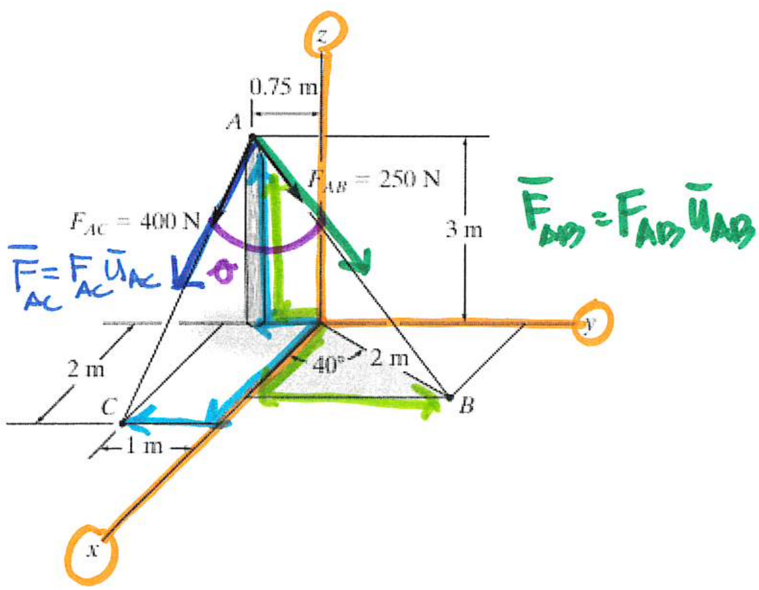
ENSC 2113 – Fall 2023

Homework #2

Work each problem on a separate piece of paper using the homework format outlined on course website (Canvas).

Submit handwritten work as a single PDF to the course website. Due Friday, September 22 by 8:45 am.

Problem #1 (20 points):



a) Write each force vector, F_{AC} and F_{AB}, as a Cartesian Vector.

b) Calculate the angle theta between F_{AC} and a line from A to B using dot product.

c) Determine the projected component of F_{AC} on a line from A to B. Express as a Cartesian Vector.

d) Determine the magnitude and direction angles of the resultant force of F_{AC} and F_{AB}.

(a) $\bar{F}_{AC} = F\bar{u}_{AC} = F\frac{\bar{r}_{AC}}{|r_{AC}|}$

① COORDINATES A(0, −0.75, 3) m
C(2, −1, 0) m

② $\bar{r}_{AC} = \{2\bar{i} - 0.25\bar{j} - 3\bar{k}\}$ m

③ $|r_{AC}| = \sqrt{(2)^2+(-0.25)^2+(-3)^2} = \sqrt{13.1}$ m

④ $\bar{u}_{AC} = \{\frac{2}{\sqrt{13.1}}\bar{i} - \frac{0.25}{\sqrt{13.1}}\bar{j} - \frac{3}{\sqrt{13.1}}\bar{k}\}$

⑤ $\bar{F}_{AC} = F\bar{u}_{AC} = 400\text{N}\,\bar{u}_{AC}$

$\boxed{\bar{F}_{AC} = \{221.3\bar{i} - 27.7\bar{j} - 332.0\bar{k}\}\text{ N}}$

$$\bar{F}_{AB} = F\,\bar{u}_{AB} = F\,\frac{\bar{r}_{AB}}{|r_{AB}|}$$

① $A(0, -0.75, 3)\,m$

$B(2\cos 40, 2\sin 40, 0)\,m = (1.532, 1.286, 0)\,m$

② $\bar{r}_{AB} = \{1.532\,\bar{i} + 2.036\,\bar{j} - 3\,\bar{k}\}\,m$

③ $|\bar{r}_{AB}| = \sqrt{(1.532)^2 + (2.036)^2 + (-3)^2} = \sqrt{15.49}\,m$

④ $\bar{u}_{AB} = \frac{\bar{r}_{AB}}{|\bar{r}_{AB}|} = \left\{\frac{1.532}{\sqrt{15.49}}\,\bar{i} + \frac{2.036}{\sqrt{15.49}}\,\bar{j} - \frac{3}{\sqrt{15.49}}\,\bar{k}\right\}$

$$\boxed{⑤\ \bar{F}_{AB} = F\bar{u}_{AB} = \{97.3\,\bar{i} + 129.3\,\bar{j} - 190.6\,\bar{k}\}\,N}$$

