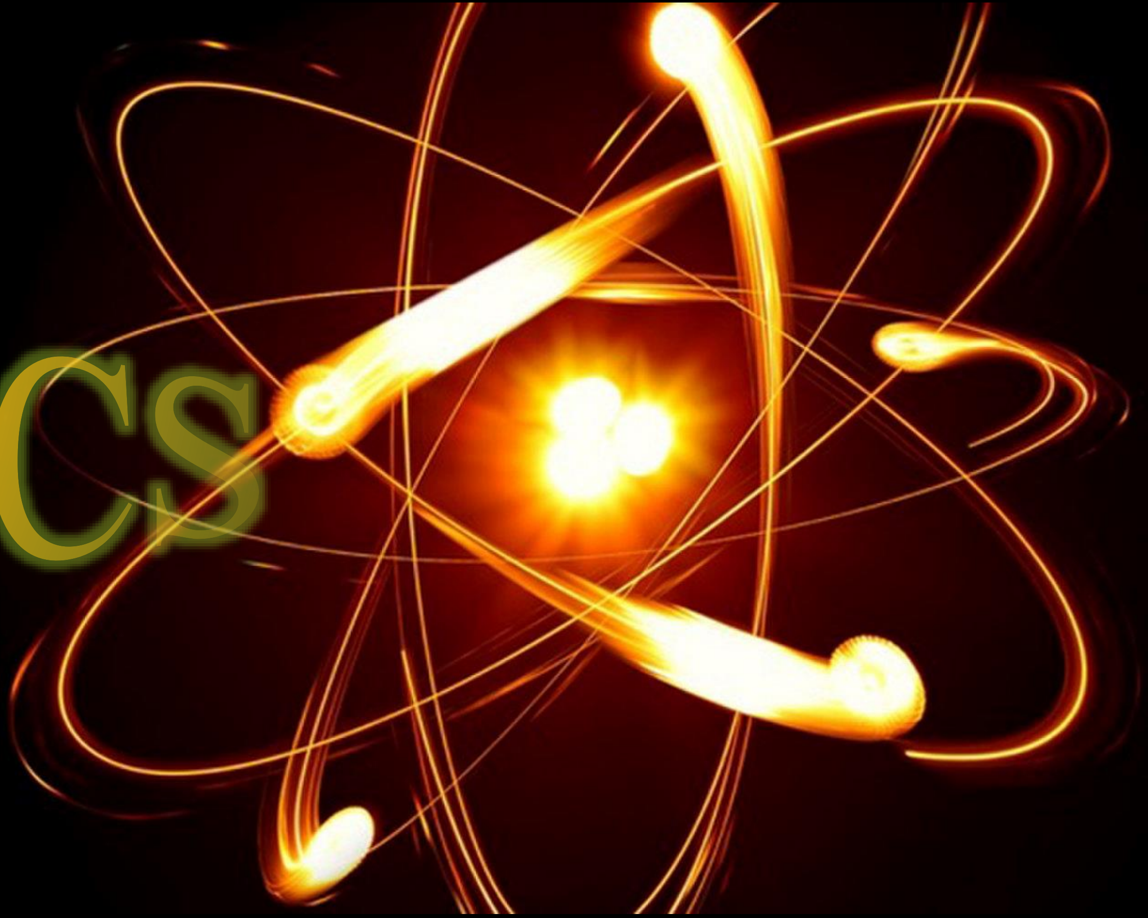
PHYSICS



Physics 1: **Mechanics and Waves**

Week 9 – Work and Energy

2023.4

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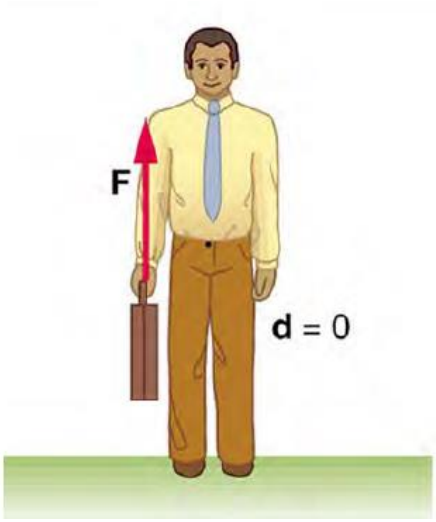
Work

A **force** exerted on an object

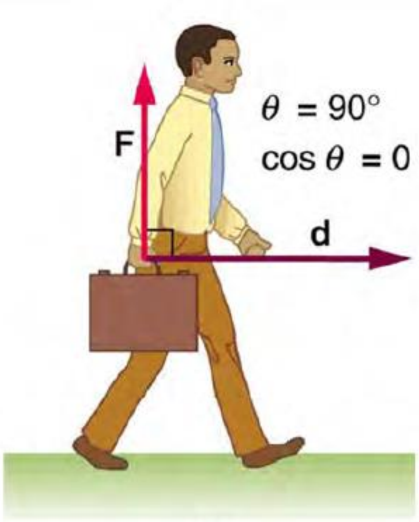
The object **moving** in the **direction of the force**

There is work done by the force in that direction

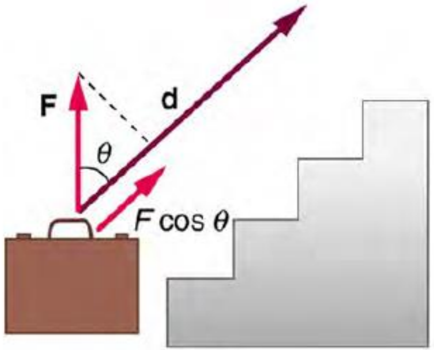
Work



no work!



no work!
(on the briefcase!)



work!

