Chapter 1

Physics and Measurement

CHAPTER OUTLINE

▲ *The workings of a mechanical clock. Complicated timepieces have been built for centuries in an effort to measure time accurately. Time is one of the basic quantities that we use in studying the motion of objects. (elektraVision/Index Stock Imagery)*

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast, the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion.

Classical physics includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A major revolution in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity not only correctly described the motion of objects moving at speeds comparable to the speed of light but also completely revolutionized the traditional concepts of space, time, and energy. The theory of relativity also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas there is a great deal of overlap among physics, chemistry, and biology. Evidence for this overlap is seen in the names of some subspecialties in science—biophysics, biochemistry, chemical physics, biotechnology, and so on. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable developments in the latter half of the 20th century were (1) unmanned planetary explorations and manned moon landings, (2) microcircuitry and high-speed computers, (3) sophisticated imaging techniques used in scientific research and medicine, and

(4) several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1.1 Standards of Length, Mass, and Time

The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout the book. Most of these quantities are *derived quantities,* in that they can be expressed as combinations of a small number of *basic quantities*. In mechanics, the three basic quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 "glitches" if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.[1] Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably. Measurements taken by different people in different places must yield the same result.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International), and its units of length, mass, and time are the *meter*, *kilogram*, and *second*, respectively. Other SI standards established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be named the *yard* and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the *meter,* defined as one ten-millionth the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris.

Many other systems for measuring length have been developed over the years, but the advantages of the French system have caused it to prevail in almost all countries and in scientific circles everywhere. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines on the bar can be determined does not meet the current requirements of science and technology. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983, **the meter (m) was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second.** In effect, this

[1] The need for assigning numerical values to various measured physical quantities was expressed by Lord Kelvin (William Thomson) as follows: "I often say that when you can measure what you are speaking about, and express it in numbers, you should know something about it, but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind. It may be the beginning of knowledge but you have scarcely in your thoughts advanced to the state of science."

Table 1.1

Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second.

Table 1.1 lists approximate values of some measured lengths. You should study this table as well as the next two tables and begin to generate an intuition for what is meant by a length of 20 centimeters, for example, or a mass of 100 kilograms or a time interval of 3.2×10^7 seconds.

Mass

The SI unit of mass, **the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland (Fig. 1.1a).

Table 1.2 lists approximate values of the masses of various objects.

Time

Before 1960, the standard of time was defined in terms of the *mean solar day* for the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The *second* was defined as $\left(\frac{1}{60}\right)\left(\frac{1}{60}\right)\left(\frac{1}{24}\right)$ of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a time standard.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an *atomic clock* (Fig. 1.1b), which uses the characteristic frequency of the cesium-133 atom as the "reference clock." **The second (s) is now defined as 9 192 631 770 times the period of vibration of radiation from the cesium atom.**[2]

[2] *Period* is defined as the time interval needed for one complete vibration.

PITFALL PREVENTION

1.1 No Commas in Numbers with Many Digits

We will use the standard scientific notation for numbers with more than three digits, in which groups of three digits are separated by spaces rather than commas. Thus, 10 000 is the same as the common American notation of 10,000. Similarly, $\pi = 3.14159265$ is written as 3.141 592 65.

Table 1.2

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

PITFALL PREVENTION

1.2 Reasonable Values

Generating intuition about typical values of quantities is important because when solving problems you must think about your end result and determine if it seems reasonable. If you are calculating the mass of a housefly and arrive at a value of 100 kg, this is *unreasonable*—there is an error somewhere.

(Courtesy of National Institute of Standards and Technology, U.S. Department of Commerce)

(a) (b)

Figure 1.1 (a) The National Standard Kilogram No. 20, an accurate copy of the International Standard Kilogram kept at Sèvres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology. (b) The nation's primary time standard is a cesium fountain atomic clock developed at the National Institute of Standards and Technology laboratories in Boulder, Colorado. The clock will neither gain nor lose a second in 20 million years.

To keep these atomic clocks—and therefore all common clocks and watches that are set to them—synchronized, it has sometimes been necessary to add leap seconds to our clocks.

Since Einstein's discovery of the linkage between space and time, precise measurement of time intervals requires that we know both the state of motion of the clock used to measure the interval and, in some cases, the location of the clock as well. Otherwise, for example, global positioning system satellites might be unable to pinpoint your location with sufficient accuracy, should you need to be rescued.

Approximate values of time intervals are presented in Table 1.3.

Table 1.3

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Table 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

In addition to SI, another system of units, the *U.S. customary system,* is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the foot (ft), slug, and second, respectively. In this text we shall use SI units because they are almost universally accepted in science and industry. We shall make some limited use of U.S. customary units in the study of classical mechanics.

In addition to the basic SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli-* and *nano-* denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^{3} m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^{3} grams (g), and 1 megavolt (MV) is 10^{6} volts (V).

1.2 Matter and Model Building

If physicists cannot interact with some phenomenon directly, they often imagine a **model** for a physical system that is related to the phenomenon. In this context, a model is a system of physical components, such as electrons and protons in an atom. Once we have identified the physical components, we make predictions about the behavior of the system, based on the interactions among the components of the system and/or the interaction between the system and the environment outside the system.

As an example, consider the behavior of *matter.* A 1-kg cube of solid gold, such as that at the left of Figure 1.2, has a length of 3.73 cm on a side. Is this cube nothing but wall-to-wall gold, with no empty space? If the cube is cut in half, the two pieces still retain their chemical identity as solid gold. But what if the pieces are cut again and again, indefinitely? Will the smaller and smaller pieces always be gold? Questions such as these can be traced back to early Greek philosophers. Two of them—Leucippus and his student Democritus—could not accept the idea that such cuttings could go on forever. They speculated that the process ultimately must end when it produces a particle

The atomic mass of lead is 207 u and that of aluminum is 27.0 u. However, the ratio of atomic masses, 207 u/27.0 u = 7.67, does not correspond to the ratio of densities, $(11.3 \times 10^3\ \text{kg/m}^3)/(2.70 \times 10^3\ \text{kg/m}^3) = 4.19$. This discrepancy is due to the difference in atomic spacings and atomic arrangements in the crystal structures of the two elements.

Quick Quiz 1.1 In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) the aluminum cam (b) the iron cam (c) Both cams have the same size.

Example 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.70 g/cm^3) has a volume of 0.200 cm^3. It is known that 27.0 g of aluminum contains 6.02×10^{23} atoms. How many aluminum atoms are contained in the cube?

Solution Because density equals mass per unit volume, the mass of the cube is

$$m = \rho V = (2.70\ \text{g}/\cancel{\text{cm}^3})(0.200\ \cancel{\text{cm}^3}) = 0.540\ \text{g}$$

To solve this problem, we will set up a ratio based on the fact that the mass of a sample of material is proportional to the number of atoms contained in the sample. This technique of solving by ratios is very powerful and should be studied and understood so that it can be applied in future problem solving. Let us express our proportionality as $m = kN$, where m is the mass of the sample, N is the number of atoms in the sample, and k is an unknown proportionality constant. We write this relationship twice, once for the actual sample of aluminum in the problem and once for a 27.0-g sample, and then we divide the first equation by the second:

$$\begin{aligned} m_{\text{sample}} &= kN_{\text{sample}} \\ m_{27.0\ \text{g}} &= kN_{27.0\ \text{g}} \end{aligned} \quad \longrightarrow \quad \frac{m_{\text{sample}}}{m_{27.0\ \text{g}}} = \frac{N_{\text{sample}}}{N_{27.0\ \text{g}}}$$

Notice that the unknown proportionality constant k cancels, so we do not need to know its value. We now substitute the values:

$$\frac{0.540\ \text{g}}{27.0\ \text{g}} = \frac{N_{\text{sample}}}{6.02 \times 10^{23}\ \text{atoms}}$$

$$N_{\text{sample}} = \frac{(0.540\ \text{g})(6.02 \times 10^{23}\ \text{atoms})}{27.0\ \text{g}}$$

$$= \boxed{1.20 \times 10^{22}\ \text{atoms}}$$

PITFALL PREVENTION

1.3 Setting Up Ratios

When using ratios to solve a problem, keep in mind that *ratios come from equations.* If you start from equations known to be correct and can divide one equation by the other as in Example 1.1 to obtain a useful ratio, you will avoid reasoning errors. So write the known equations first!

1.4 Dimensional Analysis

The word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is *length.*

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively.[3] We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is v, and in our notation the dimensions of speed are written $[v] = \text{L/T}$. As another example, the dimensions of area A are $[A] = \text{L}^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.6. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called *dimensional analysis* can be used to assist in the derivation or to check your final expression. Dimensional analysis makes use of the fact that

[3] The *dimensions* of a quantity will be symbolized by a capitalized, non-italic letter, such as L. The *symbol* for the quantity itself will be italicized, such as L for the length of an object, or t for time.

Table 1.6

Units of Area, Volume, Velocity, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position x of a car at a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.6), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol $\propto$ indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = L^1T^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$(L/T^2)^n T^m = L^1T^0$$

$$(L^n T^{m-2n}) = L^1T^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$. This result differs by a factor of $\frac{1}{2}$ from the correct expression, which is $x = \frac{1}{2}at^2$.

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

PITFALL PREVENTION

1.4 Symbols for Quantities

Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always t. Others quantities might have various symbols depending on the usage. Length may be described with symbols such as x, y, and z (for position), r (for radius), a, b, and c (for the legs of a right triangle), ℓ (for the length of an object), d (for a distance), h (for a height), etc.

Example 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{L}{T}$$

The same table gives us L/T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{L}{\cancel{T^2}}\cancel{T} = \frac{L}{T}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect*. Try it and see!)

Example 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\frac{L}{T^2} = L^n\left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

This dimensional equation is balanced under the conditions

$$n + m = \boxed{1} \qquad \text{and} \qquad m = \boxed{2}$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = \boxed{k\frac{v^2}{r}}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2.

PITFALL PREVENTION

1.5 Always Include Units

When performing calculations, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

1.5 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another, or to convert within a system, for example, from kilometers to meters. Equalities between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \cancel{\text{in.}})\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. Notice that we choose to put the unit of an inch in the denominator and it cancels with the unit in the original quantity. The remaining unit is the centimeter, which is our desired result.

Quick Quiz 1.3 The distance between two cities is 100 mi. The number of kilometers between the two cities is (a) smaller than 100 (b) larger than 100 (c) equal to 100.

Example 1.4 Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is this car exceeding the speed limit of 75.0 mi/h?

Solution We first convert meters to miles:

$$(38.0\ \cancel{\text{m}}/\text{s})\left(\frac{1\ \text{mi}}{1\ 609\ \cancel{\text{m}}}\right) = 2.36 \times 10^{-2}\ \text{mi/s}$$

Now we convert seconds to hours:

$$(2.36 \times 10^{-2}\ \text{mi}/\cancel{\text{s}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right) = 85.0\ \text{mi/h}$$

Thus, the car is exceeding the speed limit and should slow down.

What If? What if the driver is from outside the U.S. and is familiar with speeds measured in km/h? What is the speed of the car in km/h?

Answer We can convert our final answer to the appropriate units:

$$(85.0\ \cancel{\text{mi}}/\text{h})\left(\frac{1.609\ \text{km}}{1\ \cancel{\text{mi}}}\right) = 137\ \text{km/h}$$

Figure 1.3 shows the speedometer of an automobile, with speeds in both mi/h and km/h. Can you check the conversion we just performed using this photograph?

Phil Boorman/Getty Images

Figure 1.3 The speedometer of a vehicle that shows speeds in both miles per hour and kilometers per hour.

1.6 Estimates and Order-of-Magnitude Calculations

It is often useful to compute an approximate answer to a given physical problem even when little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an *order of magnitude* of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of-magnitude calculation is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of about $10^3 = 1\,000$. We use the symbol $\sim$ for "is on the order of." Thus,

$$0.008\ 6 \sim 10^{-2} \qquad 0.002\ 1 \sim 10^{-3} \qquad 720 \sim 10^{3}$$

The spirit of order-of-magnitude calculations, sometimes referred to as "guesstimates" or "ball-park figures," is given in the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle."[4] Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for

[4] E. Taylor and J. A. Wheeler, *Spacetime Physics: Introduction to Special Relativity,* 2nd ed., San Francisco, W. H. Freeman & Company, Publishers, 1992, p. 20.

final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures. The same rule applies to division. When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

QUESTIONS

1. What types of natural phenomena could serve as time standards?

2. Suppose that the three fundamental standards of the metric system were length, *density,* and time rather than length, *mass,* and time. The standard of density in this system is to be defined as that of water. What considerations about water would you need to address to make sure that the standard of density is as accurate as possible?

3. The height of a horse is sometimes given in units of "hands." Why is this a poor standard of length?

4. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m (b) 5×10^{-5} s (c) 72×10^{2} g.

5. Suppose that two quantities A and B have different dimensions. Determine which of the following arithmetic operations *could* be physically meaningful: (a) $A + B$ (b) A/B (c) $B - A$ (d) AB.

6. If an equation is dimensionally correct, does this mean that the equation must be true? If an equation is not dimensionally correct, does this mean that the equation cannot be true?

7. Do an order-of-magnitude calculation for an everyday situation you encounter. For example, how far do you walk or drive each day?

8. Find the order of magnitude of your age in seconds.

9. What level of precision is implied in an order-of-magnitude calculation?

10. Estimate the mass of this textbook in kilograms. If a scale is available, check your estimate.

11. In reply to a student's question, a guard in a natural history museum says of the fossils near his station, "When I started work here twenty-four years ago, they were eighty million years old, so you can add it up." What should the student conclude about the age of the fossils?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 1.2 Matter and Model Building

Note: Consult the endpapers, appendices, and tables in the text whenever necessary in solving problems. For this chapter, Appendix B.3 may be particularly useful. Answers to odd-numbered problems appear in the back of the book.

1. A crystalline solid consists of atoms stacked up in a repeating lattice structure. Consider a crystal as shown in Figure P1.1a. The atoms reside at the corners of cubes of side $L = 0.200$ nm. One piece of evidence for the regular arrangement of atoms comes from the flat surfaces along which a crystal separates, or cleaves, when it is broken. Suppose this crystal cleaves along a face diagonal, as shown in Figure P1.1b. Calculate the spacing d between two adjacent atomic planes that separate when the crystal cleaves.

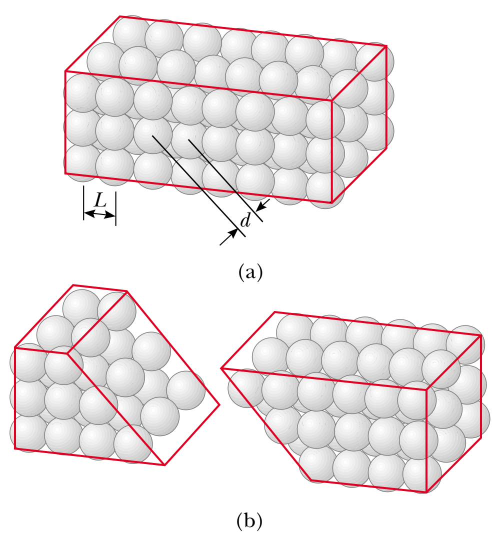


Figure P1.1

Section 1.3 Density and Atomic Mass

2. Use information on the endpapers of this book to calculate the average density of the Earth. Where does the value fit among those listed in Tables 1.5 and 14.1? Look up the density of a typical surface rock like granite in another source and compare the density of the Earth to it.

3. The standard kilogram is a platinum–iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?

4. A major motor company displays a die-cast model of its first automobile, made from 9.35 kg of iron. To celebrate its hundredth year in business, a worker will recast the model in gold from the original dies. What mass of gold is needed to make the new model?

5. What mass of a material with density ρ is required to make a hollow spherical shell having inner radius r_1 and outer radius r_2?

6. Two spheres are cut from a certain uniform rock. One has radius 4.50 cm. The mass of the other is five times greater. Find its radius.

7. Calculate the mass of an atom of (a) helium, (b) iron, and (c) lead. Give your answers in grams. The atomic masses of these atoms are 4.00 u, 55.9 u, and 207 u, respectively.

8. The paragraph preceding Example 1.1 in the text mentions that the atomic mass of aluminum is $27.0\,\text{u} = 27.0 \times 1.66 \times 10^{-27}$ kg. Example 1.1 says that 27.0 g of aluminum contains 6.02×10^{23} atoms. (a) Prove that each one of these two statements implies the other. (b) **What If?** What if it's not aluminum? Let M represent the numerical value of the mass of one atom of any chemical element in atomic mass units. Prove that M grams of the substance contains a particular number of atoms, the same number for all elements. Calculate this number precisely from the value for u quoted in the text. The number of atoms in M grams of an element is called *Avogadro's number* N_A. The idea can be extended: Avogadro's number of molecules of a chemical compound has a mass of M grams, where M atomic mass units is the mass of one molecule. Avogadro's number of atoms or molecules is called one *mole,* symbolized as 1 mol. A periodic table of the elements, as in Appendix C, and the chemical formula for a compound contain enough information to find the molar mass of the compound. (c) Calculate the mass of one mole of water, H_2O. (d) Find the molar mass of CO_2.

9. On your wedding day your lover gives you a gold ring of mass 3.80 g. Fifty years later its mass is 3.35 g. On the average, how many atoms were abraded from the ring during each second of your marriage? The atomic mass of gold is 197 u.

10. A small cube of iron is observed under a microscope. The edge of the cube is 5.00×10^{-6} cm long. Find (a) the mass of the cube and (b) the number of iron atoms in the cube. The atomic mass of iron is 55.9 u, and its density is 7.86 g/cm^3.

11. A structural I beam is made of steel. A view of its cross-section and its dimensions are shown in Figure P1.11. The density of the steel is 7.56×10^3 kg/m^3. (a) What is the mass of a section 1.50 m long? (b) Assume that the atoms are predominantly iron, with atomic mass 55.9 u. How many atoms are in this section?

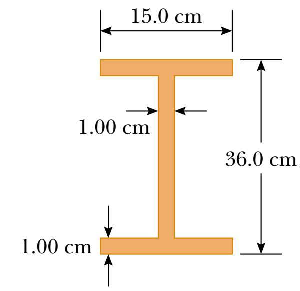


Figure P1.11

12. A child at the beach digs a hole in the sand and uses a pail to fill it with water having a mass of 1.20 kg. The mass of one molecule of water is 18.0 u. (a) Find the number of water molecules in this pail of water. (b) Suppose the quantity of water on Earth is constant at 1.32×10^{21} kg. How many of the water molecules in this pail of water are likely to have been in an equal quantity of water that once filled one particular claw print left by a Tyrannosaur hunting on a similar beach?

Section 1.4 Dimensional Analysis

13. The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position $s = ka^m t^n$, where k is a dimensionless constant. Show by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

14. Figure P1.14 shows a *frustrum of a cone.* Of the following mensuration (geometrical) expressions, which describes (a) the total circumference of the flat circular faces (b) the volume (c) the area of the curved surface? (i) $\pi(r_1 + r_2)[h^2 + (r_1 - r_2)^2]^{1/2}$ (ii) $2\pi(r_1 + r_2)$ (iii) $\pi h(r_1{}^2 + r_1 r_2 + r_2{}^2)$.

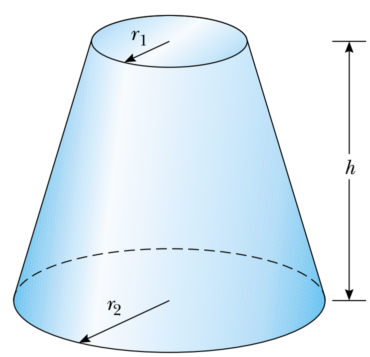


Figure P1.14

15. Which of the following equations are dimensionally correct?
(a) $v_f = v_i + ax$
(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$.

16. (a) A fundamental law of motion states that the acceleration of an object is directly proportional to the resultant force exerted on the object and inversely proportional to its mass. If the proportionality constant is defined to have no dimensions, determine the dimensions of force. (b) The newton is the SI unit of force. According to the results for (a), how can you express a force having units of newtons using the fundamental units of mass, length, and time?

17. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the magnitude of the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units $\text{kg}\cdot\text{m/s}^2$. What are the SI units of the proportionality constant G?

Section 1.5 Conversion of Units

18. A worker is to paint the walls of a square room 8.00 ft high and 12.0 ft along each side. What surface area in square meters must she cover?

19. Suppose your hair grows at the rate 1/32 in. per day. Find the rate at which it grows in nanometers per second. Because the distance between atoms in a molecule is on the order of 0.1 nm, your answer suggests how rapidly layers of atoms are assembled in this protein synthesis.

20. The volume of a wallet is 8.50 in.^{3} Convert this value to m^3, using the definition 1 in. = 2.54 cm.

21. A rectangular building lot is 100 ft by 150 ft. Determine the area of this lot in m^2.

22. An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?

23. Assume that it takes 7.00 minutes to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time interval, in hours, required to fill a 1-m^3 volume at the same rate. (1 U.S. gal = 231 in.^{3})

24. Find the height or length of these natural wonders in kilometers, meters and centimeters. (a) The longest cave system in the world is the Mammoth Cave system in central Kentucky. It has a mapped length of 348 mi. (b) In the United States, the waterfall with the greatest single drop is Ribbon Falls, which falls 1 612 ft. (c) Mount McKinley in Denali National Park, Alaska, is America's highest mountain at a height of 20 320 ft. (d) The deepest canyon in the United States is King's Canyon in California with a depth of 8 200 ft.

25. A solid piece of lead has a mass of 23.94 g and a volume of 2.10 cm^3. From these data, calculate the density of lead in SI units (kg/m^3).

26. A *section* of land has an area of 1 square mile and contains 640 acres. Determine the number of square meters in 1 acre.

27. An ore loader moves 1 200 tons/h from a mine to the surface. Convert this rate to lb/s, using 1 ton = 2 000 lb.

28. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) In the past, a federal law mandated that highway speed limits would be 55 mi/h. Use the conversion factor of part (a) to find this speed in kilometers per hour. (c) The maximum highway speed is now 65 mi/h in some places. In kilometers per hour, how much increase is this over the 55 mi/h limit?

29. At the time of this book's printing, the U.S. national debt is about \$6 trillion. (a) If payments were made at the rate of \$1 000 per second, how many years would it take to pay off the debt, assuming no interest were charged? (b) A dollar bill is about 15.5 cm long. If six trillion dollar bills were laid end to end around the Earth's equator, how many times would they encircle the planet? Take the radius of the Earth at the equator to be 6 378 km. (*Note:* Before doing any of these calculations, try to guess at the answers. You may be very surprised.)

30. The mass of the Sun is 1.99×10^{30} kg, and the mass of an atom of hydrogen, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are in the Sun?

31. One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?

32. A pyramid has a height of 481 ft and its base covers an area of 13.0 acres (Fig. P1.32). If the volume of a pyramid is given by the expression $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height, find the volume of this pyramid in cubic meters. (1 acre = 43 560 ft^2)

Sylvain Grandadam/Photo Researchers, Inc.

Figure P1.32 Problems 32 and 33.

33. The pyramid described in Problem 32 contains approximately 2 million stone blocks that average 2.50 tons each. Find the weight of this pyramid in pounds.

34. Assuming that 70% of the Earth's surface is covered with water at an average depth of 2.3 mi, estimate the mass of the water on the Earth in kilograms.

35. A hydrogen atom has a diameter of approximately 1.06×10^{-10} m, as defined by the diameter of the spherical electron cloud around the nucleus. The hydrogen nucleus has a diameter of approximately 2.40×10^{-15} m. (a) For a scale model, represent the diameter of the hydrogen atom by the length of an American football field

(100 yd = 300 ft), and determine the diameter of the nucleus in millimeters. (b) The atom is how many times larger in volume than its nucleus?

36. The nearest stars to the Sun are in the Alpha Centauri multiple-star system, about 4.0×10^{13} km away. If the Sun, with a diameter of 1.4×10^9 m, and Alpha Centauri A are both represented by cherry pits 7.0 mm in diameter, how far apart should the pits be placed to represent the Sun and its neighbor to scale?

37. The diameter of our disk-shaped galaxy, the Milky Way, is about 1.0×10^5 lightyears (ly). The distance to Messier 31, which is Andromeda, the spiral galaxy nearest to the Milky Way, is about 2.0 million ly. If a scale model represents the Milky Way and Andromeda galaxies as dinner plates 25 cm in diameter, determine the distance between the two plates.

38. The mean radius of the Earth is 6.37×10^6 m, and that of the Moon is 1.74×10^8 cm. From these data calculate (a) the ratio of the Earth's surface area to that of the Moon and (b) the ratio of the Earth's volume to that of the Moon. Recall that the surface area of a sphere is $4\pi r^2$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

39. One cubic meter ($1.00\ \text{m}^3$) of aluminum has a mass of 2.70×10^3 kg, and $1.00\ \text{m}^3$ of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.

40. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.

Section 1.6 Estimates and Order-of-Magnitude Calculations

41. Estimate the number of Ping-Pong balls that would fit into a typical-size room (without being crushed). In your solution state the quantities you measure or estimate and the values you take for them.

42. An automobile tire is rated to last for 50 000 miles. To an order of magnitude, through how many revolutions will it turn? In your solution state the quantities you measure or estimate and the values you take for them.

43. Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass on this plot? Explain your reasoning. Note that 1 acre = 43 560 ft^2.

44. Approximately how many raindrops fall on a one-acre lot during a one-inch rainfall? Explain your reasoning.

45. Compute the order of magnitude of the mass of a bathtub half full of water. Compute the order of magnitude of the mass of a bathtub half full of pennies. In your solution list the quantities you take as data and the value you measure or estimate for each.

46. Soft drinks are commonly sold in aluminum containers. To an order of magnitude, how many such containers are thrown away or recycled each year by U.S. consumers? How many tons of aluminum does this represent? In your solution state the quantities you measure or estimate and the values you take for them.

47. To an order of magnitude, how many piano tuners are in New York City? The physicist Enrico Fermi was famous for asking questions like this on oral Ph.D. qualifying examinations. His own facility in making order-of-magnitude calculations is exemplified in Problem 45.48.

Section 1.7 Significant Figures

> *Note:* Appendix B.8 on propagation of uncertainty may be useful in solving some problems in this section.

48. A rectangular plate has a length of (21.3 ± 0.2) cm and a width of (9.8 ± 0.1) cm. Calculate the area of the plate, including its uncertainty.

49. The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate the (a) area and (b) circumference of the circle and give the uncertainty in each value.

50. How many significant figures are in the following numbers? (a) 78.9 ± 0.2 (b) 3.788×10^9 (c) 2.46×10^{-6} (d) 0.005 3.

51. The radius of a solid sphere is measured to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere in kilograms per cubic meter and the uncertainty in the density.

52. Carry out the following arithmetic operations: (a) the sum of the measured values 756, 37.2, 0.83, and 2.5; (b) the product $0.003\,2 \times 356.3$; (c) the product $5.620 \times \pi$.

53. The *tropical year,* the time from vernal equinox to the next vernal equinox, is the basis for our calendar. It contains 365.242 199 days. Find the number of seconds in a tropical year.

54. A farmer measures the distance around a rectangular field. The length of the long sides of the rectangle is found to be 38.44 m, and the length of the short sides is found to be 19.5 m. What is the total distance around the field?

55. A sidewalk is to be constructed around a swimming pool that measures (10.0 ± 0.1) m by (17.0 ± 0.1) m. If the sidewalk is to measure (1.00 ± 0.01) m wide by (9.0 ± 0.1) cm thick, what volume of concrete is needed, and what is the approximate uncertainty of this volume?

Additional Problems

56. In a situation where data are known to three significant digits, we write 6.379 m = 6.38 m and 6.374 m = 6.37 m. When a number ends in 5, we arbitrarily choose to write 6.375 m = 6.38 m. We could equally well write 6.375 m = 6.37 m, "rounding down" instead of "rounding up," because we would change the number 6.375 by equal increments in both cases. Now consider an order-of-magnitude estimate, in which we consider factors rather than increments. We write 500 m $\sim 10^3$ m because 500 differs from 100 by a factor of 5 while it differs from 1 000 by only a factor of 2. We write 437 m $\sim 10^3$ m and 305 m $\sim 10^2$ m. What distance differs from 100 m and from 1 000 m by equal factors, so that we could equally well choose to represent its order of magnitude either as $\sim 10^2$ m or as $\sim 10^3$ m?

57. For many electronic applications, such as in computer chips, it is desirable to make components as small as possible to keep the temperature of the components low and to increase the speed of the device. Thin metallic coatings (films) can be used instead of wires to make electrical connections. Gold is especially useful because it does not oxidize readily. Its atomic mass is 197 u. A gold film can be no thinner than the size of a gold atom. Calculate the minimum coating thickness, assuming that a gold atom occupies a cubical volume in the film that is equal to the volume it occupies in a large piece of metal. This geometric model yields a result of the correct order of magnitude.

58. The basic function of the carburetor of an automobile is to "atomize" the gasoline and mix it with air to promote rapid combustion. As an example, assume that 30.0 cm^3 of gasoline is atomized into N spherical droplets, each with a radius of 2.00×10^{-5} m. What is the total surface area of these N spherical droplets?

59. The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.008\,00t^2$, where V is the volume in millions of cubic feet and t the time in months. Express this equation in units of cubic feet and seconds. Assign proper units to the coefficients. Assume a month is equal to 30.0 days.

60. In physics it is important to use mathematical approximations. Demonstrate that for small angles ($< 20°$)

$$\tan \alpha \approx \sin \alpha \approx \alpha = \pi\alpha'/180°$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan \alpha$ may be approximated by $\sin \alpha$ if the error is to be less than 10.0%.

61. A high fountain of water is located at the center of a circular pool as in Figure P1.61. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

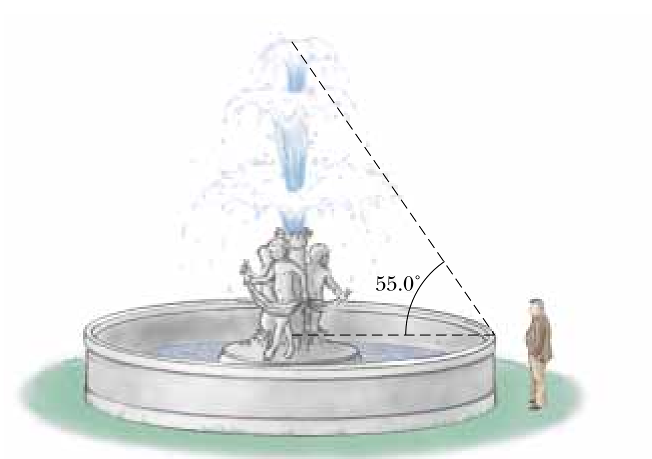


Figure P1.61

62. Collectible coins are sometimes plated with gold to enhance their beauty and value. Consider a commemorative quarter-dollar advertised for sale at \$4.98. It has a diameter of 24.1 mm, a thickness of 1.78 mm, and is completely covered with a layer of pure gold 0.180 μm thick. The volume of the plating is equal to the thickness of the layer times the area to which it is applied. The patterns on the faces of the coin and the grooves on its edge have a negligible effect on its area. Assume that the price of gold is \$10.0 per gram. Find the cost of the gold added to the coin. Does the cost of the gold significantly enhance the value of the coin?

63. There are nearly $\pi \times 10^7$ s in one year. Find the percentage error in this approximation, where "percentage error" is defined as

$$\text{Percentage error} = \frac{|\text{assumed value} - \text{true value}|}{\text{true value}} \times 100\%$$

64. Assume that an object covers an area A and has a uniform height h. If its cross-sectional area is uniform over its height, then its volume is given by $V = Ah$. (a) Show that $V = Ah$ is dimensionally correct. (b) Show that the volumes of a cylinder and of a rectangular box can be written in the form $V = Ah$, identifying A in each case. (Note that A, sometimes called the "footprint" of the object, can have any shape and the height can be replaced by average thickness in general.)

65. A child loves to watch as you fill a transparent plastic bottle with shampoo. Every horizontal cross-section is a circle, but the diameters of the circles have different values, so that the bottle is much wider in some places than others. You pour in bright green shampoo with constant volume flow rate 16.5 cm^3/s. At what rate is its level in the bottle rising (a) at a point where the diameter of the bottle is 6.30 cm and (b) at a point where the diameter is 1.35 cm?

66. One cubic centimeter of water has a mass of 1.00×10^{-3} kg. (a) Determine the mass of 1.00 m^3 of water. (b) Biological substances are 98% water. Assume that they have the same density as water to estimate the masses of a cell that has a diameter of 1.0 μm, a human kidney, and a fly. Model the kidney as a sphere with a radius of 4.0 cm and the fly as a cylinder 4.0 mm long and 2.0 mm in diameter.

67. Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

68. A creature moves at a speed of 5.00 furlongs per fortnight (not a very common unit of speed). Given that 1 furlong = 220 yards and 1 fortnight = 14 days, determine the speed of the creature in m/s. What kind of creature do you think it might be?

69. The distance from the Sun to the nearest star is about 4×10^{16} m. The Milky Way galaxy is roughly a disk of diameter $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

70. The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of these substances. Compare your results for aluminum, copper, and iron with those given in Table 1.5.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

71. (a) How many seconds are in a year? (b) If one micrometeorite (a sphere with a diameter of 1.00×10^{-6} m) strikes each square meter of the Moon each second, how many years will it take to cover the Moon to a depth of 1.00 m? To solve this problem, you can consider a cubic box on the Moon 1.00 m on each edge, and find how long it will take to fill the box.

Answers to Quick Quizzes

1.1 (a). Because the density of aluminum is smaller than that of iron, a larger volume of aluminum is required for a given mass than iron.

1.2 False. Dimensional analysis gives the units of the proportionality constant but provides no information about its numerical value. To determine its numerical value requires either experimental data or geometrical reasoning. For example, in the generation of the equation $x = \frac{1}{2}at^2$, because the factor $\frac{1}{2}$ is dimensionless, there is no way of determining it using dimensional analysis.

1.3 (b). Because kilometers are shorter than miles, a larger number of kilometers is required for a given distance than miles.

1.4 Reporting all these digits implies you have determined the location of the center of the chair's seat to the nearest $\pm 0.000\ 000\ 000\ 1$ m. This roughly corresponds to being able to count the atoms in your meter stick because each of them is about that size! It would be better to record the measurement as 1.044 m: this indicates that you know the position to the nearest millimeter, assuming the meter stick has millimeter markings on its scale.

Chapter 2

Motion in One Dimension

▲ *One of the physical quantities we will study in this chapter is the velocity of an object moving in a straight line. Downhill skiers can reach velocities with a magnitude greater than 100 km/h. (Jean Y. Rusznieswki/Getty Images)*

CHAPTER OUTLINE

As a first step in studying classical mechanics, we describe motion in terms of space and time while ignoring the agents that caused that motion. This portion of classical mechanics is called *kinematics.* (The word *kinematics* has the same root as *cinema.* Can you see why?) In this chapter we consider only motion in one dimension, that is, motion along a straight line. We first define position, displacement, velocity, and acceleration. Then, using these concepts, we study the motion of objects traveling in one dimension with a constant acceleration.

From everyday experience we recognize that motion represents a continuous change in the position of an object. In physics we can categorize motion into three types: translational, rotational, and vibrational. A car moving down a highway is an example of translational motion, the Earth's spin on its axis is an example of rotational motion, and the back-and-forth movement of a pendulum is an example of vibrational motion. In this and the next few chapters, we are concerned only with translational motion. (Later in the book we shall discuss rotational and vibrational motions.)

In our study of translational motion, we use what is called the **particle model**—we describe the moving object as a *particle* regardless of its size. In general, **a particle is a point-like object—that is, an object with mass but having infinitesimal size.** For example, if we wish to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun. As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules.

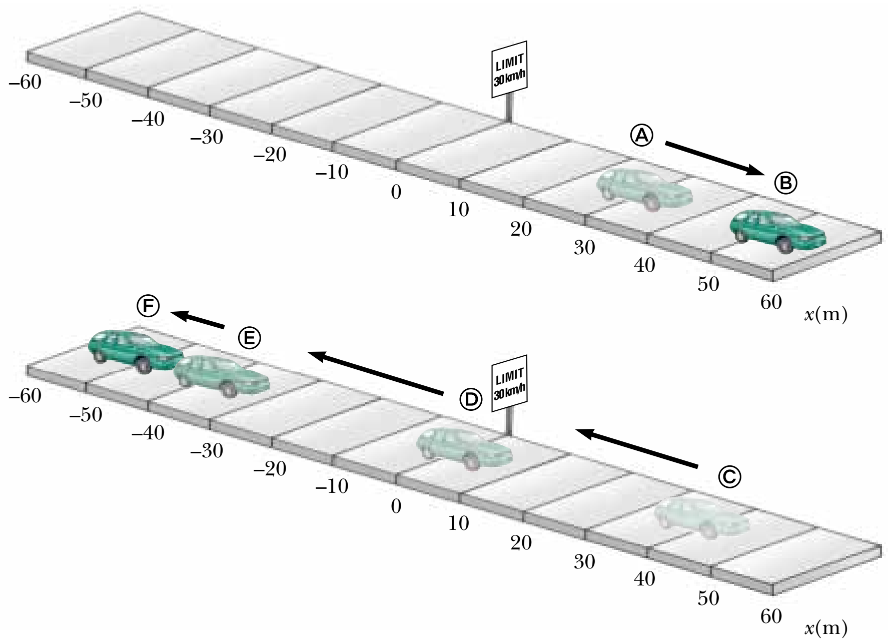
2.1 Position, Velocity, and Speed

Position

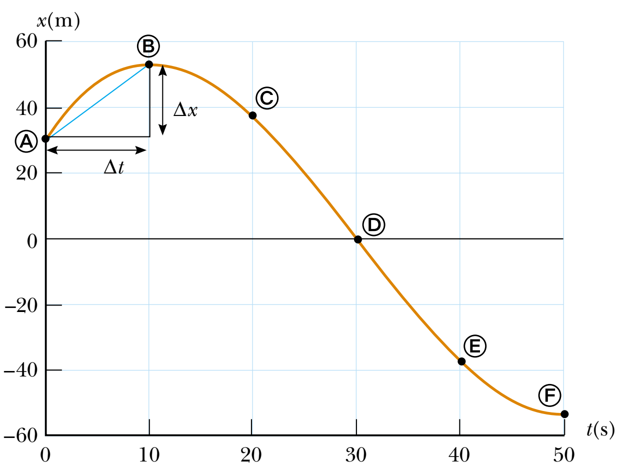
The motion of a particle is completely known if the particle's position in space is known at all times. A particle's **position** is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

Consider a car moving back and forth along the x axis as in Figure 2.1a. When we begin collecting position data, the car is 30 m to the right of a road sign, which we will use to identify the reference position $x = 0$. (Let us assume that all data in this example are known to two significant figures. To convey this information, we should report the initial position as 3.0×10^1 m. We have written this value in the simpler form 30 m to make the discussion easier to follow.) We will use the particle model by identifying some point on the car, perhaps the front door handle, as a particle representing the entire car.

We start our clock and once every 10 s note the car's position relative to the sign at $x = 0$. As you can see from Table 2.1, the car moves to the right (which we have



(a)



(b)

Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Active Figure 2.1 (a) A car moves back and forth along a straight line taken to be the x axis. Because we are interested only in the car's translational motion, we can model it as a particle. (b) Position–time graph for the motion of the "particle."

At the Active Figures link at http://www.pse6.com, you can move each of the six points Ⓐ through Ⓕ and observe the motion of the car pictorially and graphically as it follows a smooth path through the six points.

defined as the positive direction) during the first 10 s of motion, from position Ⓐ to position Ⓑ. After Ⓑ, the position values begin to decrease, suggesting that the car is backing up from position Ⓑ through position Ⓕ. In fact, at Ⓓ, 30 s after we start measuring, the car is alongside the road sign (see Figure 2.1a) that we are using to mark our origin of coordinates. It continues moving to the left and is more than 50 m to the left of the sign when we stop recording information after our sixth data point. A graphical representation of this information is presented in Figure 2.1b. Such a plot is called a *position–time graph.*

Given the data in Table 2.1, we can easily determine the change in position of the car for various time intervals. The **displacement** of a particle is defined as its change in position in some time interval. As it moves from an initial position x_i to a final position x_f, the displacement of the particle is given by $x_f - x_i$. We use the Greek letter delta (Δ) to denote the *change* in a quantity. Therefore, we write the displacement, or change in position, of the particle as

Displacement

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

It is very important to recognize the difference between displacement and distance traveled. **Distance** is the length of a path followed by a particle. Consider, for example, the basketball players in Figure 2.2. If a player runs from his own basket down the court to the other team's basket and then returns to his own basket, the *displacement* of the player during this time interval is zero, because he ended up at the same point as he started. During this time interval, however, he covered a *distance* of twice the length of the basketball court.

Displacement is an example of a vector quantity. Many other physical quantities, including position, velocity, and acceleration, also are vectors. In general, **a vector quantity requires the specification of both direction and magnitude.** By contrast, **a scalar quantity has a numerical value and no direction.** In this chapter, we use positive (+) and negative (−) signs to indicate vector direction. We can do this because the chapter deals with one-dimensional motion only; this means that any object we study can be moving only along a straight line. For example, for horizontal motion let us arbitrarily specify to the right as being the positive direction. It follows that any object always moving to the right undergoes a positive displacement $\Delta x > 0$, and any object moving to the left undergoes a negative displacement, so that $\Delta x < 0$. We shall treat vector quantities in greater detail in Chapter 3.

For our basketball player in Figure 2.2, if the trip from his own basket to the opposing basket is described by a displacement of $+28$ m, the trip in the reverse direction represents a displacement of -28 m. Each trip, however, represents a distance of 28 m, because distance is a scalar quantity. The total distance for the trip down the court and back is 56 m. Distance, therefore, is always represented as a positive number, while displacement can be either positive or negative.

There is one very important point that has not yet been mentioned. Note that the data in Table 2.1 results only in the six data points in the graph in Figure 2.1b. The smooth curve drawn through the six points in the graph is only a *possibility* of the actual motion of the car. We only have information about six instants of time—we have no idea what happened in between the data points. The smooth curve is a *guess* as to what happened, but keep in mind that it is *only* a guess.

If the smooth curve does represent the actual motion of the car, the graph contains information about the entire 50-s interval during which we watch the car move. It is much easier to see changes in position from the graph than from a verbal description or even a table of numbers. For example, it is clear that the car was covering more ground during the middle of the 50-s interval than at the end. Between positions Ⓒ and Ⓓ, the car traveled almost 40 m, but during the last 10 s, between positions Ⓔ and Ⓕ, it moved less than half that far. A common way of comparing these different motions is to divide the displacement Δx that occurs between two clock readings by the length of that particular time interval Δt. This turns out to be a very useful ratio, one that we shall use many times. This ratio has been given a special name—*average velocity*. **The average velocity $\bar{v}_x$ of a particle is defined as the**

Ken White/Allsport/Getty Images

Figure 2.2 On this basketball court, players run back and forth for the entire game. The distance that the players run over the duration of the game is nonzero. The displacement of the players over the duration of the game is approximately zero because they keep returning to the same point over and over again.

particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

Average velocity

where the subscript x indicates motion along the x axis. From this definition we see that average velocity has dimensions of length divided by time (L/T)—meters per second in SI units.

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval Δt is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), then Δx is positive and $\bar{v}_x = \Delta x/\Delta t$ is positive. This case corresponds to a particle moving in the positive x direction, that is, toward larger values of x. If the coordinate decreases in time (that is, if $x_f < x_i$) then Δx is negative and hence $\bar{v}_x$ is negative. This case corresponds to a particle moving in the negative x direction.

We can interpret average velocity geometrically by drawing a straight line between any two points on the position–time graph in Figure 2.1b. This line forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$, which is what we have defined as average velocity in Equation 2.2. For example, the line between positions Ⓐ and Ⓑ in Figure 2.1b has a slope equal to the average velocity of the car between those two times, $(52\text{ m} - 30\text{ m})/(10\text{ s} - 0) = 2.2\text{ m/s}$.

In everyday usage, the terms *speed* and *velocity* are interchangeable. In physics, however, there is a clear distinction between these two quantities. Consider a marathon runner who runs more than 40 km, yet ends up at his starting point. His total displacement is zero, so his average velocity is zero! Nonetheless, we need to be able to quantify how fast he was running. A slightly different ratio accomplishes this for us. The **average speed** of a particle, a scalar quantity, is defined as **the total distance traveled divided by the total time interval required to travel that distance:**

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} \tag{2.3}$$

Average speed

The SI unit of average speed is the same as the unit of average velocity: meters per second. However, unlike average velocity, average speed has no direction and hence carries no algebraic sign. Notice the distinction between average velocity and average speed—average velocity (Eq. 2.2) is the *displacement* divided by the time interval, while average speed (Eq. 2.3) is the *distance* divided by the time interval.

Knowledge of the average velocity or average speed of a particle does not provide information about the details of the trip. For example, suppose it takes you 45.0 s to travel 100 m down a long straight hallway toward your departure gate at an airport. At the 100-m mark, you realize you missed the rest room, and you return back 25.0 m along the same hallway, taking 10.0 s to make the return trip. The magnitude of the average *velocity* for your trip is $+75.0\text{ m}/55.0\text{ s} = +1.36\text{ m/s}$. The average *speed* for your trip is $125\text{ m}/55.0\text{ s} = 2.27\text{ m/s}$. You may have traveled at various speeds during the walk. Neither average velocity nor average speed provides information about these details.

PITFALL PREVENTION

2.1 Average Speed and Average Velocity

The magnitude of the average velocity is *not* the average speed. For example, consider the marathon runner discussed here. The magnitude of the average velocity is zero, but the average speed is clearly not zero.

Quick Quiz 2.1 Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval? (a) A particle moves in the $+x$ direction without reversing. (b) A particle moves in the $-x$ direction without reversing. (c) A particle moves in the $+x$ direction and then reverses the direction of its motion. (d) There are no conditions for which this is true.

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that

$$\Delta x = x_F - x_A = -53 \text{ m} - 30 \text{ m} = \boxed{-83 \text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}}$$

$$= \boxed{-1.7 \text{ m/s}}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1, because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Figure 2.1b, then the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ) for a total of 127 m. We find the car's average speed for this trip by dividing the distance by the total time (Eq. 2.3):

$$\text{Average speed} = \frac{127 \text{ m}}{50 \text{ s}} = \boxed{2.5 \text{ m/s}}$$

2.2 Instantaneous Velocity and Speed

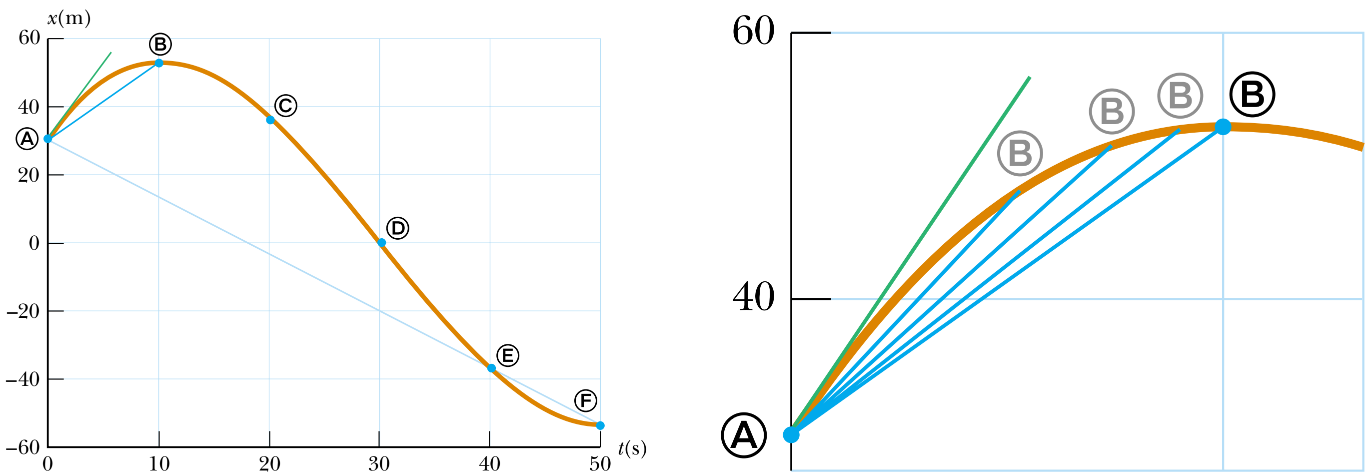
Often we need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval. For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant* you noticed the police car parked alongside the road ahead of you. In other words, you would like to be able to specify your velocity just as precisely as you can specify your position by noting what is happening at a specific clock reading—that is, at some specific instant. It may not be immediately obvious how to do this. What does it mean to talk about how fast something is moving if we "freeze time" and talk only about an individual instant? This is a subtle point not thoroughly understood until the late 1600s. At that time, with the invention of calculus, scientists began to understand how to describe an object's motion at any moment in time.

To see how this is done, consider Figure 2.3a, which is a reproduction of the graph in Figure 2.1b. We have already discussed the average velocity for the interval during which the car moved from position Ⓐ to position Ⓑ (given by the slope of the dark blue line) and for the interval during which it moved from Ⓐ to Ⓕ (represented by the slope of the light blue line and calculated in Example 2.1). Which of these two lines do you think is a closer approximation of the initial velocity of the car? The car starts out by moving to the right, which we defined to be the positive direction. Therefore, being positive, the value of the average velocity during the Ⓐ to Ⓑ interval is more representative of the initial value than is the value of the average velocity during the Ⓐ to Ⓕ interval, which we determined to be negative in Example 2.1. Now let us focus on the dark blue line and slide point Ⓑ to the left along the curve, toward point Ⓐ, as in Figure 2.3b. The line between the points becomes steeper and steeper, and as the two points become extremely close together, the line becomes a tangent line to the curve, indicated by the green line in Figure 2.3b. The slope of this tangent line

PITFALL PREVENTION

2.2 Slopes of Graphs

In any graph of physical data, the *slope* represents the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis. Remember that *a slope has units* (unless both axes have the same units). The units of slope in Figure 2.1b and Figure 2.3 are m/s, the units of velocity.



(a) (b)

Active Figure 2.3 (a) Graph representing the motion of the car in Figure 2.1. (b) An enlargement of the upper-left-hand corner of the graph shows how the blue line between positions Ⓐ and Ⓑ approaches the green tangent line as point Ⓑ is moved closer to point Ⓐ.

At the Active Figures link at* http://www.pse6.com, *you can move point Ⓑ as suggested in (b) and observe the blue line approaching the green tangent line.

represents the velocity of the car at the moment we started taking data, at point Ⓐ. What we have done is determine the *instantaneous velocity* at that moment. In other words, **the instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero:**[1]

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{2.4}$$

In calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

Instantaneous velocity

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at any time during the first 10 s in Figure 2.3, v_x is positive—the car is moving toward larger values of x. After point Ⓑ, v_x is negative because the slope is negative—the car is moving toward smaller values of x. At point Ⓑ, the slope and the instantaneous velocity are zero—the car is momentarily at rest.

From here on, we use the word *velocity* to designate instantaneous velocity. When it is *average velocity* we are interested in, we shall always use the adjective *average*.

The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity. As with average speed, instantaneous speed has no direction associated with it and hence carries no algebraic sign. For example, if one particle has an instantaneous velocity of $+25$ m/s along a given line and another particle has an instantaneous velocity of -25 m/s along the same line, both have a speed[2] of 25 m/s.

PITFALL PREVENTION

2.3 Instantaneous Speed and Instantaneous Velocity

In Pitfall Prevention 2.1, we argued that the magnitude of the average velocity is not the average speed. Notice the difference when discussing instantaneous values. The magnitude of the instantaneous velocity *is* the instantaneous speed. In an infinitesimal time interval, the magnitude of the displacement is equal to the distance traveled by the particle.

[1] Note that the displacement Δx also approaches zero as Δt approaches zero, so that the ratio looks like 0/0. As Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the slope of the line tangent to the x-versus-t curve.

[2] As with velocity, we drop the adjective for instantaneous speed: "Speed" means instantaneous speed.

Conceptual Example 2.2 The Velocity of Different Objects

Consider the following one-dimensional motions: **(A)** A ball thrown directly upward rises to a highest point and falls back into the thrower's hand. **(B)** A race car starts from rest and speeds up to 100 m/s. **(C)** A spacecraft drifts through space at constant velocity. Are there any points in the motion of these objects at which the instantaneous velocity has the same value as the average velocity over the entire motion? If so, identify the point(s).

Solution (A) The average velocity for the thrown ball is zero because the ball returns to the starting point; thus its displacement is zero. (Remember that average velocity is defined as $\Delta x/\Delta t$.) There is one point at which the instantaneous velocity is zero—at the top of the motion.

(B) The car's average velocity cannot be evaluated unambiguously with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity.

(C) Because the spacecraft's instantaneous velocity is constant, its instantaneous velocity at *any* time and its average velocity over *any* time interval are the same.

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution During the first time interval, the slope is negative and hence the average velocity is negative. Thus, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, we set $t_i = t_{\rm A} = 0$ and $t_f = t_{\rm B} = 1$ s. Using Equation 2.1, with $x = -4t + 2t^2$, we obtain for the displacement between $t = 0$ and $t = 1$ s,

$$\begin{aligned}\Delta x_{\rm A\to B} &= x_f - x_i = x_{\rm B} - x_{\rm A}\\ &= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]\\ &= \boxed{-2\text{ m}}\end{aligned}$$

To calculate the displacement during the second time interval ($t = 1$ s to $t = 3$ s), we set $t_i = t_{\rm B} = 1$ s and $t_f = t_{\rm D} = 3$ s:

$$\begin{aligned}\Delta x_{\rm B\to D} &= x_f - x_i = x_{\rm D} - x_{\rm B}\\ &= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]\\ &= \boxed{+8\text{ m}}\end{aligned}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

Solution In the first time interval, $\Delta t = t_f - t_i = t_{\rm B} - t_{\rm A} = 1$ s. Therefore, using Equation 2.2 and the displacement calculated in (a), we find that

$$\bar{v}_{x(\rm A\to B)} = \frac{\Delta x_{\rm A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

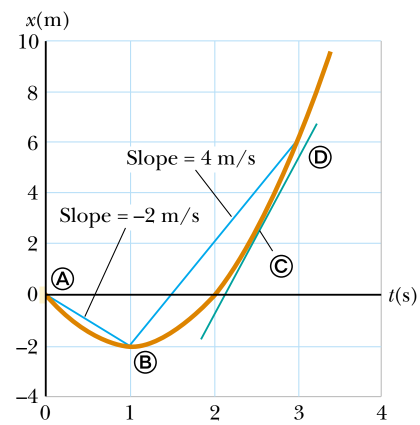


Figure 2.4 (Example 2.3) Position–time graph for a particle having an x coordinate that varies in time according to the expression $x = -4t + 2t^2$.

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(\rm B\to D)} = \frac{\Delta x_{\rm B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values are the same as the slopes of the lines joining these points in Figure 2.4.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution We can guess that this instantaneous velocity must be of the same order of magnitude as our previous results, that is, a few meters per second. By measuring the slope of the green line at $t = 2.5$ s in Figure 2.4, we find that

$$v_x = \boxed{+6\text{ m/s}}$$

[3] Simply to make it easier to read, we write the expression as $x = -4t + 2t^2$ rather than as $x = (-4.00\text{ m/s})t + (2.00\text{ m/s}^2)t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

2.3 Acceleration

In the last example, we worked with a situation in which the velocity of a particle changes while the particle is moving. This is an extremely common occurrence. (How constant is your velocity as you ride a city bus or drive on city streets?) It is possible to quantify changes in velocity as a function of time similarly to the way in which we quantify changes in position as a function of time. When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the magnitude of the velocity of a car increases when you step on the gas and decreases when you apply the brakes. Let us see how to quantify acceleration.

Suppose an object that can be modeled as a particle moving along the x axis has an initial velocity v_{xi} at time t_i and a final velocity v_{xf} at time t_f, as in Figure 2.5a.

The average acceleration $\bar{a}_x$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.6)$$

Average acceleration

As with velocity, when the motion being analyzed is one-dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). It might be easier to interpret these units if you think of them as meters per second per second. For example, suppose an object has an acceleration of $+2\ m/s^2$. You should form a mental image of the object having a velocity that is along a straight line and is increasing by 2 m/s during every interval of 1 s. If the object starts from rest, you should be able to picture it moving at a velocity of $+2$ m/s after 1 s, at $+4$ m/s after 2 s, and so on.

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the *instantaneous acceleration* as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we imagine that point Ⓐ is brought closer and closer to point Ⓑ in Figure 2.5a and we take the limit of $\Delta v_x/\Delta t$ as Δt approaches zero, we obtain the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.7)$$

Instantaneous acceleration

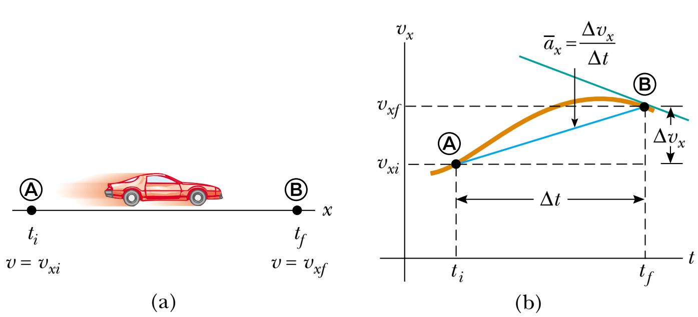


Figure 2.5 (a) A car, modeled as a particle, moving along the x axis from Ⓐ to Ⓑ has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph (rust) for the particle moving in a straight line. The slope of the blue straight line connecting Ⓐ and Ⓑ is the average acceleration in the time interval $\Delta t = t_f - t_i$.

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time,** which by definition is the slope of the velocity–time graph. The slope of the green line in Figure 2.5b is equal to the instantaneous acceleration at point Ⓑ. Thus, we see that just as the velocity of a moving particle is the slope at a point on the particle's x-t graph, the acceleration of a particle is the slope at a point on the particle's v_x-t graph. One can interpret the derivative of the velocity with respect to time as the time rate of change of velocity. If a_x is positive, the acceleration is in the positive x direction; if a_x is negative, the acceleration is in the negative x direction.

For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows. **When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.**

To help with this discussion of the signs of velocity and acceleration, we can relate the acceleration of an object to the *force* exerted on the object. In Chapter 5 we formally establish that **force is proportional to acceleration:**

$$F \propto a$$

This proportionality indicates that acceleration is caused by force. Furthermore, force and acceleration are both vectors and the vectors act in the same direction. Thus, let us think about the signs of velocity and acceleration by imagining a force applied to an object and causing it to accelerate. Let us assume that the velocity and acceleration are in the same direction. This situation corresponds to an object moving in some direction that experiences a force acting in the same direction. In this case, the object speeds up! Now suppose the velocity and acceleration are in opposite directions. In this situation, the object moves in some direction and experiences a force acting in the opposite direction. Thus, the object slows down! It is very useful to equate the direction of the acceleration to the direction of a force, because it is easier from our everyday experience to think about what effect a force will have on an object than to think only in terms of the direction of the acceleration.

PITFALL PREVENTION

2.4 Negative Acceleration

Keep in mind that *negative acceleration does not necessarily mean that an object is slowing down.* If the acceleration is negative, and the velocity is negative, the object is speeding up!

PITFALL PREVENTION

2.5 Deceleration

The word *deceleration* has the common popular connotation of *slowing down.* We will not use this word in this text, because it further confuses the definition we have given for negative acceleration.

Quick Quiz 2.2 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? (a) eastward (b) westward (c) neither of these.

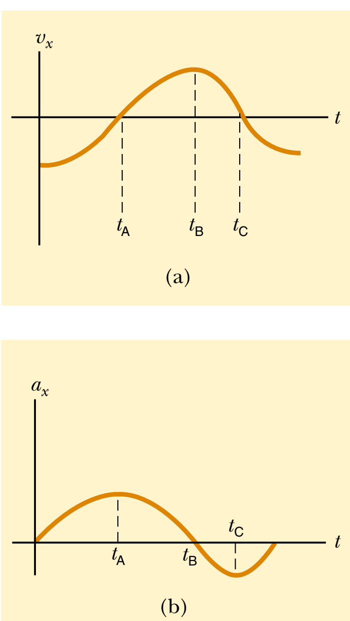


Figure 2.6 The instantaneous acceleration can be obtained from the velocity–time graph (a). At each instant, the acceleration in the a_x versus t graph (b) equals the slope of the line tangent to the v_x versus t curve (a).

From now on we shall use the term *acceleration* to mean instantaneous acceleration. When we mean average acceleration, we shall always use the adjective *average.*

Because $v_x = dx/dt$, the acceleration can also be written

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.8}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of x with respect to time.

Figure 2.6 illustrates how an acceleration–time graph is related to a velocity–time graph. The acceleration at any time is the slope of the velocity–time graph at that time. Positive values of acceleration correspond to those points in Figure 2.6a where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time t_A, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_B, when the velocity is a maximum (that is, when the slope of the v_x-t graph is zero). The acceleration is negative when the velocity is decreasing in the positive x direction, and it reaches its most negative value at time t_C.

Quick Quiz 2.3 Make a velocity–time graph for the car in Figure 2.1a. The speed limit posted on the road sign is 30 km/h. True or false? The car exceeds the speed limit at some time within the interval.

Conceptual Example 2.4 Graphical Relationships between x, v_x, and a_x

The position of an object moving along the x axis varies with time as in Figure 2.7a. Graph the velocity versus time and the acceleration versus time for the object.

Solution The velocity at any instant is the slope of the tangent to the x-t graph at that instant. Between $t = 0$ and $t = t_A$, the slope of the x-t graph increases uniformly, and so the velocity increases linearly, as shown in Figure 2.7b. Between t_A and t_B, the slope of the x-t graph is constant, and so the velocity remains constant. At t_D, the slope of the x-t graph is zero, so the velocity is zero at that instant. Between t_D and t_E, the slope of the x-t graph and thus the velocity are negative and decrease uniformly in this interval. In the interval t_E to t_F, the slope of the x-t graph is still negative, and at t_F it goes to zero. Finally, after t_F, the slope of the x-t graph is zero, meaning that the object is at rest for $t > t_F$.

The acceleration at any instant is the slope of the tangent to the v_x-t graph at that instant. The graph of acceleration versus time for this object is shown in Figure 2.7c. The acceleration is constant and positive between 0 and t_A, where the slope of the v_x-t graph is positive. It is zero between t_A and t_B and for $t > t_F$ because the slope of the v_x-t graph is zero at these times. It is negative between t_B and t_E because the slope of the v_x-t graph is negative during this interval.

Note that the sudden changes in acceleration shown in Figure 2.7c are unphysical. Such instantaneous changes cannot occur in reality.

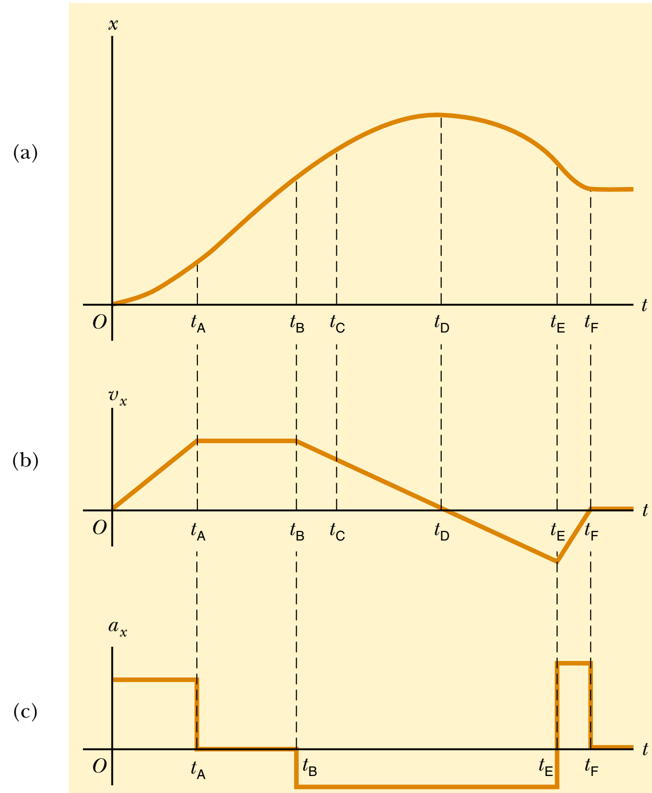


Figure 2.7 (Example 2.4) (a) Position–time graph for an object moving along the x axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Example 2.5 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A{}^2)\ \text{m/s} = [40 - 5(0)^2]\ \text{m/s} = +40\ \text{m/s}$$

$$v_{xB} = (40 - 5t_B{}^2)\ \text{m/s} = [40 - 5(2.0)^2]\ \text{m/s} = +20\ \text{m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\ \text{m/s}}{(2.0 - 0)\ \text{s}}$$

$$= \boxed{-10\ \text{m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2] \text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t \text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0) \text{ m/s}^2 = \boxed{-20 \text{ m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

Note that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.8 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Note also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.5.

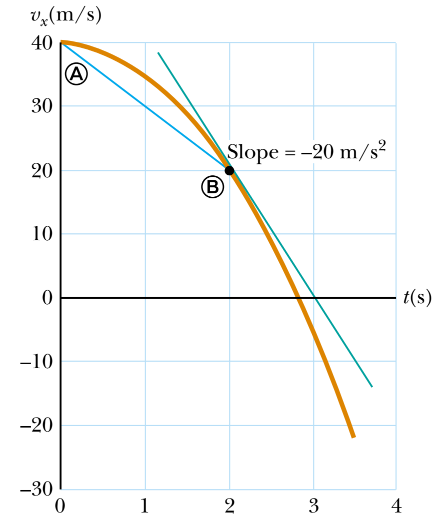


Figure 2.8 (Example 2.5) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. If you are familiar with calculus, you should recognize that there are specific rules for taking derivatives. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly. For instance, one rule tells us that the derivative of any constant is zero. As another example, suppose x is proportional to some power of t, such as in the expression

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 2.5, in which $v_x = 40 - 5t^2$, we find that the acceleration is $a_x = dv_x/dt = -10t$.

2.4 Motion Diagrams

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. It is instructive to use motion diagrams to describe the velocity and acceleration while an object is in motion.

A *stroboscopic* photograph of a moving object shows several images of the object, taken as the strobe light flashes at a constant rate. Figure 2.9 represents three sets of strobe photographs of cars moving along a straight roadway in a single direction, from left to right. The time intervals between flashes of the stroboscope are equal in each part of the diagram. In order not to confuse the two vector quantities, we use red for velocity vectors and violet for acceleration vectors in Figure 2.9. The vectors are

2.5 One-Dimensional Motion with Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the acceleration is constant. When this is the case, the average acceleration $\overline{a}_x$ over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.

If we replace $\overline{a}_x$ by a_x in Equation 2.6 and take $t_i = 0$ and t_f to be any later time t, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

or

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)} \tag{2.9}$$

This powerful expression enables us to determine an object's velocity at *any* time t if we know the object's initial velocity v_{xi} and its (constant) acceleration a_x. A velocity–time graph for this constant-acceleration motion is shown in Figure 2.10b. The graph is a straight line, the (constant) slope of which is the acceleration a_x; this is consistent with the fact that $a_x = dv_x/dt$ is a constant. Note that the slope is positive; this indicates a positive acceleration. If the acceleration were negative, then the slope of the line in Figure 2.10b would be negative.

When the acceleration is constant, the graph of acceleration versus time (Fig. 2.10c) is a straight line having a slope of zero.

Because velocity at constant acceleration varies linearly in time according to Equation 2.9, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity v_{xi} and the final velocity v_{xf}:

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)} \tag{2.10}$$

Note that this expression for average velocity applies *only* in situations in which the acceleration is constant.

We can now use Equations 2.1, 2.2, and 2.10 to obtain the position of an object as a function of time. Recalling that Δx in Equation 2.2 represents $x_f - x_i$, and recognizing that $\Delta t = t_f - t_i = t - 0 = t$, we find

$$x_f - x_i = \overline{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})t$$

Position as a function of velocity and time

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \qquad \text{(for constant } a_x\text{)} \tag{2.11}$$

This equation provides the final position of the particle at time t in terms of the initial and final velocities.

We can obtain another useful expression for the position of a particle moving with constant acceleration by substituting Equation 2.9 into Equation 2.11:

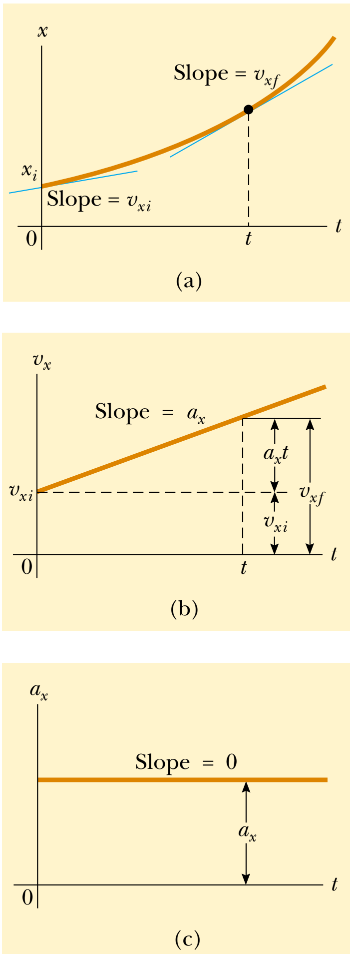
$$x_f = x_i + \tfrac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

Position as a function of time

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \text{(for constant } a_x\text{)} \tag{2.12}$$

This equation provides the final position of the particle at time t in terms of the initial velocity and the acceleration.

The position–time graph for motion at constant (positive) acceleration shown in Figure 2.10a is obtained from Equation 2.12. Note that the curve is a parabola.



Active Figure 2.10 A particle moving along the x axis with constant acceleration a_x; (a) the position–time graph, (b) the velocity–time graph, and (c) the acceleration–time graph.

At the Active Figures link at http://www.pse6.com, you can adjust the constant acceleration and observe the effect on the position and velocity graphs.

The slope of the tangent line to this curve at $t = 0$ equals the initial velocity v_{xi}, and the slope of the tangent line at any later time t equals the velocity v_{xf} at that time.

Finally, we can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation 2.9 into Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = \frac{v_{xf}^{\,2} - v_{xi}^{\,2}}{2a_x}$$

$$v_{xf}^{\,2} = v_{xi}^{\,2} + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)} \tag{2.13}$$

Velocity as a function of position

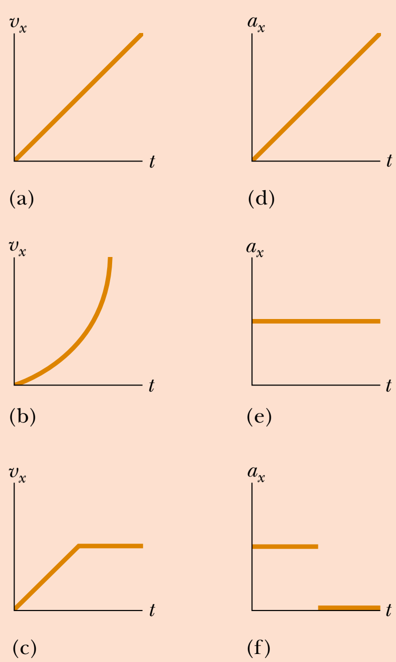
This equation provides the final velocity in terms of the acceleration and the displacement of the particle.

For motion at *zero* acceleration, we see from Equations 2.9 and 2.12 that

$$\left.\begin{aligned} v_{xf} &= v_{xi} = v_x \\ x_f &= x_i + v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration of a particle is zero, its velocity is constant and its position changes linearly with time.

Quick Quiz 2.5 In Figure 2.11, match each v_x-t graph on the left with the a_x-t graph on the right that best describes the motion.



Active Figure 2.11 (Quick Quiz 2.5) Parts (a), (b), and (c) are v_x-t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

At the Active Figures link at* http://www.pse6.com, *you can practice matching appropriate velocity vs. time graphs and acceleration vs. time graphs.

Equations 2.9 through 2.13 are **kinematic equations that may be used to solve any problem involving one-dimensional motion at constant acceleration.** Keep in mind that these relationships were derived from the definitions of velocity and

Table 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the *x* axis.

acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant.

The four kinematic equations used most often are listed in Table 2.2 for convenience. The choice of which equation you use in a given situation depends on what you know beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns. For example, suppose initial velocity v_{xi} and acceleration a_x are given. You can then find (1) the velocity at time t, using $v_{xf} = v_{xi} + a_x t$ and (2) the position at time t, using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. You should recognize that the quantities that vary during the motion are position, velocity, and time.

You will gain a great deal of experience in the use of these equations by solving a number of exercises and problems. Many times you will discover that more than one method can be used to obtain a solution. Remember that these equations of kinematics *cannot* be used in a situation in which the acceleration varies with time. They can be used only when the acceleration is constant.

Example 2.6 Entering the Traffic Flow

(A) Estimate your average acceleration as you drive up the entrance ramp to an interstate highway.

Solution This problem involves more than our usual amount of estimating! We are trying to come up with a value of a_x, but that value is hard to guess directly. The other variables involved in kinematics are position, velocity, and time. Velocity is probably the easiest one to approximate. Let us assume a final velocity of 100 km/h, so that you can merge with traffic. We multiply this value by (1 000 m/1 km) to convert kilometers to meters and then multiply by (1 h/3 600 s) to convert hours to seconds. These two calculations together are roughly equivalent to dividing by 3. In fact, let us just say that the final velocity is $v_{xf} \approx 30$ m/s. (Remember, this type of approximation and the dropping of digits when performing estimations is okay. If you were starting with U.S. customary units, you could approximate 1 mi/h as roughly 0.5 m/s and continue from there.)

Now we assume that you started up the ramp at about one third your final velocity, so that $v_{xi} \approx 10$ m/s. Finally, we assume that it takes about 10 s to accelerate from v_{xi} to v_{xf}, basing this guess on our previous experience in automobiles. We can then find the average acceleration, using Equation 2.6:

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{30 \text{ m/s} - 10 \text{ m/s}}{10 \text{ s}}$$

$$= \boxed{2 \text{ m/s}^2}$$

Granted, we made many approximations along the way, but **this type of mental effort can be surprisingly useful and often yields results that are not too different from those derived from careful measurements.** Do not be afraid to attempt making educated guesses and doing some fairly drastic number rounding to simplify estimations. Physicists engage in this type of thought analysis all the time.

(B) How far did you go during the first half of the time interval during which you accelerated?

Solution Let us assume that the acceleration is constant, with the value calculated in part (A). Because the motion takes place in a straight line and the velocity is always in the same direction, the distance traveled from the starting point is equal to the final position of the car. We can calculate the final position at 5 s from Equation 2.12:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\approx 0 + (10 \text{ m/s})(5 \text{ s}) + \frac{1}{2}(2 \text{ m/s}^2)(5 \text{ s})^2 = 50 \text{ m} + 25 \text{ m}$$

$$= \boxed{75 \text{ m}}$$

This result indicates that if you had not accelerated, your initial velocity of 10 m/s would have resulted in a 50-m movement up the ramp during the first 5 s. The additional 25 m is the result of your increasing velocity during that interval.

Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

Solution We define our x axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also note that we have no information about the change in position of the jet while it is slowing down. Equation 2.9 is the only equation in Table 2.2 that does not involve position, and so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63 \text{ m/s}}{2.0 \text{ s}}$$

$$= \boxed{-31 \text{ m/s}^2}$$

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

Solution We can now use any of the other three equations in Table 2.2 to solve for the final position. Let us choose Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63 \text{ m/s} + 0)(2.0 \text{ s})$$

$$= \boxed{63 \text{ m}}$$

If the plane travels much farther than this, it might fall into the ocean. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of the first World War. The cables are still a vital part of the operation of modern aircraft carriers.

What If? **Suppose the plane lands on the deck of the aircraft carrier with a speed higher than 63 m/s but with the same acceleration as that calculated in part (A). How will that change the answer to part (B)?**

Answer If the plane is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.11 that if v_{xi} is larger, then x_f will be larger.

If the landing deck has a length of 75 m, we can find the maximum initial speed with which the plane can land and still come to rest on the deck at the given acceleration from Equation 2.13:

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$\rightarrow v_{xi} = \sqrt{v_{xf}^2 - 2a_x(x_f - x_i)}$$

$$= \sqrt{0 - 2(-31 \text{ m/s}^2)(75 \text{ m} - 0)}$$

$$= 68 \text{ m/s}$$

Example 2.8 Watch Out for the Speed Limit!

Interactive

A car traveling at a constant speed of 45.0 m/s passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

Solution Let us model the car and the trooper as particles. A sketch (Fig. 2.12) helps clarify the sequence of events.

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set $t_B = 0$ as the time the trooper begins moving. At that instant, the car has already traveled a distance of 45.0 m because it has traveled at a constant speed of $v_x = 45.0$ m/s for 1 s. Thus, the initial position of the speeding car is $x_B = 45.0$ m.

Because the car moves with constant speed, its acceleration is zero. Applying Equation 2.12 (with $a_x = 0$) gives for the car's position at any time t:

$$x_{\text{car}} = x_B + v_{x\,\text{car}}t = 45.0 \text{ m} + (45.0 \text{ m/s})t$$

A quick check shows that at $t = 0$, this expression gives the car's correct initial position when the trooper begins to move: $x_{\text{car}} = x_B = 45.0$ m.

The trooper starts from rest at $t_B = 0$ and accelerates at 3.00 m/s^2 away from the origin. Hence, her position at any time t can be found from Equation 2.12:

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$x_{\text{trooper}} = 0 + (0)t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2$$

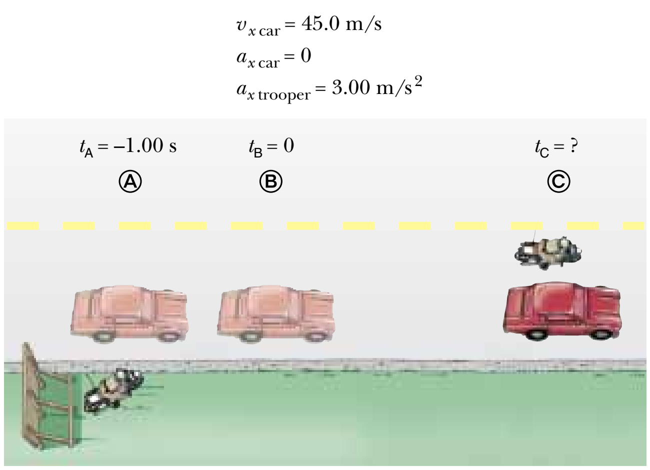


Figure 2.12 (Example 2.8) A speeding car passes a hidden trooper.

The trooper overtakes the car at the instant her position matches that of the car, which is position ©:

$$x_{\text{trooper}} = x_{\text{car}}$$

$$\tfrac{1}{2}(3.00\ \text{m/s}^2)t^2 = 45.0\ \text{m} + (45.0\ \text{m/s})t$$

This gives the quadratic equation

$$1.50t^2 - 45.0t - 45.0 = 0$$

The positive solution of this equation is $t =$ 31.0 s.

(For help in solving quadratic equations, see Appendix B.2.)

What If? **What if the trooper had a more powerful motorcycle with a larger acceleration? How would that change the time at which the trooper catches the car?**

Answer If the motorcycle has a larger acceleration, the trooper will catch up to the car sooner, so the answer for the time will be less than 31 s. Mathematically, let us cast the final quadratic equation above in terms of the parameters in the problem:

$$\tfrac{1}{2}a_x t^2 - v_{x\,\text{car}}t - x_{\text{B}} = 0$$

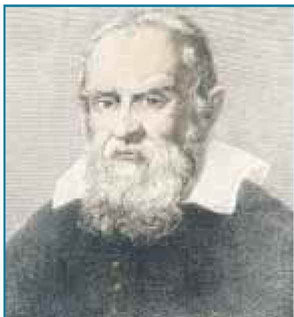
The solution to this quadratic equation is,

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_x x_{\text{B}}}}{a_x}$$

$$= \frac{v_{x\,\text{car}}}{a_x} + \sqrt{\frac{v_{x\,\text{car}}^2}{a_x^2} + \frac{2x_{\text{B}}}{a_x}}$$

where we have chosen the positive sign because that is the only choice consistent with a time $t > 0$. Because all terms on the right side of the equation have the acceleration a_x in the denominator, increasing the acceleration will decrease the time at which the trooper catches the car.

You can study the motion of the car and trooper for various velocities of the car at the Interactive Worked Example link at http://www.pse6.com.



Galileo Galilei

Italian physicist and astronomer (1564–1642)

Galileo formulated the laws that govern the motion of objects in free fall and made many other significant discoveries in physics and astronomy. Galileo publicly defended Nicholaus Copernicus's assertion that the Sun is at the center of the Universe (the heliocentric system). He published *Dialogue Concerning Two New World Systems* to support the Copernican model, a view which the Church declared to be heretical. *(North Wind)*

PITFALL PREVENTION

2.6 *g* and g

Be sure not to confuse the italicized symbol *g* for free-fall acceleration with the nonitalicized symbol g used as the abbreviation for "gram."

2.6 Freely Falling Objects

It is well known that, in the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration under the influence of the Earth's gravity. It was not until about 1600 that this conclusion was accepted. Before that time, the teachings of the great philosopher Aristotle (384–322 B.C.) had held that heavier objects fall faster than lighter ones.

The Italian Galileo Galilei (1564–1642) originated our present-day ideas concerning falling objects. There is a legend that he demonstrated the behavior of falling objects by observing that two different weights dropped simultaneously from the Leaning Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that he carried out this particular experiment, it is well established that Galileo performed many experiments on objects moving on inclined planes. In his experiments he rolled balls down a slight incline and measured the distances they covered in successive time intervals. The purpose of the incline was to reduce the acceleration; with the acceleration reduced, Galileo was able to make accurate measurements of the time intervals. By gradually increasing the slope of the incline, he was finally able to draw conclusions about freely falling objects because a freely falling ball is equivalent to a ball moving down a vertical incline.

You might want to try the following experiment. Simultaneously drop a coin and a crumpled-up piece of paper from the same height. If the effects of air resistance are negligible, both will have the same motion and will hit the floor at the same time. In the idealized case, in which air resistance is absent, such motion is referred to as *free-fall*. If this same experiment could be conducted in a vacuum, in which air resistance is truly negligible, the paper and coin would fall with the same acceleration even when the paper is not crumpled. On August 2, 1971, such a demonstration was conducted on the Moon by astronaut David Scott. He simultaneously released a hammer and a feather, and they fell together to the lunar surface. This demonstration surely would have pleased Galileo!

When we use the expression *freely falling object,* we do not necessarily refer to an object dropped from rest. **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they**

are released. Any freely falling object experiences an acceleration directed *downward*, regardless of its initial motion.

We shall denote the magnitude of the *free-fall acceleration* by the symbol g. The value of g near the Earth's surface decreases with increasing altitude. Furthermore, slight variations in g occur with changes in latitude. It is common to define "up" as the $+y$ direction and to use y as the position variable in the kinematic equations. At the Earth's surface, the value of g is approximately 9.80 m/s^2. Unless stated otherwise, we shall use this value for g when performing calculations. For making quick estimates, use $g = 10$ m/s^2.

If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one dimension under constant acceleration. Therefore, the equations developed in Section 2.5 for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than in the horizontal direction (x) and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, we always choose $a_y = -g = -9.80$ m/s^2, where the negative sign means that the acceleration of a freely falling object is downward. In Chapter 13 we shall study how to deal with variations in g with altitude.

Quick Quiz 2.6 A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

Quick Quiz 2.7 After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same.

PITFALL PREVENTION

2.7 The Sign of *g*

Keep in mind that g is a *positive number*—it is tempting to substitute -9.80 m/s^2 for g, but resist the temptation. Downward gravitational acceleration is indicated explicitly by stating the acceleration as $a_y = -g$.

PITFALL PREVENTION

2.8 Acceleration at the Top of The Motion

It is a common misconception that the acceleration of a projectile at the top of its trajectory is zero. While the velocity at the top of the motion of an object thrown upward momentarily goes to zero, *the acceleration is still that due to gravity* at this point. If the velocity and acceleration were both zero, the projectile would stay at the top!

Conceptual Example 2.9 The Daring Sky Divers

A sky diver jumps out of a hovering helicopter. A few seconds later, another sky diver jumps out, and they both fall along the same vertical line. Ignore air resistance, so that both sky divers fall with the same acceleration. Does the difference in their speeds stay the same throughout the fall? Does the vertical distance between them stay the same throughout the fall?

Solution At any given instant, the speeds of the divers are different because one had a head start. In any time interval Δt after this instant, however, the two divers increase their speeds by the same amount because they have the same acceleration. Thus, the difference in their speeds remains the same throughout the fall.

The first jumper always has a greater speed than the second. Thus, in a given time interval, the first diver covers a greater distance than the second. Consequently, the separation distance between them increases.

Example 2.10 Describing the Motion of a Tossed Ball

A ball is tossed straight up at 25 m/s. Estimate its velocity at 1-s intervals.

Solution Let us choose the upward direction to be positive. Regardless of whether the ball is moving upward or downward, its vertical velocity changes by approximately -10 m/s for every second it remains in the air. It starts out at 25 m/s. After 1 s has elapsed, it is still moving upward but at 15 m/s because its acceleration is downward (downward acceleration causes its velocity to decrease). After another second, its upward velocity has dropped to 5 m/s. Now comes the tricky part—after another half second, its velocity is zero. The ball has gone as high as it will go. After the last half of this 1-s interval, the ball is moving at -5 m/s. (The negative sign tells us that the ball is now moving in the negative direction, that is, *downward*. Its velocity has changed from $+5$ m/s to -5 m/s during that 1-s interval. The change in velocity is still $-5 \text{ m/s} - (+5 \text{ m/s}) = -10$ m/s in that second.) It continues downward, and after another 1 s has elapsed, it is falling at a velocity of -15 m/s. Finally, after another 1 s, it has reached its original starting point and is moving downward at -25 m/s.

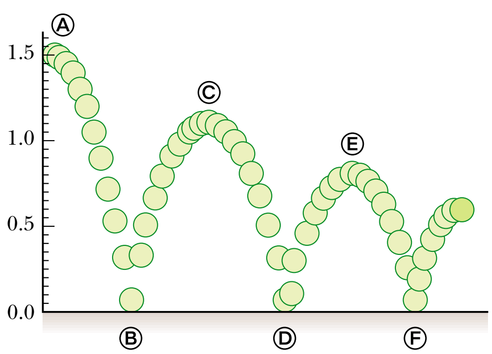
Conceptual Example 2.11 Follow the Bouncing Ball

A tennis ball is dropped from shoulder height (about 1.5 m) and bounces three times before it is caught. Sketch graphs of its position, velocity, and acceleration as functions of time, with the $+y$ direction defined as upward.

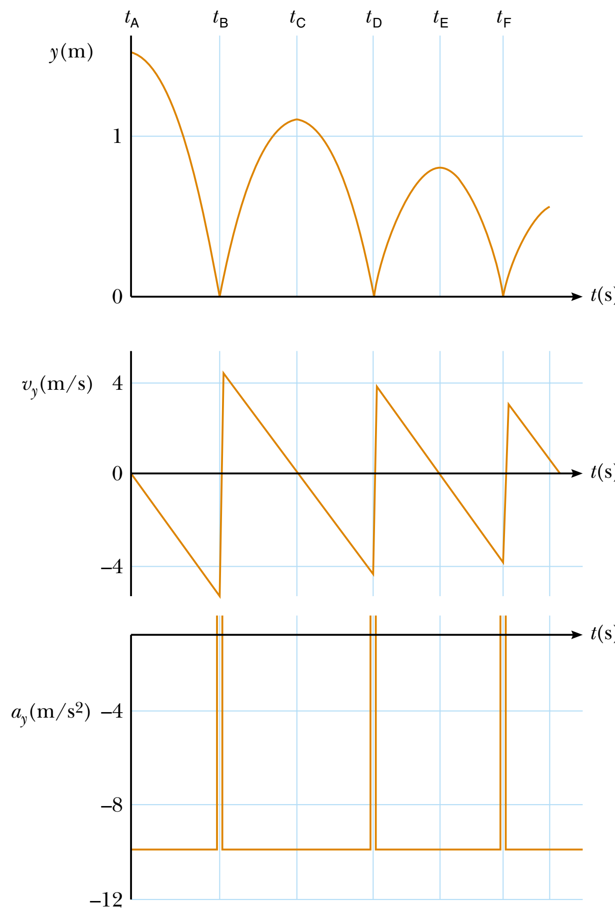
Solution For our sketch let us stretch things out horizontally so that we can see what is going on. (Even if the ball were moving horizontally, this motion would not affect its vertical motion.)

From Figure 2.13a we see that the ball is in contact with the floor at points Ⓑ, Ⓓ, and Ⓕ. Because the velocity of the ball changes from negative to positive three times during these bounces (Fig. 2.13b), the slope of the position–time graph must change in the same way. Note that the time interval between bounces decreases. Why is that?

During the rest of the ball's motion, the slope of the velocity–time graph in Fig. 2.13b should be $-9.80\ \text{m/s}^2$. The acceleration–time graph is a horizontal line at these times because the acceleration does not change when the ball is in free fall. When the ball is in contact with the floor, the velocity changes substantially during a very short time interval, and so the acceleration must be quite large and positive. This corresponds to the very steep upward lines on the velocity–time graph and to the spikes on the acceleration–time graph.



(a)



(b)

Active Figure 2.13 (Conceptual Example 2.11) (a) A ball is dropped from a height of 1.5 m and bounces from the floor. (The horizontal motion is not considered here because it does not affect the vertical motion.) (b) Graphs of position, velocity, and acceleration versus time.

At the Active Figures link at* http://www.pse6.com, *you can adjust both the value for g and the amount of "bounce" of the ball, and observe the resulting motion of the ball both pictorially and graphically.

Quick Quiz 2.8 Which values represent the ball's vertical velocity and acceleration at points Ⓐ, Ⓒ, and Ⓔ in Figure 2.13a?

(a) $v_y = 0$, $a_y = -9.80\ \text{m/s}^2$

(b) $v_y = 0$, $a_y = 9.80\ \text{m/s}^2$

(c) $v_y = 0$, $a_y = 0$

(d) $v_y = -9.80\ \text{m/s}$, $a_y = 0$

Example 2.12 Not a Bad Throw for a Rookie! Interactive

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.

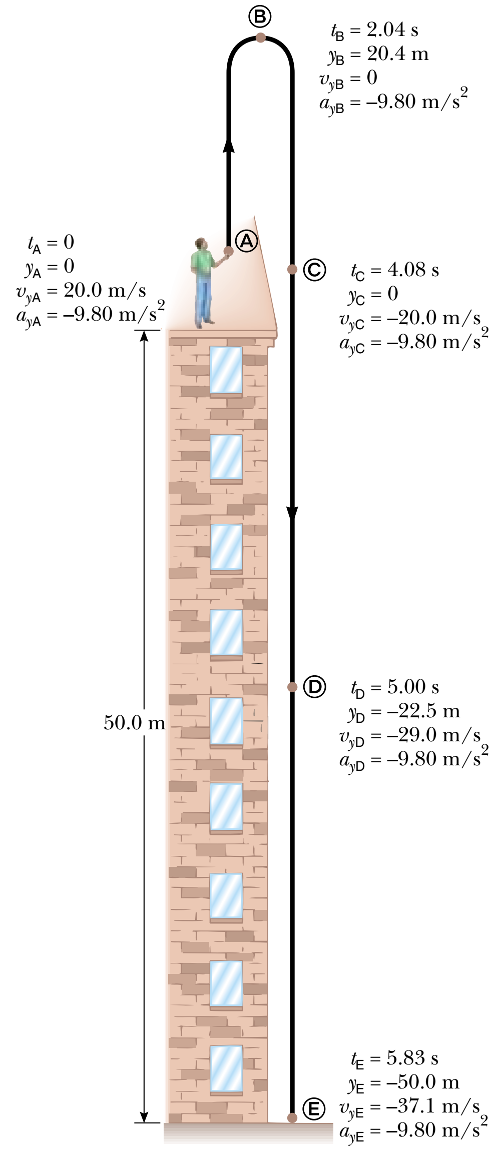


Figure 2.14 (Example 2.12) Position and velocity versus time for a freely falling stone thrown initially upward with a velocity $v_{yi} = 20.0$ m/s.

Solution (A) As the stone travels from Ⓐ to Ⓑ, its velocity must change by 20 m/s because it stops at Ⓑ. Because gravity causes vertical velocities to change by about 10 m/s for every second of free fall, it should take the stone about 2 s to go from Ⓐ to Ⓑ in our drawing. To calculate the exact time t_B at which the stone reaches maximum height, we use Equation 2.9, $v_{yB} = v_{yA} + a_y t$, noting that $v_{yB} = 0$ and setting the start of our clock readings at $t_A = 0$:

$$0 = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

$$t = t_B = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

Our estimate was pretty close.

(B) Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and because it travels for about 2 s, we expect the stone to travel about 20 m. By substituting our time into Equation 2.12, we can find the maximum height as measured from the position of the thrower, where we set $y_A = 0$:

$$y_{\text{max}} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

Our free-fall estimates are very accurate.

(C) There is no reason to believe that the stone's motion from Ⓑ to Ⓒ is anything other than the reverse of its motion from Ⓐ to Ⓑ. The motion from Ⓐ to Ⓒ is symmetric. Thus, the time needed for it to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ. When the stone is back at the height from which it was thrown (position Ⓒ), the y coordinate is again zero. Using Equation 2.12, with $y_C = 0$, we obtain

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $\boxed{t = 4.08 \text{ s},}$ which is the solution we are after. Notice that it is double the value we calculated for t_B.

(D) Again, we expect everything at Ⓒ to be the same as it is at Ⓐ, except that the velocity is now in the opposite direction. The value for t found in (c) can be inserted into Equation 2.9 to give

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$

$$= \boxed{-20.0 \text{ m/s}}$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction.

(E) For this part we ignore the first part of the motion (Ⓐ→Ⓑ) and consider what happens as the stone falls from position Ⓑ, where it has zero vertical velocity, to position Ⓓ. We define the initial time as $t_B = 0$. Because the given time for this part of the motion relative to our new zero of time is $5.00\text{ s} - 2.04\text{ s} = 2.96\text{ s}$, we estimate that the acceleration due to gravity will have changed the speed by about 30 m/s. We can calculate this from Equation 2.9, where we take $t = 2.96$ s:

$$v_{yD} = v_{yB} + a_y t = 0\text{ m/s} + (-9.80\text{ m/s}^2)(2.96\text{ s})$$
$$= \boxed{-29.0\text{ m/s}}$$

We could just as easily have made our calculation between positions Ⓐ (where we return to our original initial time $t_A = 0$) and Ⓓ:

$$v_{yD} = v_{yA} + a_y t = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)(5.00\text{ s})$$
$$= -29.0\text{ m/s}$$

To further demonstrate that we can choose different initial instants of time, let us use Equation 2.12 to find the position of the stone at $t_D = 5.00$ s (with respect to $t_A = 0$) by defining a new initial instant, $t_C = 0$:

$$y_D = y_C + v_{yC}t + \tfrac{1}{2}a_y t^2$$
$$= 0 + (-20.0\text{ m/s})(5.00\text{ s} - 4.08\text{ s})$$
$$+ \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s} - 4.08\text{ s})^2$$
$$= \boxed{-22.5\text{ m}}$$

What If? What if the building were 30.0 m tall instead of 50.0 m tall? Which answers in parts (A) to (E) would change?

Answer None of the answers would change. All of the motion takes place in the air, and the stone does not interact with the ground during the first 5.00 s. (Notice that even for a 30.0-m tall building, the stone is above the ground at $t = 5.00$ s.) Thus, the height of the building is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height of the building into any equation.

You can study the motion of the thrown ball at the Interactive Worked Example link at http://www.pse6.com.

2.7 Kinematic Equations Derived from Calculus

This section assumes the reader is familiar with the techniques of integral calculus. If you have not yet studied integration in your calculus course, you should skip this section or cover it after you become familiar with integration.

The velocity of a particle moving in a straight line can be obtained if its position as a function of time is known. Mathematically, the velocity equals the derivative of the position with respect to time. It is also possible to find the position of a particle if its velocity is known as a function of time. In calculus, the procedure used to perform this task is referred to either as *integration* or as finding the *antiderivative*. Graphically, it is equivalent to finding the area under a curve.

Suppose the v_x-t graph for a particle moving along the x axis is as shown in Figure 2.15. Let us divide the time interval $t_f - t_i$ into many small intervals, each of

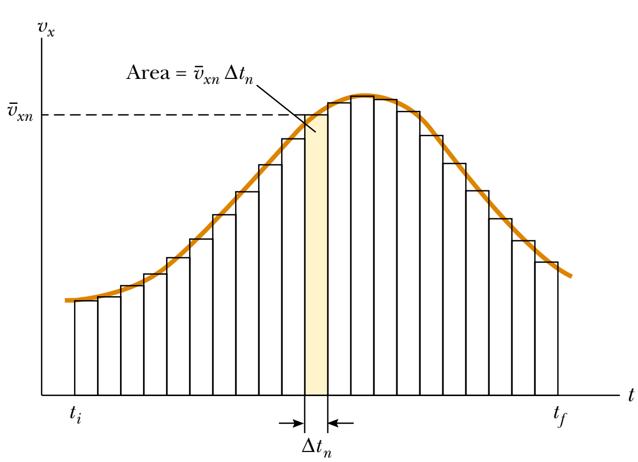


Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, while the total area under the curve is the total displacement of the particle.

SUMMARY

Take a practice test for this chapter by clicking the Practice Test link at http://www.pse6.com.

After a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x \equiv x_f - x_i \tag{2.1}$$

The **average velocity** of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t} \tag{2.2}$$

The **average speed** of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} \tag{2.3}$$

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero. By definition, this limit equals the derivative of x with respect to t, or the time rate of change of the position:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

The **instantaneous speed** of a particle is equal to the magnitude of its instantaneous velocity.

The **average acceleration** of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \tag{2.6}$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches 0. By definition, this limit equals the derivative of v_x with respect to t, or the time rate of change of the velocity:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \tag{2.7}$$

When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down. Remembering that $F \propto a$ is a useful way to identify the direction of the acceleration.

The **equations of kinematics** for a particle moving along the x axis with uniform acceleration a_x (constant in magnitude and direction) are

$$v_{xf} = v_{xi} + a_x t \tag{2.9}$$

$$x_f = x_i + \bar{v}_x t = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \tag{2.11}$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \tag{2.12}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \tag{2.13}$$

An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2.

Complicated problems are best approached in an organized manner. You should be able to recall and apply the *Conceptualize, Categorize, Analyze,* and *Finalize* steps of the General Problem-Solving Strategy when you need them.

QUESTIONS

1. The speed of sound in air is 331 m/s. During the next thunderstorm, try to estimate your distance from a lightning bolt by measuring the time lag between the flash and the thunderclap. You can ignore the time it takes for the light flash to reach you. Why?

2. The average velocity of a particle moving in one dimension has a positive value. Is it possible for the instantaneous velocity to have been negative at any time in the interval? Suppose the particle started at the origin $x = 0$. If its average velocity is positive, could the particle ever have been in the $-x$ region of the axis?

3. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?

4. Can the instantaneous velocity of an object at an instant of time ever be greater in magnitude than the average velocity over a time interval containing the instant? Can it ever be less?

5. If an object's average velocity is nonzero over some time interval, does this mean that its instantaneous velocity is never zero during the interval? Explain your answer.

6. If an object's average velocity is zero over some time interval, show that its instantaneous velocity must be zero at some time during the interval. It may be useful in your proof to sketch a graph of x versus t and to note that $v_x(t)$ is a continuous function.

7. If the velocity of a particle is nonzero, can its acceleration be zero? Explain.

8. If the velocity of a particle is zero, can its acceleration be nonzero? Explain.

9. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of A is greater than that of B? Explain.

10. Is it possible for the velocity and the acceleration of an object to have opposite signs? If not, state a proof. If so, give an example of such a situation and sketch a velocity–time graph to prove your point.

11. Consider the following combinations of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis:

Velocity	*Acceleration*
a. Positive	Positive
b. Positive	Negative
c. Positive	Zero
d. Negative	Positive
e. Negative	Negative
f. Negative	Zero
g. Zero	Positive
h. Zero	Negative

Describe what a particle is doing in each case, and give a real life example for an automobile on an east-west one-dimensional axis, with east considered the positive direction.

12. Can the equations of kinematics (Eqs. 2.9–2.13) be used in a situation where the acceleration varies in time? Can they be used when the acceleration is zero?

13. A stone is thrown vertically upward from the roof of a building. Does the position of the stone depend on the location chosen for the origin of the coordinate system? Does the stone's velocity depend on the choice of origin? Explain your answers.

14. A child throws a marble into the air with an initial speed v_i. Another child drops a ball at the same instant. Compare the accelerations of the two objects while they are in flight.

15. A student at the top of a building of height h throws one ball upward with a speed of v_i and then throws a second ball downward with the same initial speed, v_i. How do the final velocities of the balls compare when they reach the ground?

16. An object falls freely from height h. It is released at time zero and strikes the ground at time t. (a) When the object is at height $0.5h$, is the time earlier than $0.5t$, equal to $0.5t$, or later than $0.5t$? (b) When the time is $0.5t$, is the height of the object greater than $0.5h$, equal to $0.5h$, or less than $0.5h$? Give reasons for your answers.

17. You drop a ball from a window on an upper floor of a building. It strikes the ground with speed v. You now repeat the drop, but you have a friend down on the street who throws another ball upward at speed v. Your friend throws the ball upward at exactly the same time that you drop yours from the window. At some location, the balls pass each other. Is this location *at* the halfway point between window and ground, *above* this point, or *below* this point?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 2.1 Position, Velocity, and Speed

1. The position of a pinewood derby car was observed at various times; the results are summarized in the following table. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

t(s)	0	1.0	2.0	3.0	4.0	5.0
x(m)	0	2.3	9.2	20.7	36.8	57.5

2. (a) Sand dunes in a desert move over time as sand is swept up the windward side to settle in the lee side. Such "walking" dunes have been known to walk 20 feet in a year and can travel as much as 100 feet per year in particularly windy times. Calculate the average speed in each case in m/s. (b) Fingernails grow at the rate of drifting continents, on the order of 10 mm/yr. Approximately how long did it take for North America to separate from Europe, a distance of about 3 000 mi?

3. The position versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.

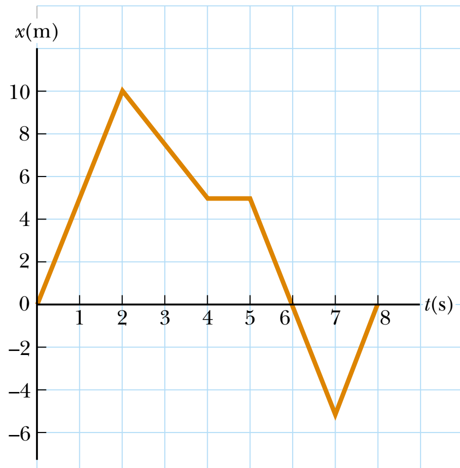


Figure P2.3 Problems 3 and 9

4. A particle moves according to the equation $x = 10t^2$ where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 to 2.10 s.

5. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. What is (a) her average speed over the entire trip? (b) her average velocity over the entire trip?

Section 2.2 Instantaneous Velocity and Speed

6. The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in meters and t is in seconds. Evaluate its position (a) at $t = 3.00$ s and (b) at $3.00\text{ s} + \Delta t$. (c) Evaluate the limit of $\Delta x/\Delta t$ as Δt approaches zero, to find the velocity at $t = 3.00$ s.

7. A position-time graph for a particle moving along the x axis is shown in Figure P2.7. (a) Find the average velocity in the time interval $t = 1.50$ s to $t = 4.00$ s. (b) Determine the instantaneous velocity at $t = 2.00$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

8. (a) Use the data in Problem 1 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this, determine the average acceleration of the car. (d) What was the initial velocity of the car?

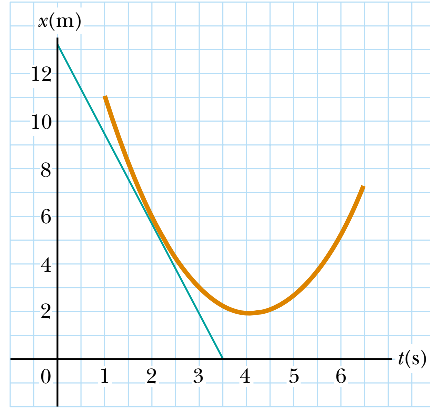


Figure P2.7

9. Find the instantaneous velocity of the particle described in Figure P2.3 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

10. A hare and a tortoise compete in a race over a course 1.00 km long. The tortoise crawls straight and steadily at its maximum speed of 0.200 m/s toward the finish line. The hare runs at its maximum speed of 8.00 m/s toward the goal for 0.800 km and then stops to tease the tortoise. How close to the goal can the hare let the tortoise approach before resuming the race, which the tortoise wins in a photo finish? Assume that, when moving, both animals move steadily at their respective maximum speeds.

Section 2.3 Acceleration

11. A 50.0-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (*Note:* 1 ms = 10^{-3} s.)

12. A particle starts from rest and accelerates as shown in Figure P2.12. Determine (a) the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and (b) the distance traveled in the first 20.0 s.

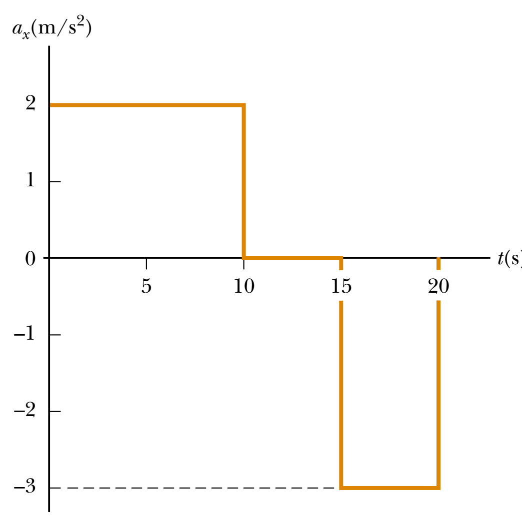


Figure P2.12

13. Secretariat won the Kentucky Derby with times for successive quarter-mile segments of 25.2 s, 24.0 s, 23.8 s, and 23.0 s. (a) Find his average speed during each quarter-mile segment. (b) Assuming that Secretariat's instantaneous speed at the finish line was the same as the average speed during the final quarter mile, find his average acceleration for the entire race. (Horses in the Derby start from rest.)

14. A velocity–time graph for an object moving along the x axis is shown in Figure P2.14. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals $t = 5.00$ s to $t = 15.0$ s and $t = 0$ to $t = 20.0$ s.

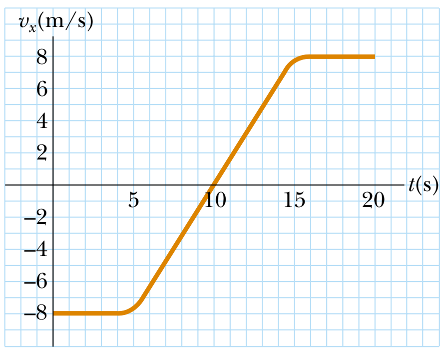


Figure P2.14

15. A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

16. An object moves along the x axis according to the equation $x(t) = (3.00t^2 - 2.00t + 3.00)$ m. Determine (a) the average speed between $t = 2.00$ s and $t = 3.00$ s, (b) the instantaneous speed at $t = 2.00$ s and at $t = 3.00$ s, (c) the average acceleration between $t = 2.00$ s and $t = 3.00$ s, and (d) the instantaneous acceleration at $t = 2.00$ s and $t = 3.00$ s.

17. Figure P2.17 shows a graph of v_x versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t = 0$ to $t = 6.00$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

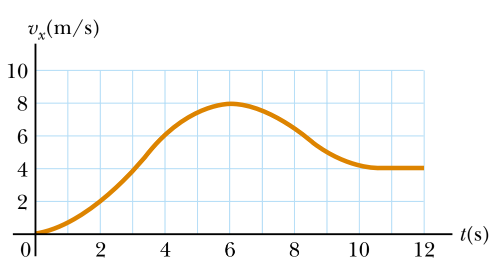


Figure P2.17

Section 2.4 Motion Diagrams

18. Draw motion diagrams for (a) an object moving to the right at constant speed, (b) an object moving to the right and speeding up at a constant rate, (c) an object moving to the right and slowing down at a constant rate, (d) an object moving to the left and speeding up at a constant rate, and (e) an object moving to the left and slowing down at a constant rate. (f) How would your drawings change if the changes in speed were not uniform; that is, if the speed were not changing at a constant rate?

Section 2.5 One-Dimensional Motion with Constant Acceleration

19. Jules Verne in 1865 suggested sending people to the Moon by firing a space capsule from a 220-m-long cannon with a launch speed of 10.97 km/s. What would have been the unrealistically large acceleration experienced by the space travelers during launch? Compare your answer with the free-fall acceleration 9.80 m/s^2.

20. A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

21. An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

22. A 745i BMW car can brake to a stop in a distance of 121 ft. from a speed of 60.0 mi/h. To brake to a stop from a speed of 80.0 mi/h requires a stopping distance of 211 ft. What is the average braking acceleration for (a) 60 mi/h to rest, (b) 80 mi/h to rest, (c) 80 mi/h to 60 mi/h? Express the answers in mi/h/s and in m/s^2.

23. A speedboat moving at 30.0 m/s approaches a no-wake buoy marker 100 m ahead. The pilot slows the boat with a constant acceleration of -3.50 m/s^2 by reducing the throttle. (a) How long does it take the boat to reach the buoy? (b) What is the velocity of the boat when it reaches the buoy?

24. Figure P2.24 represents part of the performance data of a car owned by a proud physics student. (a) Calculate from the graph the total distance traveled. (b) What distance does the car travel between the times $t = 10$ s and $t = 40$ s? (c) Draw a graph of its acceleration versus time between $t = 0$ and $t = 50$ s. (d) Write an equation for x as a function of time for each phase of the motion, represented by (i) $0a$, (ii) ab, (iii) bc. (e) What is the average velocity of the car between $t = 0$ and $t = 50$ s?

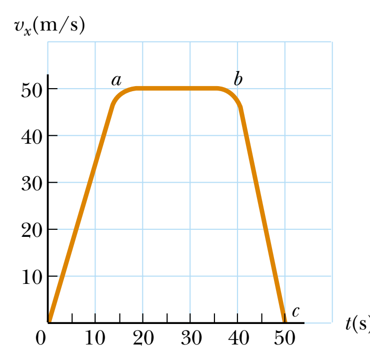


Figure P2.24

25. A particle moves along the x axis. Its position is given by the equation $x = 2 + 3t - 4t^2$ with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

26. In the Daytona 500 auto race, a Ford Thunderbird and a Mercedes Benz are moving side by side down a straightaway at 71.5 m/s. The driver of the Thunderbird realizes he must make a pit stop, and he smoothly slows to a stop over a distance of 250 m. He spends 5.00 s in the pit and then accelerates out, reaching his previous speed of 71.5 m/s after a distance of 350 m. At this point, how far has the Thunderbird fallen behind the Mercedes Benz, which has continued at a constant speed?

27. A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of $-5.00\ \text{m/s}^2$ as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long?

28. A car is approaching a hill at 30.0 m/s when its engine suddenly fails just at the bottom of the hill. The car moves with a constant acceleration of $-2.00\ \text{m/s}^2$ while coasting up the hill. (a) Write equations for the position along the slope and for the velocity as functions of time, taking $x = 0$ at the bottom of the hill, where $v_i = 30.0$ m/s. (b) Determine the maximum distance the car rolls up the hill.

29. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of $-5.60\ \text{m/s}^2$ for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

30. *Help! One of our equations is missing!* We describe constant-acceleration motion with the variables and parameters v_{xi}, v_{xf}, a_x, t, and $x_f - x_i$. Of the equations in Table 2.2, the first does not involve $x_f - x_i$. The second does not contain a_x; the third omits v_{xf} and the last leaves out t. So to complete the set there should be an equation *not* involving v_{xi}. Derive it from the others. Use it to solve Problem 29 in one step.

31. For many years Colonel John P. Stapp, USAF, held the world's land speed record. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at a speed of 632 mi/h. He and the sled were safely brought to rest in 1.40 s (Fig. P2.31). Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.

32. A truck on a straight road starts from rest, accelerating at $2.00\ \text{m/s}^2$ until it reaches a speed of 20.0 m/s. Then the truck travels for 20.0 s at constant speed until the brakes are applied, stopping the truck in a uniform manner in an additional 5.00 s. (a) How long is the truck in motion? (b) What is the average velocity of the truck for the motion described?

33. An electron in a cathode ray tube (CRT) accelerates from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) How long does the electron take to travel this 1.50 cm? (b) What is its acceleration?

34. In a 100-m linear accelerator, an electron is accelerated to 1.00% of the speed of light in 40.0 m before it coasts for 60.0 m to a target. (a) What is the electron's acceleration during the first 40.0 m? (b) How long does the total flight take?

35. Within a complex machine such as a robotic assembly line, suppose that one particular part glides along a straight track. A control system measures the average velocity of the part during each successive interval of time $\Delta t_0 = t_0 - 0$, compares it with the value v_c it should be, and switches a servo motor on and off to give the part a correcting pulse of acceleration. The pulse consists of a constant acceleration a_m applied for time interval $\Delta t_m = t_m - 0$ within the next control time interval Δt_0. As shown in Fig. P2.35, the part may be modeled as having zero acceleration when the motor is off (between t_m and t_0). A computer in the control system chooses the size of the acceleration so that the final velocity of the part will have the correct value v_c. Assume the part is initially at rest and is to have instantaneous velocity v_c at time t_0. (a) Find the required value of a_m in terms of v_c and t_m. (b) Show that the displacement Δx of the part during the time interval Δt_0 is given by $\Delta x = v_c\,(t_0 - 0.5t_m)$. For specified values of v_c and t_0, (c) what is the minimum displacement of the part? (d) What is the maximum displacement of the part? (e) Are both the minimum and maximum displacements physically attainable?

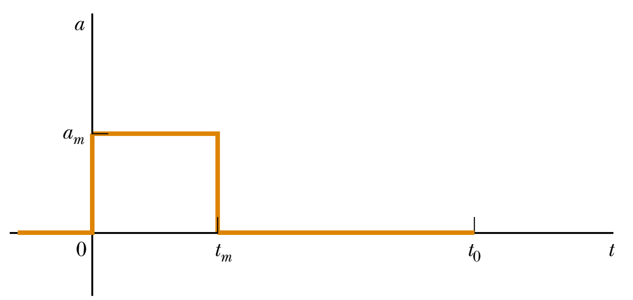
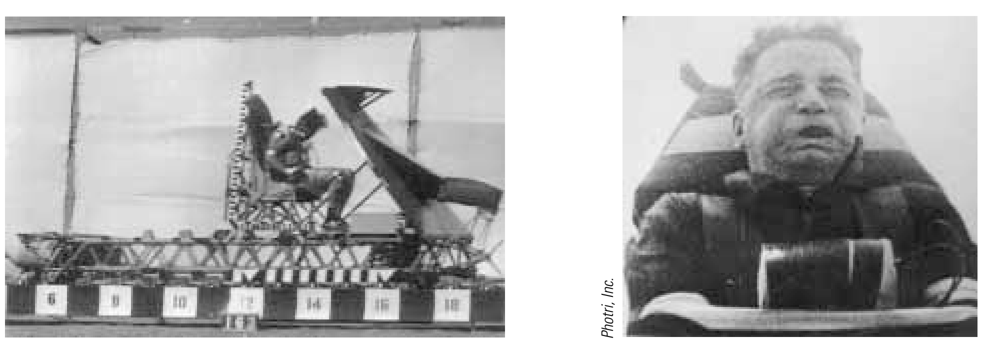


Figure P2.35



Courtesy U.S. Air Force

Photri, Inc.

Figure P2.31 *(Left)* Col. John Stapp on rocket sled. *(Right)* Col. Stapp's face is contorted by the stress of rapid negative acceleration.

36. A glider on an air track carries a flag of length ℓ through a stationary photogate, which measures the time interval Δt_d during which the flag blocks a beam of infrared light passing across the photogate. The ratio $v_d = \ell/\Delta t_d$ is the average velocity of the glider over this part of its motion. Suppose the glider moves with constant acceleration. (a) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in space. (b) Argue for or against the idea that v_d is equal to the instantaneous velocity of the glider when it is halfway through the photogate in time.

37. A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?

38. Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.

39. Solve Example 2.8, "Watch out for the Speed Limit!" by a graphical method. On the same graph plot position versus time for the car and the police officer. From the intersection of the two curves read the time at which the trooper overtakes the car.

Section 2.6 Freely Falling Objects

> *Note:* In all problems in this section, ignore the effects of air resistance.

40. A golf ball is released from rest from the top of a very tall building. Neglecting air resistance, calculate (a) the position and (b) the velocity of the ball after 1.00, 2.00, and 3.00 s.

41. *Every morning at seven o'clock*
There's twenty terriers drilling on the rock.
The boss comes around and he says, "Keep still
And bear down heavy on the cast-iron drill

And drill, ye terriers, drill." And drill, ye terriers, drill.
It's work all day for sugar in your tea
Down beyond the railway. And drill, ye terriers, drill.

The foreman's name was John McAnn.
By God, he was a blamed mean man.
One day a premature blast went off
And a mile in the air went big Jim Goff. And drill ...

Then when next payday came around
Jim Goff a dollar short was found.
When he asked what for, came this reply:
"You were docked for the time you were up in the sky." And drill...
—American folksong

What was Goff's hourly wage? State the assumptions you make in computing it.

42. A ball is thrown directly downward, with an initial speed of 8.00 m/s, from a height of 30.0 m. After what time interval does the ball strike the ground?

43. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?

44. Emily challenges her friend David to catch a dollar bill as follows. She holds the bill vertically, as in Figure P2.44, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. If his reaction time is 0.2 s, will he succeed? Explain your reasoning.

George Semple

Figure P2.44

45. In Mostar, Bosnia, the ultimate test of a young man's courage once was to jump off a 400-year-old bridge (now destroyed) into the River Neretva, 23.0 m below the bridge. (a) How long did the jump last? (b) How fast was the diver traveling upon impact with the water? (c) If the speed of sound in air is 340 m/s, how long after the diver took off did a spectator on the bridge hear the splash?

46. A ball is dropped from rest from a height h above the ground. Another ball is thrown vertically upwards from the ground at the instant the first ball is released. Determine the speed of the second ball if the two balls are to meet at a height $h/2$ above the ground.

47. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the height it reaches.

48. It is possible to shoot an arrow at a speed as high as 100 m/s. (a) If friction is neglected, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?

49. A daring ranch hand sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The constant speed of the horse is 10.0 m/s, and the distance from the limb to the level of the saddle is 3.00 m. (a) What must be the horizontal distance between the saddle and limb when the ranch hand makes his move? (b) How long is he in the air?

50. A woman is reported to have fallen 144 ft from the 17th floor of a building, landing on a metal ventilator box, which she crushed to a depth of 18.0 in. She suffered only minor injuries. Neglecting air resistance, calculate (a) the speed of the woman just before she collided with the ventilator, (b) her average acceleration while in contact with the box, and (c) the time it took to crush the box.

51. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

52. A freely falling object requires 1.50 s to travel the last 30.0 m before it hits the ground. From what height above the ground did it fall?

Section 2.7 Kinematic Equations Derived from Calculus

53. Automotive engineers refer to the time rate of change of acceleration as the "jerk." If an object moves in one dimension such that its jerk J is constant, (a) determine expressions for its acceleration $a_x(t)$, velocity $v_x(t)$, and position $x(t)$, given that its initial acceleration, velocity, and position are a_{xi}, v_{xi}, and x_i, respectively. (b) Show that $a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})$.

54. A student drives a moped along a straight road as described by the velocity-versus-time graph in Figure P2.54. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x-t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?

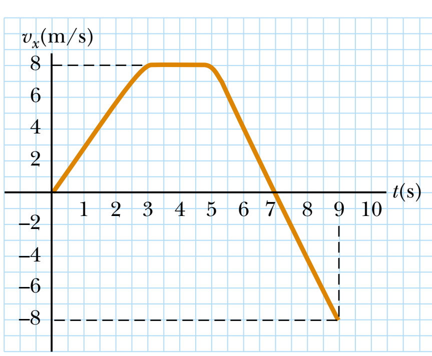


Figure P2.54

55. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by $v = (-5.00 \times 10^7)t^2 + (3.00 \times 10^5)t$, where v is in meters per second and t is in seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine the length of time the bullet is accelerated. (c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

56. The acceleration of a marble in a certain fluid is proportional to the speed of the marble squared, and is given (in SI units) by $a = -3.00v^2$ for $v > 0$. If the marble enters this fluid with a speed of 1.50 m/s, how long will it take before the marble's speed is reduced to half of its initial value?

Additional Problems

57. A car has an initial velocity v_0 when the driver sees an obstacle in the road in front of him. His reaction time is Δt_r, and the braking acceleration of the car is a. Show that the total stopping distance is

$$s_{\text{stop}} = v_0 \Delta t_r - v_0^2/2a.$$

Remember that a is a negative number.

58. The yellow caution light on a traffic signal should stay on long enough to allow a driver to either pass through the intersection or safely stop before reaching the intersection. A car can stop if its distance from the intersection is greater than the stopping distance found in the previous problem. If the car is less than this stopping distance from the intersection, the yellow light should stay on long enough to allow the car to pass entirely through the intersection. (a) Show that the yellow light should stay on for a time interval

$$\Delta t_{\text{light}} = \Delta t_r - (v_0/2a) + (s_i/v_0)$$

where Δt_r is the driver's reaction time, v_0 is the velocity of the car approaching the light at the speed limit, a is the braking acceleration, and s_i is the width of the intersection. (b) As city traffic planner, you expect cars to approach an intersection 16.0 m wide with a speed of 60.0 km/h. Be cautious and assume a reaction time of 1.10 s to allow for a driver's indecision. Find the length of time the yellow light should remain on. Use a braking acceleration of -2.00 m/s^2.

59. The Acela is the Porsche of American trains. Shown in Figure P2.59a, the electric train whose name is pronounced ah-SELL-ah is in service on the Washington-New York-Boston run. With two power cars and six coaches, it can carry 304 passengers at 170 mi/h. The carriages tilt as much as 6° from the vertical to prevent passengers from feeling pushed to the side as they go around curves. Its braking mechanism uses electric generators to recover its energy of motion. A velocity-time graph for the Acela is shown in Figure P2.59b. (a) Describe the motion of the train in each successive time interval. (b) Find the peak positive acceleration of the train in the motion graphed. (c) Find the train's displacement in miles between $t = 0$ and $t = 200$ s.

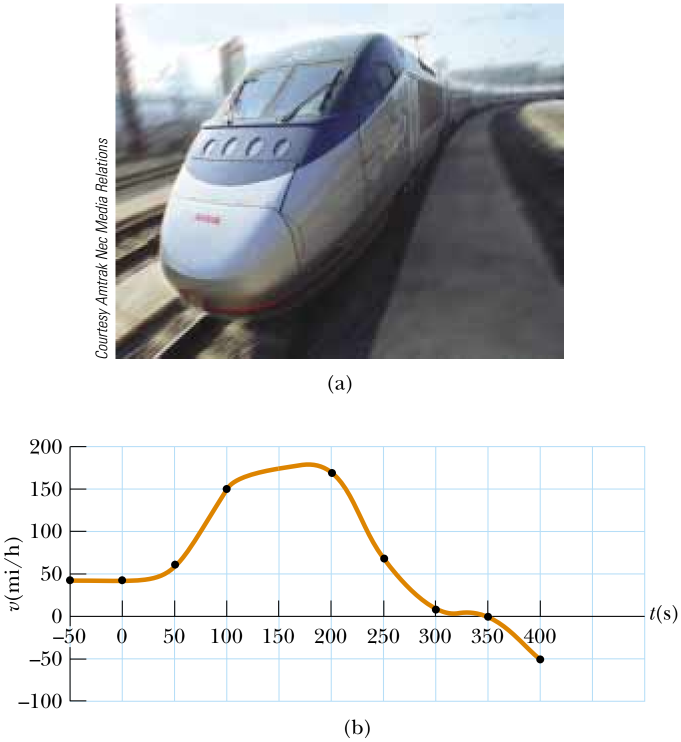


Figure P2.59 (a) The Acela—1 171 000 lb of cold steel thundering along at 150 mi/h. (b) Velocity-versus-time graph for the Acela.

60. Liz rushes down onto a subway platform to find her train already departing. She stops and watches the cars go by. Each car is 8.60 m long. The first moves past her in 1.50 s and the second in 1.10 s. Find the constant acceleration of the train.

61. A dog's hair has been cut and is now getting 1.04 mm longer each day. With winter coming on, this rate of hair growth is steadily increasing, by 0.132 mm/day every week. By how much will the dog's hair grow during 5 weeks?

62. A test rocket is fired vertically upward from a well. A catapult gives it an initial speed of 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of -9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion.)

63. A motorist drives along a straight road at a constant speed of 15.0 m/s. Just as she passes a parked motorcycle police officer, the officer starts to accelerate at 2.00 m/s^2 to overtake her. Assuming the officer maintains this acceleration, (a) determine the time it takes the police officer to reach the motorist. Find (b) the speed and (c) the total displacement of the officer as he overtakes the motorist.

64. In Figure 2.10b, the area under the velocity versus time curve and between the vertical axis and time t (vertical dashed line) represents the displacement. As shown, this area consists of a rectangle and a triangle. Compute their areas and compare the sum of the two areas with the expression on the right-hand side of Equation 2.12.

65. Setting a new world record in a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. Accelerating uniformly, Maggie took 2.00 s and Judy 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark, and by how much?

66. A commuter train travels between two downtown stations. Because the stations are only 1.00 km apart, the train never reaches its maximum possible cruising speed. During rush hour the engineer minimizes the time interval Δt between two stations by accelerating for a time interval Δt_1 at a rate $a_1 = 0.100$ m/s^2 and then immediately braking with acceleration $a_2 = -0.500$ m/s^2 for a time interval Δt_2. Find the minimum time interval of travel Δt and the time interval Δt_1.

