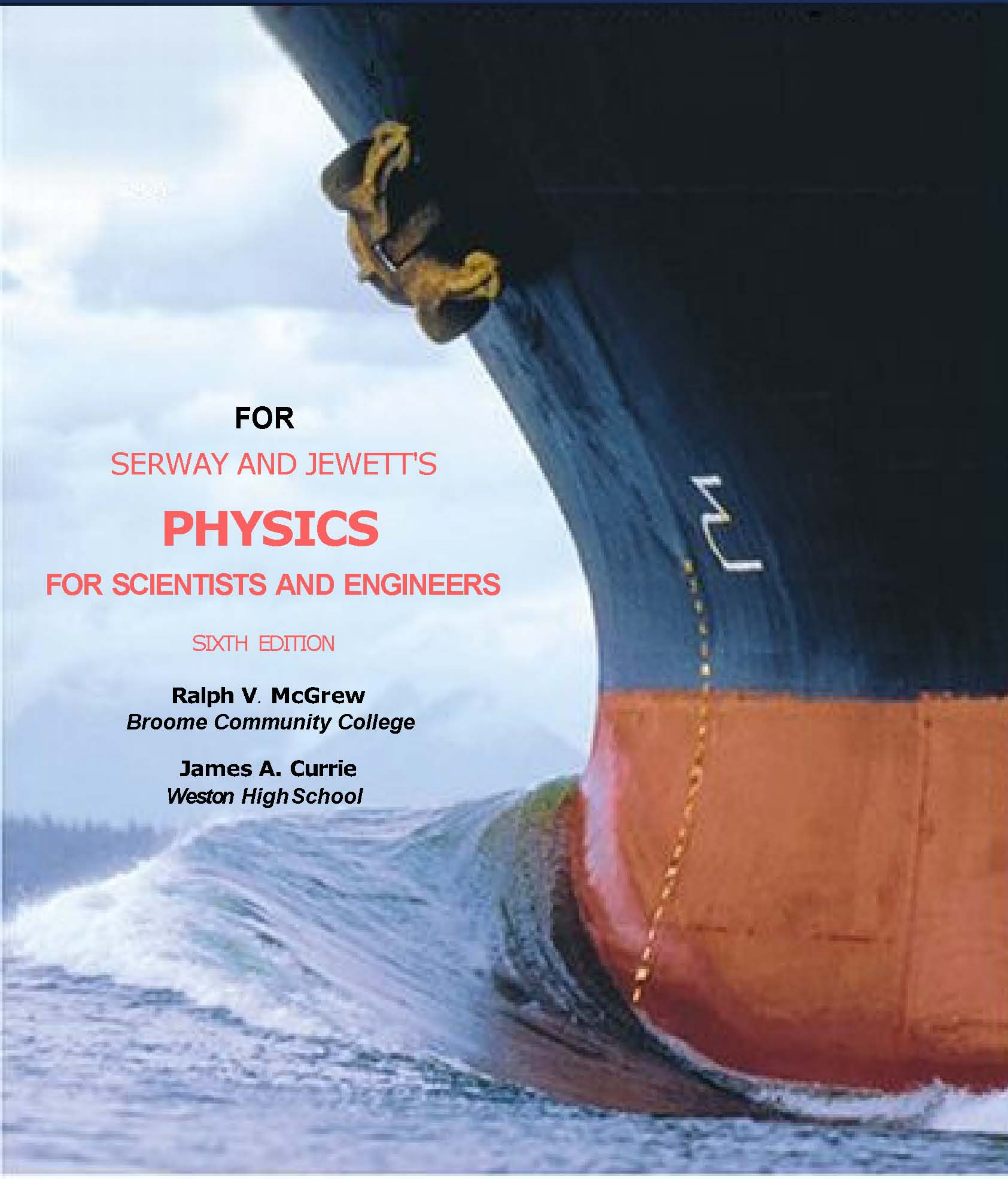
INSTRUCTOR'S SOLUTIONS MANUAL



FOR

SERWAY AND JEWETT'S

PHYSICS

FOR SCIENTISTS AND ENGINEERS

SIXTH EDITION

Ralph V. McGrew
Broome Community College

James A. Currie
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Physics and Measurement

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q1.1 Atomic clocks are based on electromagnetic waves which atoms emit. Also, pulsars are highly regular astronomical clocks.

Q1.2 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

Q1.3 People have different size hands. Defining the unit precisely would be cumbersome.

Q1.4 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

Q1.5 (b) and (d). You cannot add or subtract quantities of different dimension.

Q1.6 A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct. If an equation is not dimensionally correct, it cannot be correct.

Q1.7 If I were a runner, I might walk or run 10^1 miles per day. Since I am a college professor, I walk about 10^0 miles per day. I drive about 40 miles per day on workdays and up to 200 miles per day on vacation.

Q1.8 On February 7, 2001, I am 55 years and 39 days old.

$$55\text{ yr}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right)+39\text{ d}=20\,128\text{ d}\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)=1.74\times10^9\text{ s}\sim10^9\text{ s}.$$

Many college students are just approaching 1 Gs.

Q1.9 Zero digits. An order-of-magnitude calculation is accurate only within a factor of 10.

Q1.10 The mass of the forty-six chapter textbook is on the order of 10^0 kg .

Q1.11 With one datum known to one significant digit, we have 80 million yr + 24 yr = 80 million yr.

SOLUTIONS TO PROBLEMS

Section 1.1 **Standards of Length, Mass, and Time**

No problems in this section

Section 1.2 **Matter and Model-Building**

P1.1 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200\text{ nm}$, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$.

Section 1.3 **Density and Atomic Mass**

***P1.2** Modeling the Earth as a sphere, we find its volume as $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times 10^6\text{ m}\right)^3 = 1.08\times 10^{21}\text{ m}^3$. Its density is then $\rho = \frac{m}{V} = \frac{5.98\times 10^{24}\text{ kg}}{1.08\times 10^{21}\text{ m}^3} = \boxed{5.52\times 10^3\text{ kg/m}^3}$. This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2 000 to $3\,000\text{ kg/m}^3$. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.3 With $V = (\text{base area})(\text{height})$ $V = \left(\pi r^2\right)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\text{ kg}}{\pi(19.5\text{ mm})^2(39.0\text{ mm})}\left(\frac{10^9\text{ mm}^3}{1\text{ m}^3}\right)$$

$$\rho = \boxed{2.15\times 10^4\text{ kg/m}^3}.$$

***P1.4** Let V represent the volume of the model, the same in $\rho = \frac{m}{V}$ for both. Then $\rho_{\text{iron}} = 9.35\text{ kg}/V$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$. Next, $\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\text{ kg}}$ and $m_{\text{gold}} = 9.35\text{ kg}\left(\frac{19.3\times 10^3\text{ kg/m}^3}{7.86\times 10^3\text{ kg/m}^3}\right) = \boxed{23.0\text{ kg}}$.

P1.5 $V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$

$\rho = \frac{m}{V}$, so $m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$.

P1.6 For either sphere the volume is $V=\frac{4}{3}\pi r^3$ and the mass is $m=\rho V=\rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s}=\frac{\rho 4\pi r_\ell^3 3}{\rho 4\pi r_s^3 3}=\frac{r_\ell^3}{r_s^3}=5.$$

Then $r_\ell = r_s\sqrt[3]{5}=4.50\text{ cm}(1.71)=\boxed{7.69\text{ cm}}$.

P1.7 Use $1\text{ u}=1.66\times10^{-24}\text{ g}$.

(a) For He, $m_0=4.00\text{ u}\left(\frac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right)=\boxed{6.64\times10^{-24}\text{ g}}$.

(b) For Fe, $m_0=55.9\text{ u}\left(\frac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right)=\boxed{9.29\times10^{-23}\text{ g}}$.

(c) For Pb, $m_0=207\text{ u}\left(\frac{1.66\times10^{-24}\text{ g}}{1\text{ u}}\right)=\boxed{3.44\times10^{-22}\text{ g}}$.

***P1.8** (a) The mass of any sample is the number of atoms in the sample times the mass m_0 of one atom: $m=Nm_0$. The first assertion is that the mass of one aluminum atom is

$$m_0=27.0\text{ u}=27.0\text{ u}\times1.66\times10^{-27}\text{ kg/1 u}=4.48\times10^{-26}\text{ kg}.$$

Then the mass of 6.02×10^{23} atoms is

$$m=Nm_0=6.02\times10^{23}\times4.48\times10^{-26}\text{ kg}=0.027\,0\text{ kg}=27.0\text{ g}.$$

Thus the first assertion implies the second. Reasoning in reverse, the second assertion can be written $m=Nm_0$.

$$0.027\,0\text{ kg}=6.02\times10^{23}m_0,\text{ so } m_0=\frac{0.027\text{ kg}}{6.02\times10^{23}}=4.48\times10^{-26}\text{ kg},$$

in agreement with the first assertion.

(b) The general equation $m=Nm_0$ applied to one mole of any substance gives $M\text{ g}=NM\text{ u}$, where M is the numerical value of the atomic mass. It divides out exactly for all substances, giving $1.000\,000\,0\times10^{-3}\text{ kg}=N1.660\,540\,2\times10^{-27}\text{ kg}$. With eight-digit data, we can be quite sure of the result to seven digits. For one mole the number of atoms is

$$N=\left(\frac{1}{1.660\,540\,2}\right)10^{-3+27}=\boxed{6.022\,137\times10^{23}}.$$

(c) The atomic mass of hydrogen is 1.008 0 u and that of oxygen is 15.999 u. The mass of one molecule of H_2O is $2(1.008\,0)+15.999\text{ u}=18.0\text{ u}$. Then the molar mass is $\boxed{18.0\text{ g}}$.

(d) For CO_2 we have $12.011\text{ g}+2(15.999\text{ g})=\boxed{44.0\text{ g}}$ as the mass of one mole.

P1.9 Mass of gold abraded: $|\Delta m| = 3.80\text{ g} - 3.35\text{ g} = 0.45\text{ g} = (0.45\text{ g})\left(\frac{1\text{ kg}}{10^3\text{ g}}\right) = 4.5\times10^{-4}\text{ kg}$.

Each atom has mass $m_0 = 197\text{ u} = 197\text{ u}\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 3.27\times10^{-25}\text{ kg}$.

Now, $|\Delta m| = |\Delta N| m_0$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\text{ kg}}{3.27\times10^{-25}\text{ kg}} = 1.38\times10^{21}\text{ atoms}.$$

The rate of loss is

$$\frac{|\Delta N|}{\Delta t} = \frac{1.38\times10^{21}\text{ atoms}}{50\text{ yr}}\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{24\text{ h}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72\times10^{11}\text{ atoms/s}}.$$

P1.10 (a) $m = \rho L^3 = (7.86\text{ g/cm}^3)(5.00\times10^{-6}\text{ cm})^3 = \boxed{9.83\times10^{-16}\text{ g}} = 9.83\times10^{-19}\text{ kg}$

(b) $N = \frac{m}{m_0} = \frac{9.83\times10^{-19}\text{ kg}}{55.9\text{ u}(1.66\times10^{-27}\text{ kg/1 u})} = \boxed{1.06\times10^7\text{ atoms}}$

P1.11 (a) The cross-sectional area is

$$A = 2(0.150\text{ m})(0.010\text{ m}) + (0.340\text{ m})(0.010\text{ m})$$
$$= 6.40\times10^{-3}\text{ m}^2.$$

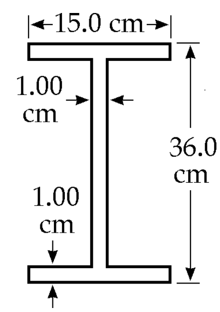


FIG. P1.11

The volume of the beam is

$$V = AL = (6.40\times10^{-3}\text{ m}^2)(1.50\text{ m}) = 9.60\times10^{-3}\text{ m}^3.$$

Thus, its mass is

$$m = \rho V = (7.56\times10^3\text{ kg/m}^3)(9.60\times10^{-3}\text{ m}^3) = \boxed{72.6\text{ kg}}.$$

(b) The mass of one typical atom is $m_0 = (55.9\text{ u})\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 9.28\times10^{-26}\text{ kg}$. Now $m = Nm_0$ and the number of atoms is $N = \frac{m}{m_0} = \frac{72.6\text{ kg}}{9.28\times10^{-26}\text{ kg}} = \boxed{7.82\times10^{26}\text{ atoms}}$.

P1.12 (a) The mass of one molecule is $m_0 = 18.0\ \text{u}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 2.99\times10^{-26}\ \text{kg}$. The number of molecules in the pail is

$$N_{\text{pail}} = \frac{m}{m_0} = \frac{1.20\ \text{kg}}{2.99\times10^{-26}\ \text{kg}} = \boxed{4.02\times10^{25}\ \text{molecules}}.$$

(b) Suppose that enough time has elapsed for thorough mixing of the hydrosphere.

$$N_{\text{both}} = N_{\text{pail}}\left(\frac{m_{\text{pail}}}{M_{\text{total}}}\right) = (4.02\times10^{25}\ \text{molecules})\left(\frac{1.20\ \text{kg}}{1.32\times10^{21}\ \text{kg}}\right),$$

or

$$N_{\text{both}} = \boxed{3.65\times10^{4}\ \text{molecules}}.$$

Section 1.4 **Dimensional Analysis**

P1.13 The term *x* has dimensions of L, *a* has dimensions of LT^{-2}, and *t* has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$\text{L} = \left(\text{LT}^{-2}\right)^m (\text{T})^n \text{ or } \text{L}^1\text{T}^0 = \text{L}^m\text{T}^{n-2m}.$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$\text{L}^1 = \text{L}^m \text{ and } \boxed{m=1}.$$

Likewise, equating terms in T, we see that $n-2m$ must equal 0. Thus, $\boxed{n=2}$. The value of *k*, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

***P1.14** (a) Circumference has dimensions of L.

(b) Volume has dimensions of L^3.

(c) Area has dimensions of L^2.

Expression (i) has dimension $\text{L}\left(\text{L}^2\right)^{1/2} = \text{L}^2$, so this must be area (c).
Expression (ii) has dimension L, so it is (a).
Expression (iii) has dimension $\text{L}\left(\text{L}^2\right) = \text{L}^3$, so it is (b). Thus, $\boxed{(\text{a}) = \text{ii};\quad (\text{b}) = \text{iii},\ (\text{c}) = \text{i}}$.

P1.15 (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are $\mathrm{m^2/s^2}$, while the units of $[v]$ are $\mathrm{m/s}$.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in $\mathrm{m^{-1}}$.

***P1.16** (a) $a \propto \dfrac{\sum F}{m}$ or $a = k\dfrac{\sum F}{m}$ represents the proportionality of acceleration to resultant force and the inverse proportionality of acceleration to mass. If k has no dimensions, we have

$$[a] = [k]\frac{[F]}{[m]},\ \frac{\mathrm{L}}{\mathrm{T}^2} = 1\frac{[\mathrm{F}]}{\mathrm{M}},\ [\mathrm{F}] = \boxed{\frac{\mathrm{M}\cdot\mathrm{L}}{\mathrm{T}^2}}.$$

(b) In units, $\dfrac{\mathrm{M}\cdot\mathrm{L}}{\mathrm{T}^2} = \dfrac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$, so $\boxed{1\ \text{newton} = 1\ \mathrm{kg}\cdot\mathrm{m/s^2}}$.

P1.17 Inserting the proper units for everything except G,

$$\left[\frac{\mathrm{kg\ m}}{\mathrm{s}^2}\right] = \frac{G[\mathrm{kg}]^2}{[\mathrm{m}]^2}.$$

Multiply both sides by $[\mathrm{m}]^2$ and divide by $[\mathrm{kg}]^2$; the units of G are $\boxed{\dfrac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}}$.

Section 1.5 **Conversion of Units**

***P1.18** Each of the four walls has area $(8.00\ \mathrm{ft})(12.0\ \mathrm{ft}) = 96.0\ \mathrm{ft}^2$. Together, they have area

$$4\left(96.0\ \mathrm{ft}^2\right)\left(\frac{1\ \mathrm{m}}{3.28\ \mathrm{ft}}\right)^2 = \boxed{35.7\ \mathrm{m}^2}.$$

P1.19 Apply the following conversion factors:

$$1\ \mathrm{in} = 2.54\ \mathrm{cm},\ 1\ \mathrm{d} = 86\,400\ \mathrm{s},\ 100\ \mathrm{cm} = 1\ \mathrm{m},\ \text{and}\ 10^9\ \mathrm{nm} = 1\ \mathrm{m}$$

$$\left(\frac{1}{32}\ \mathrm{in/day}\right)\frac{(2.54\ \mathrm{cm/in})\left(10^{-2}\ \mathrm{m/cm}\right)\left(10^9\ \mathrm{nm/m}\right)}{86\,400\ \mathrm{s/day}} = \boxed{9.19\ \mathrm{nm/s}}.$$

This means the proteins are assembled at a rate of many layers of atoms each second!

***P1.20** $8.50\ \mathrm{in}^3 = 8.50\ \mathrm{in}^3\left(\dfrac{0.025\,4\ \mathrm{m}}{1\ \mathrm{in}}\right)^3 = \boxed{1.39\times 10^{-4}\ \mathrm{m}^3}$

P1.21 *Conceptualize*: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about $A \approx (30\text{ m})(50\text{ m}) = 1\,500\text{ m}^2$.

Categorize: We model the lot as a perfect rectangle to use Area = Length × Width. Use the conversion: $1\text{ m} = 3.281\text{ ft}$.

Analyze: $A = LW = (100\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right)(150\text{ ft})\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right) = 1\,390\text{ m}^2 = \boxed{1.39\times10^3\text{ m}^2}$.

Finalize: Our calculated result agrees reasonably well with our initial estimate and has the proper units of m^2. Unit conversion is a common technique that is applied to many problems.

P1.22 (a) $V = (40.0\text{ m})(20.0\text{ m})(12.0\text{ m}) = 9.60\times10^3\text{ m}^3$

$V = 9.60\times10^3\text{ m}^3(3.28\text{ ft}/1\text{ m})^3 = \boxed{3.39\times10^5\text{ ft}^3}$

(b) The mass of the air is

$$m = \rho_{\text{air}}V = \left(1.20\text{ kg/m}^3\right)\left(9.60\times10^3\text{ m}^3\right) = 1.15\times10^4\text{ kg}.$$

The student must look up weight in the index to find

$$F_g = mg = \left(1.15\times10^4\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 1.13\times10^5\text{ N}.$$

Converting to pounds,

$$F_g = \left(1.13\times10^5\text{ N}\right)\left(1\text{ lb}/4.45\text{ N}\right) = \boxed{2.54\times10^4\text{ lb}}.$$

P1.23 (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0\text{ gal}}{420\text{ s}} = \boxed{7.14\times10^{-2}\text{ gal/s}}.$$

(b) Converting gallons first to liters, then to m^3,

$$r = \left(7.14\times10^{-2}\text{ gal/s}\right)\left(\frac{3.786\text{ L}}{1\text{ gal}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right)$$

$$r = \boxed{2.70\times10^{-4}\text{ m}^3/\text{s}}.$$

(c) At that rate, to fill a 1-m^3 tank would take

$$t = \left(\frac{1\text{ m}^3}{2.70\times10^{-4}\text{ m}^3/\text{s}}\right)\left(\frac{1\text{ h}}{3\,600}\right) = \boxed{1.03\text{ h}}.$$

***P1.24** (a) Length of Mammoth Cave $= 348\ \text{mi}\left(\dfrac{1.609\ \text{km}}{1\ \text{mi}}\right) = \boxed{560\ \text{km} = 5.60\times 10^5\ \text{m} = 5.60\times 10^7\ \text{cm}}$.

(b) Height of Ribbon Falls $= 1\,612\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{491\ \text{m} = 0.491\ \text{km} = 4.91\times 10^4\ \text{cm}}$.

(c) Height of Denali $= 20\,320\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{6.19\ \text{km} = 6.19\times 10^3\ \text{m} = 6.19\times 10^5\ \text{cm}}$.

(d) Depth of King's Canyon $= 8\,200\ \text{ft}\left(\dfrac{0.304\,8\ \text{m}}{1\ \text{ft}}\right) = \boxed{2.50\ \text{km} = 2.50\times 10^3\ \text{m} = 2.50\times 10^5\ \text{cm}}$.

P1.25 From Table 1.5, the density of lead is $1.13\times 10^4\ \text{kg/m}^3$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as mass per volume, in $\rho = \dfrac{m}{V}$. We must convert to SI units in the calculation.

$$\rho = \frac{23.94\ \text{g}}{2.10\ \text{cm}^3}\left(\frac{1\ \text{kg}}{1\,000\ \text{g}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = \boxed{1.14\times 10^4\ \text{kg/m}^3}$$

At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than $1\ \text{g/cm}^3$, and objects that float must be less dense than water.

P1.26 It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1\ \text{acre})\left(\frac{1\ \text{mi}^2}{640\ \text{acres}}\right)\left(\frac{1\,609\ \text{m}}{\text{mi}}\right)^2 = \boxed{4.05\times 10^3\ \text{m}^2}.$$

***P1.27** The weight flow rate is $1\,200\ \dfrac{\text{ton}}{\text{h}}\left(\dfrac{2\,000\ \text{lb}}{\text{ton}}\right)\left(\dfrac{1\ \text{h}}{60\ \text{min}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{667\ \text{lb/s}}$.

P1.28 $1\ \text{mi} = 1\,609\ \text{m} = 1.609\ \text{km}$; thus, to go from mph to km/h, multiply by 1.609.

(a) $1\ \text{mi/h} = \boxed{1.609\ \text{km/h}}$

(b) $55\ \text{mi/h} = \boxed{88.5\ \text{km/h}}$

(c) $65\ \text{mi/h} = 104.6\ \text{km/h}$. Thus, $\Delta v = \boxed{16.1\ \text{km/h}}$.

P1.29 (a) $$\left(\frac{6\times10^{12}\ \$}{1\,000\ \$/\text{s}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{yr}}{365\ \text{days}}\right)=\boxed{190\ \text{years}}$$

(b) The circumference of the Earth at the equator is $2\pi\left(6.378\times10^{3}\ \text{m}\right)=4.01\times10^{7}\ \text{m}$. The length of one dollar bill is 0.155 m so that the length of 6 trillion bills is $9.30\times10^{11}\ \text{m}$. Thus, the 6 trillion dollars would encircle the Earth

$$\frac{9.30\times10^{11}\ \text{m}}{4.01\times0^{7}\ \text{m}}=\boxed{2.32\times10^{4}\ \text{times}}.$$

P1.30 $$N_{\text{atoms}}=\frac{m_{\text{Sun}}}{m_{\text{atom}}}=\frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}=\boxed{1.19\times10^{57}\ \text{atoms}}$$

P1.31 $$V=At\ \text{ so }t=\frac{V}{A}=\frac{3.78\times10^{-3}\ \text{m}^{3}}{25.0\ \text{m}^{2}}=\boxed{1.51\times10^{-4}\ \text{m (or }151\ \mu\text{m)}}$$

P1.32 $$V=\frac{1}{3}Bh=\frac{\left[(13.0\ \text{acres})\left(43\,560\ \text{ft}^{2}/\text{acre}\right)\right]}{3}(481\ \text{ft})$$
$$=9.08\times10^{7}\ \text{ft}^{3},$$

or

$$V=\left(9.08\times10^{7}\ \text{ft}^{3}\right)\left(\frac{2.83\times10^{-2}\ \text{m}^{3}}{1\ \text{ft}^{3}}\right)$$
$$=\boxed{2.57\times10^{6}\ \text{m}^{3}}$$

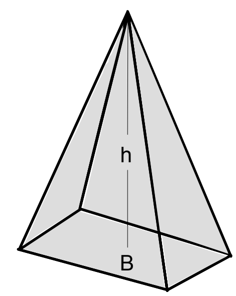


FIG. P1.32

P1.33 $$F_g=(2.50\ \text{tons/block})\left(2.00\times10^{6}\ \text{blocks}\right)(2\,000\ \text{lb/ton})=\boxed{1.00\times10^{10}\ \text{lbs}}$$

***P1.34** The area covered by water is

$$A_w=0.70A_{\text{Earth}}=(0.70)\left(4\pi R_{\text{Earth}}^{2}\right)=(0.70)(4\pi)\left(6.37\times10^{6}\ \text{m}\right)^{2}=3.6\times10^{14}\ \text{m}^{2}.$$

The average depth of the water is

$$d=(2.3\ \text{miles})(1\,609\ \text{m/1 mile})=3.7\times10^{3}\ \text{m}.$$

The volume of the water is

$$V=A_wd=\left(3.6\times10^{14}\ \text{m}^{2}\right)\left(3.7\times10^{3}\ \text{m}\right)=1.3\times10^{18}\ \text{m}^{3}$$

and the mass is

$$m=\rho V=\left(1\,000\ \text{kg/m}^{3}\right)\left(1.3\times10^{18}\ \text{m}^{3}\right)=\boxed{1.3\times10^{21}\ \text{kg}}.$$

P1.35 (a) $$d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\frac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) = \left(2.40\times10^{-15}\ \text{m}\right)\left(\frac{300\ \text{ft}}{1.06\times10^{-10}\ \text{m}}\right) = 6.79\times10^{-3}\ \text{ft, or}$$

$$d_{\text{nucleus, scale}} = \left(6.79\times10^{-3}\ \text{ft}\right)\left(304.8\ \text{mm/1 ft}\right) = \boxed{2.07\ \text{mm}}$$

(b) $$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{\frac{4\pi r_{\text{atom}}^3}{3}}{\frac{4\pi r_{\text{nucleus}}^3}{3}} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3 = \left(\frac{1.06\times10^{-10}\ \text{m}}{2.40\times10^{-15}\ \text{m}}\right)^3$$

$$= \boxed{8.62\times10^{13}\ \text{times as large}}$$

***P1.36** $$\begin{matrix}\text{scale distance}\\ \text{between}\end{matrix} = \begin{pmatrix}\text{real}\\ \text{distance}\end{pmatrix}\begin{pmatrix}\text{scale}\\ \text{factor}\end{pmatrix} = \left(4.0\times10^{13}\ \text{km}\right)\left(\frac{7.0\times10^{-3}\ \text{m}}{1.4\times10^{9}\ \text{m}}\right) = \boxed{200\ \text{km}}$$

P1.37 The scale factor used in the "dinner plate" model is

$$S = \frac{0.25\ \text{m}}{1.0\times10^{5}\ \text{lightyears}} = 2.5\times10^{-6}\ \text{m/lightyears}.$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = \left(2.0\times10^{6}\ \text{lightyears}\right)\left(2.5\times10^{-6}\ \text{m/lightyears}\right) = \boxed{5.0\ \text{m}}.$$

P1.38 (a) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^2}{4\pi r_{\text{Moon}}^2} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2 = \left(\frac{\left(6.37\times10^{6}\ \text{m}\right)\left(100\ \text{cm/m}\right)}{1.74\times10^{8}\ \text{cm}}\right)^2 = \boxed{13.4}$$

(b) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4\pi r_{\text{Earth}}^3}{3}}{\frac{4\pi r_{\text{Moon}}^3}{3}} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3 = \left(\frac{\left(6.37\times10^{6}\ \text{m}\right)\left(100\ \text{cm/m}\right)}{1.74\times10^{8}\ \text{cm}}\right)^3 = \boxed{49.1}$$

P1.39 To balance, $m_{\text{Fe}} = m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}} = \rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}{}^3 = \rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}{}^3$$

$$r_{\text{Al}} = r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3} = (2.00\ \text{cm})\left(\frac{7.86}{2.70}\right)^{1/3} = \boxed{2.86\ \text{cm}}.$$

P1.40 The mass of each sphere is

$$m_{\text{Al}} = \rho_{\text{Al}} V_{\text{Al}} = \frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3}$$

and

$$m_{\text{Fe}} = \rho_{\text{Fe}} V_{\text{Fe}} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3}.$$

Setting these masses equal,

$$\frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3} \text{ and } \boxed{r_{\text{Al}} = r_{\text{Fe}} \sqrt[3]{\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}}}.$$

Section 1.6 **Estimates and Order-of-Magnitude Calculations**

P1.41 Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4 \times 4 \times 3 = 48 \text{ m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038 \text{ m}}{2}\right)^3 = 2.87 \times 10^{-5} \text{ m}^3.$$

Therefore, one can fit about $\dfrac{48}{2.87 \times 10^{-5}} \sim \boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67 \times 10^6 \times 0.740 \sim 10^6$.

P1.42 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000 \text{ mi})(5\,280 \text{ ft/mi})(1 \text{ rev}/8 \text{ ft}) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$.

P1.43 In order to reasonably carry on photosynthesis, we might expect a blade of grass to require at least $\frac{1}{16} \text{ in}^2 = 43 \times 10^{-5} \text{ ft}^2$. Since $1 \text{ acre} = 43\,560 \text{ ft}^2$, the number of blades of grass to be expected on a quarter-acre plot of land is about

$$n = \frac{\text{total area}}{\text{area per blade}} = \frac{(0.25 \text{ acre})\left(43\,560 \text{ ft}^2/\text{acre}\right)}{43 \times 10^{-5} \text{ ft}^2/\text{blade}} = 2.5 \times 10^7 \text{ blades} \sim \boxed{10^7 \text{ blades}}.$$

P1.44 A typical raindrop is spherical and might have a radius of about 0.1 inch. Its volume is then approximately $4 \times 10^{-3}\ \text{in}^3$. Since 1 acre $= 43\,560\ \text{ft}^2$, the volume of water required to cover it to a depth of 1 inch is

$$(1\ \text{acre})(1\ \text{inch}) = (1\ \text{acre} \cdot \text{in})\left(\frac{43\,560\ \text{ft}^2}{1\ \text{acre}}\right)\left(\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right) \approx 6.3 \times 10^6\ \text{in}^3.$$

The number of raindrops required is

$$n = \frac{\text{volume of water required}}{\text{volume of a single drop}} = \frac{6.3 \times 10^6\ \text{in}^3}{4 \times 10^{-3}\ \text{in}^3} = 1.6 \times 10^9 \sim \boxed{10^9}.$$

***P1.45** Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m}) = 0.10\ \text{m}^3.$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = \left(1\,000\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 100\ \text{kg}\ \boxed{\sim 10^2\ \text{kg}}.$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = \left(8\,920\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 892\ \text{kg}\ \boxed{\sim 10^3\ \text{kg}}.$$

P1.46 The typical person probably drinks 2 to 3 soft drinks daily. Perhaps half of these were in aluminum cans. Thus, we will estimate 1 aluminum can disposal per person per day. In the U.S. there are ~250 million people, and 365 days in a year, so

$$\left(250 \times 10^6\ \text{cans/day}\right)(365\ \text{days/year}) \cong \boxed{10^{11}\ \text{cans}}$$

are thrown away or recycled each year. Guessing that each can weighs around $1/10$ of an ounce, we estimate this represents

$$\left(10^{11}\ \text{cans}\right)(0.1\ \text{oz/can})(1\ \text{lb/16 oz})(1\ \text{ton/2 000 lb}) \approx 3.1 \times 10^5\ \text{tons/year}.\ \boxed{\sim 10^5\ \text{tons}}$$

P1.47 Assume: Total population $= 10^7$; one out of every 100 people has a piano; one tuner can serve about 1 000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\ \text{tuners} \sim \left(\frac{1\ \text{tuner}}{1\,000\ \text{pianos}}\right)\left(\frac{1\ \text{piano}}{100\ \text{people}}\right)\left(10^7\ \text{people}\right) = \boxed{100}.$$

Section 1.7 **Significant Figures**

***P1.48** METHOD ONE

We treat the best value with its uncertainty as a binomial (21.3 ± 0.2) cm (9.8 ± 0.1) cm,

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right] \text{ cm}^2 .$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209 \text{ cm}^2 \pm 4 \text{ cm}^2} .$$

METHOD TWO

We add the fractional uncertainties in the data.

$$A = (21.3 \text{ cm})(9.8 \text{ cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right) = 209 \text{ cm}^2 \pm 2\% = 209 \text{ cm}^2 \pm 4 \text{ cm}^2$$

P1.49 (a)

$$\begin{aligned} \pi r^2 &= \pi(10.5 \text{ m} \pm 0.2 \text{ m})^2 \\ &= \pi\left[(10.5 \text{ m})^2 \pm 2(10.5 \text{ m})(0.2 \text{ m}) + (0.2 \text{ m})^2\right] \\ &= \boxed{346 \text{ m}^2 \pm 13 \text{ m}^2} \end{aligned}$$

(b) $2\pi r = 2\pi(10.5 \text{ m} \pm 0.2 \text{ m}) = \boxed{66.0 \text{ m} \pm 1.3 \text{ m}}$

P1.50 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

P1.51 $r = (6.50 \pm 0.20) \text{ cm} = (6.50 \pm 0.20) \times 10^{-2} \text{ m}$

$m = (1.85 \pm 0.02) \text{ kg}$

$\rho = \dfrac{m}{\left(\frac{4}{3}\right)\pi r^3}$

also,

$$\frac{\delta \rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r} .$$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta \rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103 ,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5 \times 10^{-2} \text{ m}\right)^3} = \boxed{1.61 \times 10^3 \text{ kg/m}^3}$$

and

$$\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3 \text{ kg/m}^3} = (1.6 \pm 0.2) \times 10^3 \text{ kg/m}^3 .$$

P1.52 (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ \underline{+\ 2.5?} \\ 796.\cancel{5}\cancel{3} \end{array} = \boxed{797}$$

(b) $0.003\,2(2\text{ s.f.}) \times 356.3(4\text{ s.f.}) = 1.140\,16 = (2\text{ s.f.})\ \boxed{1.1}$

(c) $5.620(4\text{ s.f.}) \times \pi(>4\text{ s.f.}) = 17.656 = (4\text{ s.f.})\ \boxed{17.66}$

***P1.53** We work to nine significant digits:

$$1\text{ yr} = 1\text{ yr}\left(\frac{365.242\,199\text{ d}}{1\text{ yr}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{31\,556\,926.0\text{ s}}.$$

P1.54 The distance around is $38.44\text{ m} + 19.5\text{ m} + 38.44\text{ m} + 19.5\text{ m} = 115.88\text{ m}$, but this answer must be rounded to 115.9 m because the distance 19.5 m carries information to only one place past the decimal. $\boxed{115.9\text{ m}}$

P1.55

$$V = 2V_1 + 2V_2 = 2(V_1 + V_2)$$
$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$
$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$
$$V = 2\left(1.70\text{ m}^3 + 0.900\text{ m}^3\right) = \boxed{5.2\text{ m}^3}$$

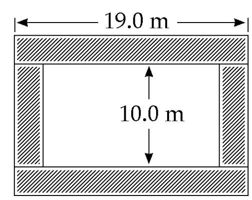


FIG. P1.55

$$\left.\begin{array}{l} \dfrac{\delta \ell_1}{\ell_1} = \dfrac{0.12\text{ m}}{19.0\text{ m}} = 0.0063 \\ \dfrac{\delta w_1}{w_1} = \dfrac{0.01\text{ m}}{1.0\text{ m}} = 0.010 \\ \dfrac{\delta t_1}{t_1} = \dfrac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011 \end{array}\right\} \frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

Additional Problems

P1.56 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}} = \frac{1\,000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2 = (100\text{ m})(1\,000\text{ m}) = 1.00 \times 10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00 \times 10^5\text{ m}^2} = \boxed{316\text{ m}}.$$

***P1.57** Consider one cubic meter of gold. Its mass from Table 1.5 is 19 300 kg. One atom of gold has mass

$$m_0 = (197\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 3.27\times10^{-25}\ \text{kg}.$$

So, the number of atoms in the cube is

$$N = \frac{19\,300\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 5.90\times10^{28}.$$

The imagined cubical volume of each atom is

$$d^3 = \frac{1\ \text{m}^3}{5.90\times10^{28}} = 1.69\times10^{-29}\ \text{m}^3.$$

So

$$d = \boxed{2.57\times10^{-10}\ \text{m}}.$$

P1.58 $$A_{\text{total}} = (N)(A_{\text{drop}}) = \left(\frac{V_{\text{total}}}{V_{\text{drop}}}\right)(A_{\text{drop}}) = \left(\frac{V_{\text{total}}}{\frac{4\pi r^3}{3}}\right)(4\pi r^2)$$

$$A_{\text{total}} = \left(\frac{3V_{\text{total}}}{r}\right) = 3\left(\frac{30.0\times10^{-6}\ \text{m}^3}{2.00\times10^{-5}\ \text{m}}\right) = \boxed{4.50\ \text{m}^2}$$

P1.59 One month is

$$1\ \text{mo} = (30\ \text{day})(24\ \text{h/day})(3\,600\ \text{s/h}) = 2.592\times10^6\ \text{s}.$$

Applying units to the equation,

$$V = \left(1.50\ \text{Mft}^3/\text{mo}\right)t + \left(0.008\,00\ \text{Mft}^3/\text{mo}^2\right)t^2.$$

Since $1\ \text{Mft}^3 = 10^6\ \text{ft}^3$,

$$V = \left(1.50\times10^6\ \text{ft}^3/\text{mo}\right)t + \left(0.008\,00\times10^6\ \text{ft}^3/\text{mo}^2\right)t^2.$$

Converting months to seconds,

$$V = \frac{1.50\times10^6\ \text{ft}^3/\text{mo}}{2.592\times10^6\ \text{s/mo}}t + \frac{0.008\,00\times10^6\ \text{ft}^3/\text{mo}^2}{\left(2.592\times10^6\ \text{s/mo}\right)^2}t^2.$$

Thus, $\boxed{V\,[\text{ft}^3] = \left(0.579\ \text{ft}^3/\text{s}\right)t + \left(1.19\times10^{-9}\ \text{ft}^3/\text{s}^2\right)t^2}$.

P1.60

α'(deg)	α(rad)	tan(α)	sin(α)	difference
15.0	0.262	0.268	0.259	3.47%
20.0	0.349	0.364	0.342	6.43%
25.0	0.436	0.466	0.423	10.2%
24.0	0.419	0.445	0.407	9.34%
24.4	0.426	0.454	0.413	9.81%
24.5	0.428	0.456	0.415	9.87%
24.6	0.429	0.458	0.416	9.98%
24.7	0.431	0.460	0.418	10.1%

$\boxed{24.6°}$

P1.61

$$2\pi r = 15.0 \text{ m}$$

$$r = 2.39 \text{ m}$$

$$\frac{h}{r} = \tan 55.0°$$

$$h = (2.39 \text{ m})\tan(55.0°) = \boxed{3.41 \text{ m}}$$

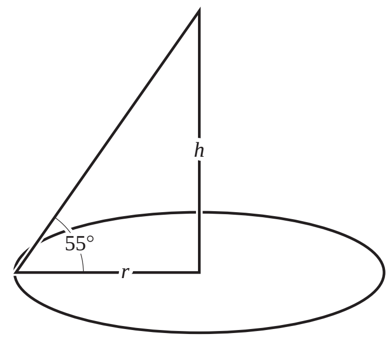


FIG. P1.61

***P1.62** Let d represent the diameter of the coin and h its thickness. The mass of the gold is

$$m = \rho V = \rho A t = \rho\left(\frac{2\pi d^2}{4} + \pi d h\right)t$$

where t is the thickness of the plating.

$$m = 19.3\left[2\pi\frac{(2.41)^2}{4} + \pi(2.41)(0.178)\right]\left(0.18\times10^{-4}\right)$$

$$= 0.003\,64 \text{ grams}$$

$$\text{cost} = 0.003\,64 \text{ grams} \times \$10/\text{gram} = \$0.036\,4 = \boxed{3.64 \text{ cents}}$$

This is negligible compared to \$4.98.

P1.63 The actual number of seconds in a year is

$$(86\,400 \text{ s/day})(365.25 \text{ day/yr}) = 31\,557\,600 \text{ s/yr}.$$

The percent error in the approximation is

$$\frac{\left|\left(\pi\times10^7 \text{ s/yr}\right) - \left(31\,557\,600 \text{ s/yr}\right)\right|}{31\,557\,600 \text{ s/yr}} \times 100\% = \boxed{0.449\%}.$$

P1.64 (a) $[V] = \mathrm{L}^3$, $[A] = \mathrm{L}^2$, $[h] = \mathrm{L}$

$$[V] = [A][h]$$

$\mathrm{L}^3 = \mathrm{L}^2\mathrm{L} = \mathrm{L}^3$. Thus, the equation is dimensionally correct.

(b) $V_{\text{cylinder}} = \pi R^2 h = \left(\pi R^2\right)h = Ah$, where $\boxed{A = \pi R^2}$

$V_{\text{rectangular object}} = \ell w h = (\ell w)h = Ah$, where $\boxed{A = \ell w}$

P1.65 (a) The speed of rise may be found from

$$v = \frac{\text{(Vol rate of flow)}}{\left(\text{Area: } \frac{\pi D^2}{4}\right)} = \frac{16.5 \text{ cm}^3/\text{s}}{\frac{\pi (6.30 \text{ cm})^2}{4}} = \boxed{0.529 \text{ cm/s}}.$$

(b) Likewise, at a 1.35 cm diameter,

$$v = \frac{16.5 \text{ cm}^3/\text{s}}{\frac{\pi (1.35 \text{ cm})^2}{4}} = \boxed{11.5 \text{ cm/s}}.$$

P1.66 (a) 1 cubic meter of water has a mass

$$m = \rho V = \left(1.00 \times 10^{-3} \text{ kg/cm}^3\right)\left(1.00 \text{ m}^3\right)\left(10^2 \text{ cm/m}\right)^3 = \boxed{1\,000 \text{ kg}}$$

(b) As a rough calculation, we treat each item as if it were 100% water.

cell: $$m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \rho\left(\frac{1}{6}\pi D^3\right) = \left(1\,000 \text{ kg/m}^3\right)\left(\frac{1}{6}\pi\right)\left(1.0 \times 10^{-6} \text{ m}\right)^3$$
$$= \boxed{5.2 \times 10^{-16} \text{ kg}}$$

kidney: $$m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \left(1.00 \times 10^{-3} \text{ kg/cm}^3\right)\left(\frac{4}{3}\pi\right)(4.0 \text{ cm})^3$$
$$= \boxed{0.27 \text{ kg}}$$

fly: $$m = \rho\left(\frac{\pi}{4}D^2 h\right) = \left(1 \times 10^{-3} \text{ kg/cm}^3\right)\left(\frac{\pi}{4}\right)(2.0 \text{ mm})^2(4.0 \text{ mm})\left(10^{-1} \text{ cm/mm}\right)^3$$
$$= \boxed{1.3 \times 10^{-5} \text{ kg}}$$

P1.67 $$V_{20 \text{ mpg}} = \frac{(10^8 \text{ cars})(10^4 \text{ mi/yr})}{20 \text{ mi/gal}} = 5.0 \times 10^{10} \text{ gal/yr}$$

$$V_{25 \text{ mpg}} = \frac{(10^8 \text{ cars})(10^4 \text{ mi/yr})}{25 \text{ mi/gal}} = 4.0 \times 10^{10} \text{ gal/yr}$$

$$\text{Fuel saved} = V_{25 \text{ mpg}} - V_{20 \text{ mpg}} = \boxed{1.0 \times 10^{10} \text{ gal/yr}}$$

P1.68 $$v = \left(5.00\frac{\text{furlongs}}{\text{fortnight}}\right)\left(\frac{220\text{ yd}}{1\text{ furlong}}\right)\left(\frac{0.914\,4\text{ m}}{1\text{ yd}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ hrs}}\right)\left(\frac{1\text{ hr}}{3\,600\text{ s}}\right) = \boxed{8.32\times10^{-4}\text{ m/s}}$$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

P1.69 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3.$$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of

$$\left(4\times10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3.$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

P1.70 The density of each material is $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$.

Al: $\rho = \dfrac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho = \dfrac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho = \dfrac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\dfrac{\text{g}}{\text{cm}^3}}$

Sn: $\rho = \dfrac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\dfrac{\text{g}}{\text{cm}^3}}$

Fe: $\rho = \dfrac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

P1.71 (a) $(3\,600\text{ s/hr})(24\text{ hr/day})(365.25\text{ days/yr}) = \boxed{3.16\times10^7\text{ s/yr}}$

(b) $V_{\text{mm}} = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi\left(5.00\times10^{-7}\text{ m}\right)^3 = 5.24\times10^{-19}\text{ m}^3$

$\dfrac{V_{\text{cube}}}{V_{\text{mm}}} = \dfrac{1\text{ m}^3}{5.24\times10^{-19}\text{ m}^3} = 1.91\times10^{18}$ micrometeorites

This would take $\dfrac{1.91\times10^{18}\text{ micrometeorites}}{3.16\times10^7\text{ micrometeorites/yr}} = \boxed{6.05\times10^{10}\text{ yr}}$.

ANSWERS TO EVEN PROBLEMS

P1.2 $5.52 \times 10^3\ \text{kg/m}^3$, between the densities of aluminum and iron, and greater than the densities of surface rocks.

P1.4 23.0 kg

P1.6 7.69 cm

P1.8 (a) and (b) see the solution, $N_A = 6.022\,137 \times 10^{23}$; (c) 18.0 g; (d) 44.0 g

P1.10 (a) 9.83×10^{-16} g; (b) 1.06×10^7 atoms

P1.12 (a) 4.02×10^{25} molecules; (b) 3.65×10^4 molecules

P1.14 (a) ii; (b) iii; (c) i

P1.16 (a) $\dfrac{\text{M} \cdot \text{L}}{\text{T}^2}$; (b) $1\ \text{newton} = 1\ \text{kg} \cdot \text{m/s}^2$

P1.18 $35.7\ \text{m}^2$

P1.20 $1.39 \times 10^{-4}\ \text{m}^3$

P1.22 (a) $3.39 \times 10^5\ \text{ft}^3$; (b) 2.54×10^4 lb

P1.24 (a) $560\ \text{km} = 5.60 \times 10^5\ \text{m} = 5.60 \times 10^7\ \text{cm}$; (b) $491\ \text{m} = 0.491\ \text{km} = 4.91 \times 10^4\ \text{cm}$; (c) $6.19\ \text{km} = 6.19 \times 10^3\ \text{m} = 6.19 \times 10^5\ \text{cm}$; (d) $2.50\ \text{km} = 2.50 \times 10^3\ \text{m} = 2.50 \times 10^5\ \text{cm}$

P1.26 $4.05 \times 10^3\ \text{m}^2$

P1.28 (a) $1\ \text{mi/h} = 1.609\ \text{km/h}$; (b) $88.5\ \text{km/h}$; (c) $16.1\ \text{km/h}$

P1.30 1.19×10^{57} atoms

P1.32 $2.57 \times 10^6\ \text{m}^3$

P1.34 1.3×10^{21} kg

P1.36 200 km

P1.38 (a) 13.4; (b) 49.1

P1.40 $r_{\text{Al}} = r_{\text{Fe}} \left(\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}} \right)^{1/3}$

P1.42 $\sim 10^7$ rev

P1.44 $\sim 10^9$ raindrops

P1.46 $\sim 10^{11}$ cans; $\sim 10^5$ tons

P1.48 $(209 \pm 4)\ \text{cm}^2$

P1.50 (a) 3; (b) 4; (c) 3; (d) 2

P1.52 (a) 797; (b) 1.1; (c) 17.66

P1.54 115.9 m

P1.56 316 m

P1.58 $4.50\ \text{m}^2$

P1.60 see the solution; 24.6°

P1.62 3.64 cents; no

P1.64 see the solution

P1.66 (a) 1 000 kg; (b) 5.2×10^{-16} kg; 0.27 kg; 1.3×10^{-5} kg

P1.68 $8.32 \times 10^{-4}\ \text{m/s}$; a snail

P1.70 see the solution

2

Motion in One Dimension

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q2.1 If I count 5.0 s between lightning and thunder, the sound has traveled $(331\ \text{m/s})(5.0\ \text{s}) = 1.7\ \text{km}$. The transit time for the light is smaller by

$$\frac{3.00 \times 10^{8}\ \text{m/s}}{331\ \text{m/s}} = 9.06 \times 10^{5}\ \text{times},$$

so it is negligible in comparison.

Q2.2 Yes. Yes, if the particle winds up in the $+x$ region at the end.

Q2.3 Zero.

Q2.4 Yes. Yes.

Q2.5 No. Consider a sprinter running a straight-line race. His average velocity would simply be the length of the race divided by the time it took for him to complete the race. If he stops along the way to tie his shoe, then his instantaneous velocity at that point would be zero.

Q2.6 We assume the object moves along a straight line. If its average velocity is zero, then the displacement must be zero over the time interval, according to Equation 2.2. The object might be stationary throughout the interval. If it is moving to the right at first, it must later move to the left to return to its starting point. Its velocity must be zero as it turns around. The graph of the motion shown to the right represents such motion, as the initial and final positions are the same. In an x vs. t graph, the instantaneous velocity at any time t is the slope of the curve at that point. At t_0 in the graph, the slope of the curve is zero, and thus the instantaneous velocity at that time is also zero.

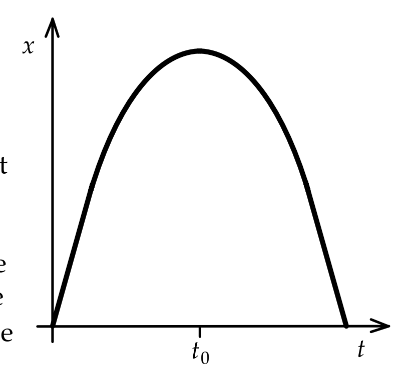


FIG. Q2.6

Q2.7 Yes. If the velocity of the particle is nonzero, the particle is in motion. If the acceleration is zero, the velocity of the particle is unchanging, or is a constant.

Q2.8 Yes. If you drop a doughnut from rest $(v = 0)$, then its acceleration is not zero. A common misconception is that immediately after the doughnut is released, both the velocity and acceleration are zero. If the acceleration were zero, then the velocity would not change, leaving the doughnut floating at rest in mid-air.

Q2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

Q2.10 Yes. Consider throwing a ball straight up. As the ball goes up, its velocity is upward $(v > 0)$, and its acceleration is directed down $(a < 0)$. A graph of v vs. t for this situation would look like the figure to the right. The acceleration is the slope of a v vs. t graph, and is always negative in this case, even when the velocity is positive.

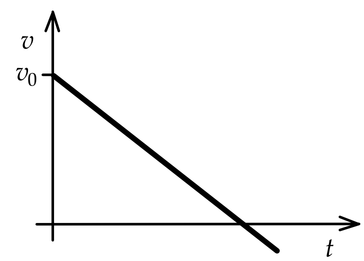


FIG. Q2.10

Q2.11 (a) Accelerating East (b) Braking East (c) Cruising East

(d) Braking West (e) Accelerating West (f) Cruising West

(g) Stopped but starting to move East

(h) Stopped but starting to move West

Q2.12 No. Constant acceleration only. Yes. Zero is a constant.

Q2.13 The position does depend on the origin of the coordinate system. Assume that the cliff is 20 m tall, and that the stone reaches a maximum height of 10 m above the top of the cliff. If the origin is taken as the top of the cliff, then the maximum height reached by the stone would be 10 m. If the origin is taken as the bottom of the cliff, then the maximum height would be 30 m.

The velocity is independent of the origin. Since the *change* in position is used to calculate the instantaneous velocity in Equation 2.5, the choice of origin is arbitrary.

Q2.14 Once the objects leave the hand, both are in free fall, and both experience the same downward acceleration equal to the free-fall acceleration, $-g$.

Q2.15 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity equal to v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

Q2.16 With $h = \frac{1}{2}gt^2$,

(a) $0.5h = \frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.

(b) The distance fallen is $0.25h = \frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

Q2.17 Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

SOLUTIONS TO PROBLEMS

Section 2.1 **Position, Velocity, and Speed**

P2.1 (a) $\bar{v} = \boxed{2.30 \text{ m/s}}$

(b) $$v = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$$

(c) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$$

***P2.2** (a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{20 \text{ ft}}{1 \text{ yr}}\left(\dfrac{1 \text{ m}}{3.281 \text{ ft}}\right)\left(\dfrac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{2 \times 10^{-7} \text{ m/s}}$ or in particularly windy times

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100 \text{ ft}}{1 \text{ yr}}\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{1 \times 10^{-6} \text{ m/s}}.$$

(b) The time required must have been

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{3\,000 \text{ mi}}{10 \text{ mm/yr}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{10^3 \text{ mm}}{1 \text{ m}}\right) = \boxed{5 \times 10^8 \text{ yr}}.$$

P2.3 (a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m}}{2 \text{ s}} = \boxed{5 \text{ m/s}}$$

(b) $$\bar{v} = \frac{5 \text{ m}}{4 \text{ s}} = \boxed{1.2 \text{ m/s}}$$

(c) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$$

(d) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$$

(e) $$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$$

P2.4 $x = 10t^2$: For
$$\begin{array}{lcccc} t(\text{s}) & = & 2.0 & 2.1 & 3.0 \\ x(\text{m}) & = & 40 & 44.1 & 90 \end{array}$$

(a) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{50 \text{ m}}{1.0 \text{ s}} = \boxed{50.0 \text{ m/s}}$$

(b) $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4.1 \text{ m}}{0.1 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

P2.5 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{\frac{d}{(5.00\ \text{m/s})} + \frac{d}{(3.00\ \text{m/s})}} = \frac{2d}{\frac{(8.00\ \text{m/s})d}{(15.0\ \text{m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0\ \text{m}^2/\text{s}^2)}{8.00\ \text{m/s}} = \boxed{3.75\ \text{m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

Section 2.2 **Instantaneous Velocity and Speed**

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.
Thus, at $t_i = 3.00\ \text{s}$: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

(b) At $t_f = 3.00\ \text{s} + \Delta t$: $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00\ \text{s}$ is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\ \text{m/s} + (3.00\ \text{m/s}^2)\Delta t\right) = \boxed{18.0\ \text{m/s}}.$$

P2.7 (a) at $t_i = 1.5\ \text{s}$, $x_i = 8.0\ \text{m}$ (Point A)
at $t_f = 4.0\ \text{s}$, $x_f = 2.0\ \text{m}$ (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

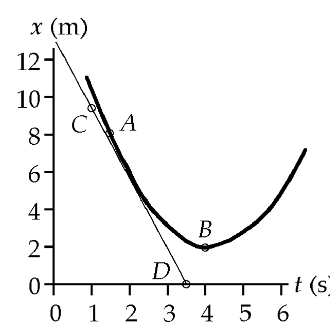


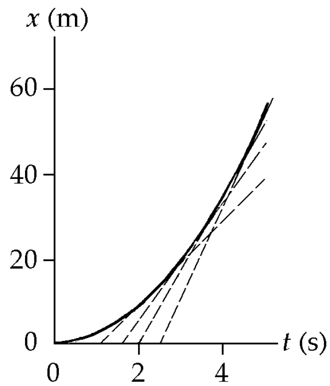
FIG. P2.7

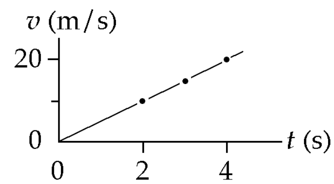
(b) The slope of the tangent line is found from points C and D. $(t_C = 1.0\ \text{s},\ x_C = 9.5\ \text{m})$ and $(t_D = 3.5\ \text{s},\ x_D = 0)$,

$$v \cong \boxed{-3.8\ \text{m/s}}.$$

(c) The velocity is zero when x is a minimum. This is at $t \cong \boxed{4\ \text{s}}$.

P2.8 (a)





(b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58\text{ m}}{2.5\text{ s}} \cong \boxed{23\text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \cong \dfrac{54\text{ m}}{3\text{ s}} \cong \boxed{18\text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \cong \dfrac{49\text{ m}}{3.4\text{ s}} \cong \boxed{14\text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \cong \dfrac{36\text{ m}}{4.0\text{ s}} \cong \boxed{9.0\text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23\text{ m/s}}{5.0\text{ s}} \cong \boxed{4.6\text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.9 (a) $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \dfrac{(5\text{ m} - 5\text{ m})}{(5\text{ s} - 4\text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\text{ m})}{(8\text{ s} - 7\text{ s})} = \boxed{+5\text{ m/s}}$

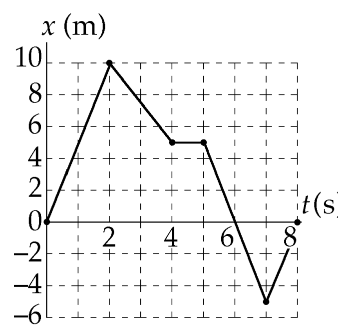


FIG. P2.9

***P2.10** Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1\,000\text{ m} - 800\text{ m}}{8\text{ m/s}} = 25\text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2\text{ m/s})(25\text{ s}) = \boxed{5.00\text{ m}}.$$

Section 2.3 **Acceleration**

P2.11 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at: \; a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}.$$

P2.12 (a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + \left(2.00 \text{ m/s}^2\right)(10.0 \text{ s}) = \boxed{20.0 \text{ m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0 \text{ m/s} + \left(3.00 \text{ m/s}^2\right)(5.00 \text{ s}) = \boxed{5.00 \text{ m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(2.00 \text{ m/s}^2\right)(10.0 \text{ s})^2 = 100 \text{ m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + 0 = 200 \text{ m}.$$

And at $t = 20.0$ s,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200 \text{ m} + (20.0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2}\left(-3.00 \text{ m/s}^2\right)(5.00 \text{ s})^2 = \boxed{262 \text{ m}}.$$

***P2.13** (a) The average speed during a time interval Δt is $\overline{v} = \dfrac{\text{distance traveled}}{\Delta t}$. During the first quarter mile segment, Secretariat's average speed was

$$\overline{v}_1 = \frac{0.250 \text{ mi}}{25.2 \text{ s}} = \frac{1\,320 \text{ ft}}{25.2 \text{ s}} = \boxed{52.4 \text{ ft/s}} \quad (35.6 \text{ mi/h}).$$

During the second quarter mile segment,

$$\overline{v}_2 = \frac{1\,320 \text{ ft}}{24.0 \text{ s}} = \boxed{55.0 \text{ ft/s}} \quad (37.4 \text{ mi/h}).$$

For the third quarter mile of the race,

$$\overline{v}_3 = \frac{1\,320 \text{ ft}}{23.8 \text{ s}} = \boxed{55.5 \text{ ft/s}} \quad (37.7 \text{ mi/h}),$$

and during the final quarter mile,

$$\overline{v}_4 = \frac{1\,320 \text{ ft}}{23.0 \text{ s}} = \boxed{57.4 \text{ ft/s}} \quad (39.0 \text{ mi/h}).$$

continued on next page

(b) Assuming that $v_f = \overline{v}_4$ and recognizing that $v_i = 0$, the average acceleration during the race was

$$\overline{a} = \frac{v_f - v_i}{\text{total elapsed time}} = \frac{57.4 \text{ ft/s} - 0}{(25.2 + 24.0 + 23.8 + 23.0) \text{ s}} = \boxed{0.598 \text{ ft/s}^2}.$$

P2.14 (a) Acceleration is the slope of the graph of v vs t.

For $0 < t < 5.00$ s, $a = 0$.

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$.

For $5.0 \text{ s} < t < 15.0$ s, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$

We can plot $a(t)$ as shown.

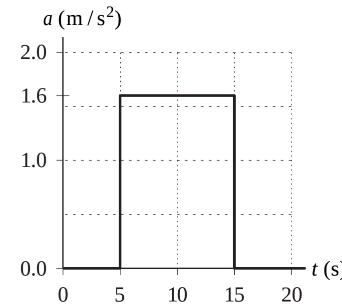


FIG. P2.14

(b) $a = \frac{v_f - v_i}{t_f - t_i}$

(i) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$$t_f = 15.0 \text{ s}$$
$$v_f = 8.00 \text{ m/s}$$
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

P2.15 $x = 2.00 + 3.00t - t^2$, $v = \frac{dx}{dt} = 3.00 - 2.00t$, $a = \frac{dv}{dt} = -2.00$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}}$

(b) $v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}}$

(c) $a = \boxed{-2.00 \text{ m/s}^2}$

P2.16 (a) At $t = 2.00\text{ s}$, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]\text{ m} = 11.0\text{ m}$.

At $t = 3.00\text{ s}$, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]\text{ m} = 24.0\text{ m}$

so

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00)\text{ m/s}$$

At $t = 2.00\text{ s}$, $v = \left[6.00(2.00) - 2.00\right]\text{ m/s} = \boxed{10.0\text{ m/s}}$.
At $t = 3.00\text{ s}$, $v = \left[6.00(3.00) - 2.00\right]\text{ m/s} = \boxed{16.0\text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\text{ m/s} - 10.0\text{ m/s}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{6.00\text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00 - 2.00) = \boxed{6.00\text{ m/s}^2}$. (This includes both $t = 2.00\text{ s}$ and $t = 3.00\text{ s}$).

P2.17 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00\text{ m/s}}{6.00\text{ s}} = \boxed{1.3\text{ m/s}^2}$

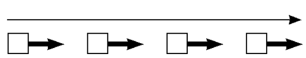
(b) Maximum positive acceleration is at $t = 3\text{ s}$, and is approximately $\boxed{2\text{ m/s}^2}$.

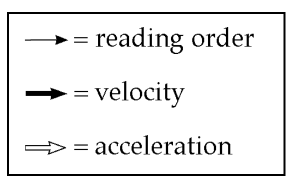
(c) $a = 0$, at $\boxed{t = 6\text{ s}}$, and also for $\boxed{t > 10\text{ s}}$.

(d) Maximum negative acceleration is at $t = 8\text{ s}$, and is approximately $\boxed{-1.5\text{ m/s}^2}$.

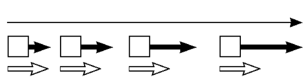
Section 2.4 Motion Diagrams

P2.18 (a)

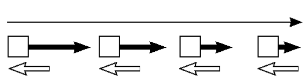




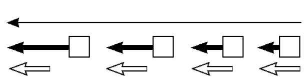
(b)



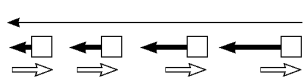
(c)



(d)



(e)



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(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.
Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.5 **One-Dimensional Motion with Constant Acceleration**

P2.19 From $v_f^2 = v_i^2 + 2ax$, we have $\left(10.97\times10^3\ \text{m/s}\right)^2 = 0 + 2a(220\ \text{m})$, so that $\boxed{a = 2.74\times10^5\ \text{m/s}^2}$ which is $\boxed{a = 2.79\times10^4 \text{ times } g}$.

P2.20 (a) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ becomes $40\ \text{m} = \frac{1}{2}\left(v_i + 2.80\ \text{m/s}\right)(8.50\ \text{s})$ which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$

P2.21 Given $v_i = 12.0\ \text{cm/s}$ when $x_i = 3.00\ \text{cm}(t = 0)$, and at $t = 2.00\ \text{s}$, $x_f = -5.00\ \text{cm}$,

$$x_f - x_i = v_i t + \frac{1}{2}at^2:\ -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2}a(2.00)^2$$
$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\ \text{cm/s}^2}.$$

***P2.22** (a) Let *i* be the state of moving at $60\ \text{mi/h}$ and *f* be at rest

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$0 = (60\ \text{mi/h})^2 + 2a_x(121\ \text{ft} - 0)\left(\frac{1\ \text{mi}}{5\,280\ \text{ft}}\right)$$
$$a_x = \frac{-3\,600\ \text{mi}}{242\ \text{h}^2}\left(\frac{5\,280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-21.8\ \text{mi/h}\cdot\text{s}}$$
$$= -21.8\ \text{mi/h}\cdot\text{s}\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-9.75\ \text{m/s}^2}.$$

(b) Similarly,

$$0 = (80\ \text{mi/h})^2 + 2a_x(211\ \text{ft} - 0)$$
$$a_x = -\frac{6\,400(5\,280)}{422(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.2\ \text{mi/h}\cdot\text{s} = -9.94\ \text{m/s}^2}.$$

(c) Let *i* be moving at $80\ \text{mi/h}$ and *f* be moving at $60\ \text{mi/h}$.

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$(60\ \text{mi/h})^2 = (80\ \text{mi/h})^2 + 2a_x(211\ \text{ft} - 121\ \text{ft})$$
$$a_x = -\frac{2\,800(5\,280)}{2(90)(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.8\ \text{mi/h}\cdot\text{s} = -10.2\ \text{m/s}^2}.$$

***P2.23** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$$x_i = 0,\ x_f = 100\text{ m},\ v_{xi} = 30\text{ m/s},\ v_{xf} = ?,\ a_x = -3.5\text{ m/s}^2,\ t = ?$$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}(-3.5\text{ m/s}^2)t^2$$

$$(1.75\text{ m/s}^2)t^2 - (30\text{ m/s})t + 100\text{ m} = 0.$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4(1.75\text{ m/s}^2)(100\text{ m})}}{2(1.75\text{ m/s}^2)} = \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s or } \boxed{4.53\text{ s}}.$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\text{ m/s} - (3.5\text{ m/s}^2)4.53\text{ s} = \boxed{14.1\text{ m/s}}$

P2.24 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\text{ m/s})(15\text{ s}) + (50\text{ m/s})(40 - 15)\text{ s}$$
$$+ \frac{1}{2}(50\text{ m/s})(10\text{ s})$$
$$\Delta x = \boxed{1\,875\text{ m}}$$

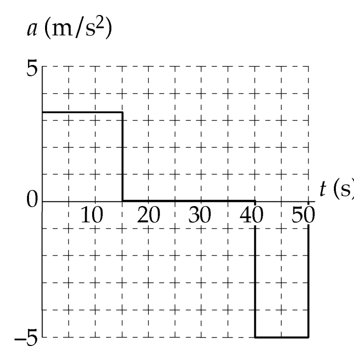

FIG. P2.24

(b) From $t = 10$ s to $t = 40$ s, displacement is

$$\Delta x = \frac{1}{2}(50\text{ m/s} + 33\text{ m/s})(5\text{ s}) + (50\text{ m/s})(25\text{ s}) = \boxed{1\,457\text{ m}}.$$

(c) $0 \le t \le 15\text{ s}$: $a_1 = \frac{\Delta v}{\Delta t} = \frac{(50 - 0)\text{ m/s}}{15\text{ s} - 0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40\text{ s}$: $\boxed{a_2 = 0}$

$40\text{ s} \le t \le 50\text{ s}$: $a_3 = \frac{\Delta v}{\Delta t} = \frac{(0 - 50)\text{ m/s}}{50\text{ s} - 40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

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(d) (i) $x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}\left(3.3\ \text{m/s}^2\right)t^2$ or $\boxed{x_1 = \left(1.67\ \text{m/s}^2\right)t^2}$

(ii) $x_2 = \frac{1}{2}(15\ \text{s})\left[50\ \text{m/s} - 0\right] + (50\ \text{m/s})(t - 15\ \text{s})$ or $\boxed{x_2 = (50\ \text{m/s})t - 375\ \text{m}}$

(iii) For $40\ \text{s} \le t \le 50\ \text{s}$,

$$x_3 = \begin{pmatrix} \text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\ \text{s} \end{pmatrix} + \frac{1}{2}a_3(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

or

$$x_3 = 375\ \text{m} + 1\,250\ \text{m} + \frac{1}{2}\left(-5.0\ \text{m/s}^2\right)(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

which reduces to

$$\boxed{x_3 = (250\ \text{m/s})t - \left(2.5\ \text{m/s}^2\right)t^2 - 4\,375\ \text{m}}.$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1\,875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

P2.25 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00\ \text{m}$, $v_i = 3.00\ \text{m/s}$, and $a = -8.00\ \text{m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}\ \text{s}$. The position at this time is:

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\frac{3}{8}\ \text{s}\right) - \left(4.00\ \text{m/s}^2\right)\left(\frac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}.$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \frac{3}{4}\ \text{s}$$

and the velocity is $v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)\left(\frac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$.

***P2.26** The time for the Ford to slow down we find from

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$t = \frac{2\Delta x}{v_{xi} + v_{xf}} = \frac{2(250\text{ m})}{71.5\text{ m/s} + 0} = 6.99\text{ s}.$$

Its time to speed up is similarly

$$t = \frac{2(350\text{ m})}{0 + 71.5\text{ m/s}} = 9.79\text{ s}.$$

The whole time it is moving at less than maximum speed is $6.99\text{ s} + 5.00\text{ s} + 9.79\text{ s} = 21.8\text{ s}$. The Mercedes travels

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t = 0 + \frac{1}{2}(71.5 + 71.5)(\text{m/s})(21.8\text{ s})$$
$$= 1\,558\text{ m}$$

while the Ford travels $250 + 350\text{ m} = 600\text{ m}$, to fall behind by $1\,558\text{ m} - 600\text{ m} = \boxed{958\text{ m}}$.

P2.27 (a) $v_i = 100\text{ m/s}$, $a = -5.00\text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ so $0 = (100)^2 - 2(5.00)\left(x_f - 0\right)$. Thus $x_f = 1\,000\text{ m}$ and $t = \boxed{20.0\text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$.

P2.28 (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0\text{ m/s}$, $a = -2.00\text{ m/s}^2$. Use these values in the general equation

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to find

$$x_f = 0 + (30.0t\text{ m/s}) + \frac{1}{2}\left(-2.00\text{ m/s}^2\right)t^2$$

when t is in seconds

$$\boxed{x_f = \left(30.0t - t^2\right)\text{ m}}.$$

To find an equation for the velocity, use $v_f = v_i + at = 30.0\text{ m/s} + \left(-2.00\text{ m/s}^2\right)t$,

$$\boxed{v_f = (30.0 - 2.00t)\text{ m/s}}.$$

(b) The distance of travel x_f becomes a maximum, $x_{\max}$, when $v_f = 0$ (turning point in the motion). Use the expressions found in part (a) for v_f to find the value of t when x_f has its maximum value:

From $v_f = (3.00 - 2.00t)\text{ m/s}$, $v_f = 0$ when $t = 15.0\text{ s}$. Then

$$x_{\max} = \left(30.0t - t^2\right)\text{m} = (30.0)(15.0) - (15.0)^2 = \boxed{225\text{ m}}.$$

P2.29 In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf} = v_{xi} - \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\ \text{s}) \end{array}\right\}.$$

So substituting for v_{xi} gives $62.4\ \text{m} = \frac{1}{2}\left[v_{xf} + \left(56.0\ \text{m/s}^2\right)(4.20\ \text{s}) + v_{xf}\right](4.20\ \text{s})$

$$14.9\ \text{m/s} = v_{xf} + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}).$$

Thus

$$v_{xf} = \boxed{3.10\ \text{m/s}}.$$

P2.30 Take any two of the standard four equations, such as $\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{array}\right\}$. Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t.$$

Thus

$$\boxed{x_f - x_i = v_{xf} t - \frac{1}{2} a_x t^2}.$$

Back in problem 29, $62.4\ \text{m} = v_{xf}(4.20\ \text{s}) - \frac{1}{2}\left(-5.60\ \text{m/s}^2\right)(4.20\ \text{s})^2$

$$v_{xf} = \frac{62.4\ \text{m} - 49.4\ \text{m}}{4.20\ \text{s}} = \boxed{3.10\ \text{m/s}}.$$

P2.31 (a) $a = \dfrac{v_f - v_i}{t} = \dfrac{632\left(\frac{5\,280}{3\,600}\right)}{1.40} = \boxed{-662\ \text{ft/s}^2} = -202\ \text{m/s}^2$

(b) $x_f = v_i t + \frac{1}{2} a t^2 = (632)\left(\dfrac{5\,280}{3\,600}\right)(1.40) - \frac{1}{2}(662)(1.40)^2 = \boxed{649\ \text{ft}} = \boxed{198\ \text{m}}$

P2.32 (a) The time it takes the truck to reach $20.0\ \text{m/s}$ is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0\ \text{m/s} - 0\ \text{m/s}}{2.00\ \text{m/s}^2} = 10.0\ \text{s}.$$

The total time is thus

$$10.0\ \text{s} + 20.0\ \text{s} + 5.00\ \text{s} = \boxed{35.0\ \text{s}}.$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100\ \text{m}.$$

With a being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400\ \text{m}.$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0\ \text{m}.$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550\ \text{m}$, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7\ \text{m/s}}$.

P2.33 We have $v_i = 2.00 \times 10^4\ \text{m/s}$, $v_f = 6.00 \times 10^6\ \text{m/s}$, $x_f - x_i = 1.50 \times 10^{-2}\ \text{m}$.

(a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$: $t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2}\ \text{m})}{2.00 \times 10^4\ \text{m/s} + 6.00 \times 10^6\ \text{m/s}} = \boxed{4.98 \times 10^{-9}\ \text{s}}$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left(6.00 \times 10^6\ \text{m/s}\right)^2 - \left(2.00 \times 10^4\ \text{m/s}\right)^2}{2(1.50 \times 10^{-2}\ \text{m})} = \boxed{1.20 \times 10^{15}\ \text{m/s}^2}$$

***P2.34** (a) $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$: $\left[0.01\left(3\times10^8 \text{ m/s}\right)\right]^2 = 0 + 2a_x(40 \text{ m})$

$$a_x = \frac{\left(3\times10^6 \text{ m/s}\right)^2}{80 \text{ m}} = \boxed{1.12\times10^{11} \text{ m/s}^2}$$

(b) We must find separately the time t_1 for speeding up and the time t_2 for coasting:

$$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_1: \quad 40 \text{ m} = \frac{1}{2}\left(3\times10^6 \text{ m/s} + 0\right)t_1$$
$$t_1 = 2.67\times10^{-5} \text{ s}$$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_2: \quad 60 \text{ m} = \frac{1}{2}\left(3\times10^6 \text{ m/s} + 3\times10^6 \text{ m/s}\right)t_2$$
$$t_2 = 2.00\times10^{-5} \text{ s}$$

total time $= \boxed{4.67\times10^{-5} \text{ s}}$.

***P2.35** (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and t_m is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between t_m and t_0 is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c(t_0 - t_m) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c\left(t_0 - \frac{1}{2}t_m\right)}.$$

(c) For constant v_c and t_0, Δx is minimized by maximizing t_m to $t_m = t_0$. Then $\Delta x_{\min} = v_c\left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}$.

(e) This is realized by having the servo motor on all the time.

(d) We maximize Δx by letting t_m approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

***P2.36** Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d .$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is not equal to v_d unless $a = 0$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is equal to v_d as determined above.

