Part I: Electricity

Chapter 26

Capacitance and Dielectrics

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LECTURE OUTLINE

- **26.1** Definition of Capacitance
- **26.2** Calculating Capacitance
- **26.3** Combinations of Capacitors
- **26.4** Energy Stored in a Charged Capacitor
- **26.5** Capacitors with Dielectrics

Circuits and Circuit Elements

- Electric circuits are the basis for the vast majority of the devices used in society.
- Circuit elements can be connected with wires to form electric circuits.
- Capacitors are one circuit element.
  – Others will be introduced in other chapters

Capacitors

- Capacitors are devices that store electric charge.
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

26.1 Definition of Capacitance

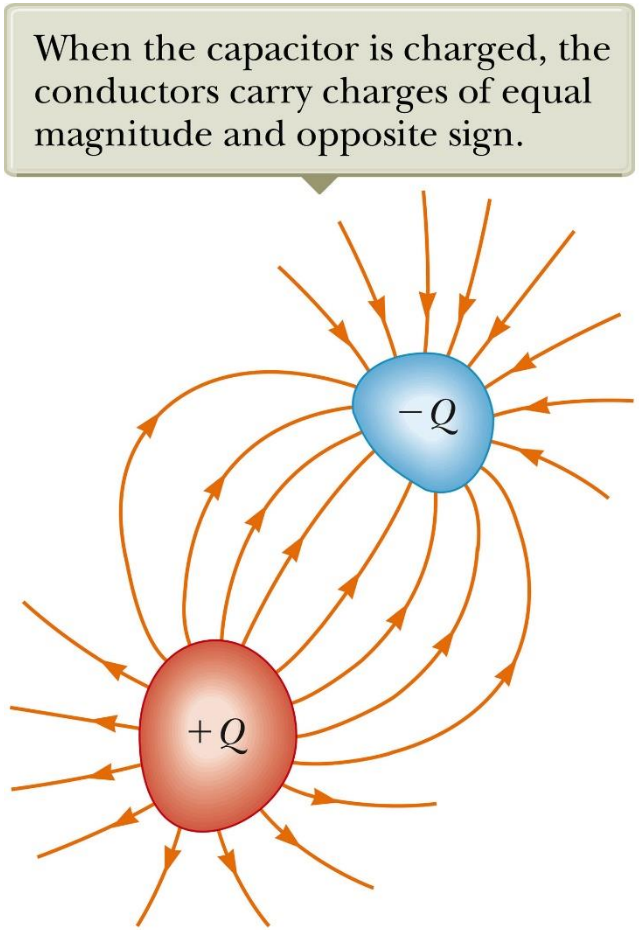
- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F)**= 1 coulomb per volt =1 C/V**.
- The farad is a large unit, typically you will see microfarads (mF) and picofarads (pF).
- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant.
- The capacitance is a measure of the capacitor's ability to store charge .
  - The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

Makeup of a Capacitor

- A capacitor consists of two conductors.
  - These conductors are called plates.
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.
- A potential difference exists between the plates due to the charge.



Capacitance:

To store charge & To store energy

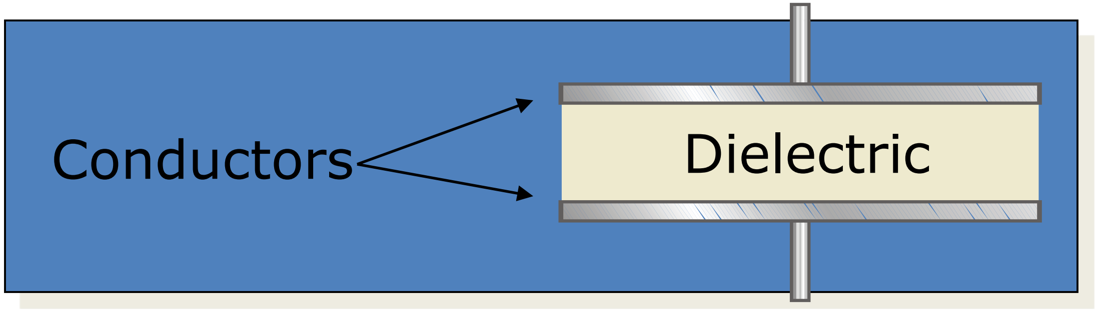
To control variation time scales in a circuit

The Capacitor

Capacitors are one of the fundamental passive components.

In its most basic form, it is composed of two plates separated by a dielectric.

The ability to store charge is the definition of **capacitance**.

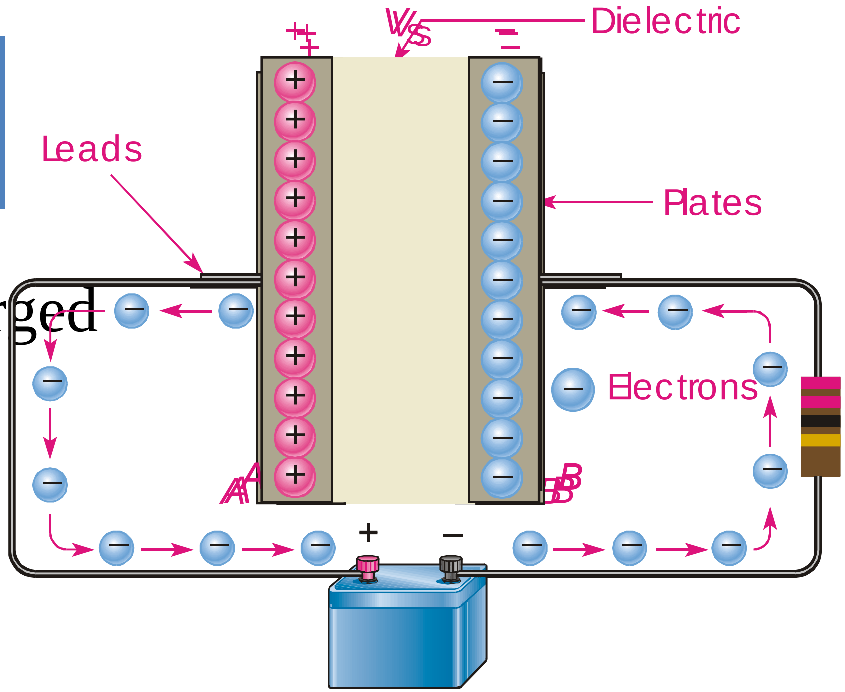


Summary

The Capacitor

The charging process…

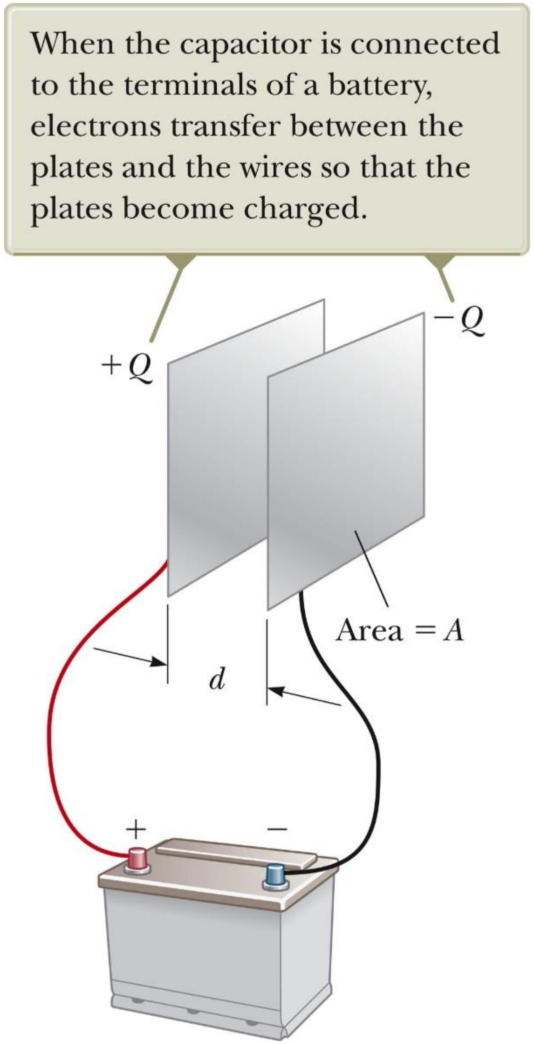
Initially uncharged
Charging
Fully charged
Source removed



A capacitor with stored charge can act as a temporary battery.

Parallel Plate Capacitor

- Each plate is connected to a terminal of the battery.
  - The battery is a source of potential difference.
- If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.



Parallel Plate Capacitor, cont

- This field applies a force on electrons in the wire just outside of the plates.
- The force causes the electrons to move onto the negative plate.
- This continues until equilibrium is achieved.
  - The plate, the wire and the terminal are all at the same potential.
- At this point, there is no field present in the wire and the movement of the electrons ceases.
- The plate is now negatively charged.
- A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged.
- In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

26.2 Calculating Capacitance

Capacitance – Isolated Sphere

- Assume a spherical charged conductor with radius *a.*
- The sphere will have the same capacitance as it would if there were a conducting sphere of infinite radius, concentric with the original sphere.
- The field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

26.2 Calculating Capacitance

Capacitance – Isolated Sphere

- To find the capacitance of the conductor, we first rewrite the capacitance as:

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

•Now letting b→∞, and substituting *R* for *a*,

$$\boxed{C = 4\pi\varepsilon_0 R \qquad \text{(isolated sphere).}}$$

•Or Assume *V* = 0 for the infinitely large shell

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / a} = \frac{R}{k_e} = 4\pi\varepsilon_o a$$

•Note, this is independent of the charge on the sphere and its potential.

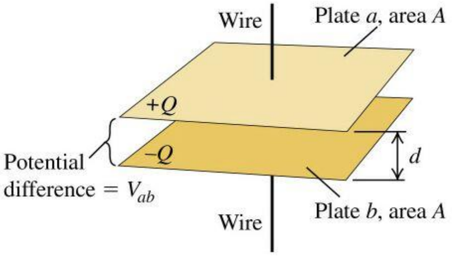
Parallel Plates- Capacitors

- The charge density on the plates is $\sigma = Q/A$.
  - A is the area of each plate, the area of each plate is equal
  - Q is the charge on each plate, equal with opposite signs
- The electric field is uniform between the plates and zero elsewhere.

- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \qquad V_{ab} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$



The capacitance is only dependent upon the geometry of the capacitor

Parallel Plates- Capacitors

$$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0} \qquad E = \frac{q}{A\varepsilon_0}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \int_-^+ E\,ds = E\int_0^d ds = Ed.$$

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$$

Parallel Plates- Capacitors
1 farad Capacitor

Example1: Given a 1 farad parallel plate capacitor having a plate separation of 1mm. What is the area of the plates?

$$C = \varepsilon_0 \frac{A}{d}$$

$$A = \frac{C\,d}{\varepsilon_0} = \frac{(1.0F)(1.0\times10^{-3}m)}{8.85\times10^{-12}F/m}$$

$$=1.1\times10^{8}m^{2}$$

This corresponds to a square of about 10km on a side!

Example

Capacitance

Find the capacitance of a 4.0 cm diameter sensor immersed in oil if the plates are separated by 0.25 mm.

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{\varepsilon_r A}{d}\right) \qquad \left(\varepsilon_r = 4.0 \text{ for oil}\right)$$

The plate area is $A = \pi r^2 = \pi\left(0.02\ \text{m}^2\right) = 1.26\times10^{-3}\ \text{m}^2$

The distance between the plates is $0.25\times10^{-3}\ \text{m}$

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{(4.0)\left(1.26\times10^{-3}\ \text{m}^2\right)}{0.25\times10^{-3}\ \text{m}}\right) = 178\ \text{pF}$$

Example, Charging the Plates in a Parallel-Plate Capacitor:

In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance $C = 0.25\ \mu F$ to the battery of potential difference $V = 12$ V. The lower capacitor plate has thickness $L = 0.50$ cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 \times 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

KEY IDEA

The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq. 25-1 ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is

$$q = CV = (0.25 \times 10^{-6}\ F)(12\ V) = 3.0 \times 10^{-6}\ C.$$

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.602 \times 10^{-19}\ C} = 1.873 \times 10^{13}\ \text{electrons}.$$

These electrons come from a volume that is the product of the face area A and the depth d we seek. Thus, from the density of conduction electrons (number per volume), we can write

$$n = \frac{N}{Ad},$$

or

$$d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28}\ \text{electrons/m}^3)} = 1.1 \times 10^{-12}\ m = 1.1\ \text{pm}. \quad \text{(Answer)}$$

In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

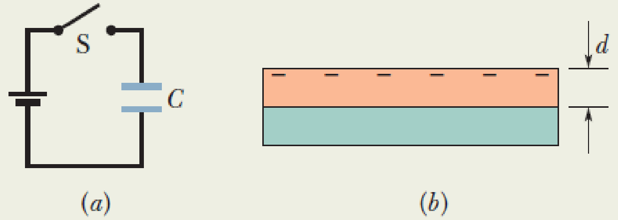
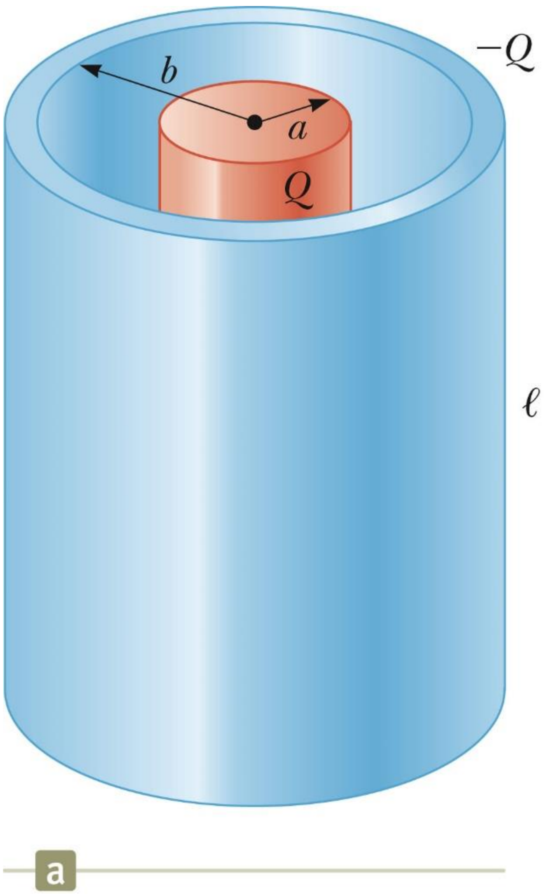


Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

The Cylindrical Capacitor

- $\Delta V = -2k_e\lambda \ln(b/a)$
- $\lambda = Q/\ell$
- The capacitance is

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)}$$



Calculating the Capacitance; A Cylindrical Capacitor :

As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder.

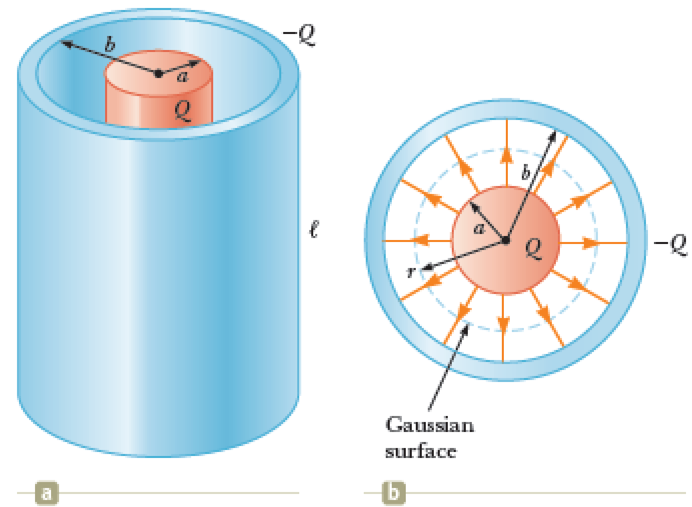


Figure 26.4 (Example 26.1) (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius a and length ℓ surrounded by a coaxial cylindrical shell of radius b. (b) End view. The electric field lines are radial. The dashed line represents the end of a cylindrical gaussian surface of radius r and length ℓ.

$$\oint \vec{E}.\,d\vec{A} = \frac{q}{\varepsilon_0}$$

$$EA = E(2\pi r L) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi\varepsilon_0 L r}.$$

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right),$$

$$\boxed{C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \quad \text{(cylindrical capacitor).}}$$

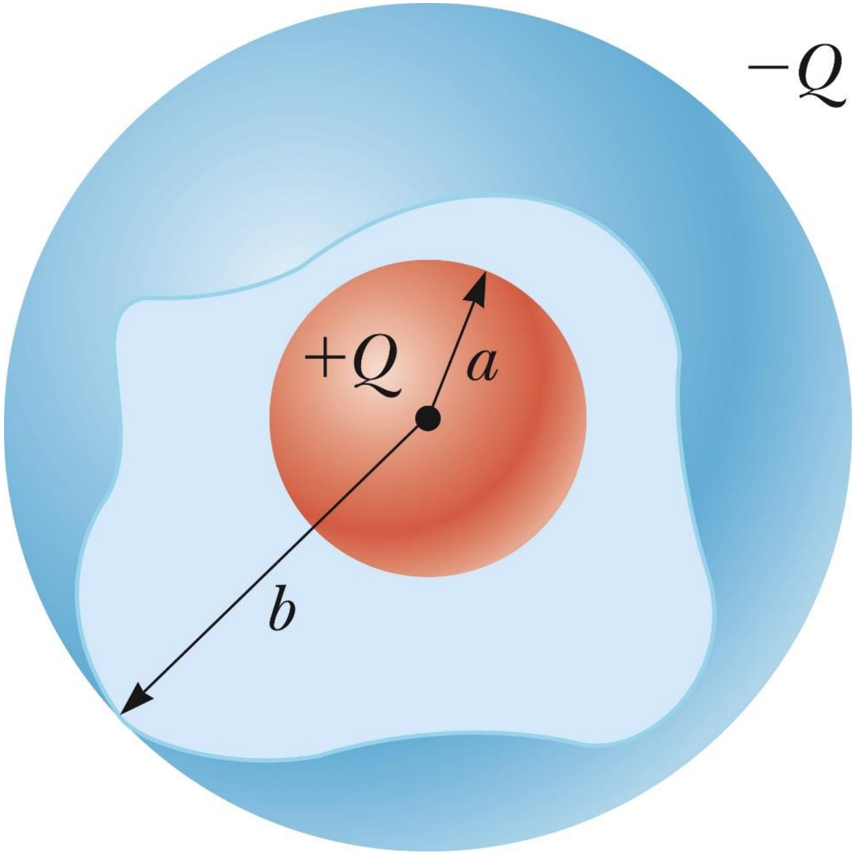
The Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b-a\right)}$$



Calculating the Capacitance; A Spherical Capacitor:

$$\oint \vec{E}.\, d\vec{A} = \frac{q}{\varepsilon_0}$$

$$EA = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$$

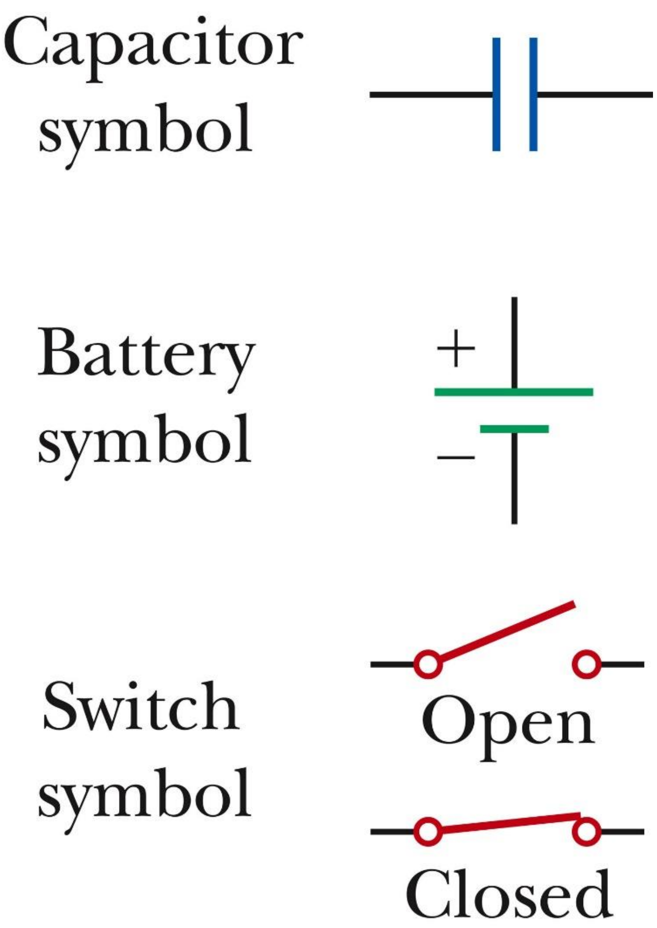
$$V = \int_{-}^{+} E\, ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab},$$

$$\boxed{C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad \text{(spherical capacitor).}}$$

26.3 Combinations of Capacitors

Circuit Symbols

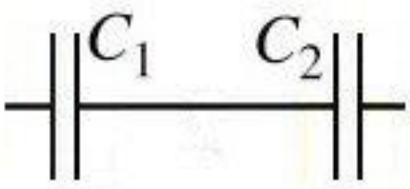
- A circuit diagram is a simplified representation of an actual circuit.
- Circuit symbols are used to represent the various elements.
- Lines are used to represent wires.
- The battery's positive terminal is indicated by the longer line.



