

Chapter 5

The Laws of Motion

- 5.1 The Concept of Force
- 5.2 Newton's First Law and Inertial Frames
- 5.3 Mass
- 5.4 Newton's Second Law
- 5.5 The Gravitational Force and Weight
- 5.6 Newton's Third Law
- 5.7 Analysis Models Using Newton's Second Law
- 5.8 Forces of Friction





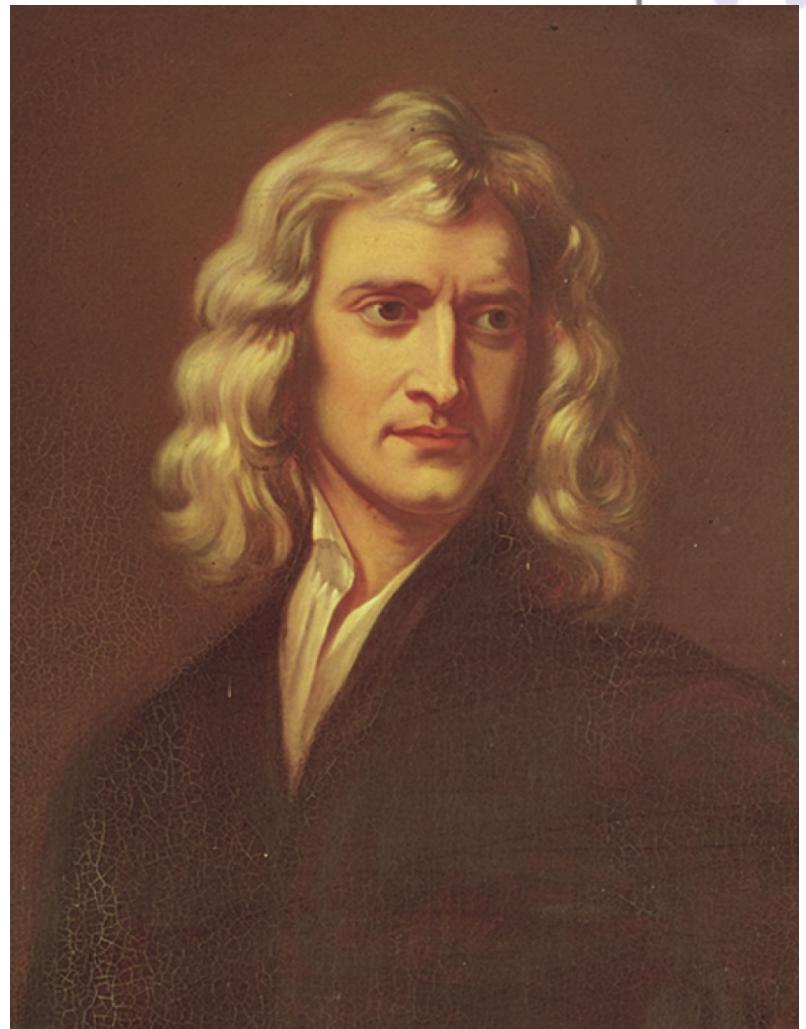
The Laws of Motion

- The description of an object in motion included its position, velocity, and acceleration.
- There was no consideration of what might influence that motion.
- Two main factors need to be addressed to answer questions about why the motion of an object will change.
 - Forces acting on the object
 - The mass of the object
- Dynamics studies the causes of motion.
- Will start with three basic laws of motion
 - Formulated by Sir Isaac Newton



Sir Isaac Newton

- 1642 – 1727
- Formulated basic laws of mechanics
- Discovered Law of Universal Gravitation
- Many observations dealing with light and optics

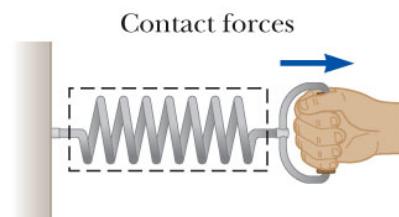


Force

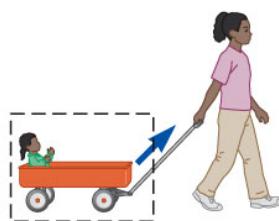


- Forces in everyday experience
 - Push on an object to move it
 - Throw or kick a ball
 - May push on an object and not be able to move it
- Forces are what cause any change in the velocity of an object.
 - Newton's definition
 - A force is that which causes an acceleration

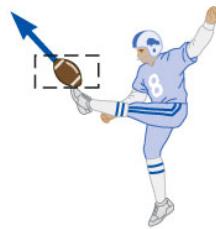
Classes of Forces



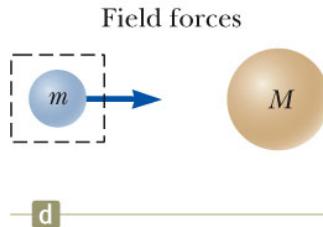
a



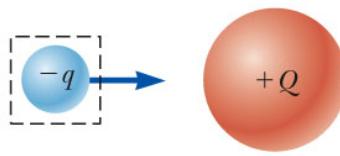
b



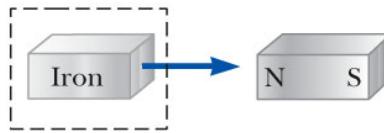
c



d



e



f

- Contact forces involve physical contact between two objects
 - Examples a, b, c
- Field forces act through empty space
 - No physical contact is required
 - Examples d, e, f



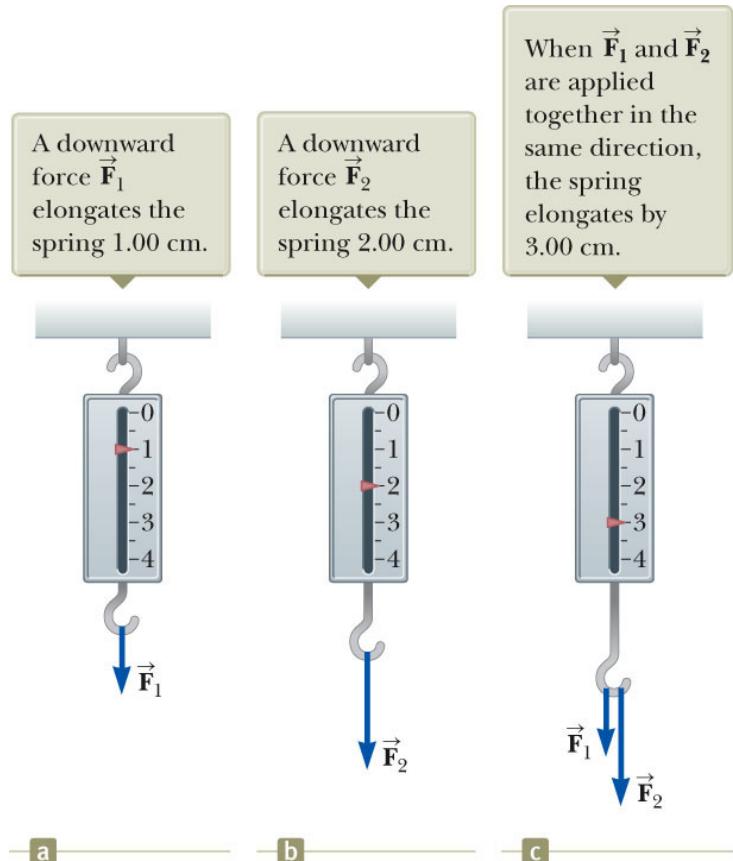
Fundamental Forces

- Gravitational force
 - Between objects
- Electromagnetic forces
 - Between electric charges
- Nuclear force
 - Between subatomic particles
- Weak forces
 - Arise in certain radioactive decay processes
- Note: These are all field forces.

More About Forces



- A spring can be used to calibrate the magnitude of a force.
- Doubling the force causes double the reading on the spring.
- When both forces are applied, the reading is three times the initial reading.

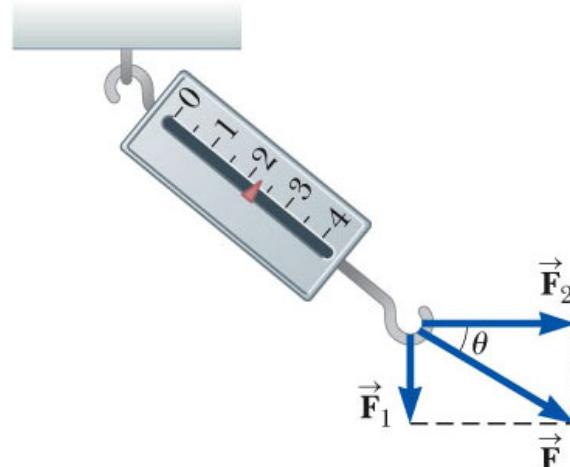




Vector Nature of Forces

- The forces are applied perpendicularly to each other.
- The resultant (or net) force is the hypotenuse.
- Forces are vectors, so you must use the rules for vector addition to find the net force acting on an object.

When \vec{F}_1 is downward and \vec{F}_2 is horizontal, the combination of the two forces elongates the spring by 2.24 cm.





Newton's First Law

- If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.
 - This is also called the *law of inertia*.
 - It defines a special set of reference frames called *inertial frames*.
 - We call this an *inertial frame of reference*.



Inertial Frames

- A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame.
 - We can consider the Earth to be such an inertial frame, although it has a small centripetal acceleration associated with its motion.

Newton's First Law – Alternative Statement



- In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.
 - Newton's First Law describes what happens in the absence of a force.
 - Does not describe zero net force
 - Also tells us that when no force acts on an object, the acceleration of the object is zero
 - Can conclude that any isolated object is either at rest or moving at a constant velocity
 - The First Law also allows the definition of **force** as ***that which causes a change in the motion of an object.***



Inertia and Mass

- The tendency of an object to resist any attempt to change its velocity is called *inertia*.
- **Mass** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity.
- Masses can be defined in terms of the accelerations produced by a given force acting on them:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

- The magnitude of the acceleration acting on an object is inversely proportional to its mass.



More About Mass

- Mass is an inherent property of an object.
- Mass is independent of the object's surroundings.
- Mass is independent of the method used to measure it.
- Mass is a scalar quantity.
 - Obeys the rules of ordinary arithmetic
- The SI unit of mass is kg.



Mass vs. Weight

- Mass and weight are two different quantities.
- Weight is equal to the magnitude of the gravitational force exerted on the object.
 - Weight will vary with location.
- Example:
 - $w_{\text{earth}} = 180 \text{ lb}$; $w_{\text{moon}} \sim 30 \text{ lb}$
 - $m_{\text{earth}} = 2 \text{ kg}$; $m_{\text{moon}} = 2 \text{ kg}$



Newton's Second Law

- When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

- Force is the cause of *changes* in motion, as measured by the acceleration.
 - Remember, an object can have motion in the absence of forces.
 - Do not interpret force as the cause of motion.

