ENSC 2113
Engineering Mechanics: Statics

Chapter 2:

Force Vectors

(Sections 2.9)

COLLEGE OF
ENGINEERING, ARCHITECTURE
AND TECHNOLOGY

Chapter 2 Outline:

2.1 Scalars and Vectors

2.2 Vector Operations

2.3 Vector Addition of Forces

2.4 Addition of a System of Coplanar Forces

2.5 Cartesian Vectors

2.6 Addition of Cartesian Vectors

2.7 Position Vectors

2.8 Force Vector Directed Along a Line

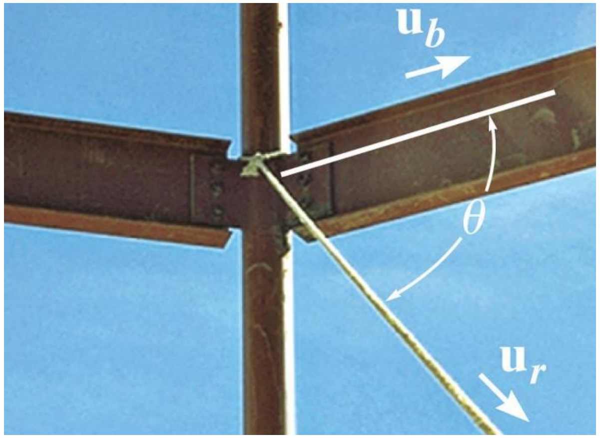
2.9 Dot Product

Chapter 2 Objectives:

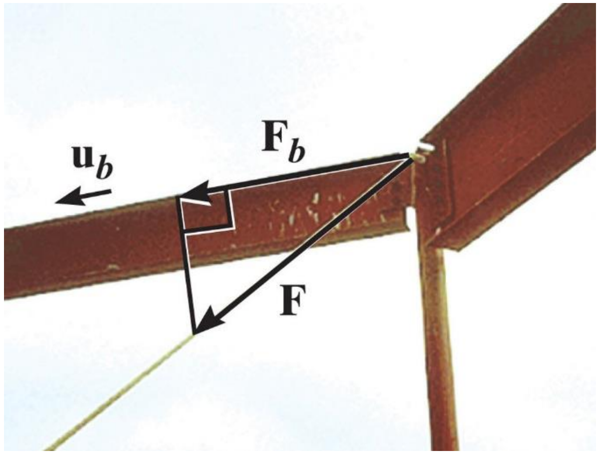
- To show how to add forces and resolve them into components using the Parallelogram Law
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another

2.9 Dot Product:

- Dot Product may be used to:
  - Find the angle between two vectors or intersecting lines



  - Find the components of a vector parallel and perpendicular to a line



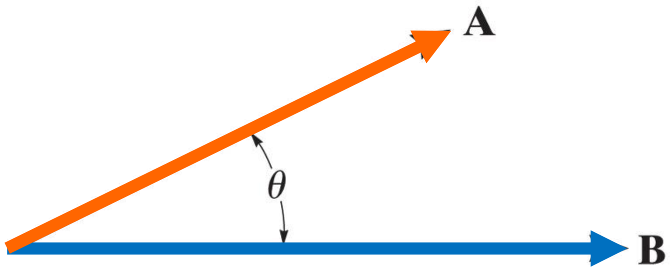
2.9 Dot Product:

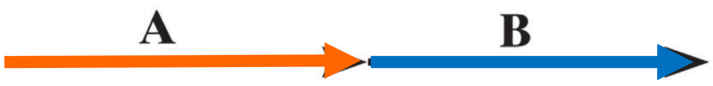
- Cartesian Vector Formulation:

  If θ=0, the vectors are aligned and directed along the same line/axis.

$$\cos\theta = \cos 0^\circ = 1$$

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$

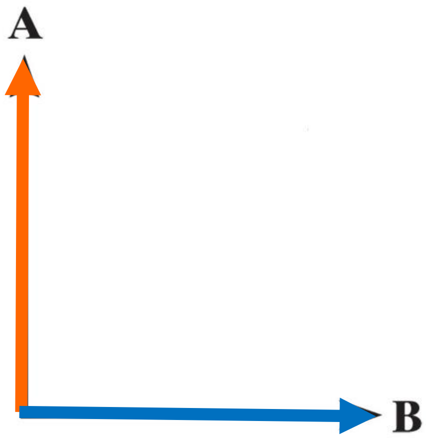




  If θ=90, the vectors are perpendicular to one another.

