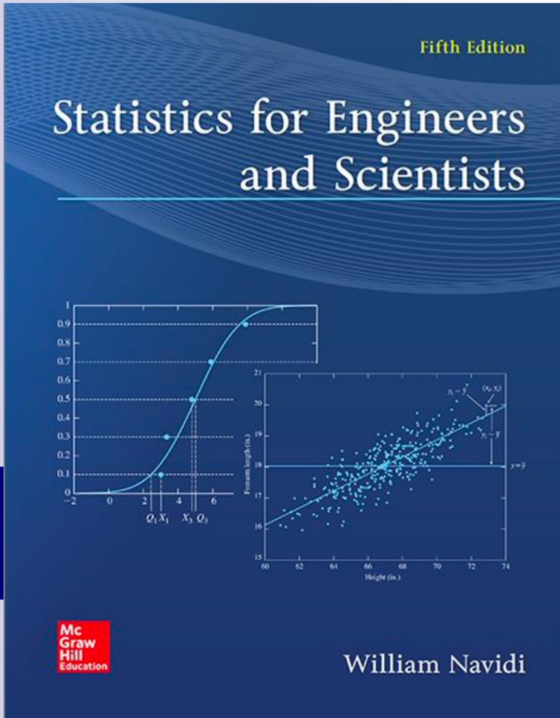


Chapter 2

Probability (part 1)

(not all sections are required)

Updated Teaching Timetable

Teaching week	Date	Teaching Contents	Requirement of students	Teaching time		Notes
				Lecture	Practice	
1	2nd September (Monday)	Ch 1: Sampling and Descriptive Statistics	Textbook, calculator	2	1	-----
2	9th September (Monday)	Ch 2: Probability	Textbook, calculator	2	1	-----
4	23rd September (Monday)	Ch 3: Propagation of Error	Textbook, calculator	2	1	**Homework 1 issued**
6	10th October (Thursday)	Ch 4: Commonly Used Distributions	Textbook, calculator	2	1	-----
7	14th October (Monday)	Ch 4: Commonly Used Distributions (continued)	Textbook, calculator	2	1	**Homework 1 due**
7	17th October (Thursday)	Ch 5: Confidence Intervals	Textbook, calculator	2	1	-----

Chapter 2 Overview

- 2-1 Basic Ideas
- 2-2 Counting Methods
- 2-3 Conditional Probability and Independence
- 2-4 Random Variables
- ~~2-5 Linear Functions of Random Variables~~
- ~~2-6 Jointly Distributed Random Variables~~

Introduction

- **Probability (P)** can be defined as the chance of an event occurring.
- It can be used to quantify what the “odds” are that a specific event will occur.
- Some examples of how probability is used everyday would be weather forecasting, “75% chance of snow” or for setting insurance rates.
- It is the foundation in which the methods of inferential statistics are built.

Sample Spaces and Probability

- A **probability experiment** is a chance process that leads to well-defined results called outcomes.
- An **outcome** is the result of a single trial of a probability experiment.
- A **sample space** is the set of all possible outcomes of a probability experiment.
- An **event** (A, B, C, etc.) consists of outcomes.
  - simple event = 1 outcome
  - compound event = more than 1 outcome

Definition

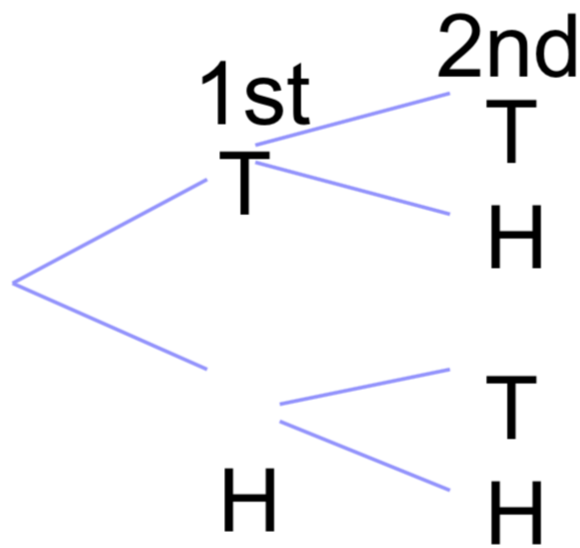
A subset of a sample space is called an **event**.