- Algebraically,

$$\vec{a} \propto \frac{\sum \vec{F}}{m} \rightarrow \sum \vec{F} = m\vec{a}$$

- With a proportionality constant of 1 and speeds much lower than the speed of light.

More About Newton's Second Law



- $\sum \vec{F}$ is the net force
 - This is the vector sum of all the forces acting on the object.
 - May also be called the total force, resultant force, or the unbalanced force.
- Newton's Second Law can be expressed in terms of components:
 - $\sum \vec{F}_x = m a_x$
 - $\sum \vec{F}_y = m a_y$
 - $\sum \vec{F}_z = m a_z$
- Remember that $m a$ is not a force.
 - The sum of the forces is equated to this product of the mass of the object and its acceleration.



Units of Force

- The SI unit of force is the **newton (N)**.
 - $1 \text{ N} = 1 \text{ kg}\cdot\text{m} / \text{s}^2$
- The US unit of force is a **pound (lb)**.
 - $1 \text{ lb} = 1 \text{ slug}\cdot\text{ft} / \text{s}^2$
- $1 \text{ N} \sim \frac{1}{4} \text{ lb}$



Example 5.1 An Accelerating Hockey Puck AM

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force \vec{F}_1 has a magnitude of 5.0 N, and is directed at $\theta = 20^\circ$ below the x axis. The force \vec{F}_2 has a magnitude of 8.0 N and its direction is $\phi = 60^\circ$ above the x axis. Determine both the magnitude and the direction of the puck's acceleration.

SOLUTION

Conceptualize Study Figure 5.4. Using your expertise in vector addition from Chapter 3, predict the approximate direction of the net force vector on the puck. The acceleration of the puck will be in the same direction.

Categorize Because we can determine a net force and we want an acceleration, this problem is categorized as one that may be solved using Newton's second law. In Section 5.7, we will formally introduce the *particle under a net force* analysis model to describe a situation such as this one.

Analyze Find the component of the net force acting on the puck in the x direction:

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos \theta + F_2 \cos \phi$$

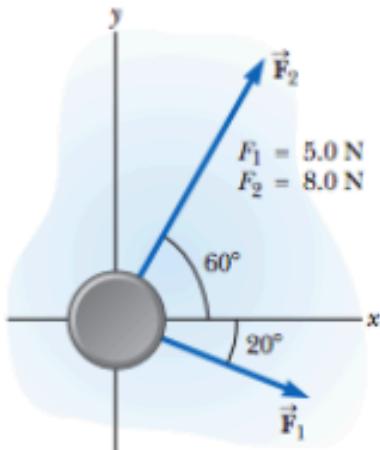


Figure 5.4

(Example 5.1) A hockey puck moving on a frictionless surface is subject to two forces \vec{F}_1 and \vec{F}_2 .



► 5.1 continued

Find the component of the net force acting on the puck in the y direction:

Use Newton's second law in component form (Eq. 5.3) to find the x and y components of the puck's acceleration:

Substitute numerical values:

Find the magnitude of the acceleration:

Find the direction of the acceleration relative to the positive x axis:

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin \theta + F_2 \sin \phi$$

$$a_x = \frac{\sum F_x}{m} = \frac{F_1 \cos \theta + F_2 \cos \phi}{m}$$

$$a_y = \frac{\sum F_y}{m} = \frac{F_1 \sin \theta + F_2 \sin \phi}{m}$$

$$a_x = \frac{(5.0 \text{ N}) \cos(-20^\circ) + (8.0 \text{ N}) \cos(60^\circ)}{0.30 \text{ kg}} = 29 \text{ m/s}^2$$

$$a_y = \frac{(5.0 \text{ N}) \sin(-20^\circ) + (8.0 \text{ N}) \sin(60^\circ)}{0.30 \text{ kg}} = 17 \text{ m/s}^2$$

$$a = \sqrt{(29 \text{ m/s}^2)^2 + (17 \text{ m/s}^2)^2} = 34 \text{ m/s}^2$$

$$\theta = \tan^{-1} \left(\frac{a_y}{a_x} \right) = \tan^{-1} \left(\frac{17}{29} \right) = 31^\circ$$

Gravitational Force



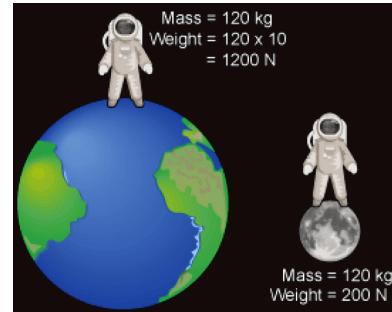
- The gravitational force, \vec{F}_g , is the force that the earth exerts on an object.
- This force is directed toward the center of the earth.
- From Newton's Second Law:
 - $\vec{F}_g = m\vec{g}$
- Its magnitude is called the weight of the object.
 - Weight = $F_g = mg$



More About Weight

- Because it is dependent on g , the weight varies with location.
 - g , and therefore the weight, is less at higher altitudes.
 - This can be extended to other planets, but the value of g varies from planet to planet, so the object's weight will vary from planet to planet.
- Weight is not an inherent property of the object.
 - The weight is a property of a *system* of items: the object and the Earth.
- Note about units:
 - Kilogram is **not** a unit of weight.

Mass vs. Weight



Mass

- is measured in kilograms
- always remains the same
- is closely related to inertia
- can NOT be measured directly

Weight

- is measured in Newtons
- can change with location
- is closely related to gravity
- can be measured directly using a scale

Gravitational Mass vs. Inertial Mass



- In Newton's Laws, the mass is the inertial mass and measures the resistance to a change in the object's motion.
- In the gravitational force, the mass is determining the gravitational attraction between the object and the Earth.
- Experiments show that gravitational mass and inertial mass have the same value.

Gravitational Mass vs. Inertial Mass



Gravitational Mass

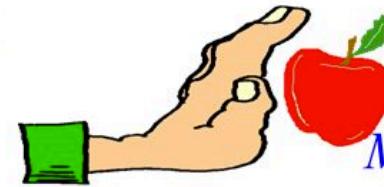
Inertial Mass

Gravitational or Inertial Mass?



A red apple is shown falling downwards under the influence of gravity, indicated by a green arrow labeled 'g' pointing vertically downwards.

$M_{\text{gravitational}}$



A hand wearing a green sleeve is shown pushing a red apple horizontally to the right.

M_{inertial}

*So, if I put an object in a gravitational field, it responds with its **gravitational mass**.*

$$m = \frac{w}{g}$$

*If I push an object, it responds to that push with its **inertial mass**.*

$$m = \frac{F}{a}$$

Question: Are these two masses the same?

There is absolutely no reason why they have to be the same, but according to the best experiments physicists, they are.



Newton's Third Law

- If two objects interact, the force \vec{F}_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force \vec{F}_{21} exerted by object 2 on object 1.
 - $\vec{F}_{12} = -\vec{F}_{21}$
 - Note on notation: \vec{F}_{AB} is the force exerted *by A on B*.

Newton's Third Law, Alternative Statement

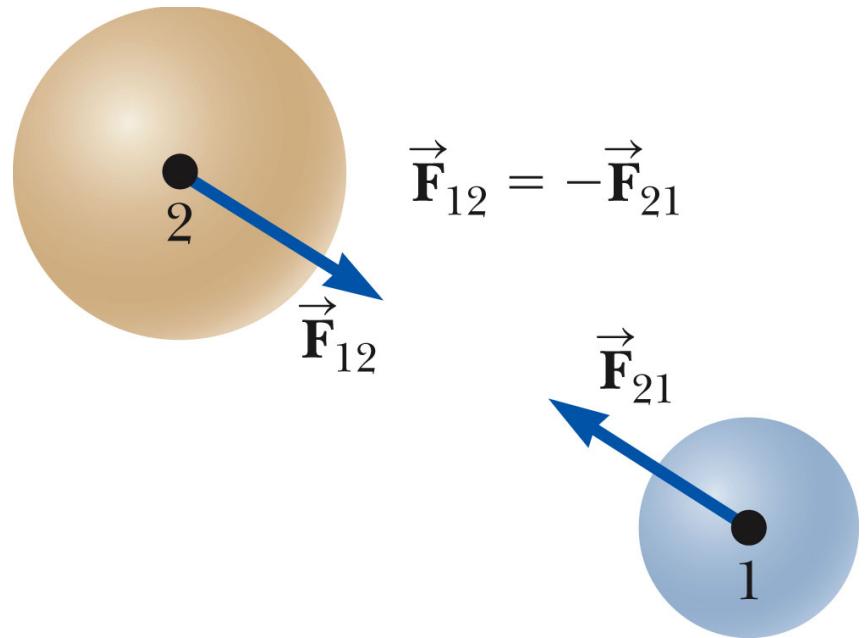


- The action force is equal in magnitude to the reaction force and opposite in direction.
 - One of the forces is the action force, the other is the reaction force.
 - The action and reaction forces must act on different objects and be of the same type.



Action-Reaction Examples, 1

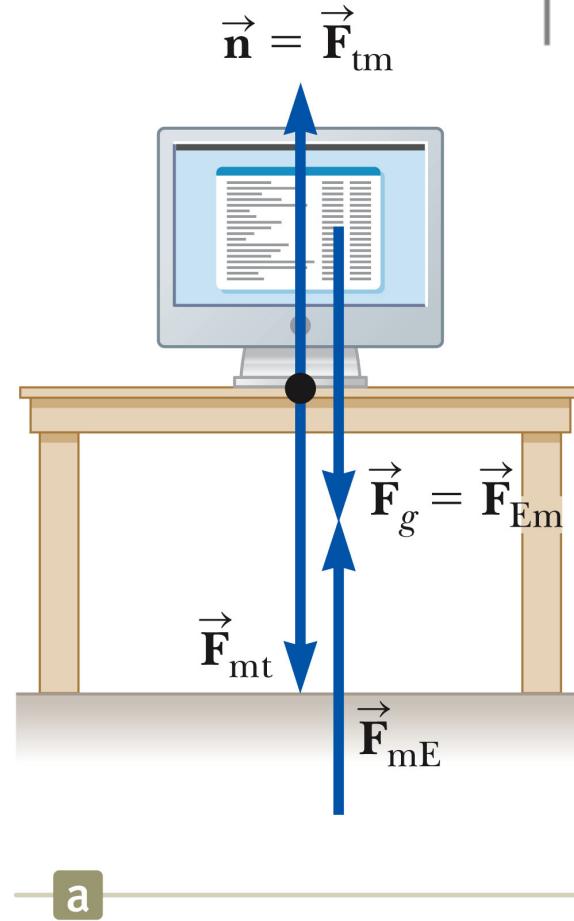
- The force \vec{F}_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to \vec{F}_{21} exerted by object 2 on object 1.
 - $\vec{F}_{12} = -\vec{F}_{21}$



Action-Reaction Examples, 2



- The normal force (table on monitor) is the reaction of the force the monitor exerts on the table.
 - Normal means perpendicular, in this case
- The action (Earth on monitor) force is equal in magnitude and opposite in direction to the reaction force, the force the monitor exerts on the Earth.





