



Physics 2: Electricity , Optics and Quanta

Week 6 - Magnetic field and Electric current

2023.10

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Structure

Magnetic field

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graph LR; A[Magnetic field] --> B[Field line]; A --> C[The effect of magnetic field on moving charges and current]; B --> D[Magnetic field generated by electric current]; B --> C; C --> E[Hall effect]; C --> F[Electric motors]; C --> G[Magnetic circuit];
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Field line

The effect of
magnetic field on
moving charges
and current

Hall effect

Electric motors

Magnetic field
generated by
electric current

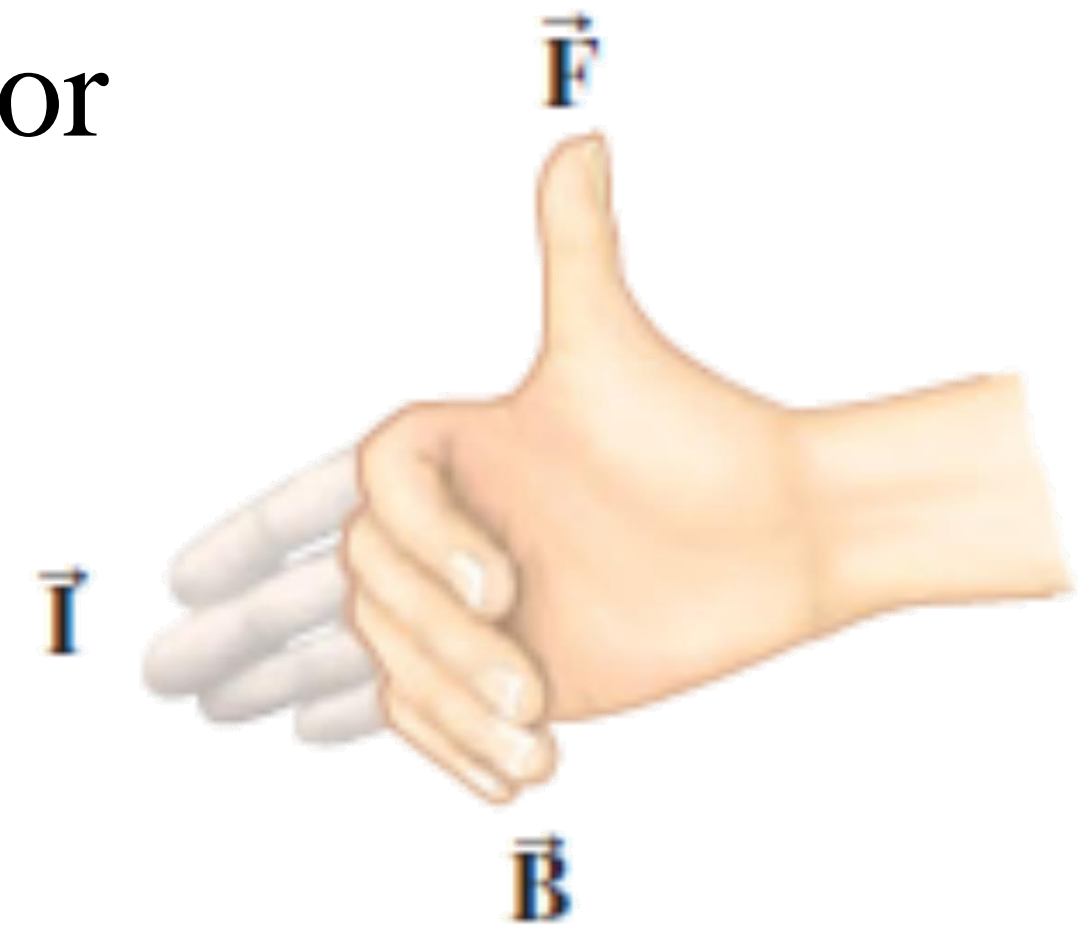
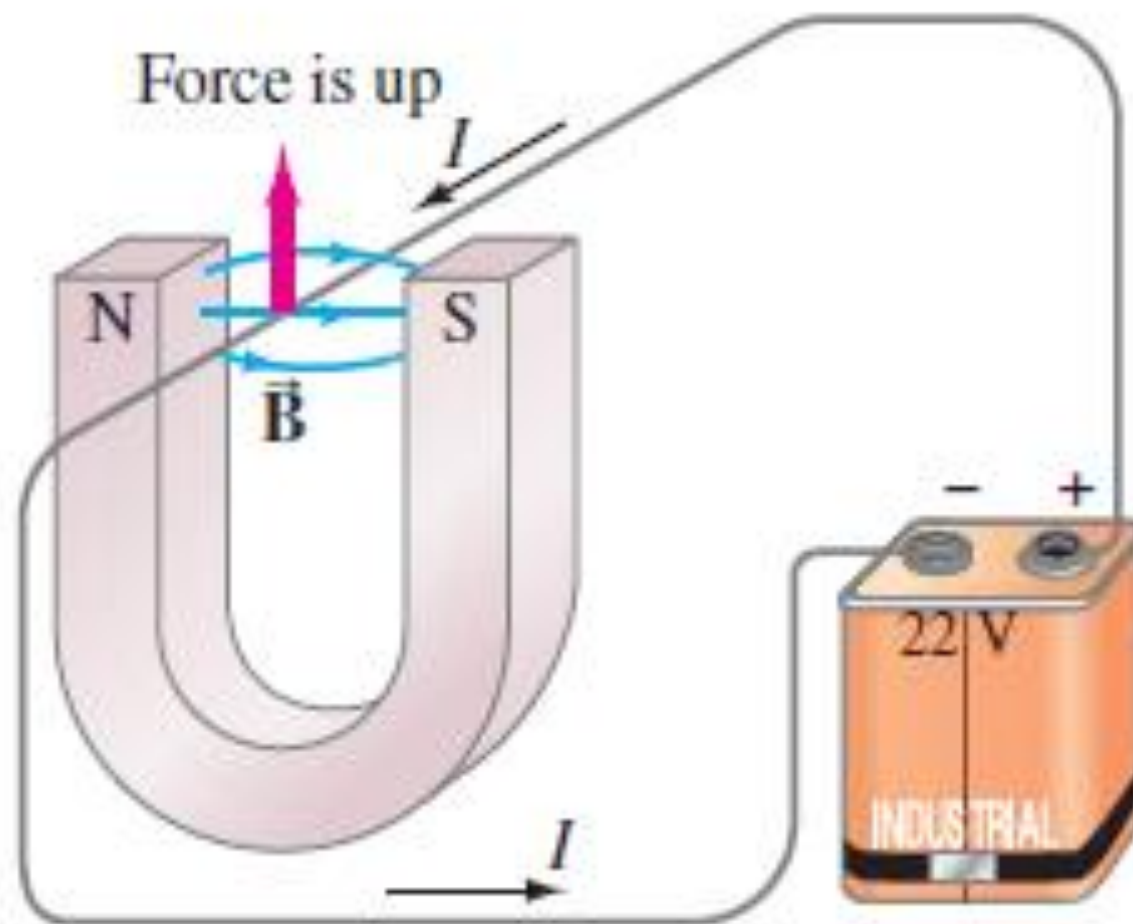
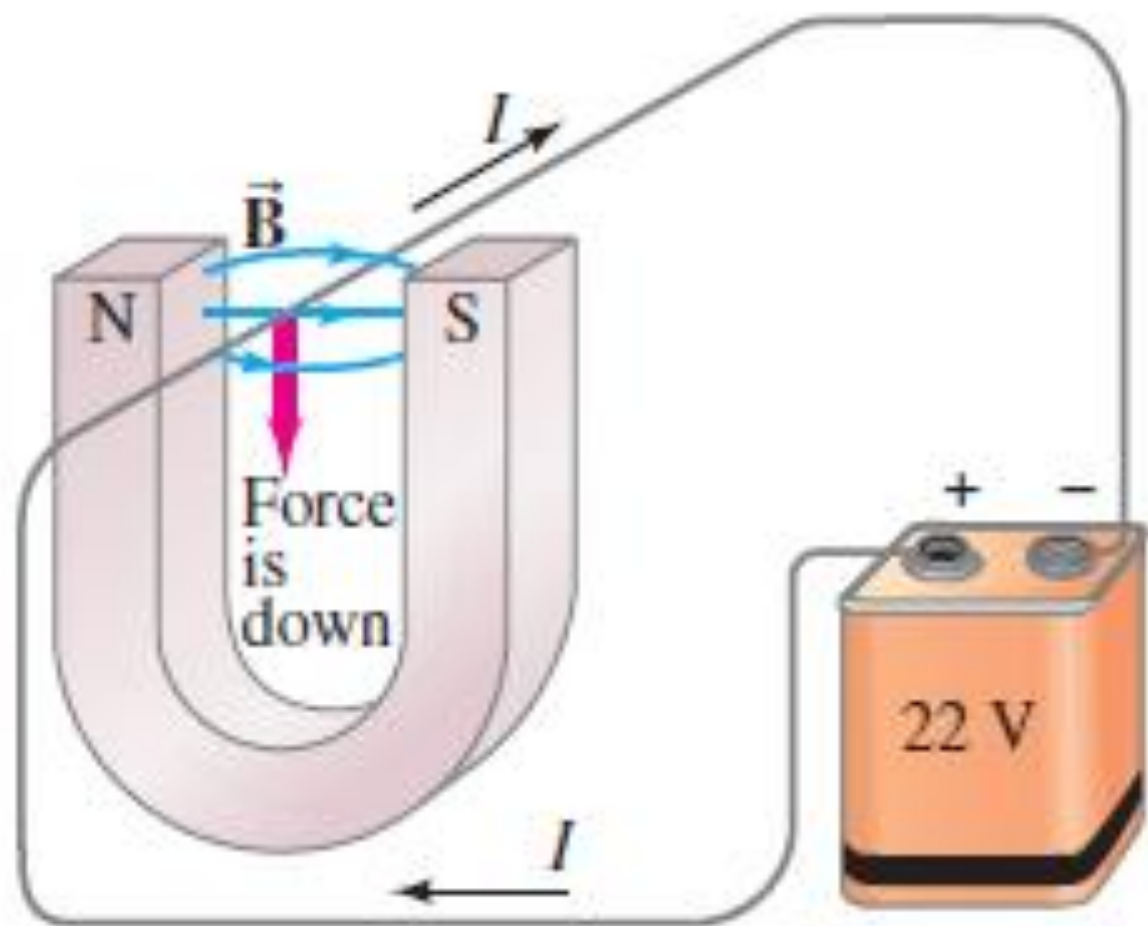
Magnetic circuit

Videos

Electric car



Magnetic force on a current-carrying conductor



Magnetic force on a current-carrying conductor

Try to derive

unknown: Forces on the conductor

known:

➤ I, B, l

➤ $F = qvB$ for charges

The charges move with the “drift” velocity: v_d

N = number of charges moving in the section of wire

A = cross section of the wire

l = length of the wire (diameter of the magnet)

n = density of charge carriers ($1/\text{m}^3$)



Magnetic force on a current-carrying conductor

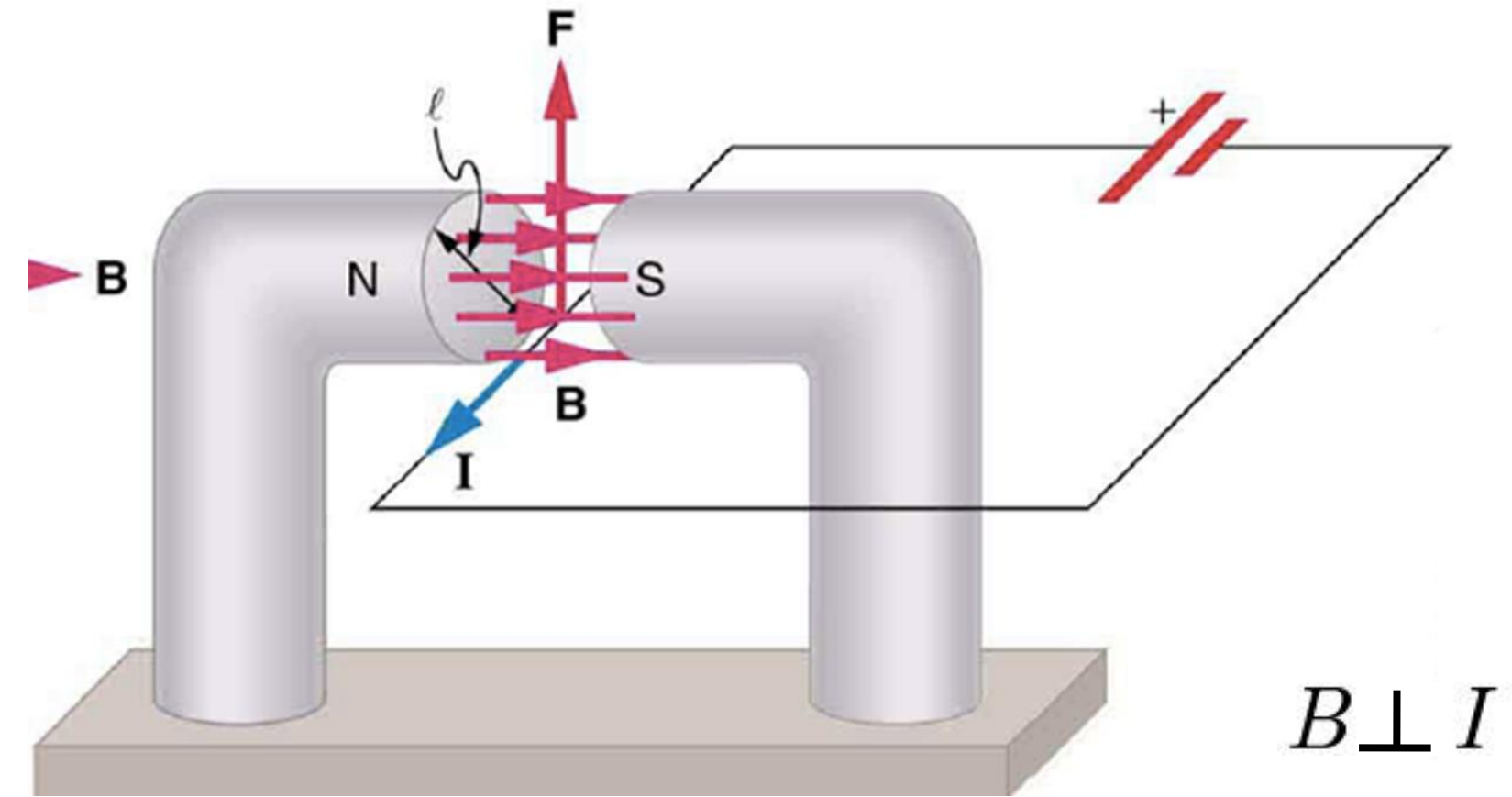
Magnetic forces on moving charges

$$F = (q v_d B) N$$

$$N = nV = nAl \quad V = Al$$

$$F = \underline{(nqAv_d)l} B$$

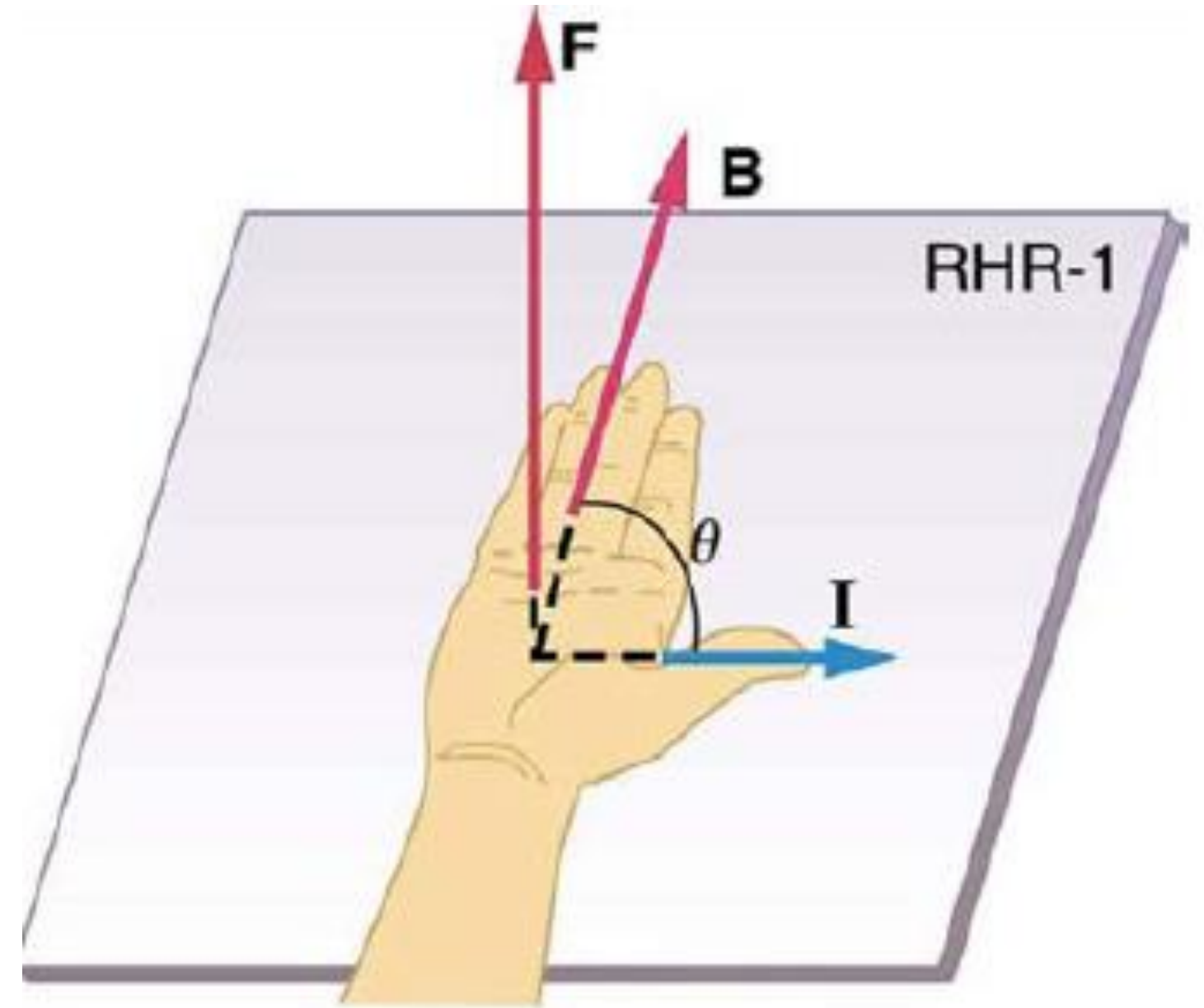
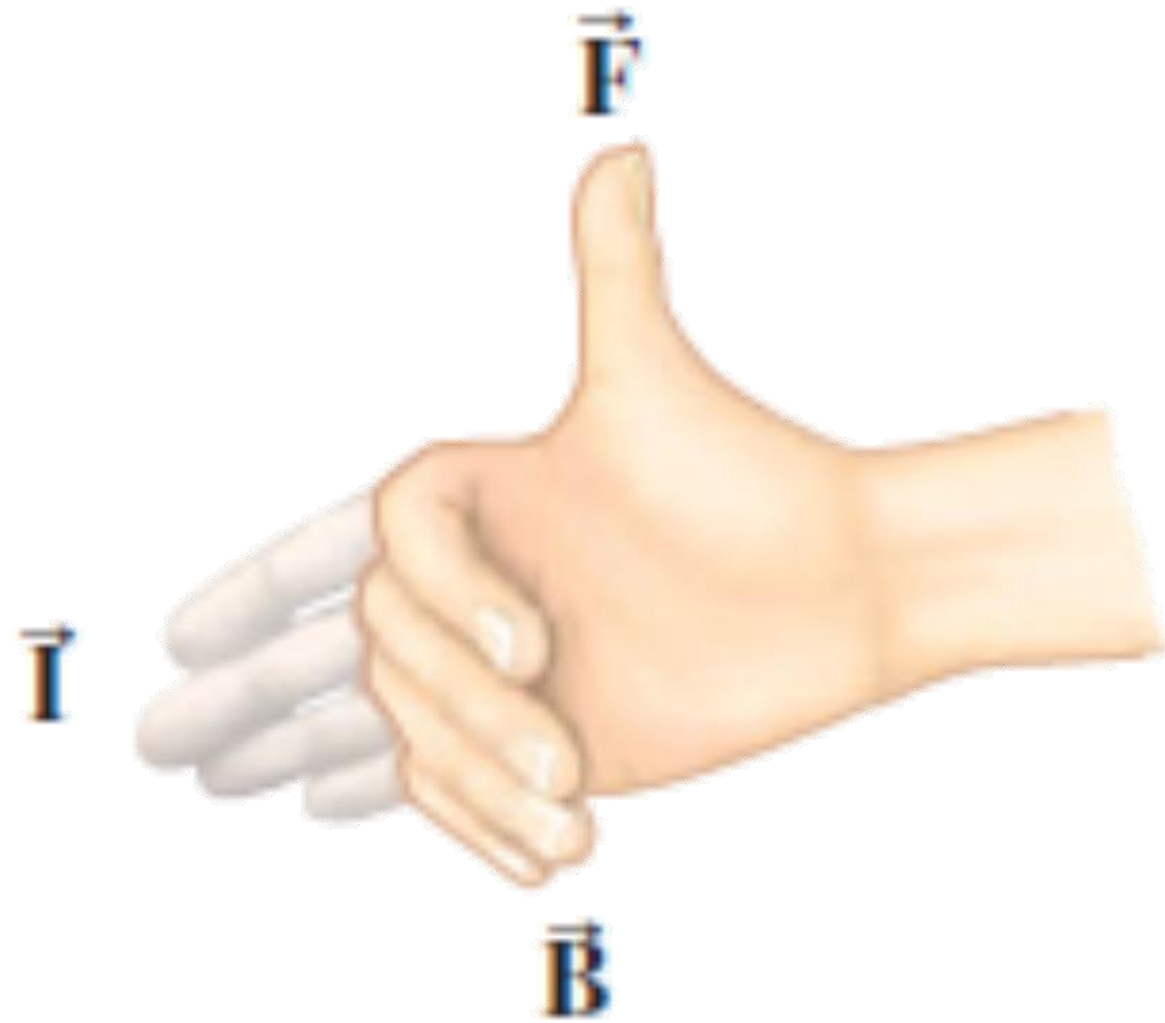
$$\Delta Q / \Delta t$$



$$\Rightarrow F = I l B \sin \theta$$

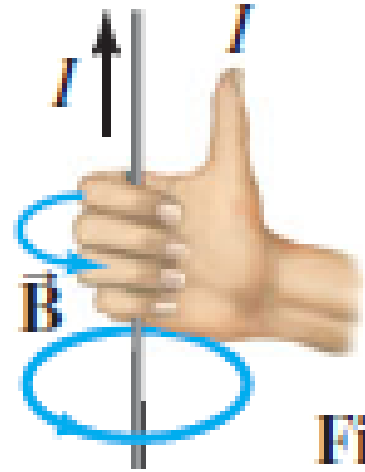
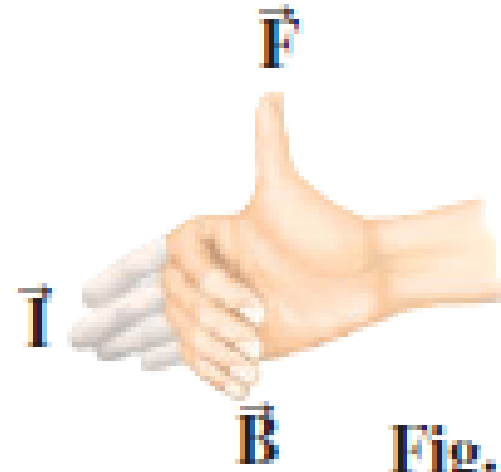
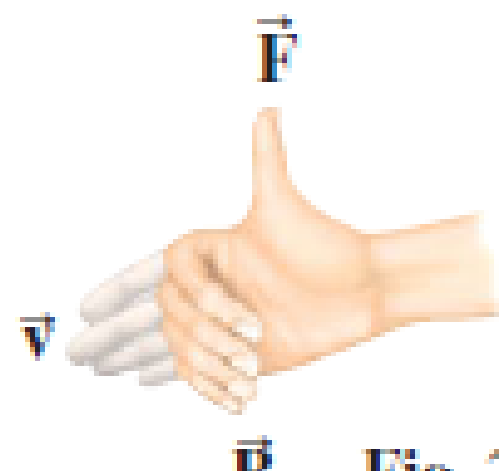
θ : angle between B and I