Work

Work is the ***product*** of the

component of the force

in the direction of motion

times the distance

through which the force acts

$$\boldsymbol{W} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{d}}$$

$$W = Fd\cos\theta$$

James Prescott Joule
(1818-1889)

Units : N· m = J (Joules)

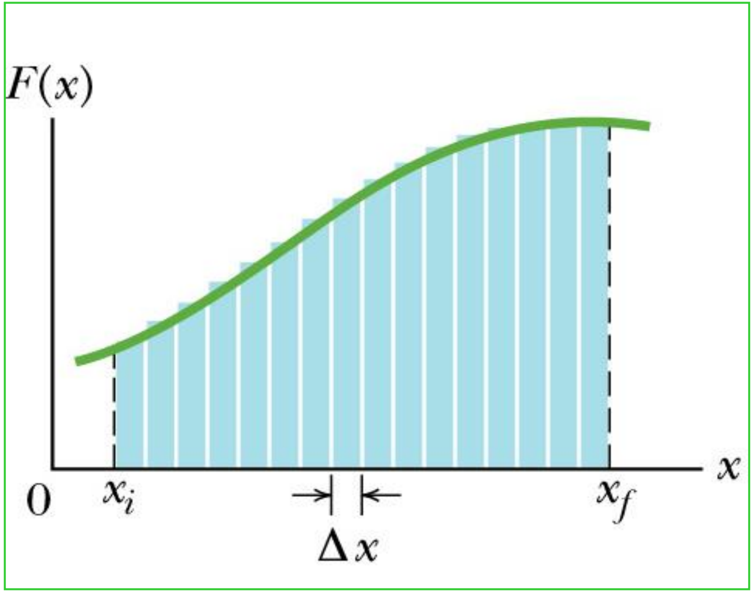
Work by variable force

Define the differential work done by any force

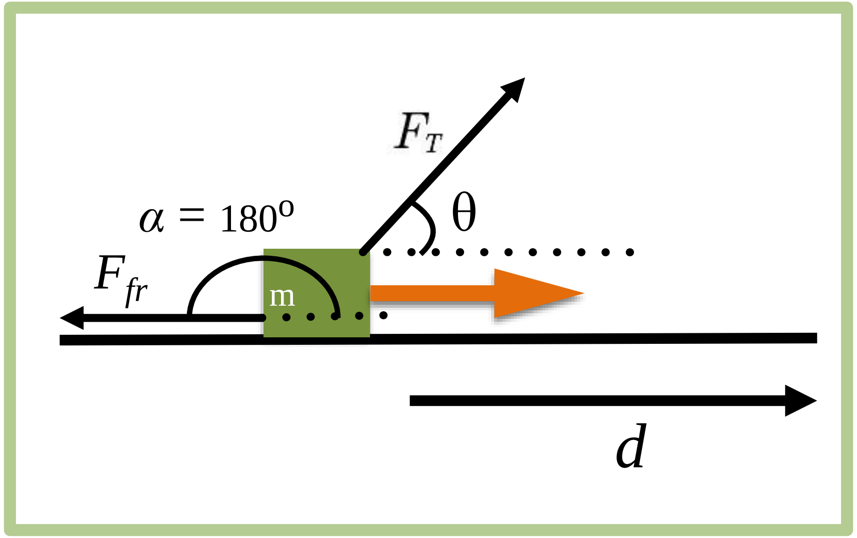
$$\mathbf{d}W = \vec{F} \cdot \mathbf{d}\vec{r}$$

The work done by a particular force is

$$\mathbf{W} = \int \mathbf{d}W = \int_i^f \vec{F} \cdot \mathbf{d}\vec{r}$$



Work



Work of F_T

$$W = F_T d \cos\theta$$

Any other work?

-- friction force does work too

$$\boxed{\cos 180^o = -1}$$

$$W = F_{fr} d \cos\alpha$$

$$\boxed{F_{fr} = \mu_k N}$$

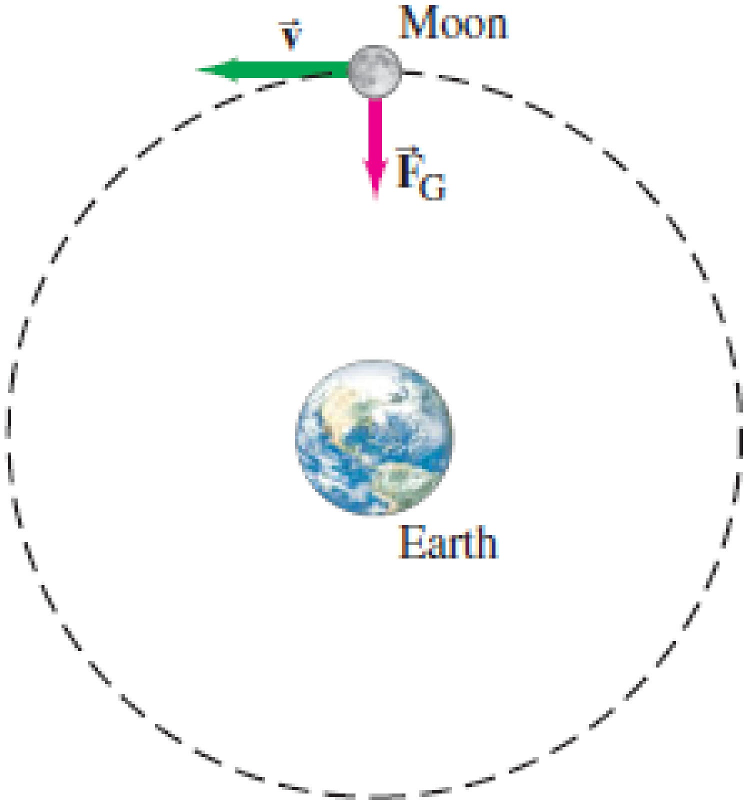
$$W = -\mu_k N d$$

Negative work!

Exercise 1

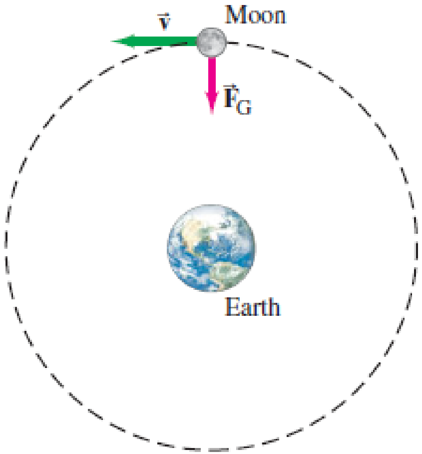
Does the Earth do work on the moon?

Why people in a satellite weightlessness?



Exercise 1

Does the Earth do work on the moon?