P2.37 (a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0,\ a = 0.500\ \text{m/s}^2,\ x_f - x_i = 9.00\ \text{m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}.$$

(b) $x_f - x_i = v_i t + \frac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\ \text{m/s},\ v_f = 0,\ x_f - x_i = 15.00\ \text{m}.$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left[0 - (3.00\ \text{m/s})^2\right]}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}.$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second:
$v_i = 3.00\ \text{m/s}$, $x_f - x_i = 8.00\ \text{m}$, $a = -0.300\ \text{m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\ \text{m/s})^2 + 2(-0.300\ \text{m/s}^2)(8.00\ \text{m}) = 4.20\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.05\ \text{m/s}}.$$

P2.38 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}\left(-2.00\ \text{m/s}^2\right)t^2.$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$
$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}.$$

***P2.39** As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and

$$x_{\text{trooper}} = 1.5t^2.$$

They intersect at

$$t = \boxed{31\ \text{s}}.$$

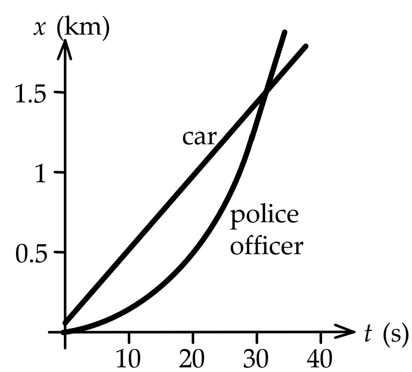


FIG. P2.39

Section 2.6 **Freely Falling Objects**

P2.40 Choose the origin ($y=0,\ t=0$) at the starting point of the ball and take upward as positive. Then $y_i=0$, $v_i=0$, and $a=-g=-9.80\ \text{m/s}^2$. The position and the velocity at time t become:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ y_f=-\frac{1}{2}gt^2=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

and

$$v_f=v_i+at:\ v_f=-gt=-\left(9.80\ \text{m/s}^2\right)t\,.$$

(a) at $t=1.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2=\boxed{-4.90\ \text{m}}$

at $t=2.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2=\boxed{-19.6\ \text{m}}$

at $t=3.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})^2=\boxed{-44.1\ \text{m}}$

(b) at $t=1.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})=\boxed{-9.80\ \text{m/s}}$

at $t=2.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})=\boxed{-19.6\ \text{m/s}}$

at $t=3.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})=\boxed{-29.4\ \text{m/s}}$

P2.41 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a=-g=-9.80\ \text{m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2=v_i^2+2a\left(y_f-y_i\right):\quad 0=v_i^2-2\left(9.80\ \text{m/s}^2\right)(1\,609\ \text{m})$$
$$v_i=178\ \text{m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ 0=(178\ \text{m/s})t-\frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2.$$

The root $t=0$ describes launch; the other root, $t=36.2\ \text{s}$, describes his flight time. His rate of pay may then be found from

$$\text{pay rate}=\frac{\$1.00}{36.2\ \text{s}}=(0.027\,6\ \$/\text{s})(3\,600\ \text{s/h})=\boxed{\$99.3/\text{h}}\,.$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.42 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -\left(4.90 \text{ m/s}^2\right)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}.$$

Solving for t,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for t, we find that $t = \boxed{1.79 \text{ s}}$.

P2.43 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0 \text{ m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$$v_f = \boxed{4.68 \text{ m/s downward}}$$

P2.44 The bill starts from rest $v_i = 0$ and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - \left(4.90 \text{ m/s}^2\right)(0.20 \text{ s})^2 = -0.20 \text{ m}.$$

This distance is about twice the distance between the center of the bill and its top edge ($\cong 8$ cm). $\boxed{\text{Thus, David will be unsuccessful}}$.

***P2.45** (a) From $\Delta y = v_i t + \frac{1}{2}at^2$ with $v_i = 0$, we have

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-23 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{2.17 \text{ s}}.$$

(b) The final velocity is $v_f = 0 + \left(-9.80 \text{ m/s}^2\right)(2.17 \text{ s}) = \boxed{-21.2 \text{ m/s}}$.

(c) The time take for the sound of the impact to reach the spectator is

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}} = \frac{23 \text{ m}}{340 \text{ m/s}} = 6.76 \times 10^{-2} \text{ s},$$

so the total elapsed time is $t_{\text{total}} = 2.17 \text{ s} + 6.76 \times 10^{-2} \text{ s} \approx \boxed{2.23 \text{ s}}$.

P2.46 At any time t, the position of the ball released from rest is given by $y_1 = h - \frac{1}{2}gt^2$. At time t, the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2}gt^2$. The time at which the first ball has a position of $y_1 = \frac{h}{2}$ is found from the first equation as $\frac{h}{2} = h - \frac{1}{2}gt^2$, which yields $t = \sqrt{\frac{h}{g}}$. To require that the second ball have a position of $y_2 = \frac{h}{2}$ at this time, use the second equation to obtain $\frac{h}{2} = v_i\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

P2.47 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00\text{ s}$, $g = 9.80\text{ m/s}^2$. Therefore,

$$v_i = gt = (9.80\text{ m/s}^2)(3.00\text{ s}) = \boxed{29.4\text{ m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\text{ m/s})(3.00\text{ s}) = \boxed{44.1\text{ m}}$$

***P2.48** (a) Consider the upward flight of the arrow.

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y(y_f - y_i) \\ 0 &= (100\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)\Delta y \\ \Delta y &= \frac{10\,000\text{ m}^2/\text{s}^2}{19.6\text{ m/s}^2} = \boxed{510\text{ m}} \end{aligned}$$

(b) Consider the whole flight of the arrow.

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ 0 &= 0 + (100\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2 \end{aligned}$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\text{ m/s}}{4.9\text{ m/s}^2} = \boxed{20.4\text{ s}}.$$

P2.49 Time to fall 3.00 m is found from Eq. 2.12 with $v_i = 0$, $3.00\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$, $t = 0.782\text{ s}$.

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82\text{ m}}$.

(b) $t = \boxed{0.782\text{ s}}$

P2.50 Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144\text{ ft},\ v_i = 0,\text{ and } a = g = 32.0\ \text{ft/s}^2.$$

Thus, $\Delta y = v_i t + \frac{1}{2}at^2 \to 144\text{ ft} = 0 + (16.0\ \text{ft/s}^2)t^2$ giving $t_{\text{fall}} = 3.00\text{ s}$. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0\ \text{ft/s}^2)(3.00\text{ s}) = \boxed{96.0\ \text{ft/s}}.$$

(b) While crushing the box, $v_i = 96.0\ \text{ft/s}$, $v_f = 0$, and $\Delta y = 18.0\text{ in.} = 1.50\text{ ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0\ \text{ft/s})^2}{2(1.50\text{ ft})} = -3.07\times10^3\ \text{ft/s}^2,\text{ or } \boxed{a = 3.07\times10^3\ \text{ft/s}^2\ \text{upward}}.$$

(c) Time to crush box: $\Delta t = \frac{\Delta y}{\overline{v}} = \frac{\Delta y}{\frac{v_f+v_i}{2}} = \frac{2(1.50\text{ ft})}{0 + 96.0\ \text{ft/s}}$ or $\boxed{\Delta t = 3.13\times10^{-2}\text{ s}}$.

P2.51 $y = 3.00t^3$: At $t = 2.00\text{ s}$, $y = 3.00(2.00)^3 = 24.0\text{ m}$ and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s}\uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t, (only positive values of t count), $\boxed{t = 7.96\text{ s}}$.

***P2.52** Consider the last 30 m of fall. We find its speed 30 m above the ground:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 30\text{ m} + v_{yi}(1.5\text{ s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(1.5\text{ s})^2$$

$$v_{yi} = \frac{-30\text{ m} + 11.0\text{ m}}{1.5\text{ s}} = -12.6\ \text{m/s}.$$

Now consider the portion of its fall above the 30 m point. We assume it starts from rest

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$(-12.6\ \text{m/s})^2 = 0 + 2(-9.8\ \text{m/s}^2)\Delta y$$

$$\Delta y = \frac{160\ \text{m}^2/\text{s}^2}{-19.6\ \text{m/s}^2} = -8.16\text{ m}.$$

Its original height was then $30\text{ m} + |-8.16\text{ m}| = \boxed{38.2\text{ m}}$.

Section 2.7 **Kinematic Equations Derived from Calculus**

P2.53 (a) $J = \frac{da}{dt} = \text{constant}$

$$da = Jdt$$

$$a = J\int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$a = \frac{dv}{dt}$$
$$dv = adt$$
$$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_it + c_2$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_it + v_i}$

$$v = \frac{dx}{dt}$$
$$dx = vdt$$
$$x = \int vdt = \int \left(\frac{1}{2}Jt^2 + a_it + v_i\right)dt$$
$$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_it^2 + v_it + c_3$$
$$x = x_i$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_it^2 + v_it + x_i}$.

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_it$

$$a^2 = a_i^2 + \left(J^2t^2 + 2Ja_it\right)$$
$$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_it\right)$$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_it + v_i$. So $(v - v_i) = \frac{1}{2}Jt^2 + a_it$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}.$$

P2.54 (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

At $t = 3\text{ s}$, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12\text{ m}$.

At $t = 5\text{ s}$, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28\text{ m}$.

At $t = 7\text{ s}$, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s}) = 36\text{ m}$.

(b) For $0 < t < 3\text{ s}$, $a = \frac{8\text{ m/s}}{3\text{ s}} = 2.67\text{ m/s}^2$.

For $3 < t < 5\text{ s}$, $a = 0$.

(c) For $5\text{ s} < t < 9\text{ s}$, $a = -\frac{16\text{ m/s}}{4\text{ s}} = \boxed{-4\text{ m/s}^2}$.

(d) At $t = 6\text{ s}$, $x = 28\text{ m} + (6\text{ m/s})(1\text{ s}) = \boxed{34\text{ m}}$.

(e) At $t = 9\text{ s}$, $x = 36\text{ m} + \frac{1}{2}(-8\text{ m/s})(2\text{ s}) = \boxed{28\text{ m}}$.

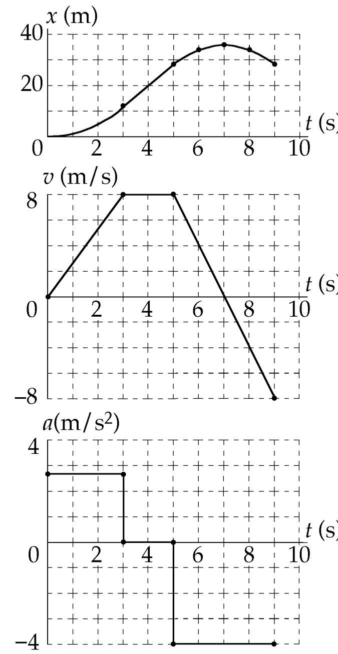


FIG. P2.54

P2.55 (a) $a = \frac{dv}{dt} = \frac{d}{dt}\left[-5.00\times10^7 t^2 + 3.00\times10^5 t\right]$

$$\boxed{a = -\left(10.0\times10^7\text{ m/s}^3\right)t + 3.00\times10^5\text{ m/s}^2}$$

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times10^7 t^2 + 3.00\times10^5 t\right)dt$$

$$x = -5.00\times10^7\frac{t^3}{3} + 3.00\times10^5\frac{t^2}{2}$$

$$\boxed{x = -\left(1.67\times10^7\text{ m/s}^3\right)t^3 + \left(1.50\times10^5\text{ m/s}^2\right)t^2}.$$

(b) The bullet escapes when $a = 0$, at $-\left(10.0\times10^7\text{ m/s}^3\right)t + 3.00\times10^5\text{ m/s}^2 = 0$

$$t = \frac{3.00\times10^5\text{ s}}{10.0\times10^7} = \boxed{3.00\times10^{-3}\text{ s}}.$$

(c) New $v = \left(-5.00\times10^7\right)\left(3.00\times10^{-3}\right)^2 + \left(3.00\times10^5\right)\left(3.00\times10^{-3}\right)$

$$v = -450\text{ m/s} + 900\text{ m/s} = \boxed{450\text{ m/s}}.$$

(d) $x = -\left(1.67\times10^7\right)\left(3.00\times10^{-3}\right)^3 + \left(1.50\times10^5\right)\left(3.00\times10^{-3}\right)^2$

$x = -0.450\text{ m} + 1.35\text{ m} = \boxed{0.900\text{ m}}$

P2.56 $a = \frac{dv}{dt} = -3.00v^2$, $v_i = 1.50\ \text{m/s}$

Solving for v, $\frac{dv}{dt} = -3.00v^2$

$$\int_{v=v_i}^{v} v^{-2}dv = -3.00\int_{t=0}^{t} dt$$

$$-\frac{1}{v} + \frac{1}{v_i} = -3.00t \text{ or } 3.00t = \frac{1}{v} - \frac{1}{v_i}.$$

When $v = \frac{v_i}{2}$, $t = \frac{1}{3.00v_i} = \boxed{0.222\ \text{s}}$.

Additional Problems

***P2.57** The distance the car travels at constant velocity, v_0, during the reaction time is $(\Delta x)_1 = v_0\Delta t_r$. The time for the car to come to rest, from initial velocity v_0, after the brakes are applied is

$$t_2 = \frac{v_f - v_i}{a} = \frac{0 - v_0}{a} = -\frac{v_0}{a}$$

and the distance traveled during this braking period is

$$(\Delta x)_2 = \bar{v}t_2 = \left(\frac{v_f + v_i}{2}\right)t_2 = \left(\frac{0 + v_0}{2}\right)\left(-\frac{v_0}{a}\right) = -\frac{v_0^2}{2a}.$$

Thus, the total distance traveled before coming to a stop is

$$s_{\text{stop}} = (\Delta x)_1 + (\Delta x)_2 = \boxed{v_0\Delta t_r - \frac{v_0^2}{2a}}.$$

***P2.58** (a) If a car is a distance $s_{\text{stop}} = v_0\Delta t_r - \frac{v_0^2}{2a}$ (See the solution to Problem 2.57) from the intersection of length s_i when the light turns yellow, the distance the car must travel before the light turns red is

$$\Delta x = s_{\text{stop}} + s_i = v_0\Delta t_r - \frac{v_0^2}{2a} + s_i.$$

Assume the driver does not accelerate in an attempt to "beat the light" (an extremely dangerous practice!). The time the light should remain yellow is then the time required for the car to travel distance Δx at constant velocity v_0. This is

$$\Delta t_{\text{light}} = \frac{\Delta x}{v_0} = \frac{v_0\Delta t_r - \frac{v_0^2}{2a} + s_i}{v_0} = \boxed{\Delta t_r - \frac{v_0}{2a} + \frac{s_i}{v_0}}.$$

(b) With $s_i = 16\ \text{m}$, $v = 60\ \text{km/h}$, $a = -2.0\ \text{m/s}^2$, and $\Delta t_r = 1.1\ \text{s}$,

$$\Delta t_{\text{light}} = 1.1\ \text{s} - \frac{60\ \text{km/h}}{2(-2.0\ \text{m/s}^2)}\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right) + \frac{16\ \text{m}}{60\ \text{km/h}}\left(\frac{1\ \text{km/h}}{0.278\ \text{m/s}}\right) = \boxed{6.23\ \text{s}}.$$

***P2.59** (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

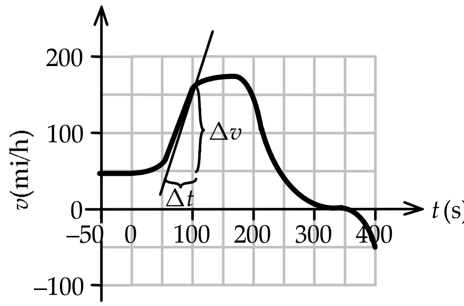


FIG. P2.59(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155-45)\ \text{mi/h}}{(100-50)\ \text{s}} = \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

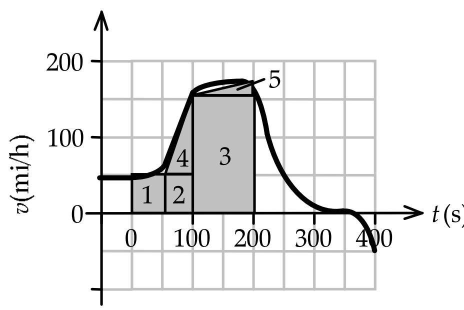


FIG. P2.59(c)

$$\begin{aligned}
\Delta x_{0\to 200\ \text{s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\
&\approx (50\ \text{mi/h})(50\ \text{s}) + (50\ \text{mi/h})(50\ \text{s}) \\
&\quad + (160\ \text{mi/h})(100\ \text{s}) \\
&\quad + \frac{1}{2}(50\ \text{s})(100\ \text{mi/h}) \\
&\quad + \frac{1}{2}(100\ \text{s})(170\ \text{mi/h} - 160\ \text{mi/h}) \\
&= 24\,000(\text{mi/h})(\text{s})
\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As $1\ \text{h} = 3\,600\ \text{s}$,

$$\Delta x_{0\to 200\ \text{s}} \approx \left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s}) = \boxed{6.7\ \text{mi}}.$$

***P2.60** Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t}=\frac{8.60\text{ m}}{1.50\text{ s}}=5.73\text{ m/s}.$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60\text{ m}}{1.10\text{ s}}=7.82\text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\text{ s})+\frac{1}{2}(1.10\text{ s})=1.30\text{ s}$$

so the acceleration is: $a_x=\frac{\Delta v_x}{\Delta t}=\frac{7.82\text{ m/s}-5.73\text{ m/s}}{1.30\text{ s}}=\boxed{1.60\text{ m/s}^2}$.

P2.61 The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_{xi}=1.04\text{ mm/d}$ and $a_x=0.132\left(\frac{\text{mm/d}}{\text{w}}\right)$. The increase in the length of the hair (i.e., displacement) during a time of $t=5.00\text{ w}=35.0\text{ d}$ is

$$\Delta x=v_{xi}t+\frac{1}{2}a_xt^2$$
$$\Delta x=(1.04\text{ mm/d})(35.0\text{ d})+\frac{1}{2}(0.132\text{ mm/d}\cdot\text{w})(35.0\text{ d})(5.00\text{ w})$$

or $\boxed{\Delta x=48.0\text{ mm}}$.

P2.62 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

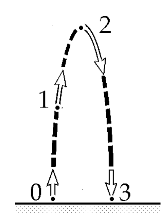


FIG. P2.62

(0 to 1) $v_f^2-(80.0)^2=2(4.00)(1\,000)$ so $v_f=120\text{ m/s}$
$120=80.0+(4.00)t$ giving $t=10.0\text{ s}$

(1 to 2) $0-(120)^2=2(-9.80)(x_f-x_i)$ giving $x_f-x_i=735\text{ m}$
$0-120=-9.80t$ giving $t=12.2\text{ s}$
This is the time of maximum height of the rocket.

(2 to 3) $v_f^2-0=2(-9.80)(-1\,735)$
$v_f=-184=(-9.80)t$ giving $t=18.8\text{ s}$

(a) $t_{\text{total}}=10+12.2+18.8=\boxed{41.0\text{ s}}$

(b) $(x_f-x_i)_{\text{total}}=\boxed{1.73\text{ km}}$

continued on next page

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.63 Distance traveled by motorist $= (15.0 \text{ m/s})t$

Distance traveled by policeman $= \frac{1}{2}(2.00 \text{ m/s}^2)t^2$

(a) intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0 \text{ s}}$

(b) $v(\text{officer}) = (2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) $x(\text{officer}) = \frac{1}{2}(2.00 \text{ m/s}^2)t^2 = \boxed{225 \text{ m}}$

P2.64 Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}.$$

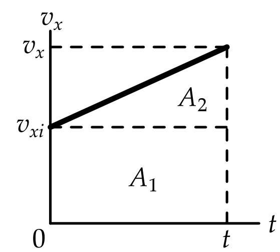


FIG. P2.64

The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

P2.65 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x = \frac{1}{2}(v + v_i)t_1,\ 100 - x = v(10.2 - t_1) \text{ and } v = v_i + at_1.$$

The first two give

$$100 = \left(10.2 - \frac{1}{2}t_1\right)v = \left(10.2 - \frac{1}{2}t_1\right)at_1$$
$$a = \frac{200}{(20.4 - t_1)t_1}.$$

$$\text{For Maggie: } a = \frac{200}{(18.4)(2.00)} = \boxed{5.43\ \text{m/s}^2}$$
$$\text{For Judy: } a = \frac{200}{(17.4)(3.00)} = \boxed{3.83\ \text{m/s}^2}$$

(b) $v = a_1 t$

$$\text{Maggie: } v = (5.43)(2.00) = \boxed{10.9\ \text{m/s}}$$
$$\text{Judy: } v = (3.83)(3.00) = \boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x = \frac{1}{2}at_1^2 + v(6.00 - t_1)$$

$$\text{Maggie: } x = \frac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\ \text{m}$$
$$\text{Judy: } x = \frac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\ \text{m}$$

Maggie is ahead by $\boxed{2.62\ \text{m}}$.

P2.66 $a_1 = 0.100\ \text{m/s}^2$

$$x = 1\,000\ \text{m} = \frac{1}{2}a_1t_1^2 + v_1t_2 + \frac{1}{2}a_2t_2^2$$

$$1\,000 = \frac{1}{2}a_1t_1^2 + a_1t_1\left(-\frac{a_1t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2$$

$$t_2 = \frac{a_1t_1}{-a_2} = \frac{12.9}{0.500} \approx 26\ \text{s}$$

$a_2 = -0.500\ \text{m/s}^2$

$t = t_1 + t_2$ and $v_1 = a_1t_1 = -a_2t_2$

$$1\,000 = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$t_1 = \sqrt{\frac{20\,000}{1.20}} = \boxed{129\ \text{s}}$$

Total time $= t = \boxed{155\ \text{s}}$

P2.67 Let the ball fall 1.50 m. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$$

$$v_{xf} = -5.42\ \text{m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$
$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00 \times 10^{-2}\ \text{m}} = +1.47 \times 10^3\ \text{m/s}^2.$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

***P2.68** (a) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We assume the package starts from rest.

$$-145\ \text{m} = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

$$t = \sqrt{\frac{2(-145\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{5.44\ \text{s}}$$

(b) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)(5.18\ \text{s})^2 = -131\ \text{m}$

$$\text{distance fallen} = |x_f| = \boxed{131\ \text{m}}$$

(c) speed $= |v_{xf}| = |v_{xi} + a_x t| = \left|0 + (-9.8\ \text{m/s}^2)5.18\ \text{s}\right| = \boxed{50.8\ \text{m/s}}$

(d) The remaining distance is

$$145\ \text{m} - 131.5\ \text{m} = 13.5\ \text{m}.$$

During deceleration,

$$v_{xi} = -50.8\ \text{m/s},\ v_{xf} = 0,\ x_f - x_i = -13.5\ \text{m}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i):$$

$$0 = (-50.8\ \text{m/s})^2 + 2a_x(-13.5\ \text{m})$$

$$a_x = \frac{-2\,580\ \text{m}^2/\text{s}^2}{2(-13.5\ \text{m})} = +95.3\ \text{m/s}^2 = \boxed{95.3\ \text{m/s}^2\ \text{upward}}.$$

P2.69 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2$,

$4.90t^2 + 2.00t - 50.0 = 0$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t = \boxed{3.00\text{ s}} \text{ after the first stone is thrown.}$$

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00$ s

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$$v_{i2} = \boxed{15.3\text{ m/s}} \text{ downward}$$

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4\text{ m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8\text{ m/s}}$ downward

P2.70 (a) $d = \frac{1}{2}(9.80)t_1^2$ $\qquad d = 336t_2$

$t_1 + t_2 = 2.40$ $\qquad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\qquad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.076\,5$ s $\qquad$ so $\qquad d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

P2.71 (a) In walking a distance Δx, in a time Δt, the length of rope ℓ is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2+h^2}}}$$

(b) $a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{\ell}\frac{dx}{dt} + v_{\text{boy}}x\frac{d}{dt}\left(\frac{1}{\ell}\right)$

$a = v_{\text{boy}}\frac{v_{\text{boy}}}{\ell} - \frac{v_{\text{boy}}x}{\ell^2}\frac{d\ell}{dt}$, but $\frac{d\ell}{dt} = v = v_{\text{boy}}\frac{x}{\ell}$

$\therefore a = \frac{v_{\text{boy}}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{\text{boy}}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{\left(x^2+h^2\right)^{3/2}}}$

(c) $\frac{v_{\text{boy}}^2}{h}$, 0

(d) v_{boy}, 0

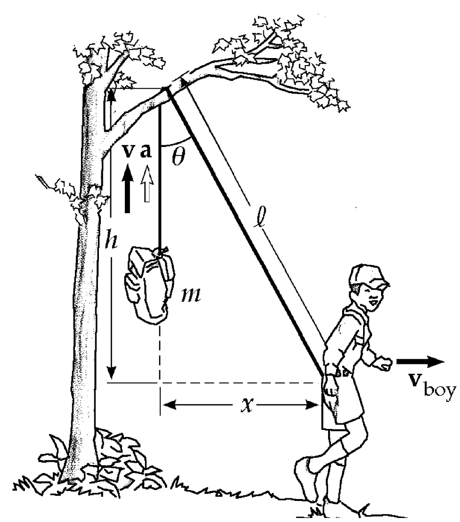


FIG. P2.71

P2.72 $h = 6.00\text{ m},\ v_{\text{boy}} = 2.00\text{ m/s}\quad v = \dfrac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\dfrac{x}{\ell} = \dfrac{v_{\text{boy}}x}{\left(x^2+h^2\right)^{1/2}}$.

However, $x = v_{\text{boy}}t : \therefore v = \dfrac{v_{\text{boy}}^2 t}{\left(v_{\text{boy}}^2t^2+h^2\right)^{1/2}} = \dfrac{4t}{\left(4t^2+36\right)^{1/2}}$.

(a)

t(s)	v(m/s)
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71

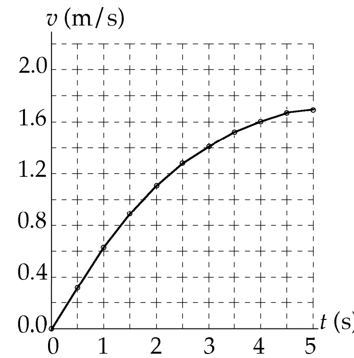


FIG. P2.72(a)

(b) From problem 2.71 above, $a = \dfrac{h^2v_{\text{boy}}^2}{\left(x^2+h^2\right)^{3/2}} = \dfrac{h^2v_{\text{boy}}^2}{\left(v_{\text{boy}}^2t^2+h^2\right)^{3/2}} = \dfrac{144}{\left(4t^2+36\right)^{3/2}}$.

t(s)	$a\left(\text{m/s}^2\right)$
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3.	0.24
3.5	0.18
4.	0.14
4.5	0.11
5	0.09

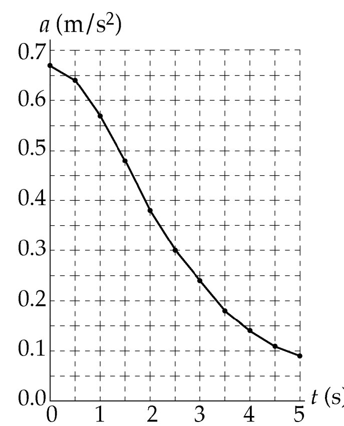


FIG. P2.72(b)

P2.73 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}\left(3.50\text{ m/s}^2\right)\left(t_k+1.00\right)^2 = \frac{1}{2}\left(4.90\text{ m/s}^2\right)\left(t_k\right)^2 = x_k$$

$$t_k + 1.00 = 1.183t_k$$

$$t_k = \boxed{5.46\text{ s}}.$$

(b) $x_k = \dfrac{1}{2}\left(4.90\text{ m/s}^2\right)(5.46\text{ s})^2 = \boxed{73.0\text{ m}}$

(c) $v_k = \left(4.90\text{ m/s}^2\right)(5.46\text{ s}) = \boxed{26.7\text{ m/s}}$

$v_s = \left(3.50\text{ m/s}^2\right)(6.46\text{ s}) = \boxed{22.6\text{ m/s}}$

P2.74

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.74

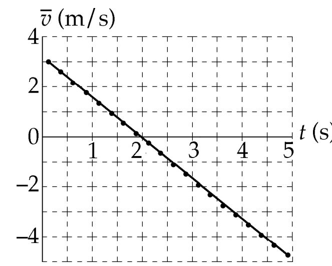


FIG. P2.74

acceleration = slope of line is constant.

$$\bar{a} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$

P2.75 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.
From the equation resulting from differentiation, we have

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v).$$

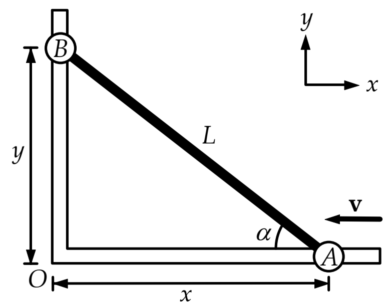


FIG. P2.75

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$. When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$.

ANSWERS TO EVEN PROBLEMS

P2.2 (a) 2×10^{-7} m/s; 1×10^{-6} m/s; (b) 5×10^{8} yr

P2.4 (a) 50.0 m/s; (b) 41.0 m/s

P2.6 (a) 27.0 m; (b) $27.0\text{ m} + (18.0\text{ m/s})\Delta t + (3.00\text{ m/s}^2)(\Delta t)^2$; (c) 18.0 m/s

P2.8 (a), (b), (c) see the solution; 4.6 m/s^2; (d) 0

P2.10 5.00 m

P2.12 (a) 20.0 m/s; 5.00 m/s; (b) 262 m

P2.14 (a) see the solution; (b) 1.60 m/s^2; 0.800 m/s^2

P2.16 (a) 13.0 m/s; (b) 10.0 m/s; 16.0 m/s; (c) 6.00 m/s^2; (d) 6.00 m/s^2

P2.18 see the solution

P2.20 (a) 6.61 m/s; (b) –0.448 m/s^2

P2.22 (a) $-21.8\text{ mi/h}\cdot\text{s} = -9.75\text{ m/s}^2$; (b) $-22.2\text{ mi/h}\cdot\text{s} = -9.94\text{ m/s}^2$; (c) $-22.8\text{ mi/h}\cdot\text{s} = -10.2\text{ m/s}^2$

P2.24 (a) 1.88 km; (b) 1.46 km; (c) see the solution; (d) (i) $x_1 = (1.67\text{ m/s}^2)t^2$; (ii) $x_2 = (50\text{ m/s})t - 375\text{ m}$; (iii) $x_3 = (250\text{ m/s})t - (2.5\text{ m/s}^2)t^2 - 4\,375\text{ m}$; (e) 37.5 m/s

P2.26 958 m

P2.28 (a) $x_f = (30.0t - t^2)$ m; $v_f = (30.0 - 2t)$ m/s; (b) 225 m

P2.30 $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; 3.10 m/s

P2.32 (a) 35.0 s; (b) 15.7 m/s

P2.34 (a) $1.12 \times 10^{11}\text{ m/s}^2$; (b) 4.67×10^{-5} s

P2.36 (a) False unless the acceleration is zero; see the solution; (b) True

P2.38 Yes; 212 m; 11.4 s

P2.40 (a) –4.90 m; –19.6 m; –44.1 m; (b) –9.80 m/s; –19.6 m/s; –29.4 m/s

P2.42 1.79 s

P2.44 No; see the solution

P2.46 The second ball is thrown at speed $v_i = \sqrt{gh}$

P2.48 (a) 510 m; (b) 20.4 s

P2.50 (a) $96.0\ \text{ft/s}$;
(b) $a = 3.07 \times 10^3\ \text{ft/s}^2$ upward;
(c) $\Delta t = 3.13 \times 10^{-2}\ \text{s}$

P2.52 38.2 m

P2.54 (a) and (b) see the solution; (c) $-4\ \text{m/s}^2$;
(d) 34 m; (e) 28 m

P2.56 0.222 s

P2.58 (a) see the solution; (b) 6.23 s

P2.60 $1.60\ \text{m/s}^2$

P2.62 (a) 41.0 s; (b) 1.73 km; (c) $-184\ \text{m/s}$

P2.64 $v_{xi}t + \frac{1}{2}a_x t^2$; displacements agree

P2.66 155 s; 129 s

P2.68 (a) 5.44 s; (b) 131 m; (c) $50.8\ \text{m/s}$;
(d) $95.3\ \text{m/s}^2$ upward

P2.70 (a) 26.4 m; (b) 6.82%

P2.72 see the solution

P2.74 see the solution; $a_x = -1.63\ \text{m/s}^2$

3

Vectors

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q3.1 No. The sum of two vectors can only be zero if they are in opposite directions and have the same magnitude. If you walk 10 meters north and then 6 meters south, you won't end up where you started.

Q3.2 No, the magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled. Two vectors in any other orientation will give a displacement less than the distance traveled. If you first walk 3 meters east, and then 4 meters south, you will have walked a total distance of 7 meters, but you will only be 5 meters from your starting point.

Q3.3 The largest possible magnitude of $\mathbf{R}=\mathbf{A}+\mathbf{B}$ is 7 units, found when **A** and **B** point in the same direction. The smallest magnitude of $\mathbf{R}=\mathbf{A}+\mathbf{B}$ is 3 units, found when **A** and **B** have opposite directions.

Q3.4 Only force and velocity are vectors. None of the other quantities requires a direction to be described.

Q3.5 If the direction-angle of **A** is between 180 degrees and 270 degrees, its components are both negative. If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs.

Q3.6 The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

Q3.7 85 miles. The magnitude of the displacement is the distance from the starting point, the 260-mile mark, to the ending point, the 175-mile mark.

Q3.8 Vectors **A** and **B** are perpendicular to each other.

Q3.9 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

Q3.10 Any vector that points along a line at 45° to the x and y axes has components equal in magnitude.

Q3.11 $A_x = B_x$ and $A_y = B_y$.

Q3.12 Addition of a vector to a scalar is not defined. Think of apples and oranges.

Q3.13 One difficulty arises in determining the individual components. The relationships between a vector and its components such as $A_x = A\cos\theta$, are based on right-triangle trigonometry. Another problem would be in determining the magnitude or the direction of a vector from its components. Again, $A = \sqrt{A_x^2 + A_y^2}$ only holds true if the two component vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, are perpendicular.

Q3.14 If the direction of a vector is specified by giving the angle of the vector measured clockwise from the positive y-axis, then the x-component of the vector is equal to the sine of the angle multiplied by the magnitude of the vector.

SOLUTIONS TO PROBLEMS

Section 3.1 **Coordinate Systems**

P3.1
$$x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$
$$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore
$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}.$$

(b) $$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$$

P3.3 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°;\ \mathbf{r} = \boxed{2.24\text{ m},\ 26.6°}$$

P3.4 (a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(2.00-[-3.00])^2+(-4.00-3.00)^2}$

$d=\sqrt{25.0+49.0}=\boxed{8.60\text{ m}}$

(b) $r_1=\sqrt{(2.00)^2+(-4.00)^2}=\sqrt{20.0}=\boxed{4.47\text{ m}}$

$\theta_1=\tan^{-1}\left(-\dfrac{4.00}{2.00}\right)=\boxed{-63.4°}$

$r_2=\sqrt{(-3.00)^2+(3.00)^2}=\sqrt{18.0}=\boxed{4.24\text{ m}}$

$\theta_2=\boxed{135°}$ measured from the $+x$ axis.

P3.5 We have $2.00=r\cos 30.0°$

$$r=\frac{2.00}{\cos 30.0°}=\boxed{2.31}$$

and $y=r\sin 30.0°=2.31\sin 30.0°=\boxed{1.15}$.

P3.6 We have $r=\sqrt{x^2+y^2}$ and $\theta=\tan^{-1}\left(\dfrac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2+y^2}=\sqrt{x^2+y^2}=\boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right)=\boxed{180°-\theta}.$$

(b) $\sqrt{(-2x)^2+(-2y)^2}=\boxed{2r}$. This point is in the third quadrant if $(x,\ y)$ is in the first quadrant or in the fourth quadrant if $(x,\ y)$ is in the second quadrant. It is at an angle of $\boxed{180°+\theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2}=\boxed{3r}$. This point is in the fourth quadrant if $(x,\ y)$ is in the first quadrant or in the third quadrant if $(x,\ y)$ is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

Section 3.2 **Vector and Scalar Quantities**

Section 3.3 **Some Properties of Vectors**

P3.7 $\tan 35.0° = \dfrac{x}{100\ \text{m}}$

$$x = (100\ \text{m})\tan 35.0° = \boxed{70.0\ \text{m}}$$

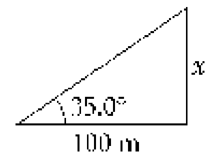


FIG. P3.7

P3.8 $R = \boxed{14\ \text{km}}$

$\theta = \boxed{65°\ \text{N of E}}$

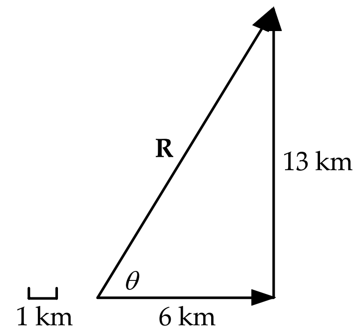


FIG. P3.8

P3.9 $-\mathbf{R} = \boxed{310\ \text{km at } 57°\ \text{S of W}}$

(Scale: 1 unit = 20 km)

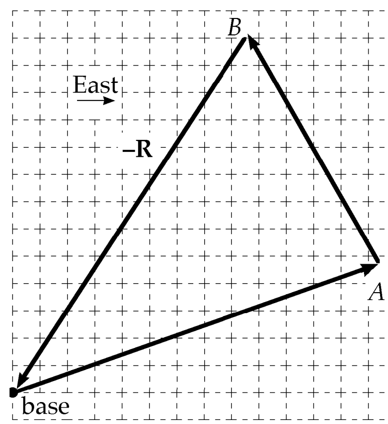


FIG. P3.9

P3.10 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector $\mathbf{A}+\mathbf{B}$ has a magnitude of $\boxed{6.1 \text{ at } 112°}$ from the x-axis.

(b) The vector difference $\mathbf{A}-\mathbf{B}$ is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector $\mathbf{A}-\mathbf{B}$ has magnitude $\boxed{14.8}$ units at an $\boxed{\text{angle of } 22°}$ from the + x-axis.

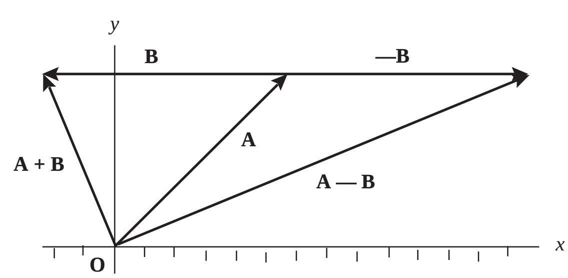


FIG. P3.10

P3.11 (a) $|\mathbf{d}| = |-10.0\hat{\mathbf{i}}| = \boxed{10.0 \text{ m}}$ since the displacement is in a straight line from point A to point B.

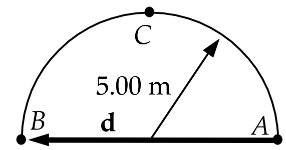


FIG. P3.11

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (*ACB*).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$

(c) If the circle is complete, **d** begins and ends at point **A**. Hence, $|\mathbf{d}| = \boxed{0}$.

P3.12 Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.

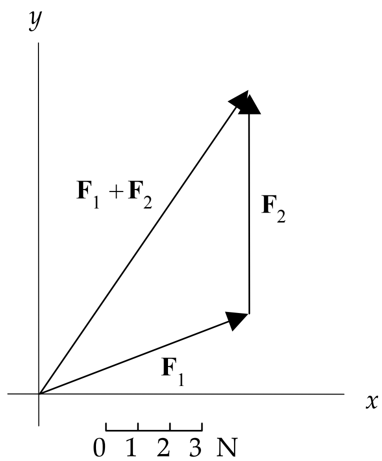


FIG. P3.12

P3.13 (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as $\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})$

$\boxed{\sim 10^3 \text{ m upward}}$.

P3.14 Your sketch should be drawn to scale, and should look somewhat like that pictured to the right. The angle from the westward direction, θ, can be measured to be $\boxed{4° \text{ N of W}}$, and the distance R from the sketch can be converted according to the scale to be $\boxed{7.9 \text{ m}}$.

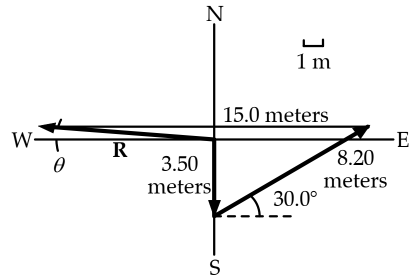


FIG. P3.14

P3.15 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\mathbf{A} + \mathbf{B}$ = 5.2 m at 60°

(b) $\mathbf{A} - \mathbf{B}$ = 3.0 m at 330°

(c) $\mathbf{B} - \mathbf{A}$ = 3.0 m at 150°

(d) $\mathbf{A} - 2\mathbf{B}$ = 5.2 m at 300°.

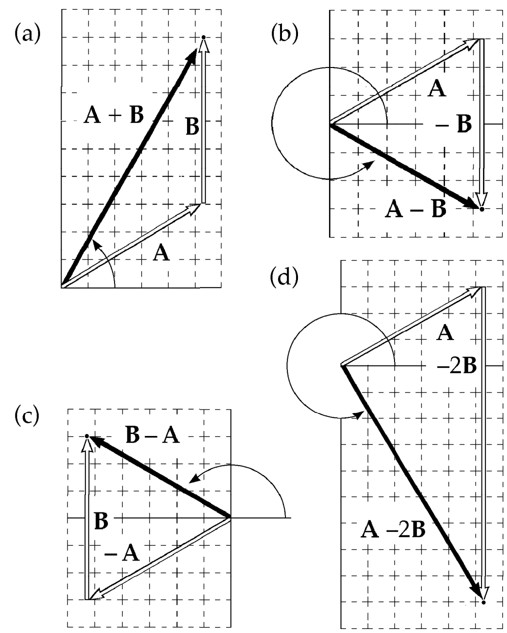


FIG. P3.15

***P3.16** The three diagrams shown below represent the graphical solutions for the three vector sums: $\mathbf{R}_1 = \mathbf{A}+\mathbf{B}+\mathbf{C}$, $\mathbf{R}_2 = \mathbf{B}+\mathbf{C}+\mathbf{A}$, and $\mathbf{R}_3 = \mathbf{C}+\mathbf{B}+\mathbf{A}$. You should observe that $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

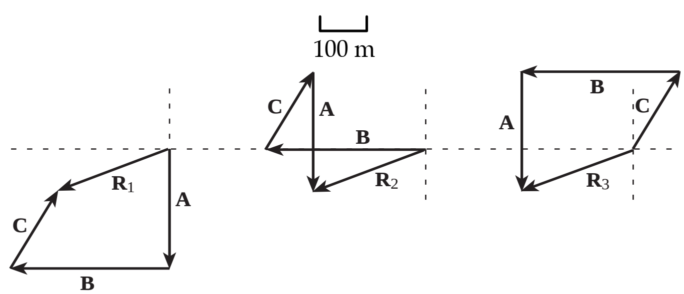
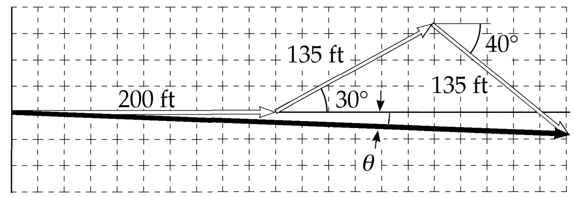


FIG. P3.16

P3.17 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be



(Scale: 1 unit = 20 ft)

FIG. P3.17

$$\boxed{d = 420 \text{ ft and } \theta = -3°}$$

Section 3.4 Components of a Vector and Unit Vectors

P3.18 Coordinates of the super-hero are:

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$

$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

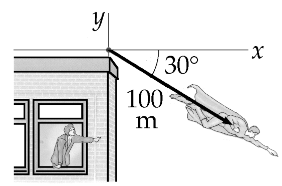


FIG. P3.18

P3.19 $A_x = -25.0$

$A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$$

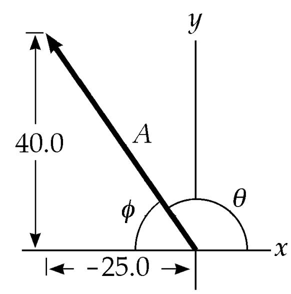


FIG. P3.19

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

P3.20 The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

P3.21 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x,\ y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x,\ y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x,\ y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

P3.22 $x = d\cos\theta = (50.0\text{ m})\cos(120) = -25.0\text{ m}$
$y = d\sin\theta = (50.0\text{ m})\sin(120) = 43.3\text{ m}$
$\mathbf{d} = \boxed{(-25.0\text{ m})\hat{\mathbf{i}} + (43.3\text{ m})\hat{\mathbf{j}}}$

***P3.23** (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

***P3.24** Let $\hat{\mathbf{i}} = \text{east}$ and $\hat{\mathbf{j}} = \text{north}$. The unicyclist's displacement is, in meters

$$280\hat{\mathbf{j}} + 220\hat{\mathbf{i}} + 360\hat{\mathbf{j}} - 300\hat{\mathbf{i}} - 120\hat{\mathbf{j}} + 60\hat{\mathbf{i}} - 40\hat{\mathbf{j}} - 90\hat{\mathbf{i}} + 70\hat{\mathbf{j}}.$$

$$\begin{aligned}\mathbf{R} &= -110\hat{\mathbf{i}} + 550\hat{\mathbf{j}}\\ &= \sqrt{(110\text{ m})^2 + (550\text{ m})^2}\text{ at }\tan^{-1}\frac{110\text{ m}}{550\text{ m}}\text{ west of north}\\ &= 561\text{ m at }11.3°\text{ west of north}.\end{aligned}$$

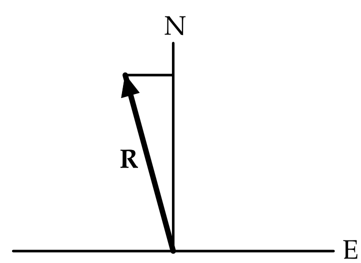


FIG. P3.24

The crow's velocity is

$$\begin{aligned}\mathbf{v} &= \frac{\Delta\mathbf{x}}{\Delta t} = \frac{561\text{ m at }11.3°\text{ W of N}}{40\text{ s}}\\ &= \boxed{14.0\text{ m/s at }11.3°\text{ west of north}}.\end{aligned}$$

P3.25 $+x$ East, $+y$ North

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° - 150 = -12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = -0.0349$$
$$\theta = -2.00°$$
$$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

P3.26 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° - 560\sin 21.0° = 527 \text{ miles}.$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}.$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

$$\text{Then } \tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11 \text{ and } \theta = 48.0°.$$

Thus, Chicago is $\boxed{788 \text{ miles at } 48.0° \text{ northeast of Dallas}}$.

P3.27 (a) See figure to the right.

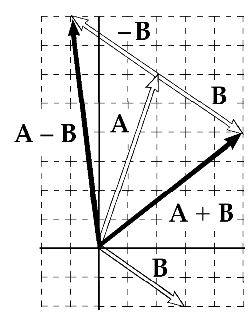


FIG. P3.27

(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$$\mathbf{C} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$

$$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$$

$$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$

$$\mathbf{D} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$

P3.28 $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{6.00}{4.00}\right) = \boxed{56.3°}$$

P3.29 We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

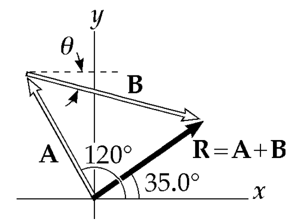


FIG. P3.29

Therefore,

$$\mathbf{B} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$
$$|\mathbf{B}| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

P3.30 $\mathbf{A} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\mathbf{B} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$:

$$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\mathbf{C} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or

$$C_x = \boxed{7.30 \text{ cm}};\ C_y = \boxed{-7.20 \text{ cm}}$$

P3.31 (a) $(\mathbf{A} + \mathbf{B}) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) + \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $(\mathbf{A} - \mathbf{B}) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

P3.32 (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47 \text{ m at } \theta = 63.4°}$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49 \text{ m at } \theta = 135°}$

P3.33 $d_1 = \left(-3.50\hat{\mathbf{j}}\right)$ m

$$d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right) \text{ m}$$

$$d_3 = \left(-15.0\hat{\mathbf{i}}\right) \text{ m}$$

$$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right) \text{ m}}$$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48 \text{ m}}.$$

The direction is $\theta = \arctan\left(\dfrac{2.30}{-9.20}\right) = \boxed{166°}$.

P3.34 Refer to the sketch

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$
$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$
$$|\mathbf{R}| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7 \text{ yards}}$$

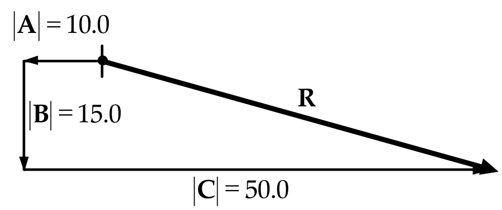


FIG. P3.34

P3.35 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$$\mathbf{F} = 120\cos(60.0°)\hat{\mathbf{i}} + 120\sin(60.0°)\hat{\mathbf{j}} - 80.0\cos(75.0°)\hat{\mathbf{i}} + 80.0\sin(75.0°)\hat{\mathbf{j}}$$

$$\mathbf{F} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

$$|\mathbf{F}| = \sqrt{39.3^2 + 181^2} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$

P3.36

East	West
x	y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2} = \boxed{4.64 \text{ m at } 78.6° \text{ N of E}}$$

P3.37 $\mathbf{A} = 3.00 \text{ m},\ \theta_A = 30.0°$ $\mathbf{B} = 3.00 \text{ m},\ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m}$ $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) \text{ m}$$

$B_x = 0,\ B_y = 3.00 \text{ m}$ so $\mathbf{B} = 3.00\hat{\mathbf{j}} \text{ m}$

$$\mathbf{A} + \mathbf{B} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right) \text{ m}}$$

P3.38 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right) \text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right) \text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right) \text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2} \text{ cm} = \boxed{10.4 \text{ cm}}$.

(b) Its angle with the y-axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.39

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$$

$$|\mathbf{B}| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$$

P3.40 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 268$ m/s. The position vector as a function of time is

$$\mathbf{P} = (268 \text{ m/s})t\hat{\mathbf{i}} + \left(7.60 \times 10^3 \text{ m}\right)\hat{\mathbf{j}}.$$

At $t = 45.0$ s, $\mathbf{P} = \left[1.21 \times 10^4\hat{\mathbf{i}} + 7.60 \times 10^3\hat{\mathbf{j}}\right]$ m. The magnitude is

$$P = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2} \text{ m} = \boxed{1.43 \times 10^4 \text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2° \text{ above the horizontal}}.$$

P3.41 (a) $\mathbf{A} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\mathbf{B} = \dfrac{\mathbf{A}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.42

$$\mathbf{R} = 75.0\cos 240°\,\hat{\mathbf{i}} + 75.0\sin 240°\,\hat{\mathbf{j}} + 125\cos 135°\,\hat{\mathbf{i}} + 125\sin 135°\,\hat{\mathbf{j}} + 100\cos 160°\,\hat{\mathbf{i}} + 100\sin 160°\,\hat{\mathbf{j}}$$

$$\mathbf{R} = -37.5\hat{\mathbf{i}} - 65.0\hat{\mathbf{j}} - 88.4\hat{\mathbf{i}} + 88.4\hat{\mathbf{j}} - 94.0\hat{\mathbf{i}} + 34.2\hat{\mathbf{j}}$$

$$\mathbf{R} = \boxed{-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}}}$$

$$\mathbf{R} = \sqrt{(-220)^2 + 57.6^2} \text{ at } \arctan\left(\frac{57.6}{220}\right) \text{ above the } -x\text{-axis}$$

$$\mathbf{R} = \boxed{227 \text{ paces at } 165°}$$

P3.43 (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$

$$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2}\text{ m} = \boxed{5.92\text{ m}}$$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{ m}}$

$$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2}\text{ m} = \boxed{19.0\text{ m}}$$

P3.44 The position vector from radar station to ship is

$$\mathbf{S} = \left(17.3\sin 136°\,\hat{\mathbf{i}} + 17.3\cos 136°\,\hat{\mathbf{j}}\right)\text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right)\text{ km}.$$

From station to plane, the position vector is

$$\mathbf{P} = \left(19.6\sin 153°\,\hat{\mathbf{i}} + 19.6\cos 153°\,\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km},$$

or

$$\mathbf{P} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)\text{ km}}.$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}.$$

P3.45 The hurricane's first displacement is $\left(\frac{41.0\text{ km}}{\text{h}}\right)(3.00\text{ h})$ at $60.0°$ N of W, and its second displacement is $\left(\frac{25.0\text{ km}}{\text{h}}\right)(1.50\text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00\text{ h})\left(-\hat{\mathbf{i}}\right)+\left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00\text{ h})\hat{\mathbf{j}}+\left(25.0\frac{\text{km}}{\text{h}}\right)(1.50\text{ h})\hat{\mathbf{j}}=61.5\text{ km}\left(-\hat{\mathbf{i}}\right)+144\text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5\text{ km})^2+(144\text{ km})^2}=\boxed{157\text{ km}}$.

P3.46 (a) $\mathbf{E}=(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{j}}$

$\mathbf{E}=\boxed{\left(15.1\hat{\mathbf{i}}+7.72\hat{\mathbf{j}}\right)\text{ cm}}$

(b) $\mathbf{F}=-(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{F}=\boxed{\left(-7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

(c) $\mathbf{G}=+(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{G}=\boxed{\left(+7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

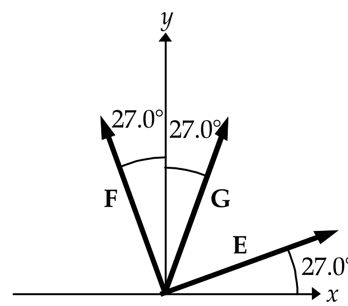


FIG. P3.46

P3.47 $A_x=-3.00$, $A_y=2.00$

(a) $\mathbf{A}=A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}=\boxed{-3.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}}$

(b) $|\mathbf{A}|=\sqrt{A_x^2+A_y^2}=\sqrt{(-3.00)^2+(2.00)^2}=\boxed{3.61}$

$\tan\theta=\frac{A_y}{A_x}=\frac{2.00}{(-3.00)}=-0.667$, $\tan^{-1}(-0.667)=-33.7°$

θ is in the 2^{nd} quadrant, so $\theta=180°+(-33.7°)=\boxed{146°}$.

(c) $\mathbf{R}_x=0$, $\mathbf{R}_y=-4.00$, $\mathbf{R}=\mathbf{A}+\mathbf{B}$ thus $\mathbf{B}=\mathbf{R}-\mathbf{A}$ and

$B_x=R_x-A_x=0-(-3.00)=3.00$, $B_y=R_y-A_y=-4.00-2.00=-6.00$.

Therefore, $\mathbf{B}=\boxed{3.00\hat{\mathbf{i}}-6.00\hat{\mathbf{j}}}$.

P3.48 Let $+x =$ East, $+y =$ North,

x	y
300	0
–175	303
0	150
125	453

(a) $\theta = \tan^{-1}\dfrac{y}{x} = \boxed{74.6^\circ \text{ N of E}}$

(b) $|\mathbf{R}| = \sqrt{x^2 + y^2} = \boxed{470 \text{ km}}$

P3.49 (a)

$$R_x = 40.0\cos 45.0^\circ + 30.0\cos 45.0^\circ = 49.5$$
$$R_y = 40.0\sin 45.0^\circ - 30.0\sin 45.0^\circ + 20.0 = 27.1$$
$$\mathbf{R} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$$

(b)

$$|\mathbf{R}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$$
$$\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7^\circ}$$

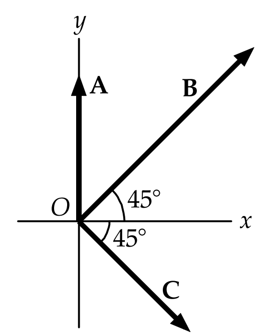


FIG. P3.49

P3.50 Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0\,.$$

Solving simultaneously,

$$\boxed{a = 5.00,\ b = 7.00}\,.$$

Therefore,

$$5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0\,.$$

Additional Problems

P3.51 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

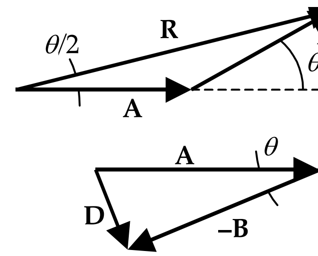


FIG. P3.51

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\frac{\theta}{2}\right) = 200A\sin\left(\frac{\theta}{2}\right)$. This gives $\tan\left(\frac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

P3.52 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint*: apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

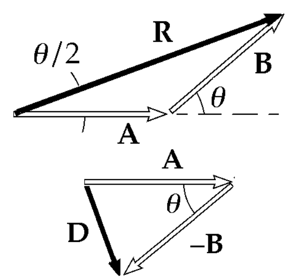


FIG. P3.52

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right)$ giving

$\boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}$.

P3.53 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)
$$\cos\theta_x = \frac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7° \text{ from } +x}$$
$$\cos\theta_y = \frac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5° \text{ from } +y}$$
$$\cos\theta_z = \frac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7° \text{ from } +z}$$

***P3.54** Take the x-axis along the tail section of the snake. The displacement from tail to head is

$$240 \text{ m}\hat{\mathbf{i}} + (420 - 240) \text{ m}\cos(180°–105°)\hat{\mathbf{i}} - 180 \text{ m}\sin 75°\,\hat{\mathbf{j}} = 287 \text{ m}\hat{\mathbf{i}} - 174 \text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2 + (174)^2}$ m $= 335$ m. From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m(h)(1 km)(3 600 s)}}{(12 \text{ km})(1\,000 \text{ m})(1 \text{ h})} = 101 \text{ s}$$
$$\text{Olaf: } \Delta t = \frac{420 \text{ m}\cdot\text{s}}{3.33 \text{ m}} = 126 \text{ s}.$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

***P3.55** The position vector from the ground under the controller of the first airplane is

$$\mathbf{r}_1 = (19.2 \text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2 \text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8 \text{ km})\hat{\mathbf{k}}$$
$$= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right) \text{ km}.$$

The second is at

$$\mathbf{r}_2 = (17.6 \text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6 \text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1 \text{ km})\hat{\mathbf{k}}$$
$$= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right) \text{ km}.$$

Now the displacement from the first plane to the second is

$$\mathbf{r}_2 - \mathbf{r}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right) \text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29 \text{ km}}.$$

***P3.56** Let A represent the distance from island 2 to island 3. The displacement is $\mathbf{A} = A$ at $159°$. Represent the displacement from 3 to 1 as $\mathbf{B} = B$ at $298°$. We have 4.76 km at $37°$ $+\mathbf{A}+\mathbf{B}=0$.

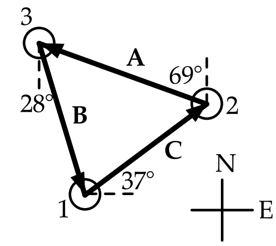


FIG. P3.56

For x-components

$$(4.76\text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$
$$3.80\text{ km} - 0.934A + 0.469B = 0$$
$$B = -8.10\text{ km} + 1.99A$$

For y-components,

$$(4.76\text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$
$$2.86\text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86\text{ km} + 0.358A - 0.883(-8.10\text{ km} + 1.99A) = 0$$
$$2.86\text{ km} + 0.358A + 7.15\text{ km} - 1.76A = 0$$
$$10.0\text{ km} = 1.40A$$
$$A = \boxed{7.17\text{ km}}$$

(b) $B = -8.10\text{ km} + 1.99(7.17\text{ km}) = \boxed{6.15\text{ km}}$

***P3.57** (a) We first express the corner's position vectors as sets of components

$$\mathbf{A} = (10\text{ m})\cos 50°\,\hat{\mathbf{i}} + (10\text{ m})\sin 50°\,\hat{\mathbf{j}} = 6.43\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}}$$
$$\mathbf{B} = (12\text{ m})\cos 30°\,\hat{\mathbf{i}} + (12\text{ m})\sin 30°\,\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 6.00\text{ m}\hat{\mathbf{j}}.$$

The horizontal width of the rectangle is

$$10.4\text{ m} - 6.43\text{ m} = 3.96\text{ m}.$$

Its vertical height is

$$7.66\text{ m} - 6.00\text{ m} = 1.66\text{ m}.$$

Its perimeter is

$$2(3.96 + 1.66)\text{ m} = \boxed{11.2\text{ m}}.$$

(b) The position vector of the distant corner is $B_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}} = \sqrt{10.4^2 + 7.66^2}\text{ m}$ at $\tan^{-1}\dfrac{7.66\text{ m}}{10.4\text{ m}} = \boxed{12.9\text{ m at }36.4°}$.

P3.58 Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\hat{\mathbf{i}} + 31.0\hat{\mathbf{j}} - 8.40\hat{\mathbf{i}} - 24.0\hat{\mathbf{j}} = \boxed{\left(3.60\hat{\mathbf{i}}\right) + \left(7.00\hat{\mathbf{j}}\right)\text{ N}}.$$

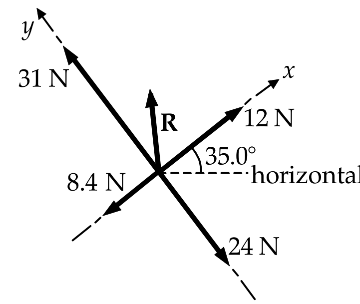


FIG. P3.58

The magnitude of the total force is

$$\sqrt{(3.60)^2 + (7.00)^2}\text{ N} = \boxed{7.87\text{ N}}$$

and the angle it makes with our $+x$-axis is given by $\tan\theta = \frac{(7.00)}{(3.60)}$, $\theta = 62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0° + 62.8° = \boxed{97.8°}$.

P3.59

$$\mathbf{d}_1 = 100\hat{\mathbf{i}}$$
$$\mathbf{d}_2 = -300\hat{\mathbf{j}}$$
$$\mathbf{d}_3 = -150\cos(30.0°)\hat{\mathbf{i}} - 150\sin(30.0°)\hat{\mathbf{j}} = -130\hat{\mathbf{i}} - 75.0\hat{\mathbf{j}}$$
$$\mathbf{d}_4 = -200\cos(60.0°)\hat{\mathbf{i}} + 200\sin(60.0°)\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 173\hat{\mathbf{j}}$$
$$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{ m}}$$
$$|\mathbf{R}| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240\text{ m}}$$
$$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$$
$$\theta = 180 + \phi = \boxed{237°}$$

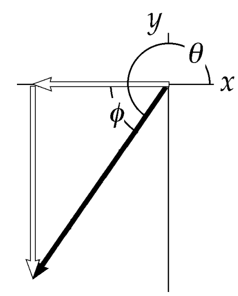


FIG. P3.59

P3.60 $$\frac{d\mathbf{r}}{dt} = \frac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{j}}\right)}{dt} = 0 + 0 - 2\hat{\mathbf{j}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{j}}}$$

The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\mathbf{r}}{dt}\text{ represents }\boxed{\text{its velocity vector}}.$$

P3.61

$$\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$$
$$\mathbf{v} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)\text{ mi/h}$$
$$|\mathbf{v}| = \boxed{390\text{ mi/h at }7.37°\text{ N of E}}$$

P3.62 (a) You start at point A: $\mathbf{r}_1 = \mathbf{r}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ to move to $\mathbf{r}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$.

Now the displacement from your current position to C is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to D is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}\left(\mathbf{r}_D - \mathbf{r}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - \mathbf{r}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0\text{ m, }16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}\left(\mathbf{r}_B - \mathbf{r}_A\right) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \dfrac{\mathbf{r}_C - \frac{(\mathbf{r}_A + \mathbf{r}_B)}{2}}{3} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3} + \dfrac{\mathbf{r}_D - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}}{4} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$

and last to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4} + \dfrac{\mathbf{r}_E - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}}{5} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

***P3.63** (a) Let T represent the force exerted by each child. The x-component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0.$$

The y-component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \mathbf{F} = 0.$$

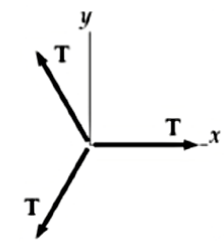


FIG. P3.63

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the total must turn clockwise by that angle, $\frac{360°}{N}$. Since each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

P3.64 (a) From the picture, $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$.

(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{|\mathbf{R}_1|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}.$$

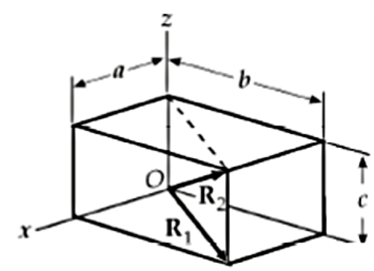


FIG. P3.64

P3.65 Since

$$\mathbf{A}+\mathbf{B}=6.00\hat{\mathbf{j}},$$

we have

$$\left(A_x+B_x\right)\hat{\mathbf{i}}+\left(A_y+B_y\right)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

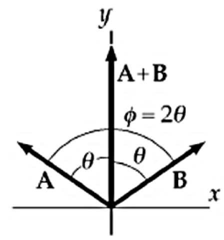


FIG. P3.65

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Eq. [2] gives

$$A_y=B_y=3.00.$$

Defining θ as the angle between either **A** or **B** and the y axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between **A** and **B** is then $\boxed{\phi=2\theta=106°}$.

***P3.66** Let θ represent the angle the x-axis makes with the horizontal. Since angles are equal if their sides are perpendicular right side to right side and left side to left side, θ is also the angle between the weight and our y axis. The x-components of the forces must add to zero:

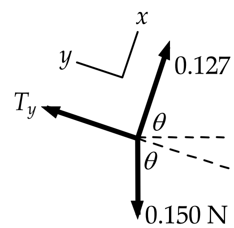


FIG. P3.66

$$-0.150 \text{ N} \sin\theta + 0.127 \text{ N} = 0 .$$

(b) $\theta = \boxed{57.9°}$

(a) The y-components for the forces must add to zero:

$$+T_y - (0.150 \text{ N})\cos 57.9° = 0 , \; T_y = \boxed{0.079\,8 \text{ N}} .$$

(c) The angle between the y axis and the horizontal is $90.0° - 57.9° = \boxed{32.1°}$.

P3.67 The displacement of point P is invariant under rotation of the coordinates. Therefore, $r = r'$ and $r^2 = (r')^2$ or, $x^2 + y^2 = (x')^2 + (y')^2$. Also, from the figure, $\beta = \theta - \alpha$

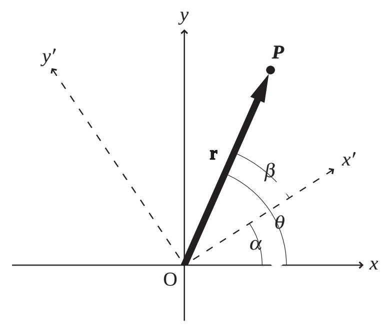


FIG. P3.67

$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right) = \tan^{-1}\left(\frac{y}{x}\right) - \alpha$$

$$\frac{y'}{x'} = \frac{\left(\frac{y}{x}\right) - \tan\alpha}{1 + \left(\frac{y}{x}\right)\tan\alpha}$$

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$$x' = x\cos\alpha + y\sin\alpha, \; y' = -x\sin\alpha + y\cos\alpha .$$

ANSWERS TO EVEN PROBLEMS

P3.2 (a) (2.17 m, 1.25 m); (–1.90 m, 3.29 m); (b) 4.55 m

P3.4 (a) 8.60 m; (b) 4.47 m at –63.4°; 4.24 m at 135°

P3.6 (a) r at $180° - \theta$; (b) $2r$ at $180° + \theta$; (c) $3r$ at $-\theta$

P3.8 14 km at 65° north of east

P3.10 (a) 6.1 at 112°; (b) 14.8 at 22°

P3.12 9.5 N at 57°

P3.14 7.9 m at 4° north of west

P3.16 see the solution

P3.18 86.6 m and –50.0 m

P3.20 1.31 km north; 2.81 km east

P3.22 $-25.0 \text{ m}\,\hat{\mathbf{i}} + 43.3 \text{ m}\,\hat{\mathbf{j}}$

P3.24 14.0 m/s at 11.3° west of north

P3.26 788 mi at 48.0° north of east

P3.28 7.21 m at 56.3°

P3.30 $\mathbf{C} = 7.30 \text{ cm}\,\hat{\mathbf{i}} - 7.20 \text{ cm}\,\hat{\mathbf{j}}$

P3.32 (a) 4.47 m at 63.4°; (b) 8.49 m at 135°

P3.34 42.7 yards

P3.36 4.64 m at 78.6°

P3.38 (a) 10.4 cm; (b) 35.5°

P3.40 1.43×10^4 m at 32.2° above the horizontal

P3.42 $-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}} = 227$ paces at 165°

P3.44 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$ km; (b) 6.31 km

P3.46 (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm;
(b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm;
(c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm

P3.48 (a) 74.6° north of east; (b) 470 km

P3.50 $a = 5.00$, $b = 7.00$

P3.52 $2\tan^{-1}\left(\frac{1}{n}\right)$

P3.54 25.4 s

P3.56 (a) 7.17 km; (b) 6.15 km

P3.58 7.87 N at 97.8° counterclockwise from a horizontal line to the right

P3.60 $(-2.00 \text{ m/s})\hat{\mathbf{j}}$; its velocity vector

P3.62 (a) (10.0 m, 16.0 m); (b) see the solution

P3.64 (a) $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$;
(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $|\mathbf{R}_2| = \sqrt{a^2 + b^2 + c^2}$

P3.66 (a) 0.079 8N; (b) 57.9°; (c) 32.1°

4

Motion in Two Dimensions

CHAPTER OUTLINE

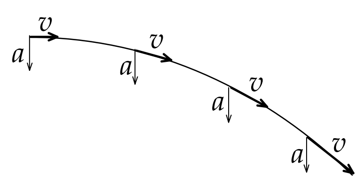
ANSWERS TO QUESTIONS

Q4.1 Yes. An object moving in uniform circular motion moves at a constant speed, but changes its direction of motion. An object cannot accelerate if its velocity is constant.

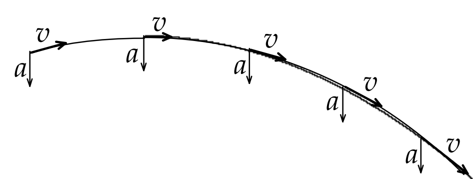
Q4.2 No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}.$$

Q4.3 (a)



(b)



Q4.4 (a) $10\hat{\mathbf{i}}$ m/s (b) $-9.80\hat{\mathbf{j}}$ m/s^2

Q4.5 The easiest way to approach this problem is to determine acceleration first, velocity second and finally position.

Vertical: In free flight, $a_y = -g$. At the top of a projectile's trajectory, $v_y = 0$. Using this, the maximum height can be found using $v_{fy}^2 = v_{iy}^2 + 2a_y\left(y_f - y_i\right)$.

Horizontal: $a_x = 0$, so v_x is always the same. To find the horizontal position at maximum height, one needs the flight time, *t*. Using the vertical information found previously, the flight time can be found using $v_{fy} = v_{iy} + a_y t$. The horizontal position is $x_f = v_{ix} t$.

If air resistance is taken into account, then the acceleration in both the *x* and *y*-directions would have an additional term due to the drag.

Q4.6 A parabola.

Q4.7 The balls will be closest together as the second ball is thrown. Yes, the first ball will always be moving faster, since its flight time is larger, and thus the vertical component of the velocity is larger. The time interval will be one second. No, since the vertical component of the motion determines the flight time.

Q4.8 The ball will have the greater speed. Both the rock and the ball will have the same vertical component of the velocity, but the ball will have the additional horizontal component.

Q4.9 (a) yes (b) no (c) no (d) yes (e) no

Q4.10 Straight up. Throwing the ball any other direction than straight up will give a nonzero speed at the top of the trajectory.

Q4.11 No. The projectile with the larger vertical component of the initial velocity will be in the air longer.

Q4.12 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q4.13 (a) no (b) yes (c) yes (d) no

Q4.14 60°. The projection angle appears in the expression for horizontal range in the function $\sin 2\theta$. This function is the same for 30° and 60°.

Q4.15 The optimal angle would be less than 45°. The longer the projectile is in the air, the more that air resistance will change the components of the velocity. Since the vertical component of the motion determines the flight time, an angle less than 45° would increase range.

Q4.16 The projectile on the moon would have both the larger range and the greater altitude. *Apollo* astronauts performed the experiment with golf balls.

Q4.17 Gravity only changes the vertical component of motion. Since both the coin and the ball are falling from the same height with the same vertical component of the initial velocity, they must hit the floor at the same time.

Q4.18 (a) no (b) yes

In the second case, the particle is continuously changing the direction of its velocity vector.

Q4.19 The racing car rounds the turn at a constant *speed* of 90 miles per hour.

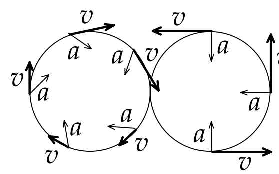
Q4.20 The acceleration cannot be zero because the pendulum does not remain at rest at the end of the arc.

Q4.21 (a) The velocity is not constant because the object is constantly changing the direction of its motion.

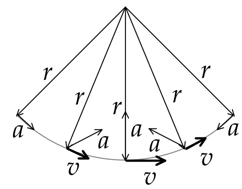
(b) The acceleration is not constant because the acceleration always points towards the center of the circle. The magnitude of the acceleration is constant, but not the direction.

Q4.22 (a) straight ahead (b) in a circle or straight ahead

Q4.23



Q4.24



Q4.25 The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are in different directions at different points in the xy plane. At a location along the x-axis, for example, $\hat{\mathbf{r}} = \hat{\mathbf{i}}$ and $\hat{\boldsymbol{\theta}} = \hat{\mathbf{j}}$, but at a point on the y-axis, $\hat{\mathbf{r}} = \hat{\mathbf{j}}$ and $\hat{\boldsymbol{\theta}} = -\hat{\mathbf{i}}$. The unit vector $\hat{\mathbf{i}}$ is equal everywhere, and $\hat{\mathbf{j}}$ is also uniform.

Q4.26 The wrench will hit at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

Q4.27 (a) The ball would move straight up and down as observed by the passenger. The ball would move in a parabolic trajectory as seen by the ground observer.

(b) Both the passenger and the ground observer would see the ball move in a parabolic trajectory, although the two observed paths would not be the same.

Q4.28 (a) g downward (b) g downward

The horizontal component of the motion does not affect the vertical acceleration.

SOLUTIONS TO PROBLEMS

Section 4.1 **The Position, Velocity, and Acceleration Vectors**

P4.1

$x(m)$	$y(m)$
0	–3 600
–3 000	0
–1 270	1 270
–4 270 m	–2 330 m

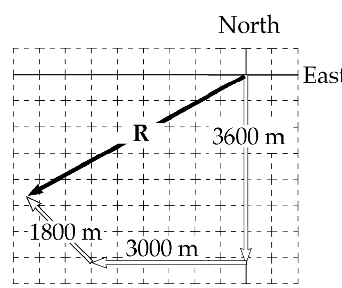


FIG. P4.1

(a) Net displacement $= \sqrt{x^2 + y^2}$

$$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$$

P4.2 (a) $\mathbf{r} = \boxed{18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\mathbf{v} = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} + \left[4.00 \text{ m/s} - \left(9.80 \text{ m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\mathbf{a} = \boxed{\left(-9.80 \text{ m/s}^2\right)\hat{\mathbf{j}}}$

(d) $\mathbf{r}(3.00 \text{ s}) = \boxed{(54.0 \text{ m})\hat{\mathbf{i}} - (32.1 \text{ m})\hat{\mathbf{j}}}$

(e) $\mathbf{v}(3.00 \text{ s}) = \boxed{(18.0 \text{ m/s})\hat{\mathbf{i}} - (25.4 \text{ m/s})\hat{\mathbf{j}}}$

(f) $\mathbf{a}(3.00 \text{ s}) = \boxed{\left(-9.80 \text{ m/s}^2\right)\hat{\mathbf{j}}}$

***P4.3** The sun projects onto the ground the x-component of her velocity:

$$5.00 \text{ m/s } \cos(-60.0°) = \boxed{2.50 \text{ m/s}}.$$

P4.4 (a) From $x = -5.00 \sin \omega t$, the x-component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00 \sin \omega t) = -5.00\omega \cos \omega t$$

and $a_x = \dfrac{dv_x}{dt} = +5.00\omega^2 \sin \omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00 \cos \omega t) = 0 + 5.00\omega \sin \omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega \sin \omega t) = 5.00\omega^2 \cos \omega t$.