Right hand rule

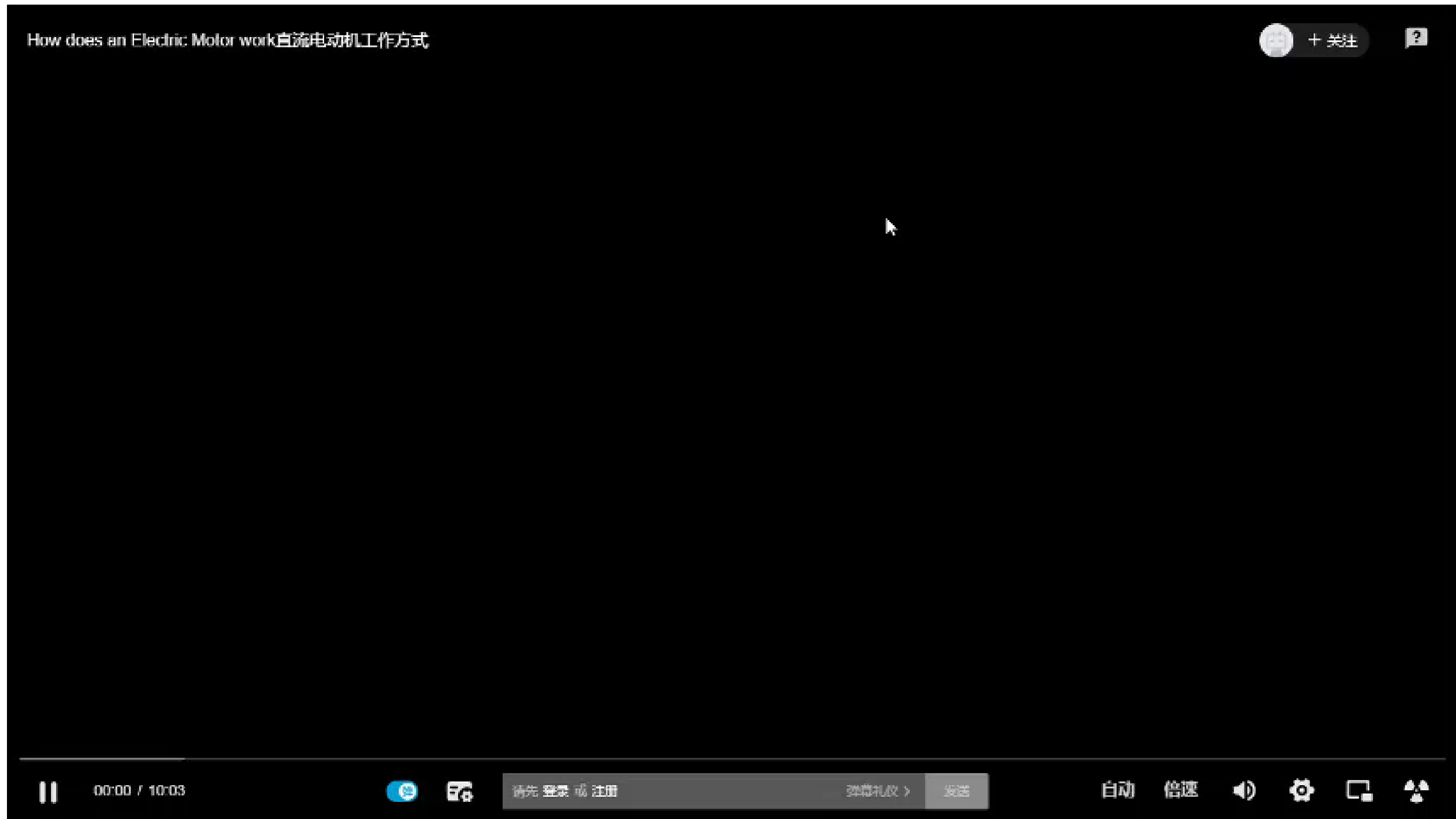


One of them is enough

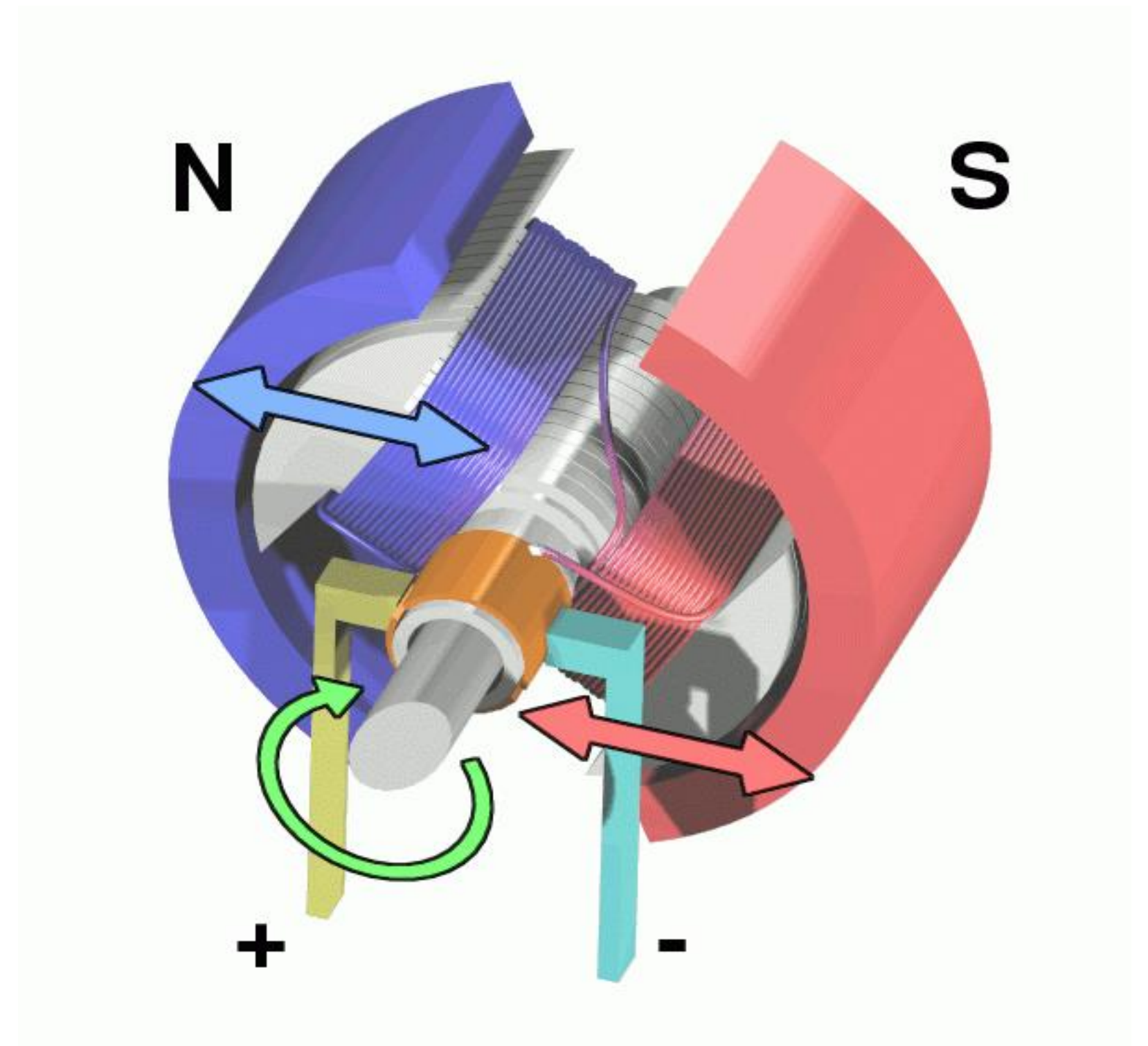
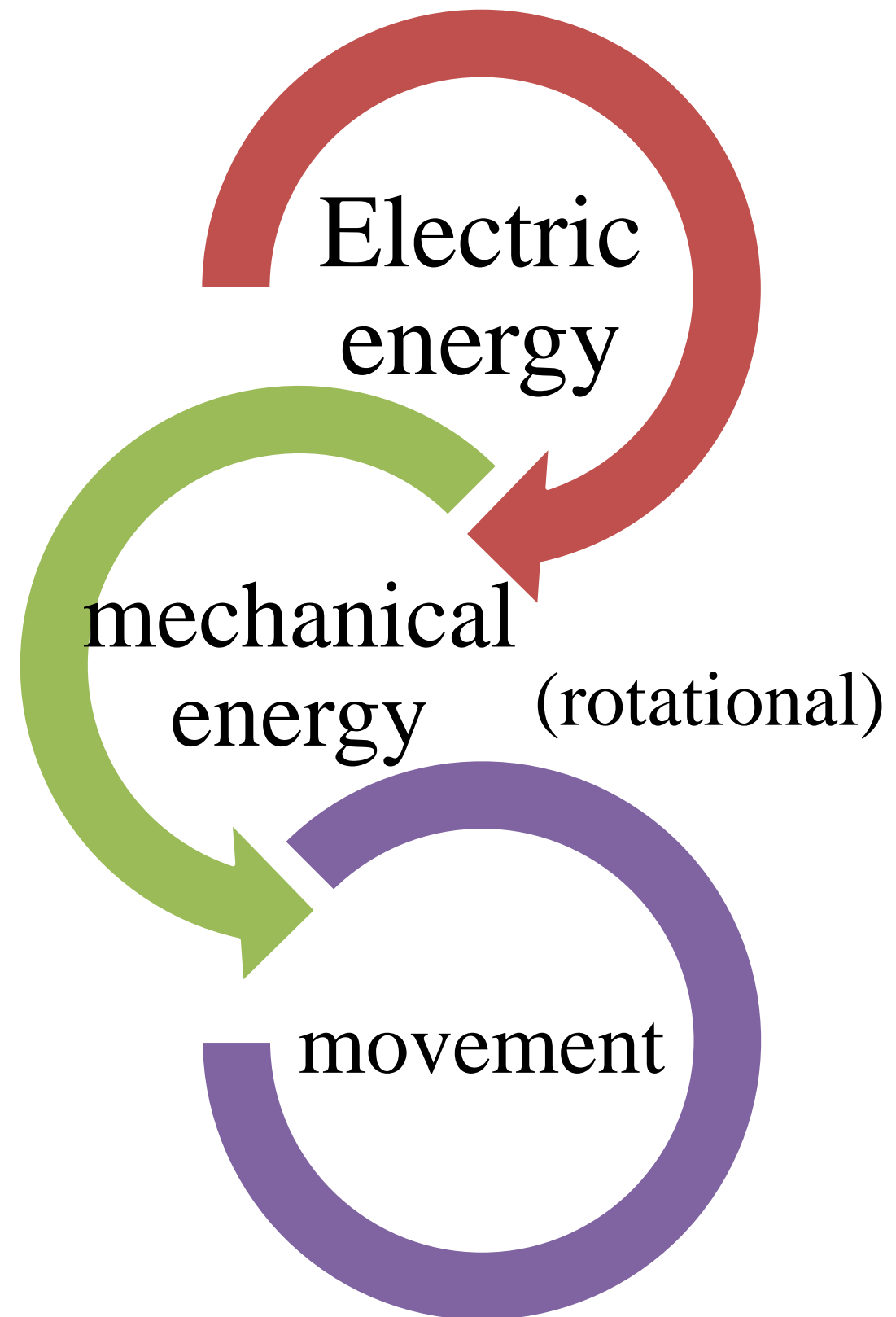
TABLE 20–1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1)	 <p>Fig. 20–8d</p>	Wrap fingers around wire with thumb pointing in direction of current I	Fingers curl in direction of \vec{B}
2. Force on electric current I due to magnetic field (RHR-2)	 <p>Fig. 20–11c</p>	Fingers first point straight along current I , then bend along magnetic field \vec{B}	Thumb points in direction of the force \vec{F}
3. Force on electric charge $+q$ due to magnetic field (RHR-3)	 <p>Fig. 20–15</p>	Fingers point along particle's velocity \vec{v} , then along \vec{B}	Thumb points in direction of the force \vec{F}

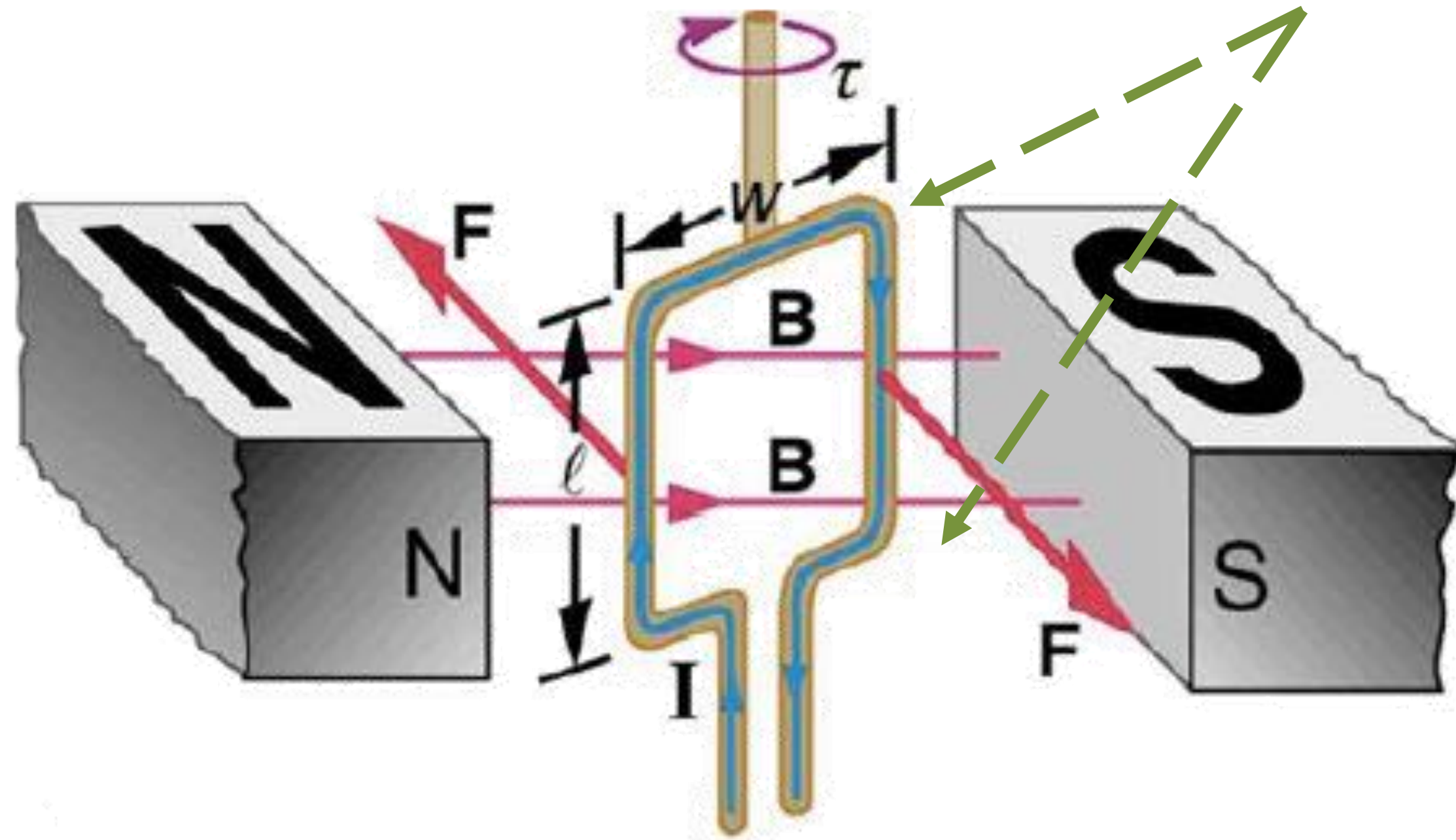
Electric motor



Electric motor



Electric motor



forces on these two parts
are vertical (**no torque**)

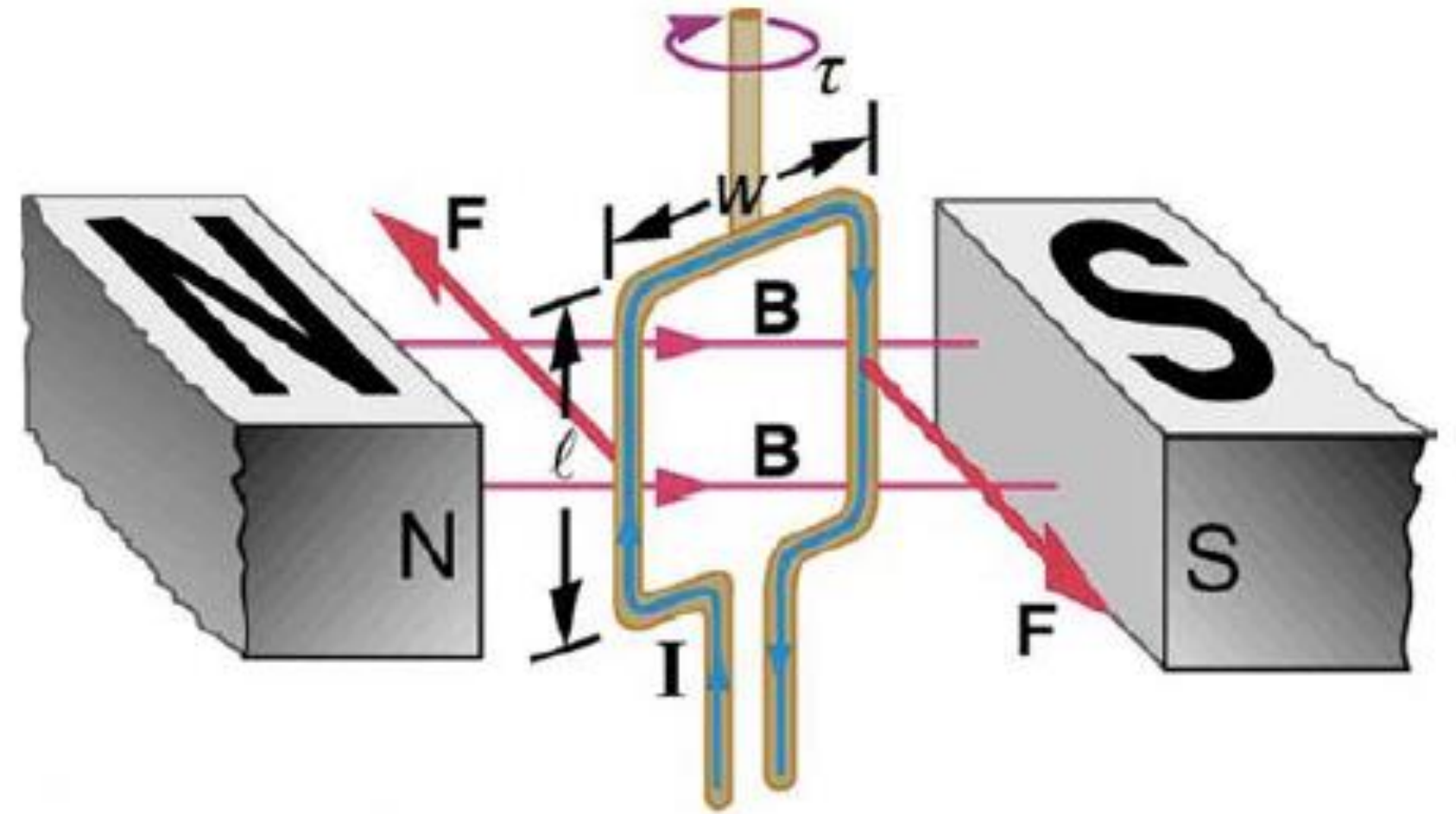
The magnetic force on the conductor creates ***torque***

Electric motor

torque: $\tau = rF \sin \theta$

r : distance from the pivot $r = w/2$

θ : angle between F and r



- The torque on each vertical segment is the same

total torque: $\tau = \frac{w}{2} F \sin \theta + \frac{w}{2} F \sin \theta = w F \sin \theta$

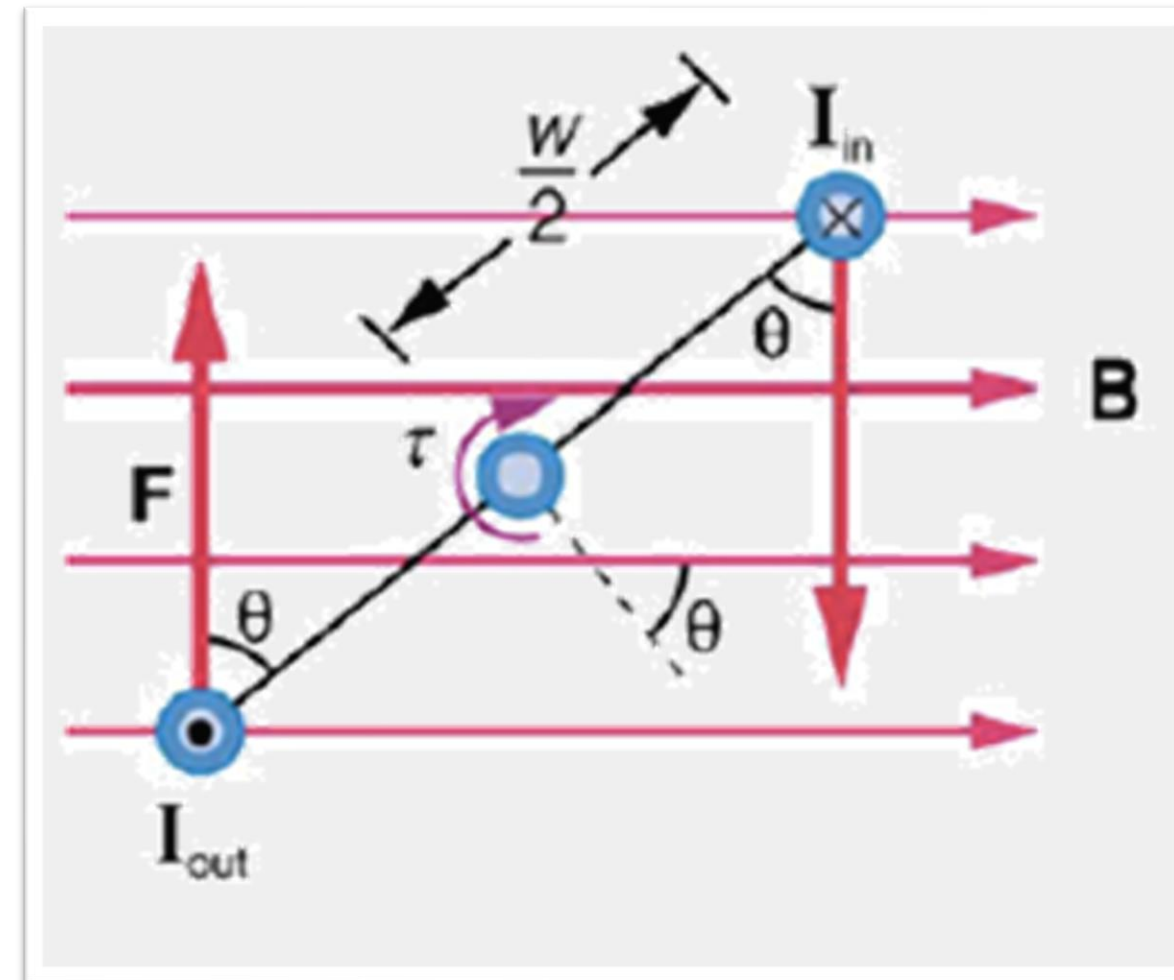
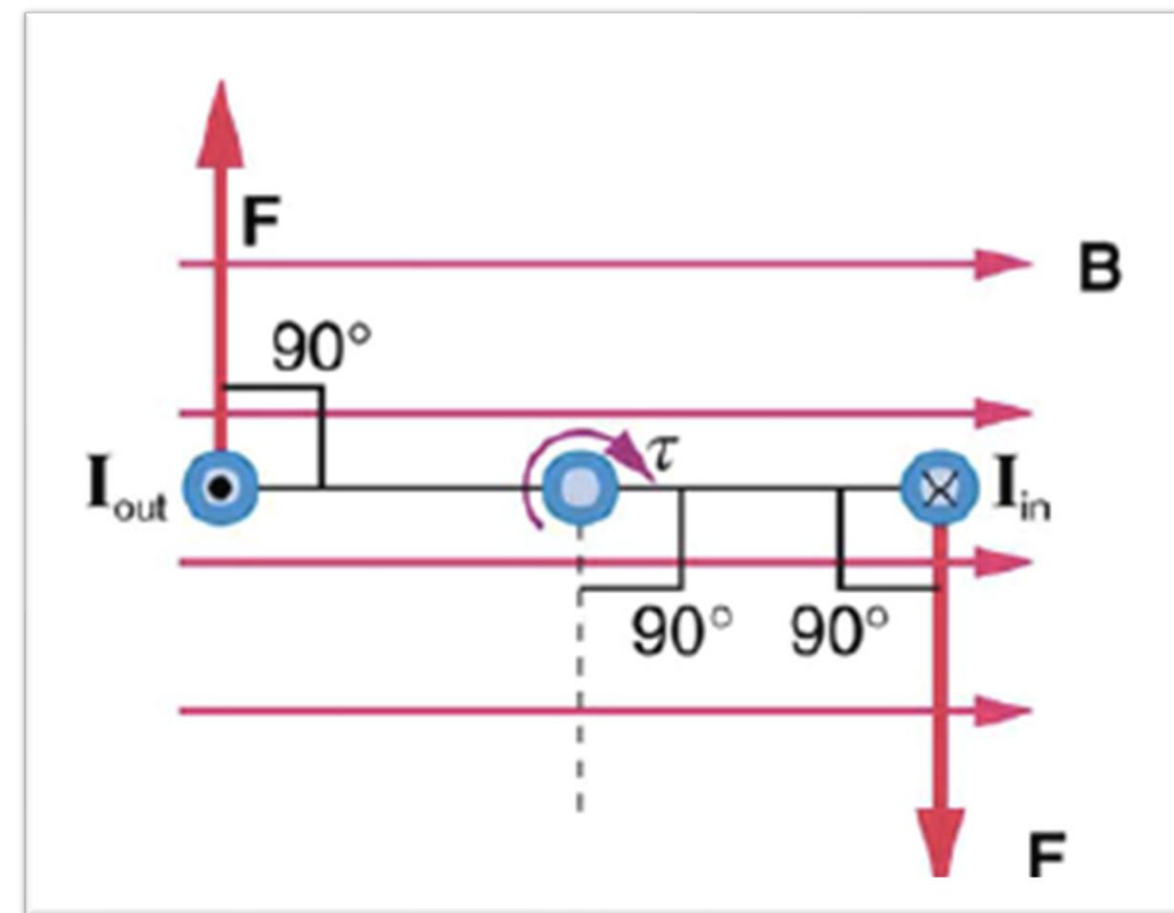
Electric motor

$$\tau_{\max} = w I l B$$

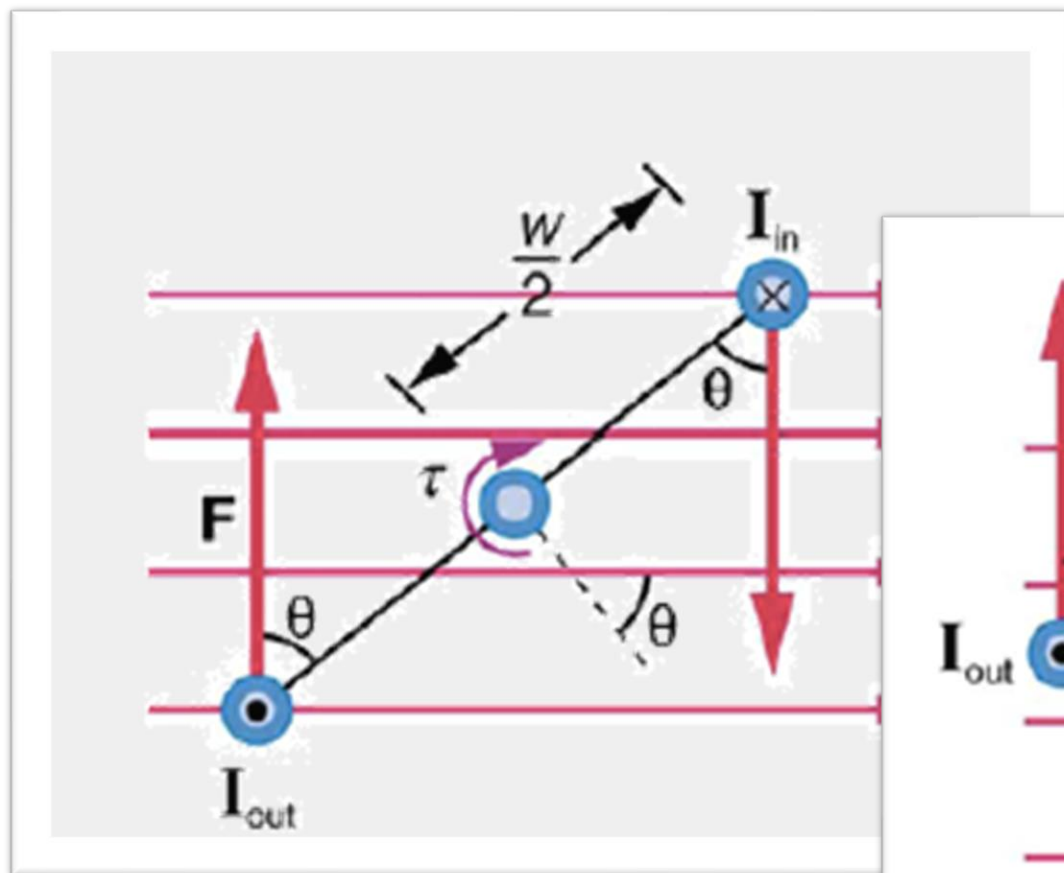
$$\tau = w F \sin \theta$$

$$F = I l B$$

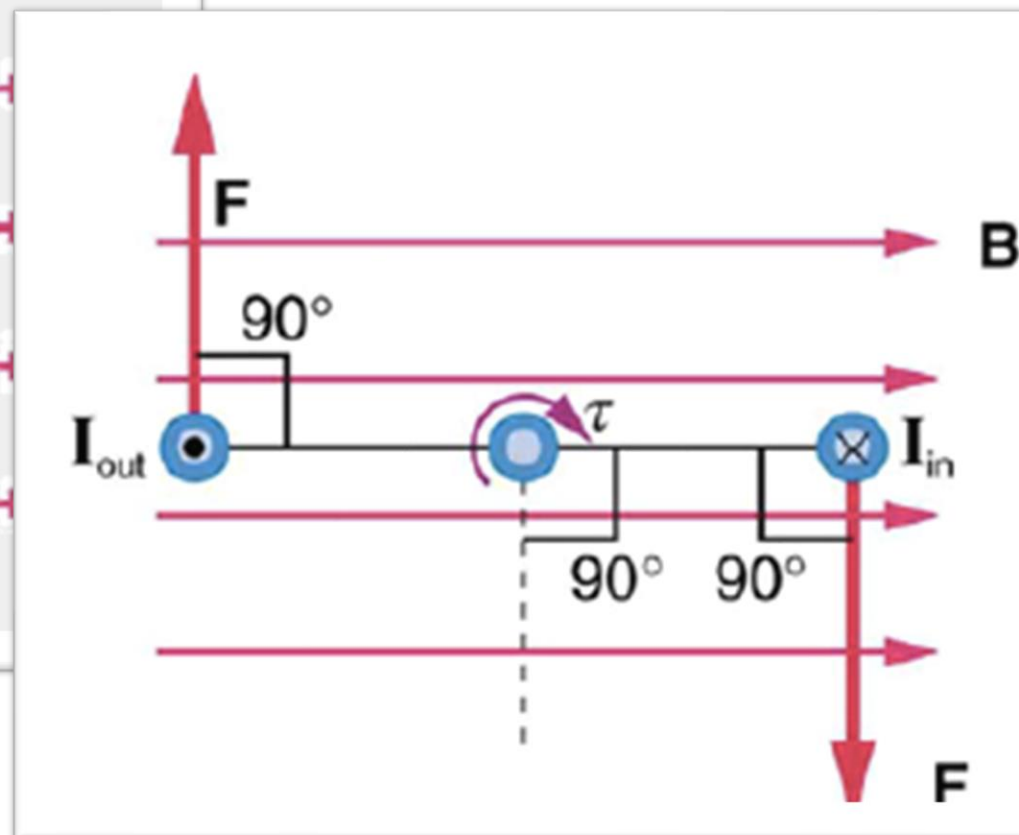
$$\Rightarrow \tau = w I l B \sin \theta$$



