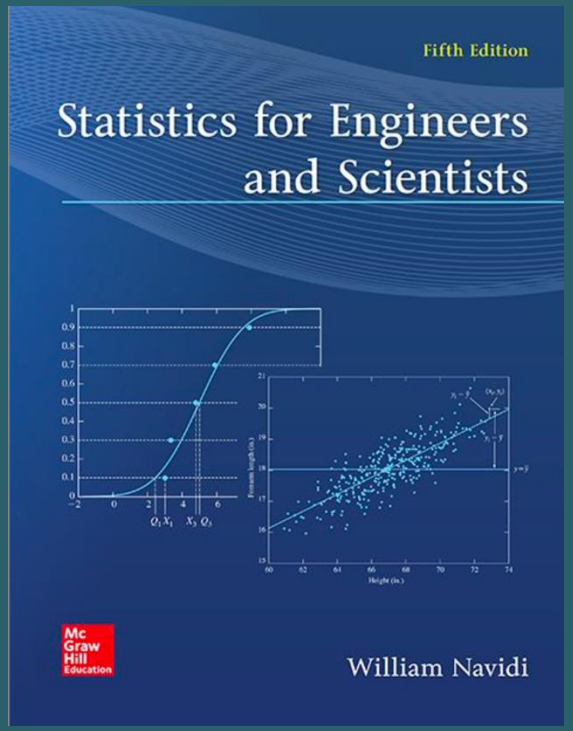


Ch 4: Commonly Used Distributions

(not all sections are required)

Ch 4: Overview (Required Sections)

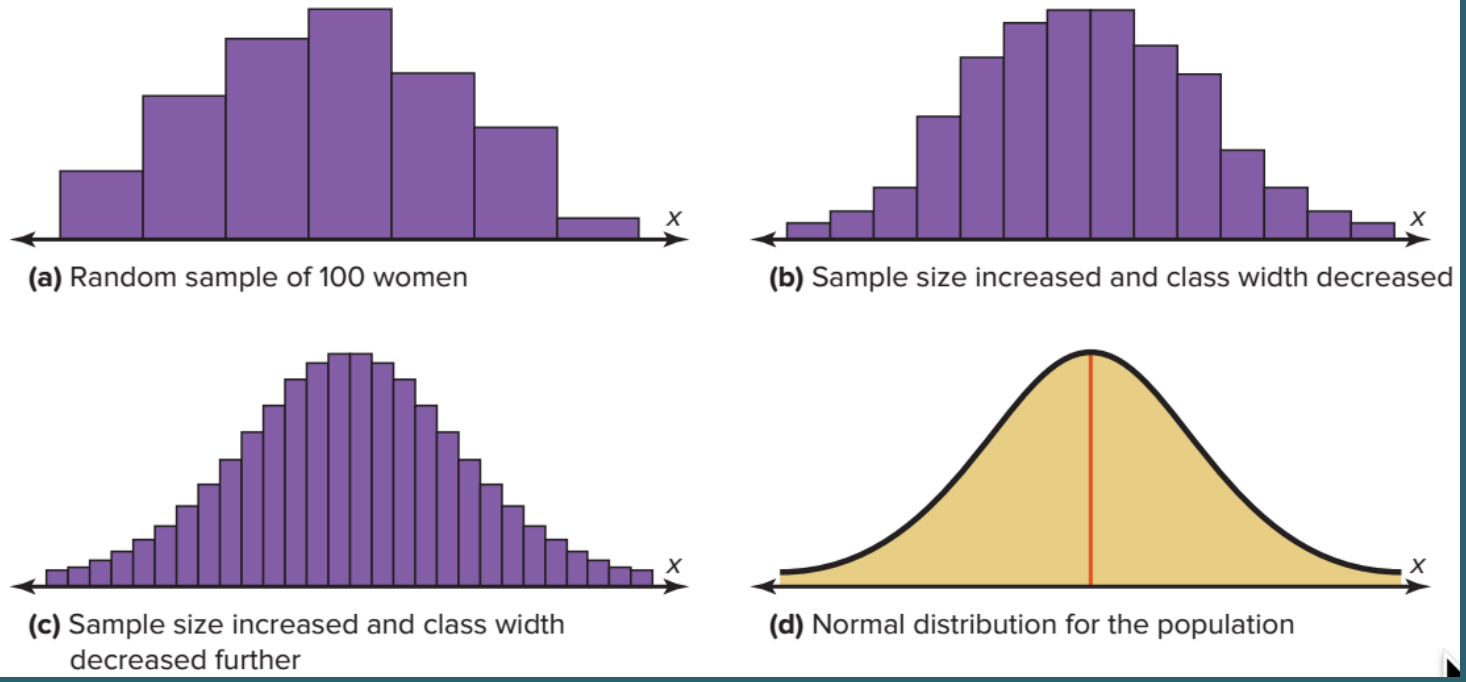
- ✓ 4-1 The Bernoulli Distribution
- ✓ 4-2 The Binomial Distribution
- ✓ 4-3 The Poisson Distribution
- **4-5 The Normal Distribution**
- 4-9 Some Principles of Point Estimation
- 4-10 Probability Plots
- 4-11 Central Limit Theorem

Introduction

- Many continuous variables have distributions that are **bell-shaped** and are called approximately **normally distributed variables**.
- The theoretical curve, called the **bell curve** or the **Gaussian distribution**, can be used to study many variables that are not normally distributed but are approximately normal.