67. A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Suppose that the maximum depth of the dent is on the order of 1 cm. Compute an order-of-magnitude estimate for the maximum acceleration of the ball while it is in contact with the pavement. State your assumptions, the quantities you estimate, and the values you estimate for them.

68. At NASA's John H. Glenn research center in Cleveland, Ohio, free-fall research is performed by dropping experiment packages from the top of an evacuated shaft 145 m high. Free fall imitates the so-called microgravity environment of a satellite in orbit. (a) What is the maximum time interval for free fall if an experiment package were to fall the entire 145 m? (b) Actual NASA specifications allow for a 5.18-s drop time interval. How far do the packages drop and (c) what is their speed at 5.18 s? (d) What constant acceleration would be required to stop an experiment package in the distance remaining in the shaft after its 5.18-s fall?

69. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What initial velocity must the second stone have if they are to hit simultaneously? (c) What is the speed of each stone at the instant the two hit the water?

70. A rock is dropped from rest into a well. The well is not really 16 seconds deep, as in Figure P2.70. (a) The sound of the splash is actually heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

71. To protect his food from hungry bears, a boy scout raises his food pack with a rope that is thrown over a tree limb at height h above his hands. He walks away from the vertical rope with constant velocity v_{boy}, holding the free end of the rope in his hands (Fig. P2.71). (a) Show that the speed v of the food pack is given by $x(x^2 + h^2)^{-1/2}\, v_{\text{boy}}$ where x

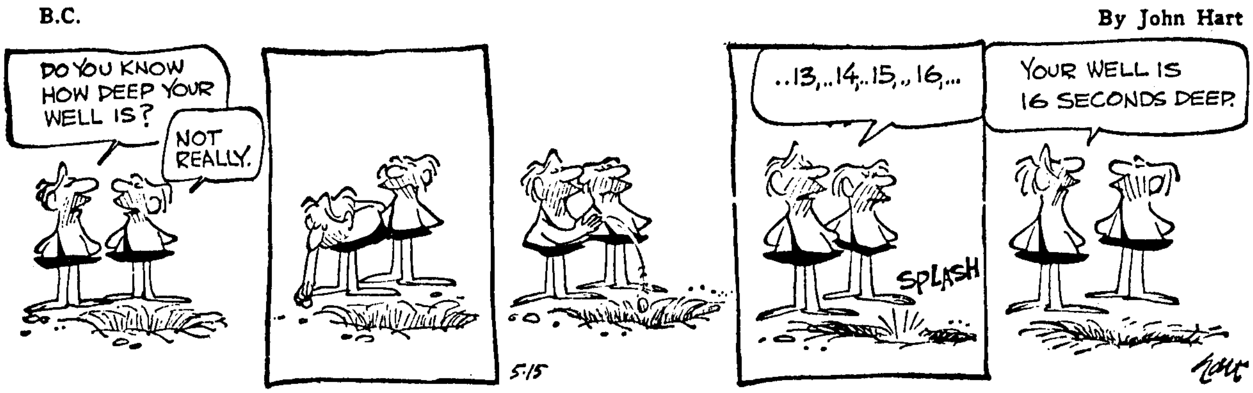


Figure P2.70

is the distance he has walked away from the vertical rope. (b) Show that the acceleration a of the food pack is $h^2(x^2 + h^2)^{-3/2}\, v_{\text{boy}}^2$. (c) What values do the acceleration a and velocity v have shortly after he leaves the point under the pack ($x = 0$)? (d) What values do the pack's velocity and acceleration approach as the distance x continues to increase?

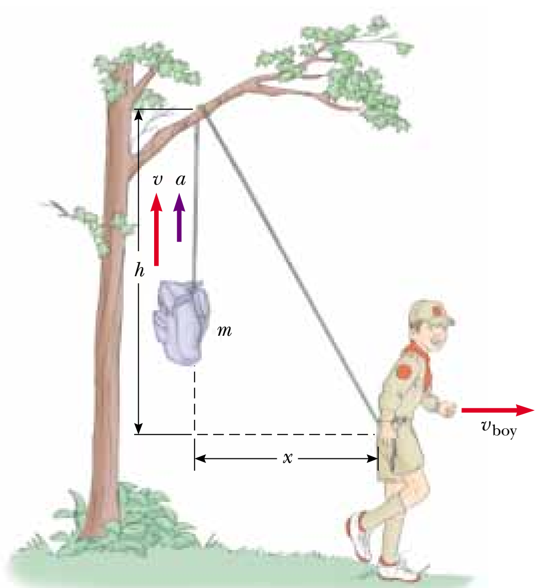


Figure P2.71 Problems 71 and 72.

72. In Problem 71, let the height h equal 6.00 m and the speed v_{boy} equal 2.00 m/s. Assume that the food pack starts from rest. (a) Tabulate and graph the speed–time graph. (b) Tabulate and graph the acceleration-time graph. Let the range of time be from 0 s to 5.00 s and the time intervals be 0.500 s.

73. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by racing with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time at which Kathy overtakes Stan, (b) the distance she travels before she catches him, and (c) the speeds of both cars at the instant she overtakes him.

74. Astronauts on a distant planet toss a rock into the air. With the aid of a camera that takes pictures at a steady rate, they record the height of the rock as a function of time as given in Table P2.74. (a) Find the average velocity of the rock in the time interval between each measurement and the next. (b) Using these average velocities to approximate instantaneous velocities at the midpoints of the time intervals, make a graph of velocity as a function of time. Does the rock move with constant acceleration? If so, plot a straight line of best fit on the graph and calculate its slope to find the acceleration.

Table P2.74

Height of a Rock versus Time

Time (s)	Height (m)	Time (s)	Height (m)
0.00	5.00	2.75	7.62
0.25	5.75	3.00	7.25
0.50	6.40	3.25	6.77
0.75	6.94	3.50	6.20
1.00	7.38	3.75	5.52
1.25	7.72	4.00	4.73
1.50	7.96	4.25	3.85
1.75	8.10	4.50	2.86
2.00	8.13	4.75	1.77
2.25	8.07	5.00	0.58
2.50	7.90		

75. Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.75. If A slides to the left with a constant speed v, find the velocity of B when $\alpha = 60.0°$.

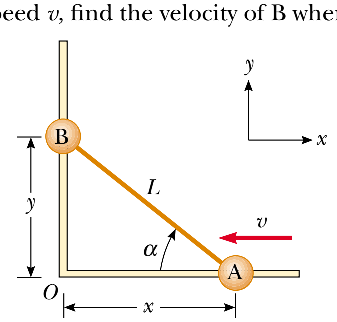


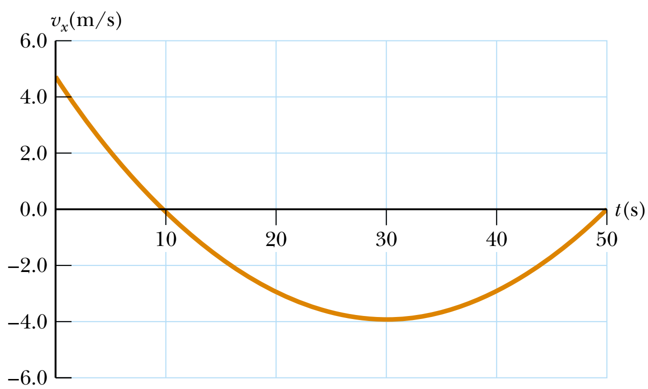
Figure P2.75

Answers to Quick Quizzes

2.1 (c). If the particle moves along a line without changing direction, the displacement and distance traveled over any time interval will be the same. As a result, the magnitude of the average velocity and the average speed will be the same. If the particle reverses direction, however, the displacement will be less than the distance traveled. In turn, the magnitude of the average velocity will be smaller than the average speed.

2.2 (b). If the car is slowing down, a force must be pulling in the direction opposite to its velocity.

2.3 False. Your graph should look something like the following. This v_x-t graph shows that the maximum speed is about 5.0 m/s, which is 18 km/h (= 11 mi/h), and so the driver was not speeding.



Answer to Quick Quiz 2.3

2.4 (c). If a particle with constant acceleration stops and its acceleration remains constant, it must begin to move again in the opposite direction. If it did not, the acceleration would change from its original constant value to zero. Choice (a) is not correct because the direction of acceleration is not specified by the direction of the velocity. Choice (b) is also not correct by counterexample—a car moving in the $-x$ direction and slowing down has a positive acceleration.

2.5 Graph (a) has a constant slope, indicating a constant acceleration; this is represented by graph (e).

Graph (b) represents a speed that is increasing constantly but not at a uniform rate. Thus, the acceleration must be increasing, and the graph that best indicates this is (d).

Graph (c) depicts a velocity that first increases at a constant rate, indicating constant acceleration. Then the velocity stops increasing and becomes constant, indicating zero acceleration. The best match to this situation is graph (f).

2.6 (e). For the entire time interval that the ball is in free fall, the acceleration is that due to gravity.

2.7 (d). While the ball is rising, it is slowing down. After reaching the highest point, the ball begins to fall and its speed increases.

2.8 (a). At the highest point, the ball is momentarily at rest, but still accelerating at $-g$.

Chapter 3

Vectors

CHAPTER OUTLINE

▲ *These controls in the cockpit of a commercial aircraft assist the pilot in maintaining control over the* velocity *of the aircraft—how fast it is traveling and in what direction it is traveling—allowing it to land safely. Quantities that are defined by both a magnitude and a direction, such as velocity, are called* vector quantities. *(Mark Wagner/Getty Images)*

In our study of physics, we often need to work with physical quantities that have both numerical and directional properties. As noted in Section 2.1, quantities of this nature are vector quantities. This chapter is primarily concerned with vector algebra and with some general properties of vector quantities. We discuss the addition and subtraction of vector quantities, together with some common applications to physical situations.

Vector quantities are used throughout this text, and it is therefore imperative that you master both their graphical and their algebraic properties.

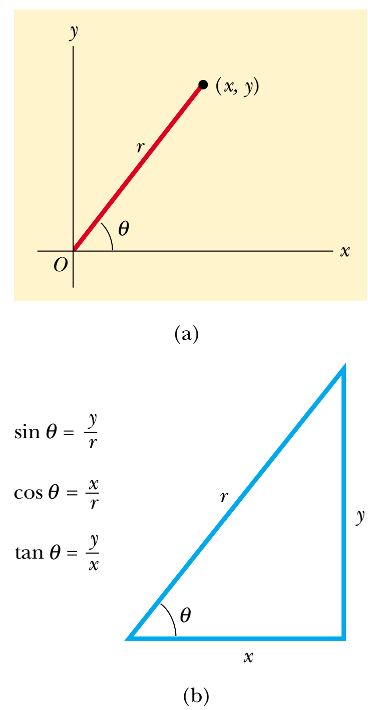
3.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. In Chapter 2, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. This description is accomplished with the use of coordinates, and in Chapter 2 we used the Cartesian coordinate system, in which horizontal and vertical axes intersect at a point defined as the origin (Fig. 3.1). Cartesian coordinates are also called *rectangular coordinates.*

Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ), as shown in Figure 3.2a. In this *polar coordinate system,* r is the distance from the origin to the point having Cartesian coordinates (x, y), and θ is the angle between a line drawn from the origin to the point and a fixed axis. This fixed axis is usually the positive x axis, and θ is usually measured counterclockwise from it. From the right triangle in Figure 3.2b, we find that $\sin\theta = y/r$ and that $\cos\theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations

$$x = r\cos\theta \tag{3.1}$$

$$y = r\sin\theta \tag{3.2}$$



Active Figure 3.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ).

At the Active Figures link at http://www.pse6.com, you can move the point and see the changes to the rectangular and polar coordinates as well as to the sine, cosine, and tangent of angle θ.

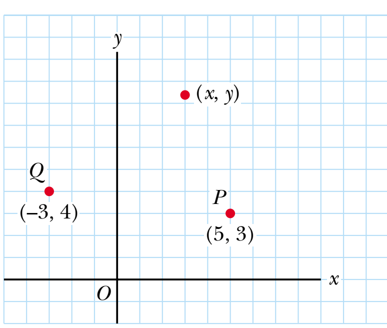


Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

Furthermore, the definitions of trigonometry tell us that

$$\tan \theta = \frac{y}{x} \tag{3.3}$$

$$r = \sqrt{x^2 + y^2} \tag{3.4}$$

Equation 3.4 is the familiar Pythagorean theorem.

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as shown in Figure 3.2a—in other words, when positive θ is an angle measured counterclockwise from the positive x axis. (Some scientific calculators perform conversions between Cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

Example 3.1 Polar Coordinates

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

Solution For the examples in this and the next two chapters we will illustrate the use of the General Problem-Solving Strategy outlined at the end of Chapter 2. In subsequent chapters, we will make fewer explicit references to this strategy, as you will have become familiar with it and should be applying it on your own. The drawing in Figure 3.3 helps us to *conceptualize* the problem. We can *categorize* this as a plug-in problem. From Equation 3.4,

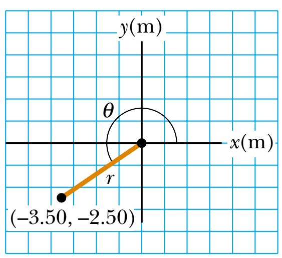
$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

and from Equation 3.3,

$$\tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216°$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$ and not 35.5°.



Active Figure 3.3 (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

At the Active Figures link at http://www.pse6.com, you can move the point in the xy plane and see how its Cartesian and polar coordinates change.

3.2 Vector and Scalar Quantities

As noted in Chapter 2, some physical quantities are scalar quantities whereas others are vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit "degrees C" or "degrees F." Temperature is therefore an example of a *scalar quantity*:

A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, speed, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a *vector quantity*:

A **vector quantity** is completely specified by a number and appropriate units plus a direction.

Another example of a vector quantity is displacement, as you know from Chapter 2. Suppose a particle moves from some point Ⓐ to some point Ⓑ along a straight path, as shown in Figure 3.4. We represent this displacement by drawing an arrow from Ⓐ to Ⓑ, with the tip of the arrow pointing away from the starting point. The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from Ⓐ to Ⓑ, such as the broken line in Figure 3.4, its displacement is still the arrow drawn from Ⓐ to Ⓑ. Displacement depends only on the initial and final positions, so the displacement vector is independent of the path taken between these two points.

In this text, we use a boldface letter, such as **A**, to represent a vector quantity. Another notation is useful when boldface notation is difficult, such as when writing on paper or on a chalkboard—an arrow is written over the symbol for the vector: $\vec{\mathbf{A}}$. The magnitude of the vector **A** is written either A or $|\mathbf{A}|$. The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity. The magnitude of a vector is *always* a positive number.

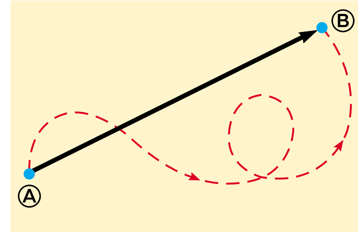


Figure 3.4 As a particle moves from Ⓐ to Ⓑ along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

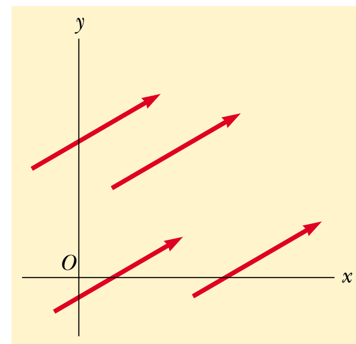


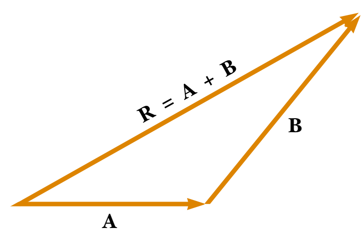
Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

3.3 Some Properties of Vectors

Equality of Two Vectors

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, $\mathbf{A} = \mathbf{B}$ only if $A = B$ and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.



Active Figure 3.6 When vector **B** is added to vector **A**, the resultant **R** is the vector that runs from the tail of **A** to the tip of **B**.

Go to the Active Figures link at http://www.pse6.com.

Adding Vectors

The rules for adding vectors are conveniently described by graphical methods. To add vector **B** to vector **A**, first draw vector **A** on graph paper, with its magnitude represented by a convenient length scale, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure 3.6. The **resultant vector** $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of **A** to the tip of **B**.

For example, if you walked 3.0 m toward the east and then 4.0 m toward the north, as shown in Figure 3.7, you would find yourself 5.0 m from where you started, measured at an angle of 53° north of east. Your total displacement is the vector sum of the individual displacements.

A geometric construction can also be used to add more than two vectors. This is shown in Figure 3.8 for the case of four vectors. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$ is the vector that completes the polygon. In other words, **R is the vector drawn from the tail of the first vector to the tip of the last vector**.

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, but as you will see in Chapter 11, the order is important

PITFALL PREVENTION

3.1 Vector Addition versus Scalar Addition

Keep in mind that $\mathbf{A} + \mathbf{B} = \mathbf{C}$ is very different from $A + B = C$. The first is a vector sum, which must be handled carefully, such as with the graphical method described here. The second is a simple algebraic addition of numbers that is handled with the normal rules of arithmetic.

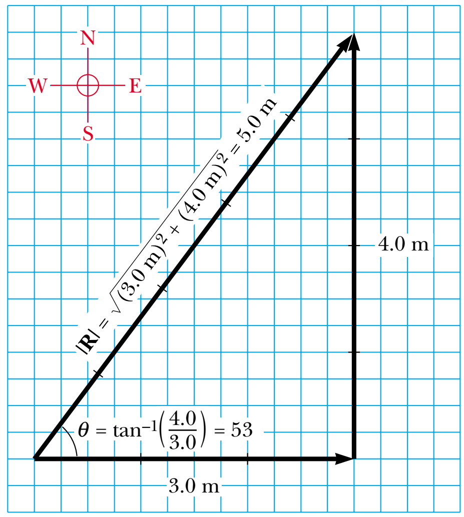


Figure 3.7 Vector addition. Walking first 3.0 m due east and then 4.0 m due north leaves you 5.0 m from your starting point.

when vectors are multiplied). This can be seen from the geometric construction in Figure 3.9 and is known as the **commutative law of addition:**

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \tag{3.5}$$

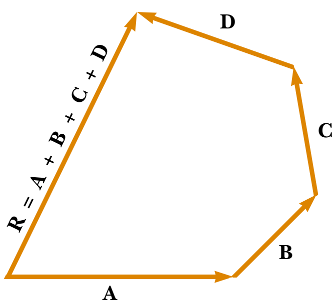


Figure 3.8 Geometric construction for summing four vectors. The resultant vector **R** is by definition the one that completes the polygon.

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 3.10. This is called the **associative law of addition:**

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \tag{3.6}$$

In summary, **a vector quantity has both magnitude and direction and also obeys the laws of vector addition** as described in Figures 3.6 to 3.10. When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because they represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

Negative of a Vector

The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is, $\mathbf{A} + (-\mathbf{A}) = 0$. The vectors **A** and $-\mathbf{A}$ have the same magnitude but point in opposite directions.

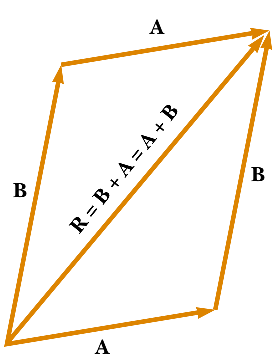


Figure 3.9 This construction shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$—in other words, that vector addition is commutative.

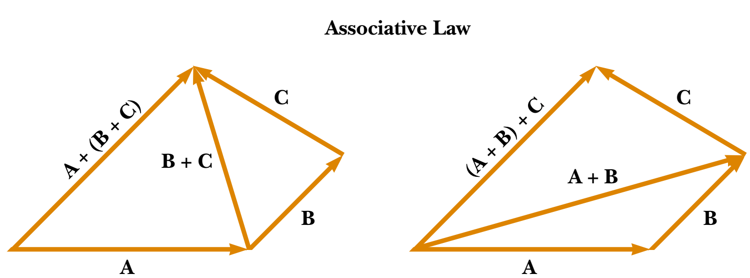


Figure 3.10 Geometric constructions for verifying the associative law of addition.

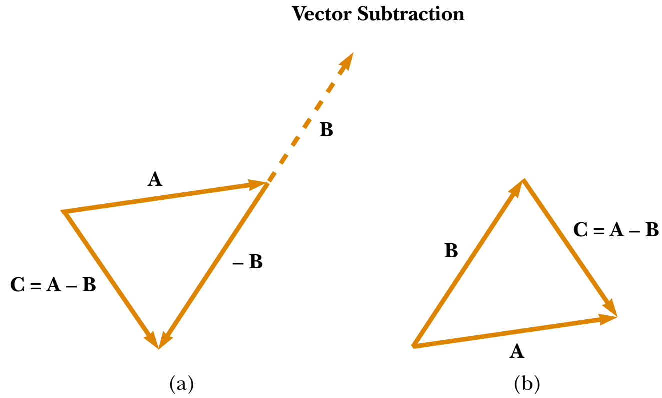


Figure 3.11 (a) This construction shows how to subtract vector **B** from vector **A**. The vector $-\mathbf{B}$ is equal in magnitude to vector **B** and points in the opposite direction. To subtract **B** from **A**, apply the rule of vector addition to the combination of **A** and $-\mathbf{B}$: Draw **A** along some convenient axis, place the tail of $-\mathbf{B}$ at the tip of **A**, and **C** is the difference $\mathbf{A} - \mathbf{B}$. (b) A second way of looking at vector subtraction. The difference vector $\mathbf{C} = \mathbf{A} - \mathbf{B}$ is the vector that we must add to **B** to obtain **A**.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector **A**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \tag{3.7}$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.11a.

Another way of looking at vector subtraction is to note that the difference $\mathbf{A} - \mathbf{B}$ between two vectors **A** and **B** is what you have to add to the second vector to obtain the first. In this case, the vector $\mathbf{A} - \mathbf{B}$ points from the tip of the second vector to the tip of the first, as Figure 3.11b shows.

Quick Quiz 3.2 The magnitudes of two vectors **A** and **B** are $A = 12$ units and $B = 8$ units. Which of the following pairs of numbers represents the *largest* and *smallest* possible values for the magnitude of the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers.

Quick Quiz 3.3 If vector **B** is added to vector **A**, under what condition does the resultant vector $\mathbf{A} + \mathbf{B}$ have magnitude $A + B$? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** are perpendicular.

Quick Quiz 3.4 If vector **B** is added to vector **A**, which *two* of the following choices must be true in order for the resultant vector to be equal to zero? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** have the same magnitude. (d) **A** and **B** are perpendicular.

Example 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 3.12a. Find the magnitude and direction of the car's resultant displacement.

Solution The vectors **A** and **B** drawn in Figure 3.12a help us to *conceptualize* the problem. We can *categorize* this as a relatively simple analysis problem in vector addition. The displacement **R** is the resultant when the two individual displacements **A** and **B** are added. We can further categorize this as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

In this example, we show two ways to *analyze* the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of **R** and its direction in Figure 3.12a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision.

The second way to solve the problem is to analyze it algebraically. The magnitude of **R** can be obtained from the law of cosines as applied to the triangle (see Appendix B.4). With $\theta = 180° - 60° = 120°$ and $R^2 = A^2 + B^2 - 2AB\cos\theta$, we find that

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$

$$= \boxed{48.2\text{ km}}$$

The direction of **R** measured from the northerly direction can be obtained from the law of sines (Appendix B.4):

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\boxed{\beta = 39.0°}$$

The resultant displacement of the car is 48.2 km in a direction 39.0° west of north.

We now *finalize* the problem. Does the angle β that we calculated agree with an estimate made by looking at Figure 3.12a or with an actual angle measured from the diagram using the graphical method? Is it reasonable that the magnitude of **R** is larger than that of both **A** and **B**? Are the units of **R** correct?

While the graphical method of adding vectors works well, it suffers from two disadvantages. First, some individuals find using the laws of cosines and sines to be awkward. Second, a triangle only results if you are adding two vectors. If you are adding three or more vectors, the resulting geometric shape is not a triangle. In Section 3.4, we explore a new method of adding vectors that will address both of these disadvantages.

What If? Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first, and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

Answer They would not change. The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant. Graphically, Figure 3.12b shows that the vectors added in the reverse order give us the same resultant vector.

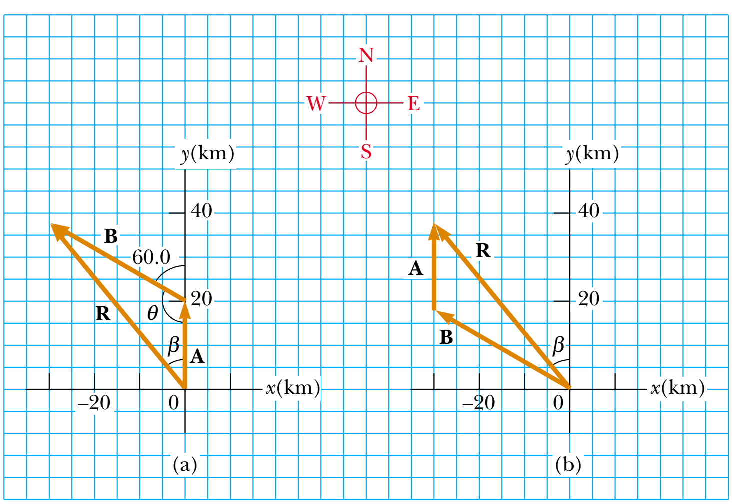


Figure 3.12 (Example 3.2) (a) Graphical method for finding the resultant displacement vector **R** = **A** + **B**. (b) Adding the vectors in reverse order (**B** + **A**) gives the same result for **R**.

Multiplying a Vector by a Scalar

If vector **A** is multiplied by a positive scalar quantity m, then the product $m\mathbf{A}$ is a vector that has the same direction as **A** and magnitude mA. If vector **A** is multiplied by a negative scalar quantity $-m$, then the product $-m\mathbf{A}$ is directed opposite **A**. For example, the vector $5\mathbf{A}$ is five times as long as **A** and points in the same direction as **A**; the vector $-\frac{1}{3}\mathbf{A}$ is one-third the length of **A** and points in the direction opposite **A**.

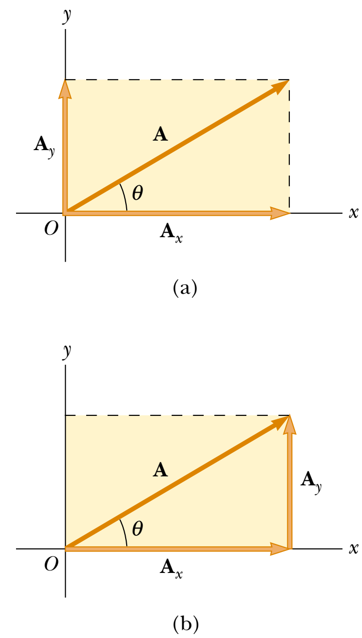


Figure 3.13 (a) A vector **A** lying in the xy plane can be represented by its component vectors $\mathbf{A}_x$ and $\mathbf{A}_y$. (b) The y component vector $\mathbf{A}_y$ can be moved to the right so that it adds to $\mathbf{A}_x$. The vector sum of the component vectors is **A**. These three vectors form a right triangle.

3.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

Consider a vector **A** lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure 3.13a. This vector can be expressed as the sum of two other vectors $\mathbf{A}_x$ and $\mathbf{A}_y$. From Figure 3.13b, we see that the three vectors form a right triangle and that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. We shall often refer to the "components of a vector **A**," written A_x and A_y (without the boldface notation). The component A_x represents the projection of **A** along the x axis, and the component A_y represents the projection of **A** along the y axis. These components can be positive or negative. The component A_x is positive if $\mathbf{A}_x$ points in the positive x direction and is negative if $\mathbf{A}_x$ points in the negative x direction. The same is true for the component A_y.

From Figure 3.13 and the definition of sine and cosine, we see that $\cos\theta = A_x/A$ and that $\sin\theta = A_y/A$. Hence, the components of **A** are

Components of the vector A

$$A_x = A\cos\theta \tag{3.8}$$

$$A_y = A\sin\theta \tag{3.9}$$

These components form two sides of a right triangle with a hypotenuse of length A. Thus, it follows that the magnitude and direction of **A** are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \tag{3.10}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3.11}$$

Note that **the signs of the components A_x and A_y depend on the angle θ**. For example, if $\theta = 120°$, then A_x is negative and A_y is positive. If $\theta = 225°$, then both A_x and A_y are negative. Figure 3.14 summarizes the signs of the components when **A** lies in the various quadrants.

When solving problems, you can specify a vector **A** either with its components A_x and A_y or with its magnitude and direction A and θ.

PITFALL PREVENTION

3.2 Component Vectors versus Components

The vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ are the *component vectors* of **A**. These should not be confused with the scalars A_x and A_y, which we shall always refer to as the *components* of **A**.

Quick Quiz 3.5 Choose the correct response to make the sentence true: A component of a vector is (a) always, (b) never, or (c) sometimes larger than the magnitude of the vector.

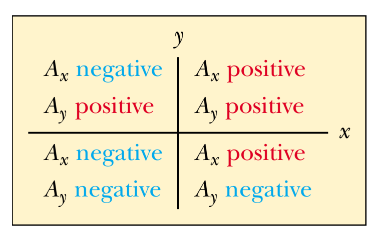


Figure 3.14 The signs of the components of a vector **A** depend on the quadrant in which the vector is located.

Suppose you are working a physics problem that requires resolving a vector into its components. In many applications it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but are still perpendicular to each other. If you choose reference axes or an angle other than the axes and angle shown in Figure 3.13, the components must be modified accordingly. Suppose a

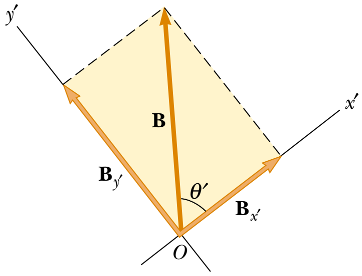


Figure 3.15 The component vectors of **B** in a coordinate system that is tilted.

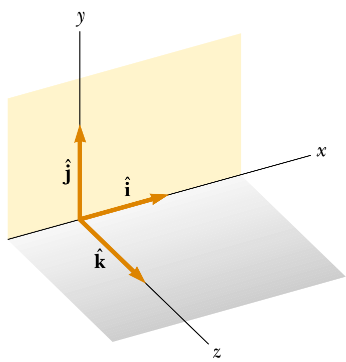
vector **B** makes an angle θ' with the x' axis defined in Figure 3.15. The components of **B** along the x' and y' axes are $B_{x'} = B\cos\theta'$ and $B_{y'} = B\sin\theta'$, as specified by Equations 3.8 and 3.9. The magnitude and direction of **B** are obtained from expressions equivalent to Equations 3.10 and 3.11. Thus, we can express the components of a vector in any coordinate system that is convenient for a particular situation.

Unit Vectors

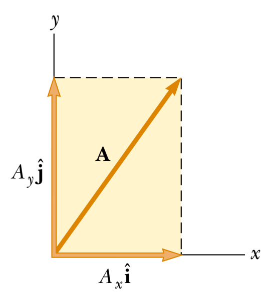
Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ to represent unit vectors pointing in the positive *x*, *y*, and *z* directions, respectively. (The "hats" on the symbols are a standard notation for unit vectors.) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure 3.16a. The magnitude of each unit vector equals 1; that is, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$.

Consider a vector **A** lying in the *xy* plane, as shown in Figure 3.16b. The product of the component A_x and the unit vector $\hat{\mathbf{i}}$ is the vector $A_x\hat{\mathbf{i}}$, which lies on the *x* axis and has magnitude $|A_x|$. (The vector $A_x\hat{\mathbf{i}}$ is an alternative representation of vector $\mathbf{A}_x$.) Likewise, $A_y\hat{\mathbf{j}}$ is a vector of magnitude $|A_y|$ lying on the *y* axis. (Again, vector $A_y\hat{\mathbf{j}}$ is an alternative representation of vector $\mathbf{A}_y$.) Thus, the unit–vector notation for the vector **A** is

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{3.12}$$



(a)



(b)

Active Figure 3.16 (a) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are directed along the *x*, *y*, and *z* axes, respectively. (b) Vector $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ lying in the *xy* plane has components A_x and A_y.

At the Active Figures link at http://www.pse6.com you can rotate the coordinate axes in 3-dimensional space and view a representation of vector A in three dimensions.

For example, consider a point lying in the *xy* plane and having Cartesian coordinates (x, y), as in Figure 3.17. The point can be specified by the **position vector r**, which in unit–vector form is given by

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \tag{3.13}$$

This notation tells us that the components of **r** are the lengths *x* and *y*.

Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector **B** to vector **A** in Equation 3.12, where vector **B** has components B_x and B_y. All we do is add the *x* and *y* components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

$$\mathbf{R} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

or

$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \tag{3.14}$$

Because $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$\begin{aligned} R_x &= A_x + B_x \\ R_y &= A_y + B_y \end{aligned} \tag{3.15}$$

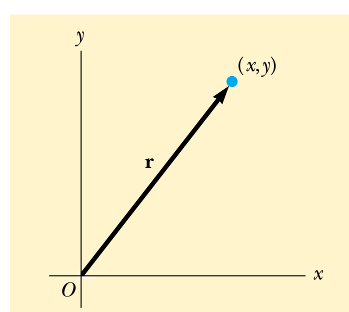


Figure 3.17 The point whose Cartesian coordinates are (x, y) can be represented by the position vector $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

We obtain the magnitude of **R** and the angle it makes with the x axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \quad (3.16)$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \quad (3.17)$$

We can check this addition by components with a geometric construction, as shown in Figure 3.18. Remember that you must note the signs of the components when using either the algebraic or the graphical method.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If **A** and **B** both have x, y, and z components, we express them in the form

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \quad (3.18)$$

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \quad (3.19)$$

The sum of **A** and **B** is

$$\mathbf{R} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}} \quad (3.20)$$

Note that Equation 3.20 differs from Equation 3.14: in Equation 3.20, the resultant vector also has a z component $R_z = A_z + B_z$. If a vector **R** has x, y, and z components, the magnitude of the vector is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. The angle θ_x that **R** makes with the x axis is found from the expression $\cos\theta_x = R_x/R$, with similar expressions for the angles with respect to the y and z axes.

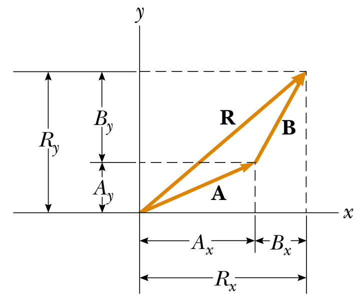


Figure 3.18 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant **R** and the components of the individual vectors.

Quick Quiz 3.6 If at least one component of a vector is a positive number, the vector cannot (a) have any component that is negative (b) be zero (c) have three dimensions.

Quick Quiz 3.7 If $\mathbf{A} + \mathbf{B} = 0$, the corresponding components of the two vectors **A** and **B** must be (a) equal (b) positive (c) negative (d) of opposite sign.

Quick Quiz 3.8 For which of the following vectors is the magnitude of the vector equal to one of the components of the vector? (a) $\mathbf{A} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ (b) $\mathbf{B} = -3\hat{\mathbf{j}}$ (c) $\mathbf{C} = +5\hat{\mathbf{k}}$

PITFALL PREVENTION

3.3 *x* and *y* Components

Equations 3.8 and 3.9 associate the cosine of the angle with the x component and the sine of the angle with the y component. This is true *only* because we measured the angle θ with respect to the x axis, so don't memorize these equations. If θ is measured with respect to the y axis (as in some problems), these equations will be incorrect. Think about which side of the triangle containing the components is adjacent to the angle and which side is opposite, and assign the cosine and sine accordingly.

PROBLEM-SOLVING HINTS

Adding Vectors

When you need to add two or more vectors, use this step-by-step procedure:

- Select a coordinate system that is convenient. (Try to reduce the number of components you need to calculate by choosing axes that line up with as many vectors as possible.)
- Draw a labeled sketch of the vectors described in the problem.
- Find the x and y components of all vectors and the resultant components (the algebraic sum of the components) in the x and y directions.
- If necessary, use the Pythagorean theorem to find the magnitude of the resultant vector and select a suitable trigonometric function to find the angle that the resultant vector makes with the x axis.

PITFALL PREVENTION

3.4 Tangents on Calculators

Generally, the inverse tangent function on calculators provides an angle between $-90°$ and $+90°$. As a consequence, if the vector you are studying lies in the second or third quadrant, the angle measured from the positive x axis will be the angle your calculator returns plus 180°.

Example 3.3 The Sum of Two Vectors

Find the sum of two vectors **A** and **B** lying in the *xy* plane and given by

$$\mathbf{A} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\ \text{m} \qquad \text{and} \qquad \mathbf{B} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\ \text{m}$$

Solution You may wish to draw the vectors to *conceptualize* the situation. We *categorize* this as a simple plug-in problem. Comparing this expression for **A** with the general expression $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$, we see that $A_x = 2.0$ m and $A_y = 2.0$ m. Likewise, $B_x = 2.0$ m and $B_y = -4.0$ m. We obtain the resultant vector **R**, using Equation 3.14:

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\hat{\mathbf{i}}\ \text{m} + (2.0 - 4.0)\hat{\mathbf{j}}\ \text{m}$$
$$= (4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}})\ \text{m}$$

or

$$R_x = 4.0\ \text{m} \qquad R_y = -2.0\ \text{m}$$

The magnitude of **R** is found using Equation 3.16:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\ \text{m})^2 + (-2.0\ \text{m})^2} = \sqrt{20}\ \text{m}$$
$$= \boxed{4.5\ \text{m}}$$

We can find the direction of **R** from Equation 3.17:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\ \text{m}}{4.0\ \text{m}} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the *x* axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\boxed{\theta = 333°}$.

Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\mathbf{d}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\mathbf{d}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm and $\mathbf{d}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find the components of the resultant displacement and its magnitude.

Solution Three-dimensional displacements are more difficult to imagine than those in two dimensions, because the latter can be drawn on paper. For this problem, let us *conceptualize* that you start with your pencil at the origin of a piece of graph paper on which you have drawn *x* and *y* axes. Move your pencil 15 cm to the right along the *x* axis, then 30 cm upward along the *y* axis, and then 12 cm *vertically away* from the graph paper. This provides the displacement described by $\mathbf{d}_1$. From this point, move your pencil 23 cm to the right parallel to the *x* axis, 14 cm parallel to the graph paper in the $-y$ direction, and then 5.0 cm vertically downward toward the graph paper. You are now at the displacement from the origin described by $\mathbf{d}_1 + \mathbf{d}_2$. From this point, move your pencil 13 cm to the left in the $-x$ direction, and (finally!) 15 cm parallel to the graph paper along the *y* axis. Your final position is at a displacement $\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$ from the origin.

Despite the difficulty in conceptualizing in three dimensions, we can *categorize* this problem as a plug-in problem due to the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way:

$$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3$$
$$= (15 + 23 - 13)\hat{\mathbf{i}}\ \text{cm} + (30 - 14 + 15)\hat{\mathbf{j}}\ \text{cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\ \text{cm}$$
$$= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\ \text{cm}$$

The resultant displacement has components $R_x = 25$ cm, $R_y = 31$ cm, and $R_z = 7.0$ cm. Its magnitude is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$= \sqrt{(25\ \text{cm})^2 + (31\ \text{cm})^2 + (7.0\ \text{cm})^2} = \boxed{40\ \text{cm}}$$

Example 3.5 Taking a Hike

Interactive

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

Solution We *conceptualize* the problem by drawing a sketch as in Figure 3.19. If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.19. Drawing the resultant **R**, we can now *categorize* this as a problem we've solved before—an addition of two vectors. This should give you a hint of the power of categorization—many new problems are very similar to problems that we have already solved if we are careful to conceptualize them.

We will *analyze* this problem by using our new knowledge of vector components. Displacement **A** has a magnitude of 25.0 km and is directed 45.0° below the positive *x* axis. From Equations 3.8 and 3.9, its components are

$$A_x = A\cos(-45.0°) = (25.0\ \text{km})(0.707) = \boxed{17.7\ \text{km}}$$

$$A_y = A\sin(-45.0°) = (25.0\ \text{km})(-0.707) = \boxed{-17.7\ \text{km}}$$

The negative value of A_y indicates that the hiker walks in the negative *y* direction on the first day. The signs of A_x and A_y also are evident from Figure 3.19.

The second displacement **B** has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B\cos 60.0° = (40.0 \text{ km})(0.500) = \boxed{20.0 \text{ km}}$$

$$B_y = B\sin 60.0° = (40.0 \text{ km})(0.866) = \boxed{34.6 \text{ km}}$$

(B) Determine the components of the hiker's resultant displacement **R** for the trip. Find an expression for **R** in terms of unit vectors.

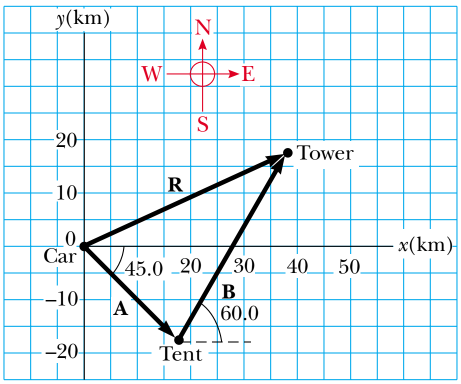


Figure 3.19 (Example 3.5) The total displacement of the hiker is the vector **R** = **A** + **B**.

Solution The resultant displacement for the trip **R** = **A** + **B** has components given by Equation 3.15:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = \boxed{37.7 \text{ km}}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = \boxed{16.9 \text{ km}}$$

In unit–vector form, we can write the total displacement as

$$\mathbf{R} = \boxed{(37.7\hat{\mathbf{i}} + 16.9\hat{\mathbf{j}}) \text{ km}}$$

Using Equations 3.16 and 3.17, we find that the vector **R** has a magnitude of 41.3 km and is directed 24.1° north of east.

Let us *finalize*. The units of **R** are km, which is reasonable for a displacement. Looking at the graphical representation in Figure 3.19, we estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of **R** in our final result. Also, both components of **R** are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.19.

Investigate this situation at the Interactive Worked Example link at **http://www.pse6.com.**

Example 3.6 Let's Fly Away!

A commuter airplane takes the route shown in Figure 3.20. First, it flies from the origin of the coordinate system shown to city A, located 175 km in a direction 30.0° north of east. Next, it flies 153 km 20.0° west of north to city B. Finally, it flies 195 km due west to city C. Find the location of city C relative to the origin.

Solution Once again, a drawing such as Figure 3.20 allows us to *conceptualize* the problem. It is convenient to choose the coordinate system shown in Figure 3.20, where the x axis points to the east and the y axis points to the north. Let us denote the three consecutive displacements by the vectors **a**, **b**, and **c**.

We can now *categorize* this problem as being similar to Example 3.5 that we have already solved. There are two primary differences. First, we are adding three vectors instead of two. Second, Example 3.5 guided us by first asking for the components in part (A). The current Example has no such guidance and simply asks for a result. We need to *analyze* the situation and choose a path. We will follow the same pattern that we did in Example 3.5, beginning with finding the components of the three vectors **a**, **b**, and **c**. Displacement **a** has a magnitude of 175 km and the components

$$a_x = a\cos(30.0°) = (175 \text{ km})(0.866) = 152 \text{ km}$$

$$a_y = a\sin(30.0°) = (175 \text{ km})(0.500) = 87.5 \text{ km}$$

Displacement **b**, whose magnitude is 153 km, has the components

$$b_x = b\cos(110°) = (153 \text{ km})(-0.342) = -52.3 \text{ km}$$

$$b_y = b\sin(110°) = (153 \text{ km})(0.940) = 144 \text{ km}$$

Finally, displacement **c**, whose magnitude is 195 km, has the components

$$c_x = c\cos(180°) = (195 \text{ km})(-1) = -195 \text{ km}$$

$$c_y = c\sin(180°) = 0$$

Therefore, the components of the position vector **R** from the starting point to city C are

$$R_x = a_x + b_x + c_x = 152 \text{ km} - 52.3 \text{ km} - 195 \text{ km}$$

$$= \boxed{-95.3 \text{ km}}$$

$$R_y = a_y + b_y + c_y = 87.5 \text{ km} + 144 \text{ km} + 0 = \boxed{232 \text{ km}}$$

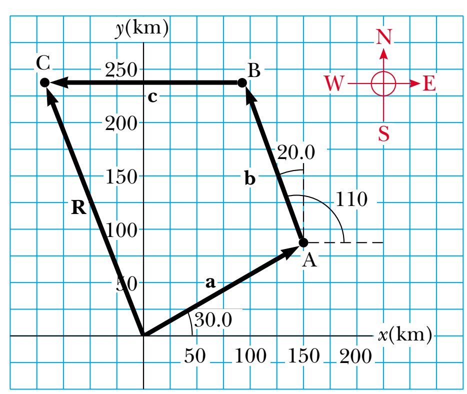


Figure 3.20 (Example 3.6) The airplane starts at the origin, flies first to city A, then to city B, and finally to city C.

In unit–vector notation, $\mathbf{R} = (-95.3\hat{\mathbf{i}} + 232\hat{\mathbf{j}})$ km. Using Equations 3.16 and 3.17, we find that the vector $\mathbf{R}$ has a magnitude of 251 km and is directed 22.3° west of north.

To *finalize* the problem, note that the airplane can reach city C from the starting point by first traveling 95.3 km due west and then by traveling 232 km due north. Or it could follow a straight-line path to C by flying a distance $R =$ 251 km in a direction 22.3° west of north.

What If? **After landing in city C, the pilot wishes to return to the origin along a single straight line. What are the components of the vector representing this displacement? What should the heading of the plane be?**

Answer The desired vector $\mathbf{H}$ (for Home!) is simply the negative of vector $\mathbf{R}$:

$$\mathbf{H} = -\mathbf{R} = (+95.3\hat{\mathbf{i}} - 232\hat{\mathbf{j}})\text{ km}$$

The heading is found by calculating the angle that the vector makes with the x axis:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-232\text{ m}}{95.3\text{ m}} = -2.43$$

This gives a heading angle of $\theta = -67.7°$, or 67.7° south of east.

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

SUMMARY

Scalar quantities are those that have only a numerical value and no associated direction. **Vector quantities** have both magnitude and direction and obey the laws of vector addition. The magnitude of a vector is *always* a positive number.

When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity. We can add two vectors $\mathbf{A}$ and $\mathbf{B}$ graphically. In this method (Fig. 3.6), the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ runs from the tail of $\mathbf{A}$ to the tip of $\mathbf{B}$.

A second method of adding vectors involves **components** of the vectors. The x component A_x of the vector $\mathbf{A}$ is equal to the projection of $\mathbf{A}$ along the x axis of a coordinate system, as shown in Figure 3.13, where $A_x = A\cos\theta$. The y component A_y of $\mathbf{A}$ is the projection of $\mathbf{A}$ along the y axis, where $A_y = A\sin\theta$. Be sure you can determine which trigonometric functions you should use in all situations, especially when θ is defined as something other than the counterclockwise angle from the positive x axis.

If a vector $\mathbf{A}$ has an x component A_x and a y component A_y, the vector can be expressed in unit–vector form as $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$. In this notation, $\hat{\mathbf{i}}$ is a unit vector pointing in the positive x direction, and $\hat{\mathbf{j}}$ is a unit vector pointing in the positive y direction. Because $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = 1$.

We can find the resultant of two or more vectors by resolving all vectors into their x and y components, adding their resultant x and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the x axis by using a suitable trigonometric function.

QUESTIONS

1. Two vectors have unequal magnitudes. Can their sum be zero? Explain.

2. Can the magnitude of a particle's displacement be greater than the distance traveled? Explain.

3. The magnitudes of two vectors $\mathbf{A}$ and $\mathbf{B}$ are $A = 5$ units and $B = 2$ units. Find the largest and smallest values possible for the magnitude of the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

4. Which of the following are vectors and which are not: force, temperature, the volume of water in a can, the ratings of a TV show, the height of a building, the velocity of a sports car, the age of the Universe?

5. A vector $\mathbf{A}$ lies in the xy plane. For what orientations of $\mathbf{A}$ will both of its components be negative? For what orientations will its components have opposite signs?

6. A book is moved once around the perimeter of a tabletop with the dimensions 1.0 m × 2.0 m. If the book ends up at its initial position, what is its displacement? What is the distance traveled?

7. While traveling along a straight interstate highway you notice that the mile marker reads 260. You travel until you reach mile marker 150 and then retrace your path to the mile marker 175. What is the magnitude of your resultant displacement from mile marker 260?

8. If the component of vector **A** along the direction of vector **B** is zero, what can you conclude about the two vectors?

9. Can the magnitude of a vector have a negative value? Explain.

10. Under what circumstances would a nonzero vector lying in the *xy* plane have components that are equal in magnitude?

11. If **A** = **B**, what can you conclude about the components of **A** and **B**?

12. Is it possible to add a vector quantity to a scalar quantity? Explain.

13. The resolution of vectors into components is equivalent to replacing the original vector with the sum of two vectors, whose sum is the same as the original vector. There are an infinite number of pairs of vectors that will satisfy this condition; we choose that pair with one vector parallel to the *x* axis and the second parallel to the *y* axis. What difficulties would be introduced by defining components relative to axes that are not perpendicular—for example, the *x* axis and a *y* axis oriented at 45° to the *x* axis?

14. In what circumstance is the *x* component of a vector given by the magnitude of the vector times the sine of its direction angle?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 3.1 Coordinate Systems

1. The polar coordinates of a point are $r = 5.50$ m and $\theta = 240°$. What are the Cartesian coordinates of this point?

2. Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine (a) the Cartesian coordinates of these points and (b) the distance between them.

3. A fly lands on one wall of a room. The lower left-hand corner of the wall is selected as the origin of a two-dimensional Cartesian coordinate system. If the fly is located at the point having coordinates (2.00, 1.00) m, (a) how far is it from the corner of the room? (b) What is its location in polar coordinates?

4. Two points in the *xy* plane have Cartesian coordinates (2.00, − 4.00) m and (− 3.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.

5. If the rectangular coordinates of a point are given by (2, *y*) and its polar coordinates are (r, 30°), determine *y* and *r*.

6. If the polar coordinates of the point (x, y) are (r, θ), determine the polar coordinates for the points: (a) ($-x$, y), (b) ($-2x$, $-2y$), and (c) ($3x$, $-3y$).

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

7. A surveyor measures the distance across a straight river by the following method: starting directly across from a tree on the opposite bank, she walks 100 m along the riverbank to establish a baseline. Then she sights across to the tree. The angle from her baseline to the tree is 35.0°. How wide is the river?

8. A pedestrian moves 6.00 km east and then 13.0 km north. Find the magnitude and direction of the resultant displacement vector using the graphical method.

9. A plane flies from base camp to lake A, 280 km away, in a direction of 20.0° north of east. After dropping off supplies it flies to lake B, which is 190 km at 30.0° west of north from lake A. Graphically determine the distance and direction from lake B to the base camp.

10. Vector **A** has a magnitude of 8.00 units and makes an angle of 45.0° with the positive *x* axis. Vector **B** also has a magnitude of 8.00 units and is directed along the negative *x* axis. Using graphical methods, find (a) the vector sum **A** + **B** and (b) the vector difference **A** − **B**.

11. A skater glides along a circular path of radius 5.00 m. If he coasts around one half of the circle, find (a) the magnitude of the displacement vector and (b) how far the person skated. (c) What is the magnitude of the displacement if he skates all the way around the circle?

12. A force $\mathbf{F}_1$ of magnitude 6.00 units acts at the origin in a direction 30.0° above the positive *x* axis. A second force $\mathbf{F}_2$ of magnitude 5.00 units acts at the origin in the direction of the positive *y* axis. Find graphically the magnitude and direction of the resultant force $\mathbf{F}_1 + \mathbf{F}_2$.

13. Arbitrarily define the "instantaneous vector height" of a person as the displacement vector from the point halfway

between his or her feet to the top of the head. Make an order-of-magnitude estimate of the total vector height of all the people in a city of population 100 000 (a) at 10 o'clock on a Tuesday morning, and (b) at 5 o'clock on a Saturday morning. Explain your reasoning.

14. A dog searching for a bone walks 3.50 m south, then runs 8.20 m at an angle 30.0° north of east, and finally walks 15.0 m west. Find the dog's resultant displacement vector using graphical techniques.

15. Each of the displacement vectors **A** and **B** shown in Fig. P3.15 has a magnitude of 3.00 m. Find graphically (a) **A** + **B**, (b) **A** − **B**, (c) **B** − **A**, (d) **A** − 2**B**. Report all angles counterclockwise from the positive x axis.

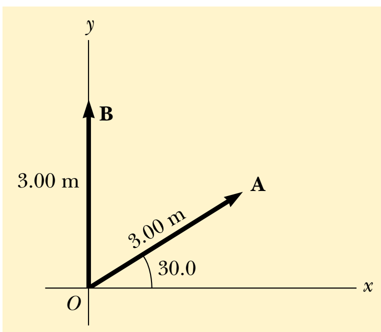


Figure P3.15 Problems 15 and 37.

16. Three displacements are **A** = 200 m, due south; **B** = 250 m, due west; **C** = 150 m, 30.0° east of north. Construct a separate diagram for each of the following possible ways of adding these vectors: $\mathbf{R}_1$ = **A** + **B** + **C**; $\mathbf{R}_2$ = **B** + **C** + **A**; $\mathbf{R}_3$ = **C** + **B** + **A**.

17. A roller coaster car moves 200 ft horizontally, and then rises 135 ft at an angle of 30.0° above the horizontal. It then travels 135 ft at an angle of 40.0° downward. What is its displacement from its starting point? Use graphical techniques.

Section 3.4 Components of a Vector and Unit Vectors

18. Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building following the path shown in Fig. P3.18.

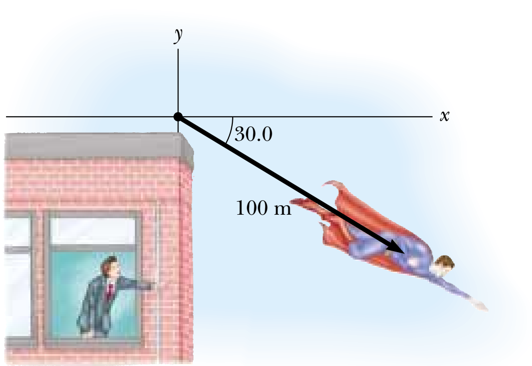


Figure P3.18

19. A vector has an x component of −25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.

20. A person walks 25.0° north of east for 3.10 km. How far would she have to walk due north and due east to arrive at the same location?

21. Obtain expressions in component form for the position vectors having the following polar coordinates: (a) 12.8 m, 150° (b) 3.30 cm, 60.0° (c) 22.0 in., 215°.

22. A displacement vector lying in the xy plane has a magnitude of 50.0 m and is directed at an angle of 120° to the positive x axis. What are the rectangular components of this vector?

23. A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her resultant displacement? (b) What is the total distance she travels?

24. In 1992, Akira Matsushima, from Japan, rode a unicycle across the United States, covering about 4 800 km in six weeks. Suppose that, during that trip, he had to find his way through a city with plenty of one-way streets. In the city center, Matsushima had to travel in sequence 280 m north, 220 m east, 360 m north, 300 m west, 120 m south, 60.0 m east, 40.0 m south, 90.0 m west (road construction) and then 70.0 m north. At that point, he stopped to rest. Meanwhile, a curious crow decided to fly the distance from his starting point to the rest location directly ("as the crow flies"). It took the crow 40.0 s to cover that distance. Assuming the velocity of the crow was constant, find its magnitude and direction.

25. While exploring a cave, a spelunker starts at the entrance and moves the following distances. She goes 75.0 m north, 250 m east, 125 m at an angle 30.0° north of east, and 150 m south. Find the resultant displacement from the cave entrance.

26. A map suggests that Atlanta is 730 miles in a direction of 5.00° north of east from Dallas. The same map shows that Chicago is 560 miles in a direction of 21.0° west of north from Atlanta. Modeling the Earth as flat, use this information to find the displacement from Dallas to Chicago.

27. Given the vectors $\mathbf{A} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\mathbf{B} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$, (a) draw the vector sum **C** = **A** + **B** and the vector difference **D** = **A** − **B**. (b) Calculate **C** and **D**, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the + x axis.

28. Find the magnitude and direction of the resultant of three displacements having rectangular components (3.00, 2.00) m, (−5.00, 3.00) m, and (6.00, 1.00) m.

29. A man pushing a mop across a floor causes it to undergo two displacements. The first has a magnitude of 150 cm and makes an angle of 120° with the positive x axis. The resultant displacement has a magnitude of 140 cm and is directed at an angle of 35.0° to the positive x axis. Find the magnitude and direction of the second displacement.

30. Vector **A** has x and y components of −8.70 cm and 15.0 cm, respectively; vector **B** has x and y components of 13.2 cm and −6.60 cm, respectively. If **A** − **B** + 3**C** = 0, what are the components of **C**?

31. Consider the two vectors $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\mathbf{B} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$. Calculate (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $|\mathbf{A} + \mathbf{B}|$, (d) $|\mathbf{A} - \mathbf{B}|$, and (e) the directions of $\mathbf{A} + \mathbf{B}$ and $\mathbf{A} - \mathbf{B}$.

32. Consider the three displacement vectors $\mathbf{A} = (3\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ m, $\mathbf{B} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$ m, and $\mathbf{C} = (-2\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ m. Use the component method to determine (a) the magnitude and direction of the vector $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$, (b) the magnitude and direction of $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C}$.

33. A particle undergoes the following consecutive displacements: 3.50 m south, 8.20 m northeast, and 15.0 m west. What is the resultant displacement?

34. In a game of American football, a quarterback takes the ball from the line of scrimmage, runs backward a distance of 10.0 yards, and then sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a forward pass 50.0 yards straight downfield perpendicular to the line of scrimmage. What is the magnitude of the football's resultant displacement?

35. The helicopter view in Fig. P3.35 shows two people pulling on a stubborn mule. Find (a) the single force that is equivalent to the two forces shown, and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons (abbreviated N).

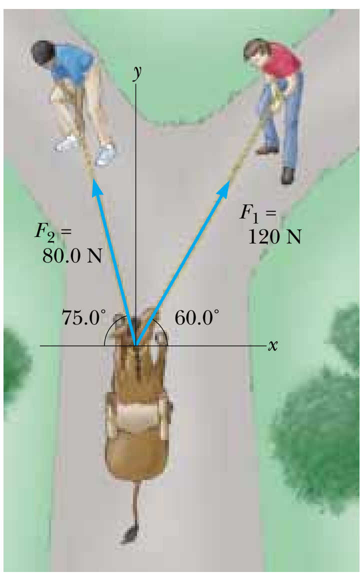


Figure P3.35

36. A novice golfer on the green takes three strokes to sink the ball. The successive displacements are 4.00 m to the north, 2.00 m northeast, and 1.00 m at 30.0° west of south. Starting at the same initial point, an expert golfer could make the hole in what single displacement?

37. Use the component method to add the vectors **A** and **B** shown in Figure P3.15. Express the resultant $\mathbf{A} + \mathbf{B}$ in unit–vector notation.

38. In an assembly operation illustrated in Figure P3.38, a robot moves an object first straight upward and then also to the east, around an arc forming one quarter of a circle of radius 4.80 cm that lies in an east-west vertical plane. The robot then moves the object upward and to the north, through a quarter of a circle of radius 3.70 cm that lies in a north-south vertical plane. Find (a) the magnitude of the total displacement of the object, and (b) the angle the total displacement makes with the vertical.

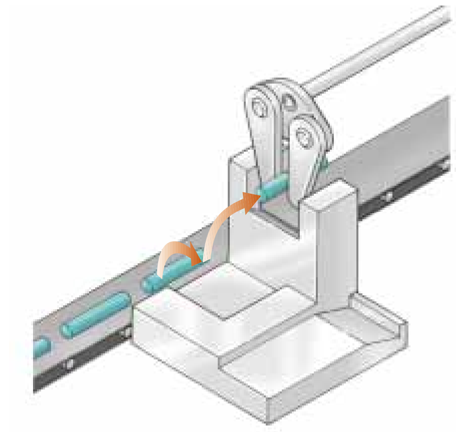


Figure P3.38

39. Vector **B** has *x*, *y*, and *z* components of 4.00, 6.00, and 3.00 units, respectively. Calculate the magnitude of **B** and the angles that **B** makes with the coordinate axes.

40. You are standing on the ground at the origin of a coordinate system. An airplane flies over you with constant velocity parallel to the *x* axis and at a fixed height of 7.60×10^3 m. At time $t = 0$ the airplane is directly above you, so that the vector leading from you to it is $\mathbf{P}_0 = (7.60 \times 10^3 \text{ m})\hat{\mathbf{j}}$. At $t = 30.0$ s the position vector leading from you to the airplane is $\mathbf{P}_{30} = (8.04 \times 10^3 \text{ m})\hat{\mathbf{i}} + (7.60 \times 10^3 \text{ m})\hat{\mathbf{j}}$. Determine the magnitude and orientation of the airplane's position vector at $t = 45.0$ s.

41. The vector **A** has *x*, *y*, and *z* components of 8.00, 12.0, and −4.00 units, respectively. (a) Write a vector expression for **A** in unit–vector notation. (b) Obtain a unit–vector expression for a vector **B** one fourth the length of **A** pointing in the same direction as **A**. (c) Obtain a unit–vector expression for a vector **C** three times the length of **A** pointing in the direction opposite the direction of **A**.

42. Instructions for finding a buried treasure include the following: Go 75.0 paces at 240°, turn to 135° and walk 125 paces, then travel 100 paces at 160°. The angles are measured counterclockwise from an axis pointing to the east, the $+x$ direction. Determine the resultant displacement from the starting point.

43. Given the displacement vectors $\mathbf{A} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ m and $\mathbf{B} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$ m, find the magnitudes of the vectors (a) $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and (b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B}$, also expressing each in terms of its rectangular components.

44. A radar station locates a sinking ship at range 17.3 km and bearing 136° clockwise from north. From the same station a rescue plane is at horizontal range 19.6 km, 153° clockwise from north, with elevation 2.20 km. (a) Write the position vector for the ship relative to the plane, letting $\hat{\mathbf{i}}$ represent east, $\hat{\mathbf{j}}$ north, and $\hat{\mathbf{k}}$ up. (b) How far apart are the plane and ship?

45. As it passes over Grand Bahama Island, the eye of a hurricane is moving in a direction 60.0° north of west with a speed of 41.0 km/h. Three hours later, the course of the hurricane suddenly shifts due north, and its speed slows to 25.0 km/h. How far from Grand Bahama is the eye 4.50 h after it passes over the island?

46. (a) Vector **E** has magnitude 17.0 cm and is directed 27.0° counterclockwise from the $+x$ axis. Express it in unit–vector notation. (b) Vector **F** has magnitude 17.0 cm and is directed 27.0° counterclockwise from the $+y$ axis. Express it in unit–vector notation. (c) Vector **G** has magnitude 17.0 cm and is directed 27.0° clockwise from the $-y$ axis. Express it in unit–vector notation.

47. Vector **A** has a negative x component 3.00 units in length and a positive y component 2.00 units in length. (a) Determine an expression for **A** in unit–vector notation. (b) Determine the magnitude and direction of **A**. (c) What vector **B** when added to **A** gives a resultant vector with no x component and a negative y component 4.00 units in length?

48. An airplane starting from airport A flies 300 km east, then 350 km at 30.0° west of north, and then 150 km north to arrive finally at airport B. (a) The next day, another plane flies directly from A to B in a straight line. In what direction should the pilot travel in this direct flight? (b) How far will the pilot travel in this direct flight? Assume there is no wind during these flights.

49. Three displacement vectors of a croquet ball are shown in Figure P3.49, where $|\mathbf{A}| = 20.0$ units, $|\mathbf{B}| = 40.0$ units, and $|\mathbf{C}| = 30.0$ units. Find (a) the resultant in unit–vector notation and (b) the magnitude and direction of the resultant displacement.

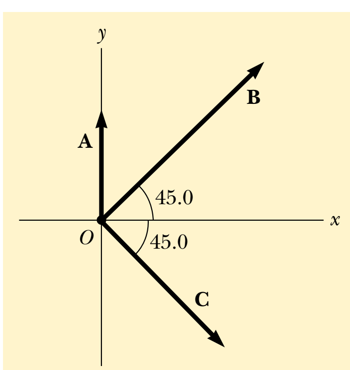


Figure P3.49

50. If $\mathbf{A} = (6.00\hat{\mathbf{i}} - 8.00\hat{\mathbf{j}})$ units, $\mathbf{B} = (-8.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ units, and $\mathbf{C} = (26.0\hat{\mathbf{i}} + 19.0\hat{\mathbf{j}})$ units, determine a and b such that $a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$.

Additional Problems

51. Two vectors **A** and **B** have precisely equal magnitudes. In order for the magnitude of **A** + **B** to be one hundred times larger than the magnitude of **A** − **B**, what must be the angle between them?

52. Two vectors **A** and **B** have precisely equal magnitudes. In order for the magnitude of **A** + **B** to be larger than the magnitude of **A** − **B** by the factor n, what must be the angle between them?

53. A vector is given by $\mathbf{R} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find (a) the magnitudes of the x, y, and z components, (b) the magnitude of **R**, and (c) the angles between **R** and the x, y, and z axes.

54. The biggest stuffed animal in the world is a snake 420 m long, constructed by Norwegian children. Suppose the snake is laid out in a park as shown in Figure P3.54, forming two straight sides of a 105° angle, with one side 240 m long. Olaf and Inge run a race they invent. Inge runs directly from the tail of the snake to its head and Olaf starts from the same place at the same time but runs along the snake. If both children run steadily at 12.0 km/h, Inge reaches the head of the snake how much earlier than Olaf?

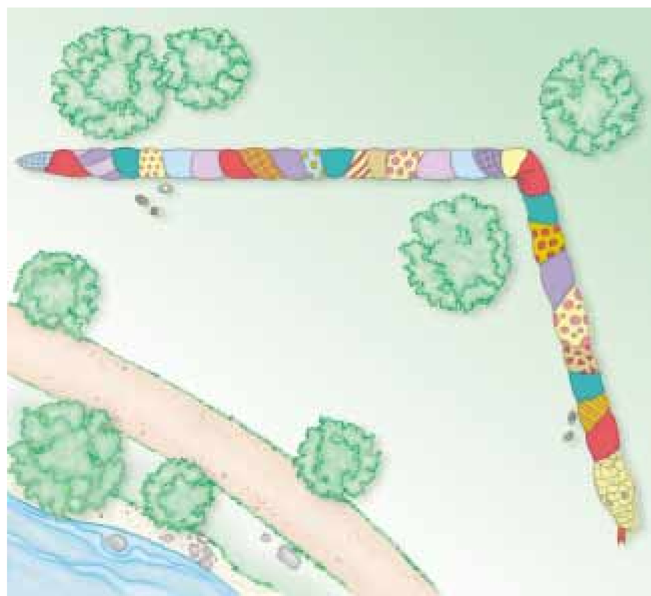


Figure P3.54

55. An air-traffic controller observes two aircraft on his radar screen. The first is at altitude 800 m, horizontal distance 19.2 km, and 25.0° south of west. The second aircraft is at altitude 1 100 m, horizontal distance 17.6 km, and 20.0° south of west. What is the distance between the two aircraft? (Place the x axis west, the y axis south, and the z axis vertical.)

56. A ferry boat transports tourists among three islands. It sails from the first island to the second island, 4.76 km away, in a direction 37.0° north of east. It then sails from the second island to the third island in a direction 69.0° west of north. Finally it returns to the first island, sailing in a direction 28.0° east of south. Calculate the distance between (a) the second and third islands (b) the first and third islands.

57. The rectangle shown in Figure P3.57 has sides parallel to the x and y axes. The position vectors of two corners are **A** = 10.0 m at 50.0° and **B** = 12.0 m at 30.0°. (a) Find the perimeter of the rectangle. (b) Find the magnitude and direction of the vector from the origin to the upper right corner of the rectangle.

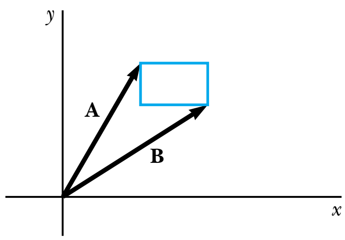


Figure P3.57

58. Find the sum of these four vector forces: 12.0 N to the right at 35.0° above the horizontal, 31.0 N to the left at 55.0° above the horizontal, 8.40 N to the left at 35.0° below the horizontal, and 24.0 N to the right at 55.0° below the horizontal. Follow these steps: Make a drawing of this situation and select the best axes for x and y so you have the least number of components. Then add the vectors by the component method.

$\boxed{59.}$ A person going for a walk follows the path shown in Fig. P3.59. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

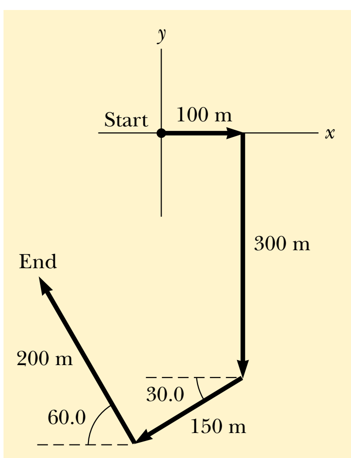


Figure P3.59

60. The instantaneous position of an object is specified by its position vector $\mathbf{r}$ leading from a fixed origin to the location of the point object. Suppose that for a certain object the position vector is a function of time, given by $\mathbf{r} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{k}}$, where r is in meters and t is in seconds. Evaluate $d\mathbf{r}/dt$. What does it represent about the object?

61. A jet airliner, moving initially at 300 mi/h to the east, suddenly enters a region where the wind is blowing at 100 mi/h toward the direction 30.0° north of east. What are the new speed and direction of the aircraft relative to the ground?

62. Long John Silver, a pirate, has buried his treasure on an island with five trees, located at the following points: (30.0 m, −20.0 m), (60.0 m, 80.0 m), (−10.0 m, −10.0 m), (40.0 m, −30.0 m), and (−70.0 m, 60.0 m), all measured relative to some origin, as in Figure P3.62. His ship's log instructs you to start at tree A and move toward tree B, but to cover only one half the distance between A and B. Then move toward tree C, covering one third the distance between your current location and C. Next move toward D, covering one fourth the distance between where you are and D. Finally move towards E, covering one fifth the distance between you and E, stop, and dig. (a) Assume that you have correctly determined the order in which the pirate labeled the trees as A, B, C, D, and E, as shown in the figure. What are the coordinates of the point where his treasure is buried? (b) **What if** you do not really know the way the pirate labeled the trees? Rearrange the order of the trees [for instance, B(30 m, −20 m), A(60 m, 80 m), E(−10 m, −10 m), C(40 m, −30 m), and D(−70 m, 60 m)] and repeat the calculation to show that the answer does not depend on the order in which the trees are labeled.

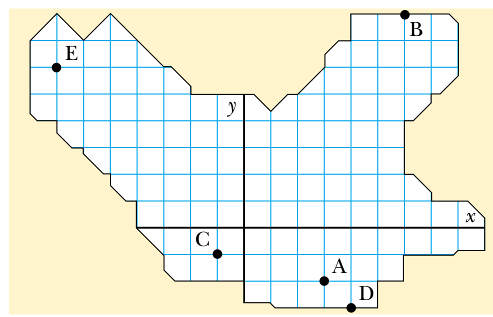


Figure P3.62

63. Consider a game in which N children position themselves at equal distances around the circumference of a circle. At the center of the circle is a rubber tire. Each child holds a rope attached to the tire and, at a signal, pulls on his rope. All children exert forces of the same magnitude F. In the case $N = 2$, it is easy to see that the net force on the tire will be zero, because the two oppositely directed force vectors add to zero. Similarly, if $N = 4$, 6, or any even integer, the resultant force on the tire must be zero, because the forces exerted by each pair of oppositely positioned children will cancel. When an odd number of children are around the circle, it is not so obvious whether the total force on the central tire will be zero. (a) Calculate the net force on the tire in the case $N = 3$, by adding the components of the three force vectors. Choose the x axis to lie along one of the ropes. (b) **What If?** Determine the net force for the general case where N is any integer, odd or even, greater than one. Proceed as follows: Assume that the total force is not zero. Then it must point in some particular direction. Let every child move one position clockwise. Give a reason that the total force must then have a direction turned clockwise by $360°/N$. Argue that the total force must nevertheless be the same as before. Explain that the contradiction proves that the magnitude of the force is zero. This problem illustrates a widely useful technique of proving a result "by symmetry"—by using a bit of the mathematics of *group theory*. The particular situation is actually encountered in physics and chemistry when an array of electric charges (ions) exerts electric forces on an atom at a central position in a molecule or in a crystal.

64. A rectangular parallelepiped has dimensions a, b, and c, as in Figure P3.64. (a) Obtain a vector expression for the face diagonal vector $\mathbf{R}_1$. What is the magnitude of this vector? (b) Obtain a vector expression for the body diagonal vector $\mathbf{R}_2$. Note that $\mathbf{R}_1$, $c\hat{\mathbf{k}}$, and $\mathbf{R}_2$ make a right triangle and prove that the magnitude of $\mathbf{R}_2$ is $\sqrt{a^2 + b^2 + c^2}$.

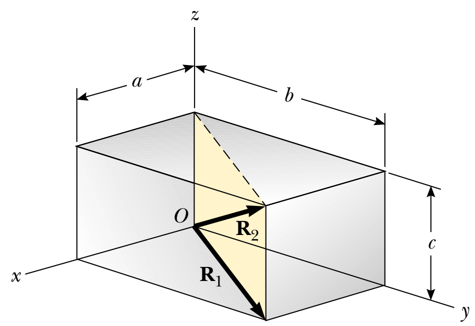


Figure P3.64

65. Vectors **A** and **B** have equal magnitudes of 5.00. If the sum of **A** and **B** is the vector $6.00\hat{\mathbf{j}}$, determine the angle between **A** and **B**.

66. In Figure P3.66 a spider is resting after starting to spin its web. The gravitational force on the spider is 0.150 newton down. The spider is supported by different tension forces in the two strands above it, so that the resultant vector force on the spider is zero. The two strands are perpendicular to each other, so we have chosen the x and y directions to be along them. The tension T_x is 0.127 newton. Find (a) the tension T_y, (b) the angle the x axis makes with the horizontal, and (c) the angle the y axis makes with the horizontal.

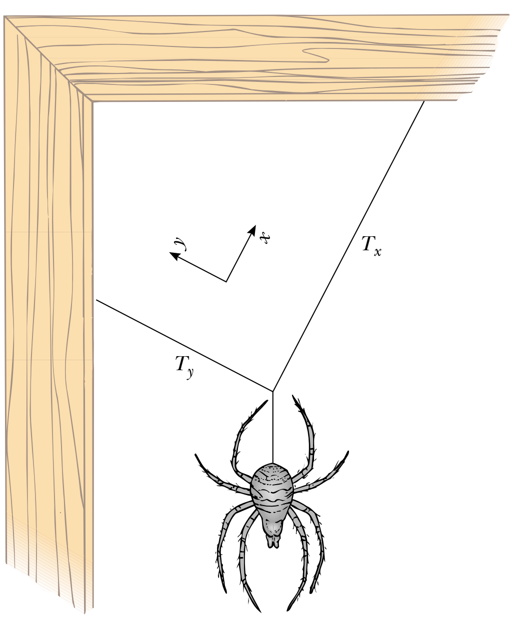


Figure P3.66

67. A point P is described by the coordinates (x, y) with respect to the normal Cartesian coordinate system shown in Fig. P3.67. Show that (x', y'), the coordinates of this point in the rotated coordinate system, are related to (x, y) and the rotation angle α by the expressions

$$x' = x \cos \alpha + y \sin \alpha$$

$$y' = -x \sin \alpha + y \cos \alpha$$

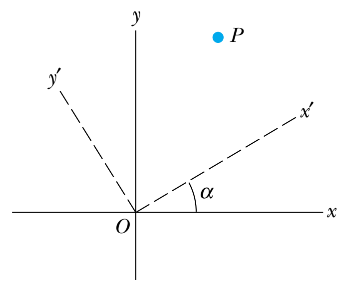


Figure P3.67

Answers to Quick Quizzes

3.1 Scalars: (a), (d), (e). None of these quantities has a direction. Vectors: (b), (c). For these quantities, the direction is necessary to specify the quantity completely.

3.2 (c). The resultant has its maximum magnitude $A + B = 12 + 8 = 20$ units when vector **A** is oriented in the same direction as vector **B**. The resultant vector has its minimum magnitude $A - B = 12 - 8 = 4$ units when vector **A** is oriented in the direction opposite vector **B**.

3.3 (a). The magnitudes will add numerically only if the vectors are in the same direction.

3.4 (b) and (c). In order to add to zero, the vectors must point in opposite directions and have the same magnitude.

3.5 (b). From the Pythagorean theorem, the magnitude of a vector is always larger than the absolute value of each component, unless there is only one nonzero component, in which case the magnitude of the vector is equal to the absolute value of that component.

3.6 (b). From the Pythagorean theorem, we see that the magnitude of a vector is nonzero if at least one component is nonzero.

3.7 (d). Each set of components, for example, the two x components A_x and B_x, must add to zero, so the components must be of opposite sign.

3.8 (c). The magnitude of **C** is 5 units, the same as the z component. Answer (b) is not correct because the magnitude of any vector is always a positive number while the y component of **B** is negative.

Chapter 4

Motion in Two Dimensions

▲ *Lava spews from a volcanic eruption. Notice the parabolic paths of embers projected into the air. We will find in this chapter that all projectiles follow a parabolic path in the absence of air resistance. (© Arndt/Premium Stock/PictureQuest)*

CHAPTER OUTLINE

In this chapter we explore the kinematics of a particle moving in two dimensions. Knowing the basics of two-dimensional motion will allow us to examine—in future chapters—a wide variety of motions, ranging from the motion of satellites in orbit to the motion of electrons in a uniform electric field. We begin by studying in greater detail the vector nature of position, velocity, and acceleration. As in the case of one-dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of these three quantities. We then treat projectile motion and uniform circular motion as special cases of motion in two dimensions. We also discuss the concept of relative motion, which shows why observers in different frames of reference may measure different positions, velocities, and accelerations for a given particle.

4.1 The Position, Velocity, and Acceleration Vectors

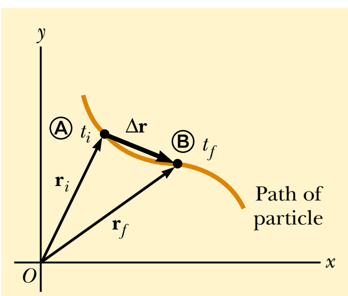


Figure 4.1 A particle moving in the *xy* plane is located with the position vector **r** drawn from the origin to the particle. The displacement of the particle as it moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$.

In Chapter 2 we found that the motion of a particle moving along a straight line is completely known if its position is known as a function of time. Now let us extend this idea to motion in the *xy* plane. We begin by describing the position of a particle by its **position vector r**, drawn from the origin of some coordinate system to the particle located in the *xy* plane, as in Figure 4.1. At time t_i the particle is at point Ⓐ, described by position vector $\mathbf{r}_i$. At some later time t_f it is at point Ⓑ, described by position vector $\mathbf{r}_f$. The path from Ⓐ to Ⓑ is not necessarily a straight line. As the particle moves from Ⓐ to Ⓑ in the time interval $\Delta t = t_f - t_i$, its position vector changes from $\mathbf{r}_i$ to $\mathbf{r}_f$. As we learned in Chapter 2, displacement is a vector, and the displacement of the particle is the difference between its final position and its initial position. We now define the **displacement vector** $\Delta\mathbf{r}$ for the particle of Figure 4.1 as being the difference between its final position vector and its initial position vector:

Displacement vector

$$\Delta\mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i \tag{4.1}$$

The direction of $\Delta\mathbf{r}$ is indicated in Figure 4.1. As we see from the figure, the magnitude of $\Delta\mathbf{r}$ is *less* than the distance traveled along the curved path followed by the particle.

As we saw in Chapter 2, it is often useful to quantify motion by looking at the ratio of a displacement divided by the time interval during which that displacement occurs, which gives the rate of change of position. In two-dimensional (or three-dimensional) kinematics, everything is the same as in one-dimensional kinematics except that we must now use full vector notation rather than positive and negative signs to indicate the direction of motion.

We define the **average velocity** of a particle during the time interval Δt as the displacement of the particle divided by the time interval:

Average velocity

$$\overline{\mathbf{v}} \equiv \frac{\Delta\mathbf{r}}{\Delta t} \tag{4.2}$$

Mark C. Burnett/Photo Researchers, Inc.

Figure 4.2 Bird's-eye view of a baseball diamond. A batter who hits a home run travels around the bases, ending up where he began. Thus, his average velocity for the entire trip is zero. His average speed, however, is not zero and is equal to the distance around the bases divided by the time interval during which he runs around the bases.

Multiplying or dividing a vector quantity by a positive scalar quantity such as Δt changes only the magnitude of the vector, not its direction. Because displacement is a vector quantity and the time interval is a positive scalar quantity, we conclude that the average velocity is a vector quantity directed along $\Delta\mathbf{r}$.

Note that the average velocity between points is *independent of the path* taken. This is because average velocity is proportional to displacement, which depends only on the initial and final position vectors and not on the path taken. As with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip because its displacement is zero. Figure 4.2 suggests such a situation in a baseball park. When a batter hits a home run, he runs around the bases and returns to home plate. Thus, his average velocity is zero during this trip. His average speed, however, is not zero.

Consider again the motion of a particle between two points in the xy plane, as shown in Figure 4.3. As the time interval over which we observe the motion becomes smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at Ⓐ. The **instantaneous velocity v** is defined as the limit of the average velocity $\Delta\mathbf{r}/\Delta t$ as Δt approaches zero:

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \tag{4.3}$$

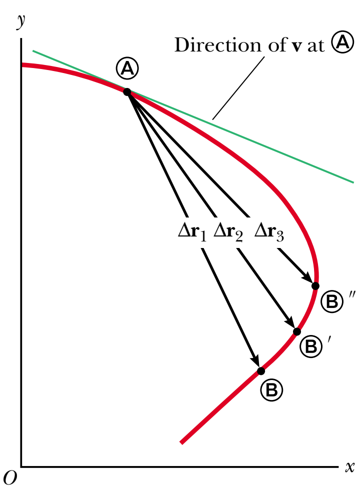


Figure 4.3 As a particle moves between two points, its average velocity is in the direction of the displacement vector $\Delta\mathbf{r}$. As the end point of the path is moved from Ⓑ to Ⓑ′ to Ⓑ″, the respective displacements and corresponding time intervals become smaller and smaller. In the limit that the end point approaches Ⓐ, Δt approaches zero, and the direction of $\Delta\mathbf{r}$ approaches that of the line tangent to the curve at Ⓐ. By definition, the instantaneous velocity at Ⓐ is directed along this tangent line.

That is, the instantaneous velocity equals the derivative of the position vector with respect to time. The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion.

The magnitude of the instantaneous velocity vector $v = |\mathbf{v}|$ is called the *speed,* which is a scalar quantity.

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from $\mathbf{v}_i$ at time t_i to $\mathbf{v}_f$ at time t_f. Knowing the velocity at these points allows us to determine the average acceleration of the particle—the **average acceleration** $\overline{\mathbf{a}}$ of a particle as it moves is defined as the change in the instantaneous velocity vector $\Delta\mathbf{v}$ divided by the time interval Δt during which that change occurs:

$$\overline{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t} \tag{4.4}$$

PITFALL PREVENTION

4.1 Vector Addition

While the vector addition discussed in Chapter 3 involves *displacement* vectors, vector addition can be applied to *any* type of vector quantity. Figure 4.4, for example, shows the addition of *velocity* vectors using the graphical approach.

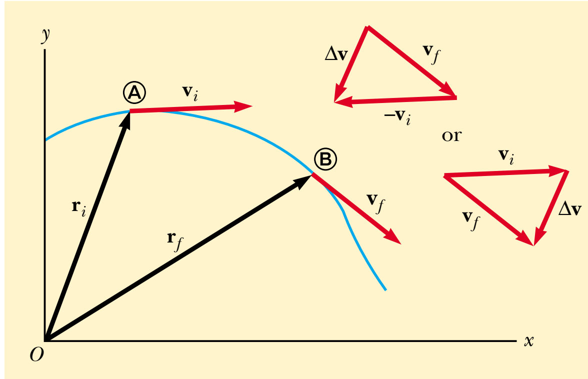


Figure 4.4 A particle moves from position Ⓐ to position Ⓑ. Its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. The vector diagrams at the upper right show two ways of determining the vector $\Delta\mathbf{v}$ from the initial and final velocities.

Because $\bar{\mathbf{a}}$ is the ratio of a vector quantity $\Delta\mathbf{v}$ and a positive scalar quantity Δt, we conclude that average acceleration is a vector quantity directed along $\Delta\mathbf{v}$. As indicated in Figure 4.4, the direction of $\Delta\mathbf{v}$ is found by adding the vector $-\mathbf{v}_i$ (the negative of $\mathbf{v}_i$) to the vector $\mathbf{v}_f$, because by definition $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$.

When the average acceleration of a particle changes during different time intervals, it is useful to define its instantaneous acceleration. The **instantaneous acceleration a** is defined as the limiting value of the ratio $\Delta\mathbf{v}/\Delta t$ as Δt approaches zero:

Instantaneous acceleration

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \tag{4.5}$$

In other words, the instantaneous acceleration equals the derivative of the velocity vector with respect to time.

It is important to recognize that various changes can occur when a particle accelerates. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Finally, both the magnitude and the direction of the velocity vector may change simultaneously.

Quick Quiz 4.1 Which of the following cannot *possibly* be accelerating? (a) An object moving with a constant speed (b) An object moving with a constant velocity (c) An object moving along a curve.

Quick Quiz 4.2 Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are (a) all three controls (b) the gas pedal and the brake (c) only the brake (d) only the gas pedal.

4.2 Two-Dimensional Motion with Constant Acceleration

In Section 2.5, we investigated one-dimensional motion in which the acceleration is constant because this type of motion is common. Let us consider now two-dimensional motion during which the acceleration remains constant in both magnitude and direction. This will also be useful for analyzing some common types of motion.

The position vector for a particle moving in the *xy* plane can be written

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \tag{4.6}$$

where x, y, and $\mathbf{r}$ change with time as the particle moves while the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ remain constant. If the position vector is known, the velocity of the particle can be obtained from Equations 4.3 and 4.6, which give

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} \quad (4.7)$$

Because $\mathbf{a}$ is assumed constant, its components a_x and a_y also are constants. Therefore, we can apply the equations of kinematics to the x and y components of the velocity vector. Substituting, from Equation 2.9, $v_{xf} = v_{xi} + a_xt$ and $v_{yf} = v_{yi} + a_yt$ into Equation 4.7 to determine the final velocity at any time t, we obtain

$$\begin{aligned}\mathbf{v}_f &= (v_{xi} + a_xt)\hat{\mathbf{i}} + (v_{yi} + a_yt)\hat{\mathbf{j}} \\ &= (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t\end{aligned}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \quad (4.8)$$

Velocity vector as a function of time

This result states that the velocity of a particle at some time t equals the vector sum of its initial velocity $\mathbf{v}_i$ and the additional velocity $\mathbf{a}t$ acquired at time t as a result of constant acceleration. It is the vector version of Equation 2.9.

Similarly, from Equation 2.12 we know that the x and y coordinates of a particle moving with constant acceleration are

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_xt^2 \qquad y_f = y_i + v_{yi}t + \tfrac{1}{2}a_yt^2$$

Substituting these expressions into Equation 4.6 (and labeling the final position vector $\mathbf{r}_f$) gives

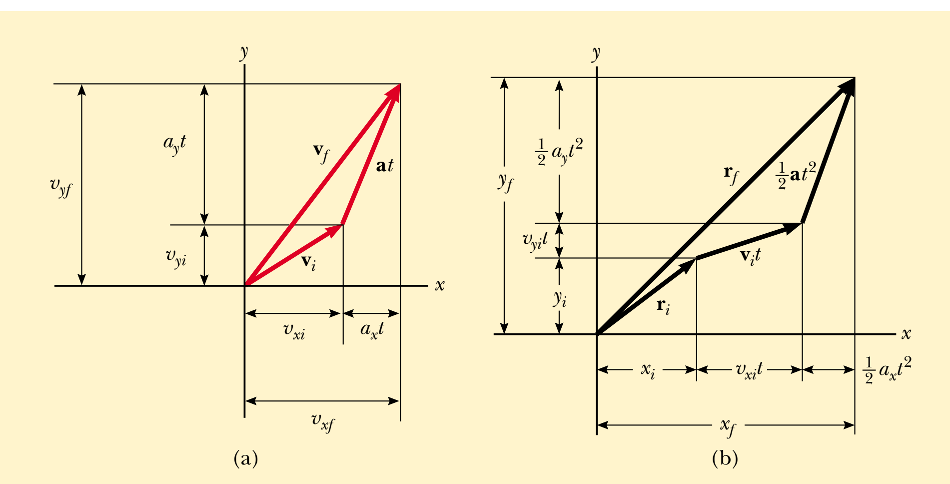
$$\begin{aligned}\mathbf{r}_f &= (x_i + v_{xi}t + \tfrac{1}{2}a_xt^2)\hat{\mathbf{i}} + (y_i + v_{yi}t + \tfrac{1}{2}a_yt^2)\hat{\mathbf{j}} \\ &= (x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}}) + (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})t + \tfrac{1}{2}(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t^2\end{aligned}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_it + \tfrac{1}{2}\mathbf{a}t^2 \quad (4.9)$$

Position vector as a function of time

which is the vector version of Equation 2.12. This equation tells us that the position vector $\mathbf{r}_f$ is the vector sum of the original position $\mathbf{r}_i$, a displacement $\mathbf{v}_it$ arising from the initial velocity of the particle and a displacement $\tfrac{1}{2}\mathbf{a}t^2$ resulting from the constant acceleration of the particle.

Graphical representations of Equations 4.8 and 4.9 are shown in Figure 4.5. Note from Figure 4.5a that $\mathbf{v}_f$ is generally not along the direction of either $\mathbf{v}_i$ or $\mathbf{a}$ because the relationship between these quantities is a vector expression. For the same reason,



Active Figure 4.5 Vector representations and components of (a) the velocity and (b) the position of a particle moving with a constant acceleration $\mathbf{a}$.

At the Active Figures link at http://www.pse6.com, you can investigate the effect of different initial positions and velocities on the final position and velocity (for constant acceleration).

from Figure 4.5b we see that $\mathbf{r}_f$ is generally not along the direction of $\mathbf{v}_i$ or $\mathbf{a}$. Finally, note that $\mathbf{v}_f$ and $\mathbf{r}_f$ are generally not in the same direction.

Because Equations 4.8 and 4.9 are vector expressions, we may write them in component form:

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \quad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases} \tag{4.8a}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \quad \begin{cases} x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \end{cases} \tag{4.9a}$$

These components are illustrated in Figure 4.5. The component form of the equations for $\mathbf{v}_f$ and $\mathbf{r}_f$ show us that two-dimensional motion at constant acceleration is equivalent to two *independent* motions—one in the x direction and one in the y direction—having constant accelerations a_x and a_y.

Example 4.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$.

(A) Determine the components of the velocity vector at any time and the total velocity vector at any time.

Solution After carefully reading the problem, we *conceptualize* what is happening to the particle. The components of the initial velocity tell us that the particle starts by moving toward the right and downward. The x component of velocity starts at 20 m/s and increases by 4.0 m/s every second. The y component of velocity never changes from its initial value of -15 m/s. We sketch a rough motion diagram of the situation in Figure 4.6. Because the particle is accelerating in the $+x$ direction, its velocity component in this direction will increase, so that the path will curve as shown in the diagram. Note that the spacing between successive images increases as time goes on because the speed is increasing. The placement of the acceleration and velocity vectors in Figure 4.6 helps us to further conceptualize the situation.

Because the acceleration is constant, we *categorize* this problem as one involving a particle moving in two dimensions with constant acceleration. To *analyze* such a problem, we use the equations developed in this section. To begin the mathematical analysis, we set $v_{xi} = 20$ m/s, $v_{yi} = -15$ m/s, $a_x = 4.0\ \text{m/s}^2$, and $a_y = 0$.

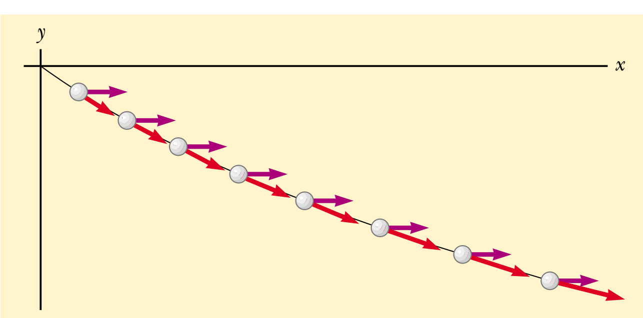


Figure 4.6 (Example 4.1) Motion diagram for the particle.

Equations 4.8a give

(1) $\quad v_{xf} = v_{xi} + a_x t = (20 + 4.0t)$ m/s

(2) $\quad v_{yf} = v_{yi} + a_y t = -15\ \text{m/s} + 0 = -15$ m/s

Therefore

$$\mathbf{v}_f = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}} = \boxed{[(20 + 4.0t)\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s}}$$

We could also obtain this result using Equation 4.8 directly, noting that $\mathbf{a} = 4.0\hat{\mathbf{i}}\ \text{m/s}^2$ and $\mathbf{v}_i = [20\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]$ m/s. To *finalize* this part, notice that the x component of velocity increases in time while the y component remains constant; this is consistent with what we predicted.

(B) Calculate the velocity and speed of the particle at $t = 5.0$ s.

Solution With $t = 5.0$ s, the result from part (A) gives

$$\mathbf{v}_f = [(20 + 4.0(5.0))\hat{\mathbf{i}} - 15\hat{\mathbf{j}}]\ \text{m/s} = \boxed{(40\hat{\mathbf{i}} - 15\hat{\mathbf{j}})\ \text{m/s}}$$

This result tells us that at $t = 5.0$ s, $v_{xf} = 40$ m/s and $v_{yf} = -15$ m/s. Knowing these two components for this two-dimensional motion, we can find both the direction and the magnitude of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis at $t = 5.0$ s, we use the fact that $\tan\theta = v_{yf}/v_{xf}$:

$$(3) \qquad \theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = \boxed{-21^\circ}$$

where the negative sign indicates an angle of 21° below the positive x axis. The speed is the magnitude of $\mathbf{v}_f$:

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = \boxed{43\ \text{m/s}}$$

To *finalize* this part, we notice that if we calculate v_i from the x and y components of $\mathbf{v}_i$, we find that $v_f > v_i$. Is this consistent with our prediction?

(C) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution Because $x_i = y_i = 0$ at $t = 0$, Equation 4.9a gives

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{(20t + 2.0t^2)\text{ m}}$$

$$y_f = v_{yi}t = \boxed{(-15t)\text{ m}}$$

Therefore, the position vector at any time t is

$$(4) \qquad \mathbf{r}_f = x_f\hat{\mathbf{i}} + y_f\hat{\mathbf{j}} = \boxed{[(20t + 2.0t^2)\hat{\mathbf{i}} - 15t\hat{\mathbf{j}}]\text{ m}}$$

(Alternatively, we could obtain $\mathbf{r}_f$ by applying Equation 4.9 directly, with $\mathbf{v}_f = (20\hat{\mathbf{i}} - 15\hat{\mathbf{j}})$ m/s and $\mathbf{a} = 4.0\hat{\mathbf{i}}$ m/s^2. Try it!) Thus, for example, at $t = 5.0$ s, $x = 150$ m, $y = -75$ m, and $\mathbf{r}_f = (150\hat{\mathbf{i}} - 75\hat{\mathbf{j}})$ m. The magnitude of the displacement of the particle from the origin at $t = 5.0$ s is the magnitude of $\mathbf{r}_f$ at this time:

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\text{ m} = 170\text{ m}$$

Note that this is *not* the distance that the particle travels in this time! Can you determine this distance from the available data?

To *finalize* this problem, let us consider a limiting case for very large values of t in the following **What If?**

What If? **What if we wait a very long time and then observe the motion of the particle? How would we describe the motion of the particle for large values of the time?**

Answer Looking at Figure 4.6, we see the path of the particle curving toward the x axis. There is no reason to assume that this tendency will change, so this suggests that the path will become more and more parallel to the x axis as time grows large. Mathematically, let us consider Equations (1) and (2). These show that the y component of the velocity remains constant while the x component grows linearly with t. Thus, when t is very large, the x component of the velocity will be much larger than the y component, suggesting that the velocity vector becomes more and more parallel to the x axis.

Equation (3) gives the angle that the velocity vector makes with the x axis. Notice that $\theta \to 0$ as the denominator (v_{xf}) becomes much larger than the numerator (v_{yf}).

Despite the fact that the velocity vector becomes more and more parallel to the x axis, the particle does not approach a limiting value of y. Equation (4) shows that both x_f and y_f continue to grow with time, although x_f grows much faster.

4.3 Projectile Motion

Assumptions of projectile motion

Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration $\mathbf{g}$ is constant over the range of motion and is directed downward,[1] and (2) the effect of air resistance is negligible.[2] With these assumptions, we find that the path of a projectile, which we call its *trajectory,* is *always* a parabola. **We use these assumptions throughout this chapter.**

To show that the trajectory of a projectile is a parabola, let us choose our reference frame such that the y direction is vertical and positive is upward. Because air resistance is neglected, we know that $a_y = -g$ (as in one-dimensional free fall) and that $a_x = 0$. Furthermore, let us assume that at $t = 0$, the projectile leaves the origin ($x_i = y_i = 0$) with speed v_i, as shown in Figure 4.7. The vector $\mathbf{v}_i$ makes an angle θ_i with the horizontal. From the definitions of the cosine and sine functions we have

$$\cos\theta_i = v_{xi}/v_i \qquad \sin\theta_i = v_{yi}/v_i$$

Therefore, the initial x and y components of velocity are

$$v_{xi} = v_i\cos\theta_i \qquad v_{yi} = v_i\sin\theta_i \tag{4.10}$$

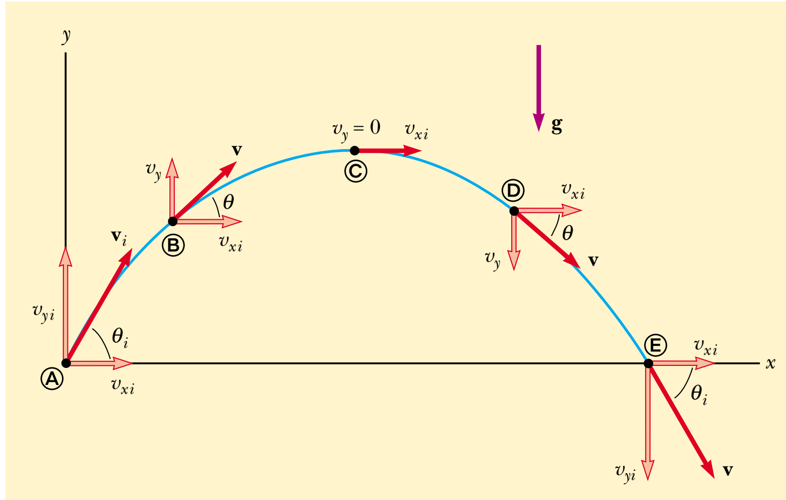
Substituting the x component into Equation 4.9a with $x_i = 0$ and $a_x = 0$, we find that

$$x_f = v_{xi}t = (v_i\cos\theta_i)t \tag{4.11}$$

[1] This assumption is reasonable as long as the range of motion is small compared with the radius of the Earth (6.4×10^6 m). In effect, this assumption is equivalent to assuming that the Earth is flat over the range of motion considered.

[2] This assumption is generally *not* justified, especially at high velocities. In addition, any spin imparted to a projectile, such as that applied when a pitcher throws a curve ball, can give rise to some very interesting effects associated with aerodynamic forces, which will be discussed in Chapter 14.

At the Active Figures link at http://www.pse6.com, you can change launch angle and initial speed. You can also observe the changing components of velocity along the trajectory of the projectile.



Active Figure 4.7 The parabolic path of a projectile that leaves the origin with a velocity $\mathbf{v}_i$. The velocity vector $\mathbf{v}$ changes with time in both magnitude and direction. This change is the result of acceleration in the negative y direction. The x component of velocity remains constant in time because there is no acceleration along the horizontal direction. The y component of velocity is zero at the peak of the path.

PITFALL PREVENTION

4.2 Acceleration at the Highest Point

As discussed in Pitfall Prevention 2.8, many people claim that the acceleration of a projectile at the topmost point of its trajectory is zero. This mistake arises from confusion between zero vertical velocity and zero acceleration. If the projectile were to experience zero acceleration at the highest point, then its velocity at that point would not change—the projectile would move horizontally at constant speed from then on! This does not happen, because the acceleration is NOT zero anywhere along the trajectory.

The Telegraph Colour Library/Getty Images

A welder cuts holes through a heavy metal construction beam with a hot torch. The sparks generated in the process follow parabolic paths.

Repeating with the y component and using $y_i = 0$ and $a_y = -g$, we obtain

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i \sin\theta_i)\,t - \tfrac{1}{2}gt^2 \qquad (4.12)$$

Next, from Equation 4.11 we find $t = x_f/(v_i \cos\theta_i)$ and substitute this expression for t into Equation 4.12; this gives

$$y = (\tan\theta_i)\,x - \left(\frac{g}{2{v_i}^2\cos^2\theta_i}\right)x^2$$

This equation is valid for launch angles in the range $0 < \theta_i < \pi/2$. We have left the subscripts off the x and y because the equation is valid for any point (x, y) along the path of the projectile. The equation is of the form $y = ax - bx^2$, which is the equation of a parabola that passes through the origin. Thus, we have shown that the trajectory of a projectile is a parabola. Note that the trajectory is completely specified if both the initial speed v_i and the launch angle θ_i are known.

The vector expression for the position vector of the projectile as a function of time follows directly from Equation 4.9, with $\mathbf{a} = \mathbf{g}$:

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{g}t^2$$

This expression is plotted in Figure 4.8, for a projectile launched from the origin, so that $\mathbf{r}_i = 0$.

The final position of a particle can be considered to be the superposition of the initial position $\mathbf{r}_i$, the term $\mathbf{v}_i t$, which is the displacement if no acceleration were present, and the term $\frac{1}{2}\mathbf{g}t^2$ that arises from the acceleration due to gravity. In other words, if there were no gravitational acceleration, the particle would continue to move along a straight path in the direction of $\mathbf{v}_i$. Therefore, the vertical distance $\frac{1}{2}\mathbf{g}t^2$ through which the particle "falls" off the straight-line path is the same distance that a freely falling object would fall during the same time interval.

In Section 4.2, we stated that two-dimensional motion with constant acceleration can be analyzed as a combination of two independent motions in the x and y directions, with accelerations a_x and a_y. Projectile motion is a special case of two-dimensional motion with constant acceleration and can be handled in this way, with zero acceleration in the x direction and $a_y = -g$ in the y direction. Thus, **when analyzing projectile motion, consider it to be the superposition of two motions:**

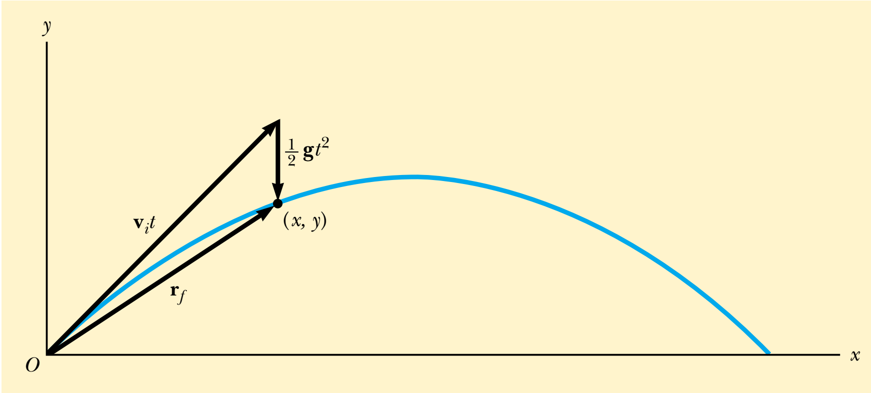


Figure 4.8 The position vector $\mathbf{r}_f$ of a projectile launched from the origin whose initial velocity at the origin is $\mathbf{v}_i$. The vector $\mathbf{v}_i t$ would be the displacement of the projectile if gravity were absent, and the vector $\frac{1}{2}\mathbf{g}t^2$ is its vertical displacement due to its downward gravitational acceleration.

(1) **constant-velocity motion in the horizontal direction** and (2) **free-fall motion in the vertical direction.** The horizontal and vertical components of a projectile's motion are completely independent of each other and can be handled separately, with time t as the common variable for both components.

Quick Quiz 4.3 Suppose you are running at constant velocity and you wish to throw a ball such that you will catch it as it comes back down. In what direction should you throw the ball relative to you? (a) straight up (b) at an angle to the ground that depends on your running speed (c) in the forward direction.

Quick Quiz 4.4 As a projectile thrown upward moves in its parabolic path (such as in Figure 4.8), at what point along its path are the velocity and acceleration vectors for the projectile perpendicular to each other? (a) nowhere (b) the highest point (c) the launch point.

Quick Quiz 4.5 As the projectile in Quick Quiz 4.4 moves along its path, at what point are the velocity and acceleration vectors for the projectile parallel to each other? (a) nowhere (b) the highest point (c) the launch point.

Example 4.2 Approximating Projectile Motion

A ball is thrown in such a way that its initial vertical and horizontal components of velocity are 40 m/s and 20 m/s, respectively. Estimate the total time of flight and the distance the ball is from its starting point when it lands.

Solution A motion diagram like Figure 4.9 helps us *conceptualize* the problem. The phrase "A ball is thrown" allows us to *categorize* this as a projectile motion problem, which we *analyze* by continuing to study Figure 4.9. The acceleration vectors are all the same, pointing downward with a magnitude of nearly 10 m/s^2. The velocity vectors change direction. Their horizontal components are all the same: 20 m/s.

Remember that the two velocity components are independent of each other. By considering the vertical motion

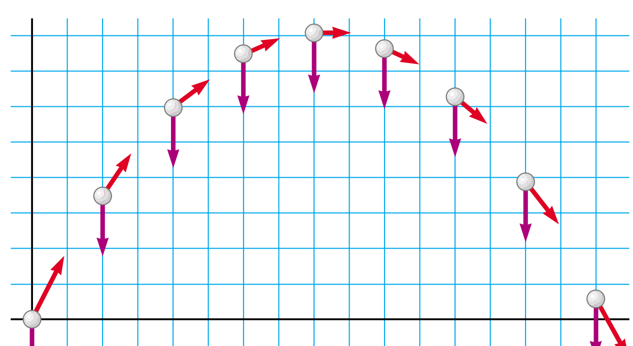


Figure 4.9 (Example 4.2) Motion diagram for a projectile.

first, we can determine how long the ball remains in the air. Because the vertical motion is free-fall, the vertical components of the velocity vectors change, second by second, from 40 m/s to roughly 30, 20, and 10 m/s in the upward direction, and then to 0 m/s. Subsequently, its velocity becomes 10, 20, 30, and 40 m/s in the downward direction. Thus it takes the ball about 4 s to go up and another 4 s to come back down, for a total time of flight of approximately 8 s.

Now we shift our analysis to the horizontal motion. Because the horizontal component of velocity is 20 m/s, and because the ball travels at this speed for 8 s, it ends up approximately 160 m from its starting point.

This is the first example that we have performed for projectile motion. In subsequent projectile motion problems, keep in mind the importance of separating the two components and of making approximations to give you rough expected results.

Horizontal Range and Maximum Height of a Projectile

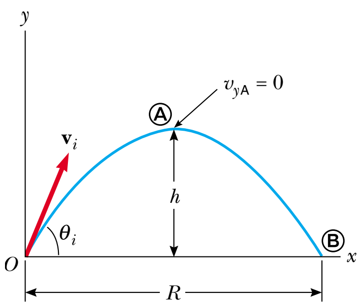


Figure 4.10 A projectile launched from the origin at $t_i = 0$ with an initial velocity $\mathbf{v}_i$. The maximum height of the projectile is h, and the horizontal range is R. At Ⓐ, the peak of the trajectory, the particle has coordinates $(R/2, h)$.

Let us assume that a projectile is launched from the origin at $t_i = 0$ with a positive v_{yi} component, as shown in Figure 4.10. Two points are especially interesting to analyze: the peak point Ⓐ, which has Cartesian coordinates $(R/2, h)$, and the point Ⓑ, which has coordinates $(R, 0)$. The distance R is called the *horizontal range* of the projectile, and the distance h is its *maximum height.* Let us find h and R in terms of v_i, θ_i, and g.

We can determine h by noting that at the peak, $v_{yA} = 0$. Therefore, we can use Equation 4.8a to determine the time t_A at which the projectile reaches the peak:

$$v_{yf} = v_{yi} + a_y t$$

$$0 = v_i \sin\theta_i - g t_A$$

$$t_A = \frac{v_i \sin\theta_i}{g}$$

Substituting this expression for t_A into the y part of Equation 4.9a and replacing $y = y_A$ with h, we obtain an expression for h in terms of the magnitude and direction of the initial velocity vector:

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \tfrac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \tag{4.13}$$

The range R is the horizontal position of the projectile at a time that is twice the time at which it reaches its peak, that is, at time $t_B = 2t_A$. Using the x part of Equation 4.9a, noting that $v_{xi} = v_{xB} = v_i \cos\theta_i$ and setting $x_B = R$ at $t = 2t_A$, we find that

$$R = v_{xi} t_B = (v_i \cos\theta_i) 2t_A$$

$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Using the identity $\sin 2\theta = 2\sin\theta\cos\theta$ (see Appendix B.4), we write R in the more compact form

$$R = \frac{v_i^2 \sin 2\theta_i}{g} \tag{4.14}$$

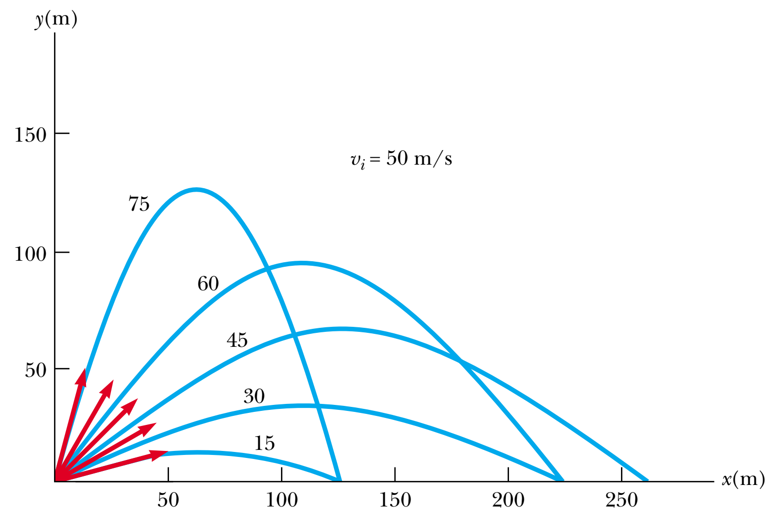
The maximum value of R from Equation 4.14 is $R_{max} = v_i^2/g$. This result follows from the fact that the maximum value of $\sin 2\theta_i$ is 1, which occurs when $2\theta_i = 90°$. Therefore, R is a maximum when $\theta_i = 45°$.

Figure 4.11 illustrates various trajectories for a projectile having a given initial speed but launched at different angles. As you can see, the range is a maximum for $\theta_i = 45°$. In addition, for any θ_i other than 45°, a point having Cartesian coordinates $(R, 0)$ can be reached by using either one of two complementary values of θ_i, such as 75° and 15°. Of course, the maximum height and time of flight for one of these values of θ_i are different from the maximum height and time of flight for the complementary value.

PITFALL PREVENTION

4.3 The Height and Range Equations

Equation 4.14 is useful for calculating R only for a symmetric path, as shown in Figure 4.10. If the path is not symmetric, *do not use this equation.* The general expressions given by Equations 4.8 and 4.9 are the *more important* results, because they give the position and velocity components of *any* particle moving in two dimensions at *any* time t.



Active Figure 4.11 A projectile launched from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_i result in the same value of R (range of the projectile).

At the Active Figures link at* http://www.pse6.com, *you can vary the projection angle to observe the effect on the trajectory and measure the flight time.

Quick Quiz 4.6 Rank the launch angles for the five paths in Figure 4.11 with respect to time of flight, from the shortest time of flight to the longest.

PROBLEM-SOLVING HINTS

Projectile Motion

We suggest that you use the following approach to solving projectile motion problems:

- Select a coordinate system and resolve the initial velocity vector into x and y components.
- Follow the techniques for solving constant-velocity problems to analyze the horizontal motion. Follow the techniques for solving constant-acceleration problems to analyze the vertical motion. The x and y motions share the same time t.

Example 4.3 The Long Jump

A long-jumper (Fig. 4.12) leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(A) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)

Solution We *conceptualize* the motion of the long-jumper as equivalent to that of a simple projectile such as the ball in Example 4.2, and *categorize* this problem as a projectile motion problem. Because the initial speed and launch angle are given, and because the final height is the same as the initial height, we further categorize this problem as satisfying the conditions for which Equations 4.13 and 4.14 can be used. This is the most direct way to *analyze* this problem, although the general methods that we have been describing will always give the correct answer. We will take the general approach and use components. Figure 4.10 provides a graphical representation of the flight of the long-jumper. As before, we set our origin of coordinates at the takeoff point and label the peak as Ⓐ and the landing point as Ⓑ. The horizontal motion is described by Equation 4.11:

$$x_f = x_B = (v_i \cos \theta_i)\, t_B = (11.0 \text{ m/s})(\cos 20.0°)\, t_B$$

The value of x_B can be found if the time of landing t_B is known. We can find t_B by remembering that $a_y = -g$ and by using the y part of Equation 4.8a. We also note that at the top of the jump the vertical component of velocity v_{yA} is zero:

$$v_{yf} = v_{yA} = v_i \sin \theta_i - g t_A$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2)\, t_A$$

$$t_A = 0.384 \text{ s}$$

Figure 4.12 (Example 4.3) Mike Powell, current holder of the world long jump record of 8.95 m.

This is the time at which the long-jumper is at the *top* of the jump. Because of the symmetry of the vertical motion, another 0.384 s passes before the jumper returns to the ground. Therefore, the time at which the jumper lands is $t_B = 2t_A = 0.768$ s. Substituting this value into the above expression for x_f gives

$$x_f = x_B = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s}) = \boxed{7.94 \text{ m}}$$

This is a reasonable distance for a world-class athlete.

(B) What is the maximum height reached?

Solution We find the maximum height reached by using Equation 4.12:

$$\begin{aligned} y_{max} = y_A &= (v_i \sin \theta_i) t_A - \tfrac{1}{2} g t_A{}^2 \\ &= (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s}) \\ &\quad - \tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2 = \boxed{0.722 \text{ m}} \end{aligned}$$

To *finalize* this problem, find the answers to parts (A) and (B) using Equations 4.13 and 4.14. The results should agree. Treating the long-jumper as a particle is an oversimplification. Nevertheless, the values obtained are consistent with experience in sports. We learn that we can model a complicated system such as a long-jumper as a particle and still obtain results that are reasonable.

Example 4.4 A Bull's-Eye Every Time

Interactive

In a popular lecture demonstration, a projectile is fired at a target T in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure 4.13. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.

Solution *Conceptualize* the problem by studying Figure 4.13. Notice that the problem asks for no numbers. The expected result must involve an algebraic argument. Because both objects are subject only to gravity, we *categorize* this problem as one involving two objects in free-fall, one moving in one dimension and one moving in two. Let us now *analyze* the problem. A collision results under the conditions stated by noting that, as soon as they are released, the projectile and the target experience the same acceleration, $a_y = -g$. Figure 4.13b shows that the initial y coordinate of the target is $x_T \tan \theta_i$ and that it falls to a position $\frac{1}{2}gt^2$ below this coordinate at time t. Therefore, the y coordinate of the target at any moment after release is

$$y_T = x_T \tan \theta_i - \tfrac{1}{2}gt^2$$

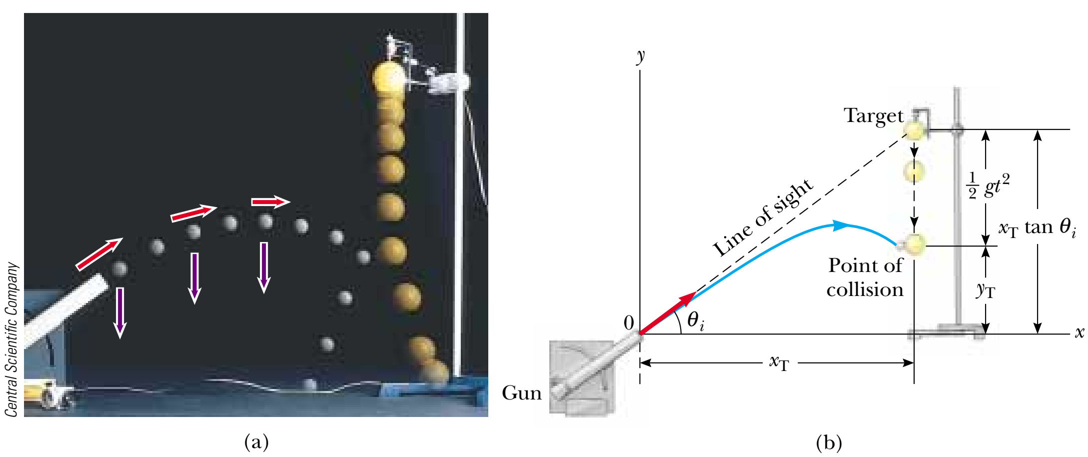


Figure 4.13 (Example 4.4) (a) Multiflash photograph of projectile–target demonstration. If the gun is aimed directly at the target and is fired at the same instant the target begins to fall, the projectile will hit the target. Note that the velocity of the projectile (red arrows) changes in direction and magnitude, while its downward acceleration (violet arrows) remains constant. (b) Schematic diagram of the projectile–target demonstration. Both projectile and target have fallen through the same vertical distance at time t, because both experience the same acceleration $a_y = -g$.

Now if we use Equation 4.9a to write an expression for the y coordinate of the projectile at any moment, we obtain

$$y_P = x_P \tan \theta_i - \tfrac{1}{2}gt^2$$

Thus, by comparing the two previous equations, we see that when the y coordinates of the projectile and target are the same, their x coordinates are the same and a collision results. That is, when $y_P = y_T$, $x_P = x_T$. You can obtain the same result, using expressions for the position vectors for the projectile and target.

To *finalize* this problem, note that a collision can result only when $v_i \sin \theta_i \geq \sqrt{gd/2}$ where d is the initial elevation of the target above the floor. If $v_i \sin \theta_i$ is less than this value, the projectile will strike the floor before reaching the target.

Investigate this situation at the Interactive Worked Example link at **http://www.pse6.com.**

Example 4.5 That's Quite an Arm!

Interactive

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s, as shown in Figure 4.14. If the height of the building is 45.0 m,

(A) how long does it take the stone to reach the ground?

Solution We *conceptualize* the problem by studying Figure 4.14, in which we have indicated the various parameters. By now, it should be natural to *categorize* this as a projectile motion problem.

To *analyze* the problem, let us once again separate motion into two components. The initial x and y components of the stone's velocity are

$$v_{xi} = v_i \cos \theta_i = (20.0 \text{ m/s}) \cos 30.0° = 17.3 \text{ m/s}$$

$$v_{yi} = v_i \sin \theta_i = (20.0 \text{ m/s}) \sin 30.0° = 10.0 \text{ m/s}$$

To find t, we can use $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ (Eq. 4.9a) with $y_i = 0$, $y_f = -45.0$ m, $a_y = -g$, and $v_{yi} = 10.0$ m/s (there is a negative sign on the numerical value of y_f because we have chosen the top of the building as the origin):

$$-45.0 \text{ m} = (10.0 \text{ m/s})t - \tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t = \boxed{4.22 \text{ s.}}$ To *finalize* this part, think: Does the negative root have any physical meaning?

(B) What is the speed of the stone just before it strikes the ground?

Solution We can use Equation 4.8a, $v_{yf} = v_{yi} + a_y t$, with $t = 4.22$ s to obtain the y component of the velocity just before the stone strikes the ground:

$$v_{yf} = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$$

Because $v_{xf} = v_{xi} = 17.3$ m/s, the required speed is

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = \boxed{35.9 \text{ m/s}}$$

To *finalize* this part, is it reasonable that the y component of the final velocity is negative? Is it reasonable that the final speed is larger than the initial speed of 20.0 m/s?

What If? **What if a horizontal wind is blowing in the same direction as the ball is thrown and it causes the ball to have a horizontal acceleration component $a_x = 0.500$ m/s^2. Which part of this example, (A) or (B), will have a different answer?**

Answer Recall that the motions in the x and y directions are independent. Thus, the horizontal wind cannot affect the vertical motion. The vertical motion determines the time of the projectile in the air, so the answer to (A) does not change. The wind will cause the horizontal velocity component to increase with time, so that the final speed will change in part (B).

We can find the new final horizontal velocity component by using Equation 4.8a:

$$v_{xf} = v_{xi} + a_x t = 17.3 \text{ m/s} + (0.500 \text{ m/s}^2)(4.22 \text{ s})$$
$$= 19.4 \text{ m/s}$$

and the new final speed:

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(19.4)^2 + (-31.4)^2} \text{ m/s} = 36.9 \text{ m/s}$$

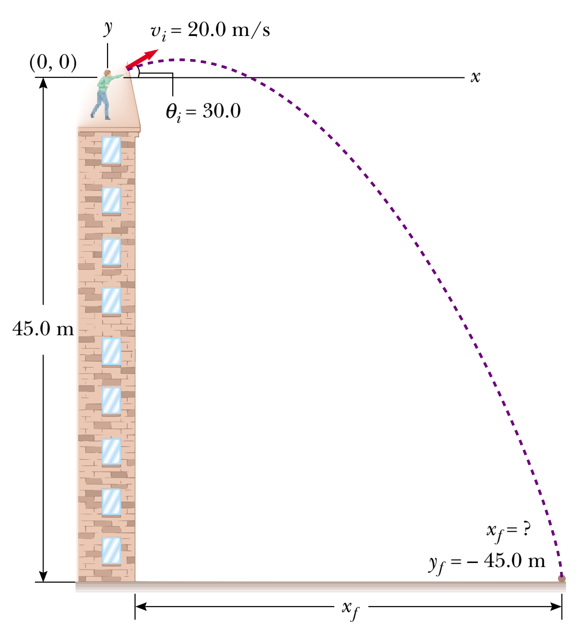


Figure 4.14 (Example 4.5) A stone is thrown from the top of a building.

Investigate this situation at the Interactive Worked Example link at **http://www.pse6.com.**

Example 4.6 The Stranded Explorers

A plane drops a package of supplies to a party of explorers, as shown in Figure 4.15. If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it is released?

Solution *Conceptualize* what is happening with the assistance of Figure 4.15. The plane is traveling horizontally when it drops the package. Because the package is in free-fall while moving in the horizontal direction, we *categorize* this as a projectile motion problem. To *analyze* the problem, we choose the coordinate system shown in Figure 4.15, in which the origin is at the point of release of the package. Consider first its horizontal motion. The only equation available for finding the position along the horizontal direction is $x_f = x_i + v_{xi}t$ (Eq. 4.9a). The initial x component of the package velocity is the same as that of the plane when the package is released: 40.0 m/s. Thus, we have

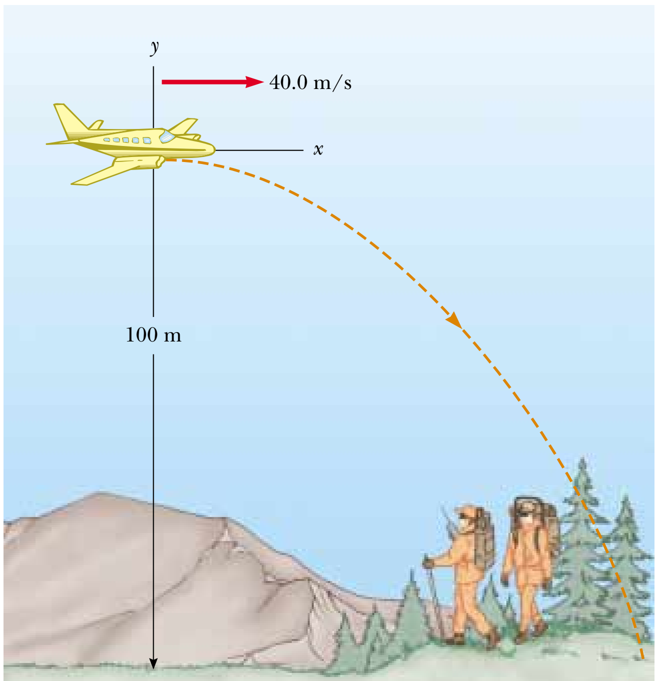


Figure 4.15 (Example 4.6) A package of emergency supplies is dropped from a plane to stranded explorers.

$$x_f = (40.0 \text{ m/s})\,t$$

If we know t, the time at which the package strikes the ground, then we can determine x_f, the distance the package travels in the horizontal direction. To find t, we use the equations that describe the vertical motion of the package. We know that, at the instant the package hits the ground, its y coordinate is $y_f = -100$ m. We also know that the initial vertical component of the package velocity v_{yi} is zero because at the moment of release, the package has only a horizontal component of velocity.

From Equation 4.9a, we have

$$y_f = -\tfrac{1}{2}gt^2$$

$$-100 \text{ m} = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

$$t = 4.52 \text{ s}$$

Substitution of this value for the time into the equation for the x coordinate gives

$$x_f = (40.0 \text{ m/s})(4.52 \text{ s}) = \boxed{181 \text{ m}}$$

The package hits the ground 181 m to the right of the drop point. To *finalize* this problem, we learn that an object dropped from a moving airplane does not fall straight down. It hits the ground at a point different from the one right below the plane where it was released. This was an important consideration for free-fall bombs such as those used in World War II.

Example 4.7 The End of the Ski Jump

A ski-jumper leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s, as shown in Figure 4.16. The landing incline below him falls off with a slope of 35.0°. Where does he land on the incline?

