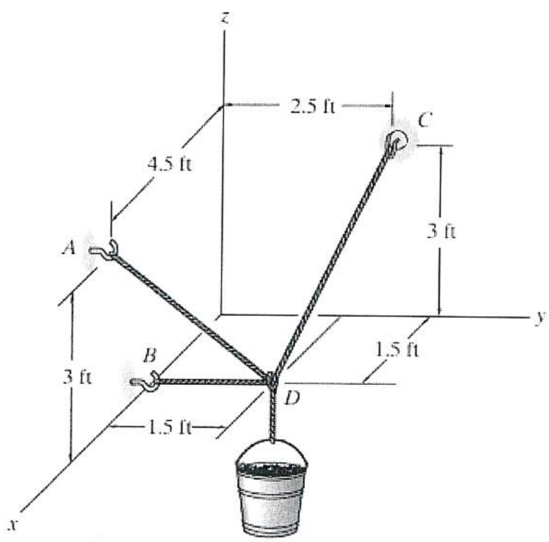
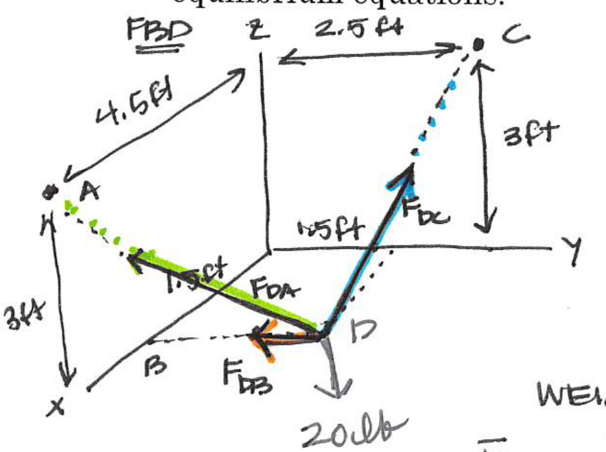
ENSC 2113 – Fall 2023

Homework #3

Problem #1 (10 pts):



If the buck and its contents have a total weight of 20 lb, determine the force in the supporting cables. DA, DB, and DC. Draw appropriate free-body diagram and use equilibrium equations.



COORDINATES

A(4.5, 0, 3) ft

B(1.5, 0, 0) ft

C(0, 2.5, 3) ft

D(1.5, 1.5, 0) ft

CARTESIAN FORCE VECTORS

WEIGHT: $\vec{W} = \{0\hat{i} + 0\hat{j} - 20\hat{k}\}$ lb

$\bar{F}_{BD} = \{0\hat{i} - F_{DB}\hat{j} + 0\hat{k}\}$

$\vec{F}_{DA}$: $r_{DA} = \{3\hat{i} - 1.5\hat{j} + 3\hat{k}\}$ ft $\quad |r_{DA}| = \sqrt{\quad} = 4.5$ ft

$\bar{u}_{DA} = \{\frac{3}{4.5}\hat{i} - \frac{1.5}{4.5}\hat{j} + \frac{3}{4.5}\hat{k}\} = \{\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\}$

$\vec{F}_{DA} = \{\frac{2}{3}F_{DA}\hat{i} - \frac{1}{3}F_{DA}\hat{j} + \frac{2}{3}F_{DA}\hat{k}\}$

$\vec{F}_{DC}$: $\bar{r}_{DC} = \{-1.5\hat{j} + 1\hat{j} + 3\hat{k}\}$ ft $\quad |r_{DC}| = \sqrt{\quad} = 3.5$ ft

$\bar{u}_{DC} = \{-\frac{1.5}{3.5}F_{DC}\hat{i} + \frac{1}{3.5}F_{DC}\hat{j} + \frac{3}{3.5}F_{DC}\hat{k}\}$

PROBLEM #1 (con't)