Series or Parallel Capacitors

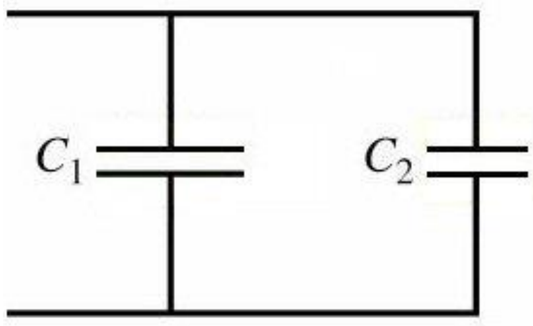
- Sometimes in order to obtain needed values of capacitance, capacitors are combined in either

Series



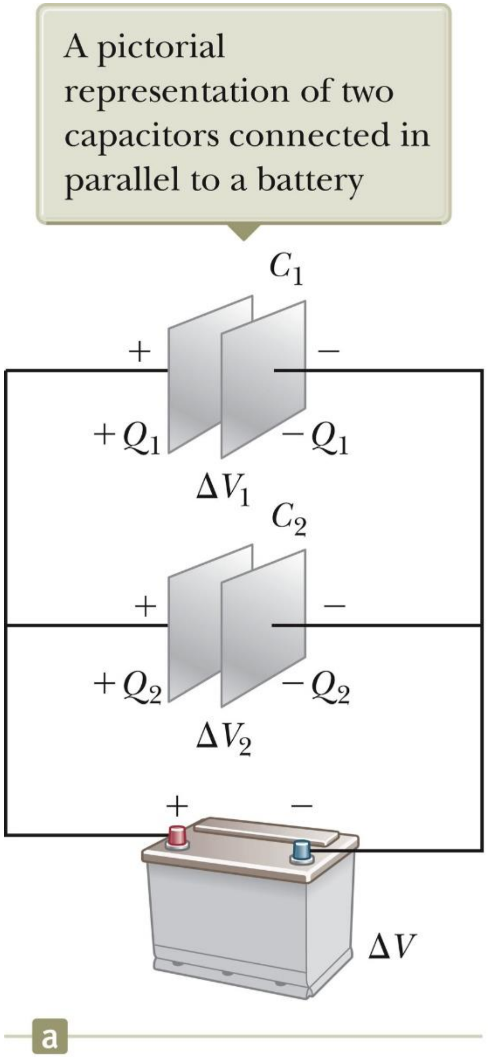
or

Parallel



Parallel Combination

•When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.



•The flow of charges ceases when the voltage across the capacitors equals that of the battery.

•The potential difference across the capacitors is the same.

- And each is equal to the voltage of the battery
- $\Delta V_1 = \Delta V_2 = \Delta V$
  - ΔV is the battery terminal voltage

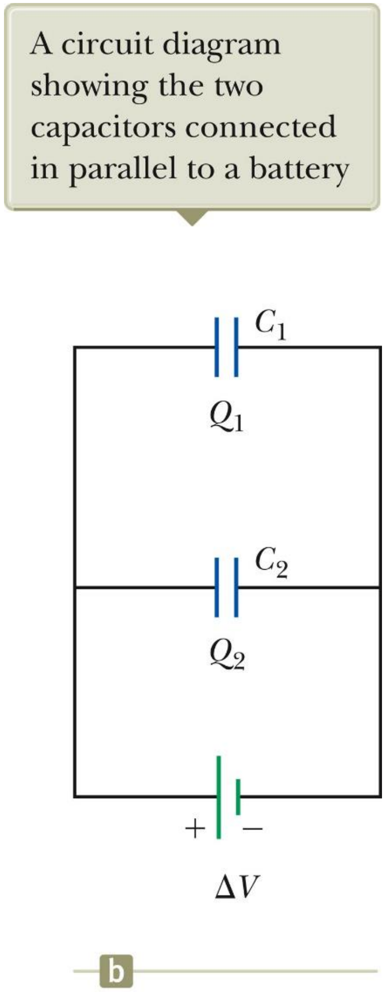
•The capacitors reach their maximum charge when the flow of charge ceases.

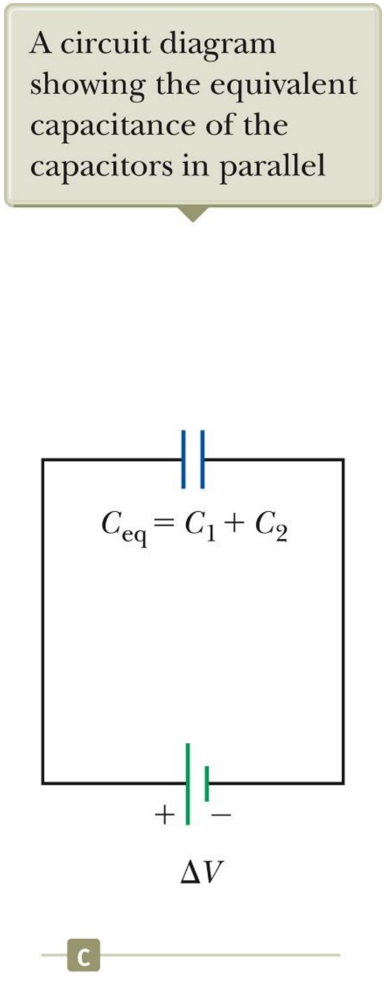
•The total charge is equal to the sum of the charges on the capacitors.

- $Q_{tot} = Q_1 + Q_2$

Parallel Combination

- The capacitors can be replaced with one capacitor with a capacitance of $C_{eq.}$
  - The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors.



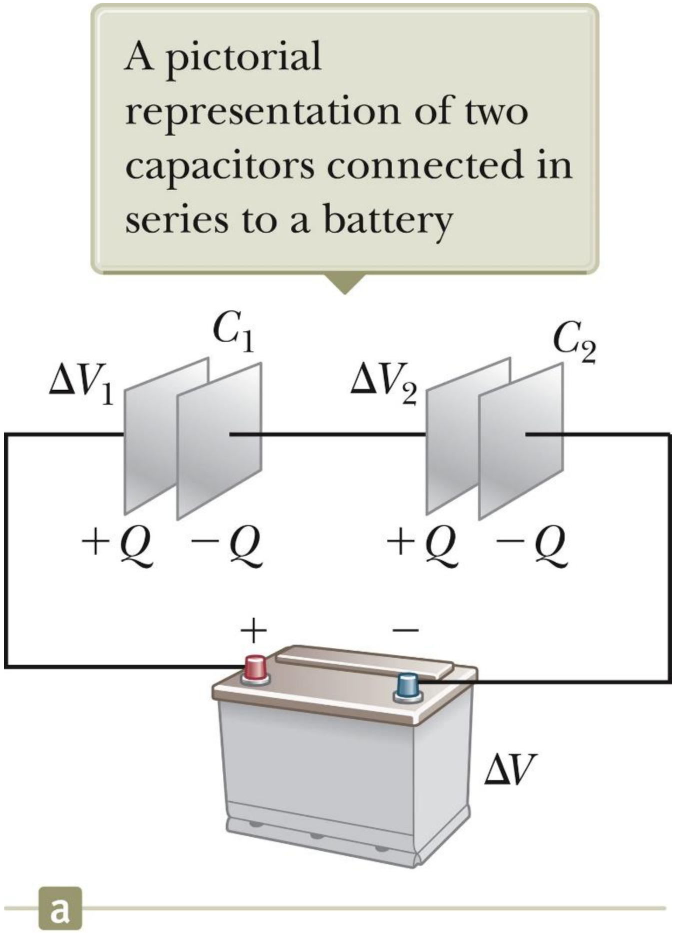


Parallel Combination

- $C_{eq} = C_1 + C_2 + C_3 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.
  - Essentially, the areas are combined

Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.
- All of the right plates gain charges of –Q and all the left plates have charges of $+Q$.



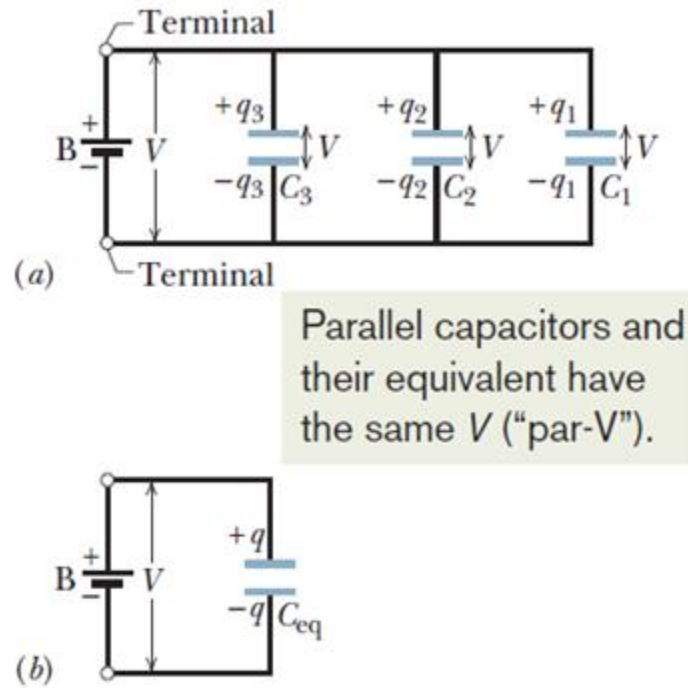
Capacitors in Parallel:

$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).}$$

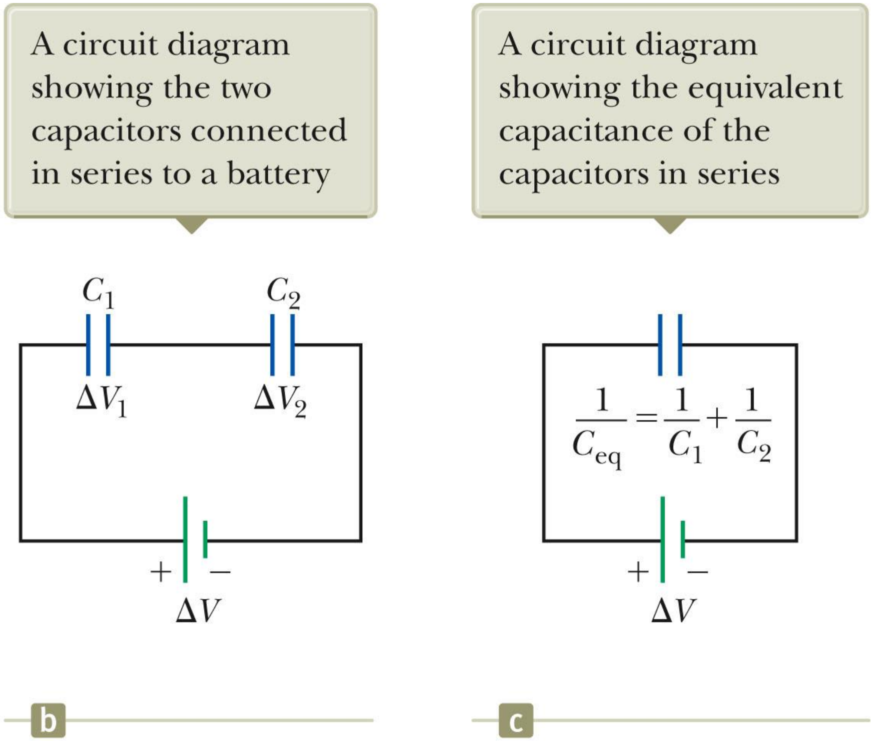


(*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

Capacitors in Series, cont.

