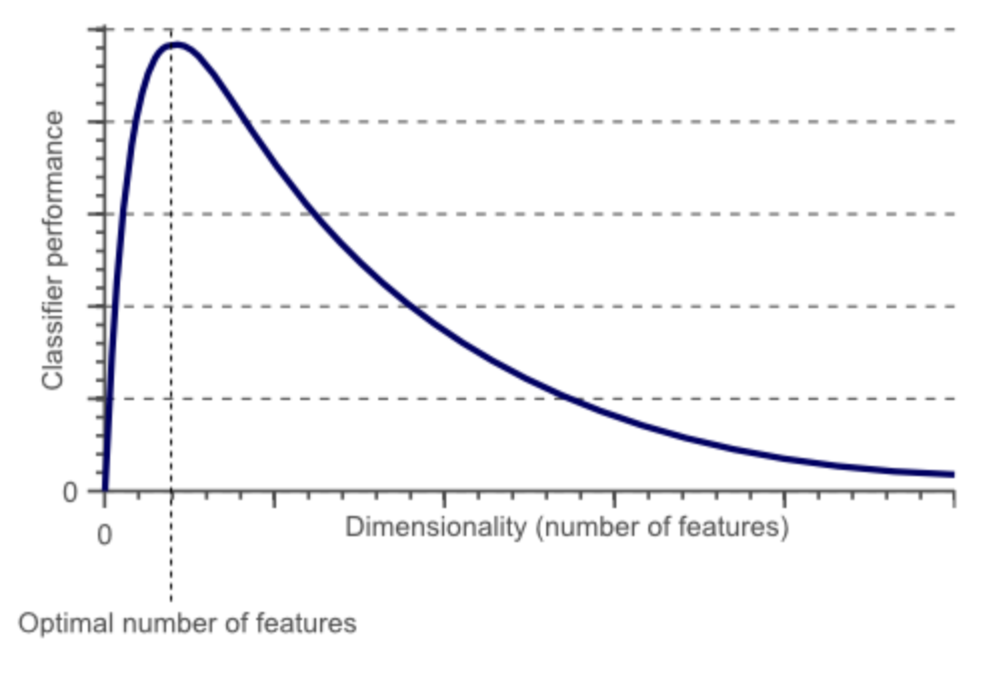
Machine Learning Approaches



Hughes Phenomenon

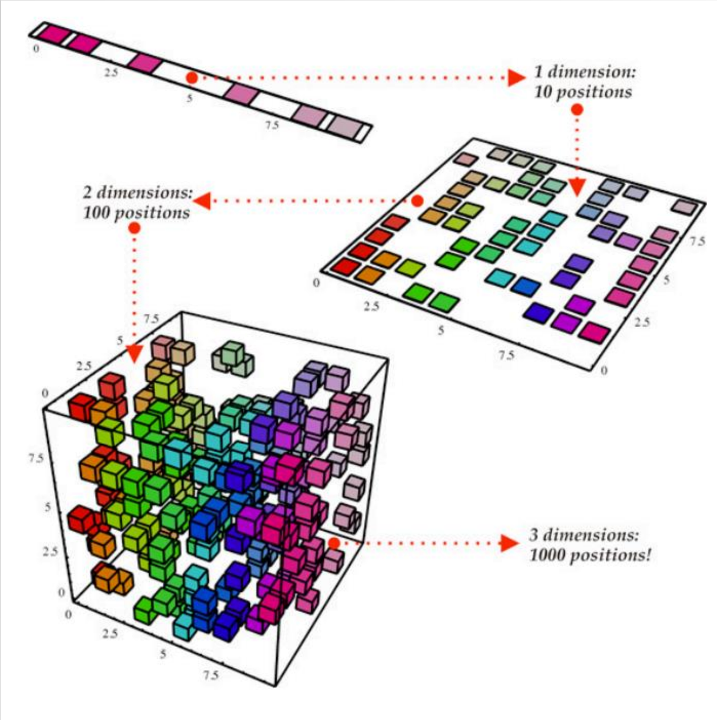
- According to Hughes phenomenon, If the number of training samples is fixed and we keep on increasing the number of dimensions then the predictive power of our machine learning model first increases, but after a certain point it tends to decrease.
- Also known as the **curse of dimensionality**