RESPONSE The gravitational force $\vec{F}_G$ exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{F}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero $(\cos 90° = 0)$. This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity.

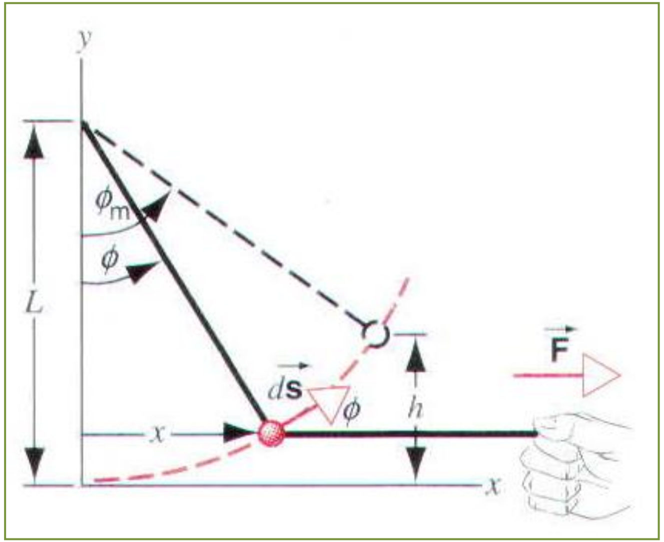
Example 1

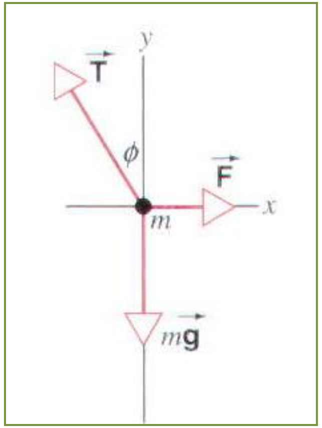
A spring hangs vertically in its *relaxed* state. A block of mass *m* is attached to the spring, but the block is *held in place* so that the spring at first does not stretch. Now the hand holding block is slowly lowered, so that the block descends at constant speed until it reaches the point at which it hangs at equilibrium with the hand removed. At this point the spring is measured to have *stretched a distance d* from its previous relaxed length. Find the *work done* on the block in this process by *(a) gravity, (b) the spring, (c) the hand.*

Example 2

A small object of mass *m* is suspended from a string of length *L*. the object is pulled sideways by a force *F* that is always horizontal, until the string finally makes an angle ϕ_m with the vertical.

The displacement is accomplished at a small constant speed. Find the work done by all forces that act on the object.

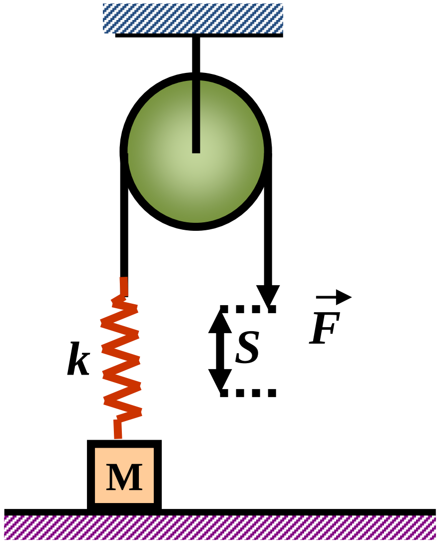




Example 3

As shown in figure, M = 2kg , k = 200N·m^{-1} , S = 0.2m , $g \approx$ 10m·s^{-2} .

A student pull the string very slowly, ignore the friction between the pulley and the string , as well as the masses of the pulley and string . At initial time, the spring is in the relaxed state. Find the work done by the force $\vec{F}$.



Work and Energy

What is Energy ?

Energy : the ability (capacity) to do work.

Positive work → **Transferring energy to object**

Negative work → **Removing energy from object**

The SI unit of energy is also Joule (N·m)

Kinetic Energy

Is a moving objects have an energy? What is it?

Applying a constant net force over a distance *d*

(acceleration = *a*) (initial speed = 0)

What is the final speed of the object?

$$v^2 = 2ad = \frac{2Fd}{m}$$

$$Fd = \frac{mv^2}{2} \equiv E_k$$

Kinetic Energy

KE or E_K $= \frac{1}{2}mv^2$

Kinetic Energy

Is a moving objects have an energy? What is it?

Applying a constant net force over a distance *d*

(acceleration = *a*) (initial speed = 0)

What is the final speed of the object?

$$Fd = \frac{mv^2}{2} \equiv E_k$$

Work-Energy Theorem $\boldsymbol{W_{total} = \Delta E_k}$

Work and kinetic energy

For a single particle:

$$\vec{F}_{\text{total}} = m\vec{a} = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}, \quad \vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

$$W_{\text{total}} = \int \vec{F}_{\text{total}} \cdot \mathrm{d}\vec{r} = m\int_i^f \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \cdot \mathrm{d}\vec{r} = m\int_i^f \vec{v} \cdot \mathrm{d}\vec{v}$$

$$\because \quad \frac{\mathrm{d}(\vec{v} \cdot \vec{v})}{\mathrm{d}t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \cdot \vec{v} + \vec{v} \cdot \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = 2\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} \cdot \vec{v}$$

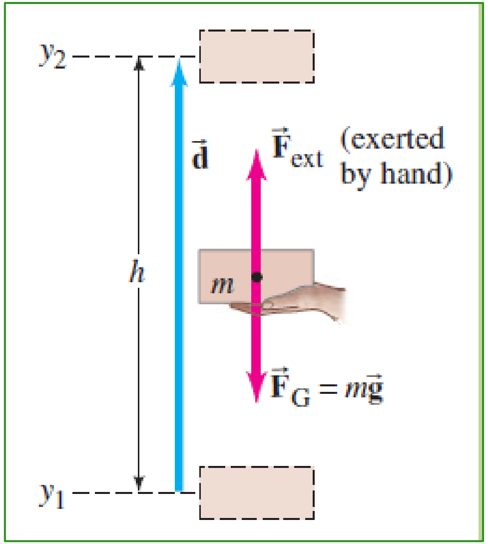
$$\vec{v} \cdot \mathrm{d}\vec{v} = \frac{1}{2}\mathrm{d}(\vec{v} \cdot \vec{v})$$

