ENSC 2113
Engineering Mechanics: Statics

Chapter 9:

Center of Gravity and Centroid

(Section 9.1)

COLLEGE OF
ENGINEERING, ARCHITECTURE
AND TECHNOLOGY

Chapter 9 Outline:

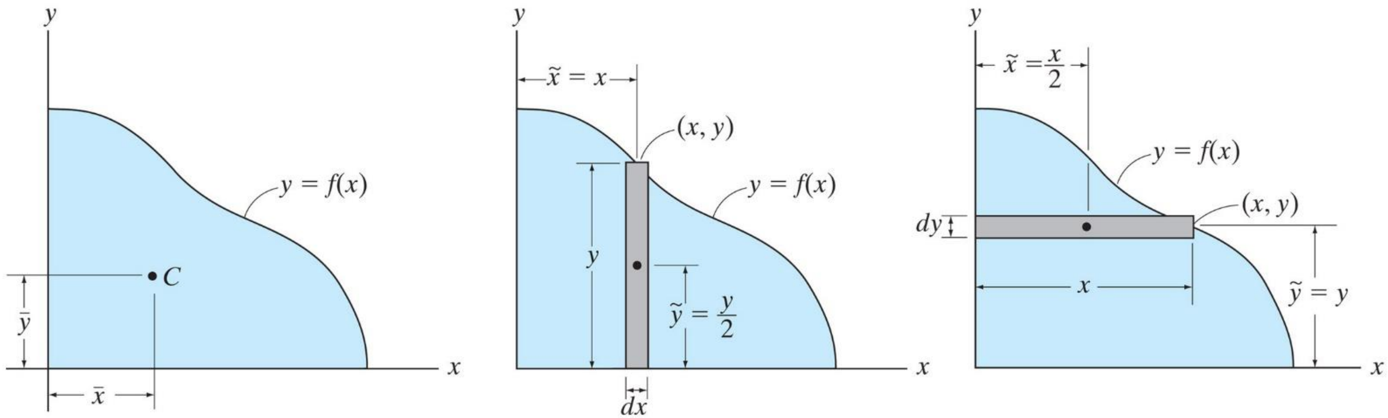
9.1 Center of Gravity, Center of Mass, and the Centroid of a Body

9.2 Composite Bodies

9.3 Theorems of Pappus and Guldinus

9.4 Resultant of a General Distributed Loading

9.5 Fluid Pressure

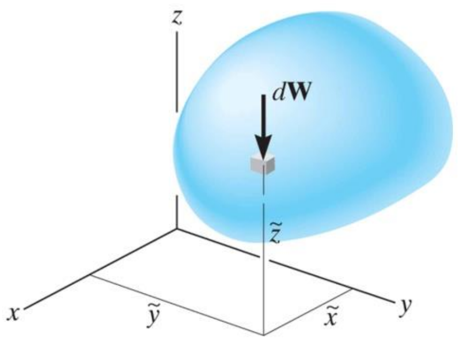


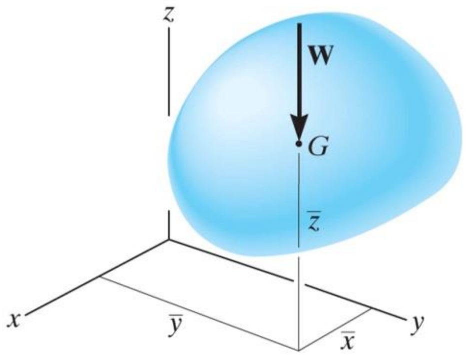
Chapter 9 Objectives:

- To discuss the concept of the center of gravity, center of mass, and the centroid.
- To show how to determine the location of the center of gravity and centroid for a body of arbitrary shape and one composed of composite parts.
- To use the theorems of Pappus and Guldinus for finding the surface area and volume for a body having axial symmetry.
- To present a method for finding the resultant of a general distributed loading to show how it applies to finding the resultant force of a pressure loading caused by a fluid.