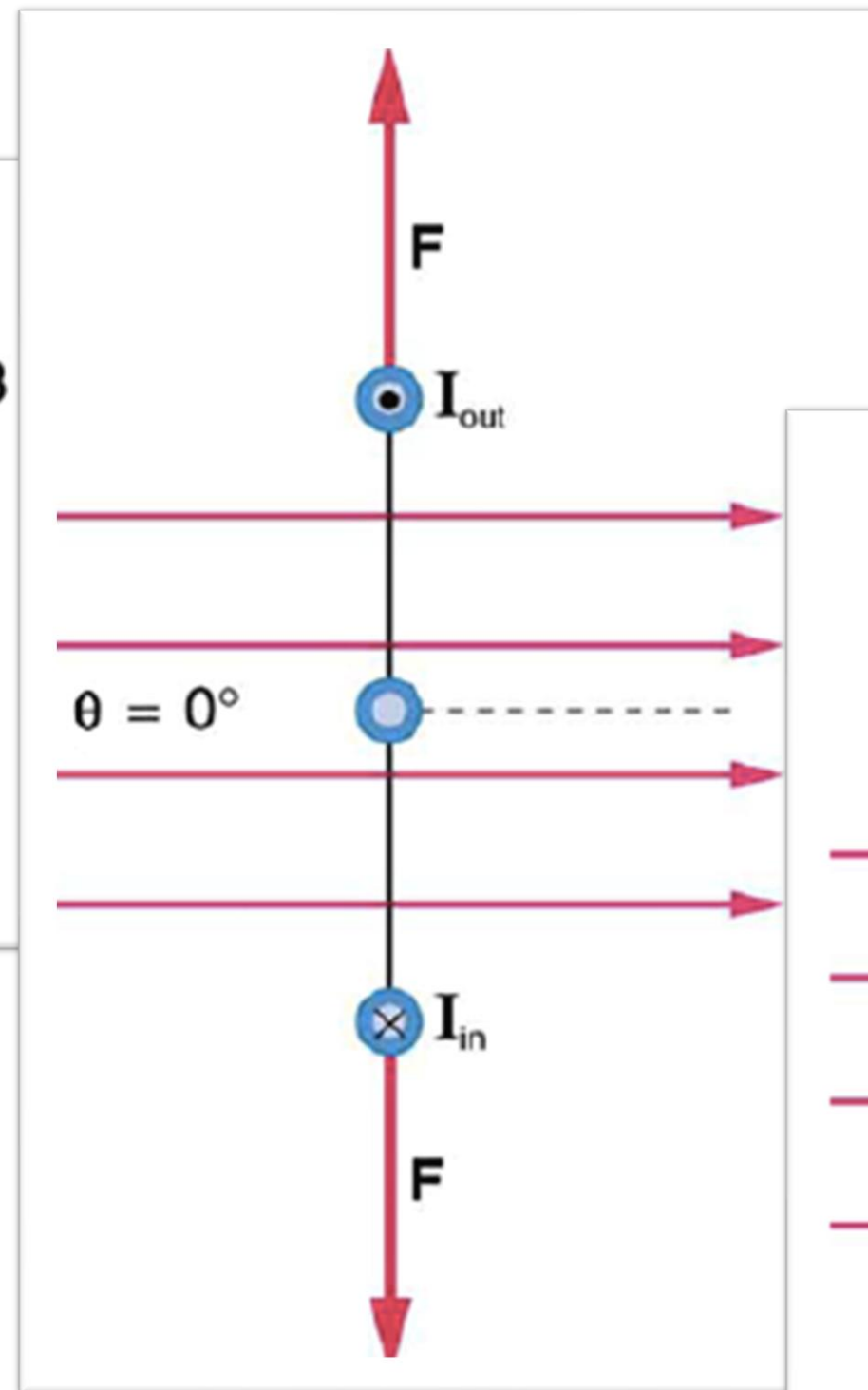
Electric motor



$$\tau = w I l B \sin \theta$$

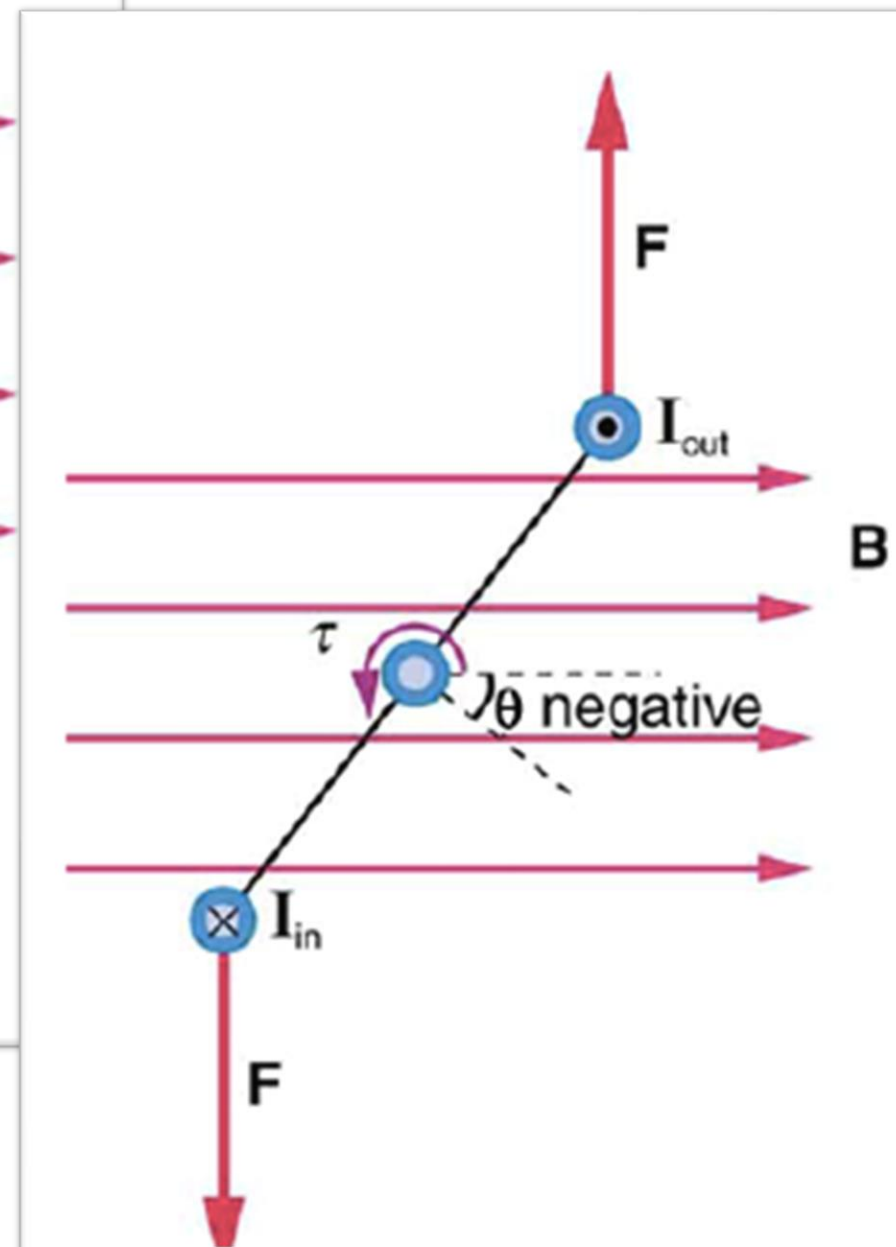


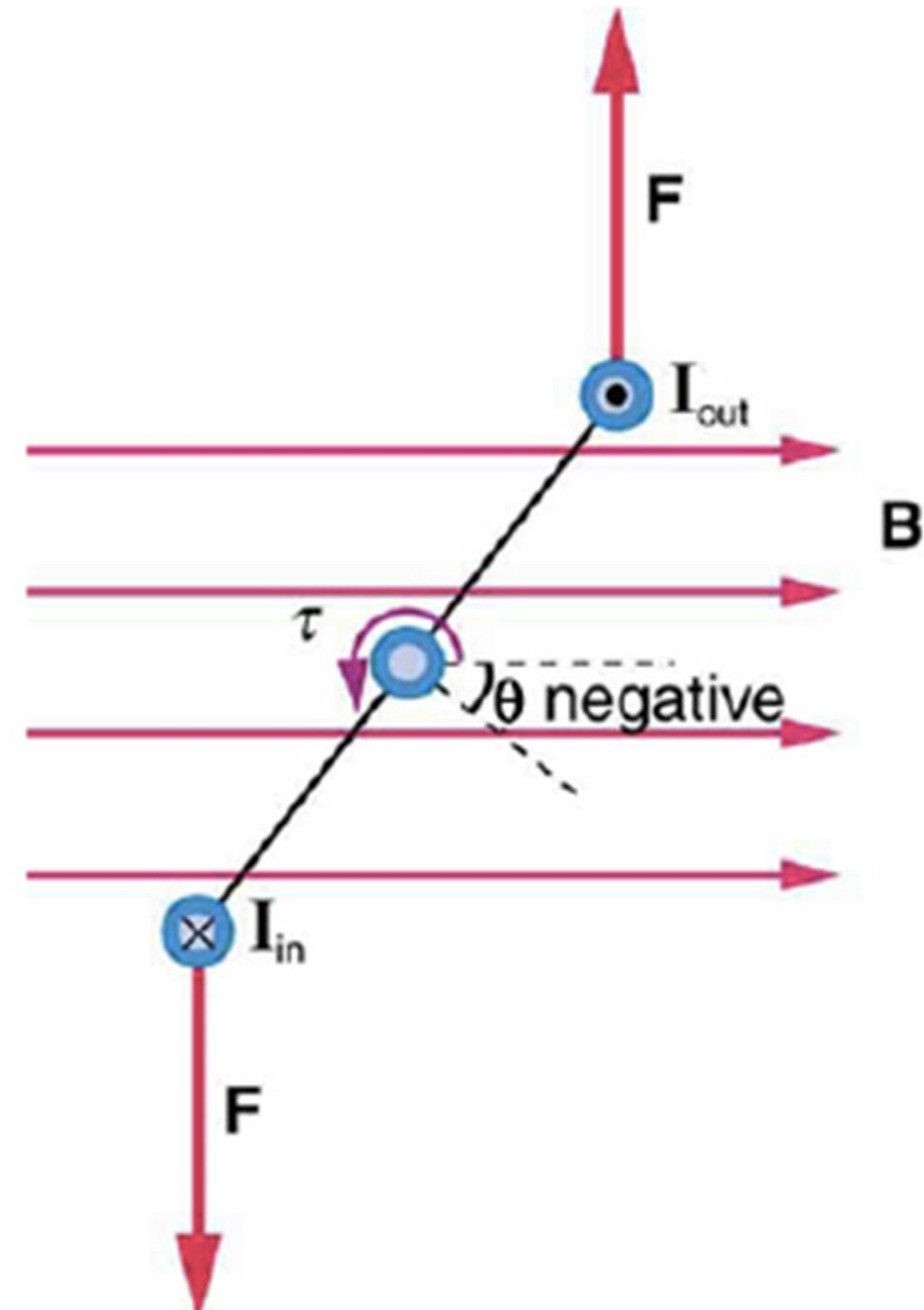
$$\tau_{\max} = w I l B$$



$$\theta = 0 \implies \tau = 0$$

motor will stop?
torque is negative



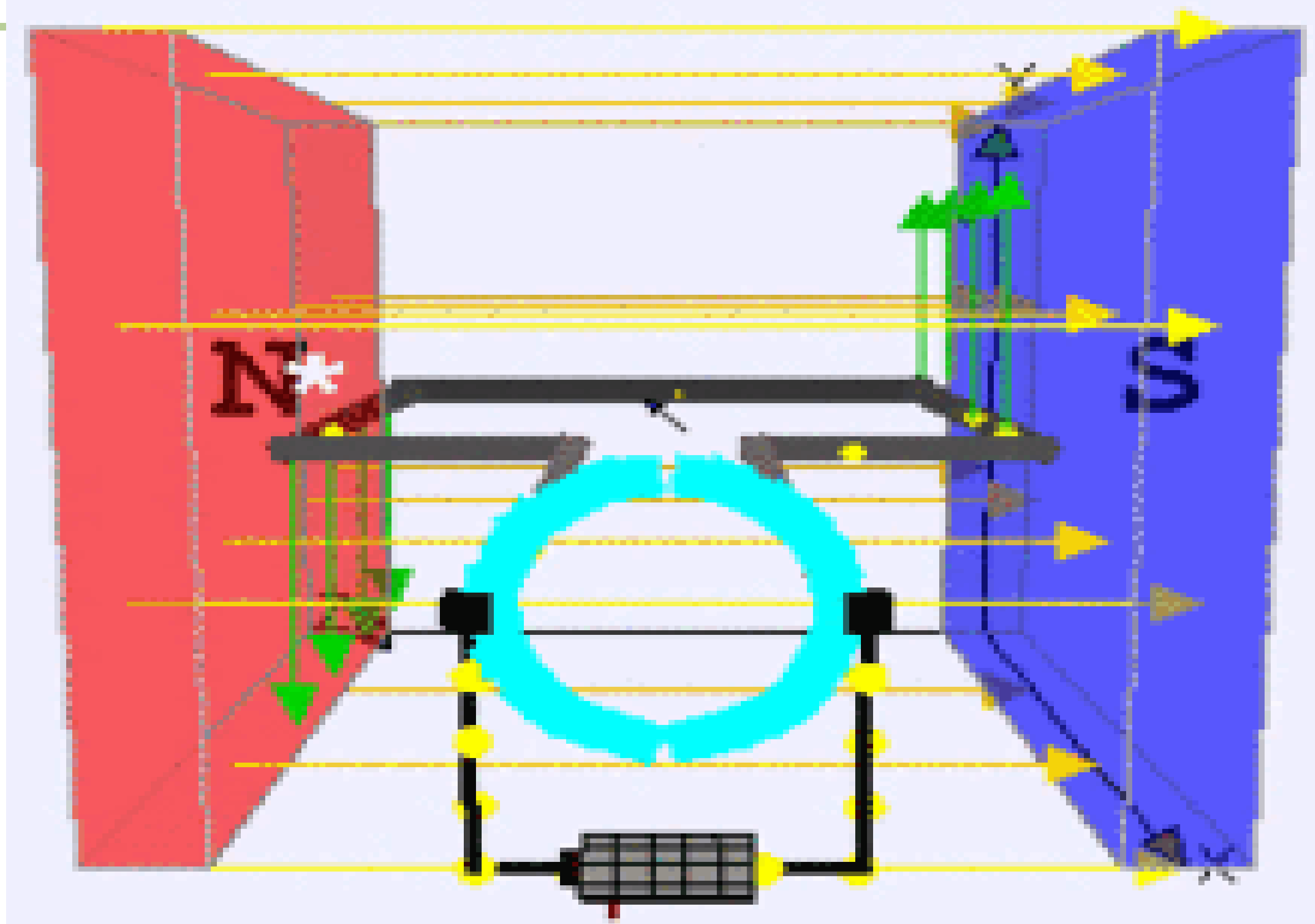


How to keep the motor going?

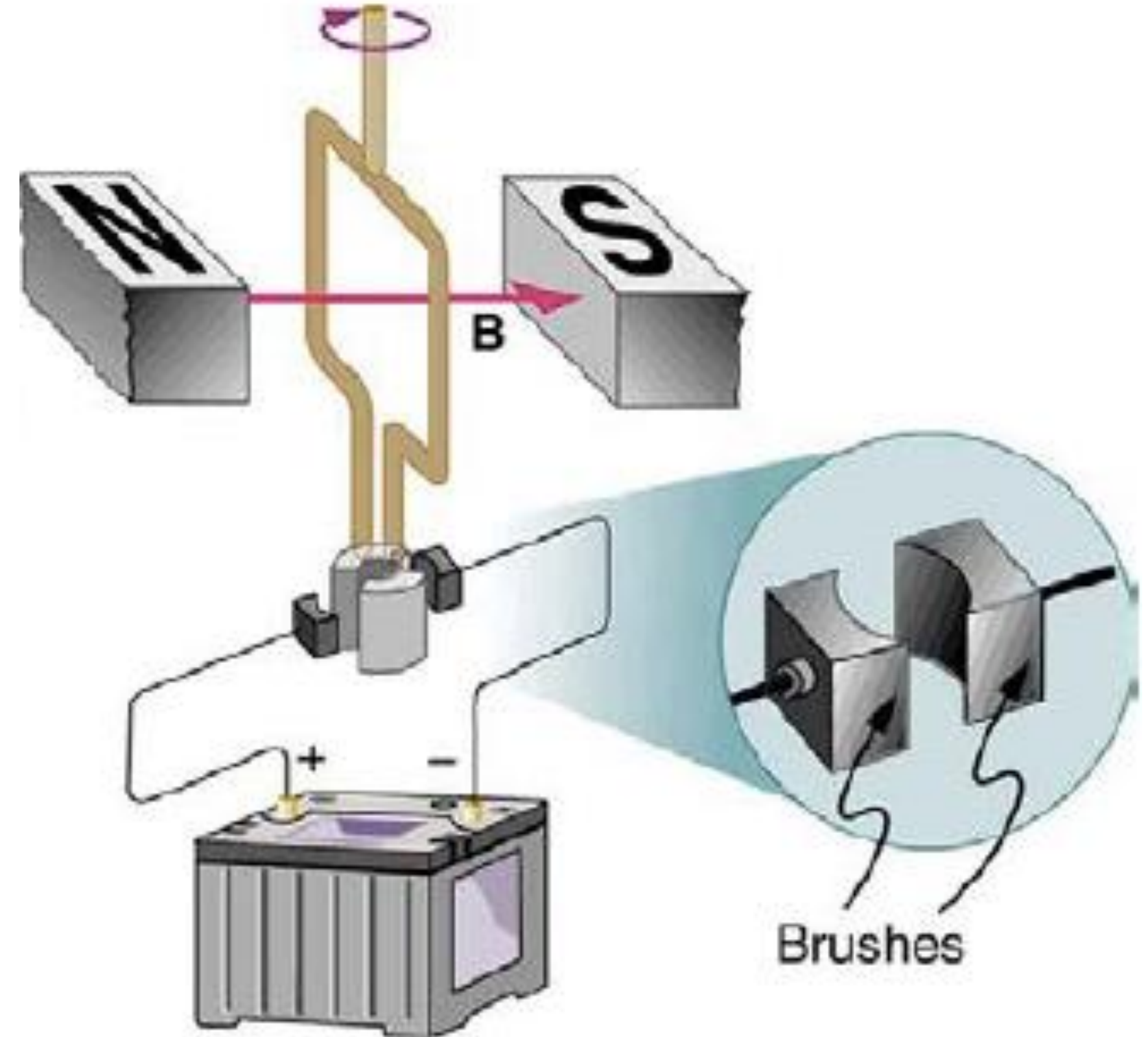
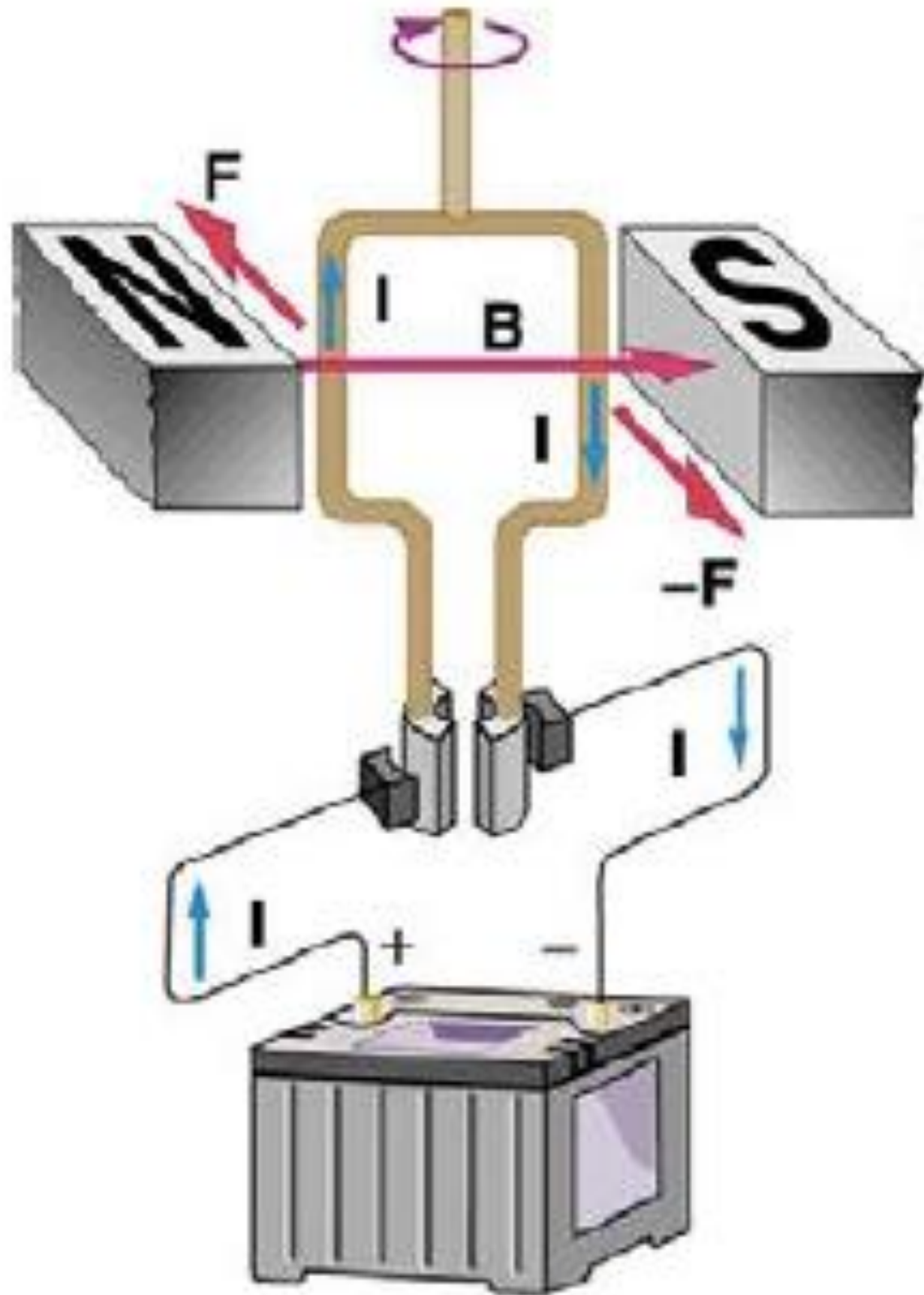
- by having the torque always in the same direction

Ways to keep the motor going?

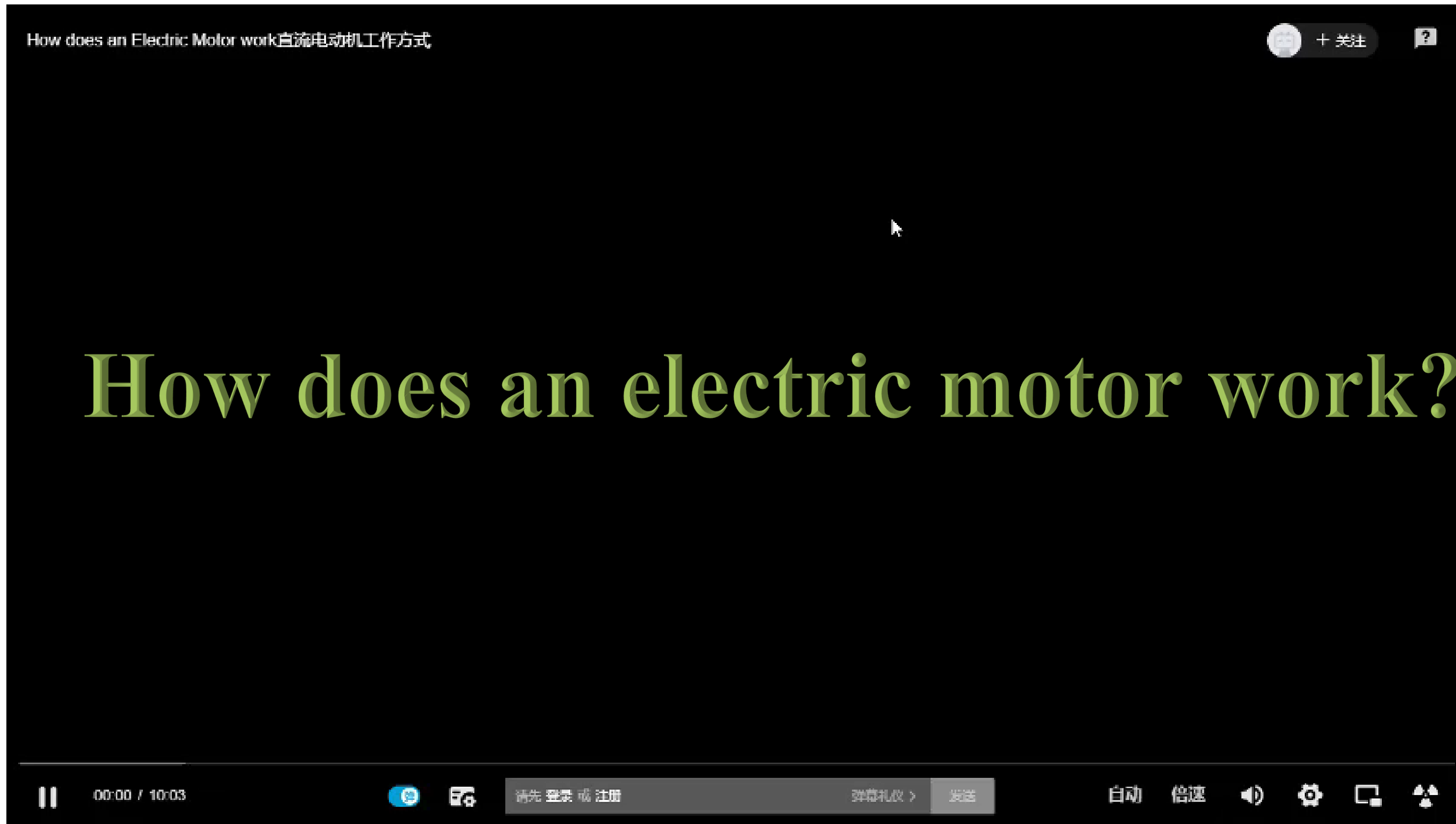
- Change the direction of :
 1. magnetic poles
 2. current



The direction of the current is reversed whenever the loop goes through $\theta = 0$



Video



Structure

Magnetic field

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graph LR; A[Magnetic field] --> B[Field line]; A --> C[The effect of magnetic field on moving charges and current]; B --> D[Hall effect]; B --> E[Electric motors]; C --> F[Magnetic field generated by electric current]; C --> G[Magnetic circuit];
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Field line