At $t = 0$, $\mathbf{v} = -5.00\omega \cos 0\hat{\mathbf{i}} + 5.00\omega \sin 0\hat{\mathbf{j}} = \boxed{\left(5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right) \text{ m/s}}$

and $\mathbf{a} = 5.00\omega^2 \sin 0\hat{\mathbf{i}} + 5.00\omega^2 \cos 0\hat{\mathbf{j}} = \boxed{\left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right) \text{ m/s}^2}$.

(b) $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00 \text{ m})\hat{\mathbf{j}} + (5.00 \text{ m})\left(-\sin \omega t\,\hat{\mathbf{i}} - \cos \omega t\,\hat{\mathbf{j}}\right)}$

$\mathbf{v} = \boxed{(5.00 \text{ m})\omega\left[-\cos \omega t\,\hat{\mathbf{i}} + \sin \omega t\,\hat{\mathbf{j}}\right]}$

$\mathbf{a} = \boxed{(5.00 \text{ m})\omega^2\left[\sin \omega t\,\hat{\mathbf{i}} + \cos \omega t\,\hat{\mathbf{j}}\right]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at } (0,\ 4.00 \text{ m})}$.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.5 (a) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t} = \frac{\left(9.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right) - \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)}{3.00} = \boxed{\left(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

(b) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)t + \frac{1}{2}\left(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)t^2$

$\boxed{x = \left(3.00t + t^2\right)\ \text{m}}$ and $\boxed{y = \left(1.50t^2 - 2.00t\right)\ \text{m}}$

P4.6 (a) $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\,\hat{\mathbf{j}}\ \text{m/s}}$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}}\ \text{m/s}^2}$$

(b) $\boxed{\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\ \text{m};\ \mathbf{v} = -12.0\hat{\mathbf{j}}\ \text{m/s}}$

P4.7 $\mathbf{v}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)$ m/s and $\mathbf{v}(20.0) = \left(20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ m/s

(a) $a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0}\ \text{m/s}^2 = \boxed{0.800\ \text{m/s}^2}$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0}\ \text{m/s}^2 = \boxed{-0.300\ \text{m/s}^2}$$

(b) $\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6° = \boxed{339°\ \text{from}\ +x\ \text{axis}}$

(c) At $t = 25.0$ s

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360\ \text{m}}$$
$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7\ \text{m}}$$
$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24\ \text{m/s}$$
$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5\ \text{m/s}$$
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

P4.8 $\mathbf{a} = 3.00\hat{\mathbf{j}}\ \text{m/s}^2$; $\mathbf{v}_i = 5.00\hat{\mathbf{i}}\ \text{m/s}$; $\mathbf{r}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$

(a) $$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2}3.00t^2\hat{\mathbf{j}}\right]\ \text{m}}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \text{m/s}}$$

(b) $t = 2.00\ \text{s}$, $\mathbf{r}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m}$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

$$\mathbf{v}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$v_f = \left|\mathbf{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \text{m/s}}$$

***P4.9** (a) For the x-component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80 \times 10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8 \times 10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4 \times 10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80 \times 10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \frac{-1.80 \times 10^7\ \text{m/s} \pm \sqrt{\left(1.8 \times 10^7\ \text{m/s}\right)^2 - 4\left(4 \times 10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\,\text{m}\right)}}{2\left(4 \times 10^{14}\ \text{m/s}^2\right)}$$

$$= \frac{-1.8 \times 10^7 \pm 1.84 \times 10^7\ \text{m/s}}{8 \times 10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation

$$t = \frac{4.39 \times 10^5\ \ \text{m/s}}{8 \times 10^{14}\ \ \text{m/s}^2} = 5.49 \times 10^{-10}\ \ \text{s}.$$

Here

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}\left(1.6 \times 10^{15}\ \text{m/s}^2\right)\left(5.49 \times 10^{-10}\ \ \text{s}\right)^2 = 2.41 \times 10^{-4}\ \text{m}.$$

So, $\boxed{\mathbf{r}_f = \left(10.0\ \hat{\mathbf{i}} + 0.241\ \hat{\mathbf{j}}\right)\ \text{mm}}$.

(b) $$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 1.80 \times 10^7\ \text{m/s}\,\hat{\mathbf{i}} + \left(8 \times 10^{14}\ \text{m/s}^2\ \hat{\mathbf{i}} + 1.6 \times 10^{15}\ \text{m/s}^2\ \hat{\mathbf{j}}\right)\left(5.49 \times 10^{-10}\ \ \text{s}\right)$$

$$= \left(1.80 \times 10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(4.39 \times 10^5\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78 \times 10^5\ \text{m/s}\right)\hat{\mathbf{j}}$$

$$= \boxed{\left(1.84 \times 10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78 \times 10^5\ \text{m/s}\right)\hat{\mathbf{j}}}$$

(c) $$\left|\mathbf{v}_f\right| = \sqrt{\left(1.84 \times 10^7\ \text{m/s}\right)^2 + \left(8.78 \times 10^5\ \text{m/s}\right)^2} = \boxed{1.85 \times 10^7\ \text{m/s}}$$

(d) $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.78 \times 10^5}{1.84 \times 10^7}\right) = \boxed{2.73°}$$

Section 4.3 **Projectile Motion**

P4.10 $x = v_{xi}t = v_i \cos\theta_i t$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

P4.11 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\ \text{m})}{v_{xi}}$$

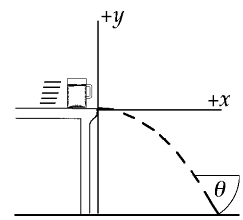


FIG. P4.11

In the same time it falls a distance of 0.860 m with acceleration downward of $9.80\ \text{m/s}^2$. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860\ \text{m} + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{v_{xi}}\right)^2.$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\ \text{m/s}^2\right)\left(1.96\ \text{m}^2\right)}{0.860\ \text{m}}} = \boxed{3.34\ \text{m/s}}.$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{3.34\ \text{m/s}}\right) = -4.11\ \text{m/s}.$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}.$$

P4.12 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}gt^2.$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}gt^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}.$$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}.$$

(b) Just before impact, the x-component of velocity is still

$$v_{xf} = v_{xi}$$

while the y-component is

$$v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}.$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{\frac{2h}{g}}}{d\sqrt{\frac{g}{2h}}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}.$$

P4.13 (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$$0 = 0 + (25.0\ \text{m/s})(\sin 70.0°)t_1 - \frac{1}{2}(9.80\ \text{m/s}^2)t_1^2$$

$$t_1 = \frac{2(25.0\ \text{m/s})\sin 70.0°}{9.80\ \text{m/s}^2} = 4.79\ \text{s}.$$

The distance to your target is

$$x_f - x_i = v_{xi}t_1 = (25.0\ \text{m/s})\cos 70.0°(4.79\ \text{s}) = 41.0\ \text{m}.$$

Now the second snowball we describe by

$$y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y t_2{}^2$$

$$0 = (25.0\ \text{m/s})\sin\theta_2 t_2 - (4.90\ \text{m/s}^2)t_2^2$$

$$t_2 = (5.10\ \text{s})\sin\theta_2$$
$$x_f - x_i = v_{xi}\,t_2$$

$$41.0\ \text{m} = (25.0\ \text{m/s})\cos\theta_2(5.10\ \text{s})\sin\theta_2 = (128\ \text{m})\sin\theta_2\cos\theta_2$$

$$0.321 = \sin\theta_2\cos\theta_2$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$ we can solve $0.321 = \frac{1}{2}\sin 2\theta_2$

$$2\theta_2 = \sin^{-1} 0.643 \text{ and } \theta_2 = \boxed{20.0°}.$$

(b) The second snowball is in the air for time $t_2 = (5.10\ \text{s})\sin\theta_2 = (5.10\ \text{s})\sin 20° = 1.75\ \text{s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79\ \text{s} - 1.75\ \text{s} = \boxed{3.05\ \text{s}}.$$

P4.14 From Equation 4.14 with $R = 15.0\ \text{m}$, $v_i = 3.00\ \text{m/s}$, $\theta_{\max} = 45.0°$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\ \text{m/s}^2}$$

P4.15 $h = \dfrac{v_i^2 \sin^2 \theta_i}{2g}$; $R = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$; $3h = R$,

so $\dfrac{3v_i^2 \sin^2 \theta_i}{2g} = \dfrac{v_i^2 (\sin 2\theta_i)}{g}$

or $\dfrac{2}{3} = \dfrac{\sin^2 \theta_i}{\sin 2\theta_i} = \dfrac{\tan \theta_i}{2}$

thus $\theta_i = \tan^{-1}\left(\dfrac{4}{3}\right) = \boxed{53.1°}$.

***P4.16** (a) To identify the maximum height we let *i* be the launch point and *f* be the highest point:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = v_i^2 \sin^2 \theta_i + 2(-g)(y_{\max} - 0)$$
$$y_{\max} = \frac{v_i^2 \sin^2 \theta_i}{2g}.$$

To identify the range we let *i* be the launch and *f* be the impact point; where *t* is not zero:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + v_i \sin \theta_i t + \frac{1}{2}(-g)t^2$$
$$t = \frac{2v_i \sin \theta_i}{g}$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$d = 0 + v_i \cos \theta_i \frac{2v_i \sin \theta_i}{g} + 0.$$

For this rock, $d = y_{\max}$

$$\frac{v_i^2 \sin^2 \theta_i}{2g} = \frac{2v_i^2 \sin \theta_i \cos \theta_i}{g}$$
$$\frac{\sin \theta_i}{\cos \theta_i} = \tan \theta_i = 4$$
$$\theta_i = \boxed{76.0°}$$

(b) Since g divides out, the answer is $\boxed{\text{the same}}$ on every planet.

(c) The maximum range is attained for $\theta_i = 45°$:

$$\frac{d_{\max}}{d} = \frac{v_i \cos 45° 2v_i \sin 45° g}{g v_i \cos 76° 2v_i \sin 76°} = 2.125 .$$

So $d_{\max} = \boxed{\dfrac{17d}{8}}$.

P4.17 (a) $x_f = v_{xi}\,t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}g\,t^2$$

$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

***P4.18** We interpret the problem to mean that the displacement from fish to bug is

$$2.00\text{ m at }30° = (2.00\text{ m})\cos 30°\,\hat{\mathbf{i}} + (2.00\text{ m})\sin 30°\,\hat{\mathbf{j}} = (1.73\text{ m})\hat{\mathbf{i}} + (1.00\text{ m})\hat{\mathbf{j}}.$$

If the water should drop 0.03 m during its flight, then the fish must aim at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement

$$(1.73\text{ m})\hat{\mathbf{i}} + (1.03\text{ m})\hat{\mathbf{j}} = 2.015\text{ m at }30.7°.$$

For the time of flight of a water drop we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$1.73\text{ m} = 0 + (v_i\cos 30.7°)t + 0$ so

$$t = \frac{1.73\text{ m}}{v_i\cos 30.7°}.$$

The vertical motion is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2.$$

The "drop on its path" is

$$-3.00\text{ cm} = \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\frac{1.73\text{ m}}{v_i\cos 30.7°}\right)^2.$$

Thus,

$$v_i = \frac{1.73\text{ m}}{\cos 30.7°}\sqrt{\frac{9.80\text{ m/s}^2}{2\times 0.03\text{ m}}} = 2.015\text{ m}\left(12.8\text{ s}^{-1}\right) = \boxed{25.8\text{ m/s}}.$$

P4.19 (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}.$$

With

$$x_f = 36.0 \text{ m}, \; v_i = 20.0 \text{ m/s, and } \theta = 53.0°$$

we find

$$y_f = (36.0 \text{ m})\tan 53.0° - \frac{\left(9.80 \text{ m/s}^2\right)(36.0 \text{ m})^2}{2(20.0 \text{ m/s})^2 \cos^2(53.0°)} = 3.94 \text{ m}.$$

The ball clears the bar by

$$(3.94 - 3.05) \text{ m} = \boxed{0.889 \text{ m}}.$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0 \text{ m/s})(\sin 53.0°)}{9.80 \text{ m/s}^2} = 1.63 \text{ s}.$$

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0 \text{ m}}{(20.0 \text{ m/s})(\cos 53.0°)} = 2.99 \text{ s}.$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P4.20 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i \left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2.$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d \tan\theta_i - \frac{g d^2}{2v_i^2 \cos^2\theta_i}}.$$

***P4.21** (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$24\ \text{m} = 0 + v_i(\cos 53°)(2.2\ \text{s}) + 0$$
$$v_i = \boxed{18.1\ \text{m/s}}.$$

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1\ \text{m/s})(\sin 53°)(2.2\ \text{s}) + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(2.2\ \text{s})^2 = 8.13\ \text{m}.$$

So it clears the parapet by $8.13\ \text{m} - 7\ \text{m} = \boxed{1.13\ \text{m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or

$$6\ \text{m} = (\tan 53°)x_f - \left(\frac{9.8\ \text{m/s}^2}{2(18.1\ \text{m/s})^2\cos^2 53°}\right)x_f^2.$$

Solving,

$$\left(0.041\,2\ \text{m}^{-1}\right)x_f^2 - 1.33x_f + 6\ \text{m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2\left(0.0412\ \text{m}^{-1}\right)}.$$

This yields two results:

$$x_f = 26.8\ \text{m or } 5.44\ \text{m}$$

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

$$26.8\ \text{m} - 24\ \text{m} = \boxed{2.79\ \text{m}}.$$

***P4.22** When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\text{ km})^2 - (2.15\text{ km})^2} = 2.437\text{ km}$$

$$y_f = x_f \tan\theta - \frac{g x_f^2}{2v_i^2\cos^2\theta_i}$$

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - \frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

$$\therefore -2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - (371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945.$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662\,,\ \boxed{\theta_i = -33.5°}$$

P4.23 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0\text{ m} = 0 + 0 + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$t = 2.86\text{ s}.$$

The extra time $3.00\text{ s} - 2.86\text{ s} = 0.143\text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance

$$(343\text{ m/s})0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(1.85 - 1.02)\ \text{m}$. From this, $v_{yi} = 4.03\ \text{m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.900 - 1.85)\ \text{m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32\ \text{m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\ \text{m/s} = 4.03\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

or $t = \boxed{0.852\ \text{s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\ \text{m} = v_{xi}(0.852\ \text{s})$$

which yields $v_{xi} = \boxed{3.29\ \text{m/s}}$.

(c) $v_{yi} = \boxed{4.03\ \text{m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(2.50 - 1.20)\ \text{m}$$

so $v_{yi} = 5.04\ \text{m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.700 - 2.50)\ \text{m}$ and $v_{yf} = -5.94\ \text{m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94\ \text{m/s} = 5.04\ \text{m/s} + (-9.80\ \text{m/s}^2)t$ and

$$\boxed{t = 1.12\ \text{s}}.$$

***P4.25** The arrow's flight time to the collision point is

$$t = \frac{x_f - x_i}{v_{xi}} = \frac{150\ \text{m}}{(45\ \text{m/s})\cos 50°} = 5.19\ \text{s}.$$

The arrow's altitude at the collision is

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ &= 0 + (45\ \text{m/s})(\sin 50°)5.19\ \text{s} + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(5.19\ \text{s})^2 = 47.0\ \text{m}. \end{aligned}$$

(a) The required launch speed for the apple is given by

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y\left(y_f - y_i\right) \\ 0 &= v_{yi}^2 + 2\left(-9.8\ \text{m/s}^2\right)(47\ \text{m} - 0) \\ v_{yi} &= \boxed{30.3\ \text{m/s}}. \end{aligned}$$

(b) The time of flight of the apple is given by

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ 0 &= 30.3\ \text{m/s} - 9.8\ \text{m/s}^2\, t \\ t &= 3.10\ \text{s}. \end{aligned}$$

So the apple should be launched after the arrow by $5.19\ \text{s} - 3.10\ \text{s} = \boxed{2.09\ \text{s}}$.

***P4.26** For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right),$$

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y-component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

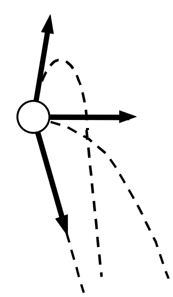


FIG. P4.26

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}.$$

Section 4.4 **Uniform Circular Motion**

P4.27 $a_c = \dfrac{v^2}{r} = \dfrac{(20.0\ \text{m/s})^2}{1.06\ \text{m}} = \boxed{377\ \text{m/s}^2}$

The mass is unnecessary information.

P4.28 $a = \dfrac{v^2}{R}$, $T = 24\ \text{h}(3\,600\ \text{s/h}) = 86\,400\ \text{s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\ \text{m})}{86\,400\ \text{s}} = 463\ \text{m/s}$$

$$a = \frac{(463\ \text{m/s})^2}{6.37\times10^6\ \text{m}} = \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}.$$

P4.29 $r = 0.500\ \text{m}$;

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\ \text{m})}{\frac{60.0\ \text{s}}{200\ \text{rev}}} = 10.47\ \text{m/s} = \boxed{10.5\ \text{m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219\ \text{m/s}^2\ \text{inward}}$$

P4.30 $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8\ \text{m/s}^2)(9.45\ \text{m})} = 16.7\ \text{m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45\ \text{m}) = 59.4\ \text{m}$. Then the rotation rate is

$$16.7\ \text{m/s}\left(\frac{1\ \text{rev}}{59.4\ \text{m}}\right) = \boxed{0.281\ \text{rev/s}}.$$

P4.31 (a) $v = r\omega$

At $8.00\ \text{rev/s}$, $v = (0.600\ \text{m})(8.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 30.2\ \text{m/s} = 9.60\pi\ \text{m/s}$.

At $6.00\ \text{rev/s}$, $v = (0.900\ \text{m})(6.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 33.9\ \text{m/s} = 10.8\pi\ \text{m/s}$.

$\boxed{6.00\ \text{rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52\times10^3\ \text{m/s}^2}$.

(c) At $6.00\ \text{rev/s}$, acceleration $= \dfrac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28\times10^3\ \text{m/s}^2}$.

P4.32 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$$a_c = g$$

so

$$\frac{v^2}{r} = g.$$

Solving for the velocity, $v = \sqrt{rg} = \sqrt{(6{,}400 + 600)(10^3\ \text{m})(8.21\ \text{m/s}^2)} = \boxed{7.58 \times 10^3\ \text{m/s}}$

$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi(7{,}000 \times 10^3\ \text{m})}{7.58 \times 10^3\ \text{m/s}} = \boxed{5.80 \times 10^3\ \text{s}}$$

$$T = 5.80 \times 10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 96.7\ \text{min}.$$

Section 4.5 **Tangential and Radial Acceleration**

P4.33 We assume the train is still slowing down at the instant in question.

$$a_c = \frac{v^2}{r} = 1.29\ \text{m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0\ \text{km/h})(10^3\ \text{m/km})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)}{15.0\ \text{s}} = -0.741\ \text{m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29\ \text{m/s}^2)^2 + (-0.741\ \text{m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

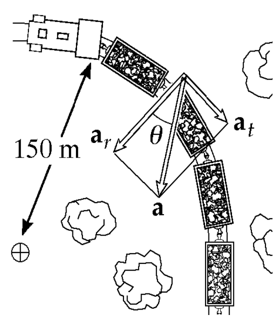


FIG. P4.33

$$\mathbf{a} = \boxed{1.48\ \text{m/s}^2\ \text{inward and } 29.9°\ \text{backward}}$$

P4.34 (a) $a_t = \boxed{0.600\ \text{m/s}^2}$

(b) $a_r = \frac{v^2}{r} = \frac{(4.00\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{0.800\ \text{m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00\ \text{m/s}^2}$

$\theta = \tan^{-1}\frac{a_r}{a_t} = \boxed{53.1°\ \text{inward from path}}$

P4.35 $r = 2.50\text{ m}$, $a = 15.0\text{ m/s}^2$

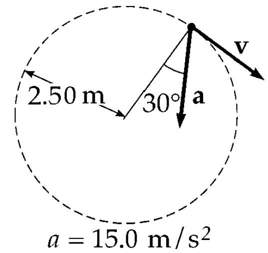


FIG. P4.35

(a) $a_c = a\cos 30.0° = \left(15.0\text{ m/s}^2\right)(\cos 30°) = \boxed{13.0\text{ m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\text{ m}\left(13.0\text{ m/s}^2\right) = 32.5\text{ m}^2/\text{s}^2$

$v = \sqrt{32.5}\text{ m/s} = \boxed{5.70\text{ m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{\left(15.0\text{ m/s}^2\right)^2 - \left(13.0\text{ m/s}^2\right)} = \boxed{7.50\text{ m/s}^2}$

P4.36 (a) See figure to the right.

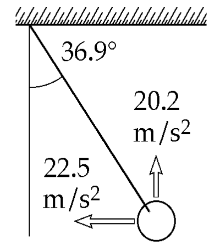


FIG. P4.36

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5\text{ m/s}^2\right)\cos(90.0°-36.9°) + \left(20.2\text{ m/s}^2\right)\cos 36.9° = \boxed{29.7\text{ m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\text{ m/s}^2\,(1.50\text{ m})} = 6.67\text{ m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67\text{ m/s at }36.9°\text{ above the horizontal}}$

***P4.37** Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

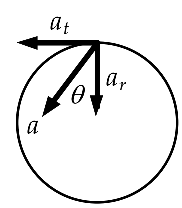


FIG. P4.37

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

and $v_{xf} = v_{xi} + a_x t$, $v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$.

Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\qquad \theta = \boxed{4.55°}$.

Section 4.6 **Relative Velocity and Relative Acceleration**

P4.38 (a)

$$\mathbf{v}_{\mathrm{H}} = 0 + \mathbf{a}_{\mathrm{H}}t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}\,(5.00\ \mathrm{s})$$

$$\mathbf{v}_{\mathrm{H}} = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$\mathbf{v}_{\mathrm{J}} = 0 + \mathbf{a}_{j}t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}\,(5.00\ \mathrm{s})$$

$$\mathbf{v}_{\mathrm{J}} = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$\mathbf{v}_{\mathrm{HJ}} = v_{\mathrm{H}} - v_{\mathrm{J}} = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$\mathbf{v}_{\mathrm{HJ}} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$\left|\mathbf{v}_{\mathrm{HJ}}\right| = \sqrt{(10.0)^2 + (25.0)^2}\ \mathrm{m/s} = \boxed{26.9\ \mathrm{m/s}}$$

(b)

$$\mathbf{r}_{\mathrm{H}} = 0 + 0 + \frac{1}{2}\mathbf{a}_{\mathrm{H}}t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}\,(5.00\ \mathrm{s})^2$$

$$\mathbf{r}_{\mathrm{H}} = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\mathbf{r}_{\mathrm{J}} = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}\,(5.00\mathrm{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\mathbf{r}_{\mathrm{HJ}} = \mathbf{r}_{\mathrm{H}} - \mathbf{r}_{\mathrm{J}} = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\mathbf{r}_{\mathrm{HJ}} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\left|\mathbf{r}_{\mathrm{HJ}}\right| = \sqrt{(25.0)^2 + (62.5)^2}\,\mathrm{m} = \boxed{67.3\ \mathrm{m}}$$

(c)

$$\mathbf{a}_{\mathrm{HJ}} = \mathbf{a}_{\mathrm{H}} - \mathbf{a}_{\mathrm{J}} = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}$$

$$\mathbf{a}_{\mathrm{HJ}} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \mathrm{m/s^2}}$$

***P4.39** $\mathbf{v}_{ce} =$ the velocity of the car relative to the earth.
$\mathbf{v}_{wc} =$ the velocity of the water relative to the car.
$\mathbf{v}_{we} =$ the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

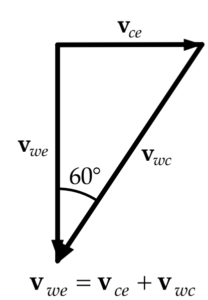


FIG. P4.39

(a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0\ \mathrm{km/h}$ or $\mathbf{v}_{wc} = \boxed{57.7\ \mathrm{km/h\ at\ 60.0°\ west\ of\ vertical}}$.

(b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \mathrm{km/h})\cos 60.0° = \boxed{28.9\ \mathrm{km/h\ downward}}.$$

P4.40 The bumpers are initially $100\ \text{m} = 0.100\ \text{km}$ apart. After time t the bumper of the leading car travels $40.0\,t$, while the bumper of the chasing car travels $60.0t$.

Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t\,,$$

yielding

$$t = 5.00 \times 10^{-3}\ \text{h} = \boxed{18.0\ \text{s}}\,.$$

P4.41 Total time in still water $t = \dfrac{d}{v} = \dfrac{2\,000}{1.20} = \boxed{1.67 \times 10^{3}\ \text{s}}$.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^{3}\ \text{s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\ \text{s}\,.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^{3} + 588 = \boxed{2.02 \times 10^{3}\ \text{s}}$.

P4.42 $v = \sqrt{150^2 + 30.0^2} = \boxed{153\ \text{km/h}}$

$\theta = \tan^{-1}\left(\dfrac{30.0}{150}\right) = \boxed{11.3^\circ\ \text{north of west}}$

P4.43 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{\frac{2L}{c}}{1 - \frac{v^2}{c^2}}}\,.$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}\,.$$

Thus, the total time for Beth is $t_2 = \dfrac{2L}{\sqrt{c^2 - v^2}} = \boxed{\dfrac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}}$.

Since $1 - \dfrac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

P4.44 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50\ \text{m/s})^2 + (9.80\ \text{m/s})^2} = \boxed{10.1\ \text{m/s}^2}$$

$$\tan\theta = \frac{2.50\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.255$$

$$\theta = \boxed{14.3°\ \text{to the south from the vertical}}$$

(b) $a = \boxed{9.80\ \text{m/s}^2\ \text{vertically downward}}$

P4.45 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}.$$

Let u represent the speed of S' relative to S. Then because there is no x-motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0\ \text{m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3}\ \text{m/s}.$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3}\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{15.3\ \text{m}}.$$

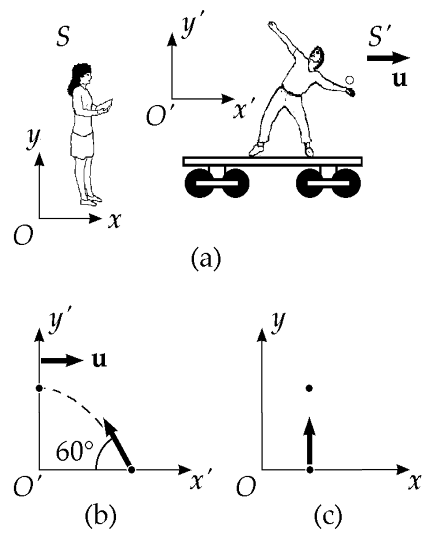


FIG. P4.45

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

***P4.46** Choose the x-axis along the 20-km distance. The y-components of the displacements of the ship and the speedboat must agree:

$$(26\ \text{km/h})t\sin(40°-15°) = (50\ \text{km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\frac{11.0}{50} = 12.7°.$$

The speedboat should head

$$15° + 12.7° = \boxed{27.7°\ \text{east of north}}.$$

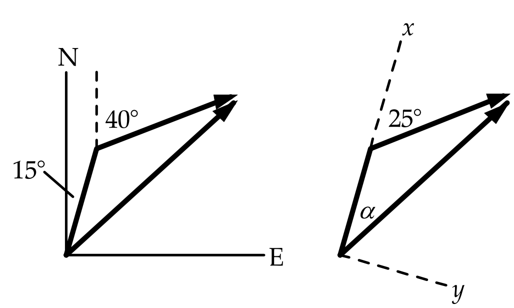


FIG. P4.46

Additional Problems

***P4.47** (a) The speed at the top is $v_x = v_i \cos\theta_i = (143 \text{ m/s})\cos 45° = \boxed{101 \text{ m/s}}$.

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$
$$0 = (143 \text{ m/s} \sin 45°)^2 + 2\left(-9.8 \text{ m/s}^2\right)\left(y_f - 31\,000 \text{ ft}\right)$$
$$y_f = 31\,000 \text{ ft} + 522 \text{ m}\left(\frac{3.28 \text{ ft}}{1 \text{ m}}\right) = \boxed{3.27 \times 10^3 \text{ ft}}.$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$-101 \text{ m/s} = +101 \text{ m/s} - \left(9.8 \text{ m/s}^2\right)t$$
$$t = \boxed{20.6 \text{ s}}$$

(d) $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8\left(9.8 \text{ m/s}^2\right)4{,}130 \text{ m}} = \boxed{180 \text{ m/s}}$$

P4.48 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2\left|x(t)\right| = 2\left(v_{xi}t\right) = 2\left(v_i \cos\theta_i\right)t = \boxed{2v_i t \cos\theta_i}.$$

P4.49 After the string breaks the ball is a projectile, and reaches the ground at time t: $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$$-1.20 \text{ m} = 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$$

so $t = 0.495 \text{ s}$.

Its constant horizontal speed is $v_x = \dfrac{x}{t} = \dfrac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}$

so before the string breaks $a_c = \dfrac{v_x^2}{r} = \dfrac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = \boxed{54.4 \text{ m/s}^2}$.

P4.50 (a) $y_f = (\tan\theta_i)(x_f) - \dfrac{g}{2v_i^2\cos^2\theta_i}x_f^2$

Setting $x_f = d\cos\phi$, and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2.$$

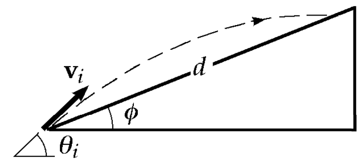


FIG. P4.50

Solving for d yields, $d = \dfrac{2v_i^2\cos\theta_i[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i]}{g\cos^2\phi}$

or $d = \boxed{\dfrac{2v_i^2\cos\theta_i\sin(\theta_i-\phi)}{g\cos^2\phi}}$.

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and $\boxed{d_{\max} = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$.

P4.51 Refer to the sketch:

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i\cos 35.0°)t$.

$\Delta y = v_{yi}t + \dfrac{1}{2}at^2$; substitution yields

$$20.0 = (v_i\sin 35.0°)t + \frac{1}{2}(-9.80)t^2.$$

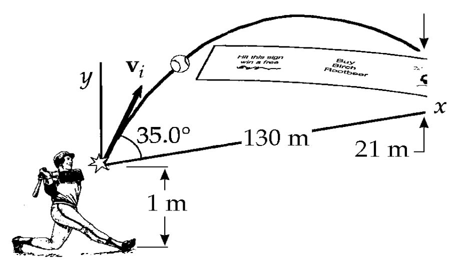


FIG. P4.51

Solving the above gives $t = \boxed{3.81\text{ s}}$.

(a) $v_i = \boxed{41.7\text{ m/s}}$

(c) $v_{yf} = v_i\sin\theta_i - gt$, $v_x = v_i\cos\theta_i$

At $t = 3.81$ s, $v_{yf} = 41.7\sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

$$v_x = (41.7\cos 35.0°) = \boxed{34.1\text{ m/s}}$$
$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7\text{ m/s}}.$$

P4.52 (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}\left(9.80\ \text{m/s}^2\right) = \frac{v^2}{1.74\times10^6\ \text{m}}$$

$$v = \sqrt{2.84\times10^6\ \text{m}^2/\text{s}^2} = \boxed{1.69\ \text{km/s}}$$

(b) $v = \dfrac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74\times10^6\ \text{m})}{1.69\times10^3\ \text{m/s}} = 6.47\times10^3\ \text{s} = \boxed{1.80\ \text{h}}$$

P4.53 (a) $a_c = \dfrac{v^2}{r} = \dfrac{(5.00\ \text{m/s})^2}{1.00\ \text{m}} = \boxed{25.0\ \text{m/s}^2}$

$a_t = g = \boxed{9.80\ \text{m/s}^2}$

(b) See figure to the right.

(c) $a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(25.0\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = \boxed{26.8\ \text{m/s}^2}$

$\phi = \tan^{-1}\left(\dfrac{a_t}{a_c}\right) = \tan^{-1}\dfrac{9.80\ \text{m/s}^2}{25.0\ \text{m/s}^2} = \boxed{21.4°}$

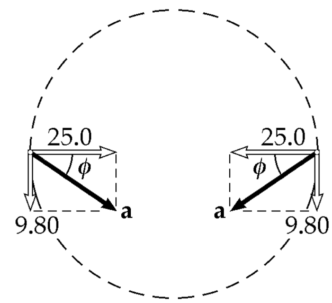


FIG. P4.53

P4.54 $x_f = v_{ix}t = v_i t\cos 40.0°$

Thus, when $x_f = 10.0\ \text{m}$, $t = \dfrac{10.0\ \text{m}}{v_i\cos 40.0°}$.

At this time, y_f should be $3.05\ \text{m} - 2.00\ \text{m} = 1.05\ \text{m}$.

Thus, $1.05\ \text{m} = \dfrac{(v_i\sin 40.0°)10.0\ \text{m}}{v_i\cos 40.0°} + \dfrac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left[\dfrac{10.0\ \text{m}}{v_i\cos 40.0°}\right]^2$.

From this, $v_i = \boxed{10.7\ \text{m/s}}$.

P4.55 The special conditions allowing use of the horizontal range equation applies.

For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}.$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

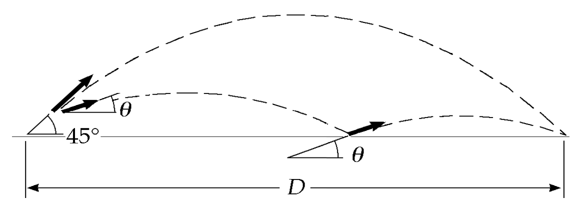


FIG. P4.55

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2.$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}.$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}.$$

The ratio of this time to that for no bounce is

$$\frac{\frac{3v_i \sin 26.6°}{g}}{\frac{2v_i \sin 45.0°}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

P4.56 Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45°$, and has a value $R = \frac{v_i^2}{g}$. Given R, this yields $v_i = \sqrt{gR}$.

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \text{ and } y = \sqrt{gR}\,t - \frac{gt^2}{2}$$

at any time, t.

At the maximum height, $v_y = 0$, giving $t = \sqrt{\frac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\left(\sqrt{\frac{R}{g}}\right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}.$$

P4.57 Choose upward as the positive y-direction and leftward as the positive x-direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20\sin 30.0°)\text{ m} = 2.10\text{ m}$$

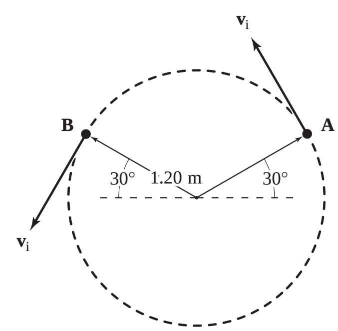


FIG. P4.57

(a) The equations of motion after release at A are

$$\begin{aligned} v_y &= v_i \sin 60.0° - gt = (1.30 - 9.80t)\text{ m/s} \\ v_x &= v_i \cos 60.0° = 0.750\text{ m/s} \\ y &= \left(2.10 + 1.30t - 4.90t^2\right)\text{ m} \\ \Delta x_A &= (0.750t)\text{ m} \end{aligned}$$

When $y = 0$, $t = \frac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800$ s. Then, $\Delta x_A = (0.750)(0.800)\text{ m} = \boxed{0.600\text{ m}}$.

(b) The equations of motion after release at point B are

$$\begin{aligned} v_y &= v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t)\text{ m/s} \\ v_x &= v_i \cos 60.0 = 0.750\text{ m/s} \\ y_i &= \left(2.10 - 1.30t - 4.90t^2\right)\text{ m}. \end{aligned}$$

When $y = 0$, $t = \frac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536$ s. Then, $\Delta x_B = (0.750)(0.536)\text{ m} = \boxed{0.402\text{ m}}$.

(c) $a_r = \frac{v^2}{r} = \frac{(1.50\text{ m/s})^2}{1.20\text{ m}} = \boxed{1.87\text{ m/s}^2\text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\,\hat{\mathbf{j}} = \boxed{9.80\text{ m/s}^2\text{ downward}}$

P4.58 The football travels a horizontal distance

$$R=\frac{v_i^2\sin(2\theta_i)}{g}=\frac{(20.0)^2\sin(60.0°)}{9.80}=35.3\text{ m}.$$

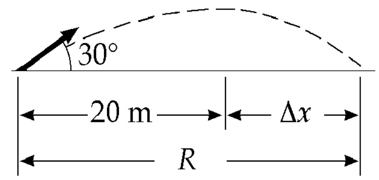


FIG. P4.58

Time of flight of ball is

$$t=\frac{2v_i\sin\theta_i}{g}=\frac{2(20.0)\sin 30.0°}{9.80}=2.04\text{ s}.$$

The receiver is Δx away from where the ball lands and $\Delta x=35.3-20.0=15.3$ m. To cover this distance in 2.04 s, he travels with a velocity

$$v=\frac{15.3}{2.04}=\boxed{7.50\text{ m/s in the direction the ball was thrown}}.$$

P4.59 (a) $\Delta y=-\frac{1}{2}gt^2;\ \Delta x=v_i t$

Combine the equations eliminating t:

$$\Delta y=-\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2.$$

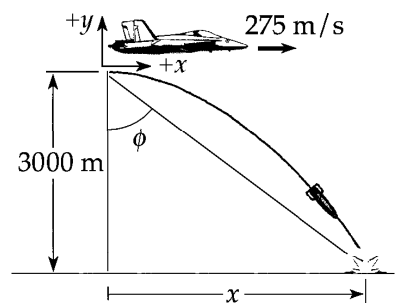


FIG. P4.59

From this, $(\Delta x)^2=\left(\frac{-2\Delta y}{g}\right)v_i^2$

thus $\Delta x=v_i\sqrt{\frac{-2\Delta y}{g}}=275\sqrt{\frac{-2(-300)}{9.80}}=6.80\times10^3=\boxed{6.80\text{ km}}$.

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{3\,000\text{ m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi=\frac{\Delta x}{\Delta y}$

therefore, $\phi=\tan^{-1}\left(\frac{\Delta x}{\Delta y}\right)=\tan^{-1}\left(\frac{6\,800}{3\,000}\right)=\boxed{66.2°}$.

***P4.60** (a) We use the approximation mentioned in the problem. The time to travel 200 m horizontally is $t = \frac{\Delta x}{v_x} = \frac{200\ \text{m}}{1{,}000\ \text{m/s}} = 0.200\ \text{s}$. The bullet falls by

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(0.2\ \text{s})^2 = \boxed{-0.196\ \text{m}}.$$

(b) The telescope axis must point below the barrel axis by $\theta = \tan^{-1}\frac{0.196\ \text{m}}{200\ \text{m}} = \boxed{0.0561°}$.

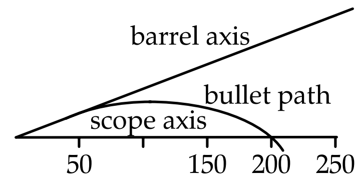


FIG. P4.60(b)

(c) $t = \frac{50.0\ \text{m}}{1\,000\ \text{m/s}} = 0.050\,0\ \text{s}$. The bullet falls by only

$$\Delta y = \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(0.05\ \text{s})^2 = -0.0122\ \text{m}.$$

At range $50\ \text{m} = \frac{1}{4}(200\ \text{m})$, the scope axis points to a location $\frac{1}{4}(19.6\ \text{cm}) = 4.90\ \text{cm}$ above the barrel axis, so the sharpshooter must $\boxed{\text{aim low}}$ by $4.90\ \text{cm} - 1.22\ \text{cm} = \boxed{3.68\ \text{cm}}$.

(d) $t = \frac{150\ \text{m}}{1\,000\ \text{m/s}} = 0.150\ \text{s}$

$\Delta y = \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(0.15\ \text{s})^2 = 0.110\ \text{m}$

$\boxed{\text{Aim low}}$ by $\frac{150}{200}(19.6\ \text{cm}) - 11.0\ \text{cm} = \boxed{3.68\ \text{cm}}$.

(e) $t = \frac{250\ \text{m}}{1\,000\ \text{m/s}} = 0.250\ \text{s}$

$\Delta y = \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)(0.25\ \text{s})^2 = 0.306\ \text{m}$

$\boxed{\text{Aim high}}$ by $30.6\ \text{cm} - \frac{250}{200}(19.6\ \text{cm}) = \boxed{6.12\ \text{cm}}$.

(f), (g) Many marksmen have a hard time believing it, but they should aim low in both cases. As in case (a) above, the time of flight is very nearly 0.200 s and the bullet falls below the barrel axis by 19.6 cm on its way. The 0.0561° angle would cut off a 19.6-cm distance on a vertical wall at a horizontal distance of 200 m, but on a vertical wall up at 30° it cuts off distance h as shown, where $\cos 30° = 19.6\ \text{cm}/h$, $h = 22.6\ \text{cm}$. The marksman must $\boxed{\text{aim low}}$ by $22.6\ \text{cm} - 19.6\ \text{cm} = 3.03\ \text{cm}$. The answer can be obtained by considering limiting cases. Suppose the target is nearly straight above or below you. Then gravity will not cause deviation of the path of the bullet, and one must aim low as in part (c) to cancel out the sighting-in of the telescope.

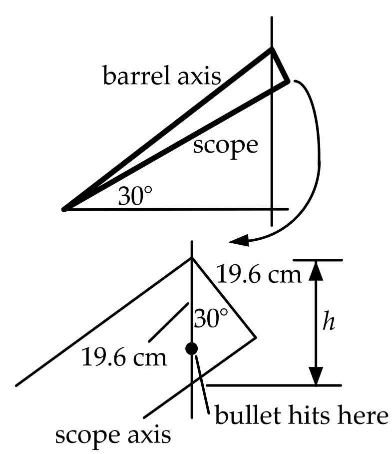


FIG. P4.60(f–g)

P4.61 (a) From Part (c), the raptor dives for $6.34-2.00=4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34)=43.4$ m forward.

$$v=\frac{\Delta d}{\Delta t}\frac{\sqrt{(197)^2+(43.4)^2}}{4.34}=\boxed{46.5\ \text{m/s}}$$

(b) $\alpha=\tan^{-1}\left(\frac{-197}{43.4}\right)=\boxed{-77.6°}$

(c) $197=\frac{1}{2}gt^2,\ \boxed{t=6.34\ \text{s}}$

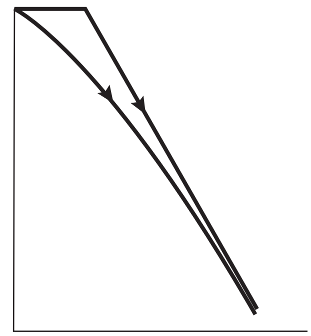


FIG. P4.61

P4.62 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b=R+0-\frac{1}{2}gt^2$$

with $x=v_i t$ so $y_b=R-\frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2+x^2=R^2.$$

We will have $y_b=y_r$ at $x=0$, but for all other x we require the ball to be above the rock surface as in $y_b>y_r$. Then $y_b^2+x^2>R^2$

$$\left(R-\frac{gx^2}{2v_i^2}\right)^2+x^2>R^2$$

$$R^2-\frac{gx^2R}{v_i^2}+\frac{g^2x^4}{4v_i^4}+x^2>R^2$$

$$\frac{g^2x^4}{4v_i^4}+x^2>\frac{gx^2R}{v_i^2}.$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1>\frac{gR}{v_i^2}$

$$\boxed{v_i>\sqrt{gR}}.$$

(b) With $v_i=\sqrt{gR}$ and $y_b=0$, we have $0=R-\frac{gx^2}{2gR}$ or $x=R\sqrt{2}$.

The distance from the rock's base is

$$x-R=\boxed{\left(\sqrt{2}-1\right)R}.$$

P4.63 (a) While on the incline

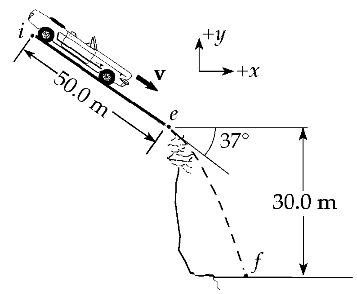


FIG. P4.63

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f - v_i = at$$
$$v_f^2 - 0 = 2(4.00)(50.0)$$
$$20.0 - 0 = 4.00t$$
$$v_f = \boxed{20.0 \text{ m/s}}$$
$$t = \boxed{5.00 \text{ s}}$$

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0\sin 37.0° = -12.0 \text{ m/s}$$
$$v_{xf} = v_{xi} \text{ since } a_x = 0$$
$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}{}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$
$$v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(c) $t_1 = 5 \text{ s};\ t_2 = \dfrac{v_{yf} - v_{yi}}{a_y} = \dfrac{-27.1 + 12.0}{-9.80} = 1.53 \text{ s}$

$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$

(d) $\Delta x = v_{xi}t_1 = 16.0(1.53) = \boxed{24.5 \text{ m}}$

P4.64 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$y = -\dfrac{1}{2}gt^2$ (3)

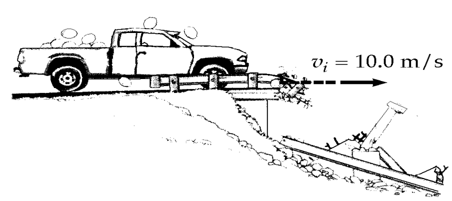


FIG. P4.64

Substitute for t from (2) into (3) $y = -\dfrac{1}{2}g\left(\dfrac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0.$$

From this, $x = 0$ or $x^3 = \dfrac{64v_i^4}{g^2}$ and $x = 4\left(\dfrac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}.$$

P4.65 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9 \text{ m/s}} \text{ and } t = 3.06 \text{ s}.$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8 \text{ m/s}.$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$
$$t = 4.52 \text{ s}$$
$$\Delta x = v_{xi} t + \frac{1}{2}a_x t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2.$$

Solving,

$$\Delta x = \boxed{360 \text{ m}}.$$

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}.$$

P4.66 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6 \text{ m})}{0.1 \text{ s}} \cong 9 \text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \cong \frac{(9 \text{ m/s})^2}{0.6 \text{ m}} \boxed{\sim 10^2 \text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P4.67 (a) $\Delta x = v_{xi}t$, $\Delta y = v_{yi}t + \frac{1}{2}gt^2$

$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and

$$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2.$$

Solving, $d = \boxed{43.2 \text{ m}}$ and $t = 2.88$ s.

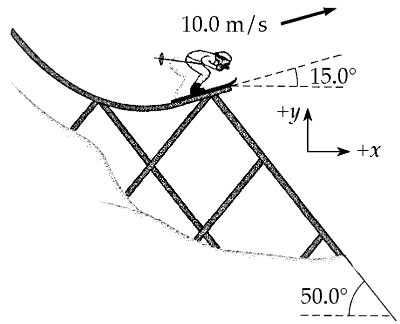


FIG. P4.67

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6 \text{ m/s}}.$$

Air resistance would decrease the values of the range and maximum height. As an airfoil, he can get some lift and increase his distance.

***P4.68** For one electron, we have

$$y = v_{iy}t,\ D = v_{ix}t + \frac{1}{2}a_x t^2 \cong \frac{1}{2}a_x t^2,\ v_{yf} = v_{yi}, \text{ and } v_{xf} = v_{xi} + a_x t \cong a_x t.$$

The angle its direction makes with the x-axis is given by

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{v_{yi}}{a_x t} = \tan^{-1}\frac{v_{yi}t}{a_x t^2} = \tan^{-1}\frac{y}{2D}.$$

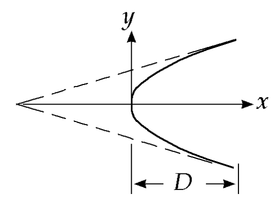


FIG. P4.68

Thus the horizontal distance from the aperture to the virtual source is $2D$. The source is at coordinate $\boxed{x = -D}$.

***P4.69** (a) The ice chest floats downstream 2 km in time t, so that $2 \text{ km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15$ min. The downstream motion is described by $d + 2 \text{ km} = (v + v_w)(t - 15 \text{ min})$. We eliminate $t = \frac{2 \text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15 \text{ min} + 2 \text{ km} = (v + v_w)\left(\frac{2 \text{ km}}{v_w} - 15 \text{ min}\right)$$

$$v(15 \text{ min}) - v_w(15 \text{ min}) + 2 \text{ km} = \frac{v}{v_w}2 \text{ km} + 2 \text{ km} - v(15 \text{ min}) - v_w(15 \text{ min})$$

$$v(30 \text{ min}) = \frac{v}{v_w}\ 2 \text{ km}$$

$$v_w = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

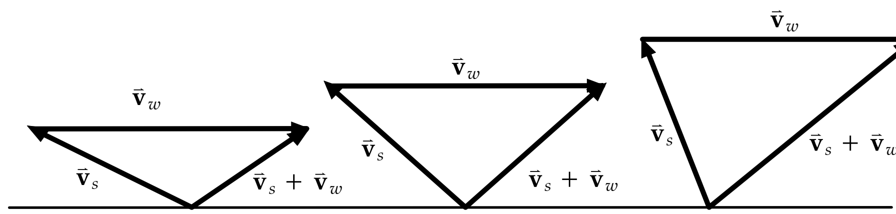
$$v_w = \frac{\Delta x}{\Delta t} = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}.$$

***P4.70** Let the river flow in the x direction.

(a) To minimize time, $\boxed{\text{swim perpendicular to the banks}}$ in the y direction. You are in the water for time t in $\Delta y = v_y t$, $t = \dfrac{80\text{ m}}{1.5\text{ m/s}} = 53.3\text{ s}$.

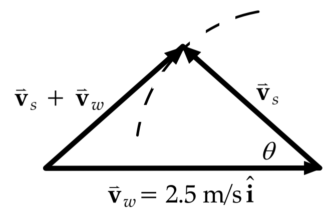
(b) The water carries you downstream by $\Delta x = v_x t = (2.50\text{ m/s})53.3\text{ s} = \boxed{133\text{ m}}$.

(c)



To minimize downstream drift, you should swim so that your resultant velocity $\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$ is perpendicular to your swimming velocity $\vec{\mathbf{v}}_s$ relative to the water. This condition is shown in the middle picture. It maximizes the angle between the resultant velocity and the shore. The angle between $\vec{\mathbf{v}}_s$ and the shore is given by $\cos\theta = \dfrac{1.5\text{ m/s}}{2.5\text{ m/s}}$,

$\boxed{\theta = 53.1°}$.



(d) Now $v_y = v_s \sin\theta = 1.5\text{ m/s } \sin 53.1° = 1.20\text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80\text{ m}}{1.2\text{ m/s}} = 66.7\text{ s}$$

$$\Delta x = v_x t = (2.5\text{ m/s} - 1.5\text{ m/s}\cos 53.1°)66.7\text{ s} = \boxed{107\text{ m}}.$$

***P4.71** Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i\cos\theta}.$$

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f.$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°.$$

$$\text{Range (at } \theta_H) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3 \text{ m from enemy ship}$$

$$3.07\times10^3 - 2\,500 - 300 = 270 \text{ m from shore.}$$

$$\text{Range (at } \theta_L) = \frac{v_i^2\sin 2\theta_L}{g} = 6.28\times10^3 \text{ m from enemy ship}$$

$$6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3 \text{ from shore.}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48\times10^3 \text{ m}}$ from the shore.

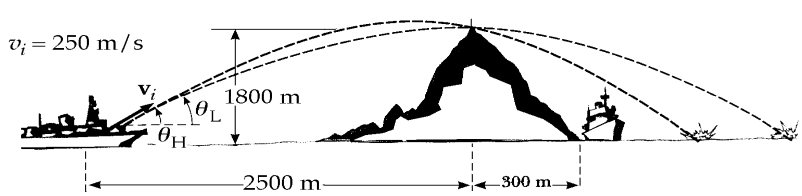


FIG. P4.71

***P4.72** We follow the steps outlined in Example 4.7, eliminating $t = \dfrac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi.$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi.$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\dfrac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2\,2\cos\theta(-\sin\theta)\sin\phi.$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\dfrac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \dfrac{\phi}{2}$.

ANSWERS TO EVEN PROBLEMS

P4.2 (a) $\mathbf{r} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$;
(b) $\mathbf{v} = 18.0\hat{\mathbf{i}} + (4.00 - 9.80t)\hat{\mathbf{j}}$;
(c) $\mathbf{a} = \left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$;
(d) $(54.0\ \text{m})\hat{\mathbf{i}} - (32.1\ \text{m})\hat{\mathbf{j}}$;
(e) $(18.0\ \text{m/s})\hat{\mathbf{i}} - (25.4\ \text{m/s})\hat{\mathbf{j}}$;
(f) $\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$

P4.4 (a) $\mathbf{v} = \left(-5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}$;
$\mathbf{a} = \left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2$;
(b) $\mathbf{r} = 4.00\ \text{m}\ \hat{\mathbf{j}}$
$+5.00\ \text{m}\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{v} = 5.00\ \text{m}\ \omega\left(-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{a} = 5.00\ \text{m}\ \omega^2\left(\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right)$;
(c) a circle of radius 5.00 m centered at $(0,\ 4.00\ \text{m})$

P4.6 (a) $\mathbf{v} = -12.0t\,\hat{\mathbf{j}}\ \text{m/s}$; $\mathbf{a} = -12.0\hat{\mathbf{j}}\ \text{m/s}^2$;
(b) $\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)$ m; $\mathbf{v} = -12.0\hat{\mathbf{j}}\ \text{m/s}$

P4.8 (a) $\mathbf{r} = \left(5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}\right)$ m;
$\mathbf{v} = \left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \text{m/s}$;
(b) $\mathbf{r} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)$ m; 7.81 m/s

P4.10 $\left(7.23\times10^3\ \text{m},\ 1.68\times10^3\ \text{m}\right)$

P4.12 (a) $d\sqrt{\dfrac{g}{2h}}$ horizontally;
(b) $\tan^{-1}\left(\dfrac{2h}{d}\right)$ below the horizontal

P4.14 $0.600\ \text{m/s}^2$

P4.16 (a) 76.0°; (b) the same; (c) $\dfrac{17d}{8}$

P4.18 25.8 m/s

P4.20 $d\tan\theta_i - \dfrac{gd^2}{\left(2v_i^2\cos^2\theta_i\right)}$

P4.22 33.5° below the horizontal

P4.24 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) 1.12 s

P4.26 $\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)$

P4.28 0.033 7 m/s^2 toward the center of the Earth

P4.30 0.281 rev/s

P4.32 7.58×10^3 m/s; 5.80×10^3 s

P4.34 (a) 0.600 m/s^2 forward;
(b) 0.800 m/s^2 inward;
(c) 1.00 m/s^2 forward and 53.1° inward

P4.36 (a) see the solution; (b) 29.7 m/s^2;
(c) 6.67 m/s at 36.9° above the horizontal

P4.38 (a) 26.9 m/s; (b) 67.3 m;
(c) $\left(2.00\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)$ m/s^2

P4.40 18.0 s

P4.42 153 km/h at 11.3° north of west

P4.44 (a) 10.1 m/s^2 at 14.3° south from the vertical; (b) 9.80 m/s^2 vertically downward

P4.46 27.7° east of north

P4.48 $2v_i t\cos\theta_i$

P4.50 (a) see the solution;
(b) $\theta_i = 45° + \frac{\phi}{2}$; $d_{\max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}$

P4.52 (a) 1.69 km/s; (b) 6.47×10^3 s

P4.54 10.7 m/s

P4.56 $\frac{R}{2}$

P4.58 7.50 m/s in the direction the ball was thrown

P4.60 (a) 19.6 cm; (b) 0.056 1°;
(c) aim low 3.68 cm; (d) aim low 3.68 cm;
(e) aim high 6.12 cm; (f) aim low;
(g) aim low

P4.62 (a) $\sqrt{gR}$; (b) $\left(\sqrt{2}-1\right)R$

P4.64 (18.8 m; −17.3 m)

P4.66 see the solution; $\sim 10^2$ m/s^2

P4.68 $x=-D$

P4.70 (a) at 90° to the bank; (b) 133 m;
(c) upstream at 53.1° to the bank; (d) 107 m

P4.72 see the solution

5

The Laws of Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q5.1 (a) The force due to gravity of the earth pulling down on the ball—the reaction force is the force due to gravity of the ball pulling up on the earth. The force of the hand pushing up on the ball—reaction force is ball pushing down on the hand.

(b) The only force acting on the ball in free-fall is the gravity due to the earth -the reaction force is the gravity due to the ball pulling on the earth.

Q5.2 The resultant force is zero, as the acceleration is zero.

Q5.3 Mistake one: The car might be momentarily at rest, in the process of (suddenly) reversing forward into backward motion. In this case, the forces on it add to a (large) backward resultant.

Mistake two: There are no cars in interstellar space. If the car is remaining at rest, there are some large forces on it, including its weight and some force or forces of support.

Mistake three: The statement reverses cause and effect, like a politician who thinks that his getting elected was the reason for people to vote for him.

Q5.4 When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap. (Both performers won Academy Awards.)

Q5.5 First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

Q5.6 It would be smart for the explorer to gently push the rock back into the storage compartment. Newton's 3rd law states that the rock will apply the same size force on her that she applies on it. The harder she pushes on the rock, the larger her resulting acceleration.

Q5.7 The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

Q5.8 While a football is in flight, the force of gravity and air resistance act on it. When a football is in the process of being kicked, the foot pushes forward on the ball and the ball pushes backward on the foot. At this time and while the ball is in flight, the Earth pulls down on the ball (gravity) and the ball pulls up on the Earth. The moving ball pushes forward on the air and the air backward on the ball.

Q5.9 It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight.

Some physics teachers demonstrate this by asking a beefy student to pull on the ends of a cord supporting a can of soup at its center. Some get two burly young men to pull on opposite ends of a strong rope, while the smallest person in class gleefully mashes the center of the rope down to the table. Point out the beauty of sagging suspension-bridge cables. With a laser and an optical lever, demonstrate that the mayor makes the courtroom table sag when he sits on it, and the judge bends the bench. Give them "I make the floor sag" buttons, available to instructors using this manual. Estimate the cost of an infinitely strong cable, and the truth will always win.

Q5.10 As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the iron is moving upward, the lifter can declare that she has thrown it, just by letting go of it for a moment, so our answer applies also to this case.

Q5.11 As the sand leaks out, the acceleration increases. With the same driving force, a decrease in the mass causes an increase in the acceleration.

Q5.12 As the rocket takes off, it burns fuel, pushing the gases from the combustion out the back of the rocket. Since the gases have mass, the total remaining mass of the rocket, fuel, and oxidizer decreases. With a constant thrust, a decrease in the mass results in an increasing acceleration.

Q5.13 The friction of the road pushing on the tires of a car causes an automobile to move. The push of the air on the propeller moves the airplane. The push of the water on the oars causes the rowboat to move.

Q5.14 As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. In the second case, the action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. The third action is the force of the glove on the ball; the reaction is the force of the ball on the glove. The fourth action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms 'action' and 'reaction.'

Q5.15 The tension in the rope must be 9 200 N. Since the rope is moving at a constant speed, then the resultant force on it must be zero. The 49ers are pulling with a force of 9 200 N. If the 49ers were winning with the rope steadily moving in their direction or if the contest was even, then the tension would still be 9 200 N. In all of these case, the acceleration is zero, and so must be the resultant force on the rope. To win the tug-of-war, a team must exert a larger force on the ground than their opponents do.

Q5.16 The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

Q5.17 This statement contradicts Newton's 3rd law. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The wall temporarily exerted on the locomotive a force greater than the force that the wall could exert without breaking.

Q5.18 The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

Q5.19 The resultant force doesn't always add to zero. If it did, nothing could ever accelerate. If we choose a single object as our system, action and reaction forces can never add to zero, as they act on different objects.

Q5.20 An object cannot exert a force on itself. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

Q5.21 To get the box to slide, you must push harder than the maximum static frictional force. Once the box is moving, you need to push with a force equal to the kinetic frictional force to maintain the box's motion.

Q5.22 The stopping distance will be the same if the mass of the truck is doubled. The stopping distance will decrease by a factor of four if the initial speed is cut in half.

Q5.23 If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Anti-lock brakes work by "pumping" the brakes (much more rapidly that you can) to minimize skidding of the tires on the road.

Q5.24 With friction, it takes longer to come down than to go up. On the way up, the frictional force and the component of the weight down the plane are in the same direction, giving a large acceleration. On the way down, the forces are in opposite directions, giving a relatively smaller acceleration. If the incline is frictionless, it takes the same amount of time to go up as it does to come down.

Q5.25 (a) The force of static friction between the crate and the bed of the truck causes the crate to accelerate. Note that the friction force on the crate is in the direction of its motion relative to the ground (but opposite to the direction of possible sliding motion of the crate relative to the truck bed).

(b) It is most likely that the crate would slide forward relative to the bed of the truck.

Q5.26 In Question 25, part (a) is an example of such a situation. Any situation in which friction is the force that accelerates an object from rest is an example. As you pull away from a stop light, friction is the force that accelerates forward a box of tissues on the level floor of the car. At the same time, friction of the ground on the tires of the car accelerates the car forward.

SOLUTIONS TO PROBLEMS

The following problems cover Sections 5.1–5.6.

Section 5.1 **The Concept of Force**

Section 5.2 **Newton's First Law and Inertial Frames**

Section 5.3 **Mass**

Section 5.4 **Newton's Second Law**

Section 5.5 **The Gravitational Force and Weight**

Section 5.6 **Newton's Third Law**

P5.1 For the same force F, acting on different masses

$$F = m_1 a_1$$

and

$$F = m_2 a_2$$

(a) $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \boxed{\dfrac{1}{3}}$

(b) $F = (m_1 + m_2)a = 4m_1 a = m_1\left(3.00 \text{ m/s}^2\right)$

$a = \boxed{0.750 \text{ m/s}^2}$

***P5.2** $v_f = 880 \text{ m/s}$, $m = 25.8 \text{ kg}$, $x_f = 6 \text{ m}$

$v_f^2 = 2ax_f = 2x_f\left(\dfrac{F}{m}\right)$

$F = \dfrac{mv_f^2}{2x_f} = \boxed{1.66 \times 10^6 \text{ N forward}}$

P5.3 $m = 3.00 \text{ kg}$

$\mathbf{a} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right) \text{ m/s}^2$

$\sum \mathbf{F} = m\mathbf{a} = \boxed{\left(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right) \text{N}}$

$\left|\sum \mathbf{F}\right| = \sqrt{(6.00)^2 + (15.0)^2} \text{ N} = \boxed{16.2 \text{ N}}$

P5.4 $F_g = \text{weight of ball} = mg$

$v_{\text{release}} = v$ and time to accelerate $= t$:

$$\mathbf{a} = \frac{\Delta v}{\Delta t} = \frac{v}{t} = \frac{v}{t}\hat{\mathbf{i}}$$

(a) Distance $x = \overline{v}t$:

$$x = \left(\frac{v}{2}\right)t = \boxed{\frac{vt}{2}}$$

(b) $\mathbf{F}_p - F_g\hat{\mathbf{j}} = \dfrac{F_g v}{gt}\hat{\mathbf{i}}$

$$\mathbf{F}_p = \boxed{\frac{F_g v}{gt}\hat{\mathbf{i}} + F_g\hat{\mathbf{j}}}$$

P5.5 $m = 4.00\text{ kg}$, $\mathbf{v}_i = 3.00\hat{\mathbf{i}}\text{ m/s}$, $\mathbf{v}_8 = \left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\text{ m/s}$, $t = 8.00\text{ s}$

$\mathbf{a} = \dfrac{\Delta\mathbf{v}}{t} = \dfrac{5.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}}{8.00}\text{ m/s}^2$

$\mathbf{F} = m\mathbf{a} = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\text{ N}}$

$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59\text{ N}}$

P5.6 (a) Let the x-axis be in the original direction of the molecule's motion.

$$v_f = v_i + at: \quad -670\text{ m/s} = 670\text{ m/s} + a\left(3.00 \times 10^{-13}\text{ s}\right)$$

$$a = \boxed{-4.47 \times 10^{15}\text{ m/s}^2}$$

(b) For the molecule, $\sum\mathbf{F} = m\mathbf{a}$. Its weight is negligible.

$$\mathbf{F}_{\text{wall on molecule}} = 4.68 \times 10^{-26}\text{ kg}\left(-4.47 \times 10^{15}\text{ m/s}^2\right) = -2.09 \times 10^{-10}\text{ N}$$

$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09 \times 10^{-10}\text{ N}}$$

P5.7 (a) $\sum F = ma$ and $v_f^2 = v_i^2 + 2ax_f$ or $a = \dfrac{v_f^2 - v_i^2}{2x_f}$.

Therefore,

$$\sum F = m\frac{\left(v_f^2 - v_i^2\right)}{2x_f}$$

$$\sum F = 9.11\times10^{-31}\ \text{kg}\frac{\left[\left(7.00\times10^5\ \text{m/s}^2\right)^2 - \left(3.00\times10^5\ \text{m/s}^2\right)^2\right]}{2(0.050\,0\ \text{m})} = \boxed{3.64\times10^{-18}\ \text{N}}.$$

(b) The weight of the electron is

$$F_g = mg = \left(9.11\times10^{-31}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 8.93\times10^{-30}\ \text{N}$$

The accelerating force is $\boxed{4.08\times10^{11}\ \text{times the weight of the electron.}}$

P5.8 (a) $F_g = mg = 120\ \text{lb} = (4.448\ \text{N/lb})(120\ \text{lb}) = \boxed{534\ \text{N}}$

(b) $m = \dfrac{F_g}{g} = \dfrac{534\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{54.5\ \text{kg}}$

P5.9 $F_g = mg = 900\ \text{N}$, $m = \dfrac{900\ \text{N}}{9.80\ \text{m/s}^2} = 91.8\ \text{kg}$

$$\left(F_g\right)_{\text{on Jupiter}} = 91.8\ \text{kg}\left(25.9\ \text{m/s}^2\right) = \boxed{2.38\ \text{kN}}$$

P5.10 Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right).$$

For a person whose mass is 88.7 kg, the change in weight is

$$\Delta F_g = 88.7\ \text{kg}(9.809\,5 - 9.780\,8) = \boxed{2.55\ \text{N}}.$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

P5.11 (a) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)$ N

$$\sum \mathbf{F} = m\mathbf{a}: \quad 20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}} = 5.00\mathbf{a}$$

$$\mathbf{a} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2$$

or

$$\boxed{a = 5.00 \text{ m/s}^2 \text{ at } \theta = 36.9°}$$

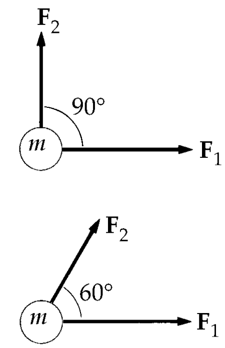


FIG. P5.11

(b) $F_{2x} = 15.0\cos 60.0° = 7.50$ N
$F_{2y} = 15.0\sin 60.0° = 13.0$ N

$$\mathbf{F}_2 = \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right) \text{ N}$$

$$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right) \text{ N} = m\mathbf{a} = 5.00\mathbf{a}$$

$$\mathbf{a} = \boxed{\left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right) \text{ m/s}^2 = 6.08 \text{ m/s}^2 \text{ at } 25.3°}$$

P5.12 We find acceleration:

$$\mathbf{r}_f - \mathbf{r}_i = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$4.20 \text{ m}\hat{\mathbf{i}} - 3.30 \text{ m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\mathbf{a}(1.20 \text{ s})^2 = 0.720 \text{ s}^2\mathbf{a}$$

$$\mathbf{a} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right) \text{ m/s}^2.$$

Now $\sum \mathbf{F} = m\mathbf{a}$ becomes

$$\mathbf{F}_g + \mathbf{F}_2 = m\mathbf{a}$$

$$\mathbf{F}_2 = 2.80 \text{ kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right) \text{ m/s}^2 + (2.80 \text{ kg})\left(9.80 \text{ m/s}^2\right)\hat{\mathbf{j}}$$

$$\mathbf{F}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right) \text{ N}}.$$

P5.13 (a) You and the earth exert equal forces on each other: $m_y g = M_e a_e$. If your mass is 70.0 kg,

$$a_e = \frac{(70.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)}{5.98 \times 10^{24} \text{ kg}} = \boxed{\sim 10^{-22} \text{ m/s}^2}.$$

(b) You and the planet move for equal times intervals according to $x = \frac{1}{2}at^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2x_y}{a_y}} = \sqrt{\frac{2x_e}{a_e}}$$

$$x_e = \frac{a_e}{a_y}x_y = \frac{m_y}{m_e}x_y = \frac{70.0 \text{ kg}(0.500 \text{ m})}{5.98 \times 10^{24} \text{ kg}}\boxed{\sim 10^{-23} \text{ m}}.$$

P5.14 $\sum \mathbf{F} = m\mathbf{a}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\mathbf{a}$

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right) \text{ N} = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum \mathbf{F} = \sqrt{(42.0)^2 + (1.00)^2} \text{ N at } \tan^{-1}\left(\frac{1.00}{42.0}\right) \text{ below the } -x\text{-axis}$$

$$\sum \mathbf{F} = 42.0 \text{ N at } 181° = m\left(3.75 \text{ m/s}^2\right)\hat{\mathbf{a}}.$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) $\therefore$ $\boxed{\hat{\mathbf{a}} \text{ is at } 181°}$ counterclockwise from the x-axis

(b) $m = \dfrac{42.0 \text{ N}}{3.75 \text{ m/s}^2} = \boxed{11.2 \text{ kg}}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 0 + \left(3.75 \text{ m/s}^2 \text{ at } 181°\right)10.0 \text{ s}$ so $\mathbf{v}_f = 37.5 \text{ m/s at } 181°$

$\mathbf{v}_f = 37.5 \text{ m/s } \cos 181° \hat{\mathbf{i}} + 37.5 \text{ m/s } \sin 181° \hat{\mathbf{j}}$ so $\mathbf{v}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right) \text{ m/s}}$

(c) $\left|\mathbf{v}_f\right| = \sqrt{37.5^2 + 0.893^2} \text{ m/s} = \boxed{37.5 \text{ m/s}}$

P5.15 (a) $\boxed{15.0 \text{ lb up}}$

(b) $\boxed{5.00 \text{ lb up}}$

(c) $\boxed{0}$

Section 5.7 **Some Applications of Newton's Laws**

P5.16 $v_x = \dfrac{dx}{dt} = 10t$, $v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10$, $a_y = \dfrac{dv_y}{dt} = 18t$

At $t = 2.00$ s, $a_x = 10.0 \text{ m/s}^2$, $a_y = 36.0 \text{ m/s}^2$

$\sum F_x = ma_x$: $3.00 \text{ kg}\left(10.0 \text{ m/s}^2\right) = 30.0 \text{ N}$

$\sum F_y = ma_y$: $3.00 \text{ kg}\left(36.0 \text{ m/s}^2\right) = 108 \text{ N}$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112 \text{ N}}$$

P5.17

$$m = 1.00 \text{ kg}$$
$$mg = 9.80 \text{ N}$$
$$\tan\alpha = \frac{0.200 \text{ m}}{25.0 \text{ m}}$$
$$\alpha = 0.458°$$

Balance forces,

$$2T\sin\alpha = mg$$
$$T = \frac{9.80 \text{ N}}{2\sin\alpha} = \boxed{613 \text{ N}}$$

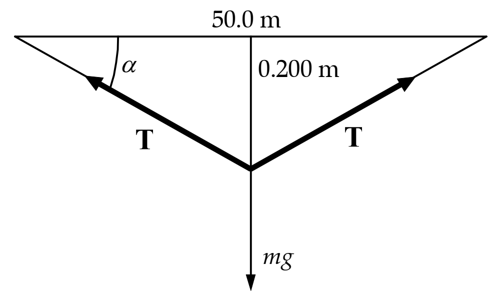


FIG. P5.17

P5.18

$$T_3 = F_g \quad (1)$$

$$T_1\sin\theta_1 + T_2\sin\theta_2 = F_g \quad (2)$$

$$T_1\cos\theta_1 = T_2\cos\theta_2 \quad (3)$$

Eliminate T_2 and solve for T_1

$$T_1 = \frac{F_g\cos\theta_2}{(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)} = \frac{F_g\cos\theta_2}{\sin(\theta_1+\theta_2)}$$
$$T_3 = F_g = \boxed{325 \text{ N}}$$
$$T_1 = F_g\left(\frac{\cos 25.0°}{\sin 85.0°}\right) = \boxed{296 \text{ N}}$$
$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = 296 \text{ N}\left(\frac{\cos 60.0°}{\cos 25.0°}\right) = \boxed{163 \text{ N}}$$

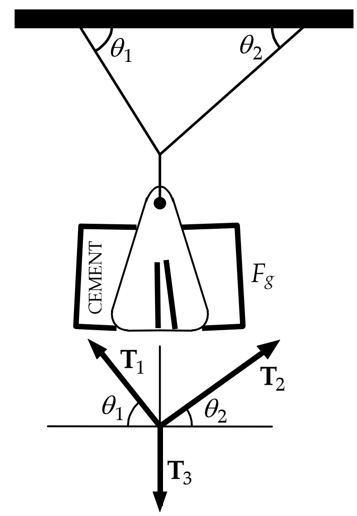


FIG. P5.18

P5.19 See the solution for T_1 in Problem 5.18.

P5.20 (a) An explanation proceeding from fundamental physical principles will be best for the parents and for you. Consider forces on the bit of string touching the weight hanger as shown in the free-body diagram:

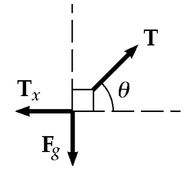


FIG. P5.20

Horizontal Forces: $\sum F_x = ma_x$: $-T_x + T\cos\theta = 0$

Vertical Forces: $\sum F_y = ma_y$: $-F_g + T\sin\theta = 0$

You need only the equation for the vertical forces to find that the tension in the string is given by $T = \boxed{\dfrac{F_g}{\sin\theta}}$. The force the child feels gets smaller, changing from T to $T\cos\theta$, while the counterweight hangs on the string. On the other hand, the kite does not notice what you are doing and the tension in the main part of the string stays constant. You do not need a level, since you learned in physics lab to sight to a horizontal line in a building. Share with the parents your estimate of the experimental uncertainty, which you make by thinking critically about the measurement, by repeating trials, practicing in advance and looking for variations and improvements in technique, including using other observers. You will then be glad to have the parents themselves repeat your measurements.

(b) $$T = \frac{F_g}{\sin\theta} = \frac{0.132\ \text{kg}\left(9.80\ \text{m/s}^2\right)}{\sin 46.3°} = \boxed{1.79\ \text{N}}$$

P5.21 (a) Isolate either mass

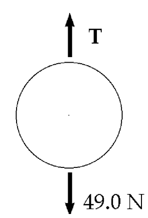


FIG. P5.21(a)

$$T + mg = ma = 0$$
$$|T| = |mg|.$$

The scale reads the tension T,

so

$$T = mg = 5.00\ \text{kg}\left(9.80\ \text{m/s}^2\right) = \boxed{49.0\ \text{N}}.$$

(b) Isolate the pulley

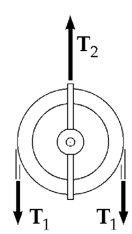


FIG. P5.21(b)

$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\ \text{N}}.$$

(c) $\sum \mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$

Take the component along the incline

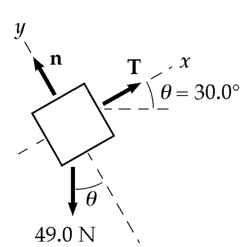


FIG. P5.21(c)

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$
$$= \boxed{24.5\ \text{N}}.$$

P5.22 The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x-axis is chosen to be parallel to the plane, then the free body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction) we have

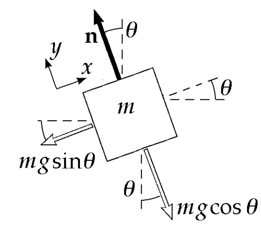


FIG. P5.22

$$\sum F_y = n - mg\cos\theta = 0:\ n = mg\cos\theta$$
$$\sum F_x = -mg\sin\theta = ma:\ a = -g\sin\theta$$

(a) When $\theta = 15.0°$

$$a = \boxed{-2.54\ \text{m/s}^2}$$

(b) Starting from rest

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2ax_f$$
$$\left|v_f\right| = \sqrt{2ax_f} = \sqrt{2\left(-2.54\ \text{m/s}^2\right)(-2.00\ \text{m})} = \boxed{3.18\ \text{m/s}}$$

P5.23 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North.

