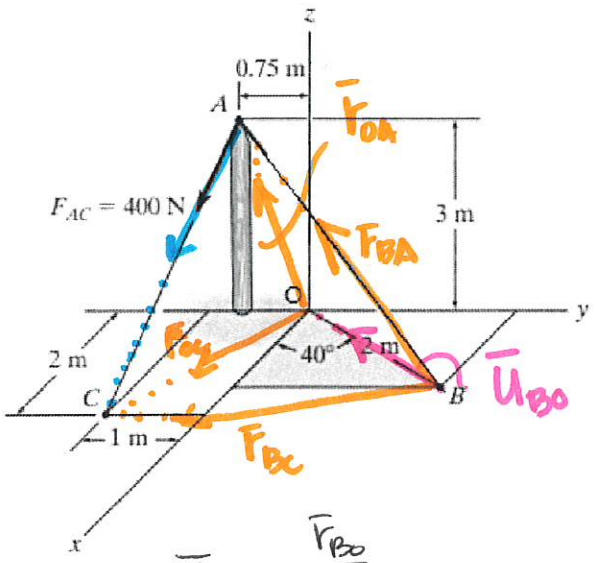
Chinese Name: KEY English Name: _____

ID#: _____

ENSC 2113 – FALL 2023 – EXAM #1

EACH PROBLEM IS WORTH THE POINTS INDICATED. BOX YOUR ANSWERS AND PROVIDE PROPER UNITS, WHERE APPLICABLE. CALCULATIONS AND FREE BODY DIAGRAMS MUST BE SHOWN THAT SUPPORT THE ANSWER TO RECEIVE CREDIT.

1. Determine the moment as a Cartesian vector created by the 400 N force (F_{AC}) about a line that passes from point B to the origin (point O). Use the cross-product method and show all matrix calculations. 30 POINTS



$\bar{M} = M\bar{u}_{BO}$ WHERE

$M =$ | • UNT VECTOR LINE B→O $\bar{u}_{BO}$ | • POSITION VECTOR FROM BO TO FORCE $\bar{r}_{BA}, \bar{r}_{BC}, \bar{r}_{OA}, \bar{r}_{OC}$ | • FORCE VECTOR $\bar{F}_{AC}$ |

COORDINATES

O(0,0,0)m B($2\cos 40, 2\sin 40, 0$)m = (1.53, 1.29, 0)m

A(0,-.75,3)m C(2,-1,0)m

UNIT VECTOR $\bar{u}_{BO} = \frac{\bar{r}_{BO}}{|r_{BO}|}$

$\bar{r}_{BO} = \{-1.53\mathbf{i} - 1.29\mathbf{j} + 0\mathbf{k}\}$m $|\bar{r}| = 2$m $\bar{u}_{BO} = \{-\frac{1.53}{2}\mathbf{i} - \frac{1.29}{2}\mathbf{j} + 0\mathbf{k}\}$

POSITION VECTOR FROM LINE BO → FORCE

$\bar{r}_{BA} = \{-1.53\mathbf{i} - 2.04\mathbf{j} + 3\mathbf{k}\}$m $\bar{r}_{OA} = \{0\mathbf{i} - 0.75\mathbf{j} + 3\mathbf{k}\}$m

$\bar{r}_{BC} = \{0.47\mathbf{i} - 2.29\mathbf{j} + 0\mathbf{k}\}$m $\bar{r}_{OC} = \{2\mathbf{i} - 1\mathbf{j} + 0\mathbf{k}\}$m ← I'M USING THIS ONE ☺

FORCE VECTOR $\bar{F}_{AC} = F_{AC}\bar{u}_{AC} = F_{AC}\frac{\bar{r}_{AC}}{|r_{AC}|}$ WHERE $F = 400$N

$\bar{r}_{AC} = \{2\mathbf{i} - 0.25\mathbf{j} - 3\mathbf{k}\}$m $|\bar{r}_{AC}| = \sqrt{13}$m $\bar{u}_{AC} = \{\frac{2}{\sqrt{13}}\mathbf{i} - \frac{.25}{\sqrt{13}}\mathbf{j} - \frac{3}{\sqrt{13}}\mathbf{k}\}$