Forces on the Object

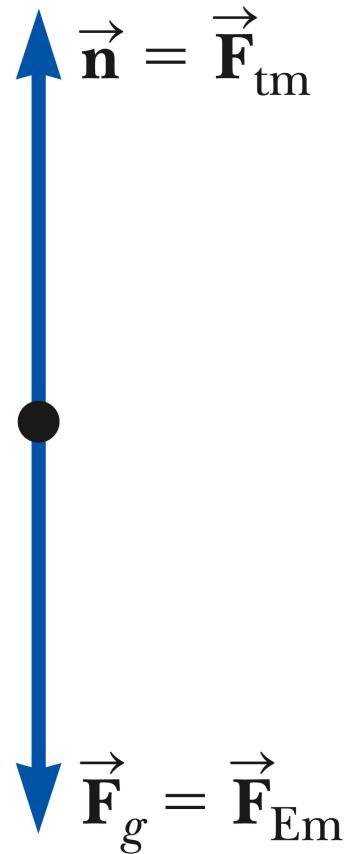
- In a free body diagram, you want the forces acting on a particular object.
 - Model the object as a particle
- The normal force and the force of gravity are the forces that act on the monitor.





Free Body Diagram

- The most important step in solving problems involving Newton's Laws is to draw the free body diagram.
- Be sure to include only the forces acting on the object of interest.
- Include any field forces acting on the object.
- Do not assume the normal force equals the weight.



Free Body Diagrams and the Particle Model



- The particle model is used by representing the object as a dot in the free body diagram.
- The forces that act on the object are shown as being applied to the dot.
- The free body helps isolate only those forces acting on the object and eliminate the other forces from the analysis.

Analysis Models Using Newton's Second Law



- Assumptions

- Objects can be modeled as particles.
- Interested only in the external forces acting on the object
 - Can neglect reaction forces
- Initially dealing with frictionless surfaces
- Masses of strings or ropes are negligible.

Analysis Model: The Particle in Equilibrium



- If the acceleration of an object that can be modeled as a particle is zero, the object is said to be in **equilibrium**.

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

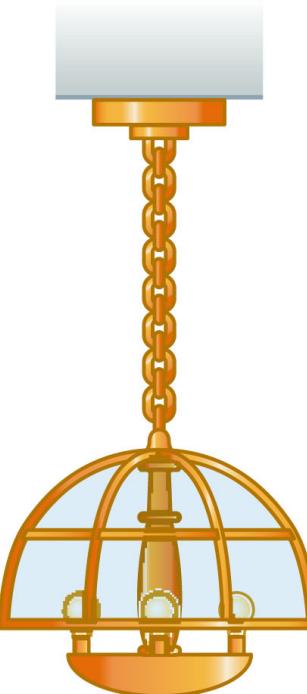
- The model is the *particle in equilibrium model*.
- Mathematically, the net force acting on the object is zero.

Equilibrium, Example



- A lamp is suspended from a chain of negligible mass.
- The forces acting on the lamp are:
 - the downward force of gravity
 - the upward tension in the chain
- Applying equilibrium gives

$$\sum F_y = 0 \rightarrow T - F_g = 0 \rightarrow T = F_g$$



Analysis Model: The Particle Under a Net Force

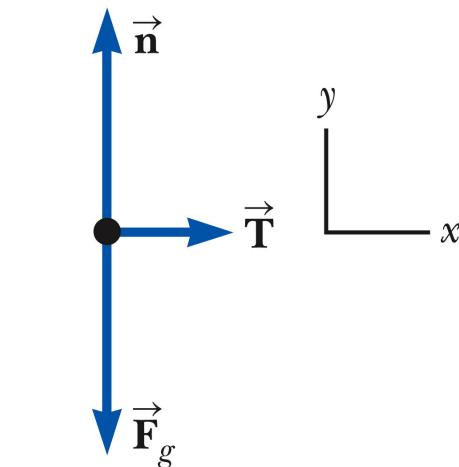


- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it.
 - Model is *particle under a net force model*.
- Draw a free-body diagram.
- Apply Newton's Second Law in component form.

Newton's Second Law, Example 1



- Forces acting on the crate:
 - A tension, acting through the rope, is the magnitude of force \vec{T}
 - The gravitational force, \vec{F}_g
 - The normal force, \vec{n} , exerted by the floor



Newton's Second Law, Example cont.



- Apply Newton's Second Law in component form:

$$\sum F_x = T = ma_x$$

$$\sum F_y = n - F_g = 0 \rightarrow n = F_g$$

- Solve for the unknown(s)
- If the tension is constant, then a is constant and the kinematic equations can be used to more fully describe the motion of the crate.

Note About the Normal Force



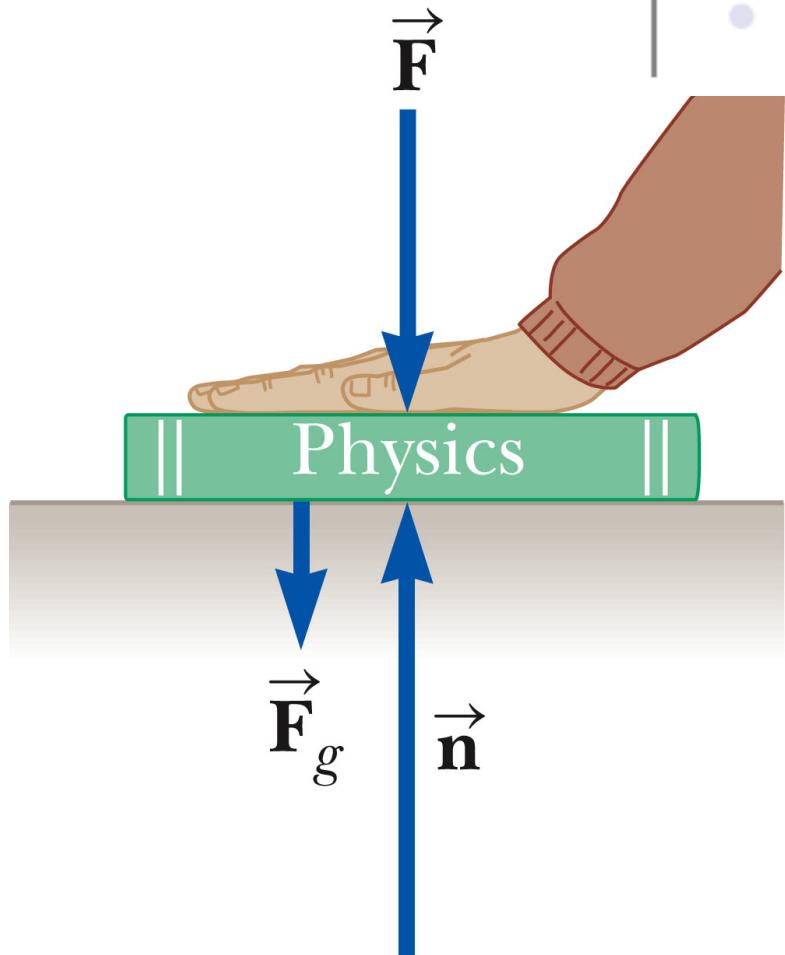
- The normal force is **not** always equal to the gravitational force of the object.

- For example, in this case

$$\sum F_y = n - F_g - F = 0$$

$$\text{and } n = mg + F$$

- \vec{n} may also be less than \vec{F}_g



Problem-Solving Hints – Applying Newton's Laws



- 1 Conceptualize
 - Draw a diagram
 - Choose a convenient coordinate system for each object
- 2 Categorize
 - Is the model a particle in equilibrium?
 - If so, $\sum \vec{F} = 0$
 - Is the model a particle under a net force?
 - If so, $\sum \vec{F} = m a$

Problem-Solving Hints – Applying Newton's Laws, cont.



•3 Analyze

- Draw free-body diagrams for each object
- Include only forces acting on the object
- Find components along the coordinate axes
- Be sure units are consistent
- Apply the appropriate equation(s) in component form
- Solve for the unknown(s)

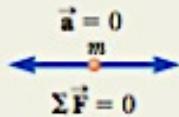
•4 Finalize

- Check your results for consistency with your free-body diagram
- Check extreme values

Analysis Model Particle in Equilibrium

Imagine an object that can be modeled as a particle. If it has several forces acting on it so that the forces all cancel, giving a net force of zero, the object will have an acceleration of zero. This condition is mathematically described as

$$\sum \vec{F} = 0 \quad (5.8)$$



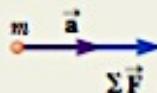
Examples

- a chandelier hanging over a dining room table
- an object moving at terminal speed through a viscous medium (Chapter 6)
- a steel beam in the frame of a building (Chapter 12)
- a boat floating on a body of water (Chapter 14)

Analysis Model Particle Under a Net Force

Imagine an object that can be modeled as a particle. If it has one or more forces acting on it so that there is a net force on the object, it will accelerate in the direction of the net force. The relationship between the net force and the acceleration is

$$\sum \vec{F} = m \vec{a} \quad (5.2)$$



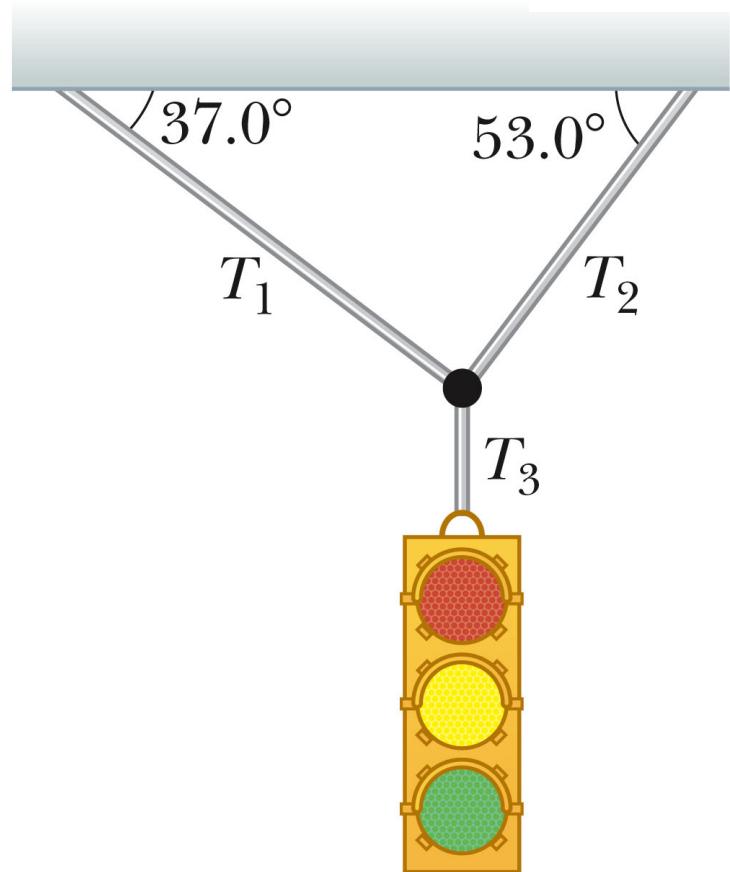
Examples

- a crate pushed across a factory floor
- a falling object acted upon by a gravitational force
- a piston in an automobile engine pushed by hot gases (Chapter 22)
- a charged particle in an electric field (Chapter 23)