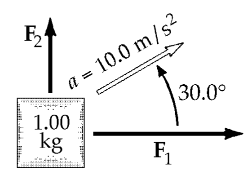


FIG. P5.23

$$\sum \mathbf{F} = m\mathbf{a} = 1.00\ \text{kg}\left(10.0\ \text{m/s}^2\right)\ \text{at}\ 30.0°$$
$$(5.00\ \text{N})\hat{\mathbf{j}} + \mathbf{F}_1 = (10.0\ \text{N})\angle 30.0° = (5.00\ \text{N})\hat{\mathbf{j}} + (8.66\ \text{N})\hat{\mathbf{i}}$$
$$\therefore F_1 = \boxed{8.66\ \text{N (East)}}$$

***P5.24** First, consider the block moving along the horizontal. The only force in the direction of movement is T. Thus, $\sum F_x = ma$

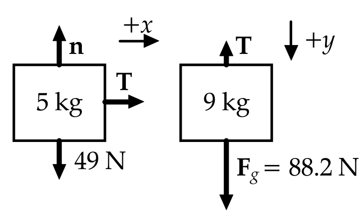


FIG. P5.24

$$T = (5\ \text{kg})a \qquad (1)$$

Next consider the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$

$$88.2\ \text{N} - T = (9\ \text{kg})a \qquad (2)$$

Note that both blocks must have the same magnitude of acceleration. Equations (1) and (2) can be added to give $88.2\ \text{N} = (14\ \text{kg})a$. Then

$$\boxed{a = 6.30\ \text{m/s}^2\ \text{and}\ T = 31.5\ \text{N}}.$$

P5.25 After it leaves your hand, the block's speed changes only because of one component of its weight:

$$\sum F_x = ma_x \quad -mg\sin 20.0° = ma$$
$$v_f^2 = v_i^2 + 2a(x_f - x_i).$$

Taking $v_f = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\text{ m}}.$$

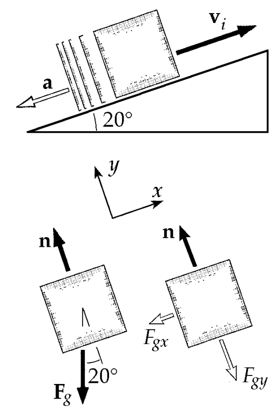


FIG. P5.25

P5.26 $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 55.0°$

(a) $\sum F_x = m_2 g\sin\theta - T = m_2 a$

and

$$T - m_1 g = m_1 a$$
$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57\text{ m/s}^2}$$

(b) $T = m_1(a+g) = \boxed{26.7\text{ N}}$

(c) Since $v_i = 0$, $v_f = at = (3.57\text{ m/s}^2)(2.00\text{ s}) = \boxed{7.14\text{ m/s}}$.

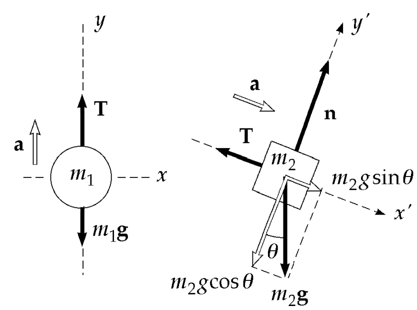


FIG. P5.26

***P5.27** We assume the vertical bar is in compression, pushing up on the pin with force A, and the tilted bar is in tension, exerting force B on the pin at $-50°$.

$$\sum F_x = 0: \quad -2\,500\text{ N}\cos 30° + B\cos 50° = 0$$
$$\boxed{B = 3.37\times 10^3\text{ N}}$$
$$\sum F_y = 0: \quad -2\,500\text{ N}\sin 30° + A - 3.37\times 10^3\text{ N}\sin 50° = 0$$
$$\boxed{A = 3.83\times 10^3\text{ N}}$$

Positive answers confirm that

$\boxed{B\text{ is in tension and }A\text{ is in compression.}}$

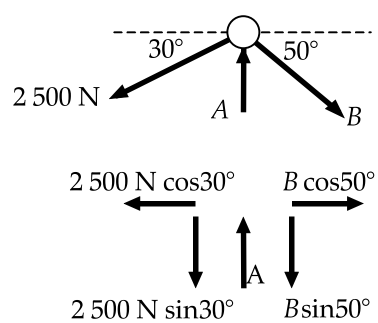


FIG. P5.27

P5.28 First, consider the 3.00 kg rising mass. The forces on it are the tension, T, and its weight, 29.4 N. With the upward direction as positive, the second law becomes

$$\sum F_y = ma_y: T - 29.4 \text{ N} = (3.00 \text{ kg})a \quad (1)$$

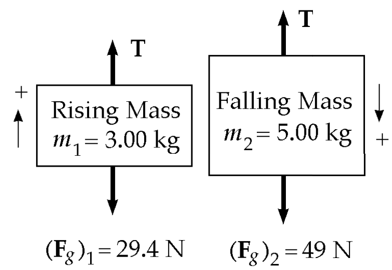


FIG. P5.28

The forces on the falling 5.00 kg mass are its weight and T, and its acceleration is the same as that of the rising mass. Calling the positive direction down for this mass, we have

$$\sum F_y = ma_y: 49 \text{ N} - T = (5.00 \text{ kg})a \quad (2)$$

Equations (1) and (2) can be solved simultaneously by adding them:

$$T - 29.4 \text{ N} + 49.0 \text{ N} - T = (3.00 \text{ kg})a + (5.00 \text{ kg})a$$

(b) This gives the acceleration as

$$a = \frac{19.6 \text{ N}}{8.00 \text{ kg}} = \boxed{2.45 \text{ m/s}^2}.$$

(a) Then

$$T - 29.4 \text{ N} = (3.00 \text{ kg})(2.45 \text{ m/s}^2) = 7.35 \text{ N}.$$

The tension is

$$T = \boxed{36.8 \text{ N}}.$$

(c) Consider either mass. We have

$$y = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2.45 \text{ m/s}^2)(1.00 \text{ s})^2 = \boxed{1.23 \text{ m}}.$$

***P5.29** As the man rises steadily the pulley turns steadily and the tension in the rope is the same on both sides of the pulley. Choose man-pulley-and-platform as the system:

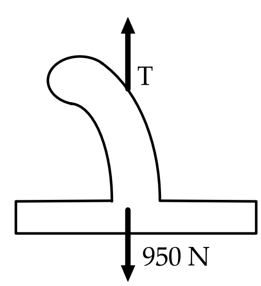


FIG. P5.29

$$\sum F_y = ma_y$$
$$+T - 950 \text{ N} = 0$$
$$T = 950 \text{ N}.$$

The worker must pull on the rope with force $\boxed{950 \text{ N}}$.

***P5.30** Both blocks move with acceleration $a=\left(\frac{m_2-m_1}{m_2+m_1}\right)g$:

$$a=\left(\frac{7\text{ kg}-2\text{ kg}}{7\text{ kg}+2\text{ kg}}\right)9.8\text{ m/s}^2=5.44\text{ m/s}^2.$$

(a) Take the upward direction as positive for m_1.

$$v_{xf}^2=v_{xi}^2+2a_x\left(x_f-x_i\right):\quad 0=\left(-2.4\text{ m/s}\right)^2+2\left(5.44\text{ m/s}^2\right)\left(x_f-0\right)$$

$$x_f=-\frac{5.76\text{ m}^2/\text{s}^2}{2\left(5.44\text{ m/s}^2\right)}=-0.529\text{ m}$$

$$x_f=\boxed{0.529\text{ m below its initial level}}$$

(b) $$v_{xf}=v_{xi}+a_xt:\quad v_{xf}=-2.40\text{ m/s}+\left(5.44\text{ m/s}^2\right)(1.80\text{ s})$$

$$v_{xf}=\boxed{7.40\text{ m/s upward}}$$

P5.31 Forces acting on 2.00 kg block:

$$T-m_1g=m_1a \quad (1)$$

Forces acting on 8.00 kg block:

$$F_x-T=m_2a \quad (2)$$

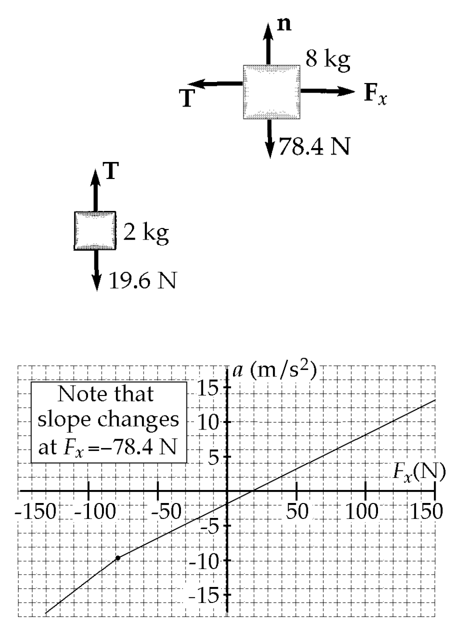


FIG. P5.31

(a) Eliminate T and solve for a:

$$a=\frac{F_x-m_1g}{m_1+m_2}$$

$$\boxed{a>0\text{ for }F_x>m_1g=19.6\text{ N}}.$$

(b) Eliminate a and solve for T:

$$T=\frac{m_1}{m_1+m_2}\left(F_x+m_2g\right)$$

$$\boxed{T=0\text{ for }F_x\le -m_2g=-78.4\text{ N}}.$$

(c)

F_x, N	–100	–78.4	–50.0	0	50.0	100
a_x, m/s²	–12.5	–9.80	–6.96	–1.96	3.04	8.04

***P5.32** (a) For force components along the incline, with the upward direction taken as positive,

$$\sum F_x = ma_x: \quad -mg\sin\theta = ma_x$$

$$a_x = -g\sin\theta = -\left(9.8\ \text{m/s}^2\right)\sin 35° = -5.62\ \text{m/s}^2 .$$

For the upward motion,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

$$0 = (5\ \text{m/s})^2 + 2\left(-5.62\ \text{m/s}^2\right)\left(x_f - 0\right)$$

$$x_f = \frac{25\ \text{m}^2/\text{s}^2}{2\left(5.62\ \text{m/s}^2\right)} = \boxed{2.22\ \text{m}}.$$

(b) The time to slide down is given by

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$0 = 2.22\ \text{m} + 0 + \frac{1}{2}\left(-5.62\ \text{m/s}^2\right)t^2$$

$$t = \sqrt{\frac{2(2.22\ \text{m})}{5.62\ \text{m/s}^2}} = 0.890\ \text{s}.$$

For the second particle,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$0 = 10\ \text{m} + v_{xi}(0.890\ \text{s}) + \left(-5.62\ \text{m/s}^2\right)(0.890\ \text{s})^2$$

$$v_{xi} = \frac{-10\ \text{m} + 2.22\ \text{m}}{0.890\ \text{s}} = -8.74\ \text{m/s}$$

$$\text{speed} = \boxed{8.74\ \text{m/s}}.$$

P5.33 First, we will compute the needed accelerations:

(1) Before it starts to move: $a_y = 0$

(2) During the first 0.800 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{1.20\ \text{m/s} - 0}{0.800\ \text{s}} = 1.50\ \text{m/s}^2$

(3) While moving at constant velocity: $a_y = 0$

(4) During the last 1.50 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{0 - 1.20\ \text{m/s}}{1.50\ \text{s}} = -0.800\ \text{m/s}^2$

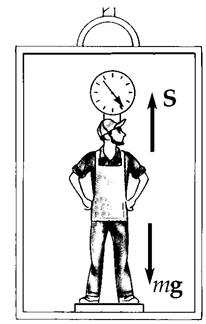


FIG. P5.33

Newton's second law is: $\sum F_y = ma_y$

$$+S - (72.0\ \text{kg})(9.80\ \text{m/s}^2) = (72.0\ \text{kg})a_y$$

$$S = 706\ \text{N} + (72.0\ \text{kg})a_y\,.$$

(a) When $a_y = 0$, $S = \boxed{706\ \text{N}}$.

(b) When $a_y = 1.50\ \text{m/s}^2$, $S = \boxed{814\ \text{N}}$.

(c) When $a_y = 0$, $S = \boxed{706\ \text{N}}$.

(d) When $a_y = -0.800\ \text{m/s}^2$, $S = \boxed{648\ \text{N}}$.

P5.34 (a) Pulley P_1 has acceleration a_2.
Since m_1 moves *twice* the distance P_1 moves in the same time, m_1 has twice the acceleration of P_1, i.e., $\boxed{a_1 = 2a_2}$.

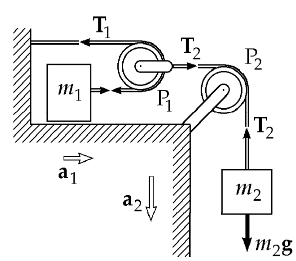


FIG. P5.34

(b) From the figure, and using

$$\sum F = ma: \quad m_2 g - T_2 = m_2 a_2 \qquad (1)$$

$$T_1 = m_1 a_1 = 2m_1 a_2 \qquad (2)$$

$$T_2 - 2T_1 = 0 \qquad (3)$$

Equation (1) becomes $m_2 g - 2T_1 = m_2 a_2$. This equation combined with Equation (2) yields

$$\frac{T_1}{m_1}\left(2m_1 + \frac{m_2}{2}\right) = m_2 g$$

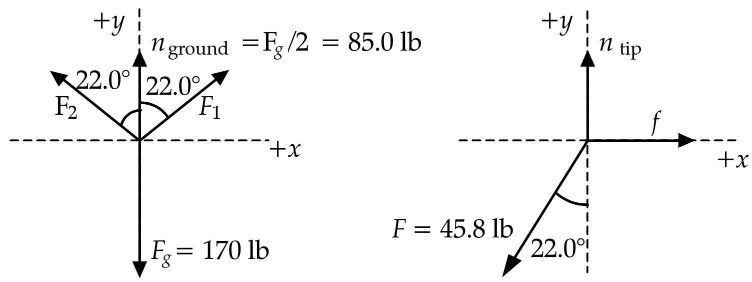
$$\boxed{T_1 = \frac{m_1 m_2}{2m_1 + \frac{1}{2}m_2} g} \text{ and } \boxed{T_2 = \frac{m_1 m_2}{m_1 + \frac{1}{4}m_2} g}.$$

(c) From the values of T_1 and T_2 we find that

$$a_1 = \frac{T_1}{m_1} = \boxed{\frac{m_2 g}{2m_1 + \frac{1}{2}m_2}} \text{ and } a_2 = \frac{1}{2}a_1 = \boxed{\frac{m_2 g}{4m_1 + m_2}}.$$

Section 5.8 **Forces of Friction**

***P5.35**



Free-Body Diagram of Person   Free-Body Diagram of Crutch Tip

FIG. P5.35

From the free-body diagram of the person,

$$\sum F_x = F_1 \sin(22.0°) - F_2 \sin(22.0°) = 0,$$

which gives

$$F_1 = F_2 = F.$$

Then, $\sum F_y = 2F\cos 22.0° + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb})\sin 22.0° = 0,$$

or

$$f = 17.2 \text{ lb}.$$

$$\sum F_y = n_{\text{tip}} - (45.8 \text{ lb})\cos 22.0° = 0,$$

which gives

$$n_{\text{tip}} = 42.5 \text{ lb}.$$

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{\max} = \mu_s n_{\text{tip}}$ and $\mu_s = \dfrac{f}{n_{\text{tip}}} = \dfrac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}$.

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}.$$

P5.36 For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$ i.e.,

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$

$$\mu_s = \frac{75.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.306}$$

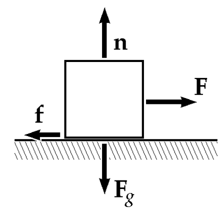


FIG. P5.36

and

$$\mu_k = \frac{60.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.245}.$$

P5.37 $\sum F_y = ma_y$: $\quad +n - mg = 0$

$$f_s \le \mu_s n = \mu_s mg$$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x: \quad -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g.$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0 \text{ mi/h} = 22.4 \text{ m/s}$, $v_f = 0$

$$v_f^2 = v_i^2 + 2a(x_f - x_i): \quad -v_i^2 = -2\mu_s g x_f$$

(a) $$x_f = \frac{v_i^2}{2\mu g}$$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.100)(9.80 \text{ m/s}^2)} = \boxed{256 \text{ m}}$$

(b) $$x_f = \frac{v_i^2}{2\mu g}$$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.600)(9.80 \text{ m/s}^2)} = \boxed{42.7 \text{ m}}$$

P5.38 If all the weight is on the rear wheels,

(a) $F = ma$: $\mu_s mg = ma$
But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s g t^2}{2}$$

so $\mu_s = \dfrac{2\Delta x}{gt^2}$:

$$\mu_s = \frac{2(0.250 \text{ mi})(1\,609 \text{ m/mi})}{(9.80 \text{ m/s}^2)(4.96 \text{ s})^2} = \boxed{3.34}.$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

***P5.39** (a) The person pushes backward on the floor. The floor pushes forward on the person with a force of friction. This is the only horizontal force on the person. If the person's shoe is on the point of slipping the static friction force has its maximum value.

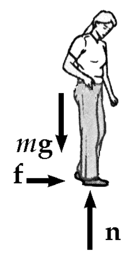


FIG. P5.39

$$\sum F_x = ma_x: \qquad f = \mu_s n = ma_x$$
$$\sum F_y = ma_y: \qquad n - mg = 0$$
$$ma_x = \mu_s mg \qquad a_x = \mu_s g = 0.5(9.8 \text{ m/s}^2) = 4.9 \text{ m/s}^2$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad 3 \text{ m} = 0 + 0 + \frac{1}{2}(4.9 \text{ m/s}^2)t^2$$
$$t = \boxed{1.11 \text{ s}}$$

(b) $x_f = \dfrac{1}{2}\mu_s g t^2$, $t = \sqrt{\dfrac{2x_f}{\mu_s g}} = \sqrt{\dfrac{2(3 \text{ m})}{(0.8)(9.8 \text{ m/s}^2)}} = \boxed{0.875 \text{ s}}$

P5.40 $m_{\text{suitcase}} = 20.0 \text{ kg}$, $F = 35.0 \text{ N}$

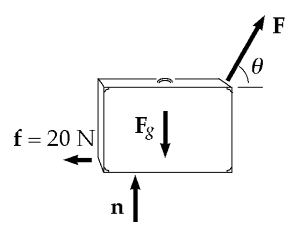


FIG. P5.40

$$\sum F_x = ma_x: \quad -20.0 \text{ N} + F\cos\theta = 0$$
$$\sum F_y = ma_y: \quad +n + F\sin\theta - F_g = 0$$

(a) $F\cos\theta = 20.0 \text{ N}$

$$\cos\theta = \frac{20.0 \text{ N}}{35.0 \text{ N}} = 0.571$$

$\boxed{\theta = 55.2°}$

(b) $n = F_g - F\sin\theta = [196 - 35.0(0.821)] \text{ N}$

$\boxed{n = 167 \text{ N}}$

P5.41 $m = 3.00 \text{ kg}$, $\theta = 30.0°$, $x = 2.00 \text{ m}$, $t = 1.50 \text{ s}$

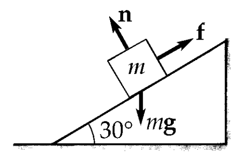


FIG. P5.41

(a) $x = \frac{1}{2}at^2$:

$$2.00 \text{ m} = \frac{1}{2}a(1.50 \text{ s})^2$$
$$a = \frac{4.00}{(1.50)^2} = \boxed{1.78 \text{ m/s}^2}$$

$\sum \mathbf{F} = \mathbf{n} + \mathbf{f} + m\mathbf{g} = m\mathbf{a}$:

$$\text{Along } x\text{:} \quad 0 - f + mg\sin 30.0° = ma$$
$$f = m(g\sin 30.0° - a)$$
$$\text{Along } y\text{:} \quad n + 0 - mg\cos 30.0° = 0$$
$$n = mg\cos 30.0°$$

(b) $\mu_k = \frac{f}{n} = \frac{m(g\sin 30.0° - a)}{mg\cos 30.0°}$, $\mu_k = \tan 30.0° - \frac{a}{g\cos 30.0°} = \boxed{0.368}$

(c) $f = m(g\sin 30.0° - a)$, $f = 3.00(9.80\sin 30.0° - 1.78) = \boxed{9.37 \text{ N}}$

(d) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

where

$$x_f - x_i = 2.00 \text{ m}$$

$$v_f^2 = 0 + 2(1.78)(2.00) = 7.11 \text{ m}^2/\text{s}^2$$
$$v_f = \sqrt{7.11 \text{ m}^2/\text{s}^2} = \boxed{2.67 \text{ m/s}}$$

***P5.42** First we find the coefficient of friction:

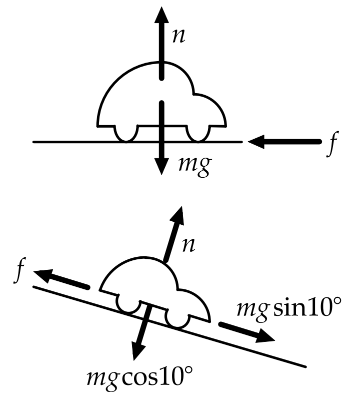


FIG. P5.42

$$\sum F_y = 0: \qquad +n - mg = 0$$
$$f = \mu_s n = \mu_s mg$$
$$\sum F_x = ma_x: \qquad v_f^2 = v_i^2 + 2a_x \Delta x = 0$$

$$-\mu_s mg = -\frac{mv_i^2}{2\Delta x}$$
$$\mu_s = \frac{v_i^2}{2g\Delta x} = \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(123\ \text{ft})} = 0.981$$

Now on the slope

$$\sum F_y = 0: \qquad +n - mg\cos 10° = 0$$
$$f_s = \mu_s n = \mu_s mg\cos 10°$$
$$\sum F_x = ma_x: \quad -\mu_s mg\cos 10° + mg\sin 10° = -\frac{mv_i^2}{2\Delta x}$$
$$\Delta x = \frac{v_i^2}{2g(\mu_s \cos 10° - \sin 10°)}$$
$$= \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(0.981\cos 10° - \sin 10°)} = \boxed{152\ \text{ft}}.$$

P5.43 $T - f_k = 5.00a$ (for 5.00 kg mass)

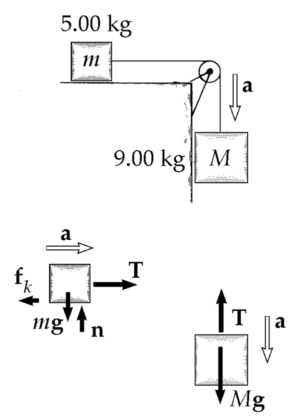


FIG. P5.43

$9.00g - T = 9.00a$ (for 9.00 kg mass)

Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$
$$a = 5.60\ \text{m/s}^2$$
$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80)$$
$$= \boxed{37.8\ \text{N}}$$

P5.44 Let a represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of m_1, of the acceleration $-a\hat{\mathbf{i}}$ of m_2, and of the acceleration $+a\hat{\mathbf{j}}$ of m_3. Call T_{12} the tension in the left rope and T_{23} the tension in the cord on the right.

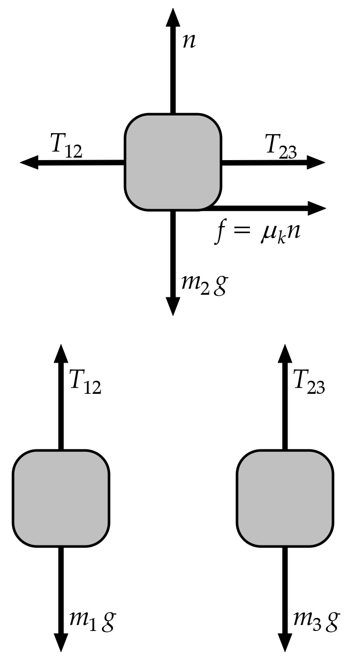


FIG. P5.44

For m_1, $\sum F_y = ma_y$ $\quad +T_{12} - m_1 g = -m_1 a$

For m_2, $\sum F_x = ma_x$ $\quad -T_{12} + \mu_k n + T_{23} = -m_2 a$

and $\sum F_y = ma_y$ $\quad n - m_2 g = 0$

for m_3, $\sum F_y = ma_y$ $\quad T_{23} - m_3 g = +m_3 a$

we have three simultaneous equations

$$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$$
$$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$$
$$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a.$$

(a) Add them up:

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}.$$

(b) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{23} = 24.2\text{ N}}.$$

P5.45 (a) See Figure to the right

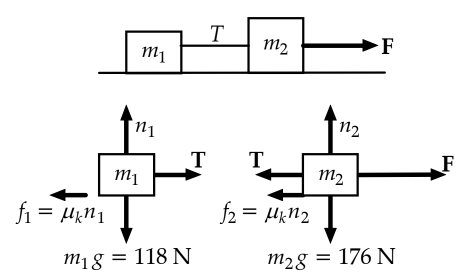


FIG. P5.45

(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)

$T - \mu m_1 g = m_1 a$ (Block #1)

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$
$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\text{ m/s}^2}$$
$$T = m_1 a + \mu m_1 g = \boxed{27.2\text{ N}}$$

P5.46 (Case 1, impending upward motion)
Setting

$$\sum F_x = 0: \quad P\cos 50.0° - n = 0$$
$$f_{s,\,\max} = \mu_s n: \quad f_{s,\,\max} = \mu_s P\cos 50.0°$$
$$= 0.250(0.643)P = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$
$$P_{\max} = \boxed{48.6\text{ N}}$$

(Case 2, impending downward motion)
As in Case 1,

$$f_{s,\,\max} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$
$$P_{\min} = \boxed{31.7\text{ N}}$$

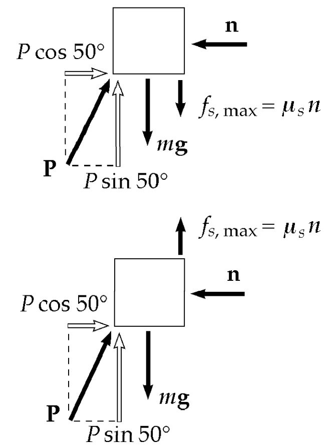


FIG. P5.46

***P5.47** When the sled is sliding uphill

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$f = \mu_k n = \mu_k mg\cos\theta$$
$$\sum F_x = ma_x: \quad +mg\sin\theta + \mu_k mg\cos\theta = ma_{\text{up}}$$
$$v_f = 0 = v_i + a_{\text{up}}t_{\text{up}}$$
$$v_i = -a_{\text{up}}t_{\text{up}}$$
$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)t_{\text{up}}$$
$$\Delta x = \frac{1}{2}\left(a_{\text{up}}t_{\text{up}} + 0\right)t_{\text{up}} = \frac{1}{2}a_{\text{up}}t_{\text{up}}^2$$

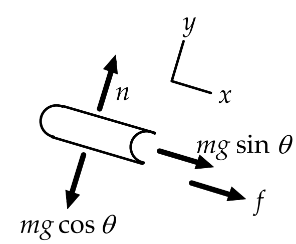


FIG. P5.47

When the sled is sliding down, the direction of the friction force is reversed:

$$mg\sin\theta - \mu_k mg\cos\theta = ma_{\text{down}}$$
$$\Delta x = \frac{1}{2}a_{\text{down}}t_{\text{down}}^2.$$

Now

$$t_{\text{down}} = 2t_{\text{up}}$$
$$\frac{1}{2}a_{\text{up}}t_{\text{up}}^2 = \frac{1}{2}a_{\text{down}}\left(2t_{\text{up}}\right)^2$$
$$a_{\text{up}} = 4a_{\text{down}}$$
$$g\sin\theta + \mu_k g\cos\theta = 4\left(g\sin\theta - \mu_k g\cos\theta\right)$$
$$5\mu_k\cos\theta = 3\sin\theta$$
$$\boxed{\mu_k = \left(\frac{3}{5}\right)\tan\theta}$$

***P5.48** Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

$$f = (f_s)_{\max} = \mu_s n.$$

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0, \text{ or } f = \frac{F_g}{2}.$$

The minimum compression force needed is then

$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5 \text{ N}}{2(0.663)} = \boxed{72.0 \text{ N}}.$$

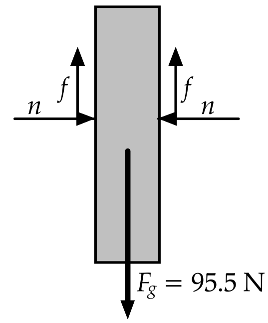


FIG. P5.48

***P5.49** (a) $n + F\sin 15° - (75 \text{ N})\cos 25° = 0$

$\therefore n = 67.97 - 0.259F$

$f_{s,\,\max} = \mu_s n = 24.67 - 0.094F$

For equilibrium: $F\cos 15° + 24.67 - 0.094F - 75\sin 25° = 0$.
This gives $\boxed{F = 8.05 \text{ N}}$.

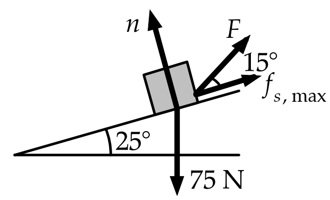


FIG. P5.49(a)

(b) $F\cos 15° - (24.67 - 0.094F) - 75\sin 25° = 0$.
This gives $\boxed{F = 53.2 \text{ N}}$.

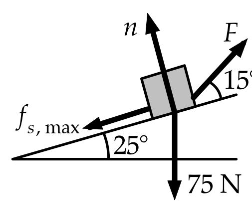


FIG. P5.49(b)

(c) $f_k = \mu_k n = 10.6 - 0.040F$. Since the velocity is constant, the net force is zero:

$$F\cos 15° - (10.6 - 0.040F) - 75\sin 25° = 0.$$

This gives $\boxed{F = 42.0 \text{ N}}$.

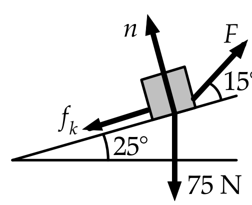


FIG. P5.49(c)

***P5.50** We must consider separately the disk when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take x and y as parallel and perpendicular to the surface of the roof:

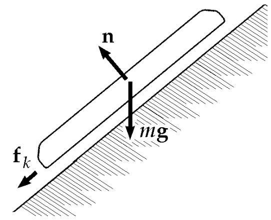


FIG. P5.50

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$
$$a_x = -\mu_k g\cos\theta - g\sin\theta = (-0.4\cos 37° - \sin 37°)9.8\ \text{m/s}^2 = -9.03\ \text{m/s}^2$$

The Frisbee goes ballistic with speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) = (15\ \text{m/s})^2 + 2(-9.03\ \text{m/s}^2)(10\ \text{m} - 0) = 44.4\ \text{m}^2/\text{s}^2$$
$$v_{xf} = 6.67\ \text{m/s}$$

For the free fall, we take x and y horizontal and vertical:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = (6.67\ \text{m/s}\ \sin 37°)^2 + 2(-9.8\ \text{m/s}^2)(y_f - 10\ \text{m}\ \sin 37°)$$
$$y_f = 6.02\ \text{m} + \frac{(4.01\ \text{m/s})^2}{19.6\ \text{m/s}^2} = \boxed{6.84\ \text{m}}$$

Additional Problems

P5.51 (a) see figure to the right

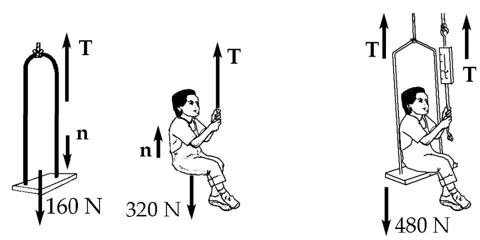


FIG. P5.51

(b) First consider Pat and the chair as the system. Note that *two* ropes support the system, and $T = 250$ N in each rope. Applying $\sum F = ma$

$$2T - 480 = ma, \text{ where } m = \frac{480}{9.80} = 49.0\ \text{kg}.$$

Solving for a gives

$$a = \frac{500 - 480}{49.0} = \boxed{0.408\ \text{m/s}^2}.$$

(c) $\sum F = ma$ on Pat:

$$\sum F = n + T - 320 = ma, \text{ where } m = \frac{320}{9.80} = 32.7\ \text{kg}$$

$$n = ma + 320 - T = 32.7(0.408) + 320 - 250 = \boxed{83.3\ \text{N}}.$$

P5.52 $\sum \mathbf{F} = m\mathbf{a}$ gives the object's acceleration

$$\mathbf{a} = \frac{\sum F}{m} = \frac{\left(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$

$$\mathbf{a} = \left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}} = \frac{d\mathbf{v}}{dt}.$$

Its velocity is

$$\int_{v_i}^{v} d\mathbf{v} = \mathbf{v} - \mathbf{v}_i = \mathbf{v} - 0 = \int_0^t \mathbf{a}\,dt$$

$$\mathbf{v} = \int_0^t \left[\left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}}\right]dt$$

$$\mathbf{v} = \left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}.$$

(a) We require $|\mathbf{v}| = 15.0\text{ m/s}$, $|\mathbf{v}|^2 = 225\text{ m}^2/\text{s}^2$

$$16.0t^2\text{ m}^2/\text{s}^4 + 1.00t^4\text{ m}^2/\text{s}^6 = 225\text{ m}^2/\text{s}^2$$

$$1.00t^4 + 16.0\text{ s}^2t^2 - 225\text{ s}^4 = 0$$

$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00\text{ s}^2$$

$$t = \boxed{3.00\text{ s}}.$$

Take $\mathbf{r}_i = 0$ at $t = 0$. The position is

$$\mathbf{r} = \int_0^t \mathbf{v}\,dt = \int_0^t \left[\left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}\right]dt$$

$$\mathbf{r} = \left(4.00\text{ m/s}^2\right)\frac{t^2}{2}\hat{\mathbf{i}} - \left(1.00\text{ m/s}^3\right)\frac{t^3}{3}\hat{\mathbf{j}}$$

at $t = 3$ s we evaluate.

(c) $\mathbf{r} = \boxed{\left(18.0\hat{\mathbf{i}} - 9.00\hat{\mathbf{j}}\right)\text{ m}}$

(b) So $|\mathbf{r}| = \sqrt{(18.0)^2 + (9.00)^2}\text{ m} = \boxed{20.1\text{ m}}$

***P5.53** (a) Situation A

$$\sum F_x = ma_x: \quad F_A + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$

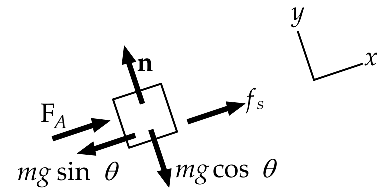


FIG. P5.53(a)

Eliminate $n = mg\cos\theta$ to solve for

$$\boxed{F_A = mg(\sin\theta - \mu_s\cos\theta)}.$$

(b) Situation B

$$\sum F_x = ma_x: \quad F_B\cos\theta + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad -F_B\sin\theta + n - mg\cos\theta = 0$$

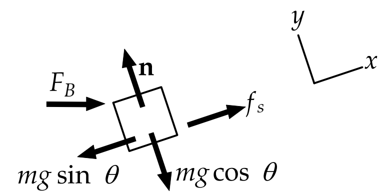


FIG. P5.53(b)

Substitute $n = mg\cos\theta + F_B\sin\theta$ to find

$$F_B\cos\theta + \mu_s mg\cos\theta + \mu_s F_B\sin\theta - mg\sin\theta = 0$$
$$\boxed{F_B = \frac{mg(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}}$$

(c) $F_A = 2\text{ kg } 9.8\text{ m/s}^2(\sin 25°–0.16\cos 25°) = 5.44\text{ N}$

$$F_B = \frac{19.6\text{ N}(0.278)}{\cos 25°+0.16\sin 25°} = 5.59\text{ N}$$

Student $\boxed{A}$ need exert less force.

(d) $$F_B = \frac{F_A}{\cos 25°+0.38\sin 25°} = \frac{F_A}{1.07}$$

Student $\boxed{B}$ need exert less force.

P5.54

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$

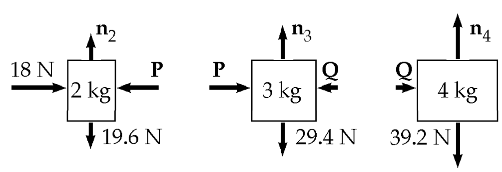


FIG. P5.54

Adding gives $18\text{ N} = (9\text{ kg})a$ so

$$a = \boxed{2.00\text{ m/s}^2}.$$

(b) $Q = 4\text{ kg}(2\text{ m/s}^2) = \boxed{8.00\text{ N net force on the 4 kg}}$

$P - 8\text{ N} = 3\text{ kg}(2\text{ m/s}^2) = \boxed{6.00\text{ N net force on the 3 kg}}$ and $P = 14\text{ N}$

$18\text{ N} - 14\text{ N} = 2\text{ kg}(2\text{ m/s}^2) = \boxed{4.00\text{ N net force on the 2 kg}}$

continued on next page

(c) From above, $Q = \boxed{8.00\text{ N}}$ and $P = \boxed{14.0\text{ N}}$.

(d) The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but lasts for so short a time that it never is associated with a large velocity. The frame of the building and your legs exert forces, small relative to the hammer blow, to bring the partition, block, and you to rest again over a time large relative to the hammer blow. This problem lends itself to interesting lecture demonstrations. One person can hold a lead brick in one hand while another hits the brick with a hammer.

P5.55 (a) First, we note that $F = T_1$. Next, we focus on the mass M and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.

Since the pulleys are not starting to rotate and are frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2T_2$, so $T_2 = \dfrac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \dfrac{Mg}{2}}$, and $\boxed{T_4 = \dfrac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(b) Since $F = T_1$, we have $\boxed{F = \dfrac{Mg}{2}}$.

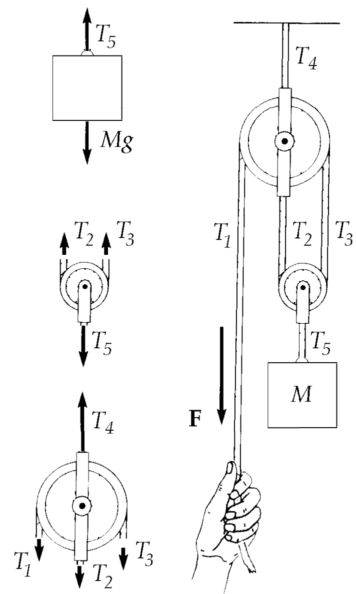


FIG. P5.55

P5.56 We find the diver's impact speed by analyzing his free-fall motion:

$$v_f^2 = v_i^2 + 2ax = 0 + 2\left(-9.80\text{ m/s}^2\right)(-10.0\text{ m}) \text{ so } v_f = -14.0\text{ m/s}.$$

Now for the 2.00 s of stopping, we have $v_f = v_i + at$:

$$0 = -14.0\text{ m/s} + a(2.00\text{ s})$$
$$a = +7.00\text{ m/s}^2.$$

Call the force exerted by the water on the diver R. Using $\sum F_y = ma$,

$$+R - 70.0\text{ kg}\left(9.80\text{ m/s}^2\right) = 70.0\text{ kg}\left(7.00\text{ m/s}^2\right)$$
$$R = \boxed{1.18\text{ kN}}.$$

P5.57 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.

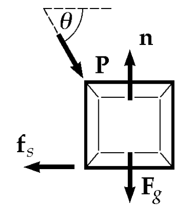


FIG. P5.57

Resolving vertically:

$$n = F_g + P\sin\theta$$

Horizontally:

$$P\cos\theta = f_s$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P(\cos\theta - \mu_s \sin\theta) \le \mu_s F_g.$$

Divide by $\cos\theta$:

$$P(1 - \mu_s \tan\theta) \le \mu_s F_g \sec\theta.$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}}.$$

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1 - 0.400\tan\theta}$

$\theta(\text{deg})$	0.00	15.0	30.0	45.0	60.0
$P(N)$	40.0	46.4	60.1	94.3	260

If the angle were 68.2° or more, the expression for P would go to infinity and motion would become impossible.

P5.58 (a) Following the in-chapter Example about a block on a frictionless incline, we have

$$a = g\sin\theta = \left(9.80\ \text{m/s}^2\right)\sin 30.0°$$

$$\boxed{a = 4.90\ \text{m/s}^2}$$

(b) The block slides distance x on the incline, with $\sin 30.0° = \dfrac{0.500\ \text{m}}{x}$

$x = 1.00\ \text{m}$: $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0 + 2\left(4.90\ \text{m/s}^2\right)(1.00\ \text{m})$

$v_f = \boxed{3.13\ \text{m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2(1.00\ \text{m})}{3.13\ \text{m/s}} = 0.639\ \text{s}$.

(c) Now in free fall $y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$:

$$-2.00 = (-3.13\ \text{m/s})\sin 30.0°\, t - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

$$\left(4.90\ \text{m/s}^2\right)t^2 + (1.56\ \text{m/s})t - 2.00\ \text{m} = 0$$

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{(1.56\ \text{m/s})^2 - 4\left(4.90\ \text{m/s}^2\right)(-2.00\ \text{m})}}{9.80\ \text{m/s}^2}$$

Only one root is physical

$$t = 0.499\ \text{s}$$

$$x_f = v_x t = \left[(3.13\ \text{m/s})\cos 30.0°\right](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

(d) total time $= t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$

(e) The mass of the block makes no difference.

P5.59 With motion impending,

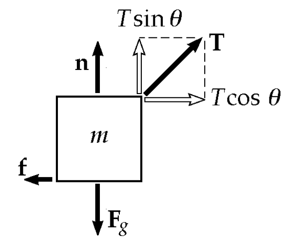


FIG. P5.59

$$n + T\sin\theta - mg = 0$$

$$f = \mu_s(mg - T\sin\theta)$$

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}.$$

To minimize T, we maximize $\cos\theta + \mu_s \sin\theta$

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0 = -\sin\theta + \mu_s\cos\theta.$$

(a) $\theta = \tan^{-1}\mu_s = \tan^{-1}0.350 = \boxed{19.3°}$

(b) $T = \dfrac{0.350(1.30\text{ kg})(9.80\text{ m/s}^2)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$

***P5.60** (a) See Figure (a) to the right.

(b) See Figure (b) to the right.

(c) For the pin,

$$\sum F_y = ma_y:\quad C\cos\theta - 357\text{ N} = 0$$

$$C = \frac{357\text{ N}}{\cos\theta}.$$

$m\mathbf{g} = (36.4\text{ kg})(9.8\text{ m/s}^2) = 357\text{ N}$

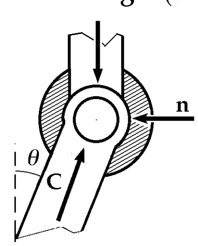


FIG. P5.60(a)

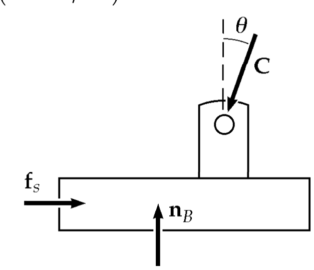


FIG. P5.60(b)

For the foot,

$$\sum F_y = ma_y:\quad +n_B - C\cos\theta = 0$$

$$n_B = \boxed{357\text{ N}}.$$

(d) For the foot with motion impending,

$$\sum F_x = ma_x:\quad +f_s - C\sin\theta_s = 0$$

$$\mu_s n_B = C\sin\theta_s$$

$$\mu_s = \frac{C\sin\theta_s}{n_B} = \frac{(357\text{ N}/\cos\theta_s)\sin\theta_s}{357\text{ N}} = \tan\theta_s.$$

(e) The maximum coefficient is

$$\mu_s = \tan\theta_s = \tan 50.2° = \boxed{1.20}.$$

P5.61 $\sum F = ma$

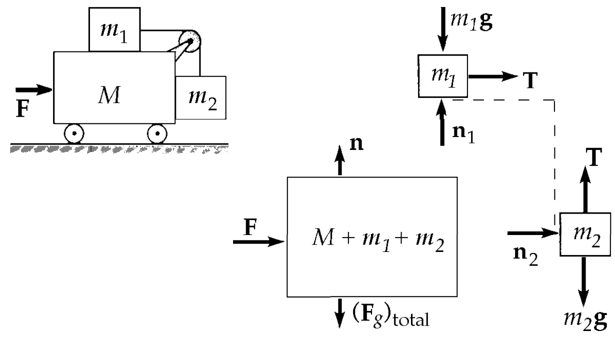


FIG. P5.61

For m_1: $T = m_1 a$

For m_2: $T - m_2 g = 0$

Eliminating T,

$$a = \frac{m_2 g}{m_1}$$

For all 3 blocks:

$$F = (M + m_1 + m_2)a = \boxed{(M + m_1 + m_2)\left(\frac{m_2 g}{m_1}\right)}$$

P5.62

t(s)	$t^2\left(s^2\right)$	x(m)
0	0	0
1.02	1.04 0	0.100
1.53	2.34 1	0.200
2.01	4.04 0	0.350
2.64	6.97 0	0.500
3.30	10.89	0.750
3.75	14.06	1.00

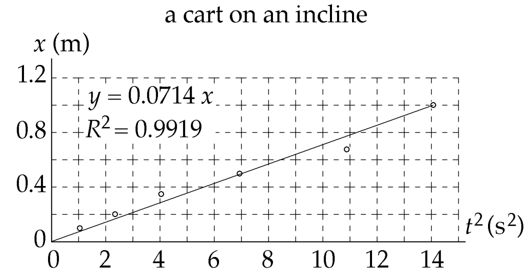


FIG. P5.62

From $x = \frac{1}{2}at^2$ the slope of a graph of x versus t^2 is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2\left(0.071\,4 \text{ m/s}^2\right) = \boxed{0.143 \text{ m/s}^2}.$$

From $a' = g\sin\theta$,

$$a' = 9.80 \text{ m/s}^2\left(\frac{1.77\,4}{127.1}\right) = 0.137 \text{ m/s}^2\text{, different by } 4\%.$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.071\,4)(4.04)}{0.350} = 18\%.$$

P5.63 (1) $m_1(a-A)=T \Rightarrow a=\dfrac{T}{m_1}+A$

(2) $MA=R_x=T \Rightarrow A=\dfrac{T}{M}$

(3) $m_2a=m_2g-T \Rightarrow T=m_2(g-a)$

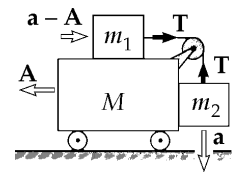


FIG. P5.63

(a) Substitute the value for a from (1) into (3) and solve for T:

$$T=m_2\left[g-\left(\frac{T}{m_1}+A\right)\right].$$

Substitute for A from (2):

$$T=m_2\left[g-\left(\frac{T}{m_1}+\frac{T}{M}\right)\right]=\boxed{m_2g\left[\frac{m_1M}{m_1M+m_2(m_1+M)}\right]}.$$

(b) Solve (3) for a and substitute value of T:

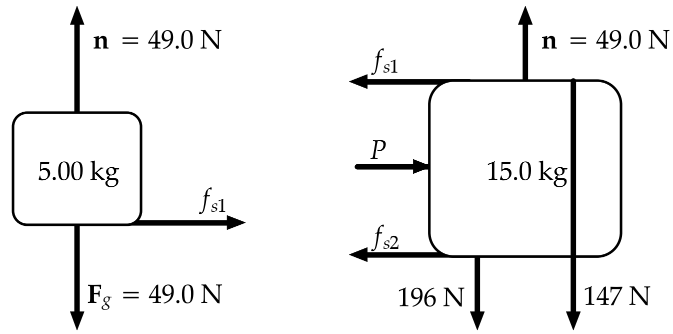
$$\boxed{a=\frac{m_2g(m_1+M)}{m_1M+m_2(M+m_1)}}.$$

(c) From (2), $A=\dfrac{T}{M}$, Substitute the value of T:

$$\boxed{A=\frac{m_1m_2g}{m_1M+m_2(m_1+M)}}.$$

(d) $$\boxed{a-A=\frac{Mm_2g}{m_1M+m_2(m_1+M)}}$$

P5.64 (a), (b) Motion impending



$f_{s1} = \mu n = 14.7\text{ N}$ $\quad f_{s2} = 0.500(196\text{ N}) = 98.0\text{ N}$

FIG. P5.64

$$P = f_{s1} + f_{s2} = 14.7\text{ N} + 98.0\text{ N} = \boxed{113\text{ N}}$$

(c) Once motion starts, kinetic friction acts.

$$112.7\text{ N} - 0.100(49.0\text{ N}) - 0.400(196\text{ N}) = (15.0\text{ kg})a_2$$

$$a_2 = \boxed{1.96\text{ m/s}^2}$$

$$0.100(49.0\text{ N}) = (5.00\text{ kg})a_1$$

$$a_1 = \boxed{0.980\text{ m/s}^2}$$

***P5.65** (a) Let x represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = \left(z^2 - h_0^2\right)^{1/2}$, $v_x = \frac{dx}{dt} = \frac{1}{2}\left(z^2 - h_0^2\right)^{-1/2}(2z)\frac{dz}{dt}$. Now $\frac{dz}{dt}$ is the rate at which string passes over the pulley, so it is equal to v_y of the counterweight.

$$v_x = z\left(z^2 - h_0^2\right)^{-1/2} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$ at release from rest, $v_y = 0$ and $a_x = ua_y$.

(c) $\sin 30.0° = \frac{80.0\text{ cm}}{z}$, $z = 1.60\text{ m}$, $u = \left(z^2 - h_0^2\right)^{-1/2} z = \left(1.6^2 - 0.8^2\right)^{-1/2}(1.6) = 1.15$.
For the counterweight

$$\sum F_y = ma_y: \quad T - 0.5\text{ kg } 9.8\text{ m/s}^2 = -0.5\text{ kg}a_y$$

$$a_y = -2T + 9.8$$

For the glider

$$\sum F_x = ma_x: \quad T\cos 30° = 1.00\text{ kg } a_x = 1.15a_y = 1.15(-2T + 9.8) = -2.31T + 11.3\text{ N}$$

$$3.18T = 11.3\text{ N}$$

$$T = \boxed{3.56\text{ N}}$$

***P5.66** The upward acceleration of the rod is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$1\times10^{-3}\ \text{m} = 0 + 0 + \frac{1}{2}a_y\left(8\times10^{-3}\ \text{s}\right)^2$$

$$a_y = 31.2\ \text{m/s}^2$$

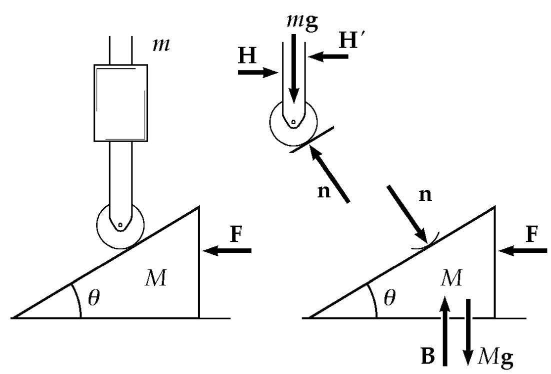


FIG. P5.66

The distance y moved by the rod and the distance x moved by the wedge in the same time are related by $\tan 15° = \frac{y}{x} \Rightarrow x = \frac{y}{\tan 15°}$. Then their speeds and accelerations are related by

$$\frac{dx}{dt} = \frac{1}{\tan 15°}\frac{dy}{dt}$$

and

$$\frac{d^2x}{dt^2} = \frac{1}{\tan 15°}\frac{d^2y}{dt^2} = \left(\frac{1}{\tan 15°}\right)31.2\ \text{m/s}^2 = 117\ \text{m/s}^2.$$

The free body diagram for the rod is shown. Here H and H' are forces exerted by the guide.

$$\sum F_y = ma_y: \qquad n\cos 15° - mg = ma_y$$

$$n\cos 15° - 0.250\ \text{kg}\left(9.8\ \text{m/s}^2\right) = 0.250\ \text{kg}\left(31.2\ \text{m/s}^2\right)$$

$$n = \frac{10.3\ \text{N}}{\cos 15°} = 10.6\ \text{N}$$

For the wedge,

$$\sum F_x = Ma_x: \quad -n\sin 15° + F = 0.5\ \text{kg}\left(117\ \text{m/s}^2\right)$$

$$F = (10.6\ \text{N})\sin 15° + 58.3\ \text{N} = \boxed{61.1\ \text{N}}$$

***P5.67** (a) Consider forces on the midpoint of the rope. It is nearly in equilibrium just before the car begins to move. Take the y-axis in the direction of the force you exert:

$$\sum F_y = ma_y: \quad -T\sin\theta + f - T\sin\theta = 0$$

$$\boxed{T = \frac{f}{2\sin\theta}}.$$

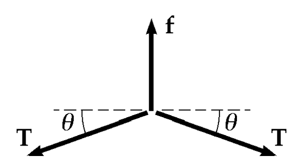


FIG. P5.67

(b) $T = \frac{100\ \text{N}}{2\sin 7°} = \boxed{410\ \text{N}}$

P5.68 Since it has a larger mass, we expect the 8.00-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string. Define up the left hand plane as positive for the 3.50-kg object and down the right hand plane as positive for the 8.00-kg object.

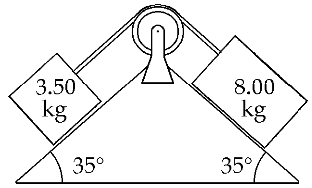


FIG. P5.68

$$\sum F_1 = m_1 a_1: \quad -m_1 g \sin 35.0° + T = m_1 a$$
$$\sum F_2 = m_2 a_2: \quad m_2 g \sin 35.0° - T = m_2 a$$

and

$$-(3.50)(9.80)\sin 35.0° + T = 3.50a$$
$$(8.00)(9.80)\sin 35.0° - T = 8.00a.$$

Adding, we obtain

$$+45.0 \text{ N} - 19.7 \text{ N} = (11.5 \text{ kg})a.$$

(b) Thus the acceleration is

$$\boxed{a = 2.20 \text{ m/s}^2}.$$

By substitution,

$$-19.7 \text{ N} + T = (3.50 \text{ kg})\left(2.20 \text{ m/s}^2\right) = 7.70 \text{ N}.$$

(a) The tension is

$$\boxed{T = 27.4 \text{ N}}.$$

P5.69 Choose the x-axis pointing down the slope.

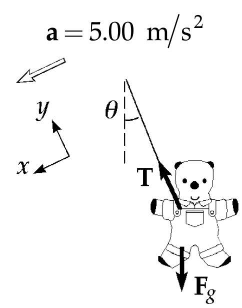


FIG. P5.69

$$v_f = v_i + at: \quad 30.0 \text{ m/s} = 0 + a(6.00 \text{ s})$$
$$a = 5.00 \text{ m/s}^2.$$

Consider forces on the toy.

$$\sum F_x = ma_x: \quad mg\sin\theta = m\left(5.00 \text{ m/s}^2\right)$$
$$\theta = \boxed{30.7°}$$
$$\sum F_y = ma_y: \quad -mg\cos\theta + T = 0$$
$$T = mg\cos\theta = (0.100)(9.80)\cos 30.7°$$
$$T = \boxed{0.843 \text{ N}}$$

***P5.70** Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta.$$

Let $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline. We have

$$\sum F_x = ma_x: \qquad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta.$$

$$\boxed{\mathbf{R} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

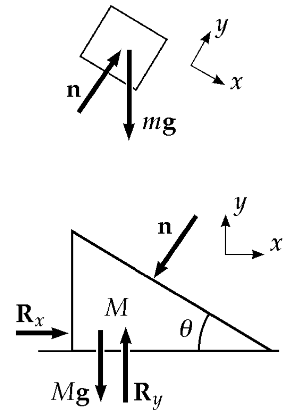


FIG. P5.70

***P5.71** Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x \quad 0.1\text{ N} = 0.2\text{ kg}\,a_x$$
$$a_x = 0.5\text{ m/s}^2.$$

Relative to the tablecloth, the acceleration of the mug is $0.5\text{ m/s}^2 - 3\text{ m/s}^2 = -2.5\text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.3\text{ m} = 0 + \frac{1}{2}\left(-2.5\text{ m/s}^2\right)t^2$$
$$t = 0.490\text{ s}.$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}\left(0.5\text{ m/s}^2\right)(0.490\text{ s})^2 = \boxed{0.060\,0\text{ m}}.$$

The tablecloth slides 36 cm over the table in this process.

P5.72 $\sum F_y = ma_y : n - mg\cos\theta = 0$

or

$$n = 8.40(9.80)\cos\theta$$
$$n = (82.3\ \text{N})\cos\theta$$

$\sum F_x = ma_x : mg\sin\theta = ma$

or

$$a = g\sin\theta$$
$$a = \left(9.80\ \text{m/s}^2\right)\sin\theta$$

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

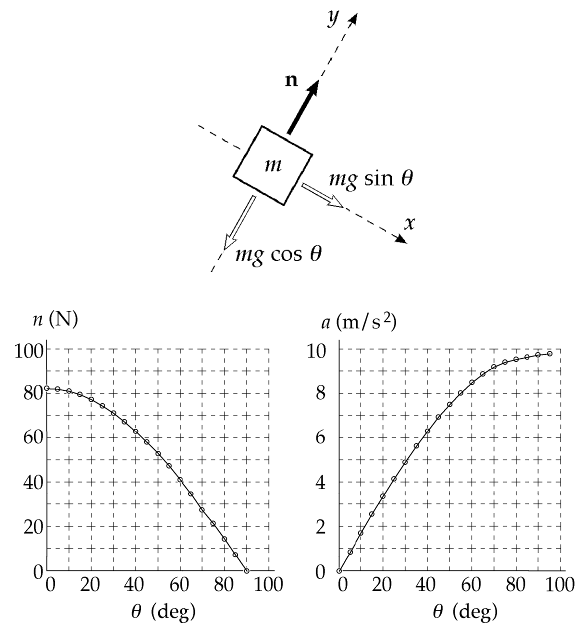


FIG. P5.72

At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

P5.73 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (other half the same, by symmetry)

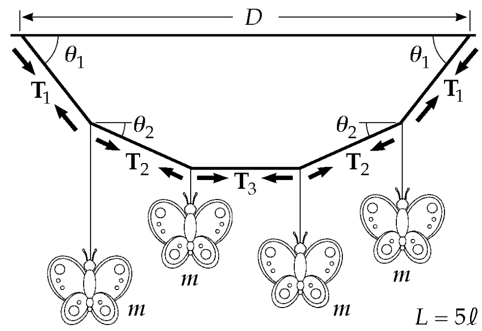


FIG. P5.69

(1) $T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$
(2) $T_1 \sin\theta_1 - T_2 \sin\theta_2 - mg = 0$
(3) $T_2 \cos\theta_2 - T_3 = 0$
(4) $T_2 \sin\theta_2 - mg = 0$

Substituting (4) into (2) for $T_2 \sin\theta_2$,

$$T_1 \sin\theta_1 - mg - mg = 0.$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}.$$

Substitute (3) into (1) for $T_2 \cos\theta_2$:

$$T_3 - T_1 \cos\theta_1 = 0,\ T_3 = T_1 \cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}.$$

From Equation (4),

$$T_2 = \frac{mg}{\sin\theta_2}.$$

(b) Divide (4) by (3):

$$\frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \frac{mg}{T_3}.$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg},\ \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}.$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}.$$

(c) D is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell$$

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

ANSWERS TO EVEN PROBLEMS

P5.2 1.66×10^{6} N forward

P5.4 (a) $\frac{vt}{2}$; (b) $\left(\frac{F_g v}{gt}\right)\hat{\mathbf{i}}+F_g\hat{\mathbf{j}}$

P5.6 (a) 4.47×10^{15} $\mathrm{m/s^2}$ away from the wall; (b) 2.09×10^{-10} N toward the wall

P5.8 (a) 534 N down; (b) 54.5 kg

P5.10 2.55 N for an 88.7 kg person

P5.12 $\left(16.3\hat{\mathbf{i}}+14.6\hat{\mathbf{j}}\right)$ N

P5.14 (a) 181°; (b) 11.2 kg; (c) 37.5 m/s; (d) $\left(-37.5\hat{\mathbf{i}}-0.893\hat{\mathbf{j}}\right)$ m/s

P5.16 112 N

P5.18 $T_1=296$ N; $T_2=163$ N; $T_3=325$ N

P5.20 (a) see the solution; (b) 1.79 N

P5.22 (a) 2.54 $\mathrm{m/s^2}$ down the incline; (b) 3.18 m/s

P5.24 see the solution; 6.30 $\mathrm{m/s^2}$; 31.5 N

P5.26 (a) 3.57 $\mathrm{m/s^2}$; (b) 26.7 N; (c) 7.14 m/s

P5.28 (a) 36.8 N; (b) 2.45 $\mathrm{m/s^2}$; (c) 1.23 m

P5.30 (a) 0.529 m; (b) 7.40 m/s upward

P5.32 (a) 2.22 m; (b) 8.74 m/s

P5.34 (a) $a_1=2a_2$;
(b) $T_1=\frac{m_1m_2g}{2m_1+\frac{m_2}{2}}$; $T_2=\frac{m_1m_2g}{m_1+\frac{m_2}{4}}$;
(c) $a_1=\frac{m_2g}{2m_1+\frac{m_2}{2}}$; $a_2=\frac{m_2g}{4m_1+m_2}$

P5.36 $\mu_s=0.306$; $\mu_k=0.245$

P5.38 (a) 3.34; (b) Time would increase

P5.40 (a) 55.2°; (b) 167 N

P5.42 152 ft

P5.44 (a) 2.31 $\mathrm{m/s^2}$ down for m_1, left for m_2 and up for m_3; (b) 30.0 N and 24.2 N

P5.46 Any value between 31.7 N and 48.6 N

P5.48 72.0 N

P5.50 6.84 m

P5.52 (a) 3.00 s; (b) 20.1 m; (c) $\left(18.0\hat{\mathbf{i}}-9.00\hat{\mathbf{j}}\right)$ m

P5.54 (a) 2.00 $\mathrm{m/s^2}$ to the right;
(b) 8.00 N right on 4 kg;
6.00 N right on 3 kg; 4 N right on 2 kg;
(c) 8.00 N between 4 kg and 3 kg;
14.0 N between 2 kg and 3 kg;
(d) see the solution

P5.56 1.18 kN

P5.58 (a) 4.90 $\mathrm{m/s^2}$; (b) 3.13 m/s at 30.0° below the horizontal; (c) 1.35 m; (d) 1.14 s; (e) No

P5.60 (a) and (b) see the solution; (c) 357 N; (d) see the solution; (e) 1.20

P5.62 see the solution; 0.143 $\mathrm{m/s^2}$ agrees with 0.137 $\mathrm{m/s^2}$

P5.64 (a) see the solution;
(b) on block one:
$49.0\text{ N }\hat{\mathbf{j}}-49.0\text{ N }\hat{\mathbf{j}}+14.7\text{ N }\hat{\mathbf{i}}$;
on block two: $-49.0\text{ N }\hat{\mathbf{j}}-14.7\text{ N }\hat{\mathbf{i}}-147\text{ N }\hat{\mathbf{j}}$
$+196\text{ N }\hat{\mathbf{j}}-98.0\text{ N }\hat{\mathbf{i}}+113\text{ N }\hat{\mathbf{i}}$;
(c) for block one: $0.980\hat{\mathbf{i}}$ $\mathrm{m/s^2}$;
for block two: 1.96 $\mathrm{m/s^2}\,\hat{\mathbf{i}}$

P5.66 61.1 N

P5.68 (a) 2.20 $\mathrm{m/s^2}$; (b) 27.4 N

P5.70 $mg\cos\theta\sin\theta$ to the right $+\left(M+m\cos^2\theta\right)g$ upward

P5.72 see the solution

6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q6.1 Mud flies off a rapidly spinning tire because the resultant force is not sufficient to keep it moving in a circular path. In this case, the force that plays a major role is the adhesion between the mud and the tire.

Q6.2 The spring will stretch. In order for the object to move in a circle, the force exerted on the object by the spring must have a size of $\frac{mv^2}{r}$. Newton's third law says that the force exerted on the object by the spring has the same size as the force exerted by the object on the spring. It is the force exerted on the spring that causes the spring to stretch.

Q6.3 Driving in a circle at a constant speed requires a centripetal acceleration but no tangential acceleration.

Q6.4 (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

Q6.5 The speed changes. The tangential force component causes tangential acceleration.

Q6.6 Consider the force required to keep a rock in the Earth's crust moving in a circle. The size of the force is proportional to the radius of the circle. If that rock is at the Equator, the radius of the circle through which it moves is about 6400 km. If the rock is at the north pole, the radius of the circle through which it moves is zero!

Q6.7 Consider standing on a bathroom scale. The resultant force on you is your actual weight minus the normal force. The scale reading shows the size of the normal force, and is your 'apparent weight.' If you are at the North or South Pole, it can be precisely equal to your actual weight. If you are at the equator, your apparent weight must be less, so that the resultant force on you can be a downward force large enough to cause your centripetal acceleration as the Earth rotates.

Q6.8 A torque is exerted by the thrust force of the water times the distance between the nozzles.

Q6.9 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

Q6.10 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

Q6.11 The ball would not behave as it would when dropped on the Earth. As the astronaut holds the ball, she and the ball are moving with the same angular velocity. The ball, however, being closer to the center of rotation, is moving with a slower tangential velocity. Once the ball is released, it acts according to Newton's first law, and simply drifts with constant velocity in the original direction of its velocity when released—it is no longer "attached" to the rotating space station. Since the ball follows a straight line and the astronaut follows a circular path, it will appear to the astronaut that the ball will "fall to the floor". But other dramatic effects will occur. Imagine that the ball is held so high that it is just slightly away from the center of rotation. Then, as the ball is released, it will move very slowly along a straight line. Thus, the astronaut may make several full rotations around the circular path before the ball strikes the floor. This will result in three obvious variations with the Earth drop. First, the time to fall will be much larger than that on the Earth, even though the feet of the astronaut are pressed into the floor with a force that suggests the same force of gravity as on Earth. Second, the ball may actually appear to bob up and down if several rotations are made while it "falls". As the ball moves in a straight line while the astronaut rotates, sometimes she is on the side of the circle on which the ball is moving toward her and other times she is on the other side, where the ball is moving away from her. The third effect is that the ball will not drop straight down to her feet. In the extreme case we have been imagining, it may actually strike the surface while she is on the opposite side, so it looks like it ended up "falling up". In the least extreme case, in which only a portion of a rotation is made before the ball strikes the surface, the ball will appear to move backward relative to the astronaut as it falls.

Q6.12 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

Q6.13 There is no such force. If the passenger slides outward across the slippery car seat, it is because the passenger is moving forward in a straight line while the car is turning under him. If the passenger pushes hard against the outside door, the door is exerting an inward force on him. No object is exerting an outward force on him, but he should still buckle his seatbelt.

Q6.14 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot's brain.

Q6.15 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

Q6.16 When you are not accelerating, the normal force and your weight are equal in size. Your body interprets the force of the floor pushing up on you as your weight. When you accelerate in an elevator, this normal force changes so that you accelerate with the elevator. In free fall, you are never weightless since the Earth's gravity and your mass do not change. It is the normal force—your apparent weight—that is zero.

Q6.17 From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2$$

where D is the coefficient of drag of the parachutist, and A is the projected area of the parachutist's body. At terminal speed,

$$a_y = \frac{dv_y}{dt} = 0 \text{ and } V_T\left(\frac{2mg}{D\rho A}\right)^{1/2}.$$

When the parachute opens, the coefficient of drag D and the effective area A both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m\frac{dv_y}{dt} = mg - \frac{D\rho A}{2}v_y^2 - \frac{L\rho A}{2}v_x^2$$

where v_y is the vertical velocity, and v_x is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

Q6.18 The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.

Q6.19 Lower air density reduces air resistance, so a tank-truck-load of fuel takes you farther.

Q6.20 Suppose the rock is moving rapidly when it enters the water. The speed of the rock decreases until it reaches terminal velocity. The acceleration, which is upward, decreases to zero as the rock approaches terminal velocity.

Q6.21 The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

SOLUTIONS TO PROBLEMS

Section 6.1 **Newton's Second Law Applied to Uniform Circular Motion**

P6.1 $m = 3.00\text{ kg}$, $r = 0.800\text{ m}$. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = 25.0(9.80) = 245\text{ N}.$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $$T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}.$$

Then $$v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \le \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3\text{ m}^2/\text{s}^2$$

and $$0 \le v \le \sqrt{65.3}$$

or $$\boxed{0 \le v \le 8.08\text{ m/s}}.$$

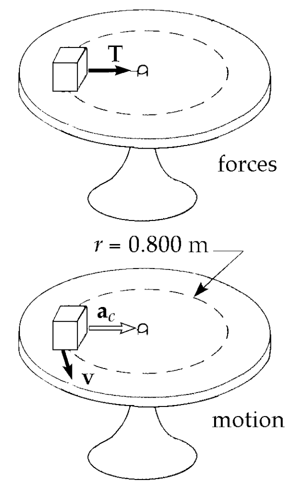


FIG. P6.1

P6.2 In $\sum F = m\frac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = 215\text{ N}.$$

Symbolically, write $\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\text{ m/s})^2$.

Dividing gives $\frac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\frac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2(130\text{ N}) = \boxed{215\text{ N}}.$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.3 (a) $$F=\frac{mv^2}{r}=\frac{\left(9.11\times 0^{-31}\text{ kg}\right)\left(2.20\times 10^{6}\text{ m/s}\right)^2}{0.530\times 10^{-10}\text{ m}}=\boxed{8.32\times 10^{-8}\text{ N inward}}$$

(b) $$a=\frac{v^2}{r}=\frac{\left(2.20\times 10^{6}\text{ m/s}\right)^2}{0.530\times 10^{-10}\text{ m}}=\boxed{9.13\times 10^{22}\text{ m/s}^2\text{ inward}}$$

P6.4 Neglecting relativistic effects. $F=ma_c=\dfrac{mv^2}{r}$

$$F=\left(2\times 1.661\times 10^{-27}\text{ kg}\right)\frac{\left(2.998\times 10^{7}\text{ m/s}\right)^2}{(0.480\text{ m})}=\boxed{6.22\times 10^{-12}\text{ N}}$$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}}=f\,\hat{\mathbf{i}}+n\hat{\mathbf{j}}+mg\left(-\hat{\mathbf{j}}\right)$

$\sum F_y=0=n-mg$

thus $n=mg$ and $\sum F_r=m\dfrac{v^2}{r}=f=\mu n=\mu mg$.

Then $\mu=\dfrac{v^2}{rg}=\dfrac{(50.0\text{ cm/s})^2}{(30.0\text{ cm})\left(980\text{ cm/s}^2\right)}=\boxed{0.085\,0}$.

P6.6 (a) $\sum F_y=ma_y$, mg_{moon} down $=\dfrac{mv^2}{r}$ down

$$v=\sqrt{g_{\text{moon}}r}=\sqrt{\left(1.52\text{ m/s}^2\right)\left(1.7\times 10^{6}\text{ m}+100\times 10^{3}\text{ m}\right)}=\boxed{1.65\times 10^{3}\text{ m/s}}$$

(b) $$v=\frac{2\pi r}{T},\ T=\frac{2\pi\left(1.8\times 10^{6}\text{ m}\right)}{1.65\times 10^{3}\text{ m/s}}=\boxed{6.84\times 10^{3}\text{ s}}=1.90\text{ h}$$

P6.7 $n=mg$ since $a_y=0$

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f=ma_c=\dfrac{mv^2}{r}$.