(b) $\cos\theta = \frac{\bar{r}_{AC} \cdot \bar{r}_{AB}}{|r_{AC}||r_{AB}|}$

$$\theta = \cos^{-1}\left[\frac{\{2\,\bar{i} - 0.25\,\bar{j} - 3\,\bar{k}\}\,m \cdot \{1.532\,\bar{i} + 2.036\,\bar{j} - 3\,\bar{k}\}\,m}{|\sqrt{13.1}\,m|\,|\sqrt{15.49}\,m|}\right]$$

$$= \cos^{-1}\left[\frac{(2)(1.532) + (-0.25)(2.036) + (-3)(-3)}{(\sqrt{13.1})(\sqrt{15.49})}\right]$$

$$= \cos^{-1}\left[\frac{11.555}{14.245}\right] = \cos^{-1}[0.811]$$

$$\boxed{\theta = 35.8^\circ}$$

(c) $(F_{AC})_{PROJ\,AB} = \bar{F}_{AC} \cdot \bar{u}_{AB} = \{221.3\,\bar{i} - 27.7\,\bar{j} - 332.0\,\bar{k}\}\,N \cdot \left\{\frac{1.532}{\sqrt{15.5}}\,\bar{i} + \frac{2}{\sqrt{15.5}}\,\bar{j} - \frac{3}{\sqrt{15.5}}\right\}$

$$= \left[221.3\left(\frac{1.532}{\sqrt{15.49}}\right) + (-27.7)\left(\frac{2}{\sqrt{15.49}}\right) + (-332)\left(-\frac{3}{\sqrt{15.49}}\right)\right]N$$

$$= [86.14 + (-14.33) + (253.07)]\,N$$

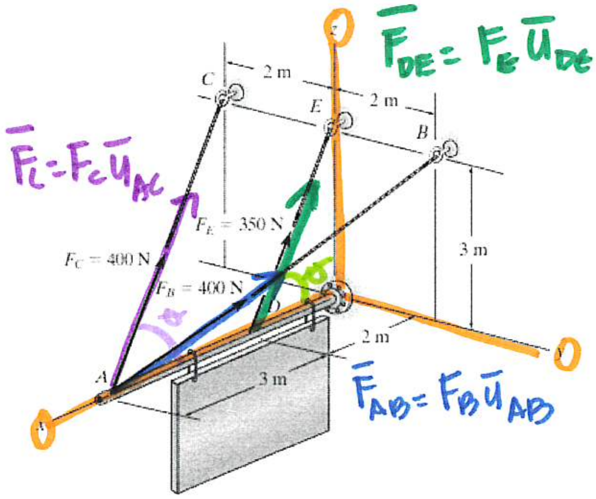
$$= 324.88\,N$$

$$(\vec{F}_{AC})_{PROJ\,AB} = (F_{AC})_{PROJ}\,\{u_{AB}\}$$

$$= 324.88\,N\left\{\frac{1.532}{\sqrt{15.49}}\,\bar{i} + \frac{2.036}{\sqrt{15.49}}\,\bar{j} - \frac{3}{\sqrt{15.49}}\,\bar{k}\right\}$$

$$\boxed{= \{126.5\,\bar{i} + 168.1\,\bar{j} - 247.6\,\bar{k}\}\,N}$$

Problem #2 (25 points):



a) Write each force vector, F_{AC}, F_{AB}, and F_{DE}, as a Cartesian Vector.
b) Calculate the angle theta between F_{AC} and a line from A to D using dot product.
c) Calculate the angle theta between F_{DE} and a line from D to A using dot product.
d) Determine the projected component of F_{AB} on a line from A to D. Express as a Cartesian Vector.
e) Determine the magnitude and direction angles of the resultant force of F_{AC} and F_{AB}.