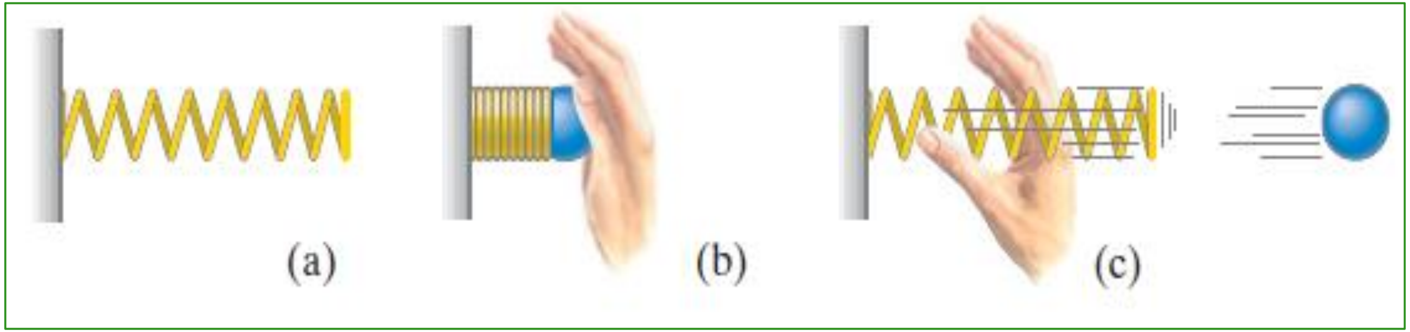
$$\therefore W_{\text{total}} = m\int_i^f \frac{1}{2}\mathrm{d}(\vec{v} \cdot \vec{v}) = \frac{1}{2}m\int_{v_i}^{v_f} \mathrm{d}v^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Potential Energy

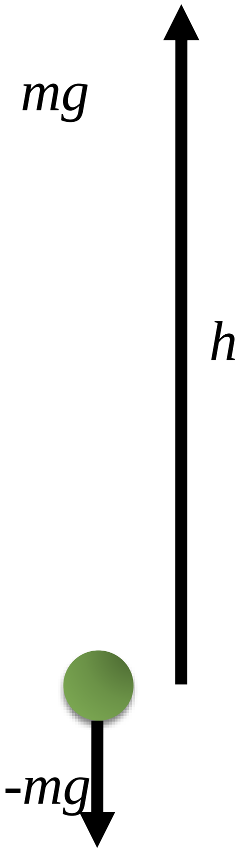
1. Gravitational Potential Energy

2. Potential Energy of Elastic Spring





Gravitational Potential Energy - **work done by gravity**



If the ball goes up: $W_{gr} = -mgh$

Gravity 'removes' kinetic energy

transforms it into potential energy E_p

$\boxed{\Delta E_P = mgh}$ $\Delta E_k = -mgh$

If the ball falls down: it gains kinetic energy

$\Delta E_k = mgh$ $\Delta E_p = -mgh$

In either case $\boxed{\Delta E_k + \Delta E_p} = 0$ **Conservation of mechanical energy**

Conservation of Mechanical Energy

$$\Delta E_k + \Delta E_p = 0$$

$$\Delta\boxed{(E_k + E_p)} = 0$$

mechanical energy

Conservation of mechanical energy

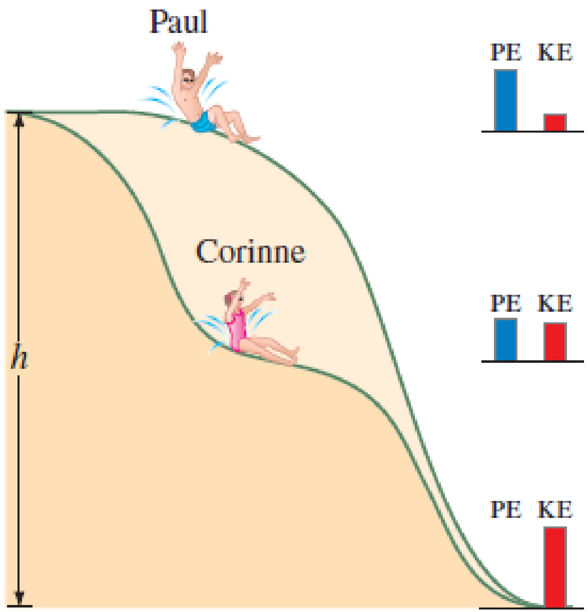
Under what condition??

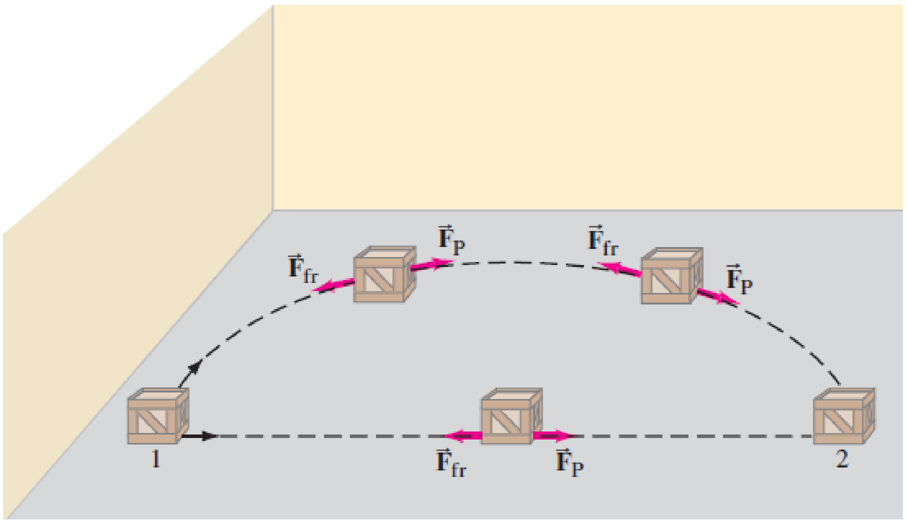
The force exerted on the object should be conservative force

The total work done by this force in moving a particle between two points is independent of the path.