EQUILIBRIUM

$\Sigma F_x \swarrow = 0 \qquad 0 + 0 + \frac{2}{3} F_{DA} - \frac{1.5}{3.5} F_{DC} = 0$

$$F_{DA} = \frac{3}{2} \frac{1.5}{3.5} F_{DC} = \frac{4.5}{7} F_{DC}$$

$\Sigma F_y \rightarrow = 0 \qquad 0 - F_{DB} - \frac{1}{3} F_{DA} + \frac{1}{3.5} F_{DC} = 0$

$$F_{DB} = -\frac{1}{3} F_{DA} + \frac{1}{3.5} F_{DC}$$

$$= -\frac{1}{3} \frac{4.5}{7} F_{DC} + \frac{1}{3.5} F_{DC}$$

$$F_{DB} = 0.0714 F_{DC}$$

$\Sigma F_z \uparrow = 0 \qquad -20\,lb + 0 + \frac{2}{3} F_{DA} + \frac{3}{3.5} F_{DC} = 0$

$$\frac{3}{3.5} F_{DC} = 20\,lb - \frac{2}{3} F_{DA} = 20\,lb - \frac{2}{3} \frac{4.5}{7} F_{DC}$$

$$\frac{3}{3.5} F_{DC} + \frac{2}{3} \frac{4.5}{7} F_{DC} = 20\,lb$$

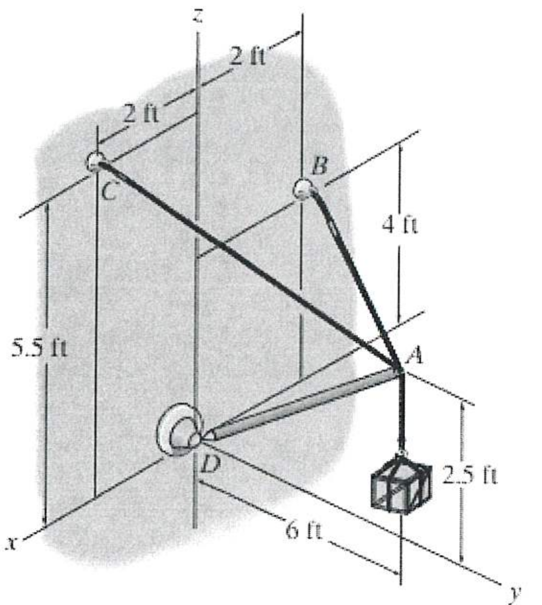
$$1.2857 F_{DC} = 20\,lb$$

$$\boxed{F_{DC} = 15.56\,lb}$$

$$\boxed{F_{DB} = 1.11\,lb}$$

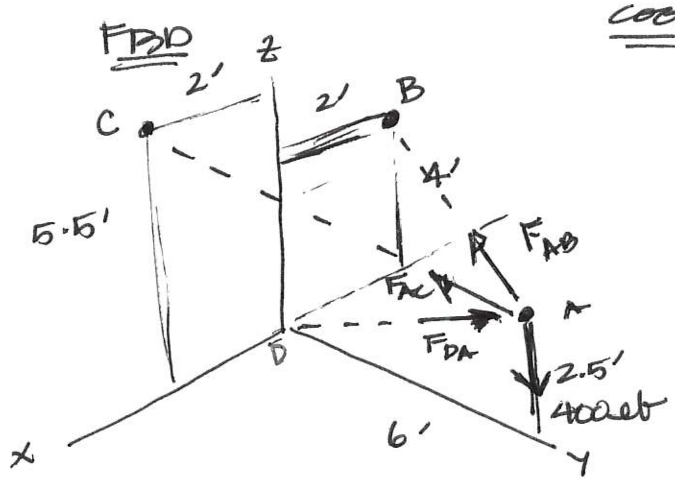
$$\boxed{F_{DA} = 10\,lb}$$

Problem #2 (10 pts):



Determine the tension developed in cables AB and AC and the force developed along strut AD for equilibrium of the 400-lb crate. Draw the appropriate free-body diagram and use equilibrium equations.

