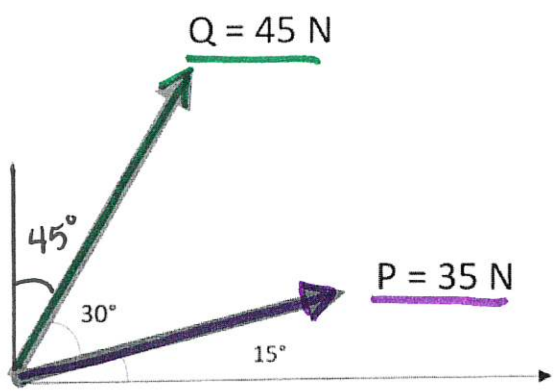
ENSC 2113 – Fall 2023

Homework #1

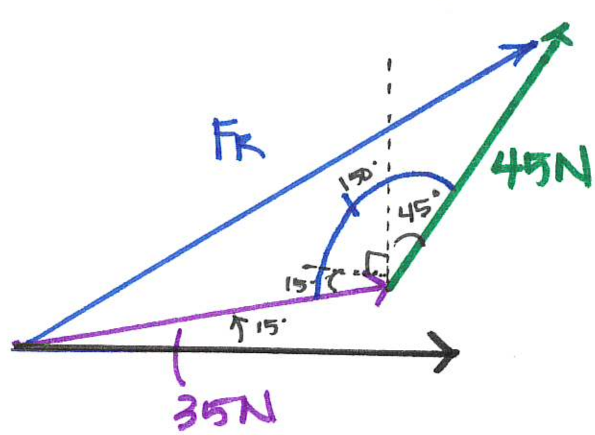
Work each problem on a separate piece of paper using the homework format outlined on course website (Canvas).

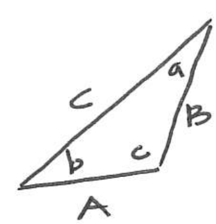
Submit handwritten work as a single PDF to the course website. Due Friday, September 15 by 8:45 am.

Problem #1 (10 pts):



Calculate the resultant of the two forces using the **parallelogram law**. Show all diagrams and work.





$C^2 = A^2 + B^2 - 2AB\cos C$

$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin C}$

ANGLE C = $15° + 90° + 45° = \underline{\underline{150°}}$