Introduction

FIGURE 6–1
Histograms and Normal Model for the Distribution of Heights of Adult Women



(a) Random sample of 100 women

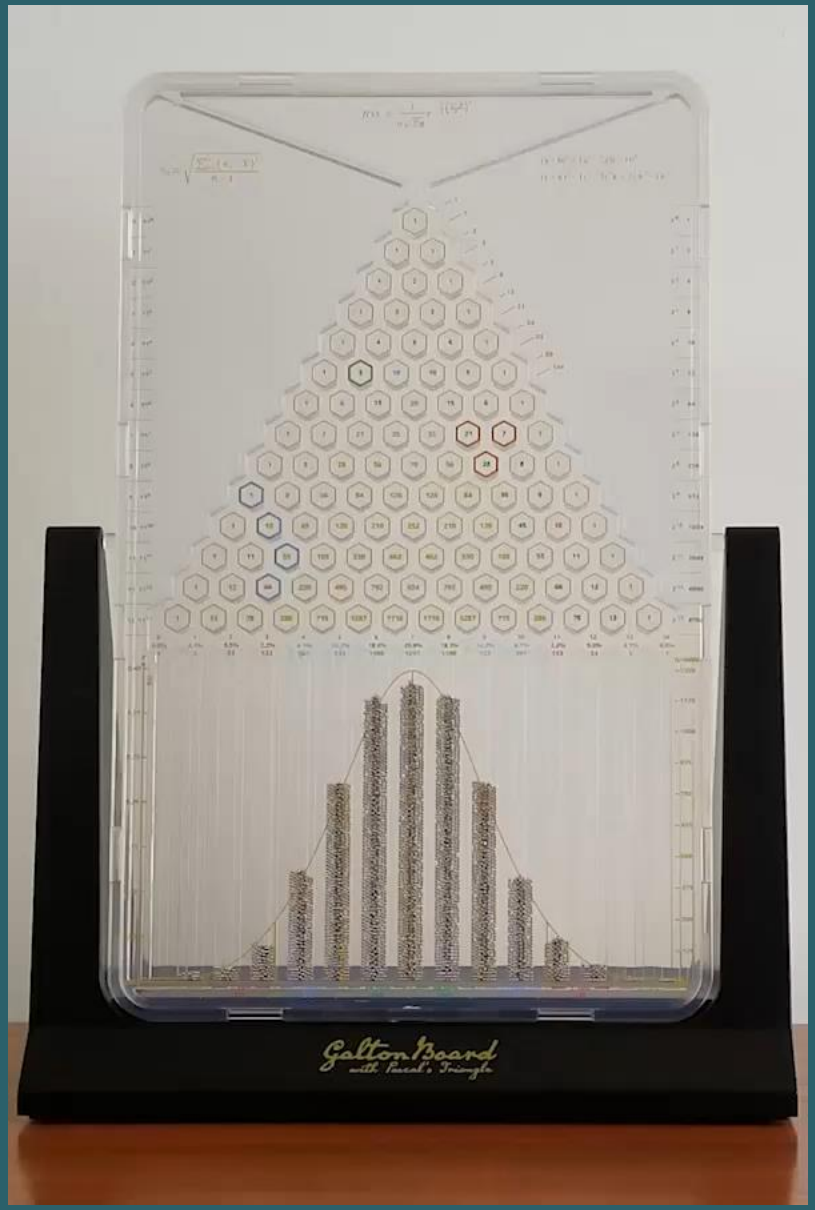
(b) Sample size increased and class width decreased

(c) Sample size increased and class width decreased further

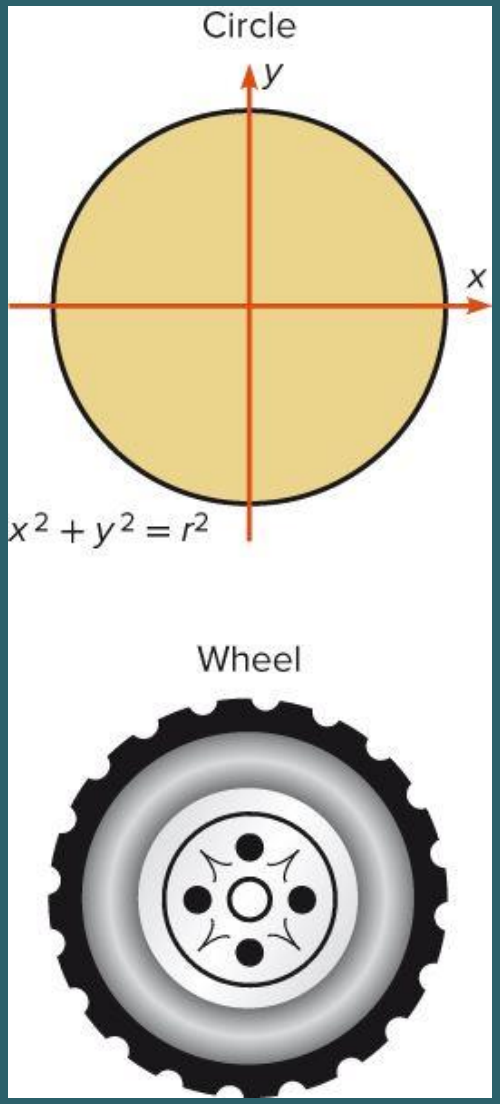
(d) Normal distribution for the population

- No variable fits a normal distribution perfectly, since a normal distribution is a **theoretical distribution**.
- However, a normal distribution can be used to describe many variables, because the deviations from a normal distribution are very small.