FBD



COORDINATES

A(0, 6, 2.5) ft

B(−2, 0, 4) ft

C(2, 0, 5.5) ft

D(0, 0, 0) ft

CARTESIAN VECTORS

$Wt = \{0\hat{i} + 0\hat{j} - 400\hat{k}\}$ lb

$\vec{F}_{DA}$: $\bar{r}_{DA} = \{0\hat{i} + 6\hat{j} + 2.5\hat{k}\}$ ft

$|r_{DA}| = \sqrt{} = 6.5$ ft

$\bar{F}_{DA} = \{0\hat{i} + \frac{6}{6.5} F_{DA}\hat{j} + \frac{2.5}{6.5} F_{DA}\hat{k}\}$

$\vec{F}_{AC}$: $\bar{r}_{AC} = \{2\hat{i} - 6\hat{j} + 3\hat{k}\}$ ft $|\bar{r}_{AC}| = \sqrt{} = 7$ ft

$\bar{F}_{AC} = \{\frac{2}{7} F_{AC}\hat{i} - \frac{6}{7} F_{AC}\hat{j} + \frac{3}{7} F_{AC}\}$

$\vec{F}_{AB}$: $\bar{r}_{AB} = \{-2\hat{i} - 6\hat{j} + 1.5\hat{k}\}$ ft $|\bar{r}_{AB}| = 6.5$ ft

$\bar{F}_{AB} = \{-\frac{2}{6.5} F_{AB}\hat{i} - \frac{6}{6.5} F_{AB}\hat{j} + \frac{1.5}{6.5} F_{AB}\hat{k}\}$

PROBLEM #2 (con't)

EQUILIBRIUM

$\Sigma F_x \swarrow = 0 \qquad 0 + 0 + \frac{2}{7} F_{AC} - \frac{2}{6.5} F_{AB} = 0$

$$F_{AC} = \frac{7}{2}\,\frac{2}{6.5} F_{AB} = \frac{7}{6.5} F_{AB}$$

$\Sigma F_y \rightarrow = 0 \qquad 0 + \frac{6}{6.5} F_{DA} - \frac{6}{7} F_{AC} - \frac{6}{6.5} F_{AB} = 0$

$$\frac{6}{6.5} F_{DA} = \frac{6}{7} F_{AC} + \frac{6}{6.5} F_{AB}$$

$$= \frac{6}{7}\left(\frac{7}{6.5} F_{AB}\right) + \frac{6}{6.5} F_{AB}$$

$$F_{DA} = \frac{6.5}{6}\left[\frac{6}{6.5} F_{AB} + \frac{6}{6.5} F_{AB}\right]$$

$$F_{DA} = 2 F_{AB}$$

$\Sigma F_z \uparrow = 0 \qquad -400 + \frac{2.5}{6.5} F_{DA} + \frac{3}{7} F_{AC} + \frac{1.5}{6.5} F_{AB} = 0$

$$\frac{2.5}{6.5}(2F_{AB}) + \frac{3}{7}\left(\frac{7}{6.5} F_{AB}\right) + \frac{1.5}{6.5} F_{AB} = 400\text{ lb}$$