The effect of
magnetic field on
moving charges
and current

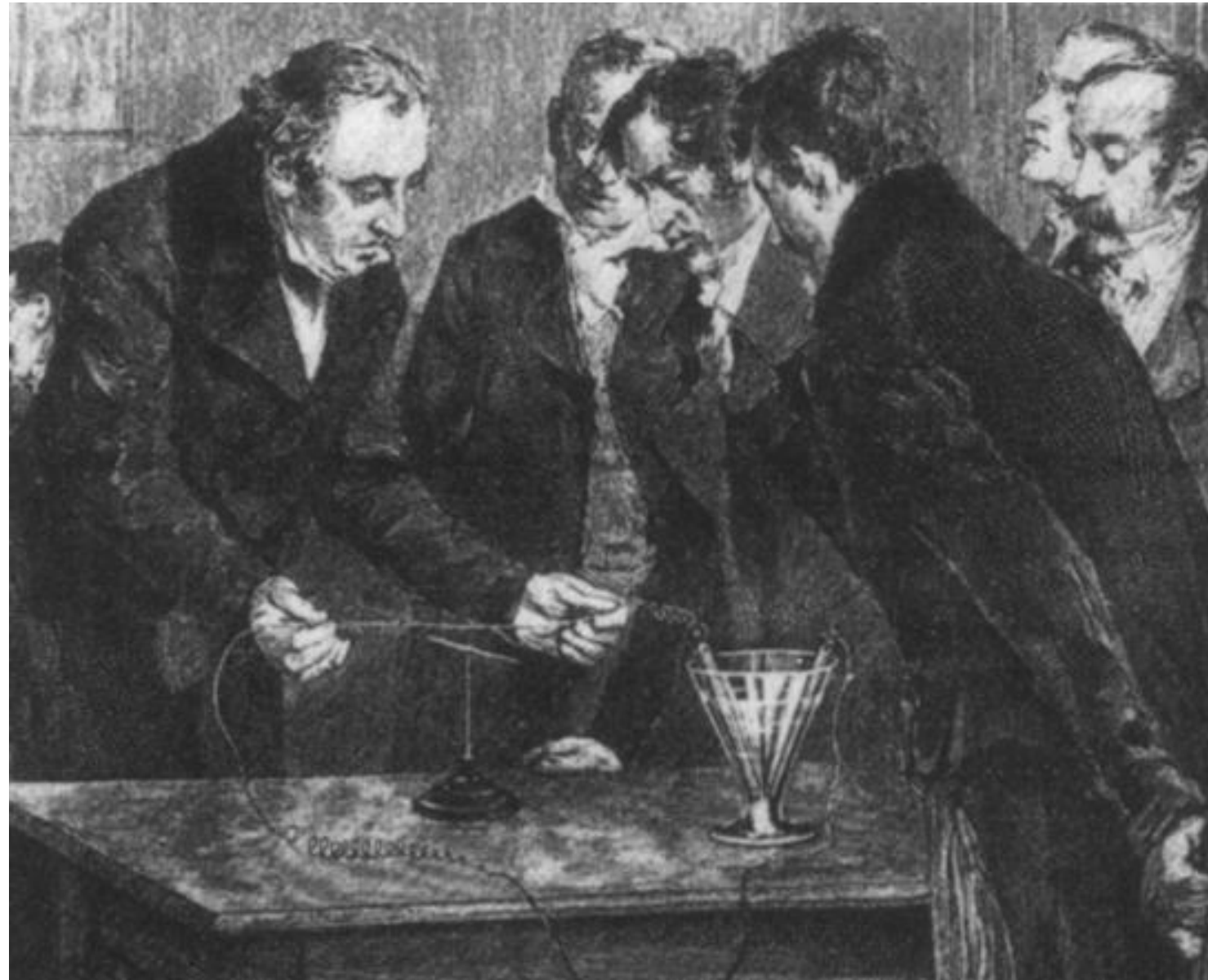
Hall effect

Electric motors

Magnetic field
generated by
electric current

Magnetic circuit

An electric current induces a magnetic field

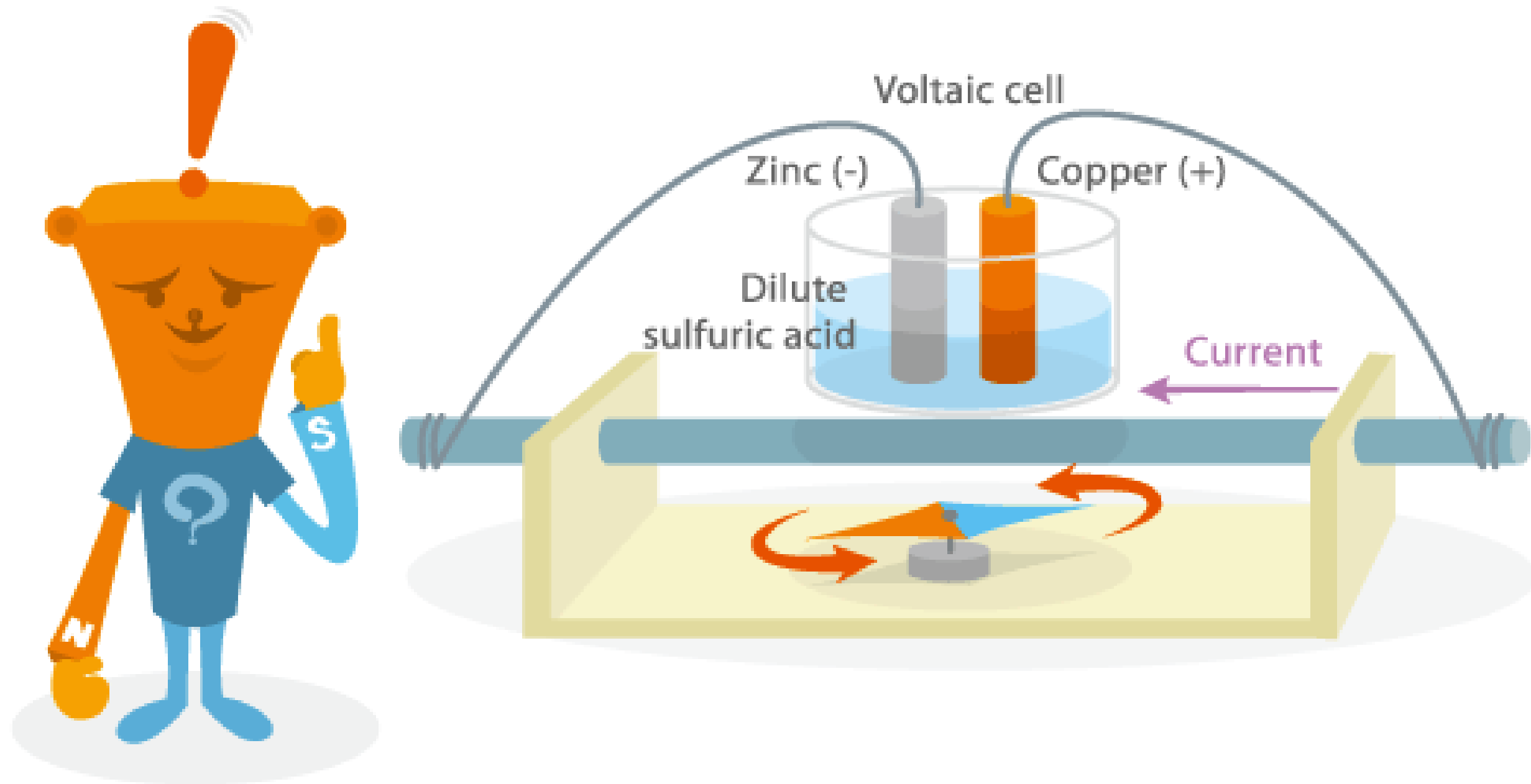


Hans Christian Ørsted (1820)

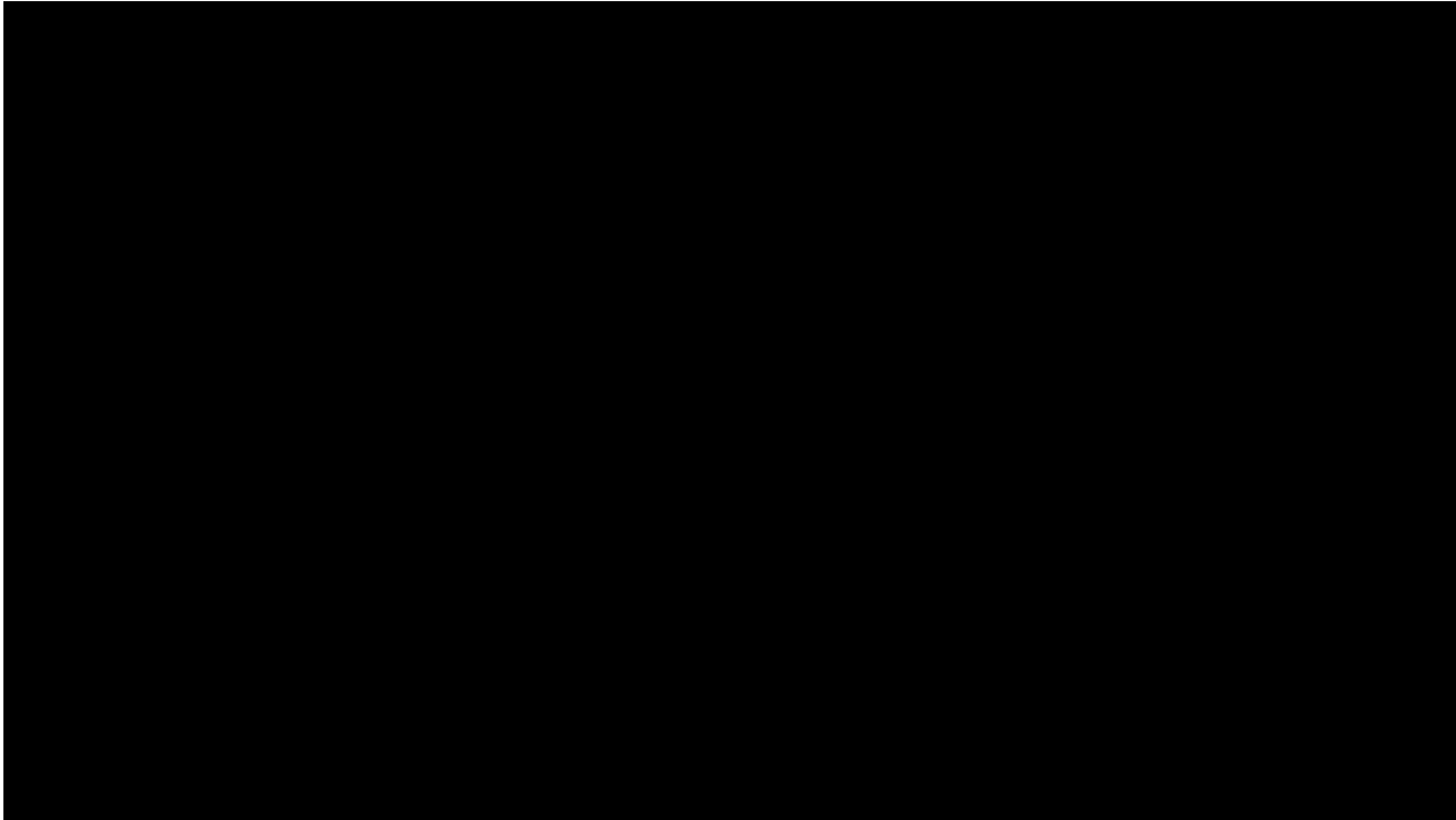
An electric current induces a magnetic field

Oersted's experiment (magnetic effect of current)

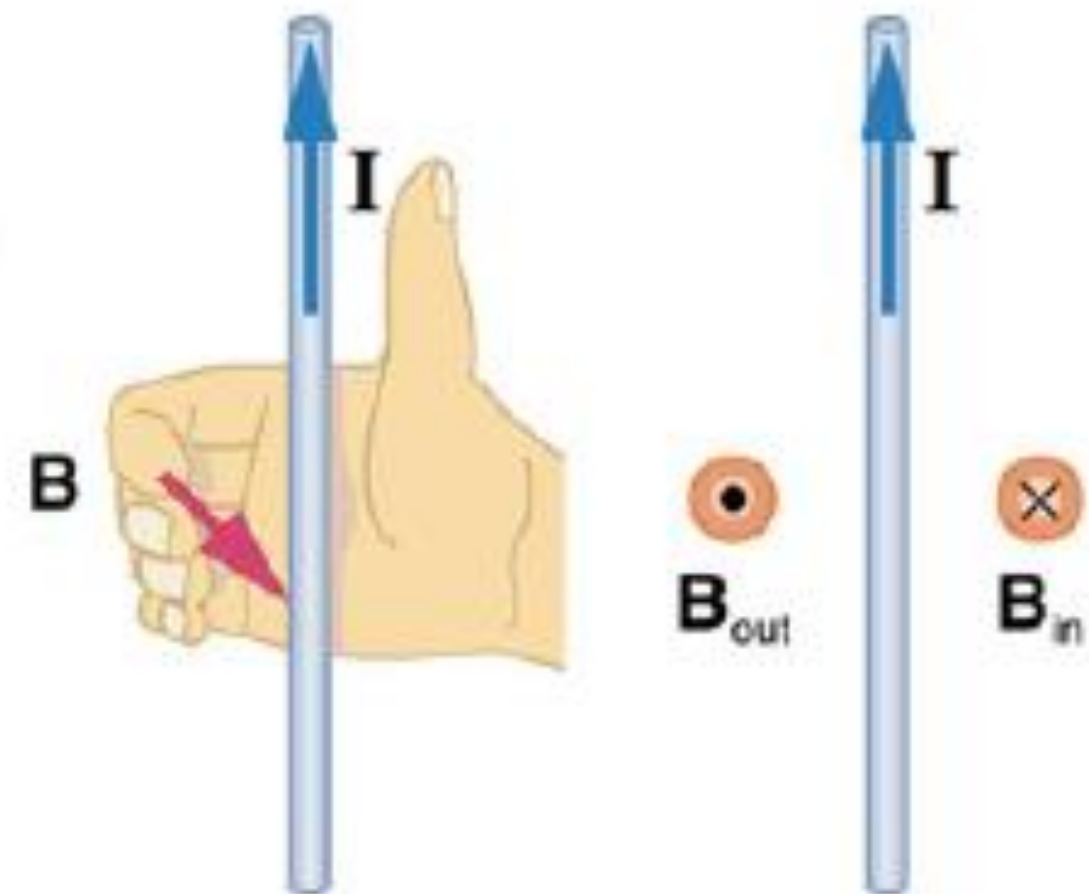
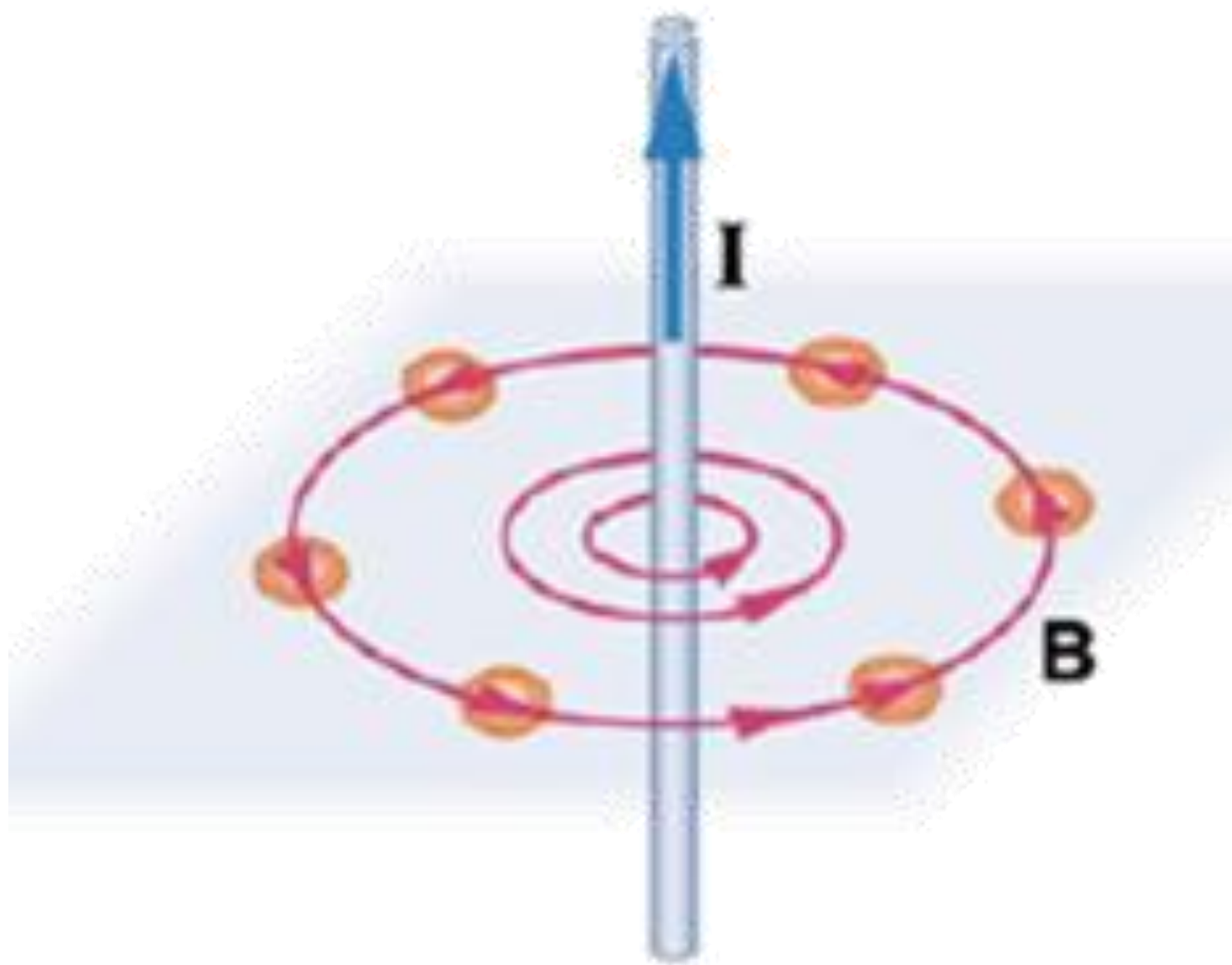
When a current from a Voltaic cell flows through a conductor, a nearby magnetic needle moves.



Oersted Experiment

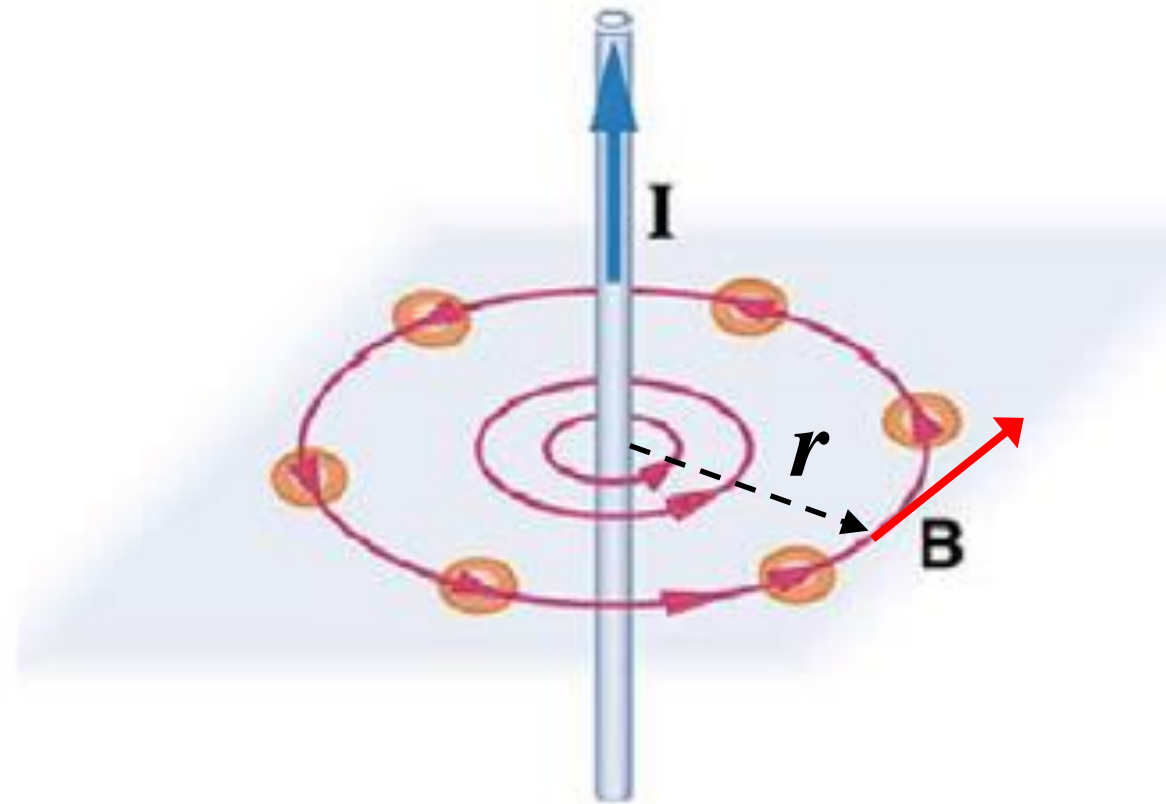


Right hand rule



Ampere's law: Magnetic field produced by a current

long, straight wire



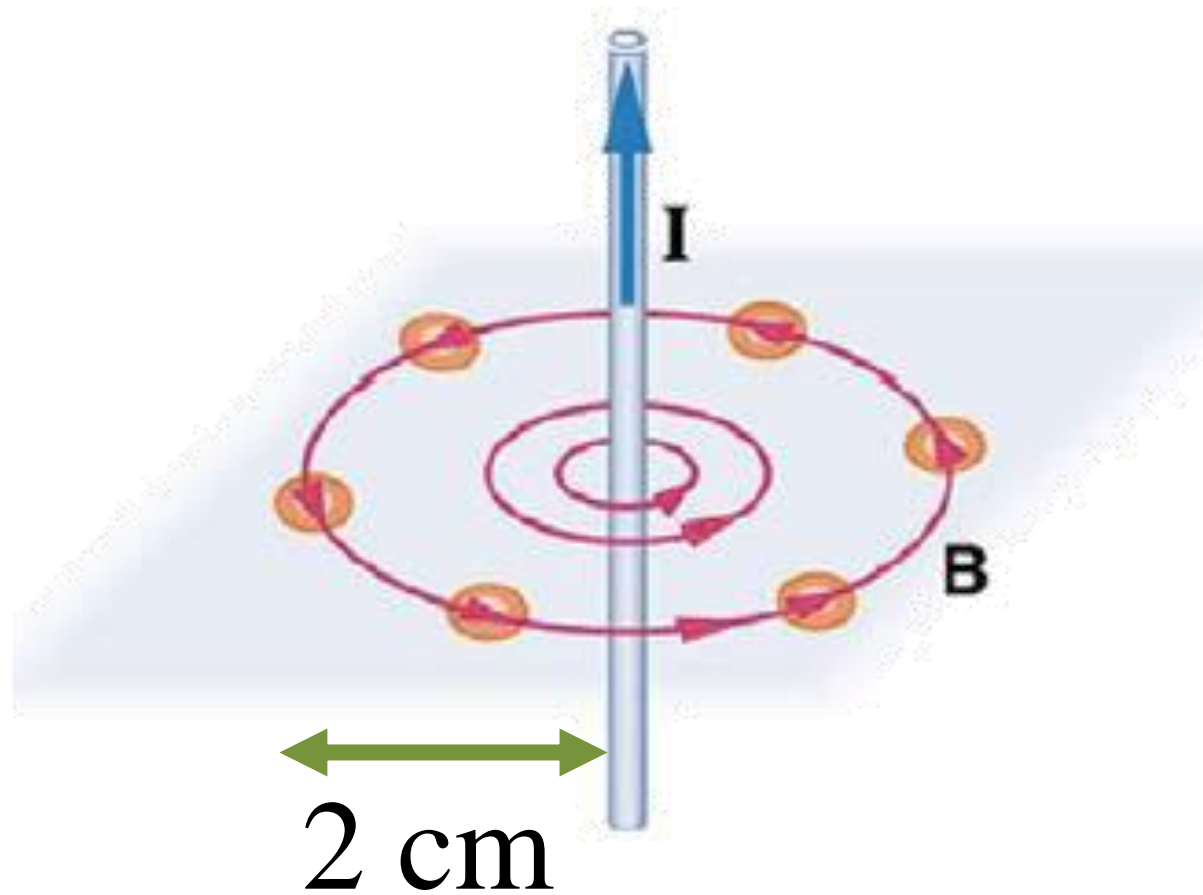
$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$
$$(\text{N/A}^2)$$

permeability of free space

Example

How much current is needed to produce a magnetic field equal to that of the Earth, at 2 cm from a long wire ?



$$B = \frac{\mu_0 I}{2\pi r}$$

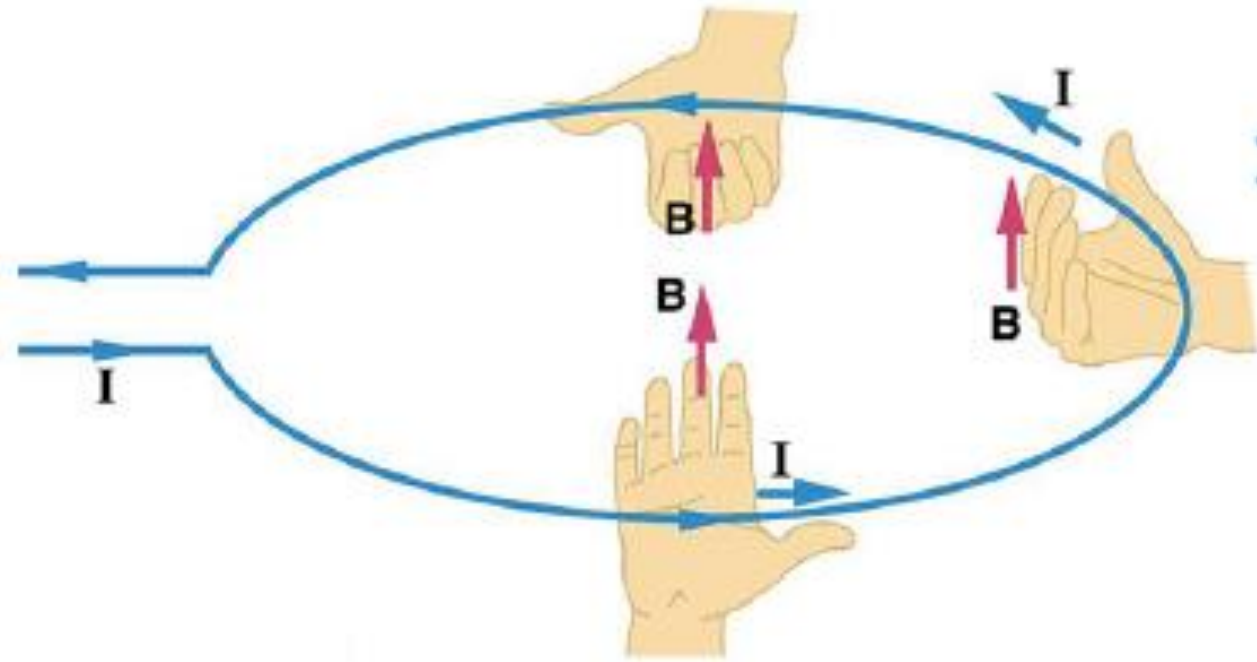
$$B = 5 \times 10^{-5} \text{ T}$$

$$r = 2 \times 10^{-2} \text{ m}$$

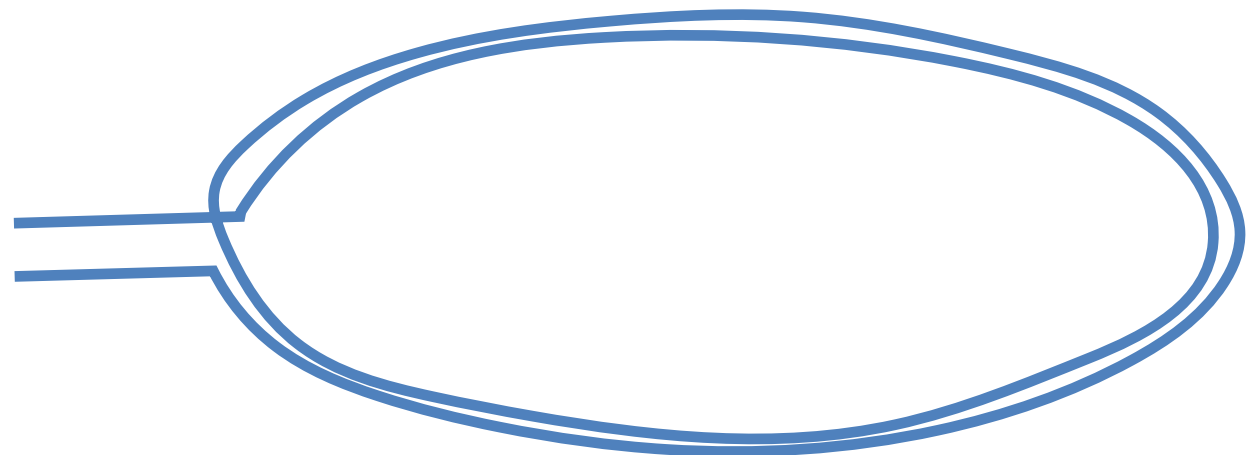
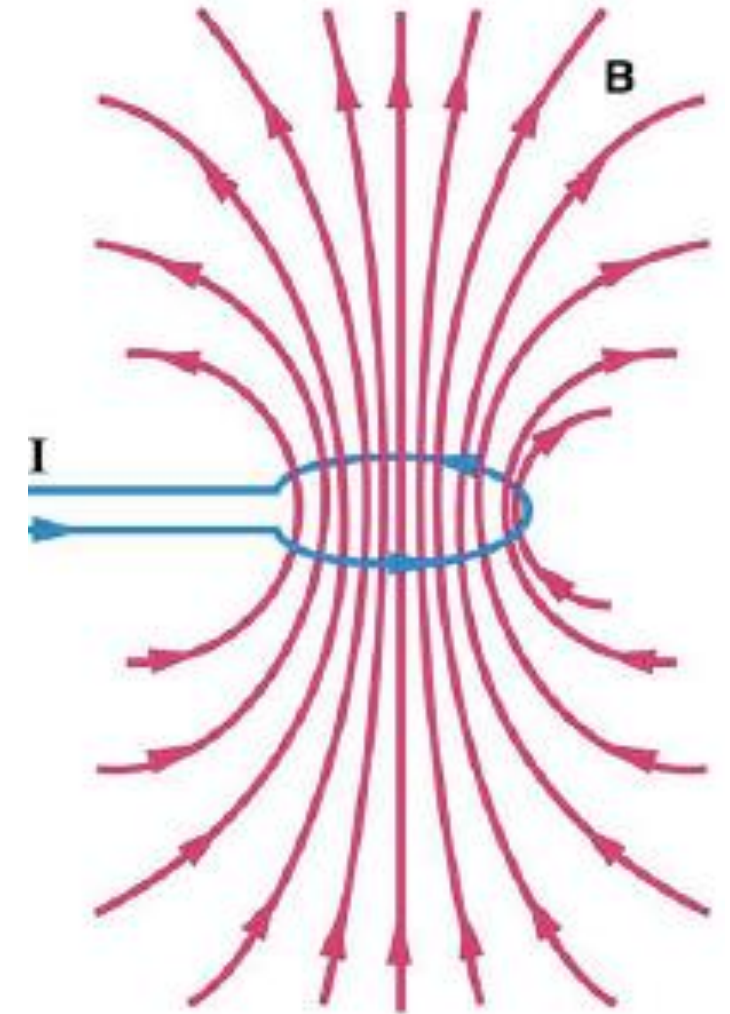
$$I = \frac{2\pi r B}{\mu_0}$$

$$= \frac{2\pi \times 2 \times 10^{-2} \text{ m} \times 5 \times 10^{-5} \text{ T}}{4\pi \times 10^{-7}} = 5 \text{ A}$$