Introduction – Galton Board



Introduction



- A circle can be used to represent many physical objects, such as a wheel or a gear.
- Even though it is not possible to manufacture a wheel that is perfectly round, the equation and the properties of a circle can be used to study many aspects of the wheel, such as area, velocity, and acceleration.

Normal Distribution (p.242)

The mathematical equation for the normal distribution is:

$$y = \frac{e^{-(X-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$$

where

$e \approx 2.718$

$\pi \approx 3.14$

$\mu = population\ mean$

$\sigma = population\ standard\ deviation$

Normal distribution is determined by the fixed values of the mean μ and standard deviation σ.

Normal Distribution…

- The Normal Distribution equation may look formidable, but in applied statistics, **tables or technology** is used for specific problems instead of the equation.
- Another important consideration in applied statistics is that the area under a normal distribution curve is used more often than the values on the y axis.
- Therefore, when a normal distribution is pictured, the y axis is sometimes omitted.

Normal Distribution…

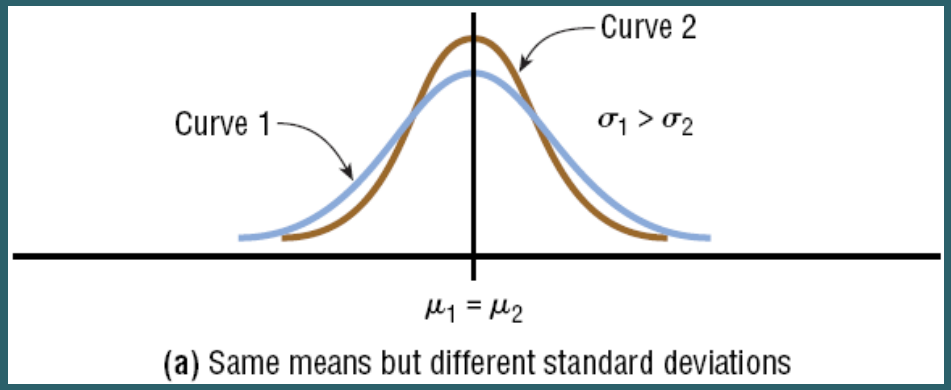
- The shape and position of the normal distribution curve depend on two parameters, **the mean** and **the standard deviation**.
- Each normally distributed variable has its own normal distribution curve, which depends on the values of the variable's mean and standard deviation.

Normal Distribution…

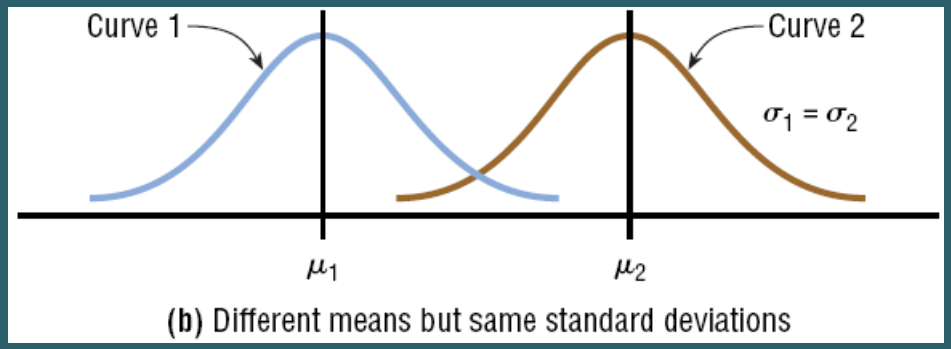
- If X is a random variable whose probability density function is normal with mean μ and variance σ^2, we write **$X \sim N(\mu, \sigma^2)$**.
- If **$X \sim N(\mu, \sigma^2)$**, then the mean and variance of X are given by

$$\mu_X = \mu$$
$$\sigma_X^2 = \sigma^2$$

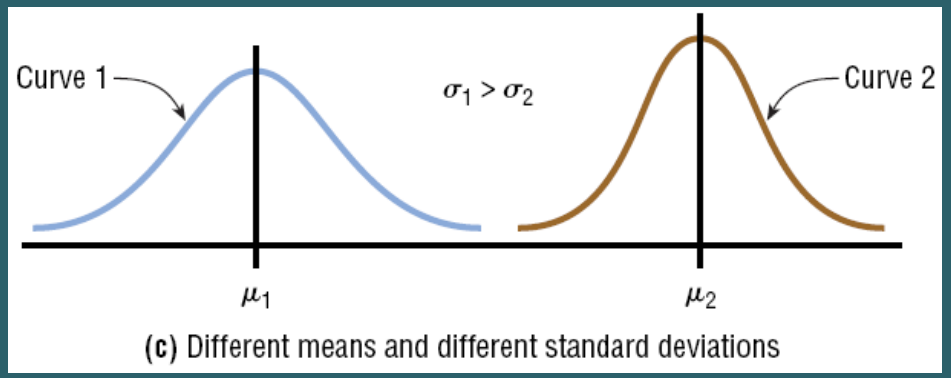
Normal Distribution…



(a) Same means but different standard deviations



(b) Different means but same standard deviations



(c) Different means and different standard deviations

Properties of the Normal Distribution

- The normal distribution curve is bell-shaped.
- The mean, median, and mode are equal and located at the centre of the distribution.
- The normal distribution curve is unimodal (i.e., it has only one mode).
- The curve is symmetrical about the mean, which is equivalent to saying that its shape is the same on both sides of a vertical line passing through the centre.