Equilibrium, Example 2



- Example 5.4
- Conceptualize the traffic light
 - Assume cables don't break
 - Nothing is moving
- Categorize as an equilibrium problem
 - No movement, so acceleration is zero
 - Model as a particle in equilibrium

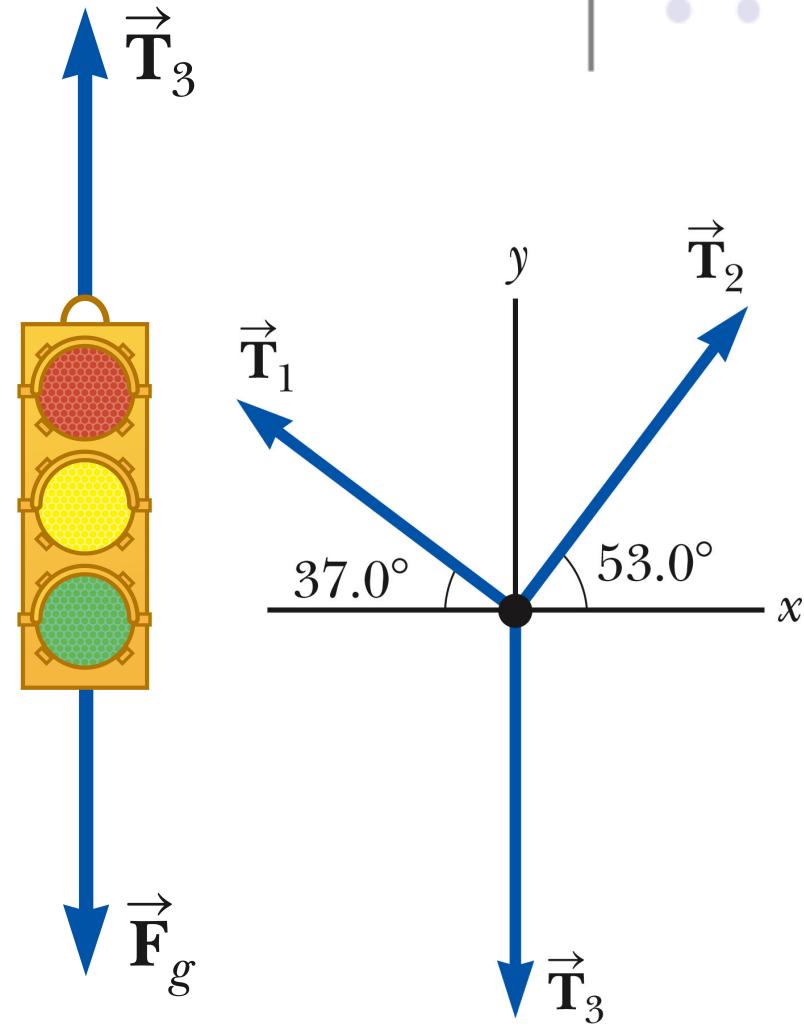


Equilibrium, Example 2, cont.



- Analyze

- Construct a diagram for the forces acting on the light
- Construct a free body diagram for the knot where the three cables are joined
 - The knot is a convenient point to choose since all the forces of interest act along lines passing through the knot.
- Apply equilibrium equations to the knot





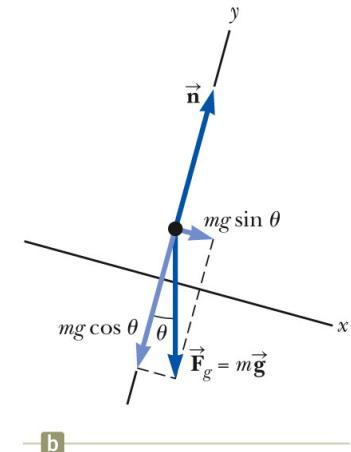
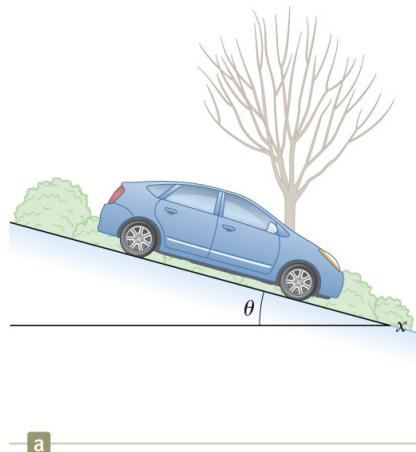
Equilibrium, Example 2, final

- Analyze, cont.
 - Find T_3 from applying equilibrium in the y-direction to the light
 - Find T_1 and T_2 from applying equilibrium in the x- and y-directions to the knot
- Finalize
 - Think about different situations and see if the results are reasonable.

Inclined Planes



- Categorize as a particle under a net force since it accelerates.
- Forces acting on the object:
 - The normal force acts perpendicular to the plane.
 - The gravitational force acts straight down.
- Choose the coordinate system with x along the incline and y perpendicular to the incline.
- Replace the force of gravity with its components.
- Apply the model of a particle under a net force to the x -direction and a particle in equilibrium to the y -direction.



Example 5.6

The Runaway Car

AM



A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

- (A) Find the acceleration of the car, assuming the driveway is frictionless.

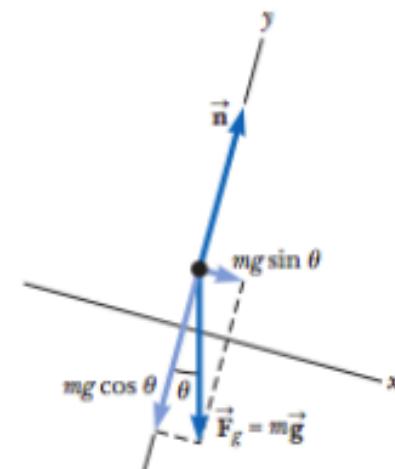
SOLUTION

Conceptualize Use Figure 5.11a to conceptualize the situation. From everyday experience, we know that a car on an icy incline will accelerate down the incline. (The same thing happens to a car on a hill with its brakes not set.)

Categorize We categorize the car as a *particle under a net force* because it accelerates. Furthermore, this example belongs to a very common category of problems in which an object moves under the influence of gravity on an inclined plane.

Analyze Figure 5.11b shows the free-body diagram for the car. The only forces acting on the car are the normal force \vec{n} exerted by the inclined plane, which acts perpendicular to the plane, and the gravitational force $\vec{F}_g = m\vec{g}$, which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with x along the incline and y perpendicular to it as in Figure 5.11b. With these axes, we represent the gravitational force by a component of magnitude $mg \sin \theta$ along the positive x axis and one of magnitude $mg \cos \theta$ along the negative y axis. Our choice of axes results in the car being modeled as a particle under a net force in the x direction and a particle in equilibrium in the y direction.

Apply these models to the car:



a

b

Figure 5.11 (Example 5.6) (a) A car on a frictionless incline. (b) The free-body diagram for the car. The black dot represents the position of the center of mass of the car. We will learn about center of mass in Chapter 9.

Solve Equation (1) for a_x :

$$(1) \quad \sum F_x = mg \sin \theta = ma_x$$

$$(2) \quad \sum F_y = n - mg \cos \theta = 0$$

$$(3) \quad a_x = g \sin \theta$$



(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d . How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

SOLUTION

Conceptualize Imagine the car is sliding down the hill and you use a stopwatch to measure the entire time interval until it reaches the bottom.

Categorize This part of the problem belongs to kinematics rather than to dynamics, and Equation (3) shows that the acceleration a_x is constant. Therefore, you should categorize the car in this part of the problem as a particle under constant acceleration.

Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, choose Equation 2.16 from the particle under constant acceleration model, $x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2$:

Solve for t :

$$d = \frac{1}{2}a_xt^2$$

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g \sin \theta}}$$

Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

$$(5) \quad v_{xf} = \sqrt{2a_xd} = \sqrt{2gd \sin \theta}$$

Finalize We see from Equations (4) and (5) that the time t at which the car reaches the bottom and its final speed v_{xf} are independent of the car's mass, as was its acceleration. Notice that we have combined techniques from Chapter 2 with new techniques from this chapter in this example. As we learn more techniques in later chapters, this process of combining analysis models and information from several parts of the book will occur more often. In these cases, use the General Problem-Solving Strategy to help you identify what analysis models you will need.

WHAT IF? What previously solved problem does this situation become if $\theta = 90^\circ$?

Answer Imagine θ going to 90° in Figure 5.11. The inclined plane becomes vertical, and the car is an object in free fall! Equation (3) becomes

$$a_x = g \sin \theta = g \sin 90^\circ = g$$

which is indeed the free-fall acceleration. (We find $a_x = g$ rather than $a_x = -g$ because we have chosen positive x to be downward in Fig. 5.11.) Notice also that the condition $n = mg \cos \theta$ gives us $n = mg \cos 90^\circ = 0$. That is consistent with the car falling downward *next to* the vertical plane, in which case there is no contact force between the car and the plane.



Multiple Objects

- When two or more objects are connected or in contact, Newton's laws may be applied to the system as a whole and/or to each individual object.
- Whichever you use to solve the problem, the other approach can be used as a check.



Example 5.7

One Block Pushes Another AM

Two blocks of masses m_1 and m_2 , with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as in Figure 5.12a. A constant horizontal force \vec{F} is applied to m_1 as shown.

- (A) Find the magnitude of the acceleration of the system.

SOLUTION

Conceptualize Conceptualize the situation by using Figure 5.12a and realize that both blocks must experience the *same* acceleration because they are in contact with each other and remain in contact throughout the motion.

Categorize We categorize this problem as one involving a *particle under a net force* because a force is applied to a system of blocks and we are looking for the acceleration of the system.