Solution We can *conceptualize* this problem based on observations of winter Olympic ski competitions. We observe the skier to be airborne for perhaps 4 s and go a distance of about 100 m horizontally. We should expect the value of d, the distance traveled along the incline, to be of the same order of magnitude. We *categorize* the problem as that of a particle in projectile motion.

To *analyze* the problem, it is convenient to select the beginning of the jump as the origin. Because $v_{xi} = 25.0$ m/s and $v_{yi} = 0$, the x and y component forms of Equation 4.9a are

$$(1) \qquad x_f = v_{xi}t = (25.0 \text{ m/s})\,t$$

$$(2) \qquad y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

From the right triangle in Figure 4.16, we see that the jumper's x and y coordinates at the landing point are $x_f = d\cos 35.0°$ and $y_f = -d\sin 35.0°$. Substituting these relationships into (1) and (2), we obtain

$$(3) \qquad d\cos 35.0° = (25.0 \text{ m/s})\,t$$

$$(4) \qquad -d\sin 35.0° = -\tfrac{1}{2}(9.80 \text{ m/s}^2)\,t^2$$

Solving (3) for t and substituting the result into (4), we find that $d = 109$ m. Hence, the x and y coordinates of the point at which the skier lands are

$$x_f = d\cos 35.0° = (109 \text{ m})\cos 35.0° = \boxed{89.3 \text{ m}}$$

$$y_f = -d\sin 35.0° = -(109 \text{ m})\sin 35.0° = \boxed{-62.5 \text{ m}}$$

To *finalize* the problem, let us compare these results to our expectations. We expected the horizontal distance to be on the order of 100 m, and our result of 89.3 m is indeed on

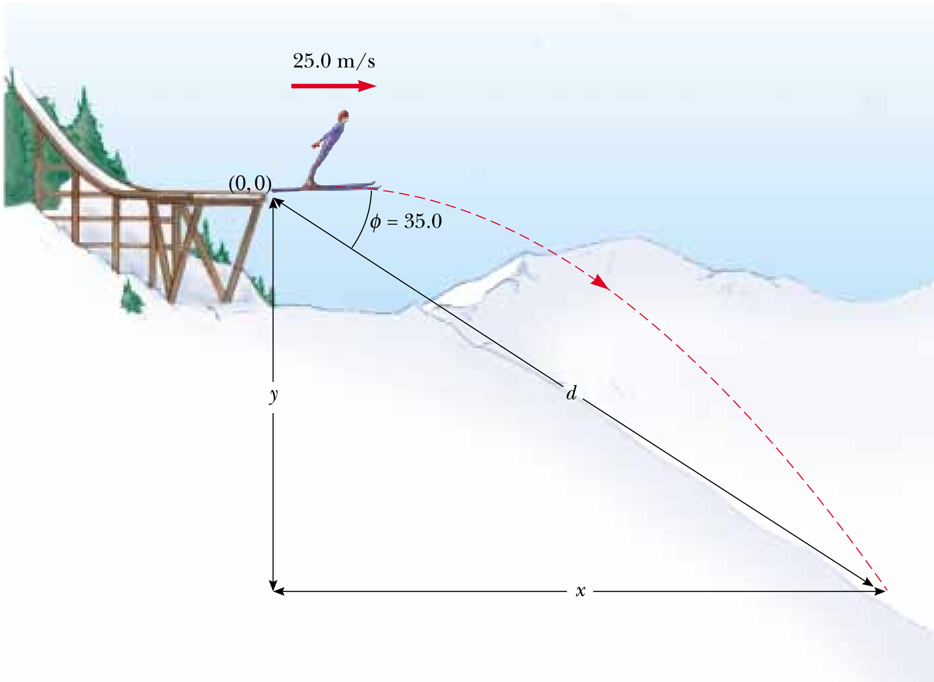


Figure 4.16 (Example 4.7) A ski jumper leaves the track moving in a horizontal direction.

this order of magnitude. It might be useful to calculate the time interval that the jumper is in the air and compare it to our estimate of about 4 s.

What If? **Suppose everything in this example is the same except that the ski jump is curved so that the jumper is projected upward at an angle from the end of the track. Is this a better design in terms of maximizing the length of the jump?**

Answer If the initial velocity has an upward component, the skier will be in the air longer, and should therefore travel further. However, tilting the initial velocity vector upward will reduce the horizontal component of the initial velocity. Thus, angling the end of the ski track upward at a *large* angle may actually *reduce* the distance. Consider the extreme case. The skier is projected at 90° to the horizontal, and simply goes up and comes back down at the end of the ski track! This argument suggests that there must be an optimal angle between 0 and 90° that represents a balance between making the flight time longer and the horizontal velocity component smaller.

We can find this optimal angle mathematically. We modify equations (1) through (4) in the following way, assuming that the skier is projected at an angle θ with respect to the horizontal:

$$\text{(1) and (3)} \quad \rightarrow \quad x_f = (v_i \cos\theta)t = d\cos\phi$$

$$\text{(2) and (4)} \quad \rightarrow \quad y_f = (v_i \sin\theta)t - \tfrac{1}{2}gt^2 = -d\sin\phi$$

If we eliminate the time t between these equations and then use differentiation to maximize d in terms of θ, we arrive (after several steps—see Problem 72!) at the following equation for the angle θ that gives the maximum value of d:

$$\theta = 45° - \frac{\phi}{2}$$

For the slope angle in Figure 4.16, $\phi = 35.0°$; this equation results in an optimal launch angle of $\theta = 27.5°$. Notice that for a slope angle of $\phi = 0°$, which represents a horizontal plane, this equation gives an optimal launch angle of $\theta = 45°$, as we would expect (see Figure 4.11).

4.4 Uniform Circular Motion

Figure 4.17a shows a car moving in a circular path with *constant speed v*. Such motion is called **uniform circular motion,** and occurs in many situations. It is often surprising to students to find that **even though an object moves at a constant speed in a circular path, it still has an acceleration.** To see why, consider the defining equation for average acceleration, $\overline{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ (Eq. 4.4).

Note that the acceleration depends on *the change in the velocity vector.* Because velocity is a vector quantity, there are two ways in which an acceleration can occur, as mentioned in Section 4.1: by a change in the *magnitude* of the velocity and/or by a change in the *direction* of the velocity. The latter situation occurs for an object moving with constant speed in a circular path. The velocity vector is always tangent to the

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4.4 Acceleration of a Particle in Uniform Circular Motion

Remember that acceleration in physics is defined as a change in the *velocity,* not a change in the *speed* (contrary to the everyday interpretation). In circular motion, the velocity vector is changing in direction, so there is indeed an acceleration.

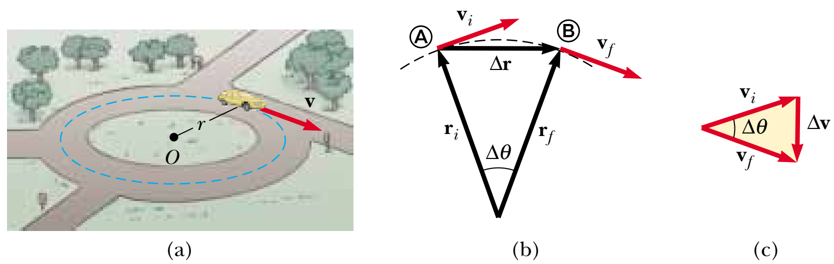

Figure 4.17 (a) A car moving along a circular path at constant speed experiences uniform circular motion. (b) As a particle moves from Ⓐ to Ⓑ, its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$, which is toward the center of the circle for small $\Delta\mathbf{r}$.

path of the object and perpendicular to the radius of the circular path. We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An acceleration of this nature is called a **centripetal acceleration** (*centripetal* means *center-seeking*), and its magnitude is

Centripetal acceleration

$$a_c = \frac{v^2}{r} \tag{4.15}$$

where r is the radius of the circle. The subscript on the acceleration symbol reminds us that the acceleration is centripetal.

First note that the acceleration must be perpendicular to the path followed by the object, which we will model as a particle. If this were not true, there would be a component of the acceleration parallel to the path and, therefore, parallel to the velocity vector. Such an acceleration component would lead to a change in the speed of the particle along the path. But this is inconsistent with our setup of the situation—the particle moves with constant speed along the path. Thus, for *uniform* circular motion, the acceleration vector can only have a component perpendicular to the path, which is toward the center of the circle.

To derive Equation 4.15, consider the diagram of the position and velocity vectors in Figure 4.17b. In addition, the figure shows the vector representing the change in position $\Delta\mathbf{r}$. The particle follows a circular path, part of which is shown by the dotted curve. The particle is at Ⓐ at time t_i, and its velocity at that time is $\mathbf{v}_i$; it is at Ⓑ at some later time t_f, and its velocity at that time is $\mathbf{v}_f$. Let us also assume that $\mathbf{v}_i$ and $\mathbf{v}_f$ differ only in direction; their magnitudes are the same (that is, $v_i = v_f = v$, because it is *uniform* circular motion). In order to calculate the acceleration of the particle, let us begin with the defining equation for average acceleration (Eq. 4.4):

$$\overline{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta\mathbf{v}}{\Delta t}$$

In Figure 4.17c, the velocity vectors in Figure 4.17b have been redrawn tail to tail. The vector $\Delta\mathbf{v}$ connects the tips of the vectors, representing the vector addition $\mathbf{v}_f = \mathbf{v}_i + \Delta\mathbf{v}$. In both Figures 4.17b and 4.17c, we can identify triangles that help us analyze the motion. The angle $\Delta\theta$ between the two position vectors in Figure 4.17b is the same as the angle between the velocity vectors in Figure 4.17c, because the velocity vector $\mathbf{v}$ is always perpendicular to the position vector $\mathbf{r}$. Thus, the two triangles are *similar*. (Two triangles are similar if the angle between any two sides is the same for both triangles and if the ratio of the lengths of these sides is the same.) This enables us to write a relationship between the lengths of the sides for the two triangles:

$$\frac{|\Delta\mathbf{v}|}{v} = \frac{|\Delta\mathbf{r}|}{r}$$

where $v = v_i = v_f$ and $r = r_i = r_f$. This equation can be solved for $|\Delta\mathbf{v}|$ and the expression so obtained can be substituted into $\overline{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ to give the magnitude of the average acceleration over the time interval for the particle to move from Ⓐ to Ⓑ:

$$|\overline{\mathbf{a}}| = \frac{|\Delta\mathbf{v}|}{\Delta t} = \frac{v}{r}\frac{|\Delta\mathbf{r}|}{\Delta t}$$

Now imagine that points Ⓐ and Ⓑ in Figure 4.17b become extremely close together. As Ⓐ and Ⓑ approach each other, Δt approaches zero, and the ratio $|\Delta\mathbf{r}|/\Delta t$ approaches the speed v. In addition, the average acceleration becomes the instantaneous acceleration at point Ⓐ. Hence, in the limit $\Delta t \to 0$, the magnitude of the acceleration is

$$a_c = \frac{v^2}{r}$$

Thus, in uniform circular motion the acceleration is directed inward toward the center of the circle and has magnitude v^2/r.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the **period** T, which is defined as the time required for one complete revolution. In the time interval T the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v = 2\pi r/T$, it follows that

Period of circular motion

$$T \equiv \frac{2\pi r}{v} \tag{4.16}$$

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4.5 Centripetal Acceleration is not Constant

We derived the magnitude of the centripetal acceleration vector and found it to be constant for uniform circular motion. But *the centripetal acceleration vector is not constant.* It always points toward the center of the circle, but continuously changes direction as the object moves around the circular path.

Quick Quiz 4.7 Which of the following correctly describes the centripetal acceleration vector for a particle moving in a circular path? (a) constant and always perpendicular to the velocity vector for the particle (b) constant and always parallel to the velocity vector for the particle (c) of constant magnitude and always perpendicular to the velocity vector for the particle (d) of constant magnitude and always parallel to the velocity vector for the particle.

Quick Quiz 4.8 A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path. The centripetal acceleration of the particle has changed by a factor of (a) 0.25 (b) 0.5 (c) 2 (d) 4 (e) impossible to determine

Example 4.8 The Centripetal Acceleration of the Earth

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

Solution We *conceptualize* this problem by bringing forth our familiar mental image of the Earth in a circular orbit around the Sun. We will simplify the problem by modeling the Earth as a particle and approximating the Earth's orbit as circular (it's actually slightly elliptical). This allows us to *categorize* this problem as that of a particle in uniform circular motion. When we begin to *analyze* this problem, we realize that we do not know the orbital speed of the Earth in Equation 4.15. With the help of Equation 4.16, however, we can recast Equation 4.15 in terms of the period of the Earth's orbit, which we know is one year:

$$a_c = \frac{v^2}{r} = \frac{\left(\dfrac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(1.496 \times 10^{11}\ \text{m})}{(1\ \text{yr})^2}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right)^2$$

$$= \boxed{5.93 \times 10^{-3}\ \text{m/s}^2}$$

To *finalize* this problem, note that this acceleration is much smaller than the free-fall acceleration on the surface of the Earth. An important thing we learned here is the technique of replacing the speed v in terms of the period T of the motion.

4.5 Tangential and Radial Acceleration

Let us consider the motion of a particle along a smooth curved path where the velocity changes both in direction and in magnitude, as described in Figure 4.18. In this situation, the velocity vector is always tangent to the path; however, the acceleration vector **a** is at some angle to the path. At each of three points Ⓐ, Ⓑ, and Ⓒ in Figure 4.18, we draw dashed circles that represent a portion of the actual path at each point. The radius of the circles is equal to the radius of curvature of the path at each point.

As the particle moves along the curved path in Figure 4.18, the direction of the total acceleration vector **a** changes from point to point. This vector can be resolved into two components, based on an origin at the center of the dashed circle: a radial component a_r along the radius of the model circle, and a tangential component a_t perpendicular to this radius. The *total* acceleration vector **a** can be written as the vector sum of the component vectors:

Total acceleration

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \tag{4.17}$$

The tangential acceleration component causes the change in the speed of the particle. This component is parallel to the instantaneous velocity, and is given by

Tangential acceleration

$$a_t = \frac{d|\mathbf{v}|}{dt} \tag{4.18}$$

The radial acceleration component arises from the change in direction of the velocity vector and is given by

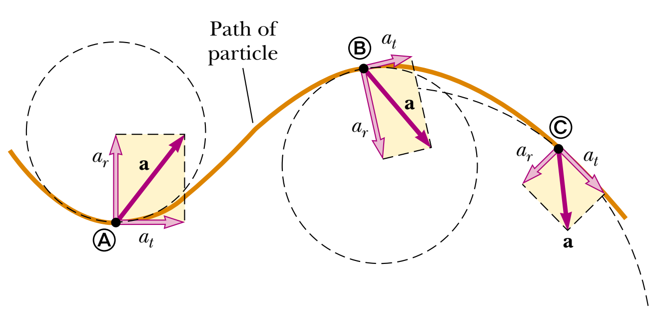
Radial acceleration

$$a_r = -a_c = -\frac{v^2}{r} \tag{4.19}$$

where r is the radius of curvature of the path at the point in question. We recognize the radial component of the acceleration as the centripetal acceleration discussed in Section 4.4. The negative sign indicates that the direction of the centripetal acceleration is toward the center of the circle representing the radius of curvature, which is opposite the direction of the radial unit vector $\hat{\mathbf{r}}$, which always points away from the center of the circle.

Because $\mathbf{a}_r$ and $\mathbf{a}_t$ are perpendicular component vectors of **a**, it follows that the magnitude of **a** is $a = \sqrt{a_r^2 + a_t^2}$. At a given speed, a_r is large when the radius of curvature is small (as at points Ⓐ and Ⓑ in Fig. 4.18) and small when r is large (such as at point Ⓒ). The direction of $\mathbf{a}_t$ is either in the same direction as **v** (if v is increasing) or opposite **v** (if v is decreasing).

In uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always completely radial, as we described in Section 4.4. In other words, uniform circular motion is a special case of motion along a general curved path. Furthermore, if the direction of **v** does not change, then there is no radial acceleration and the motion is one-dimensional (in this case, $a_r = 0$, but a_t may not be zero).



Active Figure 4.18 The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector **v** (always tangent to the path) changes in direction and magnitude, the components of the acceleration **a** are a tangential component a_t and a radial component a_r.

***At the Active Figures link* at http://www.pse6.com, you can study the acceleration components of a roller coaster car.**

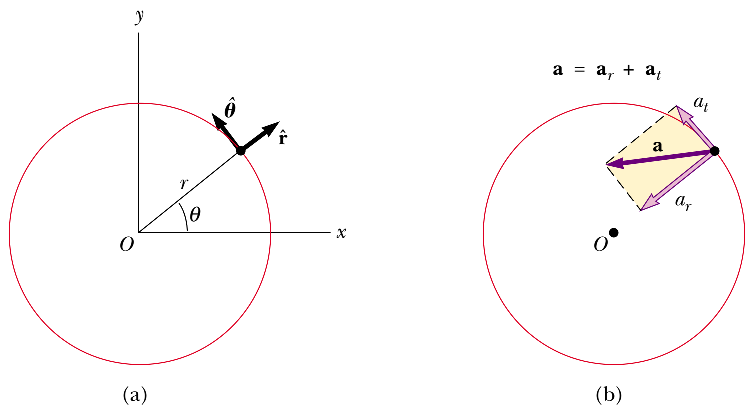


Figure 4.19 (a) Descriptions of the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$. (b) The total acceleration **a** of a particle moving along a curved path (which at any instant is part of a circle of radius r) is the sum of radial and tangential component vectors. The radial component vector is directed toward the center of curvature. If the tangential component of acceleration becomes zero, the particle follows uniform circular motion.

It is convenient to write the acceleration of a particle moving in a circular path in terms of unit vectors. We do this by defining the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ shown in Figure 4.19a, where $\hat{\mathbf{r}}$ is a unit vector lying along the radius vector and directed radially outward from the center of the circle and $\hat{\boldsymbol{\theta}}$ is a unit vector tangent to the circle. The direction of $\hat{\boldsymbol{\theta}}$ is in the direction of increasing θ, where θ is measured counterclockwise from the positive x axis. Note that both $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ "move along with the particle" and so vary in time. Using this notation, we can express the total acceleration as

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r = \frac{d|\mathbf{v}|}{dt}\,\hat{\boldsymbol{\theta}} - \frac{v^2}{r}\,\hat{\mathbf{r}} \tag{4.20}$$

These vectors are described in Figure 4.19b.

Quick Quiz 4.9 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors parallel? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Quick Quiz 4.10 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors perpendicular everywhere along the path? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Example 4.9 Over the Rise

A car exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction of the total acceleration vector for the car at this instant?

Solution *Conceptualize* the situation using Figure 4.20a. Because the car is moving along a curved path, we can *categorize* this as a problem involving a particle experiencing both tangential and radial acceleration. Now we recognize that this is a relatively simple plug-in problem. The radial acceleration is given by Equation 4.19. With $v = 6.00$ m/s and $r = 500$ m, we find that

$$a_r = -\frac{v^2}{r} = -\frac{(6.00\ \text{m/s})^2}{500\ \text{m}} = \boxed{-0.0720\ \text{m/s}^2}$$

The radial acceleration vector is directed straight downward

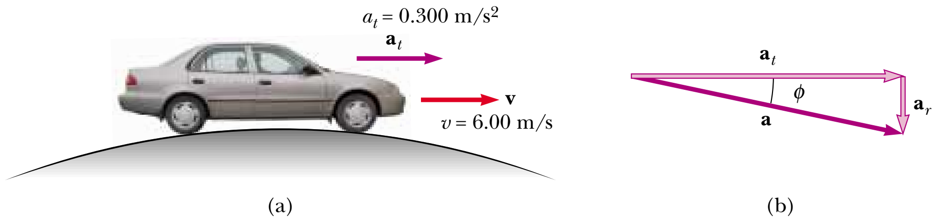


Figure 4.20 (Example 4.9) (a) A car passes over a rise that is shaped like a circle. (b) The total acceleration vector **a** is the sum of the tangential and radial acceleration vectors $\mathbf{a}_t$ and $\mathbf{a}_r$.

while the tangential acceleration vector has magnitude 0.300 m/s² and is horizontal. Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of **a** is

$$a = \sqrt{{a_r}^2 + {a_t}^2} = \sqrt{(-0.0720)^2 + (0.300)^2} \text{ m/s}^2$$

$$= \boxed{0.309 \text{ m/s}^2}$$

If ϕ is the angle between **a** and the horizontal, then

$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.0720 \text{ m/s}^2}{0.300 \text{ m/s}^2}\right) = \boxed{-13.5°}$$

This angle is measured downward from the horizontal. (See Figure 4.20b.)

4.6 Relative Velocity and Relative Acceleration

In this section, we describe how observations made by different observers in different frames of reference are related to each other. We find that observers in different frames of reference may measure different positions, velocities, and accelerations for a given particle. That is, two observers moving relative to each other generally do not agree on the outcome of a measurement.

As an example, consider two observers watching a man walking on a moving beltway at an airport in Figure 4.21. The woman standing on the moving beltway will see the man moving at a normal walking speed. The woman observing from the stationary floor will see the man moving with a higher speed, because the beltway speed combines with his walking speed. Both observers look at the same man and arrive at different values for his speed. Both are correct; the difference in their measurements is due to the relative velocity of their frames of reference.

Suppose a person riding on a skateboard (observer A) throws a ball in such a way that it appears in this person's frame of reference to move first straight upward and

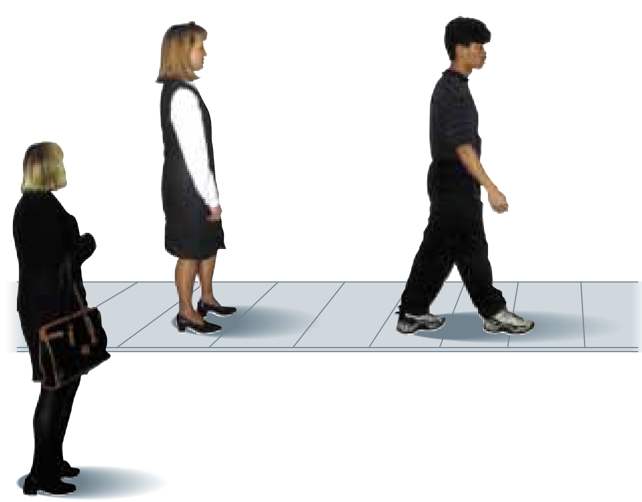


Figure 4.21 Two observers measure the speed of a man walking on a moving beltway. The woman standing on the beltway sees the man moving with a slower speed than the woman observing from the stationary floor.

The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.24. The quantity $\mathbf{v}_{br}$ is due north, $\mathbf{v}_{rE}$ is due east, and the vector sum of the two, $\mathbf{v}_{bE}$, is at an angle θ, as defined in Figure 4.24. Thus, we can find the speed v_{bE} of the boat relative to Earth by using the Pythagorean theorem:

$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10.0)^2 + (5.00)^2}\ \text{km/h}$$
$$= 11.2\ \text{km/h}$$

The direction of $\mathbf{v}_{bE}$ is

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6°$$

The boat is moving at a speed of 11.2 km/h in the direction 26.6° east of north relative to Earth. To *finalize* the problem, note that the speed of 11.2 km/h is faster than your boat speed of 10.0 km/h. The current velocity adds to yours to give you a larger speed. Notice in Figure 4.24 that your resultant velocity is at an angle to the direction straight across the river, so you will end up downstream, as we predicted.

Example 4.11 Which Way Should We Head?

If the boat of the preceding example travels with the same speed of 10.0 km/h relative to the river and is to travel due north, as shown in Figure 4.25, what should its heading be?

Solution This example is an extension of the previous one, so we have already *conceptualized* and *categorized* the problem. The *analysis* now involves the new triangle shown in Figure 4.25. As in the previous example, we know $\mathbf{v}_{rE}$ and the magnitude of the vector $\mathbf{v}_{br}$, and we want $\mathbf{v}_{bE}$ to be directed across the river. Note the difference between the triangle in Figure 4.24 and the one in Figure 4.25—the hypotenuse in Figure 4.25 is no longer $\mathbf{v}_{bE}$. Therefore, when we use the Pythagorean theorem to find $\mathbf{v}_{bE}$ in this situation, we obtain

$$v_{bE} = \sqrt{v_{br}^2 - v_{rE}^2} = \sqrt{(10.0)^2 - (5.00)^2}\ \text{km/h} = 8.66\ \text{km/h}$$

Now that we know the magnitude of $\mathbf{v}_{bE}$, we can find the direction in which the boat is heading:

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = 30.0°$$

To *finalize* this problem, we learn that the boat must head upstream in order to travel directly northward across the river. For the given situation, the boat must steer a course 30.0° west of north.

What If? **Imagine that the two boats in Examples 4.10 and 4.11 are racing across the river. Which boat arrives at the opposite bank first?**

Answer In Example 4.10, the velocity of 10 km/h is aimed directly across the river. In Example 4.11, the velocity that is directed across the river has a magnitude of only 8.66 km/h. Thus, the boat in Example 4.10 has a larger velocity component directly across the river and will arrive first.

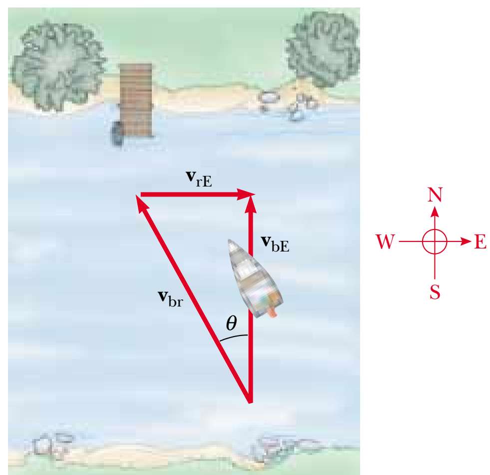


Figure 4.25 (Example 4.11) To move directly across the river, the boat must aim upstream.

SUMMARY

If a particle moves with *constant* acceleration $\mathbf{a}$ and has velocity $\mathbf{v}_i$ and position $\mathbf{r}_i$ at $t = 0$, its velocity and position vectors at some later time t are

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \tag{4.8}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \tag{4.9}$$

For two-dimensional motion in the xy plane under constant acceleration, each of these vector expressions is equivalent to two component expressions—one for the motion in the x direction and one for the motion in the y direction.

Projectile motion is one type of two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$. It is useful to think of projectile motion as the superposition of two motions: (1) constant-velocity motion in the x direction and

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

(2) free-fall motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80 \text{ m/s}^2$.

A particle moving in a circle of radius r with constant speed v is in **uniform circular motion.** It undergoes a radial acceleration $\mathbf{a}_r$ because the direction of $\mathbf{v}$ changes in time. The magnitude of $\mathbf{a}_r$ is the **centripetal acceleration** a_c:

$$a_c = \frac{v^2}{r} \tag{4.19}$$

and its direction is always toward the center of the circle.

If a particle moves along a curved path in such a way that both the magnitude and the direction of $\mathbf{v}$ change in time, then the particle has an acceleration vector that can be described by two component vectors: (1) a radial component vector $\mathbf{a}_r$ that causes the change in direction of $\mathbf{v}$ and (2) a tangential component vector $\mathbf{a}_t$ that causes the change in magnitude of $\mathbf{v}$. The magnitude of $\mathbf{a}_r$ is v^2/r, and the magnitude of $\mathbf{a}_t$ is $d|\mathbf{v}|/dt$.

The velocity $\mathbf{v}$ of a particle measured in a fixed frame of reference S can be related to the velocity $\mathbf{v}'$ of the same particle measured in a moving frame of reference S' by

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \tag{4.22}$$

where $\mathbf{v}_0$ is the velocity of S' relative to S.

QUESTIONS

1. Can an object accelerate if its speed is constant? Can an object accelerate if its velocity is constant?
2. If you know the position vectors of a particle at two points along its path and also know the time it took to move from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.
3. Construct motion diagrams showing the velocity and acceleration of a projectile at several points along its path if (a) the projectile is launched horizontally and (b) the projectile is launched at an angle θ with the horizontal.
4. A baseball is thrown with an initial velocity of $(10\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ m/s. When it reaches the top of its trajectory, what are (a) its velocity and (b) its acceleration? Neglect the effect of air resistance.
5. A baseball is thrown such that its initial x and y components of velocity are known. Neglecting air resistance, describe how you would calculate, at the instant the ball reaches the top of its trajectory, (a) its position, (b) its velocity, and (c) its acceleration. How would these results change if air resistance were taken into account?
6. A spacecraft drifts through space at a constant velocity. Suddenly a gas leak in the side of the spacecraft gives it a constant acceleration in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, so that the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft in this situation?
7. A ball is projected horizontally from the top of a building. One second later another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second ball? What will be the time interval between when the balls hit the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?
8. A rock is dropped at the same instant that a ball, at the same initial elevation, is thrown horizontally. Which will have the greater speed when it reaches ground level?
9. Determine which of the following moving objects obey the equations of projectile motion developed in this chapter. (a) A ball is thrown in an arbitrary direction. (b) A jet airplane crosses the sky with its engines thrusting the plane forward. (c) A rocket leaves the launch pad. (d) A rocket moving through the sky after its engines have failed. (e) A stone is thrown under water.
10. How can you throw a projectile so that it has zero speed at the top of its trajectory? So that it has nonzero speed at the top of its trajectory?
11. Two projectiles are thrown with the same magnitude of initial velocity, one at an angle θ with respect to the level ground and the other at angle $90° - \theta$. Both projectiles will strike the ground at the same distance from the projection point. Will both projectiles be in the air for the same time interval?
12. A projectile is launched at some angle to the horizontal with some initial speed v_i, and air resistance is negligible. Is the projectile a freely falling body? What is its acceleration in the vertical direction? What is its acceleration in the horizontal direction?
13. State which of the following quantities, if any, remain constant as a projectile moves through its parabolic trajectory: (a) speed, (b) acceleration, (c) horizontal component of velocity, (d) vertical component of velocity.

14. A projectile is fired at an angle of 30° from the horizontal with some initial speed. Firing the projectile at what other angle results in the same horizontal range if the initial speed is the same in both cases? Neglect air resistance.

15. The maximum range of a projectile occurs when it is launched at an angle of 45.0° with the horizontal, if air resistance is neglected. If air resistance is not neglected, will the optimum angle be greater or less than 45.0°? Explain.

16. A projectile is launched on the Earth with some initial velocity. Another projectile is launched on the Moon with the same initial velocity. Neglecting air resistance, which projectile has the greater range? Which reaches the greater altitude? (Note that the free-fall acceleration on the Moon is about $1.6\ \text{m/s}^2$.)

17. A coin on a table is given an initial horizontal velocity such that it ultimately leaves the end of the table and hits the floor. At the instant the coin leaves the end of the table, a ball is released from the same height and falls to the floor. Explain why the two objects hit the floor simultaneously, even though the coin has an initial velocity.

18. Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.

19. Correct the following statement: "The racing car rounds the turn at a constant velocity of 90 miles per hour."

20. At the end of a pendulum's arc, its velocity is zero. Is its acceleration also zero at that point?

21. An object moves in a circular path with constant speed v. (a) Is the velocity of the object constant? (b) Is its acceleration constant? Explain.

22. Describe how a driver can steer a car traveling at constant speed so that (a) the acceleration is zero or (b) the magnitude of the acceleration remains constant.

23. An ice skater is executing a figure eight, consisting of two equal, tangent circular paths. Throughout the first loop she increases her speed uniformly, and during the second loop she moves at a constant speed. Draw a motion diagram showing her velocity and acceleration vectors at several points along the path of motion.

24. Based on your observation and experience, draw a motion diagram showing the position, velocity, and acceleration vectors for a pendulum that swings in an arc carrying it from an initial position 45° to the right of the central vertical line to a final position 45° to the left of the central vertical line. The arc is a quadrant of a circle, and you should use the center of the circle as the origin for the position vectors.

25. What is the fundamental difference between the unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ and the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$?

26. A sailor drops a wrench from the top of a sailboat's mast while the boat is moving rapidly and steadily in a straight line. Where will the wrench hit the deck? (Galileo posed this question.)

27. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity. (a) Describe the path of the ball as seen by the passenger. Describe the path as seen by an observer standing by the tracks outside the train. (b) How would these observations change if the train were accelerating along the track?

28. A passenger on a train that is moving with constant velocity drops a spoon. What is the acceleration of the spoon relative to (a) the train and (b) the Earth?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 4.1 The Position, Velocity, and Acceleration Vectors

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Let the positive x axis point east.

2. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates as functions of time are given by the following expressions:

$$x = (18.0\ \text{m/s})t$$

and

$$y = (4.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

(a) Write a vector expression for the ball's position as a function of time, using the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. By taking derivatives, obtain expressions for (b) the velocity vector $\mathbf{v}$ as a function of time and (c) the acceleration vector $\mathbf{a}$ as a function of time. Next use unit-vector notation to write expressions for (d) the position, (e) the velocity, and (f) the acceleration of the golf ball, all at $t = 3.00$ s.

3. When the Sun is directly overhead, a hawk dives toward the ground with a constant velocity of 5.00 m/s at 60.0° below the horizontal. Calculate the speed of her shadow on the level ground.

4. The coordinates of an object moving in the xy plane vary with time according to the equations $x = -(5.00\ \text{m})\sin(\omega t)$ and $y = (4.00\ \text{m}) - (5.00\ \text{m})\cos(\omega t)$, where ω is a constant and t is in seconds. (a) Determine the components of velocity and components of acceleration at $t = 0$. (b) Write expressions for the position vector, the velocity vector, and the acceleration vector at any time $t > 0$. (c) Describe the path of the object in an xy plot.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_i = (3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s and is at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

6. The vector position of a particle varies in time according to the expression $\mathbf{r} = (3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}})$ m. (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at $t = 1.00$ s.

7. A fish swimming in a horizontal plane has velocity $\mathbf{v}_i = (4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}})$ m/s at a point in the ocean where the position relative to a certain rock is $\mathbf{r}_i = (10.0\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to unit vector $\hat{\mathbf{i}}$? (c) If the fish maintains constant acceleration, where is it at $t = 25.0$ s, and in what direction is it moving?

8. A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\hat{\mathbf{j}}$ m/s^2 and an initial velocity of $\mathbf{v}_i = 500\hat{\mathbf{i}}$ m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

9. It is not possible to see very small objects, such as viruses, using an ordinary light microscope. An electron microscope can view such objects using an electron beam instead of a light beam. Electron microscopy has proved invaluable for investigations of viruses, cell membranes and subcellular structures, bacterial surfaces, visual receptors, chloroplasts, and the contractile properties of muscles. The "lenses" of an electron microscope consist of electric and magnetic fields that control the electron beam. As an example of the manipulation of an electron beam, consider an electron traveling away from the origin along the x axis in the xy plane with initial velocity $\mathbf{v}_i = v_i\hat{\mathbf{i}}$. As it passes through the region $x = 0$ to $x = d$, the electron experiences acceleration $\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$, where a_x and a_y are constants. For the case $v_i = 1.80 \times 10^7$ m/s, $a_x = 8.00 \times 10^{14}$ m/s^2 and $a_y = 1.60 \times 10^{15}$ m/s^2, determine at $x = d = 0.0100$ m (a) the position of the electron, (b) the velocity of the electron, (c) the speed of the electron, and (d) the direction of travel of the electron (i.e., the angle between its velocity and the x axis).

Section 4.3 Projectile Motion

Note: Ignore air resistance in all problems and take $g = 9.80$ m/s^2 at the Earth's surface.

10. To start an avalanche on a mountain slope, an artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on the mountainside 42.0 s after firing. What are the x and y coordinates of the shell where it explodes, relative to its firing point?

11. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.860 m, (a) with what velocity did the mug leave the counter, and (b) what was the direction of the mug's velocity just before it hit the floor?

12. In a local bar, a customer slides an empty beer mug down the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor at distance d from the base of the counter. The height of the counter is h. (a) With what velocity did the mug leave the counter, and (b) what was the direction of the mug's velocity just before it hit the floor?

13. One strategy in a snowball fight is to throw a snowball at a high angle over level ground. While your opponent is watching the first one, a second snowball is thrown at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second snowball be thrown to arrive at the same point as the first? (b) How many seconds later should the second snowball be thrown after the first to arrive at the same time?

14. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

15. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

16. A rock is thrown upward from the level ground in such a way that the maximum height of its flight is equal to its horizontal range d. (a) At what angle θ is the rock thrown? (b) **What If?** Would your answer to part (a) be different on a different planet? (c) What is the range d_{max} the rock can attain if it is launched at the same speed but at the optimal angle for maximum range?

17. A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

18. The small archerfish (length 20 to 25 cm) lives in brackish waters of southeast Asia from India to the Philippines. This aptly named creature captures its prey by shooting a stream of water drops at an insect, either flying or at rest. The bug falls into the water and the fish gobbles it up. The archerfish has high accuracy at distances of 1.2 m to 1.5 m, and it sometimes makes hits at distances up to 3.5 m. A groove in the roof of its mouth, along with a curled tongue, forms a tube that enables the fish to impart high velocity to the water in its mouth when it suddenly closes its gill flaps. Suppose the archerfish shoots at a target

2.00 m away, at an angle of 30.0° above the horizontal. With what velocity must the water stream be launched if it is not to drop more than 3.00 cm vertically on its path to the target?

19. A place-kicker must kick a football from a point 36.0 m (about 40 yards) from the goal, and half the crowd hopes the ball will clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar? (b) Does the ball approach the crossbar while still rising or while falling?

20. A firefighter, a distance d from a burning building, directs a stream of water from a fire hose at angle θ_i above the horizontal as in Figure P4.20. If the initial speed of the stream is v_i, at what height h does the water strike the building?

Frederick McKinney/Getty Images

Figure P4.20

21. A playground is on the flat roof of a city school, 6.00 m above the street below. The vertical wall of the building is 7.00 m high, to form a meter-high railing around the playground. A ball has fallen to the street below, and a passerby returns it by launching it at an angle of 53.0° above the horizontal at a point 24.0 meters from the base of the building wall. The ball takes 2.20 s to reach a point vertically above the wall. (a) Find the speed at which the ball was launched. (b) Find the vertical distance by which the ball clears the wall. (c) Find the distance from the wall to the point on the roof where the ball lands.

22. A dive bomber has a velocity of 280 m/s at an angle θ below the horizontal. When the altitude of the aircraft is 2.15 km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ.

23. A soccer player kicks a rock horizontally off a 40.0-m high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

24. A basketball star covers 2.80 m horizontally in a jump to dunk the ball (Fig. P4.24). His motion through space can be modeled precisely as that of a particle at his *center of mass,* which we will define in Chapter 9. His center of mass is at elevation 1.02 m when he leaves the floor. It reaches a maximum height of 1.85 m above the floor, and is at elevation 0.900 m when he touches down again. Determine (a) his time of

Jed Jacobsohn/Allsport/Getty Images

Bill Lee/Dembinsky Photo Associates

Figure P4.24

flight (his "hang time"), (b) his horizontal and (c) vertical velocity components at the instant of takeoff, and (d) his takeoff angle. (e) For comparison, determine the hang time of a whitetail deer making a jump with center-of-mass elevations $y_i = 1.20$ m, $y_{max} = 2.50$ m, $y_f = 0.700$ m.

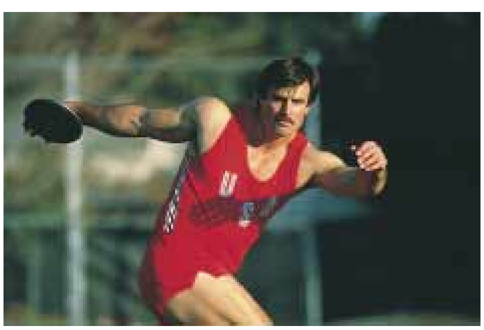
25. An archer shoots an arrow with a velocity of 45.0 m/s at an angle of 50.0° with the horizontal. An assistant standing on the level ground 150 m downrange from the launch point throws an apple straight up with the minimum initial speed necessary to meet the path of the arrow. (a) What is the initial speed of the apple? (b) At what time after the arrow launch should the apple be thrown so that the arrow hits the apple?

26. A fireworks rocket explodes at height h, the peak of its vertical trajectory. It throws out burning fragments in all directions, but all at the same speed v. Pellets of solidified metal fall to the ground without air resistance. Find the smallest angle that the final velocity of an impacting fragment makes with the horizontal.

Section 4.4 Uniform Circular Motion

Note: Problems 8, 10, 12, and 16 in Chapter 6 can also be assigned with this section.

27. The athlete shown in Figure P4.27 rotates a 1.00-kg discus along a circular path of radius 1.06 m. The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.



Sam Sargent/Liaison International

Figure P4.27

28. From information on the endsheets of this book, compute the radial acceleration of a point on the surface of the Earth at the equator, due to the rotation of the Earth about its axis.

29. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

30. As their booster rockets separate, Space Shuttle astronauts typically feel accelerations up to $3g$, where $g = 9.80$ m/s^2. In their training, astronauts ride in a device where they experience such an acceleration as a centripetal acceleration. Specifically, the astronaut is fastened securely at the end of a mechanical arm that then turns at constant speed in a horizontal circle. Determine the rotation rate, in revolutions per second, required to give an astronaut a centripetal acceleration of $3.00g$ while in circular motion with radius 9.45 m.

31. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater speed for the stone at the end of the sling? (b) What is the centripetal acceleration of the stone at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?

32. The astronaut orbiting the Earth in Figure P4.32 is preparing to dock with a Westar VI satellite. The satellite is in a circular orbit 600 km above the Earth's surface, where the free-fall acceleration is 8.21 m/s^2. Take the radius of the Earth as 6 400 km. Determine the speed of the satellite and the time interval required to complete one orbit around the Earth.

Courtesy of NASA

Figure P4.32

Section 4.5 Tangential and Radial Acceleration

33. A train slows down as it rounds a sharp horizontal turn, slowing from 90.0 km/h to 50.0 km/h in the 15.0 s that it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume it continues to slow down at this time at the same rate.

34. An automobile whose speed is increasing at a rate of 0.600 m/s^2 travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the centripetal acceleration component, and (c) the magnitude and direction of the total acceleration.

35. Figure P4.35 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. At this instant, find (a) the radial acceleration, (b) the speed of the particle, and (c) its tangential acceleration.

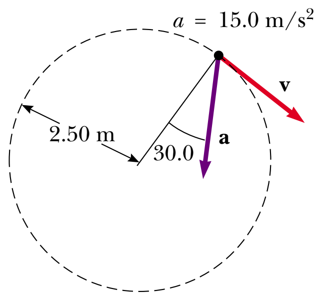


Figure P4.35

36. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point on its way up, its total acceleration is $(-22.5\hat{\mathbf{i}} + 20.2\hat{\mathbf{j}})$ m/s^2. At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

37. A race car starts from rest on a circular track. The car increases its speed at a constant rate a_t as it goes once around the track. Find the angle that the total acceleration of the car makes—with the radius connecting the center of the track and the car—at the moment the car completes the circle.

Section 4.6 Relative Velocity and Relative Acceleration

38. Heather in her Corvette accelerates at the rate of $(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s^2, while Jill in her Jaguar accelerates at $(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ m/s^2. They both start from rest at the origin of an xy coordinate system. After 5.00 s, (a) what is Heather's speed with respect to Jill, (b) how far apart are they, and (c) what is Heather's acceleration relative to Jill?

39. A car travels due east with a speed of 50.0 km/h. Raindrops are falling at a constant speed vertically with respect to the Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical. Find the velocity of the rain with respect to (a) the car and (b) the Earth.

40. How long does it take an automobile traveling in the left lane at 60.0 km/h to pull alongside a car traveling in the same direction in the right lane at 40.0 km/h if the cars' front bumpers are initially 100 m apart?

41. A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

42. The pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/h. If there is a wind of 30.0 km/h toward the north, find the velocity of the airplane relative to the ground.

43. Two swimmers, Alan and Beth, start together at the same point on the bank of a wide stream that flows with a speed v. Both move at the same speed $c(c > v)$, relative to the water. Alan swims downstream a distance L and then upstream the same distance. Beth swims so that her motion relative to the Earth is perpendicular to the banks of the stream. She swims the distance L and then back the same distance, so that both swimmers return to the starting point. Which swimmer returns first? (*Note:* First guess the answer.)

44. A bolt drops from the ceiling of a train car that is accelerating northward at a rate of 2.50 m/s^2. What is the acceleration of the bolt relative to (a) the train car? (b) the Earth?

45. A science student is riding on a flatcar of a train traveling along a straight horizontal track at a constant speed of 10.0 m/s. The student throws a ball into the air along a path that he judges to make an initial angle of 60.0° with the horizontal and to be in line with the track. The student's professor, who is standing on the ground nearby, observes the ball to rise vertically. How high does she see the ball rise?

46. A Coast Guard cutter detects an unidentified ship at a distance of 20.0 km in the direction 15.0° east of north. The ship is traveling at 26.0 km/h on a course at 40.0° east of north. The Coast Guard wishes to send a speedboat to intercept the vessel and investigate it. If the speedboat travels 50.0 km/h, in what direction should it head? Express the direction as a compass bearing with respect to due north.

Additional Problems

47. *The "Vomit Comet."* In zero-gravity astronaut training and equipment testing, NASA flies a KC135A aircraft along a parabolic flight path. As shown in Figure P4.47, the aircraft climbs from 24 000 ft to 31 000 ft, where it enters the zero-g parabola with a velocity of 143 m/s nose-high at 45.0° and exits with velocity 143 m/s at 45.0° nose-low. During this portion of the flight the aircraft and objects inside its padded cabin are in free fall—they have gone ballistic. The aircraft then pulls out of the dive with an upward acceleration of 0.800g, moving in a vertical circle with radius 4.13 km. (During this portion of the flight, occupants of the plane perceive an acceleration of 1.8g.) What are the aircraft (a) speed and (b) altitude at the top of the maneuver? (c) What is the time spent in zero gravity? (d) What is the speed of the aircraft at the bottom of the flight path?

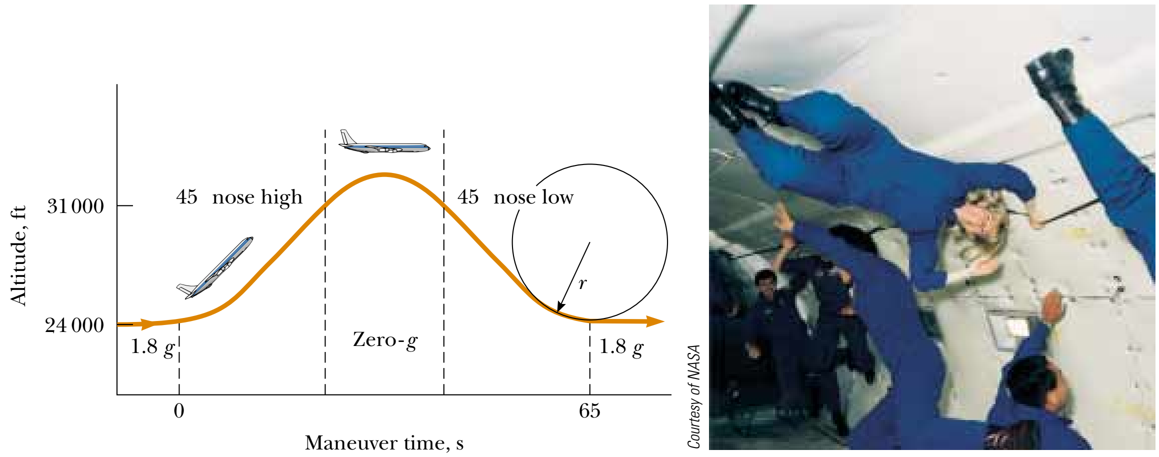


Figure P4.47

48. As some molten metal splashes, one droplet flies off to the east with initial velocity v_i at angle θ_i above the horizontal, and another droplet to the west with the same speed at the same angle above the horizontal, as in Figure P4.48. In terms of v_i and θ_i, find the distance between them as a function of time.

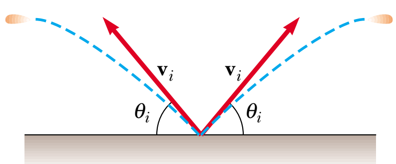


Figure P4.48

49. A ball on the end of a string is whirled around in a horizontal circle of radius 0.300 m. The plane of the circle is 1.20 m above the ground. The string breaks and the ball lands 2.00 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the radial acceleration of the ball during its circular motion.

50. A projectile is fired up an incline (incline angle ϕ) with an initial speed v_i at an angle θ_i with respect to the horizontal ($\theta_i > \phi$), as shown in Figure P4.50. (a) Show that the projectile travels a distance d up the incline, where

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$$

(b) For what value of θ_i is d a maximum, and what is that maximum value?

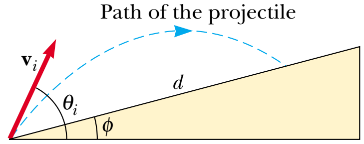


Figure P4.50

51. Barry Bonds hits a home run so that the baseball just clears the top row of bleachers, 21.0 m high, located 130 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time at which the ball reaches the cheap seats, and (c) the velocity components and the speed of the ball when it passes over the top row. Assume the ball is hit at a height of 1.00 m above the ground.

52. An astronaut on the surface of the Moon fires a cannon to launch an experiment package, which leaves the barrel moving horizontally. (a) What must be the muzzle speed of the package so that it travels completely around the Moon and returns to its original location? (b) How long does this trip around the Moon take? Assume that the free-fall acceleration on the Moon is one-sixth that on the Earth.

53. A pendulum with a cord of length $r = 1.00$ m swings in a vertical plane (Fig. P4.53). When the pendulum is in the two horizontal positions $\theta = 90.0°$ and $\theta = 270°$, its speed is 5.00 m/s. (a) Find the magnitude of the radial acceleration and tangential acceleration for these positions. (b) Draw vector diagrams to determine the direc-

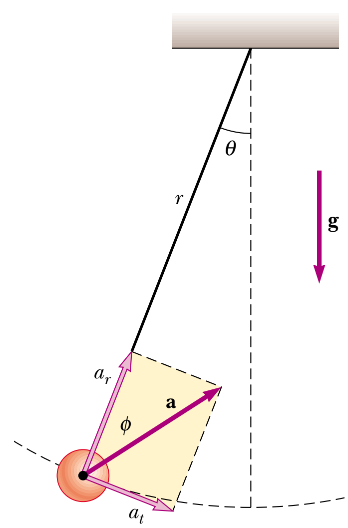


Figure P4.53

tion of the total acceleration for these two positions. (c) Calculate the magnitude and direction of the total acceleration.

54. A basketball player who is 2.00 m tall is standing on the floor 10.0 m from the basket, as in Figure P4.54. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m.

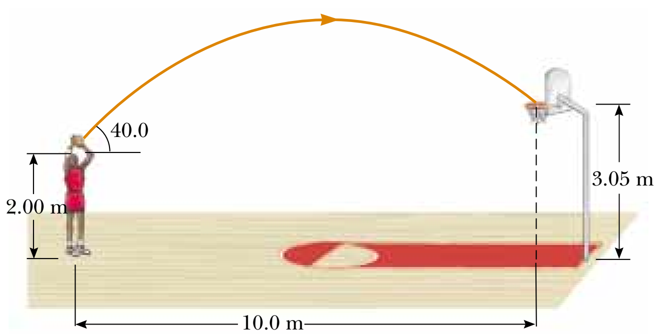


Figure P4.54

55. When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infield, on the theory that the ball arrives sooner that way. Suppose that the angle at which a bounced ball leaves the ground is the same as the angle at which the outfielder threw it, as in Figure P4.55, but that the ball's speed after the bounce is one half of what it was before the bounce. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the fielder throw the ball to make it go the same distance D with one bounce (blue path) as a ball thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

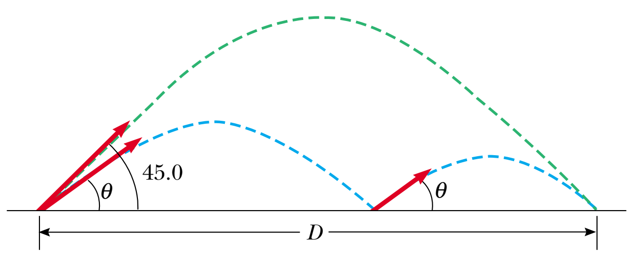


Figure P4.55

56. A boy can throw a ball a maximum horizontal distance of R on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.

57. A stone at the end of a sling is whirled in a vertical circle of radius 1.20 m at a constant speed $v_0 = 1.50$ m/s as in Figure P4.57. The center of the sling is 1.50 m above the ground. What is the range of the stone if it is released when the sling is inclined at 30.0° with the horizontal (a) at Ⓐ? (b) at Ⓑ? What is the acceleration of the stone (c) just before it is released at Ⓐ? (d) just after it is released at Ⓐ?

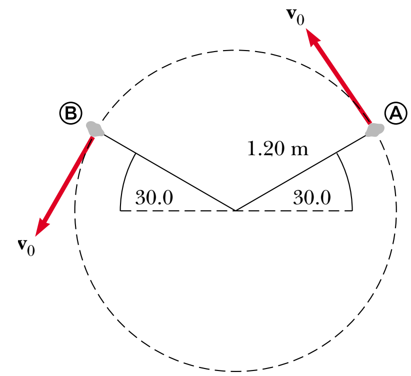


Figure P4.57

58. A quarterback throws a football straight toward a receiver with an initial speed of 20.0 m/s, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the quarterback. In what direction and with what constant speed should the receiver run in order to catch the football at the level at which it was thrown?

59. Your grandfather is copilot of a bomber, flying horizontally over level terrain, with a speed of 275 m/s relative to the ground, at an altitude of 3 000 m. (a) The bombardier releases one bomb. How far will it travel horizontally between its release and its impact on the ground? Neglect the effects of air resistance. (b) Firing from the people on the ground suddenly incapacitates the bombardier before he can call, "Bombs away!" Consequently, the pilot maintains the plane's original course, altitude, and speed through a storm of flak. Where will the plane be when the bomb hits the ground? (c) The plane has a telescopic bomb sight set so that the bomb hits the target seen in the sight at the time of release. At what angle from the vertical was the bomb sight set?

60. A high-powered rifle fires a bullet with a muzzle speed of 1.00 km/s. The gun is pointed horizontally at a large bull's eye target—a set of concentric rings—200 m away. (a) How far below the extended axis of the rifle barrel does a bullet hit the target? The rifle is equipped with a telescopic sight. It is "sighted in" by adjusting the axis of the telescope so that it points precisely at the location where the bullet hits the target at 200 m. (b) Find the angle between the telescope axis and the rifle barrel axis. When shooting at a target at a distance other than 200 m, the marksman uses the telescopic sight, placing its crosshairs to "aim high" or "aim low" to compensate for the different range. Should she aim high or low, and approximately how far from the bull's eye, when the target is at a distance of (c) 50.0 m, (d) 150 m, or (e) 250 m? *Note:* The trajectory of the bullet is everywhere so nearly horizontal that it is a good approximation to model the bullet as fired horizontally in each case. **What if** the target is uphill or downhill? (f) Suppose the target is 200 m away, but the sight line to the target is above the horizontal by 30°. Should the marksman aim high, low, or right on? (g) Suppose the target is downhill by 30°. Should the marksman aim high, low, or right on? Explain your answers.

61. A hawk is flying horizontally at 10.0 m/s in a straight line, 200 m above the ground. A mouse it has been carrying struggles free from its grasp. The hawk continues on its path at the same speed for 2.00 seconds before attempting to retrieve its prey. To accomplish the retrieval, it dives in a straight line at constant speed and recaptures the mouse 3.00 m above the ground. (a) Assuming no air resistance, find the diving speed of the hawk. (b) What angle did the hawk make with the horizontal during its descent? (c) For how long did the mouse "enjoy" free fall?

62. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\mathbf{v}_i$ as in Figure P4.62. (a) What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? (b) With this initial speed, how far from the base of the rock does the ball hit the ground?

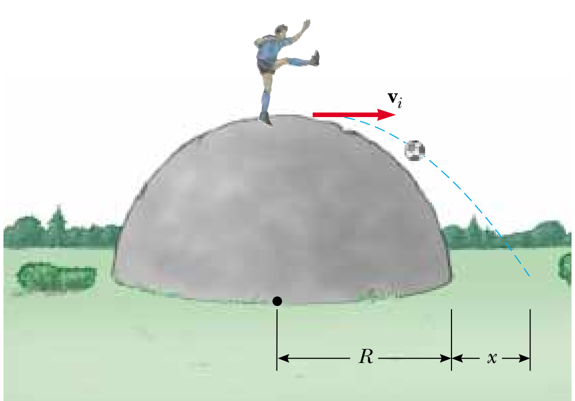


Figure P4.62

63. A car is parked on a steep incline overlooking the ocean, where the incline makes an angle of 37.0° below the horizontal. The negligent driver leaves the car in neutral, and the parking brakes are defective. Starting from rest at $t = 0$, the car rolls down the incline with a constant acceleration of 4.00 m/s^2, traveling 50.0 m to the edge of a vertical cliff. The cliff is 30.0 m above the ocean. Find (a) the speed of the car when it reaches the edge of the cliff and the time at which it arrives there, (b) the velocity of the car when it lands in the ocean, (c) the total time interval that the car is in motion, and (d) the position of the car when it lands in the ocean, relative to the base of the cliff.

64. A truck loaded with cannonball watermelons stops suddenly to avoid running over the edge of a washed-out bridge (Fig. P4.64). The quick stop causes a number of melons to fly off the truck. One melon rolls over the edge with an initial speed $v_i = 10.0$ m/s in the horizontal direction. A cross-section of the bank has the shape of the bottom half of a parabola with its vertex at the edge of the road, and with the equation $y^2 = 16x$, where x and y are measured in meters. What are the x and y coordinates of the melon when it splatters on the bank?

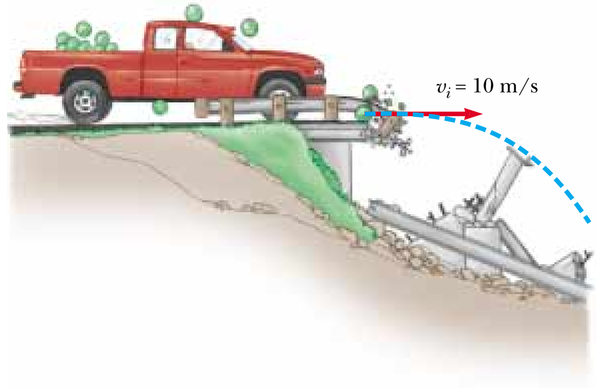


Figure P4.64

65. The determined coyote is out once more in pursuit of the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15.0 m/s^2 (Fig. P4.65). The coyote starts at rest 70.0 m from the brink of a cliff at the instant the roadrunner zips past him in the direction of the cliff. (a) If the roadrunner moves with constant speed, determine the minimum speed he must have in order to reach the cliff before the coyote. At the edge of the cliff, the roadrunner escapes by making a sudden turn, while the coyote continues straight ahead. His skates remain horizontal and continue to operate while he is in flight, so that the coyote's acceleration while in the air is $(15.0\hat{\mathbf{i}} - 9.80\hat{\mathbf{j}})$ m/s^2. (b) If the cliff is 100 m above the flat floor of a canyon, determine where the coyote lands in the canyon. (c) Determine the components of the coyote's impact velocity.

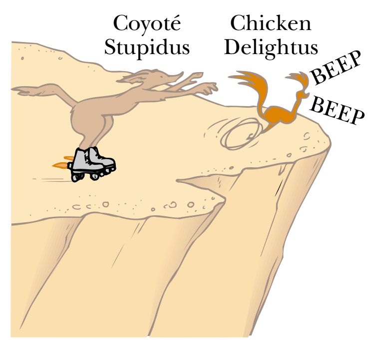


Figure P4.65

66. Do not hurt yourself; do not strike your hand against anything. Within these limitations, describe what you do to give your hand a large acceleration. Compute an order-of-magnitude estimate of this acceleration, stating the quantities you measure or estimate and their values.

67. A skier leaves the ramp of a ski jump with a velocity of 10.0 m/s, 15.0° above the horizontal, as in Figure P4.67. The slope is inclined at 50.0°, and air resistance is negligible. Find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before the landing. (How do you think the results might be affected if air resistance were included? Note that jumpers lean forward in the shape of an airfoil, with their hands at their sides, to increase their distance. Why does this work?)

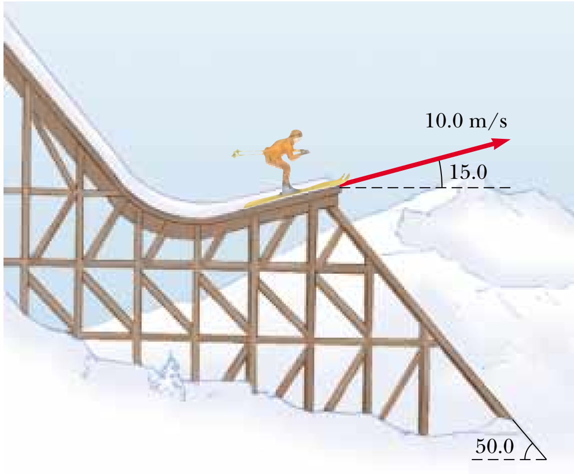


Figure P4.67

68. In a television picture tube (a cathode ray tube) electrons are emitted with velocity $\mathbf{v}_i$ from a source at the origin of coordinates. The initial velocities of different electrons make different angles θ with the x axis. As they move a distance D along the x axis, the electrons are acted on by a constant electric field, giving each a constant acceleration $\mathbf{a}$ in the x direction. At $x = D$ the electrons pass through a circular aperture, oriented perpendicular to the x axis. At the aperture, the velocity imparted to the electrons by the electric field is much larger than $\mathbf{v}_i$ in magnitude. Show that velocities of the electrons going through the aperture radiate from a certain point on the x axis, which is not the origin. Determine the location of this point. This point is called a *virtual source,* and it is important in determining where the electron beam hits the screen of the tube.

69. A fisherman sets out upstream from Metaline Falls on the Pend Oreille River in northwestern Washington State. His small boat, powered by an outboard motor, travels at a constant speed v in still water. The water flows at a lower constant speed v_w. He has traveled upstream for 2.00 km when his ice chest falls out of the boat. He notices that the chest is missing only after he has gone upstream for another 15.0 minutes. At that point he turns around and heads back downstream, all the time traveling at the same speed relative to the water. He catches up with the floating ice chest just as it is about to go over the falls at his starting point. How fast is the river flowing? Solve this problem in two ways. (a) First, use the Earth as a reference frame. With respect to the Earth, the boat travels upstream at speed $v - v_w$ and downstream at $v + v_w$. (b) A second much simpler and more elegant solution is obtained by using the water as the reference frame. This approach has important applications in many more complicated problems; examples are calculating the motion of rockets and satellites and analyzing the scattering of subatomic particles from massive targets.

70. The water in a river flows uniformly at a constant speed of 2.50 m/s between parallel banks 80.0 m apart. You are to deliver a package directly across the river, but you can swim only at 1.50 m/s. (a) If you choose to minimize the time you spend in the water, in what direction should you head? (b) How far downstream will you be carried? (c) **What If?** If you choose to minimize the distance downstream that the river carries you, in what direction should you head? (d) How far downstream will you be carried?

71. An enemy ship is on the east side of a mountain island, as shown in Figure P4.71. The enemy ship has maneuvered to within 2 500 m of the 1 800-m-high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the western shoreline is horizontally 300 m from the peak, what are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?

72. In the **What If?** section of Example 4.7, it was claimed that the maximum range of a ski-jumper occurs for a launch angle θ given by

$$\theta = 45° - \frac{\phi}{2}$$

where ϕ is the angle that the hill makes with the horizontal in Figure 4.16. Prove this claim by deriving the equation above.

Answers to Quick Quizzes

4.1 (b). An object moving with constant velocity has $\Delta\mathbf{v} = 0$, so, according to the definition of acceleration, $\mathbf{a} = \Delta\mathbf{v}/\Delta t = 0$. Choice (a) is not correct because a particle can move at a constant speed and change direction. This possibility also makes (c) an incorrect choice.

4.2 (a). Because acceleration occurs whenever the velocity changes in any way—with an increase or decrease in

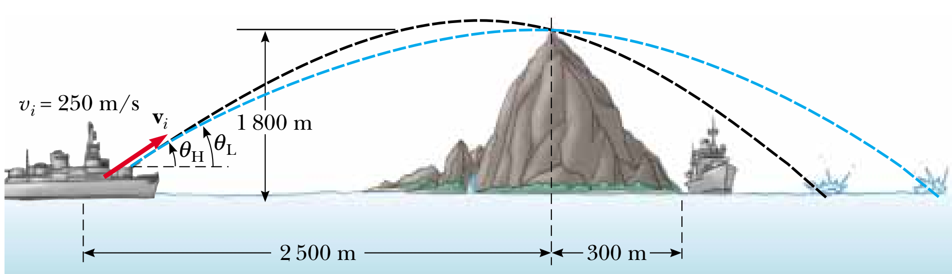


Figure P4.71

speed, a change in direction, or both—all three controls are accelerators. The gas pedal causes the car to speed up; the brake pedal causes the car to slow down. The steering wheel changes the direction of the velocity vector.

4.3 (a). You should simply throw it straight up in the air. Because the ball is moving along with you, it will follow a parabolic trajectory with a horizontal component of velocity that is the same as yours.

4.4 (b). At only one point—the peak of the trajectory—are the velocity and acceleration vectors perpendicular to each other. The velocity vector is horizontal at that point and the acceleration vector is downward.

4.5 (a). The acceleration vector is always directed downward. The velocity vector is never vertical if the object follows a path such as that in Figure 4.8.

4.6 15°, 30°, 45°, 60°, 75°. The greater the maximum height, the longer it takes the projectile to reach that altitude and then fall back down from it. So, as the launch angle increases, the time of flight increases.

4.7 (c). We cannot choose (a) or (b) because the centripetal acceleration vector is not constant—it continuously changes in direction. Of the remaining choices, only (c) gives the correct perpendicular relationship between $\mathbf{a}_c$ and $\mathbf{v}$.

4.8 (d). Because the centripetal acceleration is proportional to the square of the speed, doubling the speed increases the acceleration by a factor of 4.

4.9 (b). The velocity vector is tangent to the path. If the acceleration vector is to be parallel to the velocity vector, it must also be tangent to the path. This requires that the acceleration vector have no component perpendicular to the path. If the path were to change direction, the acceleration vector would have a radial component, perpendicular to the path. Thus, the path must remain straight.

4.10 (d). The velocity vector is tangent to the path. If the acceleration vector is to be perpendicular to the velocity vector, it must have no component tangent to the path. On the other hand, if the speed is changing, there *must* be a component of the acceleration tangent to the path. Thus, the velocity and acceleration vectors are never perpendicular in this situation. They can only be perpendicular if there is no change in the speed.

4.11 (c). Passenger A sees the coffee pouring in a “normal” parabolic path, just as if she were standing on the ground pouring it. The stationary observer B sees the coffee moving in a parabolic path that is extended horizontally due to the constant horizontal velocity of 60 mi/h.

Chapter 5

The Laws of Motion

▲ *A small tugboat exerts a force on a large ship, causing it to move. How can such a small boat move such a large object? (Steve Raymer/CORBIS)*

CHAPTER OUTLINE

In Chapters 2 and 4, we described motion in terms of position, velocity, and acceleration without considering what might cause that motion. Now we consider the cause—what might cause one object to remain at rest and another object to accelerate? The two main factors we need to consider are the forces acting on an object and the mass of the object. We discuss the three basic laws of motion, which deal with forces and masses and were formulated more than three centuries ago by Isaac Newton. Once we understand these laws, we can answer such questions as "What mechanism changes motion?" and "Why do some objects accelerate more than others?"

5.1 The Concept of Force

Everyone has a basic understanding of the concept of force from everyday experience. When you push your empty dinner plate away, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word *force* is associated with muscular activity and some change in the velocity of an object. Forces do not always cause motion, however. For example, as you sit reading this book, a gravitational force acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a large boulder and not be able to move it.

What force (if any) causes the Moon to orbit the Earth? Newton answered this and related questions by stating that forces are what cause any change in the velocity of an object. The Moon's velocity is not constant because it moves in a nearly circular orbit around the Earth. We now know that this change in velocity is caused by the gravitational force exerted by the Earth on the Moon. Because only a force can cause a change in velocity, we can think of force as *that which causes an object to accelerate.* In this chapter, we are concerned with the relationship between the force exerted on an object and the acceleration of that object.

An object accelerates due to an external force

What happens when several forces act simultaneously on an object? In this case, the object accelerates only if the net force acting on it is not equal to zero. The **net force** acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the *total force*, the *resultant force*, or the *unbalanced force.*) **If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.** That is, if the net force acting on the object is zero, the object either remains at rest or continues to move with constant velocity. When the velocity of an object is constant (including when the object is at rest), the object is said to be in **equilibrium.**

Definition of equilibrium

When a coiled spring is pulled, as in Figure 5.1a, the spring stretches. When a stationary cart is pulled sufficiently hard that friction is overcome, as in Figure 5.1b, the cart moves. When a football is kicked, as in Figure 5.1c, it is both deformed and set in motion. These situations are all examples of a class of forces called *contact forces.* That is, they involve physical contact between two objects. Other examples of contact forces are the force exerted by gas molecules on the walls of a container and the force exerted by your feet on the floor.

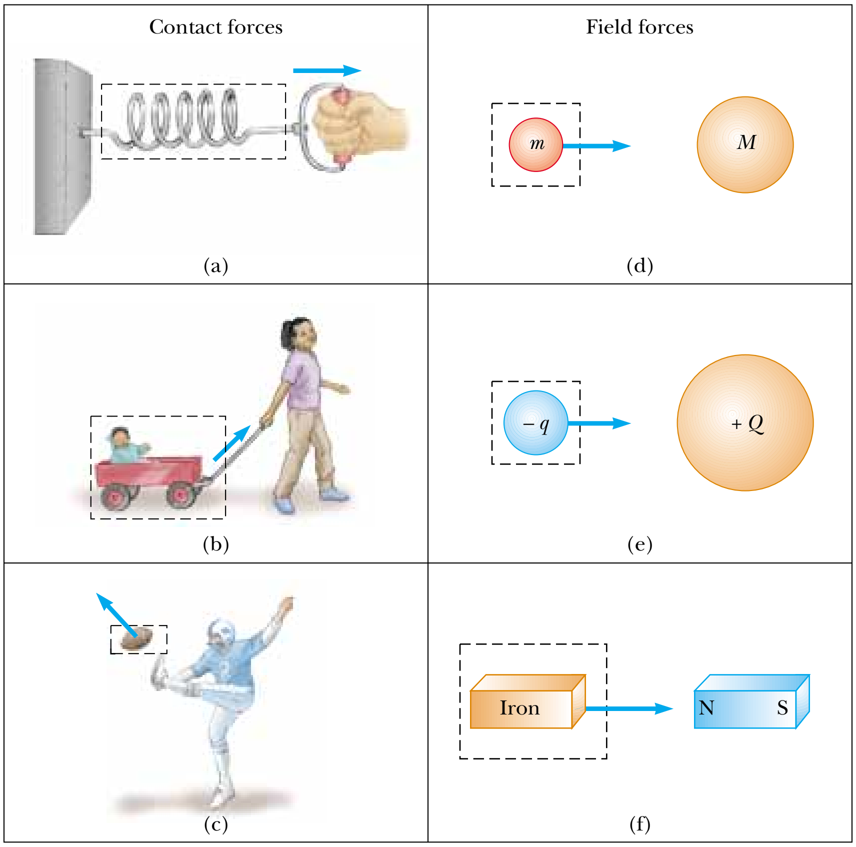


Figure 5.1 Some examples of applied forces. In each case a force is exerted on the object within the boxed area. Some agent in the environment external to the boxed area exerts a force on the object.

Another class of forces, known as *field forces*, do not involve physical contact between two objects but instead act through empty space. The gravitational force of attraction between two objects, illustrated in Figure 5.1d, is an example of this class of force. This gravitational force keeps objects bound to the Earth and the planets in orbit around the Sun. Another common example of a field force is the electric force that one electric charge exerts on another (Fig. 5.1e). These charges might be those of the electron and proton that form a hydrogen atom. A third example of a field force is the force a bar magnet exerts on a piece of iron (Fig. 5.1f).

The distinction between contact forces and field forces is not as sharp as you may have been led to believe by the previous discussion. When examined at the atomic level, all the forces we classify as contact forces turn out to be caused by electric (field) forces of the type illustrated in Figure 5.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. The only known *fundamental* forces in nature are all field forces: (1) *gravitational forces* between objects, (2) *electromagnetic forces* between electric charges, (3) *nuclear forces* between subatomic particles, and (4) *weak forces* that arise in certain radioactive decay processes. In classical physics, we are concerned only with gravitational and electromagnetic forces.

Measuring the Strength of a Force

It is convenient to use the deformation of a spring to measure force. Suppose we apply a vertical force to a spring scale that has a fixed upper end, as shown in Figure 5.2a. The spring elongates when the force is applied, and a pointer on the scale reads the value of the applied force. We can calibrate the spring by defining a reference force $\mathbf{F}_1$ as the force that produces a pointer reading of 1.00 cm. (Because force is a vector

Isaac Newton,
English physicist and mathematician (1642–1727)

Isaac Newton was one of the most brilliant scientists in history. Before the age of 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and the Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today. *(Giraudon/Art Resource)*

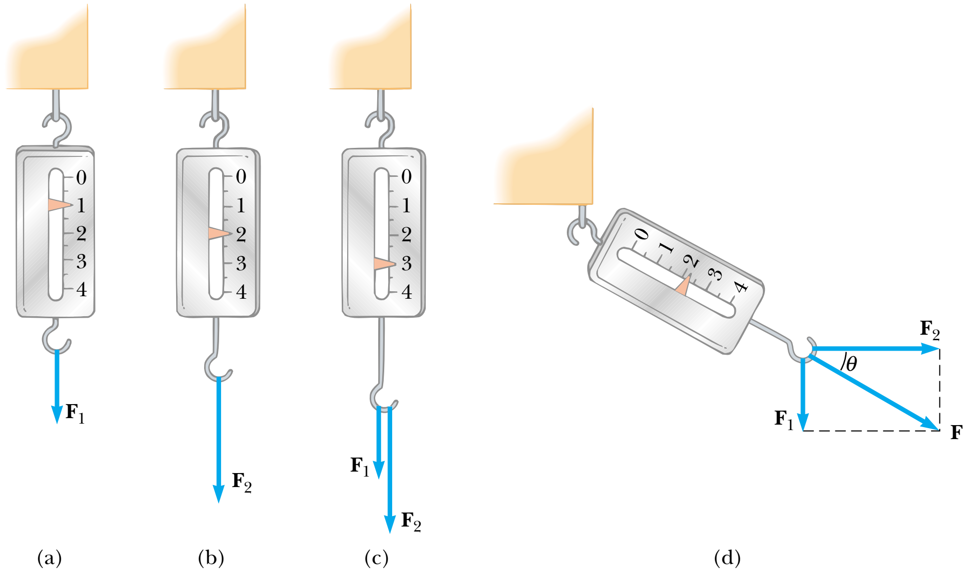


Figure 5.2 The vector nature of a force is tested with a spring scale. (a) A downward force $\mathbf{F}_1$ elongates the spring 1.00 cm. (b) A downward force $\mathbf{F}_2$ elongates the spring 2.00 cm. (c) When $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied simultaneously, the spring elongates by 3.00 cm. (d) When $\mathbf{F}_1$ is downward and $\mathbf{F}_2$ is horizontal, the combination of the two forces elongates the spring $\sqrt{(1.00 \text{ cm})^2 + (2.00 \text{ cm})^2} = 2.24$ cm.

quantity, we use the bold-faced symbol $\mathbf{F}$.) If we now apply a different downward force $\mathbf{F}_2$ whose magnitude is twice that of the reference force $\mathbf{F}_1$, as seen in Figure 5.2b, the pointer moves to 2.00 cm. Figure 5.2c shows that the combined effect of the two collinear forces is the sum of the effects of the individual forces.

Now suppose the two forces are applied simultaneously with $\mathbf{F}_1$ downward and $\mathbf{F}_2$ horizontal, as illustrated in Figure 5.2d. In this case, the pointer reads $\sqrt{5.00 \text{ cm}^2} = 2.24$ cm. The single force $\mathbf{F}$ that would produce this same reading is the sum of the two vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, as described in Figure 5.2d. That is, $|\mathbf{F}| = \sqrt{F_1^2 + F_2^2} = 2.24$ units, and its direction is $\theta = \tan^{-1}(-0.500) = -26.6°$. **Because forces have been experimentally verified to behave as vectors, you must use the rules of vector addition to obtain the net force on an object.**

5.2 Newton's First Law and Inertial Frames

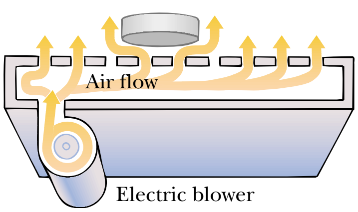


Figure 5.3 On an air hockey table, air blown through holes in the surface allow the puck to move almost without friction. If the table is not accelerating, a puck placed on the table will remain at rest.

We begin our study of forces by imagining some situations. Imagine placing a puck on a perfectly level air hockey table (Fig. 5.3). You expect that it will remain where it is placed. Now imagine your air hockey table is located on a train moving with constant velocity. If the puck is placed on the table, the puck again remains where it is placed. If the train were to accelerate, however, the puck would start moving along the table, just as a set of papers on your dashboard falls onto the front seat of your car when you step on the gas.

As we saw in Section 4.6, a moving object can be observed from any number of reference frames. **Newton's first law of motion,** sometimes called the *law of inertia,* defines a special set of reference frames called *inertial frames.* This law can be stated as follows:

Newton's first law

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

Such a reference frame is called an **inertial frame of reference**. When the puck is on the air hockey table located on the ground, you are observing it from an inertial reference frame—there are no horizontal interactions of the puck with any other objects and you observe it to have zero acceleration in that direction. When you are on the train moving at constant velocity, you are also observing the puck from an inertial reference frame. **Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame**. When the train accelerates, however, you are observing the puck from a **noninertial reference frame** because you and the train are accelerating relative to the inertial reference frame of the surface of the Earth. While the puck appears to be accelerating according to your observations, we can identify a reference frame in which the puck has zero acceleration. For example, an observer standing outside the train on the ground sees the puck moving with the same velocity as the train had before it started to accelerate (because there is almost no friction to "tie" the puck and the train together). Thus, Newton's first law is still satisfied even though your observations say otherwise.

Inertial frame of reference

A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame, and for our purposes we can consider the Earth as being such a frame. The Earth is not really an inertial frame because of its orbital motion around the Sun and its rotational motion about its own axis, both of which result in centripetal accelerations. However, these accelerations are small compared with g and can often be neglected. For this reason, we assume that the Earth is an inertial frame, as is any other frame attached to it.

Let us assume that we are observing an object from an inertial reference frame. (We will return to observations made in noninertial reference frames in Section 6.3.) Before about 1600, scientists believed that the natural state of matter was the state of rest. Observations showed that moving objects eventually stopped moving. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments and concluded that it is not the nature of an object to stop once set in motion: rather, it is its nature to *resist changes in its motion.* In his words, "Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed." For example, a spacecraft drifting through empty space with its engine turned off will keep moving forever—it would *not* seek a "natural state" of rest.

PITFALL PREVENTION

5.1 Newton's First Law

Newton's first law does *not* say what happens for an object with *zero net force,* that is, multiple forces that cancel; it says what happens *in the absence of a force.* This is a subtle but important difference that allows us to define force as that which causes a change in the motion. The description of an object under the effect of forces that balance is covered by Newton's second law.

Given our assumption of observations made from inertial reference frames, we can pose a more practical statement of Newton's first law of motion:

> In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

Another statement of Newton's first law

In simpler terms, we can say that **when no force acts on an object, the acceleration of the object is zero.** If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any *isolated object* (one that does not interact with its environment) is either at rest or moving with constant velocity. The tendency of an object to resist any attempt to change its velocity is called **inertia.**

Quick Quiz 5.1 Which of the following statements is most correct? (a) It is possible for an object to have motion in the absence of forces on the object. (b) It is possible to have forces on an object in the absence of motion of the object. (c) Neither (a) nor (b) is correct. (d) Both (a) and (b) are correct.

5.3 Mass

Imagine playing catch with either a basketball or a bowling ball. Which ball is more likely to keep moving when you try to catch it? Which ball has the greater tendency to remain motionless when you try to throw it? The bowling ball is more resistant to changes in its velocity than the basketball—how can we quantify this concept?

Definition of mass

Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity, and as we learned in Section 1.1, the SI unit of mass is the kilogram. The greater the mass of an object, the less that object accelerates under the action of a given applied force.

To describe mass quantitatively, we begin by experimentally comparing the accelerations a given force produces on different objects. Suppose a force acting on an object of mass m_1 produces an acceleration $\mathbf{a}_1$, and the *same force* acting on an object of mass m_2 produces an acceleration $\mathbf{a}_2$. The ratio of the two masses is defined as the *inverse* ratio of the magnitudes of the accelerations produced by the force:

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1} \qquad (5.1)$$

For example, if a given force acting on a 3-kg object produces an acceleration of 4 m/s^2, the same force applied to a 6-kg object produces an acceleration of 2 m/s^2. If one object has a known mass, the mass of the other object can be obtained from acceleration measurements.

Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it. Also, **mass is a scalar quantity** and thus obeys the rules of ordinary arithmetic. That is, several masses can be combined in simple numerical fashion. For example, if you combine a 3-kg mass with a 5-kg mass, the total mass is 8 kg. We can verify this result experimentally by comparing the accelerations that a known force gives to several objects separately with the acceleration that the same force gives to the same objects combined as one unit.

Mass and weight are different quantities

Mass should not be confused with weight. **Mass and weight are two different quantities.** The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location (see Section 5.5). For example, a person who weighs 180 lb on the Earth weighs only about 30 lb on the Moon. On the other hand, the mass of an object is the same everywhere: an object having a mass of 2 kg on the Earth also has a mass of 2 kg on the Moon.

5.4 Newton's Second Law

Newton's first law explains what happens to an object when no forces act on it. It either remains at rest or moves in a straight line with constant speed. Newton's second law answers the question of what happens to an object that has a nonzero resultant force acting on it.

Imagine performing an experiment in which you push a block of ice across a frictionless horizontal surface. When you exert some horizontal force $\mathbf{F}$ on the block, it moves with some acceleration $\mathbf{a}$. If you apply a force twice as great, you find that the acceleration of the block doubles. If you increase the applied force to 3$\mathbf{F}$, the acceleration triples, and so on. From such observations, we conclude that **the acceleration of an object is directly proportional to the force acting on it.**

The acceleration of an object also depends on its mass, as stated in the preceding section. We can understand this by considering the following experiment. If you apply a force $\mathbf{F}$ to a block of ice on a frictionless surface, the block undergoes some acceleration $\mathbf{a}$. If the mass of the block is doubled, the same applied force produces an acceleration $\mathbf{a}/2$. If the mass is tripled, the same applied force produces an acceleration $\mathbf{a}/3$, and so on. According to this observation, we conclude that **the magnitude of the acceleration of an object is inversely proportional to its mass.**

These observations are summarized in **Newton's second law:**

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Thus, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:[1]

Newton's second law

$$\sum \mathbf{F} = m\mathbf{a} \tag{5.2}$$

In both the textual and mathematical statements of Newton's second law above, we have indicated that the acceleration is due to the *net force* $\Sigma\mathbf{F}$ acting on an object. The **net force** on an object is the vector sum of all forces acting on the object. In solving a problem using Newton's second law, it is imperative to determine the correct net force on an object. There may be many forces acting on an object, but there is only one acceleration.

Note that Equation 5.2 is a vector expression and hence is equivalent to three component equations:

Newton's second law—component form

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{5.3}$$

Quick Quiz 5.2 An object experiences no acceleration. Which of the following *cannot* be true for the object? (a) A single force acts on the object. (b) No forces act on the object. (c) Forces act on the object, but the forces cancel.

Quick Quiz 5.3 An object experiences a net force and exhibits an acceleration in response. Which of the following statements is *always* true? (a) The object moves in the direction of the force. (b) The acceleration is in the same direction as the velocity. (c) The acceleration is in the same direction as the force. (d) The velocity of the object increases.

Quick Quiz 5.4 You push an object, initially at rest, across a frictionless floor with a constant force for a time interval Δt, resulting in a final speed of v for the object. You repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed v? (a) $4\,\Delta t$ (b) $2\,\Delta t$ (c) Δt (d) $\Delta t/2$ (e) $\Delta t/4$.

Unit of Force

The SI unit of force is the **newton**, which is defined as the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s^2. From this definition and Newton's second law, we see that the newton can be expressed in terms of the following fundamental units of mass, length, and time:

Definition of the newton

$$1\,\text{N} \equiv 1\,\text{kg}\cdot\text{m/s}^2 \tag{5.4}$$

[1] Equation 5.2 is valid only when the speed of the object is much less than the speed of light. We treat the relativistic situation in Chapter 39.

PITFALL PREVENTION

5.2 Force is the Cause of Changes in Motion

Force *does not* cause motion. We can have motion in the absence of forces, as described in Newton's first law. Force is the cause of *changes* in motion, as measured by acceleration.

PITFALL PREVENTION

5.3 $m\mathbf{a}$ is Not a Force

Equation 5.2 does *not* say that the product $m\mathbf{a}$ is a force. All forces on an object are added vectorially to generate the net force on the left side of the equation. This net force is then equated to the product of the mass of the object and the acceleration that results from the net force. Do *not* include an "$m\mathbf{a}$ force" in your analysis of the forces on an object.

Table 5.1

Units of Mass, Acceleration, and Force[a]

System of Units	Mass	Acceleration	Force
SI	kg	m/s^2	$N = kg \cdot m/s^2$
U.S. customary	slug	ft/s^2	$lb = slug \cdot ft/s^2$

[a] 1 N = 0.225 lb.

In the U.S. customary system, the unit of force is the **pound**, which is defined as the force that, when acting on a 1-slug mass,[2] produces an acceleration of 1 ft/s^2:

$$1\ \text{lb} \equiv 1\ \text{slug} \cdot \text{ft/s}^2 \tag{5.5}$$

A convenient approximation is that $1\ \text{N} \approx \frac{1}{4}$ lb.

The units of mass, acceleration, and force are summarized in Table 5.1.

Example 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the horizontal, frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\mathbf{F}_1$ has a magnitude of 5.0 N, and the force $\mathbf{F}_2$ has a magnitude of 8.0 N. Determine both the magnitude and the direction of the puck's acceleration.

Solution *Conceptualize* this problem by studying Figure 5.4. Because we can determine a net force and we want an acceleration, we *categorize* this problem as one that may be solved using Newton's second law. To *analyze* the problem, we resolve the force vectors into components. The net force acting on the puck in the x direction is

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$
$$= (5.0\ \text{N})(0.940) + (8.0\ \text{N})(0.500) = 8.7\ \text{N}$$

The net force acting on the puck in the y direction is

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$
$$= (5.0\ \text{N})(-0.342) + (8.0\ \text{N})(0.866) = 5.2\ \text{N}$$

Now we use Newton's second law in component form to find the x and y components of the puck's acceleration:

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\ \text{N}}{0.30\ \text{kg}} = 29\ \text{m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\ \text{N}}{0.30\ \text{kg}} = 17\ \text{m/s}^2$$

The acceleration has a magnitude of

$$a = \sqrt{(29)^2 + (17)^2}\ \text{m/s}^2 = \boxed{34\ \text{m/s}^2}$$

and its direction relative to the positive x axis is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{30°}$$

To *finalize* the problem, we can graphically add the vectors in Figure 5.4 to check the reasonableness of our answer. Because the acceleration vector is along the direction of the resultant force, a drawing showing the resultant force helps us check the validity of the answer. (Try it!)

What If? Suppose three hockey sticks strike the puck simultaneously, with two of them exerting the forces shown in Figure 5.4. The result of the three forces is that the hockey puck shows *no* acceleration. What must be the components of the third force?

Answer If there is zero acceleration, the net force acting on the puck must be zero. Thus, the three forces must cancel. We have found the components of the combination of the first two forces. The components of the third force must be of equal magnitude and opposite sign in order that all of the components add to zero. Thus, $F_{3x} = -8.7$ N, $F_{3y} = -5.2$ N.

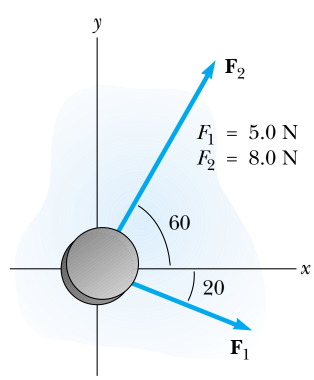


Figure 5.4 (Example 5.1) A hockey puck moving on a frictionless surface accelerates in the direction of the resultant force $\mathbf{F}_1 + \mathbf{F}_2$.

[2] The *slug* is the unit of mass in the U.S. customary system and is that system's counterpart of the SI unit the *kilogram*. Because most of the calculations in our study of classical mechanics are in SI units, the slug is seldom used in this text.

5.5 The Gravitational Force and Weight

We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force** $\mathbf{F}_g$. This force is directed toward the center of the Earth,[3] and its magnitude is called the **weight** of the object.

We saw in Section 2.6 that a freely falling object experiences an acceleration $\mathbf{g}$ acting toward the center of the Earth. Applying Newton's second law $\Sigma\mathbf{F} = m\mathbf{a}$ to a freely falling object of mass m, with $\mathbf{a} = \mathbf{g}$ and $\Sigma\mathbf{F} = \mathbf{F}_g$, we obtain

$$\mathbf{F}_g = m\mathbf{g} \tag{5.6}$$

Thus, the weight of an object, being defined as the magnitude of $\mathbf{F}_g$, is equal to mg.

Because it depends on g, weight varies with geographic location. Because g decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level. For example, a 1 000-kg palette of bricks used in the construction of the Empire State Building in New York City weighed 9 800 N at street level, but weighed about 1 N less by the time it was lifted from sidewalk level to the top of the building. As another example, suppose a student has a mass of 70.0 kg. The student's weight in a location where $g = 9.80 \text{ m/s}^2$ is $F_g = mg = 686$ N (about 150 lb). At the top of a mountain, however, where $g = 9.77 \text{ m/s}^2$, the student's weight is only 684 N. Therefore, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight!

Because weight is proportional to mass, we can compare the masses of two objects by measuring their weights on a spring scale. At a given location (at which two objects are subject to the same value of g), the ratio of the weights of two objects equals the ratio of their masses.

Equation 5.6 quantifies the gravitational force on the object, but notice that this equation does not require the object to be moving. Even for a stationary object, or an object on which several forces act, Equation 5.6 can be used to calculate the magnitude of the gravitational force. This results in a subtle shift in the interpretation of m in the equation. The mass m in Equation 5.6 is playing the role of determining the strength of the gravitational attraction between the object and the Earth. This is a completely different role from that previously described for mass, that of measuring the resistance to changes in motion in response to an external force. Thus, we call m in this type of equation the **gravitational mass.** Despite this quantity being different in behavior from inertial mass, it is one of the experimental conclusions in Newtonian dynamics that gravitational mass and inertial mass have the same value.

Quick Quiz 5.5 A baseball of mass m is thrown upward with some initial speed. A gravitational force is exerted on the ball (a) at all points in its motion (b) at all points in its motion except at the highest point (c) at no points in its motion.

Quick Quiz 5.6 Suppose you are talking by interplanetary telephone to your friend, who lives on the Moon. He tells you that he has just won a newton of gold in a contest. Excitedly, you tell him that you entered the Earth version of the same contest and also won a newton of gold! Who is richer? (a) You (b) Your friend (c) You are equally rich.

[3] This statement ignores the fact that the mass distribution of the Earth is not perfectly spherical.

PITFALL PREVENTION

5.4 "Weight of an Object"

We are familiar with the everyday phrase, the "weight of an object." However, weight is not an inherent property of an object, but rather a measure of the gravitational force between the object and the Earth. Thus, weight is a property of a *system* of items—the object and the Earth.

PITFALL PREVENTION

5.5 Kilogram is Not a Unit of Weight

You may have seen the "conversion" 1 kg = 2.2 lb. Despite popular statements of weights expressed in kilograms, the kilogram is not a unit of *weight*, it is a unit of *mass*. The conversion statement is not an equality; it is an *equivalence* that is only valid on the surface of the Earth.

Courtesy of NASA

The life-support unit strapped to the back of astronaut Edwin Aldrin weighed 300 lb on the Earth. During his training, a 50-lb mock-up was used. Although this effectively simulated the reduced weight the unit would have on the Moon, it did not correctly mimic the unchanging mass. It was just as difficult to accelerate the unit (perhaps by jumping or twisting suddenly) on the Moon as on the Earth.

Conceptual Example 5.2 How Much Do You Weigh in an Elevator?

You have most likely had the experience of standing in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force having a magnitude that is greater than your weight. Thus, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are* you heavier?

Solution No, your weight is unchanged. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force that you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

5.6 Newton's Third Law

If you press against a corner of this textbook with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin is a little larger. This simple experiment illustrates a general principle of critical importance known as **Newton's third law:**

Newton's third law

If two objects interact, the force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \tag{5.7}$$

When it is important to designate forces as interactions between two objects, we will use this subscript notation, where $\mathbf{F}_{ab}$ means "the force exerted *by* a *on* b." The third law, which is illustrated in Figure 5.5a, is equivalent to stating that **forces always occur in pairs,** or that **a single isolated force cannot exist.** The force that object 1 exerts on object 2 may be called the *action force* and the force of object 2 on object 1 the *reaction force.* In reality, either force can be labeled the action or reaction force. **The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects and must be of the same type.** For example, the force acting on a freely falling projectile is the gravitational force exerted by the Earth on the projectile $\mathbf{F}_g = \mathbf{F}_{Ep}$ (E = Earth, p = projectile), and the magnitude of this force is mg. The reaction to this force is the gravitational force exerted by the projectile on the Earth $\mathbf{F}_{pE} = -\mathbf{F}_{Ep}$. The reaction force $\mathbf{F}_{pE}$ must accelerate the Earth toward the projectile just as the action force $\mathbf{F}_{Ep}$ accelerates the projectile toward the Earth. However, because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.

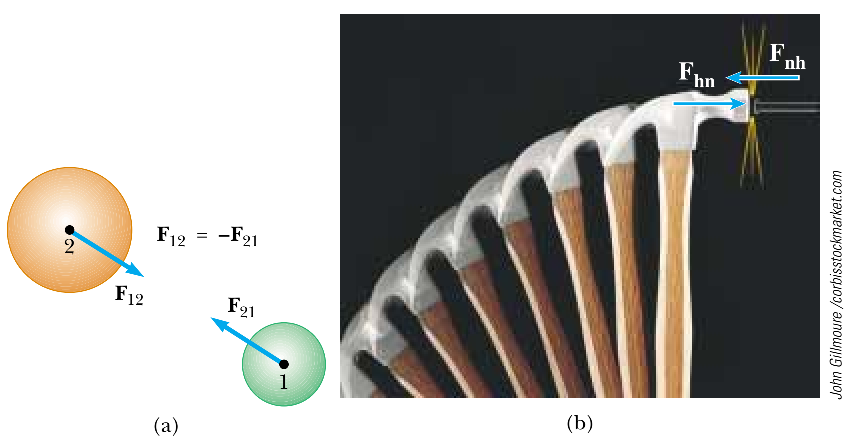


Figure 5.5 Newton's third law. (a) The force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{21}$ exerted by object 2 on object 1. (b) The force $\mathbf{F}_{hn}$ exerted by the hammer on the nail is equal in magnitude and opposite to the force $\mathbf{F}_{nh}$ exerted by the nail on the hammer.

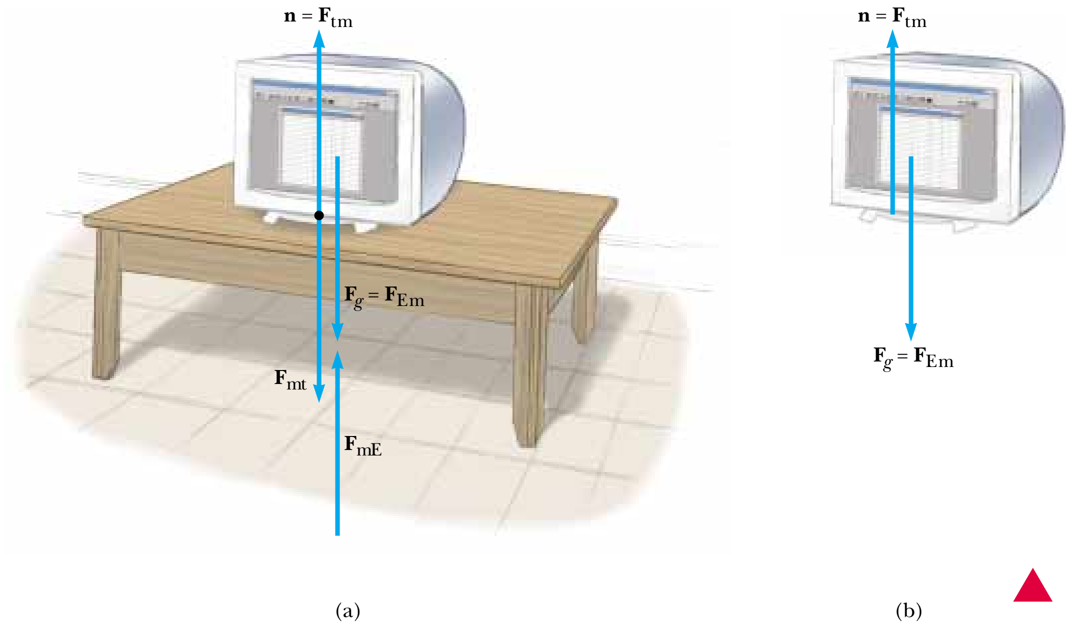


Figure 5.6 (a) When a computer monitor is at rest on a table, the forces acting on the monitor are the normal force $\mathbf{n}$ and the gravitational force $\mathbf{F}_g$. The reaction to $\mathbf{n}$ is the force $\mathbf{F}_{mt}$ exerted by the monitor on the table. The reaction to $\mathbf{F}_g$ is the force $\mathbf{F}_{mE}$ exerted by the monitor on the Earth. (b) The free-body diagram for the monitor.

Another example of Newton's third law is shown in Figure 5.5b. The force $\mathbf{F}_{hn}$ exerted by the hammer on the nail (the action) is equal in magnitude and opposite the force $\mathbf{F}_{nh}$ exerted by the nail on the hammer (the reaction). This latter force stops the forward motion of the hammer when it strikes the nail.

You experience the third law directly if you slam your fist against a wall or kick a football with your bare foot. You can feel the force back on your fist or your foot. You should be able to identify the action and reaction forces in these cases.

The Earth exerts a gravitational force $\mathbf{F}_g$ on any object. If the object is a computer monitor at rest on a table, as in Figure 5.6a, the reaction force to $\mathbf{F}_g = \mathbf{F}_{Em}$ is the force exerted by the monitor on the Earth $\mathbf{F}_{mE} = -\mathbf{F}_{Em}$. The monitor does not accelerate because it is held up by the table. The table exerts on the monitor an upward force $\mathbf{n} = \mathbf{F}_{tm}$, called the **normal force**.[4] This is the force that prevents the monitor from falling through the table; it can have any value needed, up to the point of breaking the table. From Newton's second law, we see that, because the monitor has zero acceleration, it follows that $\Sigma\mathbf{F} = \mathbf{n} - m\mathbf{g} = 0$, or $n = mg$. The normal force balances the gravitational force on the monitor, so that the net force on the monitor is zero. The reaction to $\mathbf{n}$ is the force exerted by the monitor downward on the table, $\mathbf{F}_{mt} = -\mathbf{F}_{tm} = -\mathbf{n}$.

Definition of normal force

Note that the forces acting on the monitor are $\mathbf{F}_g$ and $\mathbf{n}$, as shown in Figure 5.6b. The two reaction forces $\mathbf{F}_{mE}$ and $\mathbf{F}_{mt}$ are exerted on objects other than the monitor. Remember, **the two forces in an action–reaction pair always act on two different objects.**

Figure 5.6 illustrates an extremely important step in solving problems involving forces. Figure 5.6a shows many of the forces in the situation—those acting on the monitor, one acting on the table, and one acting on the Earth. Figure 5.6b, by contrast, shows only the forces acting on *one object*, the monitor. This is a critical drawing called a **free-body diagram**. When analyzing an object subject to forces, we are interested in the net force acting on one object, which we will model as a particle. Thus, a free-body diagram helps us to isolate only those forces on the object and eliminate the other forces from our analysis. The free-body diagram can be simplified further by representing the object (such as the monitor) as a particle, by simply drawing a dot.

PITFALL PREVENTION

5.6 *n* Does Not Always Equal *mg*

In the situation shown in Figure 5.6 and in many others, we find that $n = mg$ (the normal force has the same magnitude as the gravitational force). However, this is *not* generally true. If an object is on an incline, if there are applied forces with vertical components, or if there is a vertical acceleration of the system, then $n \neq mg$. *Always* apply Newton's second law to find the relationship between n and mg.

PITFALL PREVENTION

5.7 Newton's Third Law

This is such an important and often misunderstood concept that it will be repeated here in a Pitfall Prevention. Newton's third law action and reaction forces act on *different* objects. Two forces acting on the same object, even if they are equal in magnitude and opposite in direction, *cannot* be an action–reaction pair.

[4] *Normal* in this context means *perpendicular.*

PITFALL PREVENTION

5.8 Free-body Diagrams

The *most important* step in solving a problem using Newton's laws is to draw a proper sketch—the free-body diagram. Be sure to draw only those forces that act on the object that you are isolating. Be sure to draw *all* forces acting on the object, including any field forces, such as the gravitational force.

Quick Quiz 5.7 If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude? (a) the fly (b) the bus (c) the same force is experienced by both.

Quick Quiz 5.8 If a fly collides with the windshield of a fast-moving bus, which object experiences the greater acceleration: (a) the fly (b) the bus (c) the same acceleration is experienced by both.

Quick Quiz 5.9 Which of the following is the reaction force to the gravitational force acting on your body as you sit in your desk chair? (a) The normal force exerted by the chair (b) The force you exert downward on the seat of the chair (c) Neither of these forces.

Quick Quiz 5.10 In a free-body diagram for a single object, you draw (a) the forces acting on the object and the forces the object exerts on other objects, or (b) only the forces acting on the object.

Conceptual Example 5.3 You Push Me and I'll Push You

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

(A) Who moves away with the higher speed?

Solution This situation is similar to what we saw in Quick Quizzes 5.7 and 5.8. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are an action–reaction pair, and so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.) Therefore, the boy, having the smaller mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

(B) Who moves farther while their hands are in contact?

Solution Because the boy has the greater acceleration and, therefore, the greater average velocity, he moves farther during the time interval in which the hands are in contact.

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Rock climbers at rest are in equilibrium and depend on the tension forces in ropes for their safety.

5.7 Some Applications of Newton's Laws

In this section we apply Newton's laws to objects that are either in equilibrium ($\mathbf{a} = 0$) or accelerating along a straight line under the action of constant external forces. Remember that **when we apply Newton's laws to an object, we are interested only in external forces that act on the object.** We assume that the objects can be modeled as particles so that we need not worry about rotational motion. For now, we also neglect the effects of friction in those problems involving motion; this is equivalent to stating that the surfaces are *frictionless.* (We will incorporate the friction force in problems in Section 5.8.)

We usually neglect the mass of any ropes, strings, or cables involved. In this approximation, the magnitude of the force exerted at any point along a rope is the same at all points along the rope. In problem statements, the synonymous terms *light* and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problems. When a rope attached to an object is pulling on the object, the rope exerts a force $\mathbf{T}$ on the object, and the magnitude T of that force is called the **tension** in the rope. Because it is the magnitude of a vector quantity, tension is a scalar quantity.

Objects in Equilibrium

If the acceleration of an object that can be modeled as a particle is zero, the particle is in **equilibrium**. Consider a lamp suspended from a light chain fastened to the ceiling, as in Figure 5.7a. The free-body diagram for the lamp (Figure 5.7b) shows that the forces acting on the lamp are the downward gravitational force $\mathbf{F}_g$ and the upward force $\mathbf{T}$ exerted by the chain. If we apply the second law to the lamp, noting that $\mathbf{a} = 0$, we see that because there are no forces in the x direction, $\Sigma F_x = 0$ provides no helpful information. The condition $\Sigma F_y = ma_y = 0$ gives

$$\sum F_y = T - F_g = 0 \qquad \text{or} \qquad T = F_g$$

Again, note that $\mathbf{T}$ and $\mathbf{F}_g$ are *not* an action–reaction pair because they act on the same object—the lamp. The reaction force to $\mathbf{T}$ is $\mathbf{T}'$, the downward force exerted by the lamp on the chain, as shown in Figure 5.7c. The ceiling exerts on the chain a force $\mathbf{T}''$ that is equal in magnitude to the magnitude of $\mathbf{T}'$ and points in the opposite direction.

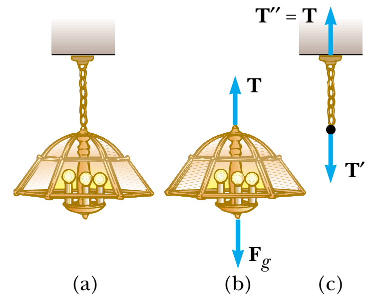


Figure 5.7 (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the gravitational force $\mathbf{F}_g$ and the force $\mathbf{T}$ exerted by the chain. (c) The forces acting on the chain are the force $\mathbf{T}'$ exerted by the lamp and the force $\mathbf{T}''$ exerted by the ceiling.

Objects Experiencing a Net Force

If an object that can be modeled as a particle experiences an acceleration, then there must be a nonzero net force acting on the object. Consider a crate being pulled to the right on a frictionless, horizontal surface, as in Figure 5.8a. Suppose you are asked to find the acceleration of the crate and the force the floor exerts on it. First, note that the horizontal force $\mathbf{T}$ being applied to the crate acts through the rope. The magnitude of $\mathbf{T}$ is equal to the tension in the rope. The forces acting on the crate are illustrated in the free-body diagram in Figure 5.8b. In addition to the force $\mathbf{T}$, the free-body diagram for the crate includes the gravitational force $\mathbf{F}_g$ and the normal force $\mathbf{n}$ exerted by the floor on the crate.

We can now apply Newton's second law in component form to the crate. The only force acting in the x direction is $\mathbf{T}$. Applying $\Sigma F_x = ma_x$ to the horizontal motion gives

$$\sum F_x = T = ma_x \qquad \text{or} \qquad a_x = \frac{T}{m}$$

No acceleration occurs in the y direction. Applying $\Sigma F_y = ma_y$ with $a_y = 0$ yields

$$n + (-F_g) = 0 \qquad \text{or} \qquad n = F_g$$

That is, the normal force has the same magnitude as the gravitational force but acts in the opposite direction.

If $\mathbf{T}$ is a constant force, then the acceleration $a_x = T/m$ also is constant. Hence, the constant-acceleration equations of kinematics from Chapter 2 can be used to obtain the crate's position x and velocity v_x as functions of time. Because $a_x = T/m =$ constant, Equations 2.9 and 2.12 can be written as

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}\left(\frac{T}{m}\right)t^2$$

In the situation just described, the magnitude of the normal force $\mathbf{n}$ is equal to the magnitude of $\mathbf{F}_g$, but this is not always the case. For example, suppose a book is lying on a table and you push down on the book with a force $\mathbf{F}$, as in Figure 5.9. Because the book is at rest and therefore not accelerating, $\Sigma F_y = 0$, which gives $n - F_g - F = 0$, or $n = F_g + F$. In this situation, the normal force is *greater* than the force of gravity. Other examples in which $n \neq F_g$ are presented later.

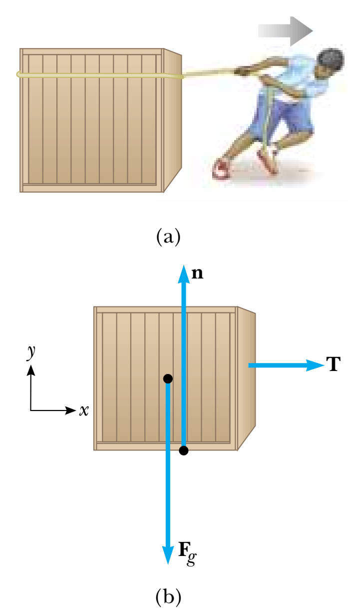


Figure 5.8 (a) A crate being pulled to the right on a frictionless surface. (b) The free-body diagram representing the external forces acting on the crate.

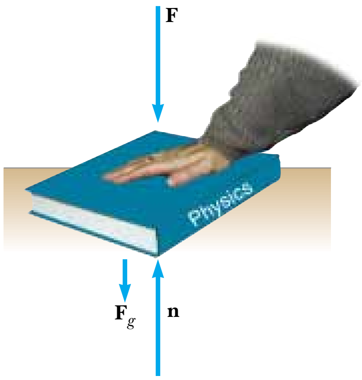


Figure 5.9 When one object pushes downward on another object with a force **F**, the normal force **n** is greater than the gravitational force: $n = F_g + F$.

PROBLEM-SOLVING HINTS

Applying Newton's Laws

The following procedure is recommended when dealing with problems involving Newton's laws:

- Draw a simple, neat diagram of the system to help *conceptualize* the problem.
- *Categorize* the problem: if any acceleration component is zero, the particle is in equilibrium in this direction and $\Sigma F = 0$. If not, the particle is undergoing an acceleration, the problem is one of nonequilibrium in this direction, and $\Sigma F = ma$.
- *Analyze* the problem by isolating the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw *separate* free-body diagrams for each object. *Do not* include in the free-body diagram forces exerted by the object on its surroundings.
- Establish convenient coordinate axes for each object and find the components of the forces along these axes. Apply Newton's second law, $\Sigma\mathbf{F} = m\mathbf{a}$, in component form. Check your dimensions to make sure that all terms have units of force.
- Solve the component equations for the unknowns. Remember that you must have as many independent equations as you have unknowns to obtain a complete solution.
- *Finalize* by making sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

Example 5.4 A Traffic Light at Rest

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?

Solution We *conceptualize* the problem by inspecting the drawing in Figure 5.10a. Let us assume that the cables do not break so that there is no acceleration of any sort in this problem in any direction. This allows us to *categorize* the problem as one of equilibrium. Because the acceleration of the system is zero, we know that the net force on the light and the net force on the knot are both zero. To *analyze* the

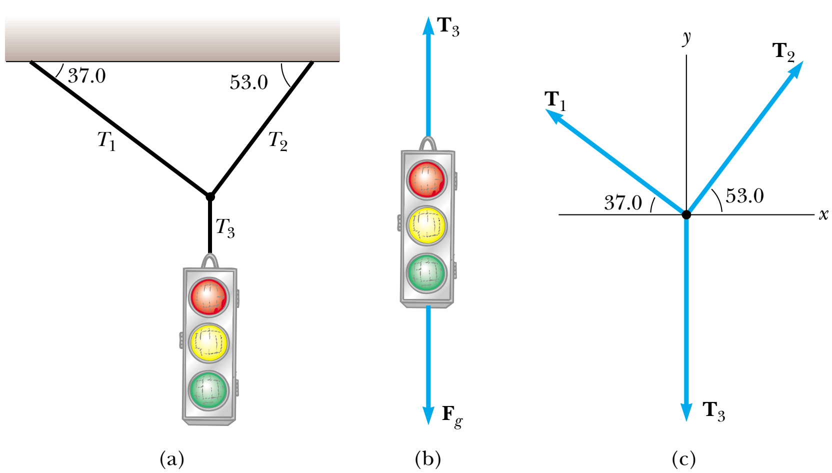


Figure 5.10 (Example 5.4) (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot where the three cables are joined.

problem, we construct two free-body diagrams—one for the traffic light, shown in Figure 5.10b, and one for the knot that holds the three cables together, as in Figure 5.10c. This knot is a convenient object to choose because all the forces of interest act along lines passing through the knot.

With reference to Figure 5.10b, we apply the equilibrium condition in the y direction, $\Sigma F_y = 0 \rightarrow T_3 - F_g = 0$. This leads to $T_3 = F_g = 122$ N. Thus, the upward force $\mathbf{T}_3$ exerted by the vertical cable on the light balances the gravitational force.

Next, we choose the coordinate axes shown in Figure 5.10c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{T}_3$	0	-122 N

Knowing that the knot is in equilibrium ($\mathbf{a} = 0$) allows us to write

(1) $$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

(2) $$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$$

From (1) we see that the horizontal components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must be equal in magnitude, and from (2) we see that the sum of the vertical components of $\mathbf{T}_1$ and $\mathbf{T}_2$ must balance the downward force $\mathbf{T}_3$, which is equal in magnitude to the weight of the light. We solve (1) for T_2 in terms of T_1 to obtain

(3) $$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$$

This value for T_2 is substituted into (2) to yield

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 122 \text{ N} = 0$$

$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33T_1 = 97.4 \text{ N}$$

Both of these values are less than 100 N (just barely for T_2), so the cables will not break. Let us *finalize* this problem by imagining a change in the system, as in the following **What If?**

What If? **Suppose the two angles in Figure 5.10a are equal. What would be the relationship between T_1 and T_2?**

Answer We can argue from the symmetry of the problem that the two tensions T_1 and T_2 would be equal to each other. Mathematically, if the equal angles are called θ, Equation (3) becomes

$$T_2 = T_1\left(\frac{\cos \theta}{\cos \theta}\right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of θ, we cannot find the values of T_1 and T_2. However, the tensions will be equal to each other, regardless of the value of θ.

Conceptual Example 5.5 Forces Between Cars in a Train

Train cars are connected by *couplers,* which are under tension as the locomotive pulls the train. As you move through the train from the locomotive to the last car, does the tension in the couplers increase, decrease, or stay the same as the train speeds up? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from the locomotive to the last car? (Assume that only the brakes on the wheels of the engine are applied.)

Solution As the train speeds up, tension decreases from front to back. The coupler between the locomotive and the first car must apply enough force to accelerate the rest of the cars. As you move back along the train, each coupler is accelerating less mass behind it. The last coupler has to accelerate only the last car, and so it is under the least tension.

When the brakes are applied, the force again decreases from front to back. The coupler connecting the locomotive to the first car must apply a large force to slow down the rest of the cars, but the final coupler must apply a force large enough to slow down *only* the last car.

Example 5.6 The Runaway Car

A car of mass m is on an icy driveway inclined at an angle θ, as in Figure 5.11a.

(A) Find the acceleration of the car, assuming that the driveway is frictionless.

Solution *Conceptualize* the situation using Figure 5.11a. From everyday experience, we know that a car on an icy incline will accelerate down the incline. (It will do the same thing as a car on a hill with its brakes not set.) This allows us to *categorize* the situation as a nonequilibrium problem—that is, one in which an object accelerates. Figure 5.11b shows the free-body diagram for the car, which we can use to *analyze* the problem. The only forces acting on the car are the normal force $\mathbf{n}$ exerted by the inclined plane, which acts perpendicular to the plane, and the gravitational force $\mathbf{F}_g = m\mathbf{g}$, which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with x along the incline and y perpendicular to it, as in Figure 5.11b. (It is possible to solve the problem with "standard" horizontal and vertical axes. You may want to try this, just for practice.) Then, we replace the gravitational force by a component of magnitude $mg \sin \theta$ along the positive x axis and one of magnitude $mg \cos \theta$ along the negative y axis.

Now we apply Newton's second law in component form, noting that $a_y = 0$:

(1) $$\sum F_x = mg \sin \theta = ma_x$$

(2) $$\sum F_y = n - mg \cos \theta = 0$$

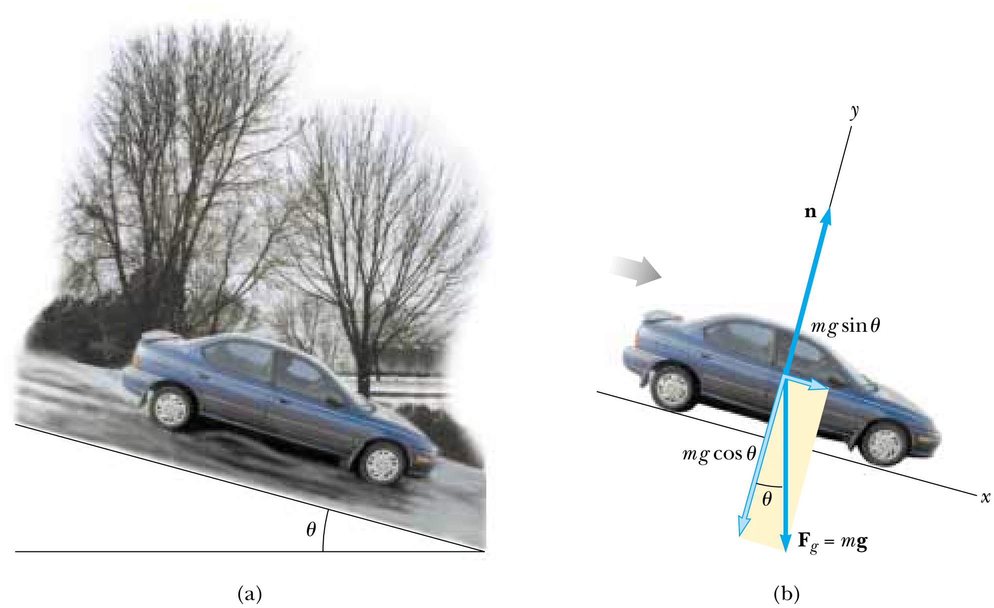


Figure 5.11 (Example 5.6) (a) A car of mass m sliding down a frictionless incline. (b) The free-body diagram for the car. Note that its acceleration along the incline is $g \sin\theta$.

Solving (1) for a_x, we see that the acceleration along the incline is caused by the component of $\mathbf{F}_g$ directed down the incline:

$$(3) \qquad a_x = \boxed{g \sin\theta}$$

To *finalize* this part, note that this acceleration component is independent of the mass of the car! It depends only on the angle of inclination and on g.

From (2) we conclude that the component of $\mathbf{F}_g$ perpendicular to the incline is balanced by the normal force; that is, $n = mg \cos\theta$. This is another example of a situation in which the normal force is *not* equal in magnitude to the weight of the object.

(B) Suppose the car is released from rest at the top of the incline, and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom, and what is the car's speed as it arrives there?

Solution *Conceptualize* by imagining that the car is sliding down the hill and you are operating a stop watch to measure the entire time interval until it reaches the bottom. This part of the problem belongs to kinematics rather than to dynamics, and Equation (3) shows that the acceleration a_x is constant. Therefore you should *categorize* this problem as that of a particle undergoing constant acceleration. Apply Equation 2.12, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, to *analyze* the car's motion. Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, we obtain

$$d = \tfrac{1}{2}a_x t^2$$

$$(4) \qquad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g \sin\theta}}}$$

Using Equation 2.13, with $v_{xi} = 0$, we find that

$$v_{xf}^{\,2} = 2a_x d$$

$$(5) \qquad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd \sin\theta}}$$

To *finalize* this part of the problem, we see from Equations (4) and (5) that the time t at which the car reaches the bottom and its final speed v_{xf} are independent of the car's mass, as was its acceleration. Note that we have combined techniques from Chapter 2 with new techniques from the present chapter in this example. As we learn more and more techniques in later chapters, this process of combining information from several parts of the book will occur more often. In these cases, use the General Problem-Solving Strategy to help you identify what techniques you will need.

What If? (A) What previously solved problem does this become if $\theta = 90°$? (B) What problem does this become if $\theta = 0$?

Answer (A) Imagine θ going to 90° in Figure 5.11. The inclined plane becomes vertical, and the car is an object in free-fall! Equation (3) becomes

$$a_x = g \sin\theta = g \sin 90^\circ = g$$

which is indeed the free-fall acceleration. (We find $a_x = g$ rather than $a_x = -g$ because we have chosen positive x to be downward in Figure 5.11.) Notice also that the condition

$n = mg \cos\theta$ gives us $n = mg \cos 90° = 0$. This is consistent with the fact that the car is falling downward *next to* the vertical plane but there is no interaction force between the car and the plane. Equations (4) and (5) give us $t = \sqrt{\frac{2d}{g \sin 90°}} = \sqrt{\frac{2d}{g}}$ and $v_{xf} = \sqrt{2gd \sin 90°} = \sqrt{2gd}$, both of which are consistent with a falling object.

(B) Imagine θ going to 0 in Figure 5.11. In this case, the inclined plane becomes horizontal, and the car is on a horizontal surface. Equation (3) becomes

$$a_x = g \sin\theta = g \sin 0 = 0$$

which is consistent with the fact that the car is at rest in equilibrium. Notice also that the condition $n = mg \cos\theta$ gives us $n = mg \cos 0 = mg$, which is consistent with our expectation.

Equations (4) and (5) give us $t = \sqrt{\frac{2d}{g \sin 0}} \to \infty$ and $v_{xf} = \sqrt{2gd \sin 0} = 0$. These results agree with the fact that the car does not accelerate, so it will never achieve a nonzero final velocity, and it will take an infinite amount of time to reach the bottom of the "hill"!

Example 5.7 One Block Pushes Another

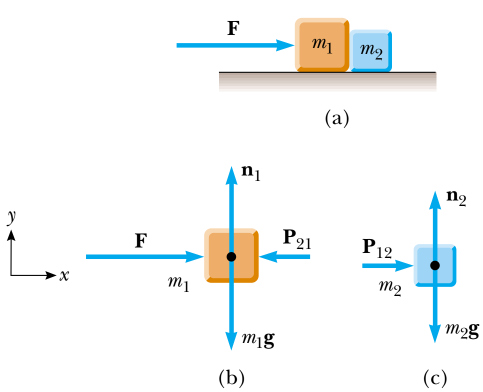
Two blocks of masses m_1 and m_2, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface, as in Figure 5.12a. A constant horizontal force $\mathbf{F}$ is applied to m_1 as shown. **(A)** Find the magnitude of the acceleration of the system.

Solution *Conceptualize* the situation using Figure 5.12a and realizing that both blocks must experience the *same* acceleration because they are in contact with each other and remain in contact throughout the motion. We *categorize* this as a Newton's second law problem because we have a force applied to a system and we are looking for an acceleration. To *analyze* the problem, we first address the combination of two blocks as a system. Because $\mathbf{F}$ is the only external horizontal force acting on the system, we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)\,a_x$$

$$(1) \qquad a_x = \boxed{\frac{F}{m_1 + m_2}}$$

To *finalize* this part, note that this would be the same acceleration as that of a single object of mass equal to the combined masses of the two blocks in Figure 5.12a and subject to the same force.



Active Figure 5.12 (Example 5.7) A force is applied to a block of mass m_1, which pushes on a second block of mass m_2. (b) The free-body diagram for m_1. (c) The free-body diagram for m_2.

At the Active Figures link at* http://www.pse6.com, *you can study the forces involved in this two-block system.

(B) Determine the magnitude of the contact force between the two blocks.

Solution *Conceptualize* by noting that the contact force is internal to the system of two blocks. Thus, we cannot find this force by modeling the whole system (the two blocks) as a single particle. We must now treat each of the two blocks individually by *categorizing* each as a particle subject to a net force. To *analyze* the situation, we first construct a free-body diagram for each block, as shown in Figures 5.12b and 5.12c, where the contact force is denoted by $\mathbf{P}$. From Figure 5.12c we see that the only horizontal force acting on m_2 is the contact force $\mathbf{P}_{12}$ (the force exerted by m_1 on m_2), which is directed to the right. Applying Newton's second law to m_2 gives

$$(2) \qquad \sum F_x = P_{12} = m_2 a_x$$

Substituting the value of the acceleration a_x given by (1) into (2) gives

$$(3) \qquad P_{12} = m_2 a_x = \boxed{\left(\frac{m_2}{m_1 + m_2}\right) F}$$

To *finalize* the problem, we see from this result that the contact force P_{12} is *less* than the applied force F. This is consistent with the fact that the force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

To *finalize* further, it is instructive to check this expression for P_{12} by considering the forces acting on m_1, shown in Figure 5.12b. The horizontal forces acting on m_1 are the applied force $\mathbf{F}$ to the right and the contact force $\mathbf{P}_{21}$ to the left (the force exerted by m_2 on m_1). From Newton's third law, $\mathbf{P}_{21}$ is the reaction to $\mathbf{P}_{12}$, so $P_{21} = P_{12}$. Applying Newton's second law to m_1 gives

$$(4) \qquad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

Substituting into (4) the value of a_x from (1), we obtain

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right) F$$

This agrees with (3), as it must.

What If? **Imagine that the force F in Figure 5.12 is applied toward the left on the right-hand block of mass m_2. Is the magnitude of the force P_{12} the same as it was when the force was applied toward the right on m_1?**

Answer With the force applied toward the left on m_2, the contact force must accelerate m_1. In the original situation, the contact force accelerates m_2. Because $m_1 > m_2$, this will require more force, so the magnitude of $\mathbf{P}_{12}$ is greater than in the original situation.

Example 5.8 Weighing a Fish in an Elevator

A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator, as illustrated in Figure 5.13. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

Solution *Conceptualize* by noting that the reading on the scale is related to the extension of the spring in the scale, which is related to the force on the end of the spring as in Figure 5.2. Imagine that a string is hanging from the end of the spring, so that the magnitude of the force exerted on the spring is equal to the tension T in the string. Thus, we are looking for T. The force $\mathbf{T}$ pulls down on the string and pulls up on the fish. Thus, we can *categorize* this problem as one of analyzing the forces and acceleration associated with the fish by means of Newton's second law. To *analyze* the problem, we inspect the free-body diagrams for the fish in Figure 5.13 and note that the external forces acting on the fish are the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the force $\mathbf{T}$ exerted by the scale. If the elevator is either at rest or moving at constant velocity, the fish does not accelerate, and so $\sum F_y = T - F_g = 0$ or $T = F_g = mg$. (Remember that the scalar mg is the weight of the fish.)

If the elevator moves with an acceleration $\mathbf{a}$ relative to an observer standing outside the elevator in an inertial frame (see Fig. 5.13), Newton's second law applied to the fish gives the net force on the fish:

$$(1) \qquad \sum F_y = T - mg = ma_y$$

where we have chosen upward as the positive y direction. Thus, we conclude from (1) that the scale reading T is greater than the fish's weight mg if $\mathbf{a}$ is upward, so that a_y is positive, and that the reading is less than mg if $\mathbf{a}$ is downward, so that a_y is negative.