(a) $\bar{F}_{AC} = 400N\,\bar{u}_{AC}$ A(5,0,0)m C(0,-2,3)m

$\bar{r}_{AC} = \{-5\bar{i} - 2\bar{j} + 3\bar{k}\}m$ $|\bar{r}_{AC}| = \sqrt{38m^2} = 6.16m$

$\bar{u}_{AC} = \{\frac{-5}{\sqrt{38}}\bar{i} - \frac{2}{\sqrt{38}}\bar{j} + \frac{3}{\sqrt{38}}\bar{k}\}$ $F_C = 400N$

$$\boxed{\bar{F}_{AC} = \{-324.4\bar{i} - 129.8\bar{j} + 194.7\bar{k}\}N}$$

$\bar{F}_{AB} = 400N\,\bar{u}_{AB}$ A(5,0,0)m B(0,2,3)m

$\bar{r}_{AB} = \{-5\bar{i} + 2\bar{j} + 3\bar{k}\}m$ $|\bar{r}_{AB}| = \sqrt{38m^2} = 6.16m$

$\bar{u}_{AB} = \{-\frac{5}{\sqrt{38}}\bar{i} + \frac{2}{\sqrt{38}}\bar{j} + \frac{3}{\sqrt{38}}\bar{k}\}$ $F_B = 400N$

$$\boxed{\bar{F}_{AB} = \{-324.4\bar{i} + 129.8\bar{j} + 194.7\bar{k}\}N}$$

$\bar{F}_{DE} = 350N\,\bar{u}_{DE}$ D(2,0,0)m E(0,0,3)m

$\bar{r}_{DE} = \{-2\bar{i} + 0\bar{j} + 3\bar{k}\}m$ $|\bar{r}_{DE}| = \sqrt{13m^2} = 3.61m$

$\bar{F}_{DE} = 350N\{\frac{-2}{\sqrt{13}}\bar{i} + \frac{3}{\sqrt{13}}\bar{k}\} = \boxed{\{-194\bar{i} + 0\bar{j} + 291.2\bar{k}\}N}$

(b) $\theta = \cos^{-1} \dfrac{\{\bar{r}_{AC}\} \cdot \{\bar{r}_{AD}\}}{|\bar{r}_{AC}||\bar{r}_{AD}|} = \dfrac{\{-5\bar{i} - 2\bar{j} + 3\bar{k}\}_m \cdot \{-3\bar{i} + 0\bar{j} + 0\bar{k}\}_m}{|6.16\,m|\,|3\,m|}$

$= \cos^{-1} \left[\dfrac{(-5)(-3) + (-2)(0) + (3)(0)}{(6.16)(3)} \right] = 0.81169$

$\boxed{= 35.7^\circ}$

(c) $\theta = \cos^{-1} \dfrac{\{\bar{r}_{DE}\} \cdot \{\bar{r}_{DA}\}}{|\bar{r}_{DE}| \cdot |\bar{r}_{DA}|} = \dfrac{\{-2\bar{i} + 0\bar{j} + 3\bar{k}\} \cdot \{3\bar{i} + 0\bar{j} + 0\bar{k}\}}{|3.61|\,|3|}$

$= \cos^{-1} \left[\dfrac{(-2)(3) + (0)(0) + (3)(0)}{(3.61)(3)} \right] = -0.554$

$\boxed{= 123.6^\circ}$

(d) $(F_{AB})_{PROJ_{AD}} = \bar{F}_{AB} \cdot \bar{u}_{AD} = \{-324.4\bar{i} + 129.8\bar{j} + 194.7\bar{k}\}N \cdot \{-1\bar{i} + 0\bar{j} + 0\bar{k}\}$