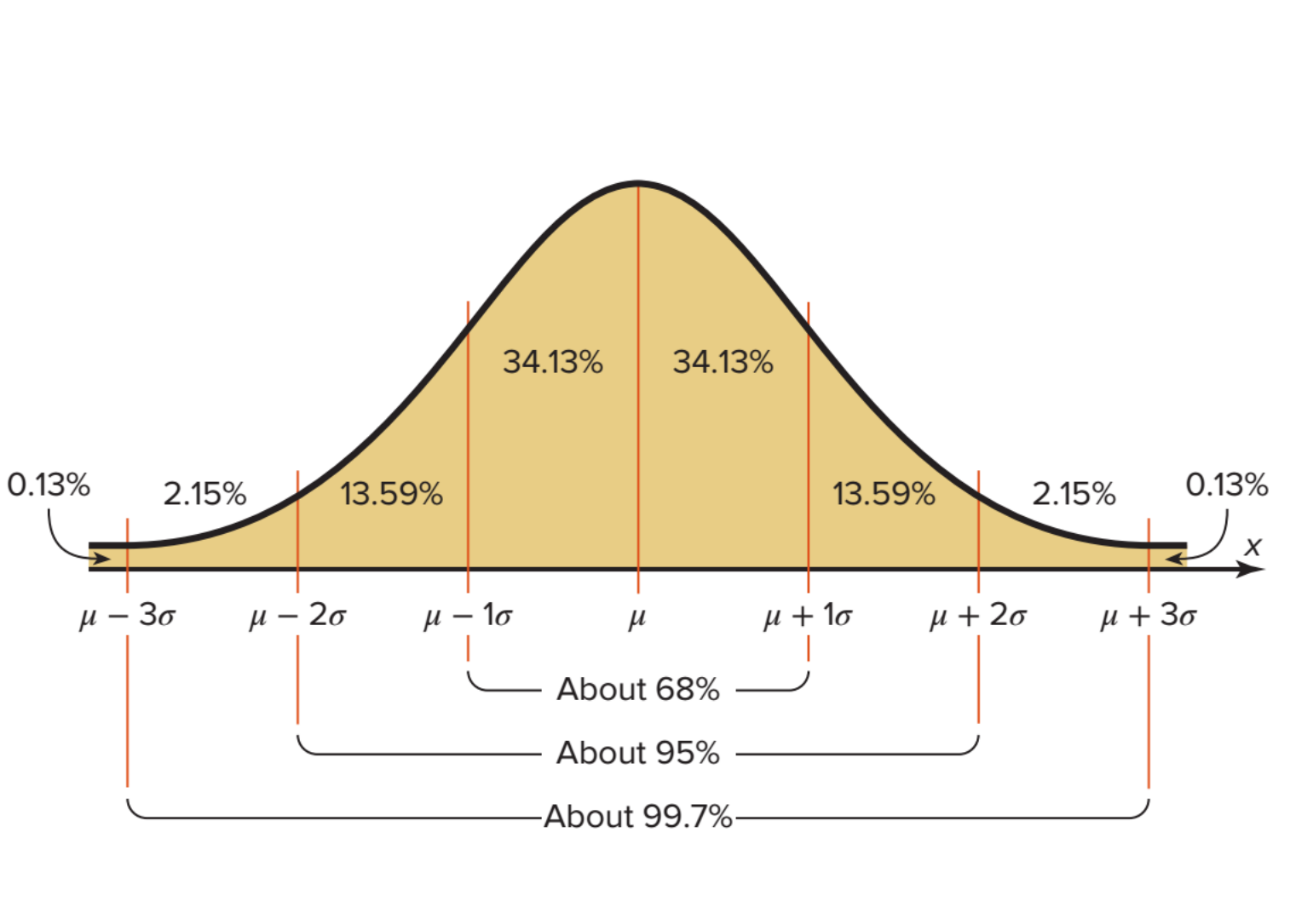
Properties of the Normal Distribution…

- The curve is continuous—there are no gaps or holes. For each value of X, here is a corresponding value of Y.
- The curve never touches the x axis. Theoretically, no matter how far in either direction the curve extends, it never meets the x axis—but it gets increasingly closer.

Properties of the Normal Distribution…

- The total area under the normal distribution curve is equal to 1.00 or 100%.
- The area under the normal curve that lies within
  - 1 standard deviation of the mean is approximately **0.68 (68%)**.
  - 2 standard deviations of the mean is approximately **0.95 (95%)**.
  - 3 standard deviations of the mean is approximately **0.997 (99.7%)**.