Magnetic field from a circular current loop



all elements of wire
produce a field

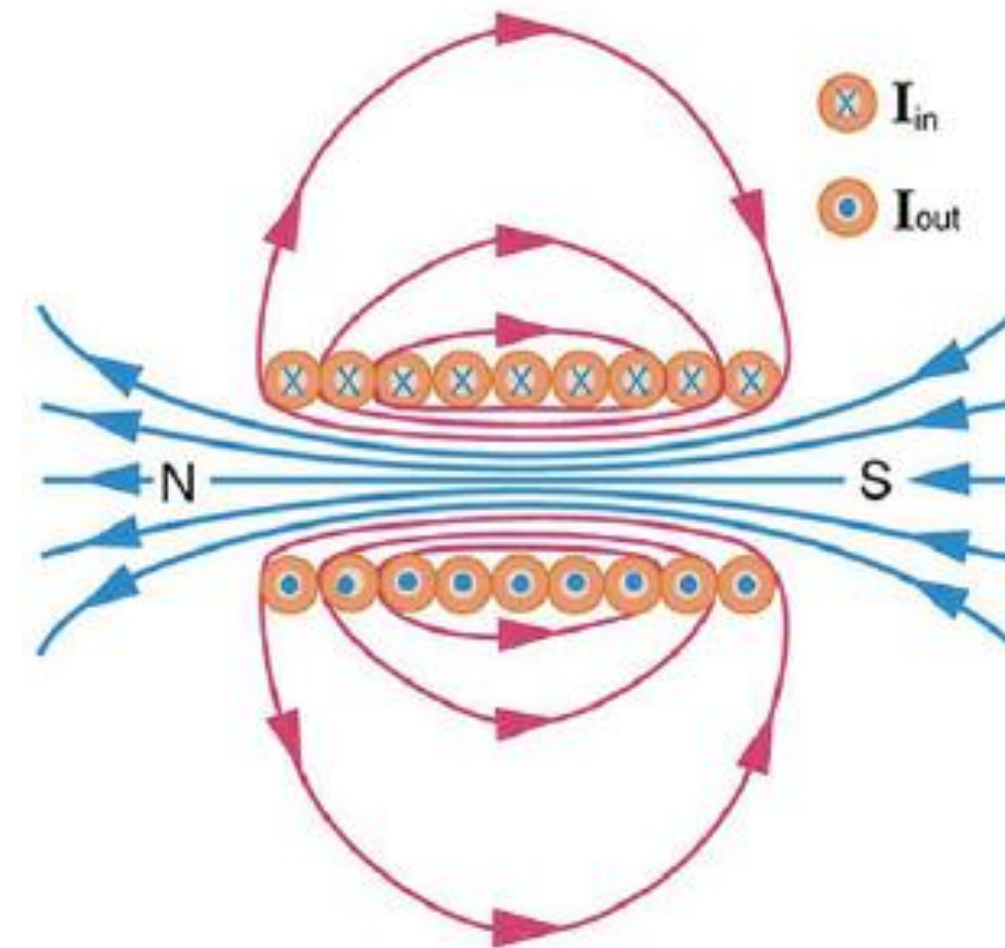
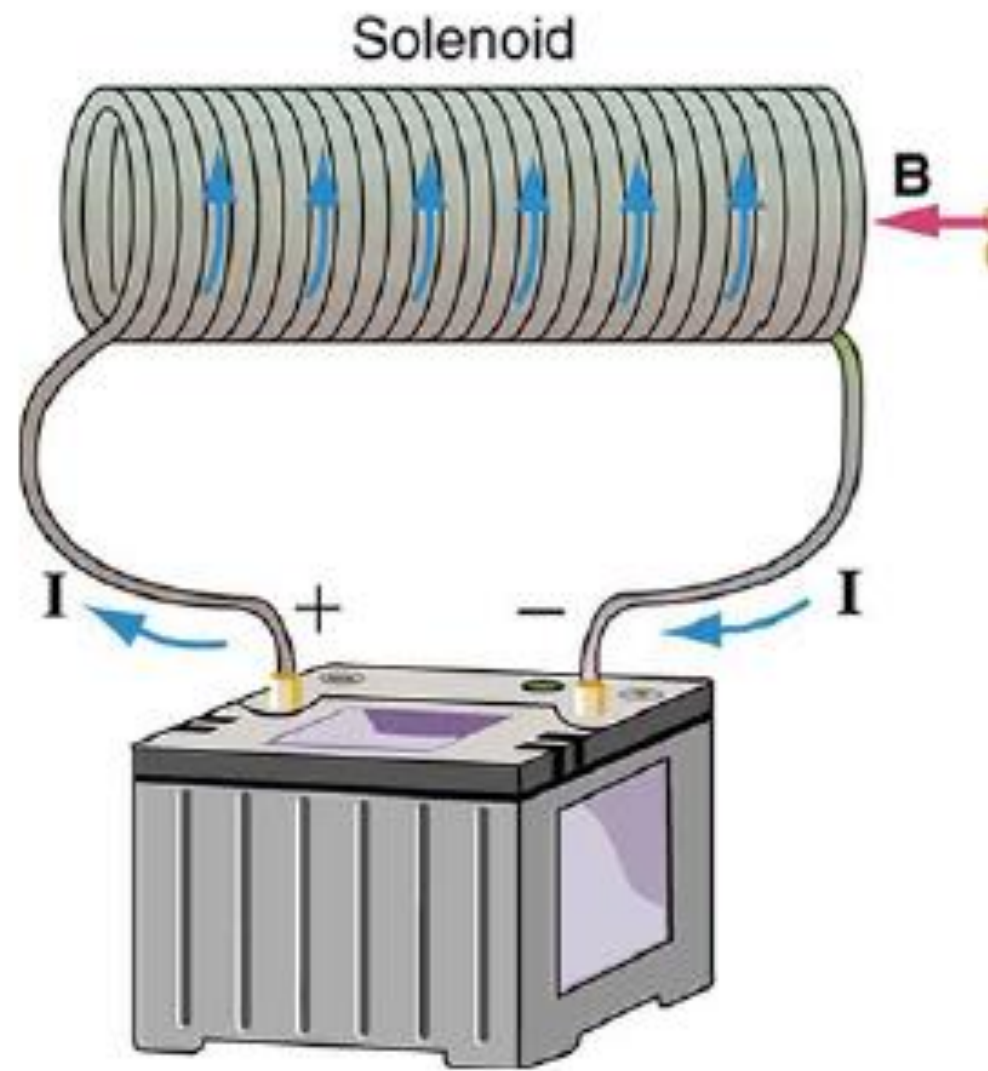


for N loops: $B = N \frac{\mu_0 I}{2R}$

at the center:

$$B = \frac{\mu_0 I}{2R}$$

Solenoid - many loops of wire along one axis



n is the *number* of turns
per unit of length

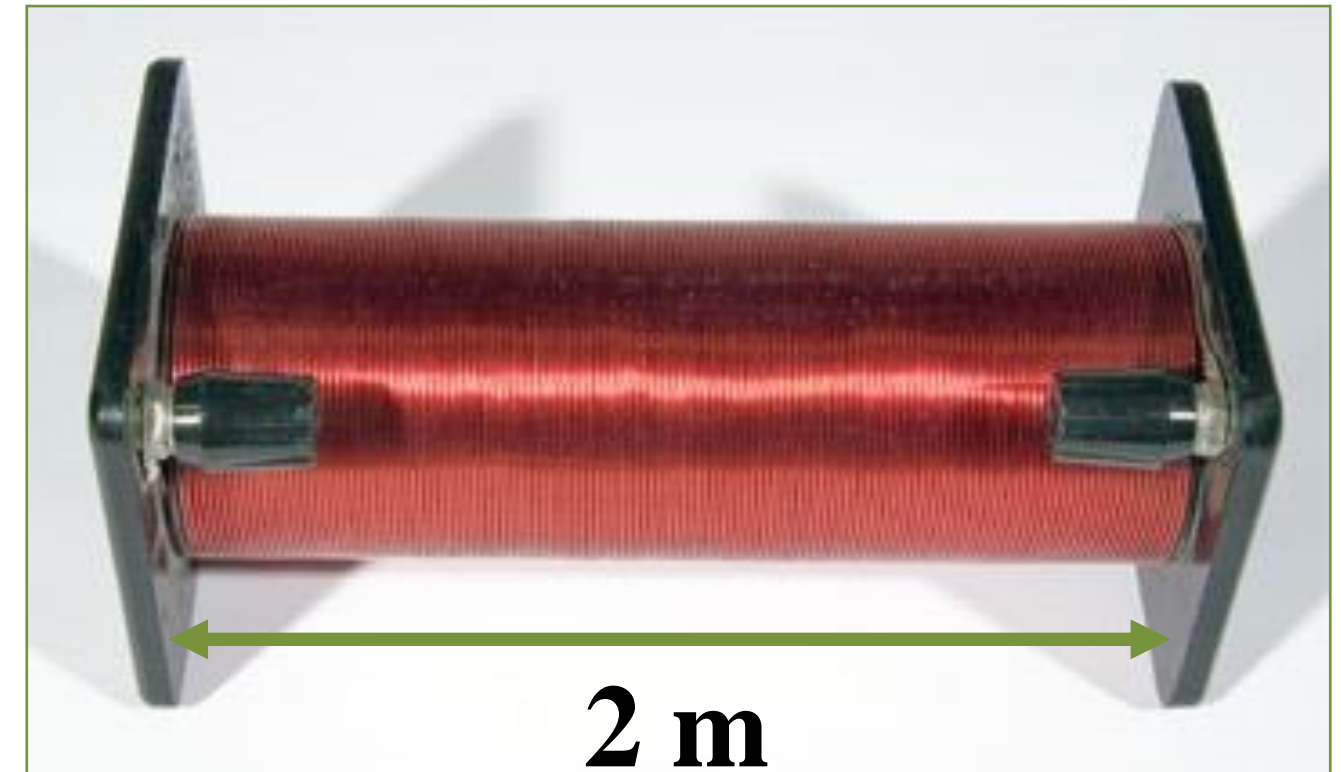
The magnetic field is almost
constant (uniform) inside:

$$B = \mu_0 n I$$

Solenoid

2000 loops \rightarrow 1000 loops/m

- current = 1600 A
very large current!



What is the field **B** inside ?

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times 1000 \text{ loops/m} \times 1600 \text{ A} = 2.01 \text{ T}$$

Magnetic force

1. An electric current produces a magnetic field

$$B = \frac{\mu_0 I}{2\pi r}$$

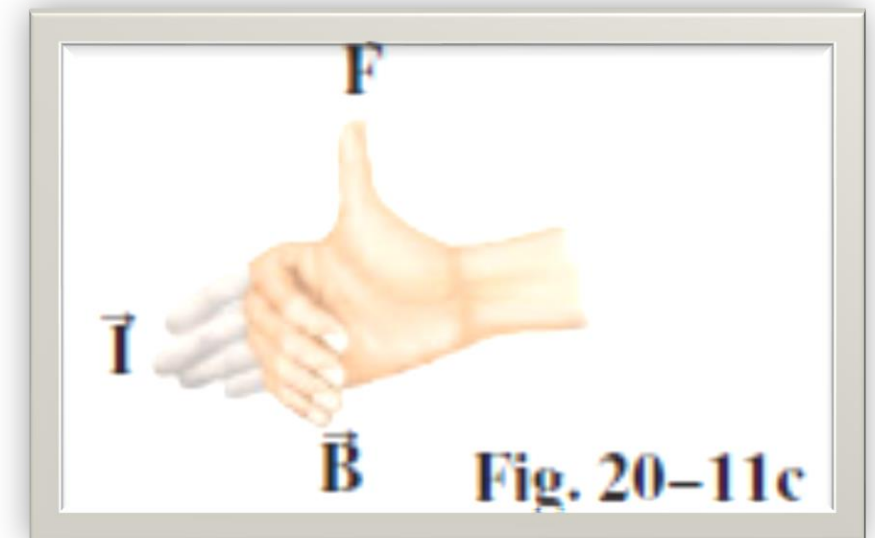
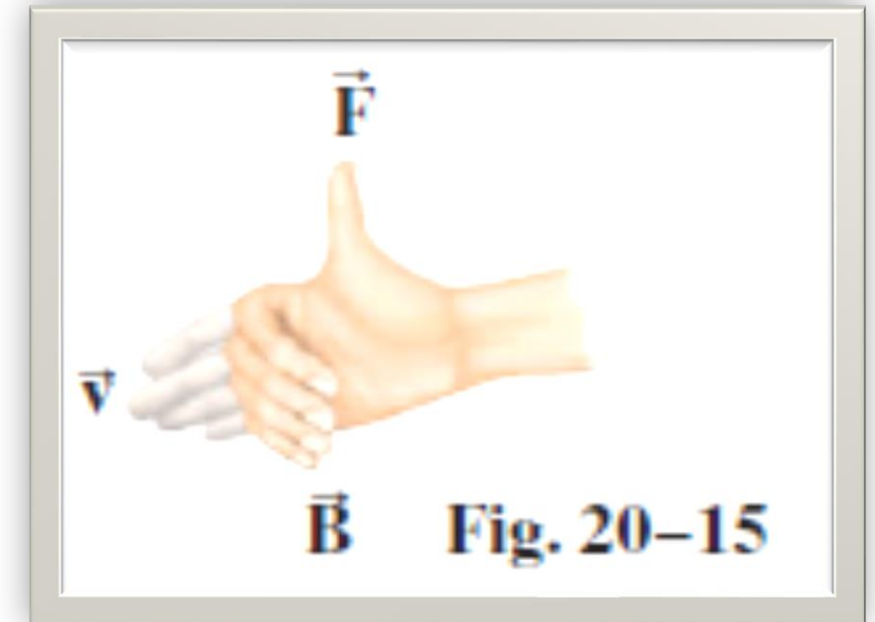
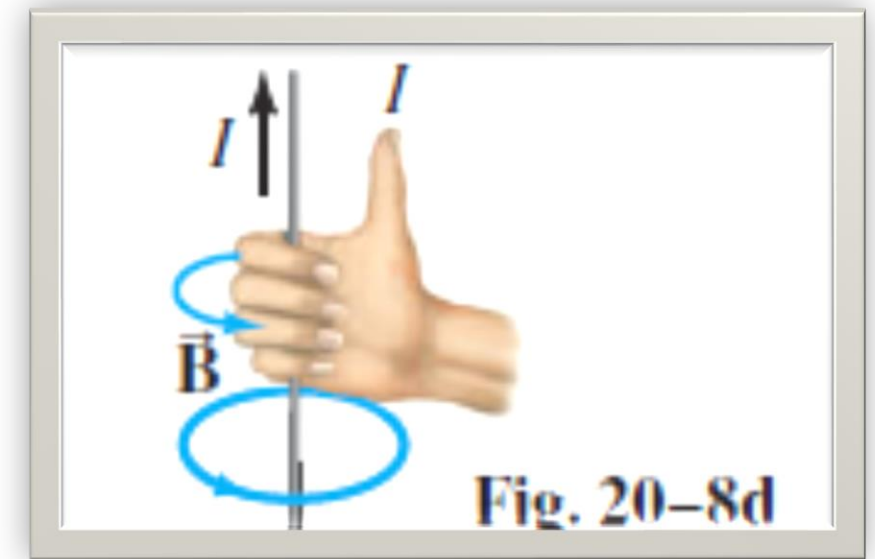
2. A moving charge in a magnetic field will be subject to a magnetic force

$$F = qvB$$

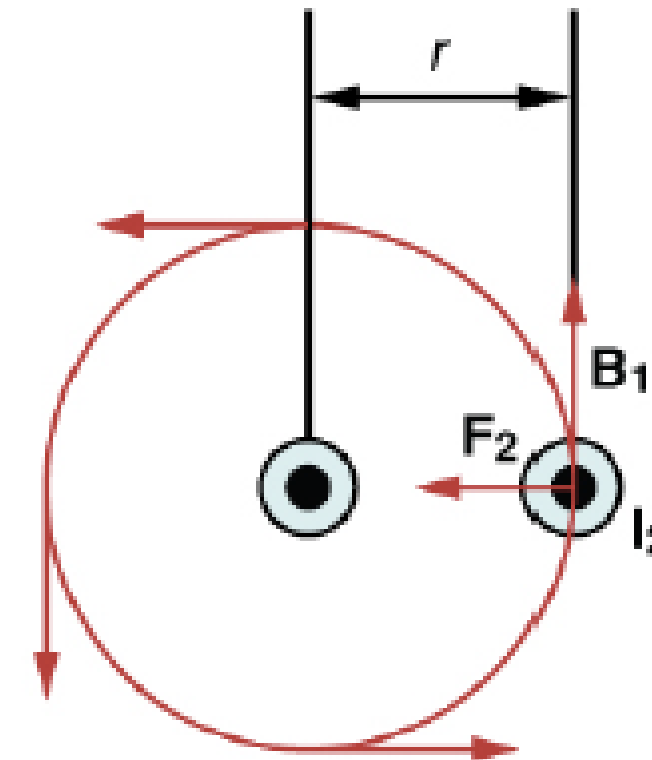
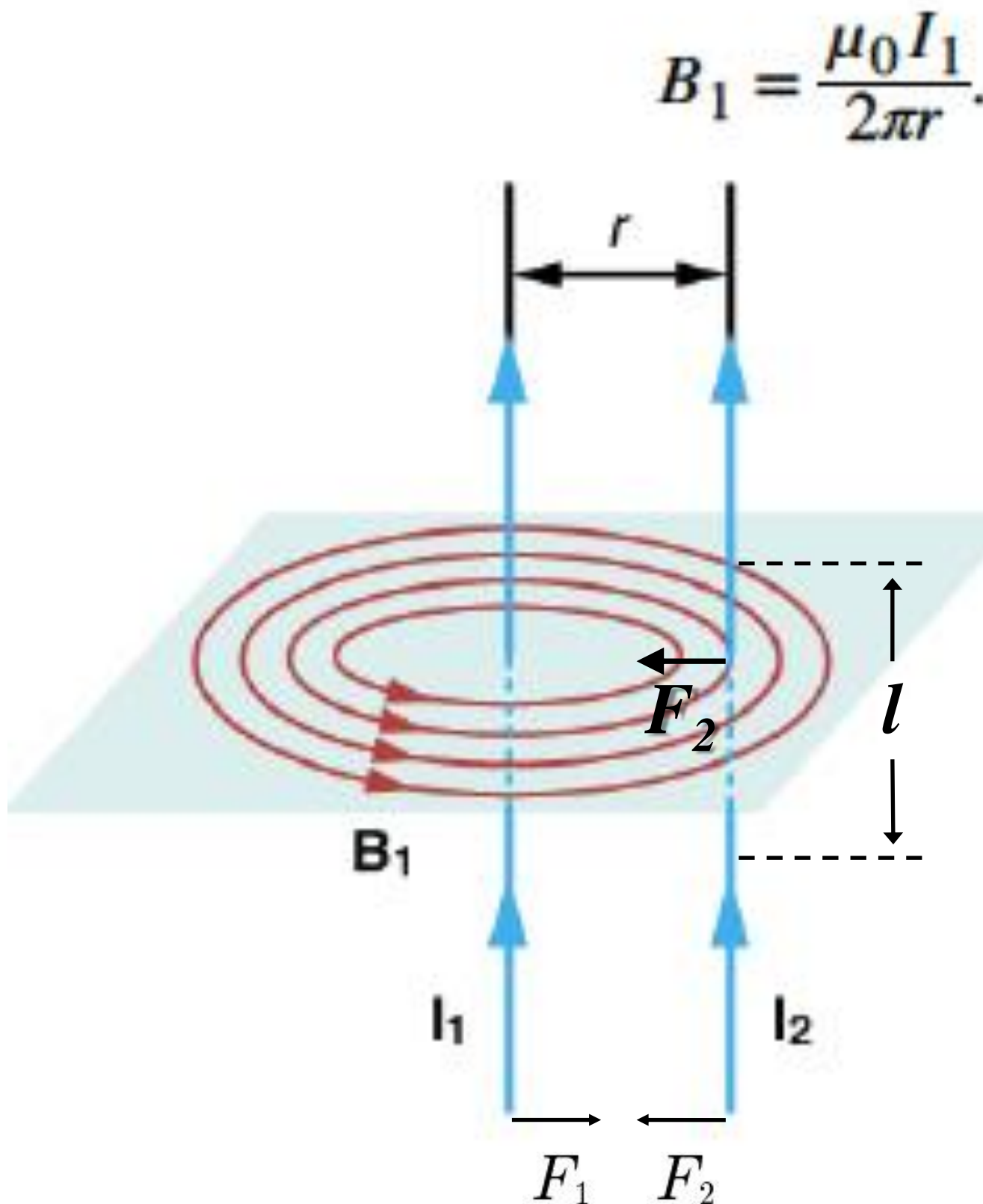
3. An electric current is made of moving charges

$$F = IlB$$

→ A wire carrying electric current will exert a force on another wire carrying electric current!



Magnetic force on Wire 2 from a parallel wire (Wire 1)

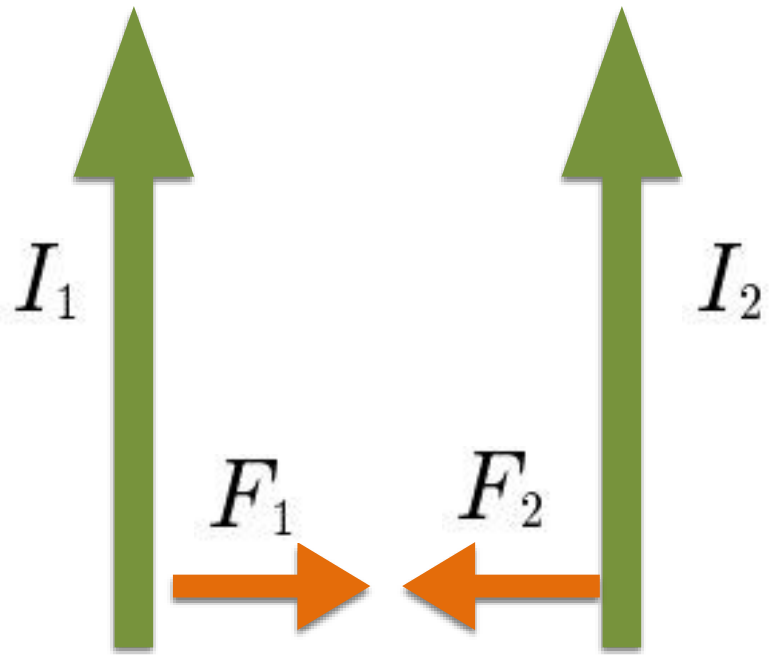


length l : $F_2 = I l B_1 \sin \theta$ $\sin \theta = 1$

**force per
unit length**

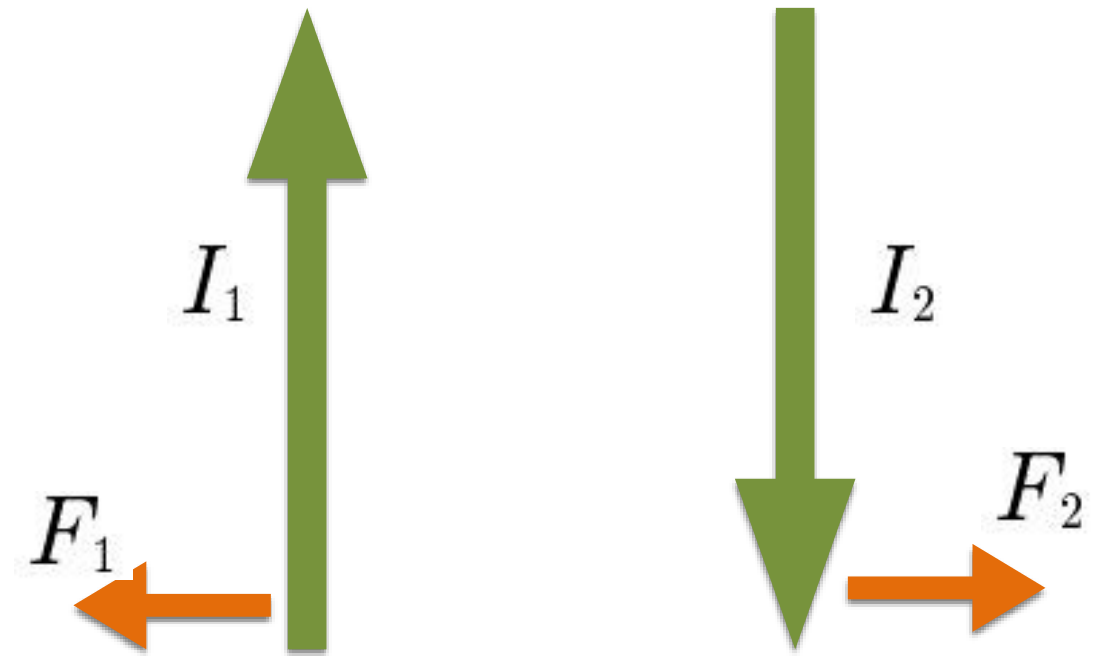
$$\frac{F_2}{l} = -\frac{F_1}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Magnetic force between two wires



**currents in the same
direction**

→ attractive force

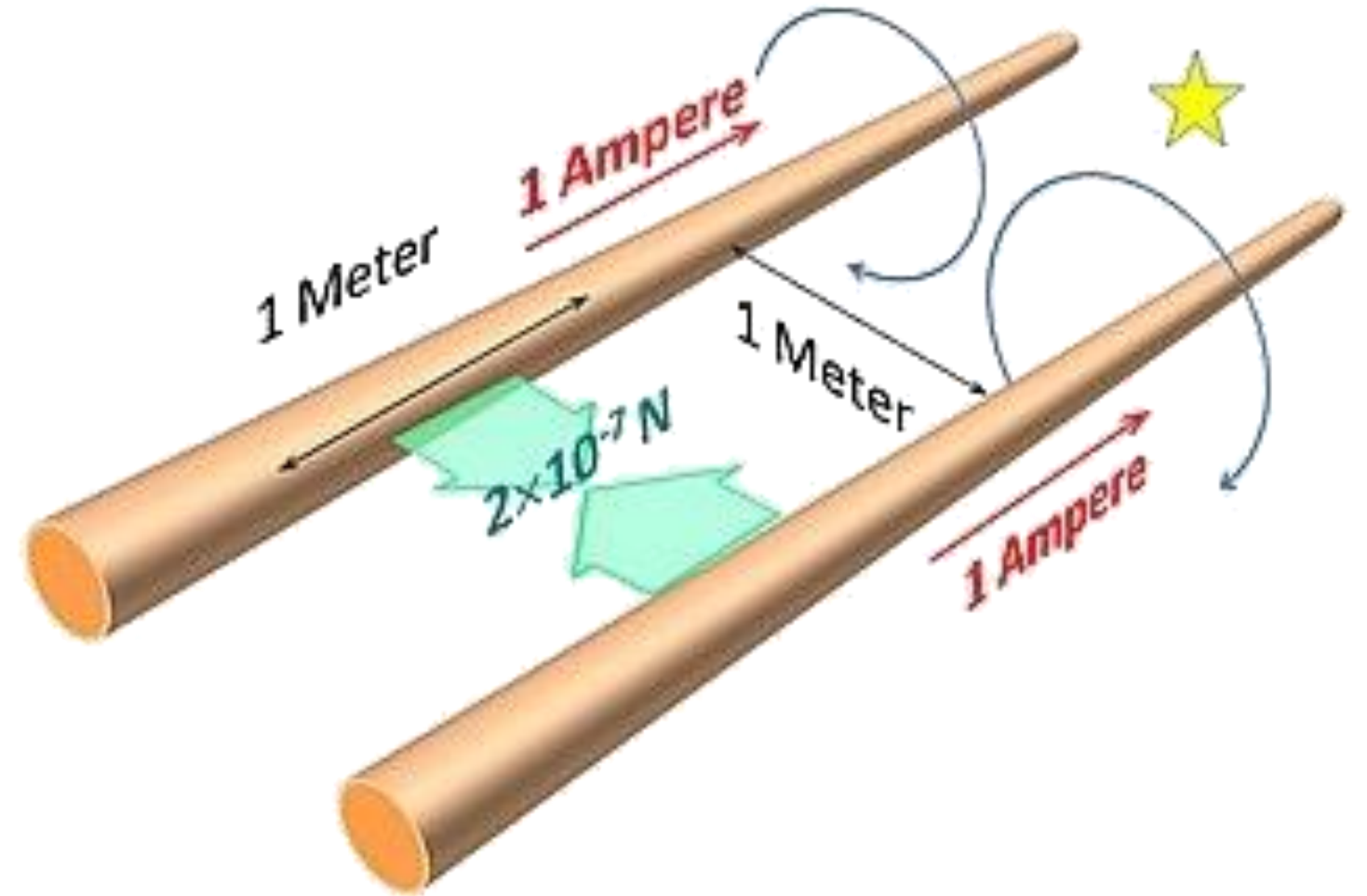


currents in opposite directions

→ repulsive force

Official definition of the Ampere

The *ampere* is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.