$= \left[(-324.4)(-1) + (129.8)(0) + (194.7)(0) \right]$

$= 324.4\,N$

$(\vec{F}_{AB})_{PROJ} = (F_{AB})_{PROJ}\, \bar{u}_{AD}$

$= 324.4N\, \{-1\bar{i} + 0\bar{j} + 0\bar{k}\}$

$\boxed{= \{-324.4\bar{i} + 0\bar{j} + 0\bar{k}\}N}$

(e) $\vec{R} = \vec{F}_{AC} + \vec{F}_{AB} = \{-324.4\bar{i} - 129.8\bar{j} + 194.7\bar{k}\}N + \{-324.4\bar{i} + 129.8\bar{j} + 194.7\bar{k}\}$

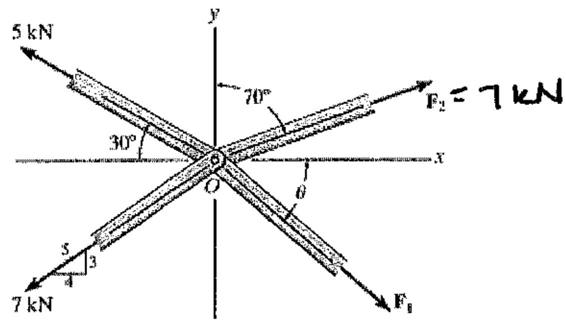
$= \{-648.8\bar{i} + 0\bar{j} + 389.4\bar{k}\}N$

$|R| = \sqrt{(-648.8)^2 + (389.4)^2} = \boxed{756.7\,N}$

$\alpha = \cos^{-1} \dfrac{-648.8\,N}{756.7\,N}$ $\quad \beta = \cos^{-1} \dfrac{0\,N}{756.7\,N}$ $\quad \gamma = \cos^{-1} \dfrac{389.4\,N}{756.7\,N}$

$\boxed{\alpha = 149^\circ \qquad \beta = 90^\circ \qquad \gamma = 59^\circ}$

Problem #3 (10 points):



The members of a truss are pin connected at joint O. Determine the magnitude of F1 and its angle θ for equilibrium using equilibrium equations. Set $F_2 = 7$ kN.

$$\overrightarrow{\Sigma F_x} = -5\text{kN}\cos 30 - \tfrac{4}{5}(7\text{kN}) + 7\text{kN}\sin 70 + F_1\cos\theta = 0$$

$$F_1\cos\theta = 3.352\text{ kN} \quad ①$$

$$\Sigma F_y \uparrow = 5\text{kN}\sin 30 - \tfrac{3}{5}(7\text{kN}) + 7\text{kN}\cos 70 - F_1\sin\theta = 0$$

$$F_1\sin\theta = 0.694\text{kN} \quad ②$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{②}{①} \qquad \frac{\cancel{F_1}\sin\theta}{\cancel{F_1}\cos\theta} = \frac{0.694\text{kN}}{3.352\text{kN}}$$

$$\tan\theta = 0.207$$

$$\theta = \tan^{-1} 0.207$$

$$\boxed{\theta = 11.7°}$$

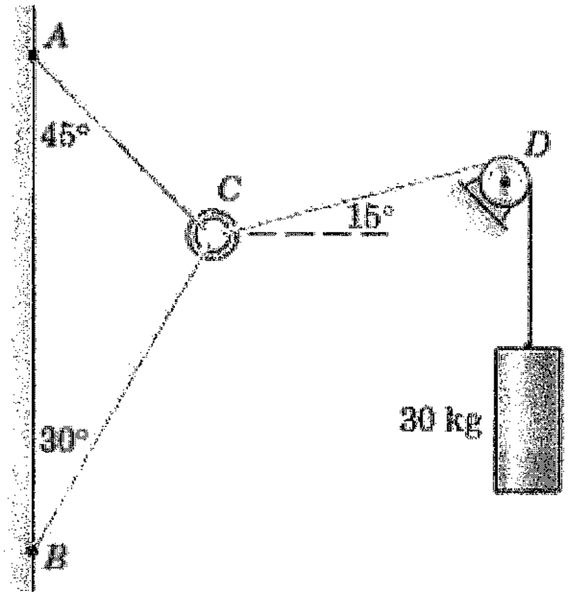
$$F_1 \sin 11.7° = 0.694\text{kN}$$

$$\boxed{F_1 = 3.42\text{ kN}}$$

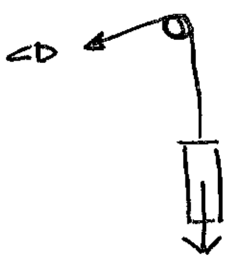
to check: $F_1 \cos 11.7° = 3.352\text{kN}$

$$F_1 = 3.42\text{ kN} \checkmark$$

Problem #4 (10 points):