$\bar{F}_{AC} = \{221.88\mathbf{i} - 27.74\mathbf{j} - 332.82\mathbf{k}\}$N

MATRIX

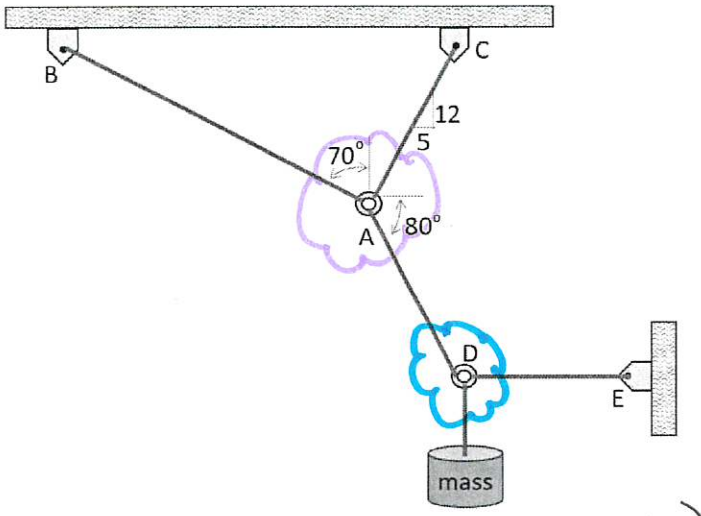
$$M = \begin{vmatrix} -\frac{1.53}{2} & -\frac{1.29}{2} & 0 \\ 2 & -1 & 0 \\ 221.88 & -27.74 & -332.8 \end{vmatrix} = (-\tfrac{1.53}{2})\left[(-1)(-332.8) - 0\right] - (-\tfrac{1.29}{2})\left[2(-332.8) - 0\right] + 0$$

$= -684$ Nm

$\bar{M} = (-684\text{Nm})\{-\frac{1.53}{2}\mathbf{i} - \frac{1.29}{2}\mathbf{j}\}$

$\bar{M} = \{523.2\mathbf{i} + 441.1\mathbf{j} + 0\mathbf{k}\}$Nm

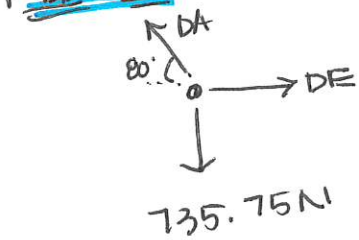
2. A 75 kg lamp is suspended in a system as shown. Utilizing equilibrium equations, determine the tension force in each of the supporting cables. Draw all pertinent free-body diagrams. 30 POINTS



FBDS

$75kg(9.81 m/s^2) = 735.75N$

FBD @ D

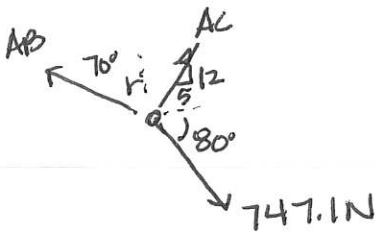


EQUILIBRIUM

$\Sigma F_y \uparrow = 0 \quad DA\sin 80 - 735.75 = 0$ $\quad \boxed{DA = 747.1N}$

$\Sigma F_x \rightarrow = 0 \quad DE - 747.1\cos 80 = 0$ $\quad \boxed{DE = 129.7N}$

FBD @ A



EQUILIBRIUM

$\Sigma F_x \rightarrow = 0 \quad -AB\sin 70 + \frac{5}{13}AC + 747.1\cos 80 = 0$ ①