Dimensionality Reduction

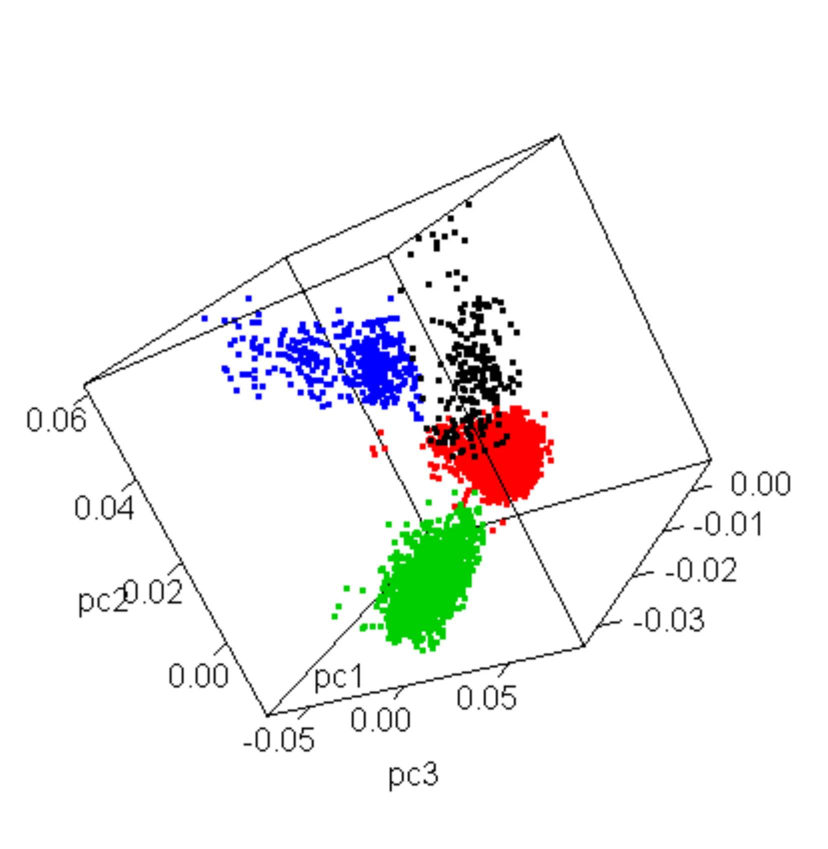
Dimensionality reduction, or dimension reduction, is the transformation of data **from a high-dimensional** space **into a low-dimensional** space so that the low-dimensional representation retains some meaningful properties of the original data, ideally close to its intrinsic dimension.

- **Applications**
  1. The most frequent use case for dimensionality reduction is **data visualization**
     - humans can only interpret on a plot the maximum of three dimensions.
  2. Dimensionality reduction **removes redundant** or highly correlated features;
  3. It also **reduces the noise** in the data
- Widely used **techniques** of dimensionality reduction
  - Principal Component Analysis (PCA)
  - Uniform Manifold Approximation and Projection (UMAP)
  - Autoencoders



Principal Components Analysis (PCA)

- Principal Components Analysis (PCA) is a technique that finds underlying variables (known as principal components) that best differentiate your data points.
- Principal components are **dimensions** along which your **data points are most spread out**.



Applications of PCA

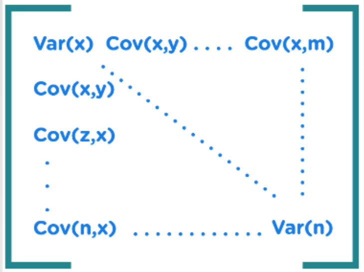
- Visualize multidimensional data.
- Reduce the number of dimensions in data.
- Help resize an image.
- Used in finance to analyze stock data and forecast returns.
- Find patterns in the high-dimensional datasets.

How PCA Works?

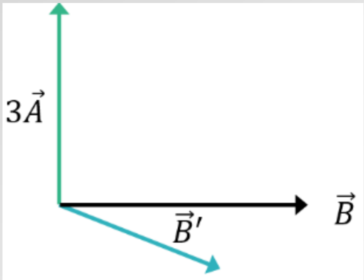
1. **Normalize** the data
   - Standardize the data before performing PCA.
   - This will ensure that each feature has a mean = 0 and variance = 1.

$$Z = \frac{x - \mu}{\sigma}$$

2. Build the **covariance matrix**
   - Construct a square matrix to express the correlation between two or more features in a multidimensional dataset.