if a particle travels in **any closed loop**, the net work done by a *conservative force* is **zero.**

Conservative and Nonconservative Forces



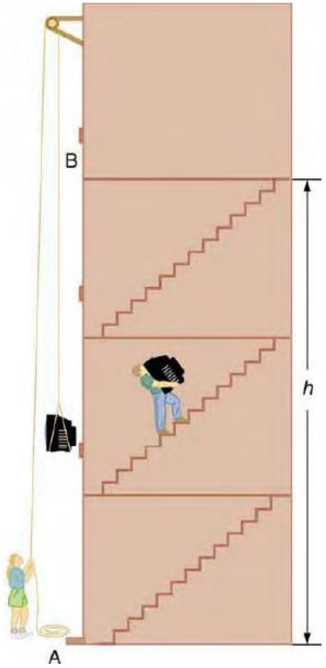


Whether or not work done depends on the path

Conservative and Nonconservative Forces

The work done by gravity / change in potential energy does not depend on the path taken

$$\Delta E_P = mgh$$



The gravitational force is a 'conservative force'

Friction is a 'nonconservative force'

No nonconservative force does work

⇓

Conservation of mechanical energy

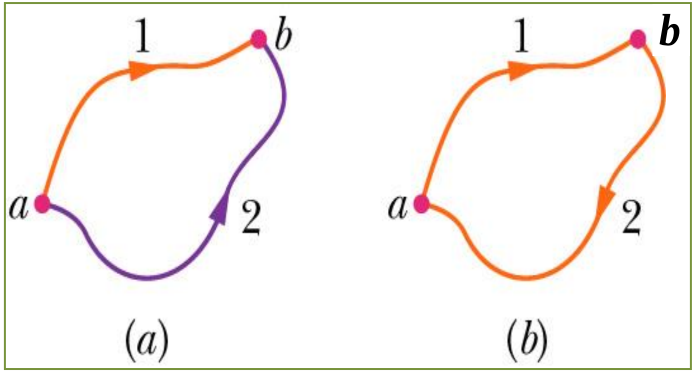
Conservative force

If a work done by the force on any system as it moves between two points is

- ***independent of the path*** following by the system between the two points;
- or if the work done by the force on any system ***around any closed path is zero.***

$$\int_{a(1)}^{b} \vec{F} \cdot \mathrm{d}\vec{r} = \int_{a(2)}^{b} \vec{F} \cdot \mathrm{d}\vec{r}$$

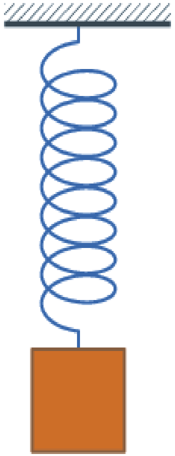
$$\boxed{\oint \vec{F} \cdot \mathrm{d}\vec{r} = 0}$$



