Chapter 2

Motion in One Dimension

2.1 Position, Velocity, and Speed

2.2 Instantaneous Velocity and Speed

2.3 Analysis Model: Particle Under Constant Velocity

2.4 Acceleration

2.6 Analysis Model: Particle Under Constant Acceleration

2.7 Freely Falling Objects

Dynamics

- The branch of physics involving the motion of an object and the relationship between that motion and other physics concepts
- *Kinematics* is a part of dynamics
  - In kinematics, you are interested in the *description* of motion
  - *Not* concerned with the cause of the motion

Quantities in Motion

- Any motion involves three concepts
  - Displacement
  - Velocity
  - Acceleration
- These concepts can be used to study objects in motion

2.1 Position, Velocity, and Speed

Position:

- Defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
  - Defines a starting point for the motion
  - The motion of a particle is completely known if the particle's position in space is known at all times.

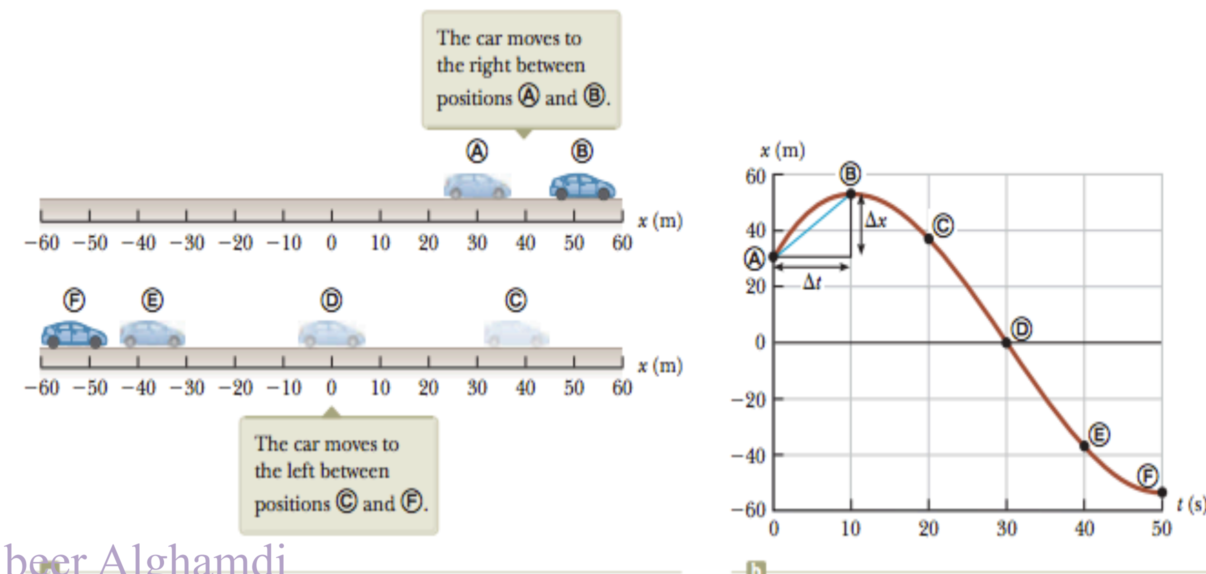


Table 2.1 Position of the Car at Various Times

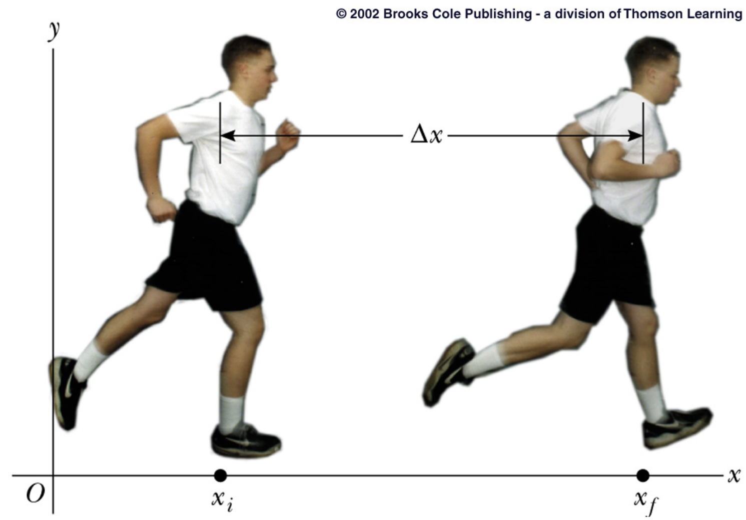
Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Displacement:

- Defined as the change in position in some time interval.

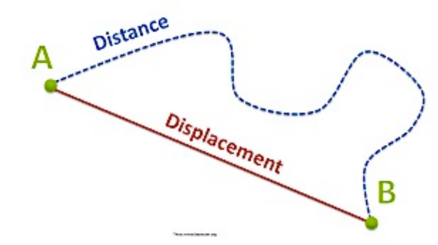
$$\Delta x \equiv x_f - x_i$$

  - f stands for final , i stands for initial and delta (Δ) to denote the change in a quantity
  - – May be represented as Δy if vertical
  - – Units are meters (m) in SI, centimeters (cm) in cgs or feet (ft) in US Customary



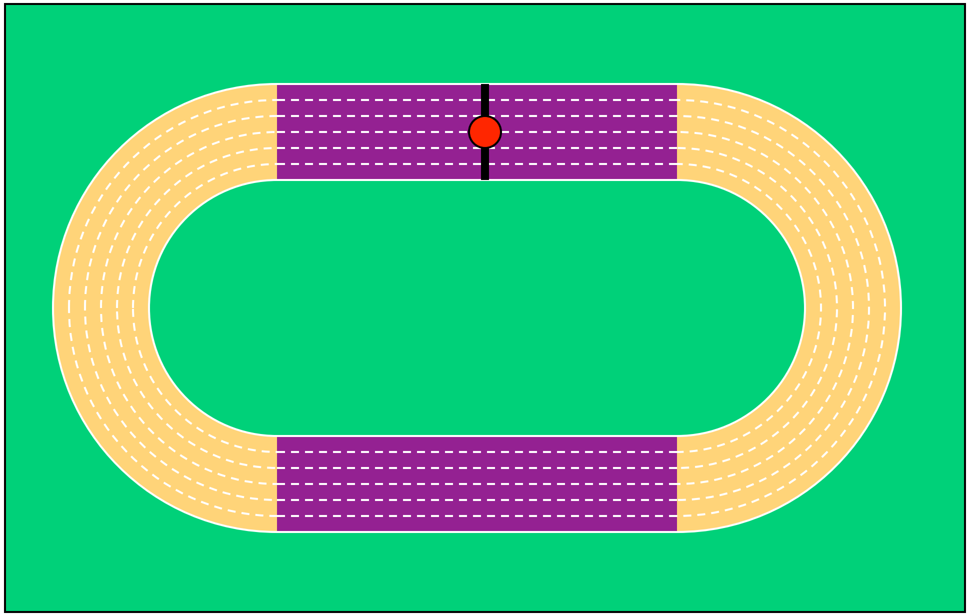
Differences between distance and displacement:

Sr. no.	Distance	Displacement
01	Distance is the length of the path travelled by a body while moving from an initial position to a final position.	Displacement is the shortest distance between the initial position and the final position of the body.
02	Distance is a scalar quantity.	Displacement is a vector quantity.
03	Distance measured is always positive.	Displacement can be positive or negative depending on the reference point.
04	The total distance covered is equal to the algebraic sum of all the distances travelled in different directions.	The net displacement is the vector sum of the individual displacements in different directions.
05	There is always a distance covered whenever there is a motion.	Displacement will be zero if the body comes back to its initial position.
06	Unit: metre (m)	Unit: metre (m)



Displacement vs. Distance

- An athlete runs around a track that is 100 meters long three times, then stops.
  - What is the athlete's distance and displacement?



