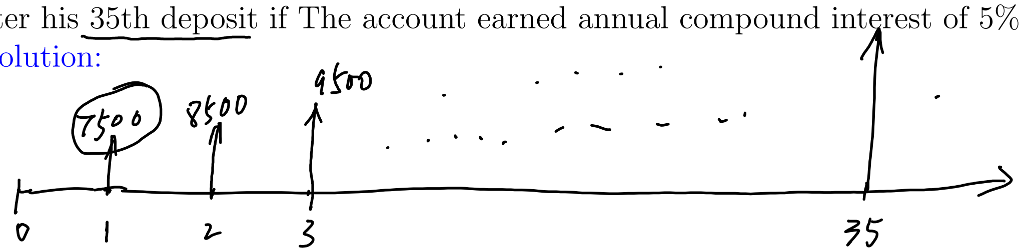
Instructions:

Please attempt every problem. You must support every solution with an appropriate amount of work and/or description. Unsupported answers may receive a score of 0. Good luck!

1. (5 pts) On Juan's 26th birthday, he deposited \$7,500 in a retirement account. Each year thereafter he deposited \$1,000 more than the previous year. Using a gradient series factor, determine how much was in the account immediately after his 35th deposit if The account earned annual compound interest of 5%.

   Solution:



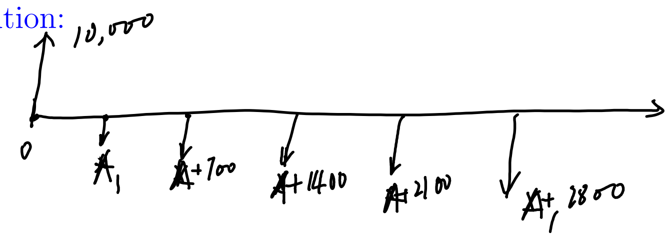
   The composite series consists of a uniform series with $A_t = \$7500$ for $t = 1, 2, \ldots, 35$ and a gradient series with $G = \$1000$. Hence, at $t = 35$, there are

$$
\begin{aligned}
F_c &= F_u + F_g \\
&= 7500\left[\frac{(1+0.05)^{35}-1}{0.05}\right] + 1000\left[\frac{(1+0.05)^{35}-(1+35\times 0.05)}{0.05^2}\right] \\
&= \$1,783,808.45
\end{aligned}
$$

   in the account immediately after his 35th deposit. +5

2. (8 pts) John borrows \$10,000 at 18% compounded annually. He pays off the loan over a 5-year period with annual payments. Each successive payment is \$700 greater than the previous payment. How much was the first payment?

   Solution:



   Assume that the first payment is A_1. Then the composite series consists of a uniform series with $A_t = A_1$ for $t = 1, 2, 3, 4, 5$ and a gradient series with

$G = \$700$. To payoff the loan, the present worth of the composite series needs to be \$10,000 at $i = 18\%$ compounded annually. Hence, we have

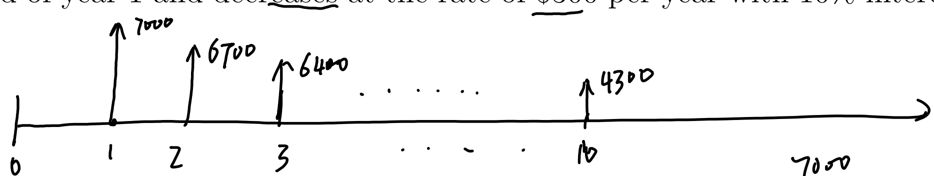
$$\begin{aligned} P_c &= P_u + P_g \\ &= A_1 \frac{(1+0.18)^5 - 1}{0.18(1+0.18)^5} + 700 \frac{1 - (1 + 5 \times 0.18)(1+0.18)^{-5}}{0.18^2} \\ &= 10000 \end{aligned}$$