For example, if the weight of the fish is 40.0 N and $\mathbf{a}$ is upward, so that $a_y = +2.00 \text{ m/s}^2$, the scale reading from (1) is

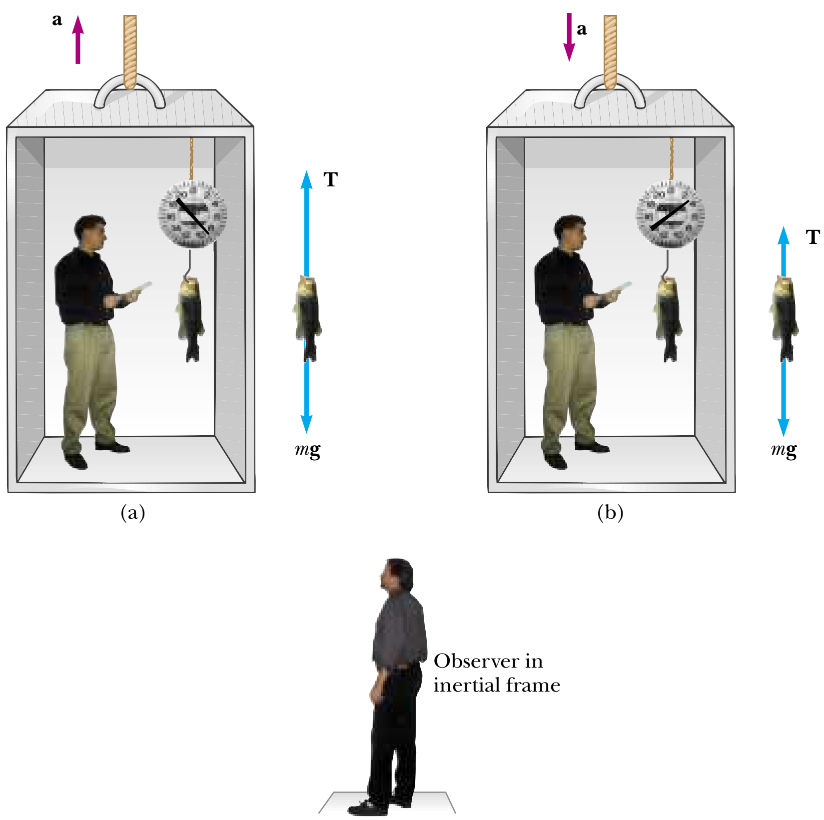


Figure 5.13 (Example 5.8) Apparent weight versus true weight. (a) When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish. (b) When the elevator accelerates downward, the spring scale reads a value less than the weight of the fish.

$$(2) \quad T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right)$$

$$= F_g\left(\frac{a_y}{g} + 1\right) = (40.0 \text{ N})\left(\frac{2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right)$$

$$= \boxed{48.2 \text{ N}}$$

If **a** is downward so that $a_y = -2.00 \text{ m/s}^2$, then (2) gives us

$$T = F_g\left(\frac{a_y}{g} + 1\right) = (40.0 \text{ N})\left(\frac{-2.00 \text{ m/s}^2}{9.80 \text{ m/s}^2} + 1\right)$$

$$= \boxed{31.8 \text{ N}}$$

To *finalize* this problem, take this advice—if you buy a fish in an elevator, make sure the fish is weighed while the elevator is either at rest or accelerating downward! Furthermore, note that from the information given here, one cannot determine the direction of motion of the elevator.

What If? Suppose the elevator cable breaks, so that the elevator and its contents are in free-fall. What happens to the reading on the scale?

Answer If the elevator falls freely, its acceleration is $a_y = -g$. We see from (2) that the scale reading T is zero in this case; that is, the fish *appears* to be weightless.

Example 5.9 The Atwood Machine

Interactive

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in Figure 5.14a, the arrangement is called an *Atwood machine.* The device is sometimes used in the laboratory to measure the free-fall acceleration. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight cord.

Solution *Conceptualize* the situation pictured in Figure 5.14a—as one object moves upward, the other object moves downward. Because the objects are connected by an inextensible string, their accelerations must be of equal magnitude. The objects in the Atwood machine are subject to the gravitational force as well as to the forces exerted by the strings connected to them—thus, we can *categorize* this as a Newton's second law problem. To *analyze* the situation, the free-body diagrams for the two objects are shown in Figure 5.14b. Two forces act on each object: the upward force **T** exerted by the string and the downward gravitational force. In problems such as this in which the pulley is modeled as massless and frictionless, the tension in the string on both sides of the pulley is the same. If the pulley has mass and/or is subject to friction, the tensions on either side are not the same and the situation requires techniques we will learn in Chapter 10.

We must be very careful with signs in problems such as this. In Figure 5.14a, notice that if object 1 accelerates upward, then object 2 accelerates downward. Thus, for consistency with signs, if we define the upward direction as positive for object 1, we must define the downward direction as positive for object 2. With this sign convention, both objects accelerate in the same direction as defined by the choice of sign. Furthermore, according to this sign convention, the y component of the net force exerted on object 1 is $T - m_1g$, and the y component of the net force exerted on object 2 is $m_2g - T$. Notice that we have chosen the signs of the forces to be consistent with the choices of signs for up and down for each object. If we assume that $m_2 > m_1$, then m_1 must accelerate upward, while m_2 must accelerate downward.

When Newton's second law is applied to object 1, we obtain

$$(1) \qquad \sum F_y = T - m_1g = m_1a_y$$

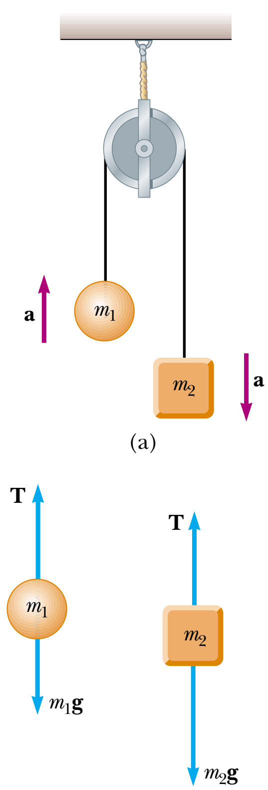
Similarly, for object 2 we find

$$(2) \qquad \sum F_y = m_2g - T = m_2a_y$$

When (2) is added to (1), T cancels and we have

$$-m_1g + m_2g = m_1a_y + m_2a_y$$

$$(3) \qquad a_y = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2}\right)g}$$



Active Figure 5.14 (Example 5.9) The Atwood machine. (a) Two objects ($m_2 > m_1$) connected by a massless inextensible cord over a frictionless pulley. (b) Free-body diagrams for the two objects.

At the Active Figures link at http://www.pse6.com, you can adjust the masses of the objects on the Atwood machine and observe the motion.

The acceleration given by (3) can be interpreted as the ratio of the magnitude of the unbalanced force on the system $(m_2 - m_1)g$, to the total mass of the system $(m_1 + m_2)$, as expected from Newton's second law.

When (3) is substituted into (1), we obtain

$$(4) \qquad T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

Finalize this problem with the following **What If?**

What If? (A) Describe the motion of the system if the objects have equal masses, that is, $m_1 = m_2$.

(B) Describe the motion of the system if one of the masses is much larger than the other, $m_1 \gg m_2$.

Answer (A) If we have the same mass on both sides, the system is balanced and it should not accelerate. Mathematically, we see that if $m_1 = m_2$, Equation (3) gives us $a_y = 0$.
(B) In the case in which one mass is infinitely larger than the other, we can ignore the effect of the smaller mass. Thus, the larger mass should simply fall as if the smaller mass were not there. We see that if $m_1 \gg m_2$, Equation (3) gives us $a_y = -g$.

Investigate these limiting cases at the Interactive Worked Example link at http://www.pse6.com.

Example 5.10 Acceleration of Two Objects Connected by a Cord — Interactive

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in Figure 5.15a. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.

Solution *Conceptualize* the motion in Figure 5.15. If m_2 moves down the incline, m_1 moves upward. Because the objects are connected by a cord (which we assume does not stretch), their accelerations have the same magnitude. We can identify forces on each of the two objects and we are looking for an acceleration, so we *categorize* this as a Newton's second-law problem. To *analyze* the problem, consider the free-body diagrams shown in Figures 5.15b and 5.15c. Applying Newton's second law in component form to the ball, choosing the upward direction as positive, yields

$$(1) \qquad \sum F_x = 0$$

$$(2) \qquad \sum F_y = T - m_1g = m_1a_y = m_1a$$

Note that in order for the ball to accelerate upward, it is necessary that $T > m_1g$. In (2), we replaced a_y with a because the acceleration has only a y component.

For the block it is convenient to choose the positive x' axis along the incline, as in Figure 5.15c. For consistency with our choice for the ball, we choose the positive direction to be down the incline. Applying Newton's second law in component form to the block gives

$$(3) \qquad \sum F_{x'} = m_2g\sin\theta - T = m_2a_{x'} = m_2a$$

$$(4) \qquad \sum F_{y'} = n - m_2g\cos\theta = 0$$

In (3) we replaced $a_{x'}$ with a because the two objects have accelerations of equal magnitude a. Equations (1) and (4) provide no information regarding the acceleration. However, if we solve (2) for T and then substitute this value for T into (3) and solve for a, we obtain

$$(5) \qquad a = \frac{m_2g\sin\theta - m_1g}{m_1 + m_2}$$

When this expression for a is substituted into (2), we find

$$(6) \qquad T = \frac{m_1m_2g(\sin\theta + 1)}{m_1 + m_2}$$

To *finalize* the problem, note that the block accelerates down the incline only if $m_2\sin\theta > m_1$. If $m_1 > m_2\sin\theta$,

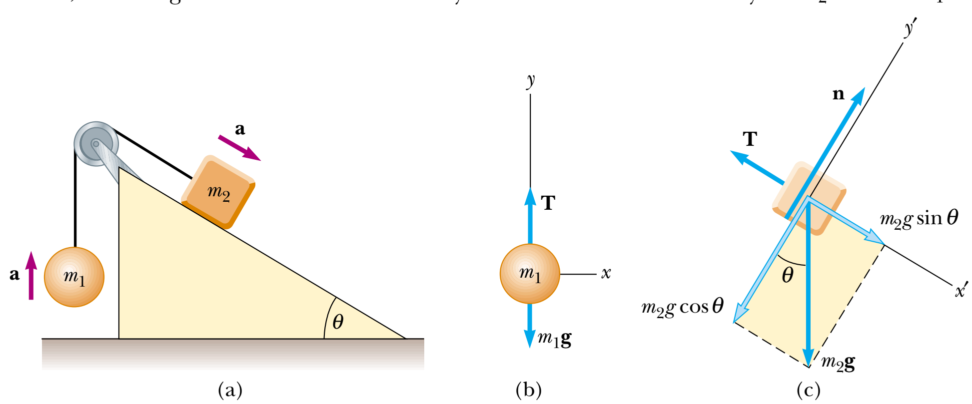


Figure 5.15 (Example 5.10) (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) Free-body diagram for the ball. (c) Free-body diagram for the block. (The incline is frictionless.)

then the acceleration is up the incline for the block and downward for the ball. Also note that the result for the acceleration (5) can be interpreted as the magnitude of the net force acting on the system divided by the total mass of the system; this is consistent with Newton's second law.

What If? (A) **What happens in this situation if the angle $\theta = 90°$?**

(B) **What happens if the mass $m_1 = 0$?**

Answer (A) If $\theta = 90°$, the inclined plane becomes vertical and there is no interaction between its surface and m_2. Thus, this problem becomes the Atwood machine of Example 5.9. Letting $\theta \rightarrow 90°$ in Equations (5) and (6) causes them to reduce to Equations (3) and (4) of Example 5.9!

(B) If $m_1 = 0$, then m_2 is simply sliding down an inclined plane without interacting with m_1 through the string. Thus, this problem becomes the sliding car problem in Example 5.6. Letting $m_1 \rightarrow 0$ in Equation (5) causes it to reduce to Equation (3) of Example 5.6!

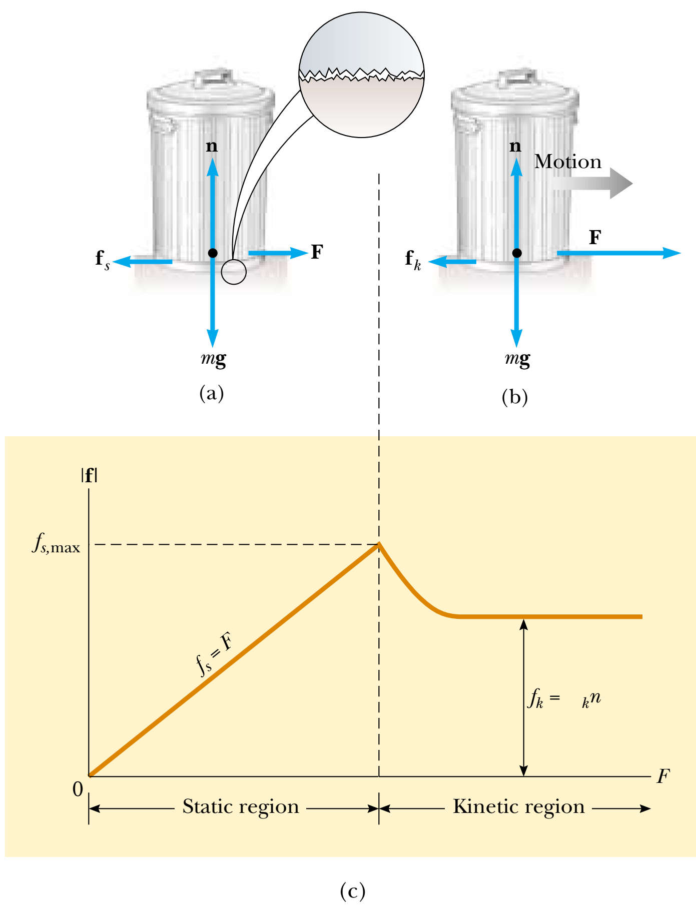
***Investigate these limiting cases at the Interactive Worked Example link at* http://www.pse6.com.**

5.8 Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a **force of friction.** Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Imagine that you are working in your garden and have filled a trash can with yard clippings. You then try to drag the trash can across the surface of your concrete patio, as in Figure 5.16a. This is a *real* surface, not an idealized, frictionless surface. If we apply an external horizontal force $\mathbf{F}$ to the trash can, acting to the right, the trash can remains stationary if $\mathbf{F}$ is small. The force that counteracts $\mathbf{F}$ and keeps the trash can from moving acts to the left and is called the **force of static friction $\mathbf{f}_s$**. As long as the trash can is not moving, $f_s = F$. Thus, if $\mathbf{F}$ is increased, $\mathbf{f}_s$ also increases. Likewise, if $\mathbf{F}$ decreases, $\mathbf{f}_s$ also

Force of static friction



Active Figure 5.16 The direction of the force of friction $\mathbf{f}$ between a trash can and a rough surface is opposite the direction of the applied force $\mathbf{F}$. Because both surfaces are rough, contact is made only at a few points, as illustrated in the "magnified" view. (a) For small applied forces, the magnitude of the force of static friction equals the magnitude of the applied force. (b) When the magnitude of the applied force exceeds the magnitude of the maximum force of static friction, the trash can breaks free. The applied force is now larger than the force of kinetic friction and the trash can accelerates to the right. (c) A graph of friction force versus applied force. Note that $f_{s,\max} > f_k$.

At the Active Figures link at* http://www.pse6.com *you can vary the applied force on the trash can and practice sliding it on surfaces of varying roughness. Note the effect on the trash can's motion and the corresponding behavior of the graph in (c).

decreases. Experiments show that the friction force arises from the nature of the two surfaces: because of their roughness, contact is made only at a few locations where peaks of the material touch, as shown in the magnified view of the surface in Figure 5.16a.

At these locations, the friction force arises in part because one peak physically blocks the motion of a peak from the opposing surface, and in part from chemical bonding ("spot welds") of opposing peaks as they come into contact. If the surfaces are rough, bouncing is likely to occur, further complicating the analysis. Although the details of friction are quite complex at the atomic level, this force ultimately involves an electrical interaction between atoms or molecules.

If we increase the magnitude of **F**, as in Figure 5.16b, the trash can eventually slips. When the trash can is on the verge of slipping, f_s has its maximum value $f_{s,\max}$, as shown in Figure 5.16c. When F exceeds $f_{s,\max}$, the trash can moves and accelerates to the right. When the trash can is in motion, the friction force is less than $f_{s,\max}$ (Fig. 5.16c). We call the friction force for an object in motion the **force of kinetic friction** $\mathbf{f}_k$. The net force $F - f_k$ in the x direction produces an acceleration to the right, according to Newton's second law. If $F = f_k$, the acceleration is zero, and the trash can moves to the right with constant speed. If the applied force is removed, the friction force acting to the left provides an acceleration of the trash can in the $-x$ direction and eventually brings it to rest, again consistent with Newton's second law.

Force of kinetic friction

Experimentally, we find that, to a good approximation, both $f_{s,\max}$ and f_k are proportional to the magnitude of the normal force. The following empirical laws of friction summarize the experimental observations:

- The magnitude of the force of static friction between any two surfaces in contact can have the values

$$f_s \le \mu_s n \qquad (5.8)$$

where the dimensionless constant μ_s is called the **coefficient of static friction** and n is the magnitude of the normal force exerted by one surface on the other. The equality in Equation 5.8 holds when the surfaces are on the verge of slipping, that is, when $f_s = f_{s,\max} \equiv \mu_s n$. This situation is called *impending motion.* The inequality holds when the surfaces are not on the verge of slipping.

- The magnitude of the force of kinetic friction acting between two surfaces is

$$f_k = \mu_k n \qquad (5.9)$$

where μ_k is the **coefficient of kinetic friction.** Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations in this text.

PITFALL PREVENTION

5.9 The Equal Sign is Used in Limited Situations

In Equation 5.8, the equal sign is used *only* in the case in which the surfaces are just about to break free and begin sliding. Do not fall into the common trap of using $f_s = \mu_s n$ in *any* static situation.

PITFALL PREVENTION

5.10 Friction Equations

Equations 5.8 and 5.9 are *not* vector equations. They are relationships between the *magnitudes* of the vectors representing the friction and normal forces. Because the friction and normal forces are perpendicular to each other, the vectors cannot be related by a multiplicative constant.

Table 5.2

Coefficients of Friction[a]

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

[a] All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s. Typical values range from around 0.03 to 1.0. Table 5.2 lists some reported values.
- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
- The coefficients of friction are nearly independent of the area of contact between the surfaces. We might expect that placing an object on the side having the most area might increase the friction force. While this provides more points in contact, as in Figure 5.16a, the weight of the object is spread out over a larger area, so that the individual points are not pressed so tightly together. These effects approximately compensate for each other, so that the friction force is independent of the area.

PITFALL PREVENTION

5.11 The Direction of the Friction Force

Sometimes, an incorrect statement about the friction force between an object and a surface is made—"the friction force on an object is opposite to its motion or impending motion"—rather than the correct phrasing, "the friction force on an object is opposite to its motion or impending motion *relative to the surface.*" Think carefully about Quick Quiz 5.12.

Quick Quiz 5.11 You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book? (a) downward (b) upward (c) out from the wall (d) into the wall.

Quick Quiz 5.12 A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it, not sliding at all. What is the direction of the friction force exerted by the truck on the crate? (a) to the west (b) to the east (c) No friction force exists because the crate is not sliding.

Quick Quiz 5.13 You place your physics book on a wooden board. You raise one end of the board so that the angle of the incline increases. Eventually, the book starts sliding on the board. If you maintain the angle of the board at this value, the book (a) moves at constant speed (b) speeds up (c) slows down (d) none of these.

Quick Quiz 5.14 You are playing with your daughter in the snow. She sits on a sled and asks you to slide her across a flat, horizontal field. You have a choice of (a) pushing her from behind, by applying a force downward on her shoulders at 30° below the horizontal (Fig. 5.17a), or (b) attaching a rope to the front of the sled and pulling with a force at 30° above the horizontal (Fig 5.17b). Which would be easier for you and why?

Figure 5.17 (Quick Quiz 5.14) A father pushes his daughter on a sled either by (a) pushing down on her shoulders, or (b) pulling up on a rope.

Conceptual Example 5.11 Why Does the Sled Accelerate?

A horse pulls a sled along a level, snow-covered road, causing the sled to accelerate, as shown in Figure 5.18a. Newton's third law states that the sled exerts a force of equal magnitude and opposite direction on the horse. In view of this, how can the sled accelerate—don't the forces cancel? Under what condition does the system (horse plus sled) move with constant velocity?

Solution Remember that the forces described in Newton's third law act on *different* objects—the horse exerts a force on the sled, and the sled exerts an equal-magnitude and oppositely directed force on the horse. Because we are interested only in the motion of the sled, we do not consider the forces it exerts on the horse. When determining the motion of an object, you must add only the forces on that object. (This is the principle behind drawing a free-body diagram.) The horizontal forces exerted on the sled are the forward force $\mathbf{T}$ exerted by the horse and the backward force of friction $\mathbf{f}_{\text{sled}}$ between sled and snow (see Fig. 5.18b). When the forward force on the sled exceeds the backward force, the sled accelerates to the right.

The horizontal forces exerted on the horse are the forward force $\mathbf{f}_{\text{horse}}$ exerted by the Earth and the backward tension force $\mathbf{T}$ exerted by the sled (Fig. 5.18c). The resultant of these two forces causes the horse to accelerate.

The force that accelerates the system (horse plus sled) is the net force $\mathbf{f}_{\text{horse}} - \mathbf{f}_{\text{sled}}$. When $\mathbf{f}_{\text{horse}}$ balances $\mathbf{f}_{\text{sled}}$, the system moves with constant velocity.

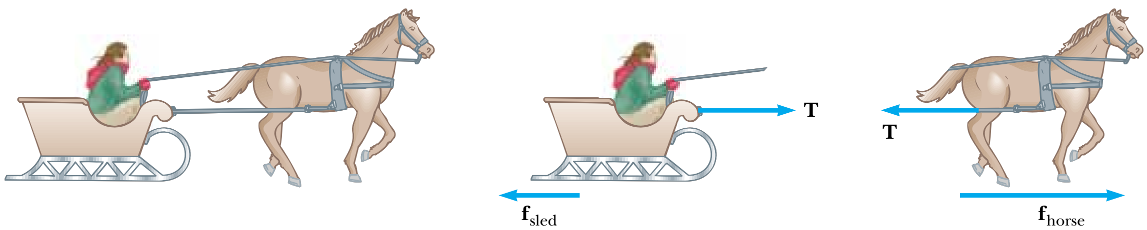


Figure 5.18 (Conceptual Example 5.11)

Example 5.12 Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure 5.19. The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s.

Solution *Conceptualizing* from the free body diagram in Figure 5.19, we see that we can *categorize* this as a Newton's second law problem. To *analyze* the problem, note that the only forces acting on the block are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of static friction $\mathbf{f}_s$. These forces balance when the block is not moving. When we choose x to be parallel to the plane and y perpendicular to it, Newton's second law applied to the block for this balanced situation gives

$$(1) \qquad \sum F_x = mg \sin\theta - f_s = ma_x = 0$$

$$(2) \qquad \sum F_y = n - mg\cos\theta = ma_y = 0$$

We can eliminate mg by substituting $mg = n/\cos\theta$ from (2) into (1) to find

$$(3) \qquad f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline angle is increased until the block is on the verge of slipping, the force of static friction has reached its maximum value $\mu_s n$. The angle θ in this situation is the critical angle θ_c, and (3) becomes

$$\mu_s n = n\tan\theta_c$$

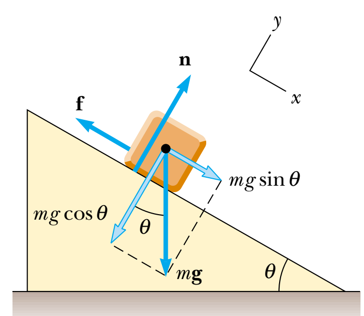


Figure 5.19 (Example 5.12) The external forces exerted on a block lying on a rough incline are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of friction $\mathbf{f}$. For convenience, the gravitational force is resolved into a component along the incline $mg\sin\theta$ and a component perpendicular to the incline $mg\cos\theta$.

$$\mu_s = \tan\theta_c$$

For example, if the block just slips at $\theta_c = 20.0°$, then we find that $\mu_s = \tan 20.0° = 0.364$.

To *finalize* the problem, note that once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k n$. However, if θ is reduced to a value less than θ_c, it may be possible to find an angle θ_c' such that the block moves down the incline with constant speed ($a_x = 0$). In this case, using (1) and (2) with f_s replaced by f_k gives

$$\mu_k = \tan\theta_c'$$

where $\theta_c' < \theta_c$.

Example 5.13 The Sliding Hockey Puck

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Solution *Conceptualize* the problem by imagining that the puck in Figure 5.20 slides to the right and eventually comes to rest. To *categorize* the problem, note that we have forces identified in Figure 5.20, but that kinematic variables are provided in the text of the problem. Thus, we must combine the techniques of Chapter 2 with those of this chapter. (We assume that the friction force is constant, which will result in a constant horizontal acceleration.) To *analyze* the situation, note that the forces acting on the puck after it is in motion are shown in Figure 5.20. First, we find the acceleration algebraically in terms of the coefficient of kinetic friction, using Newton's second law. Knowing the acceleration of the puck and the distance it travels, we can then use the equations of kinematics to find the numerical value of the coefficient of kinetic friction.

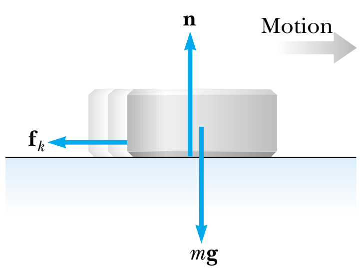


Figure 5.20 (Example 5.13) *After* the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\mathbf{g}$, the normal force $\mathbf{n}$, and the force of kinetic friction $\mathbf{f}_k$.

Defining rightward and upward as our positive directions, we apply Newton's second law in component form to the puck and obtain

$$(1) \qquad \sum F_x = -f_k = ma_x$$

$$(2) \qquad \sum F_y = n - mg = 0 \qquad (a_y = 0)$$

But $f_k = \mu_k n$, and from (2) we see that $n = mg$. Therefore, (1) becomes

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left in Figure 5.20; because the velocity of the puck is to the right, this means that the puck is slowing down. The acceleration is independent of the mass of the puck and is constant because we assume that μ_k remains constant.

Because the acceleration is constant, we can use Equation 2.13, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_f = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(115 \text{ m})} = \boxed{0.117}$$

To *finalize* the problem, note that μ_k is dimensionless, as it should be, and that it has a low value, consistent with an object sliding on ice.

Example 5.14 Acceleration of Two Connected Objects When Friction Is Present

A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley, as shown in Figure 5.21a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.

Solution *Conceptualize* the problem by imagining what happens as $\mathbf{F}$ is applied to the block. Assuming that $\mathbf{F}$ is not large enough to lift the block, the block will slide to the right and the ball will rise. We can identify forces and we want an acceleration, so we *categorize* this as a Newton's second law problem, one that includes the friction force. To *analyze* the problem, we begin by drawing free-body diagrams for the two objects, as shown in Figures 5.21b and 5.21c. Next, we apply Newton's second law in component form to each object and use Equation 5.9, $f_k = \mu_k n$. Then we can solve for the acceleration in terms of the parameters given.

The applied force $\mathbf{F}$ has x and y components $F\cos\theta$ and $F\sin\theta$, respectively. Applying Newton's second law to both objects and assuming the motion of the block is to the right, we obtain

Motion of block: (1) $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

(2) $\sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$

Motion of ball: $\sum F_x = m_2 a_x = 0$

(3) $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$

Because the two objects are connected, we can equate the magnitudes of the x component of the acceleration of the block and the y component of the acceleration of the ball. From Equation 5.9 we know that $f_k = \mu_k n$, and from (2) we know that $n = m_1 g - F\sin\theta$ (in this case n is *not* equal to $m_1 g$); therefore,

$$(4) \qquad f_k = \mu_k(m_1 g - F\sin\theta)$$

That is, the friction force is reduced because of the positive y component of $\mathbf{F}$. Substituting (4) and the value of T from (3) into (1) gives

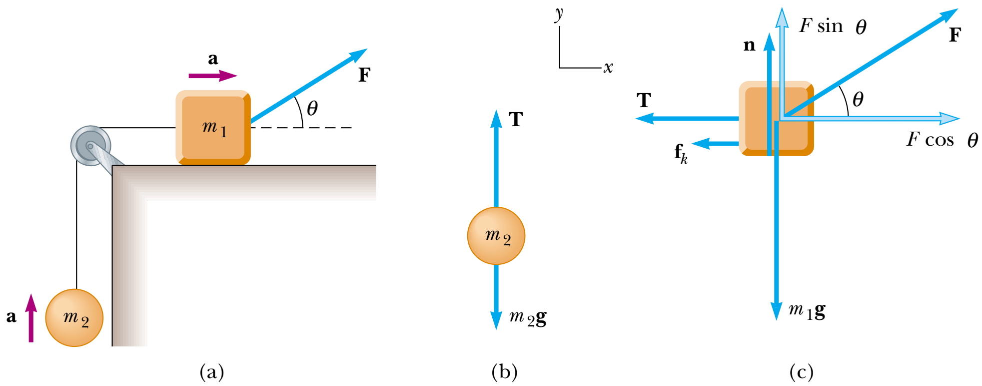


Figure 5.21 (Example 5.14) (a) The external force **F** applied as shown can cause the block to accelerate to the right. (b) and (c) The free-body diagrams assuming that the block accelerates to the right and the ball accelerates upward. The magnitude of the force of kinetic friction in this case is given by $f_k = \mu_k n = \mu_k(m_1 g - F \sin \theta)$.

$$F \cos \theta - \mu_k(m_1 g - F \sin \theta) - m_2(a + g) = m_1 a$$

Solving for a, we obtain

$$(5) \qquad a = \boxed{\frac{F(\cos \theta + \mu_k \sin \theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}}$$

To *finalize* the problem, note that the acceleration of the block can be either to the right or to the left,[5] depending on the sign of the numerator in (5). If the motion is to the left, then we must reverse the sign of f_k in (1) because the force of kinetic friction must oppose the motion of the block relative to the surface. In this case, the value of a is the same as in (5), with the two plus signs in the numerator changed to minus signs.

This is the final chapter in which we will explicitly show the steps of the General Problem-Solving Strategy in all worked examples. We will refer to them explicitly in occasional examples in future chapters, but you should use the steps in *all* of your problem solving.

[5] Equation 5 shows that when $\mu_k m_1 > m_2$, there is a range of values of F for which no motion occurs at a given angle θ.

Application Automobile Antilock Braking Systems (ABS)

If an automobile tire is rolling and not slipping on a road surface, then the maximum friction force that the road can exert on the tire is the force of static friction $\mu_s n$. One must use static friction in this situation because at the point of contact between the tire and the road, no sliding of one surface over the other occurs if the tire is not skidding. However, if the tire starts to skid, the friction force exerted on it is reduced to the force of kinetic friction $\mu_k n$. Thus, to maximize the friction force and minimize stopping distance, the wheels must maintain pure rolling motion and not skid. An additional benefit of maintaining wheel rotation is that directional control is not lost as it is in skidding. Unfortunately, in emergency situations drivers typically press down as hard as they can on the brake pedal, "locking the brakes." This stops the wheels from rotating, ensuring a skid and reducing the friction force from the static to the kinetic value. To address this problem, automotive engineers have developed antilock braking systems (ABS). The purpose of the ABS is to help typical drivers (whose tendency is to lock the wheels in an emergency) to better maintain control of their automobiles and minimize stopping distance. The system briefly releases the brakes when a wheel is just about to stop turning. This maintains rolling contact between the tire and the pavement. When the brakes are released momentarily, the stopping distance is greater than it would be if the brakes were being applied continuously. However, through the use of computer control, the "brake-off" time is kept to a minimum. As a result, the stopping distance is much less than what it would be if the wheels were to skid.

Let us model the stopping of a car by examining real data. In an issue of *AutoWeek*,[6] the braking performance for a Toyota Corolla was measured. These data correspond to the braking force acquired by a highly trained, professional driver. We begin by assuming constant acceleration. (Why do we need to make this assumption?) The magazine provided the initial speed and stopping distance in non-SI units, which we show in the left and middle sections of Table 5.3. After converting these values to SI, we use $v_f^2 = v_i^2 + 2ax$ to determine the acceleration at different speeds, shown in the right section. These do not vary greatly, and so our assumption of constant acceleration is reasonable.

[6] *AutoWeek* magazine, **48:**22–23, 1998.

Table 5.3

Data for a Toyota Corolla:				
Initial Speed		Stopping Distance		Acceleration
(mi/h)	(m/s)	(ft)	(m)	(m/s^2)
30	13.4	34	10.4	−8.63
60	26.8	143	43.6	−8.24
80	35.8	251	76.5	−8.38

Table 5.4

Stopping Distance With and Without Skidding		
	Stopping Distance	
Initial Speed (mi/h)	no skid (m)	skidding (m)
30	10.4	13.9
60	43.6	55.5
80	76.5	98.9

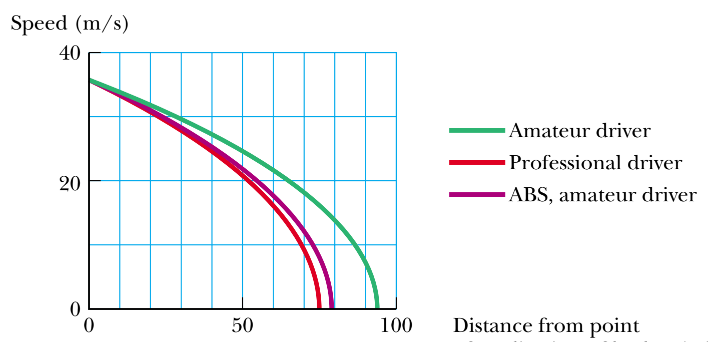


Figure 5.22 This plot of vehicle speed versus distance from the location at which the brakes were applied shows that an antilock braking system (ABS) approaches the performance of a trained professional driver.

We take an average value of acceleration of $-8.4\ m/s^2$, which is approximately $0.86g$. We then calculate the coefficient of friction from $\Sigma F = \mu_s mg = ma$, which gives $\mu_s = 0.86$ for the Toyota. This is lower than the rubber-on-concrete value given in Table 5.2. Can you think of any reasons for this?

We now estimate the stopping distance of the car if the wheels were skidding. From Table 5.2, we see that the difference between the coefficients of static and kinetic friction for rubber against concrete is about 0.2. Let us assume that our coefficients differ by the same amount, so that $\mu_k \approx 0.66$. This allows us to estimate the stopping distances when the wheels are locked and the car skids across the pavement, as shown in the third column of Table 5.4. The results illustrate the advantage of not allowing the wheels to skid.

Because an ABS keeps the wheels rotating, the higher coefficient of static friction is maintained between the tires and road. This approximates the technique of a professional driver who is able to maintain the wheels at the point of maximum friction force. Let us estimate the ABS performance by assuming that the magnitude of the acceleration is not quite as good as that achieved by the professional driver but instead is reduced by 5%.

Figure 5.22 is a plot of vehicle speed versus distance from where the brakes were applied (at an initial speed of $80.0\ mi/h = 35.8\ m/s$) for the three cases of amateur driver, professional driver, and estimated ABS performance (amateur driver). This shows that a markedly shorter distance is necessary for stopping without locking the wheels compared to skidding. In addition a satisfactory value of stopping distance is achieved when the ABS computer maintains tire rotation.

SUMMARY

Take a practice test for this chapter by clicking the Practice Test link at http://www.pse6.com.

An **inertial frame of reference** is one we can identify in which an object that does not interact with other objects experiences zero acceleration. Any frame moving with constant velocity relative to an inertial frame is also an inertial frame. **Newton's first law** states that it is possible to find such a frame, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force acting on an object equals the product of its mass and its acceleration: $\Sigma\mathbf{F} = m\mathbf{a}$. If the object is either stationary or moving with constant velocity, then the object is in equilibrium and the force vectors must cancel each other.

The **gravitational force** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration: $\mathbf{F}_g = m\mathbf{g}$. The **weight** of an object is the magnitude of the gravitational force acting on the object.

Newton's third law states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1. Thus, an isolated force cannot exist in nature.

The **maximum force of static friction** $\mathbf{f}_{s,\max}$ between an object and a surface is proportional to the normal force acting on the object. In general, $f_s \leq \mu_s n$, where μ_s is the **coefficient of static friction** and n is the magnitude of the normal force. When an object slides over a surface, the direction of the **force of kinetic friction** $\mathbf{f}_k$ is opposite the direction of motion of the object relative to the surface and is also proportional to the magnitude of the normal force. The magnitude of this force is given by $f_k = \mu_k n$, where μ_k is the **coefficient of kinetic friction.**

QUESTIONS

1. A ball is held in a person's hand. (a) Identify all the external forces acting on the ball and the reaction to each. (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Neglect air resistance.)

2. If a car is traveling westward with a constant speed of 20 m/s, what is the resultant force acting on it?

3. What is wrong with the statement "Because the car is at rest, there are no forces acting on it"? How would you correct this sentence?

4. In the motion picture *It Happened One Night* (Columbia Pictures, 1934), Clark Gable is standing inside a stationary bus in front of Claudette Colbert, who is seated. The bus suddenly starts moving forward and Clark falls into Claudette's lap. Why did this happen?

5. A passenger sitting in the rear of a bus claims that she was injured as the driver slammed on the brakes, causing a suitcase to come flying toward her from the front of the bus. If you were the judge in this case, what disposition would you make? Why?

6. A space explorer is moving through space far from any planet or star. She notices a large rock, taken as a specimen from an alien planet, floating around the cabin of the ship. Should she push it gently or kick it toward the storage compartment? Why?

7. A rubber ball is dropped onto the floor. What force causes the ball to bounce?

8. While a football is in flight, what forces act on it? What are the action–reaction pairs while the football is being kicked and while it is in flight?

9. The mayor of a city decides to fire some city employees because they will not remove the obvious sags from the cables that support the city traffic lights. If you were a lawyer, what defense would you give on behalf of the employees? Who do you think would win the case in court?

10. A weightlifter stands on a bathroom scale. He pumps a barbell up and down. What happens to the reading on the bathroom scale as this is done? **What if** he is strong enough to actually *throw* the barbell upward? How does the reading on the scale vary now?

11. Suppose a truck loaded with sand accelerates along a highway. If the driving force on the truck remains constant, what happens to the truck's acceleration if its trailer leaks sand at a constant rate through a hole in its bottom?

12. As a rocket is fired from a launching pad, its speed and acceleration increase with time as its engines continue to operate. Explain why this occurs even though the thrust of the engines remains constant.

13. What force causes an automobile to move? A propeller-driven airplane? A rowboat?

14. Identify the action–reaction pairs in the following situations: a man takes a step; a snowball hits a girl in the back; a baseball player catches a ball; a gust of wind strikes a window.

15. In a contest of National Football League behemoths, teams from the Rams and the 49ers engage in a tug-of-war, pulling in opposite directions on a strong rope. The Rams exert a force of 9 200 N and they are winning, making the center of the rope move steadily toward themselves. Is it possible to know the tension in the rope from the information stated? Is it larger or smaller than 9 200 N? How hard are the 49ers pulling on the rope? Would it change your answer if the 49ers were winning or if the contest were even? The stronger team wins by exerting a larger force—on what? Explain your answers.

16. Twenty people participate in a tug-of-war. The two teams of ten people are so evenly matched that neither team wins. After the game they notice that a car is stuck in the mud. They attach the tug-of-war rope to the bumper of the car, and all the people pull on the rope. The heavy car has just moved a couple of decimeters when the rope breaks. Why did the rope break in this situation when it did not break when the same twenty people pulled on it in a tug-of-war?

17. "When the locomotive in Figure Q5.17 broke through the wall of the train station, the force exerted by the locomotive on the wall was greater than the force the wall could exert on the locomotive." Is this statement true or in need of correction? Explain your answer.

18. An athlete grips a light rope that passes over a low-friction pulley attached to the ceiling of a gym. A sack of sand precisely equal in weight to the athlete is tied to the other end of the rope. Both the sand and the athlete are initially at rest. The athlete climbs the rope, sometimes speeding up and slowing down as he does so. What happens to the sack of sand? Explain.

19. If the action and reaction forces are always equal in magnitude and opposite in direction to each other, then doesn't the net vector force on any object necessarily add up to zero? Explain your answer.

20. Can an object exert a force on itself? Argue for your answer.

21. If you push on a heavy box that is at rest, you must exert some force to start its motion. However, once the box is sliding, you can apply a smaller force to maintain that motion. Why?

Roger Viollet, Mill Valley, CA, University Science Books, 1982

Figure Q5.17

22. The driver of a speeding empty truck slams on the brakes and skids to a stop through a distance d. (a) If the truck carried a load that doubled its mass, what would be the truck's "skidding distance"? (b) If the initial speed of the truck were halved, what would be the truck's skidding distance?

23. Suppose you are driving a classic car. Why should you avoid slamming on your brakes when you want to stop in the shortest possible distance? (Many cars have antilock brakes that avoid this problem.)

24. A book is given a brief push to make it slide up a rough incline. It comes to a stop and slides back down to the starting point. Does it take the same time to go up as to come down? **What if** the incline is frictionless?

25. A large crate is placed on the bed of a truck but not tied down. (a) As the truck accelerates forward, the crate remains at rest relative to the truck. What force causes the crate to accelerate forward? (b) If the driver slammed on the brakes, what could happen to the crate?

26. Describe a few examples in which the force of friction exerted on an object is in the direction of motion of the object.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging   □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com   = computer useful in solving problem

= paired numerical and symbolic problems

Sections 5.1 through 5.6

1. A force **F** applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2. (a) What is the value of the ratio m_1/m_2? (b) If m_1 and m_2 are combined, find their acceleration under the action of the force **F**.

2. The largest-caliber antiaircraft gun operated by the German air force during World War II was the 12.8-cm Flak 40. This weapon fired a 25.8-kg shell with a muzzle speed of 880 m/s. What propulsive force was necessary to attain the muzzle speed within the 6.00-m barrel? (Assume the shell moves horizontally with constant acceleration and neglect friction.)

3. A 3.00-kg object undergoes an acceleration given by $\mathbf{a} = (2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ m/s^2. Find the resultant force acting on it and the magnitude of the resultant force.

4. The gravitational force on a baseball is $-F_g\hat{\mathbf{j}}$. A pitcher throws the baseball with velocity $v\hat{\mathbf{i}}$ by uniformly accelerating it straight forward horizontally for a time interval $\Delta t = t - 0 = t$. If the ball starts from rest, (a) through what distance does it accelerate before its release? (b) What force does the pitcher exert on the ball?

5. To model a spacecraft, a toy rocket engine is securely fastened to a large puck, which can glide with negligible friction over a horizontal surface, taken as the xy plane. The 4.00-kg puck has a velocity of $300\hat{\mathbf{i}}$ m/s at one instant. Eight seconds later, its velocity is to be $(800\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}})$ m/s. Assuming the rocket engine exerts a constant horizontal force, find (a) the components of the force and (b) its magnitude.

6. The average speed of a nitrogen molecule in air is about 6.70×10^2 m/s, and its mass is 4.68×10^{-26} kg. (a) If it takes 3.00×10^{-13} s for a nitrogen molecule to hit a wall and rebound with the same speed but moving in the opposite direction, what is the average acceleration of the molecule during this time interval? (b) What average force does the molecule exert on the wall?

7. An electron of mass 9.11×10^{-31} kg has an initial speed of 3.00×10^5 m/s. It travels in a straight line, and its speed increases to 7.00×10^5 m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the force exerted on the electron and (b) compare this force with the weight of the electron, which we neglected.

8. A woman weighs 120 lb. Determine (a) her weight in newtons (N) and (b) her mass in kilograms (kg).

9. If a man weighs 900 N on the Earth, what would he weigh on Jupiter, where the acceleration due to gravity is 25.9 m/s^2?

10. The distinction between mass and weight was discovered after Jean Richer transported pendulum clocks from Paris to French Guyana in 1671. He found that they ran slower there quite systematically. The effect was reversed when the clocks returned to Paris. How much weight would you personally lose in traveling from Paris, where $g = 9.809\,5$ m/s^2, to Cayenne, where $g = 9.780\,8$ m/s^2? [We will consider how the free-fall acceleration influences the period of a pendulum in Section 15.5.]

11. Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a 5.00-kg object. If $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the accelerations in (a) and (b) of Figure P5.11.

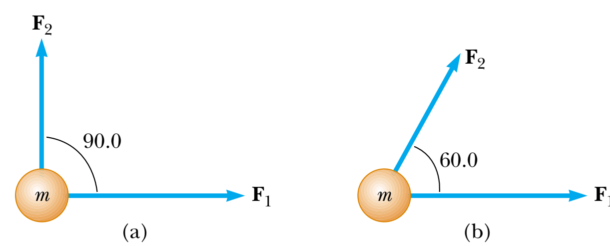


Figure P5.11

12. Besides its weight, a 2.80-kg object is subjected to one other constant force. The object starts from rest and in 1.20 s experiences a displacement of $(4.20\hat{\mathbf{i}} - 3.30\hat{\mathbf{j}})$ m, where the direction of $\hat{\mathbf{j}}$ is the upward vertical direction. Determine the other force.

13. You stand on the seat of a chair and then hop off. (a) During the time you are in flight down to the floor, the Earth is lurching up toward you with an acceleration of what order of magnitude? In your solution explain your logic. Model the Earth as a perfectly solid object. (b) The Earth moves up through a distance of what order of magnitude?

14. Three forces acting on an object are given by $\mathbf{F}_1 = (-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}})$ N, $\mathbf{F}_2 = (5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ N, and $\mathbf{F}_3 = (-45.0\hat{\mathbf{i}})$ N. The object experiences an acceleration of magnitude 3.75 m/s^2. (a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed after 10.0 s? (d) What are the velocity components of the object after 10.0 s?

15. A 15.0-lb block rests on the floor. (a) What force does the floor exert on the block? (b) If a rope is tied to the block and run vertically over a pulley, and the other end is attached to a free-hanging 10.0-lb weight, what is the force exerted by the floor on the 15.0-lb block? (c) If we replace the 10.0-lb weight in part (b) with a 20.0-lb weight, what is the force exerted by the floor on the 15.0-lb block?

Section 5.7 Some Applications of Newton's Laws

16. A 3.00-kg object is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on this object at $t = 2.00$ s.

17. The distance between two telephone poles is 50.0 m. When a 1.00-kg bird lands on the telephone wire midway between the poles, the wire sags 0.200 m. Draw a free-body diagram of the bird. How much tension does the bird produce in the wire? Ignore the weight of the wire.

18. A bag of cement of weight 325 N hangs from three wires as suggested in Figure P5.18. Two of the wires make angles $\theta_1 = 60.0°$ and $\theta_2 = 25.0°$ with the horizontal. If the system is in equilibrium, find the tensions T_1, T_2, and T_3 in the wires.

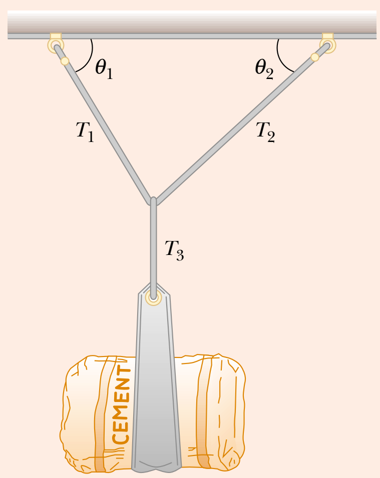


Figure P5.18 Problems 18 and 19.

19. A bag of cement of weight F_g hangs from three wires as shown in Figure P5.18. Two of the wires make angles θ_1 and θ_2 with the horizontal. If the system is in equilibrium, show that the tension in the left-hand wire is

$$T_1 = F_g \cos\theta_2 / \sin(\theta_1 + \theta_2)$$

20. You are a judge in a children's kite-flying contest, and two children will win prizes for the kites that pull most strongly and least strongly on their strings. To measure string tensions, you borrow a weight hanger, some slotted weights, and a protractor from your physics teacher, and use the following protocol, illustrated in Figure P5.20: Wait for a child to get her kite well controlled, hook the hanger onto the kite string about 30 cm from her hand, pile on weight until that section of string is horizontal, record the mass required, and record the angle between the horizontal and the string running up to the kite. (a) Explain how this method works. As you construct your explanation, imagine that the children's parents ask you about your method, that they might make false assumptions about your ability without concrete evidence, and that your explanation is an opportunity to give them confidence in your evaluation technique. (b) Find the string tension if the mass is 132 g and the angle of the kite string is 46.3°.

Figure P5.20

21. The systems shown in Figure P5.21 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Neglect the masses of the pulleys and strings, and assume the incline in part (c) is frictionless.)

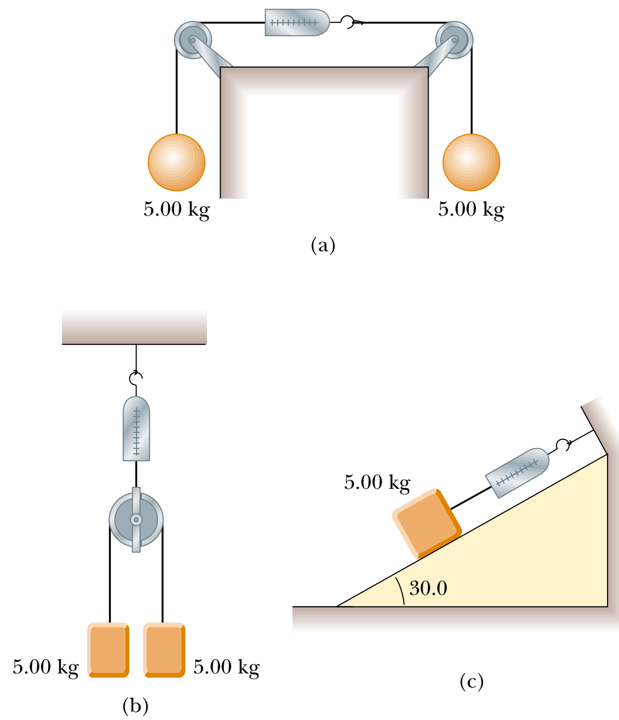


Figure P5.21

22. Draw a free-body diagram of a block which slides down a frictionless plane having an inclination of $\theta = 15.0°$ (Fig. P5.22). The block starts from rest at the top and the length of the incline is 2.00 m. Find (a) the acceleration of the block and (b) its speed when it reaches the bottom of the incline.

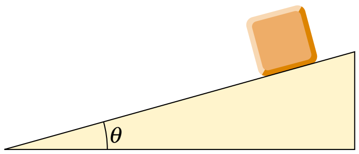


Figure P5.22 Problems 22 and 25.

23. A 1.00-kg object is observed to have an acceleration of 10.0 m/s^2 in a direction 30.0° north of east (Fig. P5.23). The force $\mathbf{F}_2$ acting on the object has a magnitude of 5.00 N and is directed north. Determine the magnitude and direction of the force $\mathbf{F}_1$ acting on the object.

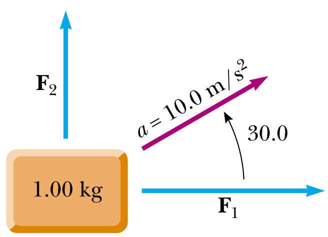


Figure P5.23

24. A 5.00-kg object placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging 9.00-kg object, as in Figure P5.24. Draw free-body diagrams of both objects. Find the acceleration of the two objects and the tension in the string.

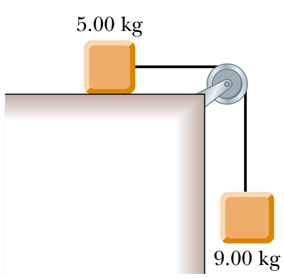


Figure P5.24 Problems 24 and 43.

25. A block is given an initial velocity of 5.00 m/s up a frictionless 20.0° incline (Fig. P5.22). How far up the incline does the block slide before coming to rest?

26. Two objects are connected by a light string that passes over a frictionless pulley, as in Figure P5.26. Draw free-body diagrams of both objects. If the incline is frictionless and if $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, and $\theta = 55.0°$, find (a) the accelerations of the objects, (b) the tension in the string, and (c) the speed of each object 2.00 s after being released from rest.

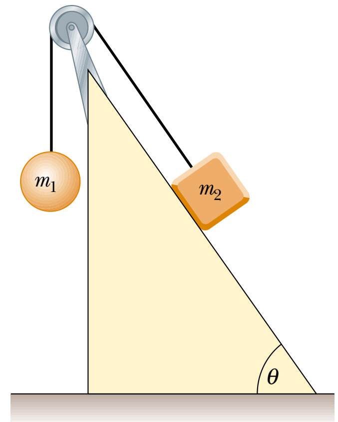


Figure P5.26

27. A tow truck pulls a car that is stuck in the mud, with a force of 2 500 N as in Fig. P5.27. The tow cable is under tension and therefore pulls downward and to the left on the pin at its upper end. The light pin is held in equilibrium by forces exerted by the two bars A and B. Each bar is a *strut*: that is, each is a bar whose weight is small compared to the forces it exerts, and which exerts forces only through hinge pins at its ends. Each strut exerts a force directed parallel to its length. Determine the force of tension or compression in each strut. Proceed as follows: Make a guess as to which way (pushing or pulling) each force acts on the top pin. Draw a free-body diagram of the pin. Use the condition for equilibrium of the pin to translate the free-body diagram into equations. From the equations calculate the forces exerted by struts A and B. If you obtain a positive answer, you correctly guessed the direction of the force. A negative answer means the direction should be reversed, but the absolute value correctly gives the magnitude of the force. If a strut pulls on a pin, it is in tension. If it pushes, the strut is in compression. Identify whether each strut is in tension or in compression.

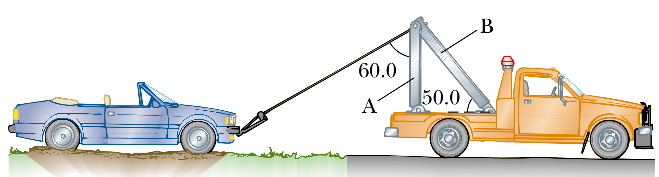


Figure P5.27

28. Two objects with masses of 3.00 kg and 5.00 kg are connected by a light string that passes over a light frictionless pulley to form an Atwood machine, as in Figure 5.14a. Determine (a) the tension in the string, (b) the acceleration of each object, and (c) the distance each object will move in the first second of motion if they start from rest.

29. In Figure P5.29, the man and the platform together weigh 950 N. The pulley can be modeled as frictionless. Determine how hard the man has to pull on the rope to lift himself steadily upward above the ground. (Or is it impossible? If so, explain why.)

Figure P5.29

30. In the Atwood machine shown in Figure 5.14a, $m_1 = 2.00$ kg and $m_2 = 7.00$ kg. The masses of the pulley and string are negligible by comparison. The pulley turns without friction and the string does not stretch. The lighter object is released with a sharp push that sets it into motion at $v_i = 2.40$ m/s downward. (a) How far will m_1 descend below its initial level? (b) Find the velocity of m_1 after 1.80 seconds.

31. In the system shown in Figure P5.31, a horizontal force $\mathbf{F}_x$ acts on the 8.00-kg object. The horizontal surface is frictionless. (a) For what values of F_x does the 2.00-kg object accelerate upward? (b) For what values of F_x is the tension in the cord zero? (c) Plot the acceleration of the 8.00-kg object versus F_x. Include values of F_x from -100 N to $+100$ N.

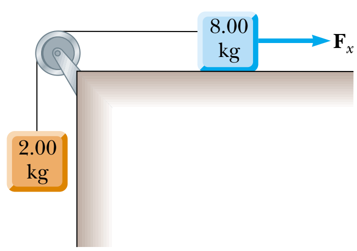


Figure P5.31

32. A frictionless plane is 10.0 m long and inclined at 35.0°. A sled starts at the bottom with an initial speed of 5.00 m/s up the incline. When it reaches the point at which it momentarily stops, a second sled is released from the top of this incline with an initial speed v_i. Both sleds reach the bottom of the incline at the same moment. (a) Determine the distance that the first sled traveled up the incline. (b) Determine the initial speed of the second sled.

33. A 72.0-kg man stands on a spring scale in an elevator. Starting from rest, the elevator ascends, attaining its maximum speed of 1.20 m/s in 0.800 s. It travels with this constant speed for the next 5.00 s. The elevator then undergoes a uniform acceleration in the negative y direction for 1.50 s and comes to rest. What does the spring scale register (a) before the elevator starts to move? (b) during the first 0.800 s? (c) while the elevator is traveling at constant speed? (d) during the time it is slowing down?

34. An object of mass m_1 on a frictionless horizontal table is connected to an object of mass m_2 through a very light pulley P_1 and a light fixed pulley P_2 as shown in Figure P5.34. (a) If a_1 and a_2 are the accelerations of m_1 and m_2, respectively, what is the relation between these accelerations? Express (b) the tensions in the strings and (c) the accelerations a_1 and a_2 in terms of the masses m_1 and m_2, and g.

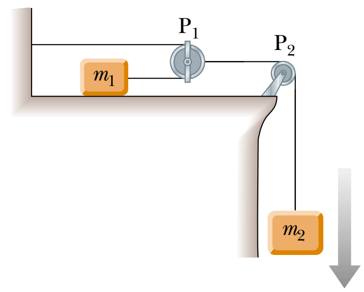


Figure P5.34

Section 5.8 Forces of Friction

35. The person in Figure P5.35 weighs 170 lb. As seen from the front, each light crutch makes an angle of 22.0° with the vertical. Half of the person's weight is supported by the crutches. The other half is supported by the vertical forces of the ground on his feet. Assuming the person is moving

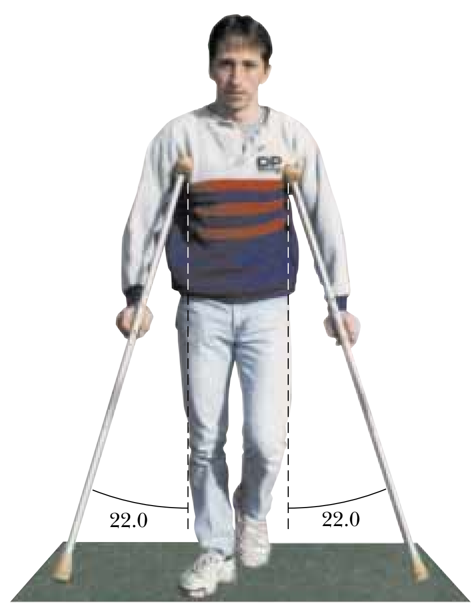


Figure P5.35

with constant velocity and the force exerted by the ground on the crutches acts along the crutches, determine (a) the smallest possible coefficient of friction between crutches and ground and (b) the magnitude of the compression force in each crutch.

36. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion. After it is in motion, a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.

37. A car is traveling at 50.0 mi/h on a horizontal highway. (a) If the coefficient of static friction between road and tires on a rainy day is 0.100, what is the minimum distance in which the car will stop? (b) What is the stopping distance when the surface is dry and $\mu_s = 0.600$?

38. Before 1960 it was believed that the maximum attainable coefficient of static friction for an automobile tire was less than 1. Then, about 1962, three companies independently developed racing tires with coefficients of 1.6. Since then, tires have improved, as illustrated in this problem. According to the 1990 Guinness Book of Records, the shortest time in which a piston-engine car initially at rest has covered a distance of one-quarter mile is 4.96 s. This record was set by Shirley Muldowney in September 1989. (a) Assume that, as in Figure P5.38, the rear wheels lifted the front wheels off the pavement. What minimum value of μ_s is necessary to achieve the record time? (b) Suppose Muldowney were able to double her engine power, keeping other things equal. How would this change affect the elapsed time?

Mike Powell/Allsport USA/Getty

Figure P5.38

39. To meet a U.S. Postal Service requirement, footwear must have a coefficient of static friction of 0.5 or more on a specified tile surface. A typical athletic shoe has a coefficient of 0.800. In an emergency, what is the minimum time interval in which a person starting from rest can move 3.00 m on a tile surface if she is wearing (a) footwear meeting the Postal Service minimum? (b) a typical athletic shoe?

40. A woman at an airport is towing her 20.0-kg suitcase at constant speed by pulling on a strap at an angle θ above the horizontal (Fig. P5.40). She pulls on the strap with a 35.0-N force, and the friction force on the suitcase is 20.0 N. Draw a free-body diagram of the suitcase. (a) What angle does the strap make with the horizontal? (b) What normal force does the ground exert on the suitcase?

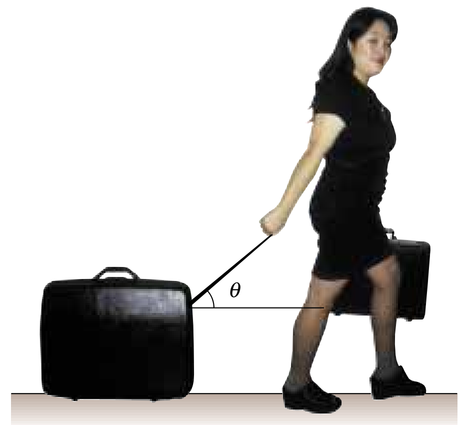


Figure P5.40

41. A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2.00 m.

42. A Chevrolet Corvette convertible can brake to a stop from a speed of 60.0 mi/h in a distance of 123 ft on a level roadway. What is its stopping distance on a roadway sloping downward at an angle of 10.0°?

43. A 9.00-kg hanging weight is connected by a string over a pulley to a 5.00-kg block that is sliding on a flat table (Fig. P5.24). If the coefficient of kinetic friction is 0.200, find the tension in the string.

44. Three objects are connected on the table as shown in Figure P5.44. The table is rough and has a coefficient of kinetic friction of 0.350. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. (a) Determine the acceleration of each object and their directions. (b) Determine the tensions in the two cords.

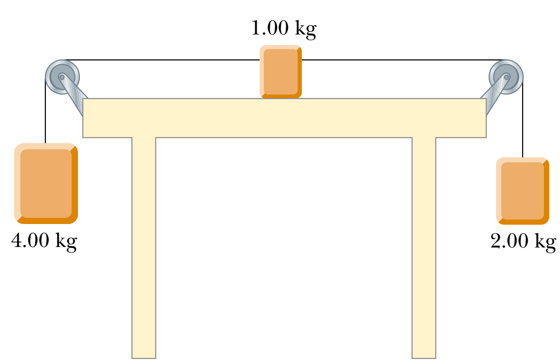


Figure P5.44

45. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force **F** (Fig. P5.45). Suppose that $F = 68.0$ N, $m_1 = 12.0$ kg, $m_2 = 18.0$ kg, and the coefficient of kinetic friction between each block and the surface is 0.100. (a) Draw a free-body diagram for each block.

(b) Determine the tension T and the magnitude of the acceleration of the system.

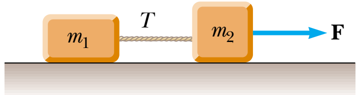


Figure P5.45

46. A block of mass 3.00 kg is pushed up against a wall by a force **P** that makes a 50.0° angle with the horizontal as shown in Figure P5.46. The coefficient of static friction between the block and the wall is 0.250. Determine the possible values for the magnitude of **P** that allow the block to remain stationary.

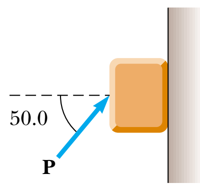


Figure P5.46

47. You and your friend go sledding. Out of curiosity, you measure the constant angle θ that the snow-covered slope makes with the horizontal. Next, you use the following method to determine the coefficient of friction μ_k between the snow and the sled. You give the sled a quick push up so that it will slide up the slope away from you. You wait for it to slide back down, timing the motion. It turns out that the sled takes twice as long to slide down as it does to reach the top point in the round trip. In terms of θ, what is the coefficient of friction?

48. The board sandwiched between two other boards in Figure P5.48 weighs 95.5 N. If the coefficient of friction between the boards is 0.663, what must be the magnitude of the compression forces (assume horizontal) acting on both sides of the center board to keep it from slipping?

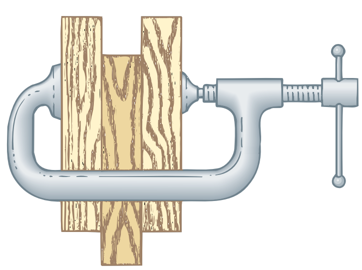


Figure P5.48

49. A block weighing 75.0 N rests on a plane inclined at 25.0° to the horizontal. A force F is applied to the object at 40.0° to the horizontal, pushing it upward on the plane. The coefficients of static and kinetic friction between the block and the plane are, respectively, 0.363 and 0.156. (a) What is the minimum value of F that will prevent the block from slipping down the plane? (b) What is the minimum value of F that will start the block moving up the plane? (c) What value of F will move the block up the plane with constant velocity?

50. **Review problem.** One side of the roof of a building slopes up at 37.0°. A student throws a Frisbee onto the roof. It strikes with a speed of 15.0 m/s and does not bounce, but slides straight up the incline. The coefficient of kinetic friction between the plastic and the roof is 0.400. The Frisbee slides 10.0 m up the roof to its peak, where it goes into free fall, following a parabolic trajectory with negligible air resistance. Determine the maximum height the Frisbee reaches above the point where it struck the roof.

Additional Problems

$\boxed{51.}$ An inventive child named Pat wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley (Fig. P5.51), Pat pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Pat's true weight is 320 N, and the chair weighs 160 N. (a) Draw free-body diagrams for Pat and the chair considered as separate systems, and another diagram for Pat and the chair considered as one system. (b) Show that the acceleration of the system is *upward* and find its magnitude. (c) Find the force Pat exerts on the chair.

Figure P5.51

52. A time-dependent force, $\mathbf{F} = (8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}})$ N, where t is in seconds, is exerted on a 2.00-kg object initially at rest. (a) At what time will the object be moving with a speed of 15.0 m/s? (b) How far is the object from its initial position when its speed is 15.0 m/s? (c) Through what total displacement has the object traveled at this time?

53. To prevent a box from sliding down an inclined plane, student A pushes on the box in the direction parallel to the incline, just hard enough to hold the box stationary. In an identical situation student B pushes on the box horizontally. Regard as known the mass m of the box, the coefficient of static friction μ_s between box and incline, and the inclination angle θ. (a) Determine the force A

has to exert. (b) Determine the force B has to exert. (c) If $m = 2.00$ kg, $\theta = 25.0°$, and $\mu_s = 0.160$, who has the easier job? (d) **What if** $\mu_s = 0.380$? Whose job is easier?

54. Three blocks are in contact with each other on a frictionless, horizontal surface, as in Figure P5.54. A horizontal force **F** is applied to m_1. Take $m_1 = 2.00$ kg, $m_2 = 3.00$ kg, $m_3 = 4.00$ kg, and $F = 18.0$ N. Draw a separate free-body diagram for each block and find (a) the acceleration of the blocks, (b) the *resultant* force on each block, and (c) the magnitudes of the contact forces between the blocks. (d) You are working on a construction project. A coworker is nailing up plasterboard on one side of a light partition, and you are on the opposite side, providing "backing" by leaning against the wall with your back pushing on it. Every blow makes your back sting. The supervisor helps you to put a heavy block of wood between the wall and your back. Using the situation analyzed in parts (a), (b), and (c) as a model, explain how this works to make your job more comfortable.

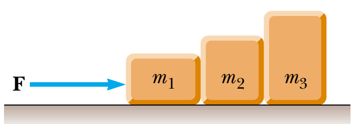


Figure P5.54

55. An object of mass M is held in place by an applied force **F** and a pulley system as shown in Figure P5.55. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5 and (b) the magnitude of **F**. *Suggestion:* Draw a free-body diagram for each pulley.

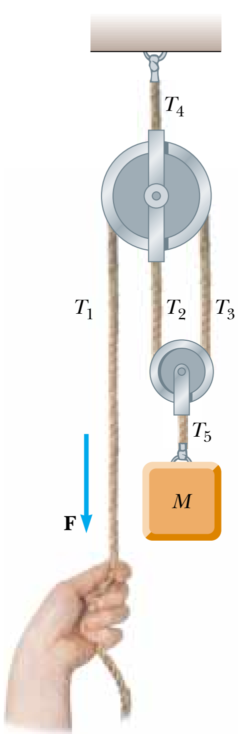


Figure P5.55

56. A high diver of mass 70.0 kg jumps off a board 10.0 m above the water. If his downward motion is stopped 2.00 s after he enters the water, what average upward force did the water exert on him?

57. A crate of weight F_g is pushed by a force **P** on a horizontal floor. (a) If the coefficient of static friction is μ_s and **P** is directed at angle θ below the horizontal, show that the minimum value of P that will move the crate is given by

$$P = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}$$

(b) Find the minimum value of P that can produce motion when $\mu_s = 0.400$, $F_g = 100$ N, and $\theta = 0°$, 15.0°, 30.0°, 45.0°, and 60.0°.

58. Review problem. A block of mass $m = 2.00$ kg is released from rest at $h = 0.500$ m above the surface of a table, at the top of a $\theta = 30.0°$ incline as shown in Figure P5.58. The frictionless incline is fixed on a table of height $H = 2.00$ m. (a) Determine the acceleration of the block as it slides down the incline. (b) What is the velocity of the block as it leaves the incline? (c) How far from the table will the block hit the floor? (d) How much time has elapsed between when the block is released and when it hits the floor? (e) Does the mass of the block affect any of the above calculations?

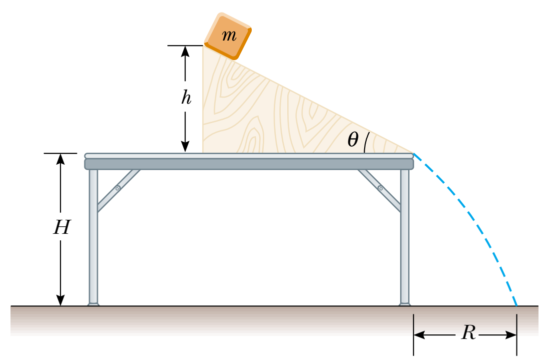


Figure P5.58 Problems 58 and 70.

59. A 1.30-kg toaster is not plugged in. The coefficient of static friction between the toaster and a horizontal countertop is 0.350. To make the toaster start moving, you carelessly pull on its electric cord. (a) For the cord tension to be as small as possible, you should pull at what angle above the horizontal? (b) With this angle, how large must the tension be?

60. Materials such as automobile tire rubber and shoe soles are tested for coefficients of static friction with an apparatus called a James tester. The pair of surfaces for which μ_s is to be measured are labeled B and C in Figure P5.60. Sample C is attached to a foot D at the lower end of a pivoting arm E, which makes angle θ with the vertical. The upper end of the arm is hinged at F to a vertical rod G, which slides freely in a guide H fixed to the frame of the apparatus and supports a load I of mass 36.4 kg. The hinge pin at F is also the axle of a wheel that can roll vertically on the frame. All of the moving parts have masses negligible in comparison to the 36.4-kg load. The pivots are nearly frictionless. The test surface B is attached to a rolling platform A. The operator slowly moves the platform to the left in the picture until the sample C suddenly slips over surface B. At the critical point where sliding motion is ready to begin, the operator notes the angle θ_s of the pivoting arm. (a) Make a free-body diagram of the pin at F. It is in equilibrium under three forces. These forces are the gravitational force on the load I, a horizontal normal force exerted by the frame, and a force of compression directed upward along the arm E. (b) Draw a free-body diagram of the foot D and sample C, considered as one system. (c) Determine the normal force that the test surface B exerts on the sample for any angle θ. (d) Show that $\mu_s = \tan \theta_s$. (e) The protractor on the tester can record angles as large as 50.2°. What is the greatest coefficient of friction it can measure?

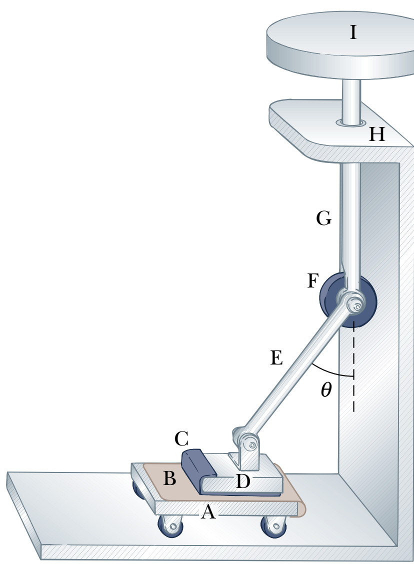


Figure P5.60

61. What horizontal force must be applied to the cart shown in Figure P5.61 in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. (*Hint:* Note that the force exerted by the string accelerates m_1.)

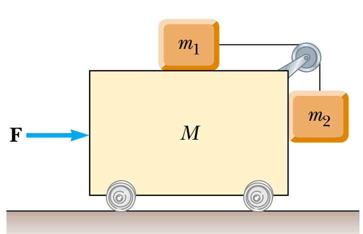


Figure P5.61 Problems 61 and 63.

62. A student is asked to measure the acceleration of a cart on a "frictionless" inclined plane as in Figure 5.11, using an air track, a stopwatch, and a meter stick. The height of the incline is measured to be 1.774 cm, and the total length of the incline is measured to be $d = 127.1$ cm. Hence, the angle of inclination θ is determined from the relation $\sin \theta = 1.774/127.1$. The cart is released from rest at the top of the incline, and its position x along the incline is measured as a function of time, where $x = 0$ refers to the initial position of the cart. For x values of 10.0 cm, 20.0 cm, 35.0 cm, 50.0 cm, 75.0 cm, and 100 cm, the measured times at which these positions are reached (averaged over five runs) are 1.02 s, 1.53 s, 2.01 s, 2.64 s, 3.30 s, and 3.75 s, respectively. Construct a graph of x versus t^2, and perform a linear least-squares fit to the data. Determine the acceleration of the cart from the slope of this graph, and compare it with the value you would get using $a' = g \sin \theta$, where $g = 9.80\ \text{m/s}^2$.

63. Initially the system of objects shown in Figure P5.61 is held motionless. All surfaces, pulley, and wheels are frictionless. Let the force **F** be zero and assume that m_2 can move only vertically. At the instant after the system of objects is released, find (a) the tension T in the string, (b) the acceleration of m_2, (c) the acceleration of M, and (d) the acceleration of m_1. (*Note:* The pulley accelerates along with the cart.)

64. One block of mass 5.00 kg sits on top of a second rectangular block of mass 15.0 kg, which in turn is on a horizontal table. The coefficients of friction between the two blocks are $\mu_s = 0.300$ and $\mu_k = 0.100$. The coefficients of friction between the lower block and the rough table are $\mu_s = 0.500$ and $\mu_k = 0.400$. You apply a constant horizontal force to the lower block, just large enough to make this block start sliding out from between the upper block and the table. (a) Draw a free-body diagram of each block, naming the forces on each. (b) Determine the magnitude of each force on each block at the instant when you have started pushing but motion has not yet started. In particular, what force must you apply? (c) Determine the acceleration you measure for each block.

65. A 1.00-kg glider on a horizontal air track is pulled by a string at an angle θ. The taut string runs over a pulley and is attached to a hanging object of mass 0.500 kg as in Fig. P5.65. (a) Show that the speed v_x of the glider and the

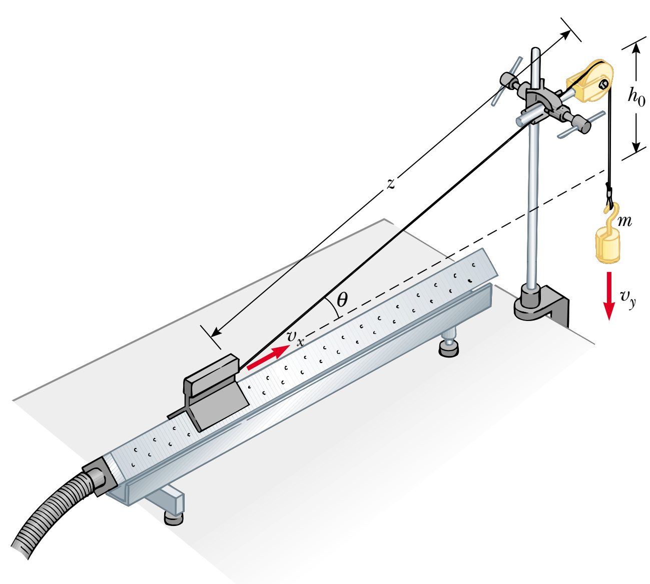


Figure P5.65

speed v_y of the hanging object are related by $v_x = uv_y$, where $u = z(z^2 - h_0^2)^{-1/2}$. (b) The glider is released from rest. Show that at that instant the acceleration a_x of the glider and the acceleration a_y of the hanging object are related by $a_x = ua_y$. (c) Find the tension in the string at the instant the glider is released for $h_0 = 80.0$ cm and $\theta = 30.0°$.

66. Cam mechanisms are used in many machines. For example, cams open and close the valves in your car engine to admit gasoline vapor to each cylinder and to allow the escape of exhaust. The principle is illustrated in Figure P5.66, showing a follower rod (also called a pushrod) of mass m resting on a wedge of mass M. The sliding wedge duplicates the function of a rotating eccentric disk on a camshaft in your car. Assume that there is no friction between the wedge and the base, between the pushrod and the wedge, or between the rod and the guide through which it slides. When the wedge is pushed to the left by the force F, the rod moves upward and does something, such as opening a valve. By varying the shape of the wedge, the motion of the follower rod could be made quite complex, but assume that the wedge makes a constant angle of $\theta = 15.0°$. Suppose you want the wedge and the rod to start from rest and move with constant acceleration, with the rod moving upward 1.00 mm in 8.00 ms. Take $m = 0.250$ kg and $M = 0.500$ kg. What force F must be applied to the wedge?

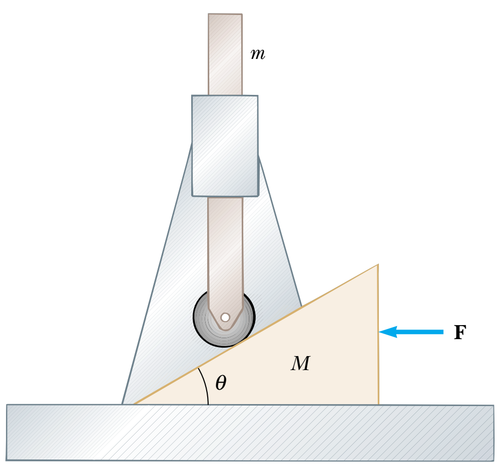


Figure P5.66

67. Any device that allows you to increase the force you exert is a kind of *machine*. Some machines, such as the prybar or the inclined plane, are very simple. Some machines do not even look like machines. An example is the following: Your car is stuck in the mud, and you can't pull hard enough to get it out. However, you have a long cable which you connect taut between your front bumper and the trunk of a stout tree. You now pull sideways on the cable at its midpoint, exerting a force f. Each half of the cable is displaced through a small angle θ from the straight line between the ends of the cable. (a) Deduce an expression for the force exerted on the car. (b) Evaluate the cable tension for the case where $\theta = 7.00°$ and $f = 100$ N.

68. Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley (Fig. P5.68). The inclines are frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.

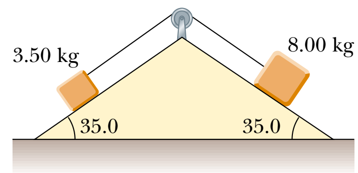


Figure P5.68

69. A van accelerates down a hill (Fig. P5.69), going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy (m = 0.100 kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle θ and (b) the tension in the string.

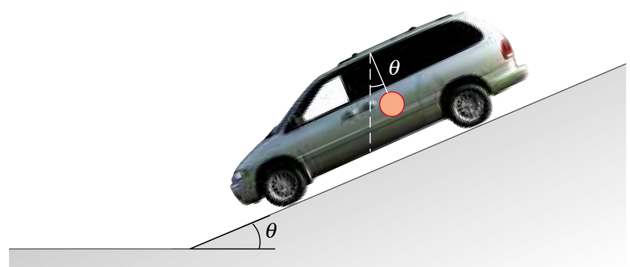


Figure P5.69

70. In Figure P5.58 the incline has mass M and is fastened to the stationary horizontal tabletop. The block of mass m is placed near the bottom of the incline and is released with a quick push that sets it sliding upward. It stops near the top of the incline, as shown in the figure, and then slides down again, always without friction. Find the force that the tabletop exerts on the incline throughout this motion.

71. A magician pulls a tablecloth from under a 200-g mug located 30.0 cm from the edge of the cloth. The cloth exerts a friction force of 0.100 N on the mug, and the cloth is pulled with a constant acceleration of 3.00 m/s^2. How far does the mug move relative to the horizontal tabletop before the cloth is completely out from under it? Note that the cloth must move more than 30 cm relative to the tabletop during the process.

72. An 8.40-kg object slides down a fixed, frictionless inclined plane. Use a computer to determine and tabulate the normal force exerted on the object and its acceleration for a series of incline angles (measured from the horizontal) ranging from 0° to 90° in 5° increments. Plot a graph of the normal force and the acceleration as functions of the incline angle. In the limiting cases of 0° and 90°, are your results consistent with the known behavior?

73. A mobile is formed by supporting four metal butterflies of equal mass m from a string of length L. The points of support are evenly spaced a distance ℓ apart as shown in Figure P5.73. The string forms an angle θ_1 with the ceiling at each end point. The center section of string is horizontal. (a) Find the tension in each section of string in terms of θ_1, m, and g. (b) Find the angle θ_2, in terms of θ_1, that the sections of string between the outside butterflies and the inside butterflies form with the horizontal. (c) Show that the distance D between the end points of the string is

$$D = \frac{L}{5}(2 \cos \theta_1 + 2 \cos [\tan^{-1}(\tfrac{1}{2} \tan \theta_1)] + 1)$$

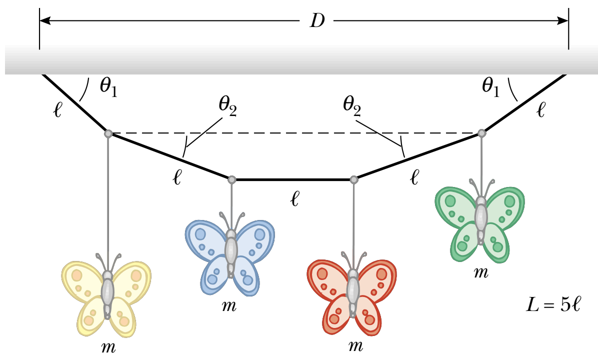


Figure P5.73

Answers to Quick Quizzes

5.1 (d). Choice (a) is true. Newton's first law tells us that motion requires no force: an object in motion continues to move at constant velocity in the absence of external forces. Choice (b) is also true. A stationary object can have several forces acting on it, but if the vector sum of all these external forces is zero, there is no net force and the object remains stationary.

5.2 (a). If a single force acts, this force constitutes the net force and there is an acceleration according to Newton's second law.

5.3 (c). Newton's second law relates only the force and the acceleration. Direction of motion is part of an object's *velocity,* and force determines the direction of acceleration, not that of velocity.

5.4 (d). With twice the force, the object will experience twice the acceleration. Because the force is constant, the acceleration is constant, and the speed of the object (starting from rest) is given by $v = at$. With twice the acceleration, the object will arrive at speed v at half the time.

5.5 (a). The gravitational force acts on the ball at *all* points in its trajectory.

5.6 (b). Because the value of g is smaller on the Moon than on the Earth, more mass of gold would be required to represent 1 newton of weight on the Moon. Thus, your friend on the Moon is richer, by about a factor of 6!

5.7 (c). In accordance with Newton's third law, the fly and bus experience forces that are equal in magnitude but opposite in direction.

5.8 (a). Because the fly has such a small mass, Newton's second law tells us that it undergoes a very large acceleration. The huge mass of the bus means that it more effectively resists any change in its motion and exhibits a small acceleration.

5.9 (c). The reaction force to your weight is an upward gravitational force on the Earth due to you.

5.10 (b). Remember the phrase "free-body." You draw *one* body (one object), free of all the others that may be interacting, and draw only the forces exerted on that object.

5.11 (b). The friction force acts opposite to the gravitational force on the book to keep the book in equilibrium. Because the gravitational force is downward, the friction force must be upward.

5.12 (b). The crate accelerates to the east. Because the only horizontal force acting on it is the force of static friction between its bottom surface and the truck bed, that force must also be directed to the east.

5.13 (b). At the angle at which the book breaks free, the component of the gravitational force parallel to the board is approximately equal to the maximum static friction force. Because the kinetic coefficient of friction is smaller than the static coefficient, at this angle, the component of the gravitational force parallel to the board is larger than the kinetic friction force. Thus, there is a net downhill force parallel to the board and the book speeds up.

5.14 (b). When pulling with the rope, there is a component of your applied force that is upward. This reduces the normal force between the sled and the snow. In turn, this reduces the friction force between the sled and the snow, making it easier to move. If you push from behind, with a force with a downward component, the normal force is larger, the friction force is larger, and the sled is harder to move.

Chapter 6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

▲ *The London Eye, a ride on the River Thames in downtown London. Riders travel in a large vertical circle for a breathtaking view of the city. In this chapter, we will study the forces involved in circular motion. (© Paul Hardy/CORBIS)*

In the preceding chapter we introduced Newton's laws of motion and applied them to situations involving linear motion. Now we discuss motion that is slightly more complicated. For example, we shall apply Newton's laws to objects traveling in circular paths. Also, we shall discuss motion observed from an accelerating frame of reference and motion of an object through a viscous medium. For the most part, this chapter consists of a series of examples selected to illustrate the application of Newton's laws to a wide variety of circumstances.

6.1 Newton's Second Law Applied to Uniform Circular Motion

In Section 4.4 we found that a particle moving with uniform speed v in a circular path of radius r experiences an acceleration that has a magnitude

$$a_c = \frac{v^2}{r}$$

The acceleration is called *centripetal acceleration* because $\mathbf{a}_c$ is directed toward the center of the circle. Furthermore, $\mathbf{a}_c$ is *always* perpendicular to $\mathbf{v}$. (If there were a component of acceleration parallel to $\mathbf{v}$, the particle's speed would be changing.)

Consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path, as illustrated in Figure 6.1. Its weight is supported by a frictionless table. Why does the ball move in a circle? According to Newton's first law, the ball tends to move in a straight line; however, the string prevents

Mike Powell / Allsport / Getty Images

An athlete in the process of throwing the hammer at the 1996 Olympic Games in Atlanta, Georgia. The force exerted by the chain causes the centripetal acceleration of the hammer. Only when the athlete releases the hammer will it move along a straight-line path tangent to the circle.

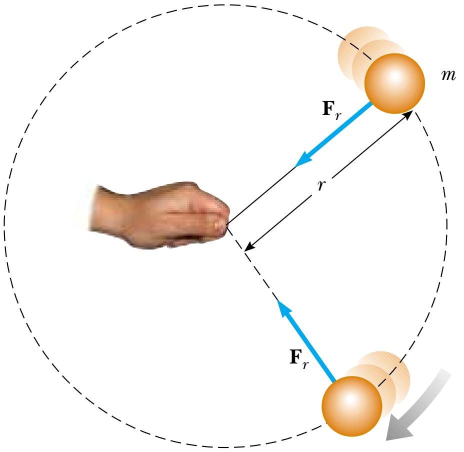


Figure 6.1 Overhead view of a ball moving in a circular path in a horizontal plane. A force $\mathbf{F}_r$ directed toward the center of the circle keeps the ball moving in its circular path.

motion along a straight line by exerting on the ball a radial force $\mathbf{F}_r$ that makes it follow the circular path. This force is directed along the string toward the center of the circle, as shown in Figure 6.1.

If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

Force causing centripetal acceleration

$$\sum F = ma_c = m\frac{v^2}{r} \tag{6.1}$$

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle. This idea is illustrated in Figure 6.2 for the ball whirling at the end of a string in a horizontal plane. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string breaks.

Figure 6.3 (Quick Quiz 6.1 and 6.2) A Ferris wheel located on the Navy Pier in Chicago, Illinois.

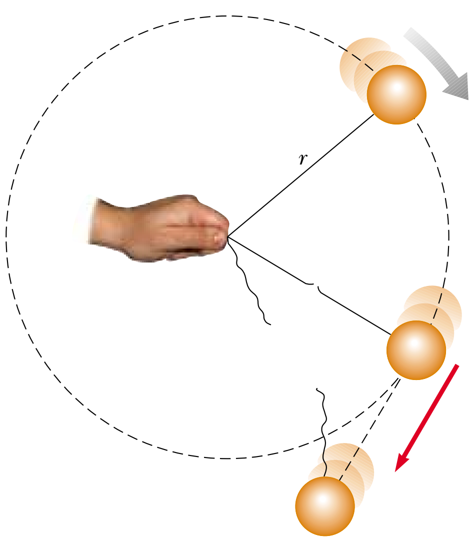
Quick Quiz 6.1 You are riding on a Ferris wheel (Fig. 6.3) that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation—it does not invert. What is the direction of your centripetal acceleration when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of your centripetal acceleration when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

Quick Quiz 6.2 You are riding on the Ferris wheel of Quick Quiz 6.1. What is the direction of the normal force exerted by the seat on you when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of the normal force exerted by the seat on you when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

PITFALL PREVENTION

6.1 Direction of Travel When the String is Cut

Study Figure 6.2 very carefully. Many students (wrongly) think that the ball will move *radially* away from the center of the circle when the string is cut. The velocity of the ball is *tangent* to the circle. By Newton's first law, the ball continues to move in the direction that it is moving just as the force from the string disappears.



Active Figure 6.2 An overhead view of a ball moving in a circular path in a horizontal plane. When the string breaks, the ball moves in the direction tangent to the circle.

At the Active Figures link at http://www.pse6.com, you can "break" the string yourself and observe the effect on the ball's motion.

Conceptual Example 6.1 Forces That Cause Centripetal Acceleration

The force causing centripetal acceleration is sometimes called a *centripetal force.* We are familiar with a variety of forces in nature—friction, gravity, normal forces, tension, and so forth. Should we add *centripetal* force to this list?

Solution No; centripetal force *should not* be added to this list. This is a pitfall for many students. Giving the force causing circular motion a name—centripetal force—leads many students to consider it as a new *kind* of force rather than a new *role* for force. A common mistake in force diagrams is to draw all the usual forces and then to add another vector for the centripetal force. But it is not a separate force—it is simply one or more of our familiar forces *acting in the role of a force that causes a circular motion.*

Consider some examples. For the motion of the Earth around the Sun, the centripetal force is *gravity.* For an object sitting on a rotating turntable, the centripetal force is *friction.* For a rock whirled horizontally on the end of a string, the magnitude of the centripetal force is the *tension* in the string. For an amusement-park patron pressed against the inner wall of a rapidly rotating circular room, the centripetal force is the *normal force* exerted by the wall. Furthermore, the centripetal force could be a combination of two or more forces. For example, as you pass through the lowest point of the Ferris wheel in Quick Quiz 6.1, the centripetal force on you is the difference between the normal force exerted by the seat and the gravitational force. We will not use the term *centripetal force* in this book after this discussion.

Example 6.2 The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure 6.4. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum.*) Find an expression for v.

Solution Conceptualize the problem with the help of Figure 6.4. We categorize this as a problem that combines equilibrium for the ball in the vertical direction with uniform circular motion in the horizontal direction. To analyze the problem, begin by letting θ represent the angle between the string and the vertical. In the free-body diagram shown, the force $\mathbf{T}$ exerted by the string is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of revolution. Because the object does not accelerate in the vertical direction, $\sum F_y = ma_y = 0$ and the upward vertical component of $\mathbf{T}$ must balance the downward gravitational force. Therefore,

$$(1) \qquad T\cos\theta = mg$$

Because the force providing the centripetal acceleration in this example is the component $T\sin\theta$, we can use Equation 6.1 to obtain

$$(2) \qquad \sum F = T\sin\theta = ma_c = \frac{mv^2}{r}$$

Dividing (2) by (1) and using $\sin\theta/\cos\theta = \tan\theta$, we eliminate T and find that

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

From the geometry in Figure 6.4, we see that $r = L\sin\theta$; therefore,

$$v = \boxed{\sqrt{Lg\sin\theta\tan\theta}}$$

Note that the speed is independent of the mass of the object.

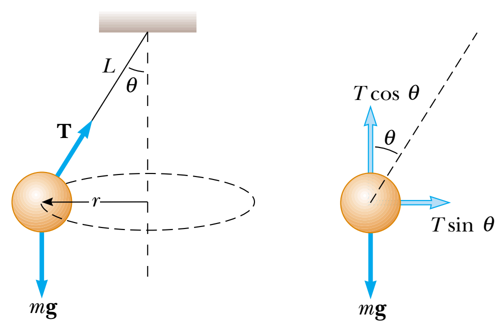


Figure 6.4 (Example 6.2) The conical pendulum and its free-body diagram.

Example 6.3 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during the motion.

Solution It makes sense that the stronger the cord, the faster the ball can twirl before the cord breaks. Also, we expect a more massive ball to break the cord at a lower speed. (Imagine whirling a bowling ball on the cord!)

Because the force causing the centripetal acceleration in this case is the force $\mathbf{T}$ exerted by the cord on the ball, Equation 6.1 yields

$$(1) \qquad T = m\frac{v^2}{r}$$

Solving for v, we have

$$v = \sqrt{\frac{Tr}{m}}$$

This shows that v increases with T and decreases with larger m, as we expect to see—for a given v, a large mass requires a large tension and a small mass needs only a small tension. The maximum speed the ball can have corresponds to the maximum tension. Hence, we find

$$v_{max} = \sqrt{\frac{T_{max}r}{m}} = \sqrt{\frac{(50.0 \text{ N})(1.50 \text{ m})}{0.500 \text{ kg}}} = \boxed{12.2 \text{ m/s}}$$

What If? **Suppose that the ball is whirled in a circle of larger radius at the same speed *v*. Is the cord more likely to break or less likely?**

Answer The larger radius means that the change in the direction of the velocity vector will be smaller for a given time interval. Thus, the acceleration is smaller and the required force from the string is smaller. As a result, the string is less likely to break when the ball travels in a circle of larger radius. To understand this argument better, let us write Equation (1) twice, once for each radius:

$$T_1 = \frac{mv^2}{r_1} \qquad T_2 = \frac{mv^2}{r_2}$$

Dividing the two equations gives us,

$$\frac{T_2}{T_1} = \frac{\left(\dfrac{mv^2}{r_2}\right)}{\left(\dfrac{mv^2}{r_1}\right)} = \frac{r_1}{r_2}$$

If we choose $r_2 > r_1$, we see that $T_2 < T_1$. Thus, less tension is required to whirl the ball in the larger circle and the string is less likely to break.

Example 6.4 What Is the Maximum Speed of the Car? Interactive

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure 6.5. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

Solution In this case, the force that enables the car to remain in its circular path is the force of static friction. (*Static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the road.) Hence, from Equation 6.1 we have

$$(1) \qquad f_s = m\frac{v^2}{r}$$

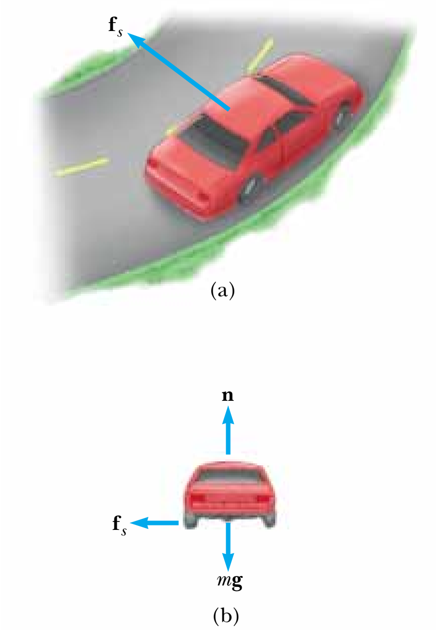


Figure 6.5 (Example 6.4) (a) The force of static friction directed toward the center of the curve keeps the car moving in a circular path. (b) The free-body diagram for the car.

The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\,max} = \mu_s n$. Because the car shown in Figure 6.5b is in equilibrium in the vertical direction, the magnitude of the normal force equals the weight ($n = mg$) and thus $f_{s,\,max} = \mu_s mg$. Substituting this value for f_s into (1), we find that the maximum speed is

$$(2) \qquad v_{max} = \sqrt{\frac{f_{s,\,max}\, r}{m}} = \sqrt{\frac{\mu_s mgr}{m}} = \sqrt{\mu_s gr}$$
$$= \sqrt{(0.500)(9.80 \text{ m/s}^2)(35.0 \text{ m})}$$
$$= \boxed{13.1 \text{ m/s}}$$

Note that the maximum speed does not depend on the mass of the car. That is why curved highways do not need multiple speed limit signs to cover the various masses of vehicles using the road.

What If? **Suppose that a car travels this curve on a wet day and begins to skid on the curve when its speed reaches only 8.00 m/s. What can we say about the coefficient of static friction in this case?**

Answer The coefficient of friction between tires and a wet road should be smaller than that between tires and a dry road. This expectation is consistent with experience with driving, because a skid is more likely on a wet road than a dry road.

To check our suspicion, we can solve (2) for the coefficient of friction:

$$\mu_s = \frac{v_{max}^2}{gr}$$

Substituting the numerical values,

$$\mu_s = \frac{v_{max}^2}{gr} = \frac{(8.00 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 0.187$$

This is indeed smaller than the coefficient of 0.500 for the dry road.

Study the relationship between the car's speed, radius of the turn, and the coefficient of static friction between road and tires at the Interactive Worked Example link at **http://www.pse6.com.**

Example 6.5 The Banked Exit Ramp **Interactive**

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 50.0 m. At what angle should the curve be banked?

Solution On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between car and road, as we saw in the previous example. However, if the road is banked at an angle θ, as in Figure 6.6, the normal force $\mathbf{n}$ has a horizontal component $n \sin\theta$ pointing toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $n_x = n \sin\theta$ causes the centripetal acceleration. Hence, Newton's second law for the radial direction gives

$$(1) \qquad \sum F_r = n \sin\theta = \frac{mv^2}{r}$$

The car is in equilibrium in the vertical direction. Thus, from $\Sigma F_y = 0$ we have

$$(2) \qquad n \cos\theta = mg$$

Dividing (1) by (2) gives

$$(3) \qquad \tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{(13.4\ \text{m/s})^2}{(50.0\ \text{m})(9.80\ \text{m/s}^2)}\right) = \boxed{20.1°}$$

If a car rounds the curve at a speed less than 13.4 m/s, friction is needed to keep it from sliding down the bank (to the left in Fig. 6.6). A driver who attempts to negotiate the curve at a speed greater than 13.4 m/s has to depend on friction to keep from sliding up the bank (to the right in Fig. 6.6). The banking angle is independent of the mass of the vehicle negotiating the curve.

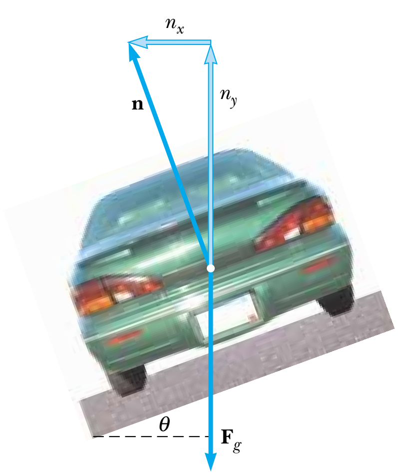


Figure 6.6 (Example 6.5) A car rounding a curve on a road banked at an angle θ to the horizontal. When friction is neglected, the force that causes the centripetal acceleration and keeps the car moving in its circular path is the horizontal component of the normal force.

What If? **What if this same roadway were built on Mars in the future to connect different colony centers; could it be traveled at the same speed?**

Answer The reduced gravitational force on Mars would mean that the car is not pressed so tightly to the roadway. The reduced normal force results in a smaller component of the normal force toward the center of the circle. This smaller component will not be sufficient to provide the centripetal acceleration associated with the original speed. The centripetal acceleration must be reduced, which can be done by reducing the speed v.

Equation (3) shows that the speed v is proportional to the square root of g for a roadway of fixed radius r banked at a fixed angle θ. Thus, if g is smaller, as it is on Mars, the speed v with which the roadway can be safely traveled is also smaller.

You can adjust the turn radius and banking angle at the Interactive Worked Example link at **http://www.pse6.com.**

Example 6.6 Let's Go Loop-the-Loop!

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in Figure 6.7a. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot **(A)** at the bottom of the loop and **(B)** at the top of the loop. Express your answers in terms of the weight of the pilot mg.

Solution To conceptualize this problem, look carefully at Figure 6.7. Based on experiences with driving over small hills in a roadway, or riding over the top of a Ferris wheel, you would expect to feel lighter at the top of the path. Similarly, you would expect to feel heavier at the bottom of the path. By looking at Figure 6.7, we expect the answer for (A) to be greater than that for (B) because at the bottom of the loop the normal and gravitational forces act in *opposite* directions, whereas at the top of the loop these two forces act in the *same* direction. The vector sum of these two forces gives the force of constant magnitude that keeps the pilot moving in a circular path at a constant speed. To yield net

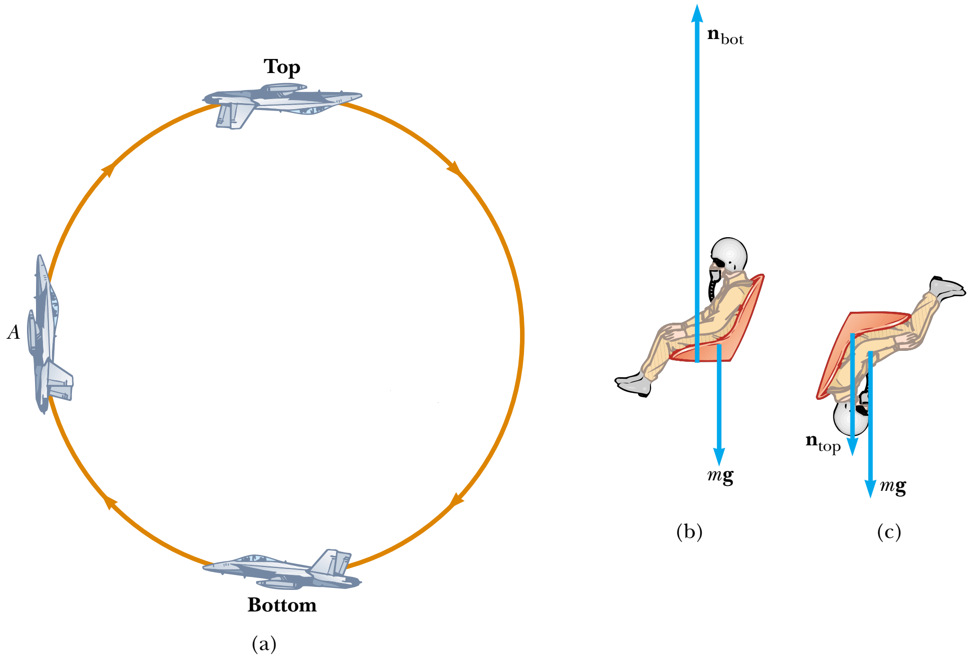


Figure 6.7 (Example 6.6) (a) An aircraft executes a loop-the-loop maneuver as it moves in a vertical circle at constant speed. (b) Free-body diagram for the pilot at the bottom of the loop. In this position the pilot experiences an apparent weight greater than his true weight. (c) Free-body diagram for the pilot at the top of the loop.

force vectors with the same magnitude, the normal force at the bottom must be greater than that at the top. Because the speed of the aircraft is constant (how likely is this?), we can categorize this as a uniform circular motion problem, complicated by the fact that the gravitational force acts at all times on the aircraft.

(A) Analyze the situation by drawing a free-body diagram for the pilot at the bottom of the loop, as shown in Figure 6.7b. The only forces acting on him are the downward gravitational force $\mathbf{F}_g = m\mathbf{g}$ and the upward force $\mathbf{n}_{bot}$ exerted by the seat. Because the net upward force that provides the centripetal acceleration has a magnitude $n_{bot} - mg$, Newton's second law for the radial direction gives

$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

Substituting the values given for the speed and radius gives

$$n_{bot} = mg\left(1 + \frac{(225 \text{ m/s})^2}{(2.70 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)}\right) = \boxed{2.91mg}$$

Hence, the magnitude of the force $\mathbf{n}_{bot}$ exerted by the seat on the pilot is *greater* than the weight of the pilot by a factor of 2.91. This means that the pilot experiences an apparent weight that is greater than his true weight by a factor of 2.91.

(B) The free-body diagram for the pilot at the top of the loop is shown in Figure 6.7c. As we noted earlier, both the gravitational force exerted by the Earth and the force $\mathbf{n}_{top}$ exerted by the seat on the pilot act downward, and so the net downward force that provides the centripetal acceleration has a magnitude $n_{top} + mg$. Applying Newton's second law yields

$$\sum F = n_{top} + mg = m\frac{v^2}{r}$$

$$n_{top} = m\frac{v^2}{r} - mg = mg\left(\frac{v^2}{rg} - 1\right)$$

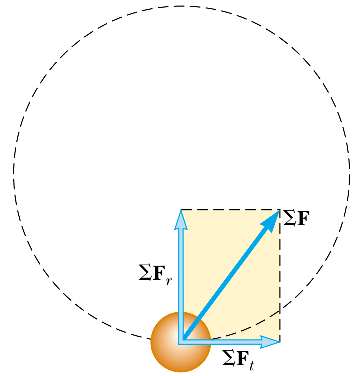
$$n_{top} = mg\left(\frac{(225 \text{ m/s})^2}{(2.70 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} - 1\right)$$

$$= \boxed{0.913mg}$$

In this case, the magnitude of the force exerted by the seat on the pilot is *less* than his true weight by a factor of 0.913, and the pilot feels lighter. To finalize the problem, note that this is consistent with our prediction at the beginning of the solution.

6.2 Nonuniform Circular Motion

In Chapter 4 we found that if a particle moves with varying speed in a circular path, there is, in addition to the radial component of acceleration, a tangential component having magnitude dv/dt. Therefore, the force acting on the particle must also have a tangential and a radial component. Because the total acceleration is $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the total force exerted on the particle is $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$, as shown in Figure 6.8. The vector $\Sigma\mathbf{F}_r$ is directed toward the center of the circle and is responsible for the centripetal acceleration. The vector $\Sigma\mathbf{F}_t$ tangent to the circle is responsible for the tangential acceleration, which represents a change in the speed of the particle with time.



Active Figure 6.8 When the force acting on a particle moving in a circular path has a tangential component ΣF_t, the particle's speed changes. The total force exerted on the particle in this case is the vector sum of the radial force and the tangential force. That is, $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$.

At the Active Figures link at http://www.pse6.com, you can adjust the initial position of the particle and compare the component forces acting on the particle to those for a child swinging on a swing set.

Quick Quiz 6.3 Which of the following is *impossible* for a car moving in a circular path? (a) the car has tangential acceleration but no centripetal acceleration. (b) the car has centripetal acceleration but no tangential acceleration. (c) the car has both centripetal acceleration and tangential acceleration.

Quick Quiz 6.4 A bead slides freely along a *horizontal*, curved wire at constant speed, as shown in Figure 6.9. Draw the vectors representing the force exerted by the wire on the bead at points Ⓐ, Ⓑ, and Ⓒ.

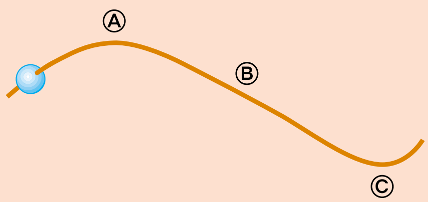


Figure 6.9 (Quick Quiz 6.4 and 6.5) A bead slides along a curved wire.

Quick Quiz 6.5 In Figure 6.9, the bead speeds up with constant tangential acceleration as it moves toward the right. Draw the vectors representing the force on the bead at points Ⓐ, Ⓑ, and Ⓒ.

Robin Smith / Getty Images

Passengers on a "corkscrew" roller coaster experience a radial force toward the center of the circular track and a tangential force due to gravity.

Chapter 7

Energy and Energy Transfer

▲ *On a wind farm, the moving air does work on the blades of the windmills, causing the blades and the rotor of an electrical generator to rotate. Energy is transferred out of the system of the windmill by means of electricity. (Billy Hustace/Getty Images)*

CHAPTER OUTLINE

A valid system may

- be a single object or particle
- be a collection of objects or particles
- be a region of space (such as the interior of an automobile engine combustion cylinder)
- vary in size and shape (such as a rubber ball, which deforms upon striking a wall)

Identifying the *need* for a system approach to solving a problem (as opposed to a particle approach) is part of the "categorize" step in the General Problem-Solving Strategy outlined in Chapter 2. Identifying the particular system and its nature is part of the "analyze" step.

No matter what the particular system is in a given problem, there is a **system boundary,** an imaginary surface (not necessarily coinciding with a physical surface) that divides the Universe into the system and the **environment** surrounding the system.

As an example, imagine a force applied to an object in empty space. We can define the object as the system. The force applied to it is an influence on the system from the environment that acts across the system boundary. We will see how to analyze this situation from a system approach in a subsequent section of this chapter.

Another example is seen in Example 5.10 (page 130). Here the system can be defined as the combination of the ball, the cube, and the string. The influence from the environment includes the gravitational forces on the ball and the cube, the normal and friction forces on the cube, and the force exerted by the pulley on the string. The forces exerted by the string on the ball and the cube are internal to the system and, therefore, are not included as an influence from the environment.

We shall find that there are a number of mechanisms by which a system can be influenced by its environment. The first of these that we shall investigate is *work.*

PITFALL PREVENTION

7.1 Identify the System

The most important step to take in solving a problem using the energy approach is to identify the appropriate system of interest. Make sure this is the *first* step you take in solving a problem.

7.2 Work Done by a Constant Force

Almost all the terms we have used thus far—velocity, acceleration, force, and so on—convey a similar meaning in physics as they do in everyday life. Now, however, we encounter a term whose meaning in physics is distinctly different from its everyday meaning—*work.*

To understand what work means to the physicist, consider the situation illustrated in Figure 7.1. A force is applied to a chalkboard eraser, and the eraser slides

Charles D. Winters

(a) (b) (c)

Figure 7.1 An eraser being pushed along a chalkboard tray.

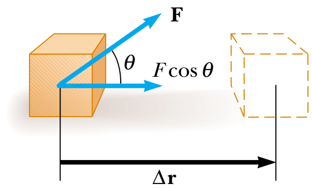


Figure 7.2 If an object undergoes a displacement $\Delta\mathbf{r}$ under the action of a constant force $\mathbf{F}$, the work done by the force is $F\Delta r\cos\theta$.

along the tray. If we want to know how effective the force is in moving the eraser, we must consider not only the magnitude of the force but also its direction. Assuming that the magnitude of the applied force is the same in all three photographs, the push applied in Figure 7.1b does more to move the eraser than the push in Figure 7.1a. On the other hand, Figure 7.1c shows a situation in which the applied force does not move the eraser at all, regardless of how hard it is pushed. (Unless, of course, we apply a force so great that we break the chalkboard tray.) So, in analyzing forces to determine the work they do, we must consider the vector nature of forces. We must also know how far the eraser moves along the tray if we want to determine the work associated with that displacement. Moving the eraser 3 m requires more work than moving it 2 cm.

Let us examine the situation in Figure 7.2, where an object undergoes a displacement along a straight line while acted on by a constant force $\mathbf{F}$ that makes an angle θ with the direction of the displacement.

Work done by a constant force

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

$$W \equiv F\,\Delta r\cos\theta \tag{7.1}$$

As an example of the distinction between this definition of work and our everyday understanding of the word, consider holding a heavy chair at arm's length for 3 min. At the end of this time interval, your tired arms may lead you to think that you have done a considerable amount of work on the chair. According to our definition, however, you have done no work on it whatsoever.[1] You exert a force to support the chair, but you do not move it. A force does no work on an object if the force does not move through a displacement. This can be seen by noting that if $\Delta r = 0$, Equation 7.1 gives $W = 0$—the situation depicted in Figure 7.1c.

Also note from Equation 7.1 that the work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of

PITFALL PREVENTION

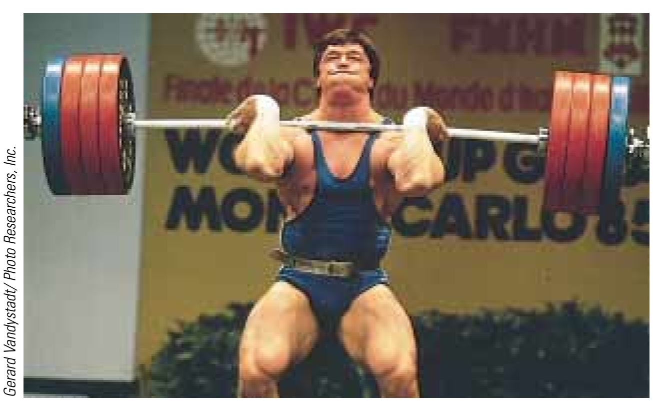
7.2 What is being Displaced?

The displacement in Equation 7.1 is that of *the point of application of the force.* If the force is applied to a particle or a non-deformable, non-rotating system, this displacement is the same as the displacement of the particle or system. For deformable systems, however, these two displacements are often not the same.

PITFALL PREVENTION

7.3 Work is Done by . . . on . . .

Not only must you identify the system, you must also identify the interaction of the system with the environment. When discussing work, always use the phrase, "the work done by . . . on . . . " After "by," insert the part of the environment that is interacting directly with the system. After "on," insert the system. For example, "the work done by the hammer on the nail" identifies the nail as the system and the force from the hammer represents the interaction with the environment. This is similar to our use in Chapter 5 of "the force exerted by . . . on . . ."



Gerard Vandystadt/Photo Researchers, Inc.

The weightlifter does no work on the weights as he holds them on his shoulders. (If he could rest the bar on his shoulders and lock his knees, he would be able to support the weights for quite some time.) Did he do any work when he raised the weights to this height?

[1] Actually, you do work while holding the chair at arm's length because your muscles are continuously contracting and relaxing; this means that they are exerting internal forces on your arm. Thus, work is being done by your body—but internally on itself rather than on the chair.

application. That is, if $\theta = 90°$, then $W = 0$ because $\cos 90° = 0$. For example, in Figure 7.3, the work done by the normal force on the object and the work done by the gravitational force on the object are both zero because both forces are perpendicular to the displacement and have zero components along an axis in the direction of $\Delta\mathbf{r}$.

The sign of the work also depends on the direction of $\mathbf{F}$ relative to $\Delta\mathbf{r}$. The work done by the applied force is positive when the projection of $\mathbf{F}$ onto $\Delta\mathbf{r}$ is in the same direction as the displacement. For example, when an object is lifted, the work done by the applied force is positive because the direction of that force is upward, in the same direction as the displacement of its point of application. When the projection of $\mathbf{F}$ onto $\Delta\mathbf{r}$ is in the direction opposite the displacement, W is negative. For example, as an object is lifted, the work done by the gravitational force on the object is negative. The factor $\cos\theta$ in the definition of W (Eq. 7.1) automatically takes care of the sign.

If an applied force $\mathbf{F}$ is in the same direction as the displacement $\Delta\mathbf{r}$, then $\theta = 0$ and $\cos 0 = 1$. In this case, Equation 7.1 gives

$$W = F\Delta r$$

Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the **newton · meter** (N · m). This combination of units is used so frequently that it has been given a name of its own: the **joule** (J).

An important consideration for a system approach to problems is to note that **work is an energy transfer.** If W is the work done on a system and W is positive, energy is transferred *to* the system; if W is negative, energy is transferred *from* the system. Thus, if a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary. This will result in a change in the energy stored in the system. We will learn about the first type of energy storage in Section 7.5, after we investigate more aspects of work.

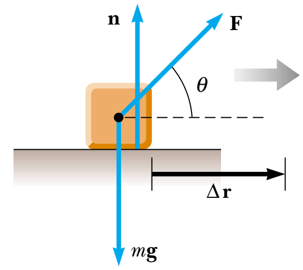


Figure 7.3 When an object is displaced on a frictionless, horizontal surface, the normal force **n** and the gravitational force $m\mathbf{g}$ do no work on the object. In the situation shown here, **F** is the only force doing work on the object.

PITFALL PREVENTION

7.4 Cause of the Displacement

We can calculate the work done by a force on an object, but that force is *not* necessarily the cause of the object's displacement. For example, if you lift an object, work is done by the gravitational force, although gravity is not the cause of the object moving upward!

Quick Quiz 7.1 The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is (a) zero (b) positive (c) negative (d) impossible to determine.

Quick Quiz 7.2 Figure 7.4 shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

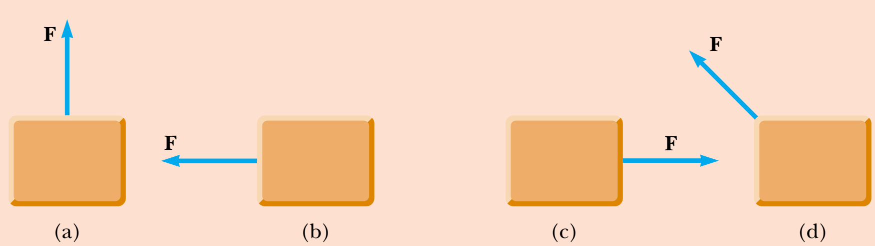


Figure 7.4 (Quick Quiz 7.2)

Example 7.1 Mr. Clean

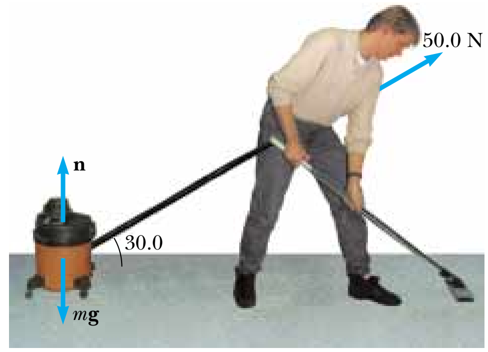
A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5a). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

Solution Figure 7.5a helps conceptualize the situation. We are given a force, a displacement, and the angle between the two vectors, so we can categorize this as a simple problem that will need minimal analysis. To analyze the situation, we identify the vacuum cleaner as the system and draw a free-body diagram as shown in Figure 7.5b. Using the definition of work (Eq. 7.1),

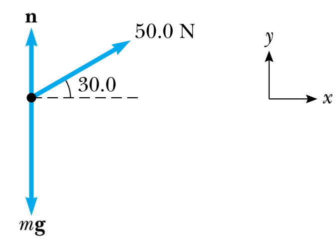
$$W = F\,\Delta r \cos\theta = (50.0\text{ N})(3.00\text{ m})(\cos 30.0°)$$

$$= 130\,\text{N}\cdot\text{m} = \boxed{130\text{ J}}$$

To finalize this problem, notice in this situation that the normal force $\mathbf{n}$ and the gravitational $\mathbf{F}_g = m\mathbf{g}$ do no work on the vacuum cleaner because these forces are perpendicular to its displacement.



(a)



(b)

Figure 7.5 (Example 7.1) (a) A vacuum cleaner being pulled at an angle of 30.0° from the horizontal. (b) Free-body diagram of the forces acting on the vacuum cleaner.

7.3 The Scalar Product of Two Vectors

Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the **scalar product** of two vectors. We write this scalar product of vectors **A** and **B** as $\mathbf{A}\cdot\mathbf{B}$. (Because of the dot symbol, the scalar product is often called the **dot product.**)

In general, the scalar product of any two vectors **A** and **B** is a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

Scalar product of any two vectors A and B

$$\boxed{\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta} \tag{7.2}$$

Note that **A** and **B** need not have the same units, as is the case with any multiplication.

Comparing this definition to Equation 7.1, we see that we can express Equation 7.1 as a scalar product:

$$W = F\,\Delta r\cos\theta = \mathbf{F}\cdot\Delta\mathbf{r} \tag{7.3}$$

In other words, $\mathbf{F}\cdot\Delta\mathbf{r}$ (read "**F** dot Δ**r**") is a shorthand notation for $F\,\Delta r\cos\theta$.

Before continuing with our discussion of work, let us investigate some properties of the dot product. Figure 7.6 shows two vectors **A** and **B** and the angle θ between them that is used in the definition of the dot product. In Figure 7.6, $B\cos\theta$ is the projection of **B** onto **A**. Therefore, Equation 7.2 means that $\mathbf{A}\cdot\mathbf{B}$ is the product of the magnitude of **A** and the projection of **B** onto **A**.[2]

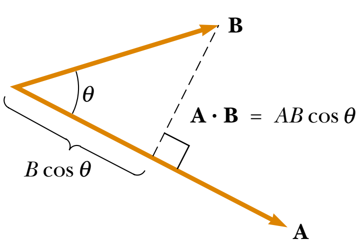


Figure 7.6 The scalar product $\mathbf{A}\cdot\mathbf{B}$ equals the magnitude of **A** multiplied by $B\cos\theta$, which is the projection of **B** onto **A**.

[2] This is equivalent to stating that $\mathbf{A}\cdot\mathbf{B}$ equals the product of the magnitude of **B** and the projection of **A** onto **B**.

From the right-hand side of Equation 7.2 we also see that the scalar product is **commutative**.[3] That is,

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

Finally, the scalar product obeys the **distributive law of multiplication,** so that

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

The dot product is simple to evaluate from Equation 7.2 when **A** is either perpendicular or parallel to **B**. If **A** is perpendicular to **B** ($\theta = 90°$), then $\mathbf{A}\cdot\mathbf{B} = 0$. (The equality $\mathbf{A}\cdot\mathbf{B} = 0$ also holds in the more trivial case in which either **A** or **B** is zero.) If vector **A** is parallel to vector **B** and the two point in the same direction ($\theta = 0$), then $\mathbf{A}\cdot\mathbf{B} = AB$. If vector **A** is parallel to vector **B** but the two point in opposite directions ($\theta = 180°$), then $\mathbf{A}\cdot\mathbf{B} = -AB$. The scalar product is negative when $90° < \theta \leq 180°$.

The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$, which were defined in Chapter 3, lie in the positive x, y, and z directions, respectively, of a right-handed coordinate system. Therefore, it follows from the definition of $\mathbf{A}\cdot\mathbf{B}$ that the scalar products of these unit vectors are

Dot products of unit vectors

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1 \tag{7.4}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0 \tag{7.5}$$

Equations 3.18 and 3.19 state that two vectors **A** and **B** can be expressed in component vector form as

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

Using the information given in Equations 7.4 and 7.5 shows that the scalar product of **A** and **B** reduces to

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \tag{7.6}$$

(Details of the derivation are left for you in Problem 6.) In the special case in which $\mathbf{A} = \mathbf{B}$, we see that

$$\mathbf{A}\cdot\mathbf{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

Quick Quiz 7.3 Which of the following statements is true about the relationship between $\mathbf{A}\cdot\mathbf{B}$ and $(-\mathbf{A})\cdot(-\mathbf{B})$? (a) $\mathbf{A}\cdot\mathbf{B} = -[(-\mathbf{A})\cdot(-\mathbf{B})]$; (b) If $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, then $(-\mathbf{A})\cdot(-\mathbf{B}) = AB\cos(\theta + 180°)$; (c) Both (a) and (b) are true. (d) Neither (a) nor (b) is true.

Quick Quiz 7.4 Which of the following statements is true about the relationship between the dot product of two vectors and the product of the magnitudes of the vectors? (a) $\mathbf{A}\cdot\mathbf{B}$ is larger than AB; (b) $\mathbf{A}\cdot\mathbf{B}$ is smaller than AB; (c) $\mathbf{A}\cdot\mathbf{B}$ could be larger or smaller than AB, depending on the angle between the vectors; (d) $\mathbf{A}\cdot\mathbf{B}$ could be equal to AB.

PITFALL PREVENTION

7.5 Work is a Scalar

Although Equation 7.3 defines the work in terms of two vectors, *work is a scalar*—there is no direction associated with it. *All* types of energy and energy transfer are scalars. This is a major advantage of the energy approach—we don't need vector calculations!

[3] This may seem obvious, but in Chapter 11 you will see another way of combining vectors that proves useful in physics and is not commutative.

Example 7.2 The Scalar Product

The vectors $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\mathbf{A}\cdot\mathbf{B}$.

Solution Substituting the specific vector expressions for $\mathbf{A}$ and $\mathbf{B}$, we find,

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\\ &= -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}}\\ &= -2(1) + 4(0) - 3(0) + 6(1)\\ &= -2 + 6 = \boxed{4}\end{aligned}$$

where we have used the facts that $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = 1$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{i}} = 0$. The same result is obtained when we use Equation 7.6 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

(B) Find the angle θ between $\mathbf{A}$ and $\mathbf{B}$.

Solution The magnitudes of $\mathbf{A}$ and $\mathbf{B}$ are

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Using Equation 7.2 and the result from part (a) we find that

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = \boxed{60.2^\circ}$$

Example 7.3 Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement $\Delta\mathbf{r} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\mathbf{F} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle.

(A) Calculate the magnitudes of the displacement and the force.

Solution We use the Pythagorean theorem:

$$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(2.0)^2 + (3.0)^2} = \boxed{3.6\text{ m}}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = \boxed{5.4\text{ N}}$$

(B) Calculate the work done by $\mathbf{F}$.

Solution Substituting the expressions for $\mathbf{F}$ and $\Delta\mathbf{r}$ into Equation 7.3 and using Equations 7.4 and 7.5, we obtain

$$\begin{aligned}W = \mathbf{F}\cdot\Delta\mathbf{r} &= [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ N}]\cdot[(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\text{ m}]\\ &= (5.0\hat{\mathbf{i}}\cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}}\cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}}\cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}\cdot 3.0\hat{\mathbf{j}})\text{ N}\cdot\text{m}\\ &= [10 + 0 + 0 + 6]\text{ N}\cdot\text{m} = \boxed{16\text{ J}}\end{aligned}$$

7.4 Work Done by a Varying Force

Consider a particle being displaced along the x axis under the action of a force that varies with position. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = F\Delta r\cos\theta$ to calculate the work done by the force because this relationship applies only when $\mathbf{F}$ is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, shown in Figure 7.7a, the x component F_x of the force is approximately constant over this small interval; for this small displacement, we can approximate the work done by the force as

$$W \approx F_x\,\Delta x$$

This is just the area of the shaded rectangle in Figure 7.7a. If we imagine that the F_x versus x curve is divided into a large number of such intervals, the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x\,\Delta x$$

If the size of the displacements is allowed to approach zero, the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the F_x curve and the x axis:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

Therefore, we can express the work done by F_x as the particle moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x \, dx \qquad (7.7)$$

This equation reduces to Equation 7.1 when the component $F_x = F \cos \theta$ is constant.