- Distance = 300 m
- Displacement = 0 m
- Why?

Vector and Scalar Quantities

- Vector quantities need both magnitude (size) and direction to completely describe them
- Scalar quantities are completely described by magnitude only

Speed

- The **average speed** of an object is defined as the total distance traveled divided by the total time elapsed

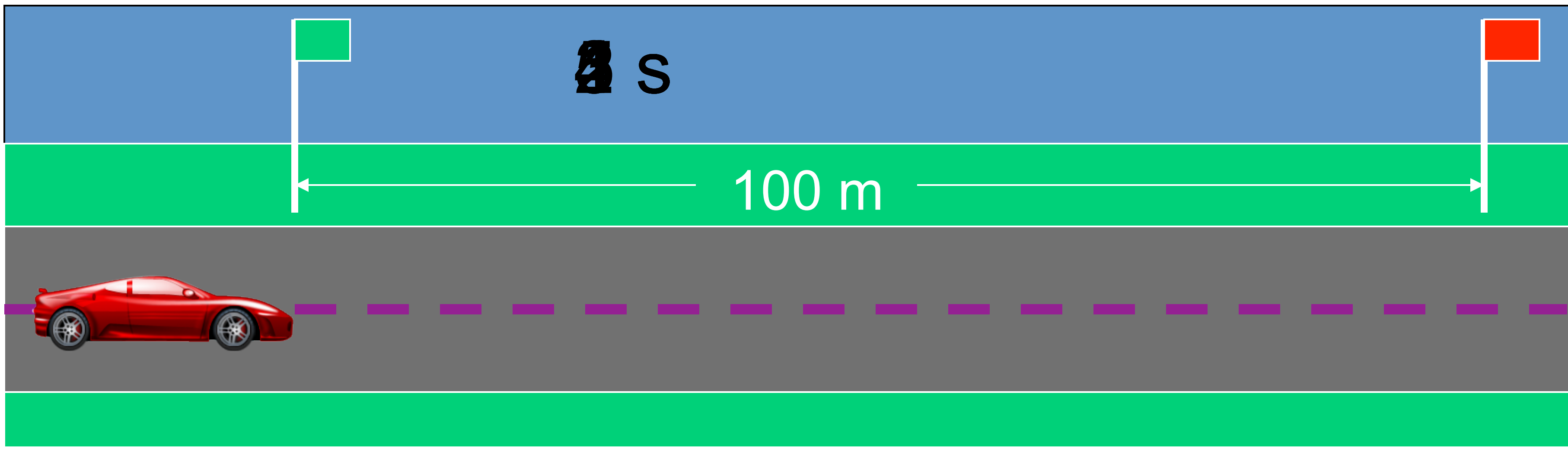
$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{d}{t}$$

  - Speed is a scalar quantity, Like distance, speed *does not* depend on direction.
  - SI units are m/s

Speed

- A car drives 100 meters in 5 seconds.



- What is the car’s average speed?
  - s = d/t
  - s = (100 m) / (5 s) = 20 m/s

Velocity:

- The average velocity $v_{x,avg}$ of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,avg} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \quad m/s$$

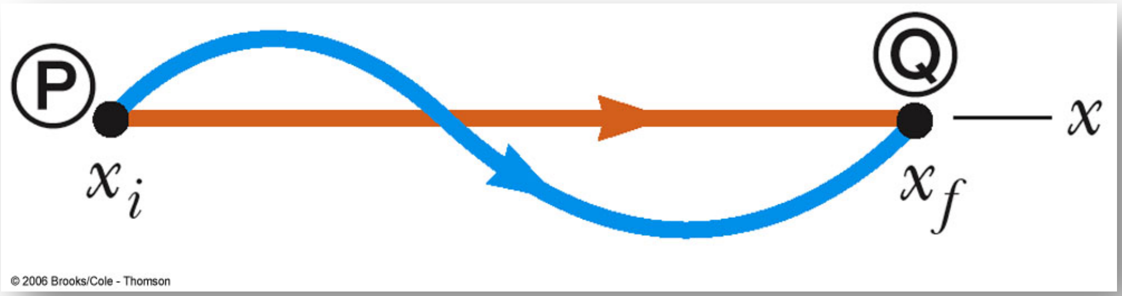
- generally use a time interval, so let $t_i = 0$

Velocity continued

- Direction will be the same as the direction of the displacement (time interval is always positive)
  - + or - is sufficient
- Units of velocity are m/s (SI), cm/s (cgs) or ft/s (US)
  - Other units may be given in a problem, but generally will need to be converted to these

SI system		English units
2.54 cm	=	1 inch
1 m	=	3.281 ft
	=	39.37 inch
1.609 km	=	1 mile

Speed vs. Velocity



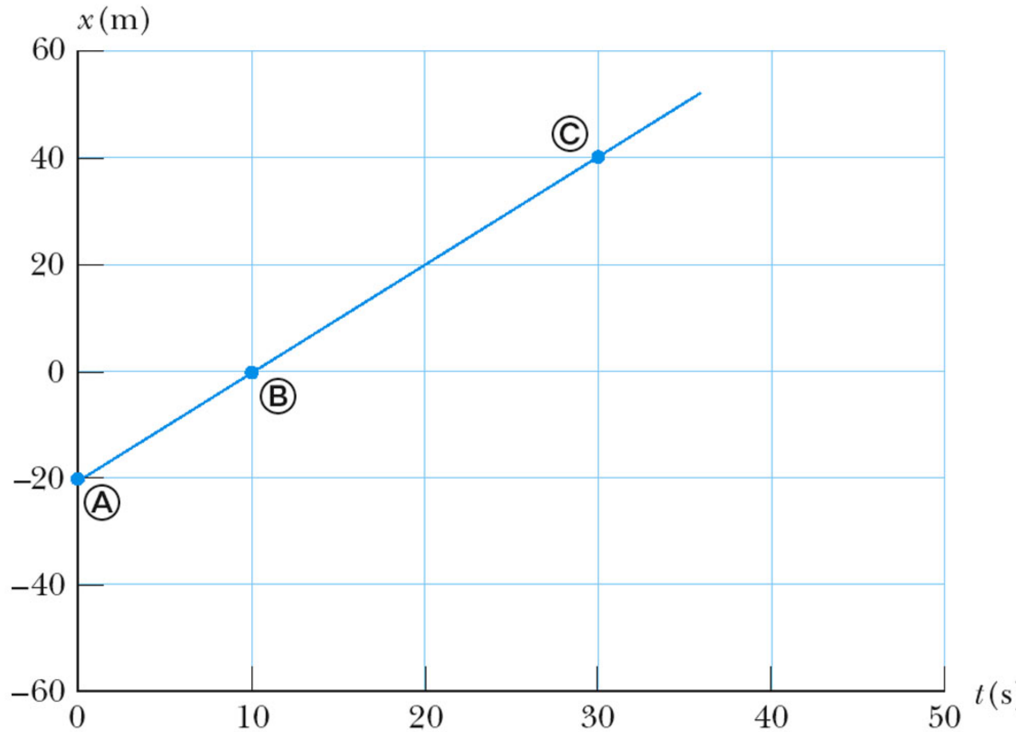
- Cars on both paths have the same average velocity since they had the same displacement in the same time interval
- The car on the blue path will have a greater average speed since the distance it traveled is larger

Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions
- An object moving with a constant velocity will have a graph that is a straight line

