Engineering Economic Analysis(EEA)
Chapter 4: Present Worth

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Learning Objectives

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4.1 Describe the eight discounted cash flow methods used in comparing investment alternatives. (Section 4.1)

4.2 Calculate a present worth (PW) converting all cash flows to a single sum equivalent at time zero for a given interest rate. (Section 4.2)

4.3 Perform an economic analysis of public investments utilizing the benefit-cost analysis for a given interest rate. (Section 4.3)

4.4 Calculate the discounted payback period (DPBP) for a given interest rate to determine how long it takes for the cumulative present worth (PW) to be positive. (Section 4.4)

4.5 Calculate the capitalized worth (CW) of an investment for a given interest rate when the planning horizon is infinitely long. (Section 4.5)

Engineering Economic Analysis

Chapter 4: Present Worth

Systematic Economic Analysis Technique

A **systematic economic analysis** usually have the following procedures:

1. Identify the investment alternatives.
2. Define the planning horizon.
3. Specify the discount rate.
4. Estimate the cash flow.
5. **Compare the alternatives.**
6. Perform supplementary analysis.
7. Select the preferred investment.

Comparing Economic Worth

This section summarizes the eight DCF methods in comparing investment alternatives.

1. The present worth (PW) method
2. The benefit-cost ratio (B/C) method
3. The discounted payback period (DPBP) method
4. The capitalized worth (CW) method
5. The annual worth (AW) method
6. The future worth (FW) method
7. The internal rate of return (IRR) method
8. The external rate of return (ERR) method

Before-Tax Versus After-Tax Analysis

In using a measure of economic worth to compare investment alternatives, you can employ either before-tax or after-tax cash flows. Be consistent, however! It is either/or but not both in the same analysis.

1. If the comparison is based on before-tax cash flows, then use a before-tax MARR; likewise,
2. the comparison is based on after-tax cash flows, then use an after-tax MARR.

Although we believe it is usually best to perform after-tax economic justifications, we will not cover tax issues until after discussing a variety of methods used to compare economic alternatives.

Equal Versus Unequal Lives

When comparing investment alternatives, they must be compared over a common time period, called the planning horizon. (Recall Principle #8: Compare investment alternatives over a common period of time.)

If the duration of the planning horizon differs from the useful lives of the alternatives, then (at the end of the planning horizon) cash flow estimates must be provided for the terminal or salvage values

1. for alternatives with lives greater than the planning horizon;
2. for alternatives having useful lives less than the planning horizon, replacement decisions must be made and cash flow estimates must be provided for the replacements.

Although we believe it is usually best to perform after-tax economic justifications, we will not cover tax issues until after discussing a variety of methods used to compare economic alternatives.

A Single Alternative

“A single alternative" is an oxymoron. If there is no choice, then there is no alternative. When we consider “a single alternative,” however, we are considering doing something versus doing nothing. Assuming that the do nothing alternative is feasible, you have two options when faced with the question “Should I invest in an opportunity or not?”: Invest, or don’t invest. In evaluating the “invest" option, we assume that the cash flow estimates reflect the differences in doing nothing versus doing something.

Engineering Economic Analysis

Chapter 4: Present Worth

- Section 4.1: Comparing Alternatives
- Section 4.2: Present Worth Calculations
- Section 4.3: Benefit-Cost Analysis
- Section 4.4: Discounted Payback Period
- Section 4.5: Capitalized Worth

Present Worth Calculations

A single alternative

Definition

The **present worth** is the value of all cash flows converted to a single sum equivalent at time zero using $i = \text{MARR}$.

When using the present worth method to evaluate whether an investment should be made, the decision depends on whether the present worth is positive. If so, then the investment is recommended.

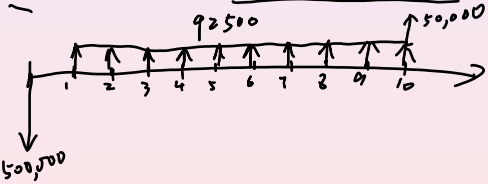
$$\boxed{\text{PW}(i) = \sum_{t=0}^{n} A_t (1+i)^{-t}} \qquad (1)$$

Example

Example (4.1)

To automatically insert electronic components in printed circuit boards for a cell phone production line, a \$500,000 surface mount placement (SMP) machine is being evaluated by a manufacturing engineer. Over the 10-year planning horizon, it is estimated that the SMP machine will produce annual after-tax cost savings of \$92,500. The engineer estimates the machine will be worth \$50,000 at the end of the 10-year period. Based on the firm's 10% after-tax MARR, should the investment be made?

Solution;



Example

The present worth of the cash flow series is

$$PW = -500,000 + 92500\left[\frac{(1+0.1)^{10}-1}{0.1\times(1+0.1)^{10}}\right] + 50,000\,(1+0.1)^{-10}$$

$$= 87649.622 > 0$$

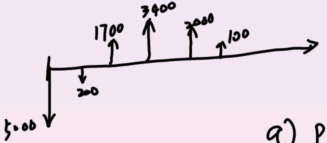
Since the PW is positive, the investment should be made.

Example

Example (4.2)

DuraTech Manufacturing is evaluating a process improvement project. The estimated receipts and disbursements associated with the project are shown below. MARR is 6%/year.

End of Year	Receipts	Disbursements
0	$ 0	$5,000
1	$ 0	$ 200
2	$2,000	$ 300
3	$4,000	$ 600
4	$3,000	$1,000
5	$1,600	$1,500



$i = MARR = 0.06$

(a) What is the present worth of this investment?

(b) What is the decision rule for judging the attractiveness of investments based on present worth?

(c) Should DuraTech implement the proposed process improvement?

a) $PW = -5000 - 200(1+0.06)^{-1} + 1700(1+0.06)^{-2} + 3400(1+0.06)^{-3} + 2000(1+0.06)^{-4} + 100(1+0.06)^{-5}$

$= \$837.933$

b) If the present worth is positive, we should make the investment.

c) Since PW > 0, DuraTech should implement the proposed process improvement.

