ENSC 2113
Engineering Mechanics: Statics

Chapter 2:

Force Vectors

(Section 2.1-2.4)

COLLEGE OF
ENGINEERING, ARCHITECTURE
AND TECHNOLOGY

Chapter 2 Outline:

2.1 Scalars and Vectors

2.2 Vector Operations

2.3 Vector Addition of Forces

2.4 Addition of a System of Coplanar Forces

2.5 Cartesian Vectors

2.6 Addition of Cartesian Vectors

2.7 Position Vectors

2.8 Force Vector Directed Along a Line

2.9 Dot Product

Chapter 2 Objectives:

- To show how to add forces and resolve them into components using the Parallelogram Law
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another

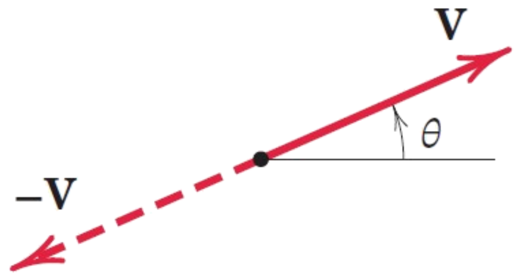
2.1 Scalars and Vectors:

- Scalar – quantity where only magnitude is associated
  - Examples: distance, time, volume density, speed, energy, and mass
  - Magnitude of a force:

$$|F| = 50N$$

2.1 Scalars and Vectors:

- Vector – quantity that has magnitude as well as direction
  - Examples: displacement, velocity, acceleration, force, moment, and momentum
  - Drawing Vectors:
    - Line Segment with an Arrowhead to Indicate Direction
    - Reference Angle θ



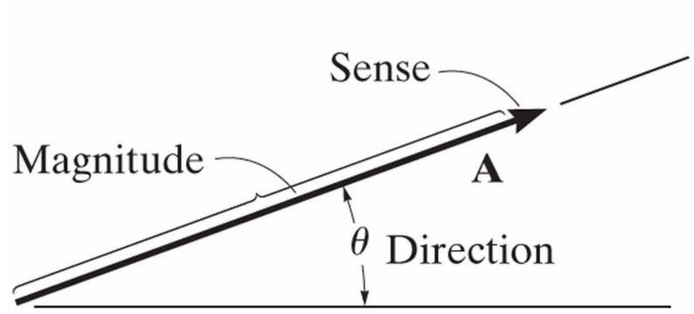
2.1 Scalars and Vectors:

Scalar: Quantity w/ magnitude (+ or -)

$$|F| = scalar$$

Vector: Quantity w/ magnitude & direction

$$\vec{F} = vector$$

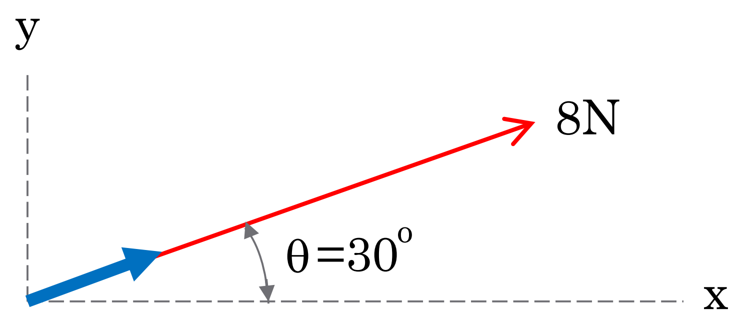


Unit Vector: Vector w/ magnitude of ONE.

Since a force has both magnitude & direction, it can be represented as a vector.

2.1 Scalars and Vectors:

- Unit Vector – vector with a magnitude of ***one*** (unity)



  - The unit vector of the 8N force is equal to one unit in the same direction of the force