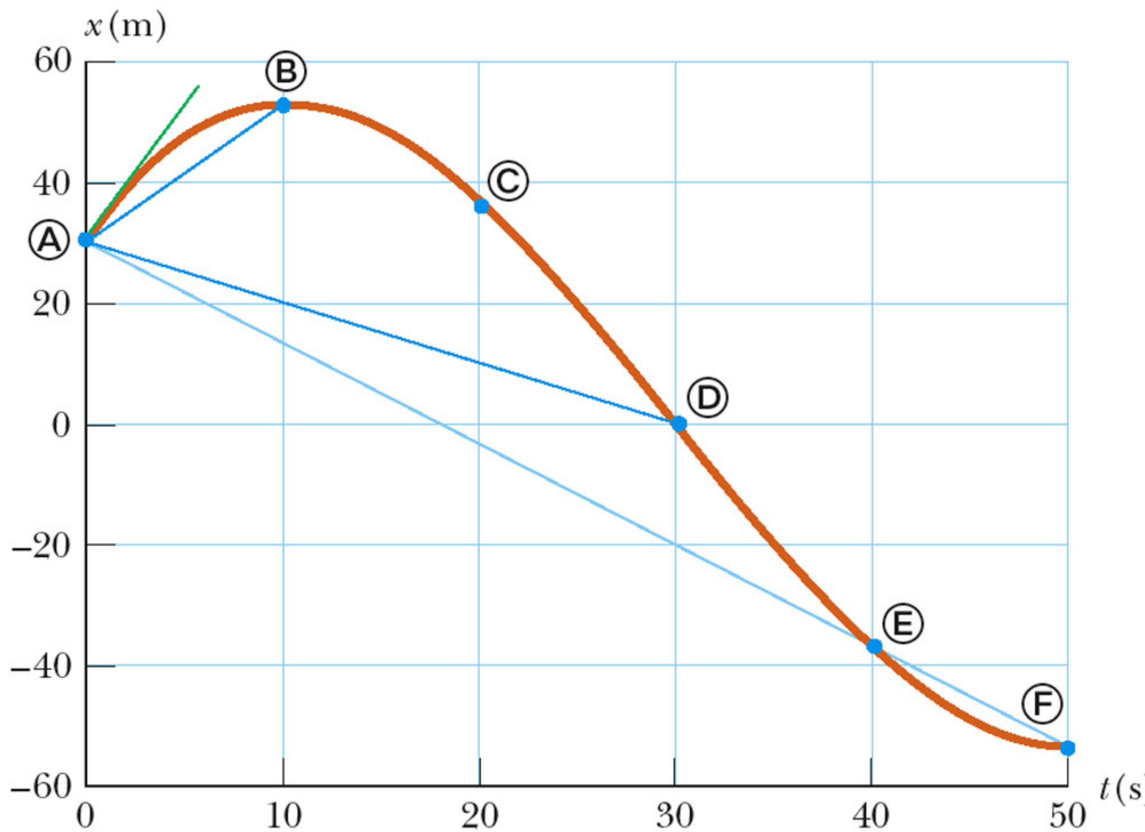
Average Velocity, Constant

- The straight line indicates constant velocity
- The slope of the line is the value of the average velocity



Average Velocity, Non Constant

- The motion is non-constant velocity
- The average velocity is the slope of the blue line joining two points



Comparison of Speed and Velocity

Sr. no	Speed	velocity
01	Speed is refers to "how fast an object is moving."	Velocity refers to "the rate at which an object changes its position."
02	Speed is a scalar quantity.	Velocity is a vector quantity.
03	Speed is the rate of motion, or the rate of change of position.	velocity is the rate of change of displacement.
04	Speed is thus the magnitude component of velocity.	Velocity contains both the magnitude and direction components.
05	Explanation : How fast my hand is moving to slapped on your face, this is speed	Explanation : When you get the slap and changes your face from right to left.. i.e the rate at which your face changes its position, this is Velocity....
06	speed= total distance/time taken	velocity= displacement(shortest root from initial to the final position) /time taken including direction.
07	Unit: km/hr like 60km/hr,	Unit: 60km/hr in east direction.

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

SOLUTION

Consult Figure 2.1 to form a mental image of the car and its motion. We model the car as a particle. From the position–time graph given in Figure 2.1b, notice that $x_{Ⓐ} = 30$ m at $t_{Ⓐ} = 0$ s and that $x_{Ⓕ} = -53$ m at $t_{Ⓕ} = 50$ s.

Use Equation 2.1 to find the displacement of the car:

$$\Delta x = x_{Ⓕ} - x_{Ⓐ} = -53 \text{ m} - 30 \text{ m} = \boxed{-83 \text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that it is the correct answer.

Use Equation 2.2 to find the car's average velocity:

$$v_{x,\text{avg}} = \frac{x_{Ⓕ} - x_{Ⓐ}}{t_{Ⓕ} - t_{Ⓐ}}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}} = \boxed{-1.7 \text{ m/s}}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1 because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Figure 2.1b, the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ), for a total of 127 m.

Use Equation 2.3 to find the car's average speed:

$$v_{\text{avg}} = \frac{127 \text{ m}}{50 \text{ s}} = \boxed{2.5 \text{ m/s}}$$

Notice that the average speed is positive, as it must be. Suppose the red-brown curve in Figure 2.1b were different so that between 0 s and 10 s it went from Ⓐ up to 100 m and then came back down to Ⓑ. The average speed of the car would change because the distance is different, but the average velocity would not change.

2.2 Instantaneous Velocity and Speed

Instantaneous Velocity

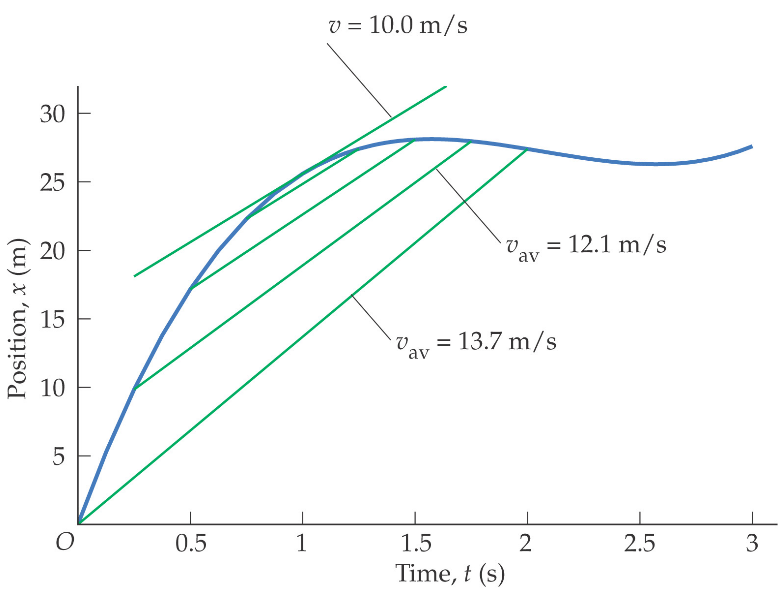
- The instantaneous speed of a particle is defined as the magnitude of its instantaneous velocity
- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