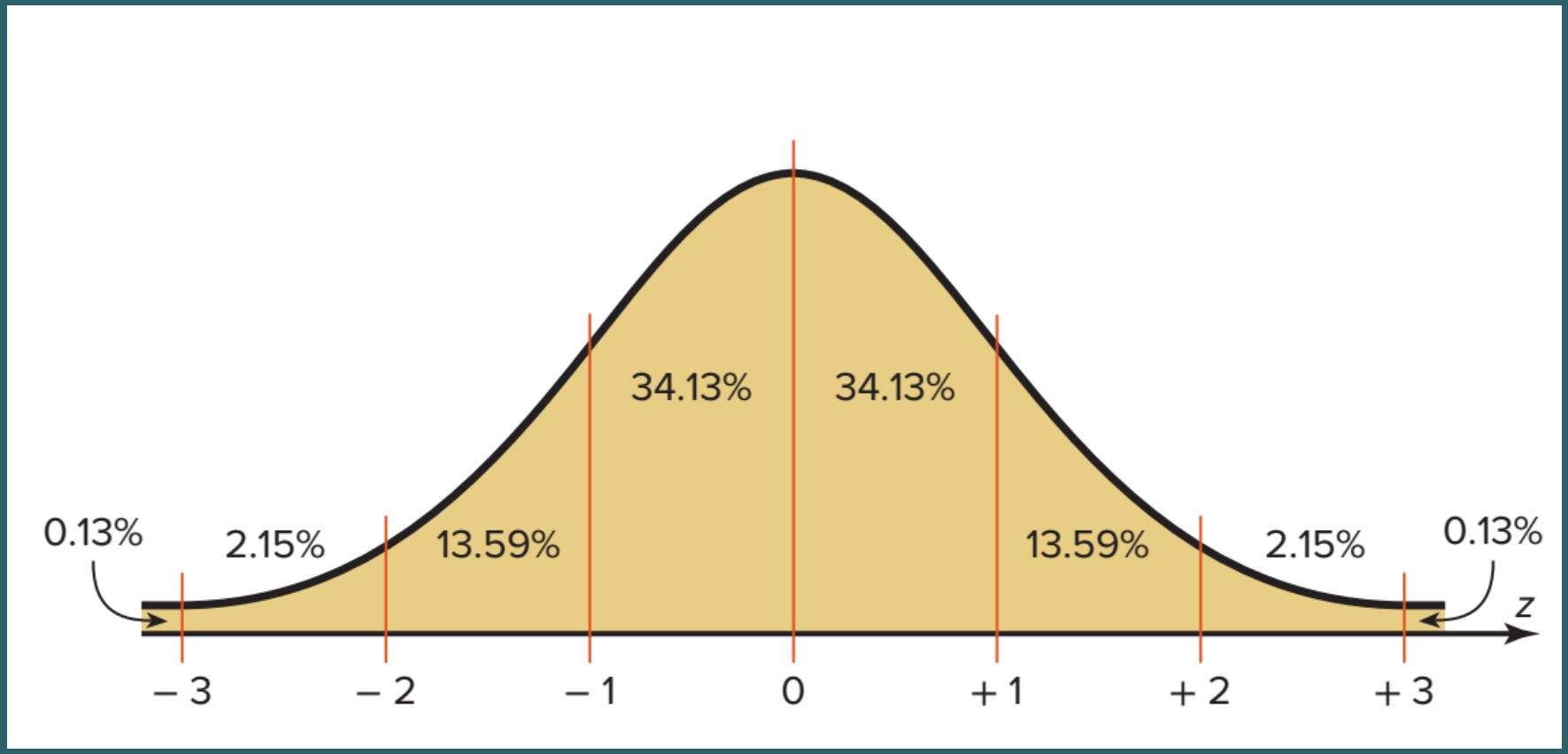
Properties of the Normal Distribution…



Standard Normal Distribution

- Since each normally distributed variable has its own mean and standard deviation, the shape and location of these curves will vary.
- In practical applications, one would have to have a table of areas under the curve for each variable. To simplify this, statisticians use the standard normal distribution.
- The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

Standard Normal Distribution



Standard Normal Distribution

The mathematical equation for a normal distribution is

$$y = \frac{e^{-(X-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$$

where $e \approx 2.718$ ($\approx$ means "is approximately equal to")

$\pi \approx 3.14$

$\mu =$ population mean

$\sigma =$ population standard deviation

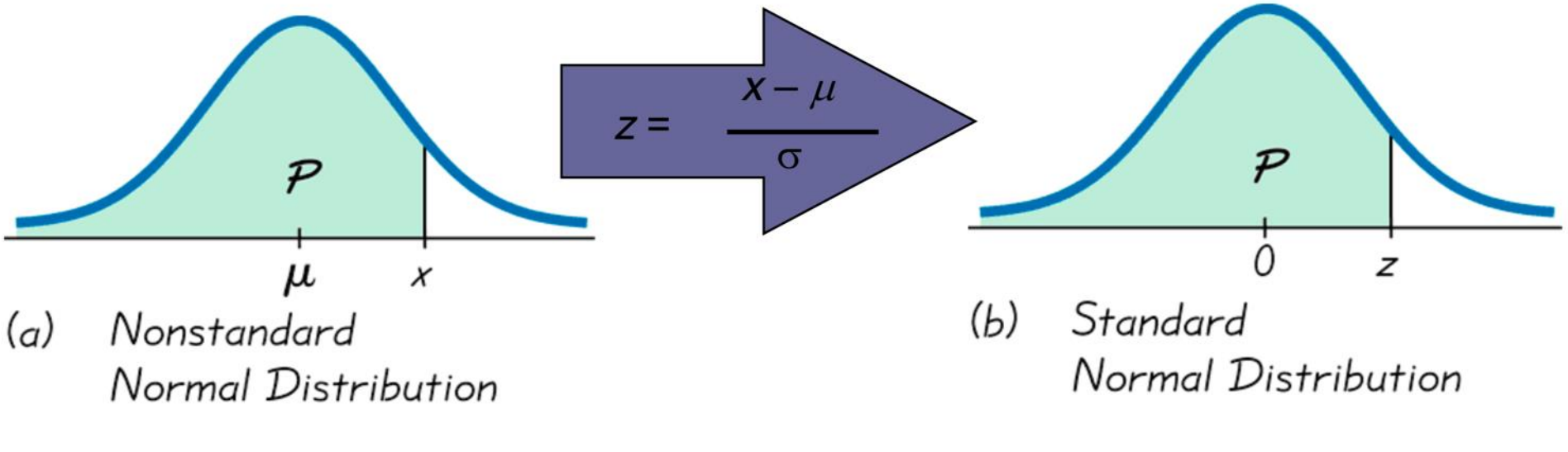
All normally distributed variables can be transformed into the standard normally distributed variable by using the formula for the standard score:

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} \quad \text{or} \quad z = \frac{X - \mu}{\sigma}$$

The formula for the standard normal distribution is

$$y = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

Converting to Standard Normal Distribution



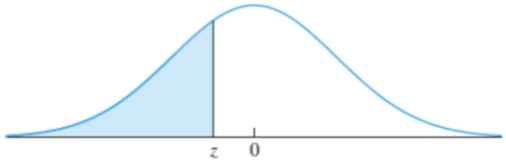
(a) Nonstandard Normal Distribution

(b) Standard Normal Distribution

Converting to Standard Normal Distribution (p.244)

- Areas under the standard normal distribution (mean 0, standard deviation 1) have been extensively tabulated.
- A typical such table, called a standard normal table, or z table, is given as **Table A.2** (in Appendix A).
- Table A.2 provides areas in the left-hand tail of the curve for values of z.
- Other areas can be calculated by subtraction or by using the fact that the total area under the curve is equal to 1.

TABLE A.2 Cumulative normal distribution (z table)

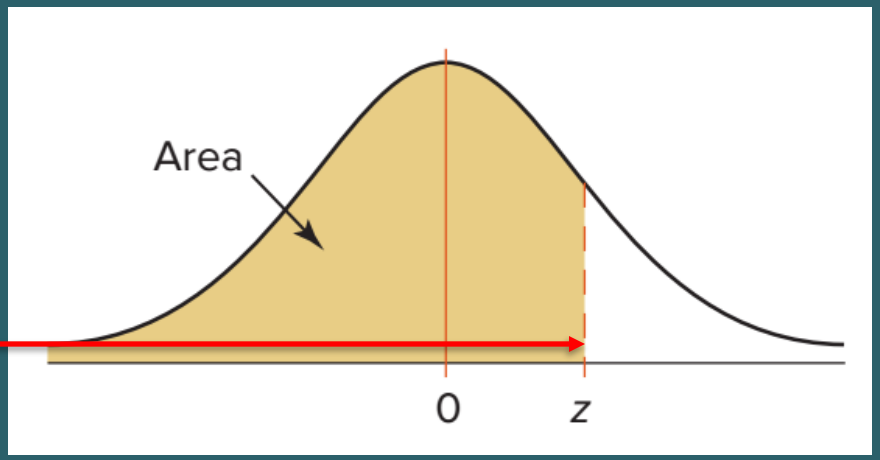


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Using Table A.2

1. It is designed only for **the standard normal distribution**, which has a mean of 0 and a standard deviation of 1.
2. It is on **two pages**, with one page for negative z-scores and the other page for positive z-scores.
3. Each value in the body of the table is a **cumulative area from the left** up to a vertical boundary above a specific z-score.



Using Table A.2…

4. When working with a graph, avoid confusion between z-scores and areas.

z-score

Distance along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table E.

Area

Region under the curve; refer to the values in the body of Table A.2.

5. The part of the z-score denoting hundredths is found across the top.

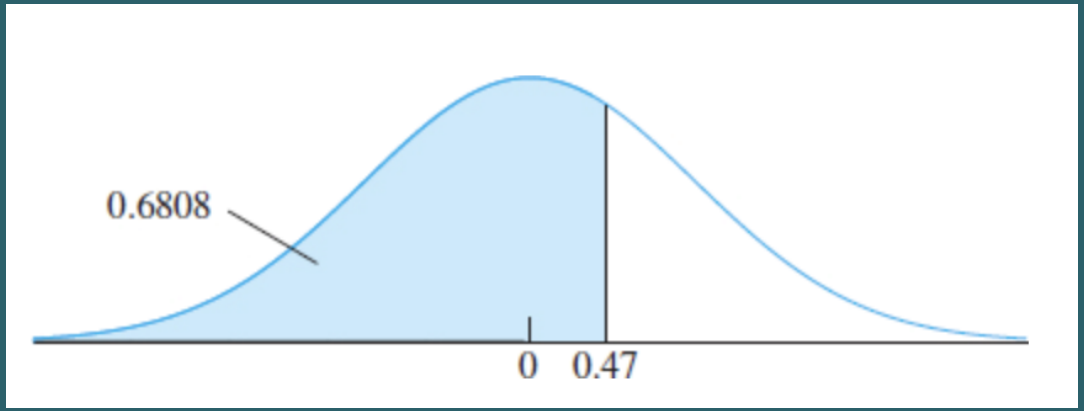
Example 4.41 (p.244)

Find the area under the normal curve to the left of z = 0.47

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599

Solution:

From the z-table, the area is **0.6808**



Example 4.42 (p.244)