Solving for A_1 yields

$$A_1 = \$2026.792.$$

Hence the first payment is \$2026.792. +8

3. (10 pts) A series of 10 end-of-year deposits is made that begins with \$7,000 at the end of year 1 and decreases at the rate of \$300 per year with 10% interest.



$$\begin{array}{r} 7000 \\ -\,2700 \\ \hline 4300 \end{array}$$

(a) What amount could be withdrawn at $t = 10$?
The composite series consists of a uniform series with $A_t = \$7000$ for $t = 1, 2, \ldots, 10$ and a gradient series with $G = -300$. Hence, at $t = 10$, we can withdraw

$$\begin{aligned} F_c &= F_u + F_g \\ &= 7000 \left[\frac{(1+0.1)^{10} - 1}{0.1} \right] - 300 \left[\frac{(1+0.1)^{10} - (1 + 10 \times 0.1)}{0.1^2} \right] \\ &= \$93749.698 \end{aligned}$$

+5

(b) What uniform annual series of deposits ($n = 10$) would result in the same accumulated balance at the end of year 10.

$$\begin{aligned} A_g &= -300 \frac{(1+0.1)^{10} - (1 + 10 \times 0.1)}{0.1(1+0.1)^{10} - 0.1} \\ &= -1117.638 \end{aligned}$$

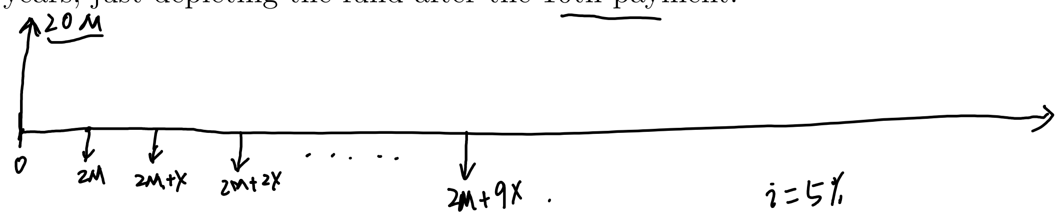
Hence,

$$A = A_1 + A_g = 7000 - 1117.638 = 5882.362.$$

A uniform annual series of deposits of \$5,882.362 would result in the same accumulated balance at the end of year 10. +5

$$A = F\left(\frac{i}{(1+i)^n - 1}\right) = 93749.698 \cdot \frac{0.1}{(1+0.1)^{10} - 1}$$

4. (12 pts) Piyush has recently inherited 20 million INR (Indian rupees) from his late Uncle Scrooge. To keep Piyush from spending his money immediately, Scrooge made arrangements for the inheritance to be deposited at the time of his death into an account paying 5%. Further arrangements instructed the bank to pay Piyush 2 million INR at the end of the 1st year, 2 $million + X$ INR at the end of the 2nd year, 2 $million + 2X$ INR at the end of the 3rd year, 2 $million + 3X$ INR at the end of the 4th year, and so on for a period of 10 years, just depleting the fund after the 10th payment.



(a) What is the value of X?

The composite series consists of a uniform series with $A_t = 2M$ INR for $t = 1, 2, \ldots, 10$ and a gradient series with $G = X$. Since the fund was depleted after the 10th payment, we have

$$\begin{aligned} P_c &= P_u + P_g \\ &= 2,000,000\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}} + X\frac{1-(1+10\times 0.05)(1+0.05)^{-10}}{0.05^2} \\ &= 20,000,000 \end{aligned}$$

Solving for X yields

$$X = 143,956.88.$$

+6

(b) How much is in the fund immediately after the 5th withdrawal?