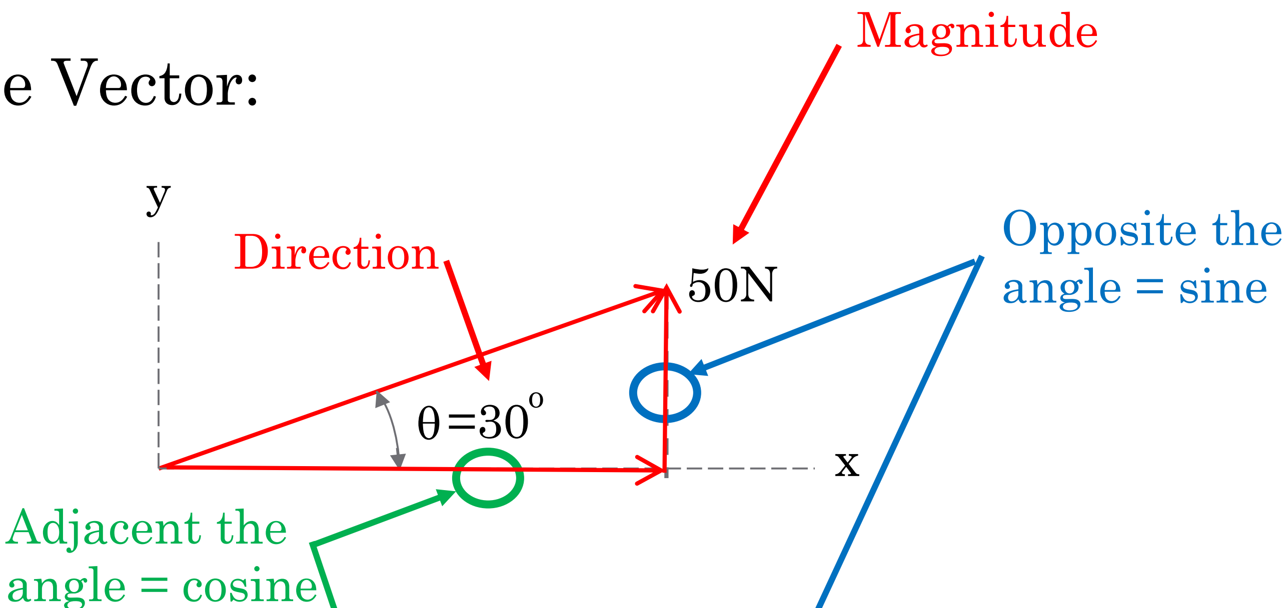
$$\vec{u} = \{\cos 30\,\hat{\imath} + \sin 30\,\hat{\jmath}\}$$

  - Magnitude of unit vector

$$|u| = \sqrt{(\cos 30)^2 + (\sin 30)^2} = 1$$

2.1 Scalars and Vectors:

- Scalar:
- Force Vector:



- Components of Vector:

$$\vec{F} = \{50\cos 30\,\hat{\imath} + 50\sin 30\,\hat{\jmath}\}N$$

$$\vec{F} = \{43.3\hat{\imath} + 35\hat{\jmath}\}N$$

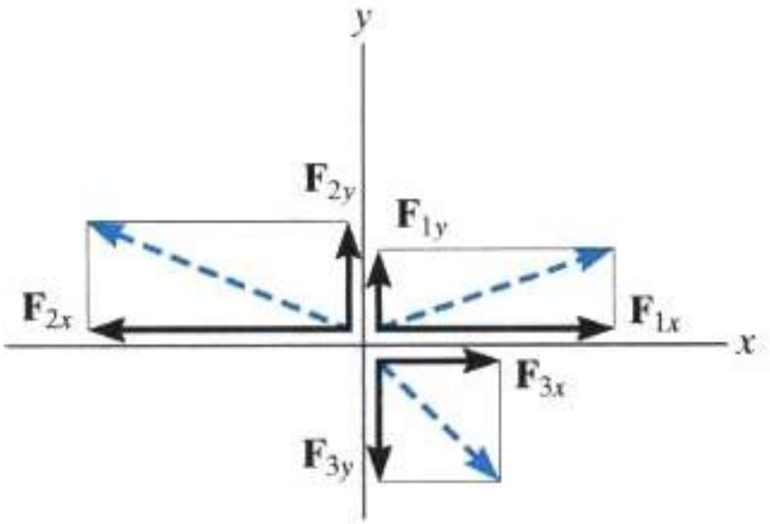
2.3-2.4 Addition of Vectors:

Resultant of Vectors: Combining two or more vectors into a single vector.

$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2}$$

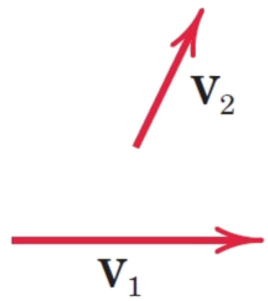
Resolution of a Vector: To describe a single vector as two or more vectors, called "*component form*"

$$\vec{F} = \{\overrightarrow{F_x}\hat{\imath} + \overrightarrow{F_y}\hat{\jmath}\}$$

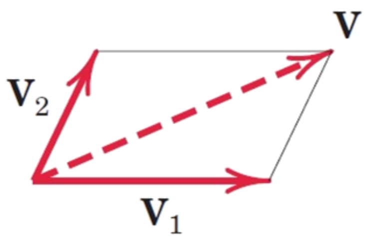


2.3-2.4 Addition of Vectors:

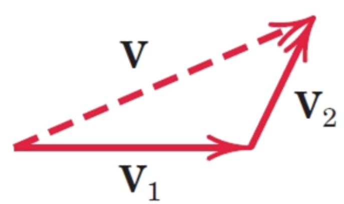
- Vector addition by parallelogram law
  - Given two vectors, V_1 and V_2