9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

- A body is composed of an infinite number of particles. Each of these particles will have a weight, ***dW*.**
- The **center of gravity** is a point which locates the resultant weight of a system of particles or a solid body
- Coordinates of the center of gravity

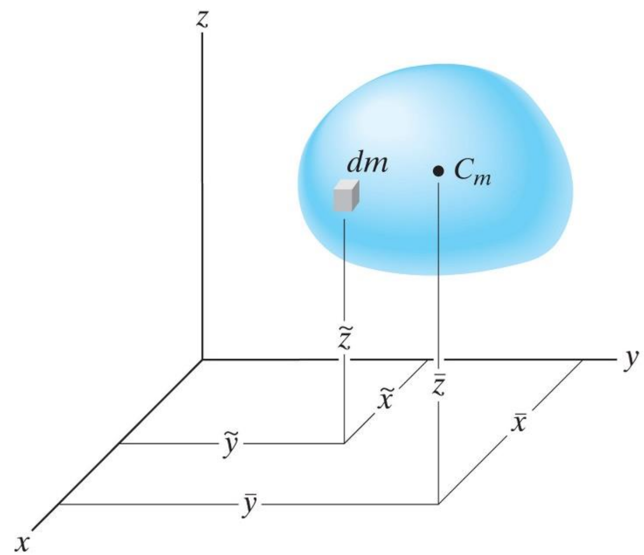




$$\bar{x} = \frac{\int \tilde{x}\ dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\ dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\ dW}{\int dW}$$

9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

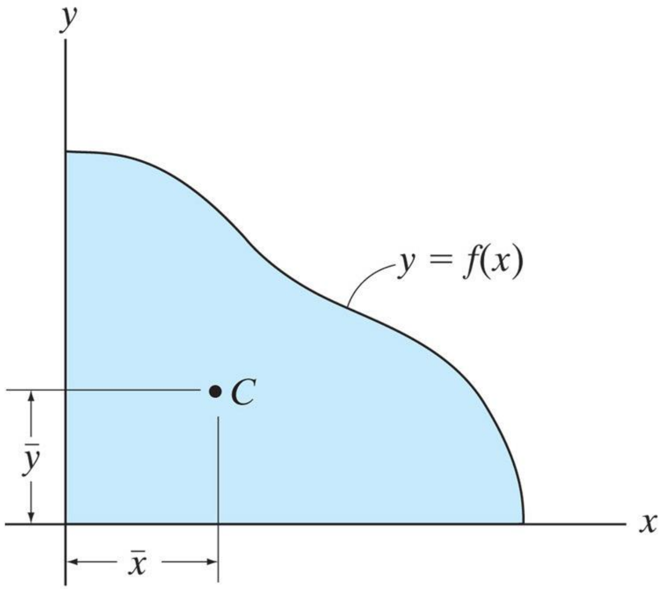
- By replacing ***W*** with a ***m*** in these equations, the coordinates of the center of mass can be found.
- Similarly, the coordinates of the centroid of volume, area, or length can be obtained by replacing ***W*** by ***V***, ***A***, or ***L***.



$$\bar{x} = \frac{\int \tilde{x}\ dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\ dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\ dm}{\int dm}$$

9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

- The **centroid** (geometric center) of an area can be determined by performing a single integration using a rectangular strip for the differential area.



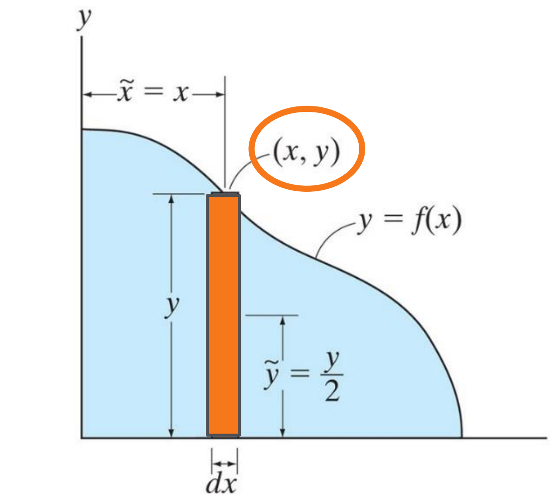
$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