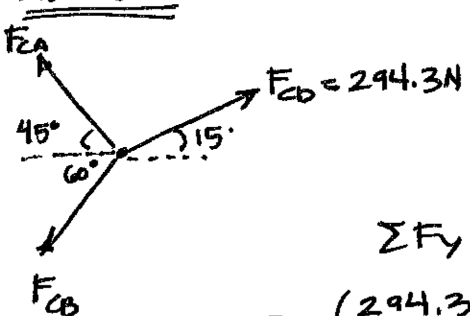
Three cables are joined at the junction of ring C. Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder using equilibrium equations. Please draw a free-body diagram as part of the solution.



$30kg\,(9.81\,m/s^2)$

$F_{CD} = 30kg\,(9.81\,m/s^2) = 294.3\,N$

FBD @ C



$\Sigma \vec{F_x} = 0$

$-F_{CA}\cos45 - F_{CB}\cos60 + 294.3N\cos15° = 0$

$F_{CA} = \dfrac{294.3N\cos15}{\cos45} - \dfrac{F_{CB}\cos60}{\cos45}$ (1)

(2)

$\Sigma F_y \uparrow = 0 \quad F_{CA}\sin45 - F_{CB}\sin60 + 294.3N(\sin15) = 0$

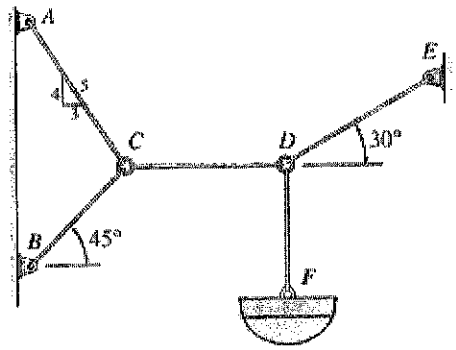
(1) → (2) $\left(\dfrac{294.3\cos15}{\cos45} - \dfrac{F_{CB}\cos60}{\cos45}\right)(\sin45) - F_{CB}\sin60 = -294.3(\sin15)$

$284.27 - 0.5F_{CB} - 0.866F_{CB} = -76.17\,N$

$1.366F_{CB} = 360.44\,N$

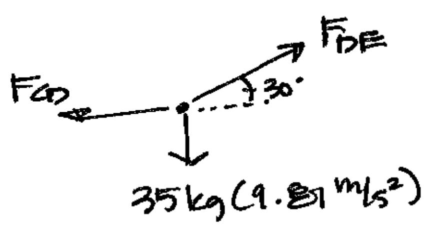
$\boxed{F_{CB} = 263.9N \quad F_{CA} = 215.4N}$

Problem #5 (10 points)



Determine the tension developed in each cord required for equilibrium of the 35-kg lamp using equilibrium equations. Draw a free-body diagram as part of your solution.

FBD @ D



$\Sigma F_y \uparrow = 0$ $\quad F_{DE}\sin 30 - 35\text{kg}(9.81\,\text{m/s}^2) = 0$