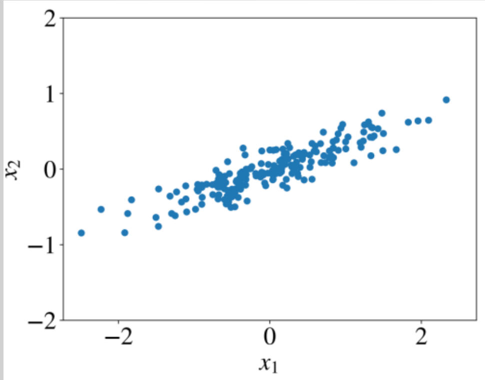
3. Find the **Eigenvectors** and Eigenvalues
   - Calculate the eigenvectors/unit vectors and eigenvalues.
   - Eigenvalues are scalars by which we multiply the eigenvector of the covariance matrix.



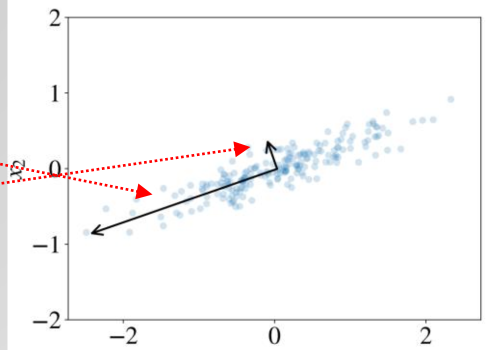
4. **Sort the eigenvectors** in highest to lowest order and select the number of principal components.

PCA Example

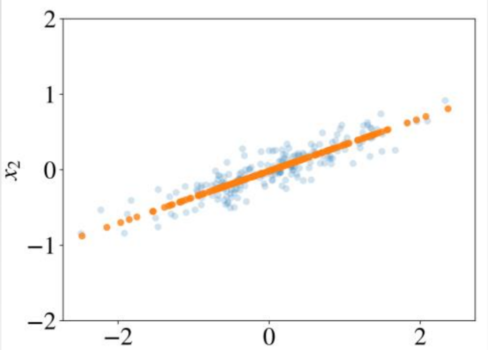
- Consider a two-dimensional data as shown in figure on the right:
  - Principal components are vectors that define a new coordinate system in which the **first axis** goes in the direction of the **highest variance** in the data.
  - The second axis is **orthogonal** to the first one and goes in the direction of the second highest variance in the data.
- If our data was three-dimensional:
  - The third axis would be orthogonal to both the first and the second axes and go in the direction of the third highest variance
- And so on for high dimensional data.