Find the area under the normal curve to the right of z = 1.38

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429

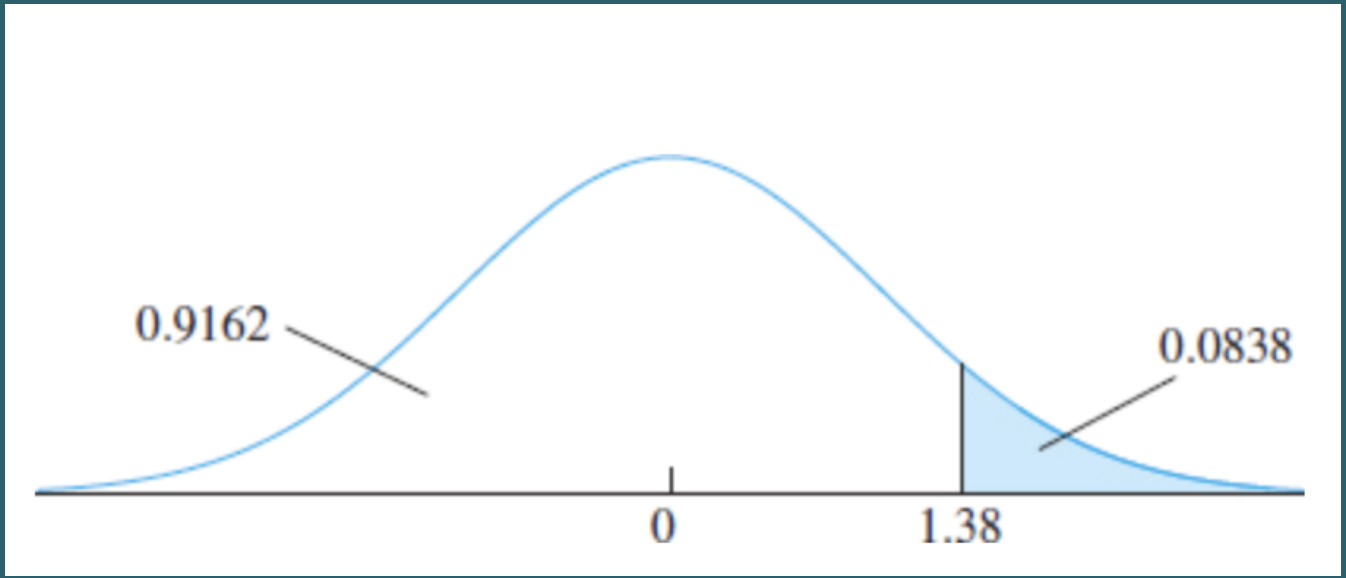
Solution:

From the z table, the area **to the left** of z = 1.38 is 0.9162

Therefore, the area to the right is **1 – 0.9162 = 0.0838**

Example 4.42 (p.244)

The area under the normal curve to the right of z = 1.38



Example 4.43 (p.245)

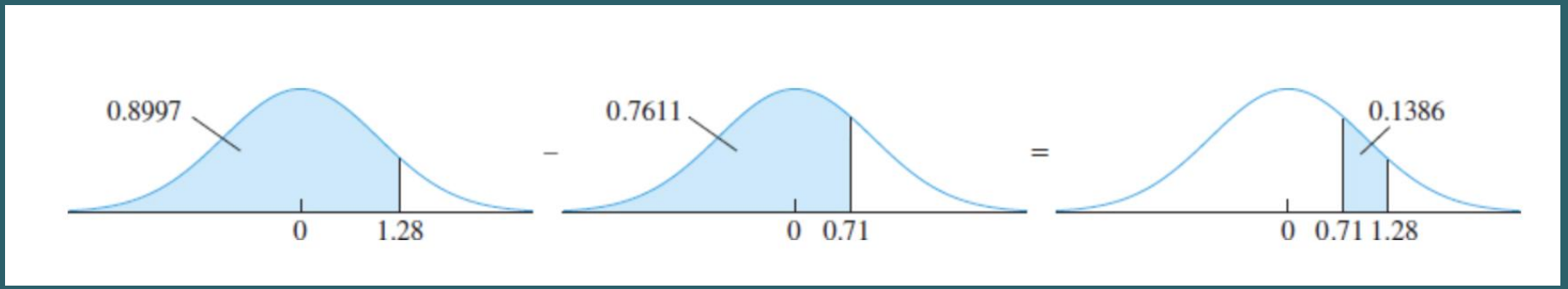
Find the area under the normal curve **between z = 0.71 and z = 1.28**.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429

Example 4.43 (p.245)

Solution:

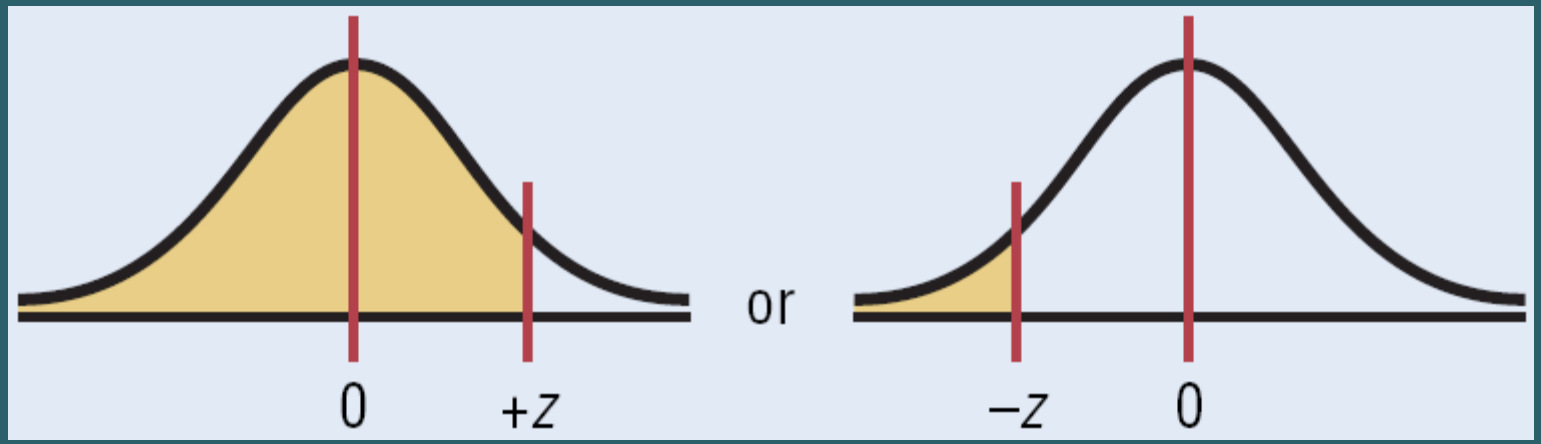
- From the z-table, the area to the left of z = 1.28 is 0.8997.
- The area to the left of z = 0.71 is 0.7611.
- The area between z = 0.71 and z = 1.28 is therefore **0.8997 – 0.7611 = 0.1386**.



