西南交大學
Southwest Jiaotong University

Physics 2: **Electricity , Optics and Quanta**

Week 4 - Kirchhoff’s rules and battery

2023.9

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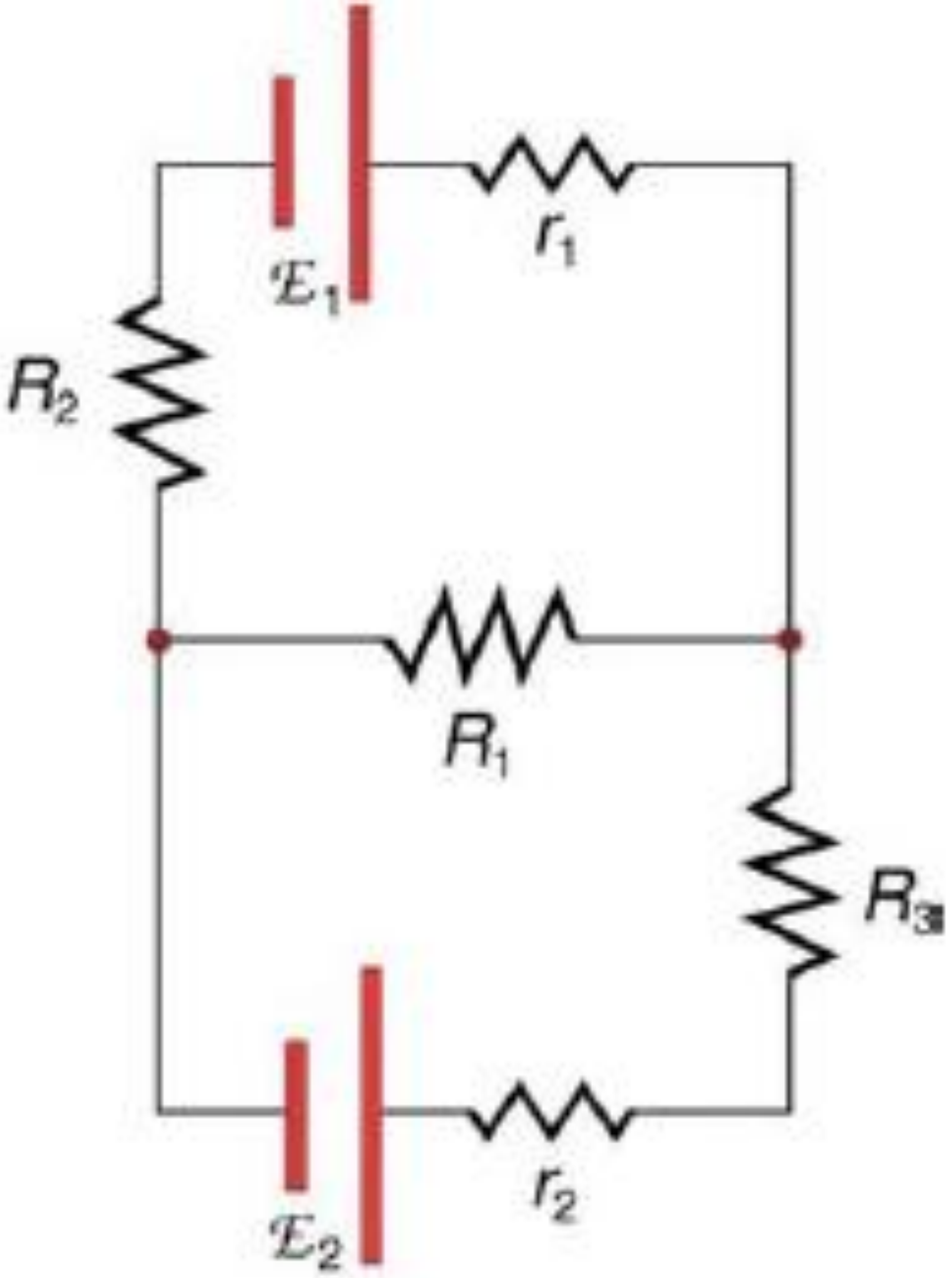
Kirchhoff’s rules

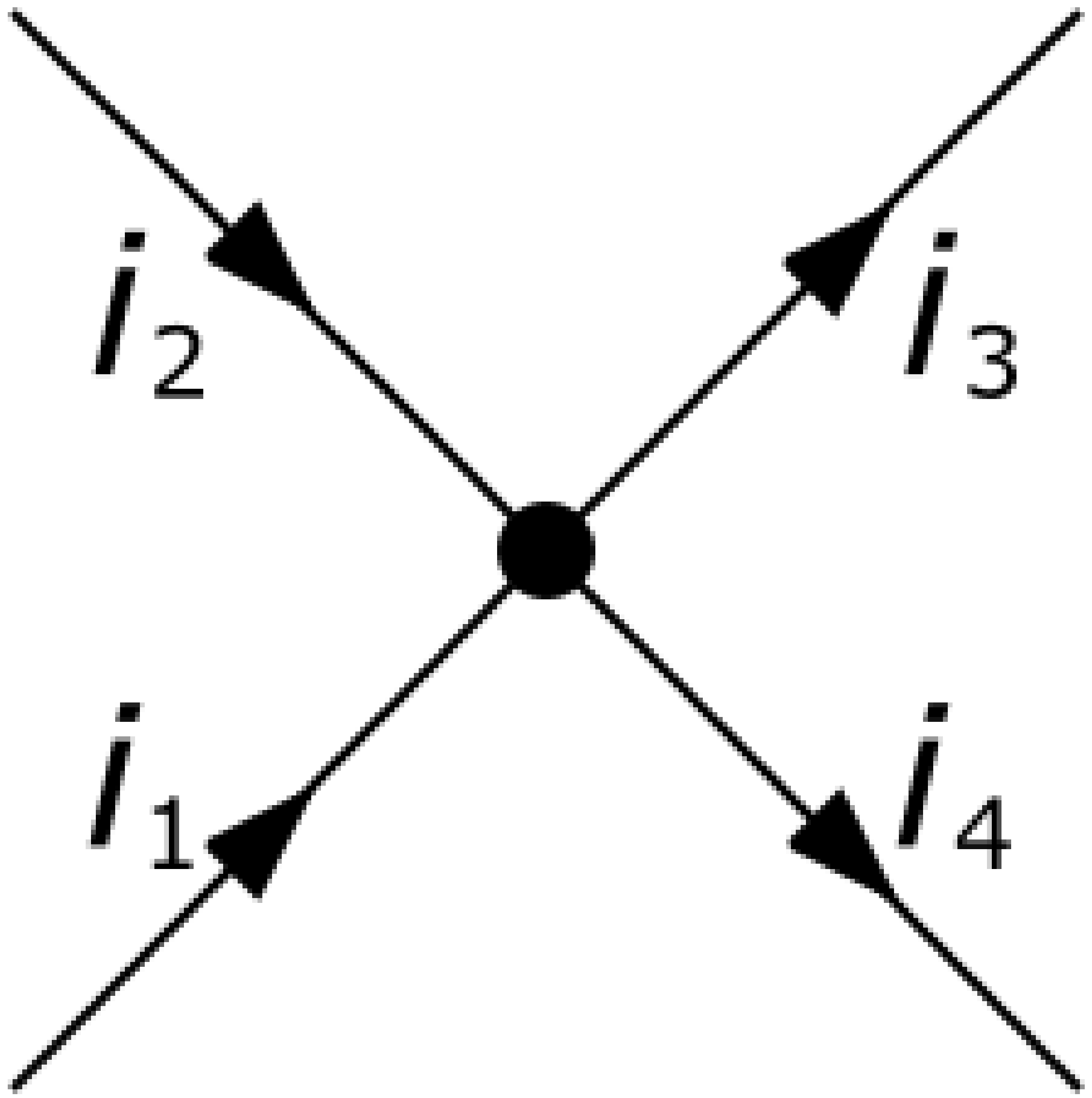


What is the current in the different resistors?

Not easy to analyze by reducing to resistors in series and parallel

- **Use Kirchhoff’s rules**
  - **junction rule**
  - **loop rule**

Kirchhoff's rules 1 - Junction rule



$$i_1 + i_2 = i_3 + i_4$$

The total current that

enters = leaves

the junction

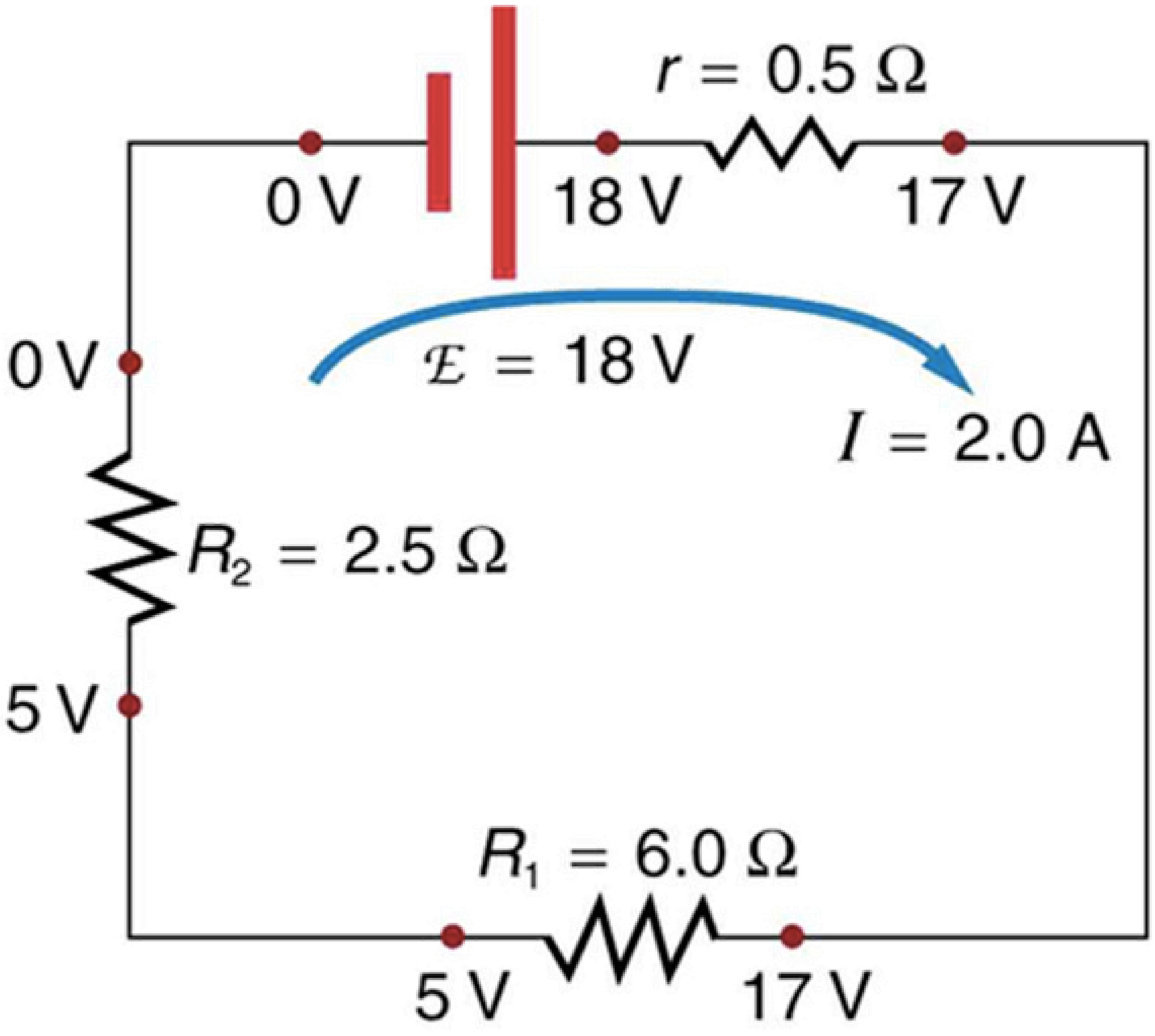
- **Conservation of charge !**

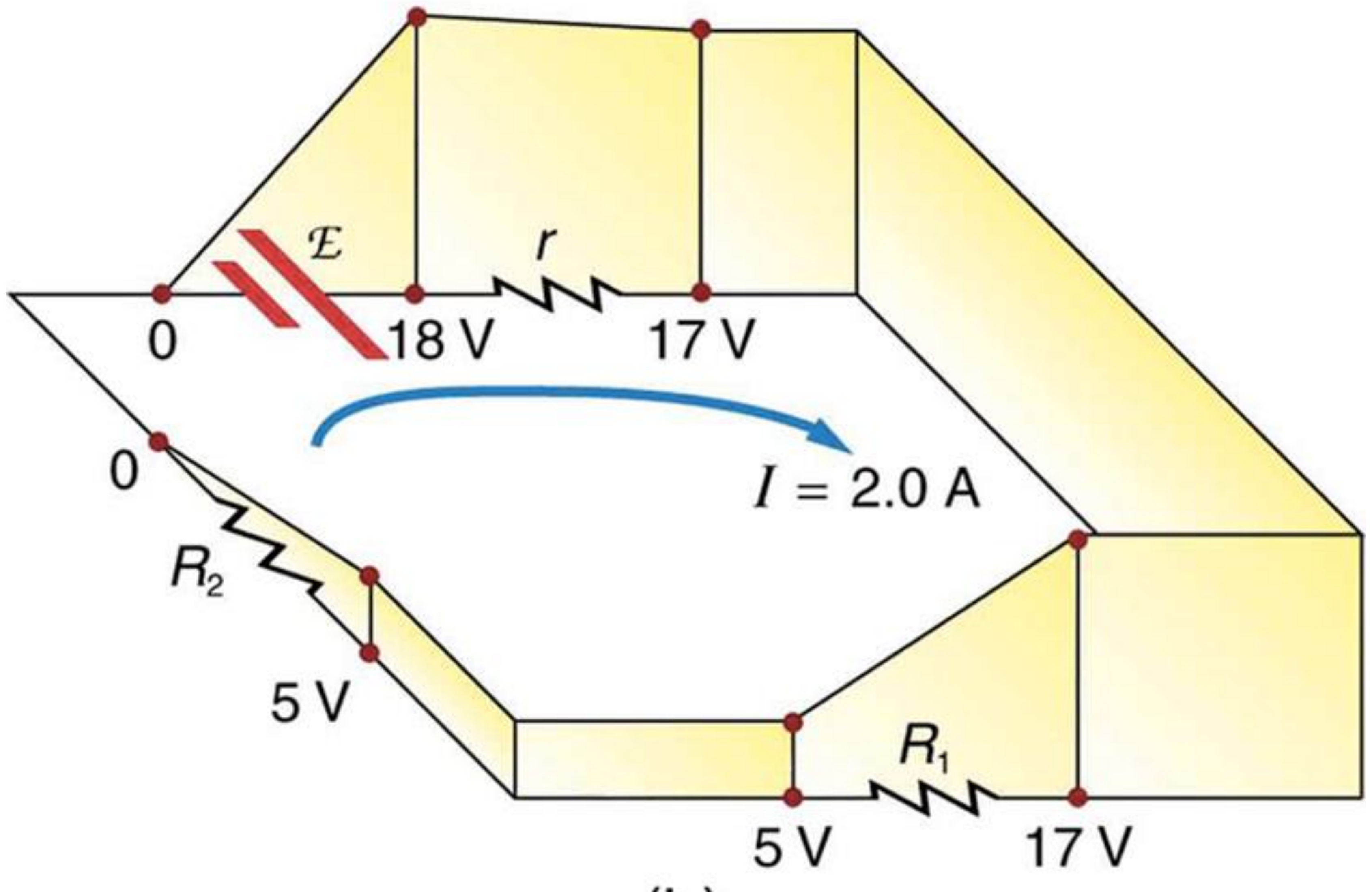
Of course, this is only true in a "steady state"