Original data

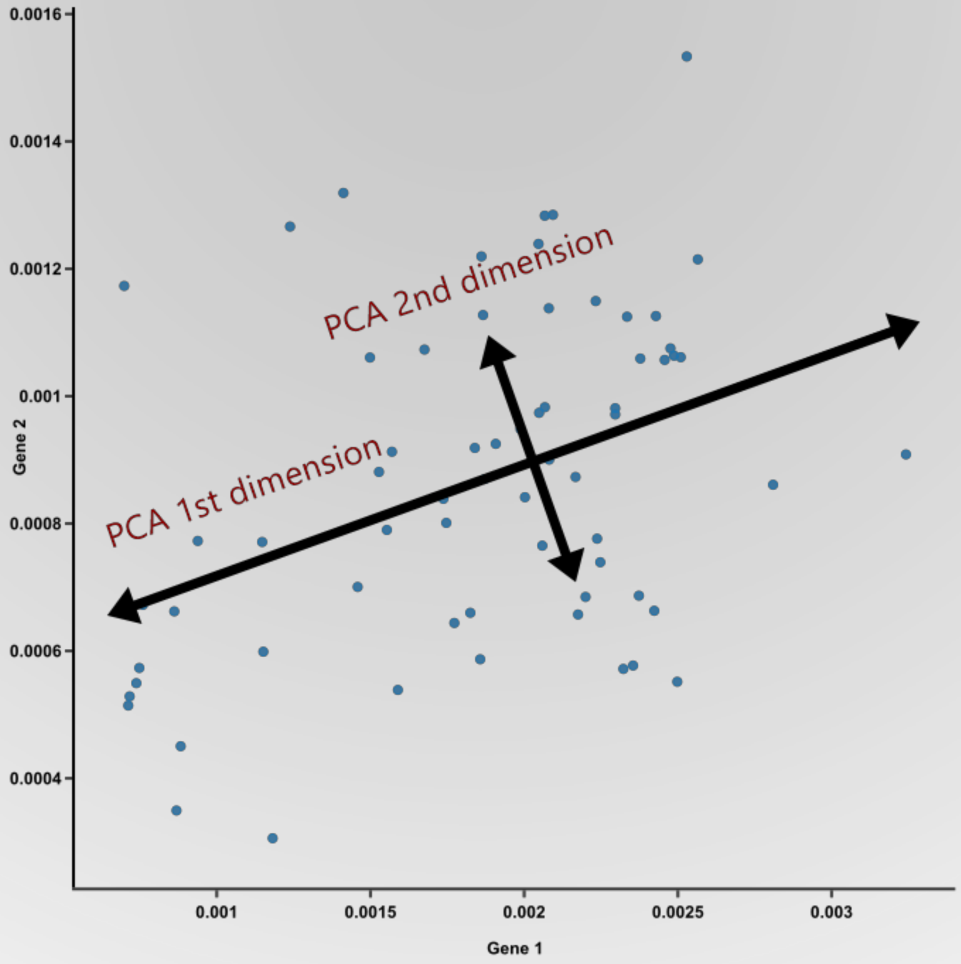


2 principal components displayed as vectors

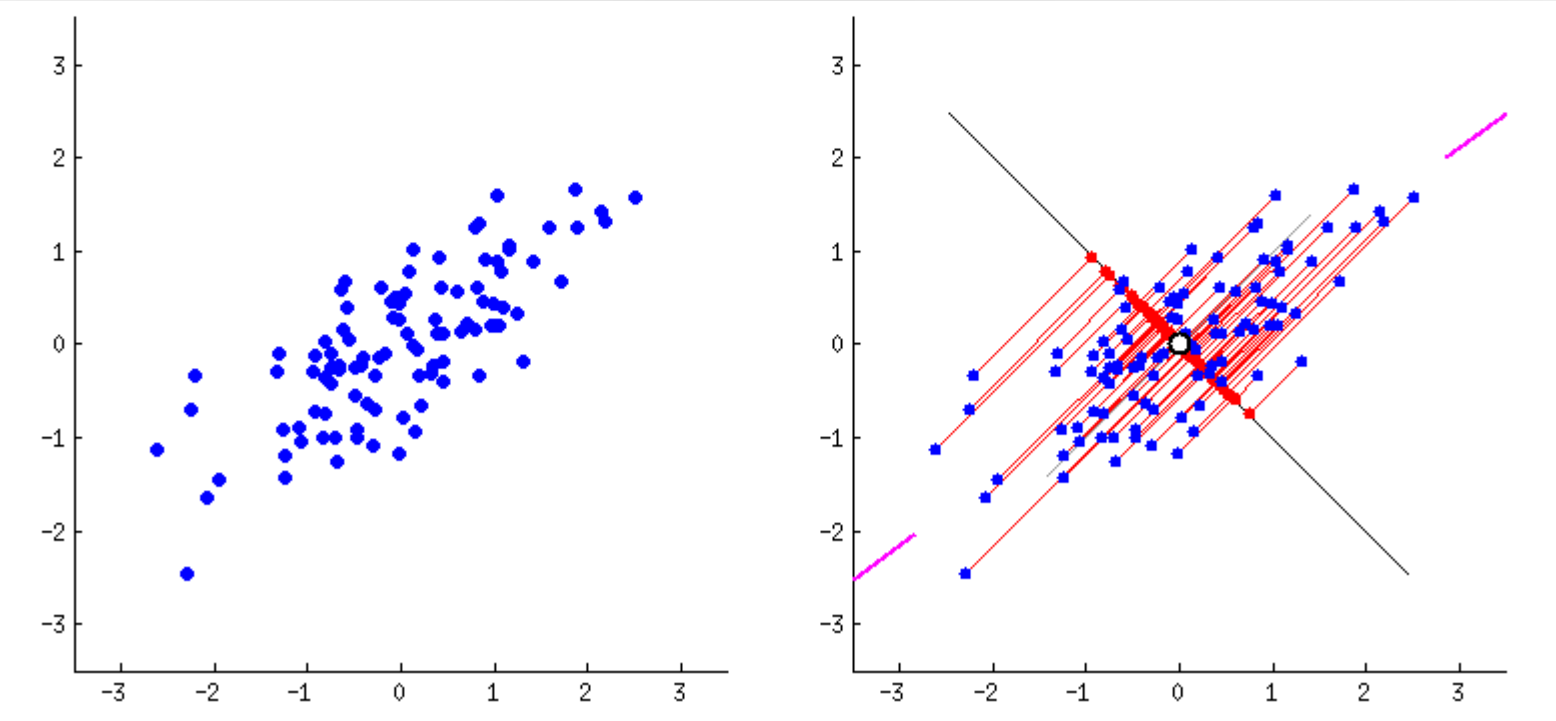


The data projected on the first principal component

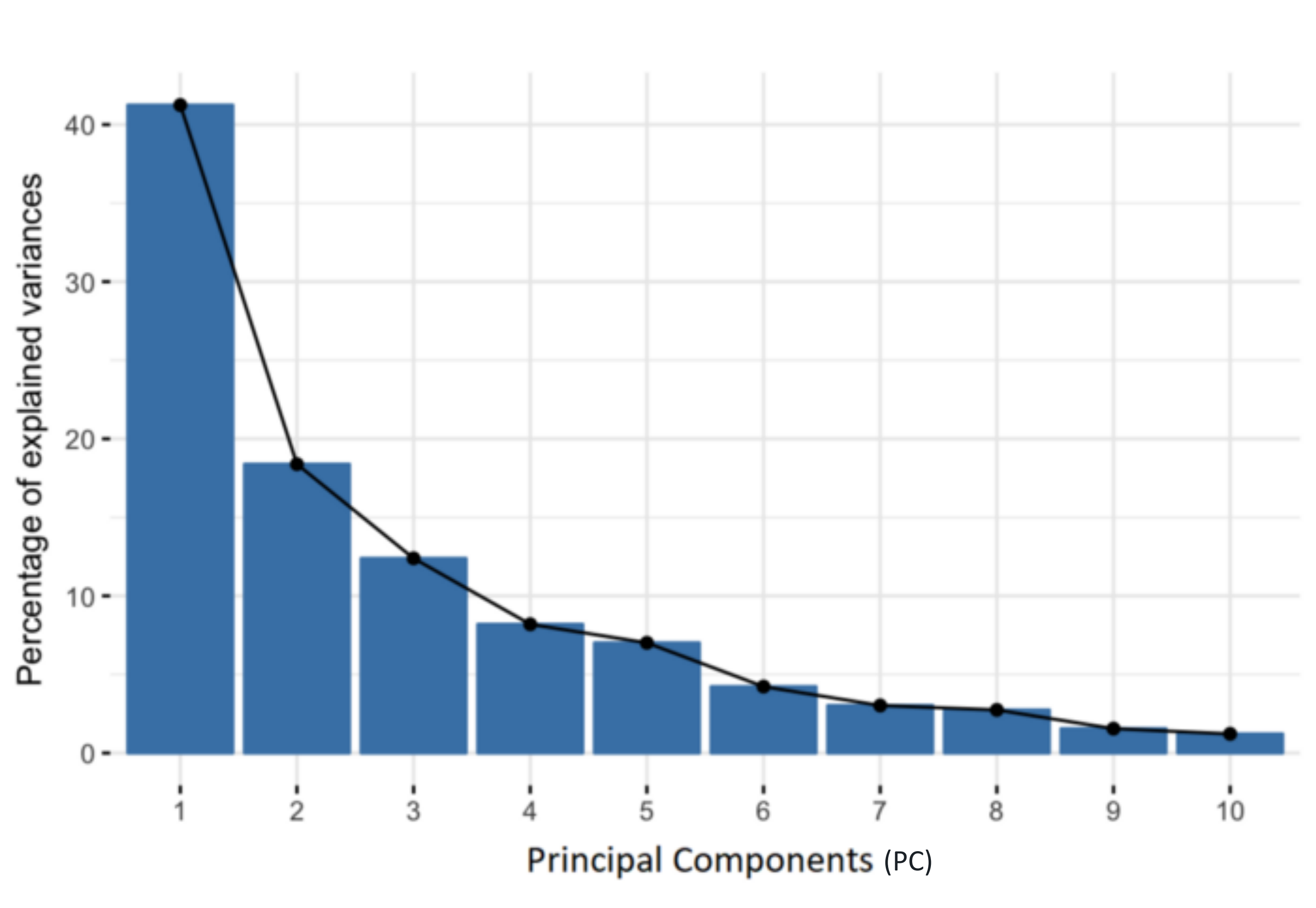
PCA Example


0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
Gene 2
0.001
0.0015
0.002
0.0025
0.003
Gene 1
PCA 2nd dimension
PCA 1st dimension

Example



Percentage of Variance (Information) for each by PC



Python Example

https://colab.research.google.com/drive/1eIddjy28cpp26pyM8qmJqdCABzG0VbBF?usp=sharing