Sample Spaces

Experiment	Sample Space
Toss a coin	Head, Tail
Roll a die	1, 2, 3, 4, 5, 6
Answer a true/false question	True, False
Toss two coins	HH, HT, TH, TT

Toss two coins

Tree diagram



Sample space

TT

TH

HT

HH

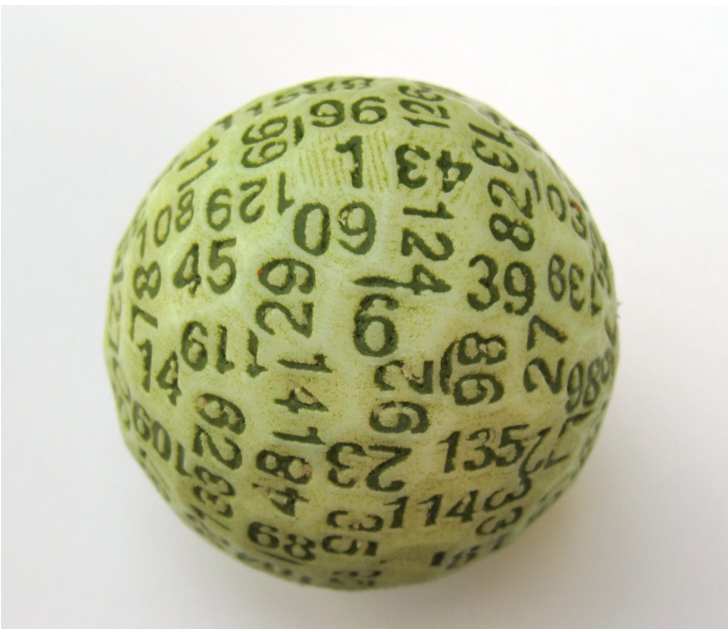
A **tree diagram** is a device consisting of line segments emanating from a starting point and also from the outcome point. It is used to determine all possible outcomes of a probability experiment.

Example: Rolling Dice

Find the sample space for rolling two dice.