Kirchhoff's rules 2 - Loop rule

$$V_{loop} = 0$$

The sum of the potential differences (positive and negative) around any loop in a circuit must be zero





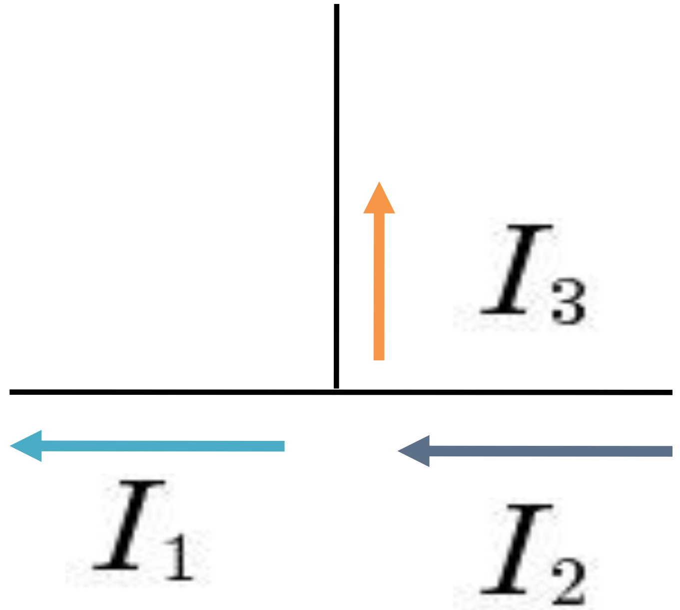
Rules for loop rule

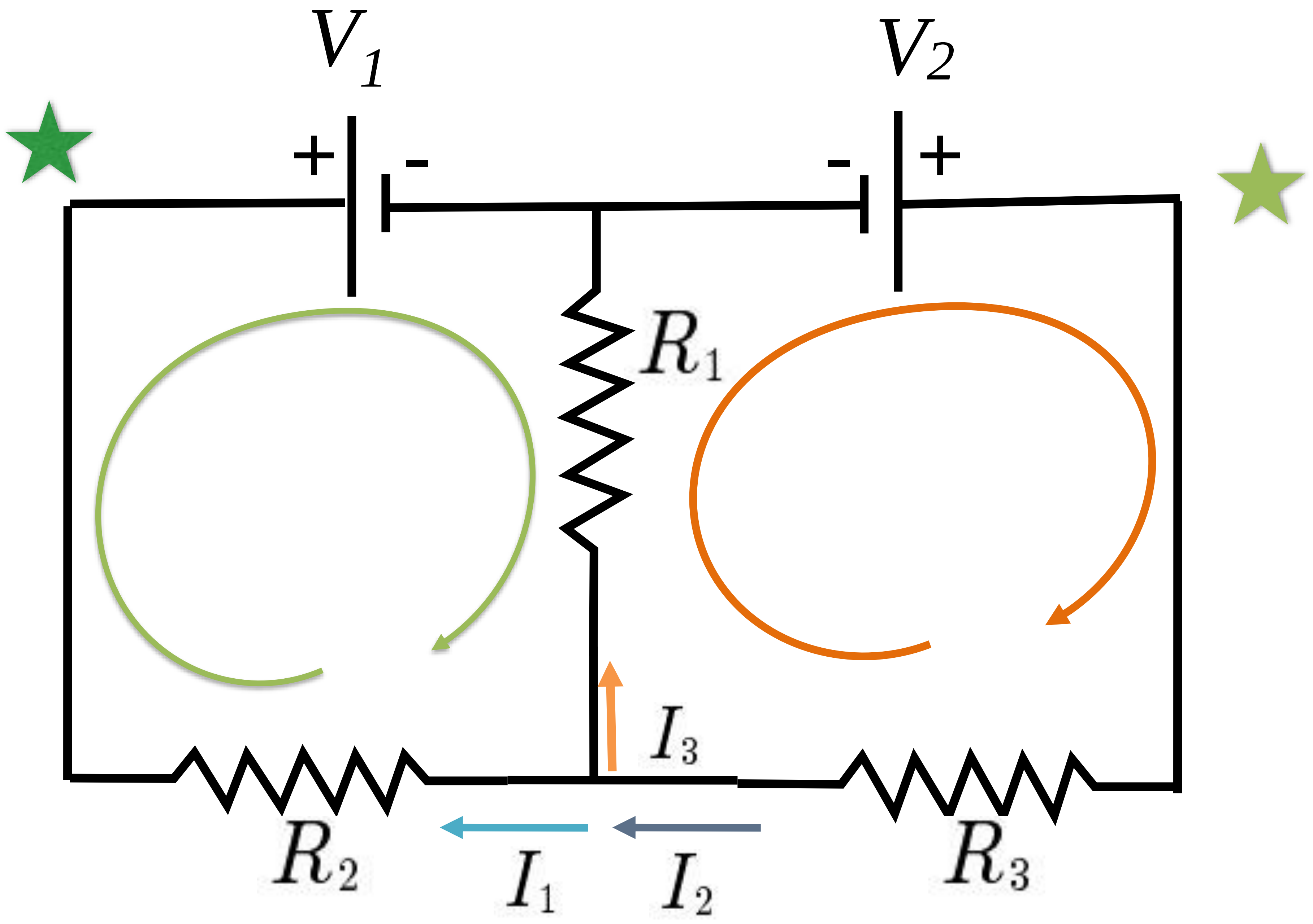
- When going across a resistor in the same direction as the current, the potential drops by IR
- When going across a resistor in the opposite direction as the current, the potential increases by IR

- When going from the negative to the positive terminal of a source of voltage, the potential increases by V
- When going from the positive to the negative terminal of a source of voltage, the potential decreases by V

Example

Junction: rule 1 !





$$I_3 = I_2 - I_1$$

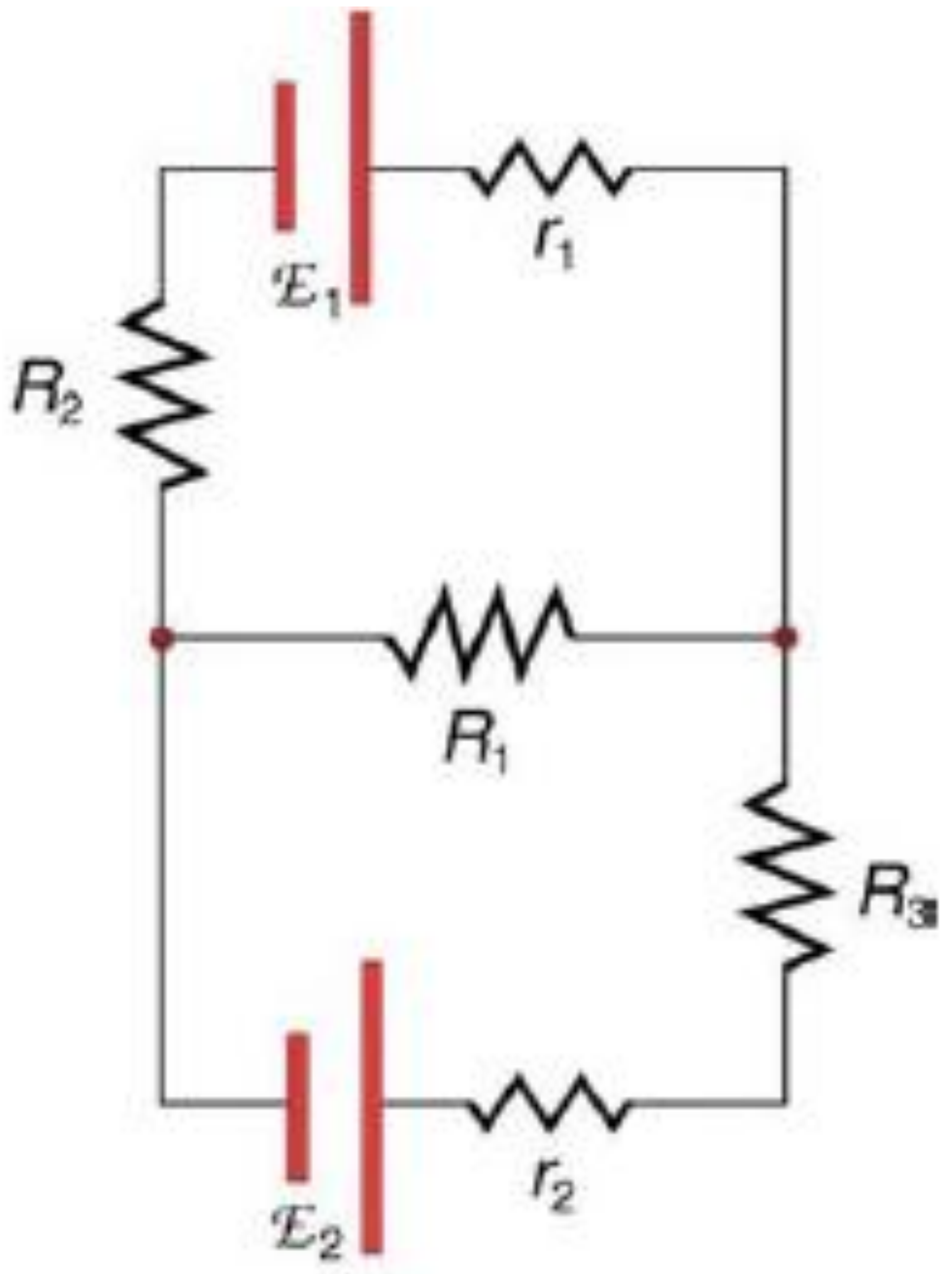
$$-V_1 + R_1 I_3 - R_2 I_1 = 0$$

$$-R_3 I_2 - R_1 I_3 + V_2 = 0$$

3 equations

3 unknowns (I_1, I_2 and I_3)

Kirchhoff's rules



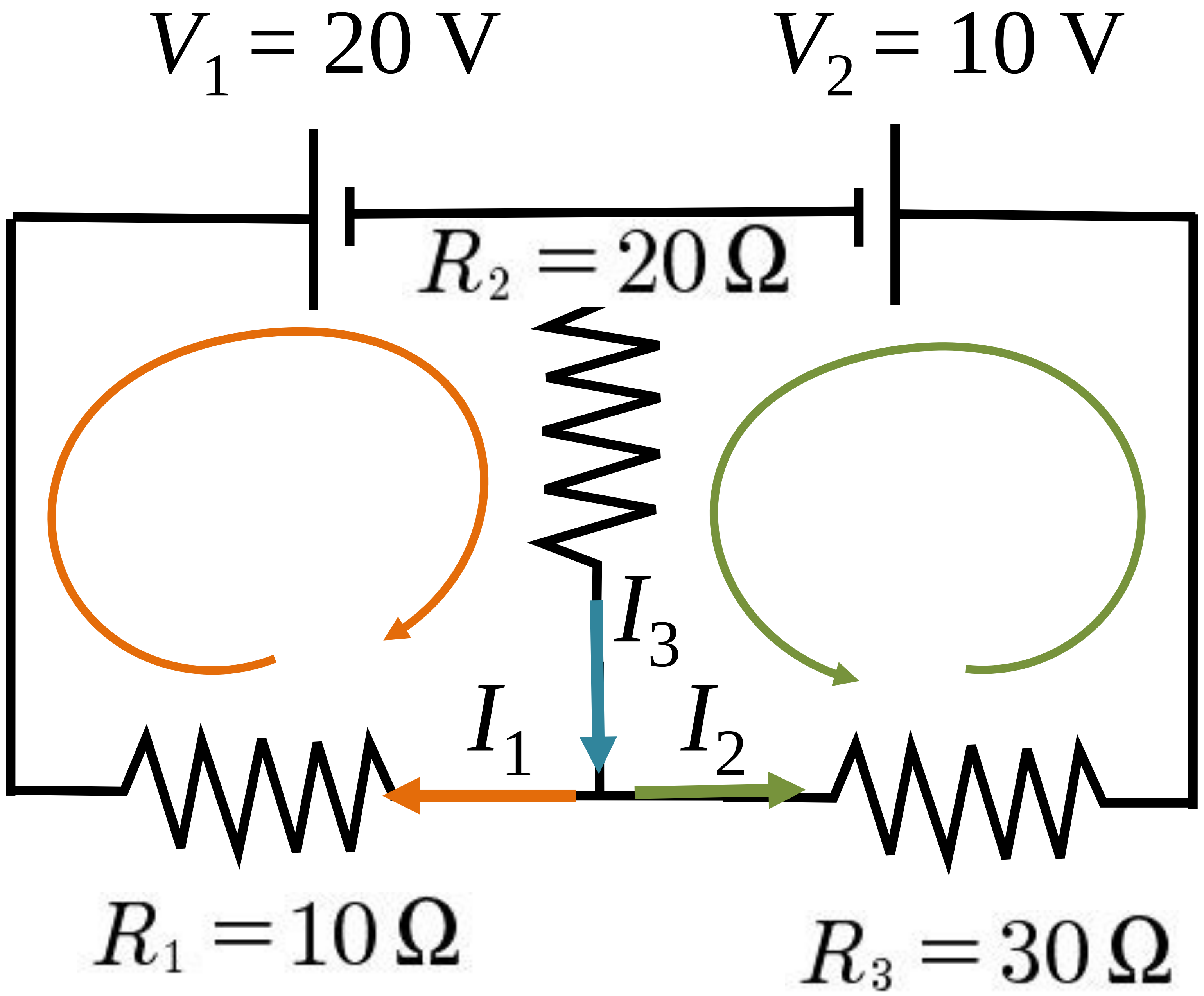
- **Use Kirchhoff's rules**
  - **junction rule – I - just define**
  - **loop rule – V - increase positive**