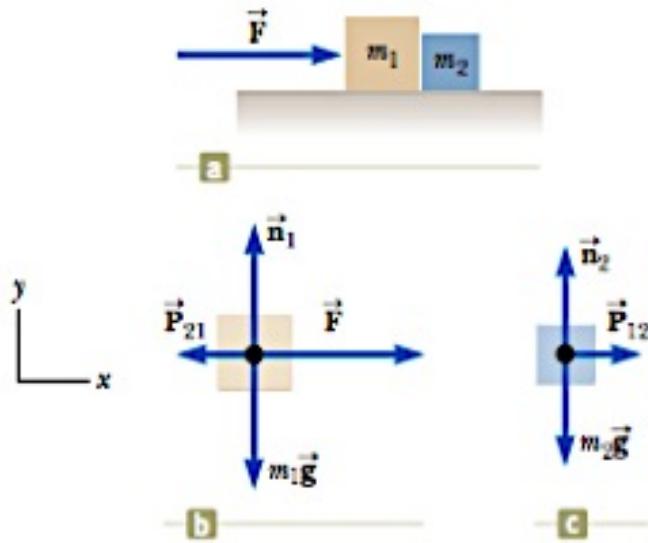


Figure 5.12 (Example 5.7) (a) A force is applied to a block of mass m_1 , which pushes on a second block of mass m_2 . (b) The forces acting on m_1 . (c) The forces acting on m_2 .



Analyze First model the combination of two blocks as a single particle under a net force. Apply Newton's second law to the combination in the x direction to find the acceleration:

$$\sum F_x = F = (m_1 + m_2)a_x$$

$$(1) \quad a_x = \frac{F}{m_1 + m_2}$$

Finalize The acceleration given by Equation (1) is the same as that of a single object of mass $m_1 + m_2$ and subject to the same force.

(B) Determine the magnitude of the contact force between the two blocks.

SOLUTION

Conceptualize The contact force is internal to the system of two blocks. Therefore, we cannot find this force by modeling the whole system (the two blocks) as a single particle.

Categorize Now consider each of the two blocks individually by categorizing each as a *particle under a net force*.

Analyze We construct a diagram of forces acting on the object for each block as shown in Figures 5.12b and 5.12c, where the contact force is denoted by \vec{P} . From Figure 5.12c, we see that the only horizontal force acting on m_2 is the contact force \vec{P}_{12} (the force exerted by m_1 on m_2), which is directed to the right.

Apply Newton's second law to m_2 :

$$(2) \quad \sum F_x = P_{12} = m_2 a_x$$

Substitute the value of the acceleration a_x given by Equation (1) into Equation (2):

$$(3) \quad P_{12} = m_2 a_x = \left(\frac{m_2}{m_1 + m_2} \right) F$$

Finalize This result shows that the contact force P_{12} is *less* than the applied force F . The force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

To finalize further, let us check this expression for P_{12} by considering the forces acting on m_1 , shown in Figure 5.12b. The horizontal forces acting on m_1 are the applied force \vec{F} to the right and the contact force \vec{P}_{21} to the left (the force exerted by m_2 on m_1). From Newton's third law, \vec{P}_{21} is the reaction force to \vec{P}_{12} , so $P_{21} = P_{12}$.

Apply Newton's second law to m_1 :

$$(4) \quad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

Solve for P_{12} and substitute the value of a_x from Equation (1):

$$P_{12} = F - m_1 a_x = F - m_1 \left(\frac{F}{m_1 + m_2} \right) = \left(\frac{m_2}{m_1 + m_2} \right) F$$

This result agrees with Equation (3), as it must.

Example 5.8

Weighing a Fish in an Elevator

AM



A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator as illustrated in Figure 5.13.

- (A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

► 5.8 continued

SOLUTION

Conceptualize The reading on the scale is related to the extension of the spring in the scale, which is related to the force on the end of the spring as in Figure 5.2. Imagine that the fish is hanging on a string attached to the end of the spring. In this case, the magnitude of the force exerted on the spring is equal to the tension T in the string. Therefore, we are looking for T . The force \vec{T} pulls down on the string and pulls up on the fish.

Categorize We can categorize this problem by identifying the fish as a *particle in equilibrium* if the elevator is not accelerating or as a *particle under a net force* if the elevator is accelerating.

Analyze Inspect the diagrams of the forces acting on the fish in Figure 5.13 and notice that the external forces acting on the fish are the downward gravitational force $\vec{F}_g = mg$ and the force \vec{T} exerted by the string. If the elevator is either at rest or moving at constant velocity, the fish is a particle in equilibrium, so $\sum F_y = T - F_g = 0$ or $T = F_g = mg$. (Remember that the scalar mg is the weight of the fish.)

Now suppose the elevator is moving with an acceleration \vec{a} relative to an observer standing outside the elevator in an inertial frame. The fish is now a particle under a net force.

Apply Newton's second law to the fish:

$$\sum F_y = T - mg = ma_y$$

$$(1) \quad T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = F_g\left(\frac{a_y}{g} + 1\right)$$

where we have chosen upward as the positive y direction. We conclude from Equation (1) that the scale reading T is greater than the fish's weight mg if \vec{a} is upward, so a_y is positive (Fig. 5.13a), and that the reading is less than mg if \vec{a} is downward, so a_y is negative (Fig. 5.13b).

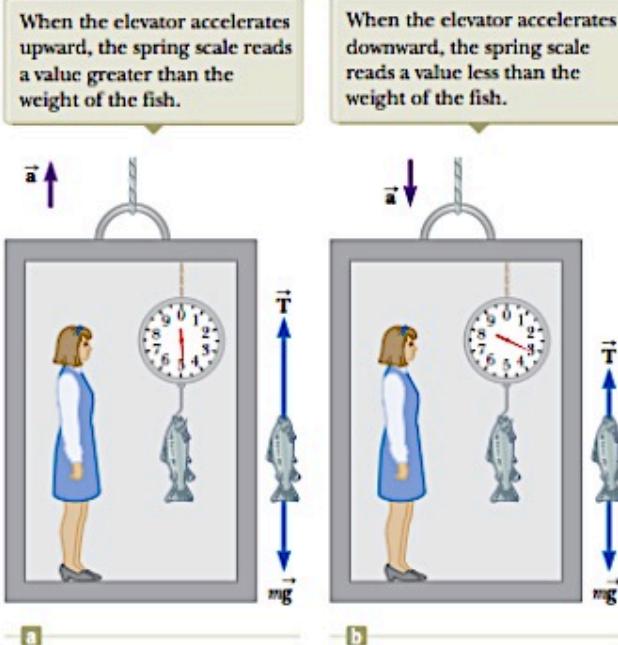


Figure 5.13 (Example 5.8) A fish is weighed on a spring scale in an accelerating elevator car.



(B) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a_y = \pm 2.00 \text{ m/s}^2$.

SOLUTION

Evaluate the scale reading from Equation (1) if \vec{a} is upward:

$$T = (40.0 \text{ N}) \left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1 \right) = 48.2 \text{ N}$$

Evaluate the scale reading from Equation (1) if \vec{a} is downward:

$$T = (40.0 \text{ N}) \left(\frac{-2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1 \right) = 31.8 \text{ N}$$

Finalize Take this advice: if you buy a fish in an elevator, make sure the fish is weighed while the elevator is either at rest or accelerating downward! Furthermore, notice that from the information given here, one cannot determine the direction of the velocity of the elevator.

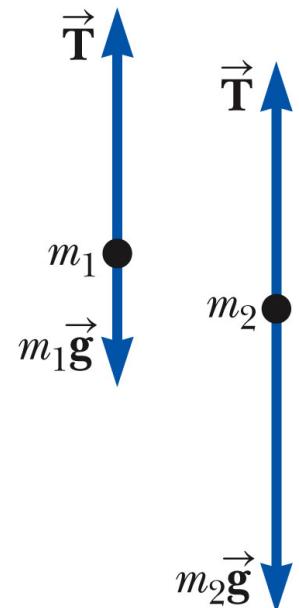
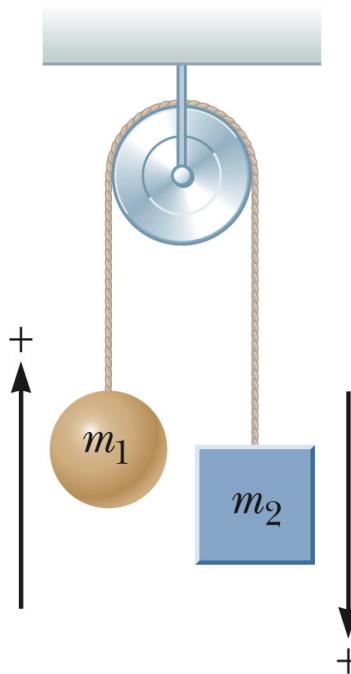
WHAT IF? Suppose the elevator cable breaks and the elevator and its contents are in free fall. What happens to the reading on the scale?

Answer If the elevator falls freely, the fish's acceleration is $a_y = -g$. We see from Equation (1) that the scale reading T is zero in this case; that is, the fish *appears* to be weightless.

Multiple Objects, Example – Atwood’s Machine



- Forces acting on the objects:
 - Tension (same for both objects, one string)
 - Gravitational force
- Each object has the same acceleration since they are connected.
- Draw the free-body diagrams
- Apply Newton’s Laws
- Solve for the unknown(s)



Exploring the Atwood' s Machine



- Vary the masses and observe the values of the tension and acceleration.
 - Note the acceleration is the same for both objects
 - The tension is the same on both sides of the pulley as long as you assume a massless, frictionless pulley.
- What if?
 - The mass of both objects is the same?
 - One of the masses is much larger than the other?

Example 5.9**The Atwood Machine AM**

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in Figure 5.14a, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to determine the value of g . Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

SOLUTION

Conceptualize Imagine the situation pictured in Figure 5.14a in action: as one object moves upward, the other object moves downward. Because the objects are connected by an inextensible string, their accelerations must be of equal magnitude.

Categorize The objects in the Atwood machine are subject to the gravitational force as well as to the forces exerted by the strings connected to them. Therefore, we can categorize this problem as one involving two *particles under a net force*.

Analyze The free-body diagrams for the two objects are shown in Figure 5.14b. Two forces act on each object: the upward force \vec{T} exerted by the string and the downward gravitational force. In problems such as this one in which the pulley is modeled as massless and frictionless, the tension in the string on both sides of the pulley is the same. If the pulley has mass or is subject to friction, the tensions on either side are not the same and the situation requires techniques we will learn in Chapter 10.

We must be very careful with signs in problems such as this one. In Figure 5.14a, notice that if object 1 accelerates upward, object 2 accelerates downward. Therefore, for consistency with signs, if we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2. With this sign convention, both objects accelerate in the same direction as defined by the choice of sign. Furthermore, according to this sign convention, the y component of the net force exerted on object 1 is $T - m_1g$, and the y component of the net force exerted on object 2 is $m_2g - T$.