Potential Energy of Elastic Spring

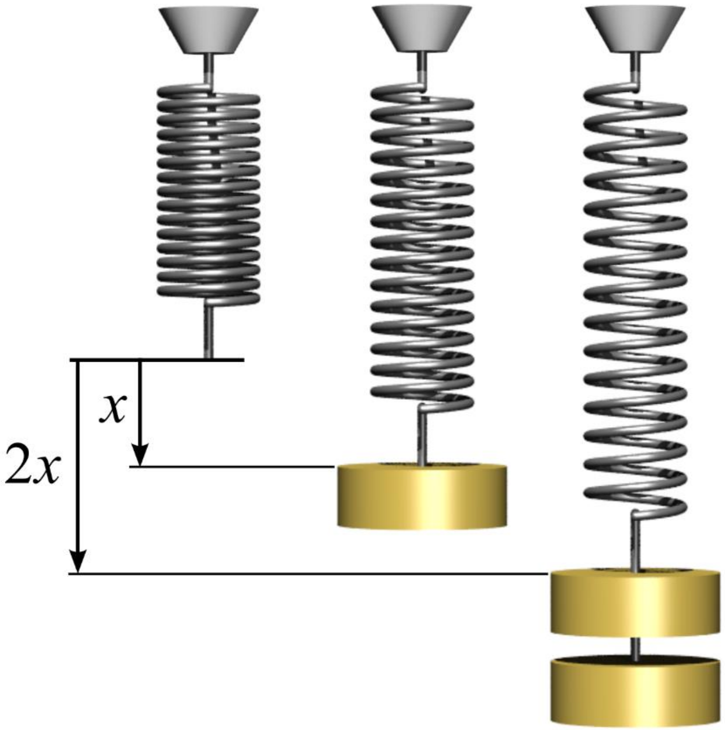
HOOKE’S LAW

$$F_s = -kx$$



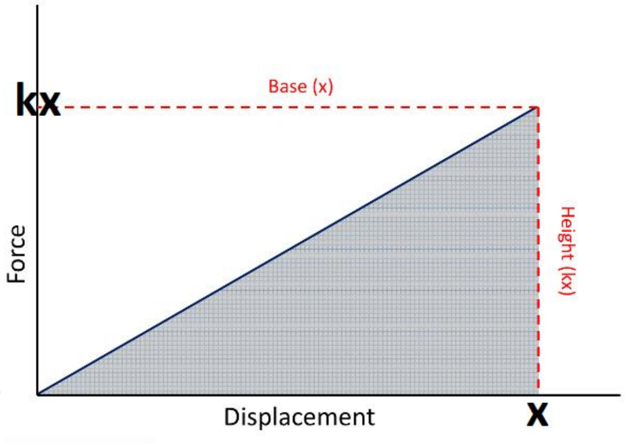
k is the spring stiffness constant

Energy : calculate method



Potential Energy of Elastic Spring

Force exerted by a spring: $F_s = - kx$



Work if compressed by a distance d - the shadow area

$$W = \frac{1}{2}kx^2$$

$$E_s = \frac{1}{2}kx^2$$

Calculus thinking

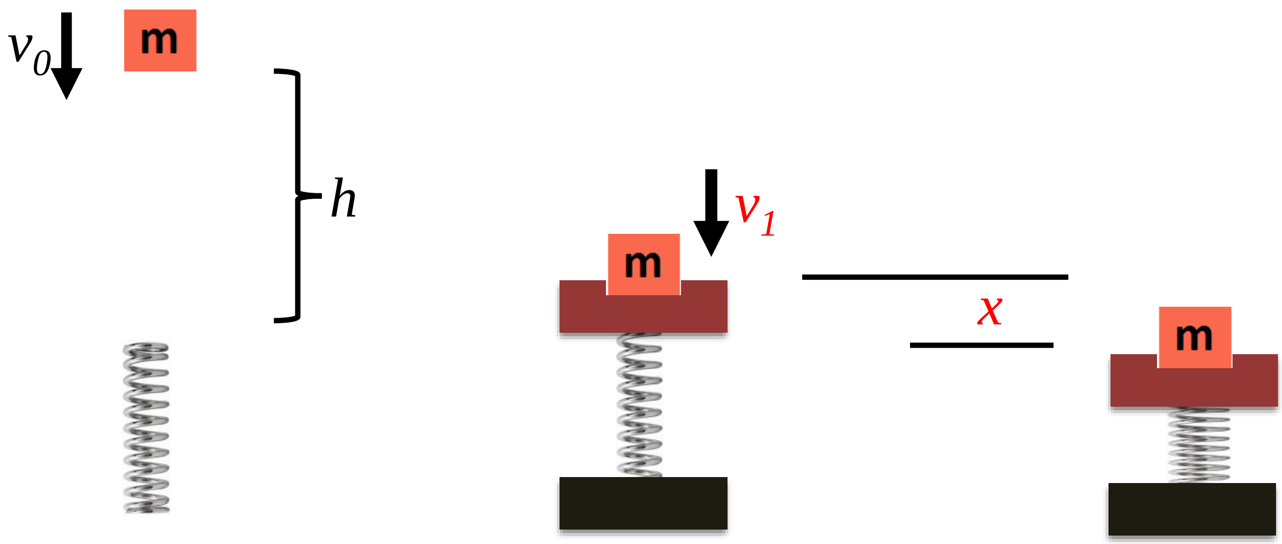
If the spring is released, the object connected on the spring gains kinetic energy:

$$\boldsymbol{E_s = \frac{1}{2}kx^2}$$

Potential energy of a spring
(spring energy)

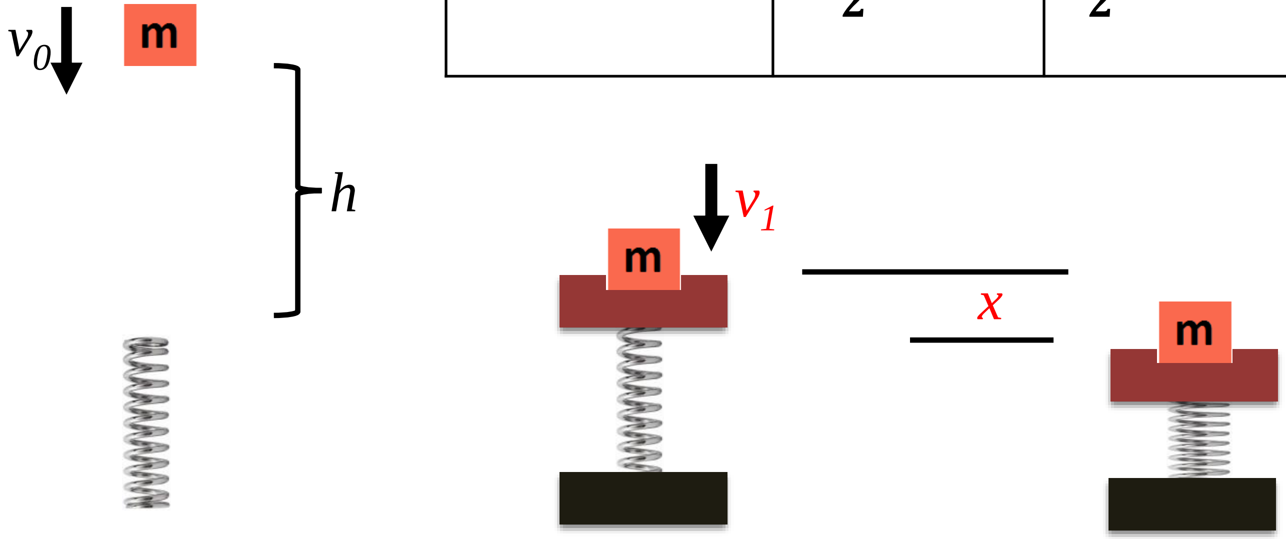
Example

E_k			
E_p			
E_s			
Total energy			



Example

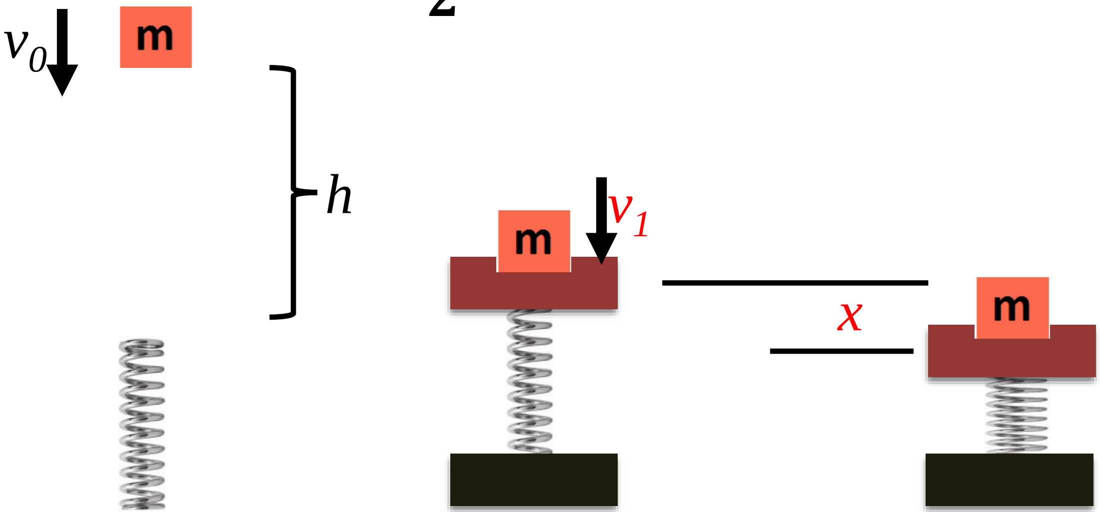
E_k	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv_1^2$	0
E_p	0	$-mgh$	$-mg(h+x)$
E_s	0	0	$\frac{1}{2}kx^2$
Total energy	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv_1^2 - mgh$	$\frac{1}{2}kx^2 - mg(h+x)$