Example

$$I_1 - I_2 = I_3$$

$$-I_2 R_3 - \mathrm{V}_2 - I_3 R_2 = 0$$

$$-I_1 R_1 - \mathrm{V}_1 - I_3 R_2 = 0$$



$$3I_1 + 2I_2 = -2$$

$$2I_1 + 5I_2 = -1$$

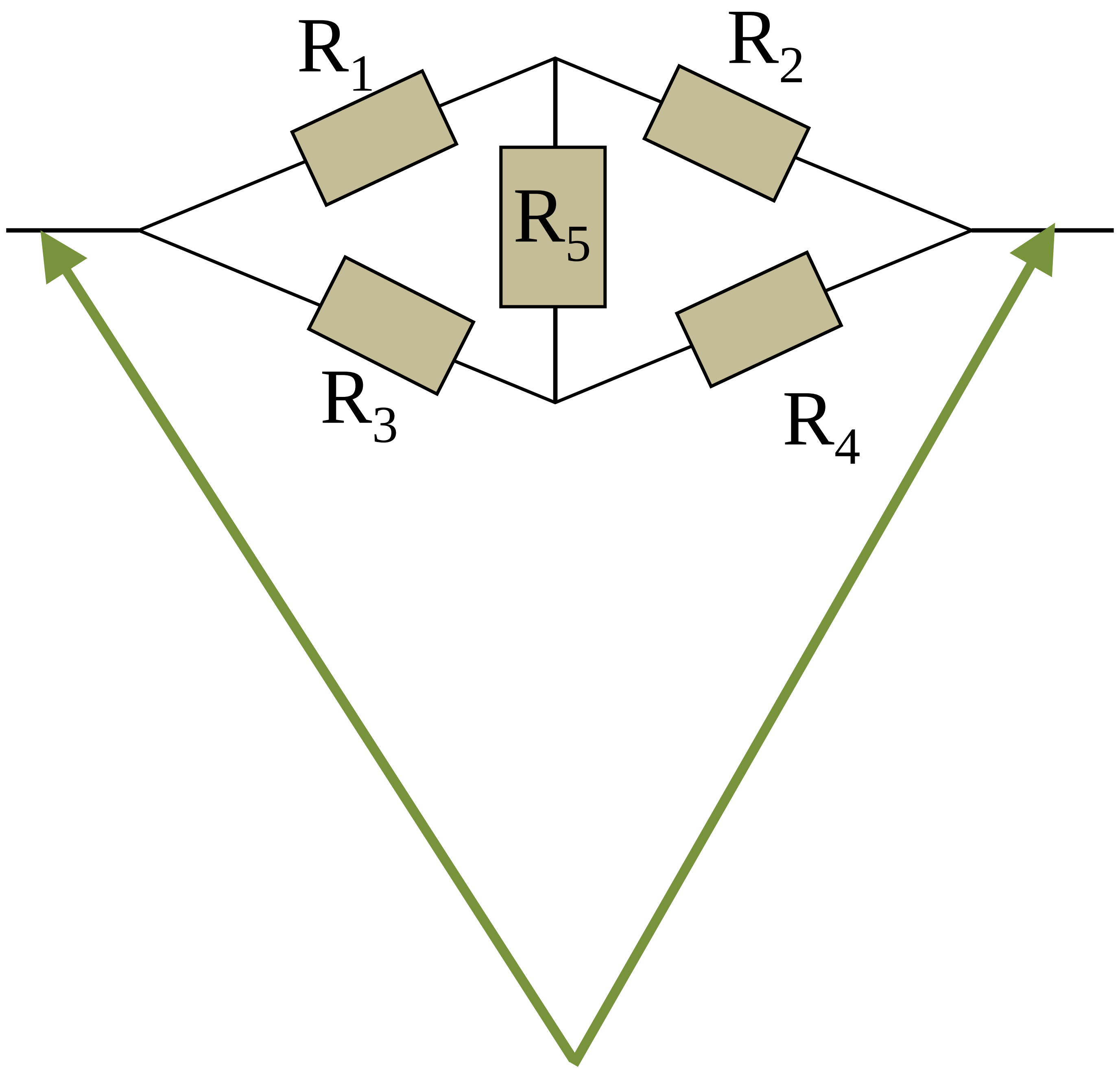
$$I_1 = -0.73 \text{ A}$$

$$I_2 = +0.09 \text{ A}$$

Equivalent resistance

A resistance bridge circuit

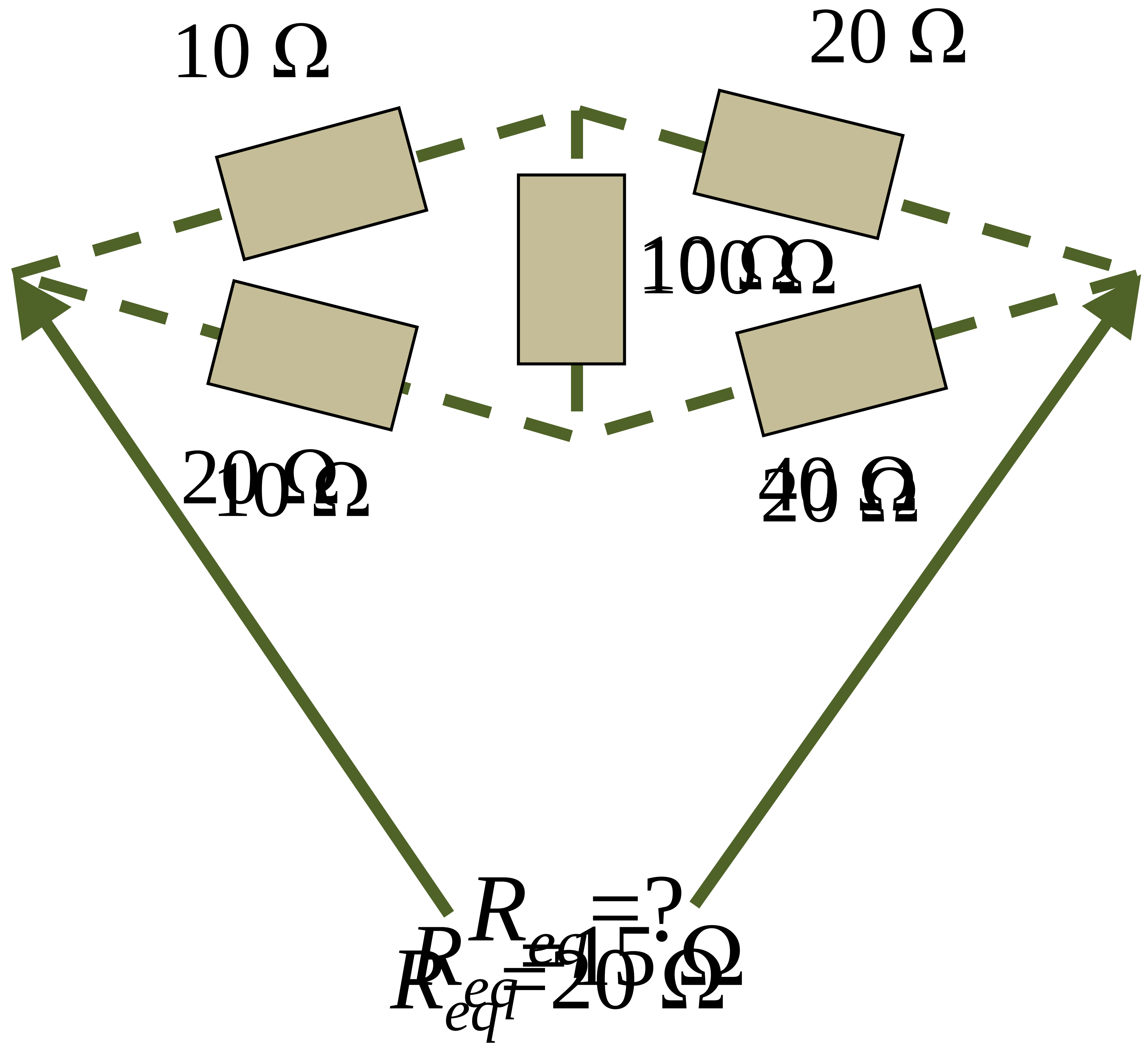
No current in R_5 ?



R_{eq}=?

Equivalent resistance

A (resistance) bridge circuit – what is the equivalent resistance?



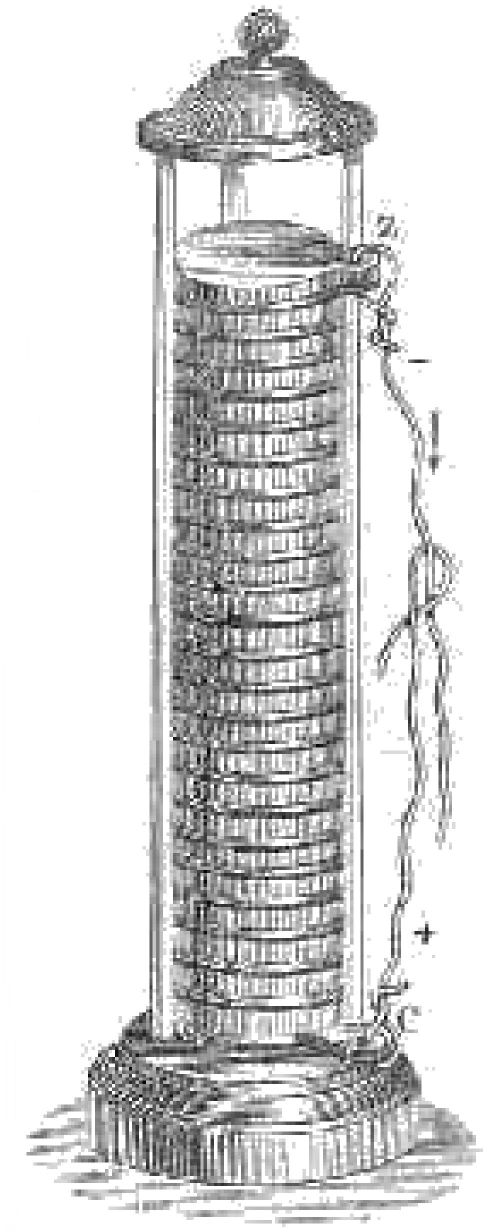
Batteries, E.M.F. and Internal Resistance

Alessandro Volta

inventor of the battery

(1799)