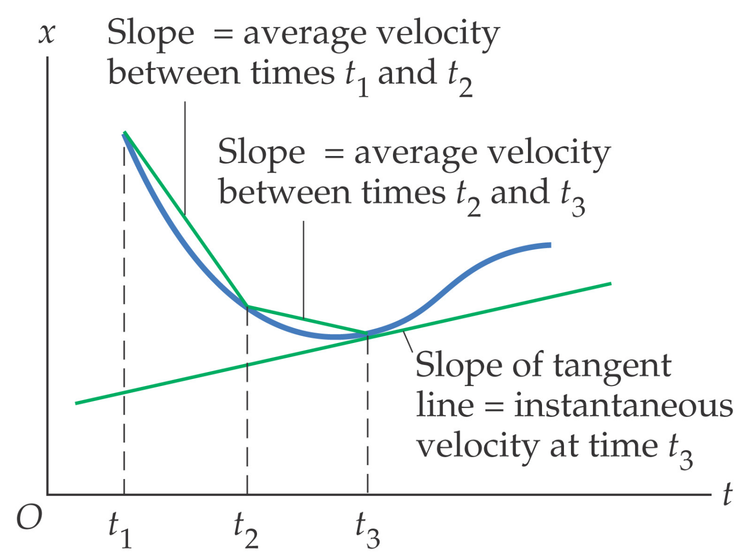
$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity indicates what is happening at every point of time

Instantaneous Velocity on a Graph

- The slope of the line tangent to the position-vs.-time graph is defined to be the instantaneous velocity at that time





Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is completely known, unlike that of the car in Figure 2.1. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

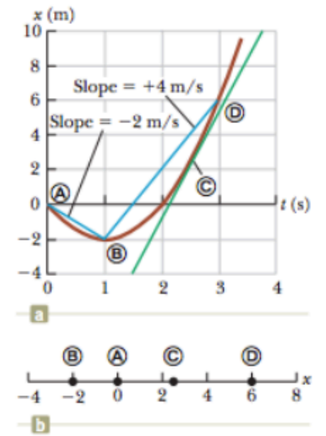


Figure 2.4 (Example 2.3) (a) Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$. (b) The particle moves in one dimension along the x axis.

SOLUTION

From the graph in Figure 2.4a, form a mental representation of the particle's motion. Keep in mind that the particle does not move in a curved path in space such as that shown by the red-brown curve in the graphical representation. The particle moves only along the x axis in one dimension as shown in Figure 2.4b. At $t = 0$, is it moving to the right or to the left?

During the first time interval, the slope is negative and hence the average velocity is negative. Therefore, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, set $t_i = t_Ⓐ = 0$ and $t_f = t_Ⓑ = 1$ s and use Equation 2.1 to find the displacement:

$$\Delta x_{Ⓐ\to Ⓑ} = x_f - x_i = x_Ⓑ - x_Ⓐ$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = \boxed{-2 \text{ m}}$$

For the second time interval ($t = 1$ s to $t = 3$ s), set $t_i = t_Ⓑ = 1$ s and $t_f = t_Ⓓ = 3$ s:

$$\Delta x_{Ⓑ\to Ⓓ} = x_f - x_i = x_Ⓓ - x_Ⓑ$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = \boxed{+8 \text{ m}}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

SOLUTION

In the first time interval, use Equation 2.2 with $\Delta t = t_f - t_i = t_Ⓑ - t_Ⓐ = 1$ s:

$$v_{x,\text{avg}\,(Ⓐ\to Ⓑ)} = \frac{\Delta x_{Ⓐ\to Ⓑ}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = \boxed{-2 \text{ m/s}}$$

In the second time interval, $\Delta t = 2$ s:

$$v_{x,\text{avg}\,(Ⓑ\to Ⓓ)} = \frac{\Delta x_{Ⓑ\to Ⓓ}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = \boxed{+4 \text{ m/s}}$$

These values are the same as the slopes of the blue lines joining these points in Figure 2.4a.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

SOLUTION

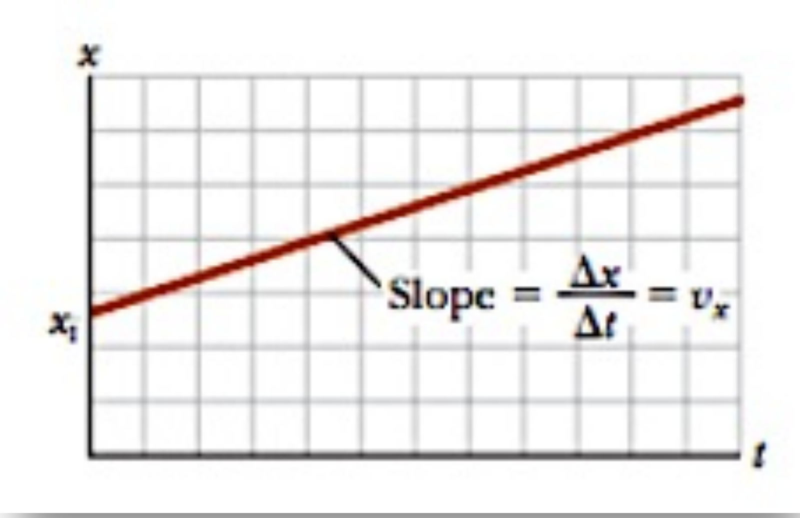
Measure the slope of the green line at $t = 2.5$ s (point Ⓒ) in Figure 2.4a:

$$v_x = \frac{10 \text{ m} - (-4 \text{ m})}{3.8 \text{ s} - 1.5 \text{ s}} = \boxed{+6 \text{ m/s}}$$

Notice that this instantaneous velocity is on the same order of magnitude as our previous results, that is, a few meters per second. Is that what you would have expected?

2.3 Analysis Model: Particle Under Constant Velocity

Uniform Velocity

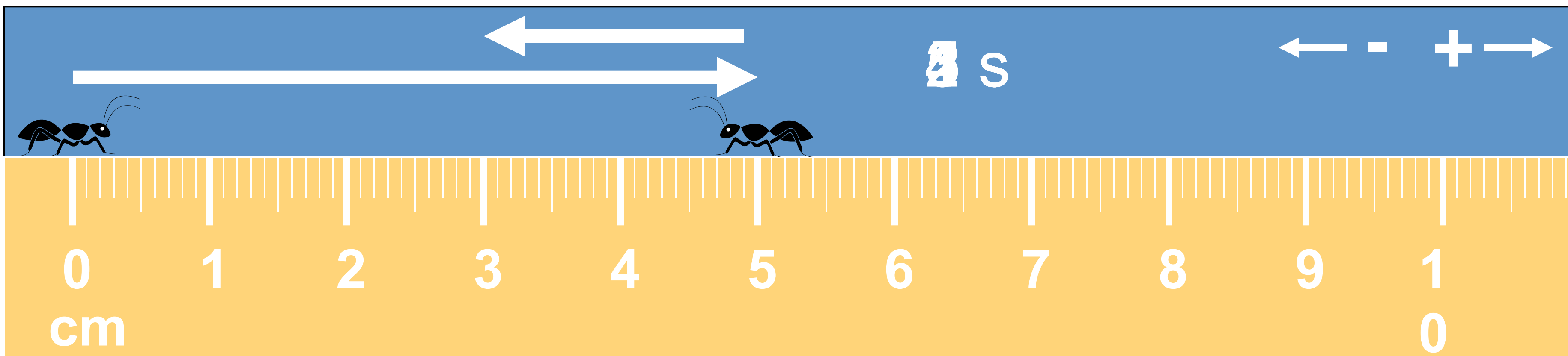


- Uniform velocity is constant velocity
- The instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Example

- Back to our ant explorer!