Let F_5 be the future worth at $t = 5$ of 20 million INR at $t = 0$ and let F_{C5} be the future worth of the first five withdrawal at $t = 5$. Hence, we have

$$\begin{aligned} F_5 &= 20,000,000(1+0.05)^5 = 25525631.25 \\ F_{C5} &= F_{u5} + F_{g5} \\ &= 2,000,000\frac{(1+0.05)^5 - 1}{0.05} + 143,956.88\frac{(1+0.05)^5 - (1+5\times 0.05)}{0.05^2} \\ &= 12,564,627.196 \end{aligned}$$

Hence, there are

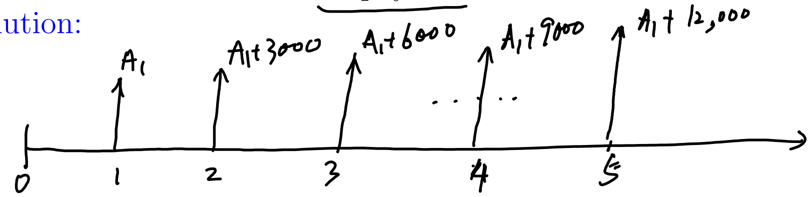
$$F_5 - F_{c5} = 25525631.25 - 12,564,627.196 = 12,961,004.054\ INR$$

in the fund immediately after the 5th withdrawal

+6

5. (8 pts) Land is purchased for \$75,000. It is agreed for the land to be paid for over a 5-year period with annual payments and using a 12% annual compound interest rate. Each payment is to be \$3,000 more than the previous payment. Determine the size of the last payment.

Solution:



Assume that the first payment is A_1. Then the composite series consists of a uniform series with $A_t = A_1$ for $t = 1, 2, 3, 4, 5$ and a gradient series with $G = \$3000$. To payoff the loan, the present worth of the composite series needs to be \$75,000 at $i = 12\%$ compounded annually. Hence, we have

$$\begin{aligned} P_c &= P_u + P_g \\ &= A_1 \frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} + 3000 \frac{1 - (1 + 5 \times 0.12)(1+0.12)^{-5}}{0.12^2} \\ &= 75,000 \end{aligned}$$

Solving for A_1 yields

$$A_1 = \$15481.946.$$

+6

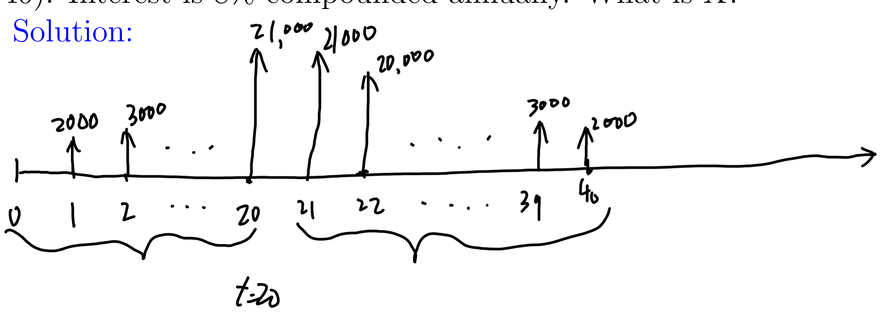
Hence, the last payment is

$$A_5 = A_1 + 4G = 15481.946 + 4 \times 3000 = \$27481.946.$$

+2

6. (10 pts) A cash flow profile starts with \$2,000 and increases by \$1,000 each year up to \$21,000 at time 20. Then, it starts again with \$21,000 at time 21 and decreases by \$1,000 each year to \$2,000 at year 40. You desire to convert it to an equivalent gradient series beginning at year 1 with \$X and continuing through year 40 with \$500 increases each year (ending at \$X + 19,500 at time 40). Interest is 8% compounded annually. What is X?