Rolling a Dice

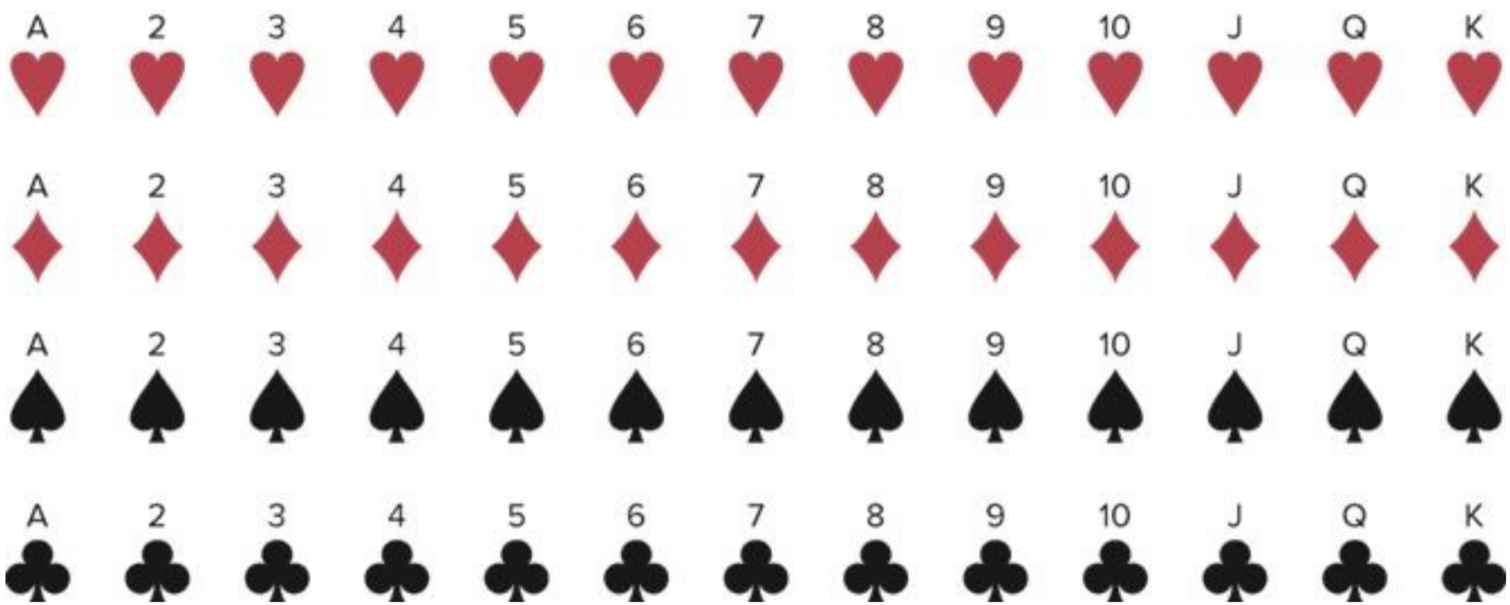
How about the sample space for rolling two 144-face dice?



Note: Historically, **dice** is the plural of **die**, but in modern English **dice** can be both the singular and the plural, so *“rolling dice”* could mean a reference to either one or more than one dice.

Example: Drawing Cards

Find the sample space for drawing one card from an ordinary deck of cards.