But the friction condition is $f\le \mu_s n$

i.e., $\dfrac{mv^2}{r}\le \mu_s mg$

$v\le\sqrt{\mu_s rg}=\sqrt{0.600(35.0\text{ m})\left(9.80\text{ m/s}^2\right)}$ $\qquad v\le\boxed{14.3\text{ m/s}}$

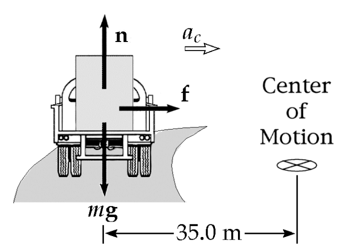


FIG. P6.7

P6.8 $$a=\frac{v^2}{r}=\frac{\left[(86.5\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)\right]^2}{61.0\ \text{m}}\left(\frac{1\ g}{9.80\ \text{m/s}^2}\right)=\boxed{0.966g}$$

P6.9 $$T\cos 5.00°=mg=(80.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)$$

(a) $T=787\ \text{N}$: $\mathbf{T}=\boxed{(68.6\ \text{N})\hat{\mathbf{i}}+(784\ \text{N})\hat{\mathbf{j}}}$

(b) $T\sin 5.00°=ma_c$: $\boxed{a_c=0.857\ \text{m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

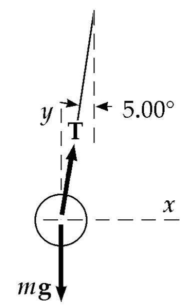


FIG. P6.9

P6.10 (b) $$v=\frac{235\ \text{m}}{36.0\ \text{s}}=\boxed{6.53\ \text{m/s}}$$

The radius is given by $\frac{1}{4}2\pi r=235\ \text{m}$

$$r=150\ \text{m}$$

(a) $$\mathbf{a}_r=\left(\frac{v^2}{r}\right)\text{ toward center}$$
$$=\frac{(6.53\ \text{m/s})^2}{150\ \text{m}}\text{ at }35.0°\text{ north of west}$$
$$=\left(0.285\ \text{m/s}^2\right)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right)+\sin 35.0°\,\hat{\mathbf{j}}\right)$$
$$=\boxed{-0.233\ \text{m/s}^2\,\hat{\mathbf{i}}+0.163\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

(c) $$\bar{\mathbf{a}}=\frac{\left(\mathbf{v}_f-\mathbf{v}_i\right)}{t}$$
$$=\frac{\left(6.53\ \text{m/s}\,\hat{\mathbf{j}}-6.53\ \text{m/s}\,\hat{\mathbf{i}}\right)}{36.0\ \text{s}}$$
$$=\boxed{-0.181\ \text{m/s}^2\,\hat{\mathbf{i}}+0.181\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

***P6.11** $F_g = mg = (4\text{ kg})(9.8\text{ m/s}^2) = 39.2\text{ N}$

$$\sin\theta = \frac{1.5\text{ m}}{2\text{ m}}$$

$$\theta = 48.6°$$

$$r = (2\text{ m})\cos 48.6° = 1.32\text{ m}$$

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a\cos 48.6° + T_b\cos 48.6° = \frac{(4\text{ kg})(6\text{ m/s})^2}{1.32\text{ m}}$$

$$T_a + T_b = \frac{109\text{ N}}{\cos 48.6°} = 165\text{ N}$$

$$\sum F_y = ma_y$$

$$+T_a\sin 48.6° - T_b\sin 48.6° - 39.2\text{ N} = 0$$

$$T_a - T_b = \frac{39.2\text{ N}}{\sin 48.6°} = 52.3\text{ N}$$

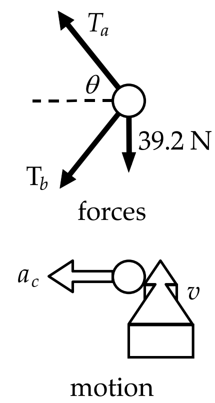


FIG. P6.11

(a) To solve simultaneously, we add the equations in T_a and T_b:

$$T_a + T_b + T_a - T_b = 165\text{ N} + 52.3\text{ N}$$

$$T_a = \frac{217\text{ N}}{2} = \boxed{108\text{ N}}$$

(b) $T_b = 165\text{ N} - T_a = 165\text{ N} - 108\text{ N} = \boxed{56.2\text{ N}}$

***P6.12** $a_c = \dfrac{v^2}{r}$. Let f represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100\,g.$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100\,g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100\cdot 9.8\text{ m/s}^2}{4\pi^2(0.021\text{ m})}\right)^{1/2} = 34.4\ \frac{1}{\text{s}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{2.06\times 10^3\text{ rev/min}}.$$

Section 6.2 **Nonuniform Circular Motion**

P6.13 $M = 40.0\text{ kg}$, $R = 3.00\text{ m}$, $T = 350\text{ N}$

(a)
$$\sum F = 2T - Mg = \frac{Mv^2}{R}$$
$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$
$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$$
$$\boxed{v = 4.81\text{ m/s}}$$

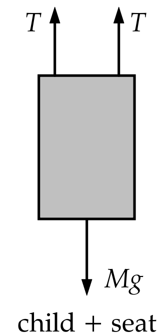


FIG. P6.13(a)

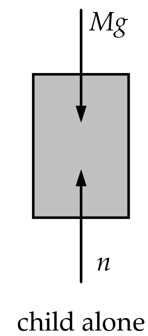


FIG. P6.13(b)

(b)
$$n - Mg = F = \frac{Mv^2}{R}$$
$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\text{ N}}$$

P6.14 (a) Consider the forces acting on the system consisting of the child *and* the seat:

$$\sum F_y = ma_y \Rightarrow 2T - mg = m\frac{v^2}{R}$$
$$v^2 = R\left(\frac{2T}{m} - g\right)$$
$$v = \boxed{\sqrt{R\left(\frac{2T}{m} - g\right)}}$$

(b) Consider the forces acting on the child alone:

$$\sum F_y = ma_y \Rightarrow n = m\left(g + \frac{v^2}{R}\right)$$

and from above, $v^2 = R\left(\frac{2T}{m} - g\right)$, so

$$n = m\left(g + \frac{2T}{m} - g\right) = \boxed{2T}.$$

P6.15 Let the tension at the lowest point be T.

$$\sum F = ma: \quad T - mg = ma_c = \frac{mv^2}{r}$$
$$T = m\left(g + \frac{v^2}{r}\right)$$
$$T = (85.0\text{ kg})\left[9.80\text{ m/s}^2 + \frac{(8.00\text{ m/s})^2}{10.0\text{ m}}\right] = 1.38\text{ kN} > 1\,000\text{ N}$$

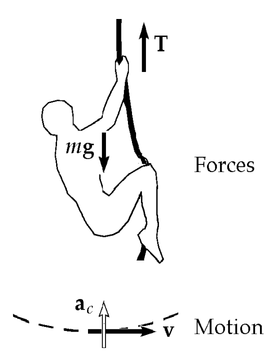


FIG. P6.15

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

P6.16 (a) $a_c = \dfrac{v^2}{r} = \dfrac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$

$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\ \text{m/s}^2}$

at an angle $\theta = \tan^{-1}\left(\dfrac{a_c}{a_t}\right) = \boxed{48.0°\ \text{inward}}$

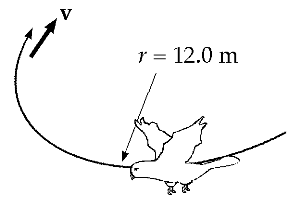


FIG. P6.16

P6.17 $\sum F_y = \dfrac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$\dfrac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{\left(9.80\ \text{m/s}^2\right)(1.00\ \text{m})} = \boxed{3.13\ \text{m/s}}$.

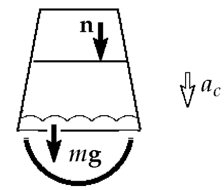


FIG. P6.17

P6.18 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\dfrac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\ \text{N}}$

P6.19 (a) $v = 20.0\ \text{m/s}$,

$n =$ force of track on roller coaster, and

$R = 10.0\ \text{m}$.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

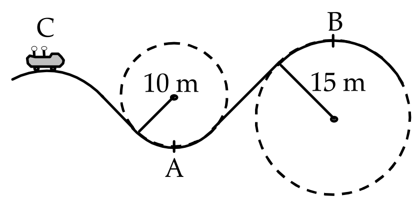


FIG. P6.19

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\ \text{kg})\left(9.80\ \text{m/s}^2\right) + \frac{(500\ \text{kg})\left(20.0\ \text{m/s}^2\right)}{10.0\ \text{m}}$$

$$n = 4\,900\ \text{N} + 20\,000\ \text{N} = \boxed{2.49 \times 10^4\ \text{N}}$$

(b) At B, $n - Mg = -\dfrac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\text{max}}^2}{R} \Rightarrow v_{\text{max}} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$$

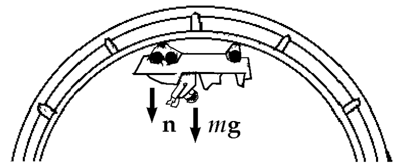


FIG. P6.20

P6.20 (a) $a_c = \dfrac{v^2}{r}$ $\quad r = \dfrac{v^2}{a_c} = \dfrac{(13.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{8.62\ \text{m}}$

(b) Let n be the force exerted by the rail.

Newton's law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg,\ \text{downward}}$$

(c) $a_c = \dfrac{v^2}{r}$ $\quad a_c = \dfrac{(13.0\ \text{m/s})^2}{20.0\ \text{m}} = \boxed{8.45\ \text{m/s}^2}$

If the force exerted by the rail is n_1

then $\quad n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since $a_c = 8.45\ \text{m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require n_1 to be positive. Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0\ \text{m})(9.80\ \text{m/s}^2)},\ v > 14.0\ \text{m/s}.$$

Section 6.3 **Motion in Accelerated Frames**

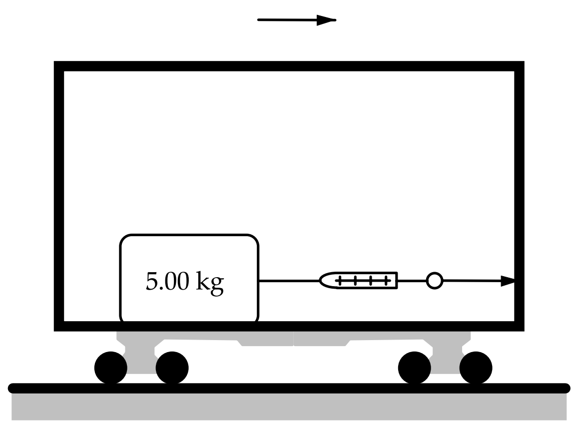


FIG. P6.21

P6.21 (a) $\sum F_x = Ma$, $a = \dfrac{T}{M} = \dfrac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force (–Ma). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.

***P6.22** We adopt the view of an inertial observer. If it is on the verge of sliding, the cup is moving on a circle with its centripetal acceleration caused by friction.

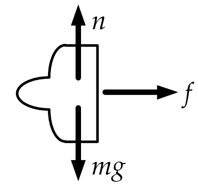


FIG. P6.22

$$\sum F_y = ma_y: \quad +n - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{r} = \mu_s n = \mu_s mg$$

$$v = \sqrt{\mu_s g r} = \sqrt{0.8\left(9.8\ \text{m/s}^2\right)(30\ \text{m})} = \boxed{15.3\ \text{m/s}}$$

If you go too fast the cup will begin sliding $\boxed{\text{straight across the dashboard to the left.}}$

P6.23 The only forces acting on the suspended object are the force of gravity $m\mathbf{g}$ and the force of tension $\mathbf{T}$, as shown in the free-body diagram. Applying Newton's second law in the x and y directions,

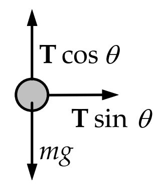


FIG. P6.23

$$\sum F_x = T\sin\theta = ma \qquad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $\quad T\cos\theta = mg \qquad (2)$

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306.$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})\left(3.00\ \text{m/s}^2\right)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}.$$

***P6.24** The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12\ \text{m})}{7.25\ \text{s}} = 0.104\ \text{m/s}.$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104\ \text{m/s})^2}{0.12\ \text{m}} = m9.01\times10^{-2}\ \text{m/s}^2.$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\frac{0.090\,1\ \text{m/s}^2}{9.8\ \text{m/s}^2} = \boxed{0.527°}.$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

P6.25 $F_{\max} = F_g + ma = 591\text{ N}$

$F_{\min} = F_g - ma = 391\text{ N}$

(a) Adding, $2F_g = 982\text{ N}$, $F_g = \boxed{491\text{ N}}$

(b) Since $F_g = mg$, $m = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Subtracting the above equations,

$2ma = 200\text{ N}$ $\quad\therefore a = \boxed{2.00\text{ m/s}^2}$

P6.26 (a)
$$\sum F_r = ma_r$$
$$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$
$$g = \frac{4\pi^2 R}{T^2}$$
$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.80\text{ m/s}^2}} = 5.07\times 10^3\text{ s} = \boxed{1.41\text{ h}}$$

(b)
$$\text{speed increase factor} = \frac{v_{\text{new}}}{v_{\text{current}}} = \frac{2\pi R}{T_{\text{new}}}\left(\frac{T_{\text{current}}}{2\pi R}\right) = \frac{T_{\text{current}}}{T_{\text{new}}} = \frac{24.0\text{ h}}{1.41\text{ h}} = \boxed{17.1}$$

***P6.27** The car moves to the right with acceleration a. We find the acceleration of a_b of the block relative to the Earth. The block moves to the right also.

$$\sum F_y = ma_y:\quad +n - mg = 0,\ n = mg,\ f = \mu_k mg$$
$$\sum F_x = ma_x:\quad +\mu_k mg = ma_b,\ a_b = \mu_k g$$

The acceleration of the block relative to the car is $a_b - a = \mu_k g - a$. In this frame the block starts from rest and undergoes displacement $-\ell$ and gains speed according to

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$
$$v_{xf}^2 = 0 + 2(\mu_k g - a)(-\ell - 0) = 2\ell(a - \mu_k g).$$

(a) $\boxed{v = \left(2\ell(a - \mu_k g)\right)^{1/2}}$ to the left

continued on next page

(b) The time for which the box slides is given by

$$\Delta x = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$
$$-\ell = \frac{1}{2}\left[0 - \left(2\ell\left(a - \mu_k g\right)\right)^{1/2}\right]t$$
$$t = \left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}.$$

The car in the Earth frame acquires finals speed $v_{xf} = v_{xi} + at = 0 + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}$. The speed of the box in the Earth frame is then

$$v_{be} = v_{bc} + v_{ce} = -\left[2\ell\left(a - \mu_k g\right)\right]^{1/2} + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}$$
$$= \frac{-(2\ell)^{1/2}\left(a - \mu_k g\right) + (2\ell)^{1/2} a}{\left(a - \mu_k g\right)^{1/2}} = \boxed{\frac{\mu_k g(2\ell)^{1/2}}{\left(a - \mu_k g\right)^{1/2}}}$$
$$= \frac{\mu_k g 2\ell}{\left[2\ell\left(a - \mu_k g\right)\right]^{1/2}} = \frac{2\mu_k g\ell}{v}.$$

***P6.28** Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g + a),\ f_k = \mu_k m(g + a)$$
$$\sum F_x = ma_x: \quad -\mu_k m(g + a) = ma_x$$

The motion across the floor is described by $L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k (g + a)t^2$.

We solve for μ_k: $vt - L = \frac{1}{2}\mu_k (g + a)t^2$, $\boxed{\frac{2(vt - L)}{(g + a)t^2} = \mu_k}$.

P6.29 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}} = 13.5\ \text{m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80)^2 + (13.5)^2}\ \text{m/s}^2 = 16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\frac{16.7}{9.80} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0\ \text{N}) = \boxed{93.8\ \text{N}}.$$

***P6.30** (a) The chunk is at radius $r = \dfrac{0.137\ \text{m} + 0.080\ \text{m}}{4} = 0.054\,2\ \text{m}$. Its speed is

$$v = \frac{2\pi r}{T} = 2\pi(0.054\,2\ \text{m})\frac{20\,000}{60\ \text{s}} = 114\ \text{m/s}$$

and its acceleration

$$a_c = \frac{v^2}{r} = \frac{(114\ \text{m/s})^2}{0.054\,2\ \text{m}} = \boxed{2.38\times10^5\ \text{m/s}^2\ \text{horizontally inward}}$$

$$= 2.38\times10^5\ \text{m/s}^2\left(\frac{g}{9.8\ \text{m/s}^2}\right) = \boxed{2.43\times10^4\,g}.$$

(b) In the frame of the turning cone, the chunk feels a horizontally outward force of $\dfrac{mv^2}{r}$. In this frame its acceleration is up along the cone, at $\tan^{-1}\dfrac{3.3\ \text{cm}}{\frac{(13.7-8)\ \text{cm}}{2}} = 49.2°$.

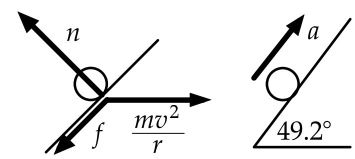


FIG. P6.30(b)

Take the y axis perpendicular to the cone:

$$\sum F_y = ma_y:\quad +n - \frac{mv^2}{r}\sin 49.2° = 0$$

$$n = (2\times10^{-3}\ \text{kg})(2.38\times10^5\ \text{m/s}^2)\sin 49.2° = \boxed{360\ \text{N}}$$

(c) $f = \mu_k n = 0.6(360\ \text{N}) = 216\ \text{N}$

$$\sum F_x = ma_x:\quad \frac{mv^2}{r}\cos 49.2° - f = ma_x$$

$$(2\times10^{-3}\ \text{kg})(2.38\times10^5\ \text{m/s}^2)\cos 49.2° - 216\ \text{N} = (2\times10^{-3}\ \text{kg})a_x$$

$$a_x = \boxed{47.5\times10^4\ \text{m/s}^2\ \text{radially up the wall of the cone}}$$

P6.31 $a_r = \left(\dfrac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.027\,6\ \text{m/s}^2$

We take the y axis along the local vertical.

$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78\ \text{m/s}^2$

$(a_{\text{net}})_x = 0.015\,8\ \text{m/s}^2$

$\theta = \arctan\dfrac{a_x}{a_y} = \boxed{0.092\,8°}$

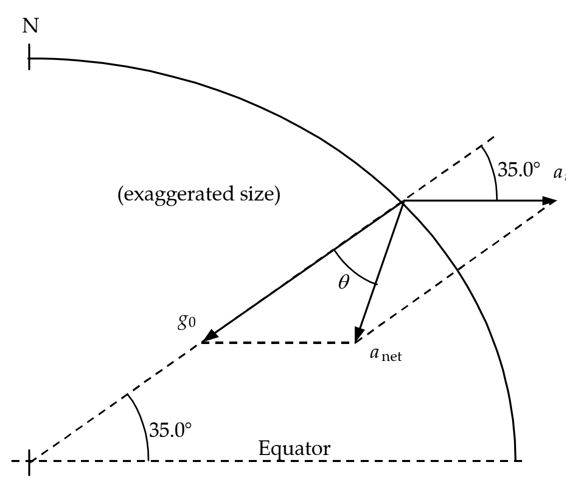


FIG. P6.31

Section 6.4 **Motion in the Presence of Resistive Forces**

P6.32 $m = 80.0\text{ kg}$, $v_T = 50.0\text{ m/s}$, $mg = \dfrac{D\rho A v_T^2}{2} \therefore \dfrac{D\rho A}{2} = \dfrac{mg}{v_T^2} = 0.314\text{ kg/m}$

(a) At $v = 30.0\text{ m/s}$

$$a = g - \frac{\frac{D\rho A v^2}{2}}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27\text{ m/s}^2\text{ downward}}$$

(b) At $v = 50.0\text{ m/s}$, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{784\text{ N directed up}}$$

(c) At $v = 30.0\text{ m/s}$

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283\text{ N}}\text{ upward}$$

P6.33 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $\qquad b = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus, $\qquad v_T = \dfrac{y}{t} = \dfrac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$

Then $\qquad b = \dfrac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = \boxed{32.7\text{ s}^{-1}}$

(b) At $t = 0$, $v = 0$ and $\qquad a = g = \boxed{9.80\text{ m/s}^2}$ down

(c) When $v = 0.150\text{ m/s}$, $\quad a = g - bv = 9.80\text{ m/s}^2 - \left(32.7\text{ s}^{-1}\right)(0.150\text{ m/s}) = \boxed{4.90\text{ m/s}^2}$ down

P6.34 (a) $\rho = \dfrac{m}{V}$, $A = 0.020\,1\text{ m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\text{ g/cm}^3\left[\frac{4}{3}\pi(8.00\text{ cm})^3\right] = 1.78\text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78\text{ kg})\left(9.80\text{ m/s}^2\right)}{0.500\left(1.20\text{ kg/m}^3\right)\left(0.020\,1\text{ m}^2\right)}} = \boxed{53.8\text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \dfrac{v_f^2}{2g} = \dfrac{(53.8\text{ m/s})^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{148\text{ m}}$

P6.35 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92\times10^3\ \text{kg/m}^3\right)\left(2.00\times10^{-2}\ \text{m}\right)^3 = 0.299\ \text{kg}.$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)\left(9.00\times10^{-2}\right) = \boxed{3.01\ \text{N}}.$$

P6.36

$$\sum F_y = ma_y$$
$$+T\cos 40.0° - mg = 0$$
$$T = \frac{(620\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{\cos 40.0°} = 7.93\times10^3\ \text{N}$$
$$\sum F_x = ma_x$$
$$-R + T\sin 40.0° = 0$$
$$R = \left(7.93\times10^3\ \text{N}\right)\sin 40.0° = 5.10\times10^3\ \text{N} = \frac{1}{2}D\rho Av^2$$
$$D = \frac{2R}{\rho Av^2} = \frac{2\left(5.10\times10^3\ \text{N}\right)\left(\frac{\text{kg m/s}^2}{\text{N}}\right)}{\left(1.20\ \text{kg/m}^2\right)\left(3.80\ \text{m}^2\right)(40.0\ \text{m/s})^2} = \boxed{1.40}$$

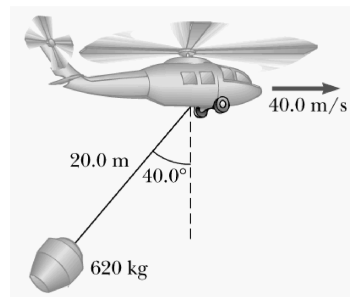


FIG. P6.36

P6.37 (a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$v = v_T\left(1 - e^{-bt/m}\right)$ $\quad$ $v = 0.632v_T$ when $e^{-bt/m} = 0.368$

or at time $\quad t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

P6.38 The resistive force is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.250)\left(1.20\ \text{kg/m}^3\right)\left(2.20\ \text{m}^2\right)(27.8\ \text{m/s})^2$$
$$R = 255\ \text{N}$$
$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

P6.39 (a) $v(t) = v_i e^{-ct}$ $\quad v(20.0\text{ s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0\text{ m/s}$.

So $5.00 = 10.0e^{-20.0c}$ and $-20.0c = \ln\left(\frac{1}{2}\right)$ $\quad c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2}\text{ s}^{-1}}$

(b) At $t = 40.0$ s $\quad v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$

(c) $v = v_i e^{-ct}$ $\quad s = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$

P6.40 $\sum F = ma$

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_0}^{v} v^{-2}dv$$

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_0}^{v} = -\frac{1}{v} + \frac{1}{v_0}$$

$$\frac{1}{v} = \frac{1}{v_0} + kt = \frac{1+v_0kt}{v_0}$$

$$v = \frac{v_0}{1+v_0kt}$$

***P6.41** (a) From Problem 40,

$$v = \frac{dx}{dt} = \frac{v_0}{1+v_0kt}$$

$$\int_0^x dx = \int_0^t v_0\frac{dt}{1+v_0kt} = \frac{1}{k}\int_0^t \frac{v_0kdt}{1+v_0kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln(1+v_0kt)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln(1+v_0kt) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln(1+v_0kt)}$$

(b) We have $\ln(1+v_0kt) = kx$

$1+v_0kt = e^{kx}$ so $v = \frac{v_0}{1+v_0kt} = \frac{v_0}{e^{kx}} = \boxed{v_0e^{-kx} = v}$

***P6.42** We write $-kmv^2 = -\frac{1}{2}D\rho Av^2$ so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\text{ kg/m}^3\right)\left(4.2\times10^{-3}\text{ m}^2\right)}{2(0.145\text{ kg})} = 5.3\times10^{-3}/\text{m}$$

$$v = v_0e^{-kx} = (40.2\text{ m/s})e^{-\left(5.3\times10^{-3}/\text{m}\right)(18.3\text{ m})} = \boxed{36.5\text{ m/s}}$$

P6.43 In $R=\frac{1}{2}D\rho Av^2$, we estimate that $D=1.00$, $\rho=1.20\ \text{kg/m}^3$, $A=(0.100\ \text{m})(0.160\ \text{m})=1.60\times10^{-2}\ \text{m}^2$ and $v=27.0\ \text{m/s}$. The resistance force is then

$$R=\frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times10^{-2}\ \text{m}^2\right)(27.0\ \text{m/s})^2=7.00\ \text{N}$$

or

$$R\sim\boxed{10^1\ \text{N}}$$

Section 6.5 **Numerical Modeling in Particle Dynamics**

Note: In some problems we compute each new position as $x(t+\Delta t)=x(t)+v(t+\Delta t)\Delta t$, rather than $x(t+\Delta t)=x(t)+v(t)\Delta t$ as quoted in the text. This method has the same theoretical validity as that presented in the text, and in practice can give quicker convergence.

P6.44 (a) At $v=v_T$, $a=0$, $-mg+bv_T=0$ $\qquad v_T=\frac{mg}{b}=\frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{3.00\times10^{-2}\ \text{kg/s}}=\boxed{0.980\ \text{m/s}}$

(b)

t(s)	x(m)	v(m/s)	F(mN)	$a\left(\text{m/s}^2\right)$
0	2	0	–29.4	–9.8
0.005	2	–0.049	–27.93	–9.31
0.01	1.999 755	–0.095 55	–26.534	–8.844 5
0.015	1.999 3	–0.139 77	–25.2	–8.40
. . . we list the result after each tenth iteration				
0.05	1.990	–0.393	–17.6	–5.87
0.1	1.965	–0.629	–10.5	–3.51
0.15	1.930	–0.770	–6.31	–2.10
0.2	1.889	–0.854	–3.78	–1.26
0.25	1.845	–0.904	–2.26	–0.754
0.3	1.799	–0.935	–1.35	–0.451
0.35	1.752	–0.953	–0.811	–0.270
0.4	1.704	–0.964	–0.486	–0.162
0.45	1.65	–0.970	–0.291	–0.096 9
0.5	1.61	–0.974	–0.174	–0.058 0
0.55	1.56	–0.977	–0.110	–0.034 7
0.6	1.51	–0.978	–0.062 4	–0.020 8
0.65	1.46	–0.979	–0.037 4	–0.012 5

Terminal velocity is never reached. The leaf is at 99.9% of v_T after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with $\Delta t=0.001\ \text{s}$, we find the fall takes 2.14 s.

P6.45 (a) When $v = v_T$, $a = 0$, $\sum F = -mg + Cv_T^2 = 0$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(4.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{2.50\times10^{-5}\ \text{kg/m}}} = \boxed{-13.7\ \text{m/s}}$$

(b)

t(s)	x(m)	v(m/s)	F(mN)	a(m/s^2)
0	0	0	– 4.704	–9.8
0.2	0	–1.96	– 4.608	–9.599 9
0.4	–0.392	–3.88	– 4.327 6	–9.015 9
0.6	–1.168	–5.683 2	–3.896 5	–8.117 8
0.8	–2.30	–7.306 8	–3.369 3	–7.019 3
1.0	–3.77	–8.710 7	–2.807 1	–5.848 1
1.2	–5.51	–9.880 3	–2.263 5	–4.715 6
1.4	–7.48	–10.823	–1.775 3	–3.698 6
1.6	–9.65	–11.563	–1.361 6	–2.836 6
1.8	–11.96	–12.13	–1.03	–2.14
2	–14.4	–12.56	–0.762	–1.59

. . . listing results after each fifth step

3	–27.4	–13.49	–0.154	–0.321
4	–41.0	–13.67	–0.029 1	–0.060 6
5	–54.7	–13.71	–0.005 42	–0.011 3

The hailstone reaches 99% of v_T after 3.3 s, 99.95% of v_T after 5.0 s, 99.99% of v_T after 6.0 s, 99.999% of v_T after 7.4 s.

P6.46 (a) At terminal velocity, $\sum F = 0 = -mg + Cv_T^2$

$$C = \frac{mg}{v_T^2} = \frac{(0.142\ \text{kg})(9.80\ \text{m/s}^2)}{(42.5\ \text{m/s})^2} = \boxed{7.70\times10^{-4}\ \text{kg/m}}$$

(b) $Cv^2 = (7.70\times10^{-4}\ \text{kg/m})(36.0\ \text{m/s})^2 = \boxed{0.998\ \text{N}}$

(c)

Elapsed Time (s)	Altitude (m)	Speed (m/s)	Resistance Force (N)	Net Force (N)	Acceleration (m/s^2)
0.000 00	0.000 00	36.000 00	–0.998 49	–2.390 09	–16.831 58
0.050 00	1.757 92	35.158 42	–0.952 35	–2.343 95	–16.506 67
…					
2.950 00	48.623 27	0.824 94	–0.000 52	–1.392 12	–9.803 69
3.000 00	48.640 00	0.334 76	–0.000 09	–1.391 69	–9.800 61
3.050 00	48.632 24	–0.155 27	0.000 02	–1.391 58	–9.799 87
…					
6.250 00	1.250 85	–26.852 97	0.555 55	–0.836 05	–5.887 69
6.300 00	–0.106 52	–27.147 36	0.567 80	–0.823 80	–5.801 44

Maximum height is about $\boxed{49\ \text{m}}$. It returns to the ground after about $\boxed{6.3\ \text{s}}$ with a speed of approximately $\boxed{27\ \text{m/s}}$.

P6.47 (a) At constant velocity $\sum F = 0 = -mg + Cv_T^2$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{0.200\text{ kg/m}}} = \boxed{-49.5\text{ m/s}}$$ with chute closed and

$$v_T = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s})}{20.0\text{ kg/m}}} = \boxed{-4.95\text{ m/s}}$$ with chute open.

(b) We use time increments of 0.1 s for $0 < t < 10\text{ s}$, then 0.01 s for $10\text{ s} < t < 12\text{ s}$, and then 0.1 s again.

time(s)	height(m)	velocity(m/s)
0	1000	0
1	995	–9.7
2	980	–18.6
4	929	–32.7
7	812	–43.7
10	674	–47.7
10.1	671	–16.7
10.3	669	–8.02
11	665	–5.09
12	659	–4.95
50	471	–4.95
100	224	–4.95
145	0	–4.95

6.48 (a) We use a time increment of 0.01 s.

time(s)	x(m)	y(m)
0	0	0
0.100	7.81	5.43
0.200	14.9	10.2
0.400	27.1	18.3
1.00	51.9	32.7
1.92	70.0	38.5
2.00	70.9	38.5
4.00	80.4	26.7
5.00	81.4	17.7
6.85	81.8	0

with θ	we find range
30.0°	86.410 m
35.0°	81.8 m
25.0°	90.181 m
20.0°	92.874 m
15.0°	93.812 m
10.0°	90.965 m
17.0°	93.732 m
16.0°	93.839 8 m
15.5°	93.829 m
15.8°	93.839 m
16.1°	93.838 m
15.9°	93.840 2 m

(b) range $= \boxed{81.8\text{ m}}$

(c) So we have maximum range at $\theta = \boxed{15.9°}$

P6.49 (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.046\,0\text{ kg})(9.80\text{ m/s}^2)}{(44.0\text{ m/s})^2} = \boxed{2.33\times 10^{-4}\text{ kg/m}}$$

(b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53\text{ m/s}}$ as that required for horizontal range of 155 m, thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s^2)	y (m)	v_y (m/s)	a_y (m/s^2)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	45.687 0	–10.565 9	0.000 0	27.451 5	–13.614 6	53.300 0	31.000 0
0.002 7	0.121 1	45.659 0	–10.552 9	0.072 7	27.415 5	–13.604 6	53.257 4	30.982 2
…								
2.501 6	90.194 6	28.937 5	–4.238 8	32.502 4	0.023 5	–9.800 0	28.937 5	0.046 6
2.504 3	90.271 3	28.926 3	–4.235 5	32.502 4	–0.002 4	–9.800 0	28.926 3	–0.004 8
2.506 9	90.348 0	28.915 0	–4.232 2	32.502 4	–0.028 4	–9.800 0	28.915 1	–0.056 3
…								
3.423 8	115.229 8	25.492 6	–3.289 6	28.397 2	–8.890 5	–9.399 9	26.998 4	–19.226 2
3.426 5	115.297 4	25.483 9	–3.287 4	28.373 6	–8.915 4	–9.397 7	26.998 4	–19.282 2
3.429 1	115.364 9	25.475 1	–3.285 1	28.350 0	–8.940 3	–9.395 4	26.998 4	–19.338 2
…								
5.151 6	154.996 8	20.843 8	–2.199 2	0.005 9	–23.308 7	–7.049 8	31.269 2	–48.195 4
5.154 3	155.052 0	20.838 0	–2.198 0	–0.055 9	–23.327 4	–7.045 4	31.279 2	–48.226 2

(c) Similarly, the initial speed is $\boxed{42\text{ m/s}}$. The motion proceeds thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s^2)	y (m)	v_y (m/s)	a_y (m/s^2)	$v=\sqrt{v_x^2+v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	28.746 2	–4.182 9	0.000 0	30.826 6	–14.610 3	42.150 0	47.000 0
0.003 5	0.100 6	28.731 6	–4.178 7	0.107 9	30.775 4	–14.594 3	42.102 6	46.967 1
…								
2.740 5	66.307 8	20.548 4	–2.137 4	39.485 4	0.026 0	–9.800 0	20.548 5	0.072 5
2.744 0	66.379 7	20.541 0	–2.135 8	39.485 5	–0.008 3	–9.800 0	20.541 0	–0.023 1
2.747 5	66.451 6	20.533 5	–2.134 3	39.485 5	–0.042 6	–9.800 0	20.533 5	–0.118 8
…								
3.146 5	74.480 5	19.715 6	–1.967 6	38.696 3	–3.942 3	–9.721 3	20.105 8	–11.307 7
3.150 0	74.549 5	19.708 7	–1.966 2	38.682 5	–3.976 4	–9.720 0	20.105 8	–11.406 7
3.153 5	74.618 5	19.701 8	–1.964 9	38.668 6	–4.010 4	–9.718 6	20.105 8	–11.505 6
…								
5.677 0	118.969 7	15.739 4	–1.254 0	0.046 5	–25.260 0	–6.570 1	29.762 3	–58.073 1
5.680 5	119.024 8	15.735 0	–1.253 3	–0.041 9	–25.283 0	–6.564 2	29.779 5	–58.103 7

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

Additional Problems

***P6.50** When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}.$$

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$.

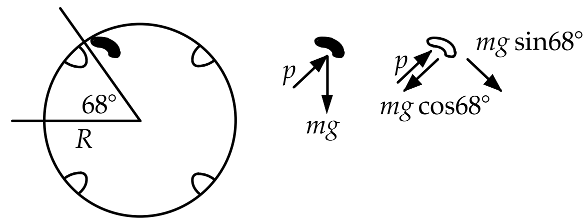


FIG. P6.50

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})\left(9.8\ \text{m/s}^2\right)\sin 68°} = 1.73\ \text{m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right) = \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}.$$

***P6.51** (a)

$$v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right) = 8.33\ \text{m/s}$$

$$\sum F_y = ma_y:\ +n - mg = -\frac{mv^2}{r}$$

$$n = m\left(g - \frac{v^2}{r}\right) = 1\,800\ \text{kg}\left[9.8\ \text{m/s}^2 - \frac{(8.33\ \text{m/s})^2}{20.4\ \text{m}}\right]$$

$$= \boxed{1.15\times 10^4\ \text{N up}}$$

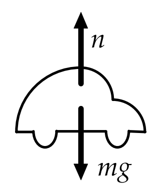


FIG. P6.51

(b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{\left(9.8\ \text{m/s}^2\right)(20.4\ \text{m})} = \boxed{14.1\ \text{m/s}} = 50.9\ \text{km/h}$$

P6.52 (a)

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $\quad mg = \frac{mv^2}{R}$

Then, $\quad v = \boxed{\sqrt{gR}}$.

***P6.53** (a) $\text{slope} = \dfrac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\,2\ \text{kg/m}}$

(b) $\text{slope} = \dfrac{R}{v^2} = \dfrac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\dfrac{1}{2}D\rho A}$

(c) $\dfrac{1}{2}D\rho A = 0.016\,2\ \text{kg/m}$

$$D = \frac{2(0.016\,2\ \text{kg/m})}{\left(1.20\ \text{kg/m}^3\right)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force $mg = 8\left(1.64\times10^{-3}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right) = 0.129\ \text{N}$ and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here $(0.016\,2\ \text{kg/m})(2.8\ \text{m/s})^2 = 0.127\ \text{N}$. The scatter percentage is $\dfrac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = 1.5\%$.

(e) The interpretation of the graph can be stated thus: For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \dfrac{1}{2}D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

P6.54 (a) While the car negotiates the curve, the accelerometer is at the angle θ.

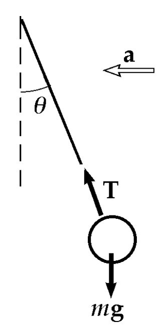


FIG. P6.54

Horizontally: $T\sin\theta = \dfrac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then $a_c = \dfrac{v^2}{r} = g\tan\theta$: $a_c = \left(9.80\ \text{m/s}^2\right)\tan 15.0°$

$a_c = \boxed{2.63\ \text{m/s}^2}$

(b) $r = \dfrac{v^2}{a_c}$ $\quad r = \dfrac{(23.0\ \text{m/s})^2}{2.63\ \text{m/s}^2} = \boxed{201\ \text{m}}$

(c) $v^2 = rg\tan\theta = (201\ \text{m})\left(9.80\ \text{m/s}^2\right)\tan 9.00°$ $\quad v = \boxed{17.7\ \text{m/s}}$

P6.55 Take x-axis up the hill

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$
$$a = \frac{T}{m}\sin\theta - g\sin\phi$$
$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$
$$T = \frac{mg\cos\phi}{\cos\theta}$$
$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$
$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

***P6.56** (a) The speed of the bag is $\frac{2\pi(7.46\text{ m})}{38\text{ s}} = 1.23\text{ m/s}$. The total force on it must add to

$$ma_c = \frac{(30\text{ kg})(1.23\text{ m/s})^2}{7.46\text{ m}} = 6.12\text{ N}$$

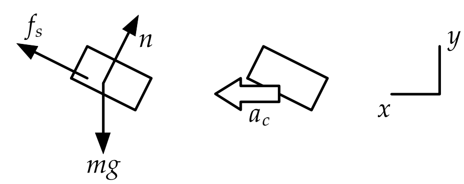


FIG. P6.56

$$\sum F_x = ma_x: \quad f_s\cos 20 - n\sin 20 = 6.12\text{ N}$$
$$\sum F_y = ma_y: \quad f_s\sin 20 + n\cos 20 - (30\text{ kg})\left(9.8\text{ m/s}^2\right) = 0$$
$$n = \frac{f_s\cos 20 - 6.12\text{ N}}{\sin 20}$$

Substitute:

$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (6.12\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 16.8\text{ N}$$
$$f_s = \boxed{106\text{ N}}$$

(b)
$$v = \frac{2\pi(7.94\text{ m})}{34\text{ s}} = 1.47\text{ m/s}$$
$$ma_c = \frac{(30\text{ kg})(1.47\text{ m/s})^2}{7.94\text{ m}} = 8.13\text{ N}$$
$$f_s\cos 20 - n\sin 20 = 8.13\text{ N}$$
$$f_s\sin 20 + n\cos 20 = 294\text{ N}$$
$$n = \frac{f_s\cos 20 - 8.13\text{ N}}{\sin 20}$$
$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (8.13\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 22.4\text{ N}$$
$$f_s = 108\text{ N}$$
$$n = \frac{(108\text{ N})\cos 20 - 8.13\text{ N}}{\sin 20} = 273\text{ N}$$
$$\mu_s = \frac{f_s}{n} = \frac{108\text{ N}}{273\text{ N}} = \boxed{0.396}$$

P6.57 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F_g' = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F_g'}$$

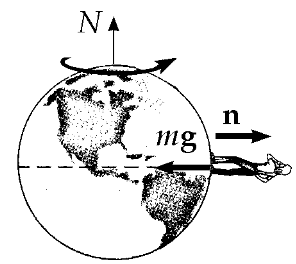


FIG. P6.57

(b) At the poles $v = 0$ and $F_g' = F_g = mg = 75.0(9.80) = \boxed{735 \text{ N}}$ down.

At the equator, $F_g' = F_g - ma_c = 735 \text{ N} - 75.0(0.033\,7) \text{ N} = \boxed{732 \text{ N}}$ down.

P6.58 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$

we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}}$.

P6.59 (a) $v = (300 \text{ mi/h})\left(\dfrac{88.0 \text{ ft/s}}{60.0 \text{ mi/h}}\right) = 440 \text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967 \text{ lb}}.$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647 \text{ lb}}.$$

Since the plane is upside down, the seat exerts this downward force.

(c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$. If we vary the aircraft's R and v such that the above is true, then the pilot feels weightless.

P6.60 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

$$n_1 = \left(m_p + m_b\right)g \text{ so } f \le \mu_{s1} n_1 = \mu_{s1}\left(m_p + m_b\right)g .$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore \left(m_p + m_b\right)\frac{v_{\max}^2}{r} = \mu_{s1}\left(m_p + m_b\right)g$$

or $v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$.

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

and $\sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$.

When the penny is about to slip on the block, $f_p = f_{p,\max} = \mu_{s2} n_2$

or $\mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

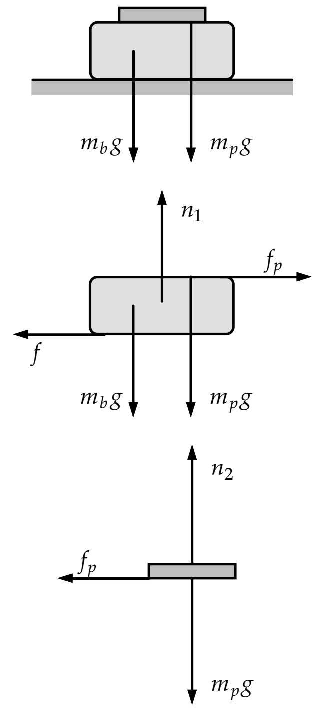


FIG. P6.60

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}.$$

P6.61 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m\left(g + a_r\right) = \boxed{455 \text{ N}}$

(c) $F_{\text{high}} = m\left(g - a_r\right) = \boxed{328 \text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397 \text{ N upward and}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \tan^{-1}\dfrac{1.58}{9.8} = \boxed{9.15° \text{ inward}}$.

P6.62 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the 3.00 m/s^2 centripetal acceleration:

$a_c = \dfrac{v^2}{r}$: $\qquad v = \sqrt{a_c r} = \sqrt{\left(3.00 \text{ m/s}^2\right)(60.0 \text{ m})} = 13.4 \text{ m/s}$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$: $\quad T = \dfrac{2\pi r}{v} = \dfrac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$

so the frequency of rotation is $\quad f = \dfrac{1}{T} = \dfrac{1}{28.1 \text{ s}} = \dfrac{1}{28.1 \text{ s}}\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$.

P6.63 (a) The mass at the end of the chain is in vertical equilibrium. Thus $T\cos\theta = mg$.

Horizontally $T\sin\theta = ma_r = \dfrac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m} = 5.17\text{ m}$$

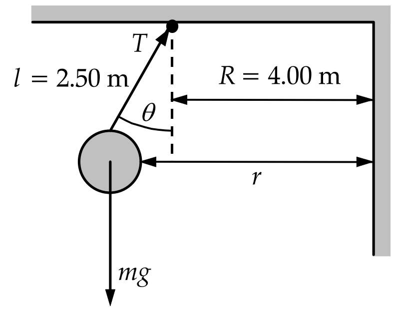


FIG. P6.63

Then $a_r = \dfrac{v^2}{5.17\text{ m}}$.

By division $\tan\theta = \dfrac{a_r}{g} = \dfrac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\ \text{m}^2/\text{s}^2$$
$$v = \boxed{5.19\ \text{m/s}}$$

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\text{ kg})\left(9.80\ \text{m/s}^2\right)}{\cos 28.0°} = \boxed{555\text{ N}}$$

P6.64 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

Thus, $v^2 = \pi Rg$ and $\boxed{v = \sqrt{\pi Rg}}$.

(b) The putty is dislodged when F, the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}.$$

P6.65 (a) $n = \dfrac{mv^2}{R}$ $\quad f - mg = 0$

$f = \mu_s n$ $\quad v = \dfrac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\text{ s}}$

$$\#\ \frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

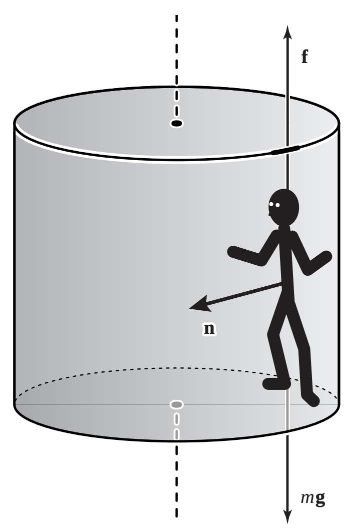


FIG. P6.65

P6.66 Let the x–axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0 \text{ m/s})(\sin 48.0°)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{ix} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi\left(6.37\times10^6 \text{ m}\right)\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285 \text{ m}}{2\pi\left(6.37\times10^6 \text{ m}\right)}\right)(360°) = 2.56\times10^{-3} \text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0°–0.002\,56° = 34.997\,4°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86\,400 \text{ s}}$, which is larger than the eastward velocity of the tee by

$$\begin{aligned}\Delta v_x = v_{fx} - v_{fi} &= \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos(\phi_i - \Delta\phi) - \cos\phi_i\right] \\ &= \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]\end{aligned}$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86\,400 \text{ s}}\sin\phi_i \sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi\left(6.37\times10^6 \text{ m}\right)}{86\,400 \text{ s}}\sin 35.0° \sin 0.002\,56° = \boxed{1.19\times10^{-2} \text{ m/s}}$$

(d) $$\Delta x = (\Delta v_x)t = \left(1.19\times10^{-2} \text{ m/s}\right)(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$$

P6.67 (a) If the car is about to slip *down* the incline, f is directed up the incline.

$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta(1+\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta(1+\mu_s\tan\theta)}.$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}}}.$$

When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}.$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1-\mu_s\tan\theta}}}.$$

(b) If $v_{\min} = \sqrt{\dfrac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

(c)

$$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1+(0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1-(0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

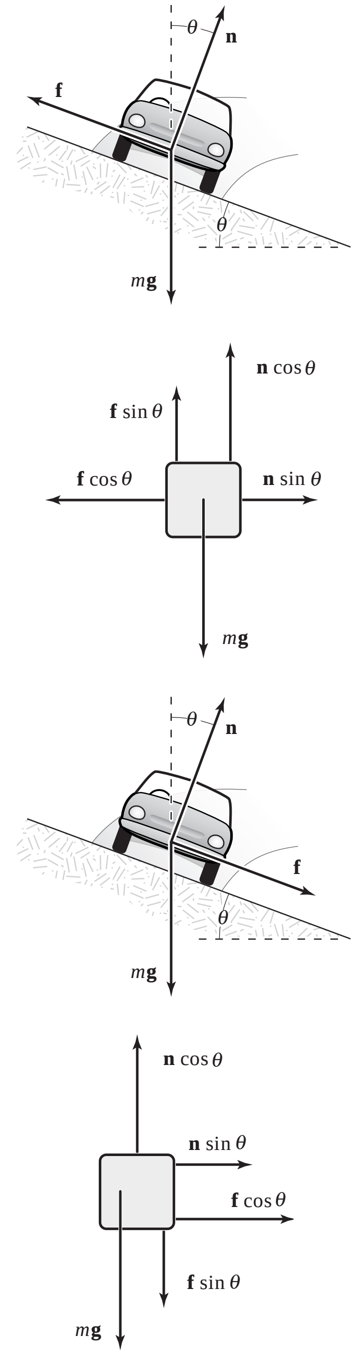


FIG. P6.67

P6.68 (a) The bead moves in a circle with radius $v = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has
an inward radial component of $n\sin\theta$
and an upward component of $n\cos\theta$

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

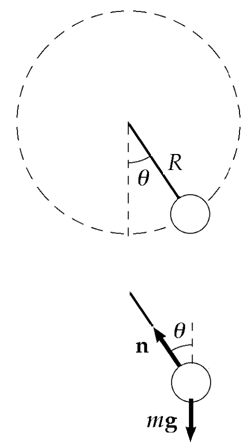


FIG. P6.68(a)

Then $\sum F_x = n\sin\theta = m\frac{v^2}{r}$ becomes $\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$

which reduces to $\frac{g\sin\theta}{\cos\theta} = \frac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \Rightarrow \theta = 0°$ (1)

and $\cos\theta = \frac{gT^2}{4\pi^2 R}$ (2)

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80 \text{ m/s}^2)(0.450 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 0.335 \text{ and } \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80 \text{ m/s}^2)(0.850 \text{ s})^2}{4\pi^2(0.150 \text{ m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

P6.69 At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

(a) $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$.

(b) $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

(c) $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1\ \text{m/s}$, and ignoring the first term:

$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2$ $\qquad v = \boxed{6.87\ \text{m/s}}$

P6.70 $v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$, $\qquad v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s $\qquad 0.500v_T = v_T\left[1 - \exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500;$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693;$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{m/s}$$

(a) $v_T = \frac{mg}{b}$ $\qquad v_T = \frac{(9.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $0.750v_T = v_T\left[1 - \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right]$ $\qquad \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

continued on next page

(c) $$\frac{dx}{dt}=\left(\frac{mg}{b}\right)\left[1-\exp\left(-\frac{bt}{m}\right)\right]; \quad \int_{x_0}^{x} dx=\int_0^t\left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]dt$$

$$x-x_0=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right)-1\right]$$

At $t=5.54$ s, $$x=9.00\text{ kg}\left(9.80\text{ m/s}^2\right)\frac{5.54\text{ s}}{1.13\text{ kg/s}}+\left(\frac{(9.00\text{ kg})^2\left(9.80\text{ m/s}^2\right)}{(1.13\text{ m/s})^2}\right)\left[\exp(-0.693)-1\right]$$

$$x=434\text{ m}+626\text{ m}(-0.500)=\boxed{121\text{ m}}$$

P6.71 $$\sum F_y=L_y-T_y-mg=L\cos 20.0°-T\sin 20.0°-7.35\text{ N}=ma_y=0$$

$$\sum F_x=L_x+T_x=L\sin 20.0°+T\cos 20.0°=m\frac{v^2}{r}$$

$$m\frac{v^2}{r}=0.750\text{ kg}\frac{(35.0\text{ m/s})^2}{(60.0\text{ m})\cos 20.0°}=16.3\text{ N}$$

$$\therefore L\sin 20.0°+T\cos 20.0°=16.3\text{ N}$$

$$L\cos 20.0°-T\sin 20.0°=7.35\text{ N}$$

$$L+T\frac{\cos 20.0°}{\sin 20.0°}=\frac{16.3\text{ N}}{\sin 20.0°}$$

$$L-T\frac{\sin 20.0°}{\cos 20.0°}=\frac{7.35\text{ N}}{\cos 20.0°}$$

$$T(\cot 20.0°+\tan 20.0°)=\frac{16.3\text{ N}}{\sin 20.0°}-\frac{7.35\text{ N}}{\cos 20.0°}$$

$$T(3.11)=39.8\text{ N}$$

$$T=\boxed{12.8\text{ N}}$$

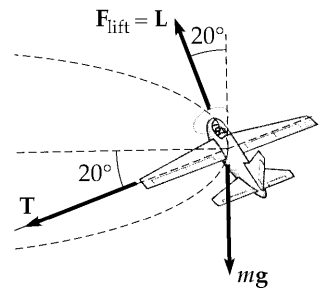
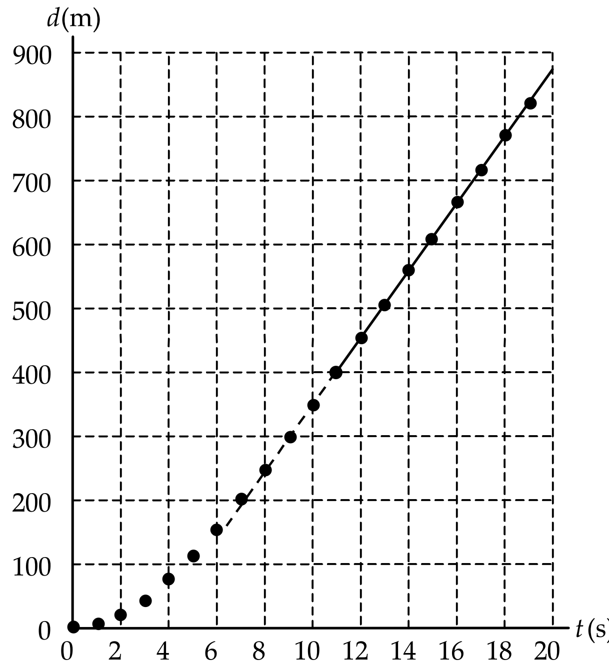


FIG. P6.71

P6.72 (a)

t(s)	d(m)
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$

***P6.73** $v = v_i - kx$ implies the acceleration is $a = \frac{dv}{dt} = 0 - k\frac{dx}{dt} = -kv$

Then the total force is $\sum F = ma = m(-kv)$

The resistive force is opposite to the velocity: $\boxed{\sum \mathbf{F} = -km\mathbf{v}}$.

ANSWERS TO EVEN PROBLEMS

P6.2 215 N horizontally inward

P6.4 6.22×10^{-12} N

P6.6 (a) 1.65 km/s; (b) 6.84×10^{3} s

P6.8 0.966 g

P6.10 (a) $\left(-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}}\right)\text{m/s}^2$; (b) 6.53 m/s; (c) $\left(-0.181\,\hat{\mathbf{i}} + 0.181\,\hat{\mathbf{j}}\right)\text{m/s}^2$

P6.12 2.06×10^{3} rev/min

P6.14 (a) $\sqrt{R\left(\frac{2T}{m} - g\right)}$; (b) $2T$ upward

P6.16 (a) 1.33 m/s^2; (b) 1.79 m/s^2 forward and 48.0° inward

P6.18 8.88 N

P6.20 (a) 8.62 m; (b) Mg downward; (c) 8.45 m/s^2, Unless they are belted in, the riders will fall from the cars.

P6.22 15.3 m/s Straight across the dashboard to the left

P6.24 0.527°

P6.26 (a) 1.41 h; (b) 17.1

P6.28 $\mu_k = \dfrac{2(vt - L)}{(g+a)t^2}$

P6.30 (a) $2.38 \times 10^5\ \text{m/s}^2$ horizontally inward $= 2.43 \times 10^4 g$; (b) 360 N inward perpendicular to the cone; (c) $47.5 \times 10^4\ \text{m/s}^2$

P6.32 (a) $6.27\ \text{m/s}^2$ downward; (b) 784 N up; (c) 283 N up

P6.34 (a) 53.8 m/s; (b) 148 m

P6.36 1.40

P6.38 $-0.212\ \text{m/s}^2$

P6.40 see the solution

P6.42 36.5 m/s

P6.44 (a) 0.980 m/s; (b) see the solution

P6.46 (a) 7.70×10^{-4} kg/m; (b) 0.998 N; (c) The ball reaches maximum height 49 m. Its flight lasts 6.3 s and its impact speed is 27 m/s.

P6.48 (a) see the solution; (b) 81.8 m; (c) 15.9°

P6.50 0.835 rev/s

P6.52 (a) $mg - \dfrac{mv^2}{R}$; (b) $v = \sqrt{gR}$

P6.54 (a) $2.63\ \text{m/s}^2$; (b) 201 m; (c) 17.7 m/s

P6.56 (a) 106 N; (b) 0.396

P6.58 (a) $m_2 g$; (b) $m_2 g$; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}$

P6.60 62.2 rev/min

P6.62 2.14 rev/min

P6.64 (a) $v = \sqrt{\pi R g}$; (b) $m \pi g$

P6.66 (a) 8.04 s; (b) 379 m/s; (c) 1.19 cm/s; (d) 9.55 cm

P6.68 (a) either 70.4° or 0°; (b) 0°

P6.70 (a) 78.3 m/s; (b) 11.1 s; (c) 121 m

P6.72 (a) and (b) see the solution; (c) 53.0 m/s

7

Energy and Energy Transfer

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q7.1 The force is perpendicular to every increment of displacement. Therefore, $\mathbf{F}\cdot\Delta\mathbf{r}=0$.

Q7.2 (a) Positive work is done by the chicken on the dirt.

(b) No work is done, although it may seem like there is.

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

Q7.3 Yes. Force times distance over which the toe is in contact with the ball. No, he is no longer applying a force. Yes, both air friction and gravity do work.

Q7.4 Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q7.5 (a) Tension (b) Air resistance

(c) Positive in increasing velocity on the downswing.
Negative in decreasing velocity on the upswing.

Q7.6 No. The vectors might be in the third and fourth quadrants, but if the angle between them is less than 90° their dot product is positive.

Q7.7 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90°<\theta<180°$.

Q7.8 If the coils of the spring are initially in contact with one another, as the load increases from zero, the graph would be an upwardly curved arc. After the load increases sufficiently, the graph will be linear, described by Hooke's Law. This linear region will be quite large compared to the first region. The graph will then be a downward curved arc as the coiled spring becomes a completely straight wire. As the load increases with a straight wire, the graph will become a straight line again, with a significantly smaller slope. Eventually, the wire would break.

Q7.9 $k'=2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q7.10 Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q7.11 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release.

Q7.12 Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

Q7.13 The longer barrel will have the higher muzzle speed. Since the accelerating force acts over a longer distance, the change in kinetic energy will be larger.

Q7.14 (a) Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

(b) If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

Q7.15 The larger engine is unnecessary. Consider a 30 minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

Q7.16 If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

Q7.17 It decreases, as the force required to lift the car decreases.

Q7.18 As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

Q7.19 The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

Q7.20 The normal force does no work because the angle between the normal force and the direction of motion is usually 90°. Static friction usually does no work because there is no distance through which the force is applied.

Q7.21 An argument for: As a glider moves along an airtrack, the only force that the track applies on the glider is the normal force. Since the angle between the direction of motion and the normal force is 90°, the work done must be zero, even if the track is not level.
Against: An airtrack has bumpers. When a glider bounces from the bumper at the end of the airtrack, it loses a bit of energy, as evidenced by a decreased speed. The airtrack does negative work.

Q7.22 Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

SOLUTIONS TO PROBLEMS

Section 7.1 **Systems and Environments**

Section 7.2 **Work Done by a Constant Force**

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.2 The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times10^{3}\text{ J}}.$$

P7.3 **Method One.**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$.
The work done by the gravitational force on Batman is

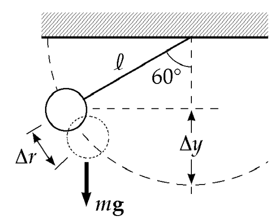


FIG. P7.3

$$W = \int_i^f F\cos\theta\,dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\,d\phi = (-80\text{ kg})\left(9.8\text{ m/s}^2\right)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^{3}\text{ J}}$$

Method Two.

The force of gravity on Batman is $mg = (80\text{ kg})\left(9.8\text{ m/s}^2\right) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is –12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}.$$

P7.4 (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

Section 7.3 **The Scalar Product of Two Vectors**

P7.5 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

P7.6
$$\mathbf{A}\cdot\mathbf{B} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$
$$\mathbf{A}\cdot\mathbf{B} = A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)$$
$$+A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)$$
$$+A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)$$
$$\mathbf{A}\cdot\mathbf{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.7 (a) $W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $$\theta = \cos^{-1}\left(\frac{\mathbf{F}\cdot\Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\frac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$$

P7.8 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$$

or $\mathbf{F}\cdot\mathbf{v} = 5.33\,\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\,\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

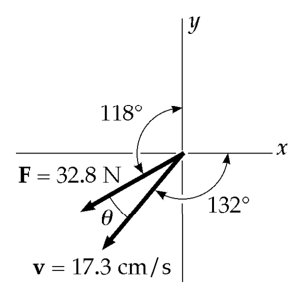


FIG. P7.8

P7.9 (a) $\mathbf{A} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\mathbf{B} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\qquad \theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{A} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\qquad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\mathbf{A} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{B} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

P7.10 $\mathbf{A}-\mathbf{B}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$$\mathbf{A}-\mathbf{B}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$$

$$\mathbf{C}\cdot(\mathbf{A}-\mathbf{B})=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$$

Section 7.4 **Work Done by a Varying Force**

P7.11 $W=\int_i^f F\,dx=$ area under curve from x_i to x_f

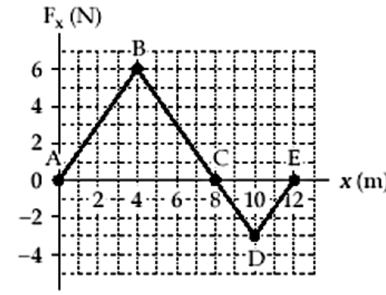


FIG. P7.11

(a) $x_i=0$ $\quad$ $x_f=8.00$ m

$$W=\text{ area of triangle } ABC=\left(\frac{1}{2}\right)AC\times\text{ altitude},$$

$$W_{0\to 8}=\left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N}=\boxed{24.0\text{ J}}$$

(b) $x_i=8.00$ m $\quad$ $x_f=10.0$ m

$$W=\text{ area of }\Delta CDE=\left(\frac{1}{2}\right)CE\times\text{ altitude},$$

$$W_{8\to 10}=\left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N})=\boxed{-3.00\text{ J}}$$

(c) $W_{0\to 10}=W_{0\to 8}+W_{8\to 10}=24.0+(-3.00)=\boxed{21.0\text{ J}}$

P7.12 $F_x=(8x-16)$ N

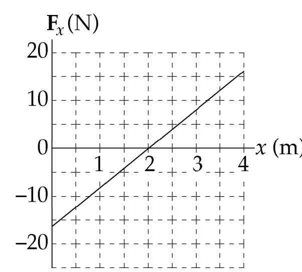


FIG. P7.12

(a) See figure to the right

(b) $W_{\text{net}}=\dfrac{-(2.00\text{ m})(16.0\text{ N})}{2}+\dfrac{(1.00\text{ m})(8.00\text{ N})}{2}=\boxed{-12.0\text{ J}}$

P7.13 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

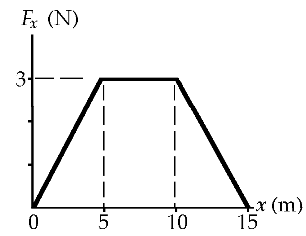


FIG. P7.13

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0) \text{ J} = \boxed{30.0 \text{ J}}$$

P7.14 $W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \int_0^{5 \text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right) \text{N} \cdot dx\hat{\mathbf{i}}$

$$\int_0^{5 \text{ m}} (4 \text{ N/m})x dx + 0 = (4 \text{ N/m}) \left.\frac{x^2}{2}\right|_0^{5 \text{ m}} = \boxed{50.0 \text{ J}}$$

P7.15 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80) \text{ N}}{2.50 \times 10^{-2} \text{ m}} = 1.57 \times 10^3 \text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938 \text{ cm}}$

(b) Work $= \frac{1}{2} ky^2$

Work $= \frac{1}{2}\left(1.57 \times 10^3 \text{ N} \cdot \text{m}\right)\left(4.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{1.25 \text{ J}}$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230 \text{ N})}{(0.400 \text{ m})} = \boxed{575 \text{ N/m}}$$

(b) Work $= F_{\text{avg}} x = \frac{1}{2}(230 \text{ N})(0.400 \text{ m}) = \boxed{46.0 \text{ J}}$

***P7.17** (a)

$$F_{\text{applied}} = k_{\text{leaf}} x_\ell + k_{\text{helper}} x_h = k_\ell x_\ell + k_h (x_\ell - y_0)$$

$$5\times10^5\ \text{N} = 5.25\times10^5\ \frac{\text{N}}{\text{m}} x_\ell + 3.60\times10^5\ \frac{\text{N}}{\text{m}}(x_\ell - 0.5\ \text{m})$$

$$x_\ell = \frac{6.8\times10^5\ \text{N}}{8.85\times10^5\ \text{N/m}} = \boxed{0.768\ \text{m}}$$

(b)

$$W = \frac{1}{2}k_\ell x_\ell^2 + \frac{1}{2}k_h x_h^2 = \frac{1}{2}\left(5.25\times10^5\ \frac{\text{N}}{\text{m}}\right)(0.768\ \text{m})^2 + \frac{1}{2}3.60\times10^5\ \frac{\text{N}}{\text{m}}(0.268\ \text{m})^2$$

$$= \boxed{1.68\times10^5\ \text{J}}$$

P7.18 (a)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right)dx\cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$W = \boxed{11.7\ \text{kJ}}$, larger by 29.6%

P7.19 $4.00\ \text{J} = \frac{1}{2}k(0.100\ \text{m})^2$

$\therefore k = 800\ \text{N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

P7.20 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x–axis in the direction of motion tangent to the cylinder.

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

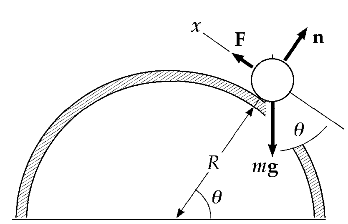


FIG. P7.20

(b)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

***P7.21** The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$= 1.5\text{ kg } 9.8\text{ m/s}^2\left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right) = \boxed{2.04\times10^{-2}\text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$
$$= \left(\frac{1\text{ m}}{1\,200\text{ N}} + \frac{1\text{ m}}{1\,800\text{ N}}\right)^{-1} = \boxed{720\text{ N/m}}$$

***P7.22** See the solution to problem 7.21.

(a) $x = mg\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)$

(b) $k = \left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)^{-1}$

P7.23 $[k] = \left[\dfrac{F}{x}\right] = \dfrac{\text{N}}{\text{m}} = \dfrac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\dfrac{\text{kg}}{\text{s}^2}}$

Section 7.5 **Kinetic Energy and the Work-Kinetic Energy Theorem**

Section 7.6 **The Non-Isolated System—Conservation of Energy**

P7.24 (a) $K_A = \dfrac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2 = \boxed{1.20\text{ J}}$

(b) $\dfrac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{(2)(7.50)}{0.600}} = \boxed{5.00\text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \dfrac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50\text{ J} - 1.20\text{ J} = \boxed{6.30\text{ J}}$

P7.25 (a) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(0.300\text{ kg})(15.0\text{ m/s})^2 = \boxed{33.8\text{ J}}$

(b) $K = \dfrac{1}{2}(0.300)(30.0)^2 = \dfrac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135\text{ J}}$

P7.26 $\mathbf{v}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$

(a) $$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0} \text{ m/s}$$
$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})\left(40.0 \text{ m}^2/\text{s}^2\right) = \boxed{60.0 \text{ J}}$$

(b) $$\mathbf{v}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$$
$$v_f^2 = \mathbf{v}_f \cdot \mathbf{v}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$$
$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$$

P7.27 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$\sum W = \Delta K$: $\quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

so $\quad (mg)(h+d)\cos 0° + \left(\bar{F}\right)(d)\cos 180° = 0 - 0$.

Thus, $\quad \bar{F} = \dfrac{(mg)(h+d)}{d} = \dfrac{(2\,100 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.12 \text{ m})}{0.120 \text{ m}} = \boxed{8.78 \times 10^5 \text{ N}}$. The force on the pile driver is $\boxed{\text{upward}}$.

P7.28 (a) $$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = \text{(area under curve from } x = 0 \text{ to } x = 5.00 \text{ m)}$$
$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$$

(b) $$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = \text{(area under curve from } x = 0 \text{ to } x = 10.0 \text{ m)}$$
$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$$

(c) $$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = \text{(area under curve from } x = 0 \text{ to } x = 15.0 \text{ m)}$$
$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$$

P7.29 (a) $$K_i + \sum W = K_f = \frac{1}{2}mv_f^2$$
$$0 + \sum W = \frac{1}{2}\left(15.0 \times 10^{-3} \text{ kg}\right)(780 \text{ m/s})^2 = \boxed{4.56 \text{ kJ}}$$

(b) $$F = \frac{W}{\Delta r \cos\theta} = \frac{4.56 \times 10^3 \text{ J}}{(0.720 \text{ m})\cos 0°} = \boxed{6.34 \text{ kN}}$$

(c) $$a = \frac{v_f^2 - v_i^2}{2x_f} = \frac{(780 \text{ m/s})^2 - 0}{2(0.720 \text{ m})} = \boxed{422 \text{ km/s}^2}$$

(d) $$\sum F = ma = \left(15 \times 10^{-3} \text{ kg}\right)\left(422 \times 10^3 \text{ m/s}^2\right) = \boxed{6.34 \text{ kN}}$$

P7.30 (a) $v_f = 0.096\left(3\times 10^8\ \text{m/s}\right) = 2.88\times 10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(9.11\times 10^{-31}\ \text{kg}\right)\left(2.88\times 10^7\ \text{m/s}\right)^2 = \boxed{3.78\times 10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78\times 10^{-16}\ \text{J}$$

$$F = \boxed{1.35\times 10^{-14}\ \text{N}}$$

(c) $\sum F = ma$; $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times 10^{-14}\ \text{N}}{9.11\times 10^{-31}\ \text{kg}} = \boxed{1.48\times 10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t \quad 2.88\times 10^7\ \text{m/s} = 0 + \left(1.48\times 10^{16}\ \text{m/s}^2\right)t$

$$t = \boxed{1.94\times 10^{-9}\ \text{s}}$$

Check: $\quad x_f = x_i + \dfrac{1}{2}\left(v_{xi} + v_{xf}\right)t$

$$0.028\ \text{m} = 0 + \frac{1}{2}\left(0 + 2.88\times 10^7\ \text{m/s}\right)t$$

$$t = 1.94\times 10^{-9}\ \text{s}$$

Section 7.7 **Situations Involving Kinetic Friction**

P7.31 $\sum F_y = ma_y$: $\quad n - 392\ \text{N} = 0$

$$n = 392\ \text{N}$$

$$f_k = \mu_k n = (0.300)(392\ \text{N}) = 118\ \text{N}$$

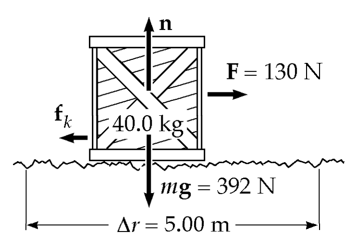


FIG. P7.31

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\ \text{J}}$

(b) $\Delta E_{\text{int}} = f_k\Delta x = (118)(5.00) = \boxed{588\ \text{J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$$\frac{1}{2}mv_f^2 - 0 = 650\ \text{J} - 588\ \text{J} + 0 + 0 = \boxed{62.0\ \text{J}}$$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\ \text{J})}{40.0\ \text{kg}}} = \boxed{1.76\ \text{m/s}}$

P7.32 (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)\left(5.00\times 10^{-2}\right)^2 - 0 = 0.625 \text{ J}$

$$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$$

so $$v_f = \sqrt{\frac{2\left(\sum W\right)}{m}} = \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b) $$\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$$

$$0 - (0.350)(2.00)(9.80)(0.050\,0) \text{ J} + 0.625 \text{ J} = \frac{1}{2}mv_f^2$$

$$0.282 \text{ J} = \frac{1}{2}(2.00 \text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$$

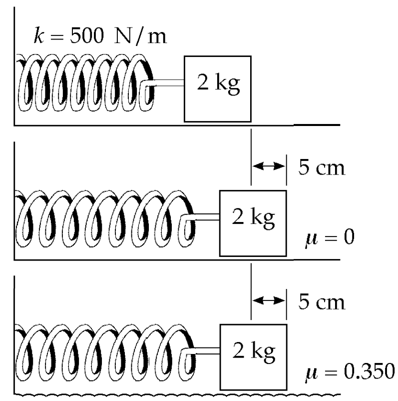


FIG. P7.32

P7.33 (a) $W_g = mg\ell\cos(90.0° + \theta)$

$$W_g = (10.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.00 \text{ m})\cos 110° = \boxed{-168 \text{ J}}$$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$$

$$\Delta E_{\text{int}} = (5.00 \text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184 \text{ J}}$$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500 \text{ J}}$

(d) $\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148 \text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65 \text{ m/s}}$$

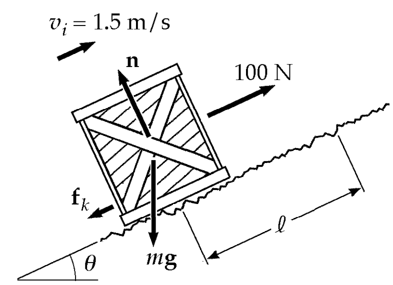


FIG. P7.33

P7.34 $\sum F_y = ma_y$: $n + (70.0 \text{ N})\sin 20.0° - 147 \text{ N} = 0$

$$n = 123 \text{ N}$$

$$f_k = \mu_k n = 0.300(123 \text{ N}) = 36.9 \text{ N}$$

(a) $W = F\Delta r\cos\theta = (70.0 \text{ N})(5.00 \text{ m})\cos 20.0° = \boxed{329 \text{ J}}$

(b) $W = F\Delta r\cos\theta = (123 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0 \text{ J}}$

(c) $W = F\Delta r\cos\theta = (147 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{\text{int}} = F\Delta x = (36.9 \text{ N})(5.00 \text{ m}) = \boxed{185 \text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W - \Delta E_{\text{int}} = 329 \text{ J} - 185 \text{ J} = \boxed{+144 \text{ J}}$

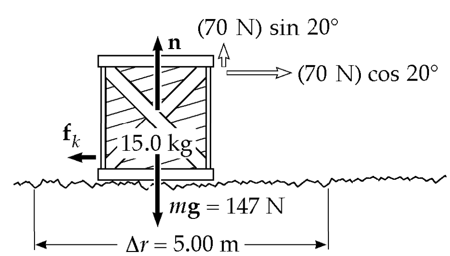


FIG. P7.34

P7.35 $v_i = 2.00\ \text{m/s}$ $\mu_k = 0.100$

$$K_i - f_k\Delta x + W_{\text{other}} = K_f: \qquad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$$

$$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x \qquad \Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\ \text{m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\ \text{m}}$$

Section 7.8 **Power**

***P7.36** $$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\ \text{kg}(0.620\ \text{m/s})^2}{2\left(21\times10^{-3}\ \text{s}\right)} = \boxed{8.01\ \text{W}}$$

P7.37 $$\text{Power} = \frac{W}{t} \qquad \mathcal{P} = \frac{mgh}{t} = \frac{(700\ \text{N})(10.0\ \text{m})}{8.00\ \text{s}} = \boxed{875\ \text{W}}$$

P7.38 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\ \text{kg})(24.6\ \text{m/s})^2 = 390\ \text{kJ}$$

with power $\mathcal{P} = \dfrac{390\,000\ \text{J}}{15.0\ \text{s}}\ \boxed{\sim 10^4\ \text{W}}$ around 30 horsepower.

P7.39 (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\ \text{m})\sin 30.0° = 30.0\ \text{m}$. Thus,

$$W_{\text{in}} = -W_g = (70.0\ \text{kg})\left(9.8\ \text{m/s}^2\right)(30.0\ \text{m}) = \boxed{2.06\times10^4\ \text{J}} = \boxed{20.6\ \text{kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathcal{P}_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times10^4\ \text{J}}{30.0\ \text{s}} = \boxed{686\ \text{W}} = 0.919\ \text{hp}.$$

P7.40 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0+1.75\ \text{m/s}}{2}\right](3.00\ \text{s}) = 2.63\ \text{m}.$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\ \text{kg})(1.75\ \text{m/s})^2 - 0 + (650\ \text{kg})g(2.63\ \text{m}) = 1.77\times10^4\ \text{J}$$

Also, $W = \bar{\mathcal{P}}t$ so $\bar{\mathcal{P}} = \dfrac{W}{t} = \dfrac{1.77\times10^4\ \text{J}}{3.00\ \text{s}} = \boxed{5.91\times10^3\ \text{W}} = 7.92\ \text{hp}.$

(b) When moving upward at constant speed $(v = 1.75\ \text{m/s})$ the applied force equals the weight $= (650\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 6.37\times10^3\ \text{N}$. Therefore,

$$\mathcal{P} = Fv = \left(6.37\times10^3\ \text{N}\right)(1.75\ \text{m/s}) = \boxed{1.11\times10^4\ \text{W}} = 14.9\ \text{hp}.$$

P7.41 *energy = power × time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\ \text{W})\left(1.00\times10^4\ \text{h}\right) = 280\ \text{kilowatt}\cdot\text{hrs}$$
$$\text{total cost} = \$17.00 + (280\ \text{kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\ \text{W})\left(1.00\times10^4\ \text{h}\right) = 1.00\times10^3\ \text{kilowatt}\cdot\text{hrs}$$
$$\#\ \text{bulb used} = \frac{1.00\times10^4\ \text{h}}{750\ \text{h/bulb}} = 13.3$$
$$\text{total cost} = 13.3(\$0.420) + \left(1.00\times10^3\ \text{kWh}\right)(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.20}$

***P7.42** (a) Burning 1 lb of fat releases energy $1\ \text{lb}\left(\frac{454\ \text{g}}{1\ \text{lb}}\right)\left(\frac{9\ \text{kcal}}{1\ \text{g}}\right)\left(\frac{4\,186\ \text{J}}{1\ \text{kcal}}\right) = 1.71\times10^7\ \text{J}.$

The mechanical energy output is $\left(1.71\times10^7\ \text{J}\right)(0.20) = nF\Delta r\cos\theta.$

Then

$$3.42\times10^6\ \text{J} = nmg\Delta y\cos 0°$$
$$3.42\times10^6\ \text{J} = n(50\ \text{kg})\left(9.8\ \text{m/s}^2\right)(80\ \text{steps})(0.150\ \text{m})$$
$$3.42\times10^6\ \text{J} = n\left(5.88\times10^3\ \text{J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times10^6\ \text{J}}{5.88\times10^3\ \text{J}} = \boxed{582}.$

This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^3\ \text{J}}{65\ \text{s}} = \boxed{90.5\ \text{W}} = 90.5\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{0.121\ \text{hp}}.$$

***P7.43** (a) The fuel economy for walking is $\frac{1\ \text{h}}{220\ \text{kcal}}\left(\frac{3\ \text{mi}}{\text{h}}\right)\left(\frac{1\ \text{kcal}}{4\,186\ \text{J}}\right)\left(\frac{1.30\times10^8\ \text{J}}{1\ \text{gal}}\right) = \boxed{423\ \text{mi/gal}}.$

(b) For bicycling $\frac{1\ \text{h}}{400\ \text{kcal}}\left(\frac{10\ \text{mi}}{\text{h}}\right)\left(\frac{1\ \text{kcal}}{4\,186\ \text{J}}\right)\left(\frac{1.30\times10^8\ \text{J}}{1\ \text{gal}}\right) = \boxed{776\ \text{mi/gal}}.$

Section 7.9 **Energy and the Automobile**

P7.44 At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $\mathcal{P}_1 = 18.3\text{ kW}$ to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$\mathcal{P}_2 = \mathcal{P}_1 + (\text{power input to move 350 kg at speed } v)$$

will be required. The additional power output needed to move 350 kg at speed v is:

$$\Delta\mathcal{P}_{\text{out}} = (\Delta f)v = (\mu_r mg)v.$$

Assuming a coefficient of rolling friction of $\mu_r = 0.016\,0$, the power output now needed from the engine is

$$\mathcal{P}_2 = \mathcal{P}_1 + (0.016\,0)(350\text{ kg})\left(9.80\text{ m/s}^2\right)(26.8\text{ m/s}) = 18.3\text{ kW} + 1.47\text{ kW}.$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{\mathcal{P}_1}{\mathcal{P}_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3+1.47}\right)(6.40\text{ km/L})$$

or $(\text{fuel economy})_2 = \boxed{5.92\text{ km/L}}$.