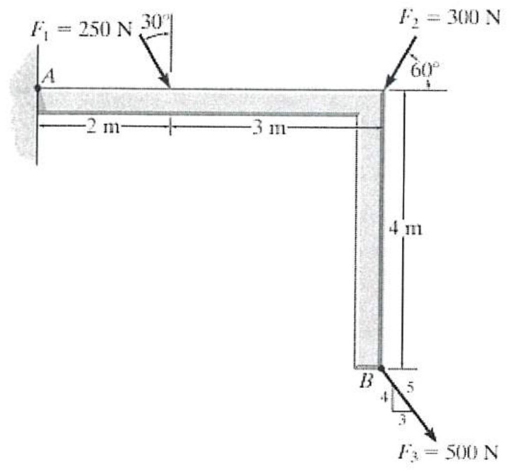
$$1.462 F_{AB} = 400\text{ lb}$$

$$\boxed{F_{AB} = 273.7\text{ lb}}$$

$$\boxed{F_{DA} = 547.4\text{ lb}}$$

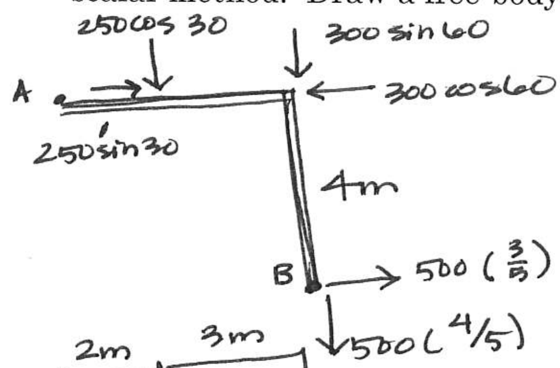
$$\boxed{F_{AC} = 294.7\text{ lb}}$$

Problem #3 (10 pts):



a) Determine the moment of each of the three forces about point A using the scalar method. Draw a free-body diagram and use right-hand rule.

b) Determine the moment of each of the three forces about point B using the scalar method. Draw a free-body diagram and use right-hand rule.



(a) $\Sigma M_A \circlearrowleft = -250\cos 30(2) - 300\sin 60(5) + 500(\tfrac{3}{5})(4) - 500(\tfrac{4}{5})(5)$

$= -433\,\text{Nm} - 1299\,\text{Nm} + 1200\,\text{Nm} - 2000\,\text{Nm}$

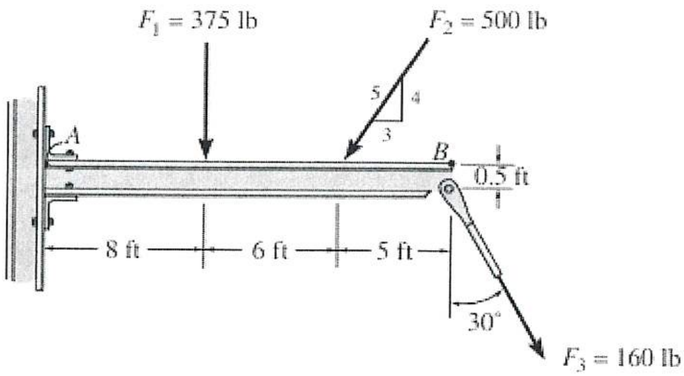
$= \boxed{-2532.1\,\text{Nm} = \boxed{2532.1\,\text{Nm} \circlearrowright}}$

(b) $\Sigma M_B \circlearrowleft = 250\cos 30(3) - 250\sin 30(4) + 300\cos 60(4)$

$= 649.5\,\text{Nm} - 500\,\text{Nm} + 600\,\text{Nm}$

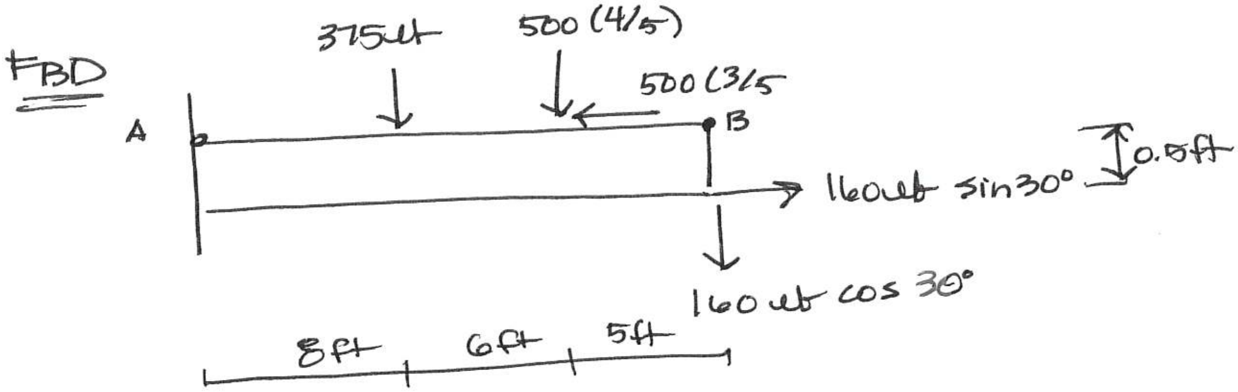
$= \boxed{749.5\,\text{Nm} = \boxed{749.5\,\text{Nm} \circlearrowleft}}$

Problem #4 (10 pts):



a) Determine the moment of each of the three forces about point A using the scalar method. Draw a free-body diagram and use right-hand rule.
b) Determine the moment of each of the three forces about point B using the scalar method. Draw a free-body diagram and use right-hand rule.