Batteries




Copyright 2015 Edison Tech center


Silver or Copper
Electrolyte
Zinc
Silver or Copper
Brine-soaked cloth or cardboard
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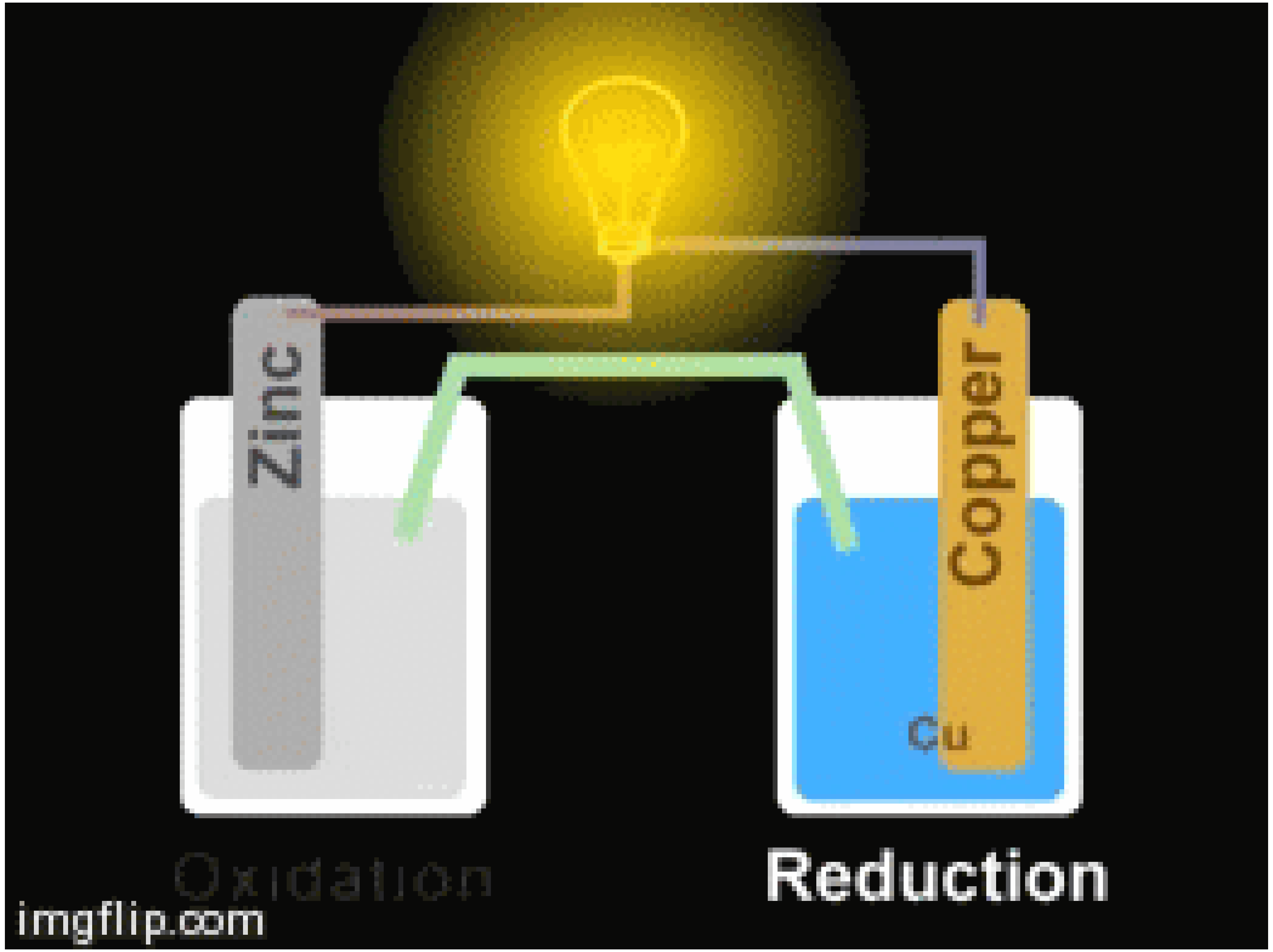
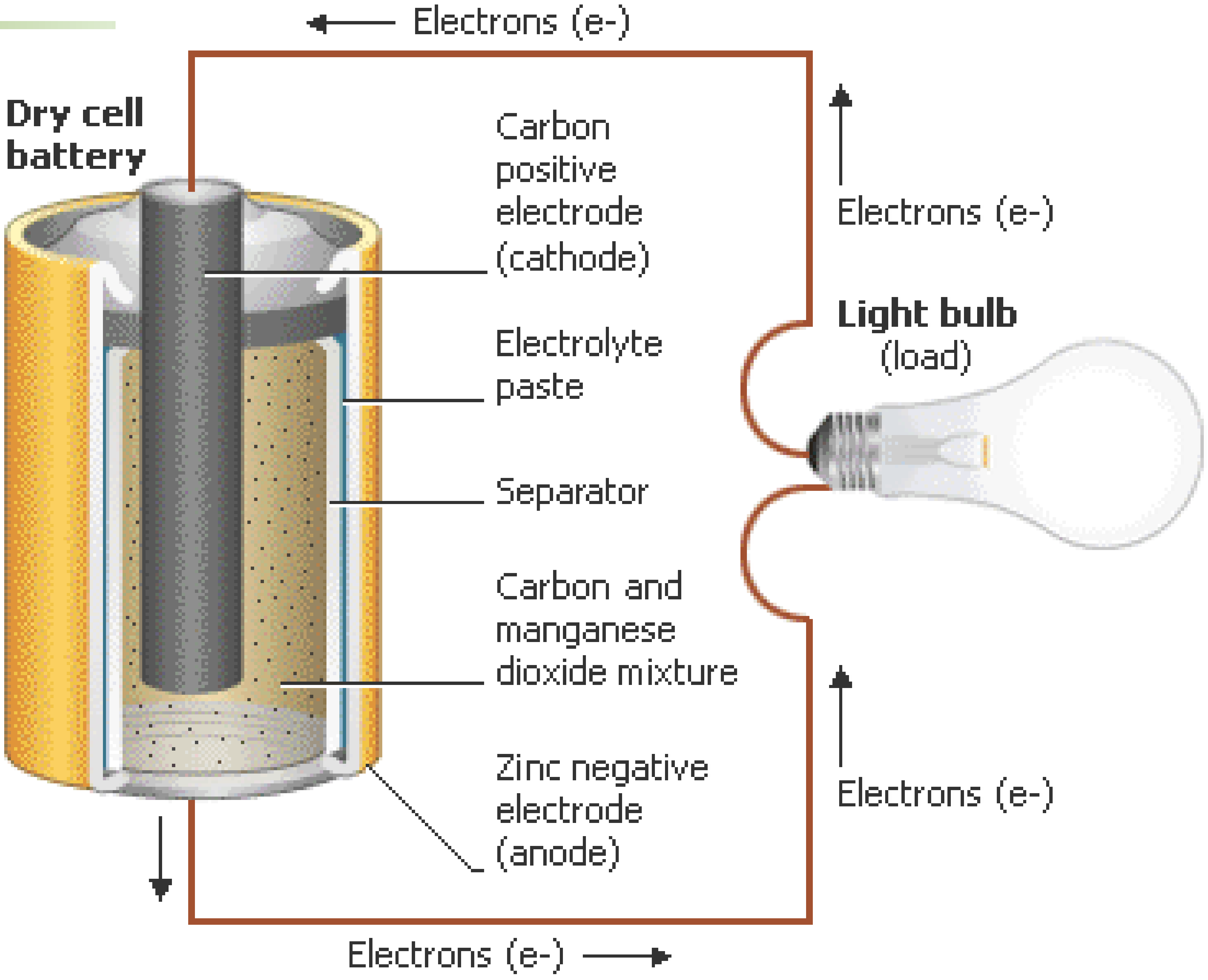
Batteries



$$Zn \longrightarrow Zn^{2+} + 2\,e^{-} \qquad Cu^{2+} + 2\,e^{-} \longrightarrow Cu$$

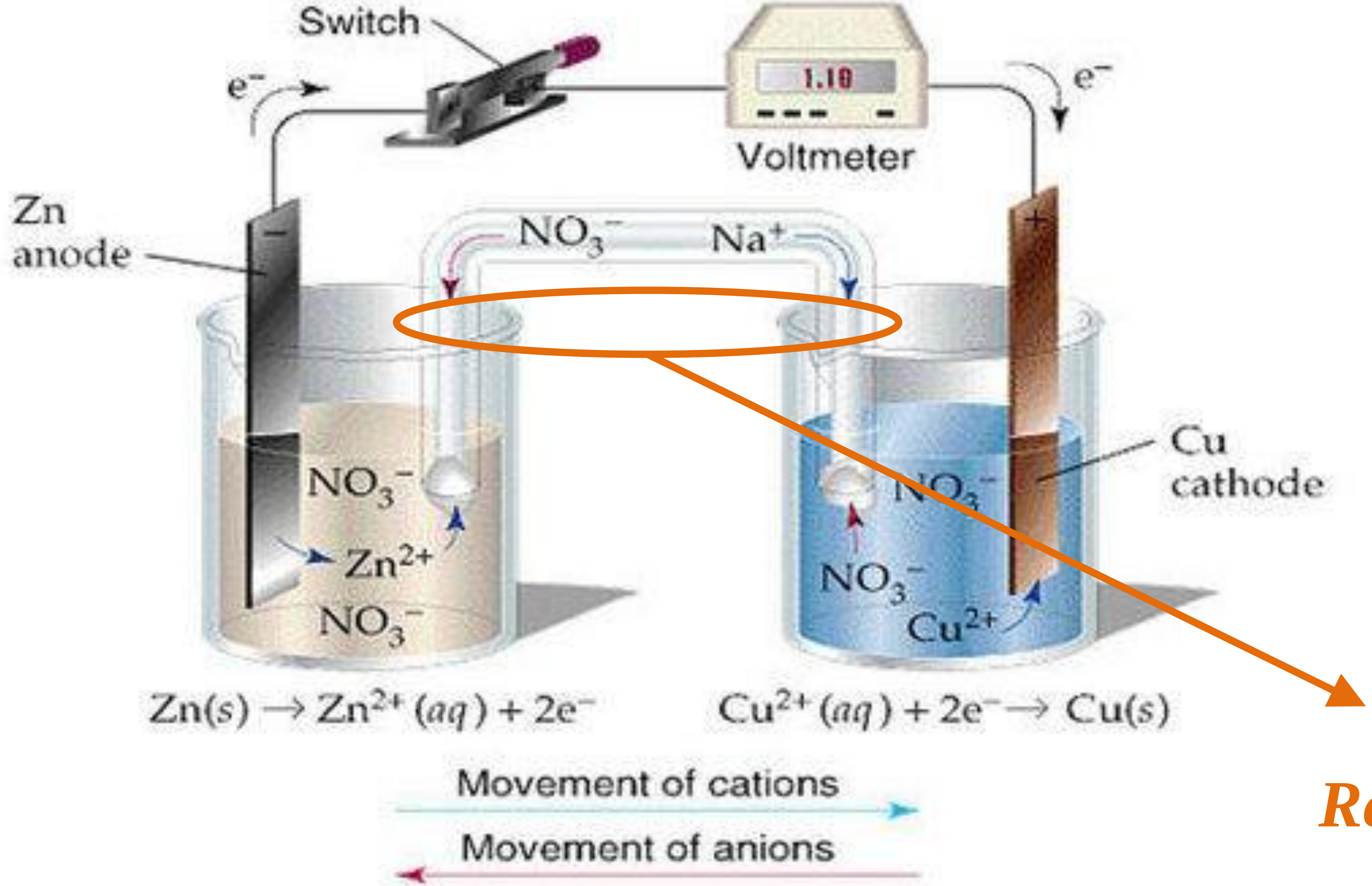
Dry cell battery



Some parameters of battery

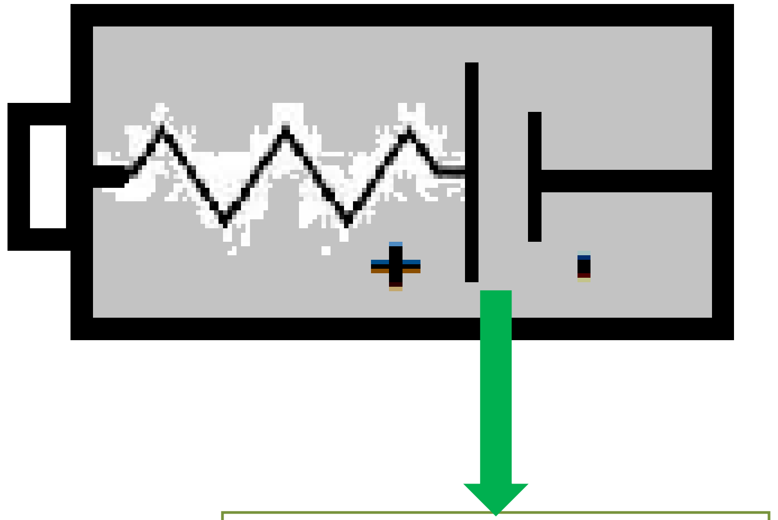
Voltage (volts)	the amount of electrical force, or pressure, at which free electrons move from the positive end of the battery to the negative end
Capacity (ampere-hours)	the number of hours the battery can supply a particular amount of electrical current before its voltage drops below a certain threshold
Power density	the amount of power a battery can deliver per unit weight
Energy density	how much energy a battery is capable of delivering, divided by the battery's volume or mass

Specs	Lithium-ion	Lead acid	Ni-MH	Ni-Cd
Specific energy (Wh/kg)	160	40	90	45
Nominal voltage (V)	Li-cobalt – 3.6V	2.1V	1.2V	1.2V
	Li-manganese – 3.8V			
	Li-phosphate – 3.2V			
	NMC – 3.6V			
Common sizes and average capacities for portable and solar lights	Custom	Custom	3xAA	3-5xAA
	2000-6000 mAh	4000-7000 mAh	2000 mAh	600 mAh
Average self-discharge rate per month (room temperature)	2-8%	2-5%	30-50%	10%
Cycle life	500-2000 (closer to 2000 for lithium phosphate batteries)	500-800	500-1000	800-1500
Typical internal resistance	$150-200\ m\Omega$	$< 50\ m\Omega$	$200-300\ m\Omega$	


Switch
e⁻
1.10
Voltmeter
e⁻
Zn
anode
NO_3^-
Na^+
Cu
cathode
NO_3^-
Zn^{2+}
NO_3^-
NO_3^-
NO_3^-
Cu^{2+}
$Zn(s) \rightarrow Zn^{2+}(aq) + 2e^-$
$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$
Movement of cations
Movement of anions

Resistance!