Example: Gender of Children

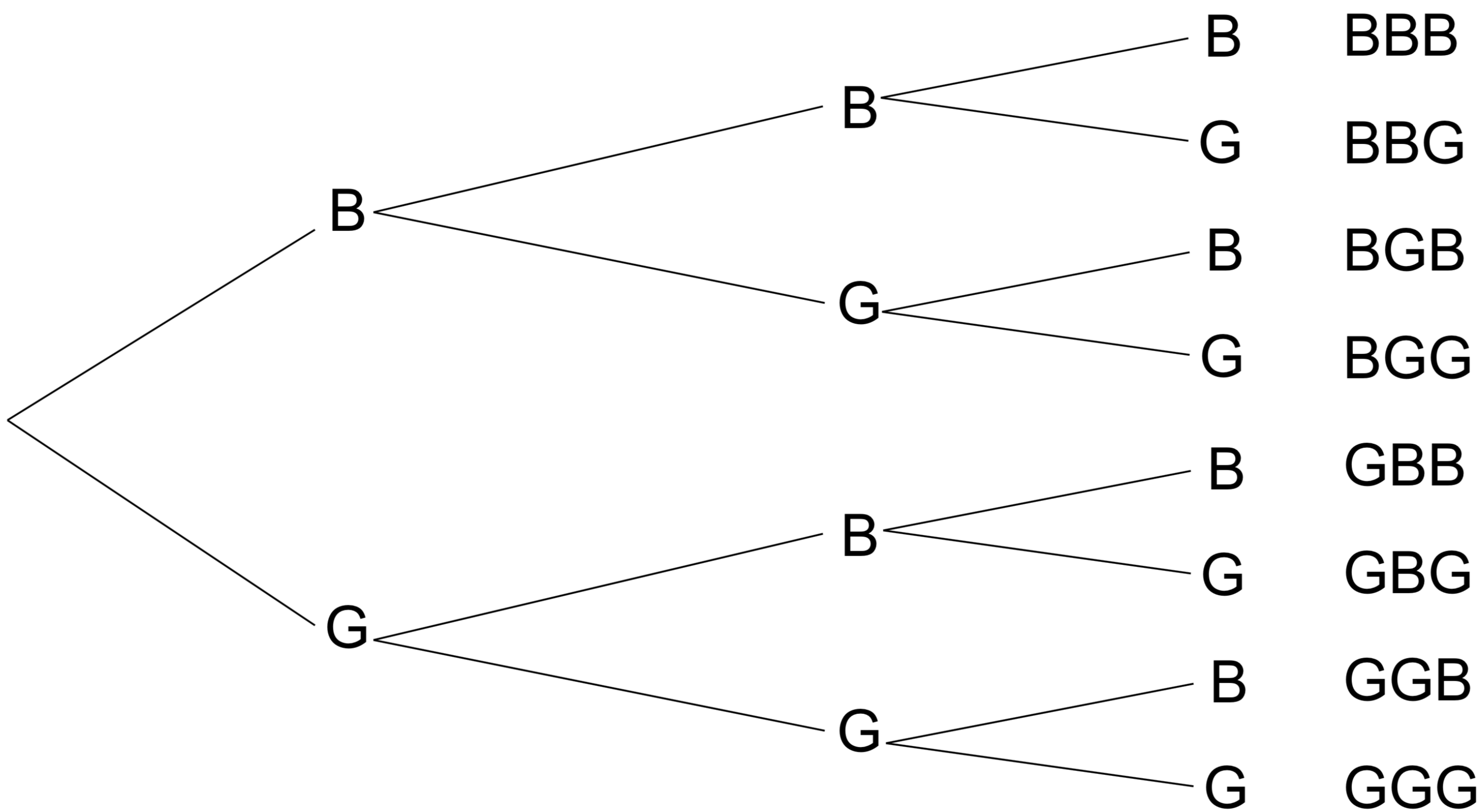
Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl.

- ✓ There could be three boys
- ✓ There could be two boys and a girl, where the girl is the youngest, the middle child, or the oldest.
- ✓ There could be one boy and two girls. Where the boy is the oldest, the middle child, or the youngest.
- ✓ The only other possibility would be that all three children are girls

BBB	BGG
BBG	GBG
BGB	GGB
GBB	GGG

Example: Gender of Children…

Use a tree diagram to find the sample space for the gender of three children in a family, as in Example 4-3



Sample Spaces and Probability

There are 3 basic interpretations of probability

1. **Classical** probability
2. **Empirical** (or relative frequency) probability
3. **Subjective** probability

Classical Probability

Classical probability uses sample spaces to determine the numerical probability that an event will happen and assumes that all outcomes in the sample space are equally likely to occur.

(Requires equally likely outcomes)

Sample Spaces and Probability

Formula for Classical Probability

The probability of any event E is

$$\boxed{\frac{\text{Number of outcomes in } E}{\text{Total number of outcomes in the sample space}}}$$

This probability is denoted by

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

where $n(E)$ is the number of outcomes in E and $n(S)$ is the number of outcomes in the sample space S.

Sample Spaces and Probability…

- Probabilities can be expressed as fractions, decimals, or where appropriate percentages.
- If you ask, “What is the probability of getting a head when a coin is tossed?” typical responses can be any of the following three.
  - “One-half (1/2)”
  - “Point five (0.5)”
  - “Fifty percent (50%)”

Example: Gender of Children

Find the probability that the family with three children would have exactly two boys.

There are three total outcomes with three boys and eight outcomes in the sample space.

The probability would be 3/8.

$$P(2\ Boys) = \frac{3}{8}$$

BBB	BGG
BBG	GBG
BGB	GGB
GBB	GGG

Example: Gender of Children…

The probability that the family would have three girls would be calculated by finding the number of outcomes with three girls.