  – Distance traveled: 7 cm
  – Displacement: +3 cm
  – Average speed: (7 cm) / (5 s) = 1.4 cm/s
  – Average velocity: (+3 cm) / (5 s) = +0.6 cm/s

2.4 Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Acceleration is the rate of change of the velocity

$$\bar{a} = a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- Units are m/s^2 (SI), cm/s^2 (cgs), and ft/s^2 (US)

Average Acceleration

- Vector quantity
- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are in the opposite directions, the speed is decreasing

Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero

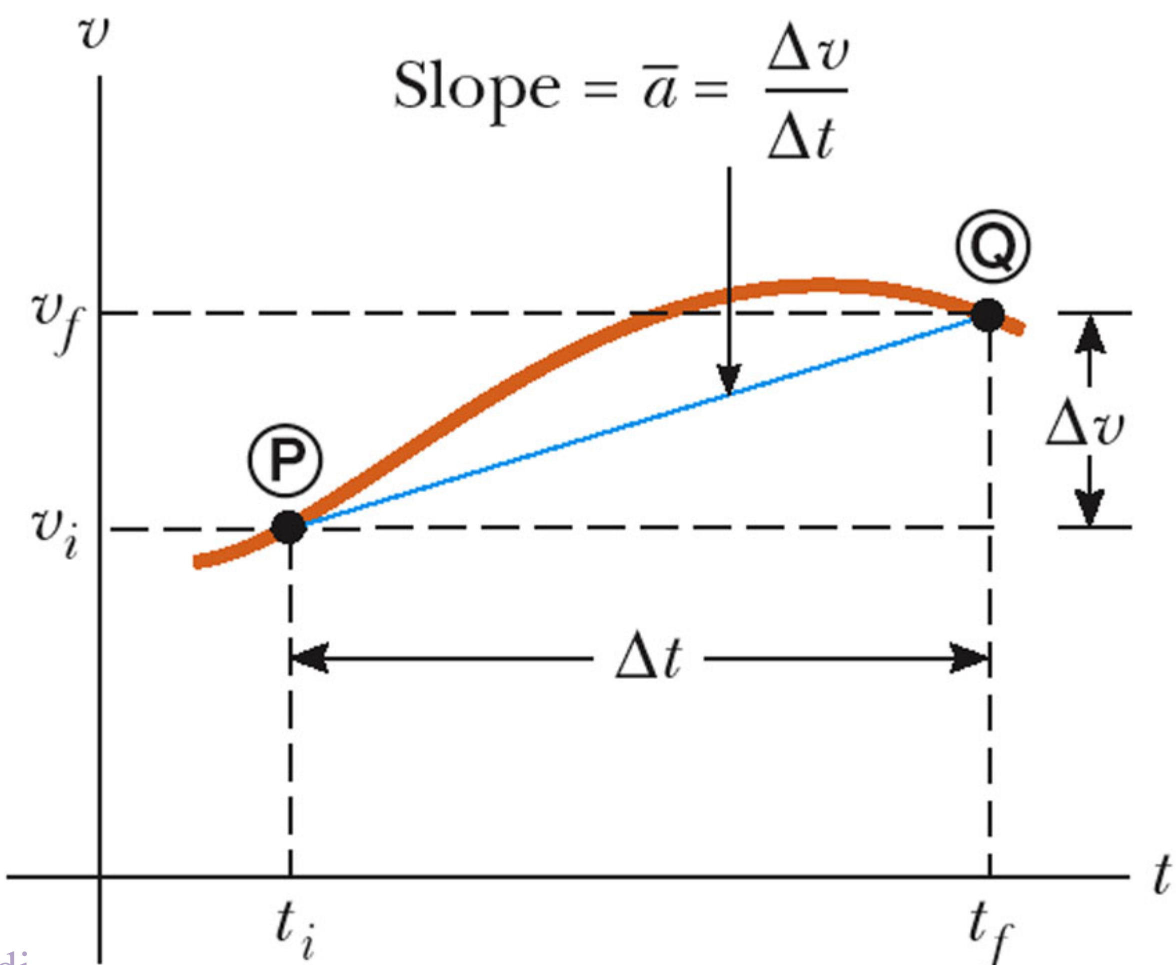
$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

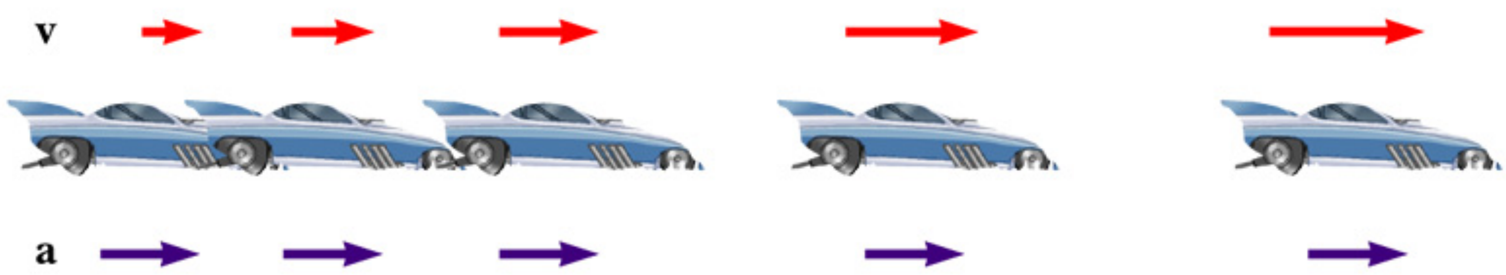
Average Acceleration



Relationship Between Acceler and Velocity

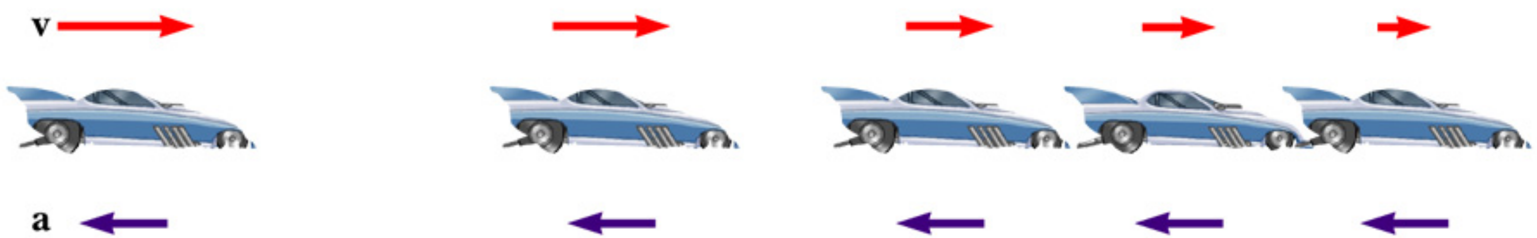
- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Relationship Between Velocity and Acceleration



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- Positive velocity and positive acceleration

Relationship Between Velocity and Acceleration



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Velocity is positive and acceleration is negative

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

SOLUTION

Think about what the particle is doing from the mathematical representation. Is it moving at $t = 0$? In which direction? Does it speed up or slow down? Figure 2.9 is a v_x–t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x–t curve is negative, we expect the acceleration to be negative.

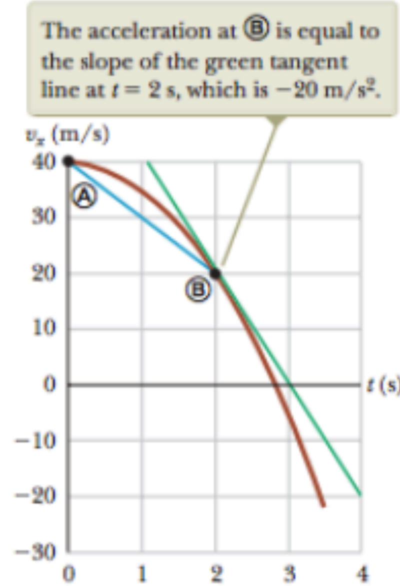


Figure 2.9 (Example 2.6) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = 40 - 5t^2$.