If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is just the work done by the net force. If we express the net force in the x direction as ΣF_x, then the total work, or *net work*, done as the particle moves from x_i to x_f is

$$\sum W = W_{\text{net}} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx \qquad (7.8)$$

If the system cannot be modeled as a particle (for example, if the system consists of multiple particles that can move with respect to each other), we cannot use Equation 7.8. This is because different forces on the system may move through different displacements. In this case, we must evaluate the work done by each force separately and then add the works algebraically.

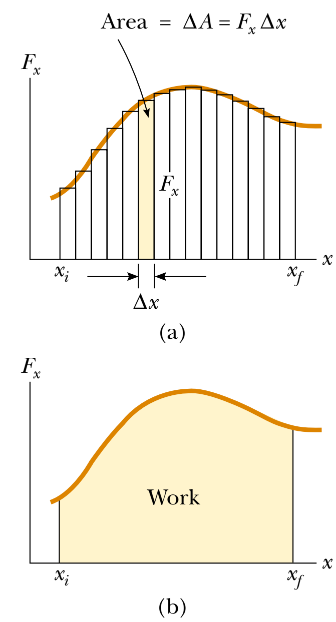


Figure 7.7 (a) The work done by the force component F_x for the small displacement Δx is $F_x \Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles. (b) The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under this curve.

Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

Solution The work done by the force is equal to the area under the curve from $x_A = 0$ to $x_C = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus the area of the triangular section from Ⓑ to Ⓒ. The area of the rectangle is $(5.0 \text{ N})(4.0 \text{ m}) = 20$ J, and the area of the triangle is $\frac{1}{2}(5.0 \text{ N})(2.0 \text{ m}) = 5.0$ J. Therefore, the total work done by the force on the particle is $\boxed{25 \text{ J.}}$

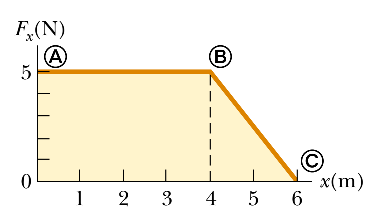


Figure 7.8 (Example 7.4) The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_B = 4.0$ m to $x_C = 6.0$ m. The net work done by this force is the area under the curve.

Example 7.5 Work Done by the Sun on a Probe

The interplanetary probe shown in Figure 7.9a is attracted to the Sun by a force given by

$$F = -\frac{1.3 \times 10^{22}}{x^2}$$

in SI units, where x is the Sun-probe separation distance. Graphically and analytically determine how much work is done by the Sun on the probe as the probe–Sun separation changes from 1.5×10^{11} m to 2.3×10^{11} m.

Graphical Solution The negative sign in the equation for the force indicates that the probe is attracted to the Sun. Because the probe is moving away from the Sun, we expect to obtain a negative value for the work done on it. A spreadsheet or other numerical means can be used to generate a graph like that in Figure 7.9b. Each small square of the grid corresponds to an area $(0.05 \text{ N})(0.1 \times 10^{11} \text{ m}) = 5 \times 10^8$ J. The work done is equal to the shaded area in Figure 7.9b. Because there are approximately 60 squares shaded, the total area (which is negative because the curve is below the x axis) is about -3×10^{10} J. This is the work done by the Sun on the probe.

Analytical Solution We can use Equation 7.7 to calculate a more precise value for the work done on the probe by the Sun. To solve this integral, we make use of the integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = -2$:

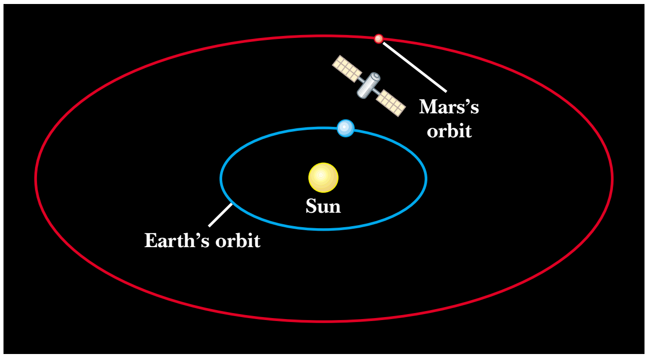
$$W = \int_{1.5\times 10^{11}}^{2.3\times 10^{11}} \left(\frac{-1.3 \times 10^{22}}{x^2} \right) dx$$

$$= (-1.3 \times 10^{22}) \int_{1.5\times 10^{11}}^{2.3\times 10^{11}} x^{-2} dx$$

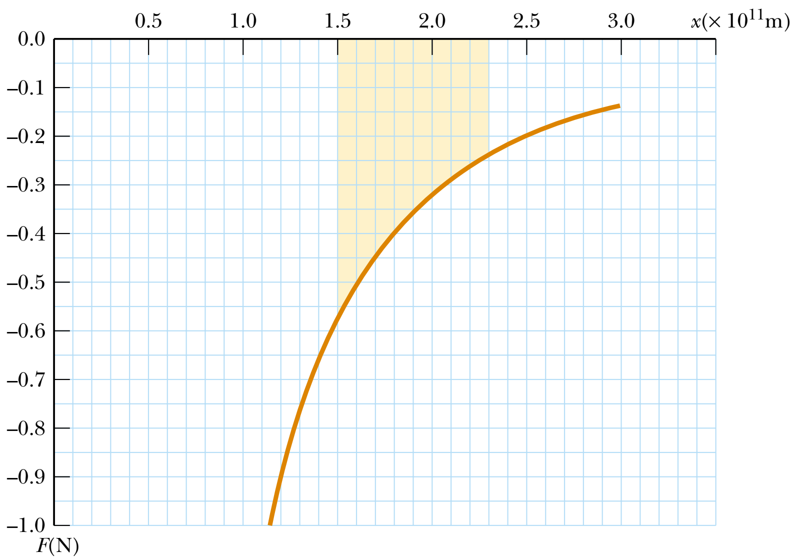
$$= (-1.3 \times 10^{22}) \left(\frac{x^{-1}}{-1} \right) \Bigg|_{1.5\times 10^{11}}^{2.3\times 10^{11}}$$

$$= (-1.3 \times 10^{22}) \left(\frac{-1}{2.3 \times 10^{11}} - \frac{-1}{1.5 \times 10^{11}} \right)$$

$$= \boxed{-3.0 \times 10^{10}\ \text{J}}$$



(a)



(b)

Figure 7.9 (Example 7.5) (a) An interplanetary probe moves from a position near the Earth's orbit radially outward from the Sun, ending up near the orbit of Mars. (b) Attractive force versus distance for the interplanetary probe.

Work Done by a Spring

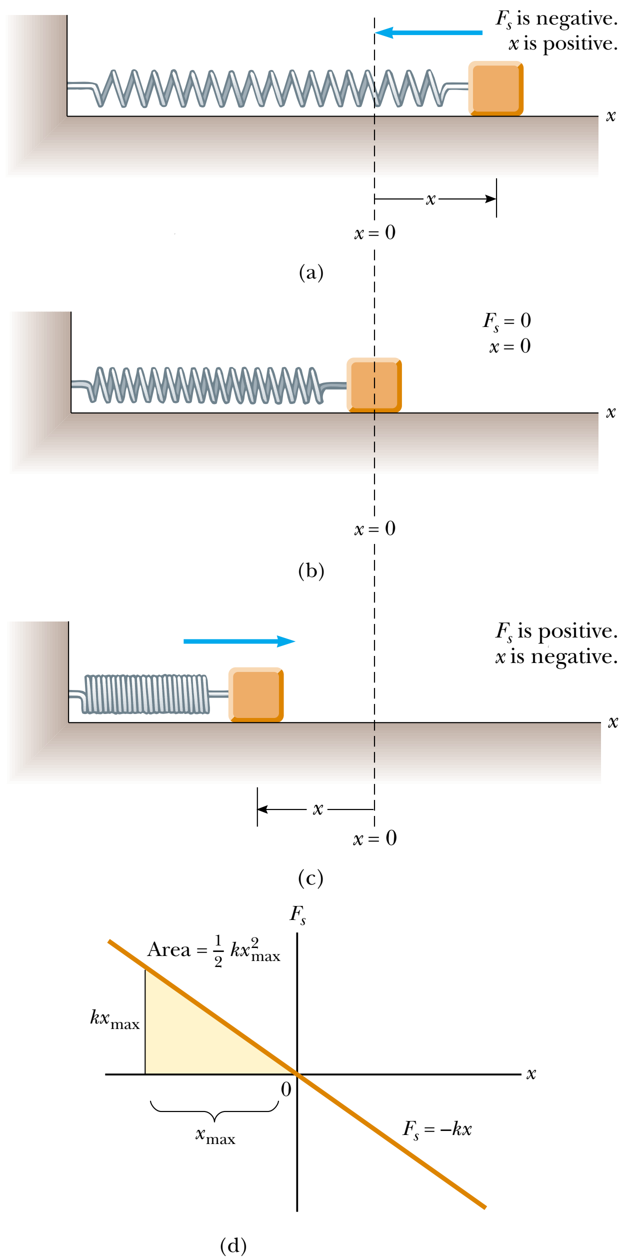
A model of a common physical system for which the force varies with position is shown in Figure 7.10. A block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

Spring force

$$F_s = -kx \tag{7.9}$$

where x is the position of the block relative to its equilibrium ($x = 0$) position and k is a positive constant called the **force constant** or the **spring constant** of the spring. In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs is known as **Hooke's law**. The value of k is a measure of the *stiffness* of the spring. Stiff springs have large k values, and soft springs have small k values. As can be seen from Equation 7.9, the units of k are N/m.

The negative sign in Equation 7.9 signifies that the force exerted by the spring is always directed *opposite* to the displacement from equilibrium. When $x > 0$ as in Figure 7.10a, so that the block is to the right of the equilibrium position, the spring force is directed to the left, in the negative x direction. When $x < 0$ as in Figure 7.10c, the block is to the left of equilibrium and the spring force is directed to the right, in the positive x direction. When $x = 0$ as in Figure 7.10b, the spring is unstretched and $F_s = 0$.



Active Figure 7.10 The force exerted by a spring on a block varies with the block's position x relative to the equilibrium position $x = 0$. (a) When x is positive (stretched spring), the spring force is directed to the left. (b) When x is zero (natural length of the spring), the spring force is zero. (c) When x is negative (compressed spring), the spring force is directed to the right. (d) Graph of F_s versus x for the block–spring system. The work done by the spring force as the block moves from $-x_{\max}$ to 0 is the area of the shaded triangle, $\frac{1}{2}kx_{\max}^2$.

At the Active Figures link at http://www.pse6.com, you can observe the block's motion for various maximum displacements and spring constants.

Because the spring force always acts toward the equilibrium position ($x = 0$), it is sometimes called a *restoring force.* If the spring is compressed until the block is at the point $-x_{\max}$ and is then released, the block moves from $-x_{\max}$ through zero to $+x_{\max}$. If the spring is instead stretched until the block is at the point $+x_{\max}$ and is then released, the block moves from $+x_{\max}$ through zero to $-x_{\max}$. It then reverses direction, returns to $+x_{\max}$, and continues oscillating back and forth.

Suppose the block has been pushed to the left to a position $-x_{\max}$ and is then released. Let us identify the block as our system and calculate the work W_s done by the spring force on the block as the block moves from $x_i = -x_{\max}$ to $x_f = 0$. Applying

Equation 7.7 and assuming the block may be treated as a particle, we obtain

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{\max}}^{0} (-kx)\, dx = \tfrac{1}{2} k x_{\max}^2 \tag{7.10}$$

where we have used the integral $\int x^n dx = x^{n+1}/(n+1)$ with $n = 1$. The work done by the spring force is positive because the force is in the same direction as the displacement of the block (both are to the right). Because the block arrives at $x = 0$ with some speed, it will continue moving, until it reaches a position $+x_{\max}$. When we consider the work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{\max}$, we find that $W_s = -\frac{1}{2}kx_{\max}^2$ because for this part of the motion the displacement is to the right and the spring force is to the left. Therefore, the *net* work done by the spring force as the block moves from $x_i = -x_{\max}$ to $x_f = x_{\max}$ is *zero.*

Figure 7.10d is a plot of F_s versus x. The work calculated in Equation 7.10 is the area of the shaded triangle, corresponding to the displacement from $-x_{\max}$ to 0. Because the triangle has base $x_{\max}$ and height $kx_{\max}$, its area is $\frac{1}{2}kx_{\max}^2$, the work done by the spring as given by Equation 7.10.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

Work done by a spring

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \tag{7.11}$$

For example, if the spring has a force constant of 80 N/m and is compressed 3.0 cm from equilibrium, the work done by the spring force as the block moves from $x_i = -3.0$ cm to its unstretched position $x_f = 0$ is 3.6×10^{-2} J. From Equation 7.11 we also see that the work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$). We shall make use of this important result in Chapter 8, in which we describe the motion of this system in greater detail.

Equations 7.10 and 7.11 describe the work done by the spring on the block. Now let us consider the work done on the spring by an *external agent* that stretches the spring very slowly from $x_i = 0$ to $x_f = x_{\max}$, as in Figure 7.11. We can calculate this work by noting that at any value of the position, the *applied force* $\mathbf{F}_{\text{app}}$ is equal in magnitude and opposite in direction to the spring force $\mathbf{F}_s$, so that $F_{\text{app}} = -(-kx) = kx$. Therefore, the work done by this applied force (the external agent) on the block–spring system is

$$W_{F_{\text{app}}} = \int_0^{x_{\max}} F_{\text{app}}\, dx = \int_0^{x_{\max}} kx\, dx = \tfrac{1}{2}kx_{\max}^2$$

This work is equal to the negative of the work done by the spring force for this displacement.

The work done by an applied force on a block–spring system between arbitrary positions of the block is

$$W_{F_{\text{app}}} = \int_{x_i}^{x_f} F_{\text{app}}\, dx = \int_{x_i}^{x_f} kx\, dx = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2 \tag{7.12}$$

Notice that this is the negative of the work done by the spring as expressed by Equation 7.11. This is consistent with the fact that the spring force and the applied force are of equal magnitude but in opposite directions.

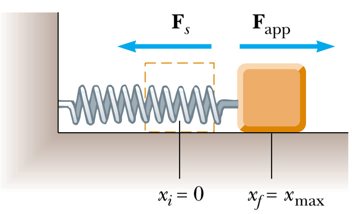


Figure 7.11 A block being pulled from $x_i = 0$ to $x_f = x_{\max}$ on a frictionless surface by a force $\mathbf{F}_{\text{app}}$. If the process is carried out very slowly, the applied force is equal in magnitude and opposite in direction to the spring force at all times.

Quick Quiz 7.5 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much work is required to load the second dart compared to that required to load the first? (a) four times as much (b) two times as much (c) the same (d) half as much (e) one-fourth as much.

Example 7.6 Measuring *k* for a Spring

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.12. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position.

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

Solution Because the object (the system) is at rest, the upward spring force balances the downward gravitational force $m\mathbf{g}$. In this case, we apply Hooke's law to give $|\mathbf{F}_s| = kd = mg$, or

$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.7 \times 10^2\ \text{N/m}}$$

(B) How much work is done by the spring as it stretches through this distance?

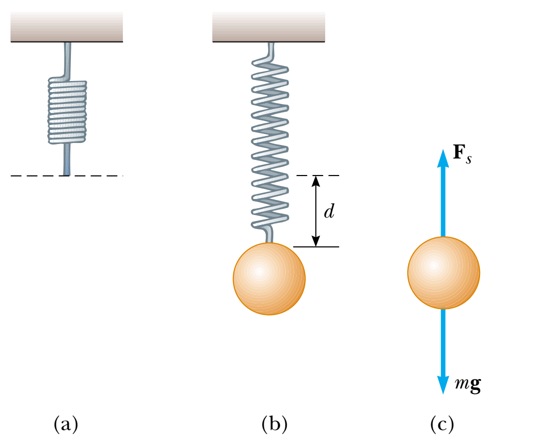


Figure 7.12 (Example 7.6) Determining the force constant k of a spring. The elongation d is caused by the attached object, which has a weight mg. Because the spring force balances the gravitational force, it follows that $k = mg/d$.

Solution Using Equation 7.11,

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$

$$= \boxed{-5.4 \times 10^{-2}\ \text{J}}$$

What If? Suppose this measurement is made on an elevator with an upward vertical acceleration a. Will the unaware experimenter arrive at the same value of the spring constant?

Answer The force $\mathbf{F}_s$ in Figure 7.12 must be larger than $m\mathbf{g}$ to produce an upward acceleration of the object. Because $\mathbf{F}_s$ must increase in magnitude, and $|\mathbf{F}_s| = kd$, the spring must extend farther. The experimenter sees a larger extension for the same hanging weight and therefore measures the spring constant to be smaller than the value found in part (A) for $\mathbf{a} = 0$.

Newton's second law applied to the hanging object gives

$$\sum F_y = |\mathbf{F}_s| - mg = ma_y$$

$$kd - mg = ma_y$$

$$d = \frac{m(g + a_y)}{k}$$

where k is the *actual* spring constant. Now, the experimenter is unaware of the acceleration, so she claims that $|\mathbf{F}_s| = k'd = mg$ where k' is the spring constant as measured by the experimenter. Thus,

$$k' = \frac{mg}{d} = \frac{mg}{\left(\dfrac{m(g + a_y)}{k}\right)} = \frac{g}{g + a_y}k$$

If the acceleration of the elevator is upward so that a_y is positive, this result shows that the measured spring constant will be smaller, consistent with our conceptual argument.

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

We have investigated work and identified it as a mechanism for transferring energy into a system. One of the possible outcomes of doing work on a system is that the system changes its speed. In this section, we investigate this situation and introduce our first type of energy that a system can possess, called *kinetic energy*.

Consider a system consisting of a single object. Figure 7.13 shows a block of mass m moving through a displacement directed to the right under the action of a net force $\Sigma\mathbf{F}$, also directed to the right. We know from Newton's second law that the block moves with an acceleration $\mathbf{a}$. If the block moves through a displacement $\Delta\mathbf{r} = \Delta x\hat{\mathbf{i}} = (x_f - x_i)\hat{\mathbf{i}}$, the work done by the net force $\Sigma\mathbf{F}$ is

$$\sum W = \int_{x_i}^{x_f} \sum F\,dx \tag{7.13}$$

Using Newton's second law, we can substitute for the magnitude of the net force $\Sigma F = ma$, and then perform the following chain-rule manipulations on the integrand:

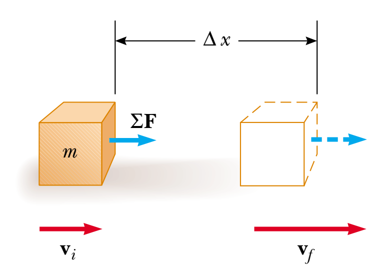


Figure 7.13 An object undergoing a displacement $\Delta\mathbf{r} = \Delta x\hat{\mathbf{i}}$ and a change in velocity under the action of a constant net force $\Sigma\mathbf{F}$.

$$\sum W = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dx}\,\frac{dx}{dt}\,dx = \int_{v_i}^{v_f} mv\,dv$$

$$\sum W = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} \tag{7.14}$$

where v_i is the speed of the block when it is at $x = x_i$ and v_f is its speed at x_f.

This equation was generated for the specific situation of one-dimensional motion, but it is a general result. It tells us that the work done by the net force on a particle of mass m is equal to the difference between the initial and final values of a quantity $\frac{1}{2}mv^2$. The quantity $\frac{1}{2}mv^2$ represents the energy associated with the motion of the particle. This quantity is so important that it has been given a special name—**kinetic energy.** Equation 7.14 states that the net work done on a particle by a net force $\Sigma\mathbf{F}$ acting on it equals the change in kinetic energy of the particle.

In general, the kinetic energy K of a particle of mass m moving with a speed v is defined as

Kinetic energy

$$K \equiv \tfrac{1}{2}mv^2 \tag{7.15}$$

Kinetic energy is a scalar quantity and has the same units as work. For example, a 2.0 kg object moving with a speed of 4.0 m/s has a kinetic energy of 16 J. Table 7.1 lists the kinetic energies for various objects.

It is often convenient to write Equation 7.14 in the form

Work–kinetic energy theorem

$$\sum W = K_f - K_i = \Delta K \tag{7.16}$$

Another way to write this is $K_f = K_i + \Sigma W$, which tells us that the final kinetic energy is equal to the initial kinetic energy plus the change due to the work done.

Equation 7.16 is an important result known as the **work–kinetic energy theorem:**

> In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.

PITFALL PREVENTION

7.6 Conditions for the Work–Kinetic Energy Theorem

The work–kinetic energy theorem is important, but limited in its application—it is not a general principle. There are many situations in which other changes in the system occur besides its speed, and there are other interactions with the environment besides work. A more general principle involving energy is conservation of energy in Section 7.6.

The work–kinetic energy theorem indicates that the speed of a particle will *increase* if the net work done on it is *positive,* because the final kinetic energy will be greater than the initial kinetic energy. The speed will *decrease* if the net work is *negative,* because the final kinetic energy will be less than the initial kinetic energy.

Table 7.1

Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^{4}	2.66×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^{3}	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^{4}	3.14×10^{10}
Automobile at 65 mi/h	2 000	29	8.4×10^{5}
Running athlete	70	10	3 500
Stone dropped from 10 m	1.0	14	98
Golf ball at terminal speed	0.046	44	45
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] Escape speed is the minimum speed an object must reach near the Earth's surface in order to move infinitely far away from the Earth.

Because we have only investigated translational motion through space so far, we arrived at the work–kinetic energy theorem by analyzing situations involving translational motion. Another type of motion is *rotational motion,* in which an object spins about an axis. We will study this type of motion in Chapter 10. The work–kinetic energy theorem is also valid for systems that undergo a change in the rotational speed due to work done on the system. The windmill in the chapter opening photograph is an example of work causing rotational motion.

The work–kinetic energy theorem will clarify a result that we have seen earlier in this chapter that may have seemed odd. In Section 7.4, we arrived at a result of zero net work done when we let a spring push a block from $x_i = -x_{\max}$ to $x_f = x_{\max}$. Notice that the speed of the block is continually changing during this process, so it may seem complicated to analyze this process. The quantity ΔK in the work–kinetic energy theorem, however, only refers to the initial and final points for the speeds—it does not depend on details of the path followed between these points. Thus, because the speed is zero at both the initial and final points of the motion, the net work done on the block is zero. We will see this concept of path independence often in similar approaches to problems.

Earlier, we indicated that work can be considered as a mechanism for transferring energy into a system. Equation 7.16 is a mathematical statement of this concept. We do work ΣW on a system and the result is a transfer of energy across the boundary of the system. The result on the system, in the case of Equation 7.16, is a change ΔK in kinetic energy. We will explore this idea more fully in the next section.

PITFALL PREVENTION

7.7 The Work–Kinetic Energy Theorem: Speed, not Velocity

The work–kinetic energy theorem relates work to a change in the *speed* of an object, not a change in its velocity. For example, if an object is in uniform circular motion, the speed is constant. Even though the velocity is changing, no work is done by the force causing the circular motion.

Quick Quiz 7.6 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much faster does the second dart leave the gun compared to the first? (a) four times as fast (b) two times as fast (c) the same (d) half as fast (e) one-fourth as fast.

Example 7.7 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

Solution We have made a drawing of this situation in Figure 7.14. We could apply the equations of kinematics to determine the answer, but let us practice the energy approach. The block is the system, and there are three external forces acting on the system. The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are horizontally displaced. Thus, the net external force acting on the block is the 12-N force. The work done by this force is

$$W = F\Delta x = (12\,\text{N})(3.0\,\text{m}) = 36\,\text{J}$$

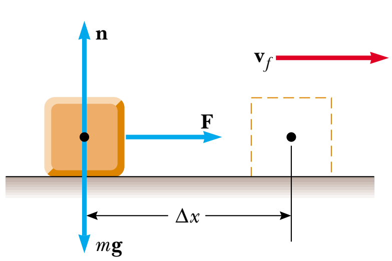


Figure 7.14 (Example 7.7) A block pulled to the right on a frictionless surface by a constant horizontal force.

Using the work–kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \tfrac{1}{2}mv_f^{\,2} - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36\,\text{J})}{6.0\,\text{kg}}} = \boxed{3.5\,\text{m/s}}$$

What If? **Suppose the magnitude of the force in this example is doubled to $F' = 2F$. The 6.0-kg block accelerates to 3.5 m/s due to this applied force while moving through a displacement $\Delta x'$. (A) How does the displacement $\Delta x'$ compare to the original displacement Δx? (B) How does the time interval $\Delta t'$ for the block to accelerate from rest to 3.5 m/s compare to the original interval Δt?**

Answer (A) If we pull harder, the block should accelerate to a higher speed in a shorter distance, so we expect $\Delta x' < \Delta x$. Mathematically, from the work–kinetic energy theorem $W = \Delta K$, we find

$$F'\Delta x' = \Delta K = F\Delta x$$

$$\Delta x' = \frac{F}{F'}\Delta x = \frac{F}{2F}\Delta x = \tfrac{1}{2}\Delta x$$

and the distance is shorter as suggested by our conceptual argument.

(B) If we pull harder, the block should accelerate to a higher speed in a shorter time interval, so we expect $\Delta t' < \Delta t$. Mathematically, from the definition of average velocity,

$$\overline{v} = \frac{\Delta x}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta x}{\overline{v}}$$

Because both the original force and the doubled force cause the same change in velocity, the average velocity $\overline{v}$ is the same in both cases. Thus,

$$\Delta t' = \frac{\Delta x'}{\overline{v}} = \frac{\frac{1}{2}\Delta x}{\overline{v}} = \frac{1}{2}\Delta t$$

and the time interval is shorter, consistent with our conceptual argument.

Conceptual Example 7.8 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp, as shown in Figure 7.15. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his statement valid?

Solution No. Suppose the refrigerator is wheeled on a dolly up the ramp at constant speed. Thus, $\Delta K = 0$. The normal force exerted by the ramp on the refrigerator is directed at 90° to the displacement and so does no work on the refrigerator. Because $\Delta K = 0$, the work–kinetic energy theorem gives

$$W_{\text{net}} = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the gravitational force equals the product of the weight mg of the refrigerator, the height h through which it is displaced, and cos 180°, or $W_{\text{by gravity}} = -mgh$. (The negative sign arises because the downward gravitational force is opposite the displacement.) Thus, the man must do the same amount of work mgh on the refrigerator, *regardless* of the length of the ramp. Although less force is required with a longer ramp, that force must act over a greater distance.

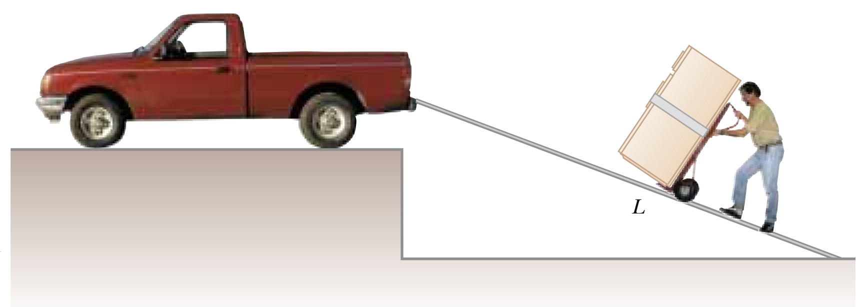


Figure 7.15 (Conceptual Example 7.8) A refrigerator attached to a frictionless wheeled dolly is moved up a ramp at constant speed.

7.6 The Nonisolated System—Conservation of Energy

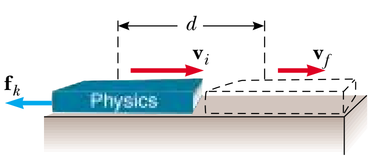


Figure 7.16 A book sliding to the right on a horizontal surface slows down in the presence of a force of kinetic friction acting to the left. The initial velocity of the book is $\mathbf{v}_i$, and its final velocity is $\mathbf{v}_f$. The normal force and the gravitational force are not included in the diagram because they are perpendicular to the direction of motion and therefore do not influence the book's speed.

We have seen examples in which an object, modeled as a particle, is acted on by various forces, resulting in a change in its kinetic energy. This very simple situation is the first example of the **nonisolated system**—a common scenario in physics problems. Physical problems for which this scenario is appropriate involve systems that interact with or are influenced by their environment, causing some kind of change in the system. If a system does not interact with its environment it is an **isolated system**, which we will study in Chapter 8.

The work–kinetic energy theorem is our first example of an energy equation appropriate for a nonisolated system. In the case of the work–kinetic energy theorem, the interaction is the work done by the external force, and the quantity in the system that changes is the kinetic energy.

In addition to kinetic energy, we now introduce a second type of energy that a system can possess. Let us imagine the book in Figure 7.16 sliding to the right on the surface of a heavy table and slowing down due to the friction force. Suppose the *surface* is the system. Then the friction force from the sliding book does work on the surface. The force on the surface is to the right and the displacement of the point of application of the force is to the right—the work is positive. But the surface is not moving after the book has stopped. Positive work has been done on the surface, yet there is no increase in the surface's kinetic energy. Is this a violation of the work–kinetic energy theorem?

It is not really a violation, because this situation does not fit the description of the conditions given for the work–kinetic energy theorem. Work is done on the system of the surface, but the result of that work is *not* an increase in kinetic energy. From your everyday experience with sliding over surfaces with friction, you can probably guess that the surface will be *warmer* after the book slides over it. (Rub your hands together briskly to experience this!) Thus, the work that was done on the surface has gone into warming the surface rather than increasing its speed. We call the energy associated with an object's temperature its **internal energy,** symbolized E_{int}. (We will define internal energy more generally in Chapter 20.) In this case, the work done on the surface does indeed represent energy transferred into the system, but it appears in the system as internal energy rather than kinetic energy.

We have now seen two methods of storing energy in a system—kinetic energy, related to motion of the system, and internal energy, related to its temperature. A third method, which we cover in Chapter 8, is *potential energy*. This is energy related to the configuration of a system in which the components of the system interact by forces. For example, when a spring is stretched, *elastic potential energy* is stored in the spring due to the force of interaction between the spring coils. Other types of potential energy include gravitational and electric.

We have seen only one way to transfer energy into a system so far—work. We mention below a few other ways to transfer energy into or out of a system. The details of these processes will be studied in other sections of the book. We illustrate these in Figure 7.17 and summarize them as follows:

Work, as we have learned in this chapter, is a method of transferring energy to a system by applying a force to the system and causing a displacement of the point of application of the force (Fig. 7.17a).

Mechanical waves (Chapters 16–18) are a means of transferring energy by allowing a disturbance to propagate through air or another medium. This is the method by which energy (which you detect as sound) leaves your clock radio through the loudspeaker and enters your ears to stimulate the hearing process (Fig. 7.17b). Other examples of mechanical waves are seismic waves and ocean waves.

Heat (Chapter 20) is a mechanism of energy transfer that is driven by a temperature difference between two regions in space. One clear example is thermal conduction, a mechanism of transferring energy by microscopic collisions. For example, a metal spoon in a cup of coffee becomes hot because fast-moving electrons and atoms in the submerged portion of the spoon bump into slower ones in the nearby part of the handle (Fig. 7.17c). These particles move faster because of the collisions and bump into the next group of slow particles. Thus, the internal energy of the spoon handle rises from energy transfer due to this bumping process.[4]

Matter transfer (Chapter 20) involves situations in which matter physically crosses the boundary of a system, carrying energy with it. Examples include filling your automobile tank with gasoline (Fig. 7.17d), and carrying energy to the rooms of your home by circulating warm air from the furnace, a process called *convection*.

PITFALL PREVENTION

7.8 Heat is not a Form of Energy

The word *heat* is one of the most misused words in our popular language. In this text, heat is a method of *transferring* energy, *not* a form of storing energy. Thus, phrases such as "heat content," "the heat of the summer," and "the heat escaped" all represent uses of this word that are inconsistent with our physics definition. See Chapter 20.

[4] The process we call heat can also proceed by convection and radiation, as well as conduction. Convection and radiation, described in Chapter 20, overlap with other types of energy transfer in our list of six.

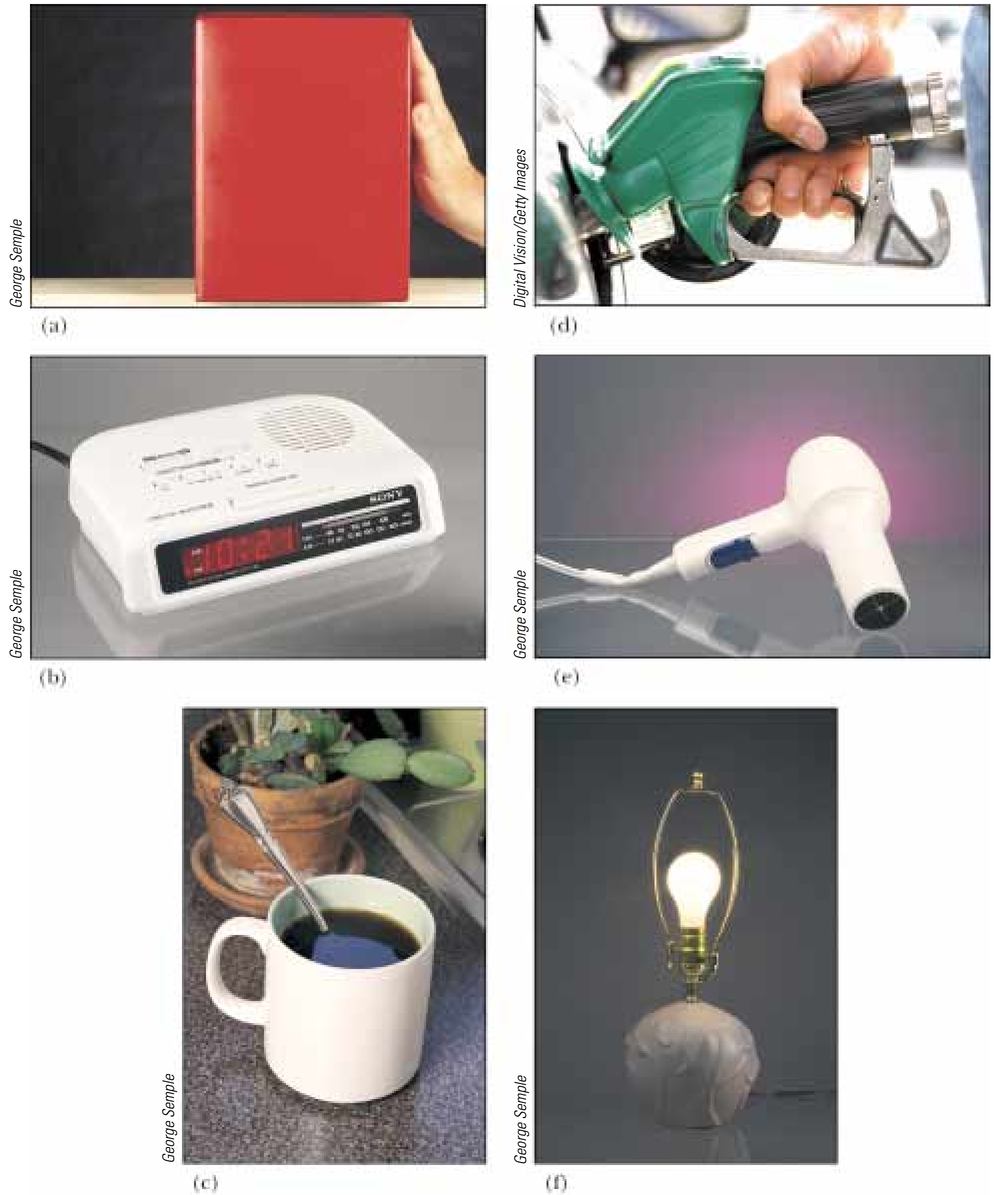


Figure 7.17 Energy transfer mechanisms. (a) Energy is transferred to the block by *work*; (b) energy leaves the radio from the speaker by *mechanical waves*; (c) energy transfers up the handle of the spoon by *heat*; (d) energy enters the automobile gas tank by *matter transfer*; (e) energy enters the hair dryer by *electrical transmission*; and (f) energy leaves the light bulb by *electromagnetic radiation*.

Electrical Transmission (Chapters 27–28) involves energy transfer by means of electric currents. This is how energy transfers into your hair dryer (Fig. 7.17e), stereo system, or any other electrical device.

Electromagnetic radiation (Chapter 34) refers to electromagnetic waves such as light, microwaves, radio waves, and so on (Fig. 7.17f). Examples of this method of transfer include cooking a baked potato in your microwave oven and light energy traveling from the Sun to the Earth through space.[5]

[5] Electromagnetic radiation and work done by field forces are the only energy transfer mechanisms that do not require molecules of the environment to be available at the system boundary. Thus, systems surrounded by a vacuum (such as planets) can only exchange energy with the environment by means of these two possibilities.

One of the central features of the energy approach is the notion that **we can neither create nor destroy energy—energy is always *conserved*.** Thus, **if the total amount of energy in a system changes, it can *only* be due to the fact that energy has crossed the boundary of the system by a transfer mechanism such as one of the methods listed above.** This is a general statement of the principle of **conservation of energy.** We can describe this idea mathematically as follows:

$$\Delta E_{\text{system}} = \sum T \tag{7.17}$$

Conservation of energy

where E_{system} is the total energy of the system, including all methods of energy storage (kinetic, internal, and potential, as discussed in Chapter 8) and T is the amount of energy transferred across the system boundary by some mechanism. Two of our transfer mechanisms have well-established symbolic notations. For work, $T_{\text{work}} = W$, as we have seen in the current chapter, and for heat, $T_{\text{heat}} = Q$, as defined in Chapter 20. The other four members of our list do not have established symbols.

This is no more complicated in theory than is balancing your checking account statement. If your account is the system, the change in the account balance for a given month is the sum of all the transfers—deposits, withdrawals, fees, interest, and checks written. It may be useful for you to think of energy as the *currency of nature*!

Suppose a force is applied to a nonisolated system and the point of application of the force moves through a displacement. Suppose further that the only effect on the system is to change its speed. Then the only transfer mechanism is work (so that ΣT in Equation 7.17 reduces to just W) and the only kind of energy in the system that changes is the kinetic energy (so that ΔE_{system} reduces to just ΔK). Equation 7.17 then becomes

$$\Delta K = W$$

which is the work–kinetic energy theorem. The work–kinetic energy theorem is a special case of the more general principle of conservation of energy. We shall see several more special cases in future chapters.

Quick Quiz 7.7 By what transfer mechanisms does energy enter and leave (a) your television set; (b) your gasoline-powered lawn mower; (c) your hand-cranked pencil sharpener?

Quick Quiz 7.8 Consider a block sliding over a horizontal surface with friction. Ignore any sound the sliding might make. If we consider the system to be the *block*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.9 If we consider the system in Quick Quiz 7.8 to be the *surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.10 If we consider the system in Quick Quiz 7.8 to be the *block and the surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

7.7 Situations Involving Kinetic Friction

Consider again the book in Figure 7.16 sliding to the right on the surface of a heavy table and slowing down due to the friction force. Work is done by the friction force because there is a force and a displacement. Keep in mind, however, that our equations for work involve the displacement *of the point of application of the force*. The friction force is spread out over the entire contact area of an object sliding on a surface, so the force is not localized at a point. In addition, the magnitudes of the friction forces at various points are constantly changing as spot welds occur, the surface and the book deform locally, and so on. The points of application of the friction force on the book are jumping all over the face of the book in contact with the surface. This means that the displacement of the point of application of the friction force (assuming we could calculate it!) is not the same as the displacement of the book.

The work–kinetic energy theorem is valid for a particle or an object that can be modeled as a particle. When an object cannot be treated as a particle, however, things become more complicated. For these kinds of situations, Newton's second law is still valid for the system, even though the work–kinetic energy theorem is not. In the case of a nondeformable object like our book sliding on the surface,[6] we can handle this in a relatively straightforward way.

Starting from a situation in which a constant force is applied to the book, we can follow a similar procedure to that in developing Equation 7.14. We start by multiplying each side of Newton's second law (x component only) by a displacement Δx of the book:

$$\left(\sum F_x\right)\Delta x = (ma_x)\Delta x \tag{7.18}$$

For a particle under constant acceleration, we know that the following relationships (Eqs. 2.9 and 2.11) are valid:

$$a_x = \frac{v_f - v_i}{t} \qquad \Delta x = \tfrac{1}{2}(v_i + v_f)t$$

where v_i is the speed at $t = 0$ and v_f is the speed at time t. Substituting these expressions into Equation 7.18 gives

$$\left(\sum F_x\right)\Delta x = m\left(\frac{v_f - v_i}{t}\right)\tfrac{1}{2}(v_i + v_f)t$$

$$\left(\sum F_x\right)\Delta x = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

This *looks* like the work–kinetic energy theorem, but *the left hand side has not been called work*. The quantity Δx is the displacement of the book—*not* the displacement of the point of application of the friction force.

Let us now apply this equation to a book that has been projected across a surface. We imagine that the book has an initial speed and slows down due to friction, the only force in the horizontal direction. The net force on the book is the kinetic friction force $\mathbf{f}_k$, which is directed opposite to the displacement Δx. Thus,

$$\left(\sum F_x\right)\Delta x = -f_k\Delta x = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \Delta K$$

$$-f_k\Delta x = \Delta K \tag{7.19}$$

which mathematically describes the decrease in kinetic energy due to the friction force.

We have generated these results by assuming that a book is moving along a straight line. An object could also slide over a surface with friction and follow a curved path. In this case, Equation 7.19 must be generalized as follows:

Change in kinetic energy due to friction

$$-f_k d = \Delta K \tag{7.20}$$

where d is the length of the path followed by an object.

If there are other forces besides friction acting on an object, the change in kinetic energy is the sum of that due to the other forces from the work–kinetic energy theorem, and that due to friction:

[6] The overall shape of the book remains the same, which is why we are saying it is nondeformable. On a microscopic level, however, there is deformation of the book's face as it slides over the surface.

$$\Delta K = -f_k d + \sum W_{\text{other forces}} \quad (7.21a)$$

or

$$K_f = K_i - f_k d + \sum W_{\text{other forces}} \quad (7.21b)$$

Now consider the larger system of the book *and* the surface as the book slows down under the influence of a friction force alone. There is no work done across the boundary of this system—the system does not interact with the environment. There are no other types of energy transfer occurring across the boundary of the system, assuming we ignore the inevitable sound the sliding book makes! In this case, Equation 7.17 becomes

$$\Delta E_{\text{system}} = \Delta K + \Delta E_{\text{int}} = 0$$

The change in kinetic energy of this book-plus-surface system is the same as the change in kinetic energy of the the book alone in Equation 7.20, because the book is the only part of the book-surface system that is moving. Thus,

$$-f_k d + \Delta E_{\text{int}} = 0$$

$$\Delta E_{\text{int}} = f_k d \quad (7.22)$$

Change in internal energy due to friction

Thus, the increase in internal energy of the system is equal to the product of the friction force and the displacement of the book.

The conclusion of this discussion is that **the result of a friction force is to transform kinetic energy into internal energy, and the increase in internal energy is equal to the decrease in kinetic energy.**

Quick Quiz 7.11 You are traveling along a freeway at 65 mi/h. Your car has kinetic energy. You suddenly skid to a stop because of congestion in traffic. Where is the kinetic energy that your car once had? (a) All of it is in internal energy in the road. (b) All of it is in internal energy in the tires. (c) Some of it has transformed to internal energy and some of it transferred away by mechanical waves. (d) All of it is transferred away from your car by various mechanisms.

Example 7.9 A Block Pulled on a Rough Surface — Interactive

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15. (This is Example 7.7, modified so that the surface is no longer frictionless.)

Solution Conceptualize this problem by realizing that the rough surface is going to apply a friction force opposite to the applied force. As a result, we expect the speed to be lower than that found in Example 7.7. The surface is rough and we are given forces and a distance, so we categorize this as a situation involving kinetic friction that must be handled by means of Equation 7.21. To analyze the problem, we have made a drawing of this situation in Figure 7.18a. We identify the block as the system, and there are four external forces interacting with the system. The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are displaced horizontally. The applied force does work just as in Example 7.7:

$$W = F\Delta x = (12 \text{ N})(3.0 \text{ m}) = 36 \text{ J}$$

In this case we must use Equation 7.21a to calculate the kinetic energy change due to friction, $\Delta K_{\text{friction}}$. Because the block is in equilibrium in the vertical direction, the normal force $\mathbf{n}$ counterbalances the gravitational force $m\mathbf{g}$, so we have $n = mg$. Hence, the magnitude of the friction force is

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0 \text{ kg})(9.80 \text{ m/s}^2) = 8.82 \text{ N}$$

The change in kinetic energy of the block due to friction is

$$\Delta K_{\text{friction}} = -f_k d = -(8.82 \text{ N})(3.0 \text{ m}) = -26.5 \text{ J}$$

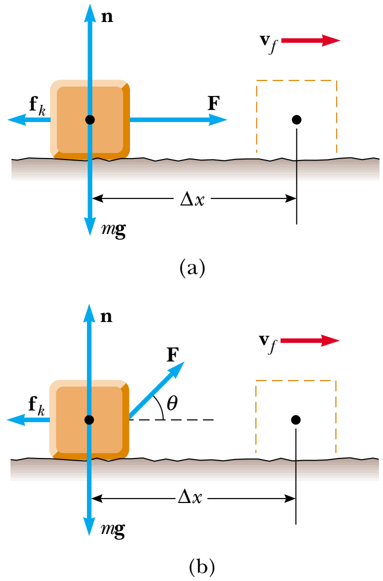


Figure 7.18 (Example 7.9) (a) A block pulled to the right on a rough surface by a constant horizontal force. (b) The applied force is at an angle θ to the horizontal.

The final speed of the block follows from Equation 7.21b:

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - f_k d + \sum W_{\text{other forces}}$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}\left(-f_k d + \sum W_{\text{other forces}}\right)}$$

$$= \sqrt{0 + \frac{2}{6.0\,\text{kg}}(-26.5\,\text{J} + 36\,\text{J})}$$

$$= \boxed{1.8\,\text{m/s}}$$

To finalize this problem note that, after covering the same distance on a frictionless surface (see Example 7.7), the speed of the block was 3.5 m/s.

(B) Suppose the force **F** is applied at an angle θ as shown in Figure 7.18b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

Solution The work done by the applied force is now

$$W = F\Delta x\cos\theta = Fd\cos\theta$$

where $\Delta x = d$ because the path followed by the block is a straight line. The block is in equilibrium in the vertical direction, so

$$\sum F_y = n + F\sin\theta - mg = 0$$

and

$$n = mg - F\sin\theta$$

Because $K_i = 0$, Equation 7.21b can be written,

$$\begin{aligned} K_f &= -f_k d + \sum W_{\text{other forces}} \\ &= -\mu_k n d + Fd\cos\theta \\ &= -\mu_k(mg - F\sin\theta)d + Fd\cos\theta \end{aligned}$$

Maximizing the speed is equivalent to maximizing the final kinetic energy. Consequently, we differentiate K_f with respect to θ and set the result equal to zero:

$$\frac{d(K_f)}{d\theta} = -\mu_k(0 - F\cos\theta)d - Fd\sin\theta = 0$$

$$\mu_k\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu_k$$

For $\mu_k = 0.15$, we have,

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = \boxed{8.5°}$$

Try out the effects of pulling the block at various angles at the Interactive Worked Example link at **http://www.pse6.com.**

Conceptual Example 7.10 Useful Physics for Safer Driving

A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. Assuming that the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

Solution Let us assume that the force of kinetic friction between the car and the road surface is constant and the same for both speeds. According to Equation 7.20, the friction force multiplied by the distance d is equal to the initial kinetic energy of the car (because $K_f = 0$). If the speed is doubled, as it is in this example, the kinetic energy is quadrupled. For a given friction force, the distance traveled is four times as great when the initial speed is doubled, and so the estimated distance that the car slides is $4d$.

Example 7.11 A Block–Spring System

Interactive

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0×10^3 N/m, as shown in Figure 7.10. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

Solution In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$. We use Equation 7.10 to find the work done by the spring with $x_{\text{max}} = x_i = -2.0\text{ cm} = -2.0 \times 10^{-2}$ m:

$$W_s = \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}(1.0\times10^3\,\text{N/m})(-2.0\times10^{-2}\,\text{m})^2 = 0.20\,\text{J}$$

Using the work–kinetic energy theorem with $v_i = 0$, we set the change in kinetic energy of the block equal to the work done on it by the spring:

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s}$$

$$= \sqrt{0 + \frac{2}{1.6\text{ kg}}(0.20\text{ J})}$$

$$= \boxed{0.50\text{ m/s}}$$

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

Solution Certainly, the answer has to be less than what we found in part (A) because the friction force retards the motion. We use Equation 7.20 to calculate the kinetic energy lost because of friction and add this negative value to the kinetic energy we calculated in the absence of friction. The kinetic energy lost due to friction is

$$\Delta K = -f_k d = -(4.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.080\text{ J}$$

In part (A), the work done by the spring was found to be 0.20 J. Therefore, the final kinetic energy in the presence of friction is

$$K_f = 0.20\text{ J} - 0.080\text{ J} = 0.12\text{ J} = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(0.12\text{ J})}{1.6\text{ kg}}} = \boxed{0.39\text{ m/s}}$$

As expected, this value is somewhat less than the 0.50 m/s we found in part (A). If the friction force were greater, then the value we obtained as our answer would have been even smaller.

What If? What if the friction force were increased to 10.0 N? What is the block's speed at $x = 0$?

Answer In this case, the loss of kinetic energy as the block moves to $x = 0$ is

$$\Delta K = -f_k d = -(10.0\text{ N})(2.0 \times 10^{-2}\text{ m}) = -0.20\text{ J}$$

which is equal in magnitude to the kinetic energy at $x = 0$ without the loss due to friction. Thus, all of the kinetic energy has been transformed by friction when the block arrives at $x = 0$ and its speed at this point is $v = 0$.

In this situation as well as that in part (B), the speed of the block reaches a maximum at some position other than $x = 0$. Problem 70 asks you to locate these positions.

Investigate the role of the spring constant, amount of spring compression, and surface friction at the Interactive Worked Example link at **http://www.pse6.com.**

7.8 Power

Consider Conceptual Example 7.8 again, which involved rolling a refrigerator up a ramp into a truck. Suppose that the man is not convinced by our argument that the work is the same regardless of the length of the ramp and sets up a long ramp with a gentle rise. Although he will do the same amount of work as someone using a shorter ramp, he will take longer to do the work simply because he has to move the refrigerator over a greater distance. While the work done on both ramps is the same, there is *something* different about the tasks—the *time interval* during which the work is done.

The time rate of energy transfer is called **power.** We will focus on work as the energy transfer method in this discussion, but keep in mind that the notion of power is valid for *any* means of energy transfer. If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the **average power** during this interval is defined as

$$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

Thus, while the same work is done in rolling the refrigerator up both ramps, less power is required for the longer ramp.

In a manner similar to the way we approached the definition of velocity and acceleration, we define the **instantaneous power** $\mathcal{P}$ as the limiting value of the average power as Δt approaches zero:

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the infinitesimal value of the work done by dW. We find from Equation 7.3 that $dW = \mathbf{F} \cdot d\mathbf{r}$. Therefore, the instantaneous power can be written

Instantaneous power

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v} \tag{7.23}$$

where we use the fact that $\mathbf{v} = d\mathbf{r}/dt$.

In general, power is defined for any type of energy transfer. Therefore, the most general expression for power is

$$\mathcal{P} = \frac{dE}{dt} \tag{7.24}$$

where dE/dt is the rate at which energy is crossing the boundary of the system by a given transfer mechanism.

The SI unit of power is joules per second (J/s), also called the **watt** (W) (after James Watt):

The watt

$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^3$$

A unit of power in the U.S. customary system is the **horsepower** (hp):

$$1 \text{ hp} = 746 \text{ W}$$

PITFALL PREVENTION

7.9 W, *W*, and watts

Do not confuse the symbol W for the watt with the italic symbol W for work. Also, remember that the watt already represents a rate of energy transfer, so that "watts per second" does not make sense. The watt is *the same as* a joule per second.

A unit of energy (or work) can now be defined in terms of the unit of power. One **kilowatt-hour** (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s. The amount of energy represented by 1 kWh is

$$1 \text{ kWh} = (10^3 \text{ W})(3\,600 \text{ s}) = 3.60 \times 10^6 \text{ J}$$

Note that a kilowatt-hour is a unit of energy, not power. When you pay your electric bill, you are buying energy, and the amount of energy transferred by electrical transmission into a home during the period represented by the electric bill is usually expressed in kilowatt-hours. For example, your bill may state that you used 900 kWh of energy during a month, and you are being charged at the rate of 10¢ per kWh. Your obligation is then $90 for this amount of energy. As another example, suppose an electric bulb is rated at 100 W. In 1.00 hour of operation, it would have energy transferred to it by electrical transmission in the amount of $(0.100 \text{ kW})(1.00 \text{ h}) = 0.100 \text{ kWh} = 3.60 \times 10^5 \text{ J}$.

Quick Quiz 7.12 An older model car accelerates from rest to speed v in 10 seconds. A newer, more powerful sports car accelerates from rest to $2v$ in the same time period. What is the ratio of the power of the newer car to that of the older car? (a) 0.25 (b) 0.5 (c) 1 (d) 2 (e) 4

Example 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion upward, as shown in Figure 7.19a.

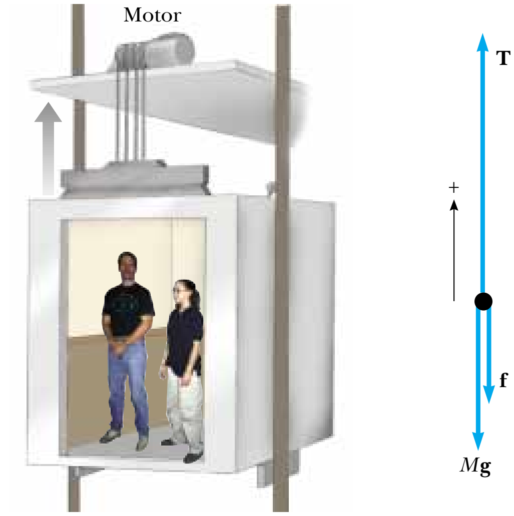
(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3.00 m/s?

Solution The motor must supply the force of magnitude T that pulls the elevator car upward. The problem states that the speed is constant, which provides the hint that $a = 0$. Therefore we know from Newton's second law that $\Sigma F_y = 0$. The free-body diagram in Figure 7.19b specifies the upward direction as positive. From Newton's second law we obtain

$$\sum F_y = T - f - Mg = 0$$

where M is the *total* mass of the system (car plus passengers), equal to 1 800 kg. Therefore,

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3 \text{ N} + (1.80 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 2.16 \times 10^4 \text{ N} \end{aligned}$$



(a) (b)

Figure 7.19 (Example 7.12) (a) The motor exerts an upward force $\mathbf{T}$ on the elevator car. The magnitude of this force is the tension T in the cable connecting the car and motor. The downward forces acting on the car are a friction force $\mathbf{f}$ and the gravitational force $\mathbf{F}_g = M\mathbf{g}$. (b) The free-body diagram for the elevator car.

Using Equation 7.23 and the fact that $\mathbf{T}$ is in the same direction as $\mathbf{v}$, we find that

$$\mathcal{P} = \mathbf{T}\cdot\mathbf{v} = Tv$$

$$= (2.16 \times 10^4\,\text{N})(3.00\,\text{m/s}) = \boxed{6.48 \times 10^4\,\text{W}}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of $1.00\,\text{m/s}^2$?

Solution We expect to obtain a value greater than we did in part (A), where the speed was constant, because the motor must now perform the additional task of accelerating the car. The only change in the setup of the problem is that in this case, $a > 0$. Applying Newton's second law to the car gives

$$\sum F_y = T - f - Mg = Ma$$

$$T = M(a + g) + f$$

$$= (1.80 \times 10^3\,\text{kg})(1.00\,\text{m/s}^2 + 9.80\,\text{m/s}^2) + 4.00 \times 10^3\,\text{N}$$

$$= 2.34 \times 10^4\,\text{N}$$

Therefore, using Equation 7.23, we obtain for the required power

$$\mathcal{P} = Tv = \boxed{(2.34 \times 10^4\,\text{N})v}$$

where v is the instantaneous speed of the car in meters per second. To compare to part (A), let $v = 3.00$ m/s, giving a power of

$$\mathcal{P} = (2.34 \times 10^4\,\text{N})(3.00\,\text{m/s}) = 7.02 \times 10^4\,\text{W}$$

This is larger than the power found in part (A), as we expect.

7.9 Energy and the Automobile

Automobiles powered by gasoline engines are very inefficient machines. Even under ideal conditions, less than 15% of the chemical energy in the fuel is used to power the vehicle. The situation is much worse than this under stop-and-go driving conditions in a city. In this section, we use the concepts of energy, power, and friction to analyze automobile fuel consumption.

Many mechanisms contribute to energy loss in an automobile. About 67% of the energy available from the fuel is lost in the engine. This energy ends up in the atmosphere, partly via the exhaust system and partly via the cooling system. (As explained in Chapter 22, energy loss from the exhaust and cooling systems is required by a fundamental law of thermodynamics.) Approximately 10% of the available energy is lost to friction in the transmission, drive shaft, wheel and axle bearings, and differential. Friction in other moving parts transforms approximately 6% of the energy to internal energy, and 4% of the energy is used to operate fuel and oil pumps and such accessories as power steering and air conditioning. This leaves a mere 13% of the available energy to propel the automobile! This energy is used mainly to balance the energy loss due to flexing of the tires and the friction caused by the air, which is more commonly referred to as *air resistance.*

Let us examine the power required to provide a force in the forward direction that balances the combination of the two friction forces. The coefficient of rolling friction μ between the tires and the road is about 0.016. For a 1 450-kg car, the weight is 14 200 N and on a horizontal roadway the force of rolling friction has a magnitude of $\mu n = \mu mg = 227$ N. As the car's speed increases, a small reduction in the normal force

Table 7.2

Friction Forces and Power Requirements for a Typical Car[a]

v(mi/h)	v(m/s)	n(N)	f_r(N)	f_a(N)	f_t(N)	$\mathcal{P} = f_t v$(kW)
0	0	14 200	227	0	227	0
20	8.9	14 100	226	48	274	2.4
40	17.9	13 900	222	192	414	7.4
60	26.8	13 600	218	431	649	17.4
80	35.8	13 200	211	767	978	35.0
100	44.7	12 600	202	1 199	1 400	62.6

[a] In this table, n is the normal force, f_r is rolling friction, f_a is air friction, f_t is total friction, and $\mathcal{P}$ is the power delivered to the wheels.

occurs as a result of decreased pressure as air flows over the top of the car. (This phenomenon is discussed in Chapter 14.) This reduction in the normal force causes a reduction in the force of rolling friction f_r with increasing speed, as the data in Table 7.2 indicate.

Now let us consider the effect of the resistive force that results from the movement of air past the car. For large objects, the resistive force f_a associated with air friction is proportional to the square of the speed (see Section 6.4) and is given by Equation 6.6:

$$f_a = \tfrac{1}{2} D\rho A v^2$$

where D is the drag coefficient, ρ is the density of air, and A is the cross-sectional area of the moving object. We can use this expression to calculate the f_a values in Table 7.2, using $D = 0.50$, $\rho = 1.20\ \text{kg/m}^3$, and $A \approx 2\ \text{m}^2$.

The magnitude of the total friction force f_t is the sum of the rolling friction force and the air resistive force:

$$f_t = f_r + f_a$$

At low speeds, rolling friction is the predominant resistive force, but at high speeds air drag predominates, as shown in Table 7.2. Rolling friction can be decreased by a reduction in tire flexing (for example, by an increase in the air pressure slightly above recommended values) and by the use of radial tires. Air drag can be reduced through the use of a smaller cross-sectional area and by streamlining the car. Although driving a car with the windows open increases air drag and thus results in a 3% decrease in mileage, driving with the windows closed and the air conditioner running results in a 12% decrease in mileage.

The total power needed to maintain a constant speed v is $f_t v$, and this is the power that must be delivered to the wheels. For example, from Table 7.2 we see that at $v = 26.8$ m/s (60 mi/h) the required power is

$$\mathcal{P} = f_t v = (649\ \text{N})(26.8\ \text{m/s}) = 17.4\ \text{kW}$$

This power can be broken down into two parts: (1) the power $f_r v$ needed to compensate for rolling friction, and (2) the power $f_a v$ needed to compensate for air drag. At $v = 26.8$ m/s, we obtain the values

$$\mathcal{P}_r = f_r v = (218\ \text{N})(26.8\ \text{m/s}) = 5.84\ \text{kW}$$

$$\mathcal{P}_a = f_a v = (431\ \text{N})(26.8\ \text{m/s}) = 11.6\ \text{kW}$$

Note that $\mathcal{P} = \mathcal{P}_r + \mathcal{P}_a$ and 67% of the power is used to compensate for air drag.

On the other hand, at $v = 44.7$ m/s(100 mi/h), $\mathcal{P}_r = 9.03$ kW, $\mathcal{P}_a = 53.6$ kW, $\mathcal{P} = 62.6$ kW and 86% of the power is associated with air drag. This shows the importance of air drag at high speeds.

Example 7.13 Gas Consumed by a Compact Car

A compact car has a mass of 800 kg, and its efficiency is rated at 18%. (That is, 18% of the available fuel energy is delivered to the wheels.) Find the amount of gasoline used to accelerate the car from rest to 27 m/s (60 mi/h). Use the fact that the energy equivalent of 1 gal of gasoline is 1.3×10^8 J.

Solution The energy required to accelerate the car from rest to a speed v is equal to its final kinetic energy, $\frac{1}{2}mv^2$:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(800 \text{ kg})(27 \text{ m/s})^2 = 2.9 \times 10^5 \text{ J}$$

If the engine were 100% efficient, each gallon of gasoline would supply 1.3×10^8 J of energy. Because the engine is only 18% efficient, each gallon delivers an energy of only $(0.18)(1.3 \times 10^8 \text{ J}) = 2.3 \times 10^7$ J. Hence, the number of gallons used to accelerate the car is

$$\text{Number of gal} = \frac{2.9 \times 10^5 \text{ J}}{2.3 \times 10^7 \text{ J/gal}} = \boxed{0.013 \text{ gal}}$$

Let us estimate that it takes 10 s to achieve the indicated speed. The distance traveled during this acceleration is

$$\Delta x = \bar{v}\Delta t = \frac{v_{xf} + v_{xi}}{2}(\Delta t) = \frac{27 \text{ m/s} + 0}{2}(10 \text{ s})$$
$$= 135 \text{ m} \approx 0.08 \text{ mi}$$

At a constant cruising speed, 0.013 gal of gasoline is sufficient to propel the car nearly 0.5 mi, over six times farther. This demonstrates the extreme energy requirements of stop-and-start driving.

Example 7.14 Power Delivered to the Wheels

Suppose the compact car in Example 7.13 has a gas mileage of 35 mi/gal at 60 mi/h. How much power is delivered to the wheels?

Solution We find the rate of gasoline consumption by dividing the speed by the gas mileage:

$$\frac{60 \text{ mi/h}}{35 \text{ mi/gal}} = 1.7 \text{ gal/h}$$

Using the fact that each gallon is equivalent to 1.3×10^8 J, we find that the total power used is

$$\mathcal{P} = (1.7 \text{ gal/h})(1.3 \times 10^8 \text{ J/gal})\left(\frac{1 \text{ h}}{3.6 \times 10^3 \text{ s}}\right)$$
$$= 62 \text{ kW}$$

Because 18% of the available power is used to propel the car, the power delivered to the wheels is $(0.18)(62 \text{ kW}) = \boxed{11 \text{ kW.}}$ This is 37% less than the 17.4-kW value obtained for the 1 450-kg car discussed in the text. Vehicle mass is clearly an important factor in power-loss mechanisms.

Example 7.15 Car Accelerating Up a Hill

Consider a car of mass m that is accelerating up a hill, as shown in Figure 7.20. An automotive engineer measures the magnitude of the total resistive force to be

$$f_t = (218 + 0.70v^2) \text{ N}$$

where v is the speed in meters per second. Determine the power the engine must deliver to the wheels as a function of speed.

Solution The forces on the car are shown in Figure 7.20, in which **F** is the force of friction from the road that propels the car; the remaining forces have their usual meaning.

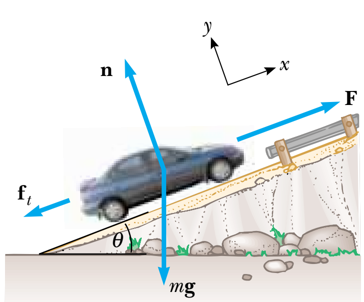


Figure 7.20 (Example 7.15) A car climbs a hill.

Applying Newton's second law to the motion along the road surface, we find that

$$\sum F_x = F - f_t - mg \sin\theta = ma$$
$$F = ma + mg\sin\theta + f_t$$
$$= ma + mg\sin\theta + (218 + 0.70v^2)$$

Therefore, the power required to move the car forward is

$$\mathcal{P} = Fv = mva + mvg\sin\theta + 218v + 0.70v^3$$

The term mva represents the power that the engine must deliver to accelerate the car. If the car moves at constant speed, this term is zero and the total power requirement is reduced. The term $mvg \sin\theta$ is the power required to provide a force to balance a component of the gravitational force as the car moves up the incline. This term would be zero for motion on a horizontal surface. The term $218v$ is the power required to provide a force to balance rolling friction, and the term $0.70v^3$ is the power needed against air drag.

If we take $m = 1\,450$ kg, $v = 27$ m/s ($= 60$ mi/h), $a = 1.0$ m/s^2, and $\theta = 10°$, then the various terms in $\mathcal{P}$ are calculated to be

$$mva = (1\,450 \text{ kg})(27 \text{ m/s})(1.0 \text{ m/s}^2)$$
$$= 39 \text{ kW} = 52 \text{ hp}$$

$$mvg \sin\theta = (1\,450 \text{ kg})(27 \text{ m/s})(9.80 \text{ m/s}^2)(\sin 10°)$$
$$= 67 \text{ kW} = 89 \text{ hp}$$
$$218v = 218(27 \text{ m/s}) = 5.9 \text{ kW} = 7.9 \text{ hp}$$
$$0.70v^3 = 0.70(27 \text{ m/s})^3 = 14 \text{ kW} = 18 \text{ hp}$$

Hence, the total power required is 126 kW or 167 hp.

Note that the power requirements for traveling at constant speed on a horizontal surface are only 20 kW, or 27 hp (the sum of the last two terms). Furthermore, if the mass were halved (as in the case of a compact car), then the power required also is reduced by almost the same factor.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

A **system** is most often a single particle, a collection of particles or a region of space. A **system boundary** separates the system from the **environment**. Many physics problems can be solved by considering the interaction of a system with its environment.

The **work** W done on a system by an agent exerting a constant force $\mathbf{F}$ on the system is the product of the magnitude Δr of the displacement of the point of application of the force and the component $F\cos\theta$ of the force along the direction of the displacement $\Delta\mathbf{r}$:

$$W \equiv F\,\Delta r\cos\theta \tag{7.1}$$

The **scalar product** (dot product) of two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined by the relationship

$$\mathbf{A}\cdot\mathbf{B} \equiv AB\cos\theta \tag{7.2}$$

where the result is a scalar quantity and θ is the angle between the two vectors. The scalar product obeys the commutative and distributive laws.

If a varying force does work on a particle as the particle moves along the x axis from x_i to x_f, the work done by the force on the particle is given by

$$W \equiv \int_{x_i}^{x_f} F_x\,dx \tag{7.7}$$

where F_x is the component of force in the x direction.

The **kinetic energy** of a particle of mass m moving with a speed v is

$$K \equiv \tfrac{1}{2}mv^2 \tag{7.15}$$

The **work–kinetic energy theorem** states that if work is done on a system by external forces and the only change in the system is in its speed, then

$$\sum W = K_f - K_i = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2} \tag{7.14, 7.16}$$

For a nonisolated system, we can equate the change in the total energy stored in the system to the sum of all the transfers of energy across the system boundary. For an isolated system, the total energy is constant—this is a statement of **conservation of energy.**

If a friction force acts, the kinetic energy of the system is reduced and the appropriate equation to be applied is

$$\Delta K = -f_k d + \sum W_{\text{other forces}} \tag{7.21a}$$

or

$$K_f = K_i - f_k d + \sum W_{\text{other forces}} \tag{7.21b}$$

The **instantaneous power** $\mathcal{P}$ is defined as the time rate of energy transfer. If an agent applies a force $\mathbf{F}$ to an object moving with a velocity $\mathbf{v}$, the power delivered by that agent is

$$\mathcal{P} \equiv \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v} \tag{7.23}$$

QUESTIONS

1. When a particle rotates in a circle, a force acts on it directed toward the center of rotation. Why is it that this force does no work on the particle?

2. Discuss whether any work is being done by each of the following agents and, if so, whether the work is positive or negative: (a) a chicken scratching the ground, (b) a person studying, (c) a crane lifting a bucket of concrete, (d) the gravitational force on the bucket in part (c), (e) the leg muscles of a person in the act of sitting down.

3. When a punter kicks a football, is he doing any work on the ball while his toe is in contact with it? Is he doing any work on the ball after it loses contact with his toe? Are any forces doing work on the ball while it is in flight?

4. Cite two examples in which a force is exerted on an object without doing any work on the object.

$\boxed{\textbf{5.}}$ As a simple pendulum swings back and forth, the forces acting on the suspended object are the gravitational force, the tension in the supporting cord, and air resistance. (a) Which of these forces, if any, does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by the gravitational force while the pendulum is swinging.

6. If the dot product of two vectors is positive, does this imply that the vectors must have positive rectangular components?

7. For what values of θ is the scalar product (a) positive and (b) negative?

8. As the load on a vertically hanging spiral spring is increased, one would not expect the F_s-versus-x graph line to remain straight, as shown in Figure 7.10d. Explain qualitatively what you would expect for the shape of this graph as the load on the spring is increased.

9. A certain uniform spring has spring constant k. Now the spring is cut in half. What is the relationship between k and the spring constant k' of each resulting smaller spring? Explain your reasoning.

$\boxed{\textbf{10.}}$ Can kinetic energy be negative? Explain.

11. Discuss the work done by a pitcher throwing a baseball. What is the approximate distance through which the force acts as the ball is thrown?

$\boxed{\textbf{12.}}$ One bullet has twice the mass of a second bullet. If both are fired so that they have the same speed, which has more kinetic energy? What is the ratio of the kinetic energies of the two bullets?

13. Two sharpshooters fire 0.30-caliber rifles using identical shells. A force exerted by expanding gases in the barrels accelerates the bullets. The barrel of rifle A is 2.00 cm longer than the barrel of rifle B. Which rifle will have the higher muzzle speed?

$\boxed{\textbf{14.}}$ (a) If the speed of a particle is doubled, what happens to its kinetic energy? (b) What can be said about the speed of a particle if the net work done on it is zero?

15. A car salesman claims that a souped-up 300-hp engine is a necessary option in a compact car, in place of the conventional 130-hp engine. Suppose you intend to drive the car within speed limits ($\leq$65 mi/h) on flat terrain. How would you counter this sales pitch?

16. Can the average power over a time interval ever be equal to the instantaneous power at an instant within the interval? Explain.

17. In Example 7.15, does the required power increase or decrease as the force of friction is reduced?

18. The kinetic energy of an object depends on the frame of reference in which its motion is measured. Give an example to illustrate this point.

19. Words given precise definitions in physics are sometimes used in popular literature in interesting ways. For example, a rock falling from the top of a cliff is said to be "gathering force as it falls to the beach below." What does the phrase "gathering force" mean, and can you repair this phrase?

20. In most circumstances, the normal force acting on an object and the force of static friction do zero work on the object. However, the reason that the work is zero is different for the two cases. Explain why each does zero work.

21. "A level air track can do no work." Argue for or against this statement.

22. Who first stated the work–kinetic energy theorem? Who showed that it is useful for solving many practical problems? Do some research to answer these questions.

PROBLEMS

1, **2**, **3** = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 7.2 Work Done by a Constant Force

$\boxed{\textbf{1.}}$ A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block.

2. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper on the cart as he moves down an aisle 50.0 m long.

$\boxed{\textbf{3.}}$ Batman, whose mass is 80.0 kg, is dangling on the free end of a 12.0-m rope, the other end of which is fixed to a tree limb above. He is able to get the rope in motion as only Batman knows how, eventually getting it to swing enough that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done by the gravitational force on Batman in this maneuver?

4. A raindrop of mass 3.35×10^{-5} kg falls vertically at constant speed under the influence of gravity and air resistance. Model the drop as a particle. As it falls 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?

Section 7.3 The Scalar Product of Two Vectors

5. Vector **A** has a magnitude of 5.00 units, and **B** has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find $\mathbf{A} \cdot \mathbf{B}$.

6. For any two vectors **A** and **B**, show that $\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$. (*Suggestion:* Write **A** and **B** in unit vector form and use Equations 7.4 and 7.5.)

Note: In Problems 7 through 10, calculate numerical answers to three significant figures as usual.

7. A force $\mathbf{F} = (6\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ N acts on a particle that undergoes a displacement $\Delta\mathbf{r} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ m. Find (a) the work done by the force on the particle and (b) the angle between **F** and $\Delta\mathbf{r}$.

8. Find the scalar product of the vectors in Figure P7.8.

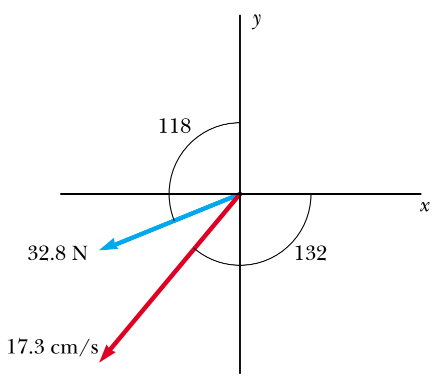


Figure P7.8

9. Using the definition of the scalar product, find the angles between (a) $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ and $\mathbf{B} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$; (b) $\mathbf{A} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$; (c) $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$.

10. For $\mathbf{A} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, and $\mathbf{C} = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, find $\mathbf{C} \cdot (\mathbf{A} - \mathbf{B})$.

Section 7.4 Work Done by a Varying Force

11. The force acting on a particle varies as in Figure P7.11. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.

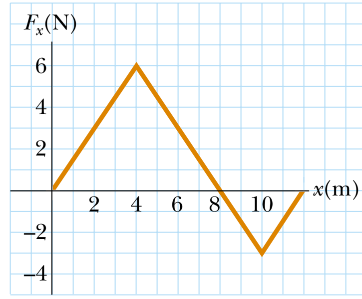


Figure P7.11

12. The force acting on a particle is $F_x = (8x - 16)$ N, where x is in meters. (a) Make a plot of this force versus x from $x = 0$ to $x = 3.00$ m. (b) From your graph, find the net work done by this force on the particle as it moves from $x = 0$ to $x = 3.00$ m.

13. A particle is subject to a force F_x that varies with position as in Figure P7.13. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 5.00$ m, (b) from $x = 5.00$ m to $x = 10.0$ m, and (c) from $x = 10.0$ m to $x = 15.0$ m. (d) What is the total work done by the force over the distance $x = 0$ to $x = 15.0$ m?

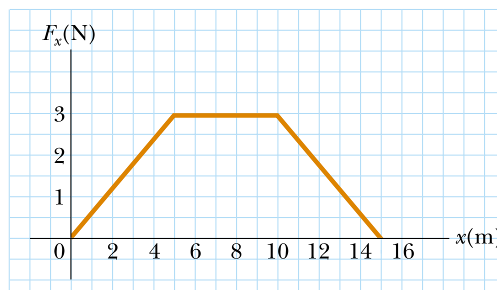


Figure P7.13 Problems 13 and 28.

14. A force $\mathbf{F} = (4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}})$ N acts on an object as the object moves in the x direction from the origin to $x = 5.00$ m. Find the work $W = \int \mathbf{F} \cdot d\mathbf{r}$ done on the object by the force.

15. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it, and (b) how much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

16. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

17. Truck suspensions often have "helper springs" that engage at high loads. One such arrangement is a leaf spring with a helper coil spring mounted on the axle, as in Figure P7.17. The helper spring engages when the main leaf spring is compressed by distance y_0, and then helps to support any additional load. Consider a leaf spring constant of 5.25×10^5 N/m, helper spring constant of 3.60×10^5 N/m, and $y_0 = 0.500$ m. (a) What is the

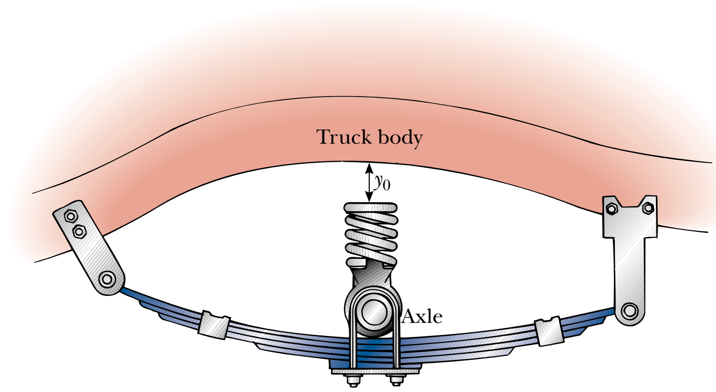


Figure P7.17

compression of the leaf spring for a load of 5.00×10^5 N? (b) How much work is done in compressing the springs?

18. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Assuming the origin is placed where the bullet begins to move, the force (in newtons) exerted by the expanding gas on the bullet is $15\,000 + 10\,000x - 25\,000x^2$, where x is in meters. (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) **What If?** If the barrel is 1.00 m long, how much work is done, and how does this value compare to the work calculated in (a)?

19. If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

20. A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder, as illustrated in Figure P7.20. (a) If the particle moves at a constant speed, show that $F = mg\cos\theta$. (*Note*: If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times.) (b) By directly integrating $W = \int \mathbf{F} \cdot d\mathbf{r}$, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.

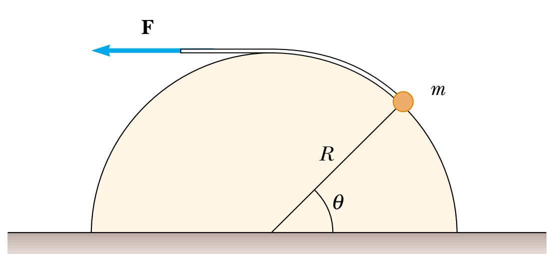


Figure P7.20

21. A light spring with spring constant 1 200 N/m is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant 1 800 N/m. An object of mass 1.50 kg is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

22. A light spring with spring constant k_1 is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant k_2. An object of mass m is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as *in series*.

23. Express the units of the force constant of a spring in SI base units.

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Section 7.6 The Nonisolated System–Conservation of Energy

24. A 0.600-kg particle has a speed of 2.00 m/s at point Ⓐ and kinetic energy of 7.50 J at point Ⓑ. What is (a) its kinetic energy at Ⓐ? (b) its speed at Ⓑ? (c) the total work done on the particle as it moves from Ⓐ to Ⓑ?

25. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) **What If?** If its speed were doubled, what would be its kinetic energy?

26. A 3.00-kg object has a velocity $(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to $(8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}})$ m/s. (*Note:* From the definition of the dot product, $v^2 = \mathbf{v} \cdot \mathbf{v}$.)

27. A 2 100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before coming into contact with the top of the beam, and it drives the beam 12.0 cm farther into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest.

28. A 4.00-kg particle is subject to a total force that varies with position as shown in Figure P7.13. The particle starts from rest at $x = 0$. What is its speed at (a) $x = 5.00$ m, (b) $x = 10.0$ m, (c) $x = 15.0$ m?

29. You can think of the work–kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, solve parts (a) and (b) separately from parts (c) and (d) to compare the predictions of the two theories. In a rifle barrel, a 15.0-g bullet is accelerated from rest to a speed of 780 m/s. (a) Find the work that is done on the bullet. (b) If the rifle barrel is 72.0 cm long, find the magnitude of the average total force that acted on it, as $F = W/(\Delta r \cos\theta)$. (c) Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (d) If the bullet has mass 15.0 g, find the total force that acted on it as $\Sigma F = ma$.

30. In the neck of the picture tube of a certain black-and-white television set, an electron gun contains two charged metallic plates 2.80 cm apart. An electric force accelerates each electron in the beam from rest to 9.60% of the speed of light over this distance. (a) Determine the kinetic energy of the electron as it leaves the electron gun. Electrons carry this energy to a phosphorescent material on the inner surface of the television screen, making it glow. For an electron passing between the plates in the electron gun, determine (b) the magnitude of the constant electric force acting on the electron, (c) the acceleration, and (d) the time of flight.

Section 7.7 Situations Involving Kinetic Friction

31. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.300, find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor system due to friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

32. A 2.00-kg block is attached to a spring of force constant 500 N/m as in Figure 7.10. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

33. A crate of mass 10.0 kg is pulled up a rough incline with an initial speed of 1.50 m/s. The pulling force is 100 N parallel to the incline, which makes an angle of 20.0° with the horizontal. The coefficient of kinetic friction is 0.400, and the crate is pulled 5.00 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate–incline system due to friction. (c) How much work is done by the 100-N force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 5.00 m?

34. A 15.0-kg block is dragged over a rough, horizontal surface by a 70.0-N force acting at 20.0° above the horizontal. The block is displaced 5.00 m, and the coefficient of kinetic friction is 0.300. Find the work done on the block by (a) the 70-N force, (b) the normal force, and (c) the gravitational force. (d) What is the increase in internal energy of the block-surface system due to friction? (e) Find the total change in the block's kinetic energy.

35. A sled of mass m is given a kick on a frozen pond. The kick imparts to it an initial speed of 2.00 m/s. The coefficient of kinetic friction between sled and ice is 0.100. Use energy considerations to find the distance the sled moves before it stops.

Section 7.8 Power

36. The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0 ms. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration.

37. A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

38. Make an order-of-magnitude estimate of the power a car engine contributes to speeding the car up to highway speed. For concreteness, consider your own car if you use one. In your solution state the physical quantities you take as data and the values you measure or estimate for them. The mass of the vehicle is given in the owner's manual. If you do not wish to estimate for a car, consider a bus or truck that you specify.

39. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. (a) How much work is required to pull him a distance of 60.0 m up a 30.0° slope (assumed frictionless) at a constant speed of 2.00 m/s? (b) A motor of what power is required to perform this task?

40. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this period? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

41. An energy-efficient lightbulb, taking in 28.0 W of power, can produce the same level of brightness as a conventional bulb operating at power 100 W. The lifetime of the energy efficient bulb is 10 000 h and its purchase price is \$17.0, whereas the conventional bulb has lifetime 750 h and costs \$0.420 per bulb. Determine the total savings obtained by using one energy-efficient bulb over its lifetime, as opposed to using conventional bulbs over the same time period. Assume an energy cost of \$0.080 0 per kilowatt-hour.

42. Energy is conventionally measured in Calories as well as in joules. One Calorie in nutrition is one kilocalorie, defined as 1 kcal = 4 186 J. Metabolizing one gram of fat can release 9.00 kcal. A student decides to try to lose weight by exercising. She plans to run up and down the stairs in a football stadium as fast as she can and as many times as necessary. Is this in itself a practical way to lose weight? To evaluate the program, suppose she runs up a flight of 80 steps, each 0.150 m high, in 65.0 s. For simplicity, ignore the energy she uses in coming down (which is small). Assume that a typical efficiency for human muscles is 20.0%. This means that when your body converts 100 J from metabolizing fat, 20 J goes into doing mechanical work (here, climbing stairs). The remainder goes into extra internal energy. Assume the student's mass is 50.0 kg. (a) How many times must she run the flight of stairs to lose one pound of fat? (b) What is her average power output, in watts and in horsepower, as she is running up the stairs?

43. For saving energy, bicycling and walking are far more efficient means of transportation than is travel by automobile. For example, when riding at 10.0 mi/h a cyclist uses food energy at a rate of about 400 kcal/h above what he would use if merely sitting still. (In exercise physiology, power is often measured in kcal/h rather than in watts. Here 1 kcal = 1 nutritionist's Calorie = 4 186 J.) Walking at 3.00 mi/h requires about 220 kcal/h. It is interesting to compare these values with the energy consumption required for travel by car. Gasoline yields about 1.30×10^8 J/gal. Find the fuel economy in equivalent miles per gallon for a person (a) walking, and (b) bicycling.

Section 7.9 Energy and the Automobile

44. Suppose the empty car described in Table 7.2 has a fuel economy of 6.40 km/liter (15 mi/gal) when traveling at 26.8 m/s (60 mi/h). Assuming constant efficiency, determine the fuel economy of the car if the total mass of passengers plus driver is 350 kg.

45. A compact car of mass 900 kg has an overall motor efficiency of 15.0%. (That is, 15% of the energy supplied by the fuel is delivered to the wheels of the car.) (a) If burning one gallon of gasoline supplies 1.34×10^8 J of energy, find the amount of gasoline used in accelerating the car from rest to 55.0 mi/h. Here you may ignore the effects of air resistance and rolling friction. (b) How many such accelerations will one gallon provide? (c) The mileage claimed for the car is 38.0 mi/gal at 55 mi/h. What power is delivered to the wheels (to overcome frictional effects) when the car is driven at this speed?

Additional Problems

46. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0°. What is the kinetic energy of the baseball at the highest point of its trajectory?

47. While running, a person dissipates about 0.600 J of mechanical energy per step per kilogram of body mass. If a 60.0-kg runner dissipates a power of 70.0 W during a race, how fast is the person running? Assume a running step is 1.50 m long.

48. The direction of any vector **A** in three-dimensional space can be specified by giving the angles α, β, and γ that the vector makes with the x, y, and z axes, respectively. If $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, (a) find expressions for $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ (these are known as *direction cosines*), and (b) show that these angles satisfy the relation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. (*Hint:* Take the scalar product of **A** with $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ separately.)

49. A 4.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval $t = 0$ to $t = 2.00$ s.

50. The spring constant of an automotive suspension spring increases with increasing load due to a spring coil that is widest at the bottom, smoothly tapering to a smaller diameter near the top. The result is a softer ride on normal road surfaces from the narrower coils, but the car does not bottom out on bumps because when the upper coils collapse, they leave the stiffer coils near the bottom to absorb the load. For a tapered spiral spring that compresses 12.9 cm with a 1 000-N load and 31.5 cm with a 5 000-N load, (a) evaluate the constants a and b in the empirical equation $F = ax^b$ and (b) find the work needed to compress the spring 25.0 cm.

51. A bead at the bottom of a bowl is one example of an object in a stable equilibrium position. When a physical system is displaced by an amount x from stable equilibrium, a restoring force acts on it, tending to return the system to its equilibrium configuration. The magnitude of the restoring force can be a complicated function of x. For example, when an ion in a crystal is displaced from its lattice site, the restoring force may not be a simple function of x. In such cases we can generally imagine the function $F(x)$ to be expressed as a power series in x, as $F(x) = -(k_1x + k_2x^2 + k_3x^3 + \ldots)$. The first term here is just Hooke's law, which describes the force exerted by a simple spring for small displacements. For small excursions from equilibrium we generally neglect the higher order terms, but in some cases it may be desirable to keep the second term as well. If we model the restoring force as $F = -(k_1x + k_2x^2)$, how much work is done in displacing the system from $x = 0$ to $x = x_{max}$ by an applied force $-F$?

52. A traveler at an airport takes an escalator up one floor, as in Figure P7.52. The moving staircase would itself carry him upward with vertical velocity component v between entry and exit points separated by height h. However, while the escalator is moving, the hurried traveler climbs the steps of the escalator at a rate of n steps/s. Assume that the height of each step is h_s. (a) Determine the amount of chemical energy converted into mechanical energy by the traveler's leg muscles during his escalator ride, given that

Ron Chapple/FPG

Figure P7.52

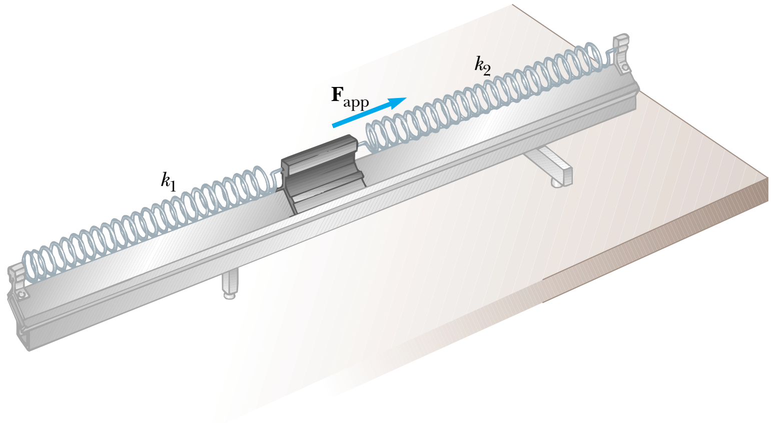


Figure P7.56

his mass is m. (b) Determine the work the escalator motor does on this person.

53. A mechanic pushes a car of mass m, doing work W in making it accelerate from rest. Neglecting friction between car and road, (a) what is the final speed of the car? During this time, the car moves a distance d. (b) What constant horizontal force did the mechanic exert on the car?

54. A 5.00-kg steel ball is dropped onto a copper plate from a height of 10.0 m. If the ball leaves a dent 3.20 mm deep, what is the average force exerted by the plate on the ball during the impact?

55. A single constant force $\mathbf{F}$ acts on a particle of mass m. The particle starts at rest at $t = 0$. (a) Show that the instantaneous power delivered by the force at any time t is $\mathcal{P} = (F^2/m)t$. (b) If $F = 20.0$ N and $m = 5.00$ kg, what is the power delivered at $t = 3.00$ s?

56. Two springs with negligible masses, one with spring constant k_1 and the other with spring constant k_2, are attached to the endstops of a level air track as in Figure P7.56. A glider attached to both springs is located between them. When the glider is in equilibrium, spring 1 is stretched by extension x_{i1} to the right of its unstretched length and spring 2 is stretched by x_{i2} to the left. Now a horizontal force $\mathbf{F}_{app}$ is applied to the glider to move it a distance x_a to the right from its equilibrium position. Show that in this process (a) the work done on spring 1 is $\frac{1}{2}k_1(x_a{}^2 + 2x_a x_{i1})$, (b) the work done on spring 2 is $\frac{1}{2}k_2(x_a{}^2 - 2x_a x_{i2})$, (c) x_{i2} is related to x_{i1} by $x_{i2} = k_1 x_{i1}/k_2$, and (d) the total work done by the force F_{app} is $\frac{1}{2}(k_1 + k_2)x_a{}^2$.

57. As the driver steps on the gas pedal, a car of mass 1 160 kg accelerates from rest. During the first few seconds of motion, the car's acceleration increases with time according to the expression

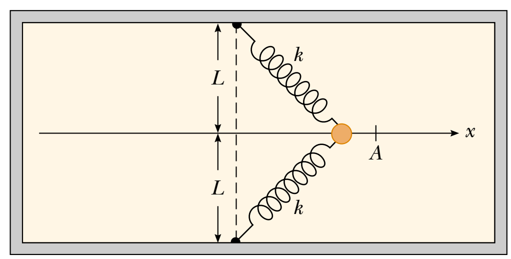
$$a = (1.16\ \mathrm{m/s^3})t - (0.210\ \mathrm{m/s^4})t^2 + (0.240\ \mathrm{m/s^5})t^3$$

(a) What work is done by the wheels on the car during the interval from $t = 0$ to $t = 2.50$ s? (b) What is the output power of the wheels at the instant $t = 2.50$ s?

58. A particle is attached between two identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P7.58, show that the force exerted by the springs on the particle is

$$\mathbf{F} = -2kx\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\hat{\mathbf{i}}$$

(b) Determine the amount of work done by this force in moving the particle from $x = A$ to $x = 0$.



Top view

Figure P7.58

59. A rocket body of mass M will fall out of the sky with terminal speed v_T after its fuel is used up. What power output must the rocket engine produce if the rocket is to fly (a) at its terminal speed straight up; (b) at three times the terminal speed straight down? In both cases assume that the mass of the fuel and oxidizer remaining in the rocket is negligible compared to M. Assume that the force of air resistance is proportional to the square of the rocket's speed.

60. **Review problem.** Two constant forces act on a 5.00-kg object moving in the xy plane, as shown in Figure P7.60. Force $\mathbf{F}_1$ is 25.0 N at 35.0°, while $\mathbf{F}_2$ is 42.0 N at 150°. At time $t = 0$, the object is at the origin and has velocity $(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}})$ m/s. (a) Express the two forces in unit-vector notation. Use unit-vector notation for your other answers. (b) Find the total force on the object. (c) Find the object's acceleration. Now, considering the instant $t = 3.00$ s, (d) find the object's velocity, (e) its location, (f) its kinetic energy from $\frac{1}{2}mv_f{}^2$, and (g) its kinetic energy from $\frac{1}{2}mv_i{}^2 + \Sigma\mathbf{F}\cdot\Delta\mathbf{r}$.

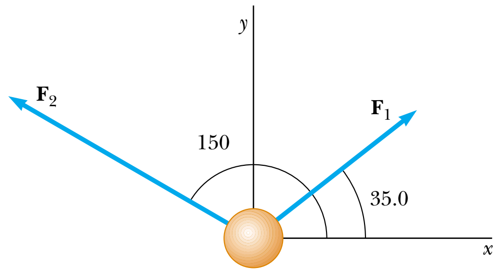


Figure P7.60

61. A 200-g block is pressed against a spring of force constant 1.40 kN/m until the block compresses the spring 10.0 cm. The spring rests at the bottom of a ramp inclined at 60.0° to the horizontal. Using energy considerations, determine how far up the incline the block moves before it stops (a) if there is no friction between the block and the ramp and (b) if the coefficient of kinetic friction is 0.400.

62. When different weights are hung on a spring, the spring stretches to different lengths as shown in the following table. (a) Make a graph of the applied force versus the extension of the spring. By least-squares fitting, determine the straight line that best fits the data. (You may not want to use all the data points.) (b) From the slope of the best-fit line, find the spring constant k. (c) If the spring is extended to 105 mm, what force does it exert on the suspended weight?

F (N)	2.0	4.0	6.0	8.0	10	12	14	16	18	20	22
L (mm)	15	32	49	64	79	98	112	126	149	175	190

63. The ball launcher in a pinball machine has a spring that has a force constant of 1.20 N/cm (Fig. P7.63). The surface on which the ball moves is inclined 10.0° with respect to the horizontal. If the spring is initially compressed 5.00 cm, find the launching speed of a 100-g ball when the plunger is released. Friction and the mass of the plunger are negligible.

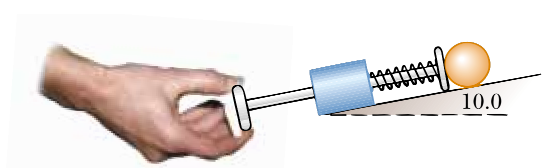


Figure P7.63

64. A 0.400-kg particle slides around a horizontal track. The track has a smooth vertical outer wall forming a circle with a radius of 1.50 m. The particle is given an initial speed of 8.00 m/s. After one revolution, its speed has dropped to 6.00 m/s because of friction with the rough floor of the track. (a) Find the energy converted from mechanical to internal in the system due to friction in one revolution. (b) Calculate the coefficient of kinetic friction. (c) What is the total number of revolutions the particle makes before stopping?

65. In diatomic molecules, the constituent atoms exert attractive forces on each other at large distances and repulsive forces at short distances. For many molecules, the Lennard-Jones law is a good approximation to the magnitude of these forces:

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right]$$

where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, we find that $F_0 = 9.60 \times 10^{-11}$ N and $\sigma = 3.50 \times 10^{-10}$ m. Determine the work done by this force if the atoms are pulled apart from $r = 4.00 \times 10^{-10}$ m to $r = 9.00 \times 10^{-10}$ m.

66. As it plows a parking lot, a snowplow pushes an ever-growing pile of snow in front of it. Suppose a car moving through the air is similarly modeled as a cylinder pushing a growing plug of air in front of it. The originally stationary air is set into motion at the constant speed v of the cylinder, as in Figure P7.66. In a time interval Δt, a new disk of air of mass Δm must be moved a distance $v\Delta t$ and hence must be given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the automobile's power loss due to air resistance is $\frac{1}{2}\rho A v^3$ and that the resistive force acting on the car is $\frac{1}{2}\rho A v^2$, where ρ is the density of air. Compare this result with the empirical expression $\frac{1}{2}D\rho A v^2$ for the resistive force.

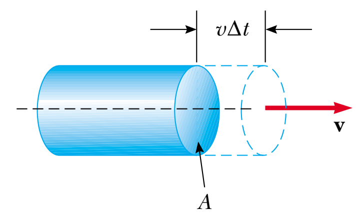


Figure P7.66

67. A particle moves along the x axis from $x = 12.8$ m to $x = 23.7$ m under the influence of a force

$$F = \frac{375}{x^3 + 3.75x}$$

where F is in newtons and x is in meters. Using numerical integration, determine the total work done by this force on the particle during this displacement. Your result should be accurate to within 2%.

68. A windmill, such as that in the opening photograph of this chapter, turns in response to a force of high-speed air resistance, $R = \frac{1}{2}D\rho A v^2$. The power available is $\mathcal{P} = Rv = \frac{1}{2}D\rho\pi r^2 v^3$, where v is the wind speed and we have assumed a circular face for the windmill, of radius r. Take the drag coefficient as $D = 1.00$ and the density of air from the front endpaper. For a home windmill with $r = 1.50$ m, calculate the power available if (a) $v = 8.00$ m/s and (b) $v = 24.0$ m/s. The power delivered to the generator is limited by the efficiency of the system, which is about 25%. For comparison, a typical home needs about 3 kW of electric power.

69. More than 2 300 years ago the Greek teacher Aristotle wrote the first book called *Physics*. Put into more precise terminology, this passage is from the end of its Section Eta:

Let $\mathcal{P}$ be the power of an agent causing motion; w, the thing moved; d, the distance covered; and Δt, the time interval required. Then (1) a power equal to $\mathcal{P}$ will in a period of time equal to Δt move $w/2$ a distance $2d$; or (2) it will move $w/2$ the given distance d in the time interval $\Delta t/2$. Also, if (3) the given power $\mathcal{P}$ moves the given object w a distance $d/2$ in time interval $\Delta t/2$, then (4) $\mathcal{P}/2$ will move $w/2$ the given distance d in the given time interval Δt.

(a) Show that Aristotle's proportions are included in the equation $\mathcal{P}\Delta t = bwd$ where b is a proportionality constant. (b) Show that our theory of motion includes this part of Aristotle's theory as one special case. In particular, describe a situation in which it is true, derive the equation representing Aristotle's proportions, and determine the proportionality constant.

70. Consider the block-spring-surface system in part (b) of Example 7.11. (a) At what position x of the block is its speed a maximum? (b) In the **What If?** section of this example, we explored the effects of an increased friction force of 10.0 N. At what position of the block does its maximum speed occur in this situation?

Answers to Quick Quizzes

7.1 (a). The force does no work on the Earth because the force is pointed toward the center of the circle and is therefore perpendicular to the direction of the displacement.

7.2 c, a, d, b. The work in (c) is positive and of the largest possible value because the angle between the force and the displacement is zero. The work done in (a) is zero because the force is perpendicular to the displacement. In (d) and (b), negative work is done by the applied force because in neither case is there a component of the force in the direction of the displacement. Situation (b) is the most negative value because the angle between the force and the displacement is 180°.

7.3 (d). Answer (a) is incorrect because the scalar product $(-\mathbf{A}) \cdot (-\mathbf{B})$ is equal to $\mathbf{A} \cdot \mathbf{B}$. Answer (b) is incorrect because $AB \cos(\theta + 180°)$ gives the negative of the correct value.

7.4 (d). Because of the range of values of the cosine function, $\mathbf{A} \cdot \mathbf{B}$ has values that range from AB to $-AB$.

7.5 (a). Because the work done in compressing a spring is proportional to the square of the compression distance x, doubling the value of x causes the work to increase fourfold.

7.6 (b). Because the work is proportional to the square of the compression distance x and the kinetic energy is proportional to the square of the speed v, doubling the compression distance doubles the speed.

7.7 (a) For the television set, energy enters by electrical transmission (through the power cord) and electromagnetic radiation (the television signal). Energy leaves by heat (from hot surfaces into the air), mechanical waves (sound from the speaker), and electromagnetic radiation (from the screen). (b) For the gasoline-powered lawn mower, energy enters by matter transfer (gasoline). Energy leaves by work (on the blades of grass), mechanical waves (sound), and heat (from hot surfaces into the air). (c) For the hand-cranked pencil sharpener, energy enters by work (from your hand turning the crank). Energy leaves by work (done on the pencil) and mechanical waves (sound).

7.8 (b). The friction force represents an interaction with the environment of the block.

7.9 (b). The friction force represents an interaction with the environment of the surface.

7.10 (a). The friction force is internal to the system, so there are no interactions with the environment.

7.11 (c). The brakes and the roadway are warmer, so their internal energy has increased. In addition, the sound of the skid represents transfer of energy away by mechanical waves.

7.12 (e). Because the speed is doubled, the kinetic energy is four times as large. This kinetic energy was attained for the newer car in the same time interval as the smaller kinetic energy for the older car, so the power is four times as large.

Chapter 8

Potential Energy

▲ *A strobe photograph of a pole vaulter. During this process, several types of energy transformations occur. The two types of potential energy that we study in this chapter are evident in the photograph.* Gravitational potential energy *is associated with the change in vertical position of the vaulter relative to the Earth.* Elastic potential energy *is evident in the bending of the pole. (©Harold E. Edgerton/Courtesy of Palm Press, Inc.)*

CHAPTER OUTLINE

In Chapter 7 we introduced the concepts of kinetic energy associated with the motion of members of a system and internal energy associated with the temperature of a system. In this chapter we introduce *potential energy*, the energy associated with the configuration of a system of objects that exert forces on each other.

The potential energy concept can be used only when dealing with a special class of forces called *conservative forces*. When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the *principle of conservation of mechanical energy*.

Potential energy is present in the Universe in various forms, including gravitational, electromagnetic, chemical, and nuclear. Furthermore, one form of energy in a system can be converted to another. For example, when a system consists of an electric motor connected to a battery, the chemical energy in the battery is converted to kinetic energy as the shaft of the motor turns. The transformation of energy from one form to another is an essential part of the study of physics, engineering, chemistry, biology, geology, and astronomy.

8.1 Potential Energy of a System

In Chapter 7, we defined a system in general, but focused our attention primarily on single particles or objects under the influence of an external force. In this chapter, we consider systems of two or more particles or objects interacting via a force that is *internal* to the system. The kinetic energy of such a system is the algebraic sum of the kinetic energies of all members of the system. There may be systems, however, in which one object is so massive that it can be modeled as stationary and its kinetic energy can be neglected. For example, if we consider a ball–Earth system as the ball falls to the ground, the kinetic energy of the system can be considered as just the kinetic energy of the ball. The Earth moves so slowly in this process that we can ignore its kinetic energy. On the other hand, the kinetic energy of a system of two electrons must include the kinetic energies of both particles.

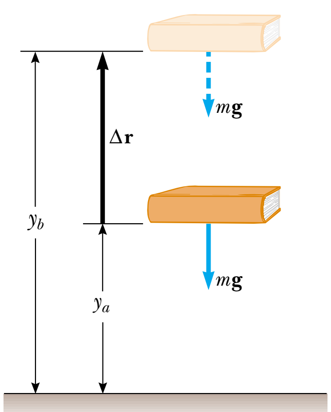


Figure 8.1 The work done by an external agent on the system of the book and the Earth as the book is lifted from a height y_a to a height y_b is equal to $mgy_b - mgy_a$.

Let us imagine a system consisting of a book and the Earth, interacting via the gravitational force. We do some work on the system by lifting the book slowly through a height $\Delta y = y_b - y_a$, as in Figure 8.1. According to our discussion of energy and energy transfer in Chapter 7, this work done on the system must appear as an increase in energy of the system. The book is at rest before we perform the work and is at rest after we perform the work. Thus, there is no change in the kinetic energy of the system. There is no reason why the temperature of the book or the Earth should change, so there is no increase in the internal energy of the system.

Because the energy change of the system is not in the form of kinetic energy or internal energy, it must appear as some other form of energy storage. After lifting the book, we could release it and let it fall back to the position y_a. Notice that the book (and, therefore, the system) will now have kinetic energy, and its source is in the work that was done in lifting the book. While the book was at the highest point, the energy of the system had the *potential* to become kinetic energy, but did not do so until the book was allowed to fall. Thus, we call the energy storage mechanism before we release the book **potential energy.** We will find that a potential energy can only be associated with specific types of forces. In this particular case, we are discussing **gravitational potential energy.**

Let us now derive an expression for the gravitational potential energy associated with an object at a given location above the surface of the Earth. Consider an external agent lifting an object of mass m from an initial height y_a above the ground to a final height y_b, as in Figure 8.1. We assume that the lifting is done slowly, with no acceleration, so that the lifting force can be modeled as being equal in magnitude to the weight of the object—the object is in equilibrium and moving at constant velocity. The work done by the external agent on the system (object and Earth) as the object undergoes this upward displacement is given by the product of the upward applied force $\mathbf{F}_{\text{app}}$ and the upward displacement $\Delta\mathbf{r} = \Delta y\hat{\mathbf{j}}$:

$$W = (\mathbf{F}_{\text{app}}) \cdot \Delta\mathbf{r} = (mg\hat{\mathbf{j}}) \cdot [(y_b - y_a)\hat{\mathbf{j}}] = mgy_b - mgy_a \tag{8.1}$$

Notice how similar this equation is to Equation 7.14 in the preceding chapter. In each equation, the work done on a system equals a difference between the final and initial values of a quantity. In Equation 7.14, the work represents a transfer of energy into the system, and the increase in energy of the system is kinetic in form. In Equation 8.1, the work represents a transfer of energy into the system, and the system energy appears in a different form, which we have called gravitational potential energy.

Thus, we can identify the quantity mgy as the gravitational potential energy U_g:

$$U_g \equiv mgy \tag{8.2}$$

Gravitational potential energy

The units of gravitational potential energy are joules, the same as those of work and kinetic energy. Potential energy, like work and kinetic energy, is a scalar quantity. Note that Equation 8.2 is valid only for objects near the surface of the Earth, where g is approximately constant.[1]

Using our definition of gravitational potential energy, Equation 8.1 can now be rewritten as

$$W = \Delta U_g \tag{8.3}$$

which mathematically describes the fact that the work done on the system in this situation appears as a change in the gravitational potential energy of the system.

The gravitational potential energy depends only on the vertical height of the object above the surface of the Earth. The same amount of work must be done on an object–Earth system whether the object is lifted vertically from the Earth or is pushed starting from the same point up a frictionless incline, ending up at the same height. This can be shown by calculating the work with a displacement having both vertical and horizontal components:

$$W = (\mathbf{F}_{\text{app}}) \cdot \Delta\mathbf{r} = (mg\hat{\mathbf{j}}) \cdot [(x_b - x_a)\hat{\mathbf{i}} + (y_b - y_a)\hat{\mathbf{j}}] = mgy_b - mgy_a$$

where there is no term involving x in the final result because $\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$.

In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, which is normally zero. The choice of reference configuration is completely arbitrary because the important quantity is the *difference* in potential energy and this difference is independent of the choice of reference configuration.

It is often convenient to choose as the reference configuration for zero potential energy the configuration in which an object is at the surface of the Earth, but this is not essential. Often, the statement of the problem suggests a convenient configuration to use.

PITFALL PREVENTION

8.1 Potential Energy Belongs to a System

Potential energy is always associated with a *system* of two or more interacting objects. When a small object moves near the surface of the Earth under the influence of gravity, we may sometimes refer to the potential energy "associated with the object" rather than the more proper "associated with the system" because the Earth does not move significantly. We will not, however, refer to the potential energy "of the object" because this clearly ignores the role of the Earth.

[1] The assumption that g is constant is valid as long as the vertical displacement is small compared with the Earth's radius.

Quick Quiz 8.1 Choose the correct answer. The gravitational potential energy of a system (a) is always positive (b) is always negative (c) can be negative or positive.

Quick Quiz 8.2 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the kinetic energy of the system, we (a) must include the kinetic energy of both the object and the Earth (b) can ignore the kinetic energy of the Earth because it is not part of the system (c) can ignore the kinetic energy of the Earth because the Earth is so massive compared to the object.

Quick Quiz 8.3 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the potential energy of the system, we identify the system as (a) both the object and the Earth (b) only the object (c) only the Earth.

Example 8.1 The Bowler and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the ball–Earth system as the ball falls. Repeat the calculation, using the top of the bowler's head as the origin of coordinates.

Solution First, we need to estimate a few values. A bowling ball has a mass of approximately 7 kg, and the top of a person's toe is about 0.03 m above the floor. Also, we shall assume the ball falls from a height of 0.5 m. Keeping nonsignificant digits until we finish the problem, we calculate the gravitational potential energy of the ball–Earth system just before the ball is released to be $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(0.5\text{ m}) = 34.3\text{ J}$. A similar calculation for when the ball reaches his toe gives $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(0.03\text{ m}) = 2.06\text{ J}$. So, the change in gravitational potential energy of the ball–Earth system is $\Delta U_g = U_f - U_i = -32.24\text{ J}$. We should probably keep only one digit because of the roughness of our estimates; thus, we estimate that the change in gravitational potential energy is -30 J. The system had 30 J of gravitational potential energy relative to the top of the toe before the ball began its fall.

When we use the bowler's head (which we estimate to be 1.50 m above the floor) as our origin of coordinates, we find that $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(-1\text{ m}) = -68.6\text{ J}$ and $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(-1.47\text{ m}) = -100.8\text{ J}$. The change in gravitational potential energy of the ball–Earth system is $\Delta U_g = U_f - U_i = -32.24\text{ J} \approx \boxed{-30\text{ J}}$. This is the same value as before, as it must be.

8.2 The Isolated System–Conservation of Mechanical Energy

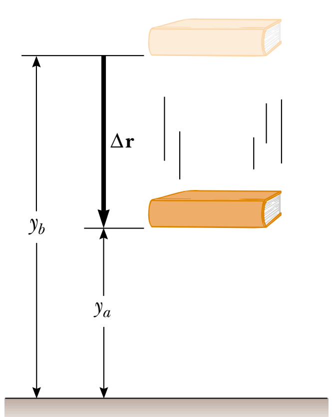


Figure 8.2 The work done by the gravitational force on the book as the book falls from y_b to a height y_a is equal to $mgy_b - mgy_a$.

The introduction of potential energy allows us to generate a powerful and universally applicable principle for solving problems that are difficult to solve with Newton's laws. Let us develop this new principle by thinking about the book–Earth system in Figure 8.1 again. After we have lifted the book, there is gravitational potential energy stored in the system, which we can calculate from the work done by the external agent on the system, using $W = \Delta U_g$.

Let us now shift our focus to the work done on the book alone by the gravitational force (Fig. 8.2) as the book falls back to its original height. As the book falls from y_b to y_a, the work done by the gravitational force on the book is

$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} = (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] = mgy_b - mgy_a \qquad (8.4)$$

From the work–kinetic energy theorem of Chapter 7, the work done on the book is equal to the change in the kinetic energy of the book:

$$W_{\text{on book}} = \Delta K_{\text{book}}$$

Therefore, equating these two expressions for the work done on the book,

$$\Delta K_{\text{book}} = mgy_b - mgy_a \tag{8.5}$$

Now, let us relate each side of this equation to the *system* of the book and the Earth. For the right-hand side,

$$mgy_b - mgy_a = -(mgy_a - mgy_b) = -(U_f - U_i) = -\Delta U_g$$

where U_g is the gravitational potential energy of the system. For the left-hand side of Equation 8.5, because the book is the only part of the system that is moving, we see that $\Delta K_{\text{book}} = \Delta K$, where K is the kinetic energy of the system. Thus, with each side of Equation 8.5 replaced with its system equivalent, the equation becomes

$$\Delta K = -\Delta U_g \tag{8.6}$$

This equation can be manipulated to provide a very important general result for solving problems. First, we bring the change in potential energy to the left side of the equation:

$$\Delta K + \Delta U_g = 0 \tag{8.7}$$

On the left, we have a sum of changes of the energy stored in the system. The right hand is zero because there are no transfers of energy across the boundary of the system—the book–Earth system is *isolated* from the environment.

We define the sum of kinetic and potential energies as **mechanical energy:**

$$E_{\text{mech}} = K + U_g$$

We will encounter other types of potential energy besides gravitational later in the text, so we can write the general form of the definition for mechanical energy without a subscript on U:

$$\boxed{E_{\text{mech}} \equiv K + U} \tag{8.8}$$

Mechanical energy of a system

where U represents the total of *all* types of potential energy.

Let us now write the changes in energy in Equation 8.7 explicitly:

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$\boxed{K_f + U_f = K_i + U_i} \tag{8.9}$$

The mechanical energy of an isolated, friction-free system is conserved.

For the gravitational situation that we have described, Equation 8.9 can be written as

$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i$$

As the book falls to the Earth, the book–Earth system loses potential energy and gains kinetic energy, such that the total of the two types of energy always remains constant.

Equation 8.9 is a statement of **conservation of mechanical energy** for an **isolated system.** An isolated system is one for which there are no energy transfers across the boundary. The energy in such a system is conserved—the sum of the kinetic and potential energies remains constant. (This statement assumes that no *nonconservative forces* act within the system; see Pitfall Prevention 8.2.)

Quick Quiz 8.4 In an isolated system, which of the following is a correct statement of the quantity that is conserved? (a) kinetic energy (b) potential energy (c) kinetic energy plus potential energy (d) both kinetic energy and potential energy.

PITFALL PREVENTION

8.2 Conditions on Equation 8.6

Equation 8.6 is true for only one of two categories of forces. These forces are called *conservative forces,* as discussed in the next section.

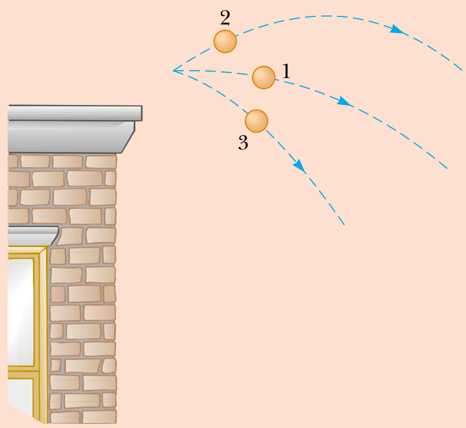
PITFALL PREVENTION

8.3 Mechanical Energy in an Isolated System

Equation 8.9 is not the only statement we can make for an isolated system. This describes conservation of *mechanical energy only* for the isolated system. We will see shortly how to include internal energy. In later chapters, we will generate new conservation statements (and associated equations) related to other conserved quantities.

Quick Quiz 8.5 A rock of mass m is dropped to the ground from a height h. A second rock, with mass $2m$, is dropped from the same height. When the second rock strikes the ground, its kinetic energy is (a) twice that of the first rock (b) four times that of the first rock (c) the same as that of the first rock (d) half as much as that of the first rock (e) impossible to determine.

Quick Quiz 8.6 Three identical balls are thrown from the top of a building, all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in Figure 8.3. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.



Active Figure 8.3 (Quick Quiz 8.6) Three identical balls are thrown with the same initial speed from the top of a building.

At the Active Figures link at http://www.pse6.com, you can throw balls at different angles from the top of the building and compare the trajectories and the speeds as the balls hit the ground.

Elastic Potential Energy

We are familiar now with gravitational potential energy; let us explore a second type of potential energy. Consider a system consisting of a block plus a spring, as shown in Figure 8.4. The force that the spring exerts on the block is given by $F_s = -kx$. In the previous chapter, we learned that the work done by an external applied force F_{app} on a system consisting of a block connected to the spring is given by Equation 7.12:

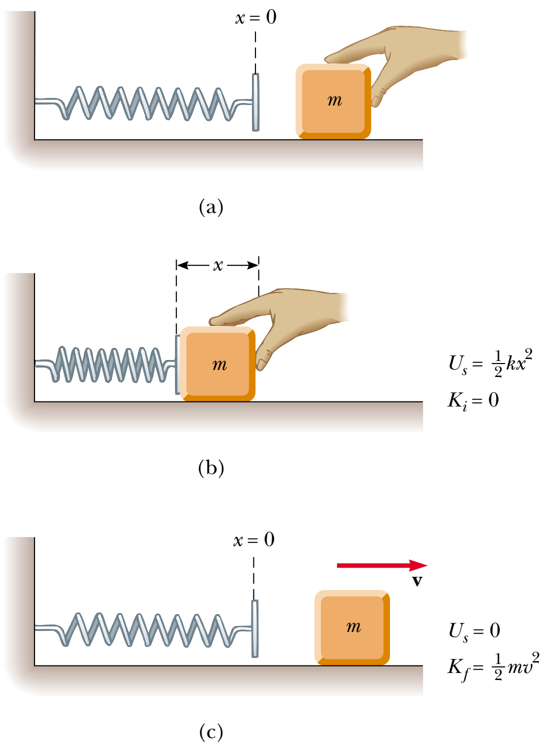
$$W_{F_{\text{app}}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \qquad (8.10)$$

In this situation, the initial and final x coordinates of the block are measured from its equilibrium position, $x = 0$. Again (as in the gravitational case), we see that the work done on the system is equal to the difference between the initial and final values of an expression related to the configuration of the system. The **elastic potential energy** function associated with the block–spring system is defined by

Elastic potential energy stored in a spring

$$U_s \equiv \frac{1}{2}kx^2 \qquad (8.11)$$

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). To visualize this, consider Figure 8.4, which shows a spring on a frictionless, horizontal surface. When a block is pushed against the spring (Fig. 8.4b) and the spring is compressed a distance x, the elastic potential energy stored in the spring is $\frac{1}{2}kx^2$.



Active Figure 8.4 (a) An undeformed spring on a frictionless horizontal surface. (b) A block of mass m is pushed against the spring, compressing it a distance x. (c) When the block is released from rest, the elastic potential energy stored in the spring is transferred to the block in the form of kinetic energy.

At the Active Figures link at http://www.pse6.com, you can compress the spring by varying amounts and observe the effect on the block's speed.

When the block is released from rest, the spring exerts a force on the block and returns to its original length. The stored elastic potential energy is transformed into kinetic energy of the block (Fig. 8.4c).

The elastic potential energy stored in a spring is zero whenever the spring is undeformed ($x = 0$). Energy is stored in the spring only when the spring is either stretched or compressed. Furthermore, the elastic potential energy is a maximum when the spring has reached its maximum compression or extension (that is, when $|x|$ is a maximum). Finally, because the elastic potential energy is proportional to x^2, we see that U_s is always positive in a deformed spring.

Quick Quiz 8.7 A ball is connected to a light spring suspended vertically, as shown in Figure 8.5. When displaced downward from its equilibrium position and released, the ball oscillates up and down. In the system of *the ball, the spring, and the Earth,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.

Quick Quiz 8.8 Consider the situation in Quick Quiz 8.7 once again. In the system of *the ball and the spring,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.

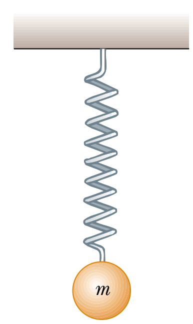


Figure 8.5 (Quick Quizzes 8.7 and 8.8) A ball connected to a massless spring suspended vertically. What forms of potential energy are associated with the system when the ball is displaced downward?

PROBLEM-SOLVING HINTS

Isolated Systems–Conservation of Mechanical Energy

We can solve many problems in physics using the principle of conservation of mechanical energy. You should incorporate the following procedure when you apply this principle:

- Define your isolated system, which may include two or more interacting particles, as well as springs or other structures in which elastic potential energy can be stored. Be sure to include all components of the system that exert forces on each other. Identify the initial and final configurations of the system.
- Identify configurations for zero potential energy (both gravitational and spring). If there is more than one force acting within the system, write an expression for the potential energy associated with each force.
- If friction or air resistance is present, mechanical energy of the system is not conserved and the techniques of Section 8.4 must be employed.
- If mechanical energy of the system is conserved, you can write the total energy $E_i = K_i + U_i$ for the initial configuration. Then, write an expression for the total energy $E_f = K_f + U_f$ for the final configuration that is of interest. Because mechanical energy is conserved, you can equate the two total energies and solve for the quantity that is unknown.

Example 8.2 Ball in Free Fall **Interactive**

A ball of mass m is dropped from a height h above the ground, as shown in Figure 8.6.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

Solution Figure 8.6 and our everyday experience with falling objects allow us to conceptualize the situation. While we can readily solve this problem with the techniques of Chapter 2, let us take an energy approach and categorize this as an energy problem for practice. To analyze the problem, we identify the system as the ball and the Earth. Because there is no air resistance and the system is isolated, we apply the principle of conservation of mechanical energy to the ball–Earth system.

At the instant the ball is released, its kinetic energy is $K_i = 0$ and the potential energy of the system is $U_i = mgh$. When the ball is at a distance y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^2$ and the potential energy relative to the ground is $U_f = mgy$. Applying Equation 8.9, we obtain

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + mgy = 0 + mgh$$

$$v_f^2 = 2g(h - y)$$

$$v_f = \boxed{\sqrt{2g(h - y)}}$$

The speed is always positive. If we had been asked to find the ball's velocity, we would use the negative value of the square root as the y component to indicate the downward motion.

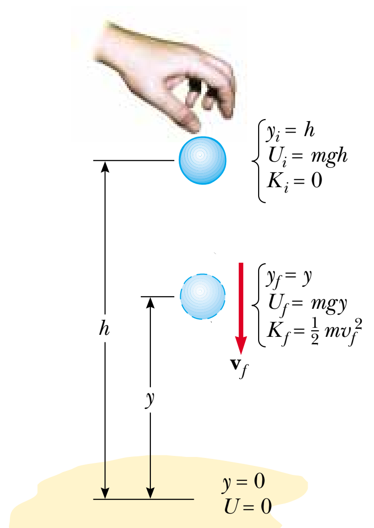


Figure 8.6 (Example 8.2) A ball is dropped from a height h above the ground. Initially, the total energy of the ball–Earth system is potential energy, equal to mgh relative to the ground. At the elevation y, the total energy is the sum of the kinetic and potential energies.

(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

Solution In this case, the initial energy includes kinetic energy equal to $\frac{1}{2}mv_i^2$ and Equation 8.9 gives

$$\tfrac{1}{2}mv_f^2 + mgy = \tfrac{1}{2}mv_i^2 + mgh$$

$$v_f{}^2 = v_i{}^2 + 2g(h - y)$$

$$v_f = \boxed{\sqrt{v_i{}^2 + 2g(h - y)}}$$

Note that this result is consistent with the expression $v_{yf}{}^2 = v_{yi}{}^2 - 2g(y_f - y_i)$ from kinematics, where $y_i = h$. Furthermore, this result is valid even if the initial velocity is at an angle to the horizontal (Quick Quiz 8.6) for two reasons: (1) energy is a scalar, and the kinetic energy depends only on the magnitude of the velocity; and (2) the change in the gravitational potential energy depends only on the change in position in the vertical direction.

What If? **What if the initial velocity v_i in part (B) were downward? How would this affect the speed of the ball at position y?**

Answer We might be tempted to claim that throwing it downward would result in it having a higher speed at y than if we threw it upward. Conservation of mechanical energy, however, depends on kinetic and potential energies, which are scalars. Thus, the direction of the initial velocity vector has no bearing on the final speed.

Compare the effect of upward, downward, and zero initial velocities at the Interactive Worked Example link at **http://www.pse6.com.**

Example 8.3 The Pendulum

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest at point Ⓐ when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

(A) Find the speed of the sphere when it is at the lowest point Ⓑ.

Solution The only force that does work on the sphere is the gravitational force. (The force applied by the cord is always perpendicular to each element of the displacement and hence does no work.) Because the pendulum–Earth system is isolated, the energy of the system is conserved. As the pendulum swings, continuous transformation between potential and kinetic energy occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point Ⓑ the pendulum has kinetic energy, but the system has lost some potential energy. At Ⓒ the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

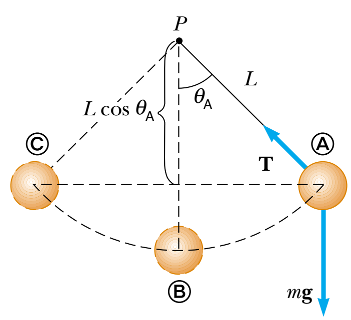


Figure 8.7 (Example 8.3) If the sphere is released from rest at the angle θ_A, it will never swing above this position during its motion. At the start of the motion, when the sphere is at position Ⓐ, the energy of the sphere–Earth system is entirely potential. This initial potential energy is transformed into kinetic energy when the sphere is at the lowest elevation Ⓑ. As the sphere continues to move along the arc, the energy again becomes entirely potential energy when the sphere is at Ⓒ.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$.

Applying the principle of conservation of mechanical energy to the system gives

$$K_B + U_B = K_A + U_A$$

$$\tfrac{1}{2}mv_B{}^2 - mgL = 0 - mgL\cos\theta_A$$

$$(1) \qquad \boxed{v_B = \sqrt{2gL(1 - \cos\theta_A)}}$$

(B) What is the tension T_B in the cord at Ⓑ?

Solution Because the tension force does no work, it does not enter into an energy equation, and we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, Newton's second law gives

$$(2) \qquad \sum F_r = mg - T_B = ma_r = -m\frac{v_B{}^2}{L}$$

Substituting Equation (1) into Equation (2) gives the tension at point Ⓑ as a function of θ_A:

$$(3) \qquad T_B = mg + 2mg(1 - \cos\theta_A) = \boxed{mg(3 - 2\cos\theta_A)}$$

From Equation (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, Equation (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$. Note also that part (A) of this example is categorized as an energy problem while part (B) is categorized as a Newton's second law problem.