Example

Example

Example

Example (4.3)

Aerotron Electronics is considering purchasing a water filtration system to assist in circuit board manufacturing. The system costs $40,000. It has an expected life of 7 years at which time its salvage value will be $7,500. Operating and maintenance expenses are estimated to be $2,000 per year. If the filtration system is not purchased, Aerotron Electronics will have to pay Bay City $12,000 per year for water purification. If the system is purchased, no water purification from Bay City will be needed. Aerotron Electronics must borrow half of the purchase price, but they cannot start repaying the loan for 2 years. The bank has agreed to three equal annual payments, with the first payment due at the end of year 2. The loan interest rate is 8% compounded annually. Aerotron Electronics' MARR is 10% compounded annually.

Should Aerotron Electronics purchase the system?

Solution:

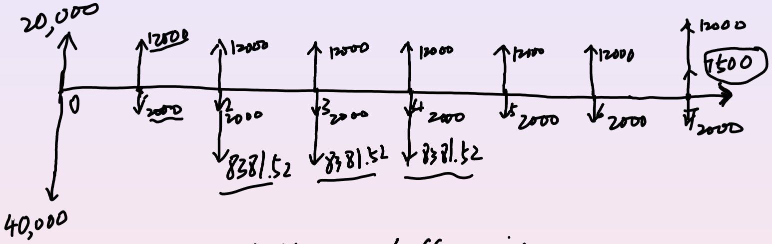
For the loan, the equal annual payments can be found as follows:

$40000 \times 0.5 = 20000$

$$20000 \times (1+0.08) \frac{0.08(1+0.08)^3}{(1+0.08)^3 - 1}$$

$$= \boxed{8381.52}$$

Example



The present worth of these cash flows is

$$PW = \underbrace{-40000 + 20000}_{\text{year } 0} + \underbrace{(12000-2000)(1+0.1)^{-1}}_{\text{PW of year 1}} + \underbrace{(12000-2000-8381.52)(1+0.1)^{-2}}_{\text{year } 2}$$

$$+ (12000-2000-8381.52)(1+0.1)^{-3} + (12000-2000-8381.52)(1+0.1)^{-4} + (12000-2000)(1+0.1)^{-5}$$

Example

$$+(12000-2000)(1+0.1)^{-6}+(12000+7500-2000)(1+0.1)^{-7}$$

$$=\boxed{13584.15}$$

Since $PW>0$, the system should be purchased.

There is another way to calculate PW.

$$PW=-40000+20000+(12000-2000)\left(\frac{(1+0.1)^{7}-1}{0.1(1+0.1)^{7}}\right),$$

$$+8381.52\left[(1+0.1)^{-2}+(1+0.1)^{-3}+(1+0.1)^{-4}\right]$$

$$+\ 7500\times(1+0.1)^{-7}$$

$$=13584.15$$

Present Worth of Multiple Alternatives

PW is easily applied when choosing the preferred alternative from among several mutually exclusive alternatives: Choose the one with the greatest PW over the planning horizon. Mathematically, the objective is

$$\max_{j} \mathrm{PW}_j = \sum_{t=0}^{n} A_{j^*t}(1 + \mathrm{MARR})^{-t} \tag{2}$$

Here the j^*th alternative has the largest present worth among these alternatives.

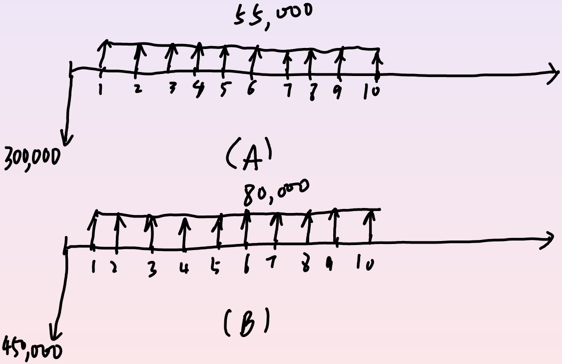
Example

Example (4.4)

Entertainment Engineers, Inc., is an Ohio-based design engineering firm that designs rides for amusement and theme parks all over the world. Two alternative designs are under consideration for a new ride called the Scream Machine at a theme park located in Florida. The two candidate designs differ in complexity, cost, and predicted revenue. The first alternative design (A) will require an investment of $300,000 and is estimated to produce after-tax revenue of $55,000 annually over a 10-year planning horizon. The second alternative design (B) will require an investment of $450,000 and is expected to generate annual after-tax revenue of $80,000. A negligible salvage value is assumed for both designs. Theme park management could decide to “do nothing"; if so, the present worth of doing nothing will be zero. An after-tax MARR of 10% is used. Which alternative design, if either, should the theme park select?

Example

Solution:



$i = MARR = 0.1$

Example

$$PW_A = -300\,000 + 55\,000 \times \left[\frac{(1+0.1)^{10} - 1}{0.1 \times (1+0.1)^{10}}\right]$$

$$= 37951.191 > 0$$

$$PW_B = -450\,000 + 80\,000 \left[\frac{(1+0.1)^{10} - 1}{0.1 \times (1+0.1)^{10}}\right]$$

$$= \boxed{41565.3685}$$

Hence $\boxed{PW_B > PW_A > 0}$

Since Alternative B has the largest PW, the there Park should select alternative B

Example

Example (4.5)

Quantum Logistics, Inc., a wholesale distributor, is considering the construction of a new warehouse to serve the southeastern geographic region near the Alabama–Georgia border. There are three cities being considered. After site visits and a budget analysis, the expected income and costs associated with locating in each of the cities have been determined. The life of the warehouse is expected to be 12 years and MARR is 15%/year.

City	Initial Cost	Net Annual Income
Lagrange	$1,260,000	$480,000
Auburn	$1,000,000	$410,000
Anniston	$1,620,000	$520,000

(a) What is the present worth of each site?

(b) What is the decision rule for determining the preferred site based on present worth ranking?

(c) Which city should be recommended?

a) $PW_1 = -1260000 + 480000 \times \left[\frac{(1+0.15)^{12}-1}{0.15(1+0.15)^{12}}\right] = 1341897.12$

$PW_2 = -1000000 + 410000 \times \left[\frac{(1+0.15)^{12}-1}{0.15(1+0.15)^{12}}\right] = 1222453.79$

$PW_3 = -1620000 + 520000 \left[\frac{(1+0.15)^{12}-1}{0.15 \times (1+0.15)^{12}}\right] = 1198721.88$

b) Choose the one with the largest PW

c) Since PW_1 is the largest, Lagrange should be recommended

Example

Example

Example

Example (4.6)

Two storage structures, given code names Y and Z, are being considered for a military base located in Sontaga. The military uses a 5%/year expected rate of return and a 24-year life for decisions of this type. The relevant characteristics for each structure are shown below.