Find the velocities at $t_i = t_Ⓐ = 0$ and $t_f = t_Ⓑ = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xⒶ} = 40 - 5t_Ⓐ^2 = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{xⒷ} = 40 - 5t_Ⓑ^2 = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

Find the average acceleration in the specified time interval $\Delta t = t_Ⓑ - t_Ⓐ = 2.0$ s:

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xⒷ} - v_{xⒶ}}{t_Ⓑ - t_Ⓐ} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= \boxed{-10 \text{ m/s}^2}$$

The negative sign is consistent with our expectations: the average acceleration, represented by the slope of the blue line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

SOLUTION

Knowing that the initial velocity at any time t is $v_{xi} = 40 - 5t^2$, find the velocity at any later time $t + \Delta t$:

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Find the change in velocity over the time interval Δt:

$$\Delta v_x = v_{xf} - v_{xi} = -10t\,\Delta t - 5(\Delta t)^2$$

To find the acceleration at any time t, divide this expression by Δt and take the limit of the result as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0}(-10t - 5\,\Delta t) = -10t$$

Substitute $t = 2.0$ s: $a_x = (-10)(2.0)\ \mathrm{m/s^2} = \boxed{-20\ \mathrm{m/s^2}}$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.

Notice that the answers to parts (A) and (B) are different. The average acceleration in part (A) is the slope of the blue line in Figure 2.9 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in part (B) is the slope of the green line tangent to the curve at point Ⓑ. Notice also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.6.

So far, we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. If you are familiar with calculus, you should recognize that there are specific rules for taking

derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t such as in the expression

$$x = At^n$$

where A and n are constants. (This expression is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.6, in which $v_x = 40 - 5t^2$, we quickly find that the acceleration is $a_x = dv_x/dt = -10t$, as we found in part (B) of the example.

2.6 Analysis Model: Particle Under Constant Acceleration

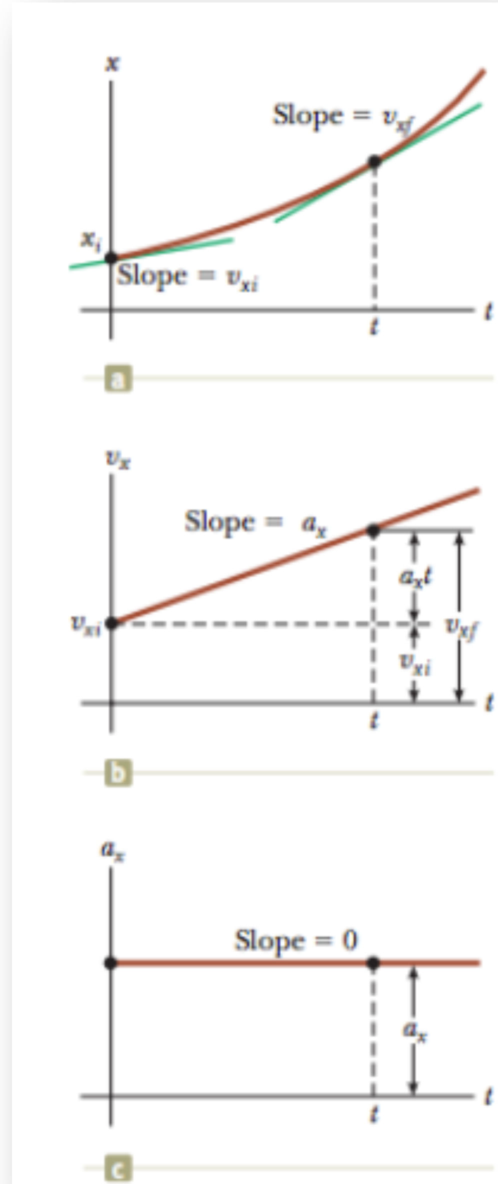
- Used in situations with uniform acceleration

$$v_f = v_i + at$$

$$x_f = x_i + \bar{v}t = \frac{1}{2}\left(v_i + v_f\right)t$$

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$



Kinematic Equations

Analysis Model **Particle Under Constant Acceleration**

Imagine a moving object that can be modeled as a particle. If it begins from position x_i and initial velocity v_{xi} and moves in a straight line with a constant acceleration a_x, its subsequent position and velocity are described by the following kinematic equations:

$$v_{xf} = v_{xi} + a_x t \tag{2.13}$$

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \tag{2.14}$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.15}$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.16}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \tag{2.17}$$

Examples

- a car accelerating at a constant rate along a straight freeway
- a dropped object in the absence of air resistance (Section 2.7)
- an object on which a constant net force acts (Chapter 5)
- a charged particle in a uniform electric field (Chapter 23)

Notes on the equations

$$\Delta x = v_{average}\, t = \left(\frac{v_o + v_f}{2}\right) t$$

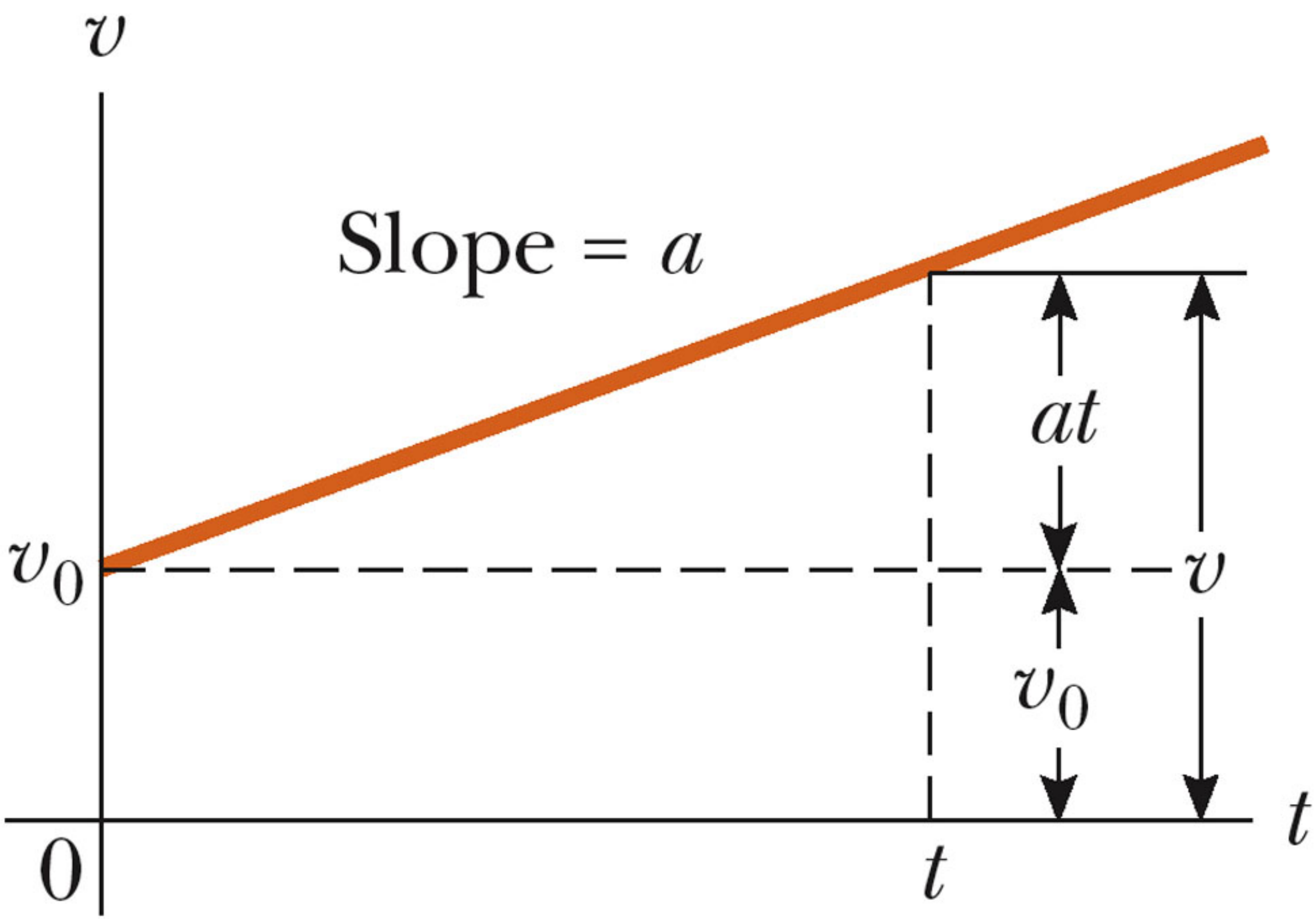
- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