  - The sum of these vectors is V and creates the diagonal of a parallelogram

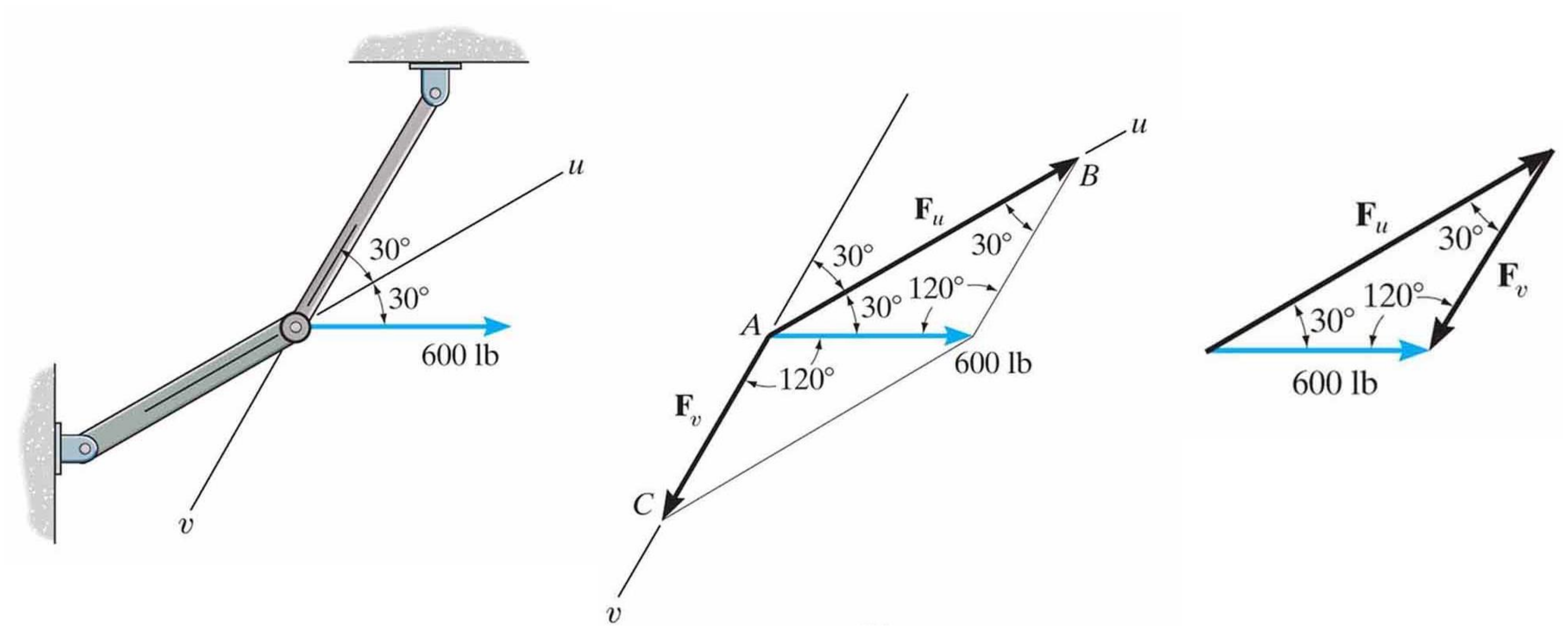


  - Either triangle may be extracted to find V



2.3-2.4 Addition of Vectors:

Parallelogram Law: This method may also be used to find component of force and is useful when the axes are not at a right angle.



Sine Law:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

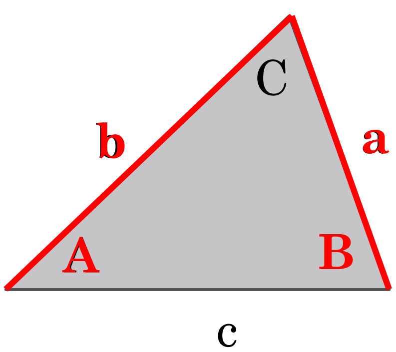
Cosine Law:

$$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

2.3-2.4 Addition of Vectors:

- Law of Sines and Law of Cosines help solve unknowns
  - Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



    - Used when two sides and at least one opposite angle is known
    - Used when two angles and at least one opposite side is known

2.3-2.4 Addition of Vectors:

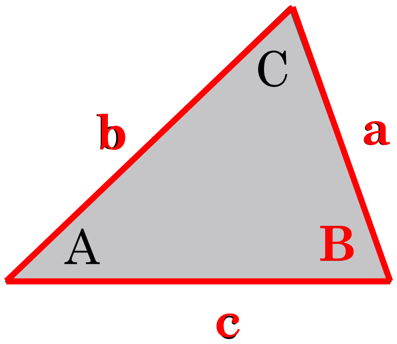
- Law of Sines and Law of Cosines help solve unknowns
  - Law of Cosines