From the particle under a net force model, apply Newton's second law to object 1:

$$(1) \quad \sum F_y = T - m_1g = m_1a_y$$

Apply Newton's second law to object 2:

$$(2) \quad \sum F_y = m_2g - T = m_2a_y$$

Add Equation (2) to Equation (1), noticing that T cancels:

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

Solve for the acceleration:

$$(3) \quad a_y = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

Substitute Equation (3) into Equation (1) to find T :

$$(4) \quad T = m_1(g + a_y) = \left(\frac{2m_1m_2}{m_1 + m_2} \right) g$$

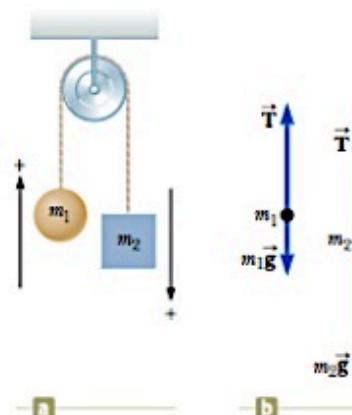


Figure 5.14 (Example 5.9) The Atwood machine. (a) Two objects connected by a massless inextensible string over a frictionless pulley. (b) The free-body diagrams for the two objects.



Finalize The acceleration given by Equation (3) can be interpreted as the ratio of the magnitude of the unbalanced force on the system $(m_2 - m_1)g$ to the total mass of the system $(m_1 + m_2)$, as expected from Newton's second law. Notice that the sign of the acceleration depends on the relative masses of the two objects.

WHAT IF? Describe the motion of the system if the objects have equal masses, that is, $m_1 = m_2$.

Answer If we have the same mass on both sides, the system is balanced and should not accelerate. Mathematically, we see that if $m_1 = m_2$, Equation (3) gives us $a_y = 0$.

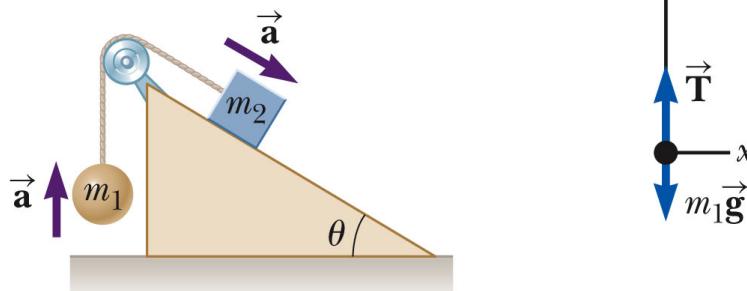
WHAT IF? What if one of the masses is much larger than the other: $m_1 \gg m_2$?

Answer In the case in which one mass is infinitely larger than the other, we can ignore the effect of the smaller mass. Therefore, the larger mass should simply fall as if the smaller mass were not there. We see that if $m_1 \gg m_2$, Equation (3) gives us $a_y = -g$.

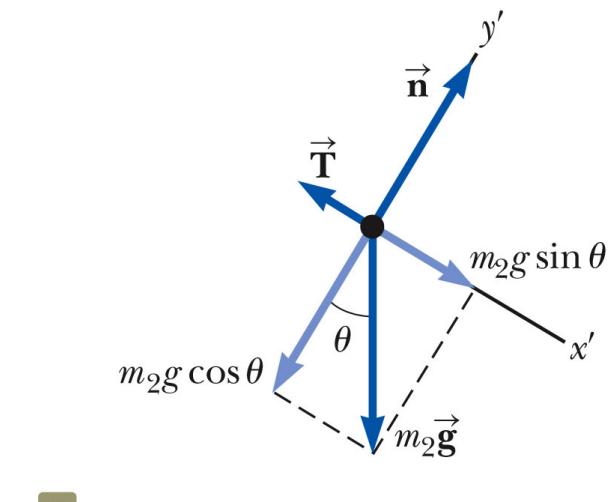
Multiple Objects, Example 2



- Draw the free-body diagram for each object
 - One cord, so tension is the same for both objects
 - Connected, so acceleration is the same for both objects
- Categorize as particles under a net force
- Apply Newton's Laws
- Solve for the unknown(s)



a b



c

Example 5.10 Acceleration of Two Objects Connected by a Cord

AM

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects and the tension in the cord.

SOLUTION

Conceptualize Imagine the objects in Figure 5.15 in motion. If m_2 moves down the incline, then m_1 moves upward. Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude. Notice the normal coordinate axes in Figure 5.15b for the ball and the “tilted” axes for the block in Figure 5.15c.

Categorize We can identify forces on each of the two objects and we are looking for an acceleration, so we categorize the objects as *particles under a net force*. For the block, this model is only valid for the x' direction. In the y' direction, we apply the *particle in equilibrium* model because the block does not accelerate in that direction.

Analyze Consider the free-body diagrams shown in Figures 5.15b and 5.15c.

Apply Newton's second law in the y direction to the ball, choosing the upward direction as positive:

$$(1) \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

For the ball to accelerate upward, it is necessary that $T > m_1 g$. In Equation (1), we replaced a_y with a because the acceleration has only a y component.

For the block, we have chosen the x' axis along the incline as in Figure 5.15c. For consistency with our choice for the ball, we choose the positive x' direction to be down the incline.

Apply the particle under a net force model to the block in the x' direction and the particle in equilibrium model in the y' direction:

$$(2) \sum F_{x'} = m_2 g \sin \theta - T = m_2 a_{x'} = m_2 a$$

$$(3) \sum F_{y'} = n - m_2 g \cos \theta = 0$$

In Equation (2), we replaced $a_{x'}$ with a because the two objects have accelerations of equal magnitude a .

Solve Equation (1) for T :

$$(4) T = m_1(g + a)$$

Substitute this expression for T into Equation (2):

$$m_2 g \sin \theta - m_1(g + a) = m_2 a$$

Solve for a :

$$(5) a = \left(\frac{m_2 \sin \theta - m_1}{m_1 + m_2} \right) g$$

Substitute this expression for a into Equation (4) to find T :

$$(6) T = \left(\frac{m_1 m_2 (\sin \theta + 1)}{m_1 + m_2} \right) g$$

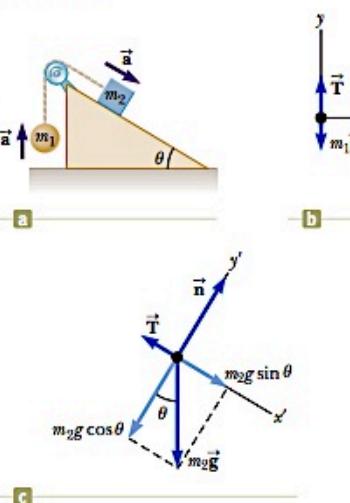


Figure 5.15 (Example 5.10) (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) The free-body diagram for the ball. (c) The free-body diagram for the block. (The incline is frictionless.)

Forces of Friction



- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion.
 - This is due to the interactions between the object and its environment.
- This resistance is called the *force of friction*.



Forces of Friction, cont.

- Friction is proportional to the normal force.
 - $f_s = \mu_s n$ (static friction) and $f_k = \mu_k n$ (kinetic friction)
 - μ is the coefficient of friction
 - These equations relate the magnitudes of the forces; they are not vector equations.
 - For static friction, the equals sign is valid only at *impeding* motion, the surfaces are on the verge of slipping.
 - Use the inequality for static friction if the surfaces are not on the verge of slipping.



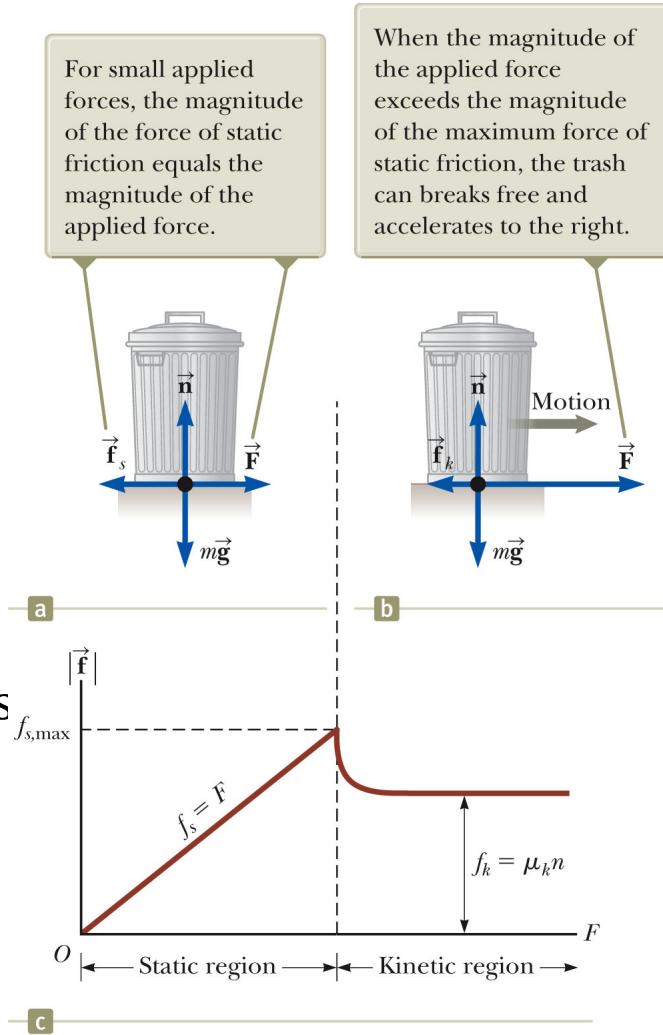
Forces of Friction, final

- The coefficient of friction depends on the **surfaces in contact**.
- The force of static friction is generally **greater** than the force of kinetic friction.
- The direction of the frictional force is **opposite** the direction of motion and parallel to the surfaces in contact.
- The coefficients of friction are nearly independent of the area of contact.