	Structure Y	Structure Z
First Cost	\$4,500	\$10,000
Estimated Life	12 years	24 years
Estimated Salvage Value	None	\$1,800
Annual Maintenance Cost	\$1,000	\$720

(a) What is the present worth of each alternatives?

(b) What is the decision rule for determining the preferred alternative based on present worth ranking?

(c) Which structure should be recommended?

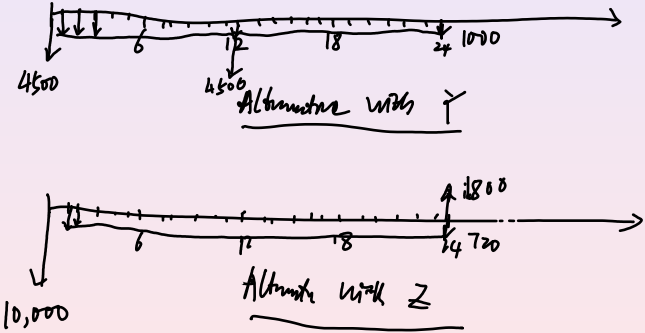
Solution:

Since the planning horizon is 24 years.

Alternative with structure Y involves two such structures back to back.

Alternative with structure Z involves only one such structure.

Example


6
12
18
24
1000
4500
4500
Alternative with Y
1800
6
12
18
24
720
10,000
Alternative with Z

Example

Hence $PW_1 = -4500 - 1000\left[\frac{(1+0.05)^{24}-1}{0.05(1+0.05)^{24}}\right] - 4500\times(1+0.05)^{-12}$

Present Worth of One-Shot Investments

Definition (One-Shot Investment)

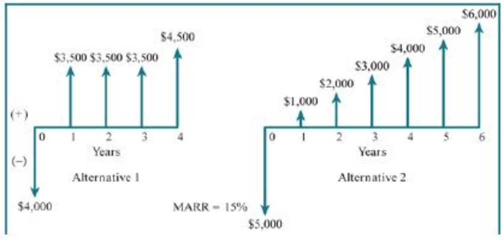
A one-shot Investment is an investment that is available only once.

When one-shot investments are being considered, the planning horizon is defined to be equal to the longest life among the investment alternatives. Then, the present worth is computed for each alternative.

Example

Example (4.7)

Consider the two cash flow diagrams given the figure below.



Both alternatives are one-shot investments. As such, we cannot predict what investment alternatives might be available in the future. However, the minimum attractive rate of return, 15%, reflects the opportunity to reinvest recovered capital. Which alternative is preferred?

Example

Example

Example

Example (4.8)

Two new opportunities are being considered for a venture capital firm. Both are one-time opportunities with no option for renewal. The firm uses a 12decisions of this type. The relevant characteristics for each option are shown below. Based on a present worth analysis, which option is preferred?

	Option 1	Option 2
Initial Investment	$100,000	$75,000
Estimated Life	12 years	9 years
Expected Annual Return	$16,500	$14,300

Example

Example

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Public Investments

Government units fund projects using money taken, usually in the form of taxes, from the public. They then provide goods or services to the public that would be unfeasible for individuals to provide on their own. While they are not in business to make a profit, it is important that they make wise investment decisions. Projects should provide benefits for the public's greater good that exceed the costs of providing those benefits. The most frequently used method in evaluating government (local, state, or federal) projects is benefit-cost analysis.

The two most common forms of benefit-cost analysis are the benefit-cost ratio (B/C) or, equivalently, a measure of benefits minus costs (B-C). With either of these two forms, both the benefits (B) and costs (C) are typically expressed as present worth or annual worth monetary figures (e.g., $), using an appropriate discount rate for government programs or projects, and an appropriate planning horizon.

Benefit-Cost Analysis for a Single Alternative

Definition

Benefit-Cost Ratio (B/C) is the ratio of the present worth of net public benefits to the present worth of net government costs using $i = MARR$.

Benefits Minus Costs (B-C) is the difference of the present worth of net public benefits and present worth of net government costs using $i = MARR$.

To illustrate the mathematics of benefit-cost analysis, if

B_{jt} = *net* public benefits (benefits minus disbenefits, expressed as money) associated with alternative j

C_{jt} = *net* government costs (costs minus residuals such as salvage values) associated with alternative j

i = appropriate interest or discount rate, and

n = length of the planning horizon,

Cost vs Disbenefit

Both costs and disbenefits represent negative impacts, but they differ in nature and purpose.

- Cost: This typically refers to the monetary expenses incurred to implement a project, such as initial capital expenses, operational costs, maintenance, and sometimes indirect expenses (like administrative or training costs). These are the direct financial outflows needed to make the project happen.
- Disbenefits, on the other hand, refer to negative outcomes or adverse effects that result from the project, which might not be directly reflected in the budget but impact stakeholders. For instance, a new highway may lead to increased noise pollution, reduced air quality, or displaced communities. These are often quantified in BCR analysis by estimating their economic impact or assigning a monetary equivalent, even though they aren't traditional expenses.

Benefit-Cost Analysis for a Single Alternative

- The B/C criterion may be expressed mathematically, using the present worth of all net benefits over the present worth of all net costs, as

$$B/C_j(i) = \frac{\sum_{t=1}^{n} B_{jt}(1+i)^{-t}}{\sum_{t=1}^{n} C_{jt}(1+i)^{-t}} \tag{3}$$

  If the B/C ratio is greater than 1, the project is desirable.

- The net benefits minus net costs criterion (B-C) is expressed as

$$(B-C)_j(i) = \sum_{t=1}^{n}(B_{jt} - C_{jt})(1+i)^{-t} \tag{4}$$

  If the difference is positive, the project is desirable.

Example

Example (4.9)

Consider a 10-year land reclamation project that will commit the government to a stream of expenditures appearing in the Cost column of Figure 4.6. Real benefits appear in the Benefit column. Discounting takes place at a rate of 7%. Is this project desirable?