- An equivalent capacitor can be found that performs the same function as the series combination.
- The charges are all the same.

$$Q_1 = Q_2 = Q$$



Capacitors in Series, final

- The potential differences add up to the battery voltage.

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \ldots$$

- The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.

Capacitors in Series:

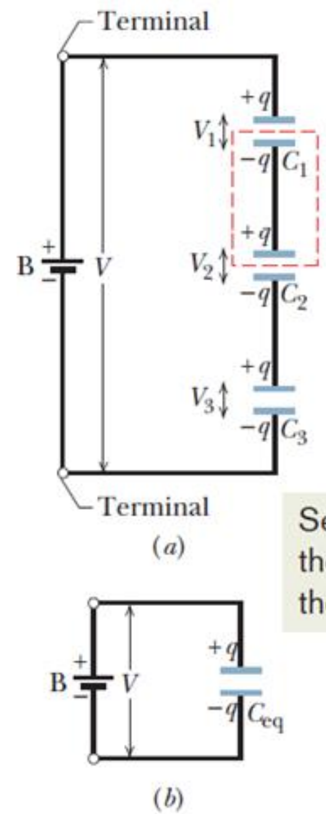
$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}).$$



Series capacitors and their equivalent have the same *q* ("seri-q").

(*a*) Three capacitors connected in series to battery B. The battery maintains potential difference *V* between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

Example, Capacitors in Parallel and in Series:

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10a, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

⇨

$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

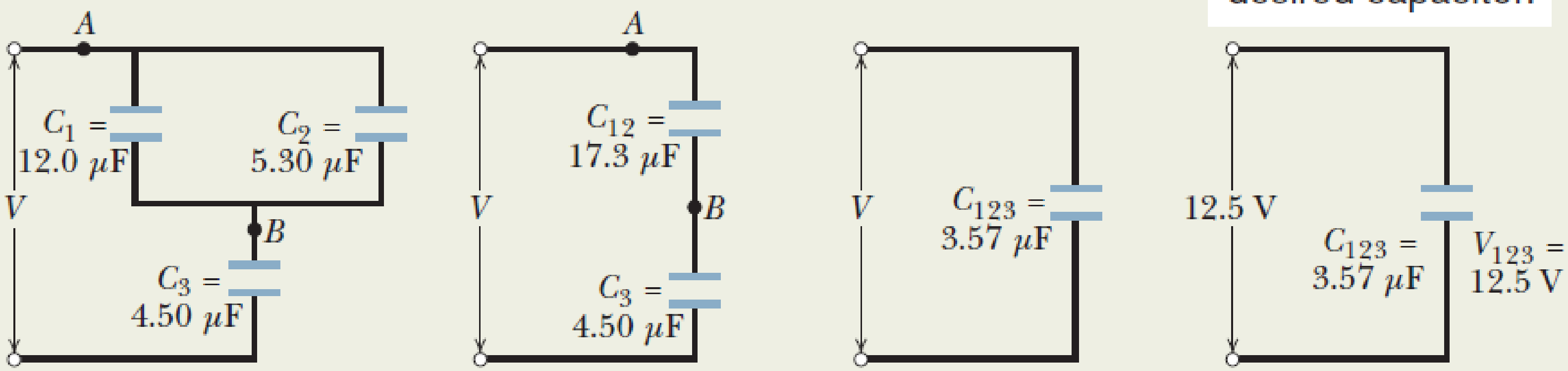
$$C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}. \qquad \text{(Answer)}$$

We first reduce the circuit to a single capacitor.

The equivalent of parallel capacitors is larger.

The equivalent of series capacitors is smaller.

Next, we work backwards to the desired capacitor.

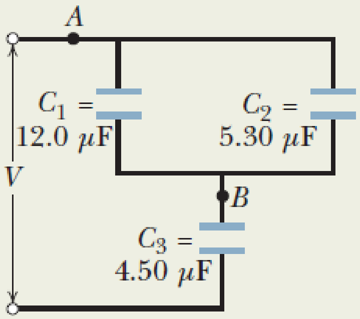


(a) (b) (c) (d)

Example, Capacitors in Parallel and in Series:

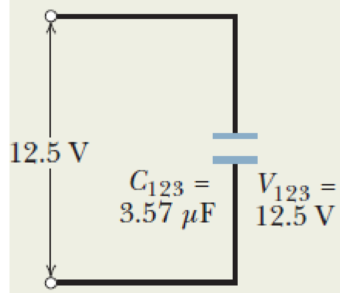
(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?

We first reduce the circuit to a single capacitor.



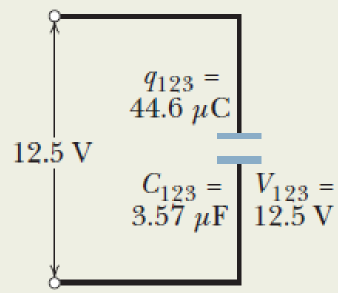
(*a*)

Next, we work backwards to the desired capacitor.



(*d*)

Applying $q = CV$ yields the charge.



(*e*)

$$q_{123} = C_{123}V = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

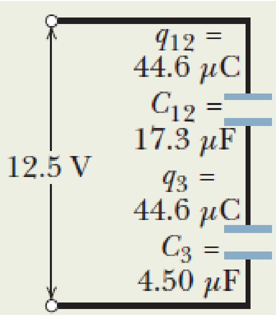
$$q_{12} = q_{123} = 44.6\ \mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

$$V_1 = V_{12} = 2.58\ \text{V},$$

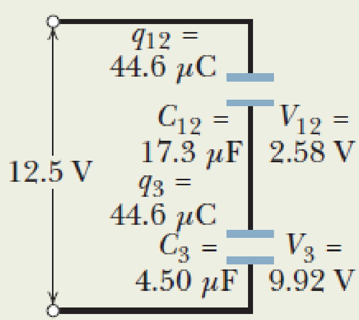
$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\ \text{V}) = 31.0\ \mu\text{C}.$$

Series capacitors and their equivalent have the same q (“seri-q”).



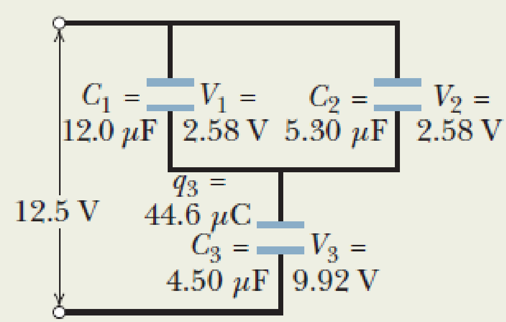
(*f*)

Applying $V = q/C$ yields the potential difference.



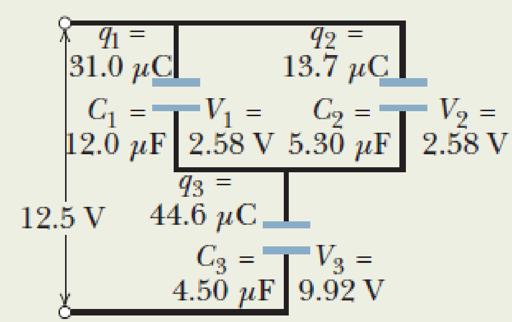
(*g*)

Parallel capacitors and their equivalent have the same V (“par-V”).



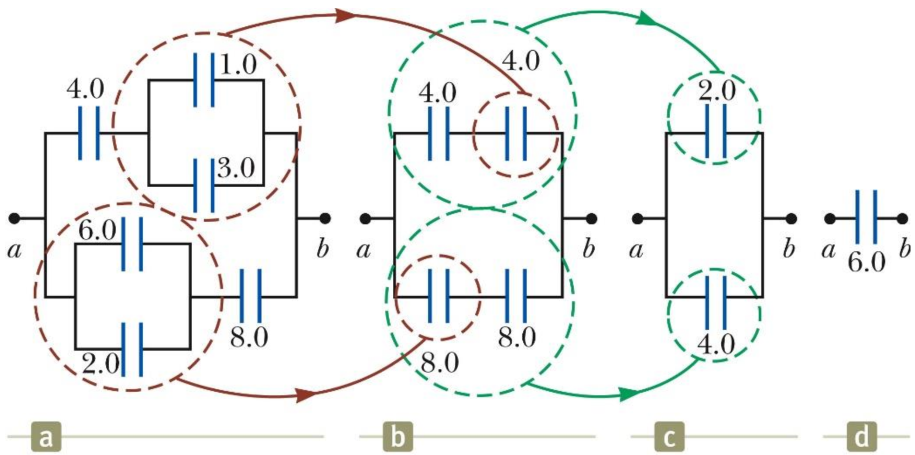
(*h*)

Applying $q = CV$ yields the charge.