FBD

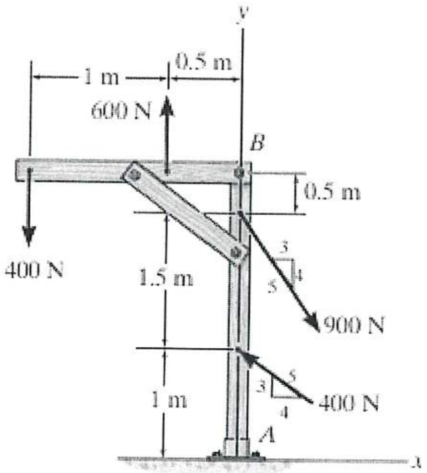


$$\Sigma M_A \circlearrowleft = -375\,lb(8\,ft) - 500\,lb\left(\tfrac{4}{5}\right)(14') + 1600 \sin 30(0.5)$$
$$-160 \cos 30(19')$$
$$= (-3000 - 5600 + 400 - 2632.7)\,lbft$$
$$= -10{,}832.7\,lbf = -10.8\,k\cdot ft$$
$$= \boxed{10.8\,k\cdot ft \circlearrowright}$$

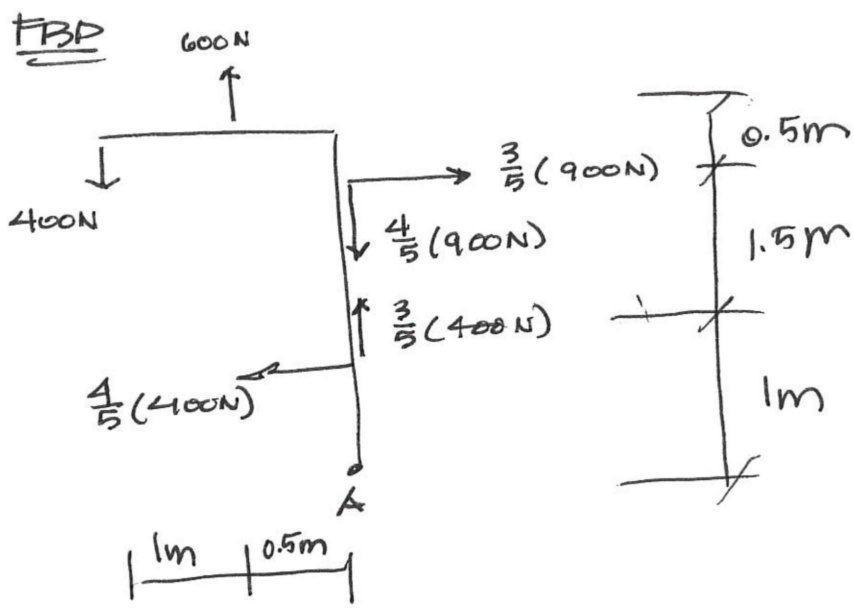
* 1 KIP = 1,000 lb
1 K = 1,000 lb

$$\Sigma M_B \circlearrowleft = 375(11) + 500\left(\tfrac{4}{5}\right)(5) + 160 \sin 30\,(0.5\,ft)$$
$$= (4125 + 2000 + 40)\,lbft$$
$$= \boxed{6{,}165\,lbft \circlearrowleft}$$

Problem #5 (10 pts):



Using scalar analysis, determine the moment created by the forces about point A. Draw the appropriate free-body diagram and use right hand rule.