Solution:



- The future worth F^1_{c20} of the first 20 cash flows at $t = 20$. The composite series consists of a uniform series with $A_t = A_1 = 2000$ for $t = 1, 2, \ldots, 20$ and a gradient series with $G = 1000$.

$$F^1_{c20} = 2000\frac{(1+0.08)^{20}-1}{0.08} + 1000\frac{(1+0.08)^{20}-(1+20\times 0.08)}{0.08^2}$$
$$= 413,548.482$$

+2

- The future worth F^2_{c40} of the 20 cash flows at $t = 21, 22, \ldots, 40$ at $t = 40$. The composite series consists of a uniform series with $A_t = A_1 = 21000$ for $t = 21, 22, \ldots, 40$ and a gradient series with $G = -1000$. Hence, we have

$$F^2_{c40} = 21000\frac{(1+0.08)^{20}-1}{0.08} - 1000\frac{(1+0.08)^{20}-(1+20\times 0.08)}{0.08^2}$$
$$= 638,976.697$$

+2

- The future worth F_{equi} of the equivalent series at $t = 40$. The composite series consists of a uniform series with $A_t = A_1 = X$ for $t = 1, 2, \ldots, 40$ and a gradient series with $G = 500$. Hence

$$F_{equi} = X\frac{(1+0.08)^{40}-1}{0.08} + 500\frac{(1+0.08)^{40}-(1+40\times 0.08)}{0.08^2}$$
$$= 259.0565X + 1,369,103.242$$

+2

- The equation. Since you desire to convert it to a equivalent gradient series. The future values at $t = 40$ of these two cash flows should have the same value. That is,

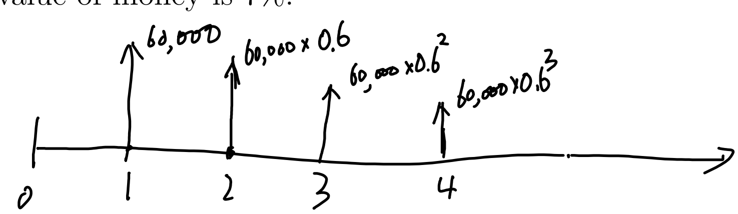
$$F^1_{c20}(1+0.08)^{20} + F^2_{c40} = F_{equi}$$
$$413,548.482 \times (1+0.08)^{20} + 638,976.697 = 259.0565X + 1,369,103.242$$

Solving for X yields

$$X = 4622.178.$$

+4

7. (5 pts) A famous high-volume calculus text generates royalties beginning with \$60,000 in the first year and declining each year by 40% of the previous year due to used sales and competition. The author is on a 4-year cycle of revision. Determine the present worth of one complete cycle of royalties if the author's time value of money is 7%.



Solution:

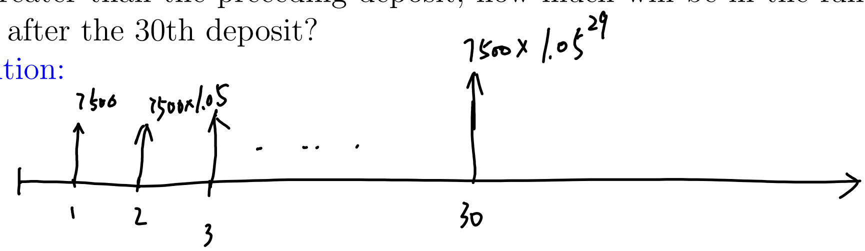
The complete cycle of royalties are a geometric cash flow series with $A_1 = 60,000$, $j = -0.4$, and $n = 4$. Hence the present worth of one complete cycle of royalties with $i = 0.07$ is

$$P = 60,000 \left[\frac{1 - (1 - 0.4)^4 (1 + 0.07)^{-4}}{\underbrace{0.07 + 0.4}_{i - j}} \right]$$
$$= 115037.717$$

+5

8. (5 pts) Suppose you make 30 annual investments in a fund that pays 5% compounded annually. If your first deposit is $7,500 and each successive deposit is 5% greater than the preceding deposit, how much will be in the fund immediately after the 30th deposit?

Solution:



These 30 annual deposits form~~s~~ a geometric cash flow series with $A_1 = 7500$, $j = 0.05$, and $n = 30$. Since the interest rate is $i = 0.05$, which is the same as j. The amount in the fund immediately after the 30th deposit is

$$F = nA_1(1 + i)^{n-1}$$
$$= 30 \times 7500 \times (1 + 0.05)^{30-1}$$
$$= 926,130.509$$

+5