Static Friction



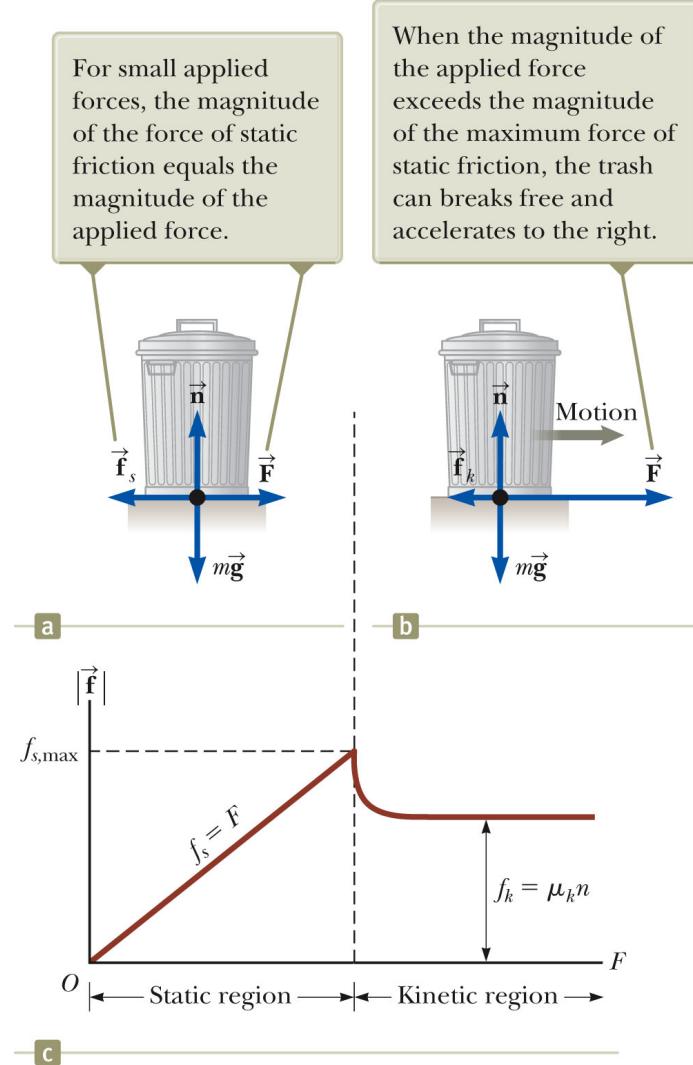
- Static friction acts to keep the object from moving.
- As long as the object is not moving, $f_s = F$
- If \vec{F} increases, so does \vec{f}_s
- If \vec{F} decreases, so does \vec{f}_s
- $f_s \leq \mu_s n$
 - Remember, the equality holds when the surfaces are on the verge of slipping.





Kinetic Friction

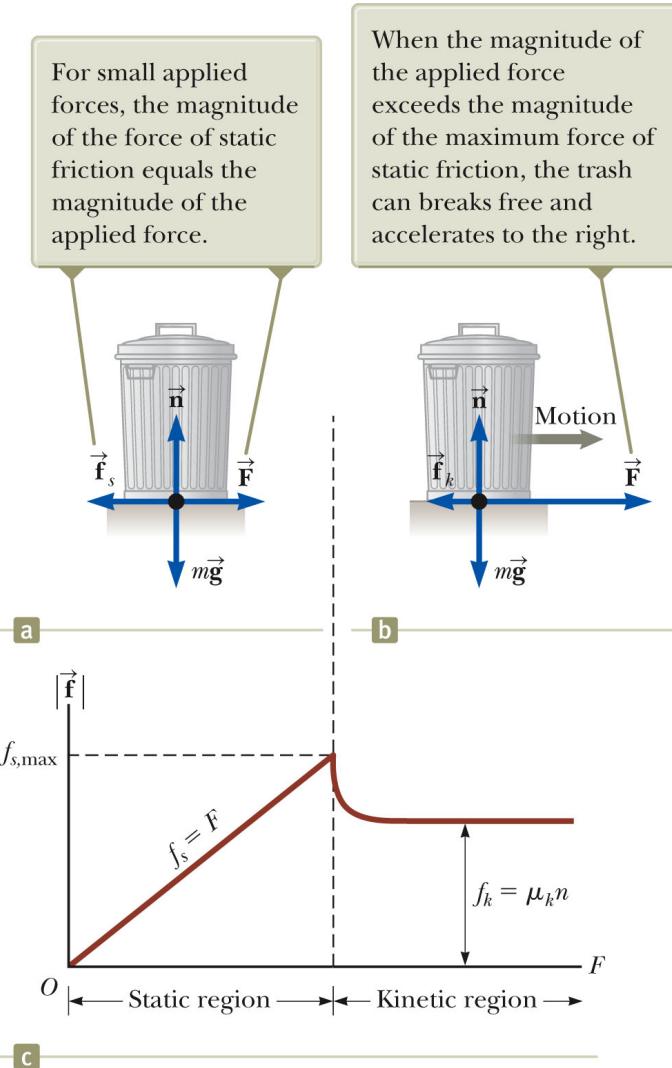
- The force of kinetic friction acts when the object is in motion.
- Although μ_k can vary with speed, we shall neglect any such variations.
- $f_k = \mu_k n$





Explore Forces of Friction

- Vary the applied force
- Note the value of the frictional force
 - Compare the values
- Note what happens when the can starts to move





Some Coefficients of Friction

TABLE 5.1

Coefficients of Friction

	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Friction in Newton's Laws Problems

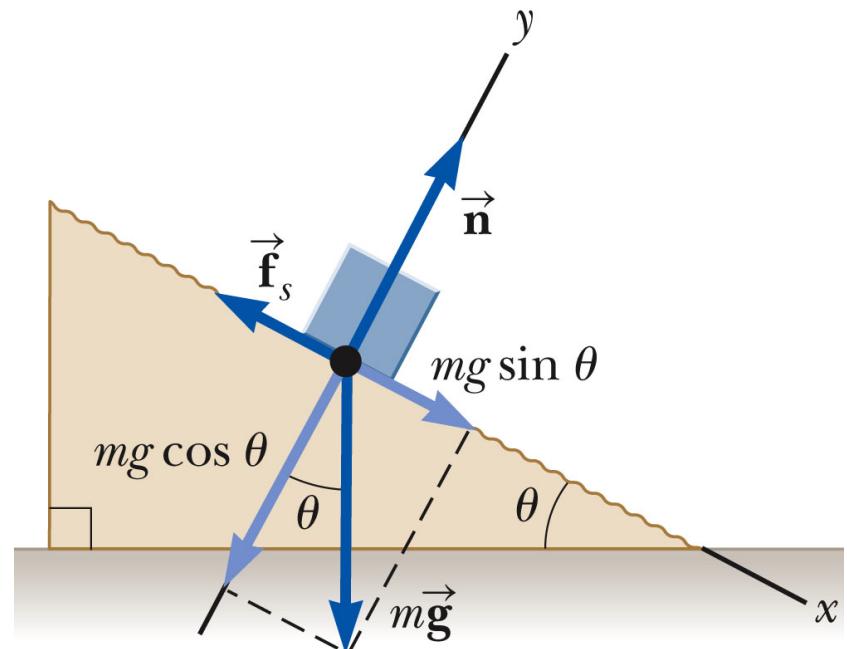


- Friction is a force, so it simply is included in the $\sum \vec{F}$ in Newton's Laws.
- The rules of friction allow you to determine the direction and magnitude of the force of friction.



Friction Example, 1

- The block is sliding down the plane, so friction acts up the plane.
- This setup can be used to experimentally determine the coefficient of friction.
- $\mu = \tan \theta$
 - For μ_s , use the angle where the block just slips.
 - For μ_k , use the angle where the block slides down at a constant speed.



Example 5.11

Experimental Determination of μ_s and μ_k AM



The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in Figure 5.18. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.

SOLUTION

Conceptualize Consider Figure 5.18 and imagine that the block tends to slide down the incline due to the gravitational force. To simulate the situation, place a coin on this book's cover and tilt the book until the coin begins to slide. Notice how this example differs from Example 5.6. When there is no friction on an incline, *any* angle of the incline will cause a stationary object to begin moving. When there is friction, however, there is no movement of the object for angles less than the critical angle.

Categorize The block is subject to various forces. Because we are raising the plane to the angle at which the block is just ready to begin to move but is not moving, we categorize the block as a *particle in equilibrium*.

Analyze The diagram in Figure 5.18 shows the forces on the block: the gravitational force \vec{mg} , the normal force \vec{n} , and the force of static friction \vec{f}_s . We choose x to be parallel to the plane and y perpendicular to it.

From the particle in equilibrium model, apply Equation 5.8 to the block in both the x and y directions:

$$(1) \sum F_x = mg \sin \theta - f_s = 0$$

$$(2) \sum F_y = n - mg \cos \theta = 0$$

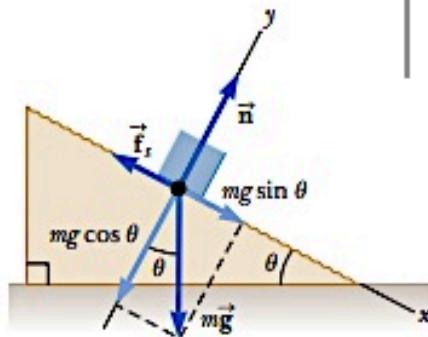


Figure 5.18 (Example 5.11) The external forces exerted on a block lying on a rough incline are the gravitational force \vec{mg} , the normal force \vec{n} , and the force of friction \vec{f}_s . For convenience, the gravitational force is resolved into a component $mg \sin \theta$ along the incline and a component $mg \cos \theta$ perpendicular to the incline.



► 5.11 continued

Substitute $mg = n/\cos \theta$ from Equation (2) into Equation (1):

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s n$. The angle θ in this situation is the critical angle θ_c . Make these substitutions in Equation (3):

We have shown, as requested, that the coefficient of static friction is related only to the critical angle. For example, if the block just slips at $\theta_c = 20.0^\circ$, we find that $\mu_s = \tan 20.0^\circ = 0.364$.

Finalize Once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. If θ is reduced to a value less than θ_c , however, it may be possible to find an angle θ'_c such that the block moves down the incline with constant speed as a particle in equilibrium again ($a_x = 0$). In this case, use Equations (1) and (2) with f_s replaced by f_k to find μ_k : $\mu_k = \tan \theta'_c$, where $\theta'_c < \theta_c$.

$$(3) \quad f_s = mg \sin \theta = \left(\frac{n}{\cos \theta} \right) \sin \theta = n \tan \theta$$

$$\mu_s n = n \tan \theta_c$$

$$\mu_s = \tan \theta_c$$

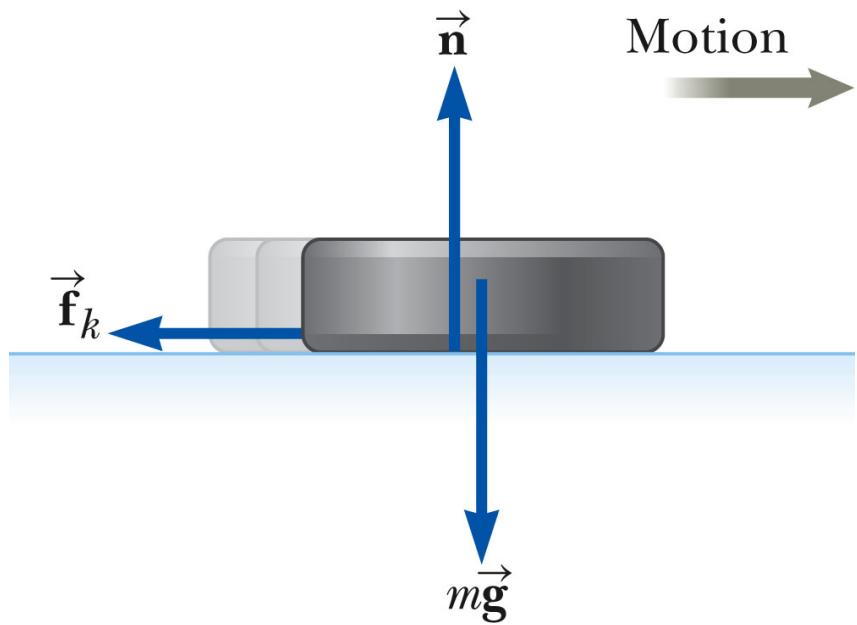


Friction, Example 2

- Draw the free-body diagram, including the force of kinetic friction.
 - Opposes the motion
 - Is parallel to the surfaces in contact
- Continue with the solution as with any Newton's Law problem.
- This example gives information about the motion which can be used to find the acceleration to use in Newton's Laws.