1	EOY	Cost	Benefit	PW Costs	PW Benefits	
2	0	$0	$0	$0	$0	
3	1	$100,000	$0	$93,458	$0	
4	2	$200,000	$0	$174,688	$0	
5	3	$300,000	$50,000	$244,889	$40,815	
6	4	$300,000	$100,000	$228,869	$76,290	
7	5	$200,000	$300,000	$142,597	$213,896	
8	6	$100,000	$400,000	$66,634	$266,537	
9	7	$50,000	$400,000	$31,137	$249,100	
10	8	$50,000	$400,000	$29,100	$232,804	
11	9	$50,000	$400,000	$27,197	$217,573	=I
12	10	$50,000	$250,000	$25,417	~~$127,087~~	=I
13		PRESENT WORTH		$1,063,987	$1,424,102	

Example

Example

Example (4.10)

A growing city of 60,000 has an intersection near the south end of the main business district. The north-south street has no traffic control at that intersection, and the east-west intersecting street has only stop signs. There have been a few accidents in recent years and it has been estimated that a traffic light can provide benefits of $55,000 per year in reduced property loss and disabling injuries, increasing by $2,500 per year. The disbenefits of the traffic light include cost of fuel and time delays, amounting to an estimated $24,000 per year, increasing by $2,000 per year. The installation cost of the traffic light is estimated to be $120,000 with maintenance and electricity running $6,000 in the first year and increasing by $300 per year over a time horizon of 10 years. It is anticipated that the intersection will need to be redesigned at that time and the traffic light removed and sold to a nearby small community for a net salvage value of $4,000. The discount rate is 7%. Is the traffic light justified?

Example

EOY	Costs	SV	Ben	Disben	PW Costs	PW SV	PW Ben	PW Disben
0	$120,000		$0	$0	$120,000		$0	$0
1	$6,000		$55,000	$24,000	$5,607		$51,402	$22,430
2	$6,300		$57,500	$26,000	$5,503		$50,223	$22,709
3	$6,600		$60,000	$28,000	$5,388		$48,978	$22,856
4	$6,900		$62,500	$30,000	$5,264		$47,681	$22,887
5	$7,200		$65,000	$32,000	$5,134		$46,344	$22,816
6	$7,500		$67,500	$34,000	$4,998		$44,978	$22,656
7	$7,800		$70,000	$36,000	$4,857		$43,592	$22,419
8	$8,100		$72,500	$38,000	$4,714		$42,196	$22,116
9	$8,400		$75,000	$40,000	$4,569		$40,795	$21,757
10	$8,700	$4,000	$77,500	$42,000	$4,423	$2,033	$39,397	$21,361
			PRESENT WORTH		$170,456	$2,033	$455,586	$223,997
				B/C=	1.375		*B-C=*	$63,166

Example: Approach 1

In this book, it is recommended that the present worth of all public benefits and disbenefits be calculated as the present worth of net public benefits (PW benefits-PW disbenefits) in the numerator, and all government costs and salvage values be calculated as net government costs (PW costs-PW salvage value) in the denominator as follows:

$$B/C = (\text{PW Benefits-PW Disbenefits})/(\text{PW Cost-PW Salvage Value}) \tag{5}$$

For the example above,

$$B/C = (455,586 - 223,997)/(170,456 - 2,033) = 1.375.$$

Because this is a number greater than 1.000, the project is worthwhile.
An equivalent analysis is to calculate B-C. If $B - C > \$0$, the project is worthwhile:

$$B - C = \$455,586 - \$223,997 - \$170,456 + \$2,033 = \$63,166.$$

Example: Approach 2

This approach is not recommended, but sometimes used in practice. In this approach, all of the "positives" (for example, public benefits and government salvage value) are put in the numerator and all of the "negatives" (government costs and public disbenefits) are put in the denominator as follows:

$$B/C = (\text{PW Benefits}+\text{PW Salvage Value})/(\text{PW Cost}+\text{PW Disbenefits}) \tag{6}$$

For the example above,

$$B/C = (455,586 + 2,033)/(170,456 + 223,997) = 1.160.$$

Because this is a number greater than 1.000, even though different from the 1.375 above, it is a worthwhile project.
An equivalent analysis is to calculate B-C. If $B - C > \$0$, the project is worthwhile:

$$B-C = (\$455,586+\$2,033)-(\$170,456+\$223,997) = \$63,166.$$

Example: Approach 3

This approach is not recommended, but sometimes used in practice. In this approach, the pure public benefits appear in the numerator and the government costs, salvage value, and public disbenefits are put in the denominator as follows:

$$B/C = (\text{PW Benefits})/(\text{PW Cost-PW Salvage Value+PW Disbenefits}) \tag{7}$$

For the example above,

$$B/C = (455,586)/(170,456 - 2,033 + 223,997) = 1.161.$$

Because this is a number greater than 1.000, even though different from the 1.375 and 1.160 above, it is a worthwhile project.
An equivalent analysis is to calculate B-C. If $B - C > \$0$, the project is worthwhile:

$$B-C = \$455,586-(\$170,456-2,033+\$223,997) = \$63,166.$$

Discussion on these three approaches.

- Approach 1 shows the recommended approach, keeping public benefits and disbenefits in the numerator and government costs and salvage values in the denominator.
- It is simply to show that one should select and stay with a consistent method of calculating B/C.
- If multiple alternatives are being considered for government investment, the use of different calculation approaches by different proposers can lead to inconsistent decisions.

Example

Example (4.11)

The Oklahoma City Zoo has proposed adding to their Web site a major segment providing a virtual tour of the grounds and animals, suitable for both routine enjoyment and educational purposes in classrooms. Survey data indicate that this will have either a neutral or positive effect upon actual zoo attendance. The Web site will be professionally done and have an initial cost of $325,000. Upkeep, refreshing the videos, and developing videos for scientific research and entertainment will cost another $80,000 per year. The zoo is expected to be in operation for an indefinite period; however, a study period of only 10 years for the Web site is to be assumed, with only a residual (salvage) value of $60,000 for the archival value being anticipated. Interest is 7%. An estimated 100,000 persons will visit the e-zoo in the first year, increasing by 30,000 each year, and they will receive, on the average, an additional $0.80 of benefit per visit when the new area is complete. On the basis of B/C analysis, should the Web site be supported for funding?