Notes on the equations

$$v = v_o + at$$

- Shows velocity as a function of acceleration and time
- Use when you don't know and aren't asked to find the displacement

Graphical Interpretation of the Equation



Notes on the equations

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

- Gives displacement as a function of time, velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity

Notes on the equations

$$v^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement
- Use when you don’t know and aren’t asked for the time

Example 2.7 Carrier Landing AM

A jet lands on an aircraft carrier at a speed of 140 mi/h ($\approx$ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

2.7 continued

Equation 2.13 is the only equation in the particle under constant acceleration model that does not involve position, so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}}$$

$$= \boxed{-32\text{ m/s}^2}$$

(B) If the jet touches down at position $x_i = 0$, what is its final position?

SOLUTION

Use Equation 2.15 to solve for the final position:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s}) = \boxed{63\text{ m}}$$

Given the size of aircraft carriers, a length of 63 m seems reasonable for stopping the jet. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of World War I. The cables are still a vital part of the operation of modern aircraft carriers.

WHAT IF? Suppose the jet lands on the deck of the aircraft carrier with a speed faster than 63 m/s but has the same acceleration due to the cable as that calculated in part (A). How will that change the answer to part (B)?

Answer If the jet is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.15 that if v_{xi} is larger, then x_f will be larger.

2.7 Free Fall

- All objects moving under the influence of gravity only are said to be in free fall
  - Free fall does not depend on the object's original motion
- All objects falling near the earth's surface fall with a constant acceleration
- The acceleration is called the acceleration due to gravity, and indicated by *g*

Acceleration due to Gravity

- Symbolized by g
- g = 9.80 m/s^2
  - When estimating, use g » 10 m/s^2
- g is always directed downward
  - toward the center of the earth
- Ignoring air resistance and assuming g doesn't vary with altitude over short vertical distances, free fall is constantly accelerated motion

$$v_f = v_i - gt$$

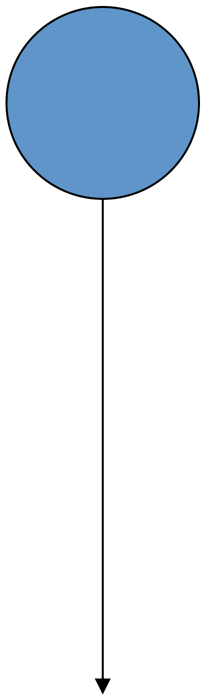
$$y_f = y_i + \bar{v}t = \frac{1}{2}\left(v_i + v_f\right)t$$

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i)$$

Free Fall – an object dropped

- Initial velocity is zero
- Let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical
- Acceleration is g = -9.80 m/s^2

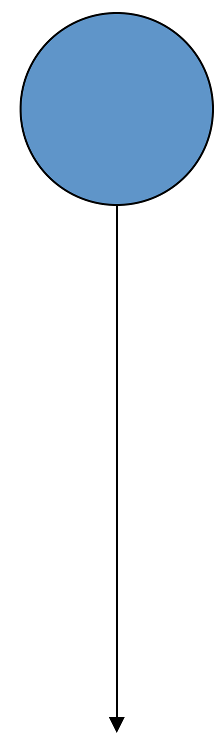


$v_i = 0$

$a = g$

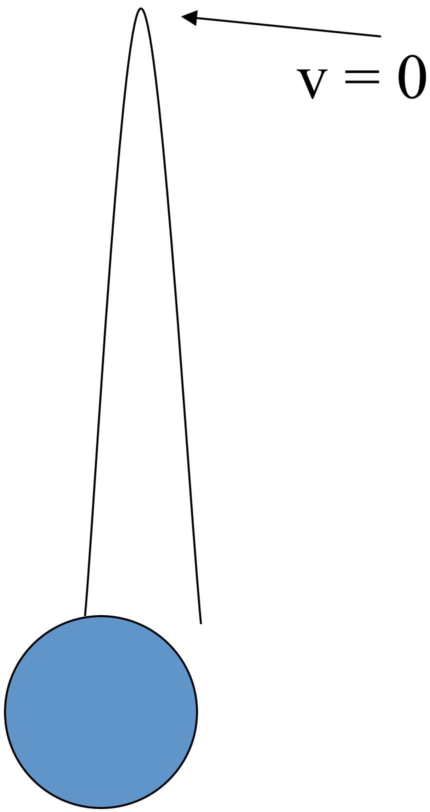
Free Fall - an object throw downward

- $a = g = -9.80\ m/s^2$
- Initial velocity $\neq 0$
  - With upward being positive, initial velocity will be negative



Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- a = g = -9.80 m/s^2 everywhere in the motion

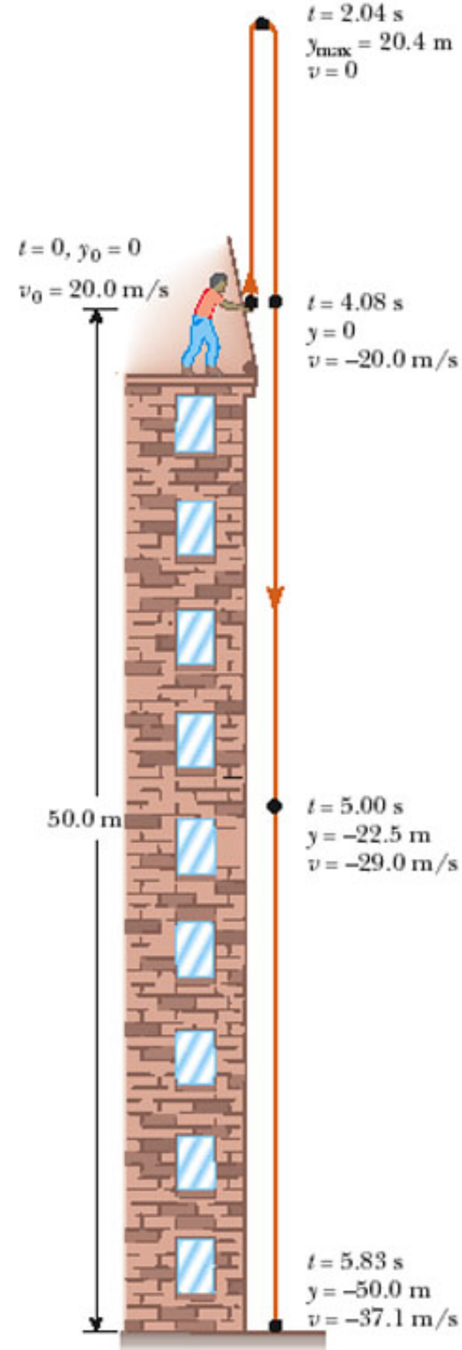


Thrown upward, cont.