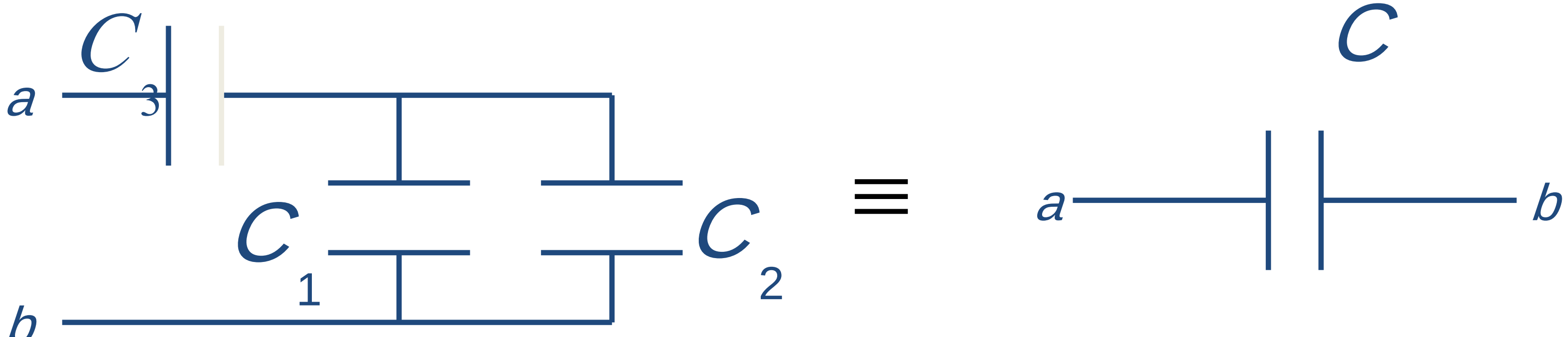
(*i*)

Equivalent Capacitance, Example



- The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors.
- These parallel combinations are in series with the capacitors next to them.
- The series combinations are in parallel and the final equivalent capacitance can be found.

Example 1



Where do we start?

Recognize that C_1 and C_2 are parallel with each other and combine these to get C_{12}

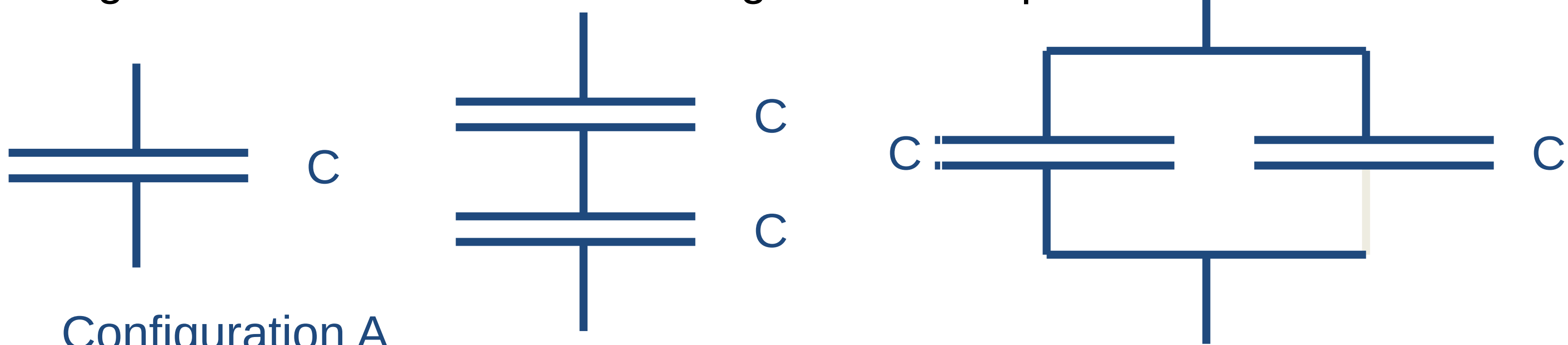
This C_{12} is then in series with with C_3

The resultant capacitance is then given by

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \quad \Rightarrow \quad C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}$$

Example 2

Three configurations are constructed using identical capacitors



Configuration A

Configuration B

Configuration C

Which of these configurations has the lowest overall capacitance?

a) Configuration A

$\boxed{\text{b) Configuration B}}$

c) Configuration C

The net capacitance for A is just C

In B, the caps are in series and the resultant is given by

$$\frac{1}{C_{net}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_{net} = \frac{C}{2}$$

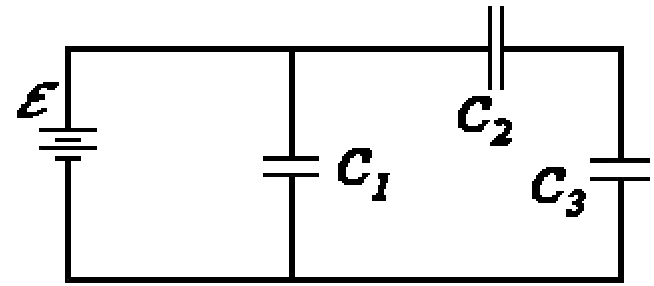
In C, the caps are in parallel and the resultant is given by

$$C_{net} = C + C = 2C$$

Example 3

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of emf $\mathcal{E}$. The capacitors obtain charges Q_1 Q_2, Q_3, and have voltages across their plates V_1, V_2, and V_3. C_{eq} is the equivalent capacitance of the circuit.

Check all of the following that apply:



c) $V_2 = V_3$

d)

b) $Q_2 = Q_3$

a) $Q_1 = Q_2$

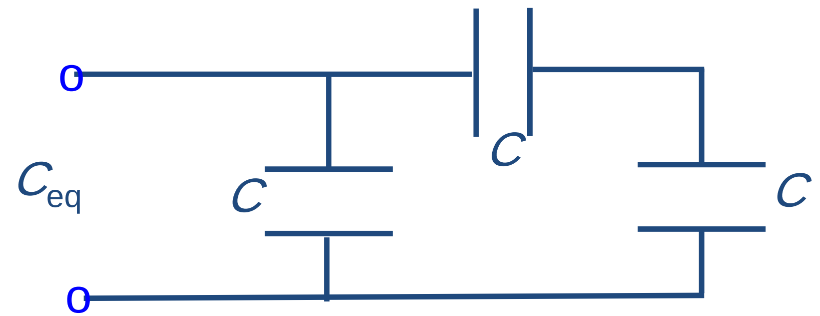
f) $C_{eq} > C_1$

e) $V_1 < V_2$

$E = V_1$

Example 4

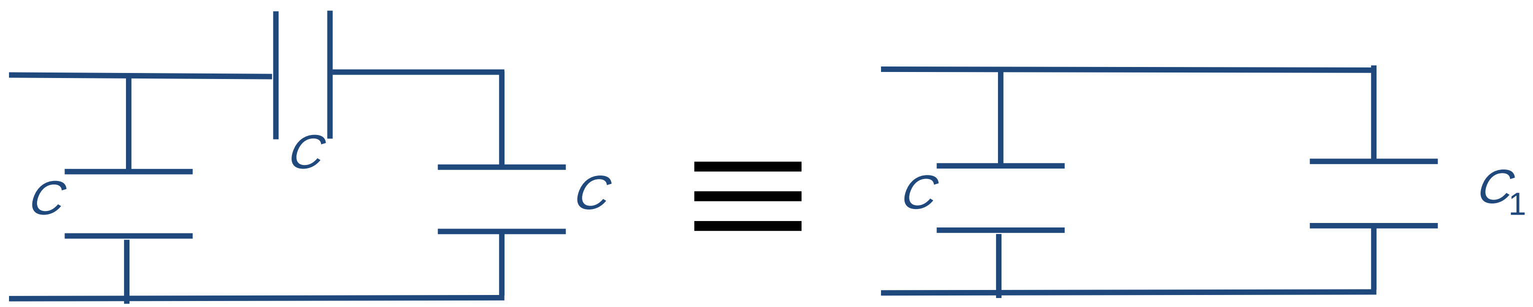
What is the equivalent capacitance, C_{eq}, of the combination shown?



(a) $C_{eq} = (3/2)C$

(b) $C_{eq} = (2/3)C$

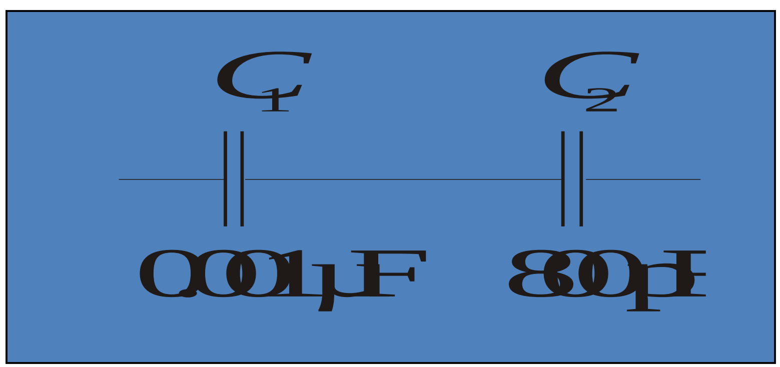
(c) $C_{eq} = 3C$



$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{2} \Rightarrow C_{eq} = C + \frac{C}{2} = \frac{3}{2}C$$

Example

If a 0.001 μF capacitor is connected in series with an 800 pF capacitor, the total capacitance is



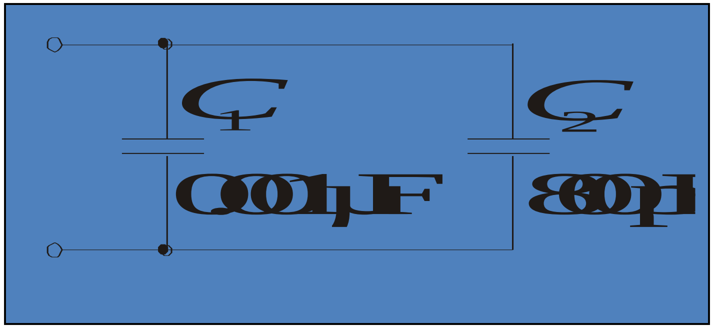
444 pF

Example

When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitors. The general equation for capacitors in parallel is

$$C_T = C_1 + C_2 + C_3 + ...C_n$$

If a 0.001 μF capacitor is connected in parallel with an 800 pF capacitor, the total capacitance is