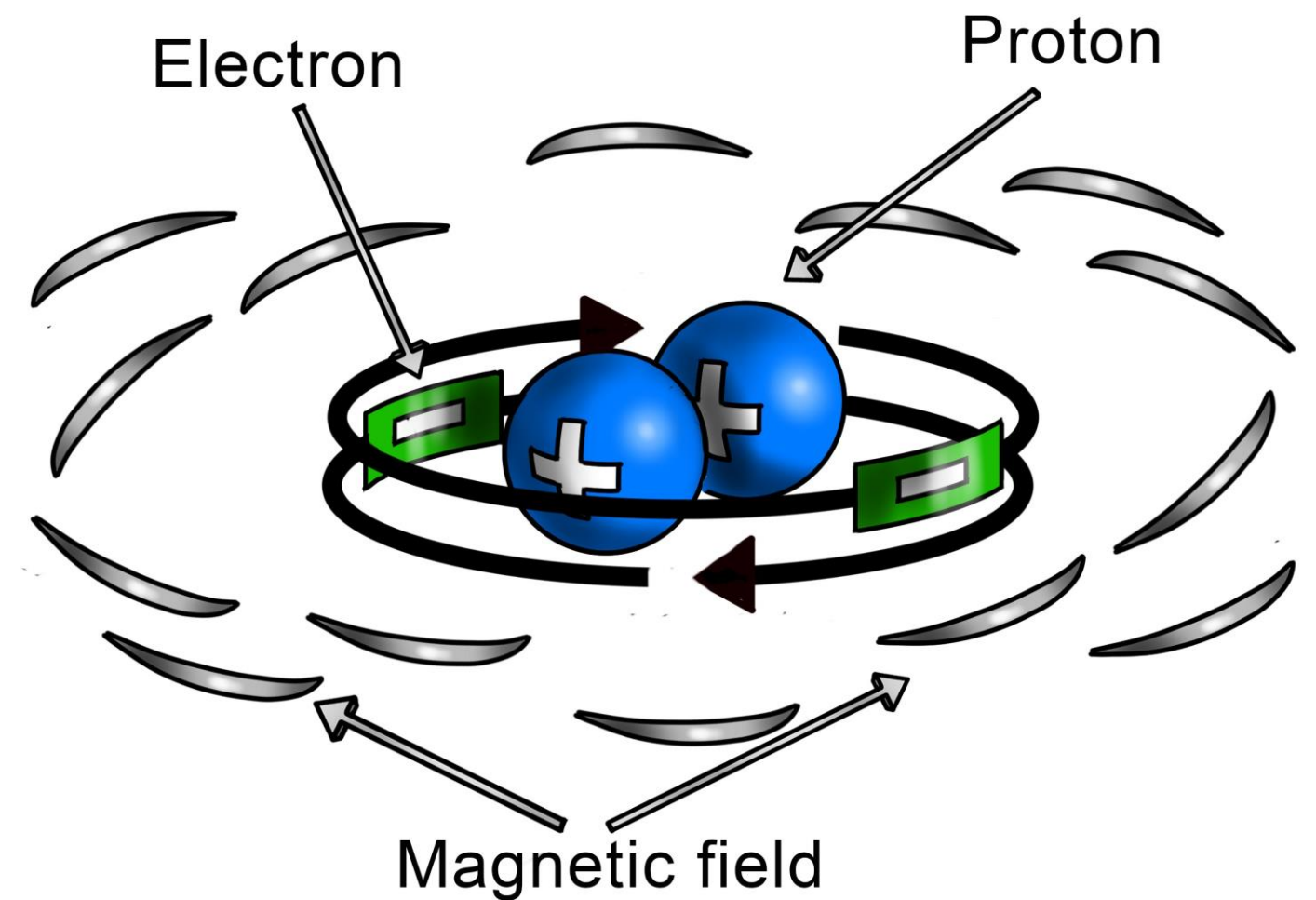
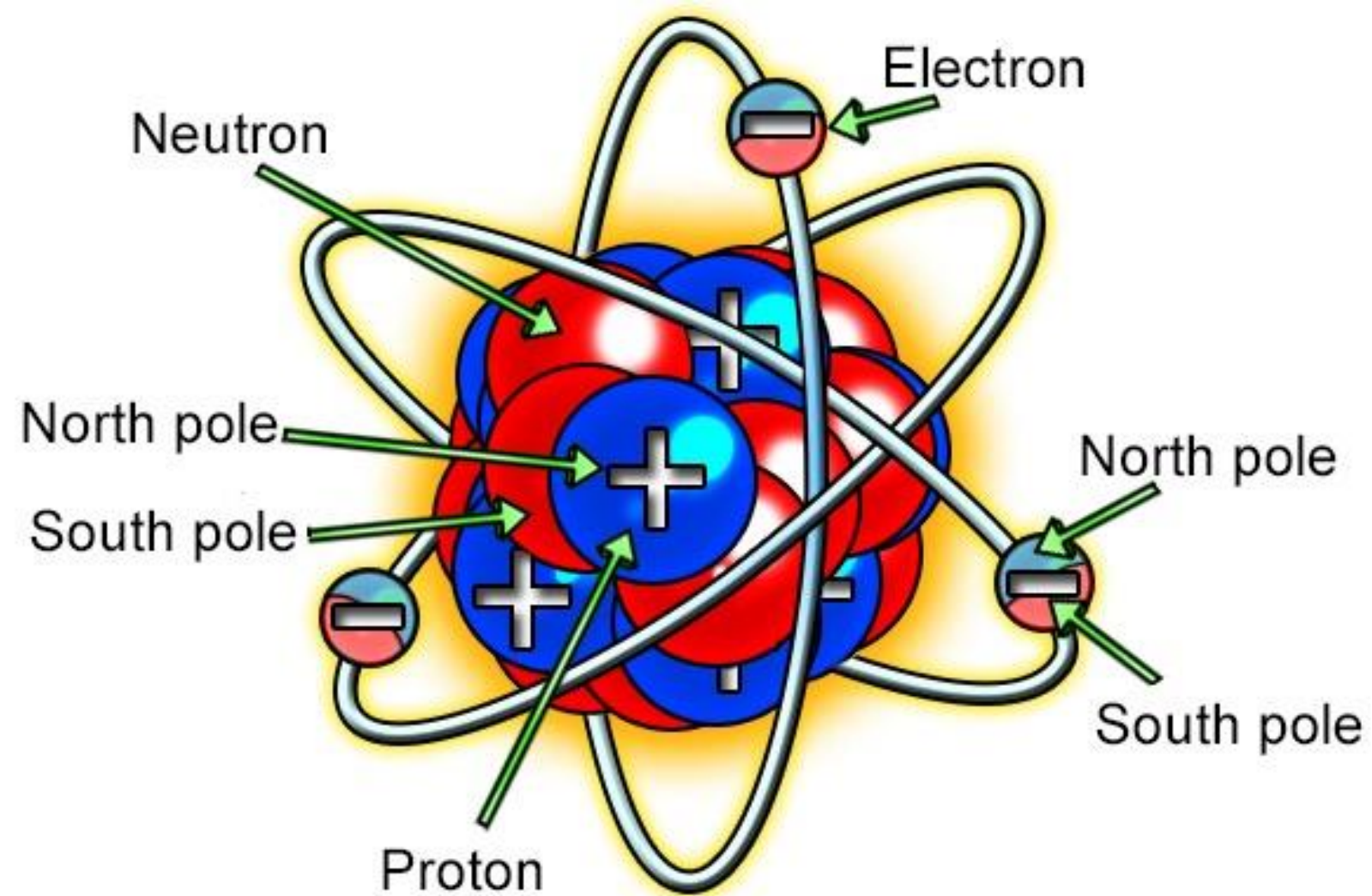
Where does magnetism come from?

Comes from electric current

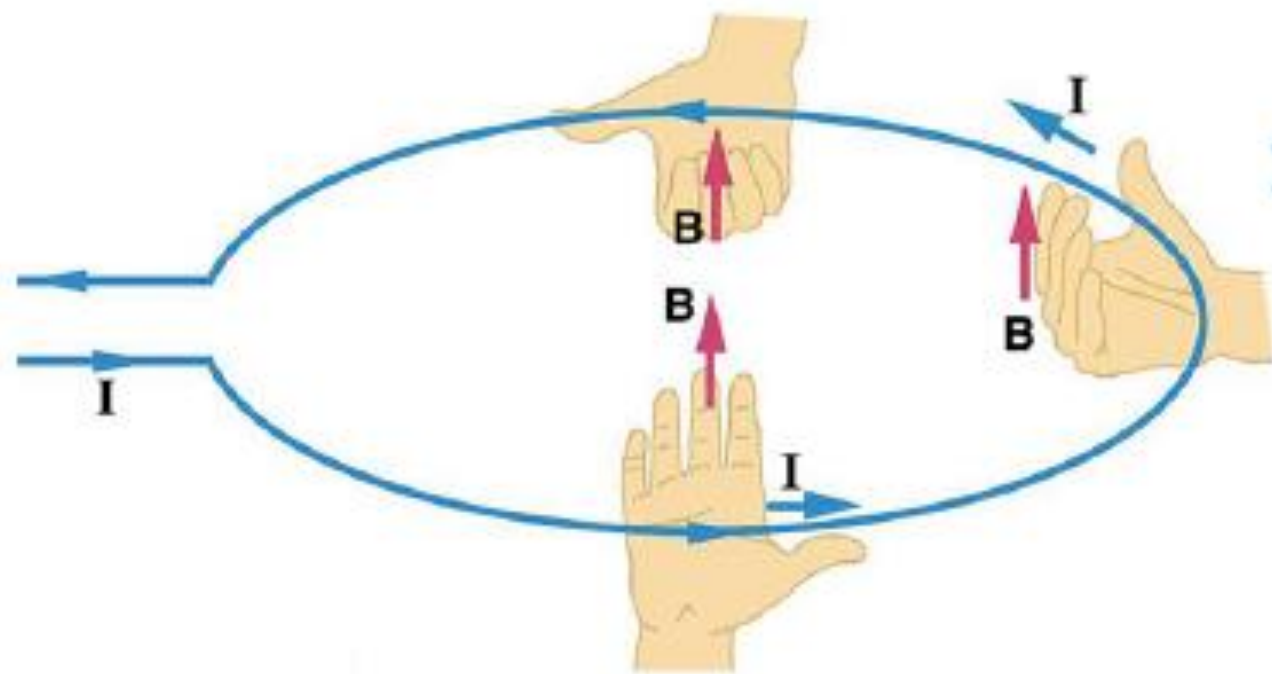


From moving charges

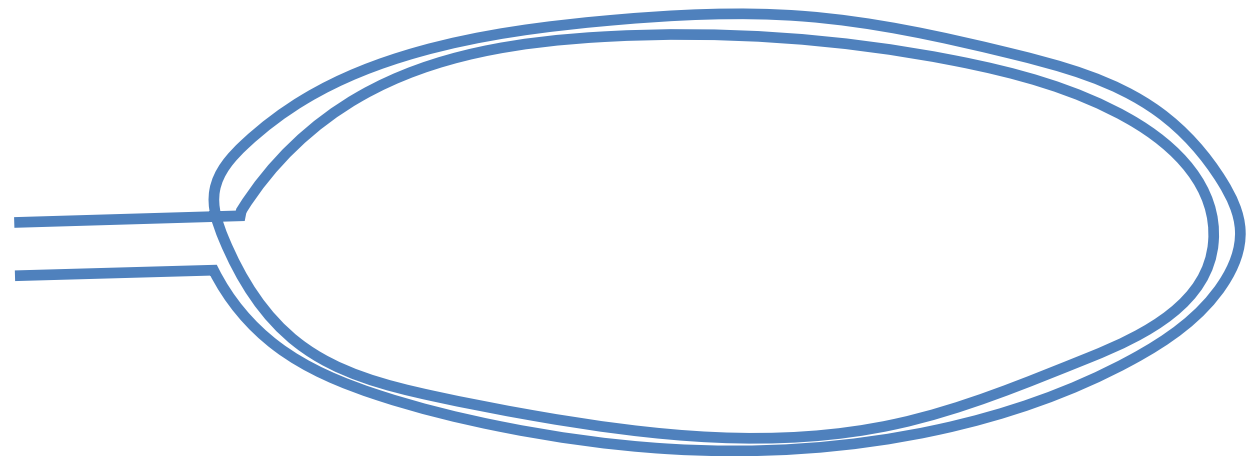
Moving electrons in atoms



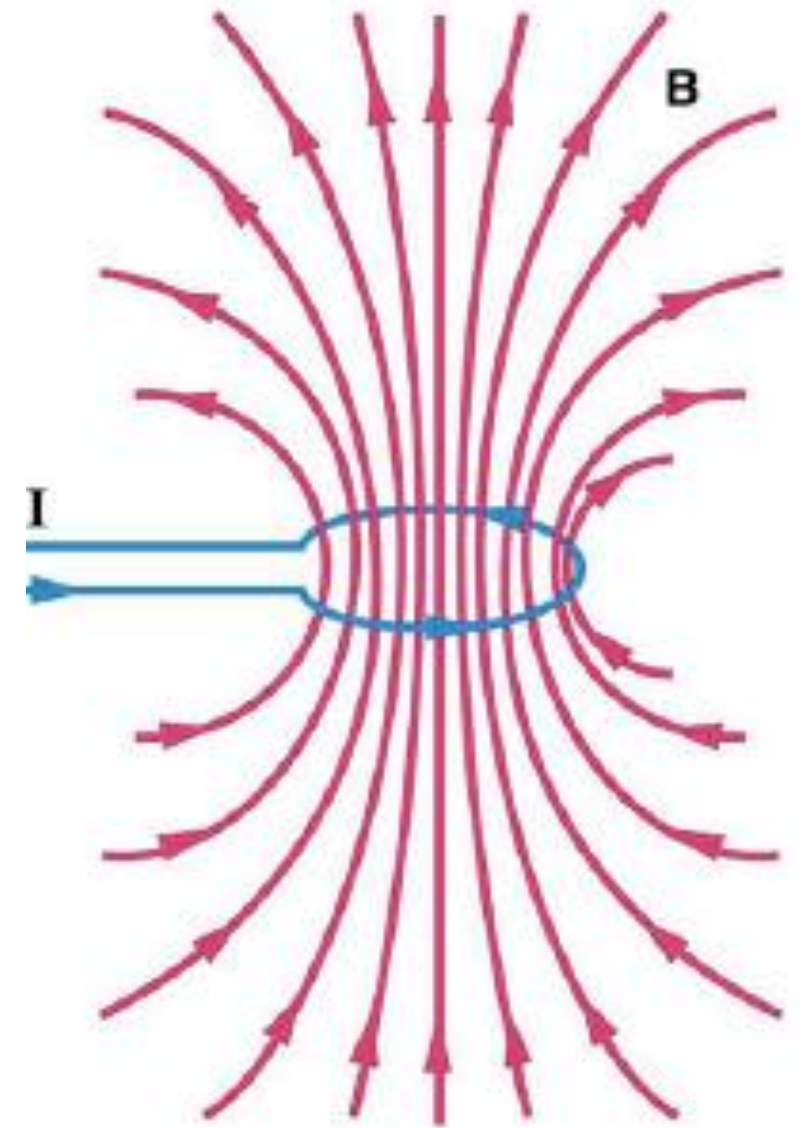
Magnetic field from a circular current loop



all elements of wire
produce a field



for N loops: $B = N \frac{\mu_0 I}{2R}$



at the center:

$$B = \frac{\mu_0 I}{2R}$$

Magnetic Field of a moving charge

moving charge = electric current



circular motion = current loop

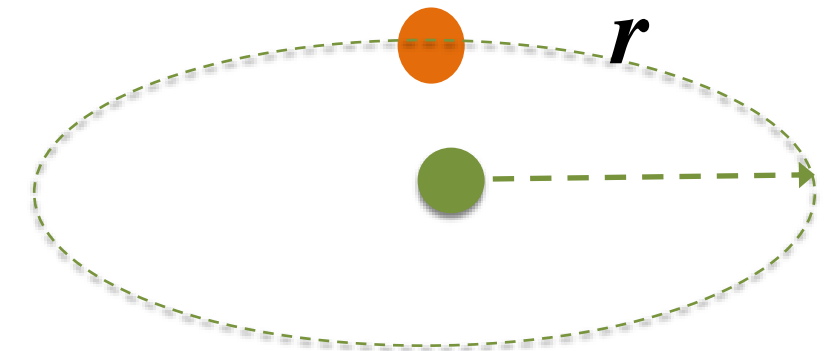
$$I = \frac{Q}{t}$$



$$I = \frac{q}{T}$$

charge on the particle

period of the circular motion



at the center:

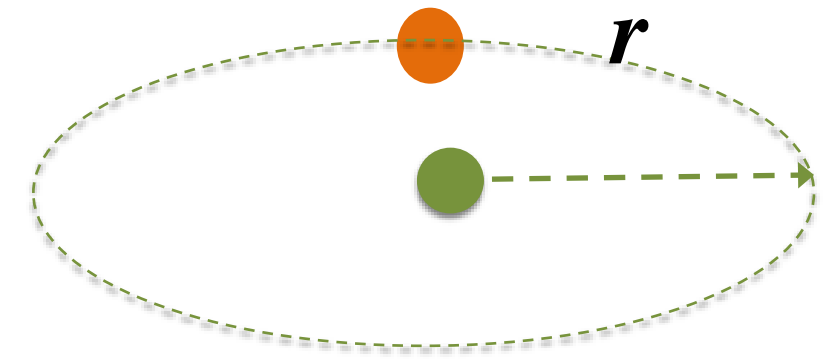
$$B = \frac{\mu_0 I}{2R}$$
$$B = \frac{\mu_0 q}{2rT}$$

What is the magnetic field at the nucleus due to the moving electron in a hydrogen atom at its ground state?

Magnetic Field of a moving charge

The magnetic field at a hydrogen nucleus due to the moving electron

$$B = \frac{\mu_0 q}{2rT}$$



electric force on electron

$$F_e = \frac{e^2}{4\pi\epsilon_0 r^2}$$

centripetal force needed by electron

$$F_c = \frac{mv^2}{r}$$

$$\left. \vphantom{\frac{mv^2}{r}} \right\} v^2 = \frac{e^2}{4\pi\epsilon_0 mr}$$

The period $T = \frac{2\pi r}{v} = \frac{2\pi r \sqrt{4\pi\epsilon_0 mr}}{e}$

$$B = \frac{\mu_0 e^2}{8\pi^{\frac{3}{2}} r^{\frac{5}{2}} \sqrt{\epsilon_0 m}}$$

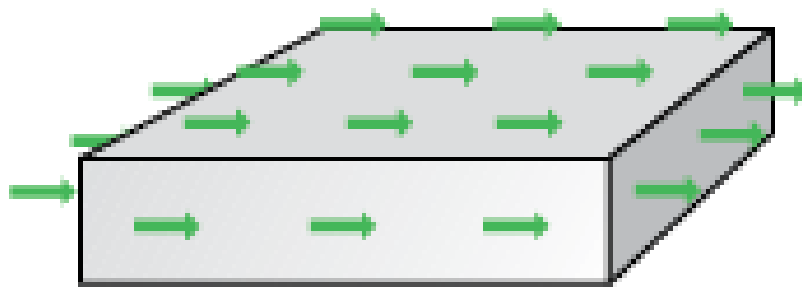
Using Bohr radius $r_0 = 0.53 \text{ \AA}$ for hydrogen ground state

$$B = 12.5 \text{ T}$$

Magnetic Field of a moving charge

- electrons “spin” on themselves
- This is like a current, that creates a magnetic field

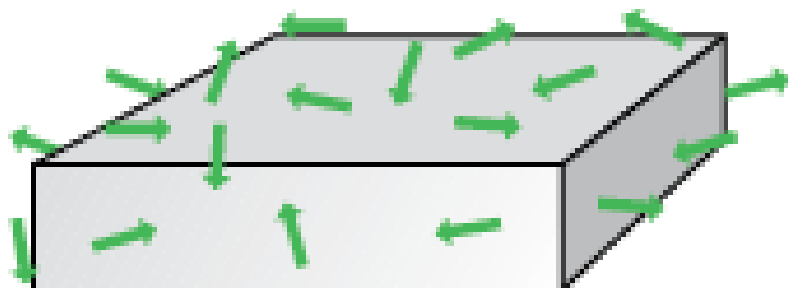
Magnetization



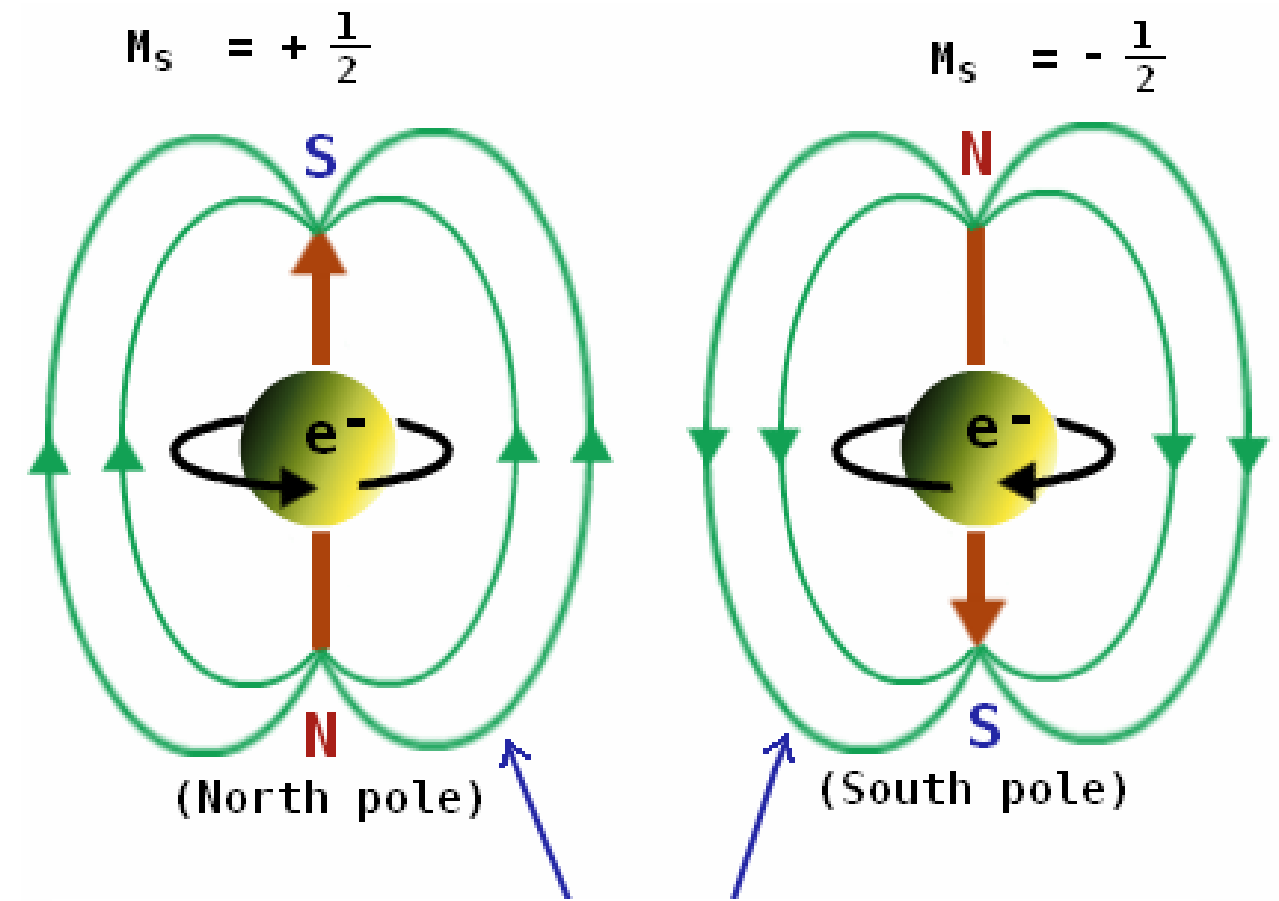
Saturation
(All magnetic domains aligned)



Partially magnetized
condition



Unmagnetized condition



*Actually a **relativistic effect**,
not really spinning*

molecular “magnets” align
under external magnetic field

Magnetic materials

$$\mu = \mu_0 \mu_r$$

Ferromagnetic materials

- Producing **strong** magnetic field after magnetization
- May make permanent magnets

Paramagnetic materials

- Weakly responds to external magnetic field
- Inducing a weak field in the **same** direction

Diamagnetic materials

- Induce an opposing magnetic field (weak)

Magnetic materials

$$\mu = \mu_0 \mu_r$$

Ferromagnetic materials

- $\mu_r \gg 1$
- Cobalt, Iron, Nickel, Gadolinium, Dysprosium,
Some compounds containing above elements

Paramagnetic materials

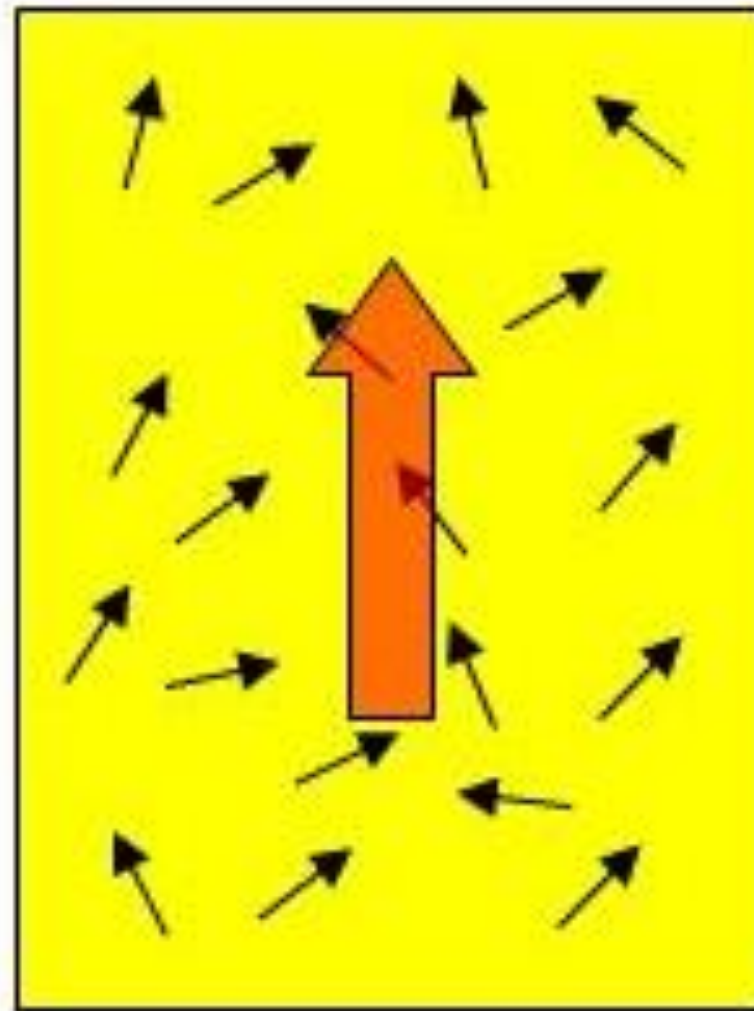
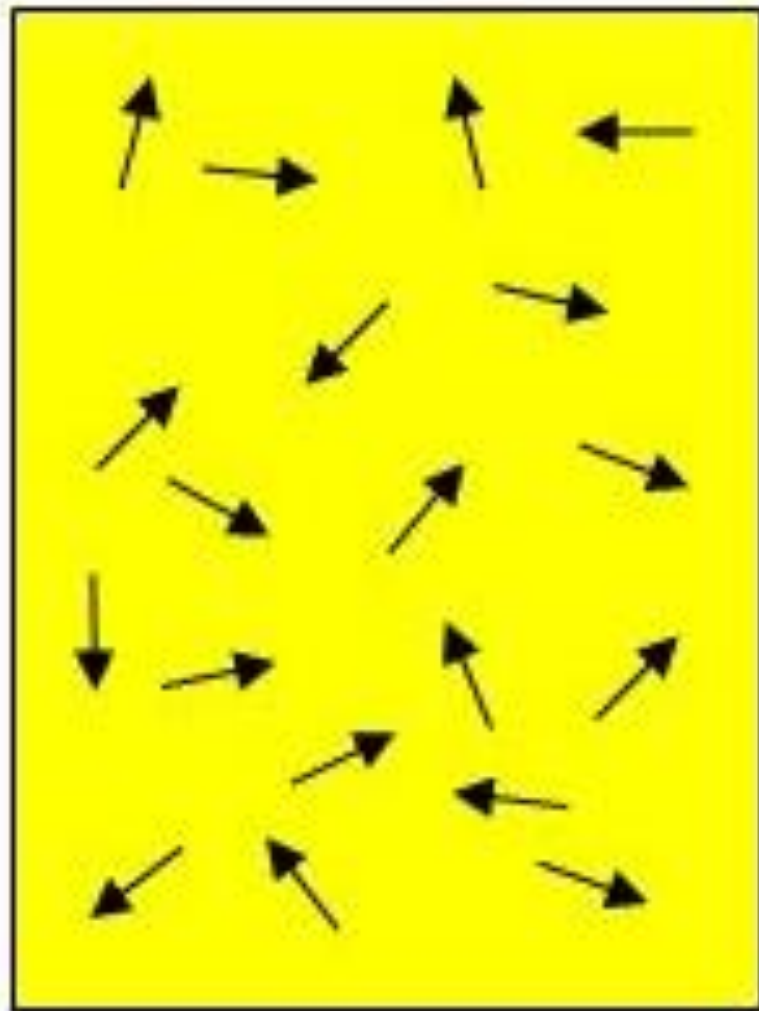
- μ_r slightly greater than 1
- Aluminum, Oxygen, Titanium, FeO