Area under the Standard Normal Distribution Curve

1. To the left of any z value:

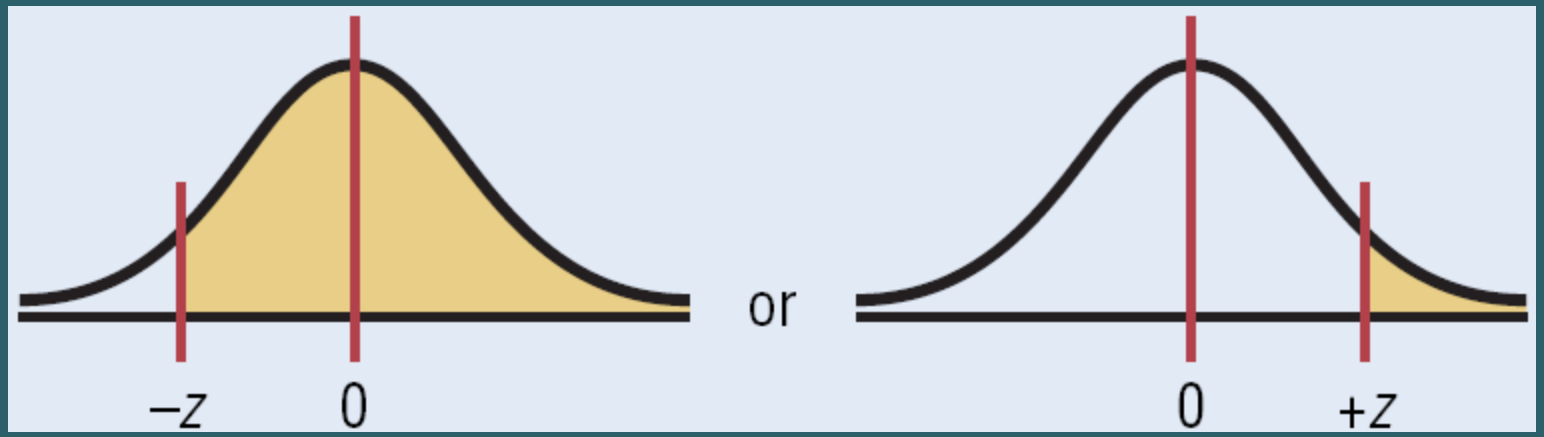
Look up the z value in the table and use the area given.



Area under the Standard Normal Distribution Curve

2. To the right of any z value:

Look up the z value and subtract the area from 1.



Area under the Standard Normal Distribution Curve

3. Between any 2 z values:

Look up both z values and subtract the corresponding areas.

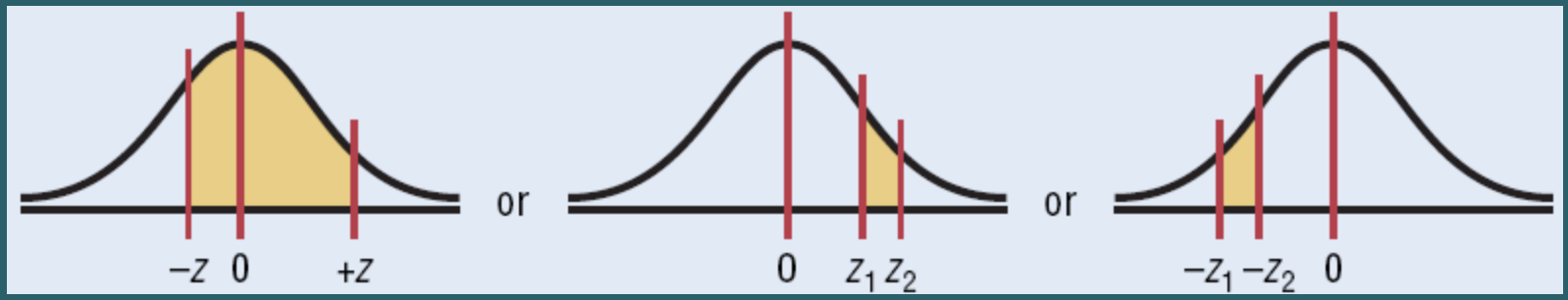


Table E (Appendix A) from "Bluman's Elementary Statistics"

Table E The Standard Normal Distribution

Cumulative Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233

End of Section 4.5