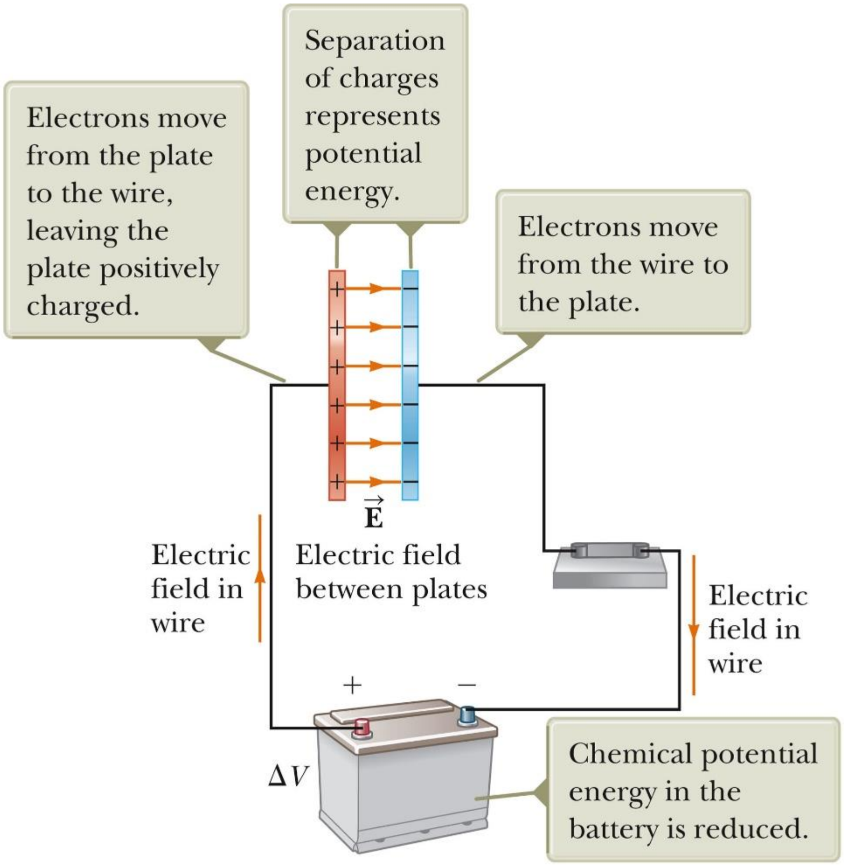


1800 pF

26.4 Energy Stored in a Charged Capacitor

- Consider the circuit to be a system.
- Before the switch is closed, the energy is stored as chemical energy in the battery.
- When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.
- The electric potential energy is related to the separation of the positive and negative charges on the plates.
- A capacitor can be described as a device that stores energy as well as charge.

Energy in a Capacitor – Overview



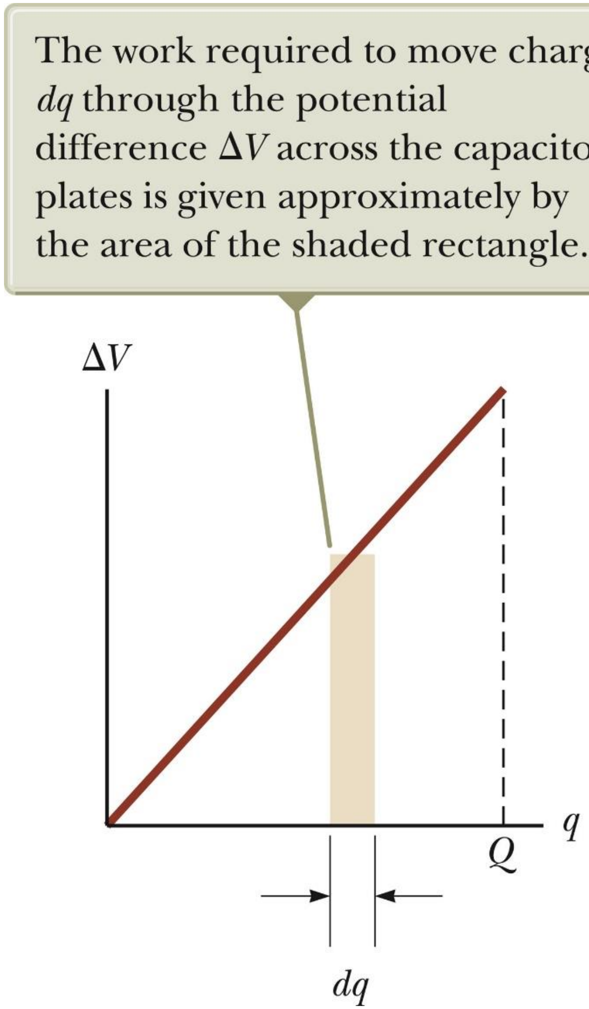
Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it.
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The work required is the area of the tan rectangle.
- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$



Energy, cont

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 \qquad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

where *U* is the stored potential energy

- This applies to a capacitor of any geometry.
- The energy stored increases as the charge increases and as the potential difference increases.
- In practice, there is a maximum voltage before discharge occurs between the plates.

Energy Density

- The energy can be considered to be stored in the electric field .
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = \frac{1}{2}(\varepsilon_o Ad)E^2$.
- It can also be expressed in terms of the energy density (energy per unit volume)

$u_E = \frac{1}{2}\varepsilon_o E^2$.

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$but\ C = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}}$$

Electric Field Energy Density

The potential energy that is stored in the capacitor can be thought of as being stored in the electric field that is in the region between the two plates of the capacitor

The quantity that is of interest is in fact the *energy density*

$$\textit{Energy Density} = u = \frac{\frac{1}{2}CV^2}{Ad}$$

where A and d are the area of the capacitor plates and their separation, respectively

Electric Field Energy Density

Using $C = \varepsilon_0 \frac{A}{d}$ and $V = E\,d$ we then have

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Even though we used the relationship for a parallel capacitor, this result holds for *all* capacitors regardless of configuration

This represents the energy density of the electric field in general

Example,
Potential Energy and Energy Density of an Electric Field:

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

KEY IDEAS

(1) An isolated sphere has capacitance given by Eq. 25-18 ($C = 4\pi\varepsilon_0 R$). (2) The energy U stored in a capacitor depends on the capacitor's charge q and capacitance C according to Eq. 25-21 ($U = q^2/2C$).

Calculation: Substituting $C = 4\pi\varepsilon_0 R$ into Eq. 25-21 gives us

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{(1.25 \times 10^{-9}\ \text{C})^2}{(8\pi)(8.85 \times 10^{-12}\ \text{F/m})(0.0685\ \text{m})}$$

$$= 1.03 \times 10^{-7}\ \text{J} = 103\ \text{nJ}. \qquad \text{(Answer)}$$

(b) What is the energy density at the surface of the sphere?

KEY IDEA

The density u of the energy stored in an electric field depends on the magnitude E of the field, according to Eq. 25-25 ($u = \frac{1}{2}\varepsilon_0 E^2$).

Calculations: Here we must first find E at the surface of the sphere, as given by Eq. 23-15:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}.$$

The energy density is then

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

$$= \frac{(1.25 \times 10^{-9}\ \text{C})^2}{(32\pi^2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0685\ \text{m})^4}$$

$$= 2.54 \times 10^{-5}\ \text{J/m}^3 = 25.4\ \mu\text{J/m}^3. \qquad \text{(Answer)}$$

Example

Suppose the capacitor shown here is charged to Q and then the battery is *disconnected*

Now suppose you pull the plates further apart so that the final separation is d_1

Which of the quantities Q, C, V, U, E change?

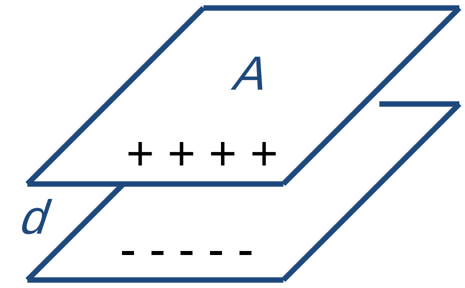
Q: Charge on the capacitor does not change

C: Capacitance Decreases

V: Voltage Increases

U: Potential Energy Increases

E: Electric Field does not change



How do these quantities change?

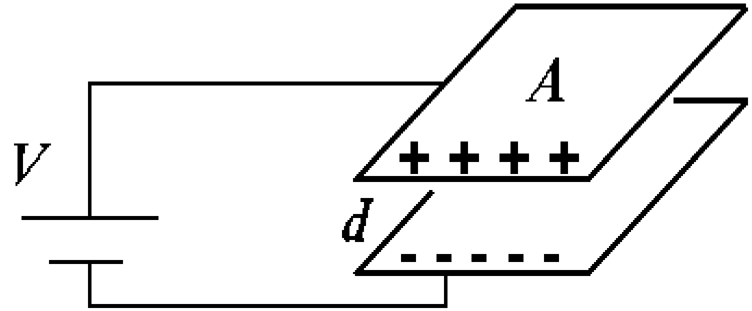
Answers: $\boxed{C_1 = \frac{d}{d_1} C}$ $\boxed{V_1 = \frac{d_1}{d} V}$ $\boxed{U_1 = \frac{d_1}{d} U}$

Example

Suppose the battery (V) is kept attached to the capacitor

Again pull the plates apart from d to d_1

Now which quantities, if any, change?



Q: Charge Decreases

C: Capacitance Decreases

V: Voltage on capacitor does not change

U: Potential Energy Decreases

E: Electric Field Decreases

How much do these quantities change?

Answers: $\boxed{Q_1 = \frac{d}{d_1} Q}$ $\boxed{C_1 = \frac{d}{d_1} C}$ $\boxed{U_1 = \frac{d}{d_1} U}$ $\boxed{E_1 = \frac{d}{d_1} E}$

Some Uses of Capacitors

- Defibrillators
  - When cardiac fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern.
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

26.5 Capacitors with Dielectrics

- A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.
  - Dielectrics include rubber, glass, and waxed paper
- With a dielectric, the capacitance becomes $C = \kappa C_{o.}$
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.
  - κ is the dielectric constant of the material.
- If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.
- If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.

Dielectrics, cont

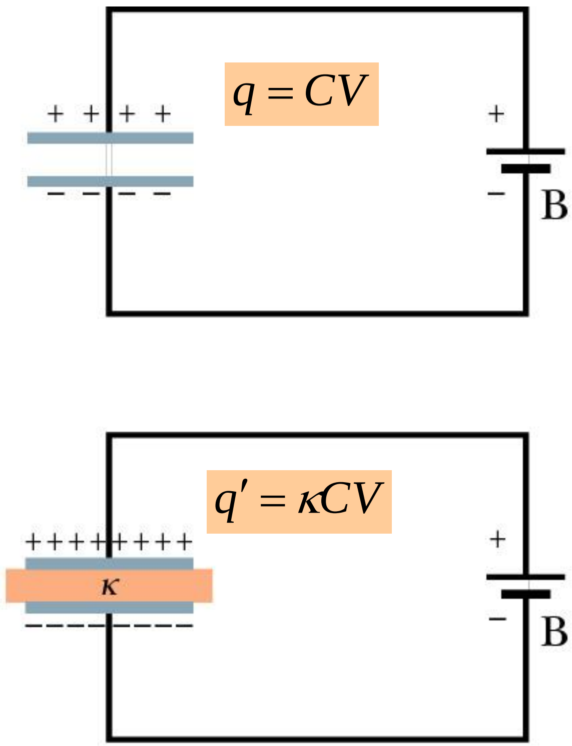
- For a parallel-plate capacitor, $C = \kappa (\varepsilon_o A) / d$
- In theory, *d* could be made very small to create a very large capacitance.
- In practice, there is a limit to *d.*
  - *d* is limited by the electric discharge that could occur though the dielectric medium separating the plates.
- For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material.