Diamagnetic materials

- μ_r slightly smaller than 1
- *most materials* ... Graphite, Gold, Silver, Water,

Paramagnetism

The partial alignment of permanent atomic magnetic moments
by a magnetic field



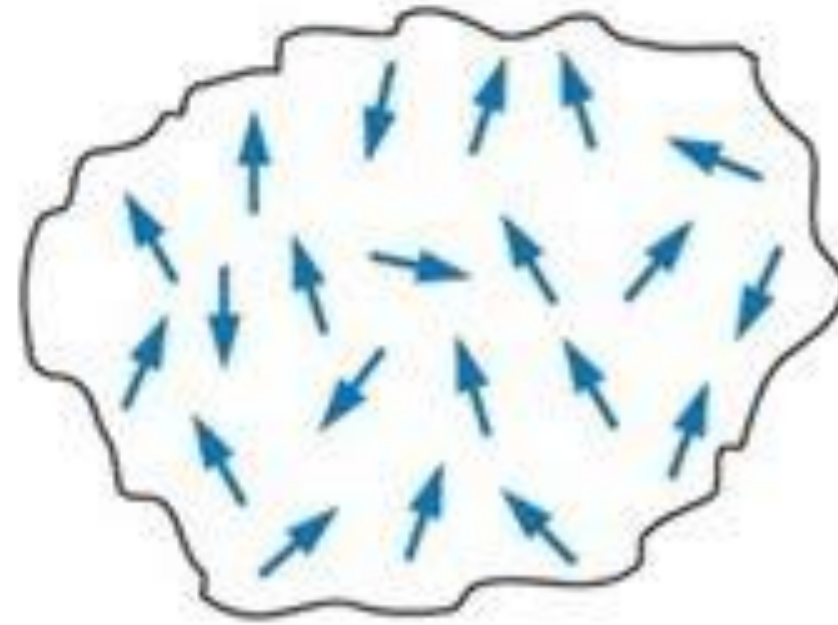
Alignment
resisted by
thermal motion



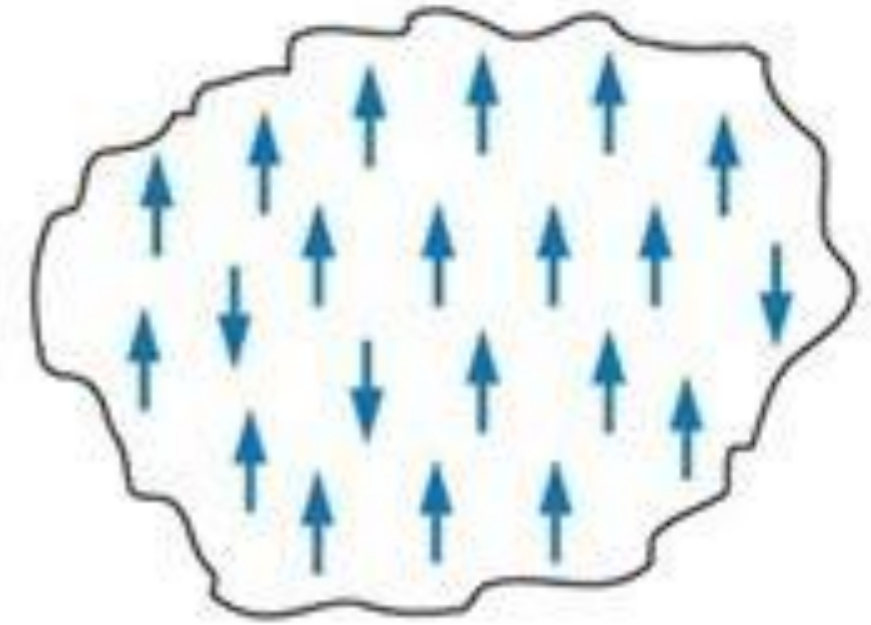
Partial
magnetization

Difference between paramagnetism and ferromagnetism

Magnetic field absent

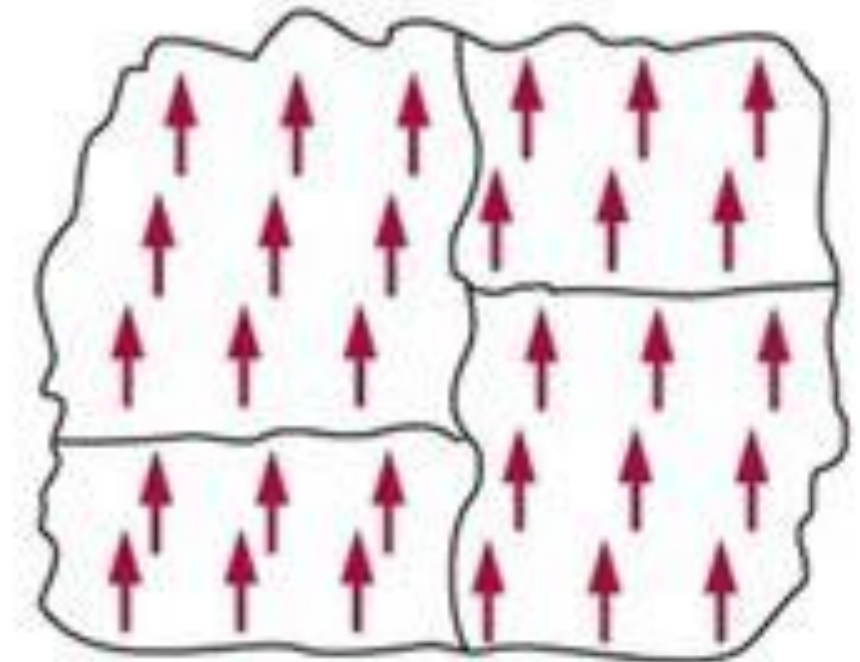
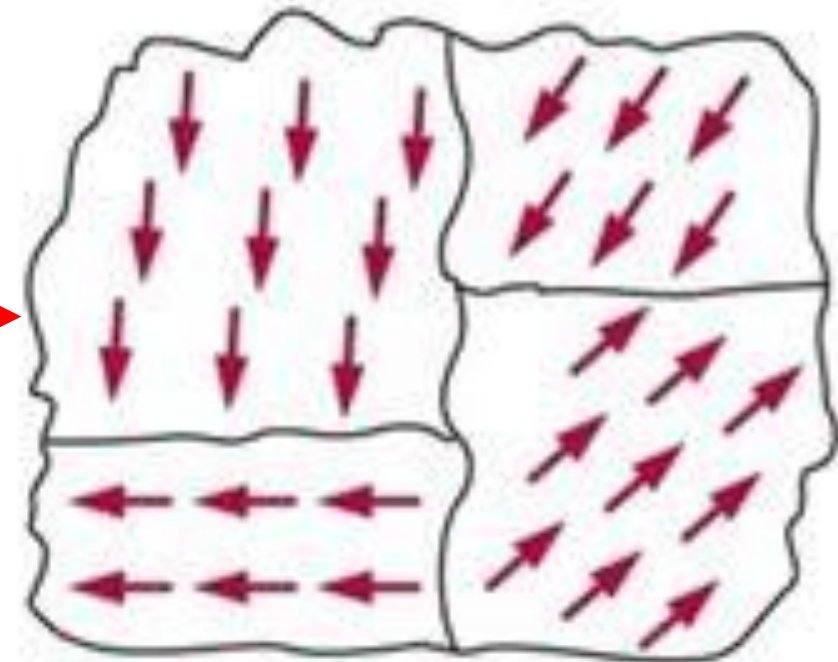


In presence of magnetic field



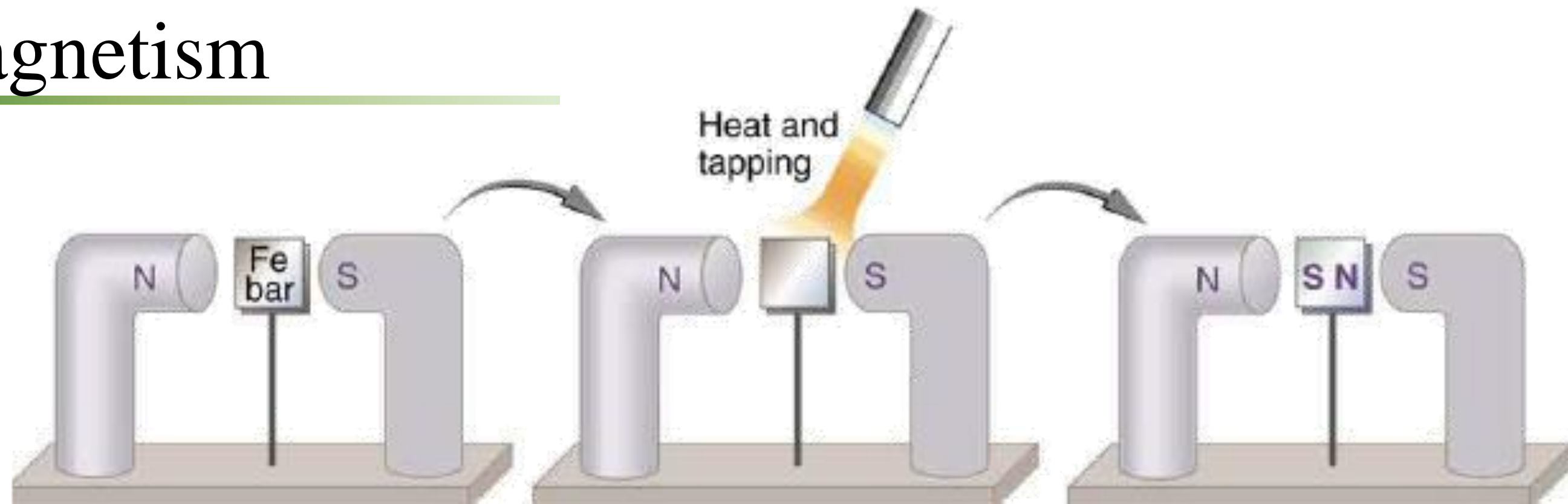
Paramagnetism

Strong interactions
between molecular
“magnets”



Ferromagnetism

Ferromagnetism



Ferromagnetism

Ferromagnetic material	Relative permeability (μ_r)
Iron (99.95% pure Fe annealed in H)	200000
Cobalt-iron (high permeability strip material)	18000
Iron (99.8% pure)	5000
Electrical steel (silicon steel)	4000
Martensitic stainless steel (annealed)	750 – 950
Carbon steel	100
Nickel	100 – 600
Martensitic stainless steel (hardened)	40 – 95
Austenitic stainless steel	1.003 – 7

$$\mu_{\text{T}} = \frac{\mu}{\mu_0}$$

$$\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

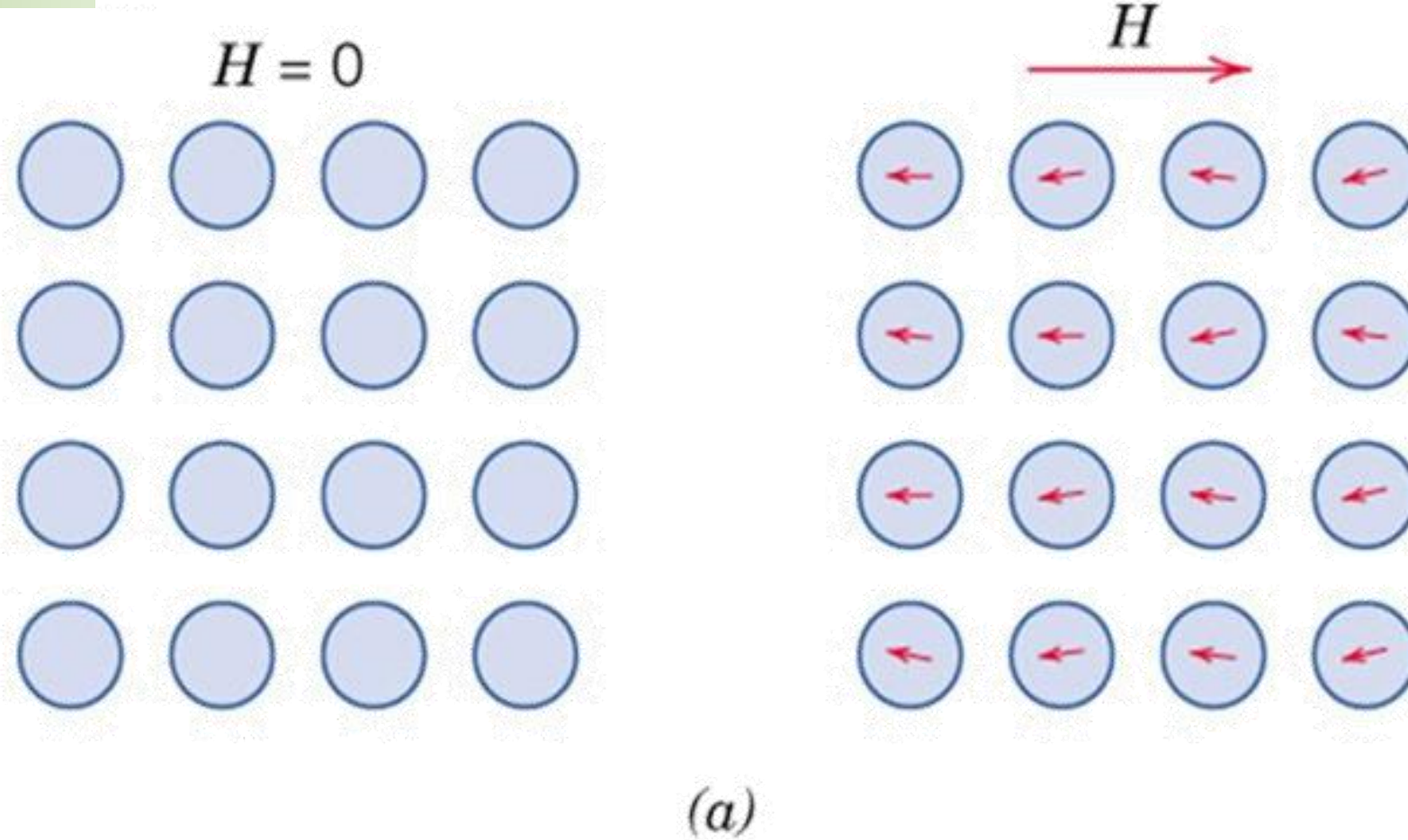
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

$$\mathbf{B} = \mu \mathbf{H}$$

Diamagnetic and Paramagnetic material

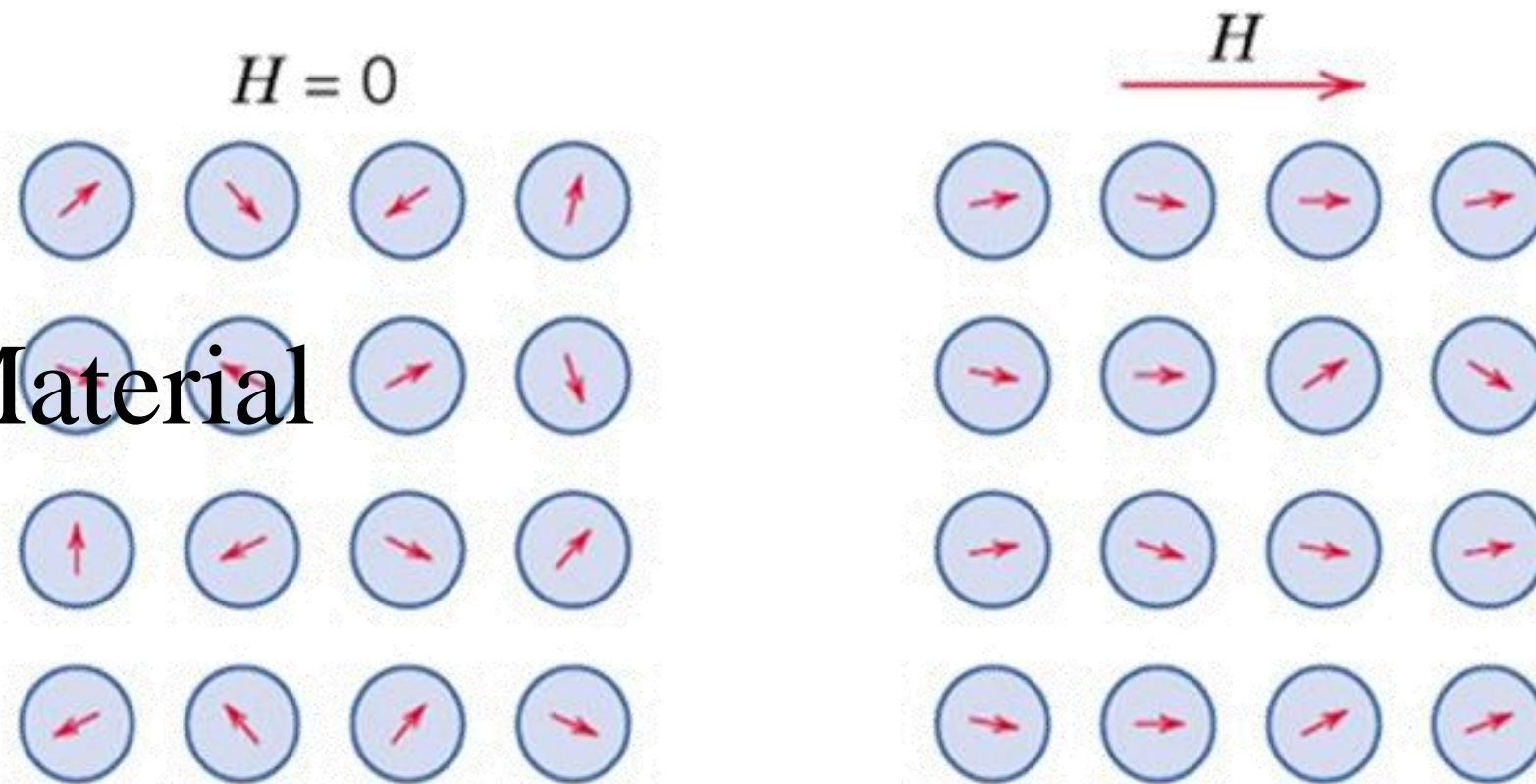
Diamagnetic Material

In presence of a field, dipoles are induced and aligned opposite to the field direction



A universal quantum effect!

Paramagnetic Material

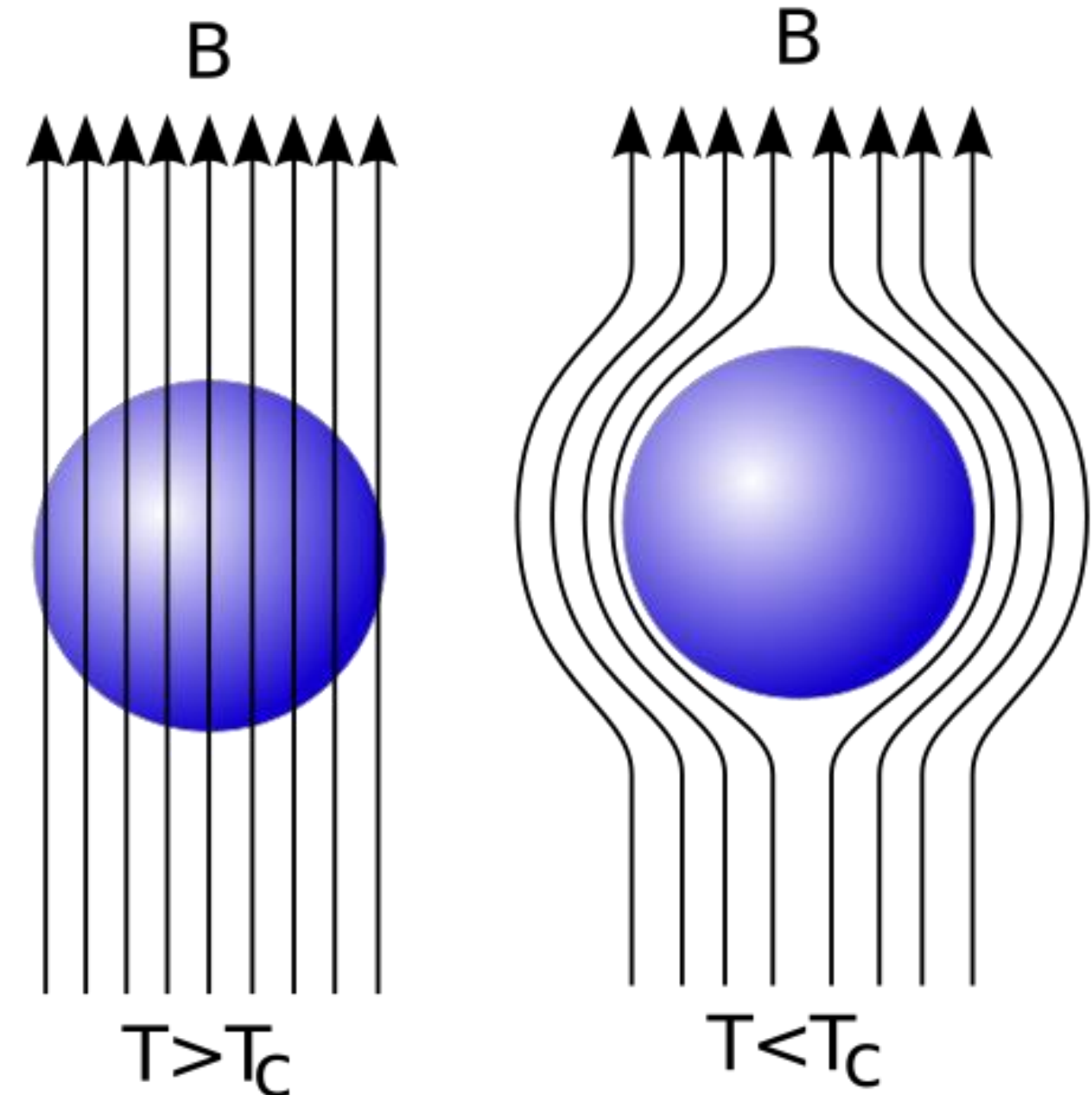


Superconductor – perfect diamagnetism

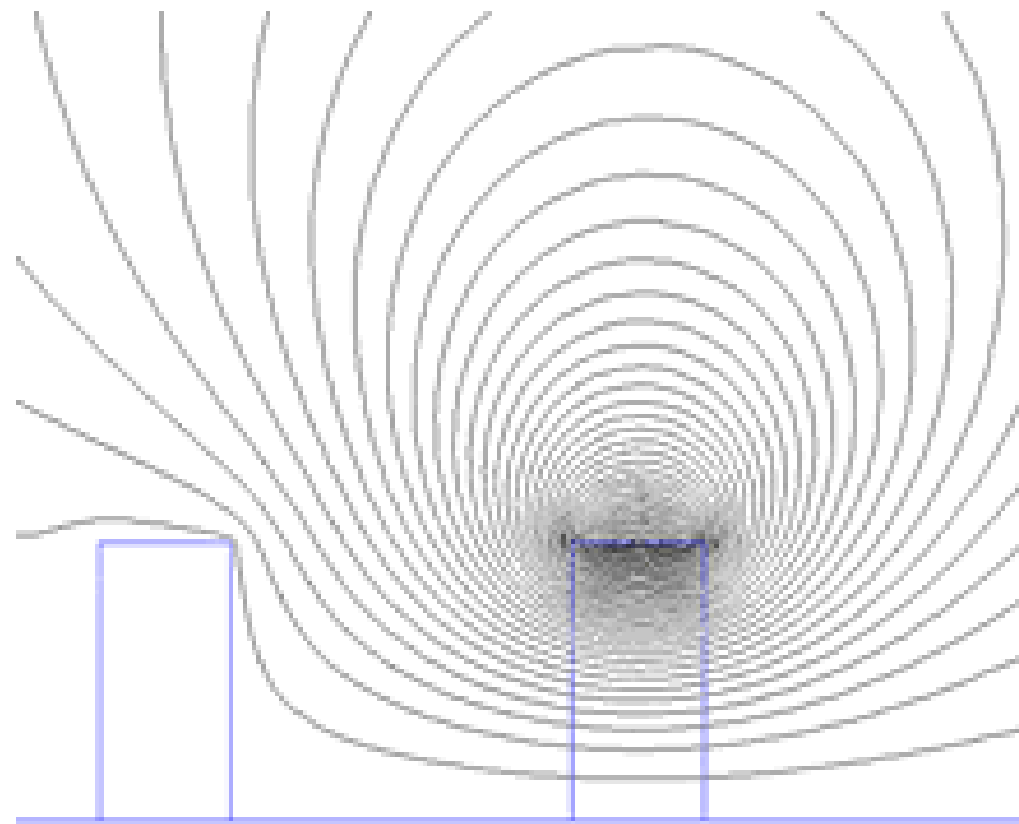
Meissner effect

All magnetic field will be **expelled**
from a superconductor during its
transition to the superconducting state