Example

Example

Benefit-Cost Analysis for Multiple Alternatives

When two or more project alternatives are being compared using a B/C ratio, the analysis should be done using the **Incremental B/C method**.

- One common, but not necessary, approach is to first order all alternatives from the lowest to the highest present worth of net costs (government costs less salvage value, if any). Alternative 1 and the second is Alternative 2, etc.
- “Do Nothing" should be considered Alternative 1 if that would be a viable decision; however, such reasons as safety, cost, or a requirement of law may preclude “Do Nothing" from consideration.

Multiple Alternatives

- The first in the list is Alternative 1, the second is Alternative 2, etc.
- Then, the present worth of the incremental benefits of Alternative 2 over Alternative 1, $\Delta B_{2-1}(i)$, are divided by the present worth of the incremental costs of Alternative 2 over Alternative 1, $\Delta C_{2-1}(i)$. That is

$$\Delta B/C_{2-1}(i) = \frac{\Delta B_{2-1}(i)}{\Delta C_{2-1}(i)} = \frac{\sum_{t=1}^{n}(B_{2t} - B_{1t})(1+i)^{-t}}{\sum_{t=1}^{n}(C_{2t} - C_{1t})(1+i)^{-t}} \tag{8}$$

- If the incremental benefits exceed the incremental costs (that is, $\Delta B/C_{2-1}(i) > 1$), Alternative 2 is the “winner.” Otherwise, Alternative 1 is the “winner." The “winner” then is compared to Alternative 3 and, again, a “winner” is selected. This continues throughout all available alternatives until a “grand winner” emerges.

Multiple Alternatives

If instead of an Incremental B/C approach a B-C approach is used, the method is very simple:

- the present worth of net benefits minus the present worth of net costs is calculated using Equation (4) for each alternative

$$(B - C)_j(i) = \sum_{t=1}^{n}(B_{jt} - C_{jt})(1 + i)^{-t}$$

  The alternative having the highest B-C value is selected.
- Of course, the incremental version of the B-C approach can also be used with a similar procedure as the B/C approach with equation (9)

$$\Delta(B - C)_{2-1}(i) = \Delta B_{2-1}(i) - \Delta C_{2-1}(i)$$
$$= \sum_{t=1}^{n}(B_{2t} - B_{1t})(1 + i)^{-t} - \sum_{t=1}^{n}(C_{2t} - C_{1t})(1 + i)^{-t} \tag{9}$$

Example

Example (4.12)

A county within which a city of 1.5 million people reside is considering proposed highway safety improvement projects to initiate in the upcoming year and they also wish to determine a ranking of proposed projects. They have a good feel for initial costs and maintenance and have already consulted many published studies to determine estimates of the benefits of reducing minor, moderate, serious, severe, critical, and fatal accidents. Benefits and costs for each alternative are shown in the table below.

Example

Projects	PW Net Public Benefits B	PW Net Govt Costs C
Do Nothing	$0	$0
Increase Site Distance	$38,000	$13,600
High Friction Surface	$116,000	$72,500
Flashing Beacons	$42,000	$14,900
Ruble Strips	$93,000	$53,000
Speed Cameras	$78,000	$35,000
Chevrons (Road Paint)	$ 105,000	$80,000

(a) Determine the overall best project based upon an Incremental B/C approach.
(b) Determine the overall best project based upon a B-C approach.
(c) Determine the ranking of these projects based upon a B-C approach.

Example

	A	B	C	D
1			PW Net	PW Net
2	Proj	Proposed	Public	Gov't
3	#	Project	Benefits *B*	Costs *C*
4	0	Do Nothing	\$0	\$0
5	1	Increase Site Distance	\$38,000	\$13,600
6				
7	1	Increase Site Distance	\$38,000	\$13,600
8	2	Flashing Beacons	\$42,000	\$14,900
9				
10	2	Flashing Beacons	\$42,000	\$14,900
11	3	Speed Cameras	\$78,000	\$35,000
12				
13	3	Speed Cameras	\$78,000	\$35,000
14	4	Rumble Strips	\$93,000	\$53,000
15				
16	3	Speed Cameras	\$78,000	\$35,000
17	5	Chevrons (Road Paint)	\$105,000	\$60,000
18				
19	5	Chevrons (Road Paint)	\$105,000	\$60,000
20	6	High Friction Surface	\$116,000	\$72,500

Example

Example

Example (4.13)

A highway is to be built connecting Maud and Bowlegs. Route A follows the old road and costs \$4 million initially and \$210,000/year thereafter. A new route, B, will cost \$6 million initially and \$180,000/year thereafter. Route C is an enhanced version of Route B with wider lanes, shoulders, and so on. Route C will cost \$9 million at first, plus \$260,000 per year to maintain. Benefits to the users, considering time, operation, and safety, are \$500,000 per year for A, \$850,000 per year for B, and \$1,000,000 per year for C. Using a 7% interest rate, a 15-year study period, and a salvage value of 50% of first cost, determine which road should be constructed.

Example

Example

Unequal Useful Lives

When faced with alternatives having unequal useful lives, the bottom line is the use of “Principal #8: Compare investment alternatives over a common period, the planning horizon.” For alternatives that have lives longer than the planning horizon, a “net residual benefit value" must be estimated to account for the benefits beyond the planning horizon. If there is any government salvage value or removal cost at the end of the planning horizon, it must also be calculated.

For alternatives that have shorter lives than the planning horizon, replacement or renewal decisions must be made, and both public net benefits and government net costs must be documented throughout the remainder of the planning horizon.

Example

Example (4.14)

A ranch has grown over the decades to become a major economic contributor to the county where it is located. The main approach from a rural road to the ranch's wide driveway has been a source of trouble for several years. The approach was designed by the county long ago, and it is the responsibility of the county to maintain the entrance from the county road to the driveway. When the road is graded, the gullies along the road sides are deepened and cleaned so the rainwater will flow nicely alongside the road that runs downhill about a mile. Unfortunately, during heavy rains the water carries away any gravel buildup, creating a large and deep depression that is hard on regular motor vehicles as well as the large trucks that must deliver to, and pick up product from, the ranch. The disbenefit to the ranch, its employees, its suppliers, truckers, and customers costs about $18,000 per year.