Internal resistance



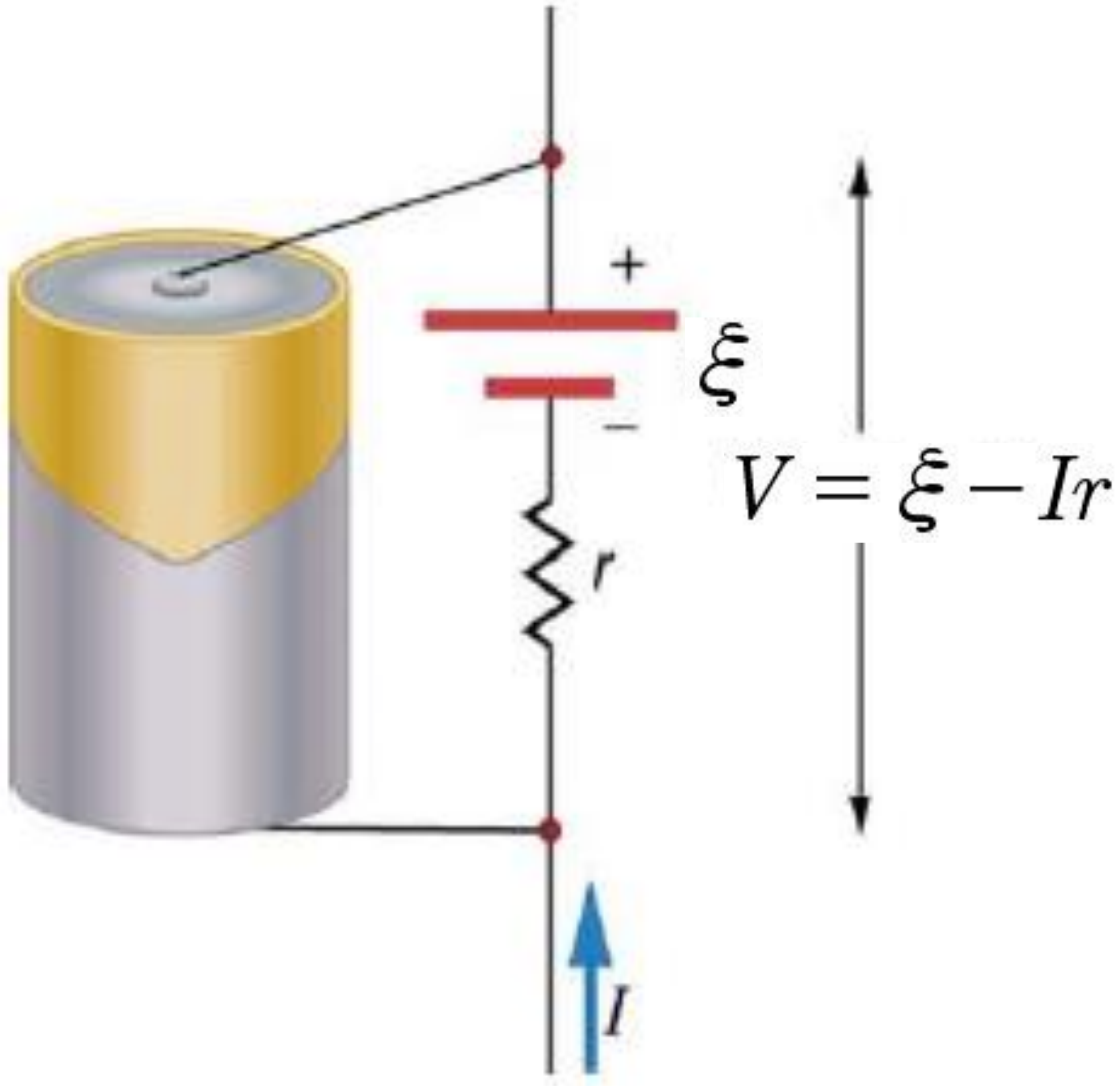
Maximum voltage the battery can provide

Internal resistance

- quality of battery
- temper-ature
- charging status
- age of battery
- current

But we may treat it as ***a constant*** *for a given battery!*

Voltage vs EMF

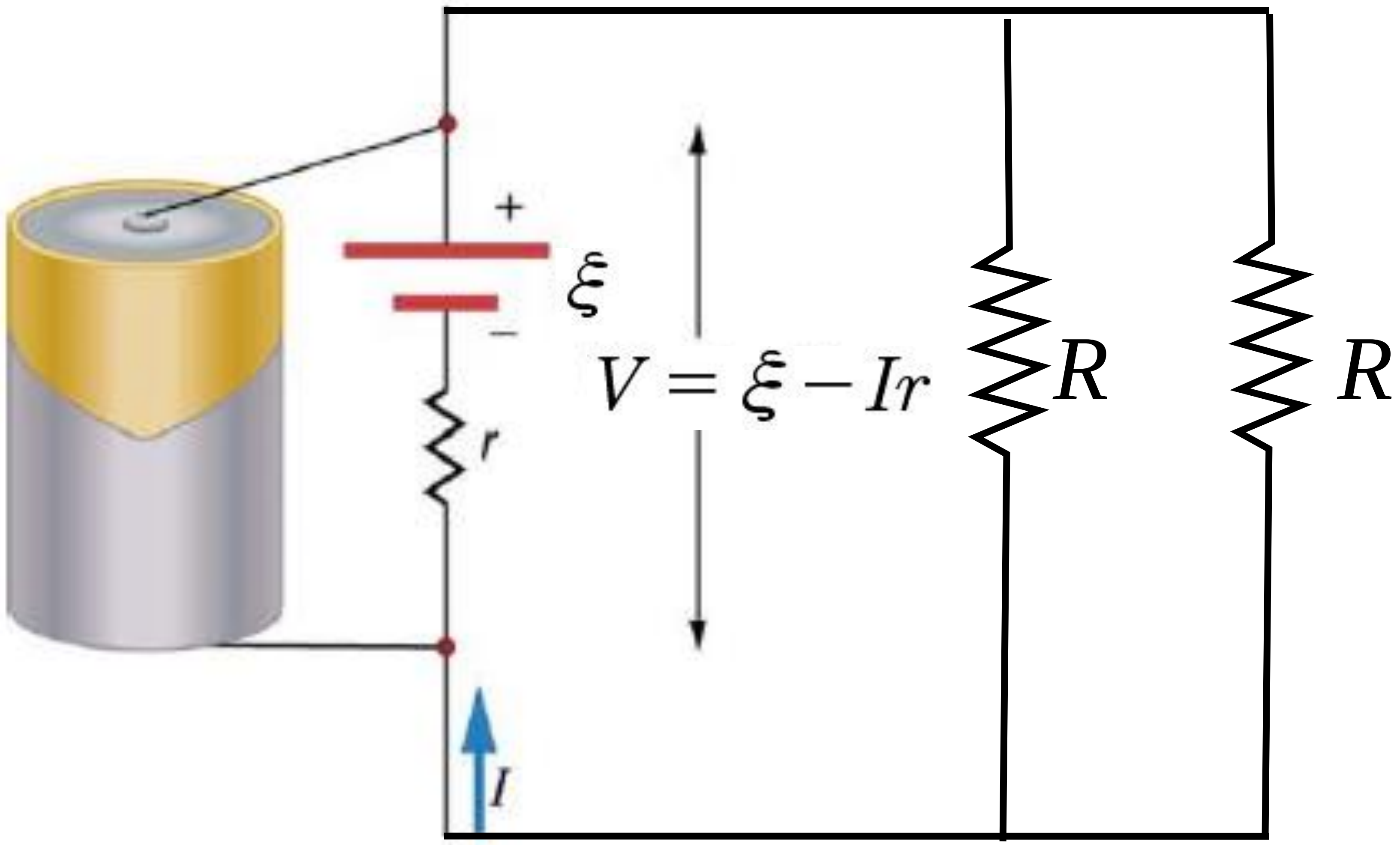


EMF ($\mathcal{E}$)= Electromotive Force

- A battery is a source of EMF in series with an ***internal resistance*** (***r***)
- The voltage at the poles of the battery depends on *r* and the current *I*

EMF = "*open circuit*" voltage

EMF



One resistance *R* :

$$I_R = \frac{\xi}{r+R}$$

$$V_R = \xi - Ir = \xi - \frac{\xi}{r+R}r$$

$$V_R = \xi\left(\frac{R}{r+R}\right)$$

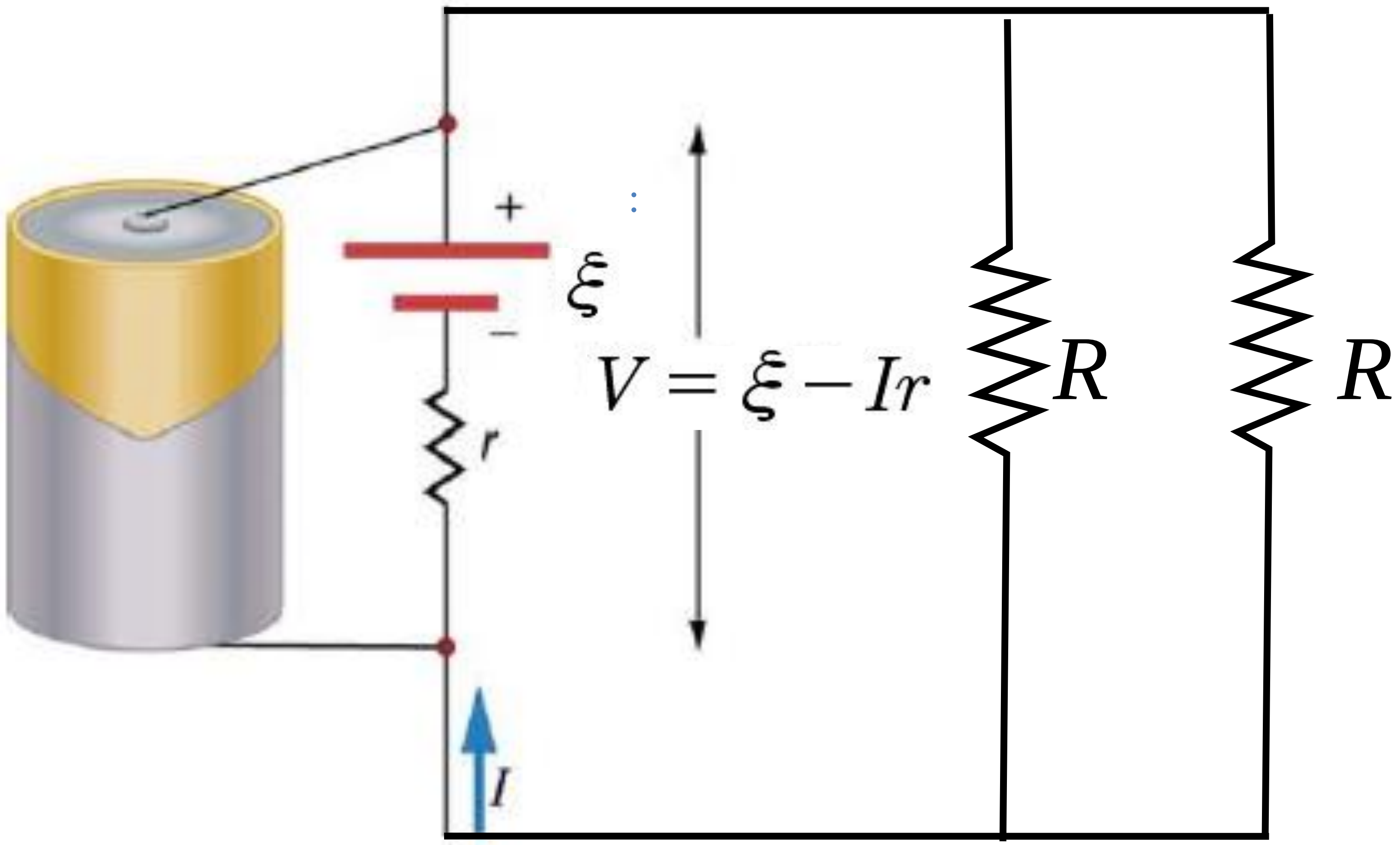
Double load :

$$I_{R/R} = \frac{\xi}{r+(R/2)}$$

$$V_{R/R} = \xi\left(\frac{R/2}{r+R/2}\right)$$

$$V_{R/R} = \xi\left(\frac{R}{2r+R}\right) < V_R$$

EMF



Power with only one resistance

$$P_R = \frac{V_R^2}{R} = \frac{\xi^2}{R}\left(\frac{R}{r+R}\right)^2$$

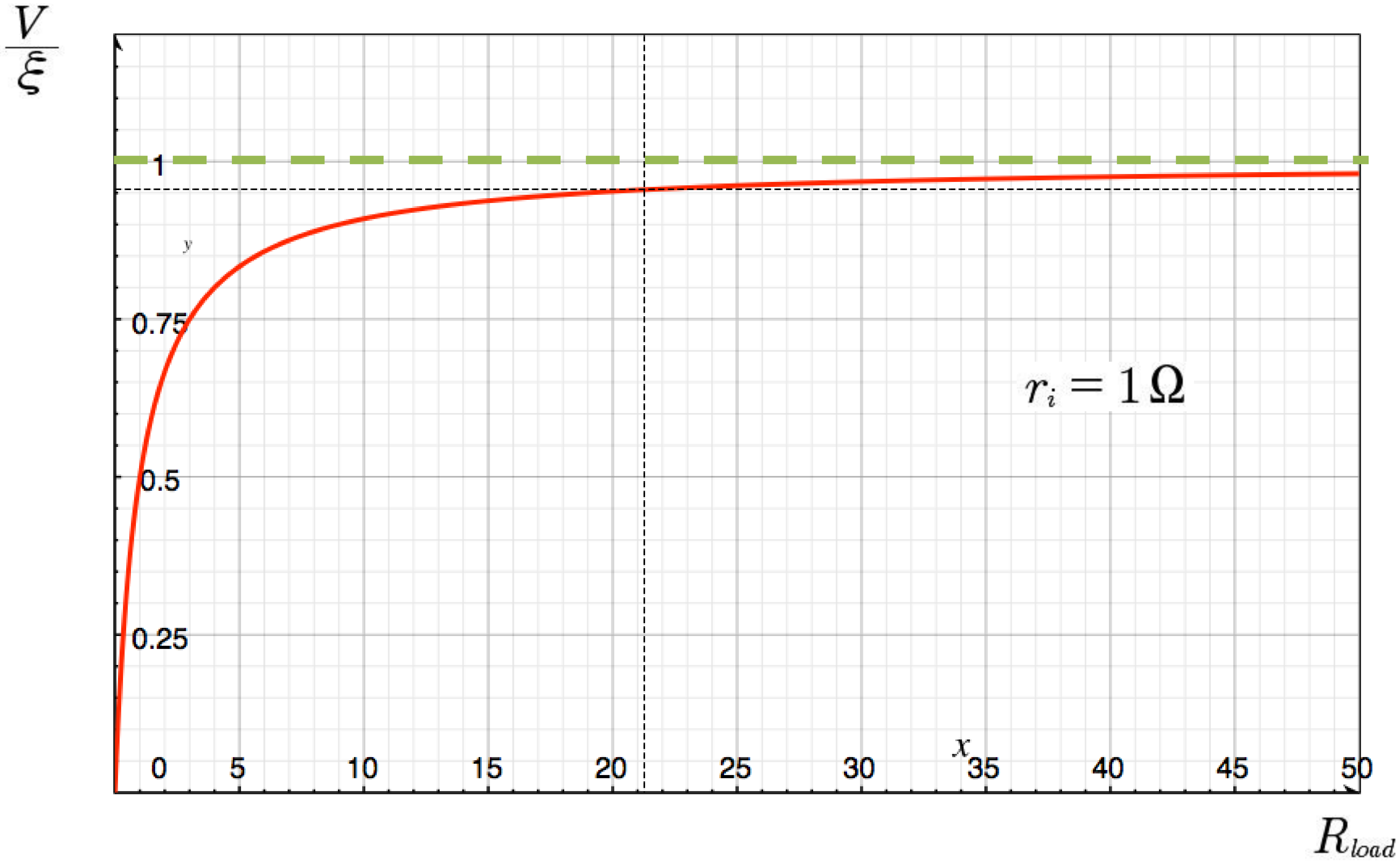
Power (on each resistor) with two resistances:

$$P_{R/R} = \frac{\xi^2}{R}\left(\frac{R}{2r+R}\right)^2 < P_R$$

Connecting a second load reduces the power in each load!