    $$a = \sqrt{b^2 + c^2 - 2bc\cos A}$$

    $$b = \sqrt{a^2 + c^2 - 2ac\cos B}$$

    $$c = \sqrt{a^2 + b^2 - 2ab\cos C}$$

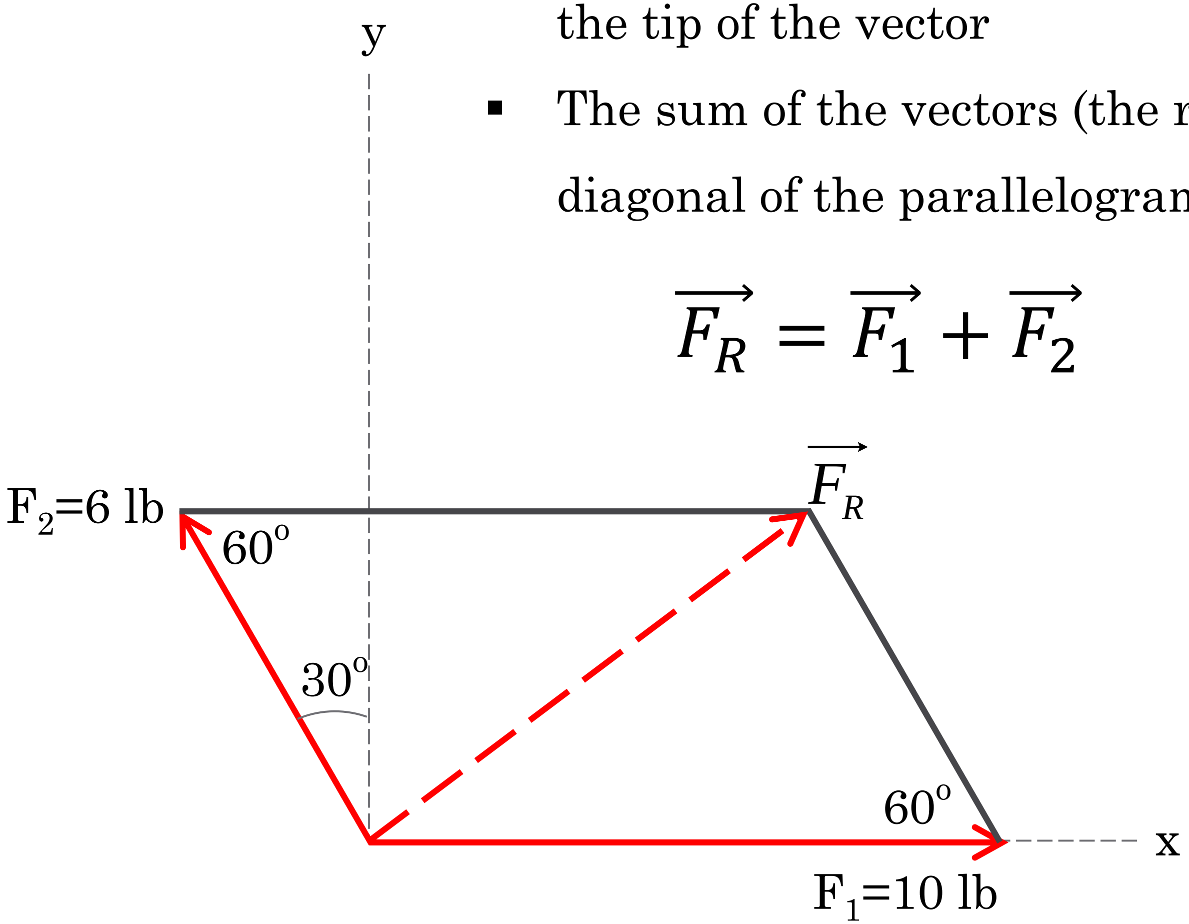
    - Used when all three sides are known
    - Used when two sides and their adjoining angle are known



2.3-2.4 Addition of Vectors:

- Example: Vector addition by parallelogram law
  - Draw lines parallel to the vectors starting at the tip of the vector
  - The sum of the vectors (the resultant) is the diagonal of the parallelogram

$$\overrightarrow{F_R} = \overrightarrow{F_1} + \overrightarrow{F_2}$$



2.3-2.4 Addition of Vectors:

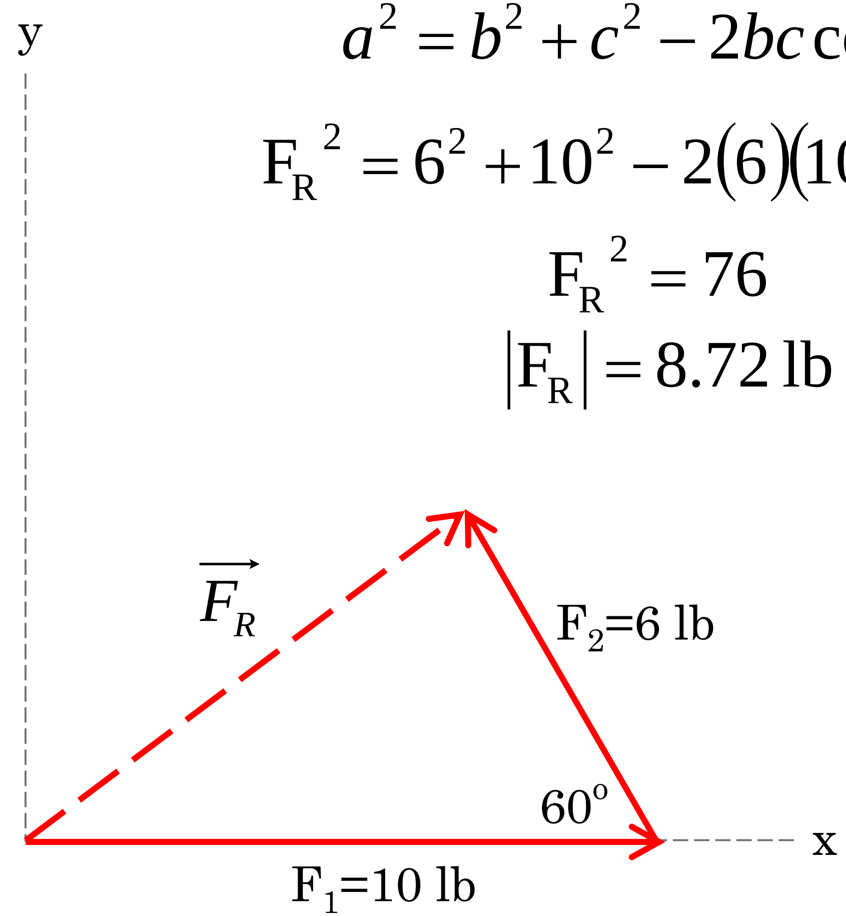
- Example: Vector addition by parallelogram law
  - Use law of cosines to solve for the resultant

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$F_R{}^2 = 6^2 + 10^2 - 2(6)(10)\cos(60)$$

$$F_R{}^2 = 76$$

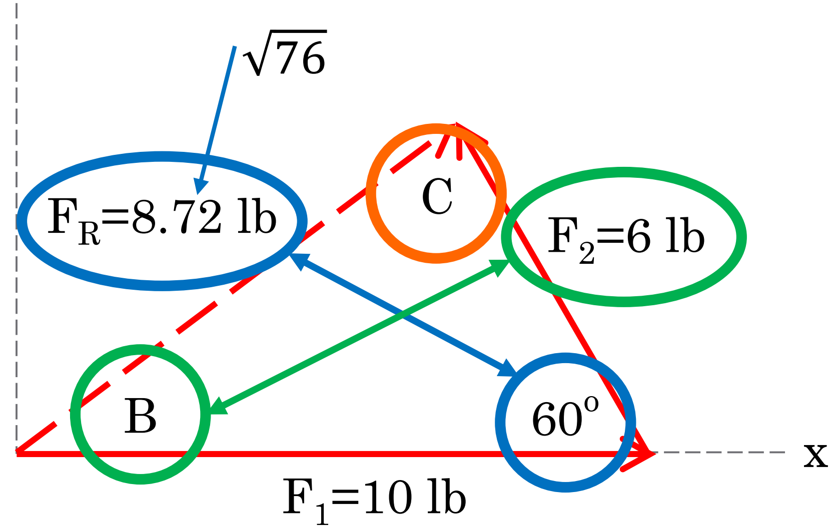
$$|F_R| = 8.72\text{ lb}$$



<u>2.3-2.4 Addition of Vectors:</u>

- Example: Vector addition by parallelogram law
  - Use law of sines to solve for angles B and C

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\frac{6}{\sin B} = \frac{8.72}{\sin 60}$$