$$F_{DE} = \frac{35\text{kg}(9.81\,\text{m/s}^2)}{\sin 30}$$

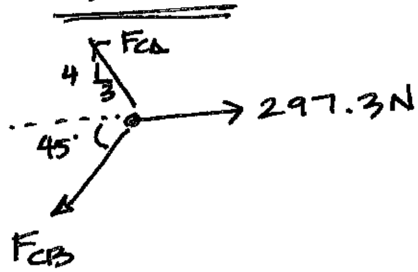
$$\boxed{F_{DE} = 686.7\text{ N}}$$

$\overrightarrow{\Sigma F_x} = 0$ $\quad -F_{CD} + 686.7\text{N}(\cos 30) = 0$

$$F_{CD} = 686.7\text{N}\cos 30$$

$$\boxed{F_{CD} = 297.3\text{ N}}$$

FBD @ C



$\Sigma F_y \uparrow = 0$ $\quad \frac{4}{5}(F_{CA}) - F_{CB}\sin 45 = 0$

$$F_{CA} = \frac{5}{4}F_{CB}\sin 45$$

$\overrightarrow{\Sigma F_x} = 0$ $\quad -\frac{3}{5}F_{CA} - F_{CB}\cos 45 + 297.3\text{N} = 0$

$$-\frac{3}{5}\left(\frac{5}{4}F_{CB}\sin 45\right) - F_{CB}\cos 45 = -297.3\text{N}$$

$$0.53F_{CB} + 0.707F_{CB} = 297.3\text{N}$$

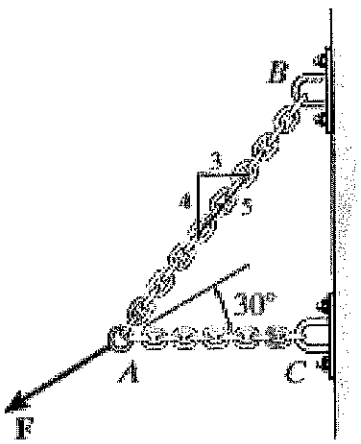
$$1.237\,F_{CB} = 297.3\text{N}$$

$$\boxed{F_{CB} = 240.3\text{ N}}$$

$$F_{CA} = \frac{5}{4}(240.3\text{N})(\sin 45)$$

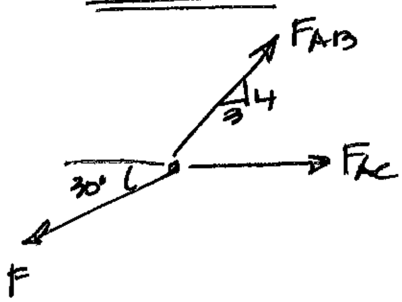
$$\boxed{F_{CA} = 212.4\text{ N}}$$

Problem #6 (10 points):



Determine the maximum force F that can be supported in the position shown if each chain can support a maximum tension of 750 lb (pounds) before it fails.

FBD @ A



$$\overrightarrow{\Sigma F_x = 0}$$

$$-F\cos 30 + F_{AC} + \tfrac{3}{5}F_{AB} = 0 \qquad (1)$$

$$\Sigma F_y \uparrow = 0$$

$$-F\sin 30 + \tfrac{4}{5}F_{AB} = 0 \qquad (2)$$

$$F_{AB} = \tfrac{5}{4}F\sin 30 = 0.625\,F$$

$$F_{AB} = 0.625\,F$$

$$(2) \longrightarrow (1)$$

$$-F\cos 30 + F_{AC} + \tfrac{3}{5}(0.625F) = 0$$

$$F_{AC} = F\cos 30 - \tfrac{3}{5}(.625F)$$

$$F_{AC} = 0.491\,F$$

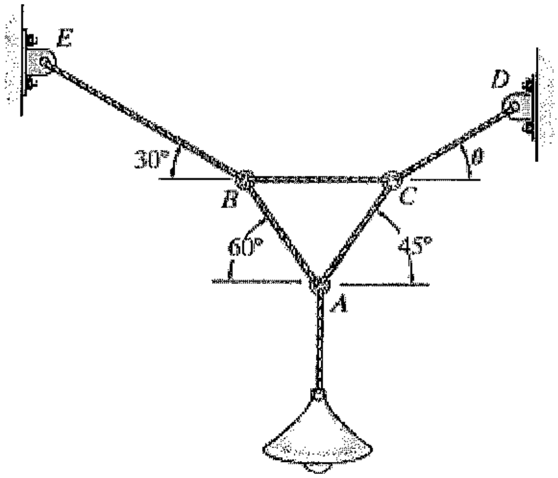
F_{AC} HAS $0.491F$ AND F_{AB} HAS $0.625F$