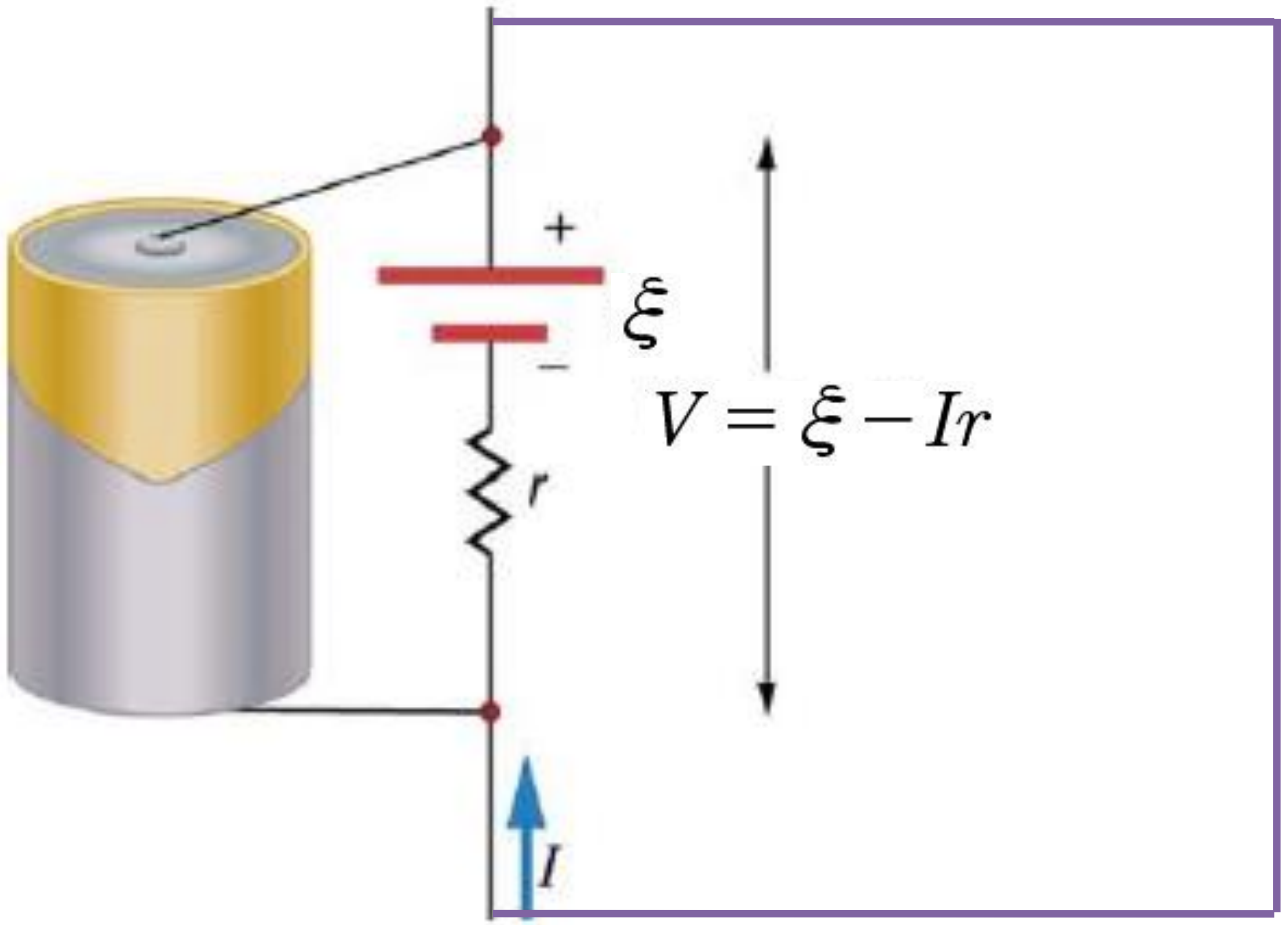
Voltage vs EMF

Output



Batteries

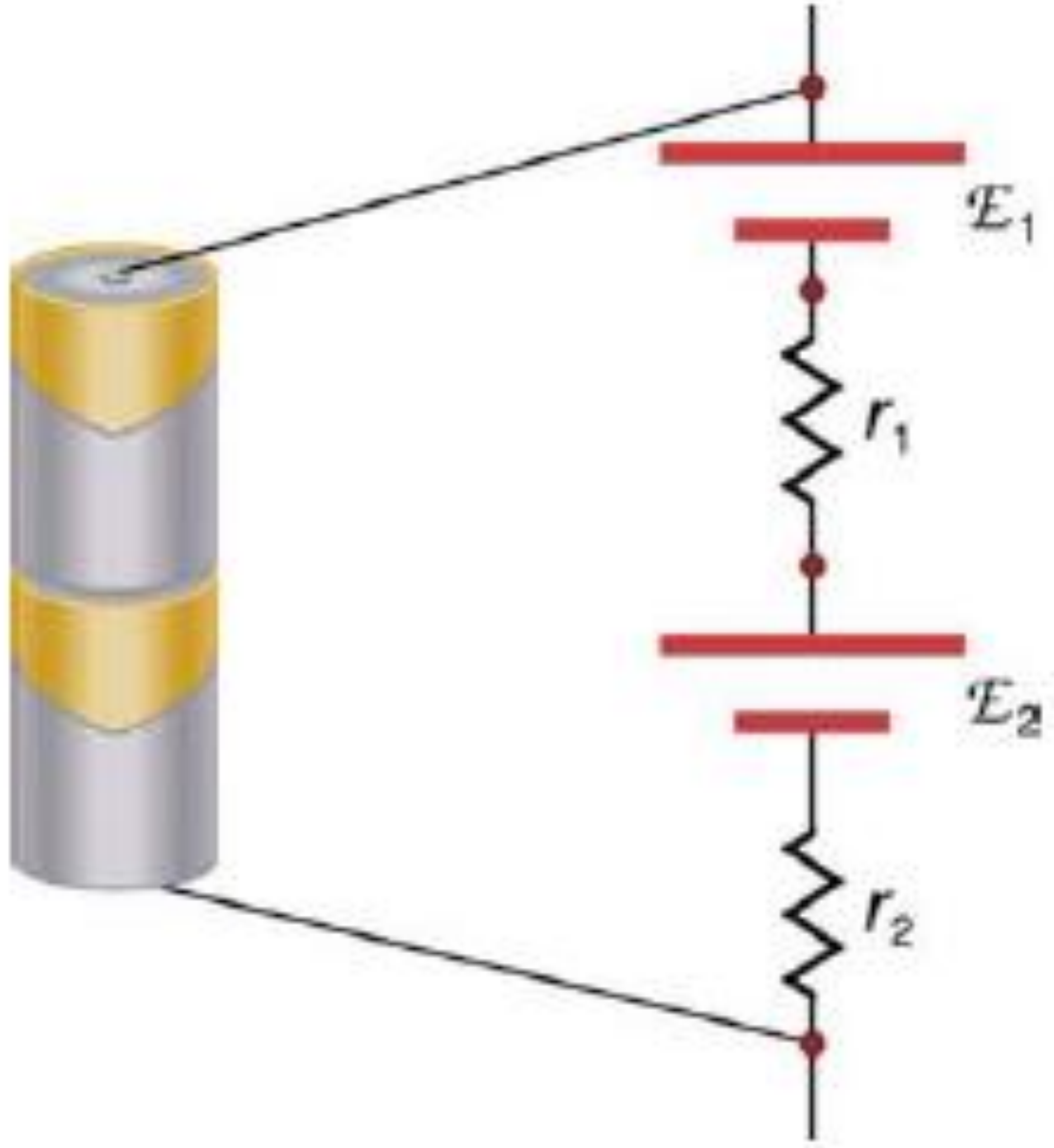
What is the maximum current you can get from a battery?