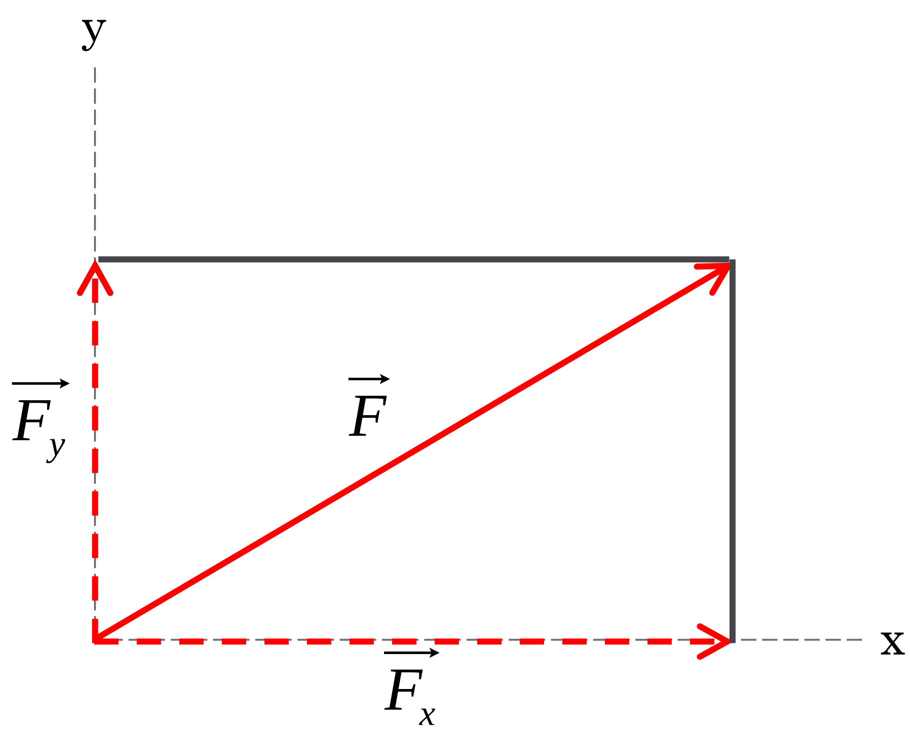
$$(6)(\sin 60) = (8.72)(\sin B)$$

$$\sin^{-1}\frac{(6)(\sin 60)}{8.72} = B$$

$$B = 36.6^{\circ}$$

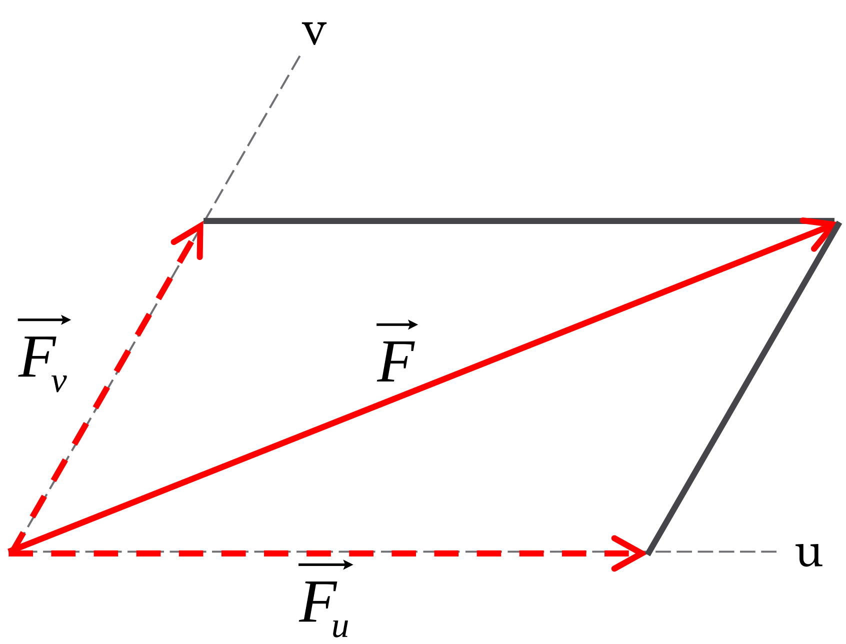
2.3-2.4 Addition of Vectors:

- Parallelogram law may be used to find rectangular components and nonrectangular components
  - Rectangular



2.3-2.4 Addition of Vectors:

- Parallelogram law may be used to find rectangular components and nonrectangular components
  - Nonrectangular

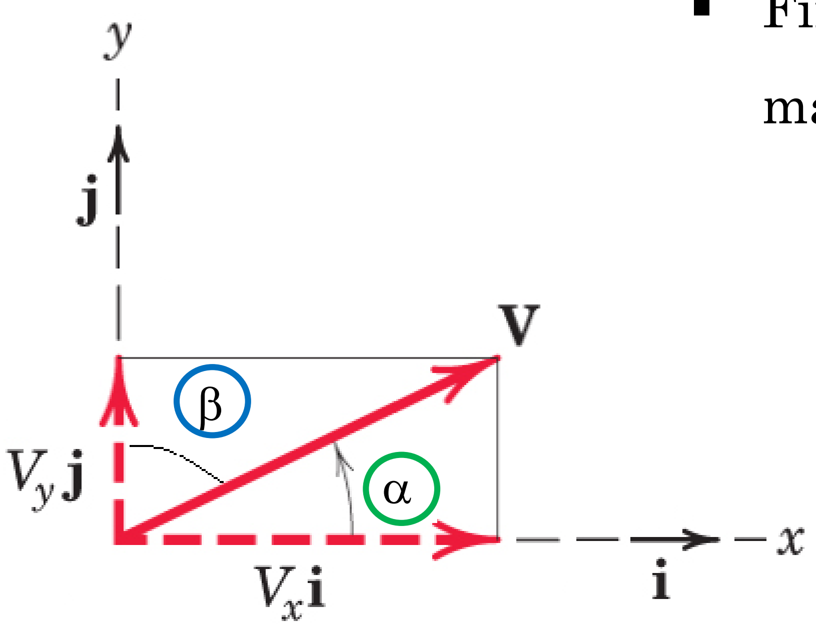


2.3-2.4 Addition of Vectors:

- Direction Angles or *Direction Cosines*
  - 2-D

    α is the angle between the vector and the positive x-axis

    β is the angle between the vector and the positive y-axis



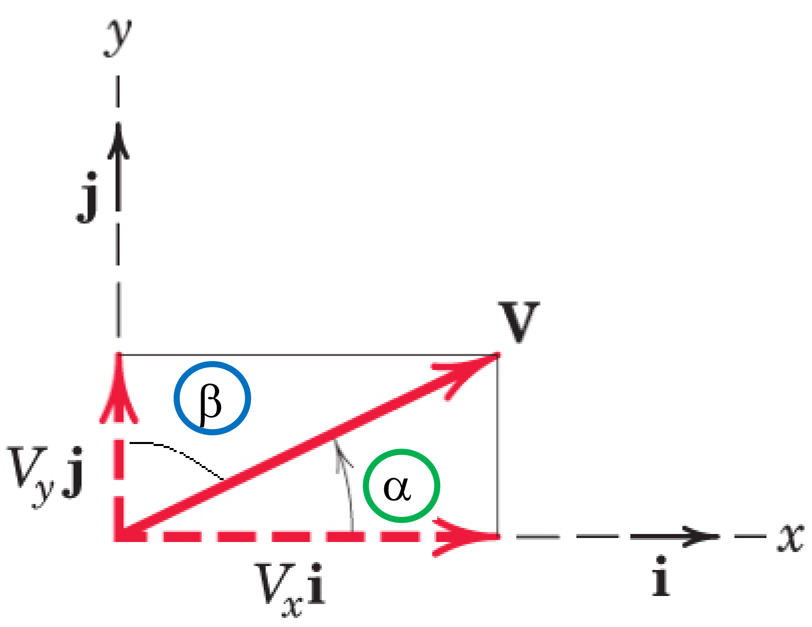
- Finding the components from the magnitude and direction angles:

$$V_x = V\cos\alpha$$
$$V_y = V\cos\beta$$
$$\vec{V} = \{V\cos\alpha\,\hat{\imath} + V\cos\beta\,\hat{\jmath}\}$$

Note: angles >90 degrees produce negative components

2.3-2.4 Addition of Vectors:

- Direction Angles or *Direction Cosines*
  - 2-D



- Finding the direction angles from the magnitude and components

$$V_x = V\cos\alpha$$

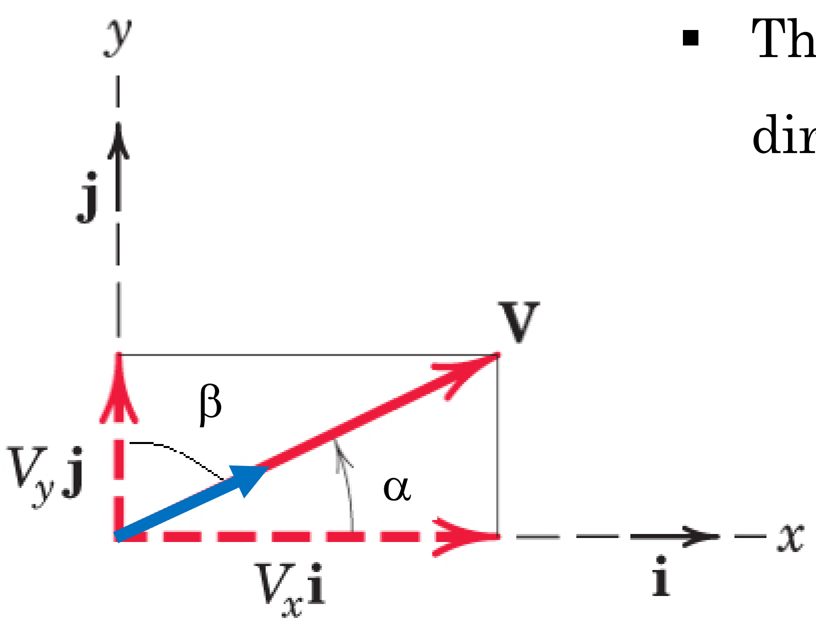
$$\alpha = cos^{-1}\frac{V_x}{V}$$

$$V_y = V\cos\beta$$

$$\beta = cos^{-1}\frac{V_y}{V}$$

2.3-2.4 Addition of Vectors:

- Unit Vector – using *Direction Cosines*
  - 2-D



- Force vector:

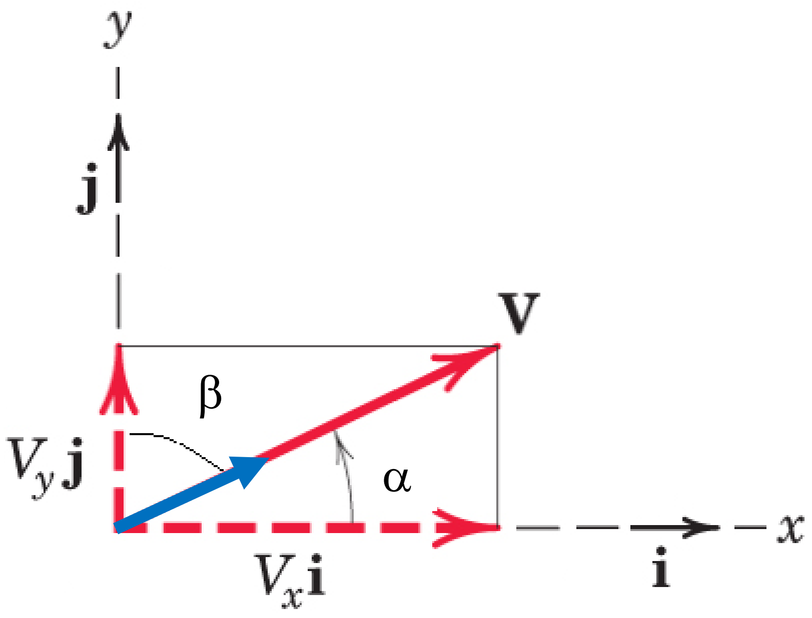
$$\vec{V} = \{V\cos\alpha\,\hat{\imath} + V\cos\beta\,\hat{\jmath}\}$$

- The unit vector gives the direction of the vector:

$$\vec{u} = \{\cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath}\}$$

2.3-2.4 Addition of Vectors:

- Unit vector – using magnitude and components
  - 2-D



- The unit vector is the vector divided by its magnitude:

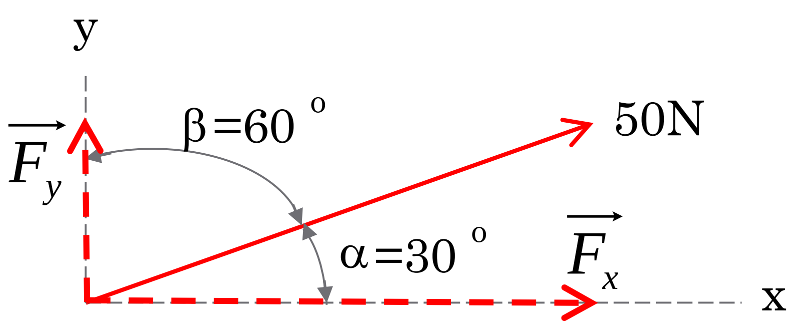
$$\vec{V} = \{V_x\hat{\imath} + V_y\hat{\jmath}\}$$

$$|V| = \left\{\sqrt{{V_x}^2 + {V_y}^2}\right\}$$

$$\vec{u} = \left\{\frac{V_x}{|V|}\hat{\imath} + \frac{V_y}{|V|}\hat{\jmath}\right\}$$

2.3-2.4 Vector Addition:

- Example: Vectors



- Finding the components from the magnitude and direction angles:

$$F_x = F\cos\alpha \;= 50\cos 30 = 43.3N$$

$$F_y = F\cos\beta \;= 50cos60 = 25N$$

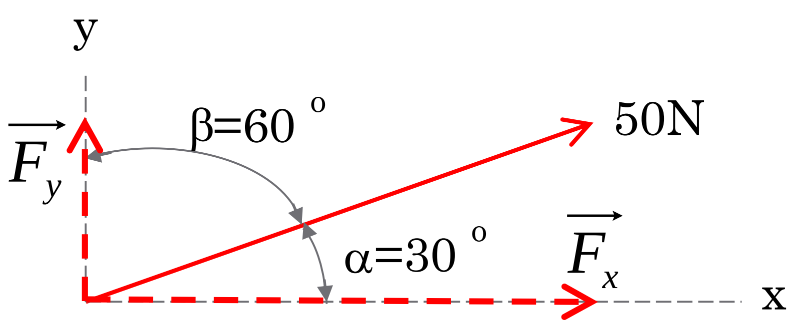
- Finding the direction angles from the magnitude and components:

$$\alpha = \cos^{-1}\frac{43.3}{50} = 30$$

$$\beta = \cos^{-1}\frac{25}{50} = 60$$

2.3-2.4 Vector Addition:

- Example: Vectors



- Unit vector from magnitude and components:

$$\vec{u} = \left\{ \frac{F_x}{F}\hat{i} + \frac{F_y}{F}\hat{j} \right\}$$

$$\vec{u} = \left\{ \frac{43.3}{50}\hat{i} + \frac{25}{50}\hat{j} \right\}$$

$$\vec{u} = \left\{ 0.866\hat{i} + 0.5\hat{j} \right\}$$

- Force magnitude

$$|F| = \sqrt{(F_x)^2 + \left(F_y\right)^2 + (F_z)^2}$$

$$|F| = 50\text{N}$$

- Force in Cartesian Vector Form:

$$\vec{F} = \left\{ 43.3\hat{i} + 25\hat{j} \right\} N$$

- Unit vector from angles:

$$\vec{u} = \left\{ \cos 30\hat{i} + \cos 60\hat{j} \right\}$$

$$\vec{u} = \left\{ 0.866\hat{i} + 0.5\hat{j} \right\}$$

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