Example

Three alternatives have been identified to lessen this inconvenience cost-effectively. The alternatives will be evaluated using the county's discount rate of i = 5horizon of 24 years with the assumption that any installation will be removed by the county at the end of the planning horizon. All three alternatives have different useful lives, and adjustments must be made to ensure each covers the full 24-year planning horizon.

- Alternative 1 is to excavate the gully at the entrance to the driveway and place a 3-foot-diameter, 45-foot long culvert pipe, covered with gravel. The first cost for the pipe, grading, placement, transportation, and supplies is $13,000. Yearly additional maintenance for the county will be $1,000. Benefits to the ranch will be $14,000 per year. The useful life will be 8 years with no salvage value, and there will be a $2,000 removal cost. Alternative 1 will be replaced at the end of years 8 and 16 to cover the 24-year planning horizon. Costs are expected to remain constant.

Example

- Alternative 2 is to excavate the gully at the entrance to the driveway and install several joints of precast concrete box culverts in standard strengths and sizes. The first cost for this is $28,000 with only $400 additional yearly maintenance and a useful life of 16 years. Removal cost is $2,800. Benefits to the ranch will be $16,000 per year. For the remaining 8 years of the planning horizon, Alternative 1 will be used and its benefits and costs will apply.
- Alternative 3 is to divert the water flow upstream of the driveway, into the ranch property, to the first of a series of ponds with large holding capacity. This would be done underground with large corrugated nonmetallic piping. Benefits to the ranch will be $16,500. Mainly due to the deep excavation, distance, leveling, connecting, and installation care necessary, this would have a first cost of $60,000 and $0 additional maintenance per year and a useful life of 24 years with a $20,000 cost of removal.

Example

- Determine the best alternative using the B-C approach
- Determine the best alternative using the Incremental B/C approach.

	A	B	C	D	E	F	G
1		Alternative 1		Alternative 2		Alternative 3	
2		Benefits/year=	$14,000	Benefits/year=	$16,000	Benefits/year=	$16,500
3		Initial cost=	$13,000	Initial cost=	$28,000	Initial cost=	$60,000
4		Add maint/yr=	$1,000	Add maint/yr=	$400	Add maint/yr=	$0
5		Removal=	$2,000	Removal=	$2,800	Removal=	$20,000
6		Useful life=	8 years	Useful life=	16 years	Useful life=	24 years
7				Change to Alt 1 at EOY 16			

Example

8	EOY	Alt 1	Alt 1	Alt 2	Alt 2	Alt 3	Alt 3
9		Benefits	Costs	Benefits	Costs	Benefits	Costs
10	0		$13,000		$28,000		$60,000
11	1	$14,000	$1,000	$16,000	$400	$16,500	$0
12	2	$14,000	$1,000	$16,000	$400	$16,500	$0
13	3	$14,000	$1,000	$16,000	$400	$16,500	$0
14	4	$14,000	$1,000	$16,000	$400	$16,500	$0
15	5	$14,000	$1,000	$16,000	$400	$16,500	$0
16	6	$14,000	$1,000	$16,000	$400	$16,500	$0
17	7	$14,000	$1,000	$16,000	$400	$16,500	$0
18	8	$14,000	$14,000	$16,000	$400	$16,500	$0
19	9	$14,000	$1,000	$16,000	$400	$16,500	$0
20	10	$14,000	$1,000	$16,000	$400	$16,500	$0
21	11	$14,000	$1,000	$16,000	$400	$16,500	$0
22	12	$14,000	$1,000	$16,000	$400	$16,500	$0
23	13	$14,000	$1,000	$16,000	$400	$16,500	$0
24	14	$14,000	$1,000	$16,000	$400	$16,500	$0
25	15	$14,000	$1,000	$16,000	$400	$16,500	$0
26	16	$14,000	$14,000	$16,000	$16,200	$16,500	$0
27	17	$14,000	$1,000	$14,000	$1,000	$16,500	$0
28	18	$14,000	$1,000	$14,000	$1,000	$16,500	$0
29	19	$14,000	$1,000	$14,000	$1,000	$16,500	$0
30	20	$14,000	$1,000	$14,000	$1,000	$16,500	$0
31	21	$14,000	$1,000	$14,000	$1,000	$16,500	$0
32	22	$14,000	$1,000	$14,000	$1,000	$16,500	$0
33	23	$14,000	$1,000	$14,000	$1,000	$16,500	$0
34	24	$14,000	$3,000	$14,000	$3,000	$16,500	$20,000

Example

Example

Example (4.15)

The state of Washington must decide among three highway alternatives to replace an old winding road. The length of the current route is 26 miles. Planners agree that the old road must be replaced or overhauled; they cannot keep it as it is.

- Alternative A is to overhaul and resurface the old road at a cost of \$2 million/mile. Resurfacing will also cost \$2 million/mile at the end of each 10-year period. Annual maintenance will cost \$10,000/mile.
- Alternative B is to cut a new road following the terrain. It will be only 22 miles long. Its first cost will be \$3 million/mile with resurfacing at 10-year intervals costing \$2 million/mile with annual maintenance at \$12,000/mile.

Example

- Alternative C also involves a new highway to be built along a 20.5-mile straight line. Its first cost will be $4 million/mile with resurfacing at 10-year intervals costing $2 million/mile and with annual maintenance costing $20,000/mile. This increase over Routes A and B is due to additional roadside bank retention efforts.

Traffic density along each of the three routes will fluctuate widely from day to day but will average 4,000 vehicles/day throughout the 365-day year. This volume is composed of 350 light trucks, 250 heavy trucks, and 80 motorcycles, and the remaining 3,320 are automobiles. The average operation cost for each of these vehicles is $0.70, $1.10, $0.30, and $0.60 per mile, respectively.

Example

There will be a time savings because of the different distances along each of the routes, as well as different speeds that each of the routes will sustain. Route A will allow heavy trucks to average 35 miles/hour, while the other vehicles can maintain 45 miles/hour. For each of Routes B and C, these numbers are 40 miles/hour for heavy trucks and 50 miles/hour for other vehicles. The cost of time for all commercial traffic is valued at \$25/hour and for noncommercial vehicles, \$10/hour. Twenty-five percent of the automobiles and all of the trucks are considered commercial.