There is only one event with three girls.

$$P(3\ Girls) = \frac{1}{8}$$

BBB	BGG
BBG	GBG
BGB	GGB
GBB	GGG

Axioms of Probability

The subject of probability is based on three commonsense rules, known as axioms.

The Axioms of Probability

1. Let S be a sample space. Then $P(S) = 1$.
2. For any event A, $0 \le P(A) \le 1$.
3. If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$.

   More generally, if $A_1, A_2, \ldots$ are mutually exclusive events, then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

Example of rolling a die (Axiom 1)

When a single die is rolled, what is the probability of getting a number less than 7?

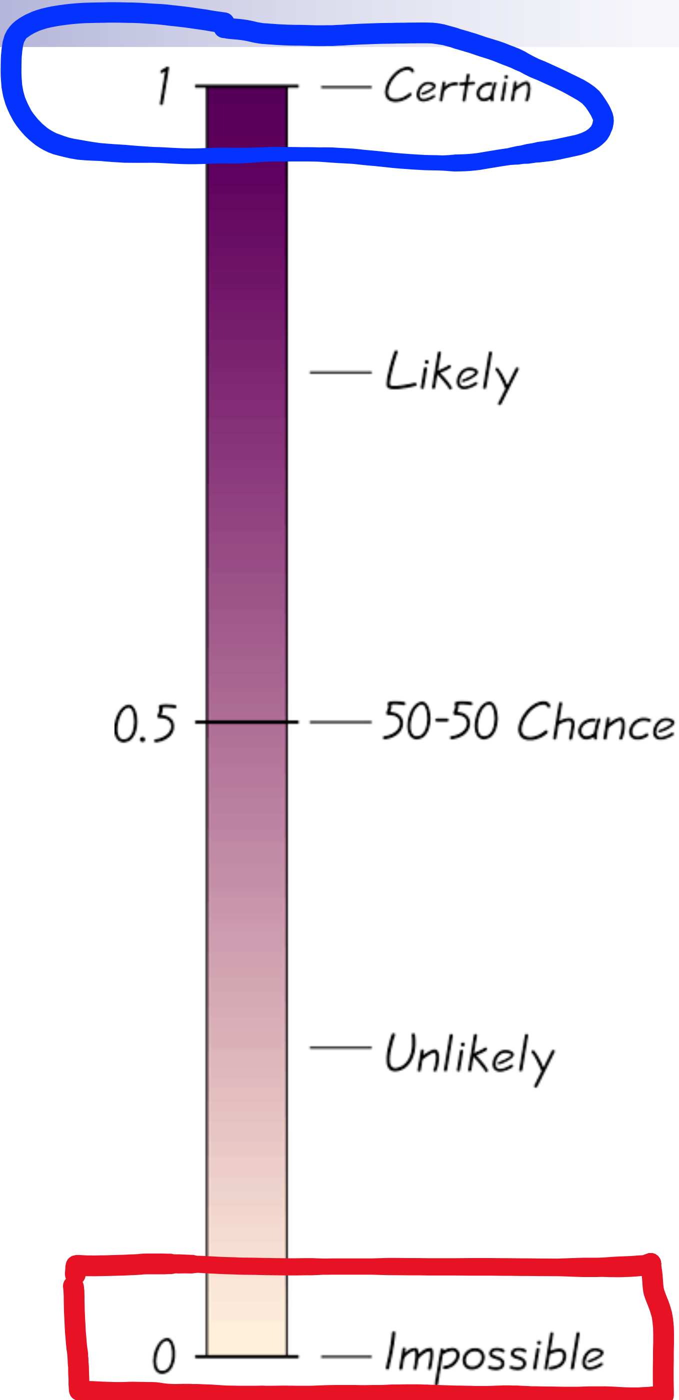
SOLUTION

Since all outcomes—1, 2, 3, 4, 5, and 6—are less than 7, the probability is

$$P(\text{number less than 7}) = \frac{6}{6} = 1$$

The event of getting a number less than 7 is certain.