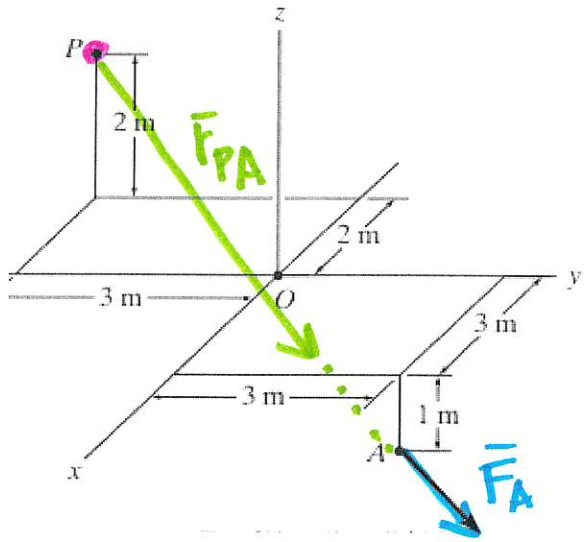
$$\Sigma M_A \circlearrowright = 400N(1.5m) - 600N(0.5m) - \tfrac{3}{5}(900N)(2.5m) + \tfrac{4}{5}(400N)(1m)$$

$$= 600Nm - 300Nm - 1350\,Nm + 320Nm$$

$$= -730\ Nm$$

$$= \boxed{730\ Nm \circlearrowright}$$

Problem #6 (10 pts):



$\mathbf{F}_A = \{3\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}\}$ kN

Using vector analysis, determine the moment of $\mathbf{F}_A$ about point P. Express the moment as a magnitude.

$$\bar{M} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \text{ (point)} \\ \text{POSITION FROM POINT} \rightarrow \text{FORCE} & & \\ \text{FORCE} & & \end{vmatrix}$$

POSITION VECTOR FROM POINT P TO FORCE @ A $\bar{r}_{PA}$

COORDINATES: $P(-2, -3, 2)$ m $\quad A(3, 3, -1)$ m

$$\bar{r}_{PA} = \{(3 - -2)\bar{i} + (3 - -3)\bar{j} + (-1-2)\bar{k}\} \text{ m}$$

tip tail $\quad \bar{r}_{PA} = \{5\bar{i} + 6\bar{j} - 3\bar{k}\}$ m

$$\bar{M}_P = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 5 & 6 & -3 \\ 3 & 6 & -4 \end{vmatrix} \begin{matrix} \\ \text{m} \\ \text{kN} \end{matrix}$$

$$= \{[(6)(-4) - (-3)(6)]\bar{i} - [(5)(-4) - (-3)(3)]\bar{j} + [5(6) - 6(3)]\bar{k}\} \text{ kNm}$$

$$= \{[-24 - (-18)]\bar{i} - [-20 - (-9)]\bar{j} + [30 - (18)]\bar{k}\} \text{ kNm}$$

CARTESIAN VECTOR

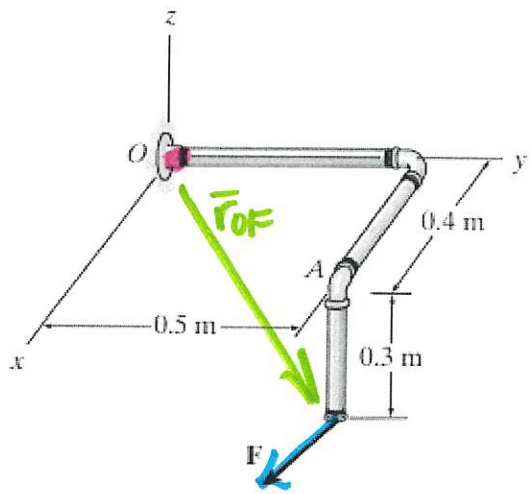
$$= \boxed{\{-6\bar{i} + 11\bar{j} + 12\bar{k}\} \text{ kNm}}$$

MAGNITUDE =

$$M_P = \sqrt{(-6)^2 + (11)^2 + (12^2)}$$

$$\boxed{M_P = 17.3 \text{ kN m}}$$

Problem #7 (10 pts):