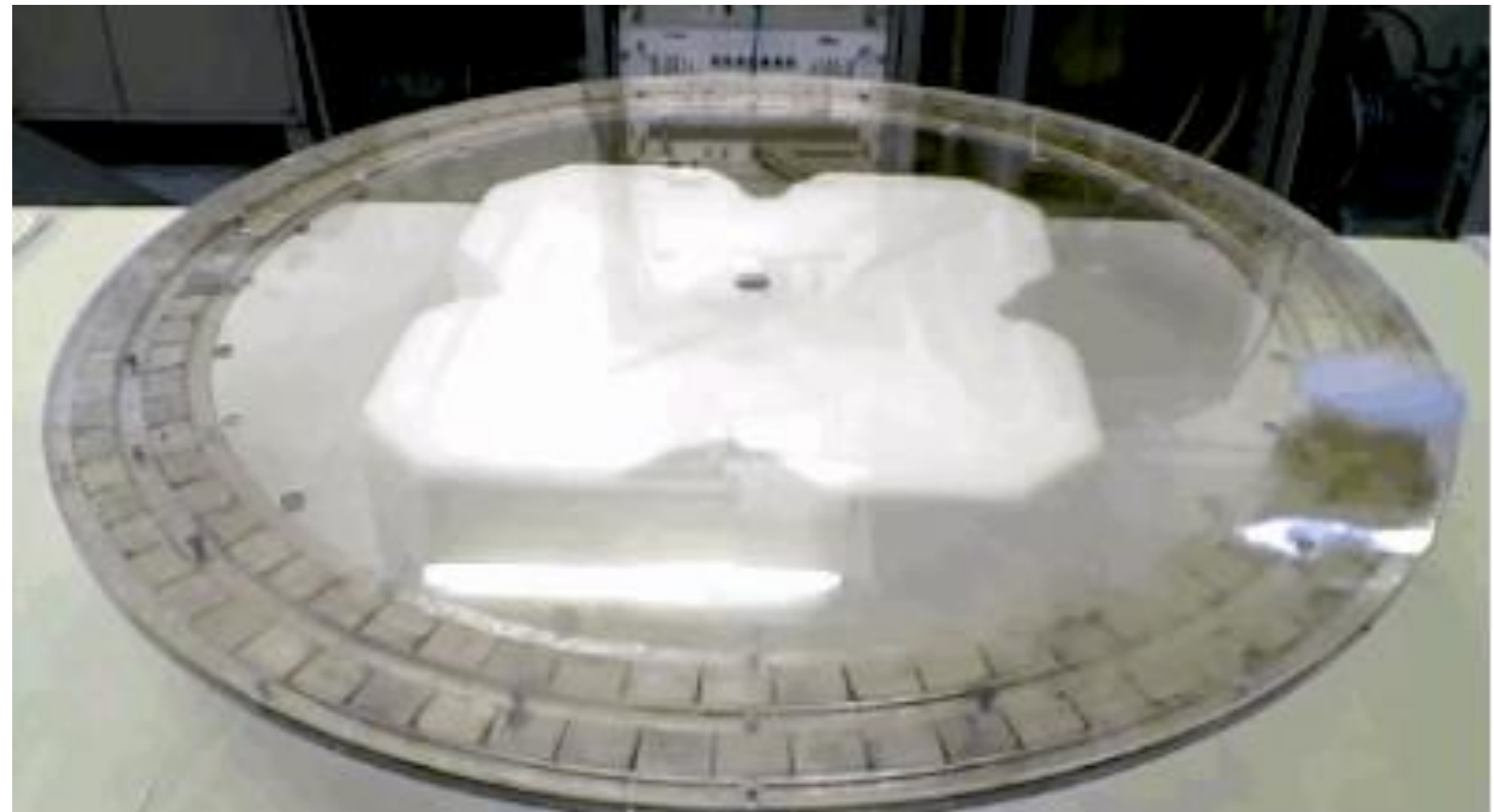
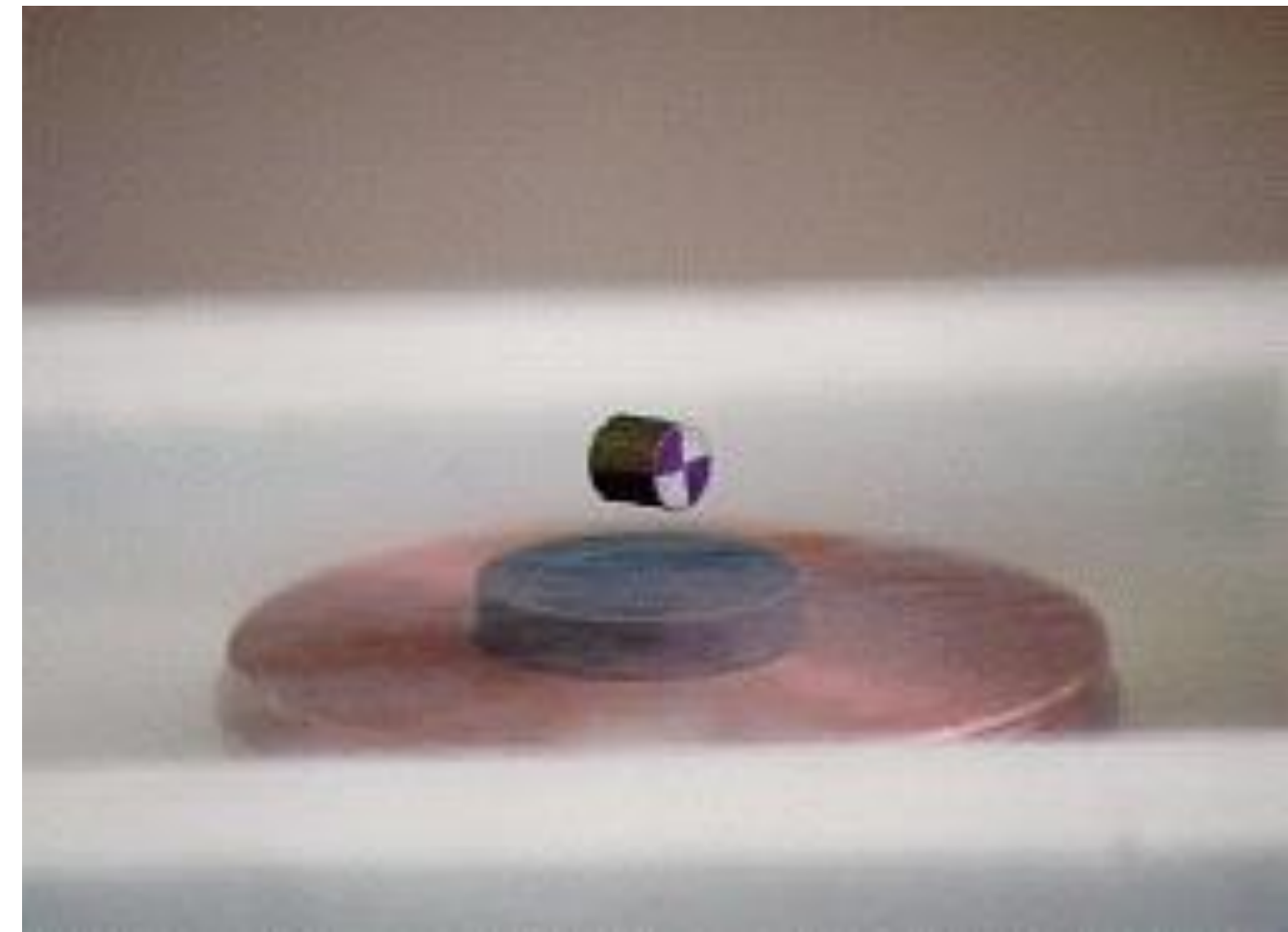
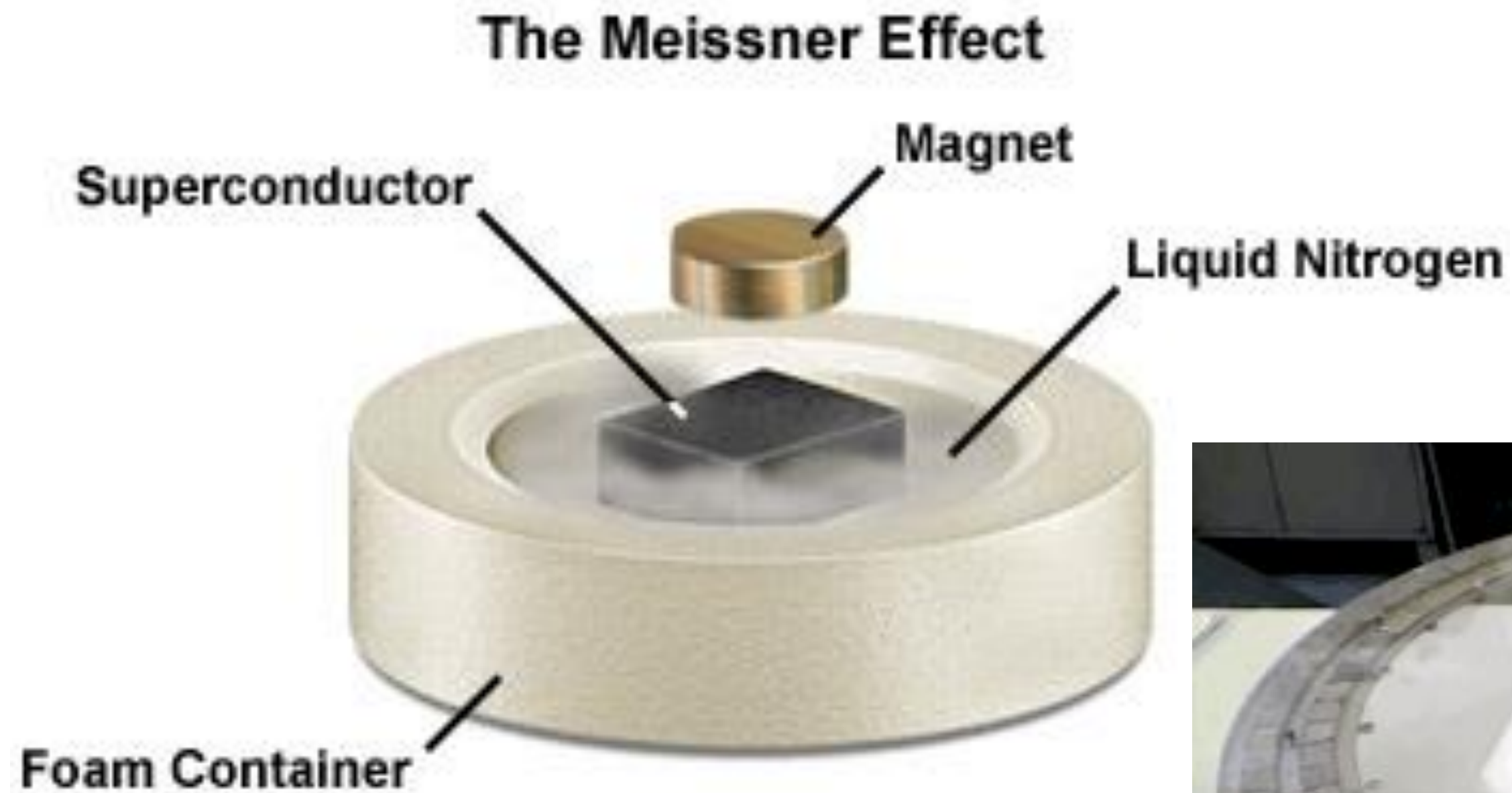
$$\mu_r = 0$$



Repulsive force



Superconducting magnetic levitation



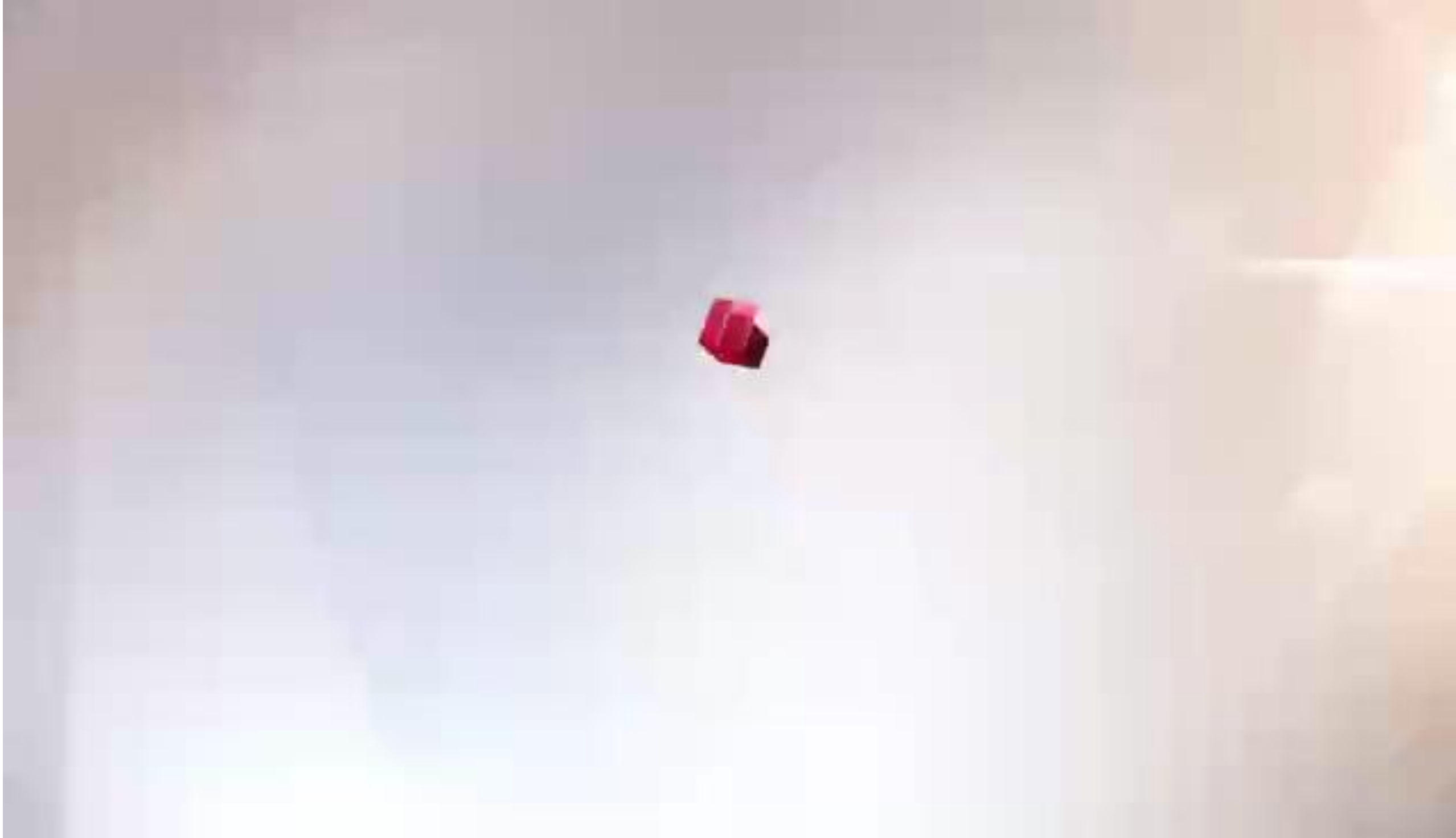


A SUPER CHUTE

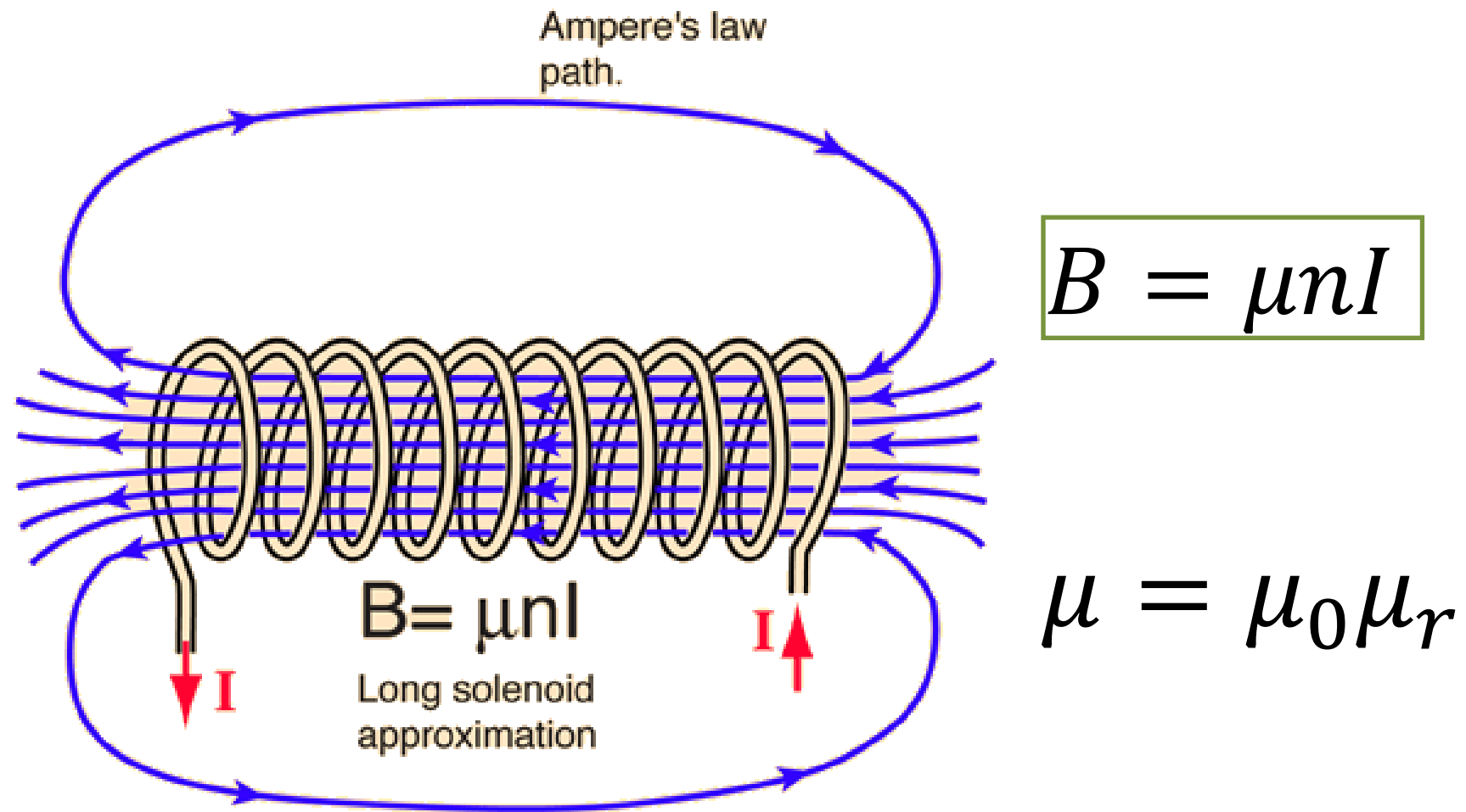
THIS RING IS THE facelift on which the future of ground transportation might be riding. Deng Ziqiang, a professor at Southwest Jiaotong University in Chengdu, China, stands in the center of a prototype designed to test the latest turn-in-maglev (magnetic levitation) train technology. Right now, the best maglev trains reach speeds exceeding 400 kilometers (250 miles) per hour. Yeah, that's fast, but researchers like Deng are confident that the train cars, which are suspended



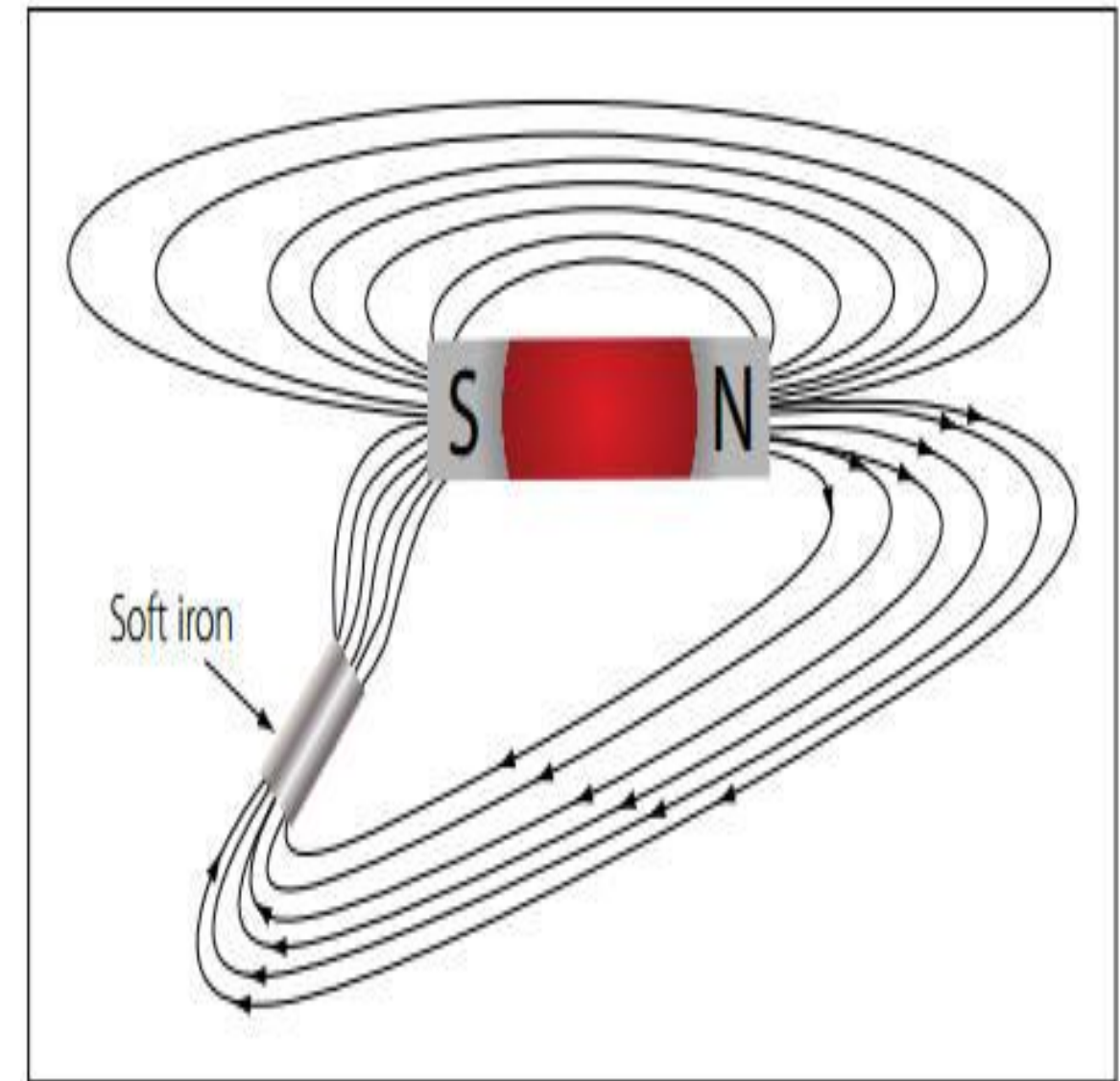
IN SWJTU



Solenoid with a ferromagnetic core

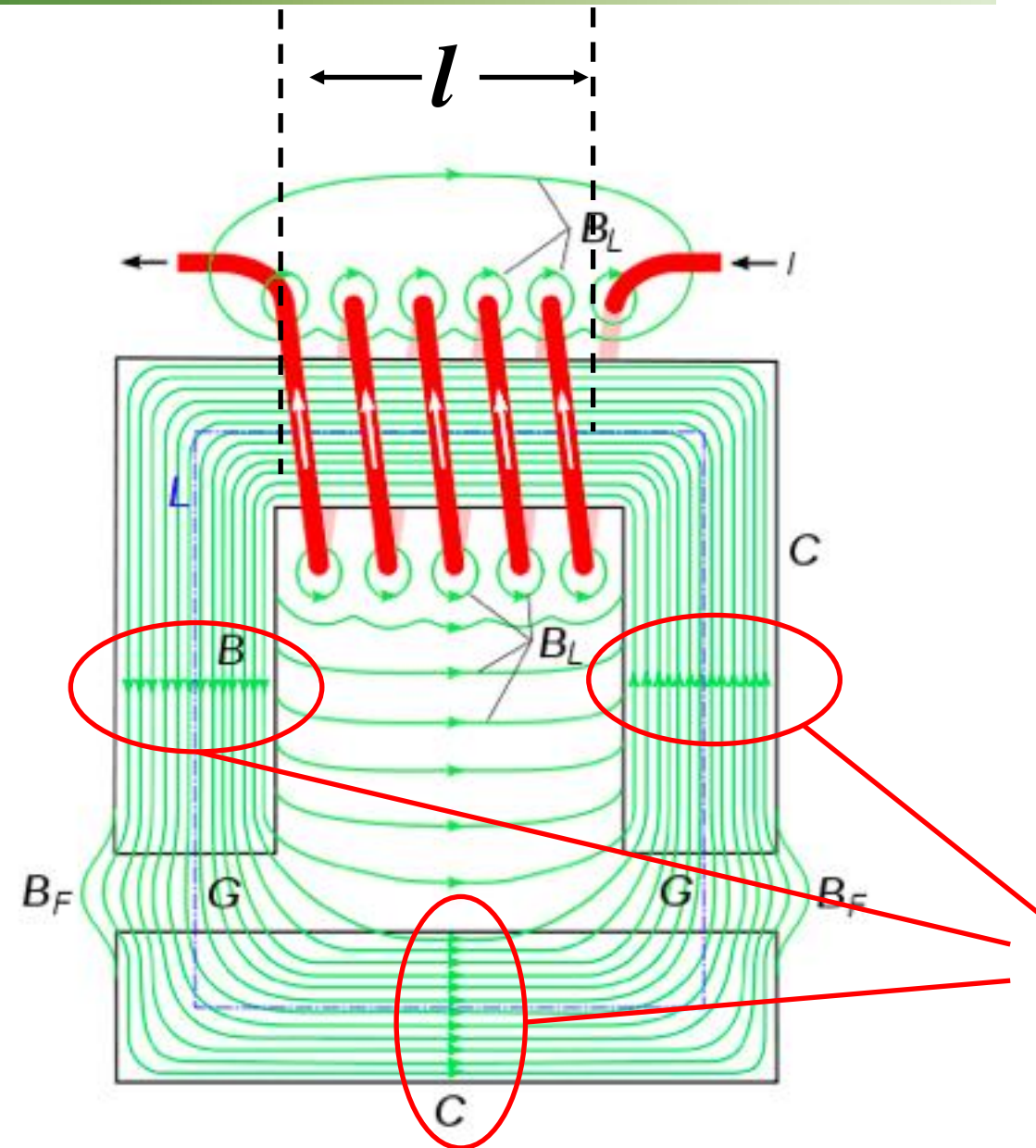


The Magnetic Fields concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent



n - the number of turns per unit of length
“**soft**” ferromagnetic materials have very large μ_r value

Magnetic circuit



Almost all the magnetic field lines are contained in the soft ferromagnetic materials (having very large μ_r value)

Same *magnetic flux* Φ



Same *current* in an *electric circuit*

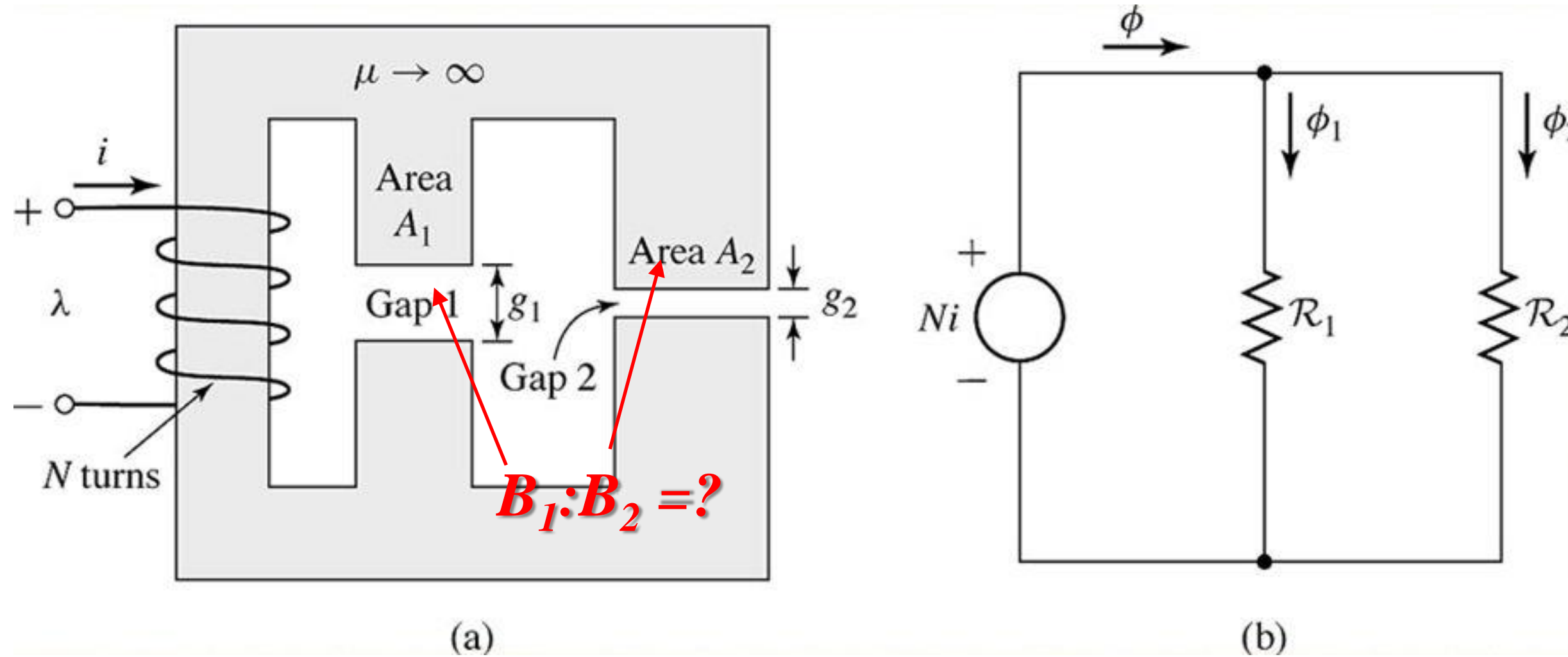
$$\Phi = \mu \frac{A}{l} NI$$

$$I = \sigma \frac{A}{l} \mathcal{E}$$

$$B = \mu n I$$

$$\frac{\Phi}{A} = \mu \frac{NI}{l}$$

Magnetic circuit vs. Electric circuit



reluctance $R_m = \frac{l}{\mu A}$

$$\frac{B_1}{B_2} = \frac{g_2}{g_1}$$

$$\frac{\Phi_1}{\Phi_2} = \frac{R_2}{R_1} = \frac{A_1 g_2}{A_2 g_1}$$

Magnetic circuit vs. Electric circuit

Magnetic				Electric		
Name	Symbol	Units		Name	Symbol	Units
Magnetomotive force (MMF)	$\mathcal{F} = \int \mathbf{H} \cdot d\mathbf{l}$	ampere-turn		Electromotive force (EMF)	$\mathcal{E} = \int \mathbf{E} \cdot d\mathbf{l}$	volt
Magnetic field	H	ampere/meter		Electric field	E	Volt/meter
Magnetic flux	Φ	weber		Electric current	I	ampere
Reluctance	R_m	1/henry		Electrical resistance	R	ohm
Permeance	$P = \frac{1}{R_m}$	henry		Electric conductance	$G = 1/R$	siemens
Magnetic flux density	B	tesla		Current density	J	ampere/meter ²
Permeability	μ	henry/meter		Electrical conductivity	σ	siemens/meter