Possible Values for Probabilities (Axiom 2)



Example of rolling a die (Axiom 3)

Outcome	1		2		3		4		5		6
Probability	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$
Sum	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6} = \frac{6}{6} = 1$

Equally Likely Outcomes (p.56)

For some experiments, a sample space can be constructed in which all the outcomes are equally likely. A simple example is the roll of a fair die, in which the sample space is {1, 2, 3, 4, 5, 6} and each of these outcomes has probability 1/6. Another type of experiment that results in equally likely outcomes is the random selection of an item from a population of items. The items in the population can be thought of as the outcomes in a sample space, and each item is equally likely to be selected.

> A population from which an item is sampled at random can be thought of as a sample space with equally likely outcomes.

If a sample space contains N equally likely outcomes, the probability of each outcome is $1/N$. This is so, because the probability of the whole sample space must be 1, and this probability is equally divided among the N outcomes. If A is an event that contains k outcomes, then $P(A)$ can be found by summing the probabilities of the k outcomes, so $P(A) = k/N$.

> If $\mathcal{S}$ is a sample space containing N equally likely outcomes, and if A is an event containing k outcomes, then
>
> $$P(A) = \frac{k}{N} \tag{2.4}$$

Exercise: Rolling Two Dice

If two dice are rolled one time, find the probability of getting a **sum of 7 or 11**.

Complementary Events

- The **complement of an event A** is the set of outcomes in the sample space that are not included in the outcomes of event A.
- The complement of A is denoted in the textbook by $\mathbf{A^c}$ (which means not A)

$$P(A) + P(A^c) = 1$$

$$P(A) = 1 - P(A^c)$$

Examples of Complementary Events

Event	Complement of the Event
Rolling a die and getting a 4	
Selecting a letter of the alphabet and getting a vowel	
Selecting a month and getting a month that begins with a J	
Selecting a day of the week and getting a weekday	

Exercise: Residence of People

If the probability that a person lives in an industrialized country of the world is 1/5 , find the probability that a person does not live in an industrialized country.

4-1 Sample Spaces and Probability

There are 3 basic interpretations of probability

1. **Classical** probability
2. **Empirical (or relative frequency) probability**
3. **Subjective** probability

Empirical Probability

Empirical Probability relies on experience and observation to determine the likelihood of outcomes.

Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = \frac{frequency\ for\ the\ class}{total\ frequencies\ in\ the\ distribution}$$

$$\boxed{P(E) = \frac{f}{n}}$$

Example: Blood Types

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

Type	Frequency
A	22
B	5
AB	2
O	21
Total	50

$$P(\mathrm{O}) = \frac{f}{n} = \frac{21}{50}$$

Example: Blood Types…

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

b. A person has type A or type B blood.

Type	Frequency
A	22
B	5
AB	2
O	21
	Total 50

$$P(\text{A or B}) = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$$

Example: Blood Types…

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

c. A person has neither type A nor type O blood.

Type	**Frequency**
A	$\cancel{22}$
B	5
AB	2
O	$\cancel{21}$
	Total 50

$$P(\text{neither A nor O})$$
$$= \frac{5}{50} + \frac{2}{50}$$
$$= \frac{7}{50}$$

Example: Blood Types…

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

d. A person does not have type AB blood.

Type	Frequency
A	22
B	5
AB	2
O	21
	Total 50

$$P(\text{not AB})$$
$$= 1 - P(\text{AB})$$
$$= 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$$

Sample Spaces and Probability

There are 3 basic interpretations of probability

1. **Classical** probability
2. **Empirical** (or relative frequency) probability
3. **Subjective probability**

Subjective Probability

Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information, or by using knowledge of the relevant circumstances.

Examples: weather forecasting, predicting outcomes of sporting events

End of Probability (part 1)