Finally, there is a significant safety factor to be included. An excessive number of accidents per year have occurred along the old winding road. Route A will reduce the number of vehicles involved in accidents to 105, and Routes B and C are expected to involve only 75 and 50 vehicles in accidents, respectively. The average cost per vehicle in an accident is estimated to be \$18,000, considering actual property damages, lost time and wages, medical expenses, and other relevant costs.

Example

The Washington planners want to compare the three alternative routes using benefit-cost criteria, specifically the popular benefit-cost ratio.

	A	B	C	D
1		Route A	Route B	Route C
2	General info			
3	Miles	26	22	20.5
4	Hvy truck speed miles/hour	35	40	40
5	Other veh speed miles/hour	45	50	50
6	Accidents/year	105	75	50
7	Cost/accident	$18,000	$18,000	$18,000
8				
9	Government-related info			
10	Discount rate	8.00%	8.00%	8.00%
11	Planning horizon in years	30	30	30
12	First Cost $/mi	$2,000,000	$3,000,000	$4,000,000
13	Resurfacing cost $/mi @10 yrs	$2,000,000	$2,000,000	$2,000,000
14	Resurfacing cost $/mi @20 yrs	$2,000,000	$2,000,000	$2,000,000
15	Maintenance $/mi-yr	$10,000	$12,000	$20,000

Example

17	Public-related info	Vehicles/day	Op cost/mile	Time cost/veh hr
18	Light commercial trucks	350	$0.70	$25
19	Heavy commercial trucks	250	$1.10	$25
20	Motorcycles	80	$0.30	$10
21	Commercial autos	830	$0.60	$25
22	Noncommercial autos	2490	$0.60	$10
23				
24	Total vehicles	4000		
25	Automobiles	3320		
26	% commercial autos	25.00%		
27	% noncommercial autos	75.00%		

Example

Example

B38 fx =($B18*$C18+$B19*$C19+$B20*$C20+$B21*$C21+$B22*$C22)*B3*365

	A	B	C	D	E
29	Summary of Annual Equivalent Government and Public Costs				
30		Route A	Route B	Route C	
31	Government				
32	First cost of highway $/year	$4,619,027	$5,862,611	$7,283,850	=-PMT(D10,D11,D12*D3)
33	Resurfacing costs $/year	$3,130,507	$2,648,890	$2,468,284	
34	Maintenance costs $/year	$260,000	$264,000	$410,000	=D15*D3
35	Total $/year	$8,009,533	$8,775,501	$10,162,134	=SUM(D32:D34)
37	Public				
38	Operating costs	$24,066,640	$20,364,080	$18,975,620	
39	Time costs	$13,335,710	$10,119,808	$9,429,821	
40	Accident costs	$1,890,000	$1,350,000	$900,000	=D7*D6
41	Total $/year	$39,292,350	$31,833,888	$29,305,441	=SUM(D38:D40)
43	Benefit B to A		$7,458,462		
44	Cost B to A		$765,968		
45	B/C Ratio B to A	Prefer B to A	9.74		
47	Benefit C to B			$2,528,447	=C41-D41
48	Cost C to B			$1,386,633	=D35-C35
49	B/C Ratio C to B		Prefer C to B	1.82	=D47/D48
51	=PMT(D10,D11,PV(D10,10,,D13*D3)+PV(D10,20,,D14*D3))				
53	=($B18*$C18+$B19*$C19+$B20*$C20+$B21*$C21+$B22*$C22)*D3*365				
55	=($B18*$D18/D5+$B19*$D19/D4+$B20*$D20/D5+$B21*$D21/D5+$B22*$D22/D5)*D3*365				

Engineering Economic Analysis

Chapter 4: Present Worth

- Section 4.1: Comparing Alternatives
- Section 4.2: Present Worth Calculations
- Section 4.3: Benefit-Cost Analysis
- Section 4.4: Discounted Payback Period
- Section 4.5: Capitalized Worth

Discounted Payback Period

Definition (Discounted Payback Period (DPBP))

The **discounted payback period (DPBP)** is the length of time required for the cumulative present worth to become positive using $i = MARR$.

We do not recommend using DPBP to identify the investment that is to be made from among a set of mutually exclusive investment alternatives. Instead, we recommend it be used as a supplemental tool. If DPBP is used as a stand-alone measure of economic worth, it is difficult to understand how one would decide if the DPBP value obtained was acceptable or not.

DPBP for a Single Alternative

Let's consider one example.

Example (4.16)

An initial \$400,000 investment in new production equipment will yield annual positive cash flows of \$150,000 in year 1. Annual cash flows will decrease by \$25,000 each year thereafter. The new equipment has a useful life of 7 years. MARR is 10% per year. Determine the DPBP of this project.

Example

Example (4.17)

To automatically insert electronic components in printed circuit boards for a cell phone production line, a $500,000 surface mount placement (SMP) machine is being evaluated by a manufacturing engineer. Over the 10-year planning horizon, it is estimated that the SMP machine will produce annual after-tax cost savings of $92,500. The engineer estimates the machine will be worth $50,000 at the end of the 10-year period. Assume that the firm's after-tax MARR is 10%. Suppose management asked the manufacturing engineer to determine how long it takes for the new SMP machine to recover fully its initial cost of $500,000, including the time value of money.

Example

	A	B	C	D	E	F
1	MARR =	10%				
2	EOY	CF	$SV_{geometric}$	$Cum(PW_{geom})$	$SV_{gradient}$	$Cum(PW_{grad})$
3	0	-$500,000	$500,000	n/a	$500,000	n/a
4	1	$92,500	$397,500	-$54,545.45	$455,000	-$2,272.73
5	2	$92,500	$316,013	-$78,295.45	$410,000	-$619.83
6	3	$92,500	$251,230	-$81,213.42	$365,000	$4,263.71
7	4	$92,500	$199,728	-$70,370.67	$320,000	$11,776.86
8	5	$92,500	$158,784	-$50,760.10	$275,000	$21,401.14
9	6	$92,500	$126,233	-$25,883.17	$230,000	$32,690.62
10	7	$92,500	$100,355	$1,826.83	$185,000	$45,262.99
11	8	$92,500	$79,782	$30,699.75	$140,000	$58,791.71
12	9	$92,500	$63,427	$59,608.94	$95,000	$72,998.98
13	10	$92,500	$50,424	$87,813.27	$50,000	$87,649.62

Example

DPBP and Salvage Value

When using DPBP, salvage values should not be ignored. However, determining salvage values for periods of use ranging from 1 year to n years tends to be a very inexact process. The need to know salvage values for all possible periods of use is a limitation of the DPBP method.