P7.45 (a) fuel needed $= \dfrac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \dfrac{\frac{1}{2}mv_f^2 - 0}{\text{eff.}\times(\text{energy content of fuel})}$

$$= \frac{\frac{1}{2}(900\text{ kg})(24.6\text{ m/s})^2}{(0.150)\left(1.34\times10^8\text{ J/gal}\right)} = \boxed{1.35\times10^{-2}\text{ gal}}$$

(b) $\boxed{73.8}$

(c) power $= \left(\dfrac{1\text{ gal}}{38.0\text{ mi}}\right)\left(\dfrac{55.0\text{ mi}}{1.00\text{ h}}\right)\left(\dfrac{1.00\text{ h}}{3\,600\text{ s}}\right)\left(\dfrac{1.34\times10^8\text{ J}}{1\text{ gal}}\right)(0.150) = \boxed{8.08\text{ kW}}$

Additional Problems

P7.46 At start, $\mathbf{v} = (40.0\text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + (40.0\text{ m/s})\sin 30.0°\,\hat{\mathbf{j}}$

At apex, $\mathbf{v} = (40.0\text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6\text{ m/s})\hat{\mathbf{i}}$

And $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(0.150\text{ kg})(34.6\text{ m/s})^2 = \boxed{90.0\text{ J}}$

P7.47 Concentration of Energy output $= (0.600\ \text{J/kg}\cdot\text{step})(60.0\ \text{kg})\left(\dfrac{1\ \text{step}}{1.50\ \text{m}}\right) = 24.0\ \text{J/m}$

$$F = (24.0\ \text{J/m})(1\ \text{N}\cdot\text{m/J}) = 24.0\ \text{N}$$
$$\mathcal{P} = Fv$$
$$70.0\ \text{W} = (24.0\ \text{N})v$$
$$v = \boxed{2.92\ \text{m/s}}$$

P7.48 (a) $\mathbf{A}\cdot\hat{\mathbf{i}} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\hat{\mathbf{i}} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \dfrac{A_x}{A}}$.

Similarly, $\boxed{\cos\beta = \dfrac{A_y}{A}}$

and $\boxed{\cos\gamma = \dfrac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\dfrac{A_x}{A}\right)^2 + \left(\dfrac{A_y}{A}\right)^2 + \left(\dfrac{A_z}{A}\right)^2 = \dfrac{A^2}{A^2} = 1$

P7.49 (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\ \text{J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathcal{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\ \text{W}}$

(d) $W = \displaystyle\int_0^{2.00} \mathcal{P}dt = \int_0^{2.00}\left(48.0t + 288t^3\right)dt = \boxed{1\,250\ \text{J}}$

***P7.50** (a) We write

$$F = ax^b$$
$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$
$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$
$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$
$$\ln 5 = b \ln 2.44$$
$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$
$$a = \frac{1\,000 \text{ N}}{(0.129 \text{ m})^{1.80}} = \boxed{4.01 \times 10^4 \text{ N/m}^{1.8} = a}$$

(b) $$W = \int_0^{0.25 \text{ m}} F dx = \int_0^{0.25 \text{ m}} 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} x^{1.8} dx$$
$$= 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \left.\frac{x^{2.8}}{2.8}\right|_0^{0.25 \text{ m}} = 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{(0.25 \text{ m})^{2.8}}{2.8}$$
$$= \boxed{294 \text{ J}}$$

***P7.51** The work done by the applied force is

$$W = \int_i^f F_{\text{applied}} dx = \int_0^{x_{\max}} -\left[-\left(k_1 x + k_2 x^2\right)\right] dx$$
$$= \int_0^{x_{\max}} k_1 x\, dx + \int_0^{x_{\max}} k_2 x^2 dx = k_1 \left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2 \left.\frac{x^3}{3}\right|_0^{x_{\max}}$$
$$= \boxed{k_1 \frac{x_{\max}^2}{2} + k_2 \frac{x_{\max}^3}{3}}$$

P7.52 (a) The work done by the traveler is $mgh_s N$ where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where $$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$

and $$\text{vertical velocity of person} = v + nh_s$$

Then, $$N = \frac{nh}{v + nh_s}$$

and the work done by the person becomes $$W_{\text{person}} = \boxed{\frac{mgnhh_s}{v + nh_s}}$$

continued on next page

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = \left[(\text{force exerted})(\text{speed})(\text{time})\right] = mgvt$$

where $t = \dfrac{h}{v + nh_s}$ as above.

Thus, $W_e = \boxed{\dfrac{mgvh}{v + nh_s}}$.

As a check, the total work done on the person's body must add up to *mgh*, the work an elevator would do in lifting him.

It does add up as follows: $\sum W = W_{\text{person}} + W_e = \dfrac{mgnhh_s}{v + nh_s} + \dfrac{mgvh}{v + nh_s} = \dfrac{mgh(nh_s + v)}{v + nh_s} = mgh$

P7.53 (a) $\Delta K = \dfrac{1}{2}mv^2 - 0 = \sum W$, so

$v^2 = \dfrac{2W}{m}$ and $v = \boxed{\sqrt{\dfrac{2W}{m}}}$

(b) $W = \mathbf{F} \cdot \mathbf{d} = F_x d \Rightarrow F_x = \boxed{\dfrac{W}{d}}$

***P7.54** During its whole motion from $y = 10.0 \text{ m}$ to $y = -3.20 \text{ mm}$, the force of gravity and the force of the plate do work on the ball. It starts and ends at rest

$$K_i + \sum W = K_f$$

$$0 + F_g \Delta y \cos 0° + F_p \Delta x \cos 180° = 0$$

$$mg(10.003\ 2 \text{ m}) - F_p(0.003\ 20 \text{ m}) = 0$$

$$F_p = \frac{5 \text{ kg}(9.8 \text{ m/s}^2)(10 \text{ m})}{3.2 \times 10^{-3} \text{ m}} = \boxed{1.53 \times 10^5 \text{ N upward}}$$

P7.55 (a) $\mathcal{P} = Fv = F(v_i + at) = F\left(0 + \dfrac{F}{m}t\right) = \boxed{\left(\dfrac{F^2}{m}\right)t}$

(b) $\mathcal{P} = \left[\dfrac{(20.0 \text{ N})^2}{5.00 \text{ kg}}\right](3.00 \text{ s}) = \boxed{240 \text{ W}}$

***P7.56** (a) $$W_1 = \int_i^f F_1 dx = \int_{x_{i1}}^{x_{i1}+x_a} k_1 x\, dx = \frac{1}{2}k_1\left[\left(x_{i1}+x_a\right)^2 - x_{i1}^2\right] = \frac{1}{2}k_1\left(x_a^2 + 2x_a x_{i1}\right)$$

(b) $$W_2 = \int_{-x_{i2}}^{-x_{i2}+x_a} k_2 x\, dx = \frac{1}{2}k_2\left[\left(-x_{i2}+x_a\right)^2 - x_{i2}^2\right] = \frac{1}{2}k_2\left(x_a^2 - 2x_a x_{i2}\right)$$

(c) Before the horizontal force is applied, the springs exert equal forces: $k_1 x_{i1} = k_2 x_{i2}$

$$x_{i2} = \frac{k_1 x_{i1}}{k_2}$$

(d) $$W_1 + W_2 = \frac{1}{2}k_1 x_a^2 + k_1 x_a x_{i1} + \frac{1}{2}k_2 x_a^2 - k_2 x_a x_{i2}$$
$$= \frac{1}{2}k_1 x_a^2 + \frac{1}{2}k_2 x_a^2 + k_1 x_a x_{i1} - k_2 x_a \frac{k_1 x_{i1}}{k_2}$$
$$= \frac{1}{2}\left(k_1 + k_2\right)x_a^2$$

***P7.57** (a) $$v = \int_0^t a\, dt = \int_0^t \left(1.16t - 0.21t^2 + 0.24t^3\right)dt$$
$$= 1.16\frac{t^2}{2} - 0.21\frac{t^3}{3} + 0.24\frac{t^4}{4}\bigg|_0^t = 0.58t^2 - 0.07t^3 + 0.06t^4$$

At $t = 0$, $v_i = 0$. At $t = 2.5$ s,

$$v_f = \left(0.58 \text{ m/s}^3\right)(2.5 \text{ s})^2 - \left(0.07 \text{ m/s}^4\right)(2.5 \text{ s})^3 + \left(0.06 \text{ m/s}^5\right)(2.5 \text{ s})^4 = 4.88 \text{ m/s}$$
$$K_i + W = K_f$$
$$0 + W = \frac{1}{2}mv_f^2 = \frac{1}{2}1\,160 \text{ kg}\left(4.88 \text{ m/s}\right)^2 = \boxed{1.38 \times 10^4 \text{ J}}$$

(b) At $t = 2.5$ s,

$$a = \left(1.16 \text{ m/s}^3\right)2.5 \text{ s} - \left(0.210 \text{ m/s}^4\right)(2.5 \text{ s})^2 + \left(0.240 \text{ m/s}^5\right)(2.5 \text{ s})^3 = 5.34 \text{ m/s}^2.$$

Through the axles the wheels exert on the chassis force

$$\sum F = ma = 1\,160 \text{ kg } 5.34 \text{ m/s}^2 = 6.19 \times 10^3 \text{ N}$$

and inject power

$$\mathscr{P} = Fv = 6.19 \times 10^3 \text{ N}\left(4.88 \text{ m/s}\right) = \boxed{3.02 \times 10^4 \text{ W}}.$$

P7.58 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

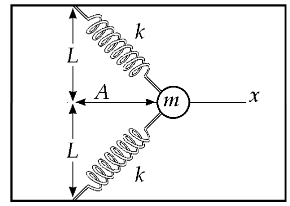


FIG. P7.58

$$\mathbf{F}=-2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}=\boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}.$$

(b) $$W=\int_i^f F_x dx \qquad W=\int_A^0 -2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)dx$$

$$W=-2k\int_A^0 x\,dx+kL\int_A^0\left(x^2+L^2\right)^{-1/2}2x\,dx \qquad W=-2k\frac{x^2}{2}\bigg|_A^0+kL\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\bigg|_A^0$$

$$W=-0+kA^2+2kL^2-2kL\sqrt{A^2+L^2} \qquad W=\boxed{2kL^2+kA^2-2kL\sqrt{A^2+L^2}}$$

***P7.59** For the rocket falling at terminal speed we have

$$\sum F=ma$$
$$+R-Mg=0$$
$$Mg=\frac{1}{2}D\rho Av_T^2$$

(a) For the rocket with engine exerting thrust T and flying up at the same speed,

$$\sum F=ma$$
$$+T-Mg-R=0$$
$$T=2Mg$$

The engine power is $\mathscr{P}=Fv=Tv_T=\boxed{2Mgv_T}$.

(b) For the rocket with engine exerting thrust T_b and flying down steadily at $3v_T$,
$R_b=\frac{1}{2}D\rho A(3v_T)^2=9Mg$

$$\sum F=ma$$
$$-T_b-Mg+9Mg=0$$
$$T_b=8Mg$$

The engine power is $\mathscr{P}=Tv=8Mg3v_T=\boxed{24Mgv_T}$.

P7.60 (a) $\mathbf{F}_1 = (25.0\text{ N})\left(\cos 35.0°\,\hat{\mathbf{i}} + \sin 35.0°\,\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$

$\mathbf{F}_2 = (42.0\text{ N})\left(\cos 150°\,\hat{\mathbf{i}} + \sin 150°\,\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$

(b) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\text{ m/s}^2}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\text{ m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})$

$\mathbf{v}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\text{ m/s}}$

(e) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \dfrac{1}{2}\mathbf{a}t^2$

$\mathbf{r}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\text{ s}) + \dfrac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})^2$

$\Delta\mathbf{r} = \mathbf{r}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\text{ m}}$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00\text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\text{ kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \mathbf{F}\cdot\Delta\mathbf{r}$

$K_f = \dfrac{1}{2}(5.00\text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})\right]$

$K_f = 55.6\text{ J} + 1\,426\text{ J} = \boxed{1.48\text{ kJ}}$

P7.61 (a) $\sum W = \Delta K: \qquad W_s + W_g = 0$

$\dfrac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90°+60°) = 0$

$\dfrac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$

$\Delta x = \boxed{4.12\text{ m}}$

(b) $\sum W = \Delta K + \Delta E_{\text{int}}: \qquad W_s + W_g - \Delta E_{\text{int}} = 0$

$\dfrac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\,\Delta x = 0$

$\dfrac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$

$\Delta x = \boxed{3.35\text{ m}}$

P7.62 (a)

F(N)	L(mm)	F(N)	L(mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

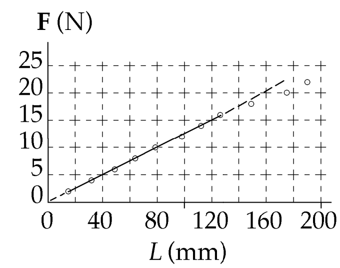


FIG. P7.62

(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \dfrac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

P7.63

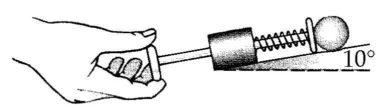


FIG. P7.63

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.050\,0\ \text{m})\sin 10.0° = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

P7.64 (a) $\Delta E_{\text{int}} = -\Delta K = -\dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\Delta E_{\text{int}} = -\dfrac{1}{2}(0.400\ \text{kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\ \text{J}}$

(b) $\Delta E_{\text{int}} = f\Delta r = \mu_k mg(2\pi r)$: $5.60\ \text{J} = \mu_k(0.400\ \text{kg})\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})$

Thus, $\mu_k = \boxed{0.152}$.

(c) After N revolutions, the object comes to rest and $K_f = 0$.

Thus, $\Delta E_{\text{int}} = -\Delta K = -0 + K_i = \dfrac{1}{2}mv_i^2$

or $\mu_k mg\left[N(2\pi r)\right] = \dfrac{1}{2}mv_i^2$.

This gives $N = \dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \dfrac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})} = \boxed{2.28\ \text{rev}}$.

P7.65 If positive F represents an outward force, (same as direction as r), then

$$W=\int_i^f \mathbf{F}\cdot d\mathbf{r}=\int_{r_i}^{r_f}\left(2F_0\sigma^{13}r^{-13}-F_0\sigma^7 r^{-7}\right)dr$$

$$W=\left.\frac{2F_0\sigma^{13}r^{-12}}{-12}-\frac{F_0\sigma^7 r^{-6}}{-6}\right|_{r_i}^{r_f}$$

$$W=\frac{-F_0\sigma^{13}\left(r_f^{-12}-r_i^{-12}\right)}{6}+\frac{F_0\sigma^7\left(r_f^{-6}-r_i^{-6}\right)}{6}=\frac{F_0\sigma^7}{6}\left[r_f^{-6}-r_i^{-6}\right]-\frac{F_0\sigma^{13}}{6}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[r_f^{-6}-r_i^{-6}\right]-1.89\times10^{-134}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[1.88\times10^{-6}-2.44\times10^{-6}\right]10^{60}-1.89\times10^{-134}\left[3.54\times10^{-12}-5.96\times10^{-8}\right]10^{120}$$

$$W=-2.49\times10^{-21}\text{ J}+1.12\times10^{-21}\text{ J}=\boxed{-1.37\times10^{-21}\text{ J}}$$

P7.66 $$\mathscr{P}\Delta t=W=\Delta K=\frac{(\Delta m)v^2}{2}$$

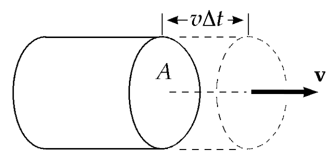


FIG. P7.66

The density is $\rho=\dfrac{\Delta m}{\text{vol}}=\dfrac{\Delta m}{A\Delta x}$.

Substituting this into the first equation and solving for $\mathscr{P}$, since $\dfrac{\Delta x}{\Delta t}=v$,

for a constant speed, we get $\boxed{\mathscr{P}=\dfrac{\rho A v^3}{2}}$.

Also, since $\mathscr{P}=Fv$, $\boxed{F=\dfrac{\rho A v^2}{2}}$.

Our model predicts the same proportionalities as the empirical equation, and gives $D=1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P7.67 We evaluate $\displaystyle\int_{12.8}^{23.7}\frac{375dx}{x^3+3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\ldots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\ldots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791\,.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

***P7.68** $\mathscr{P} = \frac{1}{2} D\rho\pi r^2 v^3$

(a) $\mathscr{P}_a = \frac{1}{2} 1\left(1.20 \text{ kg/m}^3\right)\pi(1.5 \text{ m})^2(8 \text{ m/s})^3 = \boxed{2.17\times 10^3 \text{ W}}$

(b) $\frac{\mathscr{P}_b}{\mathscr{P}_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24 \text{ m/s}}{8 \text{ m/s}}\right)^3 = 3^3 = 27$

$\mathscr{P}_b = 27\left(2.17\times 10^3 \text{ W}\right) = \boxed{5.86\times 10^4 \text{ W}}$

P7.69 (a) The suggested equation $\mathscr{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathscr{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathscr{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

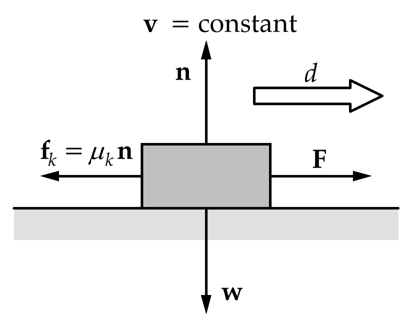


FIG. P7.69

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \mathbf{F} = m\mathbf{a}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \text{ and puts out power } \mathscr{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathscr{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

***P7.70** (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $kx_a - f_k = ma = 0$.

$\left(1.0\times 10^3 \text{ N/m}\right)x_a - 4.0 \text{ N} = 0$ $\boxed{x = -4.0\times 10^{-3} \text{ m}}$

(b) By the same logic,

$\left(1.0\times 10^3 \text{ N/m}\right)x_b - 10.0 \text{ N} = 0$ $\boxed{x = -1.0\times 10^{-2} \text{ m}}$

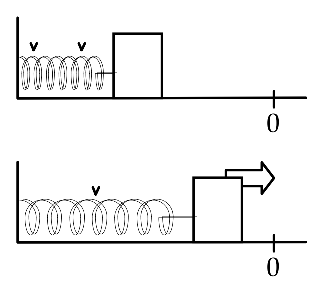


FIG. P7.70

ANSWERS TO EVEN PROBLEMS

P7.2 1.59×10^3 J

P7.4 (a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J

P7.6 see the solution

P7.8 5.33 W

P7.10 16.0

P7.12 (a) see the solution; (b) –12.0 J

P7.14 50.0 J

P7.16 (a) 575 N/m; (b) 46.0 J

P7.18 (a) 9.00 kJ; (b) 11.7 kJ, larger by 29.6%

P7.20 (a) see the solution; (b) mgR

P7.22 (a) $\frac{mg}{k_1} + \frac{mg}{k_2}$; (b) $\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

P7.24 (a) 1.20 J; (b) 5.00 m/s; (c) 6.30 J

P7.26 (a) 60.0 J; (b) 60.0 J

P7.28 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P7.30 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}$ m/s^2; (d) 1.94 ns

P7.32 (a) 0.791 m/s; (b) 0.531 m/s

P7.34 (a) 329 J; (b) 0; (c) 0; (d) 185 J; (e) 144 J

P7.36 8.01 W

P7.38 $\sim 10^4$ W

P7.40 (a) 5.91 kW; (b) 11.1 kW

P7.42 No. (a) 582; (b) 90.5 W = 0.121 hp

P7.44 5.92 km/L

P7.46 90.0 J

P7.48 (a) $\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$; (b) see the solution

P7.50 (a) $a = \frac{40.1 \text{ kN}}{m^{1.8}}$; $b = 1.80$; (b) 294 J

P7.52 (a) $\frac{mgnhh_s}{v + nh_s}$; (b) $\frac{mgvh}{v + nh_s}$

P7.54 1.53×10^5 N upward

P7.56 see the solution

P7.58 (a) see the solution; (b) $2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}$

P7.60 (a) $\mathbf{F}_1 = \left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)$ N; $\mathbf{F}_2 = \left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)$ N; (b) $\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)$ N; (c) $\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)$ m/s^2; (d) $\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)$ m/s; (e) $\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)$ m; (f) 1.48 kJ; (g) 1.48 kJ

P7.62 (a) see the solution; (b) 125 N/m ± 2%; (c) 13.1 N

P7.64 (a) 5.60 J; (b) 0.152; (c) 2.28 rev

P7.66 see the solution

P7.68 (a) 2.17 kW; (b) 58.6 kW

P7.70 (a) $x = -4.0$ mm; (b) –1.0 cm

8

Potential Energy

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q8.1 The final speed of the children will not depend on the slide length or the presence of bumps if there is no friction. If there is friction, a longer slide will result in a lower final speed. Bumps will have the same effect as they effectively lengthen the distance over which friction can do work, to decrease the total mechanical energy of the children.

Q8.2 Total energy is the sum of kinetic and potential energies. Potential energy can be negative, so the sum of kinetic plus potential can also be negative.

Q8.3 Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

Q8.4 (a) mgh is provided by the muscles.

(b) No further energy is supplied to the object-Earth system, but some chemical energy must be supplied to the muscles as they keep the weight aloft.

(c) The object loses energy mgh, giving it back to the muscles, where most of it becomes internal energy.

Q8.5 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q8.6 Three potential energy terms will appear in the expression of total mechanical energy, one for each conservative force. If you write an equation with initial energy on one side and final energy on the other, the equation contains six potential-energy terms.

Q8.7 (a) It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

Q8.8 The original kinetic energy of the skidding can be degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

Q8.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q8.10 Elastic potential energy of plates under stress plus gravitational energy is released when the plates "slip". It is carried away by mechanical waves.

Q8.11 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q8.12 Using switchbacks requires no less work, as it does not change the *change* in potential energy from top to bottom. It does, however, require less force (of static friction on the rolling drive wheels of a car) to propel the car up the gentler slope. Less power is required if the work can be done over a longer period of time.

Q8.13 There is no work done since there is no change in kinetic energy. In this case, air resistance must be negligible since the acceleration is zero.

Q8.14 There is no violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q8.15 Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

Q8.16 Gravitational energy is proportional to mass, so it doubles.

Q8.17 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q8.18 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (*K*) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

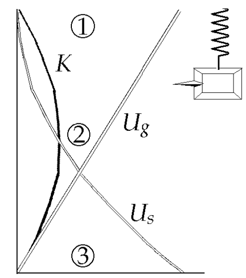


FIG. Q8.18

Q8.19 (a) Kinetic energy of the running athlete is transformed into elastic potential energy of the bent pole. This potential energy is transformed to a combination of kinetic energy and gravitational potential energy of the athlete and pole as the athlete approaches the bar. The energy is then all gravitational potential of the pole and the athlete as the athlete hopefully clears the bar. This potential energy then turns to kinetic energy as the athlete and pole fall to the ground. It immediately becomes internal energy as their macroscopic motion stops.

(b) Rotational kinetic energy of the athlete and shot is transformed into translational kinetic energy of the shot. As the shot goes through its trajectory as a projectile, the kinetic energy turns to a mix of kinetic and gravitational potential. The energy becomes internal energy as the shot comes to rest.

(c) Kinetic energy of the running athlete is transformed to a mix of kinetic and gravitational potential as the athlete becomes projectile going over a bar. This energy turns back into kinetic as the athlete falls down, and becomes internal energy as he stops on the ground.

The ultimate source of energy for all of these sports is the sun. See question 9.

Q8.20 Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

Q8.21 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

Q8.22 The ball is in neutral equilibrium.

Q8.23 The ball is in stable equilibrium when it is directly below the pivot point. The ball is in unstable equilibrium when it is vertically above the pivot.

SOLUTIONS TO PROBLEMS

Section 8.1 **Potential Energy of a System**

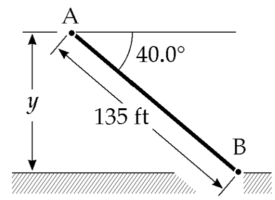


FIG. P8.1

P8.1 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}.$$

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With $135 \text{ ft} = 41.1 \text{ m}$, this height is found as:

$$y = (41.1 \text{ m})\sin 40.0° = 26.4 \text{ m}.$$

Thus,

$$U_A = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) = \boxed{2.59 \times 10^5 \text{ J}}.$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} = \boxed{-2.59 \times 10^5 \text{ J}}.$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}.$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}.$$

P8.2 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m}) = \boxed{800 \text{ J}}.$$

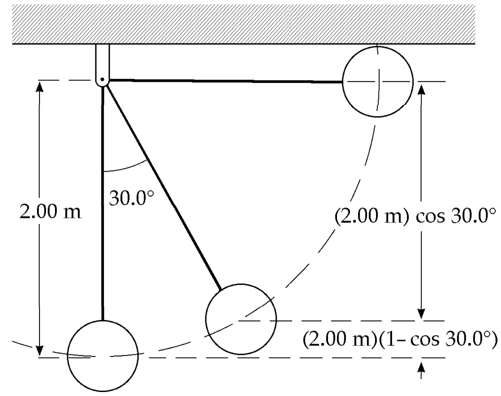


FIG. P8.2

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00 \text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1-\cos 30.0°) = \boxed{107 \text{ J}}.$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

***P8.3** The volume flow rate is the volume of water going over the falls each second:

$$3 \text{ m}(0.5 \text{ m})(1.2 \text{ m/s}) = 1.8 \text{ m}^3/\text{s}$$

The mass flow rate is $\dfrac{m}{t} = \rho\dfrac{V}{t} = \left(1\,000 \text{ kg/m}^3\right)\left(1.8 \text{ m}^3/\text{s}\right) = 1\,800 \text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathscr{P}_{\text{in}} = \dfrac{\text{energy}}{t} = \dfrac{mgy}{t} = \dfrac{m}{t}gy = (1\,800 \text{ kg/s})\left(9.8 \text{ m/s}^2\right)(5 \text{ m}) = 8.82\times10^4 \text{ J/s}$

The output power is $\mathscr{P}_{\text{useful}} = (\text{efficiency})\mathscr{P}_{\text{in}} = 0.25\left(8.82\times10^4 \text{ W}\right) = \boxed{2.20\times10^4 \text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85%, with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

Section 8.2 The Isolated System—Conservation of Mechanical Energy

***P8.4** (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump: $mgy = 36 \text{ kg}\left(9.81 \text{ m/s}^2\right)(0.25 \text{ m}) = 88.3 \text{ J}$. For all of the jumps of the children the energy is $12\left(1.05\times10^6\right)88.3 \text{ J} = \boxed{1.11\times10^9 \text{ J}}$.

(b) The seismic energy is modeled as $E = \dfrac{0.01}{100}1.11\times10^9 \text{ J} = 1.11\times10^5 \text{ J}$, making the Richter magnitude $\dfrac{\log E - 4.8}{1.5} = \dfrac{\log 1.11\times10^5 - 4.8}{1.5} = \dfrac{5.05-4.8}{1.5} = \boxed{0.2}$.

P8.5 $U_i + K_i = U_f + K_f$: $\quad mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $\quad n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)$$

$$= \boxed{0.098\,0\ \text{N downward}}$$

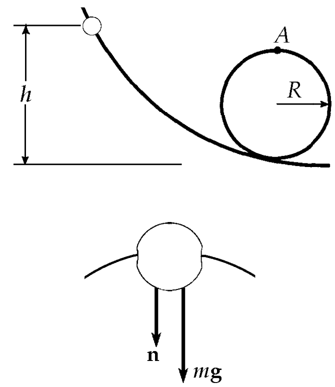


FIG. P8.5

P8.6 From leaving ground to the highest point, $\quad K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\ \text{m/s})^2 + 0 = 0 + m\left(9.80\ \text{m/s}^2\right)y$$

The mass makes no difference: $\quad \therefore y = \dfrac{(6.00\ \text{m/s})^2}{(2)\left(9.80\ \text{m/s}^2\right)} = \boxed{1.84\ \text{m}}$

***P8.7** (a) $\quad \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

$$0 + \frac{1}{2}(10\ \text{N/m})(-0.18\ \text{m})^2 = \frac{1}{2}(0.15\ \text{kg})v_f^2 + 0$$

$$v_f = (0.18\ \text{m})\sqrt{\left(\frac{10\ \text{N}}{0.15\ \text{kg}\cdot\text{m}}\right)\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right)} = \boxed{1.47\ \text{m/s}}$$

(b) $\quad K_i + U_{si} = K_f + U_{sf}$

$$0 + \frac{1}{2}(10\ \text{N/m})(-0.18\ \text{m})^2 = \frac{1}{2}(0.15\ \text{kg})v_f^2 + \frac{1}{2}(10\ \text{N/m})(0.25\ \text{m} - 0.18\ \text{m})^2$$

$$0.162\ \text{J} = \frac{1}{2}(0.15\ \text{kg})v_f^2 + 0.024\,5\ \text{J}$$

$$v_f = \sqrt{\frac{2(0.138\ \text{J})}{0.15\ \text{kg}}} = \boxed{1.35\ \text{m/s}}$$

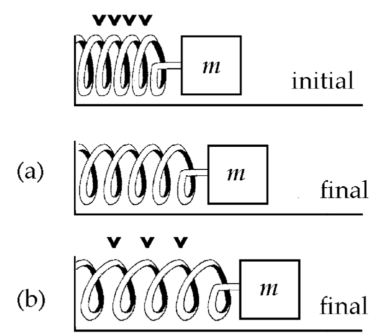


FIG. P8.7

***P8.8** The energy of the car is $E = \frac{1}{2}mv^2 + mgy$

$E = \frac{1}{2}mv^2 + mgd\sin\theta$ where d is the distance it has moved along the track.

$\mathcal{P} = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$

(a) When speed is constant,

$\mathcal{P} = mgv\sin\theta = 950\text{ kg}\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin 30° = \boxed{1.02\times10^4\text{ W}}$

(b) $\frac{dv}{dt} = a = \frac{2.2\text{ m/s} - 0}{12\text{ s}} = 0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$\mathcal{P} = mva + mgv\sin\theta = 950\text{ kg}(2.2\text{ m/s})\left(0.183\text{ m/s}^2\right) + 1.02\times10^4\text{ W} = \boxed{1.06\times10^4\text{ W}}$

(c) At the top end,

$\frac{1}{2}mv^2 + mgd\sin\theta = 950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2 + \left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right) = \boxed{5.82\times10^6\text{ J}}$

***P8.9** (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y = 0$ at the top end of the string.

$$\left(K + U_g\right)_i = \left(K + U_g\right)_f: \qquad 0 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$\left(9.8\text{ m/s}^2\right)(-2\text{ m}\cos 30°) = \frac{1}{2}v_f^2 + \left(9.8\text{ m/s}^2\right)(-2\text{ m})$$

$$v_f = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(1-\cos 30°)} = \boxed{2.29\text{ m/s}}$$

(b) Choose the initial point at $\theta = 30°$ and the final point at $\theta = 15°$:

$$0 + mg(-L\cos 30°) = \frac{1}{2}mv_f^2 + mg(-L\cos 15°)$$

$$v_f = \sqrt{2gL(\cos 15° - \cos 30°)} = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(\cos 15° - \cos 30°)} = \boxed{1.98\text{ m/s}}$$

P8.10 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$:

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or $$0 + mg(d+x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2.$$

Solving for d gives $$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}.$$

P8.11 From conservation of energy for the block-spring-Earth system,

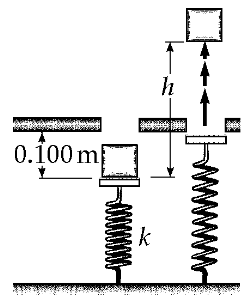


FIG. P8.11

$$U_{gt} = U_{si},$$

or

$$(0.250 \text{ kg})\left(9.80 \text{ m/s}^2\right)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$.

P8.12 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

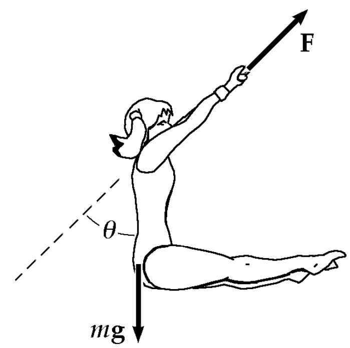


FIG. P8.12

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

(b) At the bottom of the swing, $\theta = 0°$ so

$$F = mg(3 - 2\cos\theta_i)$$
$$F = 2mg = mg(3 - 2\cos\theta_i)$$

which gives

$$\theta_i = \boxed{60.0°}.$$

P8.13 Using conservation of energy for the system of the Earth and the two objects

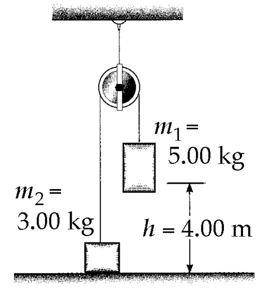


FIG. P8.13

(a) $$(5.00 \text{ kg})g(4.00 \text{ m}) = (3.00 \text{ kg})g(4.00 \text{ m}) + \frac{1}{2}(5.00 + 3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43 \text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$
$$\Delta y = 1.00 \text{ m}$$
$$y_{\max} = 4.00 \text{ m} + \Delta y = \boxed{5.00 \text{ m}}$$

P8.14 $m_1 > m_2$

(a) $$m_1 gh = \frac{1}{2}(m_1 + m_2)v^2 + m_2 gh$$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}}}$$

(b) Since m_2 has kinetic energy $\frac{1}{2}m_2 v^2$, it will rise an additional height Δh determined from

$$m_2 g\,\Delta h = \frac{1}{2}m_2 v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1 - m_2)h}{(m_1 + m_2)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\frac{2m_1 h}{m_1 + m_2}}$.

P8.15 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

$$K_i + U_{gi} = K_f + U_{gf}: \qquad \frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$
$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$
$$v_i = \boxed{5.49 \text{ m/s}}$$

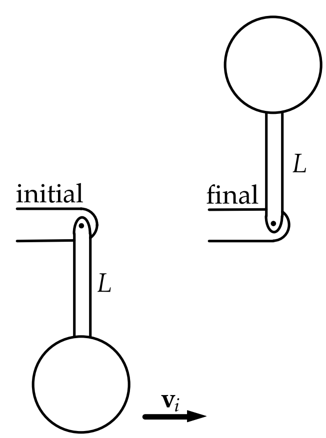


FIG. P8.15

***P8.16** efficiency $= \dfrac{\text{useful output energy}}{\text{total input energy}} = \dfrac{\text{useful output power}}{\text{total input power}}$

$$e = \frac{m_{\text{water}}gy/t}{(1/2)m_{\text{air}}\left(v^2/t\right)} = \frac{2\rho_{\text{water}}\left(v_{\text{water}}/t\right)gy}{\rho_{\text{air}}\pi r^2\left(\ell v^2/t\right)} = \frac{2\rho_{\text{w}}\left(v_{\text{w}}/t\right)gy}{\rho_{\text{a}}\pi r^2 v^3}$$

where ℓ is the length of a cylinder of air passing through the mill and v_{w} is the volume of water pumped in time t. We need inject negligible kinetic energy into the water because it starts and ends at rest.

$$\frac{v_{\text{w}}}{t} = \frac{e\rho_{\text{a}}\pi r^2 v^3}{2\rho_{\text{w}}gy} = \frac{0.275\left(1.20\ \text{kg/m}^3\right)\pi(1.15\ \text{m})^2(11\ \text{m/s})^3}{2\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)35\ \text{m}}$$

$$= 2.66\times10^{-3}\ \text{m}^3/\text{s}\left(\frac{1\,000\ \text{L}}{1\ \text{m}^3}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{160\ \text{L/min}}$$

P8.17 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$
$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times10^7\ \text{J}}.$$

P8.18 In the swing down to the breaking point, energy is conserved:

$$mgr\cos\theta = \frac{1}{2}mv^2$$

at the breaking point consider radial forces

$$\sum F_r = ma_r$$

$$+T_{\max} - mg\cos\theta = m\frac{v^2}{r}$$

Eliminate $\frac{v^2}{r} = 2g\cos\theta$

$$T_{\max} - mg\cos\theta = 2mg\cos\theta$$

$$T_{\max} = 3mg\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{T_{\max}}{3mg}\right) = \cos^{-1}\left(\frac{44.5\text{ N}}{3(2.00\text{ kg})(9.80\text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8^\circ}$$

***P8.19** (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

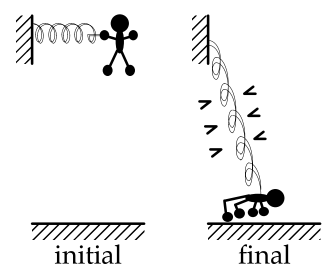


FIG. P8.19(a)

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$

$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg(y_i - y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$

$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b) $k = 3.33\frac{mg}{25.8\text{ m}}$ $\quad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\quad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

***P8.20** When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0+0+0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 **Conservative and Nonconservative Forces**

P8.21 $F_g = mg = (4.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 39.2 \text{ N}$

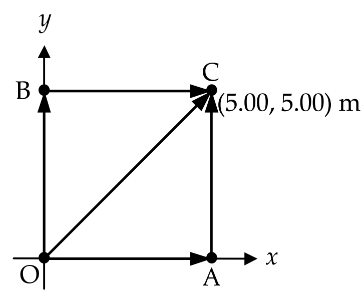


FIG. P8.21

(a) Work along OAC = work along OA + work along AC

$$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$$

$$= (39.2 \text{ N})(5.00 \text{ m}) + (39.2 \text{ N})(5.00 \text{ m})(-1)$$

$$= \boxed{-196 \text{ J}}$$

(b) W along OBC = W along OB + W along BC

$$= (39.2 \text{ N})(5.00 \text{ m})\cos 180° + (39.2 \text{ N})(5.00 \text{ m})\cos 90.0°$$

$$= \boxed{-196 \text{ J}}$$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$$= (39.2 \text{ N})\left(5.00 \times \sqrt{2} \text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196 \text{ J}}$$

The results should all be the same, since gravitational forces are conservative.

P8.22 (a) $W = \int \mathbf{F} \cdot d\mathbf{r}$ and if the force is constant, this can be written as

$$W = \mathbf{F} \cdot \int d\mathbf{r} = \boxed{\mathbf{F} \cdot \left(\mathbf{r}_f - \mathbf{r}_i\right), \text{ which depends only on end points, not path.}}$$

(b) $$W = \int \mathbf{F} \cdot d\mathbf{r} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00 \text{ N}) \int_0^{5.00 \text{ m}} dx + (4.00 \text{ N}) \int_0^{5.00 \text{ m}} dy$$

$$W = (3.00 \text{ N})x\Big|_0^{5.00 \text{ m}} + (4.00 \text{ N})y\Big|_0^{5.00 \text{ m}} = 15.0 \text{ J} + 20.0 \text{ J} = \boxed{35.0 \text{ J}}$$

The same calculation applies for all paths.

P8.23 (a) $$W_{OA} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y = 0$ $$W_{OA} = 0$$

$$W_{AC} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$$

For $x = 5.00$ m, $$W_{AC} = 125\text{ J}$$

and $$W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$$

(b) $$W_{OB} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$$

since along this path, $x = 0$, $$W_{OB} = 0$$

$$W_{BC} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}} \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

since $y = 5.00$ m, $$W_{BC} = 50.0\text{ J}$$

$$W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$$

(c) $$W_{OC} = \int \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) \cdot \left(2y\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}\right) = \int \left(2y\,dx + x^2\,dy\right)$$

Since $x = y$ along OC, $$W_{OC} = \int_0^{5.00\text{ m}} \left(2x + x^2\right)dx = \boxed{66.7\text{ J}}$$

(d) F is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

P8.24 (a) $$(\Delta K)_{A\to B} = \sum W = W_g = mg\Delta h = mg(5.00 - 3.20)$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\text{ m/s}}$$

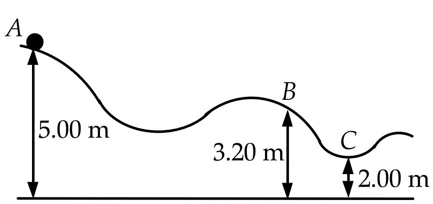


FIG. P8.24

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\text{ m/s}}$

(b) $$W_g\Big|_{A\to C} = mg(3.00\text{ m}) = \boxed{147\text{ J}}$$

P8.25 (a) $\mathbf{F} = \left(3.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)$ N

$m = 4.00$ kg

$\mathbf{r} = \left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)$ m

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00\text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$$-9.00 = \frac{4.00v^2}{2} - 4.00\left(\frac{(4.00)^2}{2}\right)$$

so $v = \sqrt{\dfrac{32.0 - 9.00}{2.00}} = \boxed{3.39\text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00\text{ J}}$

Section 8.4 **Changes in Mechanical Energy for Nonconservative Forces**

P8.26 (a) $U_f = K_i - K_f + U_i$ $\qquad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

P8.27 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher,

is $\qquad \Delta E = F\Delta r \cos 0° = F(\pi R)$.

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then $\qquad \Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$

becomes $\qquad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$

or $\qquad v_f = \sqrt{v_i^2 + 2gy_i + \dfrac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \dfrac{2(30.0)\pi(0.600)}{0.250}}$

$v_f = \boxed{26.5\text{ m/s}}$

***P8.28** The useful output energy is

$$120\text{ Wh}(1 - 0.60) = mg\left(y_f - y_i\right) = F_g\Delta y$$

$$\Delta y = \frac{120\text{ W}(3\,600\text{ s})0.40}{890\text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194\text{ m}}$$

***P8.29** As the locomotive moves up the hill at constant speed, its output power goes into internal energy plus gravitational energy of the locomotive-Earth system:

$$\mathcal{P}t = mgy + f\Delta r = mg\Delta r\sin\theta + f\Delta r \qquad \mathcal{P} = mgv_f\sin\theta + fv_f$$

As the locomotive moves on level track,

$$\mathcal{P} = fv_i \qquad 1\,000\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right) = f(27\text{ m/s}) \qquad f = 2.76\times10^4\text{ N}$$

Then also $746\,000\text{ W} = (160\,000\text{ kg})(9.8\text{ m/s}^2)v_f\left(\frac{5\text{ m}}{100\text{ m}}\right) + (2.76\times10^4\text{ N})v_f$

$$v_f = \frac{746\,000\text{ W}}{1.06\times10^5\text{ N}} = \boxed{7.04\text{ m/s}}$$

P8.30 We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k d\cos180°$$

$$0 - 0 - mg(y_i - y_f) = -f_k d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m} + 5.00\text{ m})}{5.00\text{ m}} = \boxed{2.06\text{ kN}}$$

P8.31 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $\quad m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

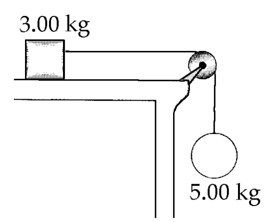


FIG. P8.31

P8.32 $\Delta E_{\text{mech}} = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E_{\text{mech}} = W_{\text{app}} - f\Delta x$, where W_{app} is the work the boy did pushing forward on the wheels.

Thus, $\quad W_{\text{app}} = (K_f - K_i) + (U_{gf} - U_{gi}) + f\Delta x$

or $\quad W_{\text{app}} = \frac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + f\Delta x$

$$W_{\text{app}} = \frac{1}{2}(47.0)\left[(6.20)^2 - (1.40)^2\right] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{\text{app}} = \boxed{168\text{ J}}$$

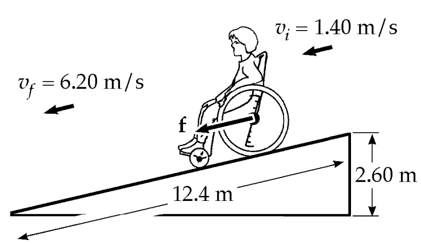


FIG. P8.32

P8.33 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

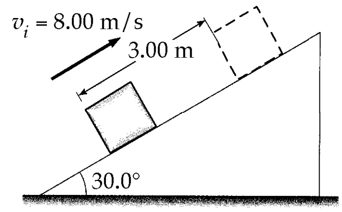


FIG. P8.33

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\cos 30.0°} = \boxed{0.679}$$

P8.34 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P8.35 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0+\frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\ \text{N/m})\left(5.00\times10^{-2}\ \text{m}\right)^2 - \left(3.20\times10^{-2}\ \text{N}\right)(0.150\ \text{m}) = \frac{1}{2}\left(5.30\times10^{-3}\ \text{kg}\right)v^2$$

$$v=\sqrt{\frac{2\left(5.20\times10^{-3}\ \text{J}\right)}{5.30\times10^{-3}\ \text{kg}}} = \boxed{1.40\ \text{m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\mathbf{F}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\ \text{N}}{8.00\ \text{N/m}} = 0.400\ \text{cm}$$

and the ball has moved

$$5.00\ \text{cm} - 0.400\ \text{cm} = \boxed{4.60\ \text{cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00\left(5.00\times10^{-2}\right)^2 - \left(3.20\times10^{-2}\right)\left(4.60\times10^{-2}\right) = \frac{1}{2}\left(5.30\times10^{-3}\right)v^2 + \frac{1}{2}8.00\left(4.00\times10^{-3}\right)^2$$

$$v = \boxed{1.79\ \text{m/s}}$$

P8.36 $\sum F_y = n - mg\cos 37.0° = 0$

$\therefore n = mg\cos 37.0° = 400\ \text{N}$

$f = \mu n = 0.250(400\ \text{N}) = 100\ \text{N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g\left(h_f - h_i\right) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times10^3$

$\Delta U_B = m_B g\left(h_f - h_i\right) = (100)(9.80)(-20.0) = -1.96\times10^4$

$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$

$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92\ \text{kJ}}$.

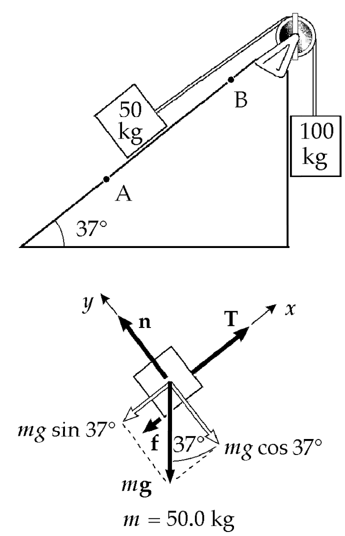


FIG. P8.36

P8.37 (a) The object moved down distance $1.20\text{ m}+x$. Choose $y=0$ at its lower point.

$$K_i+U_{gi}+U_{si}+\Delta E_{\text{mech}}=K_f+U_{gf}+U_{sf}$$

$$0+mgy_i+0+0=0+0+\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=(160\text{ N/m})x^2-(14.7\text{ N})x-17.6\text{ J}$$

$$x=\frac{14.7\text{ N}\pm\sqrt{(-14.7\text{ N})^2-4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x=\frac{14.7\text{ N}\pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x=\boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=160x^2-2.44x-2.93$$

The positive root is $x=\boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i-f\Delta x=\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)-0.700\text{ N}(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J}+14.7\text{ N}x-0.840\text{ J}-0.700\text{ N}x=160\text{ N/m}\,x^2$$

$$160x^2-14.0x-16.8=0$$

$$x=\frac{14.0\pm\sqrt{(14.0)^2-4(160)(-16.8)}}{320}$$

$$x=\boxed{0.371\text{ m}}$$

P8.38 The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})\left(9.80\text{ m/s}^2\right)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

***P8.39** (a) Let m be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then

$$\boxed{a = \frac{\mu_k gx}{L}\text{ opposite to the motion.}}$$

(b) In an incremental bit of forward motion dx, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

Section 8.5 **Relationship Between Conservative Forces and Potential Energy**

P8.40 (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

P8.41 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x + 4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\quad\quad$ $\Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\quad\quad$ $K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P8.42 $$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial x} = -\left(9x^2y - 7\right) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial y} = -\left(3x^3 - 0\right) = -3x^3$$

Thus, the force acting at the point $(x,\ y)$ is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

P8.43 $$U(r) = \frac{A}{r}$$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. The positive value indicates a force of repulsion.

Section 8.6 **Energy Diagrams and the Equilibrium of a System**

P8.44

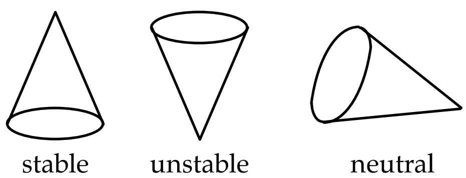


FIG. P8.44

P8.45 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

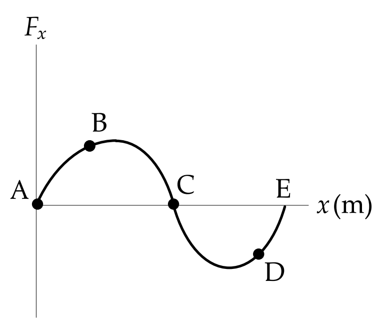


FIG. P8.45

P8.46 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6 \text{ J} \le E < 1 \text{ J}}$.

(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6 \text{ mm} \le r \le 3.6 \text{ mm}}$.

(d) $K + U = E$. Thus, $K_{max} = E - U_{min} = -3.0 \text{ J} - (-5.6 \text{ J}) = \boxed{2.6 \text{ J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5 \text{ mm}}$.

(f) $-3 \text{ J} + W = 1 \text{ J}$. Hence, the binding energy is $W = \boxed{4 \text{ J}}$.

P8.47 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2 + L^2}$, so each exerts force $k\left(\sqrt{x^2 + L^2} - L\right)$ towards its fixed end. The y-components cancel out and the x components add to:

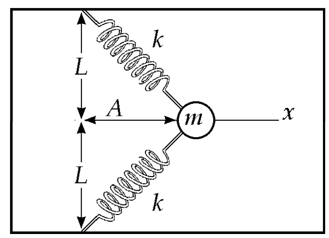


FIG. P8.47(a)

$$F_x = -2k\left(\sqrt{x^2 + L^2} - L\right)\left(\frac{x}{\sqrt{x^2 + L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right) dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2 + L^2}} dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2 + 1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

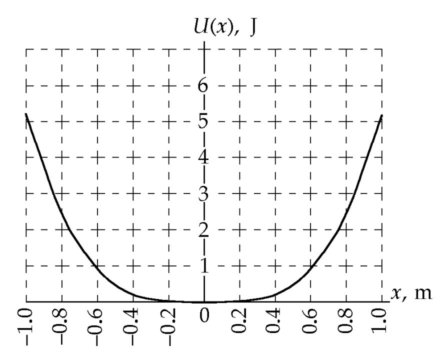


FIG. P8.47(b)

(c) $K_i + U_i + \Delta E_{mech} = K_f + U_f$

$0 + 0.400 \text{ J} + 0 = \frac{1}{2}(1.18 \text{ kg})v_f^2 + 0$

$v_f = \boxed{0.823 \text{ m/s}}$

Additional Problems

P8.48 The potential energy of the block-Earth system is mgh.

An amount of energy $\mu_k mgd\cos\theta$ is converted into internal energy due to friction on the incline. Therefore the final height $y_{\max}$ is found from

$$mgy_{\max} = mgh - \mu_k mgd\cos\theta$$

where

$$d = \frac{y_{\max}}{\sin\theta}$$

$$\therefore mgy_{\max} = mgh - \mu_k mgy_{\max}\cot\theta$$

Solving,

$$y_{\max} = \frac{h}{1+\mu_k\cot\theta}.$$

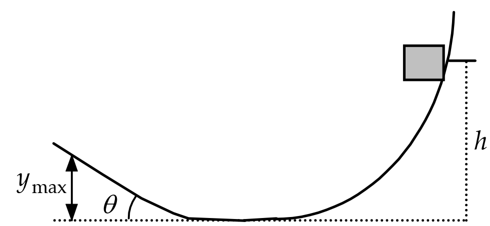


FIG. P8.48

P8.49 At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})\left(9.80\text{ m/s}^2\right)(40\times 0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\dfrac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$.

P8.50 $v = 100\text{ km/h} = 27.8\text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)\left(1.20\text{ kg/m}^3\right)\left(2.50\text{ m}^2\right)(27.8\text{ m/s})^2 = 382\text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + \left(K_f - K_i\right) + \left(U_{gf} - U_{gi}\right)$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})\left(9.80\text{ m/s}^2\right)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

P8.51 m = mass of pumpkin
R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$.
Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

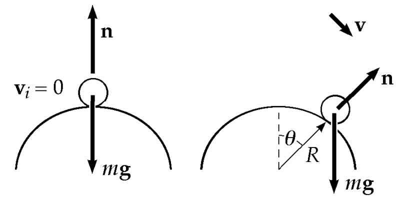


FIG. P8.51

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \frac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

P8.52 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

(b) $K_A + U_A = K_B + U_B$
$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

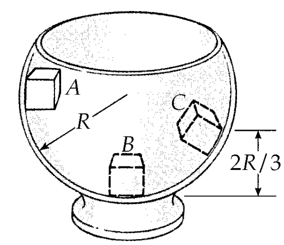


FIG. P8.52

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$
$K_C = K_A + U_A - U_C = mg(h_A - h_C)$
$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

P8.53 (a) $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b)
$$\Delta E_{\text{mech}} = \Delta K + \Delta U = K_B - K_A + U_B - U_A$$
$$= K_B + mg(h_B - h_A)$$
$$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$$
$$= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.54 The gain in internal energy due to friction represents a loss in mechanical energy that must be equal to the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx\cos\theta = \Delta K + \frac{1}{2}kx^2 - mgx\sin\theta$$

and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy.

P8.55 (a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

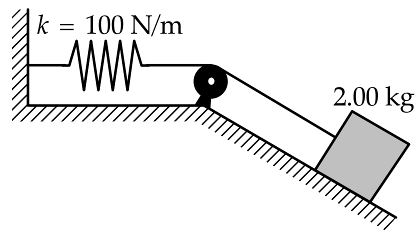


FIG. P8.55

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\text{ m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\text{ m/s}^2}$. The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

K initially increases, but then goes back to zero.

P8.56 $k = 2.50 \times 10^4 \text{ N/m}$, $m = 25.0 \text{ kg}$

$x_A = -0.100 \text{ m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(-0.100 \text{ m}) + \frac{1}{2}\left(2.50 \times 10^4 \text{ N/m}\right)(-0.100 \text{ m})^2$$

$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$:

$$0 + (25.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$$

$$x_C = \boxed{0.410 \text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$:

$$\frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) K and v are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or $$|x| = \frac{(25.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus, $K = K_{\max}$ at $x = \boxed{-9.80 \text{ mm}}$

(e) $$K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$$

or $$\frac{1}{2}(25.0 \text{ kg})v_{\max}^2 = (25.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)\left[(-0.100 \text{ m}) - (-0.009\,8 \text{ m})\right] + \frac{1}{2}\left(2.50 \times 10^4 \text{ N/m}\right)\left[(-0.100 \text{ m})^2 - (-0.009\,8 \text{ m})^2\right]$$

yielding $v_{\max} = \boxed{2.85 \text{ m/s}}$

P8.57 $$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

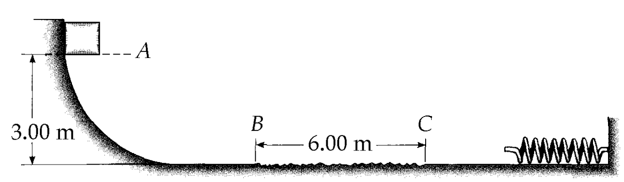


FIG. P8.57

P8.58 (a) $\mathbf{F} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x).$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x).$$

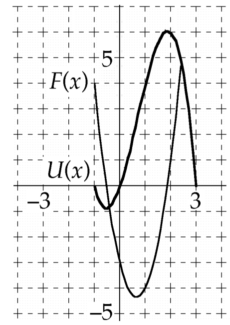


FIG. P8.58

P8.59 $(K+U)_i = (K+U)_f$

$$0 + (30.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m}) + \frac{1}{2}(250 \text{ N/m})(0.200 \text{ m})^2$$
$$= \frac{1}{2}(50.0 \text{ kg})v^2 + (20.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m})\sin 40.0°$$
$$58.8 \text{ J} + 5.00 \text{ J} = (25.0 \text{ kg})v^2 + 25.2 \text{ J}$$
$$\boxed{v = 1.24 \text{ m/s}}$$

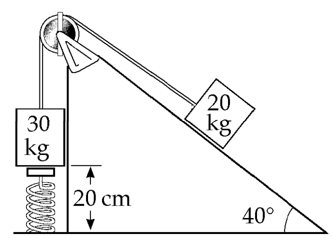


FIG. P8.59

P8.60 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$
$$\frac{1}{2}(50.0 \text{ N/m})d^2 + 0.250(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)d - \frac{1}{2}(1.00 \text{ kg})\left(3.00 \text{ m/s}^2\right) = 0$$
$$d = \frac{[-2.45 \pm 21.25] \text{ N}}{50.0 \text{ N/m}} = \boxed{0.378 \text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$
$$v = \sqrt{(3.00 \text{ m/s})^2 - \frac{2}{(1.00 \text{ kg})}(2.45 \text{ N})(2)(0.378 \text{ m})}$$
$$= \boxed{2.30 \text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2$$
$$D = \frac{9.00 \text{ J}}{2(0.250)(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)} - 2(0.378 \text{ m}) = \boxed{1.08 \text{ m}}$$

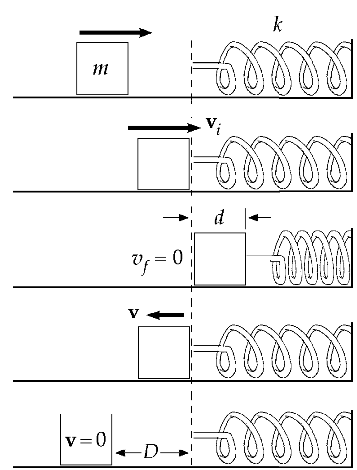


FIG. P8.60

P8.61 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

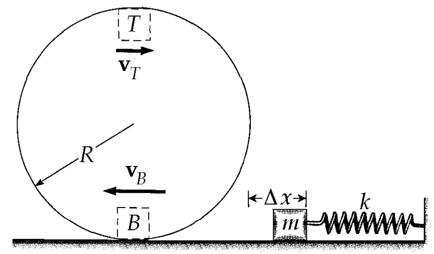


FIG. P8.61

$$\frac{1}{2}(450\ \text{N/m})(\Delta x)^2 = \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\ \text{m}}$$

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\ \text{kg})\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m}) + \frac{1}{2}(0.500\ \text{kg})v_T^2 - \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$
$$= -(7.00\ \text{N})(\pi)(1.00\ \text{m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\ \text{m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\ \text{m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

P8.62 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

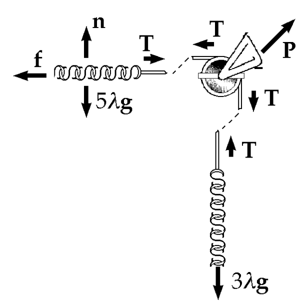


FIG. P8.62

(a) For the five meters on the table with motion impending,

$\sum F_y = 0$: $\quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$

$\qquad f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\sum F_x = 0$: $\quad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\sum F_y = 0$: $\quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

continued on next page

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \quad +n-(5-x)\lambda g = 0 \qquad n=(5-x)\lambda g$$
$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x=5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\frac{3}{2}=6.5\text{ m}$.

$$K_i+U_i+\Delta E_{\text{mech}}=K_f+U_f: \qquad 0+\left(m_1gy_1+m_2gy_2\right)_i-\int_i^f f_k dx=\left(\frac{1}{2}mv^2+mgy\right)_f$$
$$(5\lambda g)8+(3\lambda g)6.5-\int_0^5(2\lambda g-0.4x\lambda g)dx=\frac{1}{2}(8\lambda)v^2+(8\lambda g)4$$
$$40.0g+19.5g-2.00g\int_0^5 dx+0.400g\int_0^5 x\,dx=4.00v^2+32.0g$$
$$27.5g-2.00gx\big|_0^5+0.400g\frac{x^2}{2}\bigg|_0^5=4.00v^2$$
$$27.5g-2.00g(5.00)+0.400g(12.5)=4.00v^2$$
$$22.5g=4.00v^2$$
$$v=\sqrt{\frac{(22.5\text{ m})\left(9.80\text{ m/s}^2\right)}{4.00}}=\boxed{7.42\text{ m/s}}$$

P8.63 Launch speed is found from

$$mg\left(\frac{4}{5}h\right)=\frac{1}{2}mv^2: \qquad v=\sqrt{2g\left(\frac{4}{5}\right)h}$$
$$v_y=v\sin\theta$$

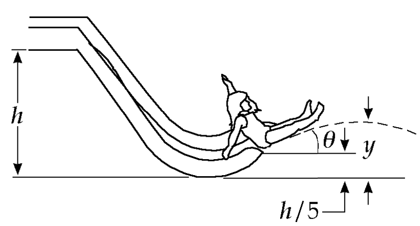


FIG. P8.63

The height y above the water (by conservation of energy for the child-Earth system) is found from

$$mgy=\frac{1}{2}mv_y^2+mg\frac{h}{5} \qquad \left(\text{since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion}\right)$$
$$y=\frac{1}{2g}v_y^2+\frac{h}{5}=\frac{1}{2g}v^2\sin^2\theta+\frac{h}{5}$$
$$y=\frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta+\frac{h}{5}=\boxed{\frac{4}{5}h\sin^2\theta+\frac{h}{5}}$$

***P8.64** (a) The length of string between glider and pulley is given by $\ell^2 = x^2 + h_0^2$. Then $2\ell \dfrac{d\ell}{dt} = 2x\dfrac{dx}{dt} + 0$.

Now $\dfrac{d\ell}{dt}$ is the rate at which string goes over the pulley: $\dfrac{d\ell}{dt} = v_y = \dfrac{x}{\ell}v_x = (\cos\theta)v_x$.

(b) $\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$

$0 + 0 + m_B g\left(y_{30} - y_{45}\right) = \dfrac{1}{2}m_A v_x^2 + \dfrac{1}{2}m_B v_y^2$

Now $y_{30} - y_{45}$ is the amount of string that has gone over the pulley, $\ell_{30} - \ell_{45}$. We have $\sin 30° = \dfrac{h_0}{\ell_{30}}$ and $\sin 45° = \dfrac{h_0}{\ell_{45}}$, so $\ell_{30} - \ell_{45} = \dfrac{h_0}{\sin 30°} - \dfrac{h_0}{\sin 45°} = 0.40\text{ m}\left(2 - \sqrt{2}\right) = 0.234\text{ m}$.
From the energy equation

$$0.5\text{ kg } 9.8\text{ m/s}^2\ 0.234\text{ m} = \frac{1}{2}1.00\text{ kg } v_x^2 + \frac{1}{2}0.500\text{ kg } v_x^2\cos^2 45°$$

$$v_x = \sqrt{\frac{1.15\text{ J}}{0.625\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

(c) $v_y = v_x\cos\theta = (1.35\text{ m/s})\cos 45° = \boxed{0.958\text{ m/s}}$

(d) The acceleration of neither glider is constant, so knowing distance and acceleration at one point is not sufficient to find speed at another point.

P8.65 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}\left(\sin 50° + \sin\phi\right)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$\left(K + U_g\right)_i + W_{\text{wind}} = \left(K + U_g\right)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg } v_i^2 + 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m}) = 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg } v_i^2 - 1.26\times10^4\text{ J} - 5.5\times10^3\text{ J} = -1.72\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg } v_i^2 + 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$
$$= 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg } v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.66 Case I: Surface is frictionless

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20\times10^2\text{ N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}v^2 = \frac{1}{2}\left(7.20\times10^2\text{ N/m}\right)\left(10^{-1}\text{ m}\right)^2 - (0.300)(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\left(10^{-1}\text{ m}\right)$$

$$\boxed{v = 0.923\text{ m/s}}$$

***P8.67** (a) $\left(K+U_g\right)_A = \left(K+U_g\right)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\text{ m/s}^2\right)6.3\text{ m}} = \boxed{11.1\text{ m/s}}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(11.1\text{ m/s})^2}{6.3\text{ m}} = \boxed{19.6\text{ m/s}^2\text{ up}}$$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\text{ kg}\left(9.8\text{ m/s}^2 + 19.6\text{ m/s}^2\right) = \boxed{2.23\times10^3\text{ N up}}$$

(d) $$W = F\Delta r\cos\theta = 2.23\times10^3\text{ N}(0.450\text{ m})\cos0° = \boxed{1.01\times10^3\text{ J}}$$

(e) $\left(K+U_g\right)_B + W = \left(K+U_g\right)_D$

$$\frac{1}{2}mv_B^2 + 0 + 1.01\times10^3\text{ J} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2 + 1.01\times10^3\text{ J} = \frac{1}{2}76\text{ kg }v_D^2 + 76\text{ kg}\left(9.8\text{ m/s}^2\right)6.3\text{ m}$$

$$\sqrt{\frac{\left(5.70\times10^3\text{ J} - 4.69\times10^3\text{ J}\right)2}{76\text{ kg}}} = v_D = \boxed{5.14\text{ m/s}}$$

(f) $\left(K+U_g\right)_D = \left(K+U_g\right)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\text{ m/s})^2}{2\left(9.8\text{ m/s}^2\right)} = \boxed{1.35\text{ m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34\text{ m} = 0 + 5.14\text{ m/s}\,t + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39\text{ s}}$$

***P8.68** If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$:

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.69 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \mathbf{F} \cdot d\mathbf{s} = F\int dx = F\sqrt{2LH - H^2}$$

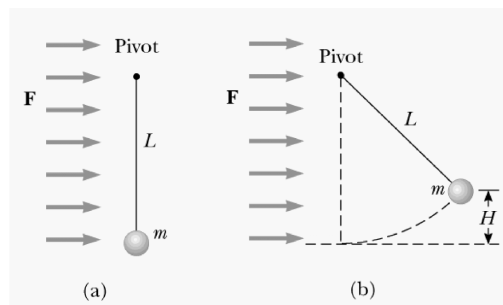


FIG. P8.69

The work-energy theorem can be written:

$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}$, or

$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$ giving $F^2 2LH - F^2H^2 = m^2g^2H^2$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = \left(F^2 + m^2g^2\right)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which would be hard to approach experimentally.

(b) $$H = \frac{2(2.00\text{ m})}{1 + \left[(2.00\text{ kg})\left(9.80\text{ m/s}^2\right)/14.7\text{ N}\right]^2} = \boxed{1.44\text{ m}}$$

continued on next page

(c) Call θ the equilibrium angle with the vertical.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg} = \dfrac{14.7 \text{ N}}{19.6 \text{ N}} = 0.750$, or $\theta = 36.9°$

Therefore, $H_{\text{eq}} = L(1-\cos\theta) = (2.00 \text{ m})(1-\cos 36.9°) = \boxed{0.400 \text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{\text{eq}} \to L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{\left(H_{\text{eq}}\right)_{\max} = L}$.

P8.70 Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call v_r the speed here. By conservation of energy

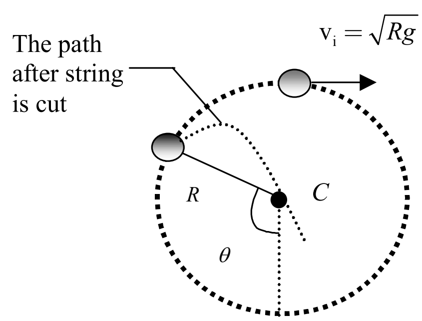


FIG. P8.70

$$K_i + U_i + \Delta E = K_r + U_r$$
$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$
$$gR + 2gR = v_r^2 + 2gR\cos\phi$$
$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are $v_x = v_r\cos\phi$ and $v_y = v_r\sin\phi$ so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi t$$
$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$
$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$
$$\cos\phi = \frac{6 \pm \sqrt{36-12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.183\,5 \qquad \phi = 79.43° \qquad \text{so } \theta = \boxed{100.6°}$$

P8.71 Applying Newton's second law at the bottom (b) and top (t) of the circle gives

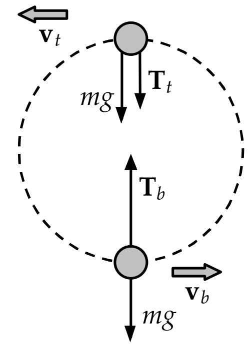


FIG. P8.71

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives $\quad T_b = T_t + 2mg + \dfrac{m\left(v_b^2 - v_t^2\right)}{R}$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\dfrac{m\left(v_b^2 - v_t^2\right)}{2} + (0 - 2mgR) = 0$ and $\dfrac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

P8.72 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

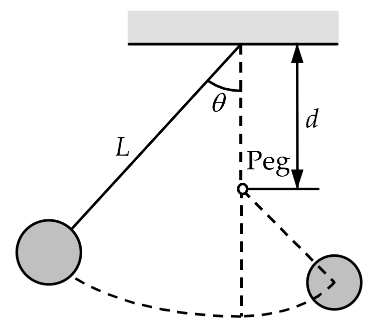


FIG. P8.72

(b) Relative to the point of suspension,

$$U_i = 0,\, U_f = -mg\left[d - (L - d)\right]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d.$$

Upon solving, we get $\boxed{d = \dfrac{3L}{5}}$.

***P8.73** (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $mgh = \frac{1}{2}mv_t^2 + mg(2R)$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}.$$

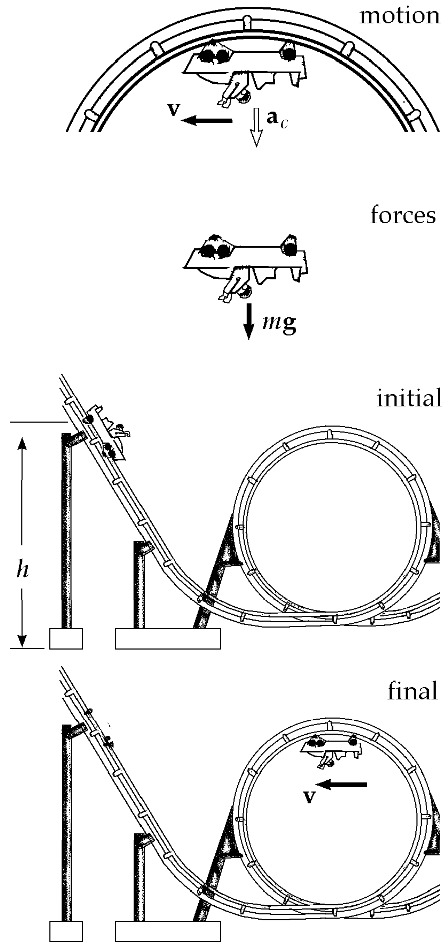


FIG. P8.73

***P8.74** (a) Conservation of energy for the sled-rider-Earth system, between A and C:

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m(9.80\ \text{m/s}^2)(9.76\ \text{m}) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(9.76\ \text{m})} = \boxed{14.1\ \text{m/s}}$$

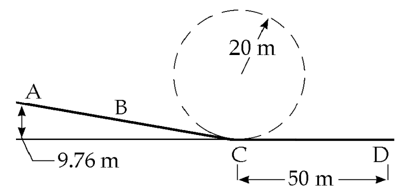


FIG. P8.74(a)

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k\Delta x = K_f + U_{gf}: \quad \frac{1}{2}(80\ \text{kg})(2.5\ \text{m/s})^2 + (80\ \text{kg})(9.80\ \text{m/s}^2)(9.76\ \text{m}) - f_k\Delta x = 0 + 0$$

$$-f_k\Delta x = \boxed{-7.90\times 10^3\ \text{J}}$$

(c) The water exerts a frictional force $\quad f_k = \dfrac{7.90\times 10^3\ \text{J}}{\Delta x} = \dfrac{7.90\times 10^3\ \text{N}\cdot\text{m}}{50\ \text{m}} = 158\ \text{N}$

and also a normal force of $\quad n = mg = (80\ \text{kg})(9.80\ \text{m/s}^2) = 784\ \text{N}$

The magnitude of the water force is $\quad \sqrt{(158\ \text{N})^2 + (784\ \text{N})^2} = \boxed{800\ \text{N}}$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\ \text{m}}{54.3\ \text{m}} = 10.4°$$

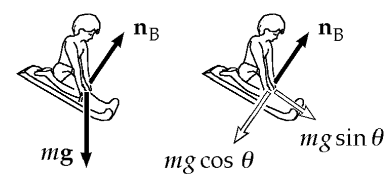


FIG. P8.74(d)

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \quad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\ \text{kg})(9.80\ \text{m/s}^2)\cos 10.4° = \boxed{771\ \text{N}}$$

(e)

$$\sum F_y = ma_y: \quad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(80.0\ \text{kg})(14.1\ \text{m/s})^2}{20\ \text{m}}$$

$$n_C = \boxed{1.57\times 10^3\ \text{N up}}$$

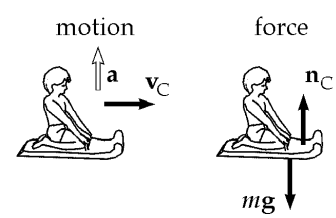


FIG. P8.74(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

ANSWERS TO EVEN PROBLEMS

P8.2 (a) 800 J; (b) 107 J; (c) 0

P8.4 (a) 1.11×10^9 J; (b) 0.2

P8.6 1.84 m

P8.8 (a) 10.2 kW; (b) 10.6 kW; (c) 5.82×10^6 J

P8.10 $d = \dfrac{kx^2}{2mg\sin\theta} - x$

P8.12 (a) see the solution; (b) 60.0°

P8.14 (a) $\sqrt{\dfrac{2(m_1 - m_2)gh}{(m_1 + m_2)}}$; (b) $\dfrac{2m_1 h}{m_1 + m_2}$

P8.16 160 L/min

P8.18 40.8°

P8.20 $\left(\dfrac{8gh}{15}\right)^{1/2}$

P8.22 (a) see the solution; (b) 35.0 J

P8.24 (a) $v_B = 5.94$ m/s; $v_C = 7.67$ m/s; (b) 147 J

P8.26 (a) $U_f = 22.0$ J; $E = 40.0$ J; (b) Yes. The total mechanical energy changes.