$\Sigma F_y \uparrow = 0 \quad AB\cos 70 + \frac{12}{13}AC - 747.1\sin 80 = 0$ ②

① $AC = \frac{13}{5}\left[AB\sin 70 - 747.1\cos 80\right]$

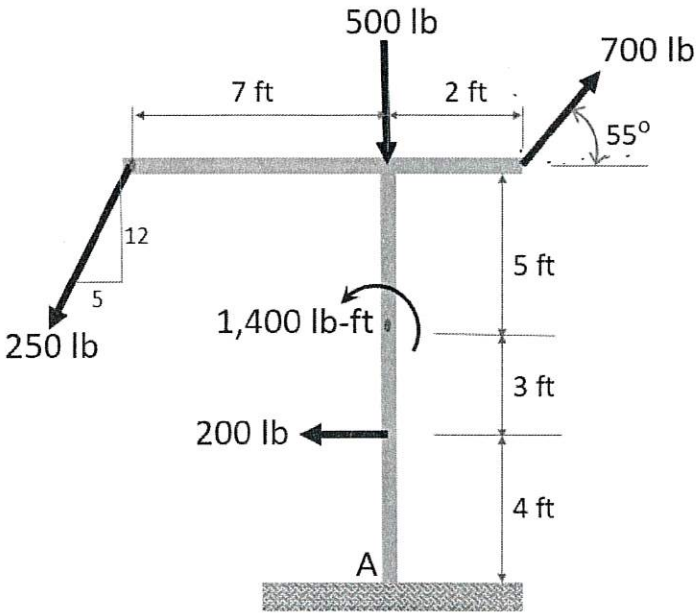
② $AB\cos 70 + \frac{12}{13}\left[\frac{13}{5}\left[AB\sin 70 - 747.1\cos 80\right]\right] - 747.1\sin 80 = 0$

$AB\cos 70 + \frac{12}{5}AB\sin 70 = \frac{12}{5}747.1\cos 80 + 747.1\sin 80$

$\boxed{AB = 403.2N}$

$\boxed{AC = 647.7N}$

3. Replace the loading system by an equivalent resultant force system and specify where the resultant's line of action vertically from point A (measured from point A above or below). 30 POINTS ↕



RESULTANT FORCE

$R_x = \Sigma F_x \rightarrow = -\frac{5}{13}(250\,lb) + 700\,lb\cos 55° - 200\,lb = 105.35\,lb \rightarrow$

$R_y = \Sigma F_y \uparrow = -\frac{12}{13}(250\,lb) - 500\,lb + 700\,lb\sin 55° = -157.36\,lb = 157.36\,lb \downarrow$

$R = \sqrt{R_x^2 + R_y^2} = 189.37\,lb$

$\theta = \tan^{-1}\frac{157.36}{105.35} = -56.2°$

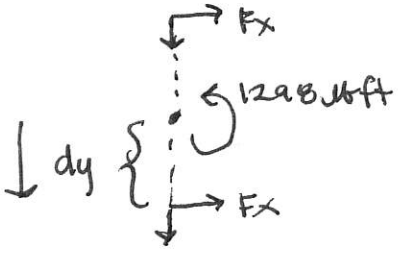
$360 - \theta = 303.8°$

$R = 189\,lb \text{ @ } -56.2° \text{ OR } 303.8°$

RESULTANT MOMENT @ A

$R_{M_A} = \Sigma M_A \circlearrowleft = \frac{5}{13}(250)(12) + \frac{12}{13}(250)(7) - 700\cos 55(12) + 700\sin 55(2)$
$+1400 + 200(4) = 1298\,lb\text{-}ft \circlearrowleft$

MOVE VERTICAL ↕ FROM A

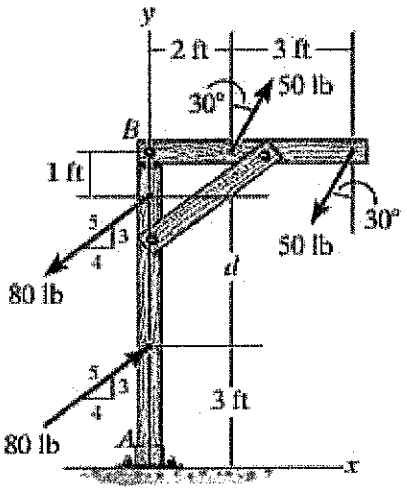


$M_A = F_x d_y$

$d_y = \frac{M_A}{F_x} = \frac{1298\,lb\text{-}ft}{105.35\,lb} = 12.32\,ft$

$d_y = 12.32\,ft \downarrow$

4. Determine the resultant couple moment given d = 4 ft. 10 POINTS



$R_M = 80lb\left(\frac{4}{5}\right)(4) - 50lb\cos 30(3)$

$= \boxed{126\ lb\,ft}$