Example 5.12

- A cubic on a frozen pond is given an initial speed of 20.0 m/s. If the cubic always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the cubic and ice.





Apply the particle under a net force model in the x direction to the puck:

$$(1) \sum F_x = -f_k = ma_x$$

Apply the particle in equilibrium model in the y direction to the puck:

$$(2) \sum F_y = n - mg = 0$$

Substitute $n = mg$ from Equation (2) and $f_k = \mu_k n$ into Equation (1):

$$\begin{aligned} -\mu_k n &= -\mu_k mg = ma_x \\ a_x &= -\mu_k g \end{aligned}$$

Apply the particle under constant acceleration model to the puck, choosing Equation 2.17 from the model, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_{xi} = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

Solve for the coefficient of kinetic friction:

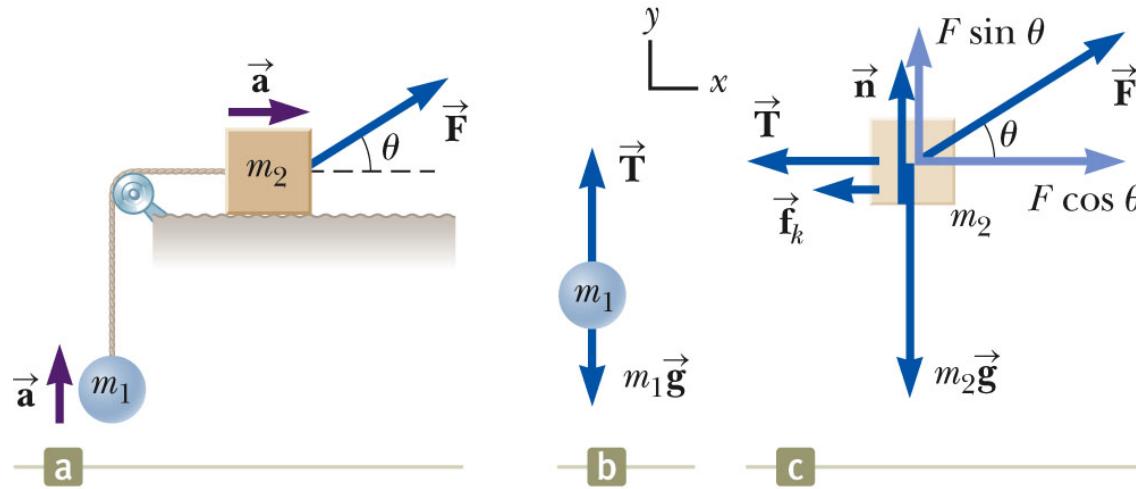
$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

Substitute the numerical values:

$$\mu_k = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(115 \text{ m})} = 0.177$$



Friction, Example 3



- Friction acts only on the object in contact with another surface.
- Draw the free-body diagrams.
- Apply Newton's Laws as in any other multiple object system problem.

Example 5.13**Acceleration of Two Connected Objects When Friction Is Present****AM**

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k . Determine the magnitude of the acceleration of the two objects.

SOLUTION

Conceptualize Imagine what happens as \vec{F} is applied to the block. Assuming \vec{F} is large enough to break the block free from static friction but not large enough to lift the block, the block slides to the right and the ball rises.

Categorize We can identify forces and we want an acceleration, so we categorize this problem as one involving two *particles under a net force*, the ball and the block. Because we assume that the block does not rise into the air due to the applied force, we model the block as a *particle in equilibrium* in the vertical direction.

Analyze First draw force diagrams for the two objects as shown in Figures 5.20b and 5.20c. Notice that the string exerts a force of magnitude T on both objects. The applied force \vec{F} has x and y components $F\cos\theta$ and $F\sin\theta$, respectively. Because the two objects are connected, we can equate the magnitudes of the x component of the acceleration of the block and the y component of the acceleration of the ball and call them both a . Let us assume the motion of the block is to the right.

Apply the particle under a net force model to the block in the horizontal direction:

Because the block moves only horizontally, apply the particle in equilibrium model to the block in the vertical direction:

Apply the particle under a net force model to the ball in the vertical direction:

Solve Equation (2) for n :

Substitute n into $f_k = \mu_k n$ from Equation 5.10:

Substitute Equation (4) and the value of T from Equation (3) into Equation (1):

Solve for a :

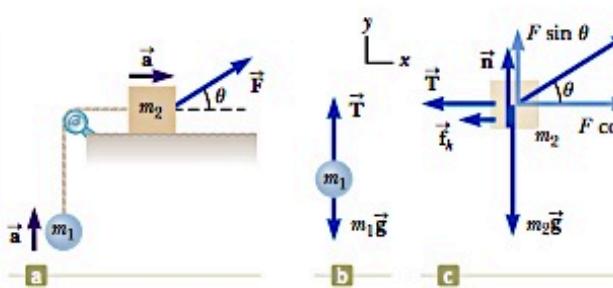


Figure 5.20 (Example 5.13) (a) The external force \vec{F} applied as shown can cause the block to accelerate to the right. (b, c) Diagrams showing the forces on the two objects, assuming the block accelerates to the right and the ball accelerates upward.

$$(1) \sum F_x = F\cos\theta - f_k - T = m_2 a_x = m_2 a$$

$$(2) \sum F_y = n + F\sin\theta - m_2 g = 0$$

$$(3) \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

$$n = m_2 g - F\sin\theta$$

$$(4) f_k = \mu_k(m_2 g - F\sin\theta)$$

$$F\cos\theta - \mu_k(m_2 g - F\sin\theta) - m_1(a + g) = m_2 a$$

$$(5) a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$

Analysis Model Summary



- Particle under a net force
 - If a particle experiences a non-zero net force, its acceleration is related to the force by Newton's Second Law.
 - May also include using a particle under constant acceleration model to relate force and kinematic information.
- Particle in equilibrium
 - If a particle maintains a constant velocity (including a value of zero), the forces on the particle balance and Newton's Second Law becomes. $\sum \vec{F} = 0$

Homework



1-Three forces, given by $F_1 = (-2.00\mathbf{i} + 2.00\mathbf{j}) \text{ N}$, $F_2 = (5.00\mathbf{i} - 3.00\mathbf{j}) \text{ N}$, and $F_3 = -45.0\mathbf{i} \text{ N}$, act on an object to give it an acceleration of magnitude $a = 3.75 \text{ m/s}^2$

2- A 3.00-kg object undergoes an acceleration given by $a = 2.00 \mathbf{i} + 5.00 \mathbf{j} \text{ m/s}^2$. Find (a) the resultant force acting on the object and (b) the magnitude of the resultant force.

3- A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2 . The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2 . (a) What is the value of the ratio m_1/m_2 ? (b) If m_1 and m_2 are combined into one object, find its acceleration under the action of the force F .

4- A block slides down a frictionless plane having an inclination of $\theta = 15.0^\circ$. The block starts from rest at the top, and the length of the incline is 2.00 m. (a) Draw a free-body diagram of the block. Find (b) the acceleration of the block and (c) its speed when it reaches the bottom of the incline.

5- The systems shown in Figure P5.28 are in equilibrium. If the spring scales are calibrated in newton's, what do they read? Ignore the masses of the pulleys and strings and assume the pulleys and the incline in Figure P5.28d are frictionless.

6- A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle θ above the horizontal (Fig. P5.60). She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. (a) Draw a freebody diagram of the suitcase. (b) What angle does the strap make with the horizontal? (c) What is the magnitude of the normal force that the ground exerts on the suitcase?

7- Figure P5.36 shows loads hanging from the ceiling of an elevator that is moving at constant velocity. Find the tension in each of the three strands of cord supporting each load.

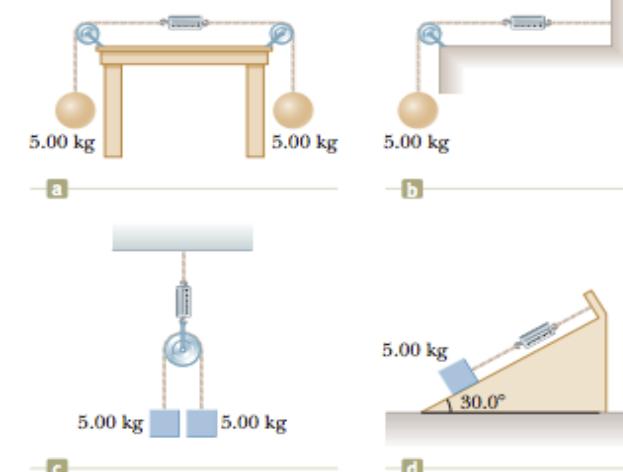


Figure P5.28



Figure P5.60

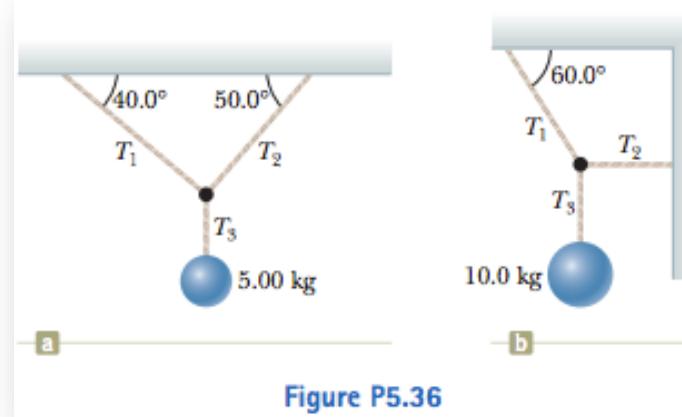


Figure P5.36