F_{AB} LIMITS . . . . $F_{AB} = 0.625F$

$$750\,lb = 0.625\,F$$

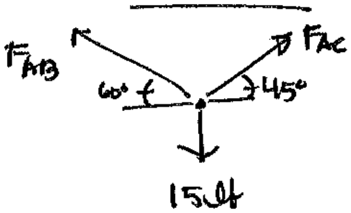
$$\boxed{F = 1200\,lb}$$

Problem #7 (15 points):



The lamp has a weigh of 15 lb and is supported by the six cords connected together as shown. Determine the tension in each cord and the angle θ for equilibrium using equilibrium equations. Draw all free-body diagrams needed as part of the solution. Cord BC is horizontal.

FBD @ A



$\overrightarrow{\Sigma F_x} = 0 \qquad -F_{AB}\cos 60 + F_{AC}\cos 45 = 0$

$F_{AB} = F_{AC}\dfrac{\cos 45}{\cos 60}$

$\Sigma F_y \uparrow = 0 \qquad F_{AB}\sin 60 + F_{AC}\sin 45 - 15\,lb = 0$

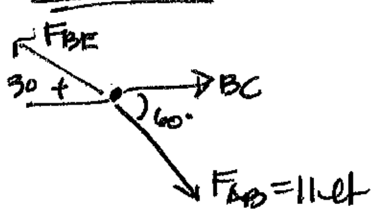
$F_{AC}\dfrac{\cos 45}{\cos 60}(\sin 60) + F_{AC}\sin 45 = 15\,lb$

$1.932 F_{AC} = 15\,lb$

$\boxed{F_{AC} = 7.76\,lb}$

$F_{AB} = (7.76\,lb)\dfrac{\cos 45}{\cos 60} \rightsquigarrow \boxed{F_{AB} = 11\,lb}$

FBD @ B



$\Sigma F_y \uparrow = 0 \qquad F_{BE}\sin 30 - 11\,lb\sin 60 = 0$

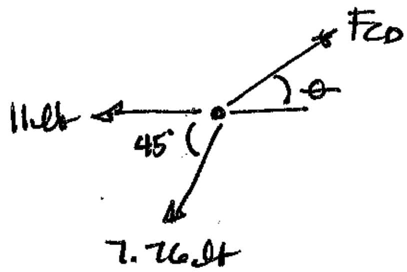
$F_{BE} = 11\,lb\dfrac{\sin 60}{\sin 30}$

$\boxed{F_{BE} = 19.1\,lb}$

$\overrightarrow{\Sigma F_x} = 0 \qquad -19.1\,lb\cos 30 + F_{BC} + 11\,lb\cos 60 = 0$

$\boxed{F_{BC} = 11\,lb}$

FBD @ C



$\overrightarrow{\Sigma F_x} = 0 \qquad -11 - 7.76\,lb\cos 45 + F_{CD}\cos\theta = 0$

$$F_{CD}\cos\theta = 16.487\,lb \quad (1)$$

$\Sigma F_y \uparrow = 0 \qquad F_{CD}\sin\theta - 7.76\,lb\sin 45 = 0$

$$F_{CD}\sin\theta = 5.487\,lb \quad (2)$$

$$\frac{\cancel{F_{CD}}\sin\theta}{\cancel{F_{CD}}\cos\theta} = \frac{5.487\,lb}{16.487\,lb}$$

$$\tan\theta = 0.3328$$

$$\theta = \tan^{-1} 0.3328$$

$$\boxed{\theta = 18.4°}$$

$$F_{CD}\cos 18.4° = 16.487\,lb$$

$$F_{CD} = \frac{16.487\,lb}{\cos 18.4°} = 17.38\,lb$$

$$\boxed{F_{CD} = 17.38\,lb}$$