P8.28 194 m

P8.30 2.06 kN up

P8.32 168 J

P8.34 (a) 24.5 m/s; (b) yes; (c) 206 m; (d) Air drag depends strongly on speed.

P8.36 3.92 kJ

P8.38 44.1 kW

P8.40 (a) $\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}$;
(b) $\Delta U = \dfrac{5A}{2} - \dfrac{19B}{3}$; $\Delta K = \dfrac{19B}{3} - \dfrac{5A}{2}$

P8.42 $\left(7 - 9x^2 y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}$

P8.44 see the solution

P8.46 (a) $r = 1.5$ mm and 3.2 mm, stable; 2.3 mm and unstable; $r \to \infty$ neutral;
(b) $-5.6\text{ J} \le E < 1\text{ J}$; (c) $0.6\text{ mm} \le r \le 3.6\text{ mm}$;
(d) 2.6 J; (e) 1.5 mm; (f) 4 J

P8.48 see the solution

P8.50 33.4 kW

P8.52 (a) 0.588 J; (b) 0.588 J; (c) 2.42 m/s;
(d) 0.196 J; 0.392 J

P8.54 0.115

P8.56 (a) 100 J; (b) 0.410 m; (c) 2.84 m/s;
(d) −9.80 mm; (e) 2.85 m/s

P8.58 (a) $\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}$; (b) 1.87; −0.535;
(c) see the solution

P8.60 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P8.62 (a) see the solution; (b) 7.42 m/s

P8.64 (a) see the solution; (b) 1.35 m/s;
(c) 0.958 m/s; (d) see the solution

P8.66 0.923 m/s

P8.68 $2m$

P8.70 100.6°

P8.72 see the solution

P8.74 (a) 14.1 m/s; (b) −7.90 J; (c) 800 N;
(d) 771 N; (e) 1.57 kN up

9

Linear Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q9.1 No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

Q9.2 The momentum doubles since it is proportional to the speed. The kinetic energy quadruples, since it is proportional to the speed-squared.

Q9.3 The momenta of two particles will only be the same if the masses of the particles of the same.

Q9.4 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

Q9.5 Provided there is some form of potential energy in the system, the parts of an isolated system can move if the system is initially at rest. Consider two air-track gliders on a horizontal track. If you compress a spring between them and then tie them together with a string, it is possible for the system to start out at rest. If you then burn the string, the potential energy stored in the spring will be converted into kinetic energy of the gliders.

Q9.6 No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both objects wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

Q9.7 Interestingly, mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

Q9.8 (a) Linear momentum is conserved since there are no external forces acting on the system.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q9.9 Momentum conservation is not violated if we make our system include the Earth along with the clay. When the clay receives an impulse backwards, the Earth receives the same size impulse forwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than the acceleration of the clay, but the planet absorbs all of the momentum that the clay loses.

Q9.10 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

Q9.11 As a ball rolls down an incline, the Earth receives an impulse of the same size and in the opposite direction as that of the ball. If you consider the Earth-ball system, momentum conservation is not violated.

Q9.12 Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T + 3m_c}{3m_T + m_c}$ times the speed of the car, the car will lose more energy.

Q9.13 The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

Q9.14 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q9.15 The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

Q9.16 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

Q9.17 (c) In this case, the impulse on the Frisbee is largest. According to Newton's third law, the impulse on the skater and thus the final speed of the skater will also be largest.

Q9.18 Usually but not necessarily. In a one-dimensional collision between two identical particles with the same initial speed, the kinetic energy of the particles will not change.

Q9.19 g downward.

Q9.20 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Q9.21 The planet is in motion around the sun, and thus has momentum and kinetic energy of its own. The spacecraft is directed to cross the planet's orbit behind it, so that the planet's gravity has a component pulling forward on the spacecraft. Since this is an elastic collision, and the velocity of the planet remains nearly unchanged, the probe must both increase speed and change direction for both momentum and kinetic energy to be conserved.

Q9.22 No—an external force of gravity acts on the moon. Yes, because its speed is constant.

Q9.23 The impulse given to the egg is the same regardless of how it stops. If you increase the impact time by dropping the egg onto foam, you will decrease the impact force.

Q9.24 Yes. A boomerang, a kitchen stool.

Q9.25 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little circle, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q9.26 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q9.27 The gun recoiled.

Q9.28 Inflate a balloon and release it. The air escaping from the balloon gives the balloon an impulse.

Q9.29 There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

SOLUTIONS TO PROBLEMS

Section 9.1 **Linear Momentum and Its Conservation**

P9.1 $m = 3.00 \text{ kg}$, $\mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

P9.2 (a) At maximum height $\mathbf{v}=0$, so $\mathbf{p}=\boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}(0.100)\text{kg}(15.0\ \text{m/s})^2=11.2\ \text{J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K=5.62\ \text{J}=\frac{1}{2}(0.100\ \text{kg})v^2$$
$$v=\sqrt{\frac{2\times5.62\ \text{J}}{0.100\ \text{kg}}}=10.6\ \text{m/s}$$

Then $\mathbf{p}=m\mathbf{v}=(0.100\ \text{kg})(10.6\ \text{m/s})\hat{\mathbf{j}}$

$$\mathbf{p}=\boxed{1.06\ \text{kg}\cdot\text{m/s}\,\hat{\mathbf{j}}}.$$

P9.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2-v_i^2=2a(x_f-x_i): \qquad 0-v_i^2=2(-9.80\ \text{m/s}^2)(0.250\ \text{m})$$
$$v_i=2.20\ \text{m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0=(5.98\times10^{24}\ \text{kg})v_e+(85.0\ \text{kg})(2.20\ \text{m/s})$$
$$v_e\sim\boxed{10^{-23}\ \text{m/s}}$$

P9.4 (a) For the system of two blocks $\Delta p=0$,

or $p_i=p_f$

Therefore, $0=Mv_m+(3M)(2.00\ \text{m/s})$

Solving gives $v_m=\boxed{-6.00\ \text{m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2=\frac{1}{2}Mv_M^2+\frac{1}{2}(3M)v_{3M}^2=\boxed{8.40\ \text{J}}$

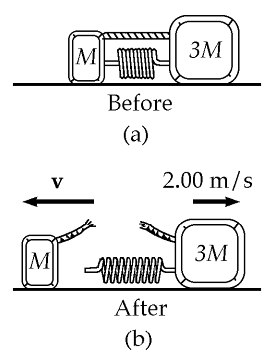


FIG. P9.4

P9.5 (a) The momentum is $p = mv$, so $v = \dfrac{p}{m}$ and the kinetic energy is $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{p}{m}\right)^2 = \boxed{\dfrac{p^2}{2m}}$.

(b) $K = \dfrac{1}{2}mv^2$ implies $v = \sqrt{\dfrac{2K}{m}}$, so $p = mv = m\sqrt{\dfrac{2K}{m}} = \boxed{\sqrt{2mK}}$.

Section 9.2 **Impulse and Momentum**

***P9.6** From the impulse-momentum theorem, $\overline{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\overline{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}.$$

Therefore, the magnitude of the needed retarding force is $\boxed{6.44 \times 10^3\text{ N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt =$ area under curve

$I = \dfrac{1}{2}(1.50 \times 10^{-3}\text{ s})(18\,000\text{ N}) = \boxed{13.5\text{ N}\cdot\text{s}}$

(b) $F = \dfrac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\text{ kN}}$

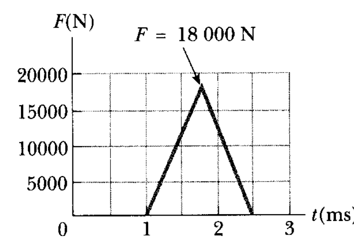


FIG. P9.7

***P9.8** The impact speed is given by $\dfrac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \dfrac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2\text{ up} - mv_1\text{ down} &= m(v_2 + v_1)\text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\text{ up} \\ &= 0.15\text{ kg}\sqrt{2(9.8\text{ m/s}^2)}\left(\sqrt{0.960\text{ m}} + \sqrt{1.25\text{ m}}\right)\text{ up} \\ &= \boxed{1.39\text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

P9.9

$$\Delta\mathbf{p} = \mathbf{F}\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00 \text{ kg})(10.0 \text{ m/s})(0.866)$$

$$= -52.0 \text{ kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0 \text{ kg}\cdot\text{m/s}}{0.200 \text{ s}} = \boxed{-260 \text{ N}}$$

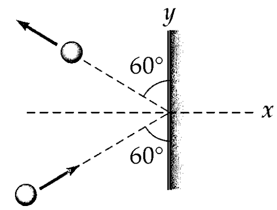


FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}} \text{ N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0 \text{ J}}$

P9.11 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $\quad (0.200 \text{ kg})(15.0 \text{ m/s})(-\cos 45.0°) + I_x = (0.200 \text{ kg})(40.0 \text{ m/s})\cos 30.0°$

$I_x = 9.05 \text{ N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$: $\quad (0.200 \text{ kg})(15.0 \text{ m/s})(-\sin 45.0°) + I_y = (0.200 \text{ kg})(40.0 \text{ m/s})\sin 30.0°$

$$\mathbf{I} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{N}\cdot\text{s}}$$

(b) $\mathbf{I} = \frac{1}{2}(0 + \mathbf{F}_m)(4.00 \text{ ms}) + \mathbf{F}_m(20.0 \text{ ms}) + \frac{1}{2}\mathbf{F}_m(4.00 \text{ ms})$

$\mathbf{F}_m \times 24.0 \times 10^{-3} \text{ s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{N}\cdot\text{s}$

$\mathbf{F}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right) \text{N}}$

P9.12 If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt, the average force during impact is given by

$$\overline{F} = \frac{m\left(0 - v_{\text{impact}}\right)}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \text{ or } \overline{F} = \frac{m\sqrt{2gh}}{\Delta t} \text{ (directed upward).}$$

Assuming a mass of 55 kg and an impact time of ≈ 1.0 s, the magnitude of this average force is

$$\left|\overline{F}\right| = \frac{(55 \text{ kg})\sqrt{2\left(9.8 \text{ m/s}^2\right)(10 \text{ m})}}{1.0 \text{ s}} = 770 \text{ N, or } \boxed{\sim 10^3 \text{ N}}.$$

P9.13 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600 \text{ kg})(25.0 \text{ m/s}) - 0}{1.00 \text{ s}} = \boxed{15.0 \text{ N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

***P9.14** (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \qquad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

(c) $I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = mv_f = 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

Section 9.3 **Collisions in One Dimension**

P9.15 $(200 \text{ g})(55.0 \text{ m/s}) = (46.0 \text{ g})v + (200 \text{ g})(40.0 \text{ m/s})$

$v = \boxed{65.2 \text{ m/s}}$

***P9.16** $(m_1v_1 + m_2v_2)_i = (m_1v_1 + m_2v_2)_f$

$22.5 \text{ g}(35 \text{ m/s}) + 300 \text{ g}(-2.5 \text{ m/s}) = 22.5 \text{ g}v_{1f} + 0$

$$v_{1f} = \frac{37.5 \text{ g}\cdot\text{m/s}}{22.5 \text{ g}} = \boxed{1.67 \text{ m/s}}$$

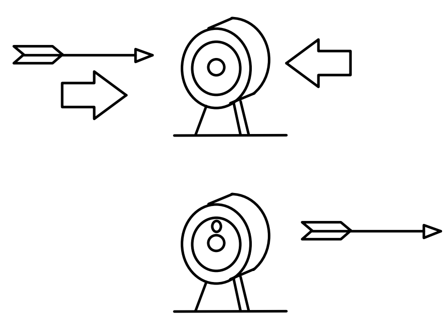


FIG. P9.16

P9.17 Momentum is conserved

$$\left(10.0\times10^{-3}\ \text{kg}\right)v=(5.01\ \text{kg})(0.600\ \text{m/s})$$

$$v=\boxed{301\ \text{m/s}}$$

P9.18 (a) $mv_{1i}+3mv_{2i}=4mv_f$ where $m=2.50\times10^4$ kg

$$v_f=\frac{4.00+3(2.00)}{4}=\boxed{2.50\ \text{m/s}}$$

(b) $$K_f-K_i=\frac{1}{2}(4m)v_f^2-\left[\frac{1}{2}mv_{1i}^2+\frac{1}{2}(3m)v_{2i}^2\right]=\left(2.50\times10^4\right)(12.5-8.00-6.00)=\boxed{-3.75\times10^4\ \text{J}}$$

P9.19 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

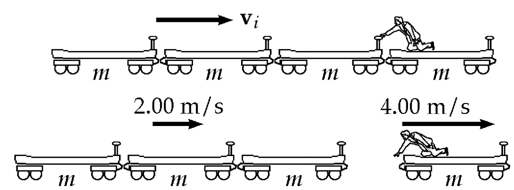


FIG. P9.19

$$(4m)v_i=(3m)(2.00\ \text{m/s})+m(4.00\ \text{m/s})$$

$$v_i=\frac{6.00\ \text{m/s}+4.00\ \text{m/s}}{4}=\boxed{2.50\ \text{m/s}}$$

(b) $$W_{\text{actor}}=K_f-K_i=\frac{1}{2}\left[(3m)(2.00\ \text{m/s})^2+m(4.00\ \text{m/s})^2\right]-\frac{1}{2}(4\ \text{m})(2.50\ \text{m/s})^2$$

$$W_{\text{actor}}=\frac{\left(2.50\times10^4\ \text{kg}\right)}{2}(12.0+16.0-25.0)(\text{m/s})^2=\boxed{37.5\ \text{kJ}}$$

(c) $\boxed{\text{The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.}}$

P9.20 v_1, speed of m_1 at B before collision.

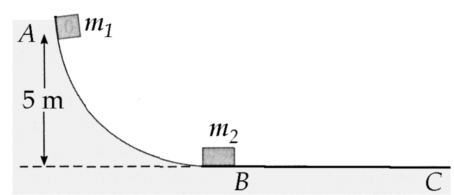


FIG. P9.20

$$\frac{1}{2}m_1v_1^2=m_1gh$$

$$v_1=\sqrt{2(9.80)(5.00)}=9.90\ \text{m/s}$$

v_{1f}, speed of m_1 at B just after collision.

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_1=-\frac{1}{3}(9.90)\ \text{m/s}=-3.30\ \text{m/s}$$

At the highest point (after collision)

$$m_1gh_{\max}=\frac{1}{2}m_1(-3.30)^2 \qquad h_{\max}=\frac{(-3.30\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)}=\boxed{0.556\ \text{m}}$$

P9.21 (a), (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0 v_g + 150 v_p = 0\text{, or } v_g = -3.33 v_p$$

Putting this into the equation (1) above gives

$$-3.33 v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

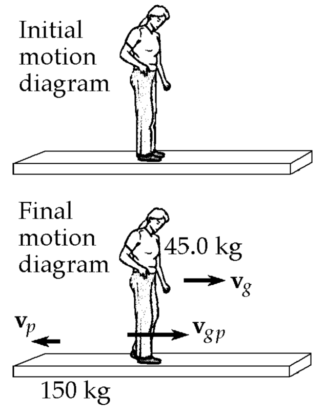


FIG. P9.21

***P9.22** For the car-truck-driver-driver system, momentum is conserved:

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $\quad (4\,000 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}} + (800 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}}) = (4\,800 \text{ kg})v_f\hat{\mathbf{i}}$

$$v_f = \frac{25\,600 \text{ kg}\cdot\text{m/s}}{4\,800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$: $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}}$

$$\mathbf{F} = \boxed{1.78\times 10^3 \text{ N}(-\hat{\mathbf{i}}) \text{ on the truck driver}}$$

For the driver of the car, $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}})$

$$\mathbf{F} = \boxed{8.89\times 10^3 \text{ N}\hat{\mathbf{i}} \text{ on the car driver}}\text{, 5 times larger.}$$

P9.23 (a) According to the Example in the chapter text, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1+m_2)^2}$$

where m_2 is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)(1.6\times 10^{-13} \text{ J}) = \boxed{4.54\times 10^{-14} \text{ J}}$

$K_n = (0.716)(1.6\times 10^{-13} \text{ J}) = \boxed{1.15\times 10^{-13} \text{ J}}$

P9.24 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

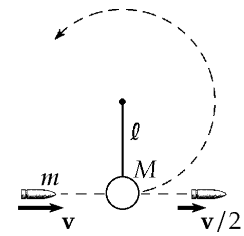


FIG. P9.24

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.25 At impact, momentum of the clay-block system is conserved, so:

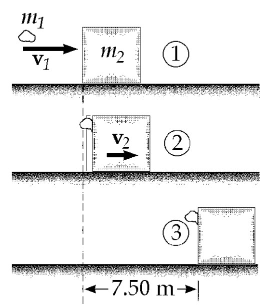


FIG. P9.25

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$

$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})$$

$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$$

$$\left(12.0 \times 10^{-3} \text{ kg}\right)v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$$

P9.26 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $\quad K_i + \Delta E_{\text{mech}} = K_f \quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - F\left(8.00 \times 10^{-2} \text{ m}\right) = 0$

For the second, $\quad p_i = p_f \quad \left(7.00 \times 10^{-3} \text{ kg}\right)v = (1.014 \text{ kg})v_f$

$$v_f = \frac{\left(7.00 \times 10^{-3}\right)v}{1.014}$$

Again, $\quad K_i + \Delta E_{\text{mech}} = K_f: \quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})v_f^2$

Substituting for v_f, $\quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})\left(\frac{7.00 \times 10^{-3} v}{1.014}\right)^2$

$$Fd = \frac{1}{2}\left(7.00 \times 10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00 \times 10^{-3}\right)^2}{1.014}v^2$$

Substituting for v, $\quad Fd = F\left(8.00 \times 10^{-2} \text{ m}\right)\left(1 - \frac{7.00 \times 10^{-3}}{1.014}\right) \qquad d = \boxed{7.94 \text{ cm}}$

***P9.27** (a) Using conservation of momentum, $\left(\sum \mathbf{p}\right)_{\text{after}} = \left(\sum \mathbf{p}\right)_{\text{before}}$, gives

$$\left[(4.0 + 10 + 3.0)\text{ kg}\right]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24\text{ m/s}$, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

Section 9.4 **Two-Dimensional Collisions**

P9.28 (a) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta = 1.54\text{ m/s} \qquad (2)$

Divide equation (2) by (1) $\tan\theta = \dfrac{1.54}{2.43} = 0.633$

From which $\boxed{\theta = 32.3°}$

Then, either (1) or (2) gives $V = \boxed{2.88\text{ m/s}}$

(b) $K_i = \dfrac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \dfrac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55 \times 10^3\text{ J}$

$K_f = \dfrac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67 \times 10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{783 J into internal energy.}}$

P9.29 $p_{xf} = p_{xi}$

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00\ \text{m/s})$$

$$0.799 v_O + 0.602 v_Y = 5.00\ \text{m/s} \qquad (1)$$

$$p_{yf} = p_{yi}$$

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602 v_O = 0.799 v_Y \qquad (2)$$

Solving (1) and (2) simultaneously,

$\boxed{v_O = 3.99\ \text{m/s}}$ and $\boxed{v_Y = 3.01\ \text{m/s}}$.

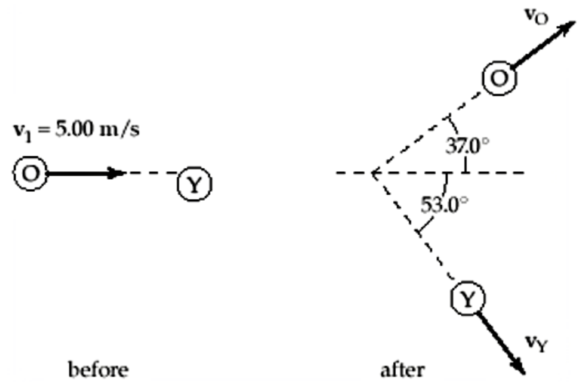


FIG. P9.29

P9.30 $p_{xf} = p_{xi}$: $\quad mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$$

$p_{yf} = p_{yi}$: $\quad mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$

$$v_O \sin\theta = v_Y \cos\theta \qquad (2)$$

From equation (2),

$$v_O = v_Y \left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

Substituting into equation (1),

$$v_Y \left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so $\quad v_Y\left(\cos^2\theta + \sin^2\theta\right) = v_i \sin\theta$, and $\boxed{v_Y = v_i \sin\theta}$.

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$.

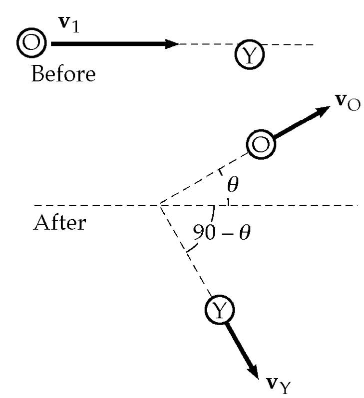


FIG. P9.30

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P9.31 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and $v_{Bi} = 8.33\ \text{m/s}$

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$ (1)

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ (2)

Solving (1) and (2) simultaneously, we find

$\boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

and $\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

P9.32 We use conservation of momentum for the system of two vehicles for both northward and eastward components.

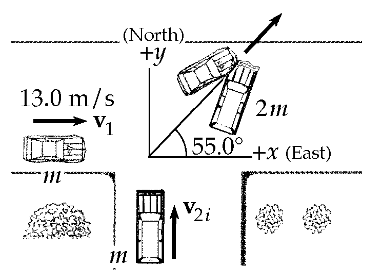


FIG. P9.32

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f\cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f\sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

P9.33 By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25 \text{ m/s}$$
$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16 \text{ m/s}$$
$$\mathbf{v}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

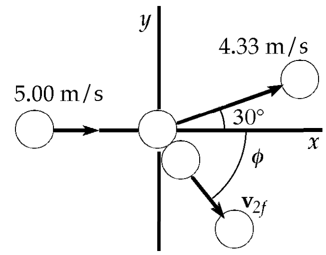


FIG. P9.33

Note that we did not need to use the fact that the collision is perfectly elastic.

P9.34 (a) $\mathbf{p}_i = \mathbf{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \qquad (1)$$
$$0 = mv\sin\theta + mv\sin\phi \qquad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \frac{v_i}{\sqrt{2}}}$

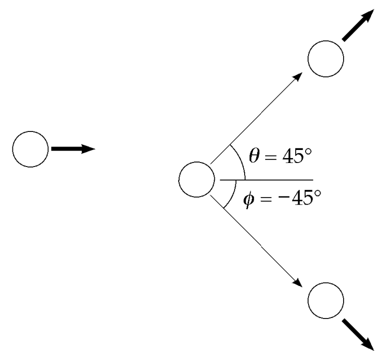


FIG. P9.34

(b) Hence, (1) gives $v_i = \dfrac{2v_i\cos\theta}{\sqrt{2}}$ $\qquad \theta = \boxed{45.0°}$ $\qquad \phi = \boxed{-45.0°}$

P9.35 $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$: $\quad 3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$

$$\mathbf{v} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right) \text{ m/s}}$$

P9.36 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $\quad -mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $\quad 0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $\quad +\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have $\quad v_{2x} = \dfrac{2v_i}{3}$

also $\quad v_{1y} = 3v_{2y}$

So the energy equation becomes $\quad 4v_i^2 = 9v_{2y}^2 + \dfrac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $\quad v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

continued on next page

(a) The object of mass m has final speed $v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\dfrac{4v_i^2}{9} + \dfrac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$ $\qquad \theta = \tan^{-1}\left(\dfrac{\sqrt{2}v_i}{3}\dfrac{3}{2v_i}\right) = \boxed{35.3°}$

P9.37 $m_0 = 17.0 \times 10^{-27}$ kg $\qquad \mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\qquad \mathbf{v}_1 = 6.00 \times 10^6\,\hat{\mathbf{j}}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\qquad \mathbf{v}_2 = 4.00 \times 10^6\,\hat{\mathbf{i}}$ m/s

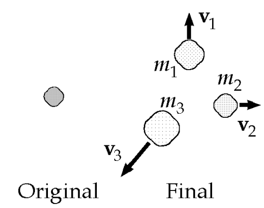


FIG. P9.37

(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\,\hat{\mathbf{j}}\right) + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\,\hat{\mathbf{i}}\right) + \left(3.60 \times 10^{-27}\right)\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{\left(-9.33 \times 10^6\,\hat{\mathbf{i}} - 8.33 \times 10^6\,\hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $E = \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}m_2v_2^2 + \dfrac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\left[\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\right)^2 + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\right)^2 + \left(3.60 \times 10^{-27}\right)\left(12.5 \times 10^6\right)^2\right]$$

$$\boxed{E = 4.39 \times 10^{-13} \text{ J}}$$

Section 9.5 **The Center of Mass**

P9.38 The x-coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0 + 0 + 0 + 0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

$$\boxed{x_{\text{CM}} = 0}$$

and the y-coordinate of the center of mass is

$$y_{\text{CM}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{\text{CM}} = 1.00 \text{ m}}$$

P9.39 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the **V**.

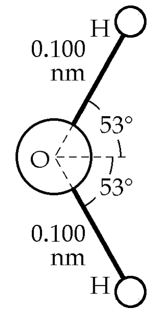


FIG. P9.39

Then $\boxed{y_{CM} = 0}$

and $x_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} =$

$$x_{CM} = \frac{0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0° + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}$$

$$\boxed{x_{CM} = 0.006\,73\text{ nm from the oxygen nucleus}}$$

***P9.40** Let the x axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{5.98 \times 10^{24}\text{ kg } 0 + 7.36 \times 10^{22}\text{ kg}\left(3.84 \times 10^{8}\text{ m}\right)}{6.05 \times 10^{24}\text{ kg}}$$

$$= \boxed{4.67 \times 10^{6}\text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^{6} m.

P9.41 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

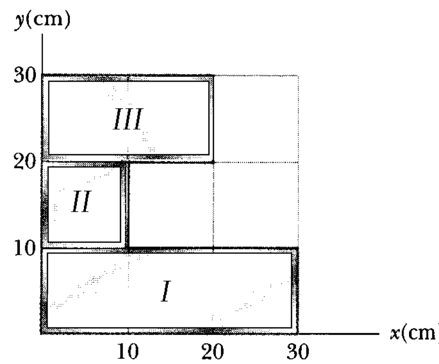


FIG. P9.41

$$A = A_1 + A_2 + A_3$$
$$M = M_1 + M_2 + M_3$$
$$\frac{M_1}{A_1} = \frac{M}{A}$$
$$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$$
$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$
$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$
$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$
$$x_{CM} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$
$$x_{CM} = \boxed{11.7\text{ cm}}$$
$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$
$$y_{CM} = \boxed{13.3\text{ cm}}$$

***P9.42** (a) Represent the height of a particle of mass *dm* within the object as *y*. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g = \int_{\text{all mass}} gy\,dm = g\int y\,dm$. For the center of mass we have $y_{CM} = \frac{1}{M}\int y\,dm$, so $U_g = gMy_{CM}$.

(b) The volume of the ramp is $\frac{1}{2}(3.6\text{ m})(15.7\text{ m})(64.8\text{ m}) = 1.83\times10^3\text{ m}^3$. Its mass is $\rho V = \left(3\,800\text{ kg/m}^3\right)\left(1.83\times10^3\text{ m}^3\right) = 6.96\times10^6\text{ kg}$. Its center of mass is above its base by one-third of its height, $y_{CM} = \frac{1}{3}15.7\text{ m} = 5.23\text{ m}$. Then

$$U_g = Mgy_{CM} = 6.96\times10^6\text{ kg}\left(9.8\text{ m/s}^2\right)5.23\text{ m} = \boxed{3.57\times10^8\text{ J}}.$$

P9.43 (a)

$$M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right]dx$$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b)

$$x_{CM} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\,x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}}\left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.44** Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the *x* axis has mass given by $\frac{dm}{rd\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L}d\theta$ where we have used the definition of radian measure. Now

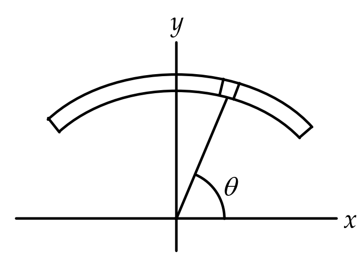


FIG. P9.44

$$y_{CM} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

Section 9.6 **Motion of a System of Particles**

P9.45 (a)
$$\mathbf{v}_{\mathrm{CM}}=\frac{\sum m_i\mathbf{v}_i}{M}=\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{M}$$
$$=\frac{(2.00\text{ kg})\left(2.00\hat{\mathbf{i}}\text{ m/s}-3.00\hat{\mathbf{j}}\text{ m/s}\right)+(3.00\text{ kg})\left(1.00\hat{\mathbf{i}}\text{ m/s}+6.00\hat{\mathbf{j}}\text{ m/s}\right)}{5.00\text{ kg}}$$
$$\mathbf{v}_{\mathrm{CM}}=\boxed{\left(1.40\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b)
$$\mathbf{p}=M\mathbf{v}_{\mathrm{CM}}=(5.00\text{ kg})\left(1.40\hat{\mathbf{i}}+2.40\hat{\mathbf{j}}\right)\text{ m/s}=\boxed{\left(7.00\hat{\mathbf{i}}+12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$$

P9.46 (a) See figure to the right.

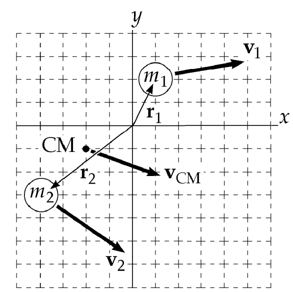


FIG. P9.46

(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{\mathrm{CM}}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_1+m_2}$$
$$\mathbf{r}_{\mathrm{CM}}=\frac{(2.00\text{ kg})(1.00\text{ m},\ 2.00\text{ m})+(3.00\text{ kg})(-4.00\text{ m},\ -3.00\text{ m})}{2.00\text{ kg}+3.00\text{ kg}}$$
$$\mathbf{r}_{\mathrm{CM}}=\boxed{\left(-2.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{\mathrm{CM}}=\frac{\mathbf{P}}{M}=\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{m_1+m_2}=\frac{(2.00\text{ kg})(3.00\text{ m/s},\ 0.50\text{ m/s})+(3.00\text{ kg})(3.00\text{ m/s},\ -2.00\text{ m/s})}{(2.00\text{ kg}+3.00\text{ kg})}$$
$$\mathbf{v}_{\mathrm{CM}}=\boxed{\left(3.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P}=M\mathbf{v}_{\mathrm{CM}}$

or as $\mathbf{P}=m_1\mathbf{v}_1+m_2\mathbf{v}_2$

Either gives $\mathbf{P}=\boxed{\left(15.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

P9.47 Let $x=$ distance from shore to center of boat
$\ell=$ length of boat
$x'=$ distance boat moves as Juliet moves toward Romeo
The center of mass stays fixed.

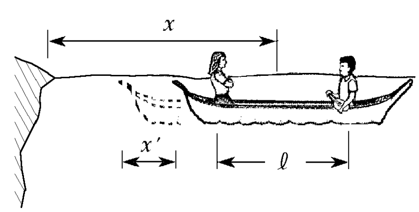


FIG. P9.47

Before: $$x_{\mathrm{CM}}=\frac{\left[M_bx+M_J\left(x-\frac{\ell}{2}\right)+M_R\left(x+\frac{\ell}{2}\right)\right]}{\left(M_B+M_J+M_R\right)}$$

After: $$x_{\mathrm{CM}}=\frac{\left[M_B(x-x')+M_J\left(x+\frac{\ell}{2}-x'\right)+M_R\left(x+\frac{\ell}{2}-x'\right)\right]}{\left(M_B+M_J+M_R\right)}$$

$$\ell\left(-\frac{55.0}{2}+\frac{77.0}{2}\right)=x'(-80.0-55.0-77.0)+\frac{\ell}{2}(55.0+77.0)$$
$$x'=\frac{55.0\ell}{212}=\frac{55.0(2.70)}{212}=\boxed{0.700\text{ m}}$$

P9.48 (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}(1.50\text{ m/s}) + 0.300\text{ kg}(-0.400\text{ m/s}) = 0.200\text{ kg } v_{1f} + 0.300\text{ kg } v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\text{ m/s}$$

Then $0.300 - 0.120 = 0.200 v_{1f} + 0.300(1.90 + v_{1f})$

$v_{1f} = -0.780\text{ m/s}$ $\quad$ $v_{2f} = 1.12\text{ m/s}$

$\boxed{\mathbf{v}_{1f} = -0.780\hat{\mathbf{i}}\text{ m/s}}$ $\quad$ $\boxed{\mathbf{v}_{2f} = 1.12\hat{\mathbf{i}}\text{ m/s}}$

(b) Before, $\mathbf{v}_{CM} = \dfrac{(0.200\text{ kg})(1.50\text{ m/s})\hat{\mathbf{i}} + (0.300\text{ kg})(-0.400\text{ m/s})\hat{\mathbf{i}}}{0.500\text{ kg}}$

$\boxed{\mathbf{v}_{CM} = (0.360\text{ m/s})\hat{\mathbf{i}}}$

Afterwards, the center of mass must move at the same velocity, as momentum of the system is conserved.

Section 9.7 **Rocket Propulsion**

P9.49 (a) Thrust $= \left|v_e \dfrac{dM}{dt}\right|$ $\quad$ Thrust $= (2.60\times10^3\text{ m/s})(1.50\times10^4\text{ kg/s}) = \boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $\quad$ $3.90\times10^7 - (3.00\times10^6)(9.80) = (3.00\times10^6)a$

$a = \boxed{3.20\text{ m/s}^2}$

***P9.50** (a) The fuel burns at a rate $\quad$ $\dfrac{dM}{dt} = \dfrac{12.7\text{ g}}{1.90\text{ s}} = 6.68\times10^{-3}\text{ kg/s}$

Thrust $= v_e \dfrac{dM}{dt}$: $\quad$ $5.26\text{ N} = v_e(6.68\times10^{-3}\text{ kg/s})$

$v_e = \boxed{787\text{ m/s}}$

(b) $v_f - v_i = v_e \ln\left(\dfrac{M_i}{M_f}\right)$: $\quad$ $v_f - 0 = (797\text{ m/s})\ln\left(\dfrac{53.5\text{ g} + 25.5\text{ g}}{53.5\text{ g} + 25.5\text{ g} - 12.7\text{ g}}\right)$

$v_f = \boxed{138\text{ m/s}}$

P9.51 $v = v_e \ln \dfrac{M_i}{M_f}$

(a) $M_i = e^{v/v_e} M_f$ $\quad$ $M_i = e^5(3.00\times10^3\text{ kg}) = 4.45\times10^5\text{ kg}$

The mass of fuel and oxidizer is $\quad$ $\Delta M = M_i - M_f = (445 - 3.00)\times10^3\text{ kg} = \boxed{442\text{ metric tons}}$

(b) $\Delta M = e^2(3.00\text{ metric tons}) - 3.00\text{ metric tons} = \boxed{19.2\text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

P9.52 (a) From Equation 9.41, $v-0=v_e\ln\left(\frac{M_i}{M_f}\right)=-v_e\ln\left(\frac{M_f}{M_i}\right)$

Now, $M_f=M_i-kt$, so $v=-v_e\ln\left(\frac{M_i-kt}{M_i}\right)=-v_e\ln\left(1-\frac{k}{M_i}t\right)$

With the definition, $T_p\equiv\frac{M_i}{k}$, this becomes

$$v(t)=\boxed{-v_e\ln\left(1-\frac{t}{T_p}\right)}$$

(b) With $v_e=1\,500$ m/s, and $T_p=144$ s, $v=-(1\,500\text{ m/s})\ln\left(1-\frac{t}{144\text{ s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

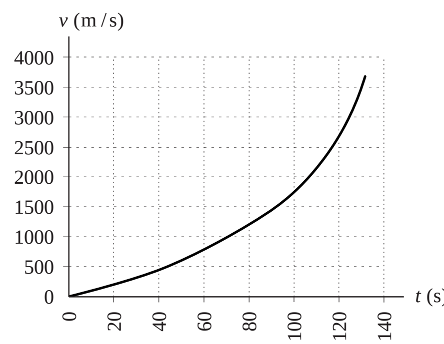


FIG. P9.52(b)

(c) $a(t)=\frac{dv}{dt}=\frac{d\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]}{dt}=-v_e\left(\frac{1}{1-\frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right)=\left(\frac{v_e}{T_p}\right)\left(\frac{1}{1-\frac{t}{T_p}}\right)$, or

$a(t)=\boxed{\frac{v_e}{T_p-t}}$

(d) With $v_e=1\,500$ m/s, and $T_p=144$ s, $a=\frac{1\,500\text{ m/s}}{144\text{ s}-t}$

$t(s)$	$a(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

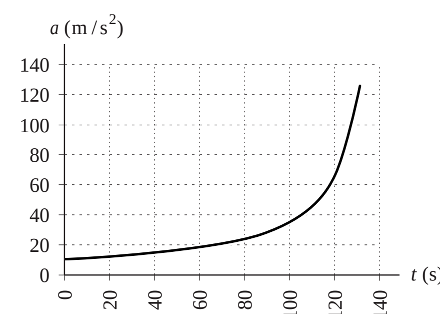


FIG. P9.52(d)

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(e) $$x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With $v_e=1\,500\ \text{m/s}=1.50\ \text{km/s}$, and $T_p=144\ \text{s}$,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

t(s)	x(km)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

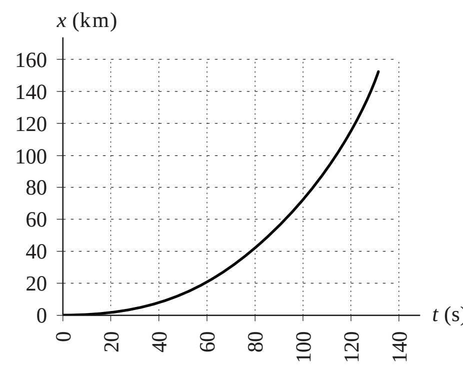

FIG. P9.52(f)

***P9.53** The thrust acting on the spacecraft is

$\sum F=ma$: $$\sum F=(3\,500\ \text{kg})\left(2.50\times10^{-6}\right)\left(9.80\ \text{m/s}^2\right)=8.58\times10^{-2}\ \text{N}$$

thrust $=\left(\dfrac{dM}{dt}\right)v_e$: $$8.58\times10^{-2}\ \text{N}=\left(\frac{\Delta M}{3\,600\ \text{s}}\right)(70\ \text{m/s})$$

$$\Delta M=\boxed{4.41\ \text{kg}}$$

Additional Problems

P9.54 (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$\mathbf{p}_i = \mathbf{p}_f$: $\quad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}$ $\quad \boxed{\mathbf{v} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}}$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2 + v_2^2 - 2v_1v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Conservation of momentum: $\quad m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)$ (1)

Conservation of energy: $\quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $\quad m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$

Factoring gives $\quad m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right)\cdot\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)\cdot\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

and with the use of the momentum equation (equation (1)),

this reduces to $\quad \left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = \left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

or $\quad \mathbf{v}_{1f} = \mathbf{v}_{2f} + \mathbf{v}_2 - \mathbf{v}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

$$\mathbf{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\mathbf{v}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2}$$

Observe that these results are the same as Equations 9.20 and 9.21, which should have been expected since this is a perfectly elastic collision in one dimension.

P9.55 (a) $(60.0 \text{ kg})4.00 \text{ m/s} = (120 + 60.0) \text{ kg} v_f$

$\mathbf{v}_f = \boxed{1.33 \text{ m/s}\,\hat{\mathbf{i}}}$

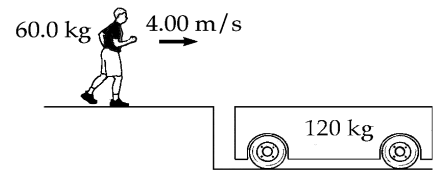

FIG. P9.55

(b) $\sum F_y = 0$: $n - (60.0 \text{ kg})9.80 \text{ m/s}^2 = 0$

$f_k = \mu_k n = 0.400(588 \text{ N}) = 235 \text{ N}$

$\mathbf{f}_k = \boxed{-235 \text{ N}\hat{\mathbf{i}}}$

(c) For the person, $p_i + I = p_f$

$mv_i + Ft = mv_f$

$(60.0 \text{ kg})4.00 \text{ m/s} - (235 \text{ N})t = (60.0 \text{ kg})1.33 \text{ m/s}$

$t = \boxed{0.680 \text{ s}}$

(d) person: $m\mathbf{v}_f - m\mathbf{v}_i = 60.0 \text{ kg}(1.33 - 4.00) \text{ m/s} = \boxed{-160 \text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

cart: $120 \text{ kg}(1.33 \text{ m/s}) - 0 = \boxed{+160 \text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33) \text{ m/s}]0.680 \text{ s} = \boxed{1.81 \text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33 \text{ m/s})0.680 \text{ s} = \boxed{0.454 \text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0 \text{ kg}(1.33 \text{ m/s})^2 - \frac{1}{2}60.0 \text{ kg}(4.00 \text{ m/s})^2 = \boxed{-427 \text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0 \text{ kg}(1.33 \text{ m/s})^2 - 0 = \boxed{107 \text{ J}}$

(i) The force exerted by the person on the cart must equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, -320 J, becomes $+320$ J of additional internal energy in this perfectly inelastic collision.

P9.56 The equation for the horizontal range of a projectile is $R = \frac{v_i^2 \sin 2\theta}{g}$. Thus, with $\theta = 45.0°$, the initial velocity is

$$v_i = \sqrt{Rg} = \sqrt{(200 \text{ m})(9.80 \text{ m/s}^2)} = 44.3 \text{ m/s}$$

$$I = \overline{F}(\Delta t) = \Delta p = mv_i - 0$$

Therefore, the magnitude of the average force acting on the ball during the impact is:

$$\overline{F} = \frac{mv_i}{\Delta t} = \frac{(46.0 \times 10^{-3} \text{ kg})(44.3 \text{ m/s})}{7.00 \times 10^{-3} \text{ s}} = \boxed{291 \text{ N}}.$$

P9.57 We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.
No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}} v_{\text{wrench}} + m_{\text{astronaut}} v_{\text{astronaut}} = 0$$

Thus $v_{\text{astronaut}} = -\dfrac{m_{\text{wrench}} v_{\text{wrench}}}{m_{\text{astronaut}}} = -\dfrac{(0.500 \text{ kg})(20.0 \text{ m/s})}{80.0 \text{ kg}} = -0.125 \text{ m/s}$

At this speed, the time to travel to the ship is

$$t = \frac{30.0 \text{ m}}{0.125 \text{ m/s}} = \boxed{240 \text{ s}} = 4.00 \text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we were told that the astronaut was not drifting away from the ship when he threw the wrench. However, this is not quite possible since he did not encounter an external force that would reduce his velocity away from the ship (there is no air friction beyond earth's atmosphere). If this were a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

P9.58 Using conservation of momentum from just before to just after the impact of the bullet with the block:

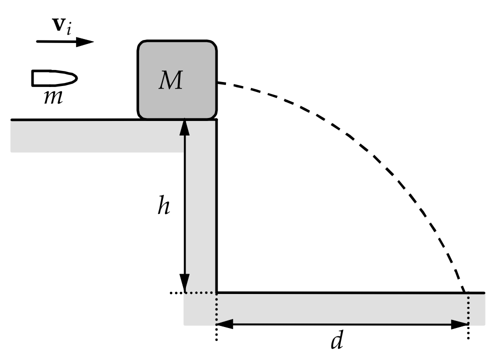

FIG. P9.58

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad (1)$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2$$

Thus, $t = \sqrt{\dfrac{2h}{g}}$ and $v_f = \dfrac{d}{t} = d\sqrt{\dfrac{g}{2h}} = \sqrt{\dfrac{gd^2}{2h}}$

Substituting into (1) from above gives $v_i = \boxed{\left(\dfrac{M+m}{m}\right)\sqrt{\dfrac{gd^2}{2h}}}$.

***P9.59** (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg}\left(-1\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)\text{ m/s}$$

$$=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}-8\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}=\boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}\left(1^2+2^2+3^2\right)\text{ m}^2/\text{s}^2=14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-0.25\hat{\mathbf{i}}+0.75\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\mathbf{v}_{2f}=\mathbf{v}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Conservation of energy:

$$14.0\text{ J}=\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+a^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}(2.67+0.333a)^2\text{ m}^2/\text{s}^2$$

$$=2.5\text{ J}+0.25a^2+5.33\text{ J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or -6.74. Either value is possible.

$$\therefore\ \boxed{a=2.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$$

$$\therefore\ \boxed{a=-6.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$$

P9.60 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$

so $v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$

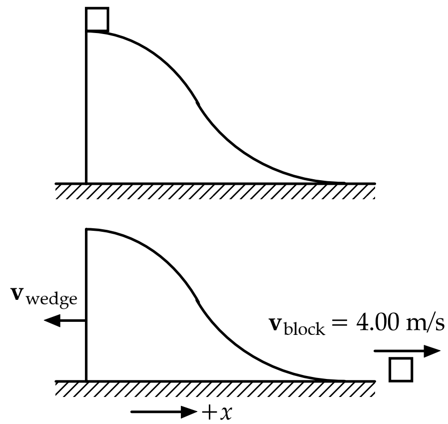

FIG. P9.60

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2$ which gives $\boxed{h = 0.952\text{ m}}$.

***P9.61** (a) Conservation of the x component of momentum for the cart-bucket-water system:

$$mv_i + 0 = (m + \rho V)v \qquad \boxed{v_i = \frac{m + \rho V}{m}v}$$

(b) Raindrops with zero x-component of momentum stop in the bucket and slow its horizontal motion. When they drip out, they carry with them horizontal momentum. Thus the cart slows with constant acceleration.

P9.62 Consider the motion of the firefighter during the three intervals:

(1) before, (2) during, and (3) after collision with the platform.

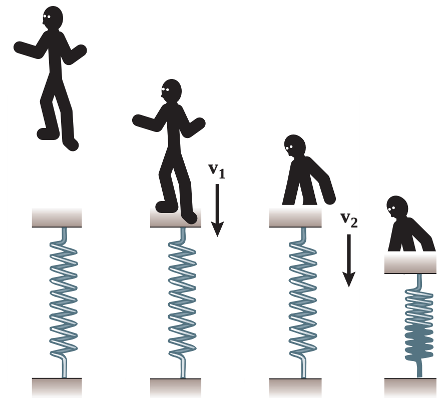

FIG. P9.62

(a) While falling a height of 4.00 m, his speed changes from $v_i = 0$ to v_1 as found from

$$\Delta E = \left(K_f + U_f\right) - \left(K_i - U_i\right), \text{ or}$$
$$K_f = \Delta E - U_f + K_i + U_i$$

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{2(-fh + mgh)}{m}} = \sqrt{\frac{2(-300(4.00) + 75.0(9.80)4.00)}{75.0}} = \boxed{6.81\ \text{m/s}}$$

(b) During the inelastic collision, momentum is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m+M} = \frac{75.0(6.81)}{75.0 + 20.0} = 5.38\ \text{m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$
$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00\ \text{m}}$$

***P9.63** (a) Each object swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.64 (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

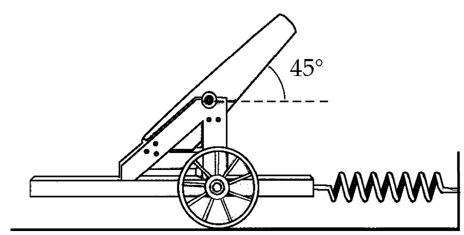

FIG. P9.64

$p_{xf} = p_{xi}$: $\quad m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5\,000)v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00\times 10^4}}\text{ m} = \boxed{1.77 \text{ m}}$$

(c) $\left|F_{s,\max}\right| = kx_{\max} \qquad \left|F_{s,\max}\right| = \left(2.00\times 10^4 \text{ N/m}\right)(1.77 \text{ m}) = \boxed{3.54\times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

P9.65 (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2) v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

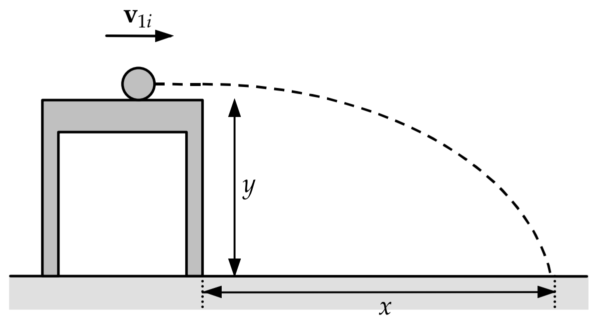

FIG. P9.65

(b) Utilizing the two equations,

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1A}t$$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{\frac{2y}{g}}}$$

From the data, $v_{1A} = \boxed{6.16 \text{ m/s}}$

Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.66** The ice cubes leave the track with speed determined by $mgy_i = \frac{1}{2}mv^2$;

$$v = \sqrt{2(9.8 \text{ m/s}^2)1.5 \text{ m}} = 5.42 \text{ m/s}.$$

Its speed at the apex of its trajectory is $5.42 \text{ m/s} \cos 40° = 4.15 \text{ m/s}$. For its collision with the wall we have

$$mv_i + F\Delta t = mv_f$$

$$0.005 \text{ kg } 4.15 \text{ m/s} + F\Delta t = 0.005 \text{ kg}\left(-\frac{1}{2}4.15 \text{ m/s}\right)$$

$$F\Delta t = -3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$$

The impulse exerted by the cube on the wall is to the right, $+3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$. Here F could refer to a large force over a short contact time. It can also refer to the average force if we interpret Δt as $\frac{1}{10}$ s, the time between one cube's tap and the next's.

$$F_{\text{av}} = \frac{3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}}{0.1 \text{ s}} = \boxed{0.312 \text{ N to the right}}$$

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

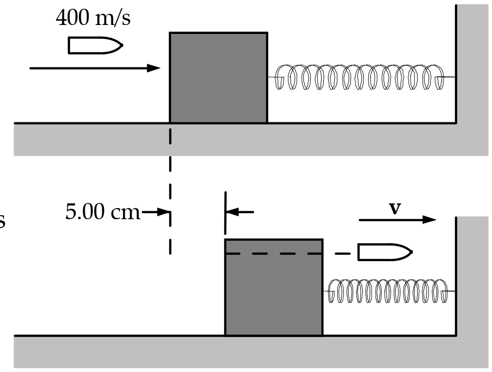

FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})(5.00\times 10^{-2}\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{(5.00\times 10^{-3}\ \text{kg})(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times 10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}(5.00\times 10^{-3}\ \text{kg})(100\ \text{m/s})^2 - \frac{1}{2}(5.00\times 10^{-3}\ \text{kg})(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})(5.00\times 10^{-2}\ \text{m})^2$$

$\Delta E = -374$ J, or there is an energy loss of $\boxed{374\ \text{J}}$.

***P9.68** The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi 1.496\times 10^{11}\ \text{m}}{3.156\times 10^{7}\ \text{s}} = 2.98\times 10^{4}\ \text{m/s}$$

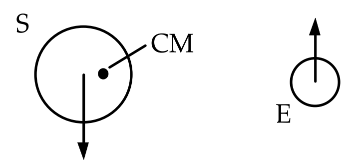

FIG. P9.68

In six months the Earth reverses its direction, to undergo momentum change

$$m_E|\Delta \mathbf{v}_E| = 2m_E v_E = 2(5.98\times 10^{24}\ \text{kg})(2.98\times 10^{4}\ \text{m/s}) = 3.56\times 10^{25}\ \text{kg}\cdot\text{m/s}.$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S|\Delta \mathbf{v}_S| = 3.56\times 10^{25}\ \text{kg}\cdot\text{m/s}$.

Then $|\Delta \mathbf{v}_S| = \dfrac{3.56\times 10^{25}\ \text{kg}\cdot\text{m/s}}{1.991\times 10^{30}\ \text{kg}} = \boxed{0.179\ \text{m/s}}$.

P9.69 (a) $\mathbf{p}_i + \mathbf{F}t = \mathbf{p}_f$: $\quad (3.00 \text{ kg})(7.00 \text{ m/s})\hat{\mathbf{j}} + (12.0 \text{ N}\hat{\mathbf{i}})(5.00 \text{ s}) = (3.00 \text{ kg})\mathbf{v}_f$

$$\mathbf{v}_f = \boxed{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) \text{ m/s}}$$

(b) $\mathbf{a} = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{t}$: $\quad \mathbf{a} = \dfrac{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}) \text{ m/s}}{5.00 \text{ s}} = \boxed{4.00\hat{\mathbf{i}} \text{ m/s}^2}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m}$: $\quad \mathbf{a} = \dfrac{12.0 \text{ N}\hat{\mathbf{i}}}{3.00 \text{ kg}} = \boxed{4.00\hat{\mathbf{i}} \text{ m/s}^2}$

(d) $\Delta\mathbf{r} = \mathbf{v}_i t + \dfrac{1}{2}\mathbf{a}t^2$: $\quad \Delta\mathbf{r} = (7.00 \text{ m/s}\,\hat{\mathbf{j}})(5.00 \text{ s}) + \dfrac{1}{2}(4.00 \text{ m/s}^2\,\hat{\mathbf{i}})(5.00 \text{ s})^2$

$$\Delta\mathbf{r} = \boxed{(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}) \text{ m}}$$

(e) $W = \mathbf{F}\cdot\Delta\mathbf{r}$: $\quad W = (12.0 \text{ N}\hat{\mathbf{i}})\cdot(50.0 \text{ m}\hat{\mathbf{i}} + 35.0 \text{ m}\hat{\mathbf{j}}) = \boxed{600 \text{ J}}$

(f) $\dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(3.00 \text{ kg})(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\cdot(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) \text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50 \text{ kg})(449 \text{ m}^2/\text{s}^2) = \boxed{674 \text{ J}}$$

(g) $\dfrac{1}{2}mv_i^2 + W = \dfrac{1}{2}(3.00 \text{ kg})(7.00 \text{ m/s})^2 + 600 \text{ J} = \boxed{674 \text{ J}}$

P9.70 We find the mass from $\quad M = 360 \text{ kg} - (2.50 \text{ kg/s})t$.

We find the acceleration from $\quad a = \dfrac{\text{Thrust}}{M} = \dfrac{v_e|dM/dt|}{M} = \dfrac{(1\,500 \text{ m/s})(2.50 \text{ kg/s})}{M} = \dfrac{3\,750 \text{ N}}{M}$

We find the velocity and position according to Euler,
from $\quad v_{\text{new}} = v_{\text{old}} + a(\Delta t)$
and $\quad x_{\text{new}} = x_{\text{old}} + v(\Delta t)$

If we take $\Delta t = 0.132$ s, a portion of the output looks like this:

Time t(s)	Total mass (kg)	Acceleration a(m/s^2)	Speed, v (m/s)	Position x(m)
0.000	360.00	10.4167	0.0000	0.0000
0.132	359.67	10.4262	1.3750	0.1815
0.264	359.34	10.4358	2.7513	0.54467
...				
65.868	195.330	19.1983	916.54	27191
66.000	195.000	19.2308	919.08	27312
66.132	194.670	19.2634	921.61	27433
...				
131.736	30.660	122.3092	3687.3	152382
131.868	30.330	123.6400	3703.5	152871
132.000	30.000	125.0000	3719.8	153362

(a) The final speed is $\quad v_f = \boxed{3.7 \text{ km/s}}$

(b) The rocket travels $\quad \boxed{153 \text{ km}}$

P9.71 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

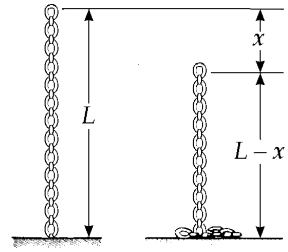

FIG. P9.71

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}.$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx.$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}.$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}.$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

P9.72 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\frac{\Delta p_x}{\Delta t} = \frac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}} = \boxed{3.75\text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\Delta t = p_{xf}$ this is $f = \frac{\Delta p_x}{\Delta t} = \boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x$: $+F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75\text{ N}}$

(d) $W = F\Delta r\cos\theta = 3.75\text{ N}(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2 = \boxed{1.41\text{ J}}$

(f) $\boxed{\text{Friction between sand and belt converts half of the input work into extra internal energy.}}$

***P9.73** $x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1\left(R+\frac{\ell}{2}\right)+m_2(0)}{m_1+m_2} = \boxed{\frac{m_1\left(R+\frac{\ell}{2}\right)}{m_1+m_2}}$

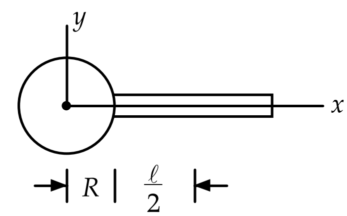

FIG. P9.73

ANSWERS TO EVEN PROBLEMS

P9.2 (a) 0; (b) $1.06\text{ kg}\cdot\text{m/s}$; upward

P9.4 (a) 6.00 m/s to the left; (b) 8.40 J

P9.6 The force is 6.44 kN

P9.8 $1.39\text{ kg}\cdot\text{m/s}$ upward

P9.10 (a) $5.40\text{ N}\cdot\text{s}$ toward the net; (b) –27.0 J

P9.12 $\sim 10^3$ N upward

P9.14 (a) and (c) see the solution; (b) small; (d) large; (e) no difference

P9.16 1.67 m/s

P9.18 (a) 2.50 m/s; (b) 3.75×10^4 J

P9.20 0.556 m

P9.22 1.78 kN on the truck driver; 8.89 kN in the opposite direction on the car driver

P9.24 $v = \frac{4M}{m}\sqrt{g\ell}$

P9.26 7.94 cm

P9.28 (a) 2.88 m/s at 32.3°; (b) 783 J becomes internal energy

P9.30 $v_Y = v_i\sin\theta$; $v_O = v_i\cos\theta$

P9.32 No; his speed was 41.5 mi/h

P9.34 (a) $v = \frac{v_i}{\sqrt{2}}$; (b) 45.0° and –45.0°

P9.36 (a) $\sqrt{2}v_i$; $\sqrt{\frac{2}{3}}v_i$; (b) 35.3°

P9.38 $(0,\ 1.00\ \text{m})$

P9.40 4.67×10^6 m from the Earth's center

P9.42 (a) see the solution; (b) 3.57×10^8 J

P9.44 0.063 5L

P9.46 (a) see the solution;
(b) $(-2.00\ \text{m},\ -1.00\ \text{m})$;
(c) $\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m/s;
(d) $\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ kg·m/s

P9.48 (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s; (b) $0.360\hat{\mathbf{i}}$ m/s

P9.50 (a) 787 m/s; (b) 138 m/s

P9.52 see the solution

P9.54 (a) $\dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$;
(b) $(v_1 - v_2)\sqrt{\dfrac{m_1 m_2}{k(m_1 + m_2)}}$;
(c) $\mathbf{v}_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2$;
$\mathbf{v}_{2f} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2$

P9.56 291 N

P9.58 $\left(\dfrac{M+m}{m}\right)\sqrt{\dfrac{gd^2}{2h}}$

P9.60 (a) -0.667 m/s; (b) 0.952 m

P9.62 (a) 6.81 m/s; (b) 1.00 m

P9.64 (a) -3.54 m/s; (b) 1.77 m; (c) 35.4 kN;
(d) No. The rails exert a vertical force to change the momentum

P9.66 0.312 N to the right

P9.68 0.179 m/s

P9.70 (a) 3.7 km/s; (b) 153 km

P9.72 (a) 3.75 N to the right; (b) 3.75 N to the right; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J;
(f) Friction between sand and belt converts half of the input work into extra internal energy.

10

Rotation of a Rigid Object About a Fixed Axis

ANSWERS TO QUESTIONS

Q10.1 1 rev/min, or $\frac{\pi}{30}$ rad/s. Into the wall (clockwise rotation). $\alpha = 0$.

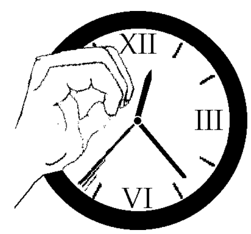

FIG. Q10.1

Q10.2 $+\hat{\mathbf{k}}$, $-\hat{\mathbf{k}}$

Q10.3 Yes, they are valid provided that ω is measured in degrees per second and α is measured in degrees per second-squared.

Q10.4 The speedometer will be inaccurate. The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

Q10.5 Smallest I is about x axis and largest I is about y axis.

Q10.6 The moment of inertia would no longer be $\frac{ML^2}{12}$ if the mass was nonuniformly distributed, nor could it be calculated if the mass distribution was not known.

Q10.7 The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

Q10.8 No horizontal force acts on the pencil, so its center of mass moves straight down.

Q10.9 You could measure the time that it takes the hanging object, m, to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.

Q10.10 You could use $\omega = \alpha t$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.

Q10.11 The angular speed ω would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

Q10.12 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In example 10.6 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.

Q10.13 Compared to an axis through the center of mass, any other parallel axis will have larger average squared distance from the axis to the particles of which the object is composed.

Q10.14 A quick flip will set the hard–boiled egg spinning faster and more smoothly. The raw egg loses mechanical energy to internal fluid friction.

Q10.15 $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = MR^2$, $I_{\text{CM}} = \frac{1}{3}MR^2$, $I_{\text{CM}} = \frac{1}{2}MR^2$

Q10.16 Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.

Q10.17 No, just as an object need not be moving to have mass.

Q10.18 No, only if its angular momentum changes.

Q10.19 Yes. Consider a pendulum at its greatest excursion from equilibrium. It is momentarily at rest, but must have an angular acceleration or it would not oscillate.

Q10.20 Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2. Thus, on the whole the moment of inertia is proportional to R^4. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

Q10.21 The moment of inertia would decrease. This would result in a higher angular speed of the earth, shorter days, and more days in the year!

Q10.22 There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of "rolling"—and so no force of kinetic friction acts to reduce *K*. Air resistance and friction associated with deformation of the ball eventually stop the ball.

Q10.23 In the frame of reference of the ground, no. Every point moves perpendicular to the line joining it to the instantaneous contact point. The contact point is not moving at all. The leading and trailing edges of the cylinder have velocities at 45° to the vertical as shown.

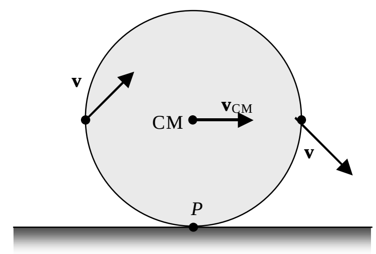

FIG. Q10.23

Q10.24 The sphere would reach the bottom first; the hoop would reach the bottom last. If each object has the same mass and the same radius, they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first.

Q10.25 To win the race, you want to decrease the moment of inertia of the wheels as much as possible. Small, light, solid disk-like wheels would be best!

SOLUTIONS TO PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

P10.1 (a)

$$\theta\big|_{t=0} = \boxed{5.00 \text{ rad}}$$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b)

$$\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$$

$$\omega\big|_{t=3.00\text{ s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{ s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

Section 10.2 **Rotational Kinematics: Rotational Motion with Constant Angular Acceleration**

***P10.2** $\omega_f = 2.51\times10^4 \text{ rev/min} = 2.63\times10^3 \text{ rad/s}$

(a) $$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{2.63\times10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.22\times10^2 \text{ rad/s}^2}$$

(b) $$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(8.22\times10^2 \text{ rad/s}^2\right)(3.2 \text{ s})^2 = \boxed{4.21\times10^3 \text{ rad}}$$

P10.3 (a) $$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(4.00 \text{ rad/s}^2\right)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$$

P10.4 $\omega_i = 2\,000 \text{ rad/s}$, $\alpha = -80.0 \text{ rad/s}^2$

(a) $$\omega_f = \omega_i + \alpha t = 2\,000 - (80.0)(10.0) = \boxed{1\,200 \text{ rad/s}}$$

(b) $$0 = \omega_i + \alpha t$$

$$t = \frac{\omega_i}{-\alpha} = \frac{2\,000}{80.0} = \boxed{25.0 \text{ s}}$$

P10.5 $$\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \ \omega_f = 0$$

(a) $$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - \frac{10\pi}{3}}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$$

(b) $$\theta_f = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$$

P10.6 $\omega_i = 3\,600 \text{ rev/min} = 3.77\times10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14\times10^2 \text{ rad}$ and $\omega_f = 0$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$0 = \left(3.77\times10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14\times10^2 \text{ rad}\right)$$

$$\alpha = \boxed{-2.26\times10^2 \text{ rad/s}^2}$$

P10.7 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $$\theta_1 = \bar{\omega} t = \frac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$$

While slowing down, $$\theta_2 = \bar{\omega} t = \frac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$$

So, $$\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$$

P10.8 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$

$$37.0\ \text{rev}\left(\frac{2\pi\,\text{rad}}{1\ \text{rev}}\right) = 98.0\ \text{rad/s}(3.00\ \text{s}) - \frac{1}{2}\alpha(3.00\ \text{s})^2$$

$232\ \text{rad} = 294\ \text{rad} - (4.50\ \text{s}^2)\alpha$: $\quad \alpha = \frac{61.5\ \text{rad}}{4.50\ \text{s}^2} = \boxed{13.7\ \text{rad/s}^2}$

P10.9 (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1\ \text{rev}}{1\ \text{day}} = \frac{2\pi\,\text{rad}}{86\,400\ \text{s}} = \boxed{7.27\times10^{-5}\ \text{rad/s}}$

(b) $\Delta t = \frac{\Delta\theta}{\omega} = \frac{107°}{7.27\times10^{-5}\ \text{rad/s}}\left(\frac{2\pi\,\text{rad}}{360°}\right) = \boxed{2.57\times10^4\ \text{s}}$ or 428 min

***P10.10** The location of the dog is described by $\theta_d = (0.750\ \text{rad/s})t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi\,\text{rad} + \frac{1}{2}0.015\ \text{rad/s}^2\,t^2.$$

We look for a solution to

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)2.09}}{0.015} = 2.88\ \text{s or } 97.1\ \text{s}$$

The dog and bone will also pass if $0.75t = \frac{2\pi}{3} - 2\pi + 0.007\,5t^2$ or if $0.75t = \frac{2\pi}{3} + 2\pi + 0.007\,5t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)(-4.19)}}{0.015} = 105\ \text{s or } -5.30\ \text{s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8\ \text{s or } 87.2\ \text{s}.$$

In order, the dog passes the bone at $\boxed{2.88\ \text{s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8\ \text{s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

Section 10.3 **Angular and Linear Quantities**

P10.11 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00\times 10^{4}\ \text{mi}}{0.250\ \text{m}}\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right) = 6.44\times 10^{7}\ \text{rad/yr}$$

$$\theta = 6.44\times 10^{7}\ \text{rad/yr}\left(\frac{1\ \text{rev}}{2\pi\,\text{rad}}\right) = 1.02\times 10^{7}\ \text{rev/yr or } \boxed{\sim 10^{7}\ \text{rev/yr}}$$

P10.12 (a) $v = r\omega;\ \omega = \dfrac{v}{r} = \dfrac{45.0\ \text{m/s}}{250\ \text{m}} = \boxed{0.180\ \text{rad/s}}$

(b) $a_r = \dfrac{v^2}{r} = \dfrac{(45.0\ \text{m/s})^2}{250\ \text{m}} = \boxed{8.10\ \text{m/s}^2\ \text{toward the center of track}}$

P10.13 Given $r = 1.00\ \text{m}$, $\alpha = 4.00\ \text{rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3° = 1.00\ \text{rad}$

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00\ \text{s}$, $\omega_f = 4.00\ \text{rad/s}^2\,(2.00\ \text{s}) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = 1.00\ \text{m}(8.00\ \text{rad/s}) = \boxed{8.00\ \text{m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00\ \text{m}(8.00\ \text{rad/s})^2 = 64.0\ \text{m/s}^2$

$a_t = r\alpha = 1.00\ \text{m}\left(4.00\ \text{rad/s}^2\right) = 4.00\ \text{m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(64.0\ \text{m/s}^2\right)^2 + \left(4.00\ \text{m/s}^2\right)^2} = \boxed{64.1\ \text{m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point P:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \dfrac{1}{2}\alpha t^2 = (1.00\ \text{rad}) + \dfrac{1}{2}\left(4.00\ \text{rad/s}^2\right)(2.00\ \text{s})^2 = \boxed{9.00\ \text{rad}}$

***P10.14** (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\text{ m}}{2}\right)76\text{ rev/min}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.605\text{ m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\text{ m/s}}{\left(\frac{0.07\text{ m}}{2}\right)} = \boxed{17.3\text{ rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\text{ m}}{2}\right)17.3\text{ rad/s} = \boxed{5.82\text{ m/s}}$$

(d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175\text{ m }7.96\text{ rad/s}\left(\frac{1}{1\text{ rad}}\right) = 1.39\text{ m/s}$$

P10.15 (a) $\omega = \frac{v}{r} = \frac{25.0\text{ m/s}}{1.00\text{ m}} = \boxed{25.0\text{ rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\text{ rad/s})^2 - 0}{2\left[(1.25\text{ rev})(2\pi\text{ rad/rev})\right]} = \boxed{39.8\text{ rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\text{ rad/s}}{39.8\text{ rad/s}^2} = \boxed{0.628\text{ s}}$

P10.16 (a) $s = \bar{v}t = (11.0\text{ m/s})(9.00\text{ s}) = 99.0\text{ m}$

$$\theta = \frac{s}{r} = \frac{99.0\text{ m}}{0.290\text{ m}} = 341\text{ rad} = \boxed{54.3\text{ rev}}$$

(b) $\omega_f = \frac{v_f}{r} = \frac{22.0\text{ m/s}}{0.290\text{ m}} = 75.9\text{ rad/s} = \boxed{12.1\text{ rev/s}}$

P10.17 (a) $\omega = 2\pi f = \dfrac{2\pi \text{ rad}}{1 \text{ rev}}\left(\dfrac{1\,200 \text{ rev}}{60.0 \text{ s}}\right) = \boxed{126 \text{ rad/s}}$

(b) $v = \omega r = (126 \text{ rad/s})\left(3.00\times10^{-2} \text{ m}\right) = \boxed{3.77 \text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2\left(8.00\times10^{-2}\right) = 1\,260 \text{ m/s}^2$ so $\mathbf{a}_r = \boxed{1.26 \text{ km/s}^2 \text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126 \text{ rad/s})\left(8.00\times10^{-2} \text{ m}\right)(2.00 \text{ s}) = \boxed{20.1 \text{ m}}$

P10.18 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m\left(1.70 \text{ m/s}^2\right)$. Its radially inward component is $\dfrac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr\left(\omega_i^2 + 2\alpha\Delta\theta\right) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi\left(1.70 \text{ m/s}^2\right).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2\left(1.70 \text{ m/s}^2\right)^2 + m^2\pi^2\left(1.70 \text{ m/s}^2\right)^2}$$

$$\mu_s = \frac{1.70 \text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

***P10.19** (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \dfrac{1}{2}gt^2$, $t = \sqrt{\dfrac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = (v_2 - v_1)t = \omega h\sqrt{\dfrac{2h}{g}} = \boxed{\omega h^{3/2}\left(\dfrac{2}{g}\right)^{1/2}}$.

(b) $\dfrac{2\pi \text{ rad}}{86\,400 \text{ s}}(50 \text{ m})^{3/2}\left(\dfrac{2 \text{ s}^2}{9.8 \text{ m}}\right)^{1/2} = \boxed{1.16 \text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

Section 10.4 **Rotational Energy**

P10.20 $m_1 = 4.00\text{ kg},\ r_1 = |y_1| = 3.00\text{ m};$

$m_2 = 2.00\text{ kg},\ r_2 = |y_2| = 2.00\text{ m};$

$m_3 = 3.00\text{ kg},\ r_3 = |y_3| = 4.00\text{ m};$

$\omega = 2.00\text{ rad/s}$ about the x-axis

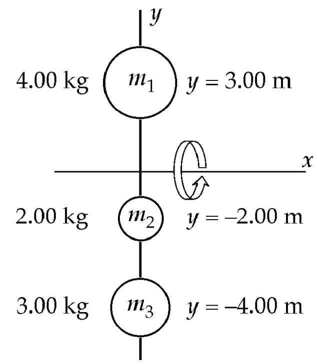

FIG. P10.20

(a) $$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b) $$v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}} \qquad K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$$

$$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}} \qquad K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$$

$$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}} \qquad K_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

P10.21 (a) $$I = \sum_j m_j r_j^2$$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

$$I = \left[\sqrt{13.0}\text{ m}\right]^2\left[3.00 + 2.00 + 2.00 + 4.00\right]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

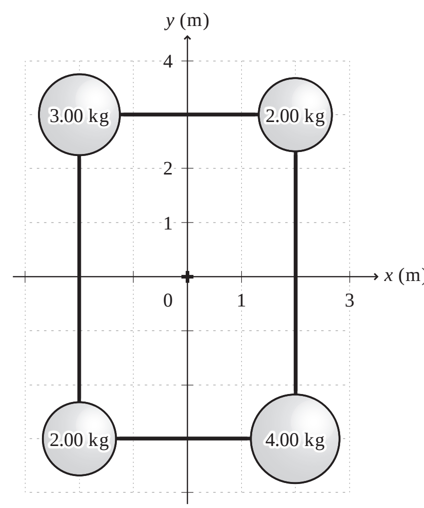

FIG. P10.21

(b) $$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)\left(6.00\text{ rad/s}\right)^2$$

$$= \boxed{2.57\times10^3\text{ J}}$$

P10.22 $I = Mx^2 + m(L-x)^2$

$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0$ (for an extremum)

$\therefore x = \frac{mL}{M+m}$

$\frac{d^2I}{dx^2} = 2m + 2M$; therefore I is minimum when the axis of rotation passes through $x = \frac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{CM} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2 = \mu L^2$$

where $\mu = \frac{Mm}{M+m}$.

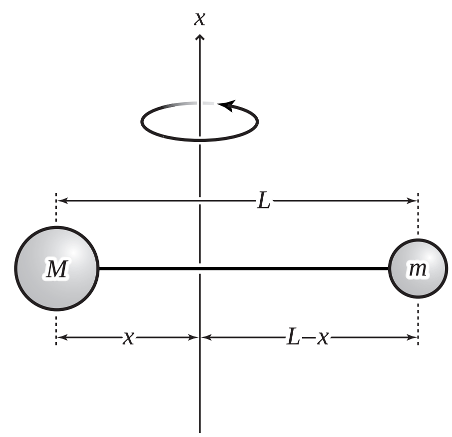

FIG. P10.22

Section 10.5 Calculation of Moments of Inertia

P10.23 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.

For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

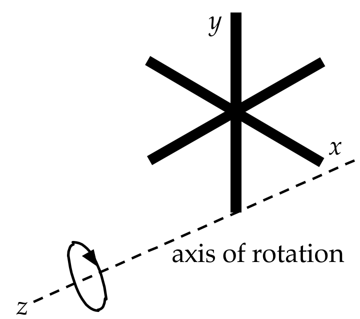

FIG. P10.23

In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\Big|_{-L/2}^{L/2} + \frac{mL}{4}x\Big|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$.

P10.24 Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m\left(R_1^2 + R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi\left[(0.305\text{ m})^2 - (0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^{3}\text{ kg/m}^3\right) = 1.44\text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2 + (0.305\text{ m})^2\right] = 8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m = \pi\left[(0.330\text{ m})^2 - (0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^{3}\text{ kg/m}^3\right) = 11.0\text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2 + (0.305\text{ m})^2\right] = 1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right) + 1.11\text{ kg}\cdot\text{m}^2 = \boxed{1.28\text{ kg}\cdot\text{m}^2}$$

P10.25 Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}.$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.26 Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi} = 0.120\text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg}\cdot\text{m}^2 \sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}.$$

P10.27 For a spherical shell $dI = \frac{2}{3}dmr^2 = \frac{2}{3}\left[\left(4\pi r^2 dr\right)\rho\right]r^2$

$$I = \int dI = \int \frac{2}{3}\left(4\pi r^2\right)r^2\rho(r)dr$$

$$I = \int_0^R \frac{2}{3}\left(4\pi r^4\right)\left(14.2 - 11.6\frac{r}{R}\right)\left(10^3 \text{ kg/m}^3\right)dr$$

$$= \left(\frac{2}{3}\right)4\pi\left(14.2\times10^3\right)\frac{R^5}{5} - \left(\frac{2}{3}\right)4\pi\left(11.6\times10^3\right)\frac{R^5}{6}$$

$$I = \frac{8\pi}{3}\left(10^3\right)R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)$$

$$M = \int dm = \int_0^R 4\pi r^2\left(14.2 - 11.6\frac{r}{R}\right)10^3\,dr$$

$$= 4\pi\times10^3\left(\frac{14.2}{3} - \frac{11.6}{4}\right)R^3$$

$$\frac{I}{MR^2} = \frac{(8\pi/3)\left(10^3\right)R^5(14.2/5 - 11.6/6)}{4\pi\times10^3 R^3 R^2(14.2/3 - 11.6/4)} = \frac{2}{3}\left(\frac{.907}{1.83}\right) = 0.330$$

$$\therefore I = \boxed{0.330MR^2}$$

***P10.28** (a) By similar triangles, $\frac{y}{x} = \frac{h}{L}$, $y = \frac{hx}{L}$. The area of the front face is $\frac{1}{2}hL$. The volume of the plate is $\frac{1}{2}hLw$. Its density is $\rho = \frac{M}{V} = \frac{M}{\frac{1}{2}hLw} = \frac{2M}{hLw}$. The mass of the ribbon is

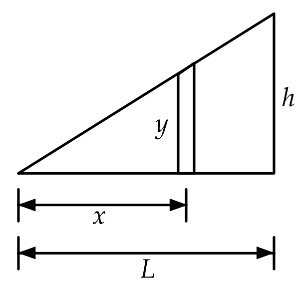

FIG. P10.28

$$dm = \rho dV = \rho ywdx = \frac{2Mywdx}{hLw} = \frac{2Mhx}{hLL}dx = \frac{2Mxdx}{L^2}.$$

The moment of inertia is

$$I = \int_{\text{all mass}} r^2 dm = \int_{x=0}^{L} x^2\frac{2Mxdx}{L^2} = \frac{2M}{L^2}\int_0^L x^3dx = \frac{2M}{L^2}\frac{L^4}{4} = \boxed{\frac{ML^2}{2}}.$$

(b) From the parallel axis theorem $I = I_{\text{CM}} + M\left(\frac{2L}{3}\right)^2 = I_{\text{CM}} + \frac{4ML^2}{9}$ and $I_h = I_{\text{CM}} + M\left(\frac{L}{3}\right)^2 = I_{\text{CM}} + \frac{ML^2}{9}$. The two triangles constitute a rectangle with moment of inertia $I_{\text{CM}} + \frac{4ML^2}{9} + I_{\text{CM}} + \frac{ML^2}{9} = \frac{1}{3}(2M)L^2$. Then $2I_{\text{CM}} = \frac{1}{9}ML^2$

$$I = I_{\text{CM}} + \frac{4ML^2}{9} = \frac{1}{18}ML^2 + \frac{8}{18}ML^2 = \boxed{\frac{1}{2}ML^2}.$$

***P10.29** We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad M_0 = \frac{4}{3}M.$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{CM} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2.$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and } K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}.$$

Section 10.6 **Torque**

P10.30 Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{par} = (100\text{ N})\cos 57.0° = 54.5\text{ N}$$

and $F_{perp} = (100\text{ N})\sin 57.0° = 83.9\text{ N}$

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is $\tau = 83.9\text{ N}(2.00\text{ m}) = \boxed{168\text{ N}\cdot\text{m}}$ (clockwise)

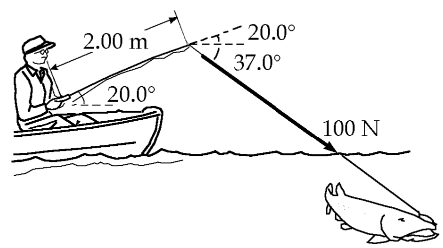

FIG. P10.30

P10.31 $\sum\tau = 0.100\text{ m}(12.0\text{ N}) - 0.250\text{ m}(9.00\text{ N}) - 0.250\text{ m}(10.0\text{ N}) = \boxed{-3.55\text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

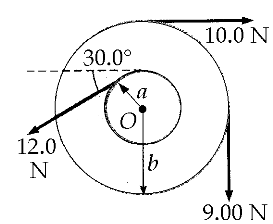

FIG. P10.31

P10.32 The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1\,500\text{ kg})(9.80\text{ m/s}^2)}{4} = 3\,680\text{ N}$$

The torque of friction can be as large as

$$\tau_{max} = f_{max}r = (\mu_s n)r = (0.800)(3\,680\text{ N})(0.300\text{ m}) = \boxed{882\text{ N}\cdot\text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

P10.33 In the previous problem we calculated the maximum torque that can be applied without skidding to be 882 N · m. This same torque is to be applied by the frictional force, *f*, between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r\text{, so } n = \frac{\tau}{\mu_k r} = \frac{882\text{ N}\cdot\text{m}}{(0.500)(0.220\text{ m})} = 8.02\times10^3\text{ N} = \boxed{8.02\text{ kN}}$$

Section 10.7 Relationship Between Torque and Angular Acceleration

P10.34 (a)

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00\text{ kg})\left(7.00\times10^{-2}\text{ m}\right)^2 = 4.90\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122\text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1\,200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03\text{ s}}$$

(b)

$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122\text{ rad/s})(1.03\text{ s})^2 = 64.7\text{ rad} = \boxed{10.3\text{ rev}}$$

P10.35 $m = 0.750\text{ kg}$, $F = 0.800\text{ N}$

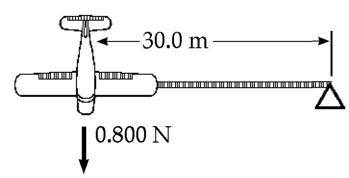

FIG. P10.35

(a) $\tau = rF = 30.0\text{ m}(0.800\text{ N}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \dfrac{\tau}{I} = \dfrac{rF}{mr^2} = \dfrac{24.0}{0.750(30.0)^2} = \boxed{0.035\,6\text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.035\,6(30.0) = \boxed{1.07\text{ m/s}^2}$

P10.36 $\omega_f = \omega_i + \alpha t$:

$$10.0\text{ rad/s} = 0 + \alpha(6.00\text{ s})$$

$$\alpha = \frac{10.00}{6.00}\text{ rad/s}^2 = 1.67\text{ rad/s}^2$$

(a) $\sum\tau = 36.0\text{ N}\cdot\text{m} = I\alpha$: $\quad I = \dfrac{\sum\tau}{\alpha} = \dfrac{36.0\text{ N}\cdot\text{m}}{1.67\text{ rad/s}^2} = \boxed{21.6\text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$:

$$0 = 10.0 + \alpha(60.0)$$

$$\alpha = -0.167\text{ rad/s}^2$$

$$|\tau| = |I\alpha| = \left(21.6\text{ kg}\cdot\text{m}^2\right)\left(0.167\text{ rad/s}^2\right) = \boxed{3.60\text{ N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\quad \theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1\text{ rad}$

During next 60.0 s $\quad \theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299\text{ rad}$

$$\theta_{\text{total}} = 329\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{52.4\text{ rev}}$$

P10.37 For m_1,

$$\sum F_y = ma_y: \quad +n - m_1 g = 0$$
$$n_1 = m_1 g = 19.6 \text{ N}$$
$$f_{k1} = \mu_k n_1 = 7.06 \text{ N}$$
$$\sum F_x = ma_x: \quad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \qquad (1)$$

For the pulley,

$$\sum \tau = I\alpha: \quad -T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$
$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})a$$
$$-T_1 + T_2 = (5.00 \text{ kg})a \qquad (2)$$

For m_2, $$+n_2 - m_2 g\cos\theta = 0$$
$$n_2 = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)(\cos 30.0°)$$
$$= 50.9 \text{ N}$$

$$f_{k2} = \mu_k n_2$$
$$= 18.3 \text{ N}: \quad -18.3 \text{ N} - T_2 + m_2 \sin\theta = m_2 a$$
$$-18.3 \text{ N} - T_2 + 29.4 \text{ N} = (6.00 \text{ kg})a \qquad (3)$$

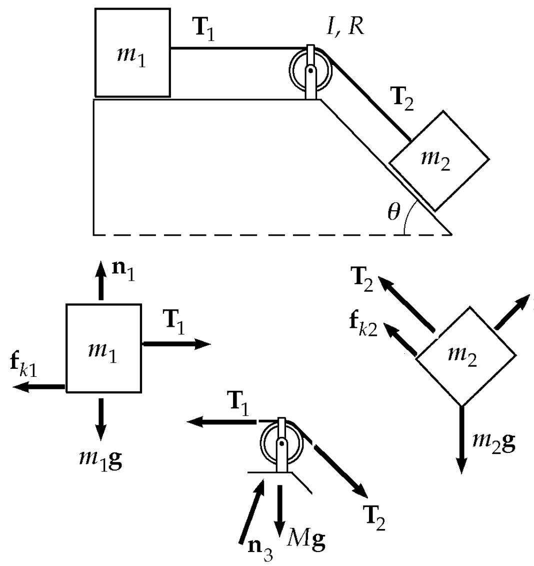

FIG. P10.37

(a) Add equations (1), (2), and (3):

$$-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} = (13.0 \text{ kg})a$$
$$a = \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}$$

(b) $$T_1 = 2.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$$
$$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) = \boxed{9.22 \text{ N}}$$

P10.38 $$I = \frac{1}{2}mR^2 = \frac{1}{2}(100 \text{ kg})(0.500 \text{ m})^2 = 12.5 \text{ kg}\cdot\text{m}^2$$
$$\omega_i = 50.0 \text{ rev/min} = 5.24 \text{ rad/s}$$
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24 \text{ rad/s}}{6.00 \text{ s}} = -0.873 \text{ rad/s}^2$$
$$\tau = I\alpha = 12.5 \text{ kg}\cdot\text{m}^2\left(-0.873 \text{ rad/s}^2\right) = -10.9 \text{ N}\cdot\text{m}$$

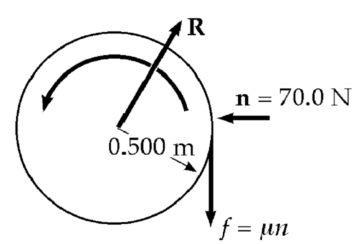

FIG. P10.38

The magnitude of the torque is given by $fR = 10.9 \text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore, $f = \dfrac{10.9 \text{ N}\cdot\text{m}}{0.500 \text{ m}}$ and $f = \mu_k n$

yields $$\mu_k = \frac{f}{n} = \frac{21.8 \text{ N}}{70.0 \text{ N}} = \boxed{0.312}$$

***P10.39** $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\text{ N}(0.230\text{ m}) + T(0.230\text{ m}) = \frac{1}{2}(80\text{ kg})\left(\frac{1.25}{2}\text{ m}\right)^2\left(-1.67\text{ rad/s}^2\right)$$

$T = \boxed{21.5\text{ N}}$

Section 10.8 **Work, Power, and Energy in Rotational Motion**

P10.40 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with $I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Therefore, $K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.04\times10^{-3}\text{ J}}$

***P10.41** The power output of the bus is $\mathcal{P} = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \mathcal{P}\Delta t = \frac{\mathcal{P}\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\mathcal{P}} = \frac{1\,600\text{ kg}(0.65\text{ m})^2\left(4\,000\cdot\frac{2\pi}{60\text{ s}}\right)^2 11.1\text{ m/s}}{4(18\cdot746\text{ W})} = \boxed{24.5\text{ km}}.$$

P10.42 Work done $= F\Delta r = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$

and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}\left(4.00\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\omega_f^2$

and from this, $\omega_f = \boxed{149\text{ rad/s}}$.

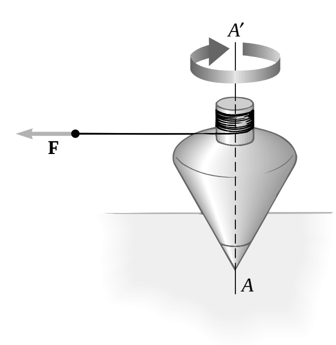

FIG. P10.42

***P10.43** (a) $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}(0.35\text{ kg})\left[(0.02\text{ m})^2 + (0.03\text{ m})^2\right] = 2.28 \times 10^{-4}\text{ kg}\cdot\text{m}^2$

$$\left(K_1 + K_2 + K_{\text{rot}} + U_{g2}\right)_i - f_k\Delta x = \left(K_1 + K_2 + K_{\text{rot}}\right)_f$$

$$\frac{1}{2}(0.850\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}(0.42\text{ kg})(0.82\text{ m/s})^2 + \frac{1}{2}\left(2.28 \times 10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{0.82\text{ m/s}}{0.03\text{ m}}\right)^2$$

$$+0.42\text{ kg}\left(9.8\text{ m/s}^2\right)(0.7\text{ m}) - 0.25(0.85\text{ kg})\left(9.8\text{ m/s}^2\right)(0.7\text{ m})$$

$$= \frac{1}{2}(0.85\text{ kg})v_f^2 + \frac{1}{2}(0.42\text{ kg})v_f^2 + \frac{1}{2}\left(2.28 \times 10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{v_f}{0.03\text{ m}}\right)^2$$

$$0.512\text{ J} + 2.88\text{ J} - 1.46\text{ J} = (0.761\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{1.94\text{ J}}{0.761\text{ kg}}} = \boxed{1.59\text{ m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\text{ m/s}}{0.03\text{ m}} = \boxed{53.1\text{ rad/s}}$

P10.44 We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$I = \frac{1}{3}1.20\text{ kg}(0.240\text{ m})^2 + \frac{2}{5}2.00\text{ kg}\left(4.00 \times 10^{-2}\text{ m}\right)^2 + 2.00\text{ kg}(0.280\text{ m})^2$$

$$I = 0.181\text{ kg}\cdot\text{m}^2$$

(a) $K_f + U_f = K_i + U_i + \Delta E$

$$\frac{1}{2}I\omega^2 + 0 = 0 + M_{\text{rod}}g\left(\frac{L}{2}\right) + M_{\text{ball}}g(L + R) + 0$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = 1.20\text{ kg}\left(9.80\text{ m/s}^2\right)(0.120\text{ m}) + 2.00\text{ kg}\left(9.80\text{ m/s}^2\right)(0.280\text{ m})$$

$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2 = \boxed{6.90\text{ J}}$$

(b) $\omega = \boxed{8.73\text{ rad/s}}$

(c) $v = r\omega = (0.280\text{ m})8.73\text{ rad/s} = \boxed{2.44\text{ m/s}}$

(d) $v_f^2 = v_i^2 + 2a\left(y_f - y_i\right)$

$$v_f = \sqrt{0 + 2\left(9.80\text{ m/s}^2\right)(0.280\text{ m})} = 2.34\text{ m/s}$$

The speed it attains in swinging is greater by $\frac{2.44}{2.34} = \boxed{1.043\,2\text{ times}}$

P10.45 (a) For the counterweight,

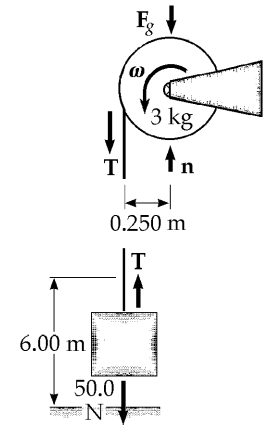

FIG. P10.45

$\sum F_y = ma_y$ becomes: $$50.0 - T = \left(\frac{50.0}{9.80}\right)a$$

For the reel $\sum \tau = I\alpha$ reads $TR = I\alpha = I\dfrac{a}{R}$

where $$I = \frac{1}{2}MR^2 = 0.093\,8 \text{ kg}\cdot\text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4 \text{ N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57 \text{ m/s}^2}$$

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$: $$v_f = \sqrt{2(7.57)6.00} = \boxed{9.53 \text{ m/s}}$$

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0 \text{ N})(6.00 \text{ m})}{5.10 \text{ kg} + \frac{0.093\,8}{(0.250)^2}}} = \boxed{9.53 \text{ m/s}}$$

P10.46 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 = 0 + m_1gh_{1i} + m_2gh_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5 \text{ kg})v^2 = 73.5 \text{ J} \Rightarrow v = \boxed{2.36 \text{ m/s}}$$

P10.47 From conservation of energy for the object-turntable-cylinder-Earth system,

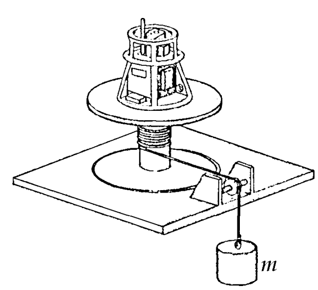

FIG. P10.47

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

P10.48 The moment of inertia of the cylinder is

$$I = \frac{1}{2}mr^2 = \frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha = \frac{\tau}{I} = \frac{(Fr)}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{\left(91.8\text{ kg}\cdot\text{m}^2\right)} = 0.817\text{ rad/s}^2.$$

At $t = 3.00$ s, we find the angular velocity

$$\omega = \omega_i + \alpha t$$

$$\omega = 0 + \left(0.817\text{ rad/s}^2\right)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(91.8\text{ kg}\cdot\text{m}^2\right)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$.

P10.49 (a) Find the velocity of the CM

$$(K+U)_i = (K+U)_f$$

$$0 + mgR = \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{\text{CM}} = R\sqrt{\frac{4g}{3R}} = \boxed{2\sqrt{\frac{Rg}{3}}}$$

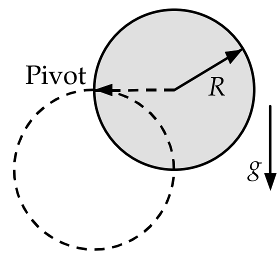

FIG. P10.49

(b) $v_L = 2v_{\text{CM}} = \boxed{4\sqrt{\frac{Rg}{3}}}$

(c) $v_{\text{CM}} = \sqrt{\frac{2mgR}{2m}} = \boxed{\sqrt{Rg}}$

***P10.50** (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}0.1\text{ kg}\left((0.015\text{ m})^2 + (0.09\text{ m})^2\right) = 4.16\times 10^{-4}\text{ kg}\cdot\text{m}^2.$$

The protruding strand has mass $\left(10^{-2}\text{ kg/m}\right)0.16\text{ m} = 1.6\times 10^{-3}\text{ kg}$ and

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = 1.6\times 10^{-3}\text{ kg}\left(\frac{1}{12}(0.16\text{ m})^2 + (0.09\text{ m} + 0.08\text{ m})^2\right)$$

$$= 4.97\times 10^{-5}\text{ kg}\cdot\text{m}^2$$

For the whole cord, $I = 4.66\times 10^{-4}\text{ kg}\cdot\text{m}^2$. In speeding up, the average power is

$$\mathcal{P} = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \frac{4.66\times 10^{-4}\text{ kg}\cdot\text{m}^2}{2(0.215\text{ s})}\left(\frac{2\,500\cdot 2\pi}{60\text{ s}}\right)^2 = \boxed{74.3\text{ W}}$$

(b) $\mathcal{P} = \tau\omega = (7.65\text{ N})(0.16\text{ m} + 0.09\text{ m})\left(\frac{2\,000\cdot 2\pi}{60\text{ s}}\right) = \boxed{401\text{ W}}$

Section 10.9 **Rolling Motion of a Rigid Object**

P10.51 (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

P10.52 $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{\left(\frac{7}{10}\right)Mv^2}$

P10.53 (a)

$$\tau = I\alpha$$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{\text{CM}} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{\text{disk}} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

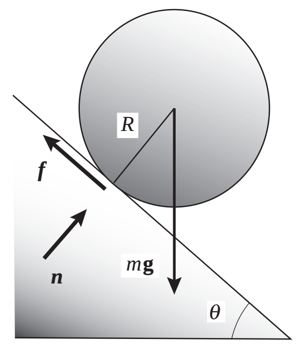

FIG. P10.53

(b)

$$Rf = I\alpha$$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{\frac{I\alpha}{R}}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.54 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{\left[1 + \left(\frac{I}{mR^2}\right)\right]}$

For a disk, $I = \frac{1}{2}mR^2$

So $v^2 = \frac{2gh}{1 + \frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.55 $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00\text{ m}}{1.50\text{ s}} = 2.00\ \text{m/s} = \frac{1}{2}\left(0 + v_f\right)$

$$v_f = 4.00\ \text{m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00\ \text{m/s}}{\left(6.38\times10^{-2}\ \text{m}\right)/2} = \frac{8.00}{6.38\times10^{-2}}\ \text{rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$\left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_i + \Delta E_{\text{mech}} = \left(K_{\text{trans}} + K_{\text{rot}} + U_g\right)_f$$

$$\left(0 + 0 + mgy_i\right) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215\text{ kg}\left(9.80\ \text{m/s}^2\right)\left[(3.00\text{ m})\sin 25.0°\right] = \frac{1}{2}(0.215\text{ kg})(4.00\ \text{m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38\times10^{-2}}\ \text{rad/s}\right)^2$$

$$2.67\text{ J} = 1.72\text{ J} + \left(7\,860\text{ s}^{-2}\right)t$$

$$I = \frac{0.951\text{ kg}\cdot\text{m}^2/\text{s}^2}{7\,860\text{ s}^{-2}} = \boxed{1.21\times10^{-4}\ \text{kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

P10.56 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

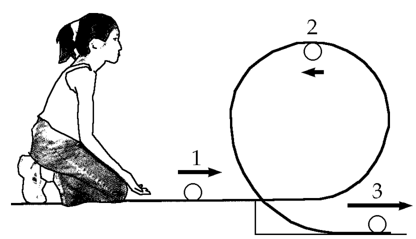

FIG. P10.56

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2$$

$$= \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03\ \text{m/s})^2 - \frac{6}{5}\left(9.80\ \text{m/s}^2\right)(0.900\text{ m})} = \boxed{2.38\ \text{m/s}}$$

The centripetal acceleration is $\frac{v_2^2}{r} = \frac{(2.38\ \text{m/s})^2}{0.450\text{ m}} = 12.6\ \text{m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03\ \text{m/s})^2 - \frac{6}{5}\left(9.80\ \text{m/s}^2\right)(-0.200\text{ m})} = \boxed{4.31\ \text{m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\ \text{m/s})^2 - 2\left(9.80\ \text{m/s}^2\right)(0.900\text{ m})} = \sqrt{-1.40\ \text{m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

Additional Problems

P10.57 $mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$

$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta$$

$$a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

Then $\left(\frac{3}{2}\frac{g}{\ell}\right)r > g\sin\theta$

for $r > \frac{2}{3}\ell$

$\therefore$ About $\boxed{\frac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

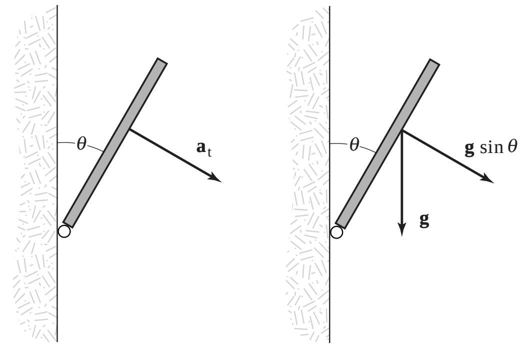

FIG. P10.57

P10.58 The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.
The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

With $$\omega = \frac{2\pi\,\text{rad}}{1\text{ rev}}\left(\frac{10^3\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \frac{1\,000\pi}{30.0}\text{ rad/s}$$

$$\mathcal{P} = 3(0.100\text{ m})^3(0.600)\left(4.00\times10^{-4}\text{ m}^2\right)\left(\frac{1\,000\pi}{30.0\text{ s}}\right)^3\rho$$

or $\mathcal{P} = \left(0.827\text{ m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\text{ kg/m}^3$,
and $\mathcal{P} = 0.827\text{ m}^5/\text{s}^3\left(1.20\text{ kg/m}^3\right) = 0.992\text{ N}\cdot\text{m/s} = \boxed{0.992\text{ W}}$

(b) In water, $\rho = 1\,000\text{ kg/m}^3$ and $\mathcal{P} = \boxed{827\text{ W}}$.

P10.59 (a) $$W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right) \quad \text{where} \quad I = \frac{1}{2}mR^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) $$t = \frac{\omega_f - 0}{\alpha} = \frac{\omega r}{a} = \frac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$$

(c) $$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2;\ \theta_i = 0;\ \omega_i = 0$$

$$\theta_f = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$$

$$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes}}$$

***P10.60** The quantity of tape is constant. Then the area of the rings you see it fill is constant. This is expressed by

$\pi r_t^2 - \pi r_s^2 = \pi r^2 - \pi r_s^2 + \pi r_2^2 - \pi r_s^2$ or $r_2 = \sqrt{r_t^2 + r_s^2 - r^2}$ is the outer radius of spool 2.

(a) Where the tape comes off spool 1, $\omega_1 = \dfrac{v}{r}$. Where the tape joins spool 2, $\omega_2 = \dfrac{v}{r_2} = v\left(r_s^2 + r_t^2 - r^2\right)^{-1/2}$.

(b) At the start, $r = r_t$ and $r_2 = r_s$ so $\omega_1 = \dfrac{v}{r_t}$ and $\omega_2 = \dfrac{v}{r_s}$. The takeup reel must spin at maximum speed. At the end, $r = r_s$ and $r_2 = r_t$ so $\omega_2 = \dfrac{v}{r_t}$ and $\omega_1 = \dfrac{v}{r_s}$. The angular speeds are just reversed.

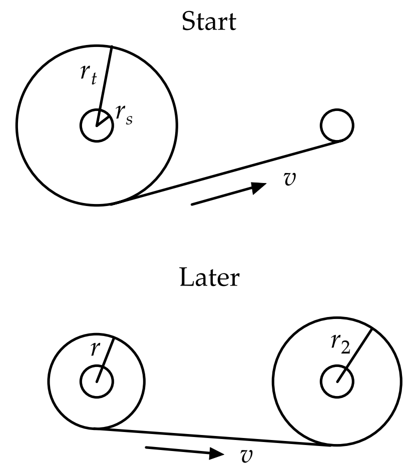

FIG. P10.60

P10.61 (a) Since only conservative forces act within the system of the rod and the Earth,

$\Delta E = 0$ so $K_f + U_f = K_i + U_i$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where $I = \dfrac{1}{3}ML^2$

Therefore, $\omega = \boxed{\sqrt{\dfrac{3g}{L}}}$

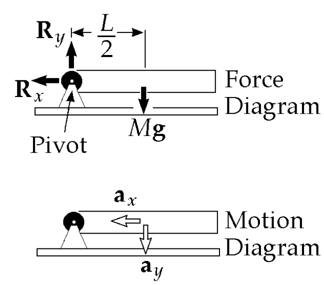

FIG. P10.61

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\alpha = \boxed{\frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\dfrac{L}{2}\right)\omega^2 = \boxed{-\dfrac{3g}{2}}$ $\quad a_y = -a_t = -r\alpha = -\alpha\left(\dfrac{L}{2}\right) = \boxed{-\dfrac{3g}{4}}$

(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$R_y - Mg = Ma_y = -\dfrac{3Mg}{4}$ $\qquad R_y = \boxed{\dfrac{Mg}{4}}$

P10.62 $\alpha = -10.0\ \text{rad/s}^2 - \left(5.00\ \text{rad/s}^3\right)t = \dfrac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt = -10.0t - 2.50t^2 = \omega - 65.0\ \text{rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2$$

(a) At $t = 3.00\ \text{s}$,

$$\omega = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)(3.00\ \text{s}) - \left(2.50\ \text{rad/s}^3\right)\left(9.00\ \text{s}^2\right) = \boxed{12.5\ \text{rad/s}}$$

(b) $$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t \left[65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2\right]dt$$

$$\theta = (65.0\ \text{rad/s})t - \left(5.00\ \text{rad/s}^2\right)t^2 - \left(0.833\ \text{rad/s}^3\right)t^3$$

At $t = 3.00\ \text{s}$,

$$\theta = (65.0\ \text{rad/s})(3.00\ \text{s}) - \left(5.00\ \text{rad/s}^2\right)9.00\ \text{s}^2 - \left(0.833\ \text{rad/s}^3\right)27.0\ \text{s}^3$$

$$\theta = \boxed{128\ \text{rad}}$$

P10.63 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.540\ \text{m})} = 3.25\ \text{m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.510\ \text{m})} = 3.16\ \text{m/s}$$

From $\omega = \dfrac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\ \text{m/s}}{0.381\ \text{m}} = 8.53\ \text{rad/s} \text{ and } \omega_2 = \frac{v_2}{r} = \frac{3.16\ \text{m/s}}{0.381\ \text{m}} = 8.29\ \text{rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\ \text{rad/s})^2 - (8.53\ \text{rad/s})^2}{4\pi} = \boxed{-0.322\ \text{rad/s}^2}$$

P10.64 At the instant it comes off the wheel, the first drop has a velocity v_1, directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}.$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$.

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{\frac{2gh_2}{R^2} - \frac{2gh_1}{R^2}}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}.$$

P10.65 $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$: $U_f = Mgh_f = 0$; $K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$

$U_i = (Mgh)_i$: $f = \mu N = \mu Mg\cos\theta$; $\omega = \frac{v}{r}$; $h = d\sin\theta$ and $I = \frac{1}{2}mr^2$

(a) $$\Delta E = E_f - E_i \text{ or } -fd = K_f + U_f - K_i - U_i$$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{\frac{v^2}{r^2}}{2} - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\frac{(\sin\theta - \mu\cos\theta)}{\frac{m}{2} + M}$$

$$v_d = \left[4gd\frac{M}{(m+2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b) $$v_f^2 = v_i^2 + 2a\Delta x, \; v_d^2 = 2ad$$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m+2M}\right)(\sin\theta - \mu\cos\theta)}$$

P10.66 (a) $E=\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\omega^2\right)$

$$E=\frac{1}{2}\cdot\frac{2}{5}\left(5.98\times10^{24}\right)\left(6.37\times10^{6}\right)^2\left(\frac{2\pi}{86\ 400}\right)^2=\boxed{2.57\times10^{29}\ \text{J}}$$

(b)
$$\frac{dE}{dt}=\frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$
$$=\frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt}$$
$$=\frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$
$$=\left(2.57\times10^{29}\ \text{J}\right)\left(\frac{-2}{86\ 400\ \text{s}}\right)\left(\frac{10\times10^{-6}\ \text{s}}{3.16\times10^{7}\ \text{s}}\right)(86\ 400\ \text{s/day})$$
$$\frac{dE}{dt}=\boxed{-1.63\times10^{17}\ \text{J/day}}$$

***P10.67** (a) $\omega_f=\omega_i+\alpha t$

$$\alpha=\frac{\omega_f-\omega_i}{t}=\frac{\frac{2\pi}{T_f}-\frac{2\pi}{T_i}}{t}=\frac{2\pi\left(T_i-T_f\right)}{T_iT_ft}$$
$$\sim\frac{2\pi\left(-10^{-3}\ \text{s}\right)}{1\ \text{d}\ 1\ \text{d}\ 100\ \text{yr}}\left(\frac{1\ \text{d}}{86\ 400\ \text{s}}\right)^2\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)=\boxed{-10^{-22}\ \text{s}^{-2}}$$

(b) The Earth, assumed uniform, has moment of inertia

$$I=\frac{2}{5}MR^2=\frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.37\times10^{6}\ \text{m}\right)^2=9.71\times10^{37}\ \text{kg}\cdot\text{m}^2$$
$$\sum\tau=I\alpha\sim9.71\times10^{37}\ \text{kg}\cdot\text{m}^2\left(-2.67\times10^{-22}\ \text{s}^{-2}\right)=\boxed{-10^{16}\ \text{N}\cdot\text{m}}$$

The negative sign indicates clockwise, to slow the planet's counterclockwise rotation.

(c) $|\tau|=Fd$. Suppose the person can exert a 900-N force.

$$d=\frac{|\tau|}{F}=\frac{2.59\times10^{16}\ \text{N}\cdot\text{m}}{900\ \text{N}}\sim\boxed{10^{13}\ \text{m}}$$

This is the order of magnitude of the size of the planetary system.

P10.68 $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{\left(\frac{31.0°}{360°}\right)\text{ rev}}{\frac{900\text{ rev}}{60\text{ s}}} = 0.005\,74\text{ s}$$

$$v = \frac{0.800\text{ m}}{0.005\,74\text{ s}} = \boxed{139\text{ m/s}}$$

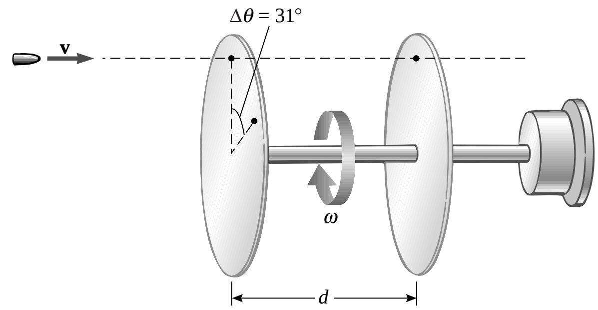

FIG. P10.68

P10.69 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \qquad (2)$$

also $\Delta y = v_i t + \dfrac{at^2}{2}$ $\qquad a = \dfrac{2y}{t^2} \qquad (3)$

and $\qquad \alpha = \dfrac{a}{R} = \dfrac{2y}{Rt^2}: \qquad (4)$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

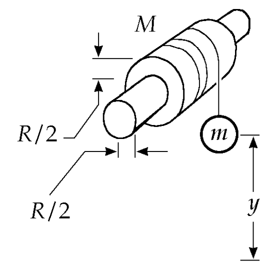

FIG. P10.69

Substituting (2), (3), (4), and (5) into (1),

we find $\qquad \tau_f = m\left(g - \dfrac{2y}{t^2}\right)R - \dfrac{5}{8}\dfrac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \dfrac{2y}{t^2}\right) - \dfrac{5}{4}\dfrac{My}{t^2}\right]}$

P10.70 (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

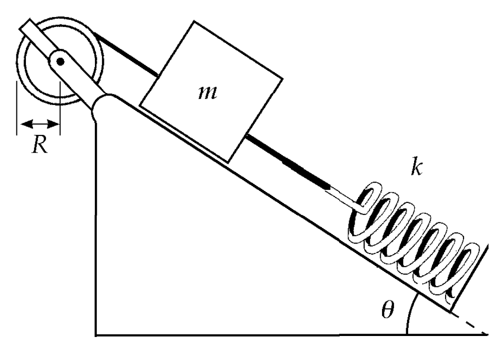

FIG. P10.70

(b) $$\omega = \sqrt{\frac{2(0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m})(\sin 37.0°) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

P10.71 (a) $m_2 g - T_2 = m_2 a$

$T_2 = m_2(g-a) = 20.0\text{ kg}\left(9.80\text{ m/s}^2 - 2.00\text{ m/s}^2\right) = \boxed{156\text{ N}}$

$T_1 - m_1 g\sin 37.0° = m_1 a$

$T_1 = (15.0\text{ kg})(9.80\sin 37.0°+2.00)\text{ m/s}^2 = \boxed{118\text{ N}}$

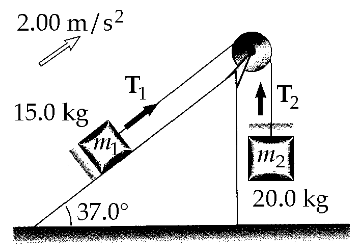

FIG. P10.71

(b) $(T_2 - T_1)R = I\alpha = I\left(\dfrac{a}{R}\right)$

$$I = \frac{(T_2-T_1)R^2}{a} = \frac{(156\text{ N} - 118\text{ N})(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

P10.72 For the board just starting to move,

$$\sum \tau = I\alpha: \qquad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

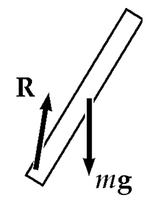

FIG. P10.72

The tangential acceleration of the end is $a_t = \ell\alpha = \dfrac{3}{2}g\cos\theta$

The vertical component is $a_y = a_t\cos\theta = \dfrac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball falling:

(a) $\dfrac{3}{2}g\cos^2\theta \ge g$ gives $\cos^2\theta \ge \dfrac{2}{3}$ so $\cos\theta \ge \sqrt{\dfrac{2}{3}}$ and $\boxed{\theta \le 35.3°}$

(b) When $\theta = 35.3°$, the cup will land underneath the release-point of the ball if $r_c = \ell\cos\theta$

When $\ell = 1.00\text{ m}$, and $\theta = 35.3°$ $r_c = 1.00\text{ m}\sqrt{\dfrac{2}{3}} = 0.816\text{ m}$

so the cup should be $(1.00\text{ m} - 0.816\text{ m}) = \boxed{0.184\text{ m from the moving end}}$

P10.73 At $t = 0$, $\omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50\text{ rad/s}$

At $t = 9.30\text{ s}$, $\omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02\times10^{-2}\text{ s}^{-1}$

(a) $$\alpha = \frac{d\omega}{dt} = \frac{d\left(\omega_0 e^{-\sigma t}\right)}{dt} = \omega_0(-\sigma)e^{-\sigma t}$$

At $t = 3.00\text{ s}$,

$$\alpha = (3.50\text{ rad/s})\left(-6.02\times10^{-2}\text{ s}^{-1}\right)e^{-3.00\left(6.02\times10^{-2}\right)} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $$\theta = \int_0^t \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}\left[e^{-\sigma t} - 1\right] = \frac{\omega_0}{\sigma}\left[1 - e^{-\sigma t}\right]$$

At $t = 2.50\text{ s}$,

$$\theta = \frac{3.50\text{ rad/s}}{\left(6.02\times10^{-2}\right)1/\text{s}}\left[1 - e^{-\left(6.02\times10^{-2}\right)(2.50)}\right] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c) As $t\to\infty$, $\theta \to \dfrac{\omega_0}{\sigma}\left(1 - e^{-\infty}\right) = \dfrac{3.50\text{ rad/s}}{6.02\times10^{-2}\text{ s}^{-1}} = 58.2\text{ rad} = \boxed{9.26\text{ rev}}$

P10.74 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t,$$

where $\omega_h = \dfrac{\pi}{6}$ rad/h

and $\theta_m = \omega_m t,$

where $\omega_m = 2\pi$ rad/h

Therefore, $\tau = -4.90\ \text{m/s}^2\left[60.0\ \text{kg}(2.70\ \text{m})\sin\left(\frac{\pi t}{6}\right) + 100\ \text{kg}(4.50\ \text{m})\sin 2\pi t\right]$

or $\tau = -794\ \text{N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right]$, where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h,

so $\tau = -794\ \text{N}\cdot\text{m}\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794\ \text{N}\cdot\text{m}}$

(ii) At 5:15, $t = 5\ \text{h} + \dfrac{15}{60}\ \text{h} = 5.25\ \text{h}$, and substitution gives:

$\tau = \boxed{-2\,510\ \text{N}\cdot\text{m}}$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2\,940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

***P10.75** (a) As the bicycle frame moves forward at speed v, the center of each wheel moves forward at the same speed and the wheels turn at angular speed $\omega = \frac{v}{R}$. The total kinetic energy of the bicycle is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}\left(m_{\text{frame}} + 2m_{\text{wheel}}\right)v^2 + 2\left(\frac{1}{2}I_{\text{wheel}}\omega^2\right) = \frac{1}{2}\left(m_{\text{frame}} + 2m_{\text{wheel}}\right)v^2 + \left(\frac{1}{2}m_{\text{wheel}}R^2\right)\left(\frac{v^2}{R^2}\right).$$

This yields

$$K = \frac{1}{2}\left(m_{\text{frame}} + 3m_{\text{wheel}}\right)v^2 = \frac{1}{2}\left[8.44\text{ kg} + 3(0.820\text{ kg})\right](3.35\text{ m/s})^2 = \boxed{61.2\text{ J}}.$$

(b) As the block moves forward with speed v, the top of each trunk moves forward at the same speed and the center of each trunk moves forward at speed $\frac{v}{2}$. The angular speed of each roller is $\omega = \frac{v}{2R}$. As in part (a), we have one object undergoing pure translation and two identical objects rolling without slipping. The total kinetic energy of the system of the stone and the trees is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}m_{\text{stone}}v^2 + 2\frac{1}{2}m_{\text{tree}}\left(\frac{v}{2}\right)^2 + 2\left(\frac{1}{2}I_{\text{tree}}\omega^2\right) = \frac{1}{2}\left(m_{\text{stone}} + \frac{1}{2}m_{\text{tree}}\right)v^2 + \left(\frac{1}{2}m_{\text{tree}}R^2\right)\left(\frac{v^2}{4R^2}\right).$$

This gives

$$K = \frac{1}{2}\left(m_{\text{stone}} + \frac{3}{4}m_{\text{tree}}\right)v^2 = \frac{1}{2}\left[844\text{ kg} + 0.75(82.0\text{ kg})\right](0.335\text{ m/s})^2 = \boxed{50.8\text{ J}}.$$

P10.76 Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

$$mg(R-r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10}{7}\frac{(R-r)(1-\cos\theta)g}{r^2}}$$

or $\omega = \boxed{\sqrt{\frac{10Rg(1-\cos\theta)}{7r^2}}}$ since $R >> r$.

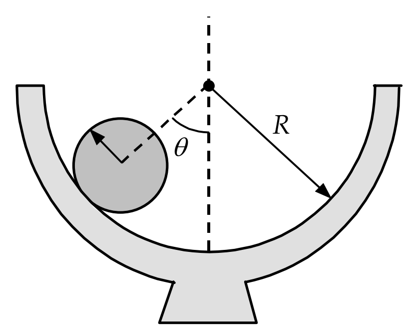

FIG. P10.76

P10.77 $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

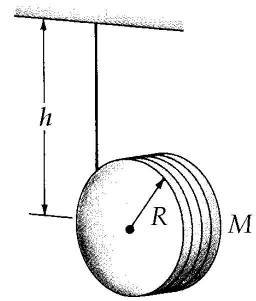

FIG. P10.77

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus $T = \boxed{\frac{Mg}{3}}$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\qquad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}: \quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f{}^2$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.78 (a) $\sum F_x = F - f = Ma$: $\sum \tau = fR = I\alpha$

Using $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$, we find $a = \boxed{\frac{2F}{3M}}$

(b) When there is no slipping, $f = \mu Mg$.

Substituting this into the torque equation of part (a), we have

$$\mu MgR = \frac{1}{2}MRa \text{ and } \mu = \boxed{\frac{F}{3Mg}}.$$

P10.79 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

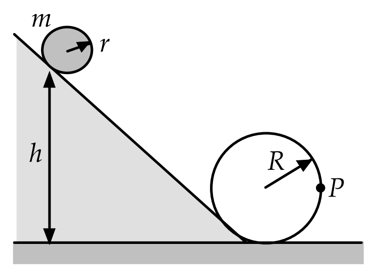

FIG. P10.79

Note that initially the center of mass of the sphere is a distance $h+r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R-r$. The conservation of energy requirement gives

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\text{min}}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\text{min}} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\text{min}} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point P, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2\text{, or}$$

$$v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point P, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{\left(\frac{10}{7}\right)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R >> r\text{)}$$

P10.80 Consider the free-body diagram shown. The sum of torques about the chosen pivot is

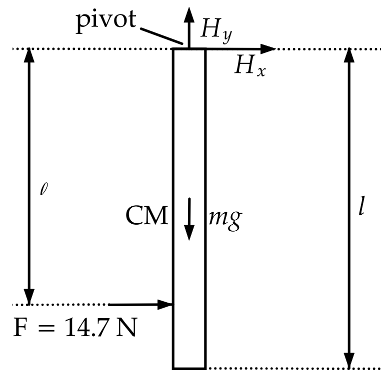

FIG. P10.80

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{CM}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{CM} \qquad (1)$$

(a) $\ell = l = 1.24\ \text{m}$: In this case, Equation (1) becomes

$$a_{CM} = \frac{3F}{2m} = \frac{3(14.7\ \text{N})}{2(0.630\ \text{kg})} = 35.0\ \text{m/s}^2$$

$\sum F_x = ma_{CM} \Rightarrow F + H_x = ma_{CM}$ or $H_x = ma_{CM} - F$

Thus, $H_x = (0.630\ \text{kg})(35.0\ \text{m/s}^2) - 14.7\ \text{N} = +7.35\ \text{N}$ or

$\mathbf{H}_x = \boxed{7.35\hat{\mathbf{i}}\ \text{N}}$.

(b) $\ell = \frac{1}{2} = 0.620\ \text{m}$: For this situation, Equation (1) yields

$$a_{CM} = \frac{3F}{4m} = \frac{3(14.7\ \text{N})}{4(0.630\ \text{kg})} = 17.5\ \text{m/s}^2.$$

Again, $\sum F_x = ma_{CM} \Rightarrow H_x = ma_{CM} - F$, so

$$H_x = (0.630\ \text{kg})(17.5\ \text{m/s}^2) - 14.7\ \text{N} = -3.68\ \text{N} \text{ or } \mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}}\ \text{N}}.$$

(c) If $H_x = 0$, then $\sum F_x = ma_{CM} \Rightarrow F = ma_{CM}$, or $a_{CM} = \frac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24\ \text{m}) = \boxed{0.827\ \text{m (from the top)}}.$$

P10.81 Let the ball have mass m and radius r. Then $I = \frac{2}{5}mr^2$. If the ball takes four seconds to go down twenty-meter alley, then $\bar{v} = 5\ \text{m/s}$. The translational speed of the ball will decrease somewhat as the ball loses energy to sliding friction and some translational kinetic energy is converted to rotational kinetic energy; but its speed will always be on the order of $5.00\ \text{m/s}$, including at the starting point.

As the ball slides, the kinetic friction force exerts a torque on the ball to increase the angular speed. When $\omega = \frac{v}{r}$, the ball has achieved pure rolling motion, and kinetic friction ceases. To determine the elapsed time before pure rolling motion is achieved, consider:

$$\sum \tau = I\alpha \Rightarrow (\mu_k mg)r = \left(\frac{2}{5}mr^2\right)\left[\frac{(5.00\ \text{m/s})/r}{t}\right] \text{ which gives}$$

$$t = \frac{2(5.00\ \text{m/s})}{5\mu_k g} = \frac{2.00\ \text{m/s}}{\mu_k g}$$

Note that the mass and radius of the ball have canceled. If $\mu_k = 0.100$ for the polished alley, the sliding distance will be given by

$$\Delta x = \bar{v}t = (5.00\ \text{m/s})\left[\frac{2.00\ \text{m/s}}{(0.100)(9.80\ \text{m/s}^2)}\right] = 10.2\ \text{m or } \Delta x \sim \boxed{10^1\ \text{m}}.$$

P10.82 Conservation of energy between apex and the point where the grape leaves the surface:

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}.$$

At the point where the grape leaves the surface, $n \to 0$.

Thus, $mg\cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g\cos\theta$.

Substituting this into Equation (1) gives

$g - g\cos\theta = \frac{7}{10}g\cos\theta$ or $\cos\theta = \frac{10}{17}$ and $\theta = \boxed{54.0°}$.

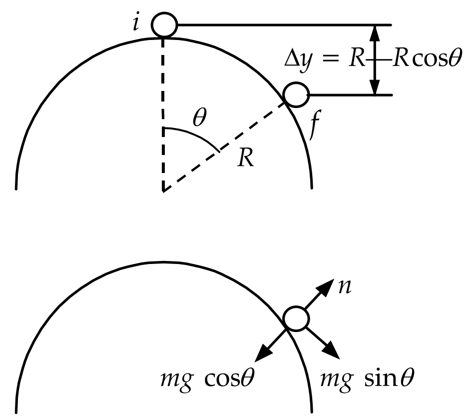

FIG. P10.82

P10.83 (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{\text{CM}}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{\text{CM}}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{\text{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.84 (a) The mass of the roll decreases as it unrolls. We have $m = \frac{Mr^2}{R^2}$ where M is the initial mass of the roll. Since $\Delta E = 0$, we then have $\Delta U_g + \Delta K_{\text{trans}} + \Delta K_{\text{rot}} = 0$. Thus, when $I = \frac{mr^2}{2}$,

$$(mgr - MgR) + \frac{mv^2}{2} + \left[\frac{mr^2}{2}\frac{\omega^2}{2}\right] = 0$$

Since $\omega r = v$, this becomes $v = \boxed{\sqrt{\frac{4g\left(R^3 - r^3\right)}{3r^2}}}$

(b) Using the given data, we find $v = \boxed{5.31 \times 10^4 \text{ m/s}}$

(c) We have assumed that $\Delta E = 0$. When the roll gets to the end, we will have an inelastic collision with the surface. $\boxed{\text{The energy goes into internal energy}}$. With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr \to 0$. Also, as r approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

P10.85 (a) $\sum F_x = F + f = Ma_{\text{CM}}$

$\sum \tau = FR - fR = I\alpha$

$FR - \left(Ma_{\text{CM}} - F\right)R = \frac{Ia_{\text{CM}}}{R}$ $\qquad \boxed{a_{\text{CM}} = \frac{4F}{3M}}$

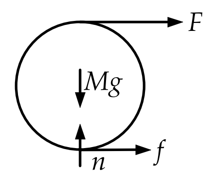

FIG. P10.85

(b) $f = Ma_{\text{CM}} - F = M\left(\frac{4F}{3M}\right) - F = \boxed{\frac{1}{3}F}$

(c) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$v_f = \boxed{\sqrt{\frac{8Fd}{3M}}}$

P10.86 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

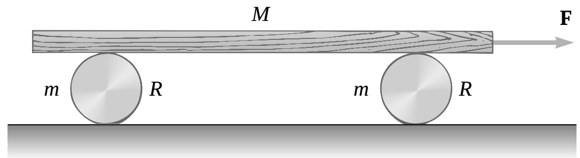

FIG. P10.86

For the plank,

$$\sum F_x = ma_x \qquad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\text{ cm})} = \frac{a_p}{(0.100\text{ m})}$$

Then for each,

$$\sum F_x = ma_x \qquad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2\frac{a_p}{10.0\text{ cm}}$$

So $\quad f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p$

Add to eliminate f_b:

$$2f_t = (1.50\text{ kg})a_p$$

(a) And $6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$

$$a_p = \frac{(6.00\text{ N})}{(7.50\text{ kg})} = \boxed{0.800\text{ m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(b) Substituting back, $2f_t = (1.50\text{ kg})0.800\text{ m/s}^2$

$$f_t = \boxed{0.600\text{ N}}$$

$$0.600\text{ N} + f_b = \frac{1}{2}\text{ kg}\left(0.800\text{ m/s}^2\right)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{0.200\text{ N forward}}$ rather than backward as we assumed.

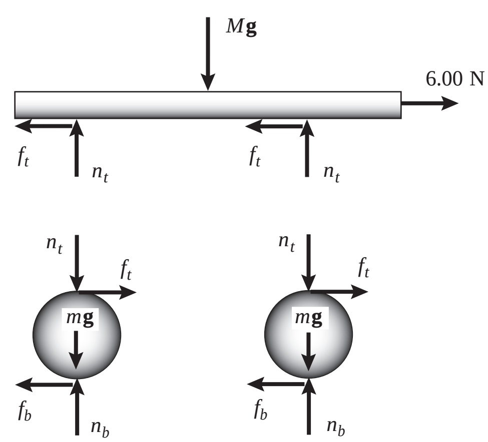

FIG. P10.86(b)

P10.87 Rolling is instantaneous rotation about the contact point P. The weight and normal force produce no torque about this point.

Now F_1 produces $\boxed{\text{a clockwise torque}}$ about P and makes the spool roll forward.

$\boxed{\text{Counterclockwise torques}}$ result from F_3 and F_4, making the spool roll to the left.

The force F_2 produces $\boxed{\text{zero torque}}$ about point P and does not cause the spool to roll. If F_2 were strong enough, it would cause the spool to slide to the right, but not roll.

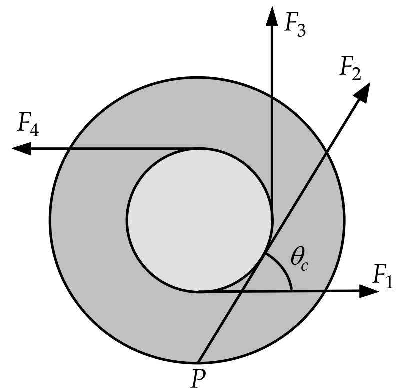

FIG. P10.87

P10.88 The force applied at the critical angle exerts zero torque about the spool's contact point with the ground and so will not make the spool roll.

From the right triangle shown in the sketch, observe that $\theta_c = 90° - \phi = 90° - (90° - \gamma) = \gamma$.

Thus, $\boxed{\cos\theta_c = \cos\gamma = \frac{r}{R}}$.

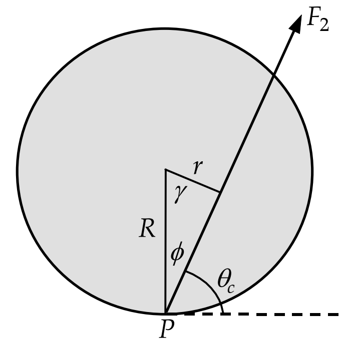

FIG. P10.88

P10.89 (a) Consider motion starting from rest over distance x along the incline:

$$(K_{\text{trans}} + K_{\text{rot}} + U)_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}} + U)_f$$

$$0 + 0 + Mgx\sin\theta + 0 = \frac{1}{2}Mv^2 + 2\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + 0$$

$$2Mgx\sin\theta = (M+2m)v^2$$

Since acceleration is constant,

$$v^2 = v_i^2 + 2ax = 0 + 2ax, \text{ so}$$

$$2Mgx\sin\theta = (M+2m)2ax$$

$$a = \boxed{\frac{Mg\sin\theta}{(M+2m)}}$$

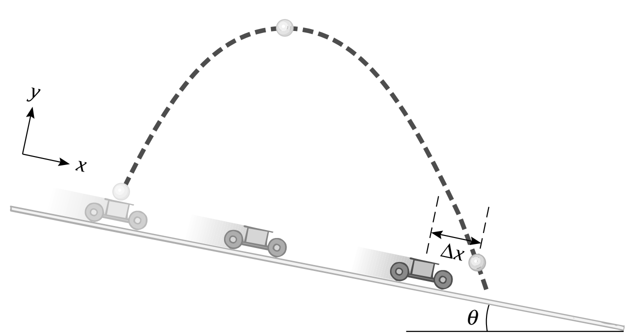

FIG. P10.88

continued on next page

(c) Suppose the ball is fired from a cart at rest. It moves with acceleration $g\sin\theta = a_x$ down the incline and $a_y = -g\cos\theta$ perpendicular to the incline. For its range along the ramp, we have

$$y - y_i = v_{yi}t - \frac{1}{2}g\cos\theta t^2 = 0 - 0$$

$$t = \frac{2v_{yi}}{g\cos\theta}$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d = 0 + \frac{1}{2}g\sin\theta\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d = \boxed{\frac{2v_{yi}^2\sin\theta}{g\cos^2\theta}}$$

(b) In the same time the cart moves

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d_c = 0 + \frac{1}{2}\left(\frac{g\sin\theta M}{(M+2m)}\right)\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d_c = \frac{2v_{yi}^2\sin\theta M}{g(M+2m)\cos^2\theta}$$

So the ball overshoots the cart by

$$\Delta x = d - d_c = \frac{2v_{yi}^2\sin\theta}{g\cos^2\theta} - \frac{2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \frac{2v_{yi}^2\sin\theta M + 4v_{yi}^2\sin\theta m - 2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \boxed{\frac{4mv_{yi}^2\sin\theta}{(M+2m)g\cos^2\theta}}$$

P10.90 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,

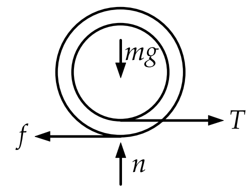

FIG. P10.90

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$
$$fR_2^2 m - TR_1 R_2 m = IT - If$$
$$f\left(I + mR_2^2\right) = T\left(I + mR_1 R_2\right)$$
$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right)T$$

Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left.}}$

ANSWERS TO EVEN PROBLEMS

P10.2 (a) 822 rad/s^2; (b) $4.21 \times 10^3 \text{ rad}$

P10.4 (a) $1.20 \times 10^2 \text{ rad/s}$; (b) 25.0 s

P10.6 -226 rad/s^2

P10.8 13.7 rad/s^2

P10.10 (a) 2.88 s; (b) 12.8 s

P10.12 (a) 0.180 rad/s;
(b) 8.10 m/s^2 toward the center of the track

P10.14 (a) 0.605 m/s; (b) 17.3 rad/s; (c) 5.82 m/s;
(d) The crank length is unnecessary

P10.16 (a) 54.3 rev; (b) 12.1 rev/s

P10.18 0.572

P10.20 (a) $92.0 \text{ kg}\cdot\text{m}^2$; 184 J;
(b) 6.00 m/s; 4.00 m/s; 8.00 m/s; 184 J

P10.22 see the solution

P10.24 $1.28 \text{ kg}\cdot\text{m}^2$

P10.26 $\sim 10^0 \text{ kg}\cdot\text{m}^2$

P10.28 $\frac{1}{2}ML^2$

P10.30 $168 \text{ N}\cdot\text{m}$ clockwise

P10.32 $882 \text{ N}\cdot\text{m}$

P10.34 (a) 1.03 s; (b) 10.3 rev

P10.36 (a) $21.6 \text{ kg}\cdot\text{m}^2$; (b) $3.60 \text{ N}\cdot\text{m}$; (c) 52.4 rev

P10.38 0.312

P10.40 $1.04 \times 10^{-3} \text{ J}$

P10.42 149 rad/s

P10.44 (a) 6.90 J; (b) 8.73 rad/s; (c) 2.44 m/s;
(d) 1.043 2 times larger

P10.46 2.36 m/s

P10.48 276 J

P10.50 (a) 74.3 W; (b) 401 W

P10.52 $\frac{7Mv^2}{10}$

P10.54 The disk; $\sqrt{\frac{4gh}{3}}$ versus $\sqrt{gh}$

P10.56 (a) $2.38\ \text{m/s}$; (b) $4.31\ \text{m/s}$;
(c) It will not reach the top of the loop.

P10.58 (a) 0.992 W; (b) 827 W

P10.60 see the solution

P10.62 (a) $12.5\ \text{rad/s}$; (b) 128 rad

P10.64 $\dfrac{g(h_2 - h_1)}{2\pi R^2}$

P10.66 (a) $2.57 \times 10^{29}\ \text{J}$; (b) $-1.63 \times 10^{17}\ \text{J/day}$

P10.68 $139\ \text{m/s}$

P10.70 (a) $\sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}$; (b) $1.74\ \text{rad/s}$

P10.72 see the solution

P10.74 (a) $-794\ \text{N}\cdot\text{m}$; $-2\,510\ \text{N}\cdot\text{m}$; 0;
$-1\,160\ \text{N}\cdot\text{m}$; $-2\,940\ \text{N}\cdot\text{m}$;
(b) see the solution

P10.76 $\sqrt{\dfrac{10Rg(1-\cos\theta)}{7r^2}}$

P10.78 see the solution

P10.80 (a) $35.0\ \text{m/s}^2$; $7.35\hat{\mathbf{i}}\ \text{N}$;
(b) $17.5\ \text{m/s}^2$; $-3.68\hat{\mathbf{i}}\ \text{N}$;
(c) At 0.827 m from the top.

P10.82 54.0°

P10.84 (a) $\sqrt{\dfrac{4g(R^3 - r^3)}{3r^2}}$; (b) $5.31 \times 10^4\ \text{m/s}$;
(c) It becomes internal energy.

P10.86 (a) $0.800\ \text{m/s}^2$; $0.400\ \text{m/s}^2$;
(b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground

P10.88 see the solution

P10.90 see the solution; to the left

11

Angular Momentum

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q11.1 No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis.

Q11.2 $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$ is a scalar quantity, since $(\mathbf{B}\times\mathbf{C})$ is a vector. Since $\mathbf{A}\cdot\mathbf{B}$ is a scalar, and the cross product between a scalar and a vector is not defined, $(\mathbf{A}\cdot\mathbf{B})\times\mathbf{C}$ is undefined.

Q11.3 (a) Down–cross–left is away from you: $-\hat{\mathbf{j}}\times\left(-\hat{\mathbf{i}}\right)=-\hat{\mathbf{k}}$

(b) Left–cross–down is toward you: $-\hat{\mathbf{i}}\times\left(-\hat{\mathbf{j}}\right)=\hat{\mathbf{k}}$

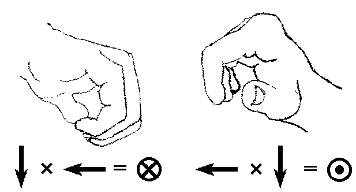

FIG. Q11.3

Q11.4 The torque about the point of application of the force is zero.

Q11.5 You cannot conclude anything about the magnitude of the angular momentum vector without first defining your axis of rotation. Its direction will be perpendicular to its velocity, but you cannot tell its direction in three-dimensional space until an axis is specified.

Q11.6 Yes. If the particles are moving in a straight line, then the angular momentum of the particles about any point on the path is zero.

Q11.7 Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

Q11.8 No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

Q11.9 There must be two rotors to balance the torques on the body of the helicopter. If it had only one rotor, the engine would cause the body of the helicopter to swing around rapidly with angular momentum opposite to the rotor.

Q11.10 The angular momentum of the particle about the center of rotation is constant. The angular momentum about any point that does not lie along the axis through the center of rotation and perpendicular to the plane of motion of the particle is not constant in time.

Q11.11 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

Q11.12 The diver leaves the platform with some angular momentum about a horizontal axis through her center of mass. When she draws up her legs, her moment of inertia decreases and her angular speed increases for conservation of angular momentum. Straightening out again slows her rotation.

Q11.13 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

Q11.14 The angular speed must increase. Since gravity does not exert a torque on the system, its angular momentum remains constant as the gas contracts.

Q11.15 Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases.

Q11.16 The turntable will rotate counterclockwise. Since the angular momentum of the mouse-turntable system is initially zero, as both are at rest, the turntable must rotate in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero.

Q11.17 Since the cat cannot apply an external torque to itself while falling, its angular momentum cannot change. Twisting in this manner changes the orientation of the cat to feet-down without changing the total angular momentum of the cat. Unfortunately, humans aren't flexible enough to accomplish this feat.

Q11.18 The angular speed of the ball must increase. Since the angular momentum of the ball is constant, as the radius decreases, the angular speed must increase.

Q11.19 Rotating the book about the axis that runs across the middle pages perpendicular to the binding—most likely where you put the rubber band—is the one that has the intermediate moment of inertia and gives unstable rotation.

Q11.20 The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

SOLUTIONS TO PROBLEMS

Section 11.1 **The Vector Product and Torque**

P11.1 $\mathbf{M}\times\mathbf{N}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6 & 2 & -1\\ 2 & -1 & -3\end{vmatrix}=\boxed{-7.00\hat{\mathbf{i}}+16.0\hat{\mathbf{j}}-10.0\hat{\mathbf{k}}}$

P11.2 (a) $\text{area}=|\mathbf{A}\times\mathbf{B}|=AB\sin\theta=(42.0\text{ cm})(23.0\text{ cm})\sin(65.0°-15.0°)=\boxed{740\text{ cm}^2}$

(b) $\mathbf{A}+\mathbf{B}=\left[(42.0\text{ cm})\cos 15.0°+(23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}+\left[(42.0\text{ cm})\sin 15.0°+(23.0\text{ cm})\sin 65.0°\right]\hat{\mathbf{j}}$

$\mathbf{A}+\mathbf{B}=(50.3\text{ cm})\hat{\mathbf{i}}+(31.7\text{ cm})\hat{\mathbf{j}}$

$\text{length}=|\mathbf{A}+\mathbf{B}|=\sqrt{(50.3\text{ cm})^2+(31.7\text{ cm})^2}=\boxed{59.5\text{ cm}}$

P11.3 (a) $\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3 & 4 & 0\\ 2 & 3 & 0\end{vmatrix}=\boxed{-17.0\hat{\mathbf{k}}}$

(b) $|\mathbf{A}\times\mathbf{B}|=|\mathbf{A}||\mathbf{B}|\sin\theta$

$17=5\sqrt{13}\sin\theta$

$\theta=\arcsin\left(\frac{17}{5\sqrt{13}}\right)=\boxed{70.6°}$

P11.4 $\mathbf{A}\cdot\mathbf{B}=-3.00(6.00)+7.00(-10.0)+(-4.00)(9.00)=-124$

$AB=\sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2}=127$

(a) $\cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)=\cos^{-1}(-0.979)=\boxed{168°}$

(b) $\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3.00 & 7.00 & -4.00\\ 6.00 & -10.0 & 9.00\end{vmatrix}=23.0\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-12.0\hat{\mathbf{k}}$

$|\mathbf{A}\times\mathbf{B}|=\sqrt{(23.0)^2+(3.00)^2+(-12.0)^2}=26.1$

$\sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right)=\sin^{-1}(0.206)=\boxed{11.9°}$ or 168°

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

***P11.5** $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = 0.450\ \text{m}(0.785\ \text{N})\sin(90°-14°)\ \text{up} \times \text{east}$

$= \boxed{0.343\ \text{N}\cdot\text{m north}}$

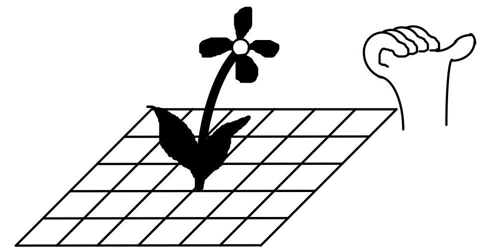

FIG. P11.5

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $\left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\right)\cdot\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 0$?

$$8 - 9 - 4 = -5 \neq 0$$

$\boxed{\text{No}}$. The cross product could not work out that way.

P11.7 $\left|\mathbf{A}\times\mathbf{B}\right| = \mathbf{A}\cdot\mathbf{B} \Rightarrow AB\sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

P11.8 (a) $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(2-9) = \boxed{(-7.00\ \text{N}\cdot\text{m})\hat{\mathbf{k}}}$

(b) The particle's position vector relative to the new axis is $1\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{j}} = 1\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

$$\boldsymbol{\tau} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0\ \text{N}\cdot\text{m})\hat{\mathbf{k}}}$$

P11.9 $\boxed{\left|\mathbf{F}_3\right| = \left|\mathbf{F}_1\right| + \left|\mathbf{F}_2\right|}$

The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\mathbf{F}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

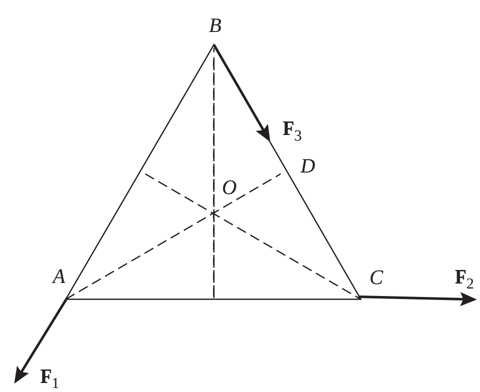

FIG. P11.9

***P11.10** $\left|\hat{\mathbf{i}} \times \hat{\mathbf{i}}\right| = 1 \cdot 1 \cdot \sin 0° = 0$

$\hat{\mathbf{j}} \times \hat{\mathbf{j}}$ and $\hat{\mathbf{k}} \times \hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right| = 1 \cdot 1 \cdot \sin 90° = 1$

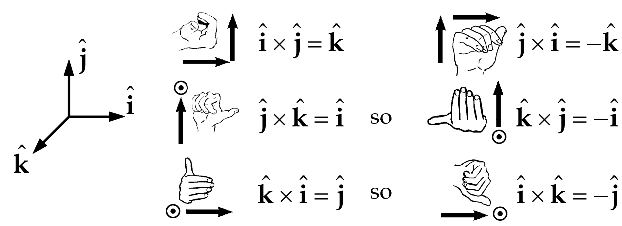

FIG. P11.10

Section 11.2 **Angular Momentum**

P11.11 $L = \sum m_i v_i r_i$

$$= (4.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m}) + (3.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m})$$

$L = 17.5 \text{ kg} \cdot \text{m}^2/\text{s}$, and

$$\boxed{\mathbf{L} = \left(17.5 \text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

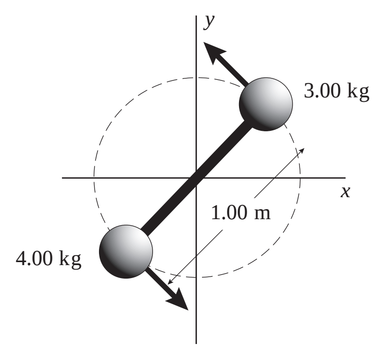

FIG. P11.11

P11.12 $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$$\mathbf{L} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right) \text{ m} \times (1.50 \text{ kg})\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\mathbf{L} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right) \text{ kg} \cdot \text{m}^2/\text{s} = \boxed{\left(-22.0 \text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.13 $\mathbf{r} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}} \text{ m}\right)$ $\qquad$ $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 5.00\hat{\mathbf{j}} \text{ m/s}$

so $\quad \mathbf{p} = m\mathbf{v} = 2.00 \text{ kg}\left(5.00\hat{\mathbf{j}} \text{ m/s}\right) = 10.0\hat{\mathbf{j}} \text{ kg} \cdot \text{m/s}$

and $\quad \mathbf{L} = \mathbf{r} \times \mathbf{p} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0 \text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

P11.14 $\sum F_x = ma_x$ $\qquad T\sin\theta = \dfrac{mv^2}{r}$

$\sum F_y = ma_y$ $\qquad T\cos\theta = mg$

So $\dfrac{\sin\theta}{\cos\theta} = \dfrac{v^2}{rg}$ $\qquad v = \sqrt{rg\dfrac{\sin\theta}{\cos\theta}}$

$L = rmv\sin 90.0°$

$L = rm\sqrt{rg\dfrac{\sin\theta}{\cos\theta}}$

$L = \sqrt{m^2 g r^3 \dfrac{\sin\theta}{\cos\theta}}$

$r = \ell\sin\theta$, so

$$L = \boxed{\sqrt{m^2 g \ell^3 \frac{\sin^4\theta}{\cos\theta}}}$$

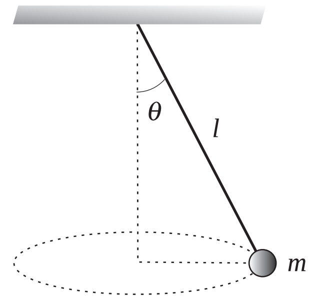

FIG. P11.14

P11.15 The angular displacement of the particle around the circle is $\theta = \omega t = \dfrac{vt}{R}$.

The vector from the center of the circle to the mass is then $R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$.

The vector from point P to the mass is

$$\mathbf{r} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\mathbf{r} = R\left[\left(1+\cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

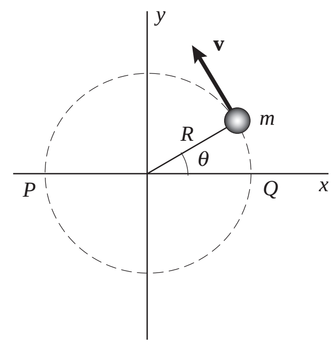

FIG. P11.15

The velocity is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So $\quad \mathbf{L} = \mathbf{r}\times m\mathbf{v}$

$$\mathbf{L} = mvR\left[(1+\cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right]\times\left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\mathbf{L} = \boxed{mvR\hat{\mathbf{k}}\left[\cos\left(\frac{vt}{R}\right)+1\right]}$$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\tau| = |\mathbf{r}\times\mathbf{F}| = 8.00\times10^{-2}\text{ m}(4.00\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{3.14\text{ N}\cdot\text{m}}.$$

(b) $|\mathbf{L}| = |\mathbf{r}\times m\mathbf{v}| + I\omega$ $\qquad |\mathbf{L}| = Rmv + \dfrac{1}{2}MR^2\left(\dfrac{v}{R}\right) = R\left(m+\dfrac{M}{2}\right)v = \boxed{(0.400\text{ kg}\cdot\text{m})v}$

(c) $\tau = \dfrac{dL}{dt} = (0.400\text{ kg}\cdot\text{m})a$ $\qquad a = \dfrac{3.14\text{ N}\cdot\text{m}}{0.400\text{ kg}\cdot\text{m}} = \boxed{7.85\text{ m/s}^2}$

P11.17 (a) $\boxed{\text{zero}}$

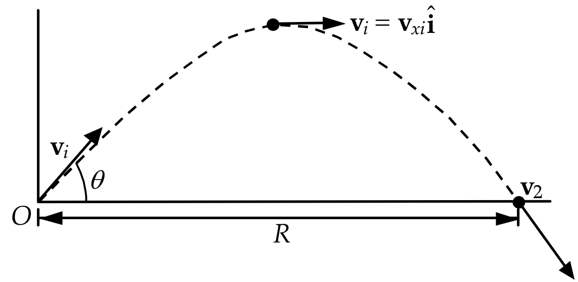

FIG. P11.17

(b) At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and}$$

$$y = h_{\max} = \frac{(v_i \sin\theta)^2}{2g}$$

$$\begin{aligned}\mathbf{L}_1 &= \mathbf{r}_1 \times m\mathbf{v}_1 \\ &= \left[\frac{v_i^2 \sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i \sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times mv_{xi}\hat{\mathbf{i}} \\ &= \boxed{\frac{-m(v_i \sin\theta)^2 v_i \cos\theta}{2g}\hat{\mathbf{k}}}\end{aligned}$$

(c) $\mathbf{L}_2 = R\hat{\mathbf{i}} \times m\mathbf{v}_2$, where $R = \dfrac{v_i^2 \sin 2\theta}{g}$

$$\begin{aligned}&= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\,\hat{\mathbf{i}} - v_i \sin\theta\,\hat{\mathbf{j}}\right) \\ &= -mRv_i \sin\theta\,\hat{\mathbf{k}} = \boxed{\frac{-mv_i^3 \sin 2\theta \sin\theta}{g}\hat{\mathbf{k}}}\end{aligned}$$

(d) The downward force of gravity exerts a torque in the $-z$ direction.

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a)
$$\mathbf{r} = (4.30\text{ km})\hat{\mathbf{k}} = \left(4.30 \times 10^3\text{ m}\right)\hat{\mathbf{k}}$$

$$\mathbf{p} = m\mathbf{v} = 12\,000\text{ kg}\left(-175\hat{\mathbf{i}}\text{ m/s}\right) = -2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \left(4.30 \times 10^3\,\hat{\mathbf{k}}\text{ m}\right) \times \left(-2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}\right) = \boxed{\left(-9.03 \times 10^9\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No}}$. $L = |\mathbf{r}||\mathbf{p}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L =$ *constant* as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero}}$. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr\sin 180° = 0$.