Dielectrics

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching.
    - This decreases d and increases C.

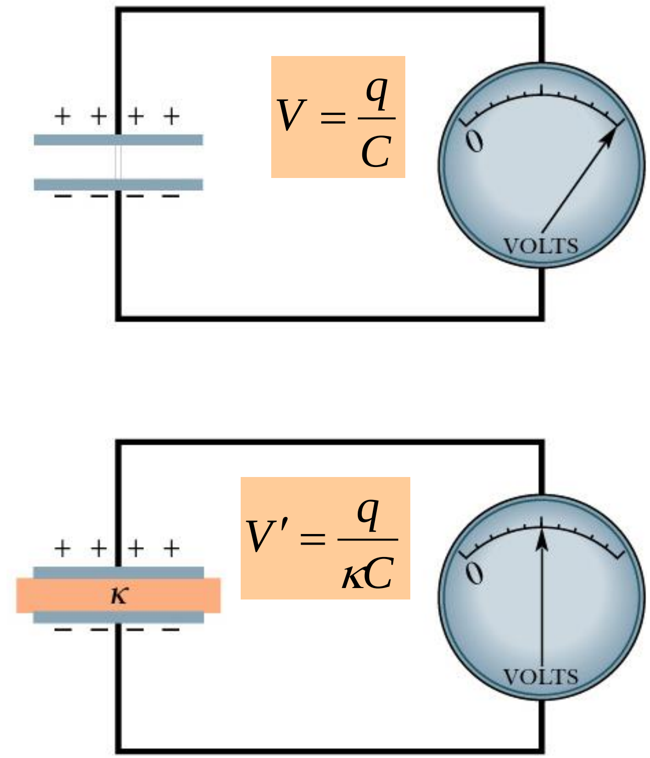
What Happens When You Insert a Dielectric?

- With battery attached, V=const, so more charge flows to the capacitor



V = a constant

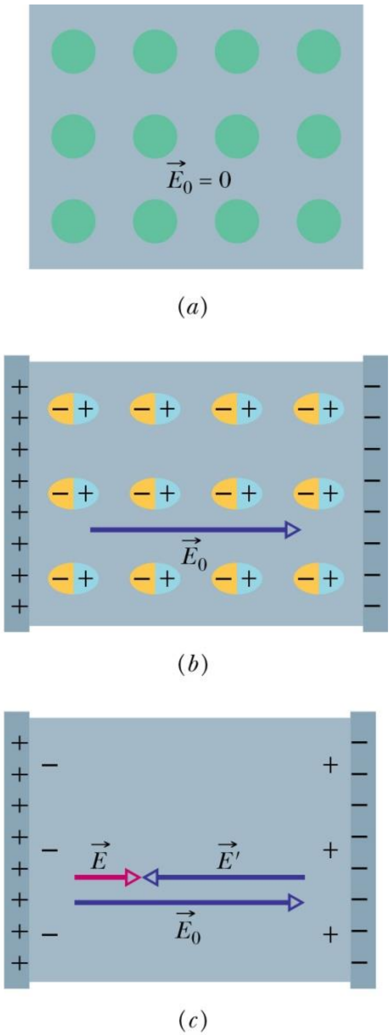
- With battery disconnected, q=const, so voltage (for given q) drops.



q = a constant

What Does the Dielectric Do?

- A dielectric material is made of molecules.
- Polar dielectrics already have a dipole moment (like the water molecule).
- Non-polar dielectrics are not naturally polar, but actually stretch in an electric field, to become polar.
- The molecules of the dielectric align with the applied electric field in a manner to oppose the electric field.
- This reduces the electric field, so that the net electric field is less than it was for a given charge on the plates.
- This lowers the potential (case b of the previous slide).
- If the plates are attached to a battery (case a of the previous slide), more charge has to flow onto the plates.



(a) (b) (c)

$$K = \frac{C}{C_0} \qquad \dots \dots \dots 1$$

$$C_0 = \frac{q}{V_0} = \frac{q}{E_0 \mathrm{d}} \qquad \dots \dots \dots \dots 2$$

$$C = \frac{q}{V} = \frac{q}{E \mathrm{d}} \qquad \dots \dots \dots \dots 3$$

From 2 and 3

$$\frac{C}{C_0} = \frac{V_0}{V} = \frac{E_0}{E} = K \qquad \dots \dots \dots \dots \dots 4$$

From Gauss law

$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad \dots \dots \dots \dots 5$$

$$E = \frac{\sigma}{\varepsilon} \qquad \dots \dots \dots \dots \dots \dots ...6$$

From 5 and 6

$$K = \frac{\varepsilon}{\varepsilon_0} \qquad \dots \dots \dots \dots \dots \dots \dots 7$$

$$\Rightarrow \quad K = \frac{C}{C_0} = \frac{V_0}{V} = \frac{E_0}{E} = \frac{\varepsilon}{\varepsilon_0}$$

Dielectrics

Most capacitors have a nonconducting material between their plates

This nonconducting material, *a dielectric*, accomplishes three things

1) Solves mechanical problem of keeping the plates separated

2) Increases the maximum potential difference allowed between the plates

3) Increases the capacitance of a given capacitor over what it would be without the dielectric

Dielectrics

Suppose we have a capacitor of value C_0 that is charged to a potential difference of V_0 and then removed from the charging source

We would then find that it has a charge of $\boldsymbol{Q = C_0 V_0}$

$$K = \frac{C}{C_o}$$

We now insert the dielectric material into the capacitor

We find that the potential difference *decreases* by a factor *K*

$$= \frac{V_o}{V} = \frac{E_0}{E} = \frac{\varepsilon}{\varepsilon_o}$$

$$\boldsymbol{V = \frac{V_0}{K}}$$

Or equivalently the capacitance has *increased* by a factor of *K*

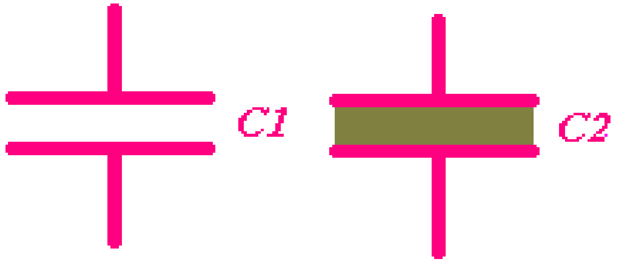
$$\mathrm{C} = \frac{\mathrm{q}}{\mathrm{V_o}}\mathrm{K}$$

$$\boldsymbol{C = K C_0}$$

This constant *K* is known as the dielectric constant and is dependent upon the material used and is a number greater than 1

Example

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected it is filled with a dielectric.



Compare the voltages of the two capacitors.

a) $V_1 > V_2$ b) $V_1 = V_2$ c) $V_1 < V_2$

We have that $Q_1 = Q_2$ and that $C_2 = KC_1$

We also have that C = Q/V or V= Q/C

Then $$V_1 = \frac{Q_1}{C_1}$$ and $$V_2 = \frac{Q_2}{C_2} = \frac{Q_1}{KC_1} = \frac{1}{K}V_1$$

Some Dielectric Constants and Dielectric Strengths

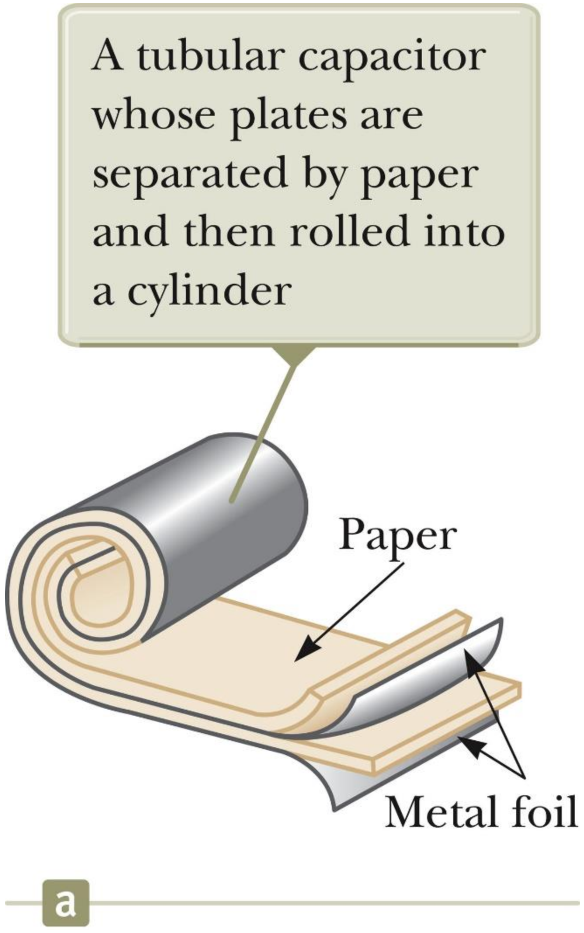
TABLE 26.1 *Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature*

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

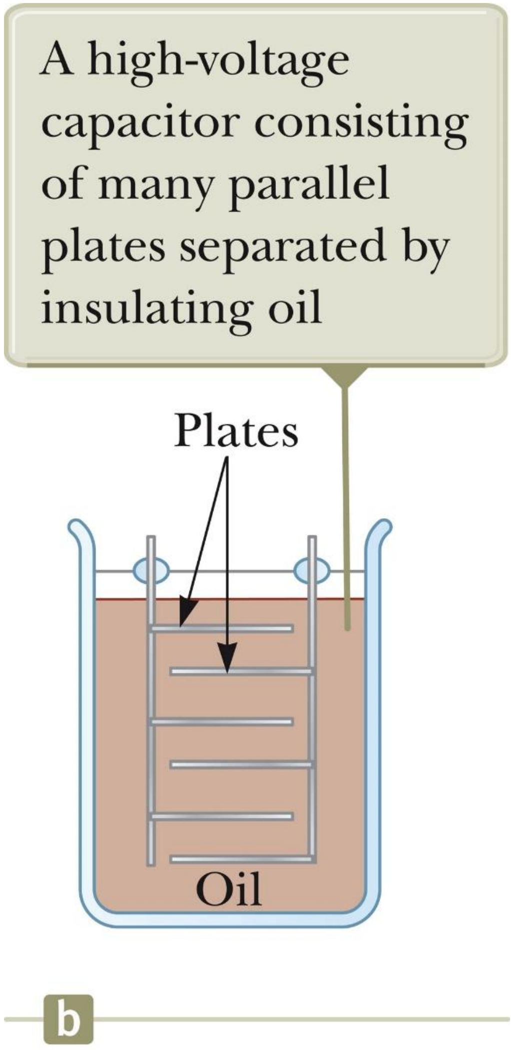
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paraffin-impregnated paper or Mylar.
- The layers are rolled into a cylinder to form a small package for the capacitor.