Example 8.4 A Grand Entrance

Interactive

You are designing an apparatus to support an actor of mass 65 kg who is to "fly" down to the stage during the performance of a play. You attach the actor's harness to a 130-kg sandbag by means of a lightweight steel cable running smoothly over two frictionless pulleys, as in Figure 8.8a. You need 3.0 m of cable between the harness and the nearest pulley so that the pulley can be hidden behind a curtain. For the apparatus to work successfully, the sandbag must

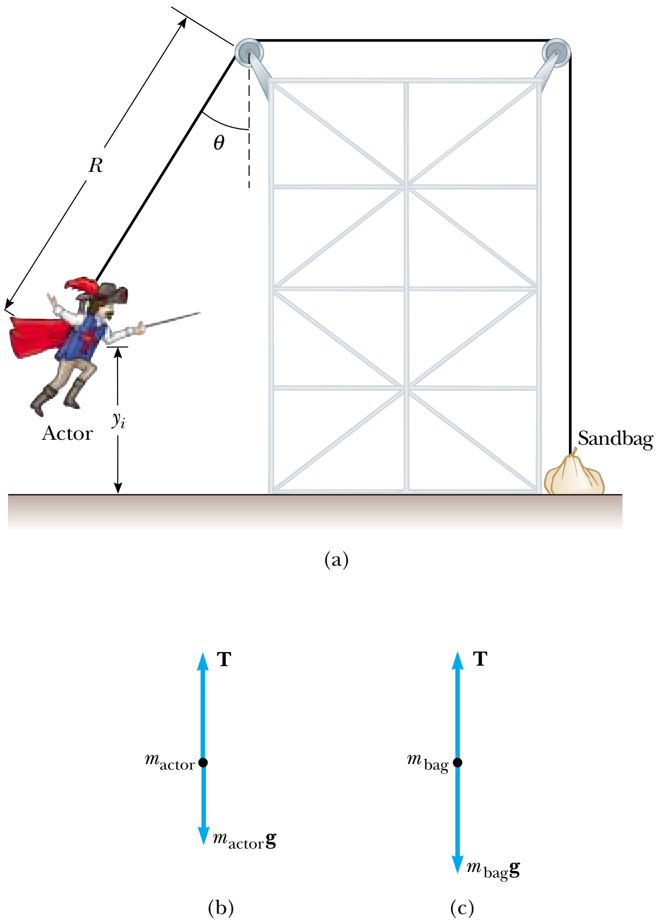


Figure 8.8 (Example 8.4) (a) An actor uses some clever staging to make his entrance. (b) Free-body diagram for the actor at the bottom of the circular path. (c) Free-body diagram for the sandbag.

never lift above the floor as the actor swings from above the stage to the floor. Let us call the initial angle that the actor's cable makes with the vertical θ. What is the maximum value θ can have before the sandbag lifts off the floor?

Solution We must use several concepts to solve this problem. To conceptualize, imagine what happens as the actor approaches the bottom of the swing. At the bottom, the cable is vertical and must support his weight as well as provide centripetal acceleration of his body in the upward direction. At this point, the tension in the cable is the highest and the sandbag is most likely to lift off the floor. Looking first at the swinging of the actor from the initial point to the lowest point, we categorize this as an energy problem involving an isolated system—the actor and the Earth. We use the principle of conservation of mechanical energy for the system to find the actor's speed as he arrives at the floor as a function of the initial angle θ and the radius R of the circular path through which he swings.

Applying conservation of mechanical energy to the actor–Earth system gives

$$K_f + U_f = K_i + U_i$$

$$(1) \qquad \tfrac{1}{2} m_{\text{actor}} v_f^{\,2} + 0 = 0 + m_{\text{actor}} g y_i$$

where y_i is the initial height of the actor above the floor and v_f is the speed of the actor at the instant before he lands. (Note that $K_i = 0$ because he starts from rest and that $U_f = 0$ because we define the configuration of the actor at the floor as having a gravitational potential energy of zero.) From the geometry in Figure 8.8a, and noting that $y_f = 0$, we see that $y_i = R - R\cos\theta = R(1 - \cos\theta)$. Using this relationship in Equation (1), we obtain

$$(2) \qquad v_f^{\,2} = 2gR(1 - \cos\theta)$$

Next, we focus on the instant the actor is at the lowest point. Because the tension in the cable is transferred as a force applied to the sandbag, we categorize the situation at this instant as a Newton's second law problem. We apply Newton's second law to the actor at the bottom of his path, using the free-body diagram in Figure 8.8b as a guide:

$$\sum F_y = T - m_{\text{actor}} g = m_{\text{actor}} \frac{v_f^{\,2}}{R}$$

$$(3) \qquad T = m_{\text{actor}} g + m_{\text{actor}} \frac{v_f^{\,2}}{R}$$

Finally, we note that the sandbag lifts off the floor when the upward force exerted on it by the cable exceeds the gravitational force acting on it; the normal force is zero when this happens. Thus, when we focus our attention on the sandbag, we categorize this part of the situation as another Newton's second law problem. A force T of the magnitude given by Equation (3) is transmitted by the cable to the sandbag. If the sandbag is to be just lifted off the floor, the normal force on it becomes zero and we require that $T = m_{\text{bag}} g$, as in Figure 8.8c. Using this condition together with Equations (2) and (3), we find that

$$m_{\text{bag}} g = m_{\text{actor}} g + m_{\text{actor}} \frac{2gR(1 - \cos\theta)}{R}$$

Solving for $\cos\theta$ and substituting in the given parameters, we obtain

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}} = \frac{3(65\text{ kg}) - 130\text{ kg}}{2(65\text{ kg})} = 0.50$$

$$\theta = 60^\circ$$

Note that we had to combine techniques from different areas of our study—energy and Newton's second law. Furthermore, we see that the length R of the cable from the actor's harness to the leftmost pulley did not appear in the final algebraic equation. Thus, the final answer is independent of R.

What If? **What if a stagehand locates the sandbag so that the cable from the sandbag to the right-hand pulley in Figure 8.8a is not vertical but makes an angle ϕ with the vertical? If the actor swings from the angle found in the solution above, will the sandbag lift off the floor? Assume that the length R remains the same.**

Answer In this situation, the gravitational force acting on the sandbag is no longer parallel to the cable. Thus, only a component of the force in the cable acts against the gravitational force, and the vertical resultant of this force component and the gravitational force should be downward. As a result, there should be a nonzero normal force to balance this resultant, and the sandbag should *not* lift off the floor.

If the sandbag is in equilibrium in the y direction and the normal force from the floor goes to zero, Newton's second law gives us $T\cos\phi = m_{\text{bag}}g$. In this case, Equation (3) gives

$$\frac{m_{\text{bag}}g}{\cos\phi} = m_{\text{actor}}g + m_{\text{actor}}\frac{v_f{}^2}{R}$$

Substituting for v_f from Equation (2) gives

$$\frac{m_{\text{bag}}g}{\cos\phi} = m_{\text{actor}}g + m_{\text{actor}}\frac{2gR(1-\cos\theta)}{R}$$

Solving for $\cos\theta$, we have

$$(4) \qquad \cos\theta = \frac{3m_{\text{actor}} - \dfrac{m_{\text{bag}}}{\cos\phi}}{2m_{\text{actor}}}$$

For $\phi = 0$, which is the situation in Figure 8.8a, $\cos\phi = 1$. For nonzero values of ϕ, the term $\cos\phi$ is smaller than 1. This makes the numerator of the fraction in Equation (4) smaller, which makes the angle θ larger. Thus, the sandbag remains on the floor if the actor swings from a larger angle. If he swings from the original angle, the sandbag remains on the floor. For example, suppose $\phi = 10°$. Then, Equation (4) gives

$$\cos\theta = \frac{3(65\text{ kg}) - \dfrac{130\text{ kg}}{\cos 10°}}{2(65\text{ kg})} = 0.48 \longrightarrow \theta = 61°$$

Thus, if he swings from 60°, he is swinging from an angle below the new maximum allowed angle, and the sandbag remains on the floor.

One factor we have not addressed is the friction force between the sandbag and the floor. If this is not large enough, the sandbag may break free and start to slide horizontally as the actor reaches some point in his swing. This will cause the length R to increase, and the actor may have a frightening moment as he begins to drop in addition to swinging!

Let the actor fly or crash without injury to people at the Interactive Worked Example link at* http://www.pse6.com. *You may choose to include the effect of friction between the sandbag and the floor.

Example 8.5 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.9a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

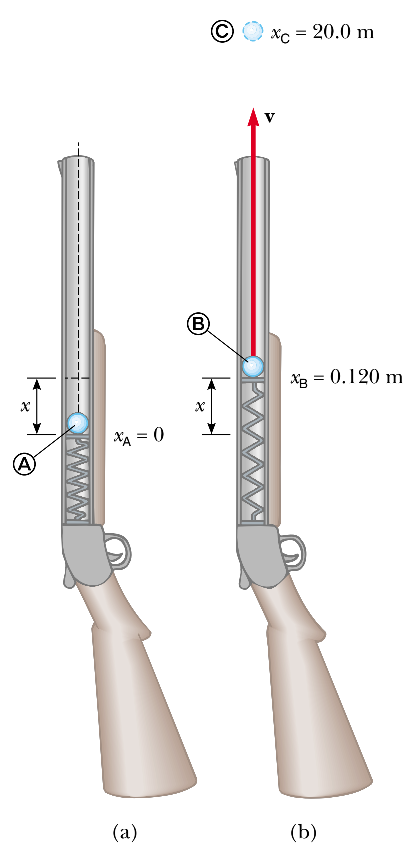


Figure 8.9 (Example 8.5) A spring-loaded popgun.

(A) Neglecting all resistive forces, determine the spring constant.

Solution Because the projectile starts from rest, its initial kinetic energy is zero. If we take the zero configuration for the gravitational potential energy of the projectile–spring–Earth system to be when the projectile is at the lowest position x_A, then the initial gravitational potential energy of the system also is zero. The mechanical energy of this system is conserved because the system is isolated.

Initially, the only mechanical energy in the system is the elastic potential energy stored in the spring of the gun, $U_{sA} = \frac{1}{2}kx^2$, where the compression of the spring is $x = 0.120$ m. The projectile rises to a maximum height $x_C = h = 20.0$ m, and so the final gravitational potential energy of the system when the projectile reaches its peak is mgh. The final kinetic energy of the projectile is zero, and the final elastic potential energy stored in the spring is zero. Because the mechanical energy of the system is conserved, we find that

$$E_C = E_A$$

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + mgh + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2} = \frac{2(0.035\,0\text{ kg})(9.80\text{ m/s}^2)(20.0\text{ m})}{(0.120\text{ m})^2}$$

$$= \boxed{953\text{ N/m}}$$

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where $x_B = 0.120$ m) as shown in Figure 8.9b.

Solution As already noted, the only mechanical energy in the system at Ⓐ is the elastic potential energy $\frac{1}{2}kx^2$. The total energy of the system as the projectile moves through the equilibrium position of the spring includes the kinetic energy of the projectile $\frac{1}{2}mv_B{}^2$ and the gravitational potential energy mgx_B of the system. Hence, the principle of conservation of mechanical energy in this case gives

$$E_B = E_A$$

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\frac{1}{2}mv_B{}^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

Solving for v_B gives

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B}$$

$$= \sqrt{\frac{(953 \text{ N/m})(0.120 \text{ m})^2}{(0.0350 \text{ kg})} - 2(9.80 \text{ m/s}^2)(0.120 \text{ m})}$$

$$= \boxed{19.7 \text{ m/s}}$$

8.3 Conservative and Nonconservative Forces

As an object moves downward near the surface of the Earth, the work done by the gravitational force on the object does not depend on whether it falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. However, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to friction forces. We can use this varying dependence on path to classify forces as either conservative or nonconservative.

Of the two forces just mentioned, the gravitational force is conservative and the friction force is nonconservative.

Conservative Forces

Conservative forces have these two equivalent properties:

Properties of a conservative force

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

PITFALL PREVENTION

8.4 Similar Equation Warning

Compare Equation 8.12 to Equation 8.3. These equations are similar except for the negative sign, which is a common source of confusion. Equation 8.3 tells us that the work done *by an outside agent* on a system causes an increase in the potential energy of the system (with no change in the kinetic or internal energy). Equation 8.12 states that work done *on a component of a system by a conservative force internal to an isolated system* causes a decrease in the potential energy of the system (with a corresponding increase in kinetic energy).

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another. As we learned in the preceding section, the work done by the gravitational force on an object moving between any two points near the Earth's surface is $W_g = mgy_i - mgy_f$. From this equation, we see that W_g depends only on the initial and final y coordinates of the object and hence is independent of the path. Furthermore, W_g is zero when the object moves over any closed path (where $y_i = y_f$).

For the case of the object–spring system, the work W_s done by the spring force is given by $W_s = \frac{1}{2}kx_i{}^2 - \frac{1}{2}kx_f{}^2$ (Eq. 7.11). Again, we see that the spring force is conservative because W_s depends only on the initial and final x coordinates of the object and is zero for any closed path.

We can associate a potential energy for a system with any conservative force acting between members of the system and can do this only for conservative forces. In the previous section, the potential energy associated with the gravitational force was defined as $U_g \equiv mgy$. In general, the work W_c done by a conservative force on an object that is a member of a system as the object moves from one position to another is equal to the initial value of the potential energy of the system minus the final value:

$$W_c = U_i - U_f = -\Delta U \tag{8.12}$$

This equation should look familiar to you. It is the general form of the equation for work done by the gravitational force (Eq. 8.4) as an object moves relative to the Earth and that for the work done by the spring force (Eq. 7.11) as the extension of the spring changes.

Nonconservative Forces

A force is **nonconservative** if it does not satisfy properties 1 and 2 for conservative forces. Nonconservative forces acting within a system cause a *change* in the mechanical energy E_{mech} of the system. We have defined mechanical energy as the sum of the kinetic and all potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases. As a result of the friction force, the temperatures of the book and surface increase. The type of energy associated with temperature is internal energy, which we introduced in Chapter 7. Only part of the book's kinetic energy is transformed to internal energy in the book. The rest appears as internal energy in the surface. (When you trip and fall while running across a gymnasium floor, not only does the skin on your knees warm up, so does the floor!) Because the force of kinetic friction transforms the mechanical energy of a system into internal energy, it is a nonconservative force.

As an example of the path dependence of the work, consider Figure 8.10. Suppose you displace a book between two points on a table. If the book is displaced in a straight line along the blue path between points Ⓐ and Ⓑ in Figure 8.10, you do a certain amount of work against the kinetic friction force to keep the book moving at a constant speed. Now, imagine that you push the book along the brown semicircular path in Figure 8.10. You perform more work against friction along this longer path than along the straight path. The work done depends on the path, so the friction force cannot be conservative.

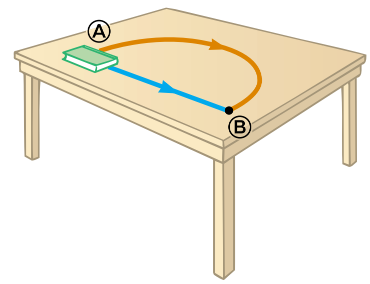


Figure 8.10 The work done against the force of kinetic friction depends on the path taken as the book is moved from Ⓐ to Ⓑ. The work is greater along the red path than along the blue path.

8.4 Changes in Mechanical Energy for Nonconservative Forces

As we have seen, if the forces acting on objects within a system are conservative, then the mechanical energy of the system is conserved. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system changes.

Consider the book sliding across the surface in the preceding section. As the book moves through a distance d, the only force that does work on it is the force of kinetic friction. This force causes a decrease in the kinetic energy of the book. This decrease was calculated in Chapter 7, leading to Equation 7.20, which we repeat here:

$$\Delta K = -f_k d \tag{8.13}$$

Suppose, however, that the book is part of a system that also exhibits a change in potential energy. In this case, $-f_k d$ is the amount by which the *mechanical* energy of the system changes because of the force of kinetic friction. For example, if the book moves on an incline that is not frictionless, there is a change in both the kinetic energy and the gravitational potential energy of the book–Earth system. Consequently,

$$\Delta E_{\text{mech}} = \Delta K + \Delta U_g = -f_k d$$

In general, if a friction force acts within a system,

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d \tag{8.14}$$

Change in mechanical energy of a system due to friction within the system

where ΔU is the change in *all* forms of potential energy. Notice that Equation 8.14 reduces to Equation 8.9 if the friction force is zero.

Quick Quiz 8.9 A block of mass m is projected across a horizontal surface with an initial speed v. It slides until it stops due to the friction force between the block and the surface. The same block is now projected across the horizontal surface with an initial speed $2v$. When the block has come to rest, how does the distance from the projection point compare to that in the first case? (a) It is the same. (b) It is twice as large. (c) It is four times as large. (d) The relationship cannot be determined.

Quick Quiz 8.10 A block of mass m is projected across a horizontal surface with an initial speed v. It slides until it stops due to the friction force between the block and the surface. The surface is now tilted at 30°, and the block is projected up the surface with the same initial speed v. Assume that the friction force remains the same as when the block was sliding on the horizontal surface. When the block comes to rest momentarily, how does the decrease in mechanical energy of the block–surface–Earth system compare to that when the block slid over the horizontal surface? (a) It is the same. (b) It is larger. (c) It is smaller. (d) The relationship cannot be determined.

PROBLEM-SOLVING HINTS

Isolated Systems–Nonconservative Forces

You should incorporate the following procedure when you apply energy methods to a system in which nonconservative forces are acting:

- Follow the procedure in the first three bullets of the Problem-Solving Hints in Section 8.2. If nonconservative forces act within the system, the third bullet should tell you to use the techniques of this section.
- Write expressions for the total initial and total final mechanical energies of the system. The difference between the total final mechanical energy and the total initial mechanical energy equals the change in mechanical energy of the system due to friction.

Example 8.6 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.11. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution Because $v_i = 0$, the initial kinetic energy of the crate–Earth system when the crate is at the top of the ramp is zero. If the y coordinate is measured from the bottom of the ramp (the final position of the crate, for which the gravitational potential energy of the system is zero) with the upward direction being positive, then $y_i = 0.500\,\text{m}$. Therefore, the total mechanical energy of the system when the crate is at the top is all potential energy:

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$
$$= (3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) = 14.7\text{ J}$$

When the crate reaches the bottom of the ramp, the potential energy of the system is zero because the elevation of

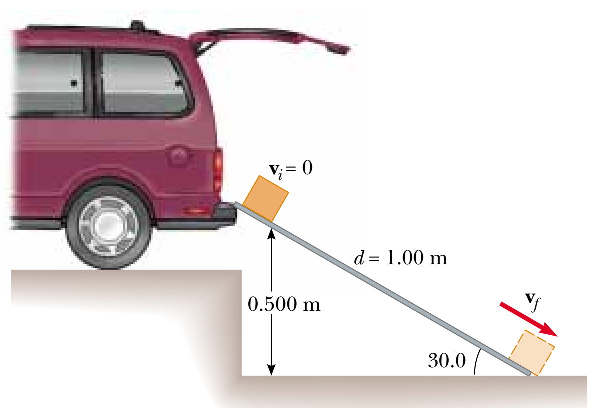


Figure 8.11 (Example 8.6) A crate slides down a ramp under the influence of gravity. The potential energy decreases while the kinetic energy increases.

the crate is $y_f = 0$. Therefore, the total mechanical energy of the system when the crate reaches the bottom is all kinetic energy:

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

We cannot say that $E_i = E_f$ because a nonconservative force reduces the mechanical energy of the system. In this case, Equation 8.14 gives $\Delta E_{\text{mech}} = -f_k d$, where d is the distance the crate moves along the ramp. (Remember that the forces normal to the ramp do no work on the crate because they are perpendicular to the displacement.) With $f_k = 5.00\text{ N}$ and $d = 1.00$ m, we have

$$(1) \qquad -f_k d = (-5.00\text{ N})(1.00\text{ m}) = -5.00\text{ J}$$

Applying Equation 8.14 gives

$$E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$

$$(2) \qquad \tfrac{1}{2}mv_f^2 = 14.7\text{ J} - 5.00\text{ J} = 9.70\text{ J}$$

$$v_f^2 = \frac{19.4\text{ J}}{3.00\text{ kg}} = 6.47\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{2.54\text{ m/s}}$$

What If? **A cautious worker decides that the speed of the crate when it arrives at the bottom of the ramp may be so large that its contents may be damaged. Therefore, he replaces the ramp with a longer one such that the new ramp makes an angle of 25° with the ground. Does this new ramp reduce the speed of the crate as it reaches the ground?**

Answer Because the ramp is longer, the friction force will act over a longer distance and transform more of the mechanical energy into internal energy. This reduces the kinetic energy of the crate, and we expect a lower speed as it reaches the ground.

We can find the length d of the new ramp as follows:

$$\sin 25° = \frac{0.500\text{ m}}{d} \longrightarrow d = \frac{0.500\text{ m}}{\sin 25°} = 1.18\text{ m}$$

Now, Equation (1) becomes

$$-f_k d = (-5.00\text{ N})(1.18\text{ m}) = -5.90\text{ J}$$

and Equation (2) becomes

$$\tfrac{1}{2}mv_f^2 = 14.7\text{ J} - 5.90\text{ J} = 8.80\text{ J}$$

leading to

$$v_f = 2.42\text{ m/s}$$

The final speed is indeed lower than in the higher-angle case.

Example 8.7 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.12. The child starts from rest at the top.

(A) Determine his speed at the bottom, assuming no friction is present.

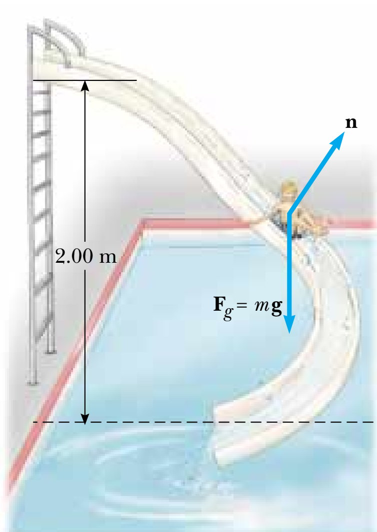


Figure 8.12 (Example 8.7) If the slide is frictionless, the speed of the child at the bottom depends only on the height of the slide.

Solution Although you have no experience on totally frictionless surfaces, you can conceptualize that your speed at the bottom of a frictionless ramp would be greater than in the situation in which friction acts. If we tried to solve this problem with Newton's laws, we would have a difficult time because the acceleration of the child continuously varies in direction due to the irregular shape of the slide. The child–Earth system is isolated and frictionless, however, so we can categorize this as a conservation of energy problem and search for a solution using the energy approach. (Note that the normal force **n** does no work on the child because this force is always perpendicular to each element of the displacement.) To analyze the situation, we measure the y coordinate in the upward direction from the bottom of the slide so that $y_i = h$, $y_f = 0$, and we obtain

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + 0 = 0 + mgh$$

$$v_f = \sqrt{2gh}$$

Note that the result is the same as it would be had the child fallen vertically through a distance h! In this example, $h = 2.00$ m, giving

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = \boxed{6.26\text{ m/s}}$$

(B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

Solution We categorize this case, with friction, as a problem in which a nonconservative force acts. Hence, mechanical energy is not conserved, and we must use Equation 8.14 to find the loss of mechanical energy due to friction:

$$\begin{aligned}\Delta E_{\text{mech}} &= (K_f + U_f) - (K_i + U_i) \\ &= (\tfrac{1}{2}mv_f^2 + 0) - (0 + mgh) = \tfrac{1}{2}mv_f^2 - mgh \\ &= \tfrac{1}{2}(20.0\text{ kg})(3.00\text{ m/s})^2 \\ &\quad - (20.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) \\ &= \boxed{-302\text{ J}}\end{aligned}$$

Again, ΔE_{mech} is negative because friction is reducing the mechanical energy of the system. (The final mechanical energy is less than the initial mechanical energy.)

What If? Suppose you were asked to find the coefficient of friction μ_k for the child on the slide. Could you do this?

Answer We can argue that the same final speed could be obtained by having the child travel down a short slide with large friction or a long slide with less friction. Thus, there does not seem to be enough information in the problem to determine the coefficient of friction.

The energy loss of 302 J must be equal to the product of the friction force and the length of the slide:

$$-f_k d = -302\text{ J}$$

We can also argue that the friction force can be expressed as $\mu_k n$, where n is the magnitude of the normal force. Thus,

$$\mu_k n d = 302\text{ J}$$

If we try to evaluate the coefficient of friction from this relationship, we run into two problems. First, there is no single value of the normal force n unless the angle of the slide relative to the horizontal remains fixed. Even if the angle were fixed, we do not know its value. The second problem is that we do not have information about the length d of the slide. Thus, we cannot find the coefficient of friction from the information given.

Example 8.8 Let's Go Skiing!

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.13. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?

Solution The system is the skier plus the Earth, and we choose as our configuration of zero potential energy that in which the skier is at the bottom of the incline. While the skier is on the frictionless incline, the mechanical energy of the system remains constant, and we find, as we did in Example 8.7, that

$$v_B = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(20.0\text{ m})} = 19.8\text{ m/s}$$

Now we apply Equation 8.14 as the skier moves along the rough horizontal surface from Ⓑ to Ⓒ. The change in mechanical energy along the horizontal surface is

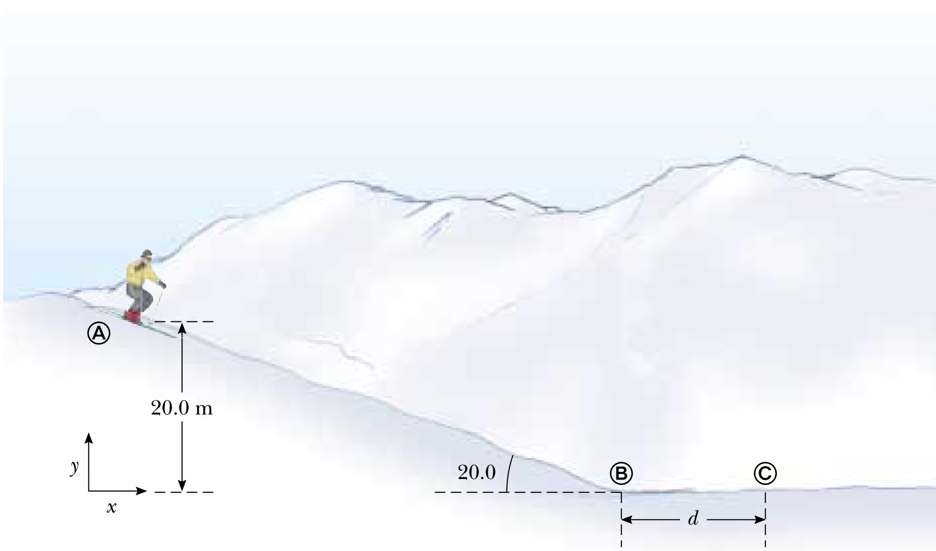


Figure 8.13 (Example 8.8) The skier slides down the slope and onto a level surface, stopping after a distance d from the bottom of the hill.

$\Delta E_{mech} = -f_k d$, where d is the horizontal distance traveled by the skier.

To find the distance the skier travels before coming to rest, we take $K_C = 0$. With $v_B = 19.8$ m/s and the friction force given by $f_k = \mu_k n = \mu_k mg$, we obtain

$$\Delta E_{mech} = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B^2 + 0) = -\mu_k mgd$$

$$d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8\text{ m/s})^2}{2(0.210)(9.80\text{ m/s}^2)} = \boxed{95.2\text{ m}}$$

Example 8.9 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity $v_A = 1.2$ m/s to the right and collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown in Figure 8.14.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution Our system in this example consists of the block and spring. All motion takes place in a horizontal plane, so we do not need to consider changes in gravitational potential energy. Before the collision, when the block is at Ⓐ, it has kinetic energy and the spring is uncompressed, so the elastic potential energy stored in the spring is zero. Thus, the total mechanical energy of the system before the collision is just $\frac{1}{2}mv_A^2$. After the collision, when the block is at Ⓒ, the spring is fully compressed; now the block is at rest and so has zero kinetic energy, while the energy stored in the spring has its maximum value $\frac{1}{2}kx^2 = \frac{1}{2}kx_{max}^2$, where the origin of coordinates $x = 0$ is chosen to be the equilibrium position of the spring and x_{max} is the maximum compression of the spring, which in this case happens to be x_C. The total mechanical energy of the system is conserved because no nonconservative forces act on objects within the system.

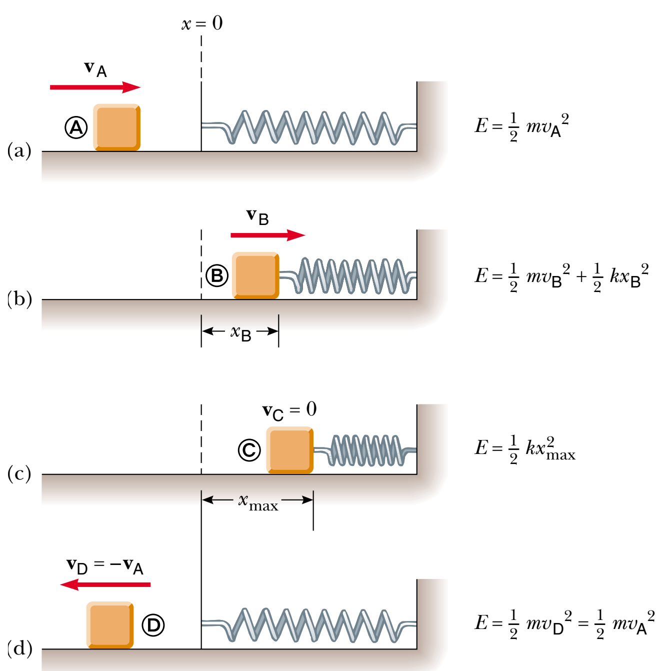


Figure 8.14 (Example 8.9) A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy of the system remains constant throughout the motion.

Because the mechanical energy of the system is conserved, the kinetic energy of the block before the collision equals the maximum potential energy stored in the fully compressed spring:

$$E_C = E_A$$

$$K_C + U_{sC} = K_A + U_{sA}$$

$$0 + \tfrac{1}{2}kx_{max}^2 = \tfrac{1}{2}mv_A^2 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0.80\text{ kg}}{50\text{ N/m}}}\,(1.2\text{ m/s}) = \boxed{0.15\text{ m}}$$

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2$ m/s, what is the maximum compression x_C in the spring?

Solution In this case, the mechanical energy of the system is *not* conserved because a friction force acts on the block. The magnitude of the friction force is

$$f_k = \mu_k n = \mu_k mg = 0.50(0.80\text{ kg})(9.80\text{ m/s}^2) = 3.92\text{ N}$$

Therefore, the change in the mechanical energy of the system due to friction as the block is displaced from the equilibrium position of the spring (where we have set our origin) to x_C is

$$\Delta E_{mech} = -f_k x_C = (-3.92x_C)$$

Substituting this into Equation 8.14 gives

$$\Delta E_{mech} = E_f - E_i = (0 + \tfrac{1}{2}kx_C^2) - (\tfrac{1}{2}mv_A^2 + 0) = -f_k x_C$$

$$\tfrac{1}{2}(50)x_C^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92x_C$$

$$25x_C^2 + 3.92x_C - 0.576 = 0$$

Solving the quadratic equation for x_C gives $x_C = 0.092$ m and $x_C = -0.25$ m. The physically meaningful root is $x_C = 0.092$ m. The negative root does not apply to this situation because the block must be to the right of the origin (positive value of x) when it comes to rest. Note that the value of 0.092 m is less than the distance obtained in the frictionless case of part (A). This result is what we expect because friction retards the motion of the system.

Example 8.10 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

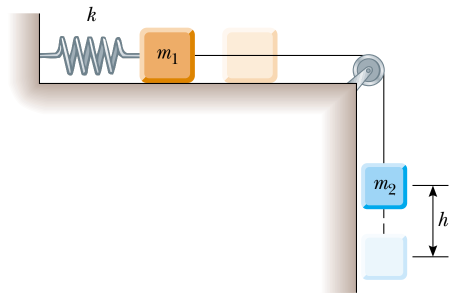


Figure 8.15 (Example 8.10) As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is lost because of friction between the sliding block and the surface.

Solution The key word *rest* appears twice in the problem statement. This suggests that the configurations associated with rest are good candidates for the initial and final configurations because the kinetic energy of the system is zero for these configurations. (Also note that because we are concerned only with the beginning and ending points of the motion, we do not need to label events with circled letters as we did in the previous two examples. Simply using i and f is sufficient to keep track of the situation.) In this situation, the system consists of the two blocks, the spring, and the Earth. We need to consider two forms of potential energy: gravitational and elastic. Because the initial and final kinetic energies of the system are zero, $\Delta K = 0$, and we can write

$$\text{(1)} \qquad \Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

where $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. As the hanging block falls a distance h, the horizontally moving block moves the same distance h to the right. Therefore, using Equation 8.14, we find that the loss in mechanical energy in the system due to friction between the horizontally sliding block and the surface is

$$\text{(2)} \qquad \Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. Therefore, we obtain

$$\text{(3)} \qquad \Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

where the coordinates have been measured from the lowest position of the falling block.

The change in the elastic potential energy of the system is that stored in the spring:

$$\text{(4)} \qquad \Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2}kh^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

This setup represents a way of measuring the coefficient of kinetic friction between an object and some surface. As you can see from the problem, sometimes it is easier to work with the changes in the various types of energy rather than the actual values. For example, if we wanted to calculate the numerical value of the gravitational potential energy associated with the horizontally sliding block, we would need to specify the height of the horizontal surface relative to the lowest position of the falling block. Fortunately, this is not necessary because the gravitational potential energy associated with the first block does not change.

8.5 Relationship Between Conservative Forces and Potential Energy

In an earlier section we found that the work done on a member of a system by a conservative force between the members does not depend on the path taken by the moving member. The work depends only on the initial and final coordinates. As a consequence, we can define a **potential energy function *U*** such that the work done by a conservative force equals the decrease in the potential energy of the system. Let us imagine a system of particles in which the configuration changes due to the motion of one particle along the x axis. The work done by a conservative force $\mathbf{F}$ as a particle moves along the x axis is[2]

[2] For a general displacement, the work done in two or three dimensions also equals $-\Delta U$, where $U = U(x, y, z)$. We write this formally as $W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = U_i - U_f$.

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \tag{8.15}$$

where F_x is the component of $\mathbf{F}$ in the direction of the displacement. That is, the work done by a conservative force acting between members of a system equals the negative of the change in the potential energy associated with that force when the configuration of the system changes, where the change in the potential energy is defined as $\Delta U = U_f - U_i$. We can also express Equation 8.15 as

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \tag{8.16}$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

The term *potential energy* implies that the system has the potential, or capability, of either gaining kinetic energy or doing work when it is released under the influence of a conservative force exerted on an object by some other member of the system. It is often convenient to establish some particular location x_i of one member of a system as representing a reference configuration and measure all potential energy differences with respect to it. We can then define the potential energy function as

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \tag{8.17}$$

The value of U_i is often taken to be zero for the reference configuration. It really does not matter what value we assign to U_i because any nonzero value merely shifts $U_f(x)$ by a constant amount and only the *change* in potential energy is physically meaningful.

If the conservative force is known as a function of position, we can use Equation 8.17 to calculate the change in potential energy of a system as an object within the system moves from x_i to x_f.

If the point of application of the force undergoes an infinitesimal displacement dx, we can express the infinitesimal change in the potential energy of the system dU as

$$dU = -F_x \, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship[3]

$$F_x = -\frac{dU}{dx} \tag{8.18}$$

Relation of force between members of a system to the potential energy of the system

That is, **the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x.**

We can easily check this relationship for the two examples already discussed. In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$, and therefore

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring (Hooke's law). Because the gravitational potential energy function is $U_g = mgy$, it follows from Equation 8.18 that $F_g = -mg$ when we differentiate U_g with respect to y instead of x.

We now see that U is an important function because a conservative force can be derived from it. Furthermore, Equation 8.18 should clarify the fact that adding a constant to the potential energy is unimportant because the derivative of a constant is zero.

[3] In three dimensions, the expression is, $\mathbf{F} = -\frac{\partial U}{\partial x}\hat{\mathbf{i}} - \frac{\partial U}{\partial y}\hat{\mathbf{j}} - \frac{\partial U}{\partial z}\hat{\mathbf{k}}$ where $\frac{\partial U}{\partial x}$ etc. are partial derivatives. In the language of vector calculus, $\mathbf{F}$ equals the negative of the *gradient* of the scalar quantity $U(x, y, z)$.

Quick Quiz 8.11 What does the slope of a graph of $U(x)$ versus x represent? (a) the magnitude of the force on the object (b) the negative of the magnitude of the force on the object (c) the x component of the force on the object (d) the negative of the x component of the force on the object.

8.6 Energy Diagrams and Equilibrium of a System

The motion of a system can often be understood qualitatively through a graph of its potential energy versus the position of a member of the system. Consider the potential energy function for a block–spring system, given by $U_s = \frac{1}{2}kx^2$. This function is plotted versus x in Figure 8.16a. (A common mistake is to think that potential energy on the graph represents height. This is clearly not the case here, where the block is only moving horizontally.) The force F_s exerted by the spring on the block is related to U_s through Equation 8.18:

$$F_s = -\frac{dU_s}{dx} = -kx$$

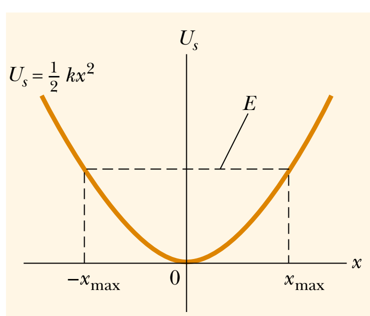
As we saw in Quick Quiz 8.11, the x component of the force is equal to the negative of the slope of the U-versus-x curve. When the block is placed at rest at the equilibrium position of the spring ($x = 0$), where $F_s = 0$, it will remain there unless some external force F_{ext} acts on it. If this external force stretches the spring from equilibrium, x is positive and the slope dU/dx is positive; therefore, the force F_s exerted by the spring is negative and the block accelerates back toward $x = 0$ when released. If the external force compresses the spring, then x is negative and the slope is negative; therefore, F_s is positive and again the mass accelerates toward $x = 0$ upon release.

Stable equilibrium

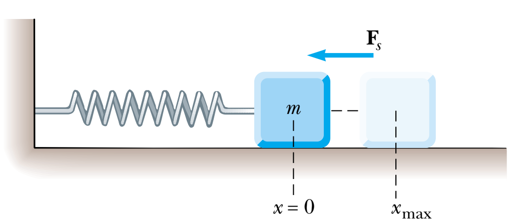
From this analysis, we conclude that the $x = 0$ position for a block–spring system is one of **stable equilibrium.** That is, any movement away from this position results in a force directed back toward $x = 0$. In general, **configurations of stable equilibrium correspond to those for which $U(x)$ is a minimum.**

From Figure 8.16 we see that if the block is given an initial displacement x_{max} and is released from rest, its total energy initially is the potential energy $\frac{1}{2}kx_{max}^2$ stored in the

At the Active Figures link at http://www.pse6.com, you can observe the block oscillate between its turning points and trace the corresponding points on the potential energy curve for varying values of k.



(a)



(b)

Active Figure 8.16 (a) Potential energy as a function of x for the frictionless block–spring system shown in (b). The block oscillates between the turning points, which have the coordinates $x = \pm x_{max}$. Note that the restoring force exerted by the spring always acts toward $x = 0$, the position of stable equilibrium.

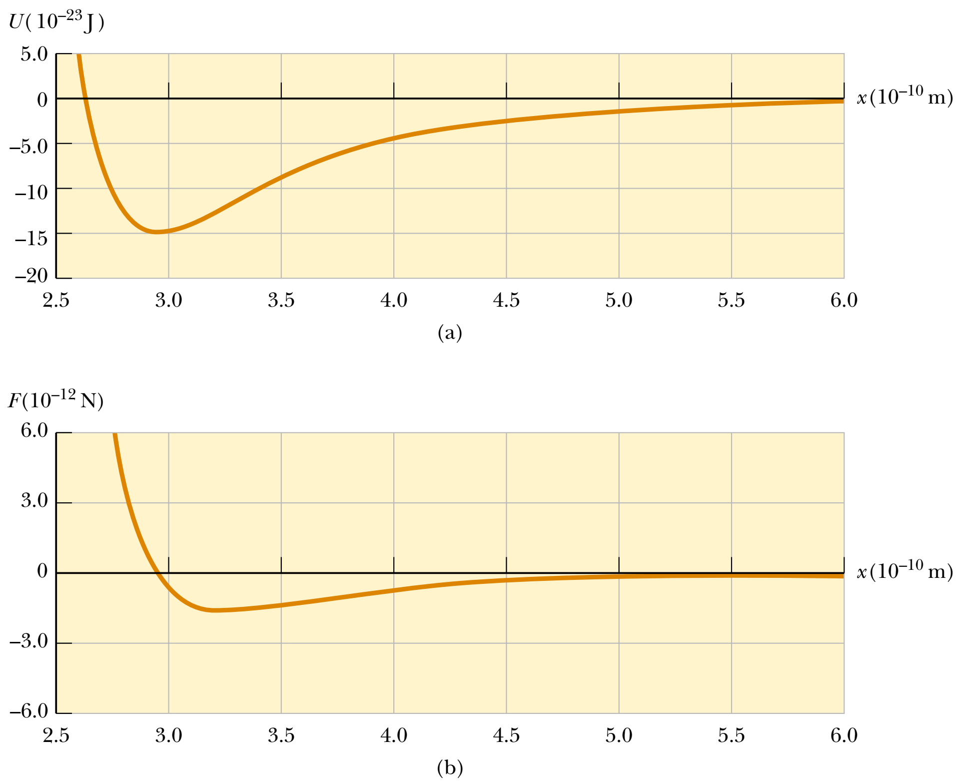


Figure 8.18 (Example 8.11) (a) Potential energy curve associated with a molecule. The distance x is the separation between the two atoms making up the molecule. (b) Force exerted on one atom by the other.

$$F_x = -\frac{dU(x)}{dx} = -4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$$= \boxed{4\epsilon\left[\frac{12\sigma^{12}}{x^{13}} - \frac{6\sigma^{6}}{x^{7}}\right]}$$

This result is graphed in Figure 8.18b. As expected, the force is positive (repulsive) at small atomic separations, zero when the atoms are at the position of stable equilibrium [recall how we found the minimum of $U(x)$], and negative (attractive) at greater separations. Note that the force approaches zero as the separation between the atoms becomes very great.

SUMMARY

Take a Practice Test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

If a particle of mass m is at a distance y above the Earth's surface, the **gravitational potential energy** of the particle–Earth system is

$$U_g \equiv mgy \tag{8.2}$$

The **elastic potential energy** stored in a spring of force constant k is

$$U_s \equiv \tfrac{1}{2}kx^2 \tag{8.11}$$

A reference configuration of the system should be chosen, and this configuration is often assigned a potential energy of zero.

A force is **conservative** if the work it does on a particle moving between two points is independent of the path the particle takes between the two points. Furthermore, a force is conservative if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be **nonconservative.**

The **total mechanical energy of a system** is defined as the sum of the kinetic energy and the potential energy:

$$E_{\text{mech}} \equiv K + U \tag{8.8}$$

If a system is isolated and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_f + U_f = K_i + U_i \tag{8.9}$$

If nonconservative forces (such as friction) act on objects inside a system, then mechanical energy is not conserved. In these situations, the difference between the total final mechanical energy and the total initial mechanical energy of the system equals the energy transformed to internal energy by the nonconservative forces.

A **potential energy function** U can be associated only with a conservative force. If a conservative force **F** acts between members of a system while one member moves along the x axis from x_i to x_f, then the change in the potential energy of the system equals the negative of the work done by that force:

$$U_f - U_i = -\int_{x_i}^{x_f} F_x\, dx \tag{8.16}$$

Systems can be in three types of equilibrium configurations when the net force on a member of the system is zero. Configurations of **stable equilibrium** correspond to those for which $U(x)$ is a minimum. Configurations of **unstable equilibrium** correspond to those for which $U(x)$ is a maximum. **Neutral equilibrium** arises where U is constant as a member of the system moves over some region.

QUESTIONS

1. If the height of a playground slide is kept constant, will the length of the slide or the presence of bumps make any difference in the final speed of children playing on it? Assume the slide is slick enough to be considered frictionless. Repeat this question assuming friction is present.
2. Explain why the total energy of a system can be either positive or negative, whereas the kinetic energy is always positive.
3. One person drops a ball from the top of a building while another person at the bottom observes its motion. Will these two people agree on the value of the gravitational potential energy of the ball–Earth system? On the change in potential energy? On the kinetic energy?
4. Discuss the changes in mechanical energy of an object–Earth system in (a) lifting the object, (b) holding the object at a fixed position, and (c) lowering the object slowly. Include the muscles in your discussion.
5. In Chapter 7, the work–kinetic energy theorem, $W = \Delta K$, was introduced. This equation states that work done on a system appears as a change in kinetic energy. This is a special-case equation, valid if there are no changes in any other type of energy such as potential or internal. Give some examples in which work is done on a system, but the change in energy of the system is not that of kinetic energy.
6. If three conservative forces and one nonconservative force act within a system, how many potential-energy terms appear in the equation that describes the system?
7. If only one external force acts on a particle, does it necessarily change the particle's (a) kinetic energy? (b) velocity?
8. A driver brings an automobile to a stop. If the brakes lock so that the car skids, where is the original kinetic energy of the car, and in what form is it after the car stops? Answer the same question for the case in which the brakes do not lock, but the wheels continue to turn.
9. You ride a bicycle. In what sense is your bicycle solar-powered?
10. In an earthquake, a large amount of energy is "released" and spreads outward, potentially causing severe damage. In what form does this energy exist before the earthquake, and by what energy transfer mechanism does it travel?
11. A bowling ball is suspended from the ceiling of a lecture hall by a strong cord. The ball is drawn away from its equilibrium position and released from rest at the tip of the demonstrator's nose as in Figure Q8.11. If the demonstrator remains stationary, explain why she is not struck by the ball on its return swing. Would this demonstrator be safe if the ball were given a push from its starting position at her nose?
12. Roads going up mountains are formed into switchbacks, with the road weaving back and forth along the face of the slope such that there is only a gentle rise on any portion of the roadway. Does this require any less work to be done by an automobile climbing the mountain compared to driving on a roadway that is straight up the slope? Why are switchbacks used?
13. As a sled moves across a flat snow-covered field at constant velocity, is any work done? How does air resistance enter into the picture?
14. You are working in a library, reshelving books. You lift a book from the floor to the top shelf. The kinetic energy of the book on the floor was zero, and the kinetic energy of the book on the top shelf is zero, so there is no change

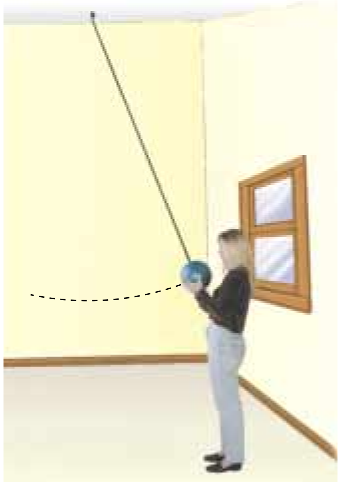


Figure Q8.11

in kinetic energy. Yet you did some work in lifting the book. Is the work–kinetic energy theorem violated?

15. A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy of the ball–Earth system a maximum?

16. A pile driver is a device used to drive objects into the Earth by repeatedly dropping a heavy weight on them. By how much does the energy of the pile driver–Earth system increase when the weight it drops is doubled? Assume the weight is dropped from the same height each time.

17. Our body muscles exert forces when we lift, push, run, jump, and so forth. Are these forces conservative?

18. A block is connected to a spring that is suspended from the ceiling. If the block is set in motion and air resistance is neglected, describe the energy transformations that occur within the system consisting of the block, Earth, and spring.

19. Describe the energy transformations that occur during (a) the pole vault (b) the shot put (c) the high jump. What is the source of energy in each case?

20. Discuss the energy transformations that occur during the operation of an automobile.

21. What would the curve of U versus x look like if a particle were in a region of neutral equilibrium?

22. A ball rolls on a horizontal surface. Is the ball in stable, unstable, or neutral equilibrium?

23. Consider a ball fixed to one end of a rigid rod whose other end pivots on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 8.1 Potential Energy of a System

1. A 1 000-kg roller coaster train is initially at the top of a rise, at point Ⓐ. It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point Ⓑ. (a) Choose point Ⓑ to be the zero level for gravitational potential energy. Find the potential energy of the roller coaster–Earth system at points Ⓐ and Ⓑ, and the change in potential energy as the coaster moves. (b) Repeat part (a), setting the zero reference level at point Ⓐ.

2. A 400-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

3. A person with a remote mountain cabin plans to install her own hydroelectric plant. A nearby stream is 3.00 m wide and 0.500 m deep. Water flows at 1.20 m/s over the brink of a waterfall 5.00 m high. The manufacturer promises only 25.0% efficiency in converting the potential energy of the water–Earth system into electric energy. Find the power she can generate. (Large-scale hydroelectric plants, with a much larger drop, are more efficient.)

Section 8.2 The Isolated System–Conservation of Mechanical Energy

4. At 11:00 A.M. on September 7, 2001, more than 1 million British school children jumped up and down for one minute. The curriculum focus of the "Giant Jump" was on earthquakes, but it was integrated with many other topics, such as exercise, geography, cooperation, testing hypotheses, and setting world records. Children built their own seismographs, which registered local effects. (a) Find the mechanical energy released in the experiment. Assume that 1 050 000 children of average mass 36.0 kg jump twelve times each, raising their centers of mass by 25.0 cm each time and briefly resting between one jump and the next. The free-fall acceleration in Britain is 9.81 m/s^2. (b) Most of the energy is converted very rapidly into internal energy within the bodies of the children and the floors of the school buildings. Of the energy that propagates into the ground, most produces high-frequency "microtremor" vibrations that are rapidly damped and cannot travel far. Assume that 0.01% of the energy is carried away by a long-range seismic wave. The magnitude of an earthquake on the Richter scale is given by

$$M = \frac{\log E - 4.8}{1.5}$$

where E is the seismic wave energy in joules. According to this model, what is the magnitude of the demonstration quake? (It did not register above background noise overseas or on the seismograph of the Wolverton Seismic Vault, Hampshire.)

$\boxed{5.}$ A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from a height $h = 3.50R$. (a) What is its speed at point Ⓐ? (b) How large is the normal force on it if its mass is 5.00 g?

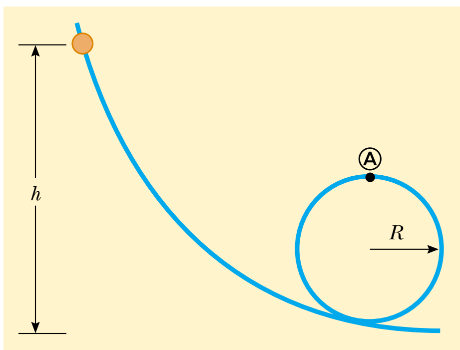


Figure P8.5

6. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground at the high jump with vertical velocity component 6.00 m/s. How far does his center of mass move up as he makes the jump?

7. A glider of mass 0.150 kg moves on a horizontal frictionless air track. It is permanently attached to one end of a massless horizontal spring, which has a force constant of 10.0 N/m both for extension and for compression. The other end of the spring is fixed. The glider is moved to compress the spring by 0.180 m and then released from rest. Calculate the speed of the glider (a) at the point where it has moved 0.180 m from its starting point, so that the spring is momentarily exerting no force and (b) at the point where it has moved 0.250 m from its starting point.

8. A loaded ore car has a mass of 950 kg and rolls on rails with negligible friction. It starts from rest and is pulled up a mine shaft by a cable connected to a winch. The shaft is inclined at 30.0° above the horizontal. The car accelerates uniformly to a speed of 2.20 m/s in 12.0 s and then continues at constant speed. (a) What power must the winch motor provide when the car is moving at constant speed? (b) What maximum power must the winch motor provide? (c) What total energy transfers out of the motor by work by the time the car moves off the end of the track, which is of length 1 250 m?

9. A simple pendulum, which we will consider in detail in Chapter 15, consists of an object suspended by a string. The object is assumed to be a particle. The string, with its top end fixed, has negligible mass and does not stretch. In the absence of air friction, the system oscillates by swinging back and forth in a vertical plane. If the string is 2.00 m long and makes an initial angle of 30.0° with the vertical, calculate the speed of the particle (a) at the lowest point in its trajectory and (b) when the angle is 15.0°.

10. An object of mass m starts from rest and slides a distance d down a frictionless incline of angle θ. While sliding, it contacts an unstressed spring of negligible mass as shown in Figure P8.10. The object slides an additional distance x as it is brought momentarily to rest by compression of the spring (of force constant k). Find the initial separation d between object and spring.

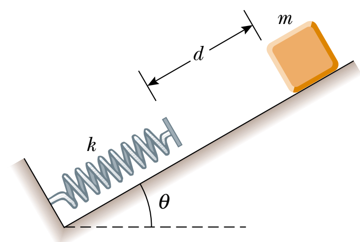


Figure P8.10

$\boxed{11.}$ A block of mass 0.250 kg is placed on top of a light vertical spring of force constant 5 000 N/m and pushed downward so that the spring is compressed by 0.100 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

12. A circus trapeze consists of a bar suspended by two parallel ropes, each of length ℓ, allowing performers to swing in a vertical circular arc (Figure P8.12). Suppose a performer with mass m holds the bar and steps off an elevated platform, starting from rest with the ropes at an angle θ_i with respect to the vertical. Suppose the size of the performer's body is small compared to the length ℓ, that she does not pump the trapeze to swing higher, and that air resistance is negligible. (a) Show that when the ropes make an angle θ with the vertical, the performer must exert a force

$$mg(3\cos\theta - 2\cos\theta_i)$$

in order to hang on. (b) Determine the angle θ_i for which

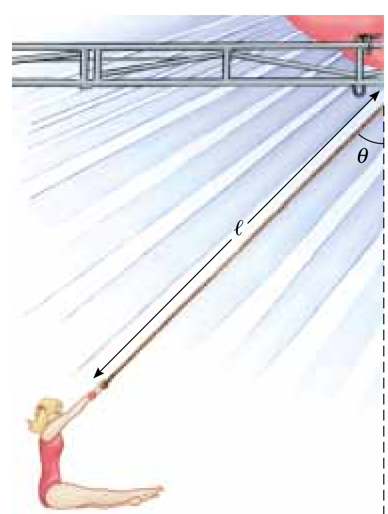


Figure P8.12

the force needed to hang on at the bottom of the swing is twice the performer's weight.

13. Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure P8.13. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground. (b) Find the maximum height to which the 3.00-kg object rises.

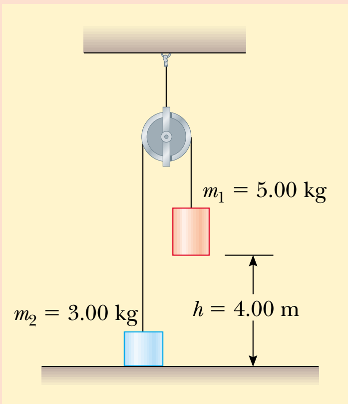


Figure P8.13 Problems 13 and 14.

14. Two objects are connected by a light string passing over a light frictionless pulley as in Figure P8.13. The object of mass m_1 is released from rest at height h. Using the principle of conservation of energy, (a) determine the speed of m_2 just as m_1 hits the ground. (b) Find the maximum height to which m_2 rises.

15. A light rigid rod is 77.0 cm long. Its top end is pivoted on a low-friction horizontal axle. The rod hangs straight down at rest with a small massive ball attached to its bottom end. You strike the ball, suddenly giving it a horizontal velocity so that it swings around in a full circle. What minimum speed at the bottom is required to make the ball go over the top of the circle?

16. Air moving at 11.0 m/s in a steady wind encounters a windmill of diameter 2.30 m and having an efficiency of 27.5%. The energy generated by the windmill is used to pump water from a well 35.0 m deep into a tank 2.30 m above the ground. At what rate in liters per minute can water be pumped into the tank?

17. A 20.0-kg cannon ball is fired from a cannon with muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the conservation of energy principle to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let $y = 0$ at the cannon.

18. A 2.00-kg ball is attached to the bottom end of a length of fishline with a breaking strength of 10 lb (44.5 N). The top end of the fishline is held stationary. The ball is released from rest with the line taut and horizontal ($\theta = 90.0°$). At what angle θ (measured from the vertical) will the fishline break?

19. A daredevil plans to bungee-jump from a balloon 65.0 m above a carnival midway (Figure P8.19). He will use a uniform elastic cord, tied to a harness around his body, to stop his fall at a point 10.0 m above the ground. Model his body as a particle and the cord as having negligible mass and obeying Hooke's force law. In a preliminary test, hanging at rest from a 5.00-m length of the cord, he finds that his body weight stretches it by 1.50 m. He will drop from rest at the point where the top end of a longer section of the cord is attached to the stationary balloon. (a) What length of cord should he use? (b) What maximum acceleration will he experience?

Gamma

Figure P8.19

20. **Review problem.** The system shown in Figure P8.20 consists of a light inextensible cord, light frictionless pulleys, and blocks of equal mass. It is initially held at rest so that the blocks are at the same height above the ground. The blocks are then released. Find the speed of block A at the moment when the vertical separation of the blocks is h.

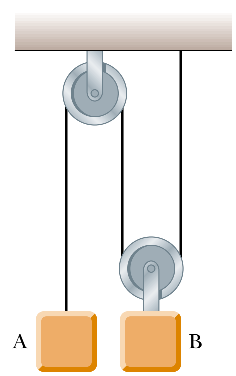


Figure P8.20

Section 8.3 Conservative and Nonconservative Forces

21. A 4.00-kg particle moves from the origin to position C, having coordinates $x = 5.00$ m and $y = 5.00$ m. One force on the particle is the gravitational force acting in the negative y direction (Fig. P8.21). Using Equation 7.3, calculate the

work done by the gravitational force in going from O to C along (a) OAC. (b) OBC. (c) OC. Your results should all be identical. Why?

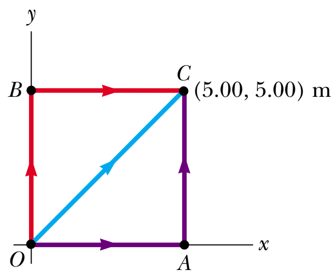


Figure P8.21 Problems 21, 22 and 23.

22. (a) Suppose that a constant force acts on an object. The force does not vary with time, nor with the position or the velocity of the object. Start with the general definition for work done by a force

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$$

and show that the force is conservative. (b) As a special case, suppose that the force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N acts on a particle that moves from O to C in Figure P8.21. Calculate the work done by $\mathbf{F}$ if the particle moves along each one of the three paths OAC, OBC, and OC. (Your three answers should be identical.)

23. A force acting on a particle moving in the xy plane is given by $\mathbf{F} = (2y\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}})$ N, where x and y are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m, as in Figure P8.21. Calculate the work done by $\mathbf{F}$ along (a) OAC, (b) OBC, (c) OC. (d) Is $\mathbf{F}$ conservative or nonconservative? Explain.

24. A particle of mass $m = 5.00$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure P8.24. Determine (a) the particle's speed at points Ⓑ and Ⓒ and (b) the net work done by the gravitational force in moving the particle from Ⓐ to Ⓒ.

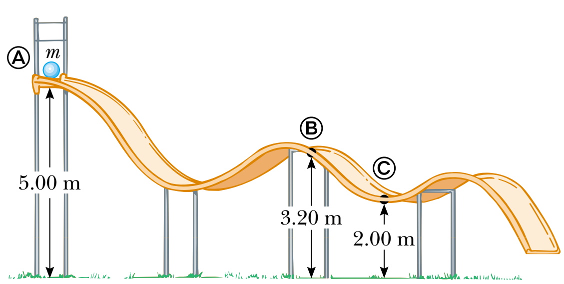


Figure P8.24

25. A single constant force $\mathbf{F} = (3\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ N acts on a 4.00-kg particle. (a) Calculate the work done by this force if the particle moves from the origin to the point having the vector position $\mathbf{r} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ m. Does this result depend on the path? Explain. (b) What is the speed of the particle at $\mathbf{r}$ if its speed at the origin is 4.00 m/s? (c) What is the change in the potential energy?

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

26. At time t_i, the kinetic energy of a particle is 30.0 J and the potential energy of the system to which it belongs is 10.0 J. At some later time t_f, the kinetic energy of the particle is 18.0 J. (a) If only conservative forces act on the particle, what are the potential energy and the total energy at time t_f? (b) If the potential energy of the system at time t_f is 5.00 J, are there any nonconservative forces acting on the particle? Explain.

27. In her hand a softball pitcher swings a ball of mass 0.250 kg around a vertical circular path of radius 60.0 cm before releasing it from her hand. The pitcher maintains a component of force on the ball of constant magnitude 30.0 N in the direction of motion around the complete path. The speed of the ball at the top of the circle is 15.0 m/s. If she releases the ball at the bottom of the circle, what is its speed upon release?

28. An electric scooter has a battery capable of supplying 120 Wh of energy. If friction forces and other losses account for 60.0% of the energy usage, what altitude change can a rider achieve when driving in hilly terrain, if the rider and scooter have a combined weight of 890 N?

29. The world's biggest locomotive is the MK5000C, a behemoth of mass 160 metric tons driven by the most powerful engine ever used for rail transportation, a Caterpillar diesel capable of 5 000 hp. Such a huge machine can provide a gain in efficiency, but its large mass presents challenges as well. The engineer finds that the locomotive handles differently from conventional units, notably in braking and climbing hills. Consider the locomotive pulling no train, but traveling at 27.0 m/s on a level track while operating with output power 1 000 hp. It comes to a 5.00% grade (a slope that rises 5.00 m for every 100 m along the track). If the throttle is not advanced, so that the power level is held steady, to what value will the speed drop? Assume that friction forces do not depend on the speed.

30. A 70.0-kg diver steps off a 10.0-m tower and drops straight down into the water. If he comes to rest 5.00 m beneath the surface of the water, determine the average resistance force exerted by the water on the diver.

31. The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

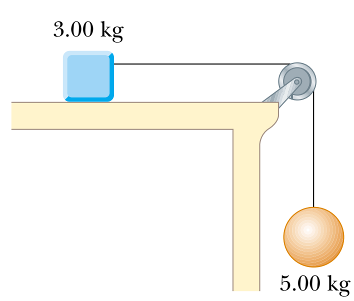


Figure P8.31

32. A boy in a wheelchair (total mass 47.0 kg) wins a race with a skateboarder. The boy has speed 1.40 m/s at the crest of a slope 2.60 m high and 12.4 m long. At the bottom of the slope his speed is 6.20 m/s. If air resistance and rolling resistance can be modeled as a constant friction force of 41.0 N, find the work he did in pushing forward on his wheels during the downhill ride.

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

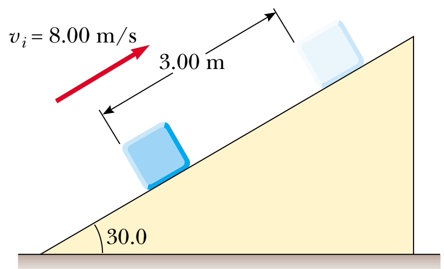


Figure P8.33

34. An 80.0-kg skydiver jumps out of a balloon at an altitude of 1 000 m and opens the parachute at an altitude of 200 m. (a) Assuming that the total retarding force on the diver is constant at 50.0 N with the parachute closed and constant at 3 600 N with the parachute open, what is the speed of the diver when he lands on the ground? (b) Do you think the skydiver will be injured? Explain. (c) At what height should the parachute be opened so that the final speed of the skydiver when he hits the ground is 5.00 m/s? (d) How realistic is the assumption that the total retarding force is constant? Explain.

35. A toy cannon uses a spring to project a 5.30-g soft rubber ball. The spring is originally compressed by 5.00 cm and has a force constant of 8.00 N/m. When the cannon is fired, the ball moves 15.0 cm through the horizontal barrel of the cannon, and there is a constant friction force of 0.032 0 N between the barrel and the ball. (a) With what speed does the projectile leave the barrel of the cannon? (b) At what point does the ball have maximum speed? (c) What is this maximum speed?

36. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P8.36. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0 kg block and incline is 0.250. Determine the change in the kinetic energy of the 50.0-kg block as it moves from Ⓐ to Ⓑ, a distance of 20.0 m.

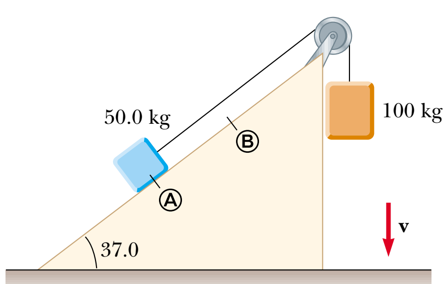


Figure P8.36

37. A 1.50-kg object is held 1.20 m above a relaxed massless vertical spring with a force constant of 320 N/m. The object is dropped onto the spring. (a) How far does it compress the spring? (b) **What If?** How far does it compress the spring if the same experiment is performed on the Moon, where $g = 1.63\ \text{m/s}^2$? (c) **What If?** Repeat part (a), but this time assume a constant air-resistance force of 0.700 N acts on the object during its motion.

38. A 75.0-kg skysurfer is falling straight down with terminal speed 60.0 m/s. Determine the rate at which the skysurfer–Earth system is losing mechanical energy.

39. A uniform board of length L is sliding along a smooth (frictionless) horizontal plane as in Figure P8.39a. The board then slides across the boundary with a rough horizontal surface. The coefficient of kinetic friction between the board and the second surface is μ_k. (a) Find the acceleration of the board at the moment its front end has traveled a distance x beyond the boundary. (b) The board stops at the moment its back end reaches the boundary, as in Figure P8.39b. Find the initial speed v of the board.

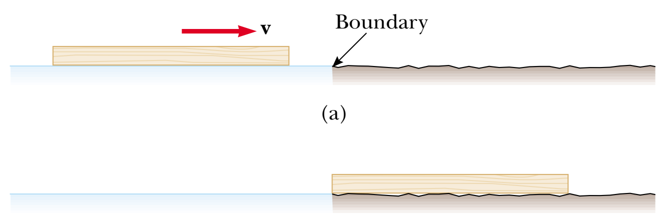


Figure P8.39

Section 8.5 Relationship Between Conservative Forces and Potential Energy

40. A single conservative force acting on a particle varies as $\mathbf{F} = (-Ax + Bx^2)\hat{\mathbf{i}}$ N, where A and B are constants and x is in meters. (a) Calculate the potential-energy function $U(x)$ associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and the change in kinetic energy as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

41. A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N describes the force, where x is in meters. As the particle moves along the x axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at $x = 5.00$ m if its speed is 3.00 m/s at $x = 1.00$ m.

42. A potential-energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).

43. The potential energy of a system of two particles separated by a distance r is given by $U(r) = A/r$, where A is a constant. Find the radial force $\mathbf{F}_r$ that each particle exerts on the other.

Section 8.6 Energy Diagrams and Equilibrium of a System

44. A right circular cone can be balanced on a horizontal surface in three different ways. Sketch these three equilibrium configurations, and identify them as positions of stable, unstable, or neutral equilibrium.

45. For the potential energy curve shown in Figure P8.45, (a) determine whether the force F_x is positive, negative, or zero at the five points indicated. (b) Indicate points of stable, unstable, and neutral equilibrium. (c) Sketch the curve for F_x versus x from $x = 0$ to $x = 9.5$ m.

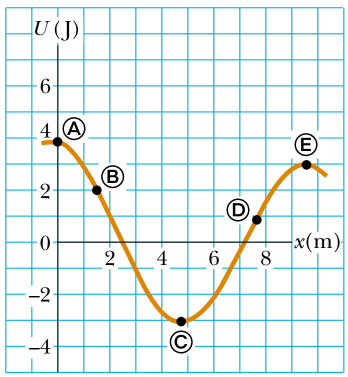


Figure P8.45

46. A particle moves along a line where the potential energy of its system depends on its position r as graphed in Figure P8.46. In the limit as r increases without bound, $U(r)$ approaches +1 J. (a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable, or neutral equilibrium. (b) The particle will be bound if the total energy of the system is in what range? Now suppose that the system has energy −3 J. Determine (c) the range of positions where the particle can be found, (d) its maximum kinetic energy, (e) the location where it has maximum kinetic energy, and (f) the *binding energy* of the system—that is, the additional energy that it would have to be given in order for the particle to move out to $r \to \infty$.

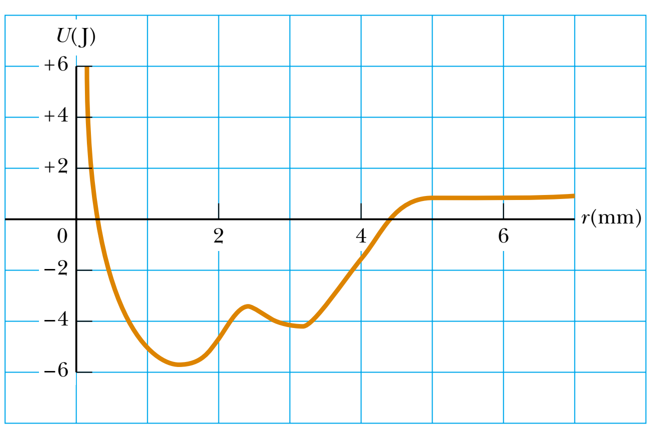
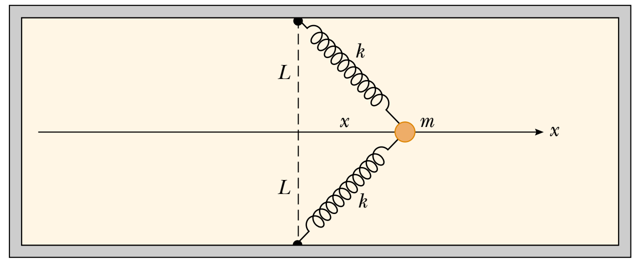


Figure P8.46

47. A particle of mass 1.18 kg is attached between two identical springs on a horizontal frictionless tabletop. The springs have force constant k and each is initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P8.47, show that the potential energy of the system is

$$U(x) = kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)$$

(*Hint*: See Problem 58 in Chapter 7.) (b) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume that $L = 1.20$ m and $k = 40.0$ N/m. (c) If the particle is pulled 0.500 m to the right and then released, what is its speed when it reaches the equilibrium point $x = 0$?



Top View

Figure P8.47

Additional Problems

48. A block slides down a curved frictionless track and then up an inclined plane as in Figure P8.48. The coefficient of kinetic friction between block and incline is μ_k. Use energy methods to show that the maximum height reached by the block is

$$y_{\max} = \frac{h}{1 + \mu_k \cot \theta}$$

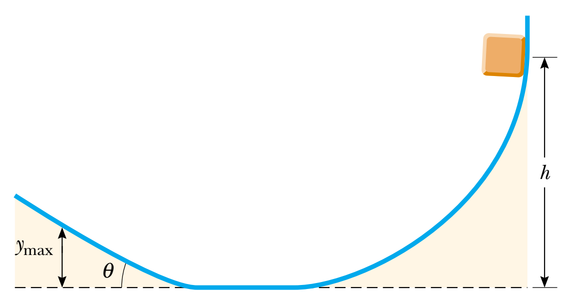


Figure P8.48

49. Make an order-of-magnitude estimate of your power output as you climb stairs. In your solution, state the physical quantities you take as data and the values you measure or estimate for them. Do you consider your peak power or your sustainable power?

50. **Review problem.** The mass of a car is 1 500 kg. The shape of the body is such that its aerodynamic drag coefficient is $D = 0.330$ and the frontal area is 2.50 m^2. Assuming that the drag force is proportional to v^2 and neglecting other sources of friction, calculate the power required to maintain a speed of 100 km/h as the car climbs a long hill sloping at 3.20°.

51. Assume that you attend a state university that started out as an agricultural college. Close to the center of the campus is a tall silo topped with a hemispherical cap. The cap is frictionless when wet. Someone has somehow balanced a pumpkin at the highest point. The line from the center of curvature of the cap to the pumpkin makes an angle $\theta_i = 0°$ with the vertical. While you happen to be standing nearby in the middle of a rainy night, a breath of wind makes the pumpkin start sliding downward from rest. It loses contact with the cap when the line from the center of the hemisphere to the pumpkin makes a certain angle with the vertical. What is this angle?

52. A 200-g particle is released from rest at point Ⓐ along the horizontal diameter on the inside of a frictionless, hemispherical bowl of radius $R = 30.0$ cm (Fig. P8.52). Calculate (a) the gravitational potential energy of the particle–Earth system when the particle is at point Ⓐ relative to point Ⓑ, (b) the kinetic energy of the particle at point Ⓑ, (c) its speed at point Ⓑ, and (d) its kinetic energy and the potential energy when the particle is at point Ⓒ.

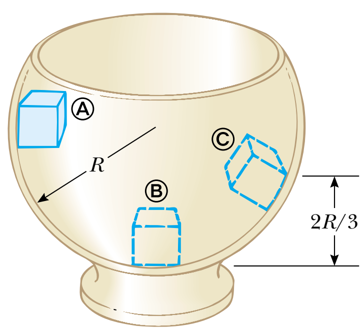


Figure P8.52 Problems 52 and 53.

53. **What If?** The particle described in Problem 52 (Fig. P8.52) is released from rest at Ⓐ, and the surface of the bowl is rough. The speed of the particle at Ⓑ is 1.50 m/s. (a) What is its kinetic energy at Ⓑ? (b) How much mechanical energy is transformed into internal energy as the particle moves from Ⓐ to Ⓑ? (c) Is it possible to determine the coefficient of friction from these results in any simple manner? Explain.

54. A 2.00-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m (Fig. P8.54). The pulley is frictionless. The block is released from rest when the spring is unstretched. The block moves 20.0 cm down the incline before coming to rest. Find the coefficient of kinetic friction between block and incline.

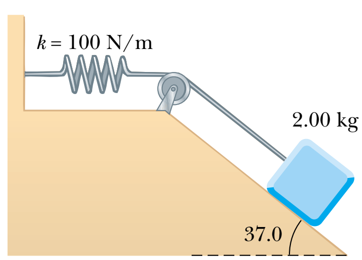


Figure P8.54 Problems 54 and 55.

55. **Review problem.** Suppose the incline is frictionless for the system described in Problem 54 (Fig. P8.54). The block is released from rest with the spring initially unstretched. (a) How far does it move down the incline before coming to rest? (b) What is its acceleration at its lowest point? Is the acceleration constant? (c) Describe the energy transformations that occur during the descent.

56. A child's pogo stick (Fig. P8.56) stores energy in a spring with a force constant of 2.50×10^4 N/m. At position Ⓐ ($x_A = -0.100$ m), the spring compression is a maximum and the child is momentarily at rest. At position Ⓑ ($x_B = 0$), the spring is relaxed and the child is moving upward. At position Ⓒ, the child is again momentarily at rest at the top of the jump. The combined mass of child and pogo stick is 25.0 kg. (a) Calculate the total energy of the child–stick–Earth system if both gravitational and elastic potential energies are zero for $x = 0$. (b) Determine x_C. (c) Calculate the speed of the child at $x = 0$. (d) Determine the value of x for which the kinetic energy of the system is a maximum. (e) Calculate the child's maximum upward speed.

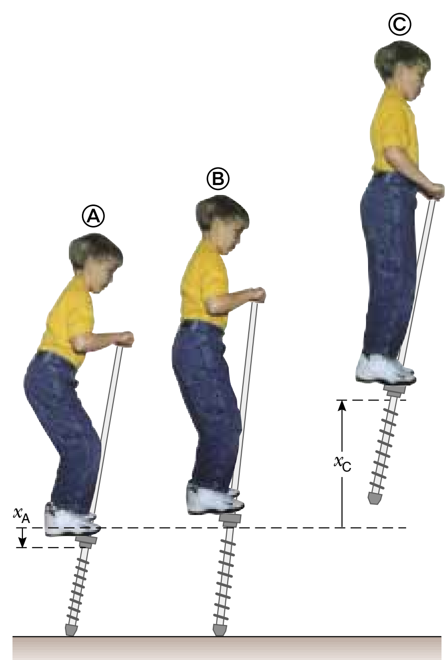


Figure P8.56

57. A 10.0-kg block is released from point Ⓐ in Figure P8.57. The track is frictionless except for the portion between points Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between Ⓑ and Ⓒ.

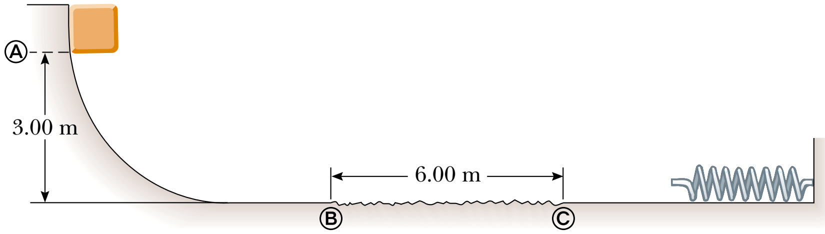


Figure P8.57

58. The potential energy function for a system is given by $U(x) = -x^3 + 2x^2 + 3x$. (a) Determine the force F_x as a function of x. (b) For what values of x is the force equal to zero? (c) Plot $U(x)$ versus x and F_x versus x, and indicate points of stable and unstable equilibrium.

59. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.59. The spring is unstretched when the system is shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

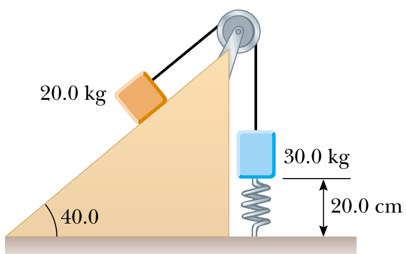


Figure P8.59

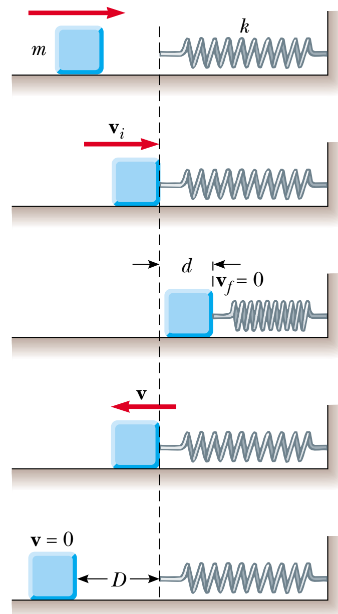


Figure P8.60

60. A 1.00-kg object slides to the right on a surface having a coefficient of kinetic friction 0.250 (Fig. P8.60). The object has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has a force constant of 50.0 N/m. The object comes to rest after the spring has been compressed a distance d. The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the object comes to rest a distance D to the left of the unstretched spring. Find (a) the distance of compression d, (b) the speed v at the unstretched position when the object is moving to the left, and (c) the distance D where the object comes to rest.

61. A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x (Fig. P8.61). The force constant of the spring is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The speed of the block at the bottom of the track is $v_B = 12.0$ m/s, and the block experiences an average friction force of 7.00 N while sliding up the track. (a) What is x? (b) What speed do you predict for the block at the top of the track? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

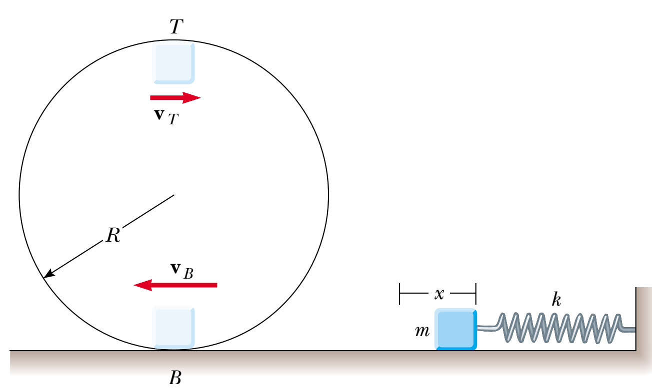


Figure P8.61

62. A uniform chain of length 8.00 m initially lies stretched out on a horizontal table. (a) If the coefficient of static friction between chain and table is 0.600, show that the chain will begin to slide off the table if at least 3.00 m of it hangs over the edge of the table. (b) Determine the speed of the chain as all of it leaves the table, given that the coefficient of kinetic friction between the chain and the table is 0.400.

63. A child slides without friction from a height h along a curved water slide (Fig. P8.63). She is launched from a height $h/5$ into the pool. Determine her maximum airborne height y in terms of h and θ.

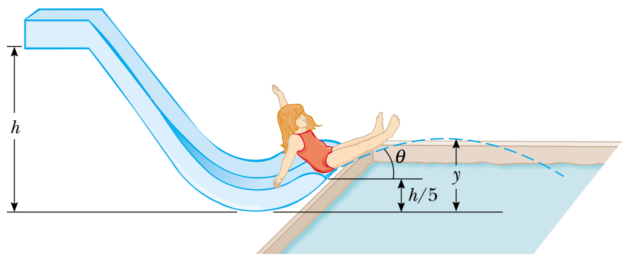


Figure P8.63

64. Refer to the situation described in Chapter 5, Problem 65. A 1.00-kg glider on a horizontal air track is pulled by a string at angle θ. The taut string runs over a light pulley at height $h_0 = 40.0$ cm above the line of motion of the glider. The other end of the string is attached to a hanging mass of 0.500 kg as in Fig. P5.65. (a) Show that the speed of the glider v_x and the speed of the hanging mass v_y are related by $v_y = v_x \cos\theta$. The glider is released from rest when $\theta = 30.0°$. Find (b) v_x and (c) v_y when $\theta = 45.0°$. (d) Explain why the answers to parts (b) and (c) to Chapter 5, Problem 65 do not help to solve parts (b) and (c) of this problem.

65. Jane, whose mass is 50.0 kg, needs to swing across a river (having width D) filled with man-eating crocodiles to save Tarzan from danger. She must swing into a wind exerting constant horizontal force **F**, on a vine having length L and initially making an angle θ with the vertical (Fig. P8.65). Taking $D = 50.0$ m, $F = 110$ N, $L = 40.0$ m, and $\theta = 50.0°$, (a) with what minimum speed must Jane begin her swing in order to just make it to the other side? (b) Once the rescue is complete, Tarzan and Jane must swing back across the river. With what minimum speed must they begin their swing? Assume that Tarzan has a mass of 80.0 kg.

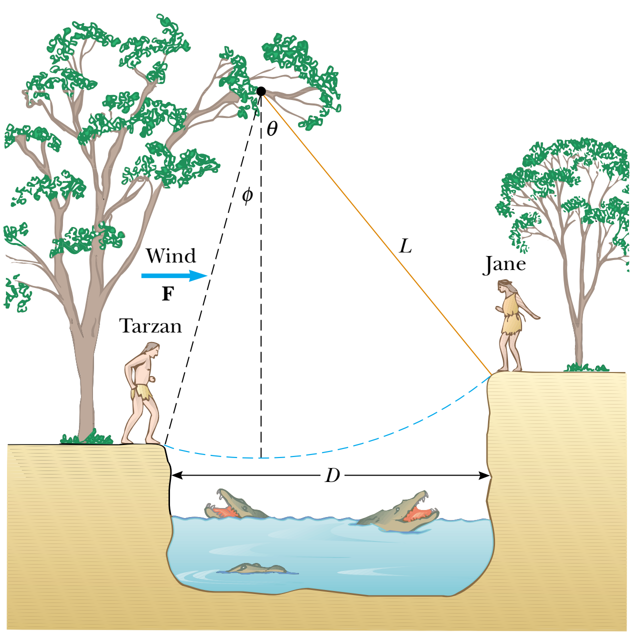


Figure P8.65

66. A 5.00-kg block free to move on a horizontal, frictionless surface is attached to one end of a light horizontal spring. The other end of the spring is held fixed. The spring is compressed 0.100 m from equilibrium and released. The speed of the block is 1.20 m/s when it passes the equilibrium position of the spring. The same experiment is now repeated with the frictionless surface replaced by a surface for which the coefficient of kinetic friction is 0.300. Determine the speed of the block at the equilibrium position of the spring.

67. A skateboarder with his board can be modeled as a particle of mass 76.0 kg, located at his center of mass (which we will study in Chapter 9). As in Figure P8.67, the skateboarder starts from rest in a crouching position at one lip of a half-pipe (point Ⓐ). The half-pipe is a dry water channel, forming one half of a cylinder of radius 6.80 m with its axis horizontal. On his descent, the skateboarder moves without friction so that his center of mass moves through one quarter of a circle of radius 6.30 m. (a) Find his speed at the bottom of the half-pipe (point Ⓑ). (b) Find his centripetal acceleration. (c) Find the normal force n_B acting on the skateboarder at point Ⓑ. Immediately after passing point Ⓑ, he stands up and raises his arms, lifting his center of mass from 0.500 m to 0.950 m above the concrete (point Ⓒ). To account for the conversion of chemical into mechanical energy, model his legs as doing work by pushing him vertically up, with a constant force equal to the normal force n_B, over a distance of 0.450 m. (You will be able to solve this problem with a more accurate model in Chapter 11.) (d) What is the work done on the skateboarder's body in this process? Next, the skateboarder glides upward with his center of mass moving in a quarter circle of radius 5.85 m. His body is horizontal when he passes point Ⓓ, the far lip of the half-pipe. (e) Find his speed at this location. At last he goes ballistic, twisting around while his center of mass moves vertically. (f) How high above point Ⓓ does he rise? (g) Over what time interval is he airborne before he touches down, 2.34 m below the level of point Ⓓ? [*Caution*: Do not try this yourself without the required skill and protective equipment, or in a drainage channel to which you do not have legal access.]

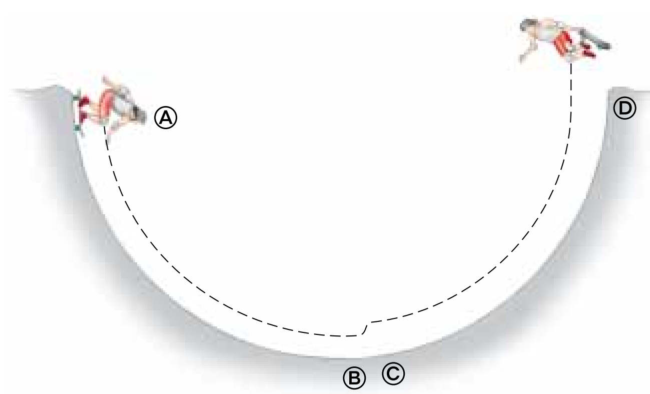


Figure P8.67

68. A block of mass M rests on a table. It is fastened to the lower end of a light vertical spring. The upper end of the spring is fastened to a block of mass m. The upper block is pushed down by an additional force $3mg$, so the spring compression is $4mg/k$. In this configuration the upper block is released from rest. The spring lifts the lower block off the table. In terms of m, what is the greatest possible value for M?

69. A ball having mass m is connected by a strong string of length L to a pivot point and held in place in a vertical position. A wind exerting constant force of magnitude F is blowing from left to right as in Figure P8.69a. (a) If the ball is released from rest, show that the maximum height H reached by the ball, as measured from its initial height, is

$$H = \frac{2L}{1 + (mg/F)^2}$$

Check that the above result is valid both for cases when $0 \leq H \leq L$ and for $L \leq H \leq 2L$. (b) Compute the value of H using the values $m = 2.00$ kg, $L = 2.00$ m, and $F = 14.7$ N. (c) Using these same values, determine the *equilibrium* height of the ball. (d) Could the equilibrium height ever be larger than L? Explain.

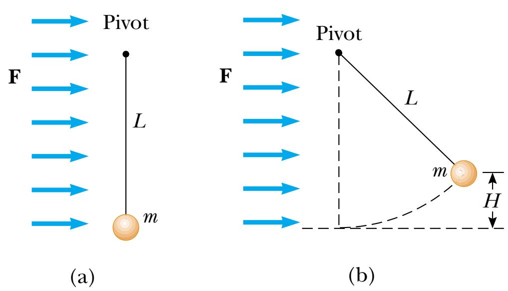


Figure P8.69

70. A ball is tied to one end of a string. The other end of the string is held fixed. The ball is set moving around a vertical circle without friction, and with speed $v_i = \sqrt{Rg}$ at the top of the circle, as in Figure P8.70. At what angle θ should the string be cut so that the ball will then travel through the center of the circle?

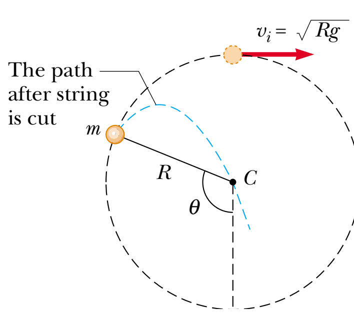


Figure P8.70

71. A ball whirls around in a vertical circle at the end of a string. If the total energy of the ball–Earth system remains constant, show that the tension in the string at the bottom is greater than the tension at the top by six times the weight of the ball.

72. A pendulum, comprising a string of length L and a small sphere, swings in the vertical plane. The string hits a peg located a distance d below the point of suspension (Fig. P8.72). (a) Show that if the sphere is released from a height below that of the peg, it will return to this height after striking the peg. (b) Show that if the pendulum is released from the horizontal position ($\theta = 90°$) and is to swing in a complete circle centered on the peg, then the minimum value of d must be $3L/5$.

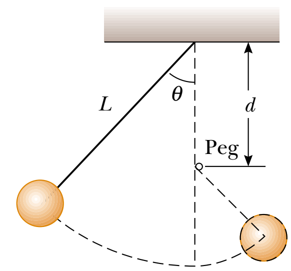


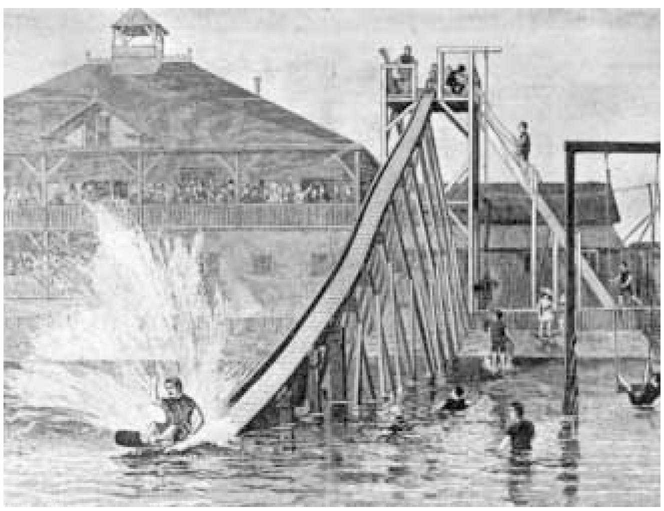
Figure P8.72

73. A roller-coaster car is released from rest at the top of the first rise and then moves freely with negligible friction. The roller coaster shown in Figure P8.73 has a circular loop of radius R in a vertical plane. (a) Suppose first that the car barely makes it around the loop: at the top of the loop the riders are upside down and feel weightless. Find the required height of the release point above the bottom of the loop in terms of R. (b) Now assume that the release point is at or above the minimum required height. Show that the normal force on the car at the bottom of the loop exceeds the normal force at the top of the loop by six times the weight of the car. The normal force on each rider follows the same rule. Such a large normal force is dangerous and very uncomfortable for the riders. Roller coasters are therefore not built with circular loops in vertical planes. Figure P6.20 and the photograph on page 157 show two actual designs.


Figure P8.73

74. **Review problem.** In 1887 in Bridgeport, Connecticut, C. J. Belknap built the water slide shown in Figure P8.74. A rider on a small sled, of total mass 80.0 kg, pushed off to start at the top of the slide (point Ⓐ) with a speed of 2.50 m/s. The chute was 9.76 m high at the top, 54.3 m long, and 0.51 m wide. Along its length, 725 wheels made

friction negligible. Upon leaving the chute horizontally at its bottom end (point Ⓒ), the rider skimmed across the water of Long Island Sound for as much as 50 m, "skipping along like a flat pebble," before at last coming to rest and swimming ashore, pulling his sled after him. According to *Scientific American*, "The facial expression of novices taking their first adventurous slide is quite remarkable, and the sensations felt are correspondingly novel and peculiar." (a) Find the speed of the sled and rider at point Ⓒ. (b) Model the force of water friction as a constant retarding force acting on a particle. Find the work done by water friction in stopping the sled and rider. (c) Find the magnitude of the force the water exerts on the sled. (d) Find the magnitude of the force the chute exerts on the sled at point Ⓑ. (e) At point Ⓒ the chute is horizontal but curving in the vertical plane. Assume its radius of curvature is 20.0 m. Find the force the chute exerts on the sled at point Ⓒ.



Engraving from Scientific American, July 1888

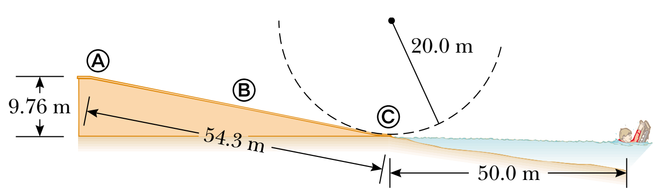


Figure P8.74

Answers to Quick Quizzes

8.1 (c). The sign of the gravitational potential energy depends on your choice of zero configuration. If the two objects in the system are closer together than in the zero configuration, the potential energy is negative. If they are farther apart, the potential energy is positive.

8.2 (c). The reason that we can ignore the kinetic energy of the massive Earth is that this kinetic energy is so small as to be essentially zero.

8.3 (a). We must include the Earth if we are going to work with gravitational potential energy.

8.4 (c). The total mechanical energy, kinetic plus potential, is conserved.

8.5 (a). The more massive rock has twice as much gravitational potential energy associated with it compared to the lighter rock. Because mechanical energy of an isolated system is conserved, the more massive rock will arrive at the ground with twice as much kinetic energy as the lighter rock.

8.6 $v_1 = v_2 = v_3$. The first and third balls speed up after they are thrown, while the second ball initially slows down but then speeds up after reaching its peak. The paths of all three balls are parabolas, and the balls take different times to reach the ground because they have different initial velocities. However, all three balls have the same speed at the moment they hit the ground because all start with the same kinetic energy and the ball–Earth system undergoes the same change in gravitational potential energy in all three cases.

8.7 (c). This system exhibits changes in kinetic energy as well as in both types of potential energy.

8.8 (a). Because the Earth is not included in the system, there is no gravitational potential energy associated with the system.

8.9 (c). The friction force must transform four times as much mechanical energy into internal energy if the speed is doubled, because kinetic energy depends on the square of the speed. Thus, the force must act over four times the distance.

8.10 (c). The decrease in mechanical energy of the system is $f_k d$, where d is the distance the block moves along the incline. While the force of kinetic friction remains the same, the distance d is smaller because a component of the gravitational force is pulling on the block in the direction opposite to its velocity.

8.11 (d). The slope of a $U(x)$-versus-x graph is by definition $dU(x)/dx$. From Equation 8.18, we see that this expression is equal to the negative of the x component of the conservative force acting on an object that is part of the system.

Chapter 9

Linear Momentum and Collisions

▲ *A moving bowling ball carries momentum, the topic of this chapter. In the collision between the ball and the pins, momentum is transferred to the pins. (Mark Cooper/Corbis Stock Market)*

CHAPTER OUTLINE

Consider what happens when a bowling ball strikes a pin, as in the opening photograph. The pin is given a large velocity as a result of the collision; consequently, it flies away and hits other pins or is projected toward the backstop. Because the average force exerted on the pin during the collision is large (resulting in a large acceleration), the pin achieves the large velocity very rapidly and experiences the force for a very short time interval. According to Newton's third law, the pin exerts a reaction force on the ball that is equal in magnitude and opposite in direction to the force exerted by the ball on the pin. This reaction force causes the ball to accelerate, but because the ball is so much more massive than the pin, the ball's acceleration is much less than the pin's acceleration.

Although F and a are large for the pin, they vary in time—a complicated situation! One of the main objectives of this chapter is to enable you to understand and analyze such events in a simple way. First, we introduce the concept of *momentum*, which is useful for describing objects in motion. Imagine that you have intercepted a football and see two players from the opposing team approaching you as you run with the ball. One of the players is the 180-lb quarterback who threw the ball; the other is a 300-lb lineman. Both of the players are running toward you at 5 m/s. However, because the two players have different masses, intuitively you know that you would rather collide with the quarterback than with the lineman. The momentum of an object is related to both its mass and its velocity. The concept of momentum leads us to a second conservation law, that of conservation of momentum. This law is especially useful for treating problems that involve collisions between objects and for analyzing rocket propulsion. In this chapter we also introduce the concept of the center of mass of a system of particles. We find that the motion of a system of particles can be described by the motion of one representative particle located at the center of mass.

9.1 Linear Momentum and Its Conservation

In the preceding two chapters we studied situations that are complex to analyze with Newton's laws. We were able to solve problems involving these situations by applying a conservation principle—conservation of energy. Consider another situation—a 60-kg archer stands on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s. From Newton's third law, we know that the force that the bow exerts on the arrow will be matched by a force in the opposite direction on the bow (and the archer). This will cause the archer to begin to slide backward on the ice. But with what speed? We cannot answer this question directly using *either* Newton's second law or an energy approach—there is not enough information.

Despite our inability to solve the archer problem using our techniques learned so far, this is a very simple problem to solve if we introduce a new quantity that describes motion, *linear momentum.* Let us apply the General Problem-Solving Strategy and *conceptualize* an isolated system of two particles (Fig. 9.1) with masses m_1 and m_2 and moving with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ at an instant of time. Because the system is isolated, the only force on one particle is that from the other particle and we can *categorize* this as a situation in which Newton's laws will be useful. If a force from particle 1 (for example, a gravitational force) acts on particle 2, then there must be a second force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1. That is, they form a Newton's third law action–reaction pair, so that $\mathbf{F}_{12} = -\mathbf{F}_{21}$. We can express this condition as

$$\mathbf{F}_{21} + \mathbf{F}_{12} = 0$$

Let us further *analyze* this situation by incorporating Newton's second law. Over some time interval, the interacting particles in the system will accelerate. Thus, replacing each force with $m\mathbf{a}$ gives

$$m_1\mathbf{a}_1 + m_2\mathbf{a}_2 = 0$$

Now we replace the acceleration with its definition from Equation 4.5:

$$m_1\frac{d\mathbf{v}_1}{dt} + m_2\frac{d\mathbf{v}_2}{dt} = 0$$

If the masses m_1 and m_2 are constant, we can bring them into the derivatives, which gives

$$\frac{d(m_1\mathbf{v}_1)}{dt} + \frac{d(m_2\mathbf{v}_2)}{dt} = 0$$

$$\frac{d}{dt}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = 0 \qquad (9.1)$$

To *finalize* this discussion, note that the derivative of the sum $m_1\mathbf{v}_1 + m_2\mathbf{v}_2$ with respect to time is zero. Consequently, this sum must be constant. We learn from this discussion that the quantity $m\mathbf{v}$ for a particle is important, in that the sum of these quantities for an isolated system is conserved. We call this quantity *linear momentum*:

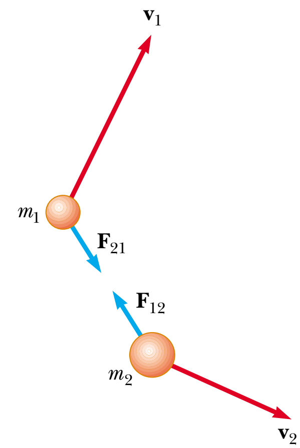


Figure 9.1 Two particles interact with each other. According to Newton's third law, we must have $\mathbf{F}_{12} = -\mathbf{F}_{21}$.

Definition of linear momentum of a particle

The **linear momentum** of a particle or an object that can be modeled as a particle of mass m moving with a velocity $\mathbf{v}$ is defined to be the product of the mass and velocity:

$$\mathbf{p} \equiv m\mathbf{v} \qquad (9.2)$$

Linear momentum is a vector quantity because it equals the product of a scalar quantity m and a vector quantity $\mathbf{v}$. Its direction is along $\mathbf{v}$, it has dimensions ML/T, and its SI unit is kg · m/s.

If a particle is moving in an arbitrary direction, $\mathbf{p}$ must have three components, and Equation 9.2 is equivalent to the component equations

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

As you can see from its definition, the concept of momentum[1] provides a quantitative distinction between heavy and light particles moving at the same velocity. For example, the momentum of a bowling ball moving at 10 m/s is much greater than that of a tennis ball moving at the same speed. Newton called the product $m\mathbf{v}$ *quantity of motion*; this is perhaps a more graphic description than our present-day word *momentum*, which comes from the Latin word for movement.

Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle. We start with Newton's second law and substitute the definition of acceleration:

$$\sum\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

[1] In this chapter, the terms *momentum* and *linear momentum* have the same meaning. Later, in Chapter 11, we shall use the term *angular momentum* when dealing with rotational motion.

In Newton's second law, the mass m is assumed to be constant. Thus, we can bring m inside the derivative notation to give us

Newton's second law for a particle

$$\sum \mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt} \tag{9.3}$$

This shows that **the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.**

This alternative form of Newton's second law is the form in which Newton presented the law and is actually more general than the form we introduced in Chapter 5. In addition to situations in which the velocity vector varies with time, we can use Equation 9.3 to study phenomena in which the mass changes. For example, the mass of a rocket changes as fuel is burned and ejected from the rocket. We cannot use $\Sigma \mathbf{F} = m\mathbf{a}$ to analyze rocket propulsion; we must use Equation 9.3, as we will show in Section 9.7.

The real value of Equation 9.3 as a tool for analysis, however, arises if we apply it to a *system* of two or more particles. As we have seen, this leads to a law of conservation of momentum for an isolated system. Just as the law of conservation of energy is useful in solving complex motion problems, the law of conservation of momentum can greatly simplify the analysis of other types of complicated motion.

Quick Quiz 9.1 Two objects have equal kinetic energies. How do the magnitudes of their momenta compare? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) not enough information to tell.

Quick Quiz 9.2 Your physical education teacher throws a baseball to you at a certain speed, and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with (a) the same speed as the baseball (b) the same momentum (c) the same kinetic energy. Rank these choices from easiest to hardest to catch.

Using the definition of momentum, Equation 9.1 can be written

$$\frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0$$

Because the time derivative of the total momentum $\mathbf{p}_{\text{tot}} = \mathbf{p}_1 + \mathbf{p}_2$ is *zero*, we conclude that the *total* momentum of the system must remain constant:

$$\mathbf{p}_{\text{tot}} = \mathbf{p}_1 + \mathbf{p}_2 = \text{constant} \tag{9.4}$$

or, equivalently,

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} \tag{9.5}$$

where $\mathbf{p}_{1i}$ and $\mathbf{p}_{2i}$ are the initial values and $\mathbf{p}_{1f}$ and $\mathbf{p}_{2f}$ the final values of the momenta for the two particles for the time interval during which the particles interact. Equation 9.5 in component form demonstrates that the total momenta in the x, y, and z directions are all independently conserved:

$$p_{ix} = p_{fx} \qquad p_{iy} = p_{fy} \qquad p_{iz} = p_{fz} \tag{9.6}$$

This result, known as the **law of conservation of linear momentum**, can be extended to any number of particles in an isolated system. It is considered one of the most important laws of mechanics. We can state it as follows:

PITFALL PREVENTION

9.1 Momentum of a *System* is Conserved

Remember that the momentum of an isolated *system* is conserved. The momentum of one particle within an isolated system is not necessarily conserved, because other particles in the system may be interacting with it. Always apply conservation of momentum to an isolated *system*.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Conservation of momentum

This law tells us that **the total momentum of an isolated system at all times equals its initial momentum.**

Notice that we have made no statement concerning the nature of the forces acting on the particles of the system. The only requirement is that the forces must be *internal* to the system.

Quick Quiz 9.3 A ball is released and falls toward the ground with no air resistance. The isolated system for which momentum is conserved is (a) the ball (b) the Earth (c) the ball and the Earth (d) impossible to determine.

Quick Quiz 9.4 A car and a large truck traveling at the same speed make a head-on collision and stick together. Which vehicle experiences the larger change in the magnitude of momentum? (a) the car (b) the truck (c) The change in the magnitude of momentum is the same for both. (d) impossible to determine.

Example 9.1 The Archer

Interactive

Let us consider the situation proposed at the beginning of this section. A 60-kg archer stands at rest on frictionless ice and fires a 0.50-kg arrow horizontally at 50 m/s (Fig. 9.2). With what velocity does the archer move across the ice after firing the arrow?

Solution We *cannot* solve this problem using Newton's second law, $\Sigma \mathbf{F} = m\mathbf{a}$, because we have no information about the force on the arrow or its acceleration. We *cannot* solve this problem using an energy approach because we do not know how much work is done in pulling the bow back or how much potential energy is stored in the bow. However, we *can* solve this problem very easily with conservation of momentum.

Let us take the system to consist of the archer (including the bow) and the arrow. The system is not isolated because the gravitational force and the normal force act on the system. However, these forces are vertical and perpendicular to the motion of the system. Therefore, there are no external forces in the horizontal direction, and we can consider the system to be isolated in terms of momentum components in this direction.

The total horizontal momentum of the system before the arrow is fired is zero ($m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = 0$), where the archer is particle 1 and the arrow is particle 2. Therefore, the total horizontal momentum after the arrow is fired must be zero; that is,

$$m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = 0$$

We choose the direction of firing of the arrow as the positive x direction. With $m_1 = 60$ kg, $m_2 = 0.50$ kg, and $\mathbf{v}_{2f} = 50\hat{\mathbf{i}}$ m/s, solving for $\mathbf{v}_{1f}$, we find the recoil velocity of the archer to be

$$\mathbf{v}_{1f} = -\frac{m_2}{m_1}\mathbf{v}_{2f} = -\left(\frac{0.50 \text{ kg}}{60 \text{ kg}}\right)(50\hat{\mathbf{i}} \text{ m/s}) = \boxed{-0.42\hat{\mathbf{i}} \text{ m/s}}$$

The negative sign for $\mathbf{v}_{1f}$ indicates that the archer is moving to the left after the arrow is fired, in the direction opposite the direction of motion of the arrow, in accordance with Newton's third law. Because the archer is much more massive than the arrow, his acceleration and consequent velocity are much smaller than the acceleration and velocity of the arrow.

What If? **What if the arrow were shot in a direction that makes an angle θ with the horizontal? How will this change the recoil velocity of the archer?**

Answer The recoil velocity should decrease in magnitude because only a component of the velocity is in the x direction.

Figure 9.2 (Example 9.1) An archer fires an arrow horizontally to the right. Because he is standing on frictionless ice, he will begin to slide to the left across the ice.

If the arrow were shot straight up, for example, there would be no recoil at all—the archer would just be pressed down into the ice because of the firing of the arrow.

Only the x component of the momentum of the arrow should be used in a conservation of momentum statement, because momentum is only conserved in the x direction. In the y direction, the normal force from the ice and the gravitational force are external influences on the system. Conservation of momentum in the x direction gives us

$$m_1 v_{1f} + m_2 v_{2f} \cos\theta = 0$$

leading to

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} \cos\theta$$

For $\theta = 0$, $\cos\theta = 1$ and this reduces to the value when the arrow is fired horizontally. For nonzero values of θ, the cosine function is less than 1 and the recoil velocity is less than the value calculated for $\theta = 0$. If $\theta = 90°$, $\cos\theta = 0$, and there is no recoil velocity v_{1f}, as we argued conceptually.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can change the mass of the archer and the mass and speed of the arrow.*

Example 9.2 Breakup of a Kaon at Rest

One type of nuclear particle, called the *neutral kaon* (K^0), breaks up into a pair of other particles called *pions* (π^+ and π^-) that are oppositely charged but equal in mass, as illustrated in Figure 9.3. Assuming the kaon is initially at rest, prove that the two pions must have momenta that are equal in magnitude and opposite in direction.

Solution The breakup of the kaon can be written

$$K^0 \longrightarrow \pi^+ + \pi^-$$

If we let $\mathbf{p}^+$ be the final momentum of the positive pion and $\mathbf{p}^-$ the final momentum of the negative pion, the final momentum of the system consisting of the two pions can be written

$$\mathbf{p}_f = \mathbf{p}^+ + \mathbf{p}^-$$

Because the kaon is at rest before the breakup, we know that $\mathbf{p}_i = 0$. Because the momentum of the isolated system (the kaon before the breakup, the two pions afterward) is conserved, $\mathbf{p}_i = \mathbf{p}_f = 0$, so that $\mathbf{p}^+ + \mathbf{p}^- = 0$, or

$$\mathbf{p}^+ = -\mathbf{p}^-$$

An important point to learn from this problem is that even though it deals with objects that are very different from those in the preceding example, the physics is identical: *linear momentum is conserved in an isolated system.*

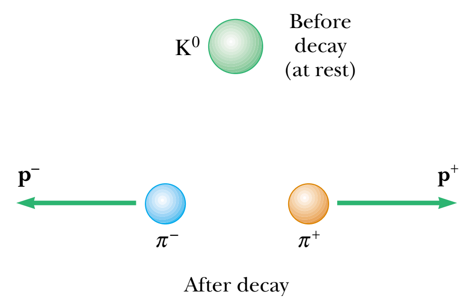


Figure 9.3 (Example 9.2) A kaon at rest breaks up spontaneously into a pair of oppositely charged pions. The pions move apart with momenta that are equal in magnitude but opposite in direction.

9.2 Impulse and Momentum

According to Equation 9.3, the momentum of a particle changes if a net force acts on the particle. Knowing the change in momentum caused by a force is useful in solving some types of problems. To build a better understanding of this important concept, let us assume that a single force $\mathbf{F}$ acts on a particle and that this force may vary with time. According to Newton's second law, $\mathbf{F} = d\mathbf{p}/dt$, or

$$d\mathbf{p} = \mathbf{F}\,dt \tag{9.7}$$

We can integrate[2] this expression to find the change in the momentum of a particle when the force acts over some time interval. If the momentum of the particle changes from $\mathbf{p}_i$ at time t_i to $\mathbf{p}_f$ at time t_f, integrating Equation 9.7 gives

[2] Note that here we are integrating force with respect to time. Compare this with our efforts in Chapter 7, where we integrated force with respect to position to find the work done by the force.

$$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F}\,dt \tag{9.8}$$

To evaluate the integral, we need to know how the force varies with time. The quantity on the right side of this equation is called the **impulse** of the force **F** acting on a particle over the time interval $\Delta t = t_f - t_i$. Impulse is a vector defined by

$$\mathbf{I} \equiv \int_{t_i}^{t_f} \mathbf{F}\,dt \tag{9.9}$$

Impulse of a force

Equation 9.8 is an important statement known as the **impulse–momentum theorem:**[3]

Impulse–momentum theorem

The impulse of the force **F** acting on a particle equals the change in the momentum of the particle.

This statement is equivalent to Newton's second law. From this definition, we see that impulse is a vector quantity having a magnitude equal to the area under the force–time curve, as described in Figure 9.4a. In this figure, it is assumed that the force varies in time in the general manner shown and is nonzero in the time interval $\Delta t = t_f - t_i$. The direction of the impulse vector is the same as the direction of the change in momentum. Impulse has the dimensions of momentum—that is, ML/T. Note that impulse is *not* a property of a particle; rather, it is a measure of the degree to which an external force changes the momentum of the particle. Therefore, when we say that an impulse is given to a particle, we mean that momentum is transferred from an external agent to that particle.

Because the force imparting an impulse can generally vary in time, it is convenient to define a time-averaged force

$$\overline{\mathbf{F}} \equiv \frac{1}{\Delta t}\int_{t_i}^{t_f} \mathbf{F}\,dt \tag{9.10}$$

where $\Delta t = t_f - t_i$. (This is an application of the mean value theorem of calculus.) Therefore, we can express Equation 9.9 as

$$\mathbf{I} \equiv \overline{\mathbf{F}}\,\Delta t \tag{9.11}$$

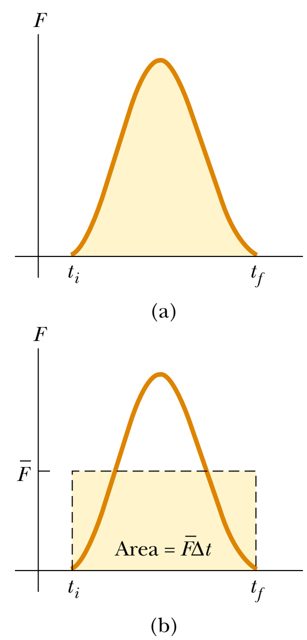


Figure 9.4 (a) A force acting on a particle may vary in time. The impulse imparted to the particle by the force is the area under the force-versus-time curve. (b) In the time interval Δt, the time-averaged force (horizontal dashed line) gives the same impulse to a particle as does the time-varying force described in part (a).

Courtesy of Saab

Airbags in automobiles have saved countless lives in accidents. The airbag increases the time interval during which the passenger is brought to rest, thereby decreasing the force on (and resultant injury to) the passenger.

[3] Although we assumed that only a single force acts on the particle, the impulse–momentum theorem is valid when several forces act; in this case, we replace **F** in Equation 9.8 with $\Sigma\mathbf{F}$.

This time-averaged force, shown in Figure 9.4b, can be interpreted as the constant force that would give to the particle in the time interval Δt the same impulse that the time-varying force gives over this same interval.

In principle, if $\mathbf{F}$ is known as a function of time, the impulse can be calculated from Equation 9.9. The calculation becomes especially simple if the force acting on the particle is constant. In this case, $\overline{\mathbf{F}} = \mathbf{F}$ and Equation 9.11 becomes

$$\mathbf{I} = \mathbf{F}\Delta t \tag{9.12}$$

In many physical situations, we shall use what is called the **impulse approximation, in which we assume that one of the forces exerted on a particle acts for a short time but is much greater than any other force present.** This approximation is especially useful in treating collisions in which the duration of the collision is very short. When this approximation is made, we refer to the force as an *impulsive force*. For example, when a baseball is struck with a bat, the time of the collision is about 0.01 s and the average force that the bat exerts on the ball in this time is typically several thousand newtons. Because this contact force is much greater than the magnitude of the gravitational force, the impulse approximation justifies our ignoring the gravitational forces exerted on the ball and bat. When we use this approximation, it is important to remember that $\mathbf{p}_i$ and $\mathbf{p}_f$ represent the momenta *immediately* before and after the collision, respectively. Therefore, in any situation in which it is proper to use the impulse approximation, the particle moves very little during the collision.

Quick Quiz 9.5 Two objects are at rest on a frictionless surface. Object 1 has a greater mass than object 2. When a constant force is applied to object 1, it accelerates through a distance d. The force is removed from object 1 and is applied to object 2. At the moment when object 2 has accelerated through the same distance d, which statements are true? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) $K_1 < K_2$ (e) $K_1 = K_2$ (f) $K_1 > K_2$.

Quick Quiz 9.6 Two objects are at rest on a frictionless surface. Object 1 has a greater mass than object 2. When a force is applied to object 1, it accelerates for a time interval Δt. The force is removed from object 1 and is applied to object 2. After object 2 has accelerated for the same time interval Δt, which statements are true? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) $K_1 < K_2$ (e) $K_1 = K_2$ (f) $K_1 > K_2$.

Quick Quiz 9.7 Rank an automobile dashboard, seatbelt, and airbag in terms of (a) the impulse and (b) the average force they deliver to a front-seat passenger during a collision, from greatest to least.

Example 9.3 Teeing Off

A golf ball of mass 50 g is struck with a club (Fig. 9.5). The force exerted by the club on the ball varies from zero, at the instant before contact, up to some maximum value and then back to zero when the ball leaves the club. Thus, the force–time curve is qualitatively described by Figure 9.4. Assuming that the ball travels 200 m, estimate the magnitude of the impulse caused by the collision.

Solution Let us use Ⓐ to denote the position of the ball when the club first contacts it, Ⓑ to denote the position of the ball when the club loses contact with the ball, and Ⓒ to denote the position of the ball upon landing. Neglecting air resistance, we can use Equation 4.14 for the range of a projectile:

$$R = x_{\text{C}} = \frac{v_{\text{B}}^2}{g}\sin 2\theta_{\text{B}}$$

Let us assume that the launch angle θ_{B} is 45°, the angle that provides the maximum range for any given launch velocity. This assumption gives $\sin 2\theta_{\text{B}} = 1$, and the launch velocity of the ball is

$$v_{\text{B}} = \sqrt{Rg} \approx \sqrt{(200\text{ m})(9.80\text{ m/s}^2)} = 44\text{ m/s}$$

Figure 9.5 (Example 9.3) A golf ball being struck by a club. Note the deformation of the ball due to the large force from the club.

Considering initial and final values of the ball's velocity for the time interval for the collision, $v_i = v_A = 0$ and $v_f = v_B$. Hence, the magnitude of the impulse imparted to the ball is

$$I = \Delta p = mv_B - mv_A = (50 \times 10^{-3}\ \text{kg})(44\ \text{m/s}) - 0$$
$$= \boxed{2.2\ \text{kg}\cdot\text{m/s}}$$

What If? **What if you were asked to find the average force on the ball during the collision with the club? Can you determine this value?**

Answer With the information given in the problem, we cannot find the average force. Considering Equation 9.11, we would need to know the time interval of the collision in order to calculate the average force. If we *assume* that the time interval is 0.01 s as it was for the baseball in the discussion after Equation 9.12, we can estimate the magnitude of the average force:

$$\bar{F} = \frac{I}{\Delta t} = \frac{2.2\ \text{kg}\cdot\text{m/s}}{0.01\ \text{s}} = 2 \times 10^2\ \text{N}$$

where we have kept only one significant figure due to our rough estimate of the time interval.

Example 9.4 How Good Are the Bumpers?

In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are $\mathbf{v}_i = -15.0\hat{\mathbf{i}}$ m/s and $\mathbf{v}_f = 2.60\hat{\mathbf{i}}$ m/s, respectively. If the collision lasts for 0.150 s, find the impulse caused by the collision and the average force exerted on the car.

Solution Let us assume that the force exerted by the wall on the car is large compared with other forces on the car so that we can apply the impulse approximation. Furthermore, we note that the gravitational force and the normal force exerted by the road on the car are perpendicular to the motion and therefore do not affect the horizontal momentum.

The initial and final momenta of the car are

$$\mathbf{p}_i = m\mathbf{v}_i = (1\,500\ \text{kg})(-15.0\hat{\mathbf{i}}\ \text{m/s})$$
$$= -2.25 \times 10^4\hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s}$$

$$\mathbf{p}_f = m\mathbf{v}_f = (1\,500\ \text{kg})(2.60\hat{\mathbf{i}}\ \text{m/s})$$
$$= 0.39 \times 10^4\hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s}$$

Hence, the impulse is equal to

Figure 9.6 (Example 9.4) (a) This car's momentum changes as a result of its collision with the wall. (b) In a crash test, much of the car's initial kinetic energy is transformed into energy associated with the damage to the car.

$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = 0.39 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s} - (-2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s})$$

$$\mathbf{I} = \boxed{2.64 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}}$$

The average force exerted by the wall on the car is

$$\overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{2.64 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}}{0.150 \text{ s}} = \boxed{1.76 \times 10^5 \hat{\mathbf{i}} \text{ N}}$$

In this problem, note that the signs of the velocities indicate the reversal of directions. What would the mathematics be describing if both the initial and final velocities had the same sign?

What If? What if the car did not rebound from the wall? Suppose the final velocity of the car is zero and the time interval of the collision remains at 0.150 s. Would this represent a larger or a smaller force by the wall on the car?

Answer In the original situation in which the car rebounds, the force by the wall on the car does two things in the time interval—it (1) stops the car and (2) causes it to move away from the wall at 2.60 m/s after the collision. If the car does not rebound, the force is only doing the first of these, stopping the car. This will require a *smaller* force.

Mathematically, in the case of the car that does not rebound, the impulse is

$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = 0 - (-2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}) = 2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}$$

The average force exerted by the wall on the car is

$$\overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{2.25 \times 10^4 \hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}}{0.150 \text{ s}} = 1.50 \times 10^5 \hat{\mathbf{i}} \text{ N}$$

which is indeed smaller than the previously calculated value, as we argued conceptually.

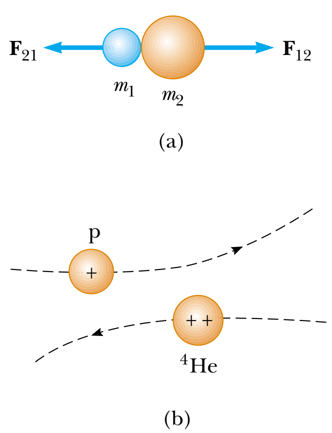


Figure 9.7 (a) The collision between two objects as the result of direct contact. (b) The "collision" between two charged particles.

9.3 Collisions in One Dimension

In this section we use the law of conservation of linear momentum to describe what happens when two particles collide. We use the term **collision** to represent an event during which two particles come close to each other and interact by means of forces. The time interval during which the velocities of the particles change from initial to final values is assumed to be short. The interaction forces are assumed to be much greater than any external forces present, so we can use the impulse approximation.

A collision may involve physical contact between two macroscopic objects, as described in Figure 9.7a, but the notion of what we mean by collision must be generalized because "physical contact" on a submicroscopic scale is ill-defined and hence meaningless. To understand this, consider a collision on an atomic scale (Fig. 9.7b), such as the collision of a proton with an alpha particle (the nucleus of a helium atom). Because the particles are both positively charged, they repel each other due to the strong electrostatic force between them at close separations and never come into "physical contact."

When two particles of masses m_1 and m_2 collide as shown in Figure 9.7, the impulsive forces may vary in time in complicated ways, such as that shown in Figure 9.4. Regardless of the complexity of the time behavior of the force of interaction, however, this force is internal to the system of two particles. Thus, the two particles form an isolated system, and the momentum of the system must be conserved. Therefore, the total momentum of an isolated system just before a collision equals the total momentum of the system just after the collision.

In contrast, the total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether or not kinetic energy is conserved is used to classify collisions as either *elastic* or *inelastic.*

Elastic collision

An **elastic collision** between two objects is one in which **the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.** Collisions between certain objects in the macroscopic world, such as billiard balls, are only *approximately* elastic because some deformation and loss of kinetic energy take place. For example, you can hear a billiard ball collision, so you know that some of the energy is being transferred away from the system by sound. An elastic collision must be perfectly silent! *Truly* elastic collisions occur between atomic and subatomic particles.

Inelastic collision

An **inelastic collision** is one in which **the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).** Inelastic collisions are of two types. When the colliding objects stick together after the collision, as happens when a meteorite collides with the Earth, the collision is called **perfectly inelastic.** When the colliding objects do not stick together, but some kinetic energy is lost, as in the case of a rubber ball colliding with a hard surface, the collision is called **inelastic** (with no modifying adverb). When the rubber ball collides with the hard surface, some of the kinetic energy of the ball is lost when the ball is deformed while it is in contact with the surface.

In most collisions, the kinetic energy of the system is *not* conserved because some of the energy is converted to internal energy and some of it is transferred away by means of sound. Elastic and perfectly inelastic collisions are limiting cases; most collisions fall somewhere between them.

In the remainder of this section, we treat collisions in one dimension and consider the two extreme cases—perfectly inelastic and elastic collisions. The important distinction between these two types of collisions is that **momentum of the system is conserved in all collisions, but kinetic energy of the system is conserved only in elastic collisions.**

PITFALL PREVENTION

9.2 Inelastic Collisions

Generally, inelastic collisions are hard to analyze unless additional information is provided. This appears in the mathematical representation as having more unknowns than equations.

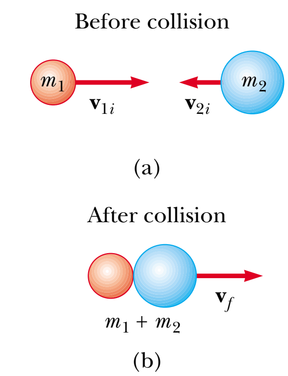
Perfectly Inelastic Collisions

Consider two particles of masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line, as shown in Figure 9.8. The two particles collide head-on, stick together, and then move with some common velocity $\mathbf{v}_f$ after the collision. Because the momentum of an isolated system is conserved in *any* collision, we can say that the total momentum before the collision equals the total momentum of the composite system after the collision:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f \tag{9.13}$$

Solving for the final velocity gives

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2} \tag{9.14}$$



Active Figure 9.8 Schematic representation of a perfectly inelastic head-on collision between two particles: (a) before collision and (b) after collision.

At the Active Figures link at http://www.pse6.com, you can adjust the masses and velocities of the colliding objects to see the effect on the final velocity.

Elastic Collisions

Consider two particles of masses m_1 and m_2 moving with initial velocities $\mathbf{v}_{1i}$ and $\mathbf{v}_{2i}$ along the same straight line, as shown in Figure 9.9. The two particles collide head-on and then leave the collision site with different velocities, $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$. If the collision is elastic, both the momentum and kinetic energy of the system are conserved. Therefore, considering velocities along the horizontal direction in Figure 9.9, we have

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \tag{9.15}$$

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 \tag{9.16}$$

Because all velocities in Figure 9.9 are either to the left or the right, they can be represented by the corresponding speeds along with algebraic signs indicating direction. We shall indicate v as positive if a particle moves to the right and negative if it moves to the left.

In a typical problem involving elastic collisions, there are two unknown quantities, and Equations 9.15 and 9.16 can be solved simultaneously to find these. An alternative approach, however—one that involves a little mathematical manipulation of Equation 9.16—often simplifies this process. To see how, let us cancel the factor $\frac{1}{2}$ in Equation 9.16 and rewrite it as

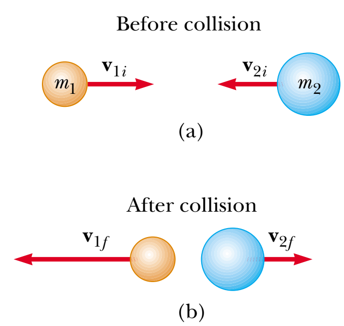
$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

and then factor both sides:

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \tag{9.17}$$

Next, let us separate the terms containing m_1 and m_2 in Equation 9.15 to obtain

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \tag{9.18}$$



Active Figure 9.9 Schematic representation of an elastic head-on collision between two particles: (a) before collision and (b) after collision.