$$R = 0 \implies (R \ll r)$$

$$\boxed{I = \frac{\xi}{r}}$$

Multiple batteries in series

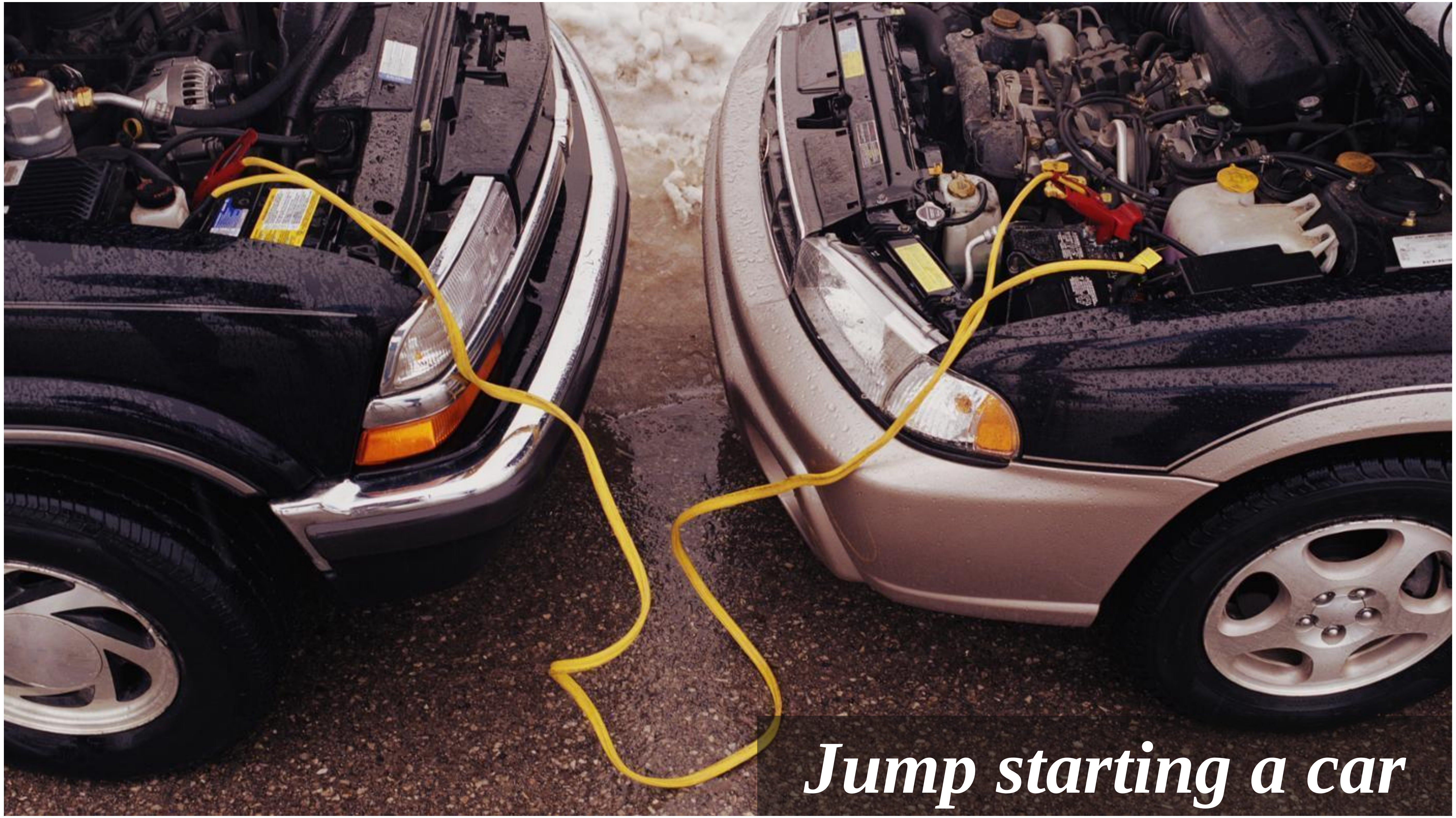


$$R_i^{tot} = r_1 + r_2 + \ldots$$

6 cells @ 1.5V

→ higher internal resistance than a 1-cell AA battery

Multiple batteries in parallel



Jump starting a car

Jump starting a car

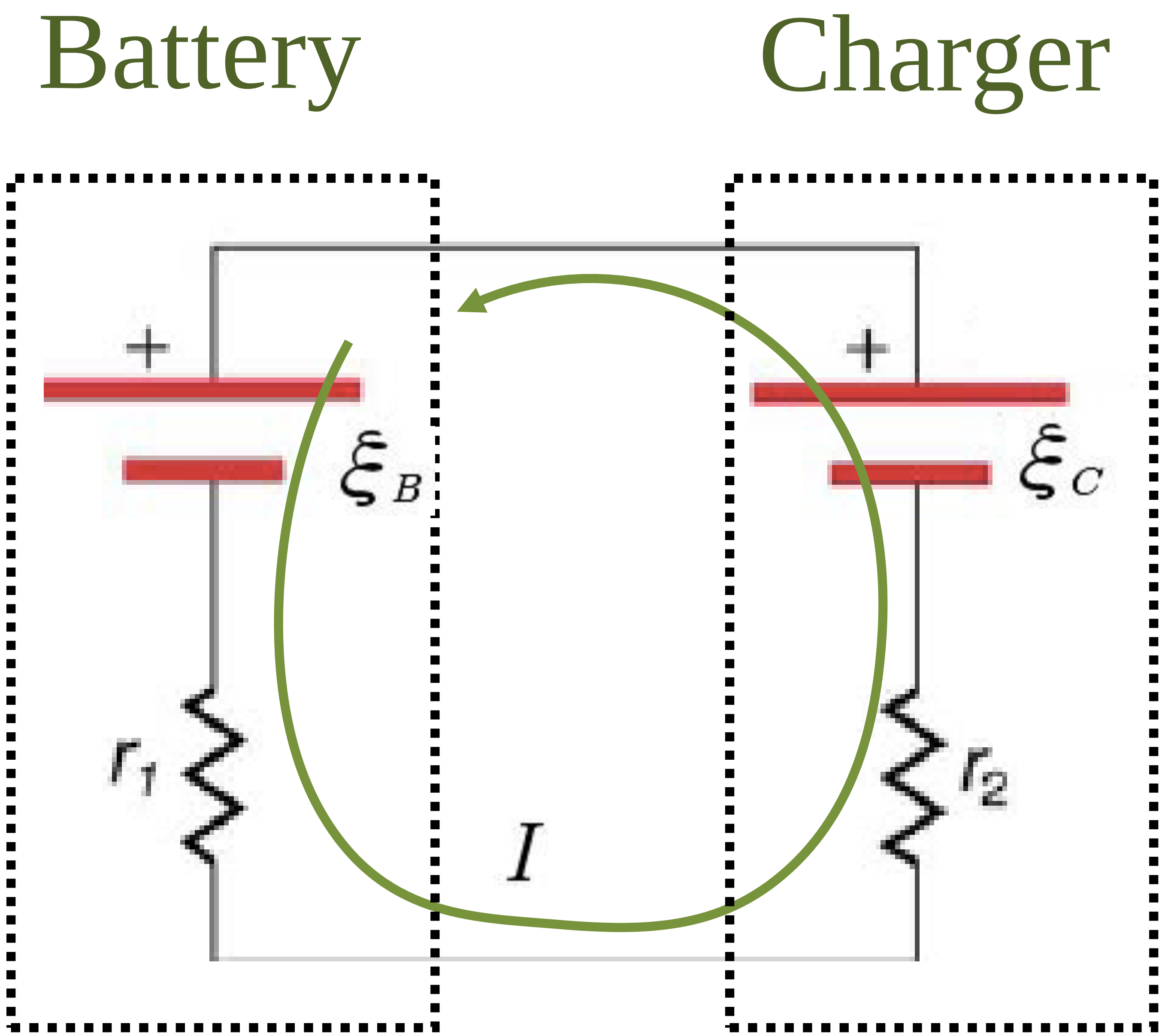
Jump starting a car

Charge a car battery

To charge the battery : what is the direction of the current?

How to calculate *I* ?

Use Kirchhoff !