Determine the moment of force F about point O using vector analysis. The force has a moment of 800 N and coordinate direction angles of $\alpha = 60^o$, $\beta = 120^o$, $\gamma = 45^o$. Express your result as a Cartesian vector.

$$\bar{M} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \quad \text{(point O)} \\ \text{POSITION } \bar{r}_{OF} & & \text{(point O} \rightarrow \text{FORCE)} \\ \text{FORCE VECTOR} & & \end{vmatrix}$$

POSITION $\vec{r}_{OF}$

COORDINATES : O(0, 0, 0) m $\qquad$ F(0.4, 0.5, −0.3) m

$$\bar{r}_{OF} = \{0.4\bar{i} + 0.5\bar{j} - 0.3\bar{k}\} \text{m}$$

FORCE VECTOR $\vec{F}$

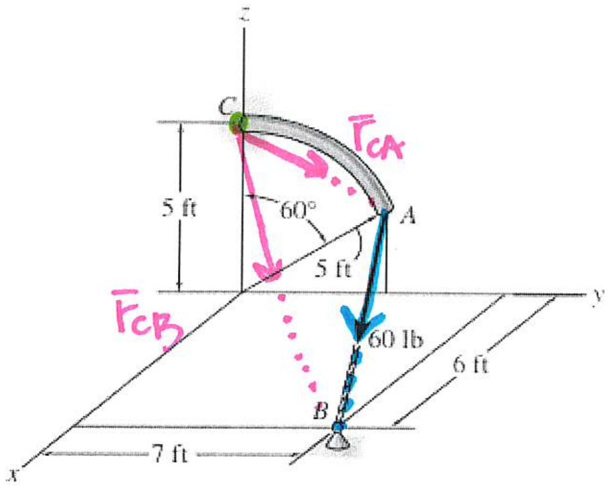
$$\bar{F} = \{800\cos 60\bar{i} + 800\cos 120\bar{j} + 800\cos 45\bar{k}\} \text{N}$$
$$= \{400\bar{i} - 400\bar{j} + 565.69\bar{k}\} \text{N}$$

$$\bar{M} = \begin{vmatrix} \bar{i} & \bar{j} & k \\ 0.4 & 0.5 & -0.3 \\ 400 & -400 & 565.69 \end{vmatrix} \begin{matrix} \\ m \\ N \end{matrix}$$

$$= \{[(0.5)(565.69) - (-0.3)(-400)]\bar{i} - [(0.4)(565.69) - (-0.3)(400)]\bar{j} + [(0.4)(-400) - 0.5(400)]\bar{k}\} \text{Nm}$$

$$= \boxed{\{+162.8\bar{i} - 346.3\bar{j} - 360\bar{k}\} \text{Nm}}$$

Problem #8 (15 pts):



The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C using vector analysis. Express your answer as a magnitude.

POSITION VECTOR FROM $C \longrightarrow A$ OR $C \longrightarrow B$

$C(0,0,5)$ ft $\quad A(0, 5\cos 30, 5\sin 30)$ ft $\quad B(6,7,0)$ ft

tip-tail

$$\bar{r}_{CA} = \{0\bar{i} + (5\cos 30)\bar{j} + (5\sin 30 - 5)\bar{k}\}\text{ ft} = \{0\bar{i} + 4.33\bar{j} - 2.5\bar{k}\}\text{ ft}$$

$$\bar{r}_{BA} = \{6\bar{i} + 7\bar{j} - 5\bar{k}\}\text{ ft}$$

FORCE $\quad \bar{F}_{AB} = F_{AB}\bar{u}_{AB} = F_{AB}\dfrac{\bar{r}_{AB}}{|\bar{r}_{AB}|}$