At the Active Figures link at http://www.pse6.com, you can adjust the masses and velocities of the colliding objects to see the effect on the final velocities.

PITFALL PREVENTION

9.3 Not a General Equation

We have spent some effort on deriving Equation 9.19, but remember that it can only be used in a very *specific* situation—a one-dimensional, elastic collision between two objects. The *general* concept is conservation of momentum (and conservation of kinetic energy if the collision is elastic) for an isolated system.

To obtain our final result, we divide Equation 9.17 by Equation 9.18 and obtain

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \tag{9.19}$$

This equation, in combination with Equation 9.15, can be used to solve problems dealing with elastic collisions. According to Equation 9.19, the *relative* velocity of the two particles before the collision, $v_{1i} - v_{2i}$, equals the negative of their relative velocity after the collision, $-(v_{1f} - v_{2f})$.

Suppose that the masses and initial velocities of both particles are known. Equations 9.15 and 9.19 can be solved for the final velocities in terms of the initial velocities because there are two equations and two unknowns:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \tag{9.20}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \tag{9.21}$$

It is important to use the appropriate signs for v_{1i} and v_{2i} in Equations 9.20 and 9.21. For example, if particle 2 is moving to the left initially, then v_{2i} is negative.

Let us consider some special cases. If $m_1 = m_2$, then Equations 9.20 and 9.21 show us that $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$. That is, the particles exchange velocities if they have equal masses. This is approximately what one observes in head-on billiard ball collisions—the cue ball stops, and the struck ball moves away from the collision with the same velocity that the cue ball had.

If particle 2 is initially at rest, then $v_{2i} = 0$, and Equations 9.20 and 9.21 become

Elastic collision: particle 2 initially at rest

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} \tag{9.22}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} \tag{9.23}$$

If m_1 is much greater than m_2 and $v_{2i} = 0$, we see from Equations 9.22 and 9.23 that $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$. That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle. An example of such a collision would be that of a moving heavy atom, such as uranium, striking a light atom, such as hydrogen.

If m_2 is much greater than m_1 and particle 2 is initially at rest, then $v_{1f} \approx -v_{1i}$ and $v_{2f} \approx 0$. That is, when a very light particle collides head-on with a very heavy particle that is initially at rest, the light particle has its velocity reversed and the heavy one remains approximately at rest.

PITFALL PREVENTION

9.4 Momentum and Kinetic Energy in Collisions

Momentum of an isolated system is conserved in *all* collisions. Kinetic energy of an isolated system is conserved *only* in elastic collisions. Why? Because there are several types of energy into which kinetic energy can transform, or be transferred out of the system (so that the system may *not* be isolated in terms of energy during the collision). However, there is only one type of momentum.

Quick Quiz 9.8 In a perfectly inelastic one-dimensional collision between two objects, what condition alone is necessary so that *all* of the original kinetic energy of the system is gone after the collision? (a) The objects must have momenta with the same magnitude but opposite directions. (b) The objects must have the same mass. (c) The objects must have the same velocity. (d) The objects must have the same speed, with velocity vectors in opposite directions.

Quick Quiz 9.9 A table-tennis ball is thrown at a stationary bowling ball. The table-tennis ball makes a one-dimensional elastic collision and bounces back along the same line. After the collision, compared to the bowling ball, the table-tennis ball has (a) a larger magnitude of momentum and more kinetic energy (b) a smaller magnitude of momentum and more kinetic energy (c) a larger magnitude of momentum and less kinetic energy (d) a smaller magnitude of momentum and less kinetic energy (e) the same magnitude of momentum and the same kinetic energy.

Example 9.5 The Executive Stress Reliever

Interactive

An ingenious device that illustrates conservation of momentum and kinetic energy is shown in Figure 9.10. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost-elastic collision between it and ball 2, ball 5 moves out, as shown in Figure 9.10b. If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1, as in Figure 9.10c?

Solution No, such movement can never occur if we assume the collisions are elastic. The momentum of the system before the collision is mv, where m is the mass of ball 1 and v is its speed just before the collision. After the collision, we would have two balls, each of mass m moving with speed $v/2$. The total momentum of the system after the collision would be $m(v/2) + m(v/2) = mv$. Thus, momentum of the system is conserved. However, the kinetic energy just before the collision is $K_i = \frac{1}{2}mv^2$ and that after the collision is $K_f = \frac{1}{2}m(v/2)^2 + \frac{1}{2}m(v/2)^2 = \frac{1}{4}mv^2$. Thus, kinetic energy of the system is *not* conserved. The only way to have both momentum and kinetic energy conserved is for one ball to move out when one ball is released, two balls to move out when two are released, and so on.

What If? **Consider what would happen if balls 4 and 5 are glued together so that they must move together. Now what happens when ball 1 is pulled out and released?**

Answer We are now forcing balls 4 and 5 to come out together. We have argued that we cannot conserve both momentum and energy in this case. However, we assumed that ball 1 stopped after striking ball 2. What if we do not make this assumption? Consider the conservation equations with the assumption that ball 1 moves after the collision. For conservation of momentum,

$$p_i = p_f$$

$$mv_{1i} = mv_{1f} + 2mv_{4,5f}$$

where $v_{4,5f}$ refers to the final speed of the ball 4–ball 5 combination. Conservation of kinetic energy gives us

$$K_i = K_f$$

$$\tfrac{1}{2}mv_{1i}{}^2 = \tfrac{1}{2}mv_{1f}{}^2 + \tfrac{1}{2}(2m)v_{4,5f}^2$$

Combining these equations, we find

$$v_{4,5f} = \tfrac{2}{3}v_{1i} \qquad v_{1f} = -\tfrac{1}{3}v_{1i}$$

Thus, balls 4 and 5 come out together and ball 1 bounces back from the collision with one third of its original speed.

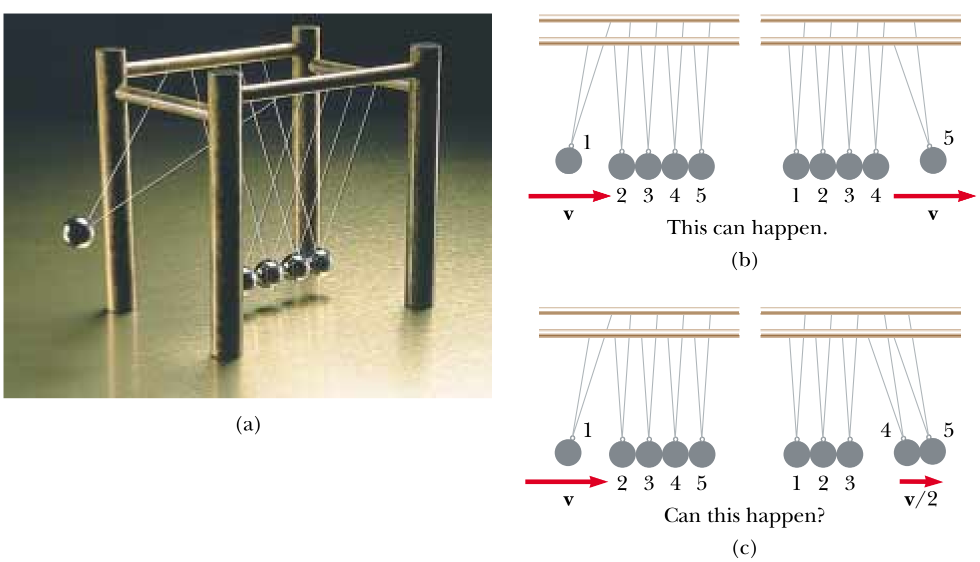


Figure 9.10 (Example 9.5) An executive stress reliever.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can "glue" balls 4 and 5 together to see the situation discussed above.*

Example 9.6 Carry Collision Insurance!

An 1 800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

Solution The phrase "become entangled" tells us that this is a perfectly inelastic collision. We can guess that the final speed is less than 20.0 m/s, the initial speed of the smaller car. The total momentum of the system (the two cars) before the collision must equal the total momentum immediately after the collision because momentum of an isolated system is conserved in any type of collision. The magnitude of the total momentum of the system before the collision is equal to that of the smaller car because the larger car is initially at rest:

$$p_i = m_1 v_i = (900 \text{ kg})(20.0 \text{ m/s}) = 1.80 \times 10^4 \text{ kg} \cdot \text{m/s}$$

After the collision, the magnitude of the momentum of the entangled cars is

$$p_f = (m_1 + m_2)v_f = (2\,700 \text{ kg})v_f$$

Equating the initial and final momenta of the system and solving for v_f, the final velocity of the entangled cars, we have

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{1.80 \times 10^4 \text{ kg} \cdot \text{m/s}}{2\,700 \text{ kg}} = 6.67 \text{ m/s}$$

Because the final velocity is positive, the direction of the final velocity is the same as the velocity of the initially moving car.

What If? Suppose we reverse the masses of the cars—a stationary 900-kg car is struck by a moving 1 800-kg car. Is the final speed the same as before?

Answer Intuitively, we can guess that the final speed will be higher, based on common experiences in driving. Mathematically, this should be the case because the system has a larger momentum if the initially moving car is the more massive one. Solving for the new final velocity, we find

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{(1\,800 \text{ kg})(20.0 \text{ m/s})}{2\,700 \text{ kg}} = 13.3 \text{ m/s}$$

which is indeed higher than the previous final velocity.

Example 9.7 The Ballistic Pendulum

The ballistic pendulum (Fig. 9.11) is an apparatus used to measure the speed of a fast-moving projectile, such as a bullet. A bullet of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. How can we determine the speed of the bullet from a measurement of h?

Solution Figure 9.11a helps to conceptualize the situation. Let configuration Ⓐ be the bullet and block before the collision, and configuration Ⓑ be the bullet and block immediately after colliding. The bullet and the block form an isolated system, so we can categorize the collision between them as a conservation of momentum problem. The collision is perfectly inelastic. To analyze the collision, we note that Equation 9.14 gives the speed of the system right after the collision when we assume the impulse approximation. Noting that $v_{2A} = 0$, Equation 9.14 becomes

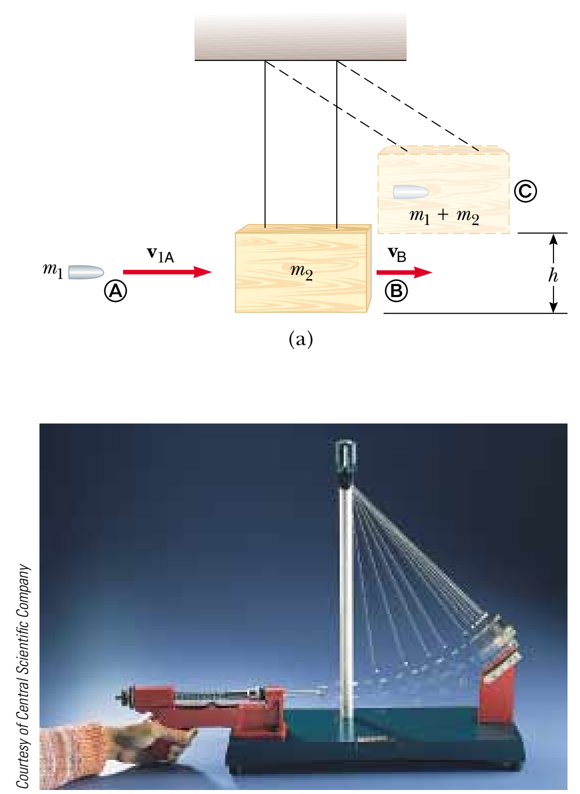
$$(1) \qquad v_B = \frac{m_1 v_{1A}}{m_1 + m_2}$$

For the process during which the bullet–block combination swings upward to height h (ending at configuration Ⓒ), we focus on a *different* system—the bullet, the block, and the Earth. This is an isolated system for energy, so we categorize this part of the motion as a conservation of mechanical energy problem:

$$K_B + U_B = K_C + U_C$$

We begin to analyze the problem by finding the total kinetic energy of the system right after the collision:

$$(2) \qquad K_B = \tfrac{1}{2}(m_1 + m_2)v_B^2$$



(b)

Figure 9.11 (Example 9.7) (a) Diagram of a ballistic pendulum. Note that $\mathbf{v}_{1A}$ is the velocity of the bullet just before the collision and $\mathbf{v}_B$ is the velocity of the bullet-block system just after the perfectly inelastic collision. (b) Multiflash photograph of a ballistic pendulum used in the laboratory.

Substituting the value of v_B from Equation (1) into Equation (2) gives

$$K_B = \frac{m_1{}^2 v_{1A}{}^2}{2(m_1 + m_2)}$$

This kinetic energy immediately after the collision is *less* than the initial kinetic energy of the bullet, as expected in an inelastic collision.

We define the gravitational potential energy of the system for configuration Ⓑ to be zero. Thus, $U_B = 0$ while $U_C = (m_1 + m_2)gh$. Conservation of energy now leads to

$$\frac{m_1{}^2 v_{1A}{}^2}{2(m_1 + m_2)} + 0 = 0 + (m_1 + m_2)gh$$

Solving for v_{1A}, we obtain

$$v_{1A} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$

To finalize this problem, note that we had to solve this problem in two steps. Each step involved a different system and a different conservation principle. Because the collision was assumed to be perfectly inelastic, some mechanical energy was converted to internal energy. It would have been *incorrect* to equate the initial kinetic energy of the incoming bullet to the final gravitational potential energy of the bullet–block–Earth combination.

Example 9.8 A Two-Body Collision with a Spring

Interactive

A block of mass $m_1 = 1.60$ kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass $m_2 = 2.10$ kg initially moving to the left with a speed of 2.50 m/s, as shown in Figure 9.12a. The spring constant is 600 N/m.

(A) Find the velocities of the two blocks after the collision.

Solution Because the spring force is conservative, no kinetic energy is converted to internal energy during the compression of the spring. Ignoring any sound made when the block hits the spring, we can model the collision as being elastic. Equation 9.15 gives us

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s}) = (1.60\text{ kg})v_{1f} + (2.10\text{ kg})v_{2f}$$

$$(1) \qquad 1.15\text{ kg}\cdot\text{m/s} = (1.60\text{ kg})v_{1f} + (2.10\text{ kg})v_{2f}$$

Equation 9.19 gives us

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$4.00\text{ m/s} - (-2.50\text{ m/s}) = -v_{1f} + v_{2f}$$

$$(2) \qquad 6.50\text{ m/s} = -v_{1f} + v_{2f}$$

Multiplying Equation (2) by 1.60 kg gives us

$$(3) \qquad 10.4\text{ kg}\cdot\text{m/s} = -(1.60\text{ kg})v_{1f} + (1.60\text{ kg})v_{2f}$$

Adding Equations (1) and (3) allows us to find v_{2f}:

$$11.55\text{ kg}\cdot\text{m/s} = (3.70\text{ kg})v_{2f}$$

$$v_{2f} = \frac{11.55\text{ kg}\cdot\text{m/s}}{3.70\text{ kg}} = \boxed{3.12\text{ m/s}}$$

Now, Equation (2) allows us to find v_{1f}:

$$6.50\text{ m/s} = -v_{1f} + 3.12\text{ m/s}$$

$$v_{1f} = \boxed{-3.38\text{ m/s}}$$

(B) During the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s, as in Figure 9.12b, determine the velocity of block 2.

Solution Because the momentum of the system of two blocks is conserved *throughout* the collision for the system of two blocks, we have, for *any* instant during the collision,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

We choose the final instant to be that at which block 1 is moving with a velocity of + 3.00 m/s:

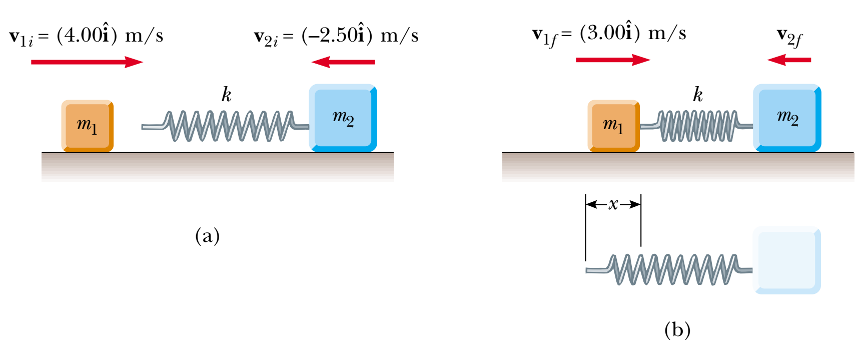


Figure 9.12 (Example 9.8) A moving block approaches a second moving block that is attached to a spring.

$$(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s}) = (1.60\text{ kg})(3.00\text{ m/s}) + (2.10\text{ kg})v_{2f}$$

$$v_{2f} = \boxed{-1.74\text{ m/s}}$$

The negative value for v_{2f} means that block 2 is still moving to the left at the instant we are considering.

(C) Determine the distance the spring is compressed at that instant.

Solution To determine the distance that the spring is compressed, shown as x in Figure 9.12b, we can use the principle of conservation of mechanical energy for the system of the spring and two blocks because no friction or other nonconservative forces are acting within the system. We choose the initial configuration of the system to be that existing just before block 1 strikes the spring and the final configuration to be that when block 1 is moving to the right at 3.00 m/s. Thus, we have

$$K_i + U_i = K_f + U_f$$

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 + 0 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 + \tfrac{1}{2}kx^2$$

Substituting the given values and the result to part (B) into this expression gives

$$x = \boxed{0.173\text{ m}}$$

(D) What is the *maximum* compression of the spring during the collision?

Solution The maximum compression would occur when the two blocks are moving with the same velocity. The conservation of momentum equation for the system can be written

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

where the initial instant is just before the collision and the final instant is when the blocks are moving with the same velocity v_f. Solving for v_f,

$$v_f = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}$$

$$= \frac{(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s})}{1.60\text{ kg} + 2.10\text{ kg}}$$

$$= 0.311\text{ m/s}$$

Now, we apply conservation of mechanical energy between these two instants as in part (C):

$$K_i + U_i = K_f + U_f$$

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 + 0 = \tfrac{1}{2}(m_1 + m_2)v_f^2 + \tfrac{1}{2}kx^2$$

Substituting the given values into this expression gives

$$x = \boxed{0.253\text{ m}}$$

At the Interactive Worked Example link at **http://www.pse6.com,** *you can change the masses and speeds of the blocks and freeze the motion at the maximum compression of the spring.*

Example 9.9 Slowing Down Neutrons by Collisions

In a nuclear reactor, neutrons are produced when an atom splits in a process called *fission.* These neutrons are moving at about 10^7 m/s and must be slowed down to about 10^3 m/s before they take part in another fission event. They are slowed down by passing them through a solid or liquid material called a *moderator.* The slowing-down process involves elastic collisions. Show that a neutron can lose most of its kinetic energy if it collides elastically with a moderator containing light nuclei, such as deuterium (in "heavy water," D_2O) or carbon (in graphite).

Solution Let us assume that the moderator nucleus of mass m_m is at rest initially and that a neutron of mass m_n and initial speed v_{ni} collides with it head-on. Because these are elastic collisions, both momentum and kinetic energy of the neutron–nucleus system are conserved. Therefore, Equations 9.22 and 9.23 can be applied to the head-on collision of a neutron with a moderator nucleus. We can represent this process by a drawing such as Figure 9.9 with $\mathbf{v}_{2i} = 0$.

The initial kinetic energy of the neutron is

$$K_{ni} = \tfrac{1}{2}m_nv_{ni}^2$$

After the collision, the neutron has kinetic energy $\tfrac{1}{2}m_nv_{nf}^2$, and we can substitute into this the value for v_{nf} given by Equation 9.22:

$$K_{nf} = \tfrac{1}{2}m_nv_{nf}^2 = \tfrac{1}{2}m_n\left(\frac{m_n - m_m}{m_n + m_m}\right)^2 v_{ni}^2$$

Therefore, the fraction f_n of the initial kinetic energy possessed by the neutron after the collision is

$$(1) \qquad f_n = \frac{K_{nf}}{K_{ni}} = \left(\frac{m_n - m_m}{m_n + m_m}\right)^2$$

From this result, we see that the final kinetic energy of the neutron is small when m_m is close to m_n and zero when $m_n = m_m$.

We can use Equation 9.23, which gives the final speed of the particle that was initially at rest, to calculate the kinetic energy of the moderator nucleus after the collision:

$$K_{mf} = \tfrac{1}{2}m_mv_{mf}^2 = \frac{2m_n^2m_m}{(m_n + m_m)^2}\,v_{ni}^2$$

Hence, the fraction f_m of the initial kinetic energy transferred to the moderator nucleus is

$$(2) \qquad f_m = \frac{K_{mf}}{K_{ni}} = \frac{4m_nm_m}{(m_n + m_m)^2}$$

Because the total kinetic energy of the system is conserved, Equation (2) can also be obtained from Equation (1) with the condition that $f_n + f_m = 1$, so that $f_m = 1 - f_n$.

Suppose that heavy water is used for the moderator. For collisions of the neutrons with deuterium nuclei in D_2O ($m_m = 2m_n$), $f_n = 1/9$ and $f_m = 8/9$. That is, 89% of the neutron's kinetic energy is transferred to the deuterium nucleus. In practice, the moderator efficiency is reduced because head-on collisions are very unlikely.

How do the results differ when graphite (^{12}C, as found in pencil lead) is used as the moderator?

9.4 Two-Dimensional Collisions

In Section 9.1, we showed that the momentum of a system of two particles is conserved when the system is isolated. For any collision of two particles, this result implies that the momentum in each of the directions x, y, and z is conserved. An important subset of collisions takes place in a plane. The game of billiards is a familiar example involving multiple collisions of objects moving on a two-dimensional surface. For such two-dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

where we use three subscripts in these equations to represent, respectively, (1) the identification of the object, (2) initial and final values, and (3) the velocity component.

Let us consider a two-dimensional problem in which particle 1 of mass m_1 collides with particle 2 of mass m_2, where particle 2 is initially at rest, as in Figure 9.13. After the collision, particle 1 moves at an angle θ with respect to the horizontal and particle 2 moves at an angle ϕ with respect to the horizontal. This is called a *glancing* collision. Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain

$$m_1v_{1i} = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi \tag{9.24}$$

$$0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi \tag{9.25}$$

where the minus sign in Equation 9.25 comes from the fact that after the collision, particle 2 has a y component of velocity that is downward. We now have two independent equations. As long as no more than two of the seven quantities in Equations 9.24 and 9.25 are unknown, we can solve the problem.

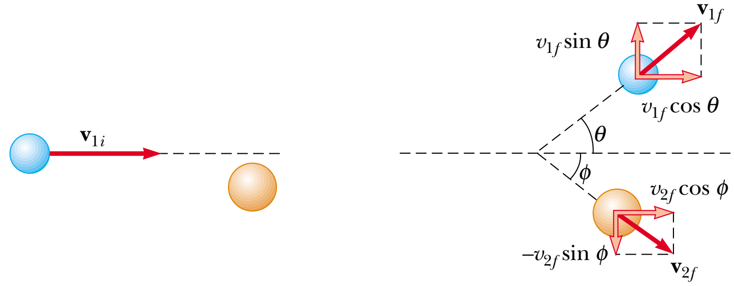
If the collision is elastic, we can also use Equation 9.16 (conservation of kinetic energy) with $v_{2i} = 0$ to give

$$\tfrac{1}{2}m_1{v_{1i}}^2 = \tfrac{1}{2}m_1{v_{1f}}^2 + \tfrac{1}{2}m_2{v_{2f}}^2 \tag{9.26}$$

PITFALL PREVENTION

9.5 Don't Use Equation 9.19

Equation 9.19, relating the initial and final relative velocities of two colliding objects, is only valid for one-dimensional elastic collisions. Do not use this equation when analyzing two-dimensional collisions.



(a) Before the collision (b) After the collision

Active Figure 9.13 An elastic glancing collision between two particles.

At the Active Figures link at http://www.pse6.com, you can adjust the speed and position of the blue particle and the masses of both particles to see the effects.

Knowing the initial speed of particle 1 and both masses, we are left with four unknowns (v_{1f}, v_{2f}, θ, and ϕ). Because we have only three equations, one of the four remaining quantities must be given if we are to determine the motion after the collision from conservation principles alone.

If the collision is inelastic, kinetic energy is *not* conserved and Equation 9.26 does *not* apply.

PROBLEM-SOLVING HINTS

Two-Dimensional Collisions

The following procedure is recommended when dealing with problems involving two-dimensional collisions between two objects:

- Set up a coordinate system and define your velocities with respect to that system. It is usually convenient to have the *x* axis coincide with one of the initial velocities.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the *x* and *y* components of the momentum of each object before and after the collision. Remember to include the appropriate signs for the components of the velocity vectors.
- Write expressions for the total momentum of the system in the *x* direction before and after the collision and equate the two. Repeat this procedure for the total momentum of the system in the *y* direction.
- If the collision is inelastic, kinetic energy of the system is *not* conserved, and additional information is probably required. If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic*, kinetic energy of the system is conserved, and you can equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

Example 9.10 Collision at an Intersection

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

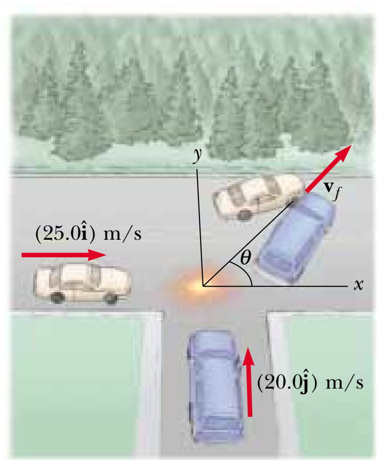


Figure 9.14 (Example 9.10) An eastbound car colliding with a northbound van.

Solution Let us choose east to be along the positive *x* direction and north to be along the positive *y* direction. Before the collision, the only object having momentum in the *x* direction is the car. Thus, the magnitude of the total initial momentum of the system (car plus van) in the *x* direction is

$$\sum p_{xi} = (1\ 500\ \text{kg})(25.0\ \text{m/s}) = 3.75 \times 10^4\ \text{kg}\cdot\text{m/s}$$

Let us assume that the wreckage moves at an angle θ and speed v_f after the collision. The magnitude of the total momentum in the *x* direction after the collision is

$$\sum p_{xf} = (4\,000 \text{ kg})v_f \cos\theta$$

Because the total momentum in the x direction is conserved, we can equate these two equations to obtain

$$(1) \qquad 3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \cos\theta$$

Similarly, the total initial momentum of the system in the y direction is that of the van, and the magnitude of this momentum is $(2\,500 \text{ kg})(20.0 \text{ m/s}) = 5.00 \times 10^4 \text{ kg}\cdot\text{m/s}$. Applying conservation of momentum to the y direction, we have

$$\sum p_{yi} = \sum p_{yf}$$

$$(2) \qquad 5.00 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \sin\theta$$

If we divide Equation (2) by Equation (1), we obtain

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{5.00 \times 10^4}{3.75 \times 10^4} = 1.33$$

$$\theta = \boxed{53.1°}$$

When this angle is substituted into Equation (2), the value of v_f is

$$v_f = \frac{5.00 \times 10^4 \text{ kg}\cdot\text{m/s}}{(4\,000 \text{ kg})\sin 53.1°} = \boxed{15.6 \text{ m/s}}$$

It might be instructive for you to draw the momentum vectors of each vehicle before the collision and the two vehicles together after the collision.

Example 9.11 Proton–Proton Collision

A proton collides elastically with another proton that is initially at rest. The incoming proton has an initial speed of 3.50×10^5 m/s and makes a glancing collision with the second proton, as in Figure 9.13. (At close separations, the protons exert a repulsive electrostatic force on each other.) After the collision, one proton moves off at an angle of 37.0° to the original direction of motion, and the second deflects at an angle of ϕ to the same axis. Find the final speeds of the two protons and the angle ϕ.

Solution The pair of protons is an isolated system. Both momentum and kinetic energy of the system are conserved in this glancing elastic collision. Because $m_1 = m_2$, $\theta = 37.0°$, and we are given that $v_{1i} = 3.50 \times 10^5$ m/s, Equations 9.24, 9.25, and 9.26 become

$$(1) \qquad v_{1f}\cos 37° + v_{2f}\cos\phi = 3.50 \times 10^5 \text{ m/s}$$

$$(2) \qquad v_{1f}\sin 37.0° - v_{2f}\sin\phi = 0$$

$$(3) \qquad v_{1f}^2 + v_{2f}^2 = (3.50 \times 10^5 \text{ m/s})^2 = 1.23 \times 10^{11} \text{ m}^2/\text{s}^2$$

We rewrite Equations (1) and (2) as follows:

$$v_{2f}\cos\phi = 3.50 \times 10^5 \text{ m/s} - v_{1f}\cos 37.0°$$

$$v_{2f}\sin\phi = v_{1f}\sin 37.0°$$

Now we square these two equations and add them:

$$v_{2f}^2\cos^2\phi + v_{2f}^2\sin^2\phi = 1.23 \times 10^{11} \text{ m}^2/\text{s}^2 - (7.00 \times 10^5 \text{ m/s})v_{1f}\cos 37.0° + v_{1f}^2\cos^2 37.0° + v_{1f}^2\sin^2 37.0°$$

$$v_{2f}^2 = 1.23 \times 10^{11} - (5.59 \times 10^5)v_{1f} + v_{1f}^2$$

Substituting into Equation (3) gives

$$v_{1f}^2 + [1.23 \times 10^{11} - (5.59 \times 10^5)v_{1f} + v_{1f}^2] = 1.23 \times 10^{11}$$

$$2v_{1f}^2 - (5.59 \times 10^5)v_{1f} = (2v_{1f} - 5.59 \times 10^5)v_{1f} = 0$$

One possibility for the solution of this equation is $v_{1f} = 0$, which corresponds to a head-on collision—the first proton stops and the second continues with the same speed in the same direction. This is not what we want. The other possibility is

$$2v_{1f} - 5.59 \times 10^5 = 0 \longrightarrow v_{1f} = \boxed{2.80 \times 10^5 \text{ m/s}}$$

From Equation (3),

$$v_{2f} = \sqrt{1.23 \times 10^{11} - v_{1f}^2} = \sqrt{1.23 \times 10^{11} - (2.80 \times 10^5)^2} = 2.12 \times 10^5 \text{ m/s}$$

and from Equation (2),

$$\phi = \sin^{-1}\left(\frac{v_{1f}\sin 37.0°}{v_{2f}}\right) = \sin^{-1}\left(\frac{(2.80 \times 10^5)\sin 37.0°}{2.12 \times 10^5}\right) = \boxed{53.0°}$$

It is interesting to note that $\theta + \phi = 90°$. This result is *not* accidental. Whenever two objects of equal mass collide elastically in a glancing collision and one of them is initially at rest, their final velocities are perpendicular to each other. The next example illustrates this point in more detail.

Example 9.12 Billiard Ball Collision

In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in Figure 9.15. If the angle to the corner pocket is 35°, at what angle θ is the cue ball deflected? Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m.

Solution Let ball 1 be the cue ball and ball 2 be the target ball. Because the target ball is initially at rest, conservation of kinetic energy (Eq. 9.16) for the two-ball system gives

$$\tfrac{1}{2}m_1 v_{1i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2$$

But $m_1 = m_2 = m$, so that

$$(1) \qquad v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

Applying conservation of momentum to the two-dimensional collision gives

$$(2) \qquad m_1\mathbf{v}_{1i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

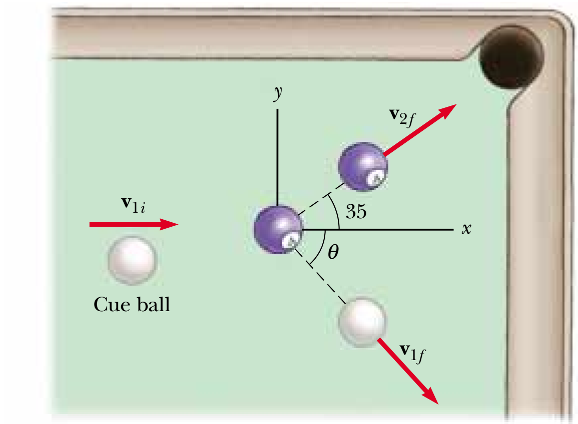


Figure 9.15 (Example 9.12) The cue ball (white) strikes the number 4 ball (blue) and sends it toward the corner pocket.

Note that because $m_1 = m_2 = m$, the masses also cancel in Equation (2). If we square both sides of Equation (2) and use the definition of the dot product of two vectors from Section 7.3, we obtain

$$v_{1i}^2 = (\mathbf{v}_{1f} + \mathbf{v}_{2f}) \cdot (\mathbf{v}_{1f} + \mathbf{v}_{2f}) = v_{1f}^2 + v_{2f}^2 + 2\mathbf{v}_{1f} \cdot \mathbf{v}_{2f}$$

Because the angle between $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$ is $\theta + 35°$, $\mathbf{v}_{1f} \cdot \mathbf{v}_{2f} = v_{1f}v_{2f}\cos(\theta + 35°)$, and so

$$(3) \qquad v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f}\cos(\theta + 35°)$$

Subtracting Equation (1) from Equation (3) gives

$$0 = 2v_{1f}v_{2f}\cos(\theta + 35°)$$

$$0 = \cos(\theta + 35°)$$

$$\theta + 35° = 90° \quad \text{or} \quad \boxed{\theta = 55°}$$

This result shows that whenever two equal masses undergo a glancing elastic collision and one of them is initially at rest, they move in perpendicular directions after the collision. The same physics describes two very different situations, protons in Example 9.11 and billiard balls in this example.

9.5 The Center of Mass

In this section we describe the overall motion of a mechanical system in terms of a special point called the **center of mass** of the system. The mechanical system can be either a group of particles, such as a collection of atoms in a container, or an extended object, such as a gymnast leaping through the air. We shall see that the center of mass of the system moves as if all the mass of the system were concentrated at that point. Furthermore, if the resultant external force on the system is $\Sigma\mathbf{F}_{\text{ext}}$ and the total mass of the system is M, the center of mass moves with an acceleration given by $\mathbf{a} = \Sigma\mathbf{F}_{\text{ext}}/M$. That is, the system moves as if the resultant external force were applied to a single particle of mass M located at the center of mass. This behavior is independent of other motion, such as rotation or vibration of the system. This is the *particle model* that was introduced in Chapter 2.

Consider a mechanical system consisting of a pair of particles that have different masses and are connected by a light, rigid rod (Fig. 9.16). The position of the center of mass of a system can be described as being the *average position* of the system's mass. The center of mass of the system is located somewhere on the line joining the two particles and is closer to the particle having the larger mass. If a single force is applied at a point on the rod somewhere between the center of mass and the less massive particle, the system rotates clockwise (see Fig. 9.16a). If the force is applied at a point on the rod somewhere between the center of mass and the more massive particle, the system rotates counterclockwise (see Fig. 9.16b). If the force is applied at the center of mass, the system moves in the direction of **F** without rotating (see Fig. 9.16c). Thus, the center of mass can be located with this procedure.

The center of mass of the pair of particles described in Figure 9.17 is located on the x axis and lies somewhere between the particles. Its x coordinate is given by

$$x_{\text{CM}} \equiv \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \tag{9.27}$$

Example 9.20 Fighting a Fire

Two firefighters must apply a total force of 600 N to steady a hose that is discharging water at the rate of 3 600 L/min. Estimate the speed of the water as it exits the nozzle.

Solution The water is exiting at 3 600 L/min, which is 60 L/s. Knowing that 1 L of water has a mass of 1 kg, we estimate that about 60 kg of water leaves the nozzle every second. As the water leaves the hose, it exerts on the hose a thrust that must be counteracted by the 600-N force exerted by the firefighters. So, applying Equation 9.42 gives

$$\text{Thrust} = \left| v_e \frac{dM}{dt} \right|$$

$$600 \text{ N} = |v_e(60 \text{ kg/s})|$$

$$v_e = \boxed{10 \text{ m/s}}$$

Firefighting is dangerous work. If the nozzle should slip from their hands, the movement of the hose due to the thrust it receives from the rapidly exiting water could injure the firefighters.

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

SUMMARY

The **linear momentum p** of a particle of mass m moving with a velocity $\mathbf{v}$ is

$$\mathbf{p} \equiv m\mathbf{v} \tag{9.2}$$

The law of **conservation of linear momentum** indicates that the total momentum of an isolated system is conserved. If two particles form an isolated system, the momentum of the system is conserved regardless of the nature of the force between them. Therefore, the total momentum of the system at all times equals its initial total momentum, or

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} \tag{9.5}$$

The **impulse** imparted to a particle by a force $\mathbf{F}$ is equal to the change in the momentum of the particle:

$$\mathbf{I} \equiv \int_{t_i}^{t_f} \mathbf{F}\, dt = \Delta\mathbf{p} \tag{9.8, 9.9}$$

This is known as the **impulse–momentum theorem.**

Impulsive forces are often very strong compared with other forces on the system and usually act for a very short time, as in the case of collisions.

When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. An **inelastic collision** is one for which the total kinetic energy of the system is not conserved. A **perfectly inelastic collision** is one in which the colliding bodies stick together after the collision. An **elastic collision** is one in which the kinetic energy of the system is conserved.

In a two- or three-dimensional collision, the components of momentum of an isolated system in each of the directions (x, y, and z) are conserved independently.

The position vector of the center of mass of a system of particles is defined as

$$\mathbf{r}_{\text{CM}} \equiv \frac{\sum_i m_i \mathbf{r}_i}{M} \tag{9.30}$$

where $M = \sum_i m_i$ is the total mass of the system and $\mathbf{r}_i$ is the position vector of the ith particle.

The position vector of the center of mass of an extended object can be obtained from the integral expression

$$\mathbf{r}_{\text{CM}} = \frac{1}{M}\int \mathbf{r}\,dm \tag{9.33}$$

The velocity of the center of mass for a system of particles is

$$\mathbf{v}_{\text{CM}} = \frac{\sum_i m_i \mathbf{v}_i}{M} \tag{9.34}$$

The total momentum of a system of particles equals the total mass multiplied by the velocity of the center of mass.

Newton's second law applied to a system of particles is

$$\sum \mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{CM}} \tag{9.38}$$

where $\mathbf{a}_{\text{CM}}$ is the acceleration of the center of mass and the sum is over all external forces. The center of mass moves like an imaginary particle of mass M under the influence of the resultant external force on the system.

QUESTIONS

1. Does a large force always produce a larger impulse on an object than a smaller force does? Explain.
2. If the speed of a particle is doubled, by what factor is its momentum changed? By what factor is its kinetic energy changed?
3. If two particles have equal kinetic energies, are their momenta necessarily equal? Explain.
4. While in motion, a pitched baseball carries kinetic energy and momentum. (a) Can we say that it carries a force that it can exert on any object it strikes? (b) Can the baseball deliver more kinetic energy to the object it strikes than the ball carries initially? (c) Can the baseball deliver to the object it strikes more momentum than the ball carries initially? Explain your answers.
5. An isolated system is initially at rest. Is it possible for parts of the system to be in motion at some later time? If so, explain how this might occur.
6. If two objects collide and one is initially at rest, is it possible for both to be at rest after the collision? Is it possible for one to be at rest after the collision? Explain.
7. Explain how linear momentum is conserved when a ball bounces from a floor.
8. A bomb, initially at rest, explodes into several pieces. (a) Is linear momentum of the system conserved? (b) Is kinetic energy of the system conserved? Explain.
9. A ball of clay is thrown against a brick wall. The clay stops and sticks to the wall. Is the principle of conservation of momentum violated in this example?
10. You are standing perfectly still, and then you take a step forward. Before the step your momentum was zero, but afterward you have some momentum. Is the principle of conservation of momentum violated in this case?
11. When a ball rolls down an incline, its linear momentum increases. Is the principle of conservation of momentum violated in this process?
12. Consider a perfectly inelastic collision between a car and a large truck. Which vehicle experiences a larger change in kinetic energy as a result of the collision?
13. A sharpshooter fires a rifle while standing with the butt of the gun against his shoulder. If the forward momentum of a bullet is the same as the backward momentum of the gun, why isn't it as dangerous to be hit by the gun as by the bullet?
14. A pole-vaulter falls from a height of 6.0 m onto a foam rubber pad. Can you calculate his speed just before he reaches the pad? Can you calculate the force exerted on him by the pad? Explain.
15. Firefighters must apply large forces to hold a fire hose steady (Fig. Q9.15). What factors related to the projection of the water determine the magnitude of the force needed to keep the end of the fire hose stationary?

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Figure Q9.15

16. A large bed sheet is held vertically by two students. A third student, who happens to be the star pitcher on the baseball team, throws a raw egg at the sheet. Explain why the egg does not break when it hits the sheet, regardless of its initial speed. (If you try this demonstration, make sure the pitcher hits the sheet near its center, and do not allow the egg to fall on the floor after being caught.)

17. A skater is standing still on a frictionless ice rink. Her friend throws a Frisbee straight at her. In which of the following cases is the largest momentum transferred to the skater? (a) The skater catches the Frisbee and holds onto it. (b) The skater catches the Frisbee momentarily, but then drops it vertically downward. (c) The skater catches the Frisbee, holds it momentarily, and throws it back to her friend.

18. In an elastic collision between two particles, does the kinetic energy of each particle change as a result of the collision?

19. Three balls are thrown into the air simultaneously. What is the acceleration of their center of mass while they are in motion?

20. A person balances a meter stick in a horizontal position on the extended index fingers of her right and left hands. She slowly brings the two fingers together. The stick remains balanced and the two fingers always meet at the 50-cm mark regardless of their original positions. (Try it!) Explain.

21. NASA often uses the gravity of a planet to "slingshot" a probe on its way to a more distant planet. The interaction of the planet and the spacecraft is a collision in which the objects do not touch. How can the probe have its speed increased in this manner?

22. The Moon revolves around the Earth. Model its orbit as circular. Is the Moon's linear momentum conserved? Is its kinetic energy conserved?

23. A raw egg dropped to the floor breaks upon impact. However, a raw egg dropped onto a thick foam rubber cushion from a height of about 1 m rebounds without breaking. Why is this possible? If you try this experiment, be sure to catch the egg after its first bounce.

24. Can the center of mass of an object be located at a position at which there is no mass? If so, give examples.

25. A juggler juggles three balls in a continuous cycle. Any one ball is in contact with his hands for one fifth of the time. Describe the motion of the center of mass of the three balls. What average force does the juggler exert on one ball while he is touching it?

26. Does the center of mass of a rocket in free space accelerate? Explain. Can the speed of a rocket exceed the exhaust speed of the fuel? Explain.

27. Early in the twentieth century, Robert Goddard proposed sending a rocket to the moon. Critics objected that in a vacuum, such as exists between the Earth and the Moon, the gases emitted by the rocket would have nothing to push against to propel the rocket. According to *Scientific American* (January 1975), Goddard placed a gun in a vacuum and fired a blank cartridge from it. (A blank cartridge contains no bullet and fires only the wadding and the hot gases produced by the burning gunpowder.) What happened when the gun was fired?

28. Explain how you could use a balloon to demonstrate the mechanism responsible for rocket propulsion.

29. On the subject of the following positions, state your own view and argue to support it. (a) The best theory of motion is that force causes acceleration. (b) The true measure of a force's effectiveness is the work it does, and the best theory of motion is that work done on an object changes its energy. (c) The true measure of a force's effect is impulse, and the best theory of motion is that impulse injected into an object changes its momentum.

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 9.1 Linear Momentum and its Conservation

1. A 3.00-kg particle has a velocity of $(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s. (a) Find its x and y components of momentum. (b) Find the magnitude and direction of its momentum.

2. A 0.100-kg ball is thrown straight up into the air with an initial speed of 15.0 m/s. Find the momentum of the ball (a) at its maximum height and (b) halfway up to its maximum height.

3. How fast can you set the Earth moving? In particular, when you jump straight up as high as you can, what is the order of magnitude of the maximum recoil speed that you give to the Earth? Model the Earth as a perfectly solid object. In your solution, state the physical quantities you take as data and the values you measure or estimate for them.

4. Two blocks of masses M and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one

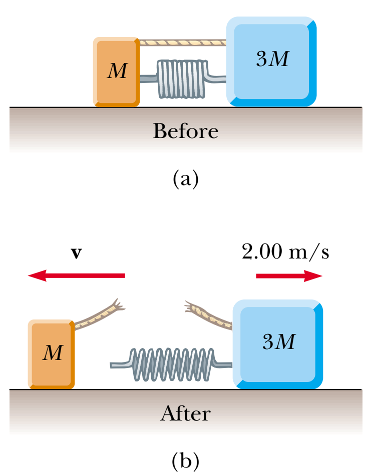


Figure P9.4

of them, and the blocks are pushed together with the spring between them (Fig. P9.4). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right with a speed of 2.00 m/s. (a) What is the speed of the block of mass M? (b) Find the original elastic potential energy in the spring if $M = 0.350$ kg.

5. (a) A particle of mass m moves with momentum p. Show that the kinetic energy of the particle is $K = p^2/2m$. (b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

Section 9.2 Impulse and Momentum

6. A friend claims that, as long as he has his seatbelt on, he can hold on to a 12.0-kg child in a 60.0 mi/h head-on collision with a brick wall in which the car passenger compartment comes to a stop in 0.050 0 s. Show that the violent force during the collision will tear the child from his arms. A child should always be in a toddler seat secured with a seat belt in the back seat of a car.

7. An estimated force–time curve for a baseball struck by a bat is shown in Figure P9.7. From this curve, determine (a) the impulse delivered to the ball, (b) the average force exerted on the ball, and (c) the peak force exerted on the ball.

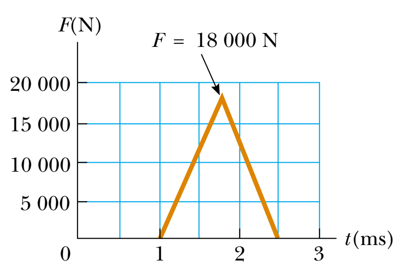


Figure P9.7

8. A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

9. A 3.00-kg steel ball strikes a wall with a speed of 10.0 m/s at an angle of 60.0° with the surface. It bounces off with the same speed and angle (Fig. P9.9). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball ?

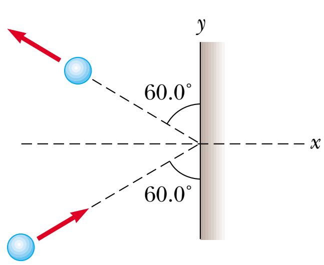


Figure P9.9

10. A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction. (a) What is the impulse delivered to the ball by the racquet? (b) What work does the racquet do on the ball?

11. In a slow-pitch softball game, a 0.200-kg softball crosses the plate at 15.0 m/s at an angle of 45.0° below the horizontal. The batter hits the ball toward center field, giving it a velocity of 40.0 m/s at 30.0° above the horizontal. (a) Determine the impulse delivered to the ball. (b) If the force on the ball increases linearly for 4.00 ms, holds constant for 20.0 ms, and then decreases to zero linearly in another 4.00 ms, what is the maximum force on the ball?

12. A professional diver performs a dive from a platform 10 m above the water surface. Estimate the order of magnitude of the average impact force she experiences in her collision with the water. State the quantities you take as data and their values.

13. A garden hose is held as shown in Figure P9.13. The hose is originally full of motionless water. What additional force is necessary to hold the nozzle stationary after the water flow is turned on, if the discharge rate is 0.600 kg/s with a speed of 25.0 m/s?

Figure P9.13

14. A glider of mass m is free to slide along a horizontal air track. It is pushed against a launcher at one end of the track. Model the launcher as a light spring of force constant k compressed by a distance x. The glider is released from rest. (a) Show that the glider attains a speed of $v = x(k/m)^{1/2}$. (b) Does a glider of large or of small mass attain a greater speed? (c) Show that the impulse imparted to the glider is given by the expression $x(km)^{1/2}$. (d) Is a greater impulse injected into a large or a small mass? (e) Is more work done on a large or a small mass?

Section 9.3 Collisions in One Dimension

15. High-speed stroboscopic photographs show that the head of a golf club of mass 200 g is traveling at 55.0 m/s just before it strikes a 46.0-g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.

16. An archer shoots an arrow toward a target that is sliding toward her with a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the 300-g target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

17. A 10.0-g bullet is fired into a stationary block of wood (m = 5.00 kg). The relative motion of the bullet stops inside the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.600 m/s. What was the original speed of the bullet?

18. A railroad car of mass 2.50×10^4 kg is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s. (a) What is the speed of the four cars after the collision? (b) How much mechanical energy is lost in the collision?

19. Four railroad cars, each of mass 2.50×10^4 kg, are coupled together and coasting along horizontal tracks at speed v_i toward the south. A very strong but foolish movie actor, riding on the second car, uncouples the front car and gives it a big push, increasing its speed to 4.00 m/s southward. The remaining three cars continue moving south, now at 2.00 m/s. (a) Find the initial speed of the cars. (b) How much work did the actor do? (c) State the relationship between the process described here and the process in Problem 18.

20. Two blocks are free to slide along the frictionless wooden track ABC shown in Figure P9.20. The block of mass m_1 = 5.00 kg is released from A. Protruding from its front end is the north pole of a strong magnet, repelling the north pole of an identical magnet embedded in the back end of the block of mass m_2 = 10.0 kg, initially at rest. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.

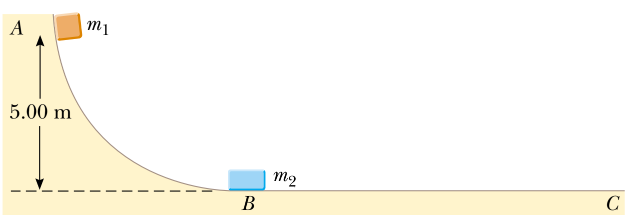


Figure P9.20

21. A 45.0-kg girl is standing on a plank that has a mass of 150 kg. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless supporting surface. The girl begins to walk along the plank at a constant speed of 1.50 m/s relative to the plank. (a) What is her speed relative to the ice surface? (b) What is the speed of the plank relative to the ice surface?

22. Most of us know intuitively that in a head-on collision between a large dump truck and a subcompact car, you are better off being in the truck than in the car. Why is this? Many people imagine that the collision force exerted on the car is much greater than that experienced by the truck. To substantiate this view, they point out that the car is crushed, whereas the truck is only dented. This idea of unequal forces, of course, is false. Newton's third law tells us that both objects experience forces of the same magnitude. The truck suffers less damage because it is made of stronger metal. But what about the two drivers? Do they experience the same forces? To answer this question, suppose that each vehicle is initially moving at 8.00 m/s and that they undergo a perfectly inelastic head-on collision. Each driver has mass 80.0 kg. Including the drivers, the total vehicle masses are 800 kg for the car and 4 000 kg for the truck. If the collision time is 0.120 s, what force does the seatbelt exert on each driver?

23. A neutron in a nuclear reactor makes an elastic head-on collision with the nucleus of a carbon atom initially at rest. (a) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? (b) If the initial kinetic energy of the neutron is 1.60×10^{-13} J, find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. (The mass of the carbon nucleus is nearly 12.0 times the mass of the neutron.)

24. As shown in Figure P9.24, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?

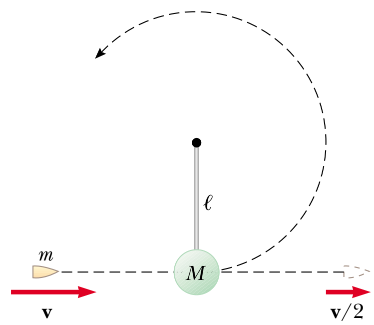


Figure P9.24

25. A 12.0-g wad of sticky clay is hurled horizontally at a 100-g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact, the block slides 7.50 m before coming to rest. If the coefficient of friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

26. A 7.00-g bullet, when fired from a gun into a 1.00-kg block of wood held in a vise, penetrates the block to a depth of 8.00 cm. **What If?** This block of wood is placed on a frictionless horizontal surface, and a second 7.00-g bullet is fired from the gun into the block. To what depth will the bullet penetrate the block in this case?

27. (a) Three carts of masses 4.00 kg, 10.0 kg, and 3.00 kg move on a frictionless horizontal track with speeds of 5.00 m/s, 3.00 m/s, and 4.00 m/s, as shown in Figure P9.27. Velcro couplers make the carts stick together after colliding. Find the final velocity of the train of three carts. (b) **What If?** Does your answer require that all the carts collide and stick together at the same time? What if they collide in a different order?

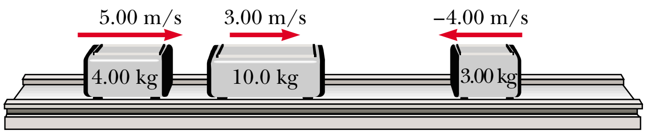


Figure P9.27

Section 9.4 Two-Dimensional Collisions

28. A 90.0-kg fullback running east with a speed of 5.00 m/s is tackled by a 95.0-kg opponent running north with a speed of 3.00 m/s. If the collision is perfectly inelastic, (a) calculate the speed and direction of the players just after the tackle and (b) determine the mechanical energy lost as a result of the collision. Account for the missing energy.

29. Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in an elastic, glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed of 5.00 m/s. After the collision, the orange disk moves along a direction that makes an angle of 37.0° with its initial direction of motion. The velocities of the two disks are perpendicular after the collision. Determine the final speed of each disk.

30. Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in an elastic, glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed v_i. After the collision, the orange disk moves along a direction that makes an angle θ with its initial direction of motion. The velocities of the two disks are perpendicular after the collision. Determine the final speed of each disk.

31. The mass of the blue puck in Figure P9.31 is 20.0% greater than the mass of the green one. Before colliding, the pucks approach each other with momenta of equal magnitudes and opposite directions, and the green puck has an initial speed of 10.0 m/s. Find the speeds of the pucks after the collision if half the kinetic energy is lost during the collision.

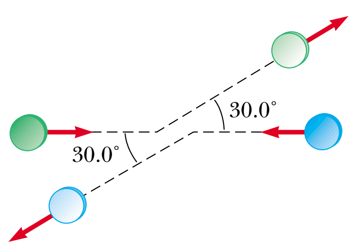


Figure P9.31

32. Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed v_{2i}. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth?

33. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves, at 4.33 m/s, at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.

34. A proton, moving with a velocity of $v_i \hat{\mathbf{i}}$, collides elastically with another proton that is initially at rest. If the two protons have equal speeds after the collision, find (a) the speed of each proton after the collision in terms of v_i and (b) the direction of the velocity vectors after the collision.

35. An object of mass 3.00 kg, moving with an initial velocity of $5.00\hat{\mathbf{i}}$ m/s, collides with and sticks to an object of mass 2.00 kg with an initial velocity of $-3.00\hat{\mathbf{j}}$ m/s. Find the final velocity of the composite object.

36. Two particles with masses m and $3m$ are moving toward each other along the x axis with the same initial speeds v_i. Particle m is traveling to the left, while particle $3m$ is traveling to the right. They undergo an elastic glancing collision such that particle m is moving downward after the collision at right angles from its initial direction. (a) Find the final speeds of the two particles. (b) What is the angle θ at which the particle $3m$ is scattered?

37. An unstable atomic nucleus of mass 17.0×10^{-27} kg initially at rest disintegrates into three particles. One of the particles, of mass 5.00×10^{-27} kg, moves along the y axis with a speed of 6.00×10^{6} m/s. Another particle, of mass 8.40×10^{-27} kg, moves along the x axis with a speed of 4.00×10^{6} m/s. Find (a) the velocity of the third particle and (b) the total kinetic energy increase in the process.

Section 9.5 The Center of Mass

38. Four objects are situated along the y axis as follows: a 2.00 kg object is at $+3.00$ m, a 3.00-kg object is at $+2.50$ m, a 2.50-kg object is at the origin, and a 4.00-kg object is at -0.500 m. Where is the center of mass of these objects?

39. A water molecule consists of an oxygen atom with two hydrogen atoms bound to it (Fig. P9.39). The angle between the two bonds is 106°. If the bonds are 0.100 nm long, where is the center of mass of the molecule?

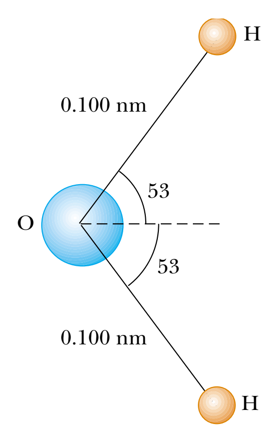


Figure P9.39

40. The mass of the Earth is 5.98×10^{24} kg, and the mass of the Moon is 7.36×10^{22} kg. The distance of separation, measured between their centers, is 3.84×10^{8} m. Locate the center of mass of the Earth–Moon system as measured from the center of the Earth.

41. A uniform piece of sheet steel is shaped as in Figure P9.41. Compute the x and y coordinates of the center of mass of the piece.

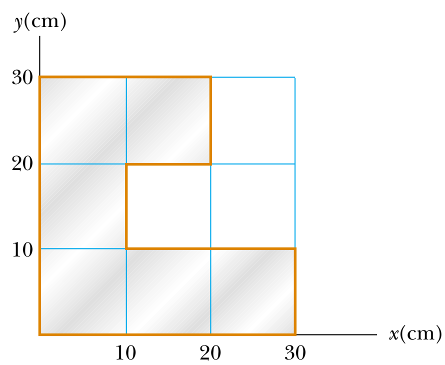


Figure P9.41

42. (a) Consider an extended object whose different portions have different elevations. Assume the free-fall acceleration is uniform over the object. Prove that the gravitational potential energy of the object–Earth system is given by $U_g = Mgy_{CM}$ where M is the total mass of the object and y_{CM} is the elevation of its center of mass above the chosen reference level. (b) Calculate the gravitational potential energy associated with a ramp constructed on level ground with stone with density 3 800 kg/m^3 and everywhere 3.60 m wide. In a side view, the ramp appears as a right triangle with height 15.7 m at the top end and base 64.8 m (Figure P9.42).

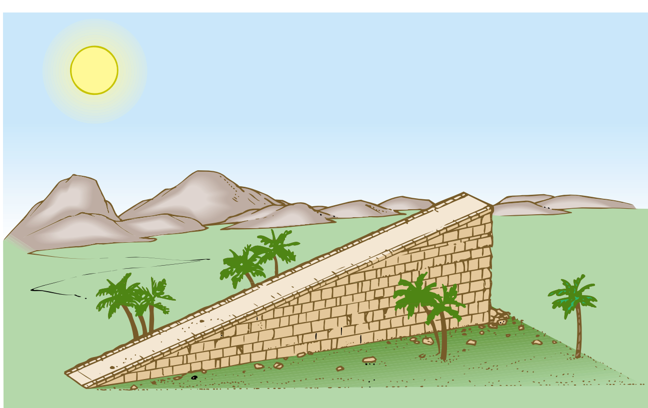


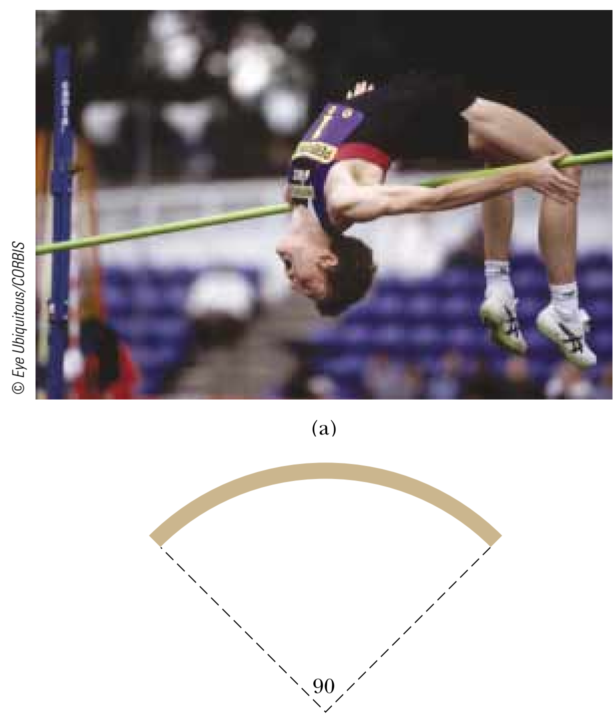
Figure P9.42

43. A rod of length 30.0 cm has linear density (mass-per-length) given by

$$\lambda = 50.0 \text{ g/m} + 20.0x \text{ g/m}^2,$$

where x is the distance from one end, measured in meters. (a) What is the mass of the rod? (b) How far from the $x = 0$ end is its center of mass?

44. In the 1968 Olympic Games, University of Oregon jumper Dick Fosbury introduced a new technique of high jumping called the "Fosbury flop." It contributed to raising the world record by about 30 cm and is presently used by nearly every world-class jumper. In this technique, the jumper goes over the bar face up while arching his back as much as possible, as in Figure P9.44a. This action places his center of mass outside his body, below his back. As his body goes over the bar, his center of mass passes below the bar. Because a given energy input implies a certain elevation for his center of mass, the action of arching his back means his body is higher than if his back were straight. As a model, consider the jumper as a thin uniform rod of length L. When the rod is straight, its center of mass is at its center. Now bend the rod in a circular arc so that it subtends an angle of 90.0° at the center of the arc, as shown in Figure P9.44b. In this configuration, how far outside the rod is the center of mass?



(a)

(b)

Figure P9.44

Section 9.6 Motion of a System of Particles

45. A 2.00-kg particle has a velocity $(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ m/s, and a 3.00-kg particle has a velocity $(1.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})$ m/s. Find (a) the velocity of the center of mass and (b) the total momentum of the system.

46. Consider a system of two particles in the xy plane: $m_1 =$ 2.00 kg is at the location $\mathbf{r}_1 = (1.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}})$ m and has a velocity of $(3.00\hat{\mathbf{i}} + 0.500\hat{\mathbf{j}})$ m/s; $m_2 = 3.00$ kg is at $\mathbf{r}_2 = (-4.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}})$ m and has velocity $(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}})$ m/s.

(a) Plot these particles on a grid or graph paper. Draw their position vectors and show their velocities. (b) Find the position of the center of mass of the system and mark it on the grid. (c) Determine the velocity of the center of mass and also show it on the diagram. (d) What is the total linear momentum of the system?

47. Romeo (77.0 kg) entertains Juliet (55.0 kg) by playing his guitar from the rear of their boat at rest in still water, 2.70 m away from Juliet, who is in the front of the boat. After the serenade, Juliet carefully moves to the rear of the boat (away from shore) to plant a kiss on Romeo's cheek. How far does the 80.0-kg boat move toward the shore it is facing?

48. A ball of mass 0.200 kg has a velocity of $150\hat{\mathbf{i}}$ m/s; a ball of mass 0.300 kg has a velocity of $-0.400\hat{\mathbf{i}}$ m/s. They meet in a head-on elastic collision. (a) Find their velocities after the collision. (b) Find the velocity of their center of mass before and after the collision.

Section 9.7 Rocket Propulsion

49. The first stage of a Saturn V space vehicle consumed fuel and oxidizer at the rate of 1.50×10^4 kg/s, with an exhaust speed of 2.60×10^3 m/s. (a) Calculate the thrust produced by these engines. (b) Find the acceleration of the vehicle just as it lifted off the launch pad on the Earth if the vehicle's initial mass was 3.00×10^6 kg. *Note:* You must include the gravitational force to solve part (b).

50. Model rocket engines are sized by thrust, thrust duration, and total impulse, among other characteristics. A size C5 model rocket engine has an average thrust of 5.26 N, a fuel mass of 12.7 g, and an initial mass of 25.5 g. The duration of its burn is 1.90 s. (a) What is the average exhaust speed of the engine? (b) If this engine is placed in a rocket body of mass 53.5 g, what is the final velocity of the rocket if it is fired in outer space? Assume the fuel burns at a constant rate.

51. A rocket for use in deep space is to be capable of boosting a total load (payload plus rocket frame and engine) of 3.00 metric tons to a speed of 10 000 m/s. (a) It has an engine and fuel designed to produce an exhaust speed of 2 000 m/s. How much fuel plus oxidizer is required? (b) If a different fuel and engine design could give an exhaust speed of 5 000 m/s, what amount of fuel and oxidizer would be required for the same task?

52. *Rocket Science.* A rocket has total mass $M_i = 360$ kg, including 330 kg of fuel and oxidizer. In interstellar space it starts from rest, turns on its engine at time $t = 0$, and puts out exhaust with relative speed $v_e = 1\,500$ m/s at the constant rate $k = 2.50$ kg/s. The fuel will last for an actual burn time of 330 kg/(2.5 kg/s) = 132 s, but define a "projected depletion time" as $T_p = M_i/k = 144$ s. (This would be the burn time if the rocket could use its payload and fuel tanks as fuel, and even the walls of the combustion chamber.) (a) Show that during the burn the velocity of the rocket is given as a function of time by

$$v(t) = -v_e \ln[1 - (t/T_p)]$$

(b) Make a graph of the velocity of the rocket as a function of time for times running from 0 to 132 s. (c) Show that the acceleration of the rocket is

$$a(t) = v_e/(T_p - t)$$

(d) Graph the acceleration as a function of time. (e) Show that the position of the rocket is

$$x(t) = v_e(T_p - t)\ln[1 - (t/T_p)] + v_e t$$

(f) Graph the position during the burn.

53. An orbiting spacecraft is described not as a "zero-g," but rather as a "microgravity" environment for its occupants and for on-board experiments. Astronauts experience slight lurches due to the motions of equipment and other astronauts, and due to venting of materials from the craft. Assume that a 3 500-kg spacecraft undergoes an acceleration of $2.50\ \mu g = 2.45 \times 10^{-5}$ m/s^2 due to a leak from one of its hydraulic control systems. The fluid is known to escape with a speed of 70.0 m/s into the vacuum of space. How much fluid will be lost in 1 h if the leak is not stopped?

Additional Problems

54. Two gliders are set in motion on an air track. A spring of force constant k is attached to the near side of one glider. The first glider, of mass m_1, has velocity $\mathbf{v}_1$, and the second glider, of mass m_2, moves more slowly, with velocity $\mathbf{v}_2$, as in Figure P9.54. When m_1 collides with the spring attached to m_2 and compresses the spring to its maximum compression x_{max}, the velocity of the gliders is $\mathbf{v}$. In terms of $\mathbf{v}_1$, $\mathbf{v}_2$, m_1, m_2, and k, find (a) the velocity $\mathbf{v}$ at maximum compression, (b) the maximum compression x_{max}, and (c) the velocity of each glider after m_1 has lost contact with the spring.

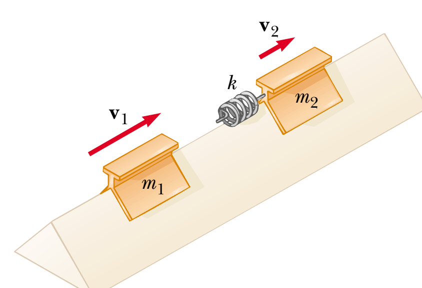


Figure P9.54

55. **Review problem.** A 60.0-kg person running at an initial speed of 4.00 m/s jumps onto a 120-kg cart initially at rest (Figure P9.55). The person slides on the cart's top surface and finally comes to rest relative to the cart. The coefficient of kinetic friction between the person and the cart is 0.400. Friction between the cart and ground can be neglected. (a) Find the final velocity of the person and cart relative to the ground. (b) Find the friction force acting on the person while he is sliding across the top surface of the cart. (c) How long does the friction force act on the person? (d) Find the change in momentum of the person and the change in momentum of the cart. (e) Determine the displacement of the person relative to the ground while he is sliding on the cart. (f) Determine the displacement of the cart relative to the ground while the person is sliding. (g) Find the change in kinetic energy of the person. (h) Find the change in kinetic energy of the cart. (i) Explain why the answers to (g) and (h) differ. (What kind of collision is this, and what accounts for the loss of mechanical energy?)

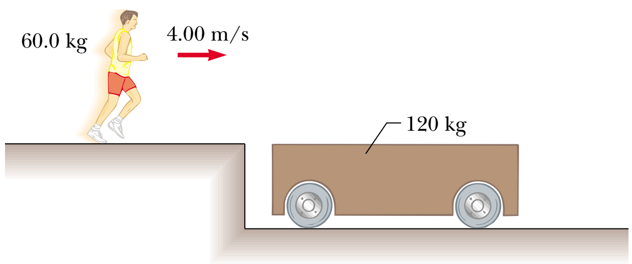


Figure P9.55

56. A golf ball ($m = 46.0$ g) is struck with a force that makes an angle of 45.0° with the horizontal. The ball lands 200 m away on a flat fairway. If the golf club and ball are in contact for 7.00 ms, what is the average force of impact? (Neglect air resistance.)

$\boxed{57.}$ An 80.0-kg astronaut is working on the engines of his ship, which is drifting through space with a constant velocity. The astronaut, wishing to get a better view of the Universe, pushes against the ship and much later finds himself 30.0 m behind the ship. Without a thruster, the only way to return to the ship is to throw his 0.500-kg wrench directly away from the ship. If he throws the wrench with a speed of 20.0 m/s relative to the ship, how long does it take the astronaut to reach the ship?

58. A bullet of mass m is fired into a block of mass M initially at rest at the edge of a frictionless table of height h (Fig. P9.58). The bullet remains in the block, and after impact the block lands a distance d from the bottom of the table. Determine the initial speed of the bullet.

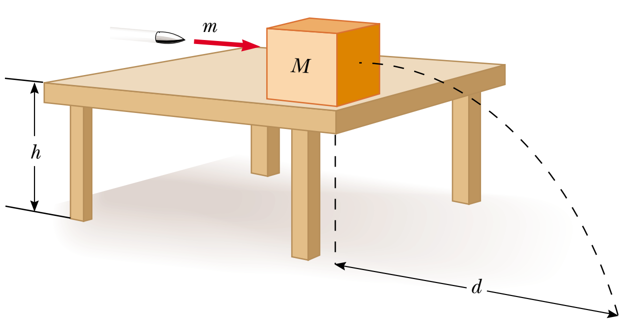


Figure P9.58

59. A 0.500-kg sphere moving with a velocity $(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} + 1.00\hat{\mathbf{k}})$ m/s strikes another sphere of mass 1.50 kg moving with a velocity $(-1.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}})$ m/s. (a) If the velocity of the 0.500-kg sphere after the collision is $(-1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 8.00\hat{\mathbf{k}})$ m/s, find the final velocity of the 1.50-kg sphere and identify the kind of collision (elastic, inelastic, or perfectly inelastic). (b) If the velocity of the 0.500-kg sphere after the collision is $(-0.250\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}})$ m/s, find the final velocity of the 1.50-kg sphere and identify the kind of collision. (c) **What If?** If the velocity of the 0.500-kg sphere after the collision is $(-1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} + a\hat{\mathbf{k}})$ m/s, find the value of a and the velocity of the 1.50-kg sphere after an elastic collision.

60. A small block of mass $m_1 = 0.500$ kg is released from rest at the top of a curve-shaped frictionless wedge of mass $m_2 = 3.00$ kg, which sits on a frictionless horizontal surface as in Figure P9.60a. When the block leaves the wedge, its velocity is measured to be 4.00 m/s to the right, as in Figure P9.60b. (a) What is the velocity of the wedge after the block reaches the horizontal surface? (b) What is the height h of the wedge?

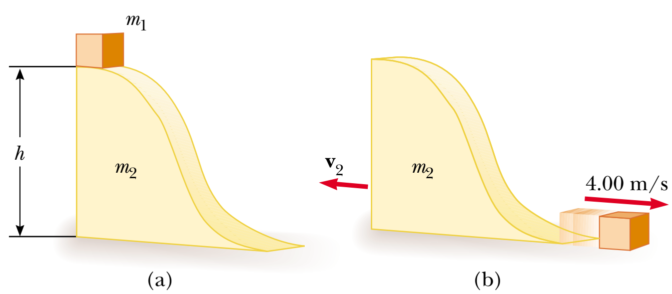


Figure P9.60

61. A bucket of mass m and volume V is attached to a light cart, completely covering its top surface. The cart is given a quick push along a straight, horizontal, smooth road. It is raining, so as the cart cruises along without friction, the bucket gradually fills with water. By the time the bucket is full, its speed is v. (a) What was the initial speed v_i of the cart? Let ρ represent the density of water. (b) **What If?** Assume that when the bucket is half full, it develops a slow leak at the bottom, so that the level of the water remains constant thereafter. Describe qualitatively what happens to the speed of the cart after the leak develops.

62. A 75.0-kg firefighter slides down a pole while a constant friction force of 300 N retards her motion. A horizontal 20.0-kg platform is supported by a spring at the bottom of the pole to cushion the fall. The firefighter starts from rest 4.00 m above the platform, and the spring constant is 4 000 N/m. Find (a) the firefighter's speed just before she collides with the platform and (b) the maximum distance the spring is compressed. (Assume the friction force acts during the entire motion.)

63. George of the Jungle, with mass m, swings on a light vine hanging from a stationary tree branch. A second vine of equal length hangs from the same point, and a gorilla of larger mass M swings in the opposite direction on it. Both vines are horizontal when the primates start from rest at the same moment. George and the gorilla meet at the lowest point of their swings. Each is afraid that one vine will break, so they grab each other and hang on. They swing upward together, reaching a point where the vines make an angle of 35.0° with the vertical. (a) Find the value of the ratio m/M. (b) **What If?** Try this at home. Tie a small magnet and a steel screw to opposite ends of a string. Hold the center of the string fixed to represent the tree branch, and reproduce a model of the motions of George and the gorilla. What changes in your analysis will make it apply to this situation? **What If?** Assume the magnet is strong, so that it noticeably attracts the screw over a distance of a few centimeters. Then the screw will be moving faster just before it sticks to the magnet. Does this make a difference?

64. A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially unstretched and with force constant $k = 2.00 \times 10^4$ N/m, as in Figure P9.64. The cannon fires a 200-kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal. (a) If the mass of the cannon and its carriage is 5 000 kg, find the recoil speed of the cannon. (b) Determine the maximum extension of the spring. (c) Find the maximum force the spring exerts on the carriage. (d) Consider the system consisting of the cannon, carriage, and shell. Is the momentum of this system conserved during the firing? Why or why not?

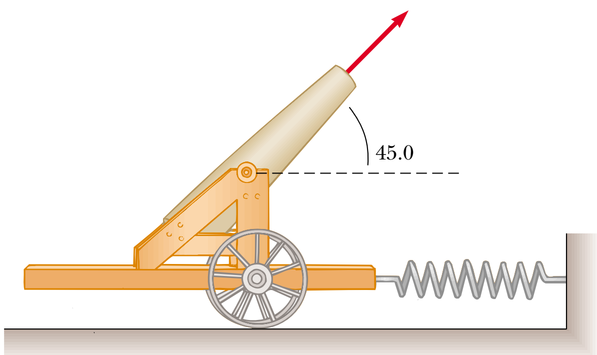


Figure P9.64

65. A student performs a ballistic pendulum experiment using an apparatus similar to that shown in Figure 9.11b. She obtains the following average data: $h = 8.68$ cm, $m_1 = 68.8$ g, and $m_2 = 263$ g. The symbols refer to the quantities in Figure 9.11a. (a) Determine the initial speed v_{1A} of the projectile. (b) The second part of her experiment is to obtain v_{1A} by firing the same projectile horizontally (with the pendulum removed from the path), by measuring its final horizontal position x and distance of fall y (Fig. P9.65). Show that the initial speed of the projectile is related to x and y through the relation

$$v_{1A} = \frac{x}{\sqrt{2y/g}}$$

What numerical value does she obtain for v_{1A} based on her measured values of $x = 257$ cm and $y = 85.3$ cm? What factors might account for the difference in this value compared to that obtained in part (a)?

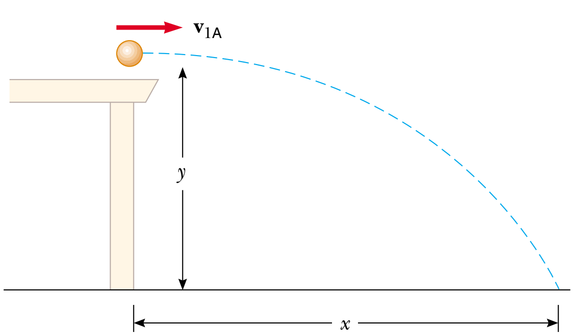


Figure P9.65

66. Small ice cubes, each of mass 5.00 g, slide down a frictionless track in a steady stream, as shown in Figure P9.66. Starting from rest, each cube moves down through a net vertical distance of 1.50 m and leaves the bottom end of the track at an angle of 40.0° above the horizontal. At the highest point of its subsequent trajectory, the cube strikes a vertical wall and rebounds with half the speed it had upon impact. If 10.0 cubes strike the wall per second, what average force is exerted on the wall?

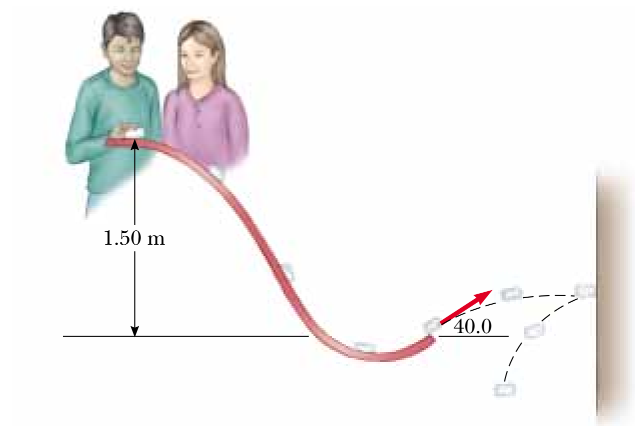


Figure P9.66

$\boxed{67.}$ A 5.00-g bullet moving with an initial speed of 400 m/s is fired into and passes through a 1.00-kg block, as in Figure P9.67. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant 900 N/m. If the block moves 5.00 cm to the right after impact, find (a) the speed at which the bullet emerges from the block and (b) the mechanical energy converted into internal energy in the collision.

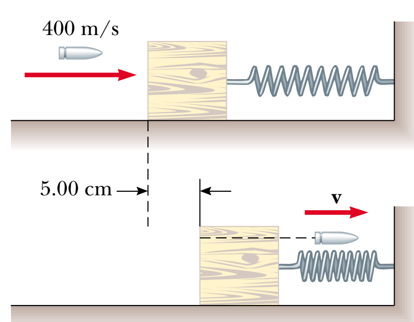


Figure P9.67

68. Consider as a system the Sun with the Earth in a circular orbit around it. Find the magnitude of the change in the velocity of the Sun relative to the center of mass of the system over a period of 6 months. Neglect the influence of other celestial objects. You may obtain the necessary astronomical data from the endpapers of the book.

69. Review problem. There are (one can say) three coequal theories of motion: Newton's second law, stating that the total force on an object causes its acceleration; the work–kinetic energy theorem, stating that the total work on an object causes its change in kinetic energy; and the impulse–momentum theorem, stating that the total impulse on an object causes its change in momentum. In this problem, you compare predictions of the three theories in one particular case. A 3.00-kg object has velocity $7.00\hat{\mathbf{j}}$ m/s. Then, a total force $12.0\hat{\mathbf{i}}$ N acts on the object for 5.00 s. (a) Calculate the object's final velocity, using the impulse–momentum theorem. (b) Calculate its acceleration from $\mathbf{a} = (\mathbf{v}_f - \mathbf{v}_i)/\Delta t$. (c) Calculate its acceleration from $\mathbf{a} = \Sigma\mathbf{F}/m$. (d) Find the object's vector displacement from $\Delta\mathbf{r} = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$. (e) Find the work done on the object from $W = \mathbf{F}\cdot\Delta\mathbf{r}$. (f) Find the final kinetic energy from $\frac{1}{2}mv_f^2 = \frac{1}{2}m\mathbf{v}_f\cdot\mathbf{v}_f$. (g) Find the final kinetic energy from $\frac{1}{2}mv_i^2 + W$.

70. A rocket has total mass $M_i = 360$ kg, including 330 kg of fuel and oxidizer. In interstellar space it starts from rest. Its engine is turned on at time $t = 0$, and it puts out exhaust with relative speed $v_e = 1\,500$ m/s at the constant rate 2.50 kg/s. The burn lasts until the fuel runs out, at time 330 kg/(2.5 kg/s) = 132 s. Set up and carry out a computer analysis of the motion according to Euler's method. Find (a) the final velocity of the rocket and (b) the distance it travels during the burn.

71. A chain of length L and total mass M is released from rest with its lower end just touching the top of a table, as in Figure P9.71a. Find the force exerted by the table on the chain after the chain has fallen through a distance x, as in Figure P9.71b. (Assume each link comes to rest the instant it reaches the table.)

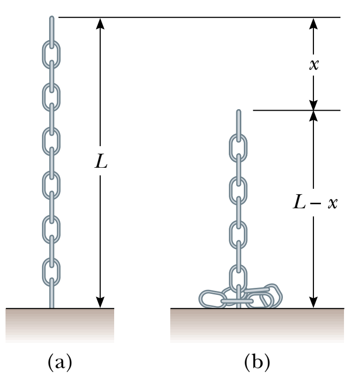


Figure P9.71

72. Sand from a stationary hopper falls onto a moving conveyor belt at the rate of 5.00 kg/s as in Figure P9.72. The conveyor belt is supported by frictionless rollers and moves at a constant speed of 0.750 m/s under the action of a constant horizontal external force $\mathbf{F}_{ext}$ supplied by the motor that drives the belt. Find (a) the sand's rate of change of momentum in the horizontal direction, (b) the force of friction exerted by the belt on the sand, (c) the external force $\mathbf{F}_{ext}$, (d) the work done by $\mathbf{F}_{ext}$ in 1 s, and (e) the kinetic energy acquired by the falling sand each second due to the change in its horizontal motion. (f) Why are the answers to (d) and (e) different?

73. A golf club consists of a shaft connected to a club head. The golf club can be modeled as a uniform rod of length ℓ and mass m_1 extending radially from the surface of a sphere of radius R and mass m_2. Find the location of the club's center of mass, measured from the center of the club head.

Answers to Quick Quizzes

9.1 (d). Two identical objects ($m_1 = m_2$) traveling at the same speed ($v_1 = v_2$) have the same kinetic energies and the same magnitudes of momentum. It also is possible, however, for particular combinations of masses and velocities to satisfy $K_1 = K_2$ but not $p_1 = p_2$. For example, a 1-kg object moving at 2 m/s has the same kinetic energy as a 4-kg object moving at 1 m/s, but the two clearly do not have the same momenta. Because we have no information about masses and speeds, we cannot choose among (a), (b), or (c).

9.2 (b), (c), (a). The slower the ball, the easier it is to catch. If the momentum of the medicine ball is the same as the momentum of the baseball, the speed of the medicine ball must be 1/10 the speed of the baseball because the medicine ball has 10 times the mass. If the kinetic energies are the same, the speed of the medicine ball must be $1/\sqrt{10}$ the speed of the baseball because of the squared speed term in the equation for K. The medicine ball is hardest to catch when it has the same speed as the baseball.

9.3 (c). The ball and the Earth exert forces on each other, so neither is an isolated system. We must include both in the system so that the interaction force is internal to the system.

9.4 (c). From Equation 9.4, if $\mathbf{p}_1 + \mathbf{p}_2 =$ constant, then it follows that $\Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 = 0$ and $\Delta\mathbf{p}_1 = -\Delta\mathbf{p}_2$. While the change in momentum is the same, the change in the *velocity* is a lot larger for the car!

9.5 (c) and (e). Object 2 has a greater acceleration because of its smaller mass. Therefore, it takes less time to travel the distance d. Even though the force applied to objects 1 and 2 is the same, the change in momentum is less for object 2 because Δt is smaller. The work $W = Fd$ done on

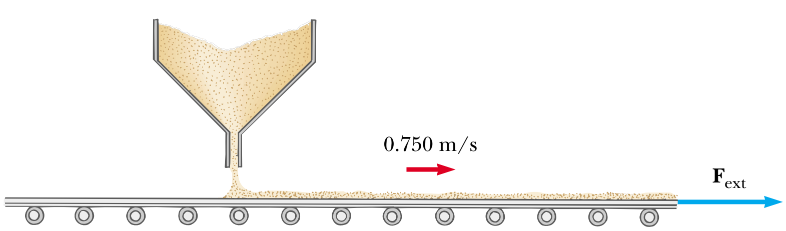


Figure P9.72

both objects is the same because both F and d are the same in the two cases. Therefore, $K_1 = K_2$.

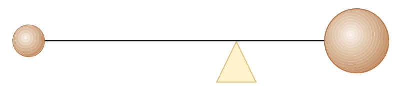
9.6 (b) and (d). The same impulse is applied to both objects, so they experience the same change in momentum. Object 2 has a larger acceleration due to its smaller mass. Thus, the distance that object 2 covers in the time interval Δt is larger than that for object 1. As a result, more work is done on object 2 and $K_2 > K_1$.

9.7 (a) All three are the same. Because the passenger is brought from the car's initial speed to a full stop, the change in momentum (equal to the impulse) is the same regardless of what stops the passenger. (b) Dashboard, seatbelt, airbag. The dashboard stops the passenger very quickly in a front-end collision, resulting in a very large force. The seatbelt takes somewhat more time, so the force is smaller. Used along with the seatbelt, the airbag can extend the passenger's stopping time further, notably for his head, which would otherwise snap forward.

9.8 (a). If all of the initial kinetic energy is transformed, then nothing is moving after the collision. Consequently, the final momentum of the system is necessarily zero and, therefore, the initial momentum of the system must be zero. While (b) and (d) *together* would satisfy the conditions, neither one *alone* does.

9.9 (b). Because momentum of the two-ball system is conserved, $\mathbf{p}_{\text{T}i} + 0 = \mathbf{p}_{\text{T}f} + \mathbf{p}_{\text{B}}$. Because the table-tennis ball bounces back from the much more massive bowling ball with approximately the same speed, $\mathbf{p}_{\text{T}f} = -\mathbf{p}_{\text{T}i}$. As a consequence, $\mathbf{p}_{\text{B}} = 2\mathbf{p}_{\text{T}i}$. Kinetic energy can be expressed as $K = p^2/2m$. Because of the much larger mass of the bowling ball, its kinetic energy is much smaller than that of the table-tennis ball.

9.10 (b). The piece with the handle will have less mass than the piece made up of the end of the bat. To see why this is so, take the origin of coordinates as the center of mass before the bat was cut. Replace each cut piece by a small sphere located at the center of mass for each piece. The sphere representing the handle piece is farther from the origin, but the product of less mass and greater distance balances the product of greater mass and less distance for the end piece:



9.11 (a). This is the same effect as the swimmer diving off the raft that we just discussed. The vessel–passengers system is isolated. If the passengers all start running one way, the speed of the vessel increases (a *small* amount!) the other way.

9.12 (b). Once they stop running, the momentum of the system is the same as it was before they started running—you cannot change the momentum of an isolated system by means of internal forces. In case you are thinking that the passengers could do this over and over to take advantage of the speed increase *while* they are running, remember that they will slow the ship down every time they return to the bow!

Chapter 10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

▲ *The Malaysian pastime of* gasing *involves the spinning of tops that can have masses up to 20 kg. Professional spinners can spin their tops so that they might rotate for hours before stopping. We will study the rotational motion of objects such as these tops in this chapter. (Courtesy Tourism Malaysia)*

When an extended object such as a wheel rotates about its axis, the motion cannot be analyzed by treating the object as a particle because at any given time different parts of the object have different linear velocities and linear accelerations. We can, however, analyze the motion by considering an extended object to be composed of a collection of particles, each of which has its own linear velocity and linear acceleration.

Rigid object

In dealing with a rotating object, analysis is greatly simplified by assuming that the object is rigid. A **rigid object** is one that is nondeformable—that is, the relative locations of all particles of which the object is composed remain constant. All real objects are deformable to some extent; however, our rigid-object model is useful in many situations in which deformation is negligible.

10.1 Angular Position, Velocity, and Acceleration

Figure 10.1 illustrates an overhead view of a rotating compact disc. The disc is rotating about a fixed axis through O. The axis is perpendicular to the plane of the figure. Let us investigate the motion of only one of the millions of "particles" making up the disc. A particle at P is at a fixed distance r from the origin and rotates about it in a circle of radius r. (In fact, *every* particle on the disc undergoes circular motion about O.) It is convenient to represent the position of P with its polar coordinates (r, θ), where r is the distance from the origin to P and θ is measured *counterclockwise* from some reference line as shown in Figure 10.1a. In this representation, the only coordinate for the particle that changes in time is the angle θ; r remains constant. As the particle moves along the circle from the reference line ($\theta = 0$), it moves through an arc of length s, as in Figure 10.1b. The arc length s is related to the angle θ through the relationship

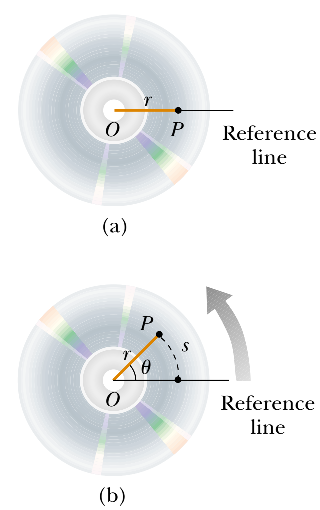


Figure 10.1 A compact disc rotating about a fixed axis through O perpendicular to the plane of the figure. (a) In order to define angular position for the disc, a fixed reference line is chosen. A particle at P is located at a distance r from the rotation axis at O. (b) As the disc rotates, point P moves through an arc length s on a circular path of radius r.

$$s = r\theta \tag{10.1a}$$

$$\theta = \frac{s}{r} \tag{10.1b}$$

Note the dimensions of θ in Equation 10.1b. Because θ is the ratio of an arc length and the radius of the circle, it is a pure number. However, we commonly give θ the artificial unit **radian** (rad), where

one radian is the angle subtended by an arc length equal to the radius of the arc.

Because the circumference of a circle is $2\pi r$, it follows from Equation 10.1b that 360° corresponds to an angle of $(2\pi r/r)$ rad $= 2\pi$ rad. (Also note that 2π rad corresponds

PITFALL PREVENTION

10.1 Remember the Radian

In rotational equations, we must use angles expressed in *radians*. Don't fall into the trap of using angles measured in degrees in rotational equations.

to one complete revolution.) Hence, 1 rad $= 360°/2\pi \approx 57.3°$. To convert an angle in degrees to an angle in radians, we use the fact that π rad $= 180°$, or

$$\theta\,(\text{rad}) = \frac{\pi}{180°}\,\theta\,(\text{deg})$$

For example, 60° equals $\pi/3$ rad and 45° equals $\pi/4$ rad.

Because the disc in Figure 10.1 is a rigid object, as the particle moves along the circle from the reference line, every other particle on the object rotates through the same angle θ. Thus, **we can associate the angle θ with the entire rigid object as well as with an individual particle.** This allows us to define the *angular position* of a rigid object in its rotational motion. We choose a reference line on the object, such as a line connecting O and a chosen particle on the object. The **angular position** of the rigid object is the angle θ between this reference line on the object and the fixed reference line in space, which is often chosen as the x axis. This is similar to the way we identify the position of an object in translational motion—the distance x between the object and the reference position, which is the origin, $x = 0$.

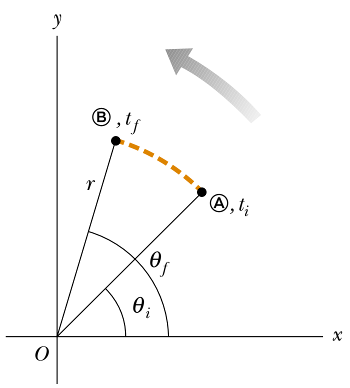


Figure 10.2 A particle on a rotating rigid object moves from Ⓐ to Ⓑ along the arc of a circle. In the time interval $\Delta t = t_f - t_i$, the radius vector moves through an angular displacement $\Delta\theta = \theta_f - \theta_i$.

As the particle in question on our rigid object travels from position Ⓐ to position Ⓑ in a time interval Δt as in Figure 10.2, the reference line of length r sweeps out an angle $\Delta\theta = \theta_f - \theta_i$. This quantity $\Delta\theta$ is defined as the **angular displacement** of the rigid object:

$$\Delta\theta \equiv \theta_f - \theta_i$$

The rate at which this angular displacement occurs can vary. If the rigid object spins rapidly, this displacement can occur in a short time interval. If it rotates slowly, this displacement occurs in a longer time interval. These different rotation rates can be quantified by introducing *angular speed*. We define the **average angular speed** $\bar{\omega}$ (Greek omega) as the ratio of the angular displacement of a rigid object to the time interval Δt during which the displacement occurs:

Average angular speed

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \tag{10.2}$$

In analogy to linear speed, the **instantaneous angular speed** ω is defined as the limit of the ratio $\Delta\theta/\Delta t$ as Δt approaches zero:

Instantaneous angular speed

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \tag{10.3}$$

Angular speed has units of radians per second (rad/s), which can be written as second^{-1} (s^{-1}) because radians are not dimensional. We take ω to be positive when θ is increasing (counterclockwise motion in Figure 10.2) and negative when θ is decreasing (clockwise motion in Figure 10.2).

Quick Quiz 10.1 A rigid object is rotating in a counterclockwise sense around a fixed axis. Each of the following pairs of quantities represents an initial angular position and a final angular position of the rigid object. Which of the sets can *only* occur if the rigid object rotates through more than 180°? (a) 3 rad, 6 rad (b) −1 rad, 1 rad (c) 1 rad, 5 rad.

Quick Quiz 10.2 Suppose that the change in angular position for each of the pairs of values in Quick Quiz 10.1 occurs in 1 s. Which choice represents the lowest average angular speed?

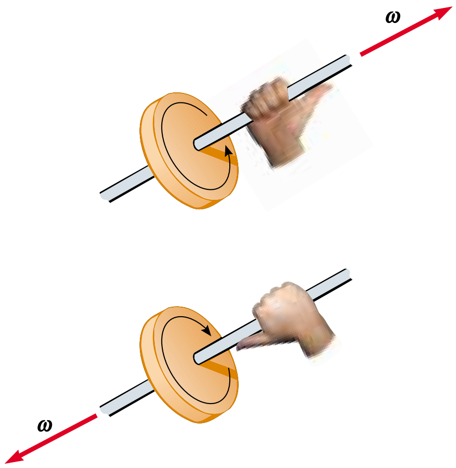


Figure 10.3 The right-hand rule for determining the direction of the angular velocity vector.

If the instantaneous angular speed of an object changes from ω_i to ω_f in the time interval Δt, the object has an angular acceleration. The **average angular acceleration** $\overline{\alpha}$ (Greek alpha) of a rotating rigid object is defined as the ratio of the change in the angular speed to the time interval Δt during which the change in the angular speed occurs:

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \tag{10.4}$$

Average angular acceleration

In analogy to linear acceleration, the **instantaneous angular acceleration** is defined as the limit of the ratio $\Delta\omega/\Delta t$ as Δt approaches zero:

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \tag{10.5}$$

Instantaneous angular acceleration

Angular acceleration has units of radians per second squared (rad/s^2), or just second^{-2} (s^{-2}). Note that α is positive when a rigid object rotating counterclockwise is speeding up or when a rigid object rotating clockwise is slowing down during some time interval.

When a rigid object is rotating about a *fixed* axis, **every particle on the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration.** That is, the quantities θ, ω, and α characterize the rotational motion of the entire rigid object as well as individual particles in the object. Using these quantities, we can greatly simplify the analysis of rigid-object rotation.

Angular position (θ), angular speed (ω), and angular acceleration (α) are analogous to linear position (x), linear speed (v), and linear acceleration (a). The variables θ, ω, and α differ dimensionally from the variables x, v, and a only by a factor having the unit of length. (See Section 10.3.)

We have not specified any direction for angular speed and angular acceleration. Strictly speaking, ω and α are the magnitudes of the angular velocity and the angular acceleration vectors[1] $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$, respectively, and they should always be positive. Because we are considering rotation about a fixed axis, however, we can use nonvector notation and indicate the directions of the vectors by assigning a positive or negative sign to ω and α, as discussed earlier with regard to Equations 10.3 and 10.5. For rotation about a fixed axis, the only direction that uniquely specifies the rotational motion is the direction along the axis of rotation. Therefore, the directions of $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are along this axis. If an object rotates in the xy plane as in Figure 10.1, the direction of $\boldsymbol{\omega}$ is out of the plane of the diagram when the rotation is counterclockwise and into the plane of the diagram when the rotation is clockwise. To illustrate this convention, it is convenient to use the *right-hand rule* demonstrated in Figure 10.3. When the four fingers of the right

[1] Although we do not verify it here, the instantaneous angular velocity and instantaneous angular acceleration are vector quantities, but the corresponding average values are not. This is because angular displacements do not add as vector quantities for finite rotations.

PITFALL PREVENTION

10.2 Specify Your Axis

In solving rotation problems, you must specify an axis of rotation. This is a new feature not found in our study of translational motion. The choice is arbitrary, but once you make it, you must maintain that choice consistently throughout the problem. In some problems, the physical situation suggests a natural axis, such as the center of an automobile wheel. In other problems, there may not be an obvious choice, and you must exercise judgement.

hand are wrapped in the direction of rotation, the extended right thumb points in the direction of $\boldsymbol{\omega}$. The direction of $\boldsymbol{\alpha}$ follows from its definition $\boldsymbol{\alpha} \equiv d\boldsymbol{\omega}/dt$. It is in the same direction as $\boldsymbol{\omega}$ if the angular speed is increasing in time, and it is antiparallel to $\boldsymbol{\omega}$ if the angular speed is decreasing in time.

Quick Quiz 10.3 A rigid object is rotating with an angular speed $\omega < 0$. The angular velocity vector $\boldsymbol{\omega}$ and the angular acceleration vector $\boldsymbol{\alpha}$ are antiparallel. The angular speed of the rigid object is (a) clockwise and increasing (b) clockwise and decreasing (c) counterclockwise and increasing (d) counterclockwise and decreasing.

10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

In our study of linear motion, we found that the simplest form of accelerated motion to analyze is motion under constant linear acceleration. Likewise, for rotational motion about a fixed axis, the simplest accelerated motion to analyze is motion under constant angular acceleration. Therefore, we next develop kinematic relationships for this type of motion. If we write Equation 10.5 in the form $d\omega = \alpha\, dt$, and let $t_i = 0$ and $t_f = t$, integrating this expression directly gives

Rotational kinematic equations

$$\omega_f = \omega_i + \alpha t \qquad \text{(for constant } \alpha\text{)} \tag{10.6}$$

where ω_i is the angular speed of the rigid object at time $t = 0$. Equation 10.6 allows us to find the angular speed ω_f of the object at any later time t. Substituting Equation 10.6 into Equation 10.3 and integrating once more, we obtain

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2 \qquad \text{(for constant } \alpha\text{)} \tag{10.7}$$

where θ_i is the angular position of the rigid object at time $t = 0$. Equation 10.7 allows us to find the angular position θ_f of the object at any later time t. If we eliminate t from Equations 10.6 and 10.7, we obtain

$$\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha(\theta_f - \theta_i) \qquad \text{(for constant } \alpha\text{)} \tag{10.8}$$

This equation allows us to find the angular speed ω_f of the rigid object for any value of its angular position θ_f. If we eliminate α between Equations 10.6 and 10.7, we obtain

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)\,t \qquad \text{(for constant } \alpha\text{)} \tag{10.9}$$

Notice that these kinematic expressions for rotational motion under constant angular acceleration are of the same mathematical form as those for linear motion under constant linear acceleration. They can be generated from the equations for linear motion by making the substitutions $x \rightarrow \theta$, $v \rightarrow \omega$, and $a \rightarrow \alpha$. Table 10.1 compares the kinematic equations for rotational and linear motion.

PITFALL PREVENTION

10.3 Just Like Translation?

Equations 10.6 to 10.9 and Table 10.1 suggest that rotational kinematics is just like translational kinematics. That is almost true, with two key differences: (1) in rotational kinematics, you must specify a rotation axis (per Pitfall Prevention 10.2); (2) in rotational motion, the object keeps returning to its original orientation—thus, you may be asked for the number of revolutions made by a rigid object. This concept has no meaning in translational motion, but is related to $\Delta\theta$, which is analogous to Δx.

Table 10.1

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Quick Quiz 10.4 Consider again the pairs of angular positions for the rigid object in Quick Quiz 10.1. If the object starts from rest at the initial angular position, moves counterclockwise with constant angular acceleration, and arrives at the final angular position with the same angular speed in all three cases, for which choice is the angular acceleration the highest?

Example 10.1 Rotating Wheel

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(A) If the angular speed of the wheel is 2.00 rad/s at $t_i = 0$, through what angular displacement does the wheel rotate in 2.00 s?

Solution We can use Figure 10.2 to represent the wheel. We arrange Equation 10.7 so that it gives us angular displacement:

$$\Delta\theta = \theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$= (2.00\text{ rad/s})(2.00\text{ s}) + \tfrac{1}{2}(3.50\text{ rad/s}^2)(2.00\text{ s})^2$$
$$= \boxed{11.0\text{ rad}} = (11.0\text{ rad})(57.3°/\text{rad}) = \boxed{630°}$$

(B) Through how many revolutions has the wheel turned during this time interval?

Solution We multiply the angular displacement found in part (A) by a conversion factor to find the number of revolutions:

$$\Delta\theta = 630°\left(\frac{1\text{ rev}}{360°}\right) = \boxed{1.75\text{ rev}}$$

(C) What is the angular speed of the wheel at $t = 2.00$ s?

Solution Because the angular acceleration and the angular speed are both positive, our answer must be greater than 2.00 rad/s. Using Equation 10.6, we find

$$\omega_f = \omega_i + \alpha t = 2.00\text{ rad/s} + (3.50\text{ rad/s}^2)(2.00\text{ s})$$
$$= \boxed{9.00\text{ rad/s}}$$

We could also obtain this result using Equation 10.8 and the results of part (A). Try it!

What If? **Suppose a particle moves along a straight line with a constant acceleration of 3.50 m/s^2. If the velocity of the particle is 2.00 m/s at $t_i = 0$, through what displacement does the particle move in 2.00 s? What is the velocity of the particle at $t = 2.00$ s?**

Answer Notice that these questions are translational analogs to parts (A) and (C) of the original problem. The mathematical solution follows exactly the same form. For the displacement,

$$\Delta x = x_f - x_i = v_i t + \tfrac{1}{2}at^2$$
$$= (2.00\text{ m/s})(2.00\text{ s}) + \tfrac{1}{2}(3.50\text{ m/s}^2)(2.00\text{ s})^2$$
$$= 11.0\text{ m}$$

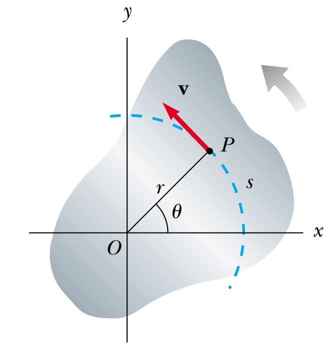
and for the velocity,

$$v_f = v_i + at = 2.00\text{ m/s} + (3.50\text{ m/s}^2)(2.00\text{ s}) = 9.00\text{ m/s}$$

Note that there is no translational analog to part (B) because translational motion is not repetitive like rotational motion.

10.3 Angular and Linear Quantities

In this section we derive some useful relationships between the angular speed and acceleration of a rotating rigid object and the linear speed and acceleration of a point in the object. To do so, we must keep in mind that when a rigid object rotates about a fixed axis, as in Figure 10.4, **every particle of the object moves in a circle whose center is the axis of rotation.**



Active Figure 10.4 As a rigid object rotates about the fixed axis through O, the point P has a tangential velocity $\mathbf{v}$ that is always tangent to the circular path of radius r.

At the Active Figures link at http://www.pse6.com, you can move point P and observe the tangential velocity as the object rotates.

Because point P in Figure 10.4 moves in a circle, the linear velocity vector $\mathbf{v}$ is always tangent to the circular path and hence is called *tangential velocity*. The magnitude of the tangential velocity of the point P is by definition the tangential speed $v = ds/dt$, where s is the distance traveled by this point measured along the circular path. Recalling that $s = r\theta$ (Eq. 10.1a) and noting that r is constant, we obtain

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

Because $d\theta/dt = \omega$ (see Eq. 10.3), we see that

$$v = r\omega \qquad (10.10)$$

That is, the tangential speed of a point on a rotating rigid object equals the perpendicular distance of that point from the axis of rotation multiplied by the angular speed. Therefore, although every point on the rigid object has the same *angular* speed, not every point has the same *tangential* speed because r is not the same for all points on the object. Equation 10.10 shows that the tangential speed of a point on the rotating object increases as one moves outward from the center of rotation, as we would intuitively expect. The outer end of a swinging baseball bat moves much faster than the handle.

We can relate the angular acceleration of the rotating rigid object to the tangential acceleration of the point P by taking the time derivative of v:

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$$

Relation between tangential and angular acceleration

$$a_t = r\alpha \qquad (10.11)$$

That is, the tangential component of the linear acceleration of a point on a rotating rigid object equals the point's distance from the axis of rotation multiplied by the angular acceleration.

In Section 4.4 we found that a point moving in a circular path undergoes a radial acceleration a_r of magnitude v^2/r directed toward the center of rotation (Fig. 10.5). Because $v = r\omega$ for a point P on a rotating object, we can express the centripetal acceleration at that point in terms of angular speed as

$$a_c = \frac{v^2}{r} = r\omega^2 \qquad (10.12)$$

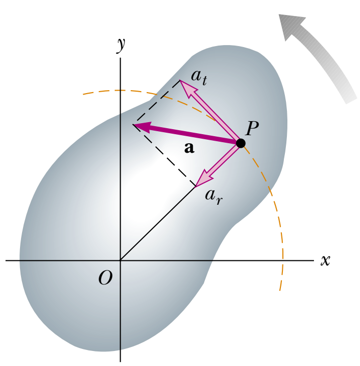


Figure 10.5 As a rigid object rotates about a fixed axis through O, the point P experiences a tangential component of linear acceleration a_t and a radial component of linear acceleration a_r. The total linear acceleration of this point is $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$.

The total linear acceleration vector at the point is $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$, where the magnitude of $\mathbf{a}_r$ is the centripetal acceleration a_c. Because $\mathbf{a}$ is a vector having a radial and a tangential component, the magnitude of $\mathbf{a}$ at the point P on the rotating rigid object is

$$a = \sqrt{a_t^{\,2} + a_r^{\,2}} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4} \qquad (10.13)$$

Quick Quiz 10.5 Andy and Charlie are riding on a merry-go-round. Andy rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Charlie, who rides on an inner horse. When the merry-go-round is rotating at a constant angular speed, Andy's angular speed is (a) twice Charlie's (b) the same as Charlie's (c) half of Charlie's (d) impossible to determine.

Quick Quiz 10.6 Consider again the merry-go-round situation in Quick Quiz 10.5. When the merry-go-round is rotating at a constant angular speed, Andy's tangential speed is (a) twice Charlie's (b) the same as Charlie's (c) half of Charlie's (d) impossible to determine.

Example 10.2 CD Player

On a compact disc (Fig. 10.6), audio information is stored in a series of pits and flat areas on the surface of the disc. The information is stored digitally, and the alternations between pits and flat areas on the surface represent binary ones and zeroes to be read by the compact disc player and converted back to sound waves. The pits and flat areas are detected by a system consisting of a laser and lenses. The length of a string of ones and zeroes representing one piece of information is the same everywhere on the disc, whether the information is near the center of the disc or near its outer edge. In order that this length of ones and zeroes always passes by the laser–lens system in the same time period, the tangential speed of the disc surface at the location of the lens must be constant. This requires, according to Equation 10.10, that the angular speed vary as the laser–lens system moves radially along the disc. In a typical compact disc player, the constant speed of the surface at the point of the laser–lens system is 1.3 m/s.

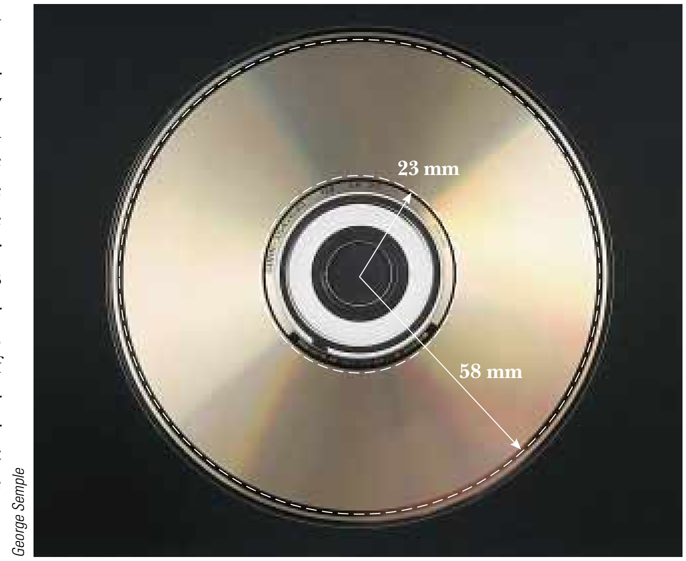


Figure 10.6 (Example 10.2) A compact disc.

(A) Find the angular speed of the disc in revolutions per minute when information is being read from the innermost first track ($r = 23$ mm) and the outermost final track ($r = 58$ mm).

Solution Using Equation 10.10, we can find the angular speed that will give us the required tangential speed at the position of the inner track,

$$\omega_i = \frac{v}{r_i} = \frac{1.3 \text{ m/s}}{2.3 \times 10^{-2} \text{ m}} = 57 \text{ rad/s}$$

$$= (57 \text{ rad/s})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right)$$

$$= \boxed{5.4 \times 10^2 \text{ rev/min}}$$

For the outer track,

$$\omega_f = \frac{v}{r_f} = \frac{1.3 \text{ m/s}}{5.8 \times 10^{-2} \text{ m}} = 22 \text{ rad/s}$$

$$= \boxed{2.1 \times 10^2 \text{ rev/min}}$$

The player adjusts the angular speed ω of the disc within this range so that information moves past the objective lens at a constant rate.

(B) The maximum playing time of a standard music CD is 74 min and 33 s. How many revolutions does the disc make during that time?

Solution We know that the angular speed is always decreasing, and we assume that it is decreasing steadily, with α constant. If $t = 0$ is the instant that the disc begins, with angular speed of 57 rad/s, then the final value of the time t is $(74 \text{ min})(60 \text{ s/min}) + 33 \text{ s} = 4\,473 \text{ s}$. We are looking for the angular displacement $\Delta\theta$ during this time interval. We use Equation 10.9:

$$\Delta\theta = \theta_f - \theta_i = \tfrac{1}{2}(\omega_i + \omega_f)t$$

$$= \tfrac{1}{2}(57 \text{ rad/s} + 22 \text{ rad/s})(4\,473 \text{ s})$$

$$= 1.8 \times 10^5 \text{ rad}$$

We convert this angular displacement to revolutions:

$$\Delta\theta = 1.8 \times 10^5 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{2.8 \times 10^4 \text{ rev}}$$

(C) What total length of track moves past the objective lens during this time?

Solution Because we know the (constant) linear velocity and the time interval, this is a straightforward calculation:

$$x_f = v_i t = (1.3 \text{ m/s})(4\,473 \text{ s}) = \boxed{5.8 \times 10^3 \text{ m}}$$

More than 5.8 km of track spins past the objective lens!

(D) What is the angular acceleration of the CD over the 4 473-s time interval? Assume that α is constant.

Solution The most direct approach to solving this problem is to use Equation 10.6 and the results to part (A). We should obtain a negative number for the angular acceleration because the disc spins more and more slowly in the positive direction as time goes on. Our answer should also be relatively small because it takes such a long time—more than an hour—for the change in angular speed to be accomplished:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{22 \text{ rad/s} - 57 \text{ rad/s}}{4\,473 \text{ s}}$$

$$= \boxed{-7.8 \times 10^{-3} \text{ rad/s}^2}$$

The disc experiences a very gradual decrease in its rotation rate, as expected.

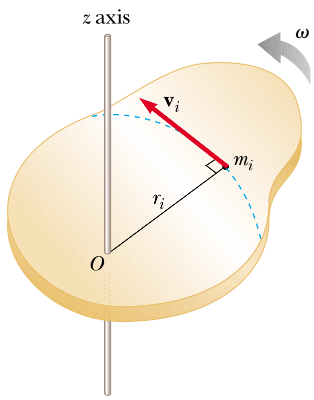


Figure 10.7 A rigid object rotating about the z axis with angular speed ω. The kinetic energy of the particle of mass m_i is $\frac{1}{2}m_i v_i^2$. The total kinetic energy of the object is called its rotational kinetic energy.

10.4 Rotational Kinetic Energy

In Chapter 7, we defined the kinetic energy of an object as the energy associated with its motion through space. An object rotating about a fixed axis remains stationary in space, so there is no kinetic energy associated with translational motion. The individual particles making up the rotating object, however, are moving through space—they follow circular paths. Consequently, there should be kinetic energy associated with rotational motion.

Let us consider an object as a collection of particles and assume that it rotates about a fixed z axis with an angular speed ω. Figure 10.7 shows the rotating object and identifies one particle on the object located at a distance r_i from the rotation axis. Each such particle has kinetic energy determined by its mass and tangential speed. If the mass of the ith particle is m_i and its tangential speed is v_i, its kinetic energy is

$$K_i = \frac{1}{2} m_i v_i^2$$

To proceed further, recall that although every particle in the rigid object has the same angular speed ω, the individual tangential speeds depend on the distance r_i from the axis of rotation according to the expression $v_i = r_i\omega$ (see Eq. 10.10). The *total* kinetic energy of the rotating rigid object is the sum of the kinetic energies of the individual particles:

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

We can write this expression in the form

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 \qquad (10.14)$$

where we have factored ω^2 from the sum because it is common to every particle. We simplify this expression by defining the quantity in parentheses as the **moment of inertia I:**

Moment of inertia

$$I \equiv \sum_i m_i r_i^2 \qquad (10.15)$$

From the definition of moment of inertia, we see that it has dimensions of ML^2 ($kg \cdot m^2$ in SI units).[2] With this notation, Equation 10.14 becomes

Rotational kinetic energy

$$K_R = \frac{1}{2} I\omega^2 \qquad (10.16)$$

Although we commonly refer to the quantity $\frac{1}{2}I\omega^2$ as **rotational kinetic energy,** it is not a new form of energy. It is ordinary kinetic energy because it is derived from a sum over individual kinetic energies of the particles contained in the rigid object. However, the mathematical form of the kinetic energy given by Equation 10.16 is convenient when we are dealing with rotational motion, provided we know how to calculate I.

It is important that you recognize the analogy between kinetic energy associated with linear motion $\frac{1}{2}mv^2$ and rotational kinetic energy $\frac{1}{2}I\omega^2$. The quantities I and ω in rotational motion are analogous to m and v in linear motion, respectively. (In fact, I takes the place of m and ω takes the place of v every time we compare a linear-motion equation with its rotational counterpart.) The moment of inertia is a measure of the resistance of an object to changes in its rotational motion, just as mass is a measure of the tendency of an object to resist changes in its linear motion.

PITFALL PREVENTION

10.4 No Single Moment of Inertia

There is one major difference between mass and moment of inertia. Mass is an inherent property of an object. The moment of inertia of an object depends on your choice of rotation axis. Thus, there is no single value of the moment of inertia for an object. There is a minimum value of the moment of inertia, which is that calculated about an axis passing through the center of mass of the object.

[2] Civil engineers use moment of inertia to characterize the elastic properties (rigidity) of such structures as loaded beams. Hence, it is often useful even in a nonrotational context.

Quick Quiz 10.7 A section of hollow pipe and a solid cylinder have the same radius, mass, and length. They both rotate about their long central axes with the same angular speed. Which object has the higher rotational kinetic energy? (a) the hollow pipe (b) the solid cylinder (c) they have the same rotational kinetic energy (d) impossible to determine.

Example 10.3 The Oxygen Molecule

Consider an oxygen molecule (O_2) rotating in the xy plane about the z axis. The rotation axis passes through the center of the molecule, perpendicular to its length. The mass of each oxygen atom is 2.66×10^{-26} kg, and at room temperature the average separation between the two atoms is $d = 1.21 \times 10^{-10}$ m. (The atoms are modeled as particles.)

(A) Calculate the moment of inertia of the molecule about the z axis.

Solution This is a straightforward application of the definition of I. Because each atom is a distance $d/2$ from the z axis, the moment of inertia about the axis is

$$I = \sum_i m_i r_i^2 = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$$

$$= \frac{(2.66 \times 10^{-26}\ \text{kg})(1.21 \times 10^{-10}\ \text{m})^2}{2}$$

$$= \boxed{1.95 \times 10^{-46}\ \text{kg}\cdot\text{m}^2}$$

This is a very small number, consistent with the minuscule masses and distances involved.

(B) If the angular speed of the molecule about the z axis is 4.60×10^{12} rad/s, what is its rotational kinetic energy?

Solution We apply the result we just calculated for the moment of inertia in the equation for K_R:

$$K_R = \tfrac{1}{2} I\omega^2$$

$$= \tfrac{1}{2}(1.95 \times 10^{-46}\ \text{kg}\cdot\text{m}^2)(4.60 \times 10^{12}\ \text{rad/s})^2$$

$$= \boxed{2.06 \times 10^{-21}\ \text{J}}$$

Example 10.4 Four Rotating Objects

Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the xy plane (Fig. 10.8). We shall assume that the radii of the spheres are small compared with the dimensions of the rods.

(A) If the system rotates about the y axis (Fig. 10.8a) with an angular speed ω, find the moment of inertia and the rotational kinetic energy about this axis.

Solution First, note that the two spheres of mass m, which lie on the y axis, do not contribute to I_y (that is, $r_i = 0$ for these spheres about this axis). Applying Equation 10.15, we obtain

$$I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 = \boxed{2Ma^2}$$

Therefore, the rotational kinetic energy about the y axis is

$$K_R = \tfrac{1}{2} I_y\omega^2 = \tfrac{1}{2}(2Ma^2)\omega^2 = \boxed{Ma^2\omega^2}$$

The fact that the two spheres of mass m do not enter into this result makes sense because they have no motion about the axis of rotation; hence, they have no rotational kinetic energy. By similar logic, we expect the moment of inertia about the x axis to be $I_x = 2mb^2$ with a rotational kinetic energy about that axis of $K_R = mb^2\omega^2$.

(B) Suppose the system rotates in the xy plane about an axis (the z axis) through O (Fig. 10.8b). Calculate the moment of inertia and rotational kinetic energy about this axis.

Solution Because r_i in Equation 10.15 is the distance between a sphere and the axis of rotation, we obtain

$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = \boxed{2Ma^2 + 2mb^2}$$

$$K_R = \tfrac{1}{2} I_z\omega^2 = \tfrac{1}{2}(2Ma^2 + 2mb^2)\omega^2 = \boxed{(Ma^2 + mb^2)\omega^2}$$

Comparing the results for parts (A) and (B), we conclude that the moment of inertia and therefore the rotational kinetic energy associated with a given angular speed depend on the axis of rotation. In part (B), we expect the result to include all four spheres and distances because all four spheres are rotating in the xy plane. Furthermore, the fact that the rotational kinetic energy in part (A) is smaller than that in part (B) indicates, based on the work–kinetic energy theorem, that it would require less work to set the system into rotation about the y axis than about the z axis.

What If? **What if the mass M is much larger than m? How do the answers to parts (A) and (B) compare?**

Answer If $M >> m$, then m can be neglected and the moment of inertia and rotational kinetic energy in part (B) become

$$I_z = 2Ma^2 \qquad \text{and} \qquad K_R = Ma^2\omega^2$$

which are the same as the answers in part (A). If the masses m of the two red spheres in Figure 10.8 are negligible, then these spheres can be removed from the figure and rotations about the y and z axes are equivalent.

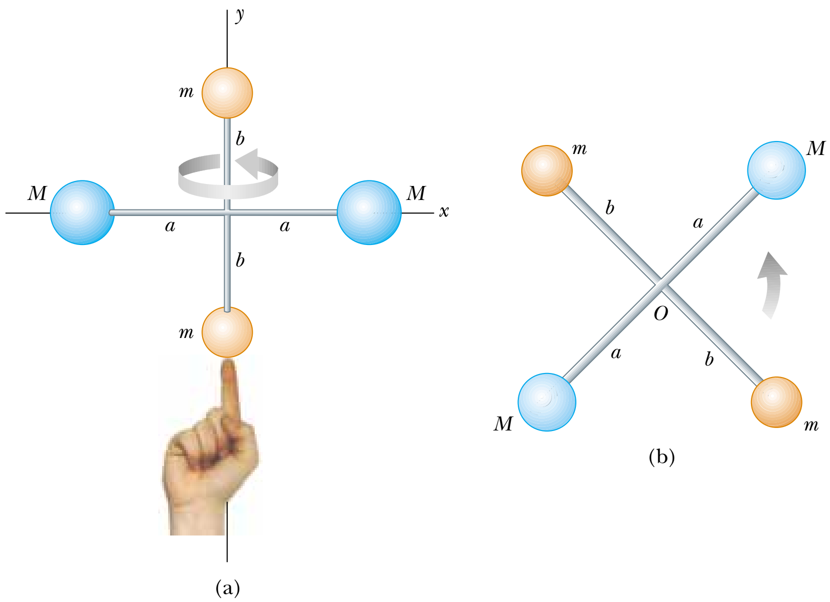


Figure 10.8 (Example 10.4) Four spheres form an unusual baton. (a) The baton is rotated about the y axis. (b) The baton is rotated about the z axis.

10.5 Calculation of Moments of Inertia

We can evaluate the moment of inertia of an extended rigid object by imagining the object to be divided into many small volume elements, each of which has mass Δm_i. We use the definition $I = \sum_i r_i^2 \Delta m_i$ and take the limit of this sum as $\Delta m_i \rightarrow 0$. In this limit, the sum becomes an integral over the volume of the object:

Moment of inertia of a rigid object

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2\, dm \tag{10.17}$$

It is usually easier to calculate moments of inertia in terms of the volume of the elements rather than their mass, and we can easily make that change by using Equation 1.1, $\rho = m/V$, where ρ is the density of the object and V is its volume. From this equation, the mass of a small element is $dm = \rho\, dV$. Substituting this result into Equation 10.17 gives

$$I = \int \rho r^2\, dV$$

If the object is homogeneous, then ρ is constant and the integral can be evaluated for a known geometry. If ρ is not constant, then its variation with position must be known to complete the integration.

The density given by $\rho = m/V$ sometimes is referred to as *volumetric mass density* because it represents mass per unit volume. Often we use other ways of expressing density. For instance, when dealing with a sheet of uniform thickness t, we can define a *surface mass density* $\sigma = \rho t$, which represents *mass per unit area*. Finally, when mass is distributed along a rod of uniform cross-sectional area A, we sometimes use *linear mass density* $\lambda = M/L = \rho A$, which is the *mass per unit length*.

Example 10.5 Uniform Thin Hoop

Find the moment of inertia of a uniform thin hoop of mass M and radius R about an axis perpendicular to the plane of the hoop and passing through its center (Fig. 10.9).

Solution Because the hoop is thin, all mass elements dm are the same distance $r = R$ from the axis, and so, applying Equation 10.17, we obtain for the moment of inertia about the z axis through O:

$$I_z = \int r^2\,dm = R^2 \int dm = \boxed{MR^2}$$

Note that this moment of inertia is the same as that of a single particle of mass M located a distance R from the axis of rotation.

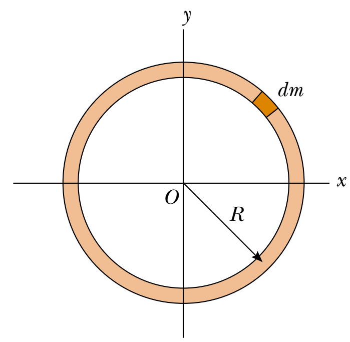


Figure 10.9 (Example 10.5) The mass elements dm of a uniform hoop are all the same distance from O.

Example 10.6 Uniform Rigid Rod

Calculate the moment of inertia of a uniform rigid rod of length L and mass M (Fig. 10.10) about an axis perpendicular to the rod (the y axis) and passing through its center of mass.

Solution The shaded length element dx in Figure 10.10 has a mass dm equal to the mass per unit length λ multiplied by dx:

$$dm = \lambda\,dx = \frac{M}{L}\,dx$$

Substituting this expression for dm into Equation 10.17, with $r^2 = x^2$, we obtain

$$I_y = \int r^2\,dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\,dx = \frac{M}{L}\int_{-L/2}^{L/2} x^2\,dx$$

$$= \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \boxed{\tfrac{1}{12}ML^2}$$

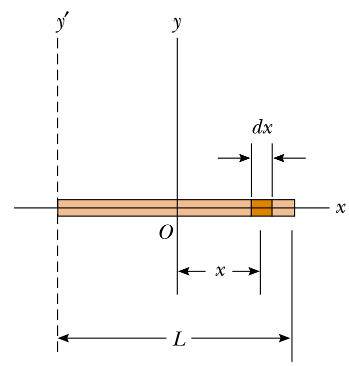


Figure 10.10 (Example 10.6) A uniform rigid rod of length L. The moment of inertia about the y axis is less than that about the y' axis. The latter axis is examined in Example 10.8.

Example 10.7 Uniform Solid Cylinder

A uniform solid cylinder has a radius R, mass M, and length L. Calculate its moment of inertia about its central axis (the z axis in Fig. 10.11).

Solution It is convenient to divide the cylinder into many cylindrical shells, each of which has radius r, thickness dr, and length L, as shown in Figure 10.11. The volume dV of each shell is its cross-sectional area multiplied by its length: $dV = L\,dA = L(2\pi r)\,dr$. If the mass per unit volume is ρ, then the mass of this differential volume element is $dm = \rho\,dV = 2\pi\rho L r\,dr$. Substituting this expression for dm into Equation 10.17, we obtain

$$I_z = \int r^2\,dm = \int r^2(2\pi\rho L r\,dr) = 2\pi\rho L\int_0^R r^3 dr = \tfrac{1}{2}\pi\rho L R^4$$

Because the total volume of the cylinder is $\pi R^2 L$, we see that $\rho = M/V = M/\pi R^2 L$. Substituting this value for ρ into the above result gives

$$\boxed{I_z = \tfrac{1}{2}MR^2}$$

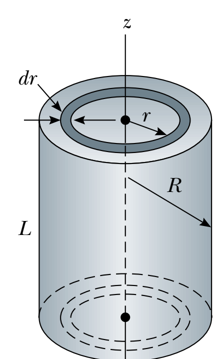


Figure 10.11 (Example 10.7) Calculating I about the z axis for a uniform solid cylinder.