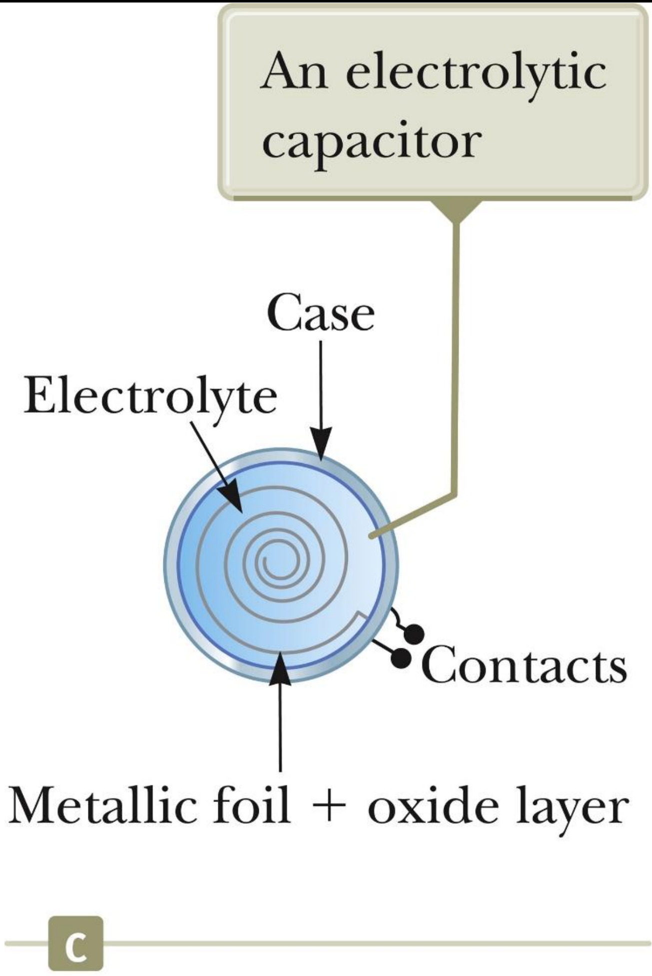
Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil.



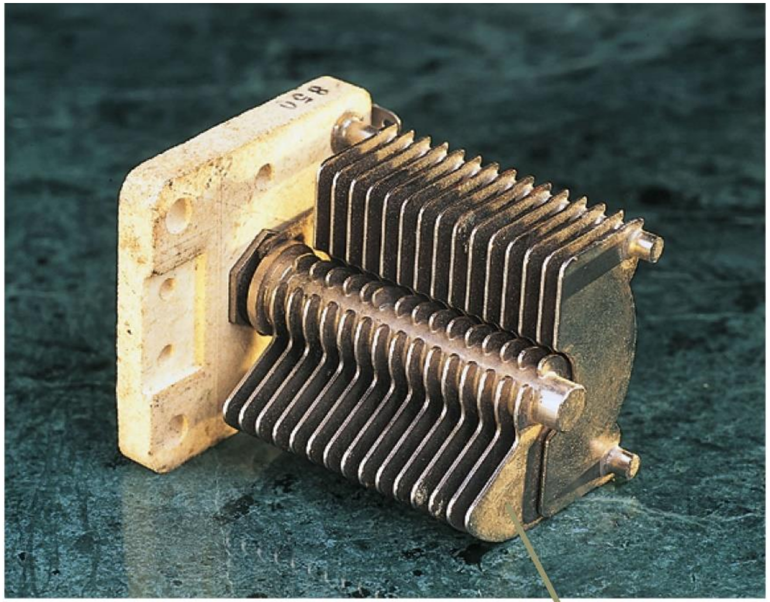
Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution.
- When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide is formed on the foil.
- This layer serves as a dielectric.
- Large values of capacitance can be obtained because the dielectric layer is very thin and the plate separation is very small.



Types of Capacitors – Variable

- Variable capacitors consist of two interwoven sets of metallic plates.
- One plate is fixed and the other is movable.
- Contain air as the dielectric
- These capacitors generally vary between 10 and 500 pF.
- Used in radio tuning circuits



When one set of metal plates is rotated so as to lie between a fixed set of plates, the capacitance of the device changes.

مثال :

وضعت مادة عازلة على شكل شريحة سمكها 0.5 cm وثابت عزلها 7 بين صفيحتي مكثف متوازي اللوحين المسافة بينهما 1 cm ومساحة كل منها $100\ cm^2$ وفرق الجهد بينهما 120 V قبل وضع المادة العازلة احسب :

١- سعة المكثف قبل ادخال المادة العازله
٢- الشحنات الحرة والمقيدة
٣- المجال الكهربائي قبل وضع المادة العازلة
٤- المجال الكهربائي بعد وضع المادة العازلة
٥- فرق الجهد بين اللوحين
٦- سعة المكثف بعد وضع المادة العازلة
٧- احسب p,D اثناء وجود مادة عازلة
٨- احسب p,D اثناء عدم وجود مادة عازلة
٩- التاثيريه
١٠- الكثافة السطحية للشحنات المستحثة على اوجه المادة العازلة

علما ان $\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N.m^2}$

الحل

-١

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(10^{-2})}{10^{-2}} = 8.9 \times 10^{-12} \text{ F}$$

$$= 8.9\, pF$$

-٢

$$q = C_0 V_0 = 8.9 \times 10^{-12} \times 100$$

$$= 8.9 \times 10^{-10} \text{ C}$$

-٣

$$\oint E_0\, dA = E_0 A = \frac{q}{\varepsilon_0}$$

$$\Longrightarrow E_0 = \frac{q}{\varepsilon_0 A} = \frac{8.9 \times 10^{-10}}{8.85 \times 10^{-12} \times 10^{-2}} = 1 \times 10^4 \left(\frac{V}{m}\right)$$

-٤

$$\oint E\, dA = EA = \frac{q}{\varepsilon}$$

$$E = \frac{q}{\varepsilon_0 KA} = \frac{E_0}{K} = \frac{1 \times 10^4}{7} = 0.143 \times 10^4 \left(\frac{V}{m}\right)$$

-٥

$$V = -\int E\, dl = E_0 l_1 + E\, l_2$$

-٦ $C = \frac{q}{V} =$

-٧ $D = K\varepsilon_0 E = 7 \times 8.85 \times 10^{-12} \times 0.143 \times 10^4 = 8.9 \times 10^{-8} \left(\frac{C}{m^2}\right)$

-

$\chi_e = K - 1 = 7 - 1 = 6$

-١٠

$$p = \varepsilon_0 E(K-1) = 8.85 \times 10^{-12} \times 0.143 \times 10^4 (7-1) = 7.6 \times 10^{-8} \left(\frac{C}{m^2}\right)$$

$$\sigma_i = \varepsilon_0 \chi_e E = 8.85 \times 10^{-12} \times 6 \times 0.143 \times 10^4 =$$

-٨

$$D_0 = \varepsilon_0 E_0 = 8.85 \times 10^{-12} \times 1 \times 10^4 = 8.9 \times 10^{-8} \left(\frac{C}{m^2}\right)$$

$$p_0 = \varepsilon_0 E_0 (K-1) = 0$$

Problems …………….
Serway Book

Quiz

1. The capacitance of a capacitor will be larger if

    a. the spacing between the plates is increased

    b. air replaces oil as the dielectric

    c. the area of the plates is increased

    d. all of the above

2. The major advantage of a mica capacitor over other types is

    a. they have the largest available capacitances

    b. their voltage rating is very high

    c. they are polarized

    d. all of the above

Quiz

3. Electrolytic capacitors are useful in applications where

   a. a precise value of capacitance is required

   b. low leakage current is required

   c. large capacitance is required

   d. all of the above

4. If a 0.015 μF capacitor is in series with a 6800 pF capacitor, the total capacitance is

   a. 1568 pF

   b. 4678 pF

   c. 6815 pF

   d. 0.022 μF

Quiz

5. Two capacitors that are initially uncharged are connected in series with a dc source. Compared to the larger capacitor, the smaller capacitor will have

a. the same charge

b. more charge

c. less voltage

d. the same voltage

6. When a capacitor is connected through a resistor to a dc voltage source, the charge on the capacitor will reach 50% of its final charge in

a. less than one time constant

b. exactly one time constant

c. greater than one time constant

d. answer depends on the amount of voltage

Quiz

7. When a capacitor is connected through a series resistor and switch to a dc voltage source, the voltage across the resistor after the switch is closed has the shape of

a. a straight line

b. a rising exponential

c. a falling exponential

d. none of the above

8. The capacitive reactance of a 100 μF capacitor to 60 Hz is

a. 6.14 kΩ

b. 265 Ω

c. 37.7 Ω

d. 26.5 Ω

Quiz

9. If an sine wave from a function generator is applied to a capacitor, the current will

a. lag voltage by 90^o

b. lag voltage by 45^o

c. be in phase with the voltage

d. none of the above

10. A switched capacitor emulates a

a. smaller capacitor

b. larger capacitor

c. battery

d. resistor

Quiz

Answers:

1. c
2. b
3. c
4. b
5. a
6. a
7. c
8. d
9. d
10. d

Summary الخلاصة

The proportionality constant *C is called the **capacitance*** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference.

The SI unit is called the *farad (F):* **1 farad (1 F)= 1 coulomb per volt =1 C/V.**

Summary الخلاصة

Capacitors are often combined in series and the question then becomes what is the equivalent capacitance?

Charge Conservation

This then means: If upper plate of C_1 gets a charge of +Q, Then the lower plate of C_1 gets a charge of -Q

If there are more than two capacitors in series, the resultant capacitance is given by

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

If there are more than two capacitors in parall, the resultant capacitance is given by

$$C_{eq} = \sum_i C_i$$

Thank You

ACKNOWLEDGEMENTS