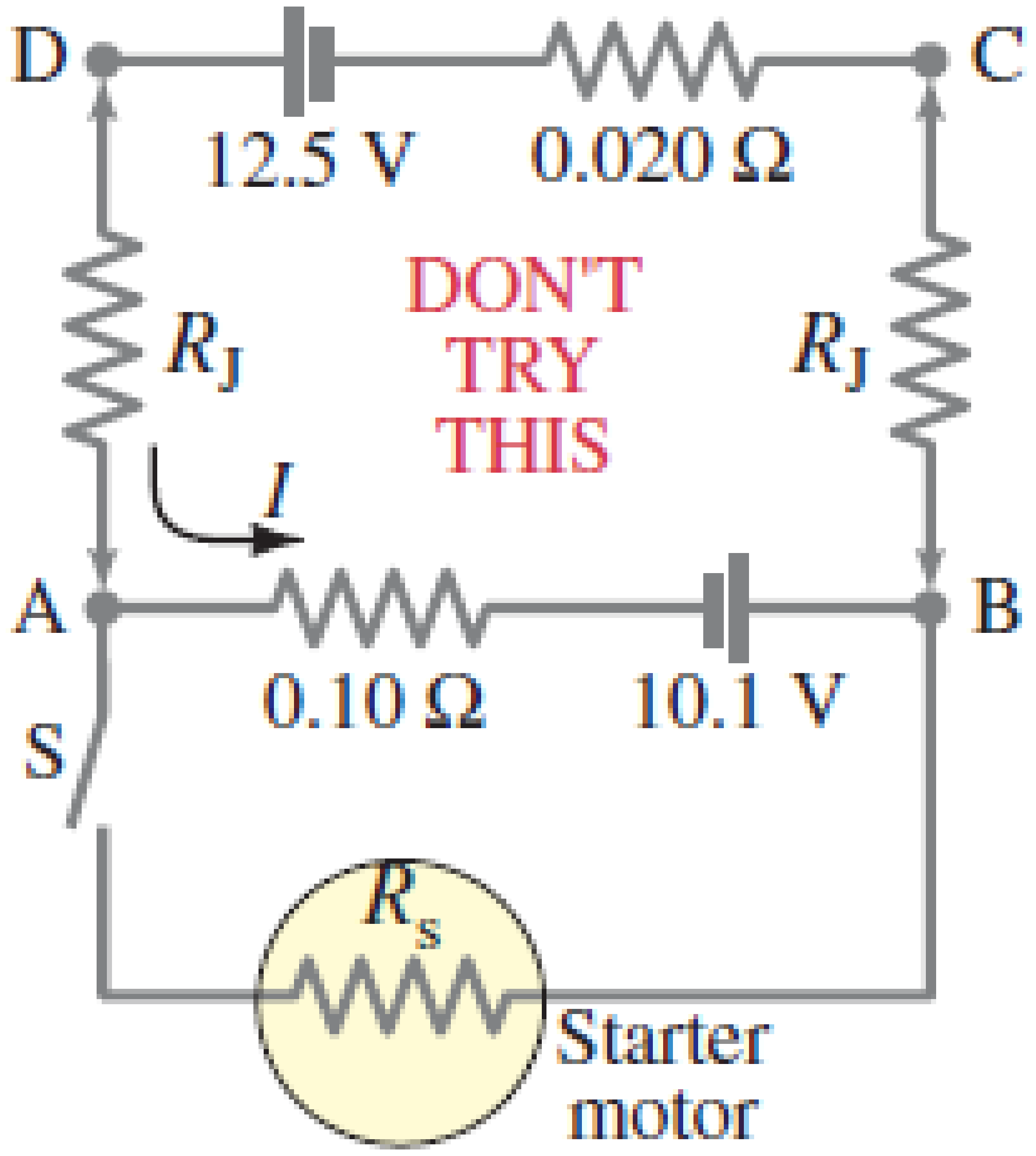
$$\Longrightarrow \xi_C > \xi_B$$

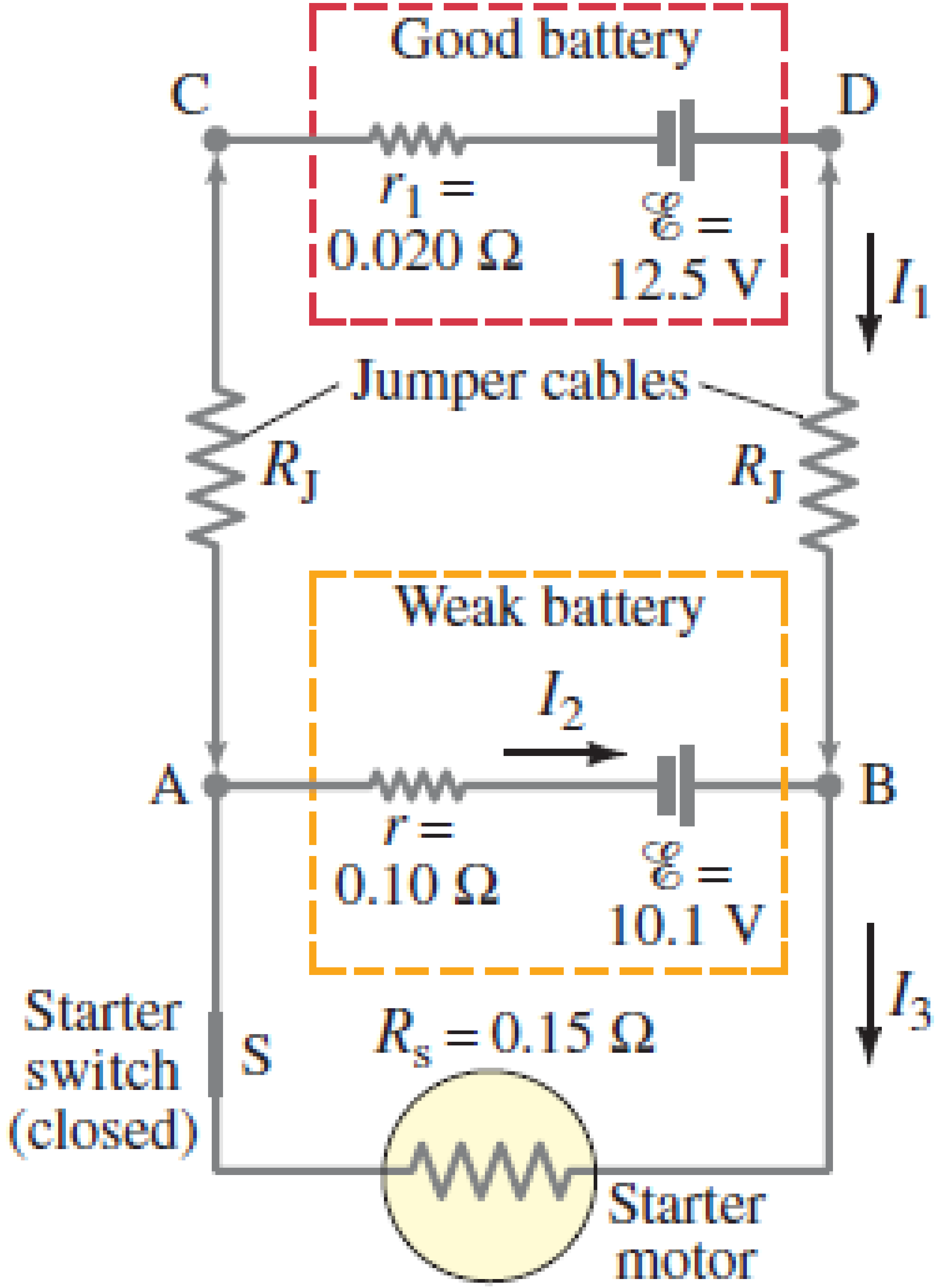
$$-\xi_B - r_1 I - r_2 I + \xi_C = 0$$

$$I = \frac{\xi_C - \xi_B}{r_1 + r_2} > 0$$

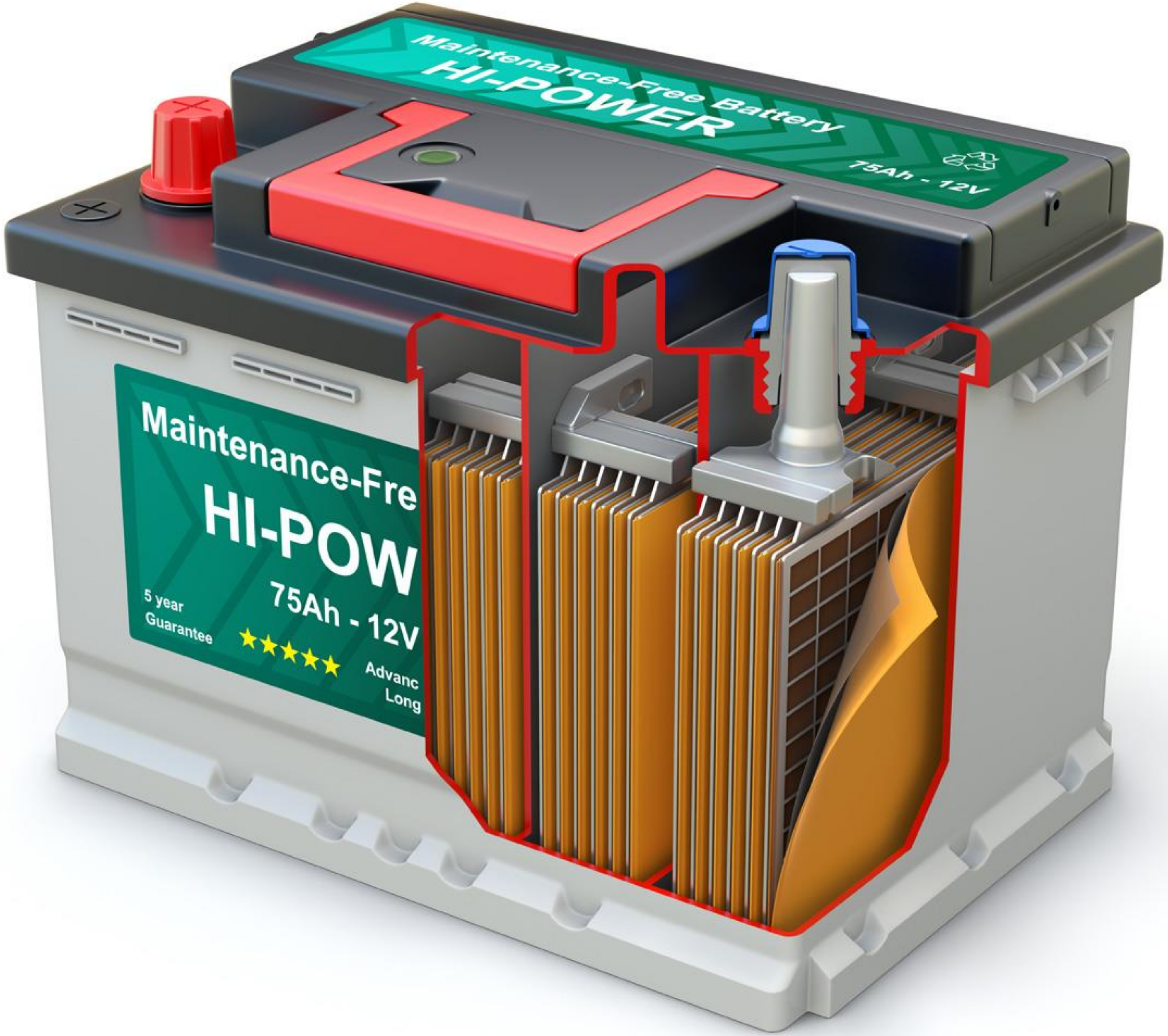
Jump starting a car

Mistakenly connected in reverse





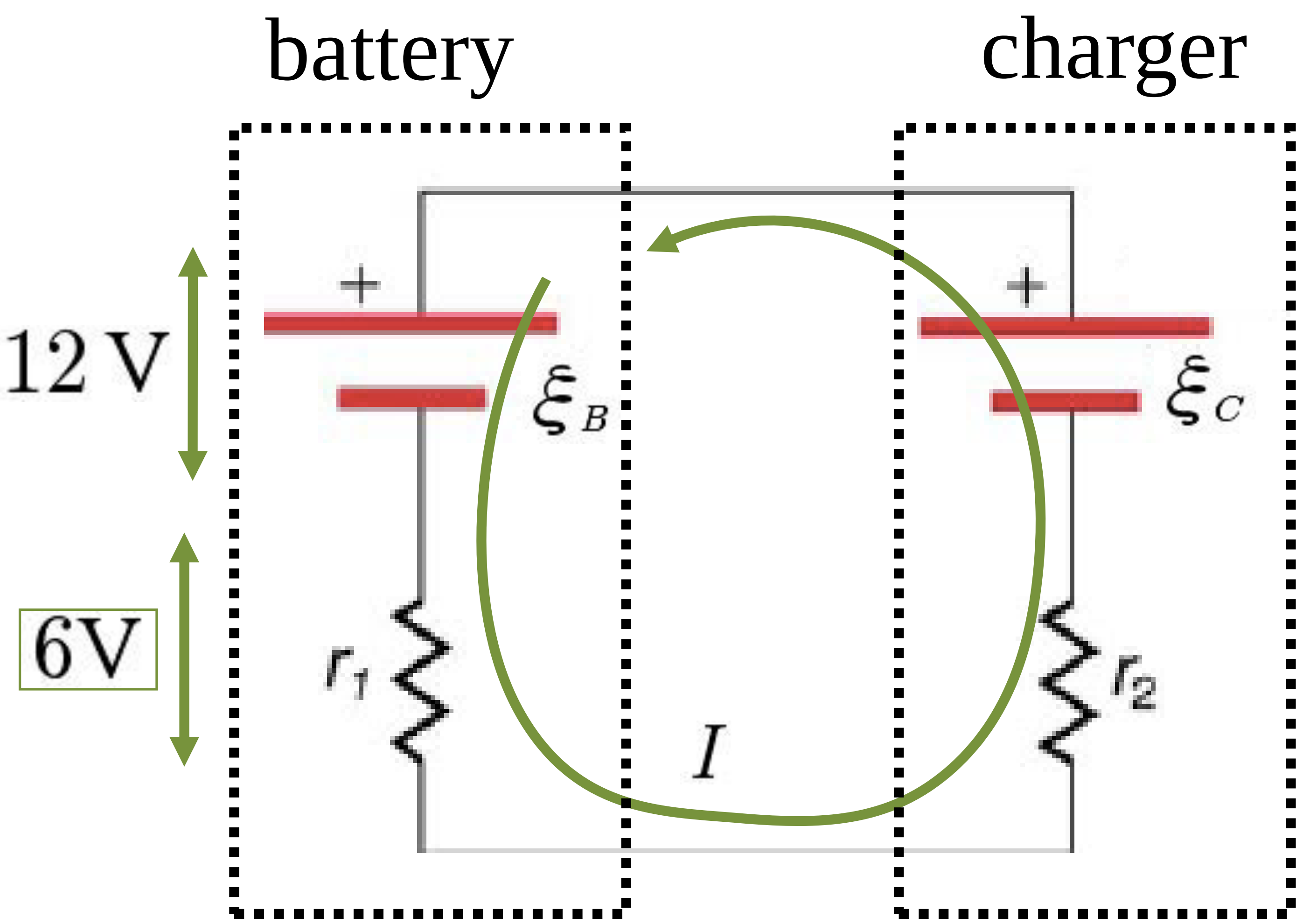
Car battery



Cells
3 series and 2 parallels

Example

Find the terminal voltage of a 12.0 V motorcycle battery having a 0.600 Ω internal resistance, if it is being charged by a current of 10.0A. (b) What is the output voltage of the battery charger?



$$\xi = 12\text{ V} \qquad I = 10\text{ A}$$

$$r_1 = 0.600\,\Omega$$

$$r_1 I = 0.600\,\Omega \times 10\text{ A} = 6\text{V}$$

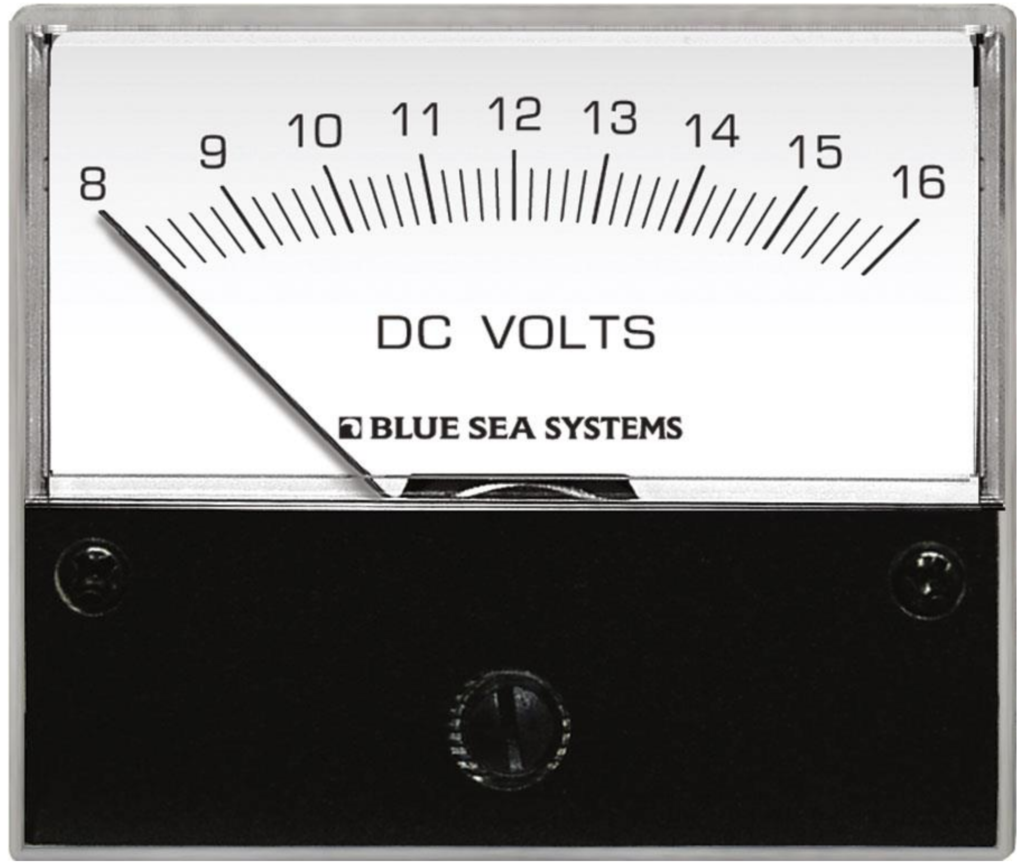
$$V = 12\text{ V} + 6\text{ V} = 18\text{ V}$$

Voltmeters and Ammeters

How to measure voltage ?

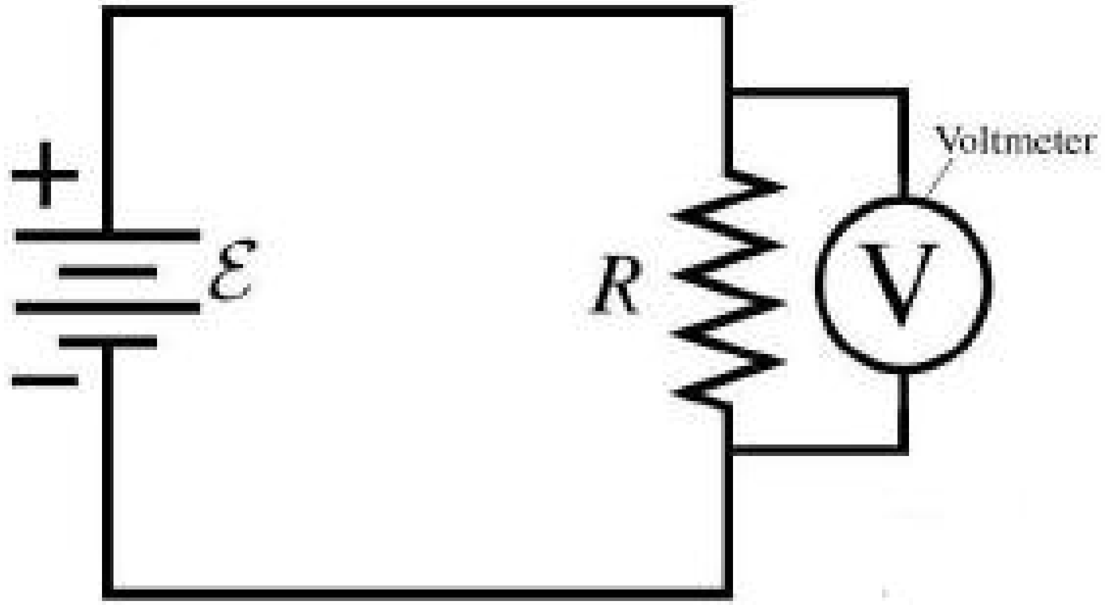
How to measure current ?

How would them be connected in the circuit?



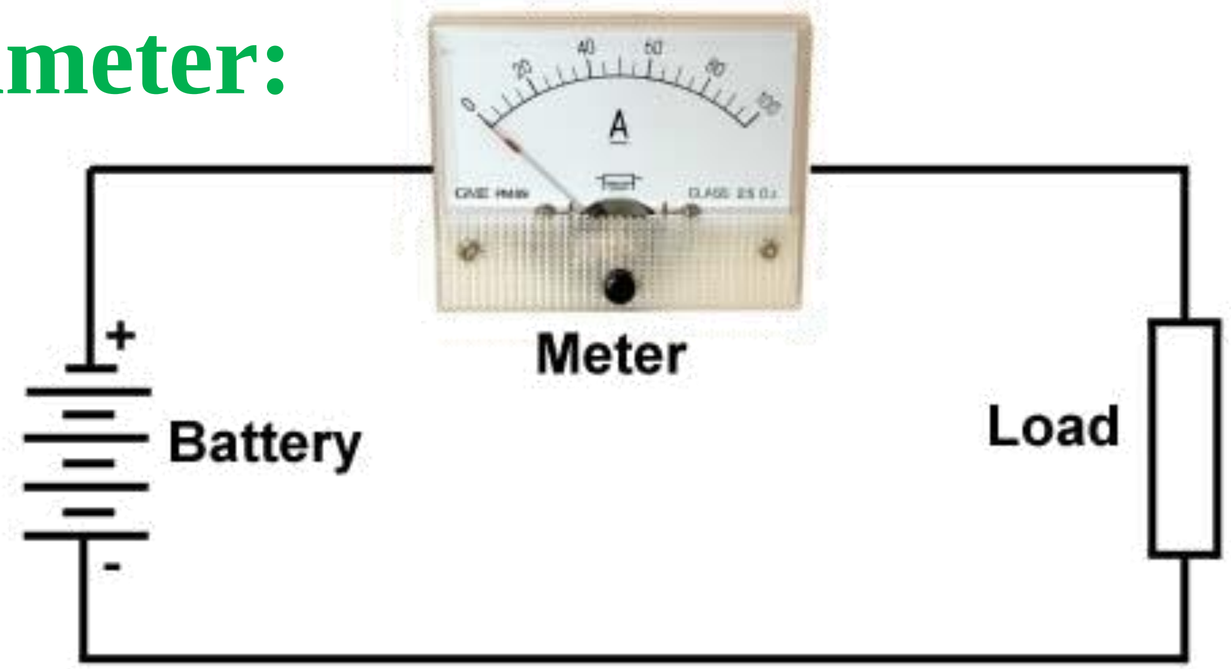
Voltmeters and Ammeters

Voltmeter:



- **large internal resistance**
- **connect in parallel**
- **does not disturb the voltage**

Ammeter:



- **small internal resistance**
- **connect in series**
- **does not disturb the current**

Digital Multimeter