What If? **What if the length of the cylinder in Figure 10.11 is increased to 2*L*, while the mass *M* and radius *R* are held fixed? How does this change the moment of inertia of the cylinder?**

Answer Note that the result for the moment of inertia of a cylinder does not depend on L, the length of the cylinder. In other words, it applies equally well to a long cylinder and a flat disk having the same mass M and radius R. Thus, the moment of inertia of the cylinder would not be affected by changing its length.

Table 10.2 gives the moments of inertia for a number of objects about specific axes. The moments of inertia of rigid objects with simple geometry (high symmetry) are relatively easy to calculate provided the rotation axis coincides with an axis of symmetry. The calculation of moments of inertia about an arbitrary axis can be cumbersome, however, even for a highly symmetric object. Fortunately, use of an important theorem, called the **parallel-axis theorem,** often simplifies the calculation. Suppose the moment of inertia about an axis through the center of mass of an object is I_{CM}. The parallel-axis theorem states that the moment of inertia about any axis parallel to and a distance D away from this axis is

Parallel-axis theorem

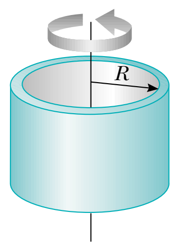
$$I = I_{CM} + MD^2 \tag{10.18}$$

To prove the parallel-axis theorem, suppose that an object rotates in the xy plane about the z axis, as shown in Figure 10.12, and that the coordinates of the center of mass are x_{CM}, y_{CM}. Let the mass element dm have coordinates x, y. Because this

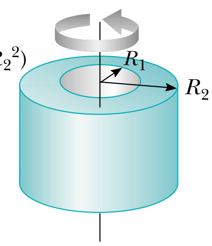
Table 10.2

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

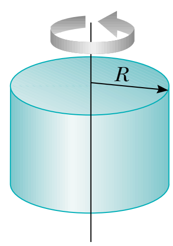
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



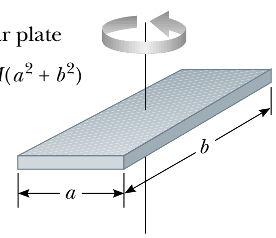
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$



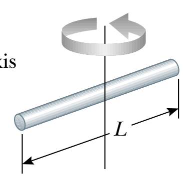
Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



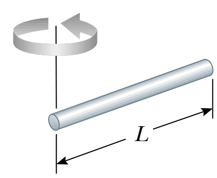
Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$



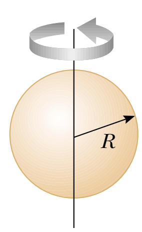
Long thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$



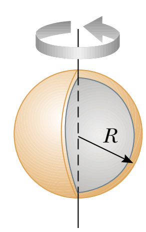
Long thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



Solid sphere
$I_{CM} = \frac{2}{5}MR^2$



Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$



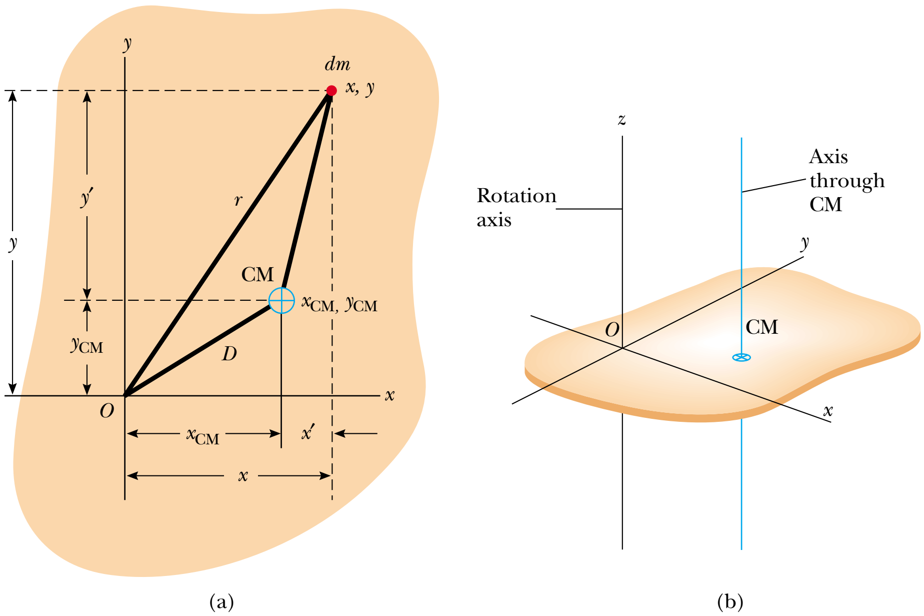


Figure 10.12 (a) The parallel-axis theorem: if the moment of inertia about an axis perpendicular to the figure through the center of mass is I_{CM}, then the moment of inertia about the z axis is $I_z = I_{CM} + MD^2$. (b) Perspective drawing showing the z axis (the axis of rotation) and the parallel axis through the CM.

element is a distance $r = \sqrt{x^2 + y^2}$ from the z axis, the moment of inertia about the z axis is

$$I = \int r^2\, dm = \int (x^2 + y^2)\, dm$$

However, we can relate the coordinates x, y of the mass element dm to the coordinates of this same element located in a coordinate system having the object's center of mass as its origin. If the coordinates of the center of mass are x_{CM}, y_{CM} in the original coordinate system centered on O, then from Figure 10.12a we see that the relationships between the unprimed and primed coordinates are $x = x' + x_{CM}$ and $y = y' + y_{CM}$. Therefore,

$$I = \int [(x' + x_{CM})^2 + (y' + y_{CM})^2]\, dm$$

$$= \int [(x')^2 + (y')^2]\, dm + 2x_{CM} \int x' dm + 2y_{CM} \int y' dm + (x_{CM}{}^2 + y_{CM}{}^2) \int dm$$

The first integral is, by definition, the moment of inertia about an axis that is parallel to the z axis and passes through the center of mass. The second two integrals are zero because, by definition of the center of mass, $\int x' dm = \int y' dm = 0$. The last integral is simply MD^2 because $\int dm = M$ and $D^2 = x_{CM}{}^2 + y_{CM}{}^2$. Therefore, we conclude that

$$I = I_{CM} + MD^2$$

Example 10.8 Applying the Parallel-Axis Theorem

Consider once again the uniform rigid rod of mass M and length L shown in Figure 10.10. Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the y' axis in Fig. 10.10).

Solution Intuitively, we expect the moment of inertia to be greater than $I_{CM} = \frac{1}{12}ML^2$ because there is mass up to a distance of L away from the rotation axis, while the farthest distance in Example 10.6 was only $L/2$. Because the distance between the center-of-mass axis and the y' axis is $D = L/2$, the parallel-axis theorem gives

$$I = I_{CM} + MD^2 = \tfrac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \boxed{\tfrac{1}{3}ML^2}$$

So, it is four times more difficult to change the rotation of a rod spinning about its end than it is to change the motion of one spinning about its center.

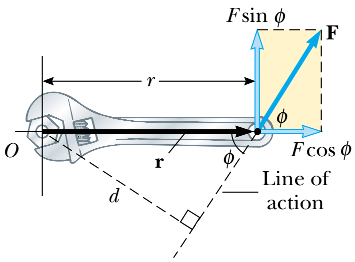


Figure 10.13 The force **F** has a greater rotating tendency about O as F increases and as the moment arm d increases. The component $F \sin \phi$ tends to rotate the wrench about O.

10.6 Torque

Why are a door's hinges and its doorknob placed near opposite edges of the door? Imagine trying to rotate a door by applying a force of magnitude F perpendicular to the door surface but at various distances from the hinges. You will achieve a more rapid rate of rotation for the door by applying the force near the doorknob than by applying it near the hinges.

If you cannot loosen a stubborn bolt with a socket wrench, what would you do in an effort to loosen the bolt? You may intuitively try using a wrench with a longer handle or slip a pipe over the existing wrench to make it longer. This is similar to the situation with the door. You are more successful at causing a change in rotational motion (of the door or the bolt) by applying the force farther away from the rotation axis.

When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called **torque** τ (Greek tau). Torque is a vector, but we will consider only its magnitude here and explore its vector nature in Chapter 11.

PITFALL PREVENTION

10.5 Torque Depends on Your Choice of Axis

Like moment of inertia, there is no unique value of the torque—its value depends on your choice of rotation axis.

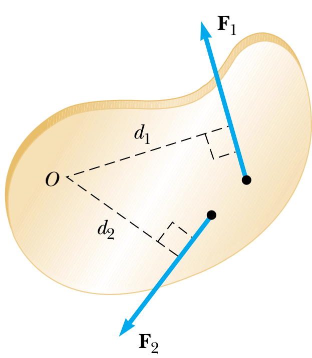
Consider the wrench pivoted on the axis through O in Figure 10.13. The applied force **F** acts at an angle ϕ to the horizontal. We define the magnitude of the torque associated with the force **F** by the expression

$$\tau \equiv rF \sin \phi = Fd \qquad (10.19)$$

where r is the distance between the pivot point and the point of application of **F** and d is the perpendicular distance from the pivot point to the line of action of **F**. (The *line of action* of a force is an imaginary line extending out both ends of the vector representing the force. The dashed line extending from the tail of **F** in Figure 10.13 is part of the line of action of **F**.) From the right triangle in Figure 10.13 that has the wrench as its hypotenuse, we see that $d = r \sin \phi$. The quantity d is called the **moment arm** (or *lever arm*) of **F**.

Moment arm

In Figure 10.13, the only component of **F** that tends to cause rotation is $F \sin \phi$, the component perpendicular to a line drawn from the rotation axis to the point of application of the force. The horizontal component $F \cos \phi$, because its line of action passes through O, has no tendency to produce rotation about an axis passing through O. From the definition of torque, we see that the rotating tendency increases as F increases and as d increases. This explains the observation that it is easier to rotate a door if we push at the doorknob rather than at a point close to the hinge. We also want to apply our push as closely perpendicular to the door as we can. Pushing sideways on the doorknob will not cause the door to rotate.



Active Figure 10.14 The force $\mathbf{F}_1$ tends to rotate the object counterclockwise about O, and $\mathbf{F}_2$ tends to rotate it clockwise.

At the Active Figures link at http://www.pse6.com, you can change the magnitudes, directions, and points of application of forces F_1 and F_2 to see how the object accelerates under the action of the two forces.

If two or more forces are acting on a rigid object, as in Figure 10.14, each tends to produce rotation about the axis at O. In this example, $\mathbf{F}_2$ tends to rotate the object clockwise and $\mathbf{F}_1$ tends to rotate it counterclockwise. We use the convention that the sign of the torque resulting from a force is positive if the turning tendency of the force is counterclockwise and is negative if the turning tendency is clockwise. For example, in Figure 10.14, the torque resulting from $\mathbf{F}_1$, which has a moment arm d_1, is positive and equal to $+F_1 d_1$; the torque from $\mathbf{F}_2$ is negative and equal to $-F_2 d_2$. Hence, the *net* torque about O is

$$\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

Torque should not be confused with force. Forces can cause a change in linear motion, as described by Newton's second law. Forces can also cause a change in rotational motion, but the effectiveness of the forces in causing this change depends on both the forces and the moment arms of the forces, in the combination that we call *torque*. Torque has units of force times length—newton · meters in SI units—and should be reported in these units. Do not confuse torque and work, which have the same units but are very different concepts.

Quick Quiz 10.8 If you are trying to loosen a stubborn screw from a piece of wood with a screwdriver and fail, should you find a screwdriver for which the handle is (a) longer or (b) fatter?

Quick Quiz 10.9 If you are trying to loosen a stubborn bolt from a piece of metal with a wrench and fail, should you find a wrench for which the handle is (a) longer (b) fatter?

Example 10.9 The Net Torque on a Cylinder

A one-piece cylinder is shaped as shown in Figure 10.15, with a core section protruding from the larger drum. The cylinder is free to rotate about the central axis shown in the drawing. A rope wrapped around the drum, which has radius R_1, exerts a force $\mathbf{T}_1$ to the right on the cylinder. A rope wrapped around the core, which has radius R_2, exerts a force $\mathbf{T}_2$ downward on the cylinder.

(A) What is the net torque acting on the cylinder about the rotation axis (which is the z axis in Figure 10.15)?

Solution The torque due to $\mathbf{T}_1$ is $-R_1T_1$. (The sign is negative because the torque tends to produce clockwise rotation.) The torque due to $\mathbf{T}_2$ is $+R_2T_2$. (The sign is positive because the torque tends to produce counterclockwise rotation.) Therefore, the net torque about the rotation axis is

$$\sum \tau = \tau_1 + \tau_2 = \boxed{R_2T_2 - R_1T_1}$$

We can make a quick check by noting that if the two forces are of equal magnitude, the net torque is negative because $R_1 > R_2$. Starting from rest with both forces of equal magnitude acting on it, the cylinder would rotate clockwise because $\mathbf{T}_1$ would be more effective at turning it than would $\mathbf{T}_2$.

(B) Suppose $T_1 = 5.0$ N, $R_1 = 1.0$ m, $T_2 = 15.0$ N, and $R_2 = 0.50$ m. What is the net torque about the rotation axis, and which way does the cylinder rotate starting from rest?

Solution Evaluating the net torque,

$$\sum \tau = (15\text{ N})(0.50\text{ m}) - (5.0\text{ N})(1.0\text{ m}) = \boxed{2.5\text{ N}\cdot\text{m}}$$

Because this torque is positive, the cylinder will begin to rotate in the counterclockwise direction.

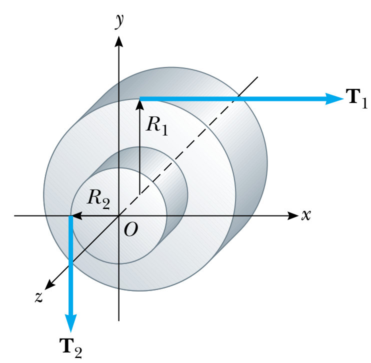


Figure 10.15 (Example 10.9) A solid cylinder pivoted about the z axis through O. The moment arm of $\mathbf{T}_1$ is R_1, and the moment arm of $\mathbf{T}_2$ is R_2.

10.7 Relationship Between Torque and Angular Acceleration

In Chapter 4, we learned that a net force on an object causes an acceleration of the object and that the acceleration is proportional to the net force (Newton's second law). In this section we show the rotational analog of Newton's second law—the angular acceleration of a rigid object rotating about a fixed axis is proportional to the net torque acting about that axis. Before discussing the more complex case of rigid-object rotation, however, it is instructive first to discuss the case of a particle moving in a circular path about some fixed point under the influence of an external force.

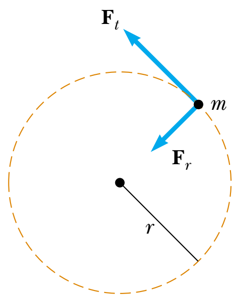


Figure 10.16 A particle rotating in a circle under the influence of a tangential force $\mathbf{F}_t$. A force $\mathbf{F}_r$ in the radial direction also must be present to maintain the circular motion.

Consider a particle of mass m rotating in a circle of radius r under the influence of a tangential force $\mathbf{F}_t$ and a radial force $\mathbf{F}_r$, as shown in Figure 10.16. The tangential force provides a tangential acceleration $\mathbf{a}_t$, and

$$F_t = ma_t$$

The magnitude of the torque about the center of the circle due to $\mathbf{F}_t$ is

$$\tau = F_t r = (ma_t)r$$

Because the tangential acceleration is related to the angular acceleration through the relationship $a_t = r\alpha$ (see Eq. 10.11), the torque can be expressed as

$$\tau = (mr\alpha)r = (mr^2)\alpha$$

Recall from Equation 10.15 that mr^2 is the moment of inertia of the particle about the z axis passing through the origin, so that

$$\tau = I\alpha \qquad (10.20)$$

That is, **the torque acting on the particle is proportional to its angular acceleration,** and the proportionality constant is the moment of inertia. Note that $\tau = I\alpha$ is the rotational analog of Newton's second law of motion, $F = ma$.

Now let us extend this discussion to a rigid object of arbitrary shape rotating about a fixed axis, as in Figure 10.17. The object can be regarded as an infinite number of mass elements dm of infinitesimal size. If we impose a Cartesian coordinate system on the object, then each mass element rotates in a circle about the origin, and each has a tangential acceleration $\mathbf{a}_t$ produced by an external tangential force $d\mathbf{F}_t$. For any given element, we know from Newton's second law that

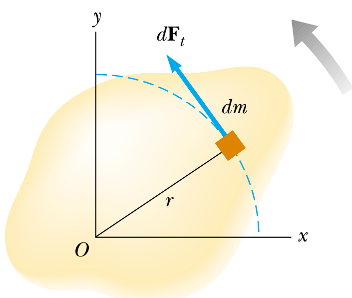


Figure 10.17 A rigid object rotating about an axis through O. Each mass element dm rotates about O with the same angular acceleration α, and the net torque on the object is proportional to α.

$$dF_t = (dm)a_t$$

The torque $d\tau$ associated with the force $d\mathbf{F}_t$ acts about the origin and is given by

$$d\tau = r\,dF_t = a_t r\,dm$$

Because $a_t = r\alpha$, the expression for $d\tau$ becomes

$$d\tau = \alpha r^2\,dm$$

Although each mass element of the rigid object may have a different linear acceleration $\mathbf{a}_t$, they all have the *same* angular acceleration α. With this in mind, we can integrate the above expression to obtain the net torque $\sum\tau$ about O due to the external forces:

$$\sum \tau = \int \alpha r^2\,dm = \alpha \int r^2\,dm$$

where α can be taken outside the integral because it is common to all mass elements. From Equation 10.17, we know that $\int r^2\,dm$ is the moment of inertia of the object about the rotation axis through O, and so the expression for $\sum\tau$ becomes

Torque is proportional to angular acceleration

$$\boxed{\sum \tau = I\alpha} \qquad (10.21)$$

Note that this is the same relationship we found for a particle moving in a circular path (see Eq. 10.20). So, again we see that the net torque about the rotation axis is proportional to the angular acceleration of the object, with the proportionality factor being I, a quantity that depends upon the axis of rotation and upon the size and shape of the object. In view of the complex nature of the system, the relationship $\sum\tau = I\alpha$ is strikingly simple and in complete agreement with experimental observations.

Finally, note that the result $\sum\tau = I\alpha$ also applies when the forces acting on the mass elements have radial components as well as tangential components. This is because the line of action of all radial components must pass through the axis of rotation, and hence all radial components produce zero torque about that axis.

Quick Quiz 10.10 You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is Δt. You replace the bit with a larger one that results in a doubling of the moment of inertia of the entire rotating mechanism of the drill. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. The time for this second bit to come to rest is (a) $4\Delta t$ (b) $2\Delta t$ (c) Δt (d) $0.5\Delta t$ (e) $0.25\Delta t$ (f) impossible to determine.

Example 10.10 Rotating Rod

A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane, as in Figure 10.18. The rod is released from rest in the horizontal position. What is the initial angular acceleration of the rod and the initial linear acceleration of its right end?

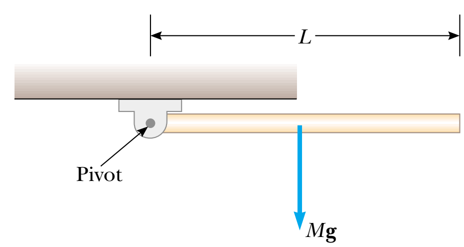


Figure 10.18 (Example 10.10) A rod is free to rotate around a pivot at the left end.

Solution We cannot use our kinematic equations to find α or a because the torque exerted on the rod varies with its angular position and so neither acceleration is constant. We have enough information to find the torque, however, which we can then use in Equation 10.21 to find the initial α and then the initial a.

The only force contributing to the torque about an axis through the pivot is the gravitational force $M\mathbf{g}$ exerted on the rod. (The force exerted by the pivot on the rod has zero torque about the pivot because its moment arm is zero.) To compute the torque on the rod, we assume that the gravitational force acts at the center of mass of the rod, as shown in Figure 10.18. The magnitude of the torque due to this force about an axis through the pivot is

$$\tau = Mg\left(\frac{L}{2}\right)$$

With $\Sigma\tau = I\alpha$, and $I = \frac{1}{3}ML^2$ for this axis of rotation (see Table 10.2), we obtain

$$(1) \qquad \alpha = \frac{\tau}{I} = \frac{Mg(L/2)}{\frac{1}{3}ML^2} = \boxed{\frac{3g}{2L}}$$

All points on the rod have this initial angular acceleration.

To find the initial linear acceleration of the right end of the rod, we use the relationship $a_t = r\alpha$ (Eq. 10.11), with $r = L$:

$$a_t = L\alpha = \boxed{\tfrac{3}{2}g}$$

What If? **What if we were to place a penny on the end of the rod and release the rod? Would the penny stay in contact with the rod?**

Answer The result for the initial acceleration of a point on the end of the rod shows that $a_t > g$. A penny will fall at acceleration g. This means that if we place a penny at the end of the rod and then release the rod, the end of the rod falls faster than the penny does! The penny does not stay in contact with the rod. (Try this with a penny and a meter stick!)

This raises the question as to the location on the rod at which we can place a penny that *will* stay in contact as both begin to fall. To find the linear acceleration of an arbitrary point on the rod at a distance $r < L$ from the pivot point, we combine (1) with Equation 10.11:

$$a_t = r\alpha = \frac{3g}{2L}r$$

For the penny to stay in contact with the rod, the limiting case is that the linear acceleration must be equal to that due to gravity:

$$a_t = g = \frac{3g}{2L}r$$

$$r = \tfrac{2}{3}L$$

Thus, a penny placed closer to the pivot than two thirds of the length of the rod will stay in contact with the falling rod while a penny farther out than this point will lose contact.

Conceptual Example 10.11 Falling Smokestacks and Tumbling Blocks

When a tall smokestack falls over, it often breaks somewhere along its length before it hits the ground, as shown in Figure 10.19. The same thing happens with a tall tower of children's toy blocks. Why does this happen?

Solution As the smokestack rotates around its base, each higher portion of the smokestack falls with a larger tangential acceleration than the portion below it. (The tangential acceleration of a given point on the smokestack is proportional to the distance of that portion from the base.) As the angular acceleration increases as the smokestack tips farther, higher portions of the smokestack experience an acceleration greater than that which could result from gravity alone; this is similar to the situation described in Example 10.10. This can happen only if these portions are being pulled downward by a force in addition to the gravitational force. The force that causes this to occur is the shear force from lower portions of the smokestack. Eventually the shear force that provides this acceleration is greater than the smokestack can withstand, and the smokestack breaks.

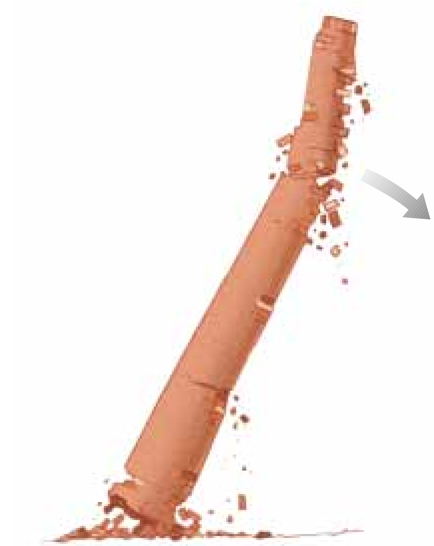


Figure 10.19 (Conceptual Example 10.11) A falling smokestack breaks at some point along its length.

Example 10.12 Angular Acceleration of a Wheel

Interactive

A wheel of radius R, mass M, and moment of inertia I is mounted on a frictionless horizontal axle, as in Figure 10.20. A light cord wrapped around the wheel supports an object of mass m. Calculate the angular acceleration of the wheel, the linear acceleration of the object, and the tension in the cord.

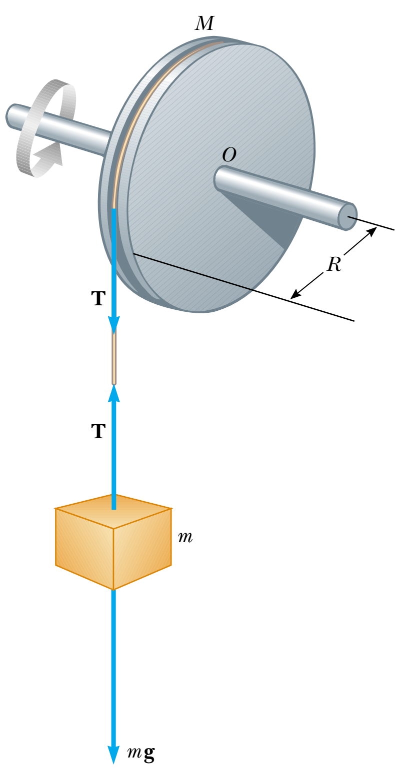


Figure 10.20 (Example 10.12) An object hangs from a cord wrapped around a wheel.

Solution The magnitude of the torque acting on the wheel about its axis of rotation is $\tau = TR$, where T is the force exerted by the cord on the rim of the wheel. (The gravitational force exerted by the Earth on the wheel and the normal force exerted by the axle on the wheel both pass through the axis of rotation and thus produce no torque.) Because $\sum \tau = I\alpha$, we obtain

$$\sum \tau = I\alpha = TR$$

$$(1) \qquad \alpha = \frac{TR}{I}$$

Now let us apply Newton's second law to the motion of the object, taking the downward direction to be positive:

$$\sum F_y = mg - T = ma$$

$$(2) \qquad a = \frac{mg - T}{m}$$

Equations (1) and (2) have three unknowns: α, a, and T. Because the object and wheel are connected by a cord that does not slip, the linear acceleration of the suspended object is equal to the tangential acceleration of a point on the rim of the wheel. Therefore, the angular acceleration α of the wheel and the linear acceleration of the object are related by $a = R\alpha$. Using this fact together with Equations (1) and (2), we obtain

$$(3) \qquad a = R\alpha = \frac{TR^2}{I} = \frac{mg - T}{m}$$

$$(4) \qquad T = \boxed{\frac{mg}{1 + (mR^2/I)}}$$

Substituting Equation (4) into Equation (2) and solving for a and α, we find that

$$(5) \qquad a = \boxed{\frac{g}{1 + (I/mR^2)}}$$

$$\alpha = \frac{a}{R} = \boxed{\frac{g}{R + (I/mR)}}$$

What If? **What if the wheel were to become very massive so that *I* becomes very large? What happens to the acceleration *a* of the object and the tension *T*?**

Answer If the wheel becomes infinitely massive, we can imagine that the object of mass m will simply hang from the cord without causing the wheel to rotate.

We can show this mathematically by taking the limit $I \rightarrow \infty$, so that Equation (5) becomes

$$a = \frac{g}{1 + (I/mR^2)} \longrightarrow 0$$

This agrees with our conceptual conclusion that the object will hang at rest. We also find that Equation (4) becomes

$$T = \frac{mg}{1 + (mR^2/I)} \longrightarrow \frac{mg}{1 + 0} = mg$$

This is consistent with the fact that the object simply hangs at rest in equilibrium between the gravitational force and the tension in the string.

At the Interactive Worked Example link at **http://www.pse6.com**, *you can change the masses of the object and the wheel as well as the radius of the wheel to see the effect on how the system moves.*

Example 10.13 Atwood's Machine Revisited

Interactive

Two blocks having masses m_1 and m_2 are connected to each other by a light cord that passes over two identical frictionless pulleys, each having a moment of inertia I and radius R, as shown in Figure 10.21a. Find the acceleration of each block and the tensions T_1, T_2, and T_3 in the cord. (Assume no slipping between cord and pulleys.)

Solution Compare this situation with the Atwood machine of Example 5.9 (p. 129). The motion of m_1 and m_2 is similar to the motion of the two blocks in that example. The primary differences are that in the present example we have two pulleys and each of the pulleys has mass. Despite these differences, the apparatus in the present example is indeed an Atwood machine.

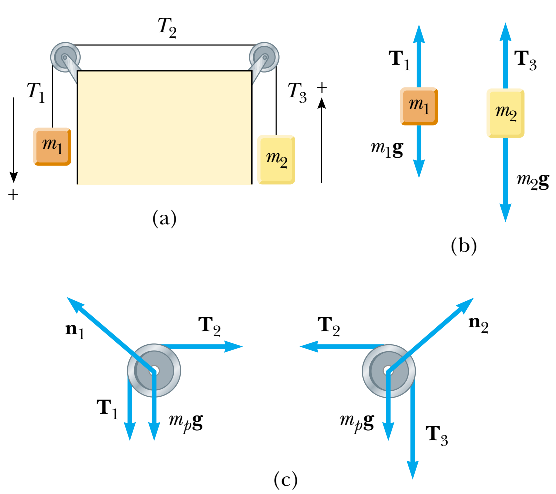


Figure 10.21 (Example 10.13) (a) Another look at Atwood's machine. (b) Free-body diagrams for the blocks. (c) Free-body diagrams for the pulleys, where $m_p\mathbf{g}$ represents the gravitational force acting on each pulley.

We shall define the downward direction as positive for m_1 and upward as the positive direction for m_2. This allows us to represent the acceleration of both masses by a single variable a and also enables us to relate a positive a to a positive (counterclockwise) angular acceleration α of the pulleys. Let us write Newton's second law of motion for each block, using the free-body diagrams for the two blocks as shown in Figure 10.21b:

$$(1) \qquad m_1g - T_1 = m_1a$$

$$(2) \qquad T_3 - m_2g = m_2a$$

Next, we must include the effect of the pulleys on the motion. Free-body diagrams for the pulleys are shown in Figure 10.21c. The net torque about the axle for the pulley on the left is $(T_1 - T_2)R$, while the net torque for the pulley on the right is $(T_2 - T_3)R$. Using the relation $\Sigma\tau = I\alpha$ for each pulley and noting that each pulley has the same angular acceleration α, we obtain

$$(3) \qquad (T_1 - T_2)R = I\alpha$$

$$(4) \qquad (T_2 - T_3)R = I\alpha$$

We now have four equations with five unknowns: α, a, T_1, T_2, and T_3. We also have a fifth equation that relates the accelerations, $a = R\alpha$. These equations can be solved simultaneously. Adding Equations (3) and (4) gives

$$(5) \qquad (T_1 - T_3)R = 2I\alpha$$

Adding Equations (1) and (2) gives

$$T_3 - T_1 + m_1g - m_2g = (m_1 + m_2)a$$

$$(6) \qquad T_1 - T_3 = (m_1 - m_2)g - (m_1 + m_2)a$$

Substituting Equation (6) into Equation (5), we have

$$[(m_1 - m_2)g - (m_1 + m_2)a]R = 2I\alpha$$

Because $\alpha = a/R$, this expression can be simplified to

$$(m_1 - m_2)g - (m_1 + m_2)a = 2I\frac{a}{R^2}$$

$$(7) \qquad a = \boxed{\frac{(m_1 - m_2)g}{m_1 + m_2 + 2(I/R^2)}}$$

Note that if $m_1 > m_2$, the acceleration is positive; this means that the left block accelerates downward, the right block accelerates upward, and both pulleys accelerate counterclockwise. If $m_1 < m_2$, the acceleration is negative and the motions are reversed. If $m_1 = m_2$, no acceleration occurs at all. You should compare these results with those found in Example 5.9.

The expression for a can be substituted into Equations (1) and (2) to give T_1 and T_3. From Equation (1),

$$\begin{aligned} T_1 &= m_1g - m_1a = m_1(g - a) \\ &= m_1\left(g - \frac{(m_1 - m_2)g}{m_1 + m_2 + 2(I/R^2)}\right) \\ &= 2m_1g\left(\frac{m_2 + (I/R^2)}{m_1 + m_2 + 2(I/R^2)}\right) \end{aligned}$$

Similarly, from Equation (2),

$$T_3 = m_2g + m_2a = 2m_2g\left(\frac{m_1 + (I/R^2)}{m_1 + m_2 + 2(I/R^2)}\right)$$

Finally, T_2 can be found from Equation (3):

$$\begin{aligned} T_2 &= T_1 - \frac{I\alpha}{R} = T_1 - \frac{Ia}{R^2} \\ &= 2m_1g\left(\frac{m_2 + (I/R^2)}{m_1 + m_2 + 2(I/R^2)}\right) \\ &\quad - \frac{I}{R^2}\left(\frac{(m_1 - m_2)g}{m_1 + m_2 + 2(I/R^2)}\right) \\ &= \frac{2m_1m_2 + (m_1 + m_2)(I/R^2)}{m_1 + m_2 + 2(I/R^2)}\, g \end{aligned}$$

What If? **What if the pulleys become massless? Does this reduce to a previously solved problem?**

Answer If the pulleys become massless, the system should behave in the same way as the massless-pulley Atwood machine that we investigated in Example 5.9. The only difference is the existence of two pulleys instead of one.

Mathematically, if $I \rightarrow 0$, Equation (7) becomes

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + 2(I/R^2)} \quad \longrightarrow \quad a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

which is the same result as Equation (3) in Example 5.9. Although the expressions for the three tensions in the present example are different from each other, all three expressions become, in the limit $I \rightarrow 0$,

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

which is the same as Equation (4) in Example 5.9.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can change the masses of the blocks and the pulleys to see the effect on the motion of the system.*

10.8 Work, Power, and Energy in Rotational Motion

Up to this point in our discussion of rotational motion in this chapter, we focused on an approach involving force, leading to a description of torque on a rigid object. We now see how an energy approach can be useful to us in solving rotational problems.

We begin by considering the relationship between the torque acting on a rigid object and its resulting rotational motion in order to generate expressions for power and a rotational analog to the work–kinetic energy theorem. Consider the rigid object pivoted at O in Figure 10.22. Suppose a single external force $\mathbf{F}$ is applied at P, where $\mathbf{F}$ lies in the plane of the page. The work done by $\mathbf{F}$ on the object as it rotates through an infinitesimal distance $ds = r\,d\theta$ is

$$dW = \mathbf{F} \cdot d\mathbf{s} = (F \sin\phi)\, r\, d\theta$$

where $F\sin\phi$ is the tangential component of $\mathbf{F}$, or, in other words, the component of the force along the displacement. Note that *the radial component of* $\mathbf{F}$ *does no work because it is perpendicular to the displacement.*

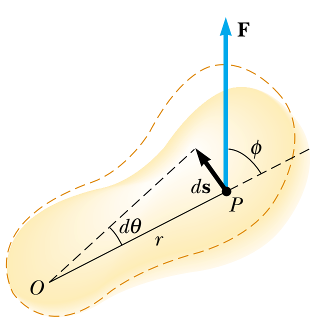


Figure 10.22 A rigid object rotates about an axis through O under the action of an external force $\mathbf{F}$ applied at P.

Because the magnitude of the torque due to **F** about O is defined as $rF\sin\phi$ by Equation 10.19, we can write the work done for the infinitesimal rotation as

$$dW = \tau\, d\theta \tag{10.22}$$

The rate at which work is being done by **F** as the object rotates about the fixed axis through the angle $d\theta$ in a time interval dt is

$$\frac{dW}{dt} = \tau\,\frac{d\theta}{dt}$$

Because dW/dt is the instantaneous power $\mathcal{P}$ (see Section 7.8) delivered by the force and $d\theta/dt = \omega$, this expression reduces to

$$\mathcal{P} = \frac{dW}{dt} = \tau\omega \tag{10.23}$$

Power delivered to a rotating rigid object

This expression is analogous to $\mathcal{P} = Fv$ in the case of linear motion, and the expression $dW = \tau\, d\theta$ is analogous to $dW = F_x\, dx$.

In studying linear motion, we found the energy approach extremely useful in describing the motion of a system. From what we learned of linear motion, we expect that when a symmetric object rotates about a fixed axis, the work done by external forces equals the change in the rotational energy.

To show that this is in fact the case, let us begin with $\Sigma\tau = I\alpha$. Using the chain rule from calculus, we can express the resultant torque as

$$\sum \tau = I\alpha = I\,\frac{d\omega}{dt} = I\,\frac{d\omega}{d\theta}\,\frac{d\theta}{dt} = I\,\frac{d\omega}{d\theta}\,\omega$$

Rearranging this expression and noting that $\Sigma\tau\, d\theta = dW$, we obtain

$$\sum \tau\, d\theta = dW = I\omega\, d\omega$$

Integrating this expression, we obtain for the total work done by the net external force acting on a rotating system

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \tfrac{1}{2}I\omega_f^{\,2} - \tfrac{1}{2}I\omega_i^{\,2} \tag{10.24}$$

Work–kinetic energy theorem for rotational motion

where the angular speed changes from ω_i to ω_f. That is, the **work–kinetic energy theorem for rotational motion** states that

> the net work done by external forces in rotating a symmetric rigid object about a fixed axis equals the change in the object's rotational energy.

In general, then, combining this with the translational form of the work–kinetic energy theorem from Chapter 7, the net work done by external forces on an object is the change in its *total* kinetic energy, which is the sum of the translational and rotational kinetic energies. For example, when a pitcher throws a baseball, the work done by the pitcher's hands appears as kinetic energy associated with the ball moving through space as well as rotational kinetic energy associated with the spinning of the ball.

In addition to the work–kinetic energy theorem, other energy principles can also be applied to rotational situations. For example, if a system involving rotating objects is isolated, the principle of conservation of energy can be used to analyze the system, as in Example 10.14 below.

Table 10.3 lists the various equations we have discussed pertaining to rotational motion, together with the analogous expressions for linear motion. The last two equations in Table 10.3, involving angular momentum L, are discussed in Chapter 11 and are included here only for the sake of completeness.

Table 10.3

Useful Equations in Rotational and Linear Motion

Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If α = constant $\begin{cases} \omega_f = \omega_i + \alpha_t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If a = constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Work $W = \int_{x_i}^{x_f} F_x\, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

Quick Quiz 10.11 A rod is attached to the shaft of a motor at the center of the rod so that the rod is perpendicular to the shaft, as in Figure 10.23a. The motor is turned on and performs work W on the rod, accelerating it to an angular speed ω. The system is brought to rest, and the rod is attached to the shaft of the motor at one end of the rod as in Figure 10.23b. The motor is turned on and performs work W on the rod. The angular speed of the rod in the second situation is (a) 4ω (b) 2ω (c) ω (d) 0.5ω (e) 0.25ω (f) impossible to determine.

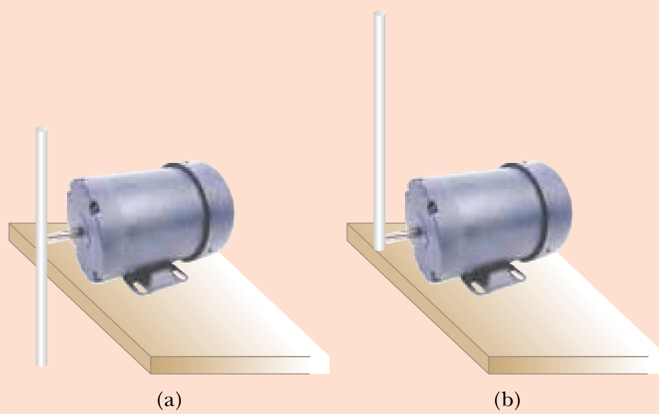


Figure 10.23 (Quick Quiz 10.11) (a) A rod is rotated about its midpoint by a motor. (b) The rod is rotated about one of its ends.

Example 10.14 Rotating Rod Revisited

Interactive

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end (Fig 10.24). The rod is released from rest in the horizontal position.

(A) What is its angular speed when it reaches its lowest position?

Solution To conceptualize this problem, consider Figure 10.24 and imagine the rod rotating downward through a quarter turn about the pivot at the left end. In this situation, the angular acceleration of the rod is not constant. Thus, the kinematic equations for rotation (Section 10.2) cannot be used to solve this problem. As we found with translational motion, however, an energy approach can make such a seemingly insoluble problem relatively easy. We categorize this as a conservation of energy problem.

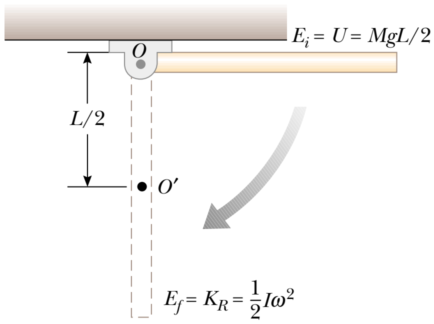


Figure 10.24 (Example 10.14) A uniform rigid rod pivoted at O rotates in a vertical plane under the action of the gravitational force.

To analyze the problem, we consider the mechanical energy of the system of the rod and the Earth. We choose the configuration in which the rod is hanging straight down as the reference configuration for gravitational potential energy and assign a value of zero for this configuration. When the rod is in the horizontal position, it has no rotational kinetic energy. The potential energy of the system in this configuration relative to the reference configuration is $MgL/2$ because the center of mass of the rod is at a height $L/2$ higher than its position in the reference configuration. When the rod reaches its lowest position, the energy is entirely rotational energy $\frac{1}{2}I\omega^2$, where I is the moment of inertia about the pivot, and the potential energy of the system is zero. Because $I = \frac{1}{3}ML^2$ (see Table 10.2) and because the system is isolated with no nonconservative forces acting, we apply conservation of mechanical energy for the system:

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}I\omega^2 + 0 = \tfrac{1}{2}(\tfrac{1}{3}ML^2)\omega^2 = 0 + \tfrac{1}{2}MgL$$

$$\omega = \boxed{\sqrt{\frac{3g}{L}}}$$

(B) Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

Solution These two values can be determined from the relationship between tangential and angular speeds. We know ω from part (A), and so the tangential speed of the center of mass is

$$v_{\text{CM}} = r\omega = \frac{L}{2}\,\omega = \boxed{\tfrac{1}{2}\sqrt{3gL}}$$

Because r for the lowest point on the rod is twice what it is for the center of mass, the lowest point has a tangential speed v equal to

$$v = 2v_{\text{CM}} = \boxed{\sqrt{3gL}}$$

To finalize this problem, note that the initial configuration in this example is the same as that in Example 10.10. In Example 10.10, however, we could only find the initial angular acceleration of the rod. We cannot use this and the kinematic equations to find the angular speed of the rod at its lowest point because the angular acceleration is not constant. Applying an energy approach in the current example allows us to find something that we cannot in Example 10.10.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can alter the mass and length of the rod and see the effect on the velocity at the lowest point.*

Example 10.15 Energy and the Atwood Machine

Consider two cylinders having different masses m_1 and m_2, connected by a string passing over a pulley, as shown in Figure 10.25. The pulley has a radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley, and the system is released from rest. Find the linear speeds of the cylinders after cylinder 2 descends through a distance h, and the angular speed of the pulley at this time.

Solution We will solve this problem by applying energy methods to an Atwood machine with a massive pulley. Because the string does not slip, the pulley rotates about the axle. We can neglect friction in the axle because the axle's radius is small relative to that of the pulley, so the frictional torque is much smaller than the torque applied by the two cylinders, provided that their masses are quite different. Consequently, the system consisting of the two cylinders, the pulley, and the Earth is isolated with no nonconservative forces acting; thus, the mechanical energy of the system is conserved.

We define the zero configuration for gravitational potential energy as that which exists when the system is released. From Figure 10.25, we see that the descent of cylinder 2 is associated with a decrease in system potential energy and the rise of cylinder 1 represents an increase in

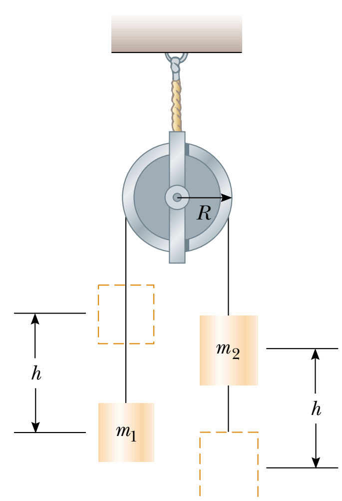


Figure 10.25 (Example 10.15) An Atwood machine.

potential energy. Because $K_i = 0$ (the system is initially at rest), we have

$$K_f + U_f = K_i + U_i$$

$$(\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}I\omega_f^2) + (m_1gh - m_2gh) = 0 + 0$$

where v_f is the same for both blocks. Because $v_f = R\omega_f$, this expression becomes

$$\left(\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}\frac{I}{R^2}v_f^2\right) = (m_2gh - m_1gh)$$

$$\tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v_f^2 = (m_2gh - m_1gh)$$

Solving for v_f, we find

$$v_f = \boxed{\left[\frac{2(m_2 - m_1)gh}{[m_1 + m_2 + (I/R^2)]}\right]^{1/2}}$$

The angular speed of the pulley at this instant is

$$\omega_f = \frac{v_f}{R} = \boxed{\frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{(m_1 + m_2 + (I/R^2))}\right]^{1/2}}$$

10.9 Rolling Motion of a Rigid Object

In this section we treat the motion of a rigid object rolling along a flat surface. In general, such motion is very complex. Suppose, for example, that a cylinder is rolling on a straight path such that the axis of rotation remains parallel to its initial orientation in space. As Figure 10.26 shows, a point on the rim of the cylinder moves in a complex path called a *cycloid.* However, we can simplify matters by focusing on the center of mass rather than on a point on the rim of the rolling object. As we see in Figure 10.26, the center of mass moves in a straight line. If an object such as a cylinder rolls without slipping on the surface (we call this *pure rolling motion*), we can show that a simple relationship exists between its rotational and translational motions.

Consider a uniform cylinder of radius R rolling without slipping on a horizontal surface (Fig. 10.27). As the cylinder rotates through an angle θ, its center of mass moves a linear distance $s = R\theta$ (see Eq. 10.1a). Therefore, the linear speed of the center of mass for pure rolling motion is given by

$$v_{\text{CM}} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega \qquad (10.25)$$

where ω is the angular speed of the cylinder. Equation 10.25 holds whenever a cylinder or sphere rolls without slipping and is the **condition for pure rolling motion.**

PITFALL PREVENTION

10.6 Equation 10.25 Looks Familiar

Equation 10.25 looks very similar to Equation 10.10, so be sure that you are clear on the difference. Equation 10.10 gives the *tangential* speed of a point on a *rotating* object located a distance r from the rotation axis if the object is rotating with angular speed ω. Equation 10.25 gives the *translational* speed of the center of mass of a *rolling* object of radius R rotating with angular speed ω.


Henry Leap and Jim Lehman

Figure 10.26 One light source at the center of a rolling cylinder and another at one point on the rim illustrate the different paths these two points take. The center moves in a straight line (green line), while the point on the rim moves in the path called a cycloid (red curve).

The **instantaneous angular speed** of a particle moving in a circular path or of a rigid object rotating about a fixed axis is

$$\omega \equiv \frac{d\theta}{dt} \tag{10.3}$$

The **instantaneous angular acceleration** of a particle moving in a circular path or a rotating rigid object is

$$\alpha \equiv \frac{d\omega}{dt} \tag{10.5}$$

When a rigid object rotates about a fixed axis, every part of the object has the same angular speed and the same angular acceleration.

If an object rotates about a fixed axis under constant angular acceleration, one can apply equations of kinematics that are analogous to those for linear motion under constant linear acceleration:

$$\omega_f = \omega_i + \alpha t \tag{10.6}$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2 \tag{10.7}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \tag{10.8}$$

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)\, t \tag{10.9}$$

A useful technique in solving problems dealing with rotation is to visualize a linear version of the same problem.

When a rigid object rotates about a fixed axis, the angular position, angular speed, and angular acceleration are related to the linear position, linear speed, and linear acceleration through the relationships

$$s = r\theta \tag{10.1a}$$

$$v = r\omega \tag{10.10}$$

$$a_t = r\alpha \tag{10.11}$$

The **moment of inertia of a system of particles** is defined as

$$I \equiv \sum_i m_i r_i^2 \tag{10.15}$$

If a rigid object rotates about a fixed axis with angular speed ω, its **rotational kinetic energy** can be written

$$K_R = \tfrac{1}{2} I\omega^2 \tag{10.16}$$

where I is the moment of inertia about the axis of rotation.

The **moment of inertia of a rigid object** is

$$I = \int r^2\, dm \tag{10.17}$$

where r is the distance from the mass element dm to the axis of rotation.

The magnitude of the **torque** associated with a force **F** acting on an object is

$$\tau = Fd \tag{10.19}$$

where d is the moment arm of the force, which is the perpendicular distance from the rotation axis to the line of action of the force. Torque is a measure of the tendency of the force to change the rotation of the object about some axis.

If a rigid object free to rotate about a fixed axis has a **net external torque** acting on it, the object undergoes an angular acceleration α, where

$$\sum \tau = I\alpha \tag{10.21}$$

The rate at which work is done by an external force in rotating a rigid object about a fixed axis, or the **power** delivered, is

$$\mathcal{P} = \tau\omega \tag{10.23}$$

If work is done on a rigid object and the only result of the work is rotation about a fixed axis, the net work done by external forces in rotating the object equals the change in the rotational kinetic energy of the object:

$$\sum W = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 \tag{10.24}$$

The **total kinetic energy** of a rigid object rolling on a rough surface without slipping equals the rotational kinetic energy about its center of mass, $\frac{1}{2}I_{CM}\omega^2$, plus the translational kinetic energy of the center of mass, $\frac{1}{2}Mv_{CM}{}^2$:

$$K = \tfrac{1}{2}I_{CM}\omega^2 + \tfrac{1}{2}Mv_{CM}{}^2 \tag{10.28}$$

QUESTIONS

1. What is the angular speed of the second hand of a clock? What is the direction of $\boldsymbol{\omega}$ as you view a clock hanging on a vertical wall? What is the magnitude of the angular acceleration vector $\boldsymbol{\alpha}$ of the second hand?
2. One blade of a pair of scissors rotates counterclockwise in the xy plane. What is the direction of $\boldsymbol{\omega}$? What is the direction of $\boldsymbol{\alpha}$ if the magnitude of the angular velocity is decreasing in time?
3. Are the kinematic expressions for θ, ω, and α valid when the angular position is measured in degrees instead of in radians?
4. If a car's standard tires are replaced with tires of larger outside diameter, will the reading of the speedometer change? Explain.
5. Suppose $a = b$ and $M > m$ for the system of particles described in Figure 10.8. About which axis (x, y, or z) does the moment of inertia have the smallest value? the largest value?
6. Suppose that the rod in Figure 10.10 has a nonuniform mass distribution. In general, would the moment of inertia about the y axis still be equal to $ML^2/12$? If not, could the moment of inertia be calculated without knowledge of the manner in which the mass is distributed?
7. Suppose that just two external forces act on a stationary rigid object and the two forces are equal in magnitude and opposite in direction. Under what condition does the object start to rotate?
8. Suppose a pencil is balanced on a perfectly frictionless table. If it falls over, what is the path followed by the center of mass of the pencil?
9. Explain how you might use the apparatus described in Example 10.12 to determine the moment of inertia of the wheel. (If the wheel does not have a uniform mass density, the moment of inertia is not necessarily equal to $\frac{1}{2}MR^2$.)
10. Using the results from Example 10.12, how would you calculate the angular speed of the wheel and the linear speed of the suspended counterweight at $t = 2$ s, if the system is released from rest at $t = 0$? Is the expression $v = R\omega$ valid in this situation?
11. If a small sphere of mass M were placed at the end of the rod in Figure 10.24, would the result for ω be greater than, less than, or equal to the value obtained in Example 10.14?
12. Explain why changing the axis of rotation of an object changes its moment of inertia.
13. The moment of inertia of an object depends on the choice of rotation axis, as suggested by the parallel-axis theorem. Argue that an axis passing through the center of mass of an object must be the axis with the smallest moment of inertia.
14. Suppose you remove two eggs from the refrigerator, one hard-boiled and the other uncooked. You wish to determine which is the hard-boiled egg without breaking the eggs. This can be done by spinning the two eggs on the floor and comparing the rotational motions. Which egg spins faster? Which rotates more uniformly? Explain.
15. Which of the entries in Table 10.2 applies to finding the moment of inertia of a long straight sewer pipe rotating about its axis of symmetry? Of an embroidery hoop rotating about an axis through its center and perpendicular to its plane? Of a uniform door turning on its hinges? Of a coin turning about an axis through its center and perpendicular to its faces?
16. Is it possible to change the translational kinetic energy of an object without changing its rotational energy?
17. Must an object be rotating to have a nonzero moment of inertia?
18. If you see an object rotating, is there necessarily a net torque acting on it?
19. Can a (momentarily) stationary object have a nonzero angular acceleration?
20. In a tape recorder, the tape is pulled past the read-and-write heads at a constant speed by the drive mechanism. Consider the reel from which the tape is pulled. As the tape is pulled from it, the radius of the roll of remaining tape decreases. How does the torque on the reel change with time? How does the angular speed of the reel change in time? If the drive mechanism is switched on so that the

tape is suddenly jerked with a large force, is the tape more likely to break when it is being pulled from a nearly full reel or from a nearly empty reel?

21. The polar diameter of the Earth is slightly less than the equatorial diameter. How would the moment of inertia of the Earth about its axis of rotation change if some mass from near the equator were removed and transferred to the polar regions to make the Earth a perfect sphere?

22. Suppose you set your textbook sliding across a gymnasium floor with a certain initial speed. It quickly stops moving because of a friction force exerted on it by the floor. Next, you start a basketball rolling with the same initial speed. It keeps rolling from one end of the gym to the other. Why does the basketball roll so far? Does friction significantly affect its motion?

23. When a cylinder rolls on a horizontal surface as in Figure 10.28, do any points on the cylinder have only a vertical component of velocity at some instant? If so, where are they?

24. Three objects of uniform density—a solid sphere, a solid cylinder, and a hollow cylinder—are placed at the top of an incline (Fig. Q10.24). They are all released from rest at the same elevation and roll without slipping. Which object reaches the bottom first? Which reaches it last? Try this at home and note that the result is independent of the masses and the radii of the objects.

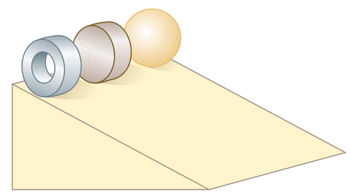


Figure Q10.24 Which object wins the race?

25. In a soap-box derby race, the cars have no engines; they simply coast down a hill to race with one another. Suppose you are designing a car for a coasting race. Do you want to use large wheels or small wheels? Do you want to use solid disk-like wheels or hoop-like wheels? Should the wheels be heavy or light?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 10.1 Angular Position, Velocity, and Acceleration

1. During a certain period of time, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at $t = 0$ (b) at $t = 3.00$ s.

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

2. A dentist's drill starts from rest. After 3.20 s of constant angular acceleration, it turns at a rate of 2.51×10^4 rev/min. (a) Find the drill's angular acceleration. (b) Determine the angle (in radians) through which the drill rotates during this period.

3. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.

4. An airliner arrives at the terminal, and the engines are shut off. The rotor of one of the engines has an initial clockwise angular speed of 2 000 rad/s. The engine's rotation slows with an angular acceleration of magnitude 80.0 rad/s^2. (a) Determine the angular speed after 10.0 s. (b) How long does it take the rotor to come to rest?

5. An electric motor rotating a grinding wheel at 100 rev/min is switched off. With constant negative angular acceleration of magnitude 2.00 rad/s^2, (a) how long does it take the wheel to stop? (b) Through how many radians does it turn while it is slowing down?

6. A centrifuge in a medical laboratory rotates at an angular speed of 3 600 rev/min. When switched off, it rotates 50.0 times before coming to rest. Find the constant angular acceleration of the centrifuge.

7. The tub of a washer goes into its spin cycle, starting from rest and gaining angular speed steadily for 8.00 s, at which time it is turning at 5.00 rev/s. At this point the person doing the laundry opens the lid, and a safety switch turns off the washer. The tub smoothly slows to rest in 12.0 s. Through how many revolutions does the tub turn while it is in motion?

8. A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel?

9. (a) Find the angular speed of the Earth's rotation on its axis. As the Earth turns toward the east, we see the sky turning toward the west at this same rate.

(b) *The rainy Pleiads wester*
And seek beyond the sea
The head that I shall dream of
That shall not dream of me.

–A. E. Housman (© Robert E. Symons)

Cambridge, England, is at longitude 0°, and Saskatoon, Saskatchewan, is at longitude 107° west. How much time elapses after the Pleiades set in Cambridge until these stars fall below the western horizon in Saskatoon?

10. A merry-go-round is stationary. A dog is running on the ground just outside its circumference, moving with a constant angular speed of 0.750 rad/s. The dog does not change his pace when he sees what he has been looking for: a bone resting on the edge of the merry-go-round one third of a revolution in front of him. At the instant the dog sees the bone ($t = 0$), the merry-go-round begins to move in the direction the dog is running, with a constant angular acceleration of 0.015 0 rad/s^2. (a) At what time will the dog reach the bone? (b) The confused dog keeps running and passes the bone. How long after the merry-go-round starts to turn do the dog and the bone draw even with each other for the second time?

Section 10.3 Angular and Linear Quantities

11. Make an order-of-magnitude estimate of the number of revolutions through which a typical automobile tire turns in 1 yr. State the quantities you measure or estimate and their values.

12. A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

13. A wheel 2.00 m in diameter lies in a vertical plane and rotates with a constant angular acceleration of 4.00 rad/s^2. The wheel starts at rest at $t = 0$, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time. At $t = 2.00$ s, find (a) the angular speed of the wheel, (b) the tangential speed and the total acceleration of the point P, and (c) the angular position of the point P.

14. Figure P10.14 shows the drive train of a bicycle that has wheels 67.3 cm in diameter and pedal cranks 17.5 cm long. The cyclist pedals at a steady angular rate of 76.0 rev/min. The chain engages with a front sprocket 15.2 cm in diameter and a rear sprocket 7.00 cm in diameter. (a) Calculate the speed of a link of the chain relative to the bicycle frame. (b) Calculate the angular speed of the bicycle wheels. (c) Calculate the speed of the bicycle relative to the road. (d) What pieces of data, if any, are not necessary for the calculations?

Figure P10.14

15. A discus thrower (Fig. P10.15) accelerates a discus from rest to a speed of 25.0 m/s by whirling it through 1.25 rev. Assume the discus moves on the arc of a circle 1.00 m in radius. (a) Calculate the final angular speed of the discus. (b) Determine the magnitude of the angular acceleration of the discus, assuming it to be constant. (c) Calculate the time interval required for the discus to accelerate from rest to 25.0 m/s.

Bruce Ayers/Stone/Getty

Figure P10.15

16. A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of a tire is 58.0 cm, find (a) the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

17. A disk 8.00 cm in radius rotates at a constant rate of 1 200 rev/min about its central axis. Determine (a) its angular speed, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.

18. A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of 1.70 m/s^2. The car makes it one quarter of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.

19. Consider a tall building located on the Earth's equator. As the Earth rotates, a person on the top floor of the building moves faster than someone on the ground with respect to an inertial reference frame, because the latter person is closer to the Earth's axis. Consequently, if an object is dropped from the top floor to the ground a distance h below, it lands east of the point vertically below where it was dropped. (a) How far to the east will the object land? Express your answer in terms of h, g, and the angular speed ω of the Earth. Neglect air resistance, and assume that the free-fall acceleration is constant over this range of heights. (b) Evaluate the eastward displacement for $h = 50.0$ m. (c) In your judgment, were we justified in ignoring this aspect of the *Coriolis effect* in our previous study of free fall?

Section 10.4 Rotational Kinetic Energy

20. Rigid rods of negligible mass lying along the y axis connect three particles (Fig. P10.20). If the system rotates about the x axis with an angular speed of 2.00 rad/s, find (a) the moment of inertia about the x axis and the total rotational kinetic energy evaluated from $\frac{1}{2}I\omega^2$ and (b) the tangential speed of each particle and the total kinetic energy evaluated from $\Sigma\frac{1}{2}m_iv_i^2$.

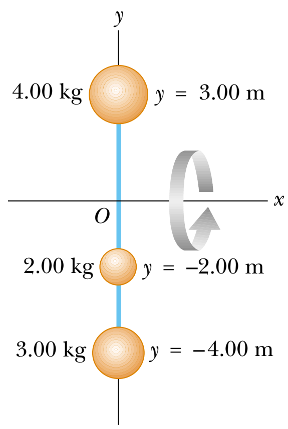


Figure P10.20

21. The four particles in Figure P10.21 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate (a) the moment of inertia of the system about the z axis and (b) the rotational kinetic energy of the system.

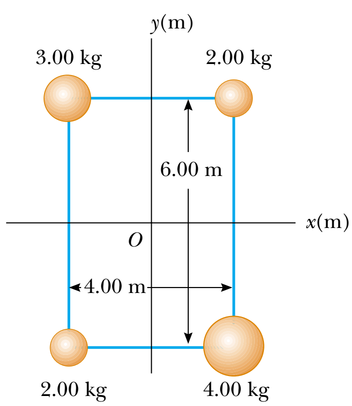


Figure P10.21

22. Two balls with masses M and m are connected by a rigid rod of length L and negligible mass as in Figure P10.22. For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the axis passes through the center of mass. Show that this moment of inertia is $I = \mu L^2$, where $\mu = mM/(m + M)$.

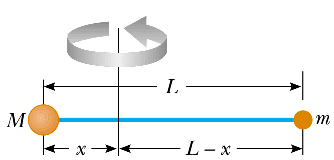


Figure P10.22

Section 10.5 Calculation of Moments of Inertia

23. Three identical thin rods, each of length L and mass m, are welded perpendicular to one another as shown in Figure P10.23. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Determine the moment of inertia of this structure.

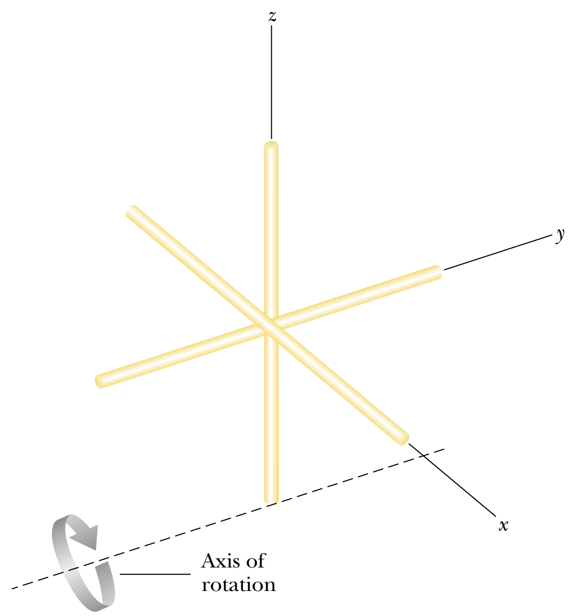


Figure P10.23

24. Figure P10.24 shows a side view of a car tire. Model it as having two sidewalls of uniform thickness 0.635 cm and a tread wall of uniform thickness 2.50 cm and width 20.0 cm. Assume the rubber has uniform density 1.10×10^3 kg/m^3. Find its moment of inertia about an axis through its center.

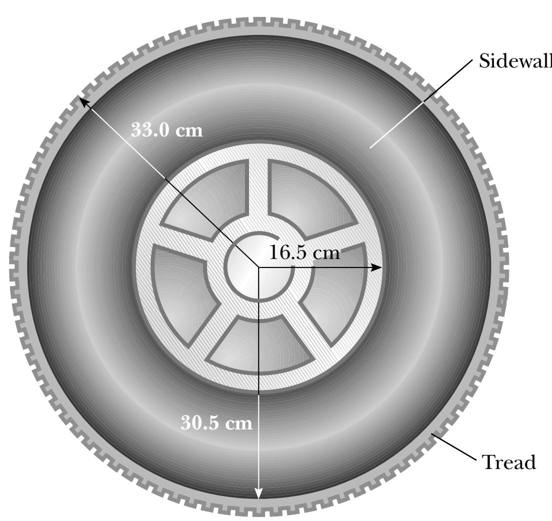


Figure P10.24

25. A uniform thin solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. Find its moment of inertia for rotation on its hinges. Is any piece of data unnecessary?

26. *Attention! About face!* Compute an order-of-magnitude estimate for the moment of inertia of your body as you stand tall and turn about a vertical axis through the top of your head and the point halfway between your ankles. In your solution state the quantities you measure or estimate and their values.

27. The density of the Earth, at any distance r from its center, is approximately

$$\rho = [14.2 - 11.6(r/R)] \times 10^3 \text{ kg/m}^3$$

where R is the radius of the Earth. Show that this density leads to a moment of inertia $I = 0.330MR^2$ about an axis through the center, where M is the mass of the Earth.

28. Calculate the moment of inertia of a thin plate, in the shape of a right triangle, about an axis that passes through one end of the hypotenuse and is parallel to the opposite leg of the triangle, as in Figure P10.28a. Let M represent the mass of the triangle and L the length of the base of the triangle perpendicular to the axis of rotation. Let h represent the height of the triangle and w the thickness of the plate, much smaller than L or h. Do the calculation in either or both of the following ways, as your instructor assigns:

(a) Use Equation 10.17. Let an element of mass consist of a vertical ribbon within the triangle, of width dx, height y, and thickness w. With x representing the location of the ribbon, show that $y = hx/L$. Show that the density of the material is given by $\rho = 2M/Lwh$. Show that the mass of the ribbon is $dm = \rho yw\,dx = 2Mx\,dx/L^2$. Proceed to use Equation 10.17 to calculate the moment of inertia.

(b) Let I represent the unknown moment of inertia about an axis through the corner of the triangle. Note that Example 9.15 demonstrates that the center of mass of the triangle is two thirds of the way along the length L, from the corner toward the side of height h. Let I_{CM} represent the moment of inertia of the triangle about an axis through the center of mass and parallel to side h. Demonstrate that $I = I_{CM} + 4ML^2/9$. Figure P10.28b shows the same object in a different orientation. Demonstrate that the moment of inertia of the triangular plate, about the y axis is $I_h = I_{CM} + ML^2/9$. Demonstrate that the sum of the moments of inertia of the triangles shown in parts (a) and (b) of the figure must be the moment of inertia of a rectangular sheet of mass $2M$ and length L, rotating like a door about an axis along its edge of height h. Use information in Table 10.2 to write down the moment of inertia of the rectangle, and set it equal to the sum of the moments of inertia of the two triangles. Solve the equation to find the moment of inertia of a triangle about an axis through its center of mass, in terms of M and L. Proceed to find the original unknown I.

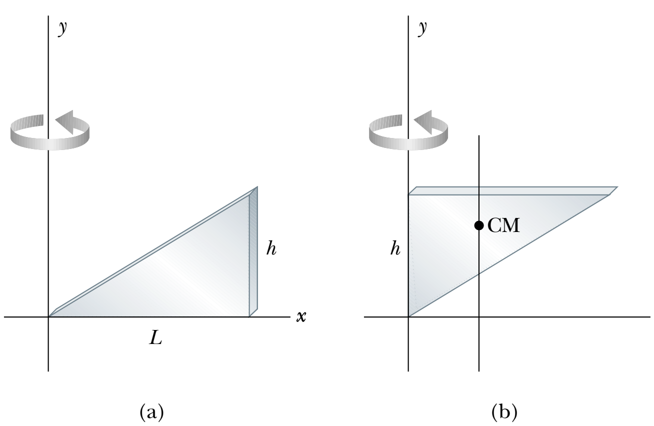


Figure P10.28

29. Many machines employ cams for various purposes, such as opening and closing valves. In Figure P10.29, the cam is a circular disk rotating on a shaft that does not pass through the center of the disk. In the manufacture of the cam, a uniform solid cylinder of radius R is first machined. Then an off-center hole of radius $R/2$ is drilled, parallel to the axis of the cylinder, and centered at a point a distance $R/2$ from the center of the cylinder. The cam, of mass M, is then slipped onto the circular shaft and welded into place. What is the kinetic energy of the cam when it is rotating with angular speed ω about the axis of the shaft?

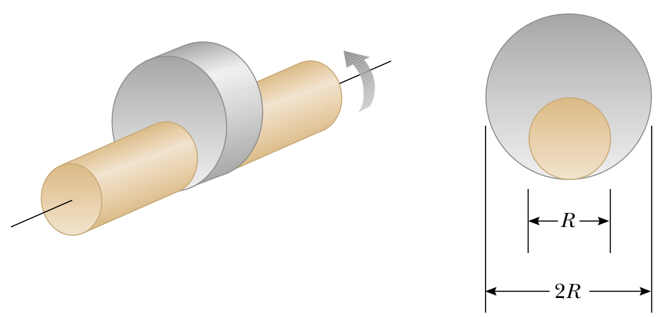


Figure P10.29

Section 10.6 Torque

30. The fishing pole in Figure P10.30 makes an angle of 20.0° with the horizontal. What is the torque exerted by the fish about an axis perpendicular to the page and passing through the fisher's hand?

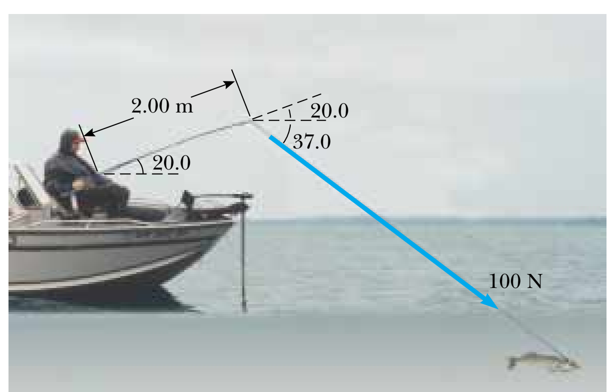


Figure P10.30

31. Find the net torque on the wheel in Figure P10.31 about the axle through O if $a = 10.0$ cm and $b = 25.0$ cm.

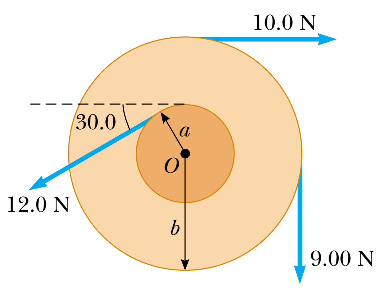


Figure P10.31

32. The tires of a 1 500-kg car are 0.600 m in diameter, and the coefficients of friction with the road surface are $\mu_s = 0.800$ and $\mu_k = 0.600$. Assuming that the weight is evenly distributed on the four wheels, calculate the maximum torque that can be exerted by the engine on a driving wheel without spinning the wheel. If you wish, you may assume the car is at rest.

33. Suppose the car in Problem 32 has a disk brake system. Each wheel is slowed by the friction force between a single brake pad and the disk-shaped rotor. On this particular car, the brake pad contacts the rotor at an average distance of 22.0 cm from the axis. The coefficients of friction between the brake pad and the disk are $\mu_s = 0.600$ and $\mu_k = 0.500$. Calculate the normal force that the pad must apply to the rotor in order to slow the car as quickly as possible.

Section 10.7 Relationship between Torque and Angular Acceleration

34. A grinding wheel is in the form of a uniform solid disk of radius 7.00 cm and mass 2.00 kg. It starts from rest and accelerates uniformly under the action of the constant torque of 0.600 N · m that the motor exerts on the wheel. (a) How long does the wheel take to reach its final operating speed of 1 200 rev/min? (b) Through how many revolutions does it turn while accelerating?

35. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane when it is in level flight. (c) Find the linear acceleration of the airplane tangent to its flight path.

36. The combination of an applied force and a friction force produces a constant total torque of 36.0 N · m on a wheel rotating about a fixed axis. The applied force acts for 6.00 s. During this time the angular speed of the wheel increases from 0 to 10.0 rad/s. The applied force is then removed, and the wheel comes to rest in 60.0 s. Find (a) the moment of inertia of the wheel, (b) the magnitude of the frictional torque, and (c) the total number of revolutions of the wheel.

37. A block of mass $m_1 = 2.00$ kg and a block of mass $m_2 = 6.00$ kg are connected by a massless string over a pulley in the shape of a solid disk having radius $R = 0.250$ m and mass $M = 10.0$ kg. These blocks are allowed to move on a fixed block-wedge of angle $\theta = 30.0°$ as in Figure P10.37. The coefficient of kinetic friction is 0.360 for both blocks. Draw free-body diagrams of both blocks and of the pulley. Determine (a) the acceleration of the two blocks and (b) the tensions in the string on both sides of the pulley.

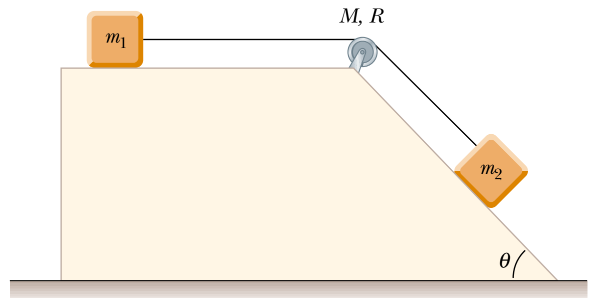


Figure P10.37

38. A potter's wheel—a thick stone disk of radius 0.500 m and mass 100 kg—is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

39. An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel, as shown in Figure P10.39. The flywheel is a solid disk with a mass of 80.0 kg and a diameter of 1.25 m. It turns on a frictionless axle. Its pulley has much smaller mass and a radius of 0.230 m. If the tension in the upper (taut) segment of the belt is 135 N and the flywheel has a clockwise angular acceleration of 1.67 rad/s^2, find the tension in the lower (slack) segment of the belt.

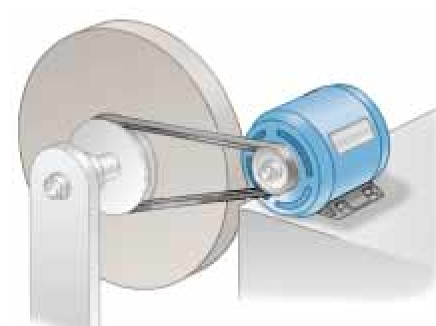


Figure P10.39

Section 10.8 Work, Power, and Energy in Rotational Motion

40. Big Ben, the Parliament tower clock in London, has an hour hand 2.70 m long with a mass of 60.0 kg, and a minute hand 4.50 m long with a mass of 100 kg (Fig. P10.40). Calculate the total rotational kinetic energy of the two hands about the axis of rotation. (You may model the hands as long, thin rods.)

Figure P10.40 Problems 40 and 74.

41. In a city with an air-pollution problem, a bus has no combustion engine. It runs on energy drawn from a large, rapidly rotating flywheel under the floor of the bus. The flywheel is spun up to its maximum rotation rate of 4 000 rev/min by an electric motor at the bus terminal. Every time the bus speeds up, the flywheel slows down slightly. The bus is equipped with regenerative braking so that the flywheel can speed up when the bus slows down. The flywheel is a uniform solid cylinder with mass 1 600 kg and radius 0.650 m. The bus body does work against air resistance and rolling resistance at the average rate of 18.0 hp as it travels with an average speed of 40.0 km/h. How far can the bus travel before the flywheel has to be spun up to speed again?

42. The top in Figure P10.42 has a moment of inertia of 4.00×10^{-4} kg·m^2 and is initially at rest. It is free to rotate about the stationary axis AA'. A string, wrapped around a peg along the axis of the top, is pulled in such a manner as to maintain a constant tension of 5.57 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?

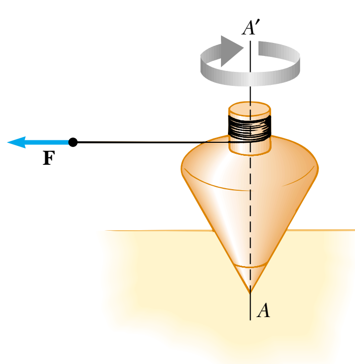


Figure P10.42

43. In Figure P10.43 the sliding block has a mass of 0.850 kg, the counterweight has a mass of 0.420 kg, and the pulley is a hollow cylinder with a mass of 0.350 kg, an inner radius of 0.020 0 m, and an outer radius of 0.030 0 m. The coefficient of kinetic friction between the block and the horizontal surface is 0.250. The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of 0.820 m/s toward the pulley when it passes through a photogate. (a) Use energy methods to predict its speed after it has moved to a second photogate, 0.700 m away. (b) Find the angular speed of the pulley at the same moment.

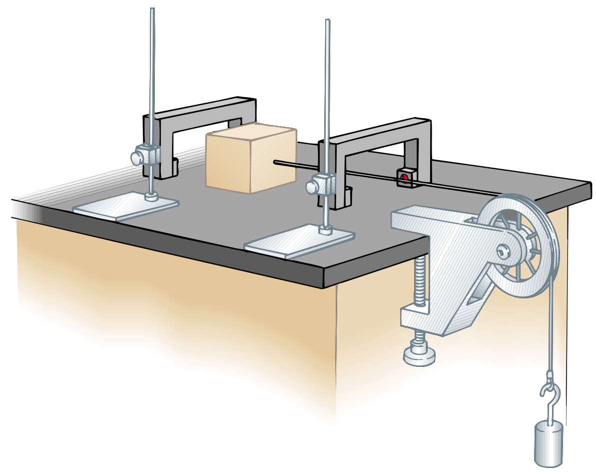


Figure P10.43

44. A cylindrical rod 24.0 cm long with mass 1.20 kg and radius 1.50 cm has a ball of diameter 8.00 cm and mass 2.00 kg attached to one end. The arrangement is originally vertical and stationary, with the ball at the top. The system is free to pivot about the bottom end of the rod after being given a slight nudge. (a) After the rod rotates through ninety degrees, what is its rotational kinetic energy? (b) What is the angular speed of the rod and ball? (c) What is the linear speed of the ball? (d) How does this compare to the speed if the ball had fallen freely through the same distance of 28 cm?

$\boxed{45.}$ An object with a weight of 50.0 N is attached to the free end of a light string wrapped around a reel of radius 0.250 m and mass 3.00 kg. The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center. The suspended object is released 6.00 m above the floor. (a) Determine the tension in the string, the acceleration of the object, and the speed with which the object hits the floor. (b) Verify your last answer by using the principle of conservation of energy to find the speed with which the object hits the floor.

46. A 15.0-kg object and a 10.0-kg object are suspended, joined by a cord that passes over a pulley with a radius of 10.0 cm and a mass of 3.00 kg (Fig. P10.46). The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest 3.00 m apart. Treat the pulley as a uniform disk, and determine the speeds of the two objects as they pass each other.

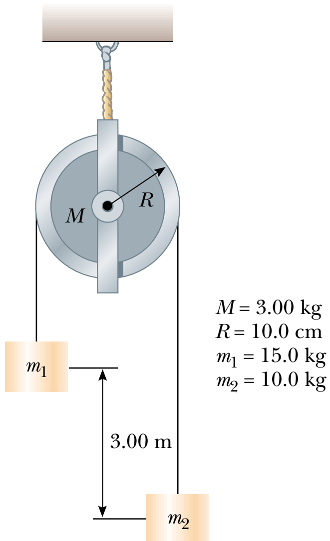


Figure P10.46

47. This problem describes one experimental method for determining the moment of inertia of an irregularly shaped object such as the payload for a satellite. Figure P10.47 shows a counterweight of mass m suspended by a cord wound around a spool of radius r, forming part of a turntable supporting the object. The turntable can rotate without friction. When the counterweight is released from rest, it descends through a distance h, acquiring a speed v. Show that the moment of inertia I of the rotating apparatus (including the turntable) is $mr^2(2gh/v^2 - 1)$.

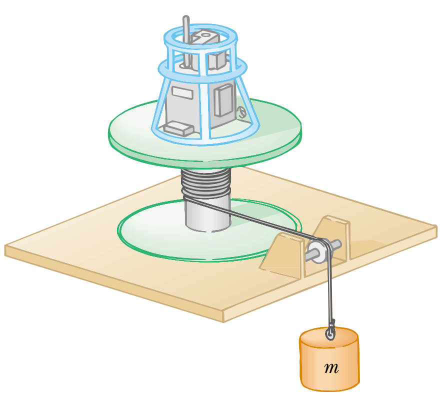


Figure P10.47

48. A horizontal 800-N merry-go-round is a solid disk of radius 1.50 m, started from rest by a constant horizontal force of 50.0 N applied tangentially to the edge of the disk. Find the kinetic energy of the disk after 3.00 s.

49. (a) A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.49). If the disk is released from rest in the position shown by the blue circle, what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) **What If?** Repeat part (a) using a uniform hoop.

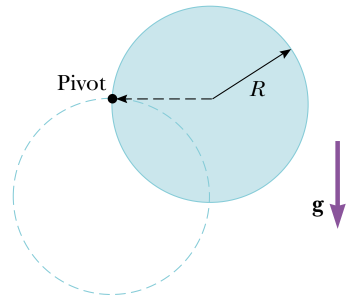


Figure P10.49

50. The head of a grass string trimmer has 100 g of cord wound in a light cylindrical spool with inside diameter 3.00 cm and outside diameter 18.0 cm, as in Figure P10.50. The cord has a linear density of 10.0 g/m. A single strand of the cord extends 16.0 cm from the outer edge of the spool. (a) When switched on, the trimmer speeds up from 0 to 2 500 rev/min in 0.215 s. (a) What average power is delivered to the head by the trimmer motor while it is accelerating? (b) When the trimmer is cutting grass, it spins at 2 000 rev/min and the grass exerts an average tangential force of 7.65 N on the outer end of the cord, which is still at a radial distance of 16.0 cm from the outer edge of the spool. What is the power delivered to the head under load?

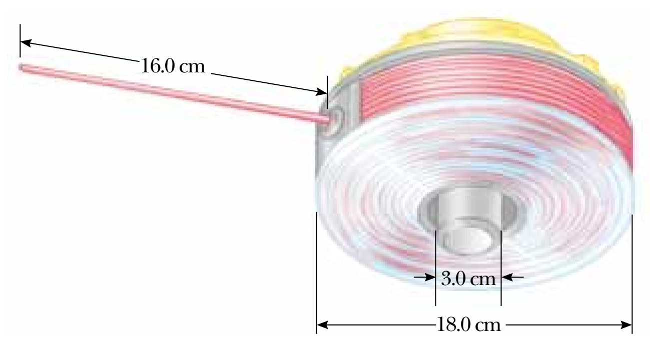


Figure P10.50

Section 10.9 Rolling Motion of a Rigid Object

51. A cylinder of mass 10.0 kg rolls without slipping on a horizontal surface. At the instant its center of mass has a speed of 10.0 m/s, determine (a) the translational kinetic energy of its center of mass, (b) the rotational kinetic energy about its center of mass, and (c) its total energy.

52. A bowling ball has mass M, radius R, and a moment of inertia of $\frac{2}{5}MR^2$. If it starts from rest, how much work must be done on it to set it rolling without slipping at a linear speed v? Express the work in terms of M and v.

53. (a) Determine the acceleration of the center of mass of a uniform solid disk rolling down an incline making angle θ with the horizontal. Compare this acceleration with that of a uniform hoop. (b) What is the minimum coefficient of

friction required to maintain pure rolling motion for the disk?

54. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height h. If they are released from rest and roll without slipping, which object reaches the bottom first? Verify your answer by calculating their speeds when they reach the bottom in terms of h.

55. A metal can containing condensed mushroom soup has mass 215 g, height 10.8 cm, and diameter 6.38 cm. It is placed at rest on its side at the top of a 3.00-m-long incline that is at 25.0° to the horizontal, and it is then released to roll straight down. Assuming mechanical energy conservation, calculate the moment of inertia of the can if it takes 1.50 s to reach the bottom of the incline. Which pieces of data, if any, are unnecessary for calculating the solution?

56. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.03 m/s on a horizontal section of a track, as shown in Figure P10.56. It rolls around the inside of a vertical circular loop 90.0 cm in diameter and finally leaves the track at a point 20.0 cm below the horizontal section. (a) Find the speed of the ball at the top of the loop. Demonstrate that it will not fall from the track. (b) Find its speed as it leaves the track. **What If?** (c) Suppose that static friction between ball and track were negligible, so that the ball slid instead of rolling. Would its speed then be higher, lower, or the same at the top of the loop? Explain.

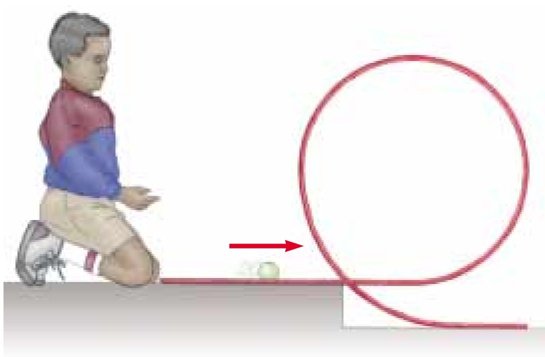
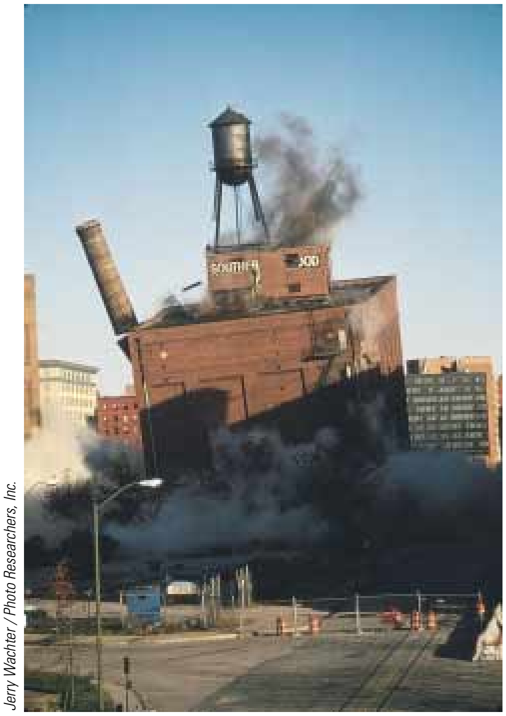


Figure P10.56

Additional Problems

57. As in Figure P10.57, toppling chimneys often break apart in mid-fall because the mortar between the bricks cannot withstand much shear stress. As the chimney begins to fall, shear forces must act on the topmost sections to accelerate them tangentially so that they can keep up with the rotation of the lower part of the stack. For simplicity, let us model the chimney as a uniform rod of length ℓ pivoted at the lower end. The rod starts at rest in a vertical position (with the frictionless pivot at the bottom) and falls over under the influence of gravity. What fraction of the length of the rod has a tangential acceleration greater than $g \sin \theta$, where θ is the angle the chimney makes with the vertical axis?



Jerry Wachter / Photo Researchers, Inc.

Figure P10.57 A building demolition site in Baltimore, MD. At the left is a chimney, mostly concealed by the building, that has broken apart on its way down. Compare with Figure 10.19.

58. **Review problem.** A mixing beater consists of three thin rods, each 10.0 cm long. The rods diverge from a central hub, separated from each other by 120°, and all turn in the same plane. A ball is attached to the end of each rod. Each ball has cross-sectional area 4.00 cm^2 and is so shaped that it has a drag coefficient of 0.600. Calculate the power input required to spin the beater at 1 000 rev/min (a) in air and (b) in water.

59. A 4.00-m length of light nylon cord is wound around a uniform cylindrical spool of radius 0.500 m and mass 1.00 kg. The spool is mounted on a frictionless axle and is initially at rest. The cord is pulled from the spool with a constant acceleration of magnitude 2.50 m/s^2. (a) How much work has been done on the spool when it reaches an angular speed of 8.00 rad/s? (b) Assuming there is enough cord on the spool, how long does it take the spool to reach this angular speed? (c) Is there enough cord on the spool?

60. A videotape cassette contains two spools, each of radius r_s, on which the tape is wound. As the tape unwinds from the first spool, it winds around the second spool. The tape moves at constant linear speed v past the heads between the spools. When all the tape is on the first spool, the tape has an outer radius r_t. Let r represent the outer radius of the tape on the first spool at any instant while the tape is being played. (a) Show that at any instant the angular speeds of the two spools are

$$\omega_1 = v/r \qquad \text{and} \qquad \omega_2 = v/(r_s^{\,2} + r_t^{\,2} - r^2)^{1/2}$$

(b) Show that these expressions predict the correct maximum and minimum values for the angular speeds of the two spools.

61. A long uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin through one end. The rod is released from rest in a vertical position, as shown in Figure P10.61. At the instant the rod is horizontal, find (a) its angular speed, (b) the magnitude of its angular acceleration, (c) the x and y components of the acceleration of its center of mass, and (d) the components of the reaction force at the pivot.

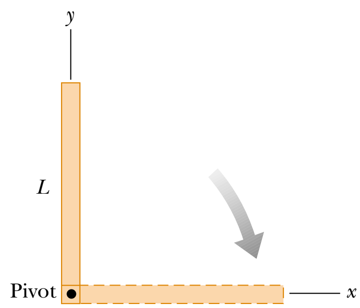


Figure P10.61

62. A shaft is turning at 65.0 rad/s at time $t = 0$. Thereafter, its angular acceleration is given by

$$\alpha = -10.0 \text{ rad/s}^2 - 5.00t \text{ rad/s}^3,$$

where t is the elapsed time. (a) Find its angular speed at $t = 3.00$ s. (b) How far does it turn in these 3 s?

63. A bicycle is turned upside down while its owner repairs a flat tire. A friend spins the other wheel, of radius 0.381 m, and observes that drops of water fly off tangentially. She measures the height reached by drops moving vertically (Fig. P10.63). A drop that breaks loose from the tire on one turn rises $h = 54.0$ cm above the tangent point. A drop that breaks loose on the next turn rises 51.0 cm above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

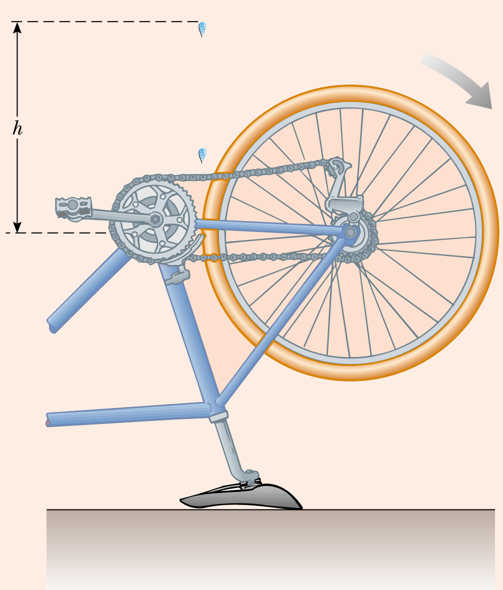


Figure P10.63 Problems 63 and 64.

64. A bicycle is turned upside down while its owner repairs a flat tire. A friend spins the other wheel, of radius R, and observes that drops of water fly off tangentially. She measures the height reached by drops moving vertically (Fig. P10.63). A drop that breaks loose from the tire on one turn rises a distance h_1 above the tangent point. A drop that breaks loose on the next turn rises a distance $h_2 < h_1$ above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

65. A cord is wrapped around a pulley of mass m and radius r. The free end of the cord is connected to a block of mass M. The block starts from rest and then slides down an incline that makes an angle θ with the horizontal. The coefficient of kinetic friction between block and incline is μ. (a) Use energy methods to show that the block's speed as a function of position d down the incline is

$$v = \sqrt{\frac{4gdM(\sin\theta - \mu\cos\theta)}{m + 2M}}$$

(b) Find the magnitude of the acceleration of the block in terms of μ, m, M, g, and θ.

66. (a) What is the rotational kinetic energy of the Earth about its spin axis? Model the Earth as a uniform sphere and use data from the endpapers. (b) The rotational kinetic energy of the Earth is decreasing steadily because of tidal friction. Find the change in one day, assuming that the rotational period decreases by 10.0 μs each year.

67. Due to a gravitational torque exerted by the Moon on the Earth, our planet's rotation period slows at a rate on the order of 1 ms/century. (a) Determine the order of magnitude of the Earth's angular acceleration. (b) Find the order of magnitude of the torque. (c) Find the order of magnitude of the size of the wrench an ordinary person would need to exert such a torque, as in Figure P10.67. Assume the person can brace his feet against a solid firmament.

Figure P10.67

68. The speed of a moving bullet can be determined by allowing the bullet to pass through two rotating paper disks mounted a distance d apart on the same axle (Fig. P10.68). From the angular displacement $\Delta\theta$ of the two bullet holes in the disks and the rotational speed of the disks, we can determine the speed v of the bullet. Find the bullet speed for the following data: $d = 80$ cm, $\omega = 900$ rev/min, and $\Delta\theta = 31.0°$.

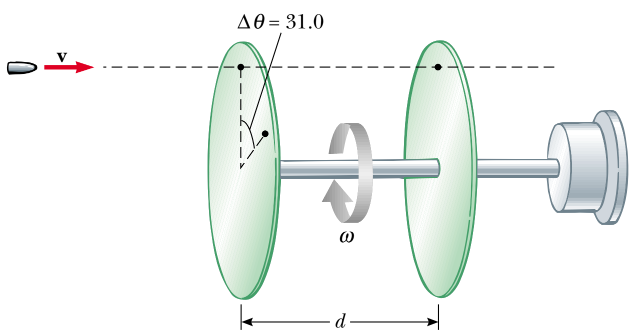


Figure P10.68

69. A uniform, hollow, cylindrical spool has inside radius $R/2$, outside radius R, and mass M (Fig. P10.69). It is mounted so that it rotates on a fixed horizontal axle. A counterweight of mass m is connected to the end of a string wound around the spool. The counterweight falls from rest at $t = 0$ to a position y at time t. Show that the torque due to the friction forces between spool and axle is

$$\tau_f = R\left[m\left(g - \frac{2y}{t^2}\right) - M\frac{5y}{4t^2}\right]$$

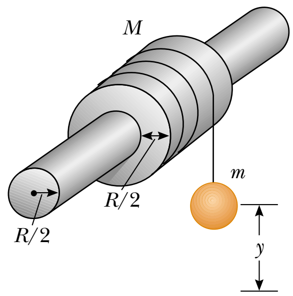


Figure P10.69

70. The reel shown in Figure P10.70 has radius R and moment of inertia I. One end of the block of mass m is connected to a spring of force constant k, and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is wound counterclockwise so that the spring stretches a distance d from its unstretched position and is then released from rest. (a) Find the angular speed of the reel when the spring is again unstretched. (b) Evaluate the angular speed numerically at this point if $I = 1.00$ kg · m^2, $R = 0.300$ m, $k = 50.0$ N/m, $m = 0.500$ kg, $d = 0.200$ m, and $\theta = 37.0°$.

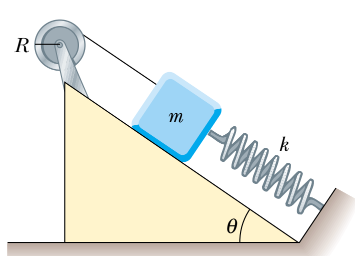


Figure P10.70

71. Two blocks, as shown in Figure P10.71, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia I. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s^2. (a) Determine T_1 and T_2, the tensions in the two parts of the string. (b) Find the moment of inertia of the pulley.

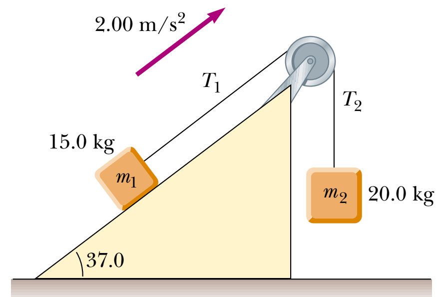


Figure P10.71

72. A common demonstration, illustrated in Figure P10.72, consists of a ball resting at one end of a uniform board of length ℓ, hinged at the other end, and elevated at an angle θ. A light cup is attached to the board at r_c so that it will catch the ball when the support stick is suddenly removed. (a) Show that the ball will lag behind the falling board when θ is less than 35.3°. (b) If the board is 1.00 m long and is supported at this limiting angle, show that the cup must be 18.4 cm from the moving end.

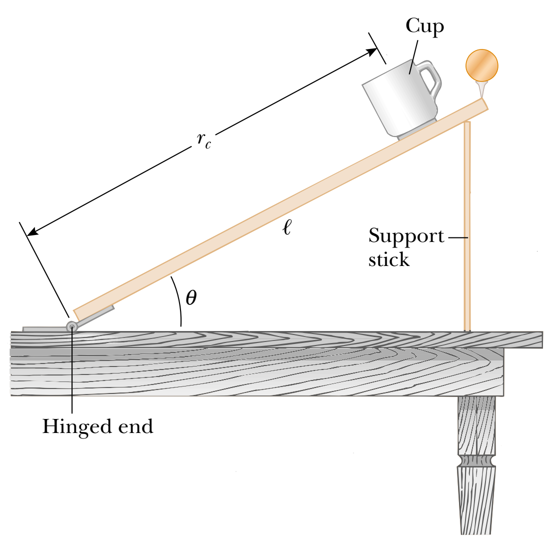


Figure P10.72

$\boxed{73.}$ As a result of friction, the angular speed of a wheel changes with time according to

$$\frac{d\theta}{dt} = \omega_0 e^{-\sigma t}$$

where ω_0 and σ are constants. The angular speed changes from 3.50 rad/s at $t = 0$ to 2.00 rad/s at $t = 9.30$ s. Use this information to determine σ and ω_0. Then determine (a) the magnitude of the angular acceleration at $t = 3.00$ s, (b) the number of revolutions the wheel makes in the first 2.50 s, and (c) the number of revolutions it makes before coming to rest.

74. The hour hand and the minute hand of Big Ben, the Parliament tower clock in London, are 2.70 m and 4.50 m long and have masses of 60.0 kg and 100 kg, respectively (see Figure P10.40). (a) Determine the total torque due to the weight of these hands about the axis of rotation when the time reads (i) 3:00 (ii) 5:15 (iii) 6:00 (iv) 8:20 (v) 9:45. (You may model the hands as long, thin uniform rods.) (b) Determine all times when the total torque about the axis of rotation is zero. Determine the times to the nearest second, solving a transcendental equation numerically.

75. (a) Without the wheels, a bicycle frame has a mass of 8.44 kg. Each of the wheels can be roughly modeled as a uniform solid disk with a mass of 0.820 kg and a radius of 0.343 m. Find the kinetic energy of the whole bicycle when it is moving forward at 3.35 m/s. (b) Before the invention of a wheel turning on an axle, ancient people moved heavy loads by placing rollers under them. (Modern people use rollers too. Any hardware store will sell you a roller bearing for a lazy susan.) A stone block of mass 844 kg moves forward at 0.335 m/s, supported by two uniform cylindrical tree trunks, each of mass 82.0 kg and radius 0.343 m. No slipping occurs between the block and the rollers or between the rollers and the ground. Find the total kinetic energy of the moving objects.

76. A uniform solid sphere of radius r is placed on the inside surface of a hemispherical bowl with much larger radius R. The sphere is released from rest at an angle θ to the vertical and rolls without slipping (Fig. P10.76). Determine the angular speed of the sphere when it reaches the bottom of the bowl.

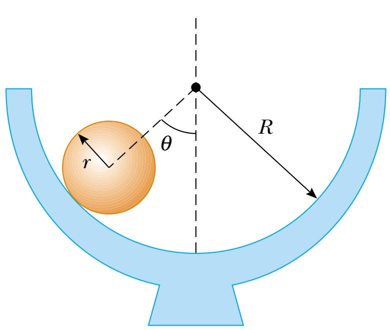


Figure P10.76

$\boxed{77.}$ A string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P10.77). Show that (a) the tension in the string is one third of the weight of the disk, (b) the magnitude of the acceleration of the center of mass is $2g/3$, and (c) the speed of the center of mass is $(4gh/3)^{1/2}$ after the disk has descended through distance h. Verify your answer to (c) using the energy approach.

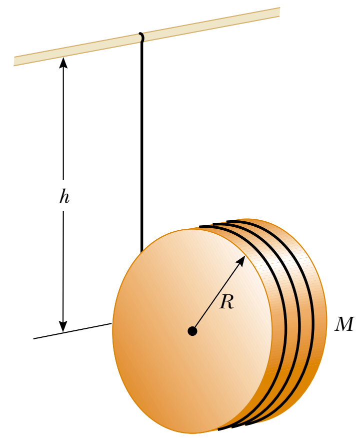


Figure P10.77

78. A constant horizontal force $\mathbf{F}$ is applied to a lawn roller in the form of a uniform solid cylinder of radius R and mass M (Fig. P10.78). If the roller rolls without slipping on the horizontal surface, show that (a) the acceleration of the center of mass is $2\mathbf{F}/3M$ and (b) the minimum coefficient of friction necessary to prevent slipping is $F/3Mg$. (*Hint:* Take the torque with respect to the center of mass.)

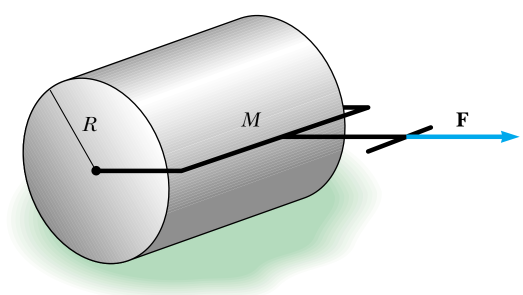


Figure P10.78

79. A solid sphere of mass m and radius r rolls without slipping along the track shown in Figure P10.79. It starts from rest with the lowest point of the sphere at height h above the bottom of the loop of radius R, much larger than r. (a) What is the minimum value of h (in terms of R) such that the sphere completes the loop? (b) What are the force components on the sphere at the point P if $h = 3R$?

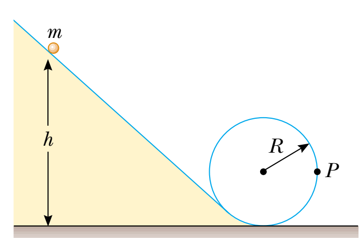


Figure P10.79

80. A thin rod of mass 0.630 kg and length 1.24 m is at rest, hanging vertically from a strong fixed hinge at its top end. Suddenly a horizontal impulsive force $(14.7\hat{\mathbf{i}})$ N is applied to it. (a) Suppose the force acts at the bottom end of the rod. Find the acceleration of its center of mass and the horizontal force the hinge exerts. (b) Suppose the force acts at the midpoint of the rod. Find the acceleration of this point and the horizontal hinge reaction. (c) Where can the impulse be applied so that the hinge will exert no horizontal force? This point is called the *center of percussion.*

81. A bowler releases a bowling ball with no spin, sending it sliding straight down the alley toward the pins. The ball continues to slide for a distance of what order of magnitude, before its motion becomes rolling without slipping? State the quantities you take as data, the values you measure or estimate for them, and your reasoning.

82. Following Thanksgiving dinner your uncle falls into a deep sleep, sitting straight up facing the television set. A naughty grandchild balances a small spherical grape at the top of his bald head, which itself has the shape of a sphere. After all the children have had time to giggle, the grape starts from rest and rolls down without slipping. It will leave contact with your uncle's scalp when the radial line joining it to the center of curvature makes what angle with the vertical?

83. (a) A thin rod of length h and mass M is held vertically with its lower end resting on a frictionless horizontal surface. The rod is then released to fall freely. Determine the speed of its center of mass just before it hits the horizontal surface. (b) **What If?** Now suppose the rod has a fixed pivot at its lower end. Determine the speed of the rod's center of mass just before it hits the surface.

84. A large, cylindrical roll of tissue paper of initial radius R lies on a long, horizontal surface with the outside end of the paper nailed to the surface. The roll is given a slight shove ($v_i \approx 0$) and commences to unroll. Assume the roll has a uniform density and that mechanical energy is conserved in the process. (a) Determine the speed of the center of mass of the roll when its radius has diminished to r. (b) Calculate a numerical value for this speed at $r = 1.00$ mm, assuming $R = 6.00$ m. (c) **What If?** What happens to the energy of the system when the paper is completely unrolled?

85. A spool of wire of mass M and radius R is unwound under a constant force $\mathbf{F}$ (Fig. P10.85). Assuming the spool is a uniform solid cylinder that doesn't slip, show that (a) the acceleration of the center of mass is $4\mathbf{F}/3M$ and (b) the force of friction is to the *right* and equal in magnitude to $F/3$. (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance d?

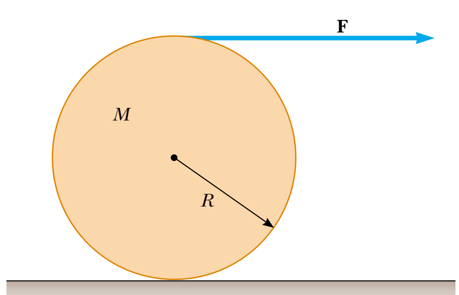


Figure P10.85

86. A plank with a mass $M = 6.00$ kg rides on top of two identical solid cylindrical rollers that have $R = 5.00$ cm and $m = 2.00$ kg (Fig. P10.86). The plank is pulled by a constant horizontal force $\mathbf{F}$ of magnitude 6.00 N applied to the end of the plank and perpendicular to the axes of the cylinders (which are parallel). The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the plank. (a) Find the acceleration of the plank and of the rollers. (b) What friction forces are acting?

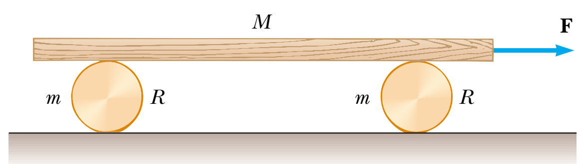


Figure P10.86

87. A spool of wire rests on a horizontal surface as in Figure P10.87. As the wire is pulled, the spool does not slip at the contact point P. On separate trials, each one of the forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_4$ is applied to the spool. For each one of these forces, determine the direction the spool will roll. Note that the line of action of $\mathbf{F}_2$ passes through P.

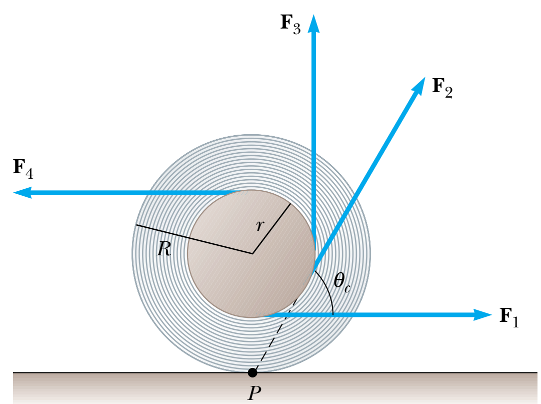


Figure P10.87 Problems 87 and 88.

88. Refer to Problem 87 and Figure P10.87. The spool of wire has an inner radius r and an outer radius R. The angle θ between the applied force and the horizontal can be varied. Show that the critical angle for which the spool does not roll is given by

$$\cos \theta_c = \frac{r}{R}$$

If the wire is held at this angle and the force increased, the spool will remain stationary until it slips along the floor.

89. In a demonstration known as the ballistics cart, a ball is projected vertically upward from a cart moving with constant velocity along the horizontal direction. The ball lands in the catching cup of the cart because both the cart a ball have the same horizontal component of velocity. **What If?** Now consider a ballistics cart on an incline making an angle θ with the horizontal as in Figure P10.89. The cart (including wheels) has a mass M and the moment of inertia of each of the two wheels is $mR^2/2$. (a) Using conservation of energy (assuming no friction between cart and axles) and assuming pure rolling motion (no slipping), show that the acceleration of the cart along the incline is

$$a_x = \left(\frac{M}{M + 2m}\right) g \sin\theta$$

(b) Note that the x component of acceleration of the ball released by the cart is $g \sin\theta$. Thus, the x component of the cart's acceleration is *smaller* than that of the ball by the factor $M/(M + 2m)$. Use this fact and kinematic equations to show that the ball overshoots the cart by an amount Δx, where

$$\Delta x = \left(\frac{4m}{M + 2m}\right)\left(\frac{\sin\theta}{\cos^2\theta}\right)\frac{v_{yi}^2}{g}$$

and v_{yi} is the initial speed of the ball imparted to it by the spring in the cart. (c) Show that the distance d that the ball travels measured along the incline is

$$d = \frac{2v_{yi}^2}{g}\frac{\sin\theta}{\cos^2\theta}$$

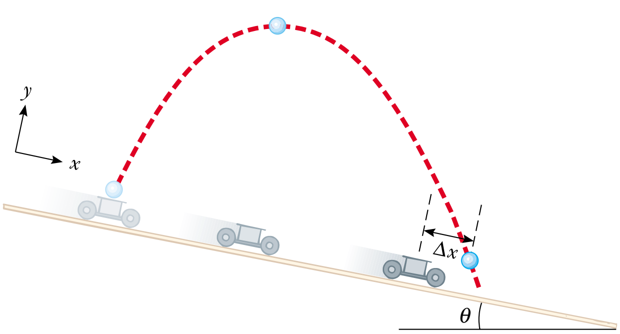


Figure P10.89

90. A spool of thread consists of a cylinder of radius R_1 with end caps of radius R_2 as in the end view shown in Figure P10.90. The mass of the spool, including the thread, is m and its moment of inertia about an axis through its center is I. The spool is placed on a rough horizontal surface so that it rolls without slipping when a force $\mathbf{T}$ acting to the right is applied to the free end of the thread. Show that the magnitude of the friction force exerted by the surface on the spool is given by

$$f = \left(\frac{I + mR_1R_2}{I + mR_2^2}\right) T$$

Determine the direction of the force of friction.

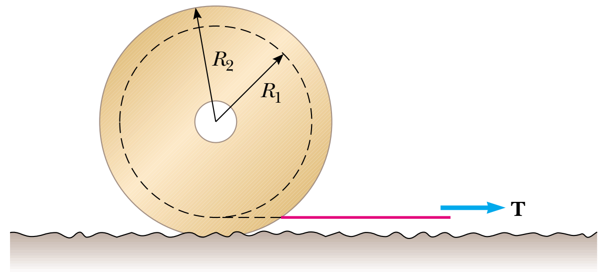


Figure P10.90

Answers to Quick Quizzes

10.1 (c). For a rotation of more than 180°, the angular displacement must be larger than $\pi = 3.14$ rad. The angular displacements in the three choices are (a) 6 rad − 3 rad = 3 rad (b) 1 rad − (−1) rad = 2 rad (c) 5 rad − 1 rad = 4 rad.

10.2 (b). Because all angular displacements occur in the same time interval, the displacement with the lowest value will be associated with the lowest average angular speed.

10.3 (b). The fact that ω is negative indicates that we are dealing with an object that is rotating in the clockwise direction. We also know that when $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are antiparallel, ω must be decreasing—the object is slowing down. Therefore, the object is spinning more and more slowly (with less and less angular speed) in the clockwise, or negative, direction.

10.4 (b). In Equation 10.8, both the initial and final angular speeds are the same in all three cases. As a result, the angular acceleration is inversely proportional to the angular displacement. Thus, the highest angular acceleration is associated with the lowest angular displacement.

10.5 (b). The system of the platform, Andy, and Charlie is a rigid object, so all points on the rigid object have the same angular speed.

10.6 (a). The tangential speed is proportional to the radial distance from the rotation axis.

10.7 (a). Almost all of the mass of the pipe is at the same distance from the rotation axis, so it has a larger moment of inertia than the solid cylinder.

10.8 (b). The fatter handle of the screwdriver gives you a larger moment arm and increases the torque that you can apply with a given force from your hand.

10.9 (a). The longer handle of the wrench gives you a larger moment arm and increases the torque that you can apply with a given force from your hand.

10.10 (b). With twice the moment of inertia and the same frictional torque, there is half the angular acceleration. With half the angular acceleration, it will require twice as long to change the speed to zero.

10.11 (d). When the rod is attached at its end, it offers four times as much moment of inertia as when attached in the center (see Table 10.2). Because the rotational kinetic energy of the rod depends on the square of the angular speed, the same work will result in half of the angular speed.

10.12 (b). All of the gravitational potential energy of the box–Earth system is transformed to kinetic energy of translation. For the ball, some of the gravitational potential energy of the ball–Earth system is transformed to rotational kinetic energy, leaving less for translational kinetic energy, so the ball moves downhill more slowly than the box does.

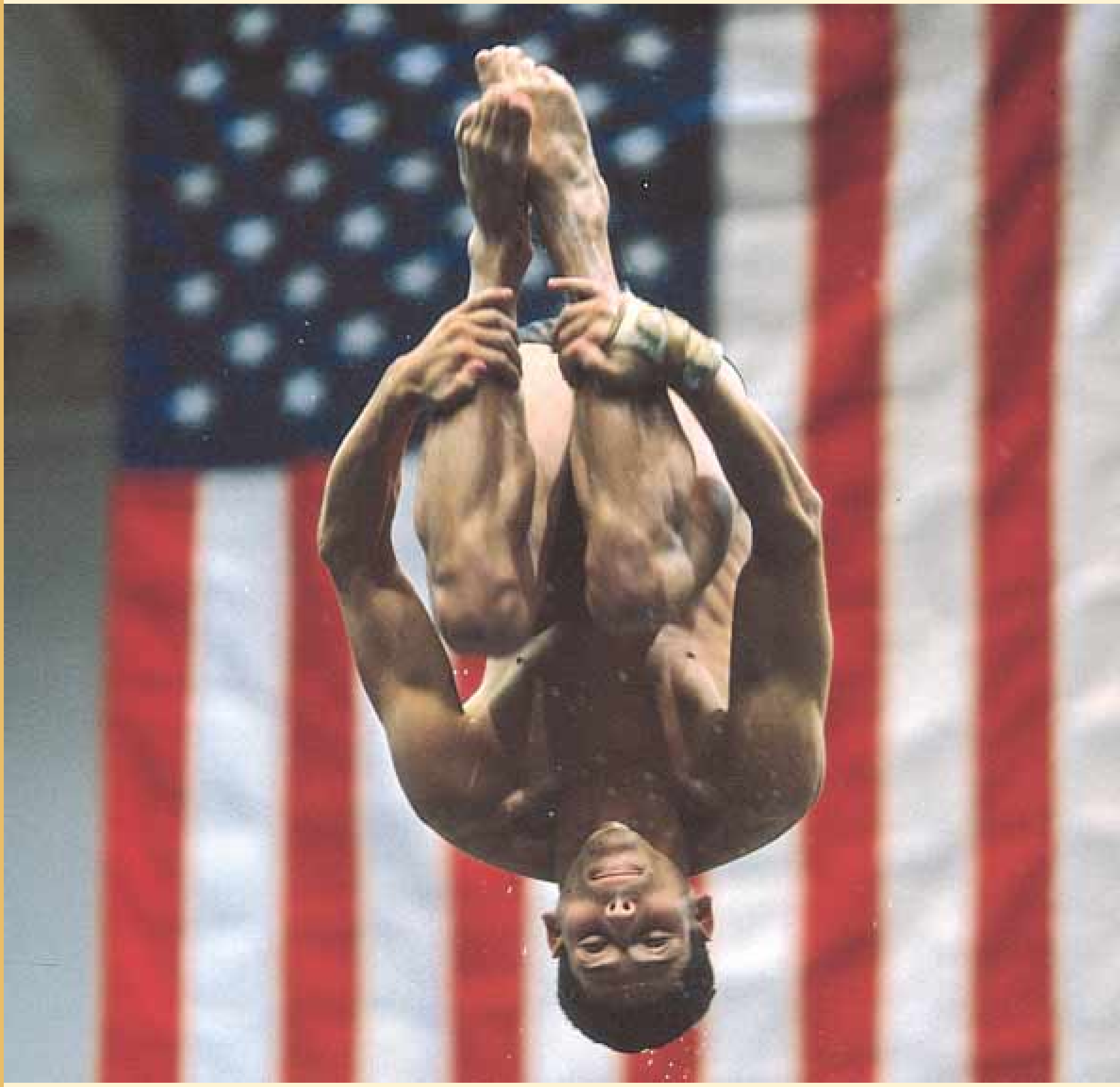
10.13 (c). In Equation 10.30, I_{CM} for a sphere is $\frac{2}{5}MR^2$. Thus, MR^2 will cancel and the remaining expression on the right-hand side of the equation is independent of mass and radius.

10.14 (a). The moment of inertia of the hollow sphere B is larger than that of sphere A. As a result, Equation 10.30 tells us that the center of mass of sphere B will have a smaller speed, so sphere A should arrive first.

Chapter 11

Angular Momentum

CHAPTER OUTLINE



▲ *Mark Ruiz undergoes a rotation during a dive at the U.S. Olympic trials in June 2000. He spins at a higher rate when he curls up and grabs his ankles due to the principle of conservation of angular momentum, as discussed in this chapter. (Otto Greule/Allsport/Getty)*

Some Physical Constants[a]		
Quantity	**Symbol**	**Value**[b]
Atomic mass unit	u	$1.660\ 538\ 73\ (13) \times 10^{-27}$ kg $931.494\ 013\ (37)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 141\ 99\ (47) \times 10^{23}$ particles/mol
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\ 008\ 99\ (37) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k_e}$	$5.291\ 772\ 083\ (19) \times 10^{-11}$ m
Boltzmann's constant	$k_B = \frac{R}{N_A}$	$1.380\ 650\ 3\ (24) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\ 310\ 215\ (18) \times 10^{-12}$ m
Coulomb constant	$k_e = \frac{1}{4\pi\epsilon_0}$	$8.987\ 551\ 788\ \ldots \times 10^{9}$ N·m²/C² (exact)
Deuteron mass	m_d	$3.343\ 583\ 09\ (26) \times 10^{-27}$ kg $2.013\ 553\ 212\ 71\ (35)$ u
Electron mass	m_e	$9.109\ 381\ 88\ (72) \times 10^{-31}$ kg $5.485\ 799\ 110\ (12) \times 10^{-4}$ u $0.510\ 998\ 902\ (21)$ MeV/c^2
Electron volt	eV	$1.602\ 176\ 462\ (63) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 176\ 462\ (63) \times 10^{-19}$ C
Gas constant	R	$8.314\ 472\ (15)$ J/K·mol
Gravitational constant	G	$6.673\ (10) \times 10^{-11}$ N·m²/kg²
Josephson frequency–voltage ratio	$\frac{2e}{h}$	$4.835\ 978\ 98\ (19) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \frac{h}{2e}$	$2.067\ 833\ 636\ (81) \times 10^{-15}$ T·m²
Neutron mass	m_n	$1.674\ 927\ 16\ (13) \times 10^{-27}$ kg $1.008\ 664\ 915\ 78\ (55)$ u $939.565\ 330\ (38)$ MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\ 783\ 17\ (20) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A (exact)
Permittivity of free space	$\epsilon_0 = \frac{1}{\mu_0 c^2}$	$8.854\ 187\ 817 \ldots \times 10^{-12}$ C²/N·m² (exact)
Planck's constant	h	$6.626\ 068\ 76\ (52) \times 10^{-34}$ J·s
	$\hbar = \frac{h}{2\pi}$	$1.054\ 571\ 596\ (82) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 621\ 58\ (13) \times 10^{-27}$ kg $1.007\ 276\ 466\ 88\ (13)$ u $938.271\ 998\ (38)$ MeV/c^2
Rydberg constant	R_H	$1.097\ 373\ 156\ 854\ 9\ (83) \times 10^{7}$ m^{-1}
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^{8}$ m/s (exact)

[a] These constants are the values recommended in 1998 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see P. J. Mohr and B. N. Taylor, *Rev. Mod. Phys.* 72:351, 2000.

[b] The numbers in parentheses for the values above represent the uncertainties of the last two digits.

Solar System Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Distance from the Sun (m)
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}
Earth	5.98×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}
Pluto	$\approx 1.4 \times 10^{22}$	$\approx 1.5 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}
Moon	7.36×10^{22}	1.74×10^{6}	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—

Physical Data Often Used[a]

Average Earth–Moon distance	3.84×10^{8} m
Average Earth–Sun distance	1.496×10^{11} m
Average radius of the Earth	6.37×10^{6} m
Density of air (20°C and 1 atm)	1.20 kg/m^3
Density of water (20°C and 1 atm)	1.00×10^{3} kg/m^3
Free-fall acceleration	9.80 m/s^2
Mass of the Earth	5.98×10^{24} kg
Mass of the Moon	7.36×10^{22} kg
Mass of the Sun	1.99×10^{30} kg
Standard atmospheric pressure	1.013×10^{5} Pa

[a] These are the values of the constants as used in the text.

Some Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{1}	deka	da
10^{-21}	zepto	z	10^{2}	hecto	h
10^{-18}	atto	a	10^{3}	kilo	k
10^{-15}	femto	f	10^{6}	mega	M
10^{-12}	pico	p	10^{9}	giga	G
10^{-9}	nano	n	10^{12}	tera	T
10^{-6}	micro	μ	10^{15}	peta	P
10^{-3}	milli	m	10^{18}	exa	E
10^{-2}	centi	c	10^{21}	zetta	Z
10^{-1}	deci	d	10^{24}	yotta	Y

Standard Abbreviations and Symbols for Units

Symbol	Unit	Symbol	Unit
A	ampere	K	kelvin
u	atomic mass unit	kg	kilogram
atm	atmosphere	kmol	kilomole
Btu	British thermal unit	L	liter
C	coulomb	lb	pound
°C	degree Celsius	ly	lightyear
cal	calorie	m	meter
d	day	min	minute
eV	electron volt	mol	mole
°F	degree Fahrenheit	N	newton
F	farad	Pa	pascal
ft	foot	rad	radian
G	gauss	rev	revolution
g	gram	s	second
H	henry	T	tesla
h	hour	V	volt
hp	horsepower	W	watt
Hz	hertz	Wb	weber
in.	inch	yr	year
J	joule	Ω	ohm

Mathematical Symbols Used in the Text and Their Meaning

Symbol	Meaning
$=$	is equal to
$\equiv$	is defined as
$\neq$	is not equal to
$\propto$	is proportional to
$\sim$	is on the order of
$>$	is greater than
$<$	is less than
$\gg (\ll)$	is much greater (less) than
$\approx$	is approximately equal to
Δx	the change in x
$\sum_{i=1}^{N} x_i$	the sum of all quantities x_i from $i = 1$ to $i = N$
$\lvert x \rvert$	the magnitude of x (always a nonnegative quantity)
$\Delta x \to 0$	Δx approaches zero
$\dfrac{dx}{dt}$	the derivative of x with respect to t
$\dfrac{\partial x}{\partial t}$	the partial derivative of x with respect to t
$\int$	integral

Conversions[a]

Length

1 in. = 2.54 cm (exact)
1 m = 39.37 in. = 3.281 ft
1 ft = 0.304 8 m
12 in. = 1 ft
3 ft = 1 yd
1 yd = 0.914 4 m
1 km = 0.621 mi
1 mi = 1.609 km
1 mi = 5 280 ft
1 μm = 10^{-6} m = 10^3 nm
1 lightyear = 9.461×10^{15} m

Area

1 m^2 = 10^4 cm^2 = 10.76 ft^2
1 ft^2 = 0.092 9 m^2 = 144 $in.^2$
1 $in.^2$ = 6.452 cm^2

Volume

1 m^3 = 10^6 cm^3 = 6.102×10^4 $in.^3$
1 ft^3 = 1 728 $in.^3$ = 2.83×10^{-2} m^3
1 L = 1 000 cm^3 = 1.057 6 qt = 0.035 3 ft^3
1 ft^3 = 7.481 gal = 28.32 L = 2.832×10^{-2} m^3
1 gal = 3.786 L = 231 $in.^3$

Mass

1 000 kg = 1 t (metric ton)
1 slug = 14.59 kg
1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2

Force

1 N = 0.224 8 lb
1 lb = 4.448 N

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s = 1.61 km/h
1 m/s = 100 cm/s = 3.281 ft/s
1 mi/min = 60 mi/h = 88 ft/s

Acceleration

1 m/s^2 = 3.28 ft/s^2 = 100 cm/s^2
1 ft/s^2 = 0.304 8 m/s^2 = 30.48 cm/s^2

Pressure

1 bar = 10^5 N/m^2 = 14.50 lb/$in.^2$
1 atm = 760 mm Hg = 76.0 cm Hg
1 atm = 14.7 lb/$in.^2$ = 1.013×10^5 N/m^2
1 Pa = 1 N/m^2 = 1.45×10^{-4} lb/$in.^2$

Time

1 yr = 365 days = 3.16×10^7 s
1 day = 24 h = 1.44×10^3 min = 8.64×10^4 s

Energy

1 J = 0.738 ft · lb
1 cal = 4.186 J
1 Btu = 252 cal = 1.054×10^3 J
1 eV = 1.6×10^{-19} J
1 kWh = 3.60×10^6 J

Power

1 hp = 550 ft · lb/s = 0.746 kW
1 W = 1 J/s = 0.738 ft · lb/s
1 Btu/h = 0.293 W

Some Approximations Useful for Estimation Problems

1 m ≈ 1 yd
1 kg ≈ 2 lb
1 N ≈ $\frac{1}{4}$ lb
1 L ≈ $\frac{1}{4}$ gal
1 m/s ≈ 2 mi/h
1 yr ≈ $\pi \times 10^7$ s
60 mi/h ≈ 100 ft/s
1 km ≈ $\frac{1}{2}$ mi

[a] See Table A.1 of Appendix A for a more complete list.

The Greek Alphabet

Alpha	Α	α	Iota	Ι	ι	Rho	Ρ	ρ
Beta	Β	β	Kappa	Κ	κ	Sigma	Σ	σ
Gamma	Γ	γ	Lambda	Λ	λ	Tau	Τ	τ
Delta	Δ	δ	Mu	Μ	μ	Upsilon	Υ	υ
Epsilon	Ε	ϵ	Nu	Ν	ν	Phi	Φ	ϕ
Zeta	Ζ	ζ	Xi	Ξ	ξ	Chi	Χ	χ
Eta	Η	η	Omicron	Ο	o	Psi	Ψ	ψ
Theta	Θ	θ	Pi	Π	π	Omega	Ω	ω

Pedagogical Color Chart

Part 1 (Chapters 1–15) : Mechanics

Displacement and position vectors

Linear ($\mathbf{v}$) and angular ($\boldsymbol{\omega}$) velocity vectors

Velocity component vectors

Force vectors ($\mathbf{F}$)

Force component vectors

Acceleration vectors ($\mathbf{a}$)

Acceleration component vectors

Linear ($\mathbf{p}$) and angular ($\mathbf{L}$) momentum vectors

Torque vectors ($\boldsymbol{\tau}$)

Linear or rotational motion directions

Springs

Pulleys

Part 4 (Chapters 23–34) : Electricity and Magnetism

Electric fields

Magnetic fields

Positive charges

Negative charges

Resistors

Batteries and other DC power supplies

Switches

Capacitors

Inductors (coils)

Voltmeters

Ammeters

AC Generators

Ground symbol

Part 5 (Chapters 35–38) : Light and Optics

Light rays

Lenses and prisms

Mirrors

Objects

Images