$$\cos\theta = \cos 90^\circ = 0$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{\imath}\cdot\hat{k} = 0$$



2.9 Dot Product:

- If two vectors are expressed in Cartesian vector form, the dot product is determined by multiplying the x, y, and z scalar components and adding the results:

$$\vec{A} = \{A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\}$$

$$\vec{B} = \{B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\}$$

$$\boxed{\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

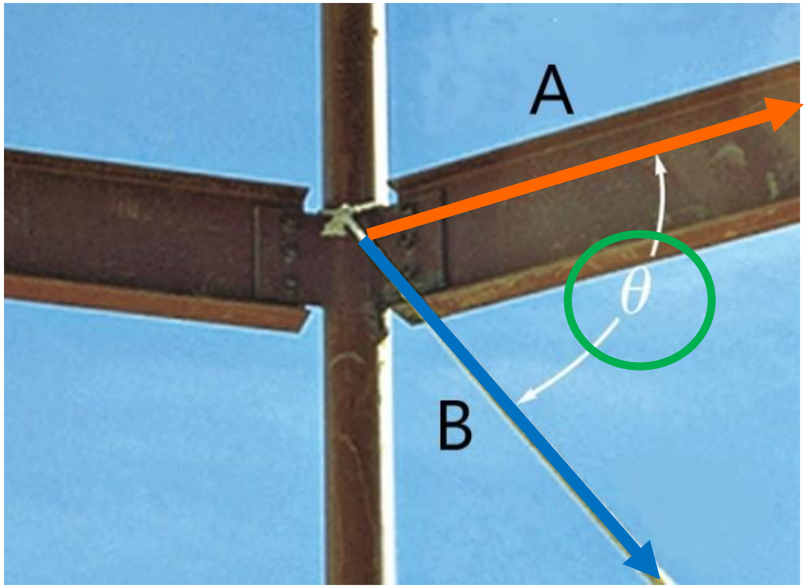
2.9 Dot Product:

- To find the angle between two vectors or intersecting lines:

$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$$

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|A||B|}\right]$$

$$0^\circ \le \theta \le 180^\circ$$

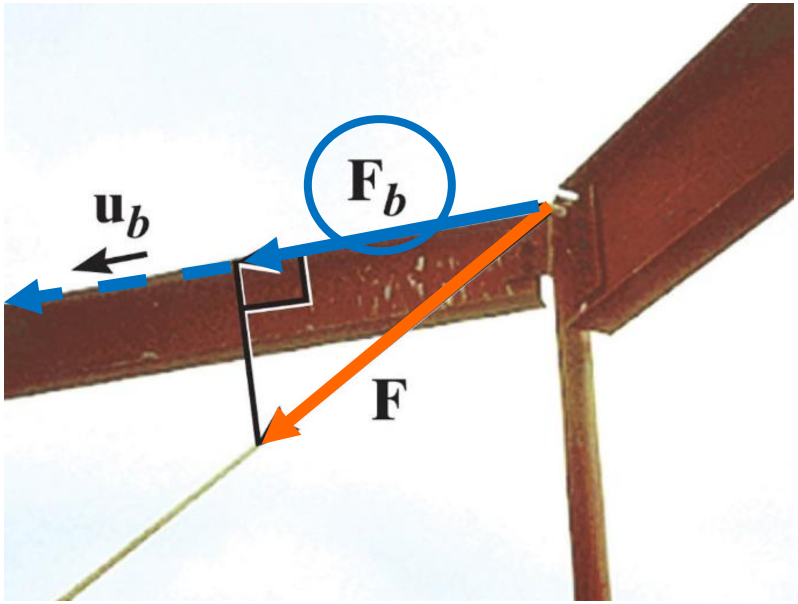


2.9 Dot Product:

- To find the magnitudes of the components of a vector parallel and perpendicular to a line