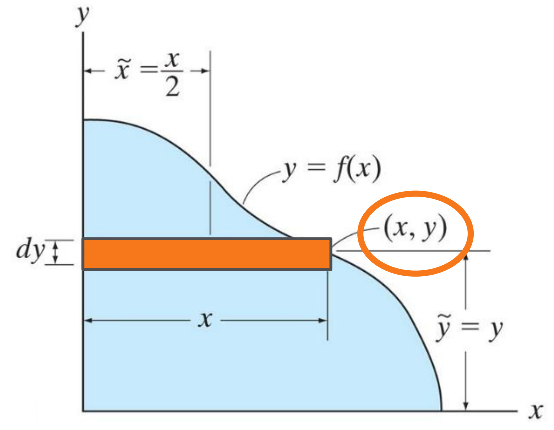
9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

Procedure:

Differential Element:

- Choose a differential element (rectangle) for integration
- Locate the element so that it touches an arbitrary point (x,y) on the curve that defines the boundary of the shape





9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

Procedure:

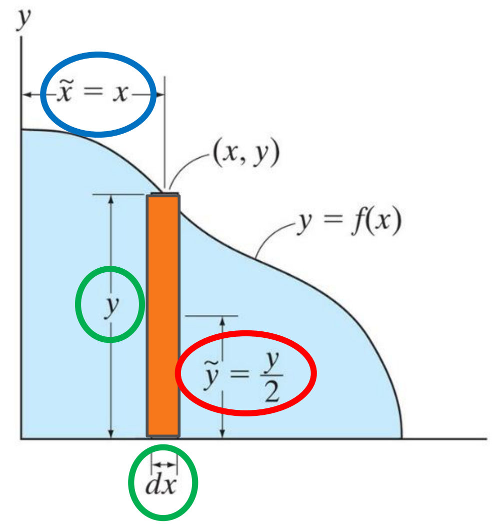
Size and Moment Arms:

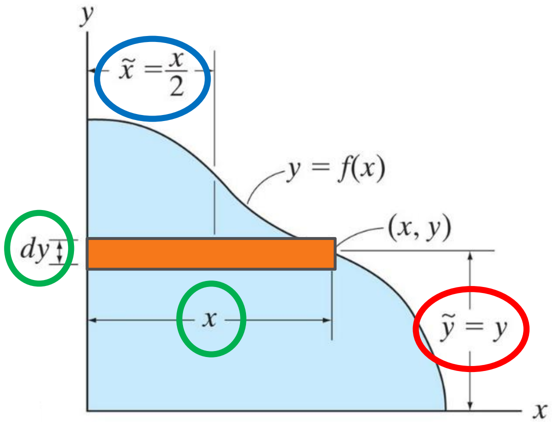
- Express the area ***dA*** of the element in terms of x or y

  dA=ydx

  dA=xdy

- Locate the centroid of the element, $(\tilde{x}, \tilde{y})$ in terms of x or y





9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

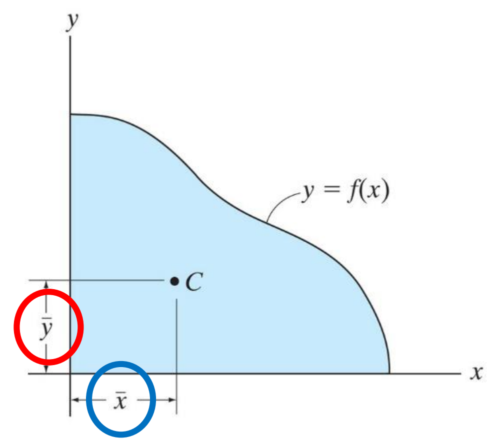
Procedure:

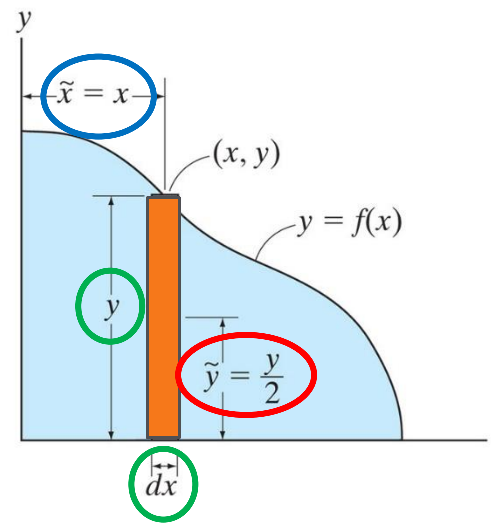
Integrations:

- Substitute into the appropriate equations
  - For a vertical slice

$$\bar{x} = \frac{\int_0^b \tilde{x} dA}{\int_0^b dA} = \frac{\int_0^b x(y d_x)}{\int_0^b y d_x}$$

$$\bar{y} = \frac{\int_0^b \tilde{y} dA}{\int_0^b dA} = \frac{\int_0^b \left(\frac{y}{2}\right)(y d_x)}{\int_0^b y d_x}$$



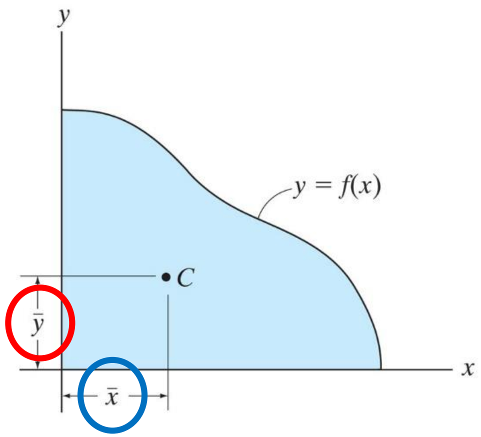


9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

Procedure:

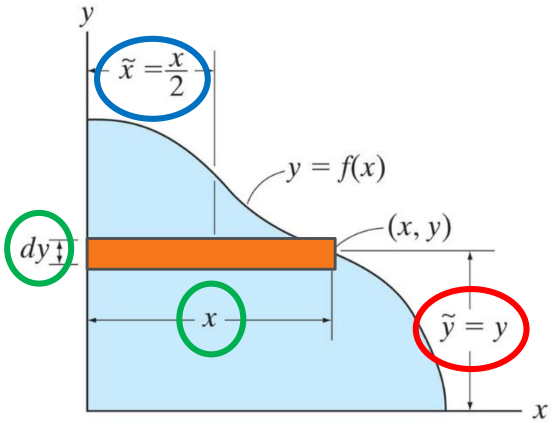
Integrations:

- Substitute into the appropriate equations
  - For a horizontal slice



$$\bar{x} = \frac{\int_0^h \tilde{x}dA}{\int_0^h dA} = \frac{\int_0^h \left(\frac{x}{2}\right)(xd_y)}{\int_0^h xd_y}$$

$$\bar{y} = \frac{\int_0^h \tilde{y}dA}{\int_0^h dA} = \frac{\int_0^h y\,(xd_y)}{\int_0^h xd_y}$$

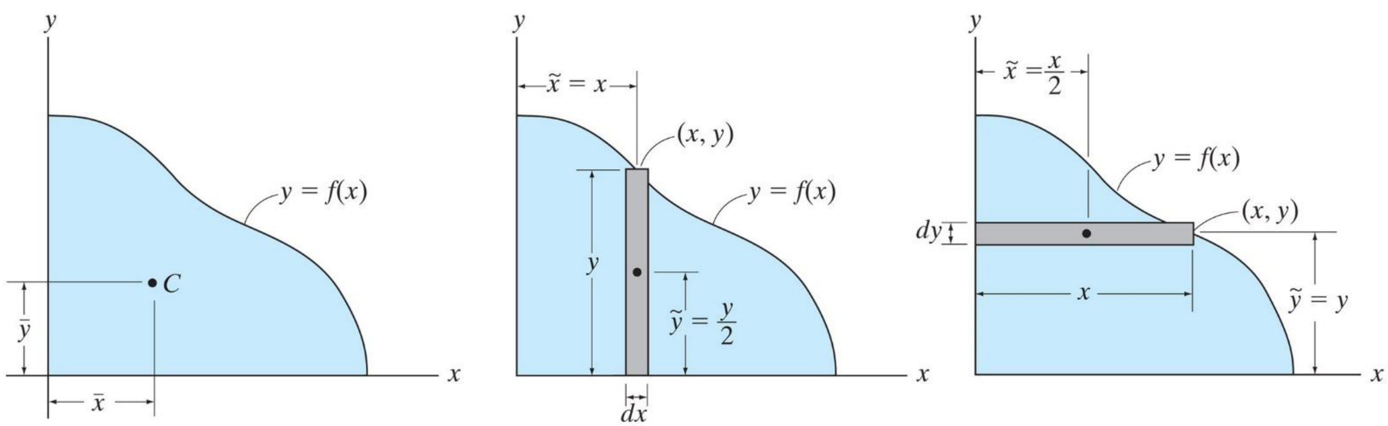


9.1 Center of Gravity, Center of Mass, and the Centroid of a Body:

Procedure:

Integrations:

- Write the function in terms of the same variable as the differential thickness of the element
- The limits of the integral are defined from the two extreme locations of the element's differential thickness
    - Vertical slice: limits are horizontal
    - Horizontal slice: limits are vertical



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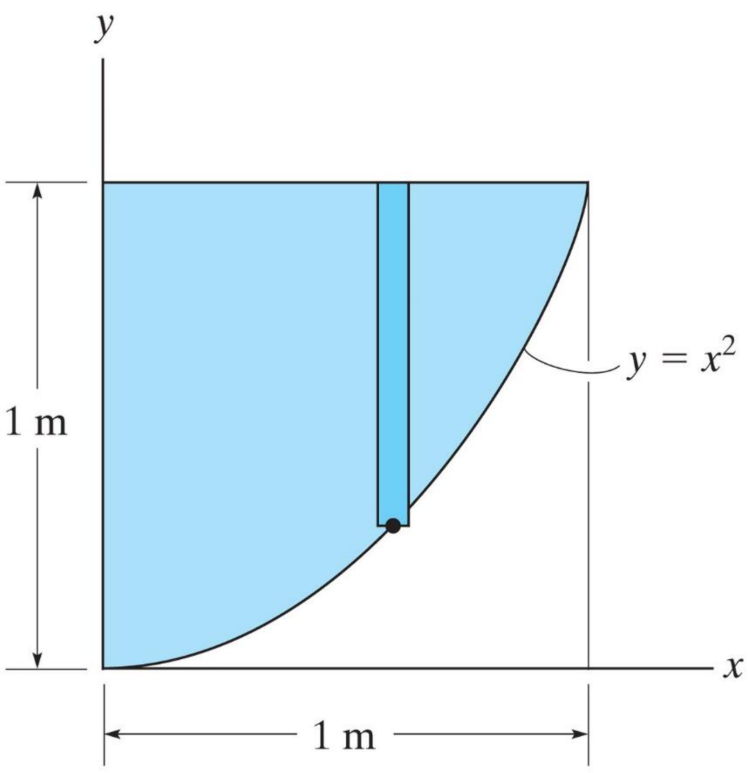
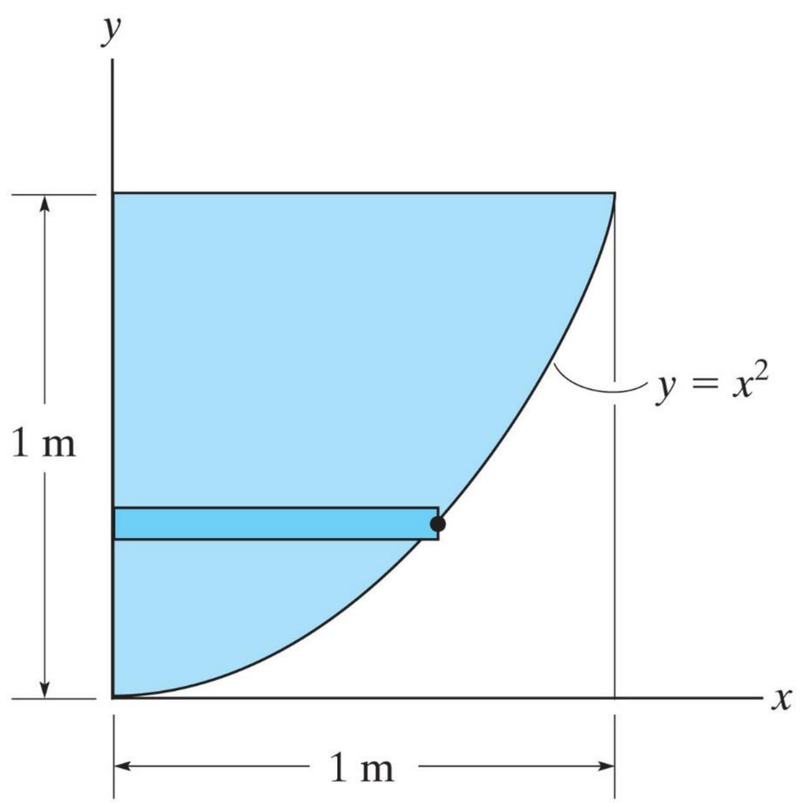
Example:

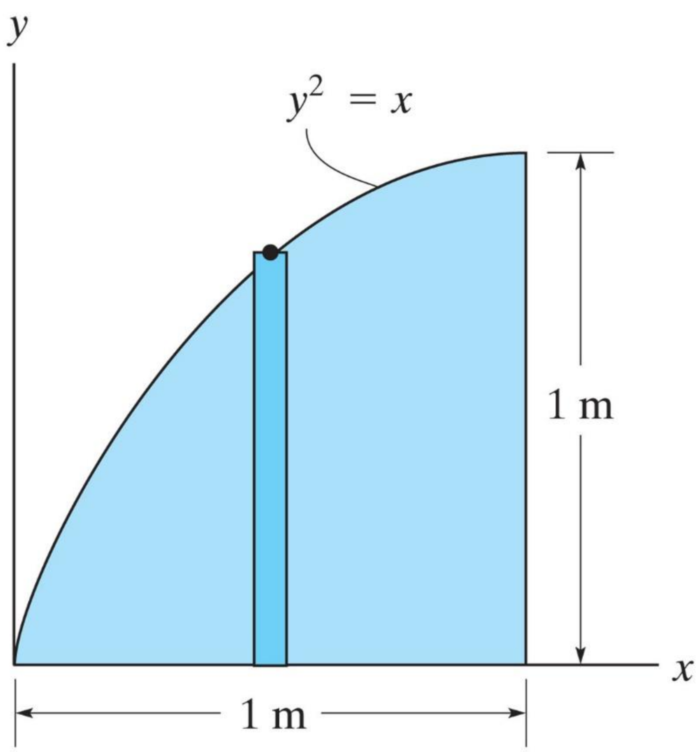
- For each image, label the size and moment arm of the differential slice and substitute into the appropriate centroidal equations:

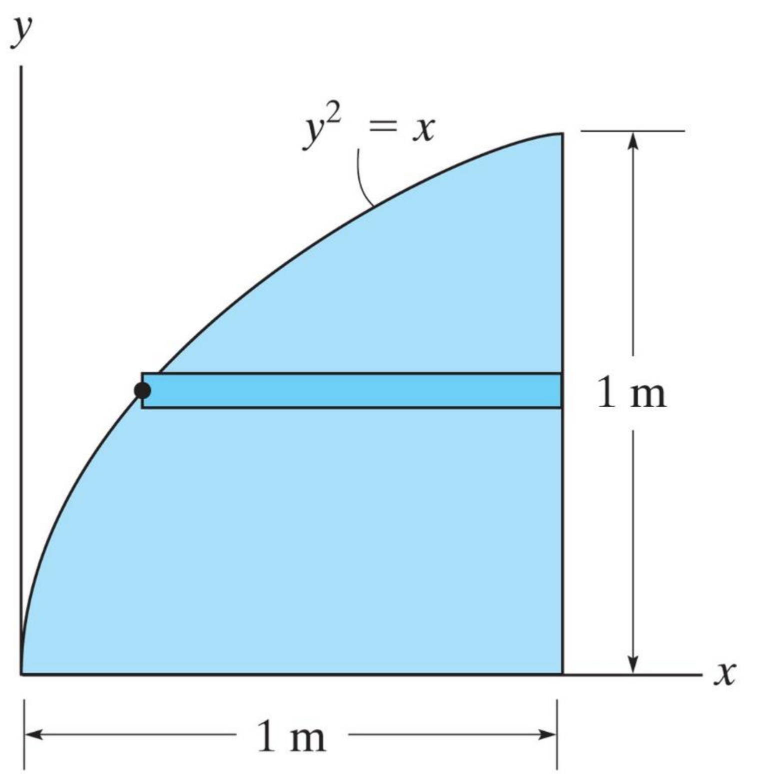




Example:

- For each image, label the size and moment arm of the differential slice and substitute into the appropriate centroidal equations:





Example:

- Calculate the centroid for the shape below. Draw and label the differential element, size, and moment arm.