$B(6,7,0)$ ft $\quad A(0, 5\cos 30, 5\sin 30)$ ft

tip-tail

$$\bar{r}_{AB} = \{6\bar{i} + (7 - 5\cos 30)\bar{j} + (0 - 5\sin 30)\bar{k}\}\text{ ft}$$

$$= \{6\bar{i} + 2.67\bar{j} - 2.5\bar{k}\}\text{ ft} \qquad |\bar{r}_{AB}| = 7.03\text{ ft}$$

$$\vec{F} = 60\text{ lb}\left\{\frac{6}{7.03}\bar{i} + \frac{2.67}{7.03}\bar{j} - \frac{2.5}{7.03}\bar{k}\right\}$$

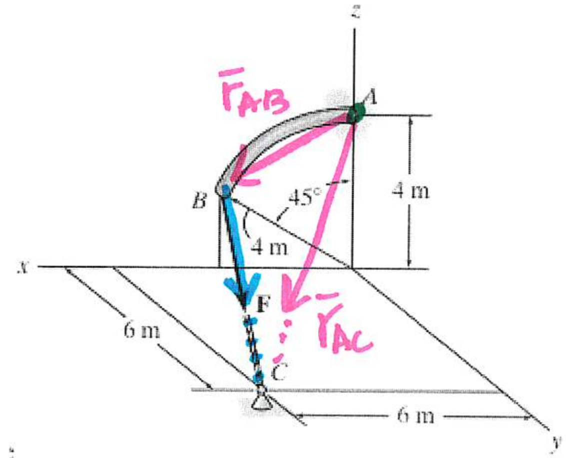
$$= \{51.23\bar{i} + 22.8\bar{j} - 21.35\bar{k}\}\text{ lb}$$

$$\bar{M} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 6 & 7 & -5 \\ 51.23 & 22.8 & -21.35 \end{vmatrix} = \{[\overset{-149.45}{7(-21.35)} - \overset{-114}{(-5)(22.8)}]\bar{i} - [\overset{-128.1}{6(-21.35)} - \overset{-256.15}{(-5)(51.23)}]\bar{j} + [\overset{136.8}{6(22.8)} - \overset{358.61}{7(51.23)}]\bar{k}\}\text{ lbft}$$

$$\bar{M} = \{-35.45\bar{i} - 128.05\bar{j} - 221.81\bar{k}\}\text{ lbft}$$

$$M = \sqrt{\quad} = \boxed{258.6\text{ lb ft}}$$

Problem #9 (15 pts):



The curved rod has a radius of 4 m. If a force of 600 N acts at its end as shown, determine the moment of this force about point A using vector analysis. Express your answer as a Cartesian Vector.

COORDINATES

A $(0, 0, 4)$ m

B $(4\sin 45, 0, 4\cos 45)$ m $= (2.83, 0, 2.83)$ m

C $(6, 6, 0)$ m

POSITION VECTOR

$\bar{r}_{AB} = \{2.83\hat{i} + 0\hat{j} - 1.17\hat{k}\}$ m

$\bar{r}_{AC} = \{6\hat{i} + 6\hat{j} - 4\hat{k}\}$ m

FORCE VECTOR

$\bar{F}_{BC} = F_{BC}\bar{u}_{BC} = F_{BC}\dfrac{\bar{r}_{BC}}{|\bar{r}_{BC}|}$

$\bar{r}_{BC} = \{3.17\hat{i} + 6\hat{j} - 2.83\hat{k}\}$ m

$|\bar{r}_{BC}| = \sqrt{} = 7.35$ m

$\bar{u}_{BC} = \left\{\frac{3.17}{7.35}\hat{i} + \frac{6}{7.35}\hat{j} - \frac{2.83}{7.35}\hat{k}\right\}$

$\bar{F}_{BC} = \{258.7\hat{i} + 489.6\hat{j} - 230.9\hat{k}\}$ lb

$$\bar{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.83 & 0 & -1.17 \\ 258.7 & 489.6 & -230.9 \end{vmatrix} = \{[0 - (-1.17)(489.6)]\hat{i} - [(2.83)(-230.9) - (-1.17)(258.7)]\hat{j} + [(2.83)(489.6) - 0]\hat{k}\} \text{ lbft}$$

$$= \boxed{\{572.8\hat{i} + 350.8\hat{j} + 1385.6\hat{k}\} \text{ lbft}}$$