  unit vector of the line is used for the dot product

  - For a force component parallel to the line B:



$$|F_b| = \vec{F} \cdot \overrightarrow{u_b}$$

$$|F_b| = |F| \cos\theta$$

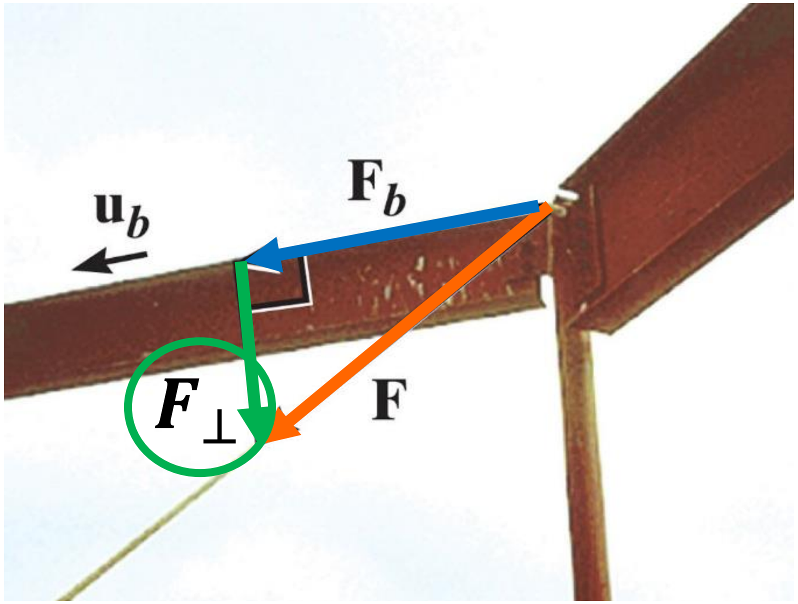
2.9 Dot Product:

- To find the magnitudes of the components of a vector parallel and perpendicular to a line
  - For a force component perpendicular to the line B:

$$\overrightarrow{F_\perp} = \vec{F} - \overrightarrow{F_b}$$

$$\overrightarrow{F_\perp} = |F| \sin\theta$$

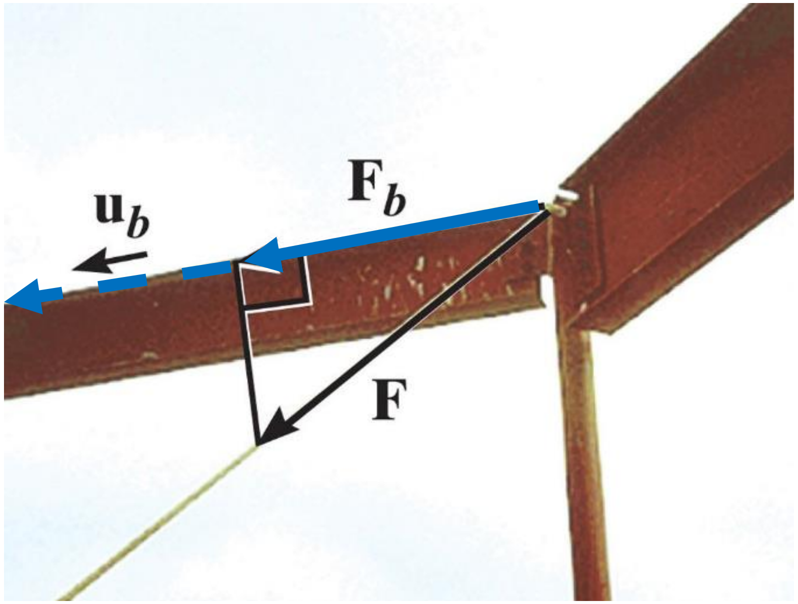
$$\overrightarrow{F_\perp} = \sqrt{F^2 - {F_b}^2}$$



2.9 Dot Product:

- To find the magnitudes of the components of a vector parallel and perpendicular to a line
  - For the vector of the component force F_b, multiply the component magnitude by the unit vector

$$\overrightarrow{F_b} = |F_b|\overrightarrow{u_b}$$



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