Example

No friction or air resistance → **energy is conserved**

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 - mgh$$
$$= \frac{1}{2}kx^2 - mg(h+x)$$



Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth's oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $\left(10^{-2}\text{ g at }0.5\text{ m/s}\right)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Energy

Kinetic Energy $$E_k = \frac{1}{2}mv^2$$

Gravitational Potential Energy $$E_p = mgh$$

Spring Potential Energy $$E_s = \frac{1}{2}kx^2$$

Mechanical Energy $$E_k + E_p + E_s$$

Conservative Energy

All forces have respective potential energy?

W_G gravitational potential energy

F_{fr} frictional potential energy?

Conservative Energy

Because potential energy is energy associated with the position or configuration of objects, potential energy can only **make sense** if it can be **stated uniquely for a given point.**

This cannot be done with nonconservative forces because the work done depends on the path taken.

Hence, **potential energy can be defined only for a *conservative force***.

Thus although potential energy is always associated with a force, **not all forces have a potential energy.**

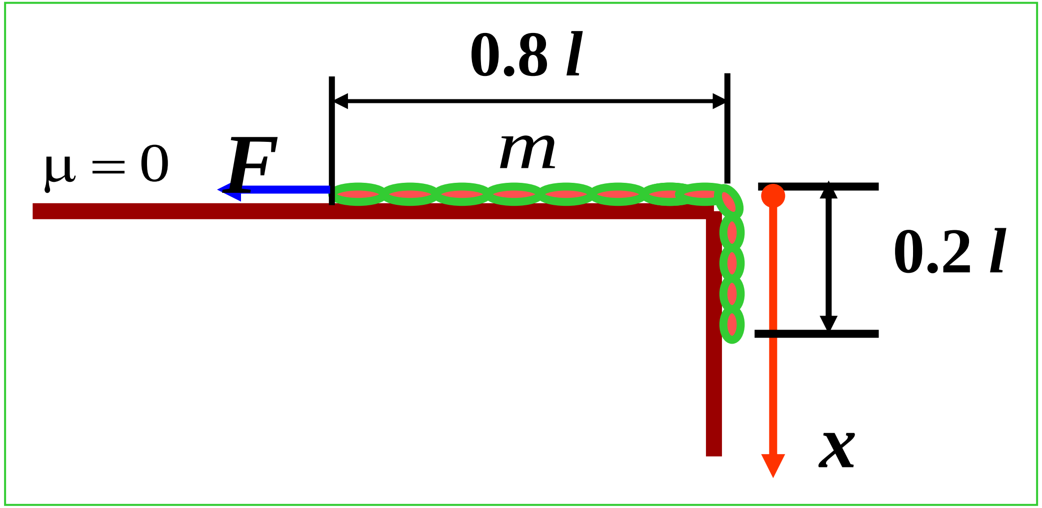
For example, there is no potential energy for friction.

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

Example 4

A uniform chain of mass *m* and length *l* is put on a frictionless horizontal surface. The length of hanging down part is 0.2 *l* , pull the chain very slowly back to the surface. Find the work done by the pulling force.



Extend work – energy theorem

$$W_{total} = \Delta E_k$$

When several forces act on an object?

What is the relationship between the work done by the nonconservative forces and the energy?

$$W_{NC} -- \ Ek_{,} \ E_p$$

$$W_{total} = W_{NC} + W_C$$

$$W_{net} = W_{NC} + W_C = \Delta E_k$$

$$\underline{} = -\Delta E_p$$

$$W_{NC} = \Delta E_k + \Delta E_p$$

Law of conservation of energy

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, transferred from one object to another, but the total amount remains constant.

Transform

Transfer

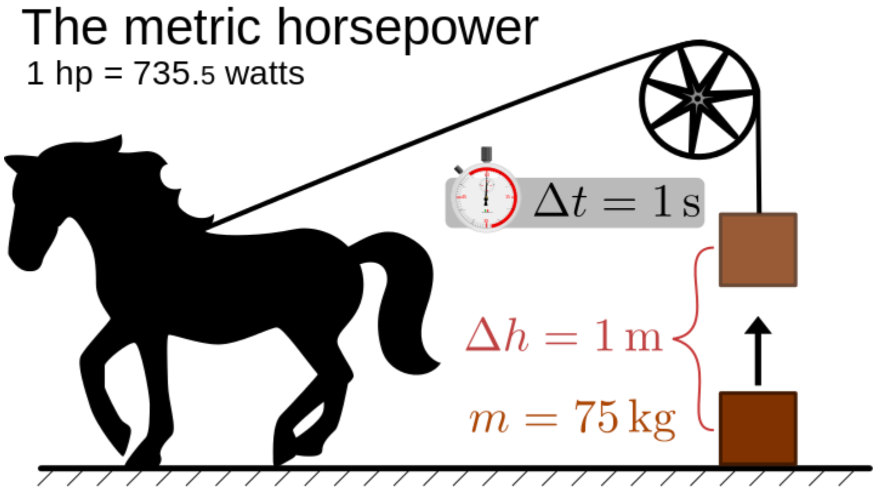
Transform

https://haokan.baidu.com/v?vid=4088022190377681925&pd=bjh&fr=bjhauthor&type=video

Power

Power is the rate at which work is done:

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\bar{v} \qquad \text{(Watts)}$$



1 horse power = 735.5 W

Power

1. Average power

$$P_{\mathrm{av}} = \frac{\Delta W}{\Delta t} \quad (\mathrm{J/s})$$

2. Instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\vec{F} \cdot \mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{F} \cdot \vec{v}$$