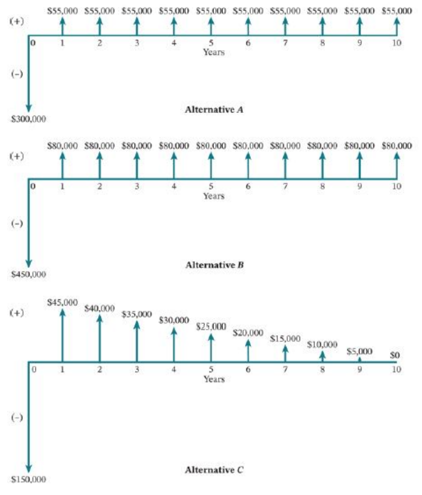
DPBP for Multiple Alternatives

We have emphasized the use of DPBP as a supplemental tool when comparing investment alternatives. We do not recommend it as the sole basis for choosing the preferred alternative. To understand why, consider the following example.

Example (4.18)

Consider the following three alternatives whose cash flow diagrams are given below. Which alternative has the smallest DPBP? Which alternative has the largest present worth?

Example


$55,000 $55,000 $55,000 $55,000 $55,000 $55,000 $55,000 $55,000 $55,000 $55,000
(+)
0 1 2 3 4 5 6 7 8 9 10
Years
(-)
$300,000
Alternative A
$80,000 $80,000 $80,000 $80,000 $80,000 $80,000 $80,000 $80,000 $80,000 $80,000
(+)
0 1 2 3 4 5 6 7 8 9 10
Years
(-)
$450,000
Alternative B
$45,000
$40,000
$35,000
$30,000
$25,000
$20,000
$15,000
$10,000
$5,000
$0
(+)
0 1 2 3 4 5 6 7 8 9 10
Years
(-)
$150,000
Alternative C

Example

Engineering Economic Analysis

Chapter 4: Present Worth

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- Section 4.5: Capitalized Worth

Capitalized Worth

Definition (Capitalized Worth)

The capitalized worth (CW) is the value of all cash flows converted to a single sum equivalent at time zero using $i = MARR$ when the planning horizon is infinitely long.

The capitalized worth method is applicable only if there is reason to believe a series of cash flows will extend indefinitely into the future. Because it does not use the same planning horizon as PW, FW, and AW, there is no reason to assume that the results will be the same as those that would occur when using a finite planning horizon.

The present worth of an infinitely long uniform series of cash flows is

$$CW = \sum_{t=1}^{\infty} A(1+i)^{-t} = \frac{A}{i} \tag{10}$$

Capitalized Worth for a Single Alternative

As with the other measures of economic worth, capitalized worth can be used in the absence of alternatives. If only costs occur, then our objective is to minimize capitalized cost.

Example (4.19)

Every 10 years, the dome of the state capitol building has to be cleaned, sandblasted, and retouched. It costs $750,000 to complete the work. Using a 5% MARR, what is the capitalized cost for refurbishing the capitol dome?

Example

Example

Example (4.20)

A new highway is to be constructed, and asphalt paving will be used. The asphalt will cost \$150 per foot, including the material and the paving operation. The asphalt is expected to last 5 years before requiring resurfacing. It is anticipated that the cost of resurfacing will remain the same per foot. Concrete drainage ditches will be installed on each side of the highway; they will each cost \$7.75 per foot to install. Ditches will have to be replaced every 15 years; the cost of replacing them will also be \$7.75 per foot. Four pipe culverts will be installed every mile; each culvert will cost \$8,000 and will last 10 years; replacement culverts will cost \$10,000 each, indefinitely. Annual maintenance of the highway will cost \$9,000 per mile. Cleaning each culvert will cost \$1,250 per year. Cleaning and maintaining each ditch will cost \$3.75 per foot per year. Determine the capitalized cost (CC) per mile of highway using a MARR of 5%.

Example

Data For Highway Construction Costs

	Initial Cost	Duration	Replacement Cost
Paving	\$150/ft	5 yrs	\$150/ft
Maintenance	\$9,000/mi	1 yr	
Ditches (install)	\$7.75/ft	15 yrs	\$7.75/ft
(cleaning/maintenance)	\$3.75/ft	1 yr	
Culverts (install + replace)	4 × \$8,000/mi	10 yrs	4 × \$10,000/mi
(cleaning)	4 × \$1,250/mi	1 yr	4 × \$1,250/mi

Example

Example

Example

CW for Multiple Alternatives

When multiple alternatives exist, the alternative having the greatest CW over the infinitely long planning horizon is recommended. Again, because capitalized worth alternatives usually involve costs, not revenues, and costs are designated with positive signs, the alternative with the smallest CC is recommended.

Example (4.21)

In a developing country, two alternatives are under consideration for delivering water from a mountainous area to an arid area in the country's southern region. A coated heavy-gauge plastic pipeline can be installed, with pumps spaced appropriately along the pipeline. Alternatively, a canal can be built; however, it will have greater water loss than the pipeline, due to evaporation and poaching along the canal route. To compensate for the water loss, the canal will have a greater carrying capacity than the pipeline.

Example

It is estimated it will cost $125 million to install the pipeline. Major replacements are planned every 15 years at a cost of $10 million. Pumping and other annual operating and maintenance costs are estimated to be $5 million. The canal will cost $200 million to construct; its annual operating and maintenance costs are anticipated to be $1 million. Major upgrades of the canal are anticipated every 10 years, at a cost of $5 million. Based on a five percent MARR and an infinitely long planning horizon, which alternative has the lowest capitalized cost?

	Initial Cost	Annual O&M	Major
Pipeline	$125 million	$5 million	$10 million every 15 years
Canal	$200 million	$1 million	$5 million every 10 years

Example

Example