- The motion may be symmetrical
  - Then $t_{up} = t_{down}$
  - Then $v = -v_i$
- The motion may not be symmetrical
  - Break the motion into various parts
    - Generally up and down

Non-symmetrical Free Fall

- Need to divide the motion into segments
- Possibilities include
  - Upward and downward portions
  - The symmetrical portion back to the release point and then the non-symmetrical portion



Example 2.10 Not a Bad Throw for a Rookie! AM

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_Ⓐ = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

SOLUTION

You most likely have experience with dropping objects or throwing them upward and watching them fall, so this problem should describe a familiar experience. To simulate this situation, toss a small object upward and notice the time interval required for it to fall to the floor. Now imagine throwing that object upward from the roof of a building. Because the stone is in free fall, it is modeled as a *particle under constant acceleration* due to gravity.

Recognize that the initial velocity is positive because the stone is launched upward. The velocity will change sign after the stone reaches its highest point, but the acceleration of the stone will *always* be downward so that it will always have a negative value. Choose an initial point just after the stone leaves the person's hand and a final point at the top of its flight.

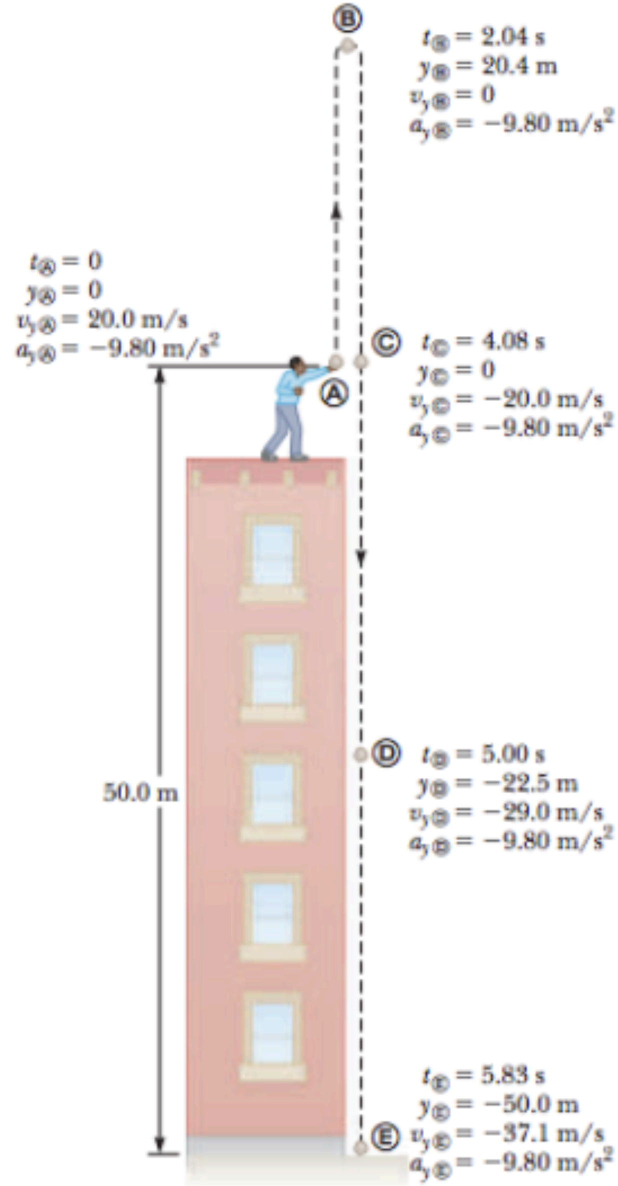


Figure 2.14 (Example 2.10) Position, velocity, and acceleration values at various times for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s. Many of the quantities in the labels for points in the motion of the stone are calculated in the example. Can you verify the other values that are not?

Use Equation 2.13 to calculate the time at which the stone reaches its maximum height:

$$v_{yf} = v_{yi} + a_y t \rightarrow t = \frac{v_{yf} - v_{yi}}{a_y}$$

Substitute numerical values:

$$t = t_Ⓑ = \frac{0 - 20.0\ m/s}{-9.80\ m/s^2} = \boxed{2.04\ s}$$

(B) Find the maximum height of the stone.

SOLUTION

As in part (A), choose the initial and final points at the beginning and the end of the upward flight.

Set $y_{Ⓐ} = 0$ and substitute the time from part (A) into Equation 2.16 to find the maximum height:

$$y_{max} = y_{Ⓑ} = y_{Ⓐ} + v_{xⒶ}t + \tfrac{1}{2}a_y t^2$$

$$y_{Ⓑ} = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2 = \boxed{20.4 \text{ m}}$$

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

SOLUTION

Choose the initial point where the stone is launched and the final point when it passes this position coming down.

Substitute known values into Equation 2.17:

$$v_{yⒸ}^2 = v_{yⒶ}^2 + 2a_y(y_{Ⓒ} - y_{Ⓐ})$$

$$v_{yⒸ}^2 = (20.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(0 - 0) = 400 \text{ m}^2/\text{s}^2$$

$$v_{yⒸ} = \boxed{-20.0 \text{ m/s}}$$

2.10 continued

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point Ⓒ. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

2.10 continued

When taking the square root, we could choose either a positive or a negative root. We choose the negative root because we know that the stone is moving downward at point Ⓒ. The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but is opposite in direction.

(D) Find the velocity and position of the stone at $t = 5.00$ s.

SOLUTION

Choose the initial point just after the throw and the final point 5.00 s later.

Calculate the velocity at Ⓓ from Equation 2.13:

$$v_{y Ⓓ} = v_{y Ⓐ} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s}) = \boxed{-29.0 \text{ m/s}}$$

Use Equation 2.16 to find the position of the stone at $t_{Ⓓ} = 5.00$ s:

$$y_{Ⓓ} = y_{Ⓐ} + v_{y Ⓐ} t + \tfrac{1}{2} a_y t^2$$
$$= 0 + (20.0 \text{ m/s})(5.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$
$$= \boxed{-22.5 \text{ m}}$$

The choice of the time defined as $t = 0$ is arbitrary and up to you to select as the problem solver. As an example of this arbitrariness, choose $t = 0$ as the time at which the stone is at the highest point in its motion. Then solve parts (C) and (D) again using this new initial instant and notice that your answers are the same as those above.

WHAT IF? What if the throw were from 30.0 m above the ground instead of 50.0 m? Which answers in parts (A) to (D) would change?

Answer None of the answers would change. All the motion takes place in the air during the first 5.00 s. (Notice that even for a throw from 30.0 m, the stone is above the ground at $t = 5.00$ s.) Therefore, the height of the throw is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height of the throw into any equation.

Home work (6,20,29,41,50&51)

1-The position of a particle moving along the x axis varies in time according to the expression $x=3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at t = 3.00 s and (b) at 3.00 s +Δt. (c) Evaluate the limit of Δx/Δt as Δt approaches zero to find the velocity at t 5 3.00 s.

2. An object moves along the x axis according to the equation x=3.00t - 2.00t + 3.00, where x is in meters and t is in seconds. Determine (a) the average speed between t = 2.00 s and t = 3.00 s, (b) the instantaneous speed at t = 2.00 s and at t = 3.00 s, (c) the average acceleration between t = 2.00 s and t = 3.00 s, and (d) the instantaneous acceleration at t = 2.00 s and t = 3.00 s. (e) At what time is the object at rest?

3. An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

4. An object moves with constant acceleration 4.00 m/s2 and over a time interval reaches a final velocity of 12.0 m/s. (a) If its initial velocity is 6.00 m/s, what is its displacement during the time interval? (b) What is the distance it travels during this interval? (c) If its initial velocity is 26.00 m/s, what is its displacement during the time interval? (d) What is the total distance it travels during the interval in part (c) ?

5. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) the ball's initial velocity and (b) the height it reaches.

6. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. At t = 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

7-A ball is thrown directly downward with an initial speed of 8.00 m/s from a height of 30.0 m. After what time interval does it strike the ground?