3. The power of the total force acting on a system

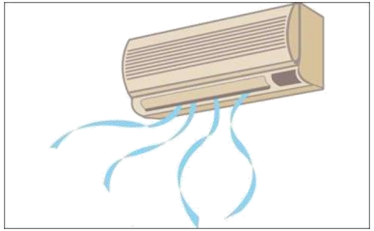
$$P_{\mathrm{total}} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\vec{F}_{\mathrm{total}} \cdot \mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{F}_{\mathrm{total}} \cdot \vec{v}$$

Power

$10^{-6}\ W$

$20 \sim 60\ W$

$200\ W$

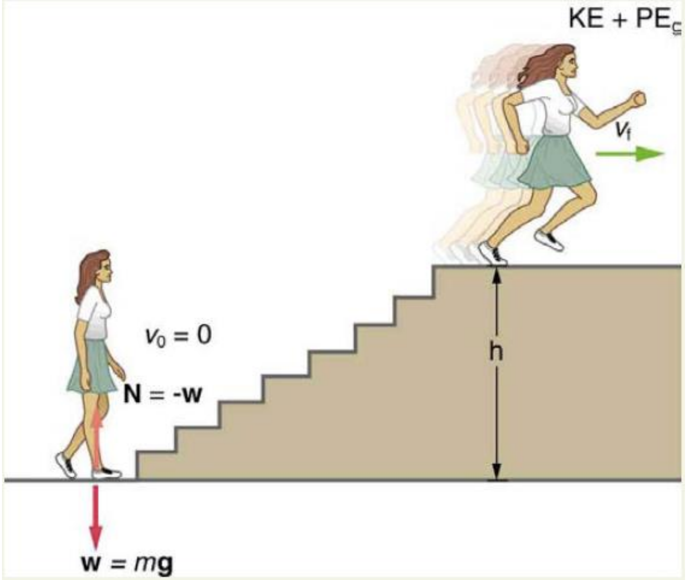


$10^{3}\ W$

$10^{5}\ W$

$10^{8}\ W$

Power



What is the power output for a 60.0 kg woman who runs up a 3.0 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.0 m/s?

Work (by her "leg force"):

$$W = KE + PE = \frac{mv_f^2}{2} + mgh$$

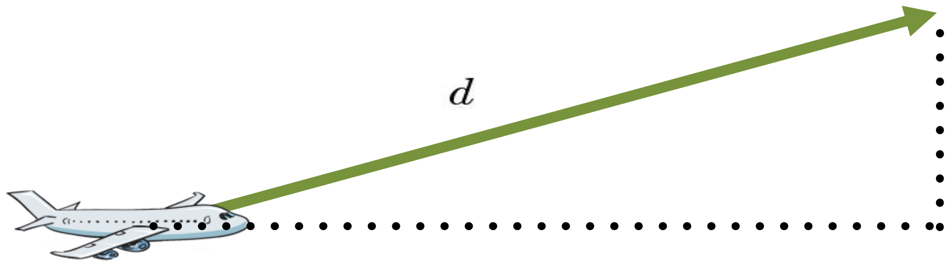
$$P = \frac{W}{t} = \frac{1}{t}\left[\frac{mv_f^2}{2} + mgh\right] = 538\ \mathrm{W}$$

Work and Energy

To see the distinction between energy and power, consider the following example. A person is limited in the work he or she can do, not only by the total energy required, but also by how fast this energy is transformed: that is, by power. For example, a person may be able to walk a long distance or climb many flights of stairs before having to stop because so much energy has been expended. On the other hand, a person who runs very quickly up stairs may feel exhausted after only a flight or two. He or she is limited in this case by power, the rate at which his or her body can transform chemical energy into mechanical energy.

Example 5

(a) How long would it take a 1.50×10^5 -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible?



Total energy gained:

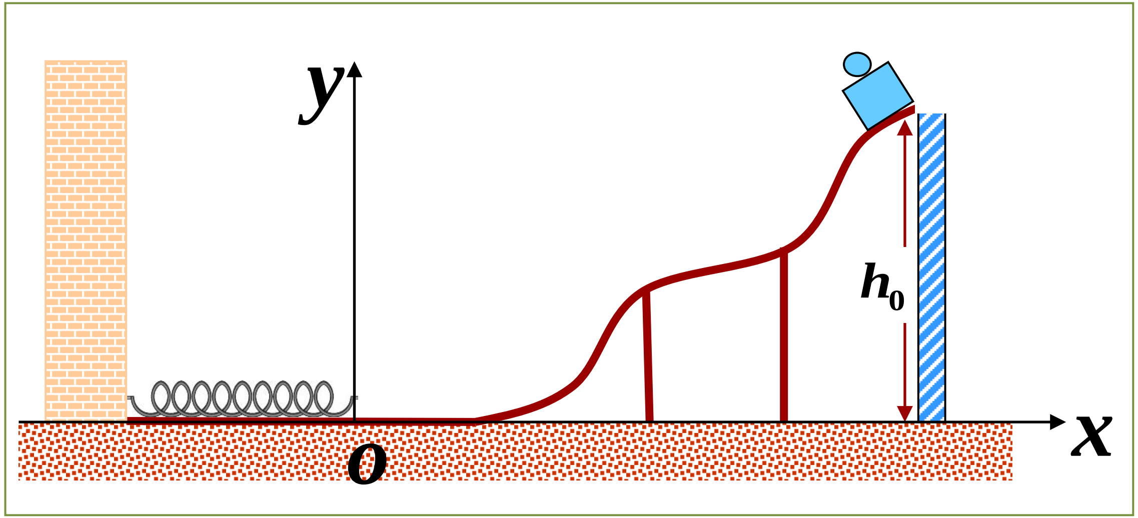
Kinetic energy + Potential energy = Power × time

Example 6

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop).

(b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

Example 7



The mass of the system *m*, the height h_0, the spring constant *k* are given. Find the maximum extent to which the spring is compressed.

Example 8

As shown in figure, on a thin staff of mass m_1 and length L, there is a small ring of mass m_2 which is connected with a light spring, the spring constant is k. At initial time, the system is static, under the application of an external force the system rotates at constant angular speed ω on a horizontal plane, the m_2 slide very slowly to the end of the staff A, at this instant, the kinetic energy of the staff is

$$\frac{1}{6}m_1L^2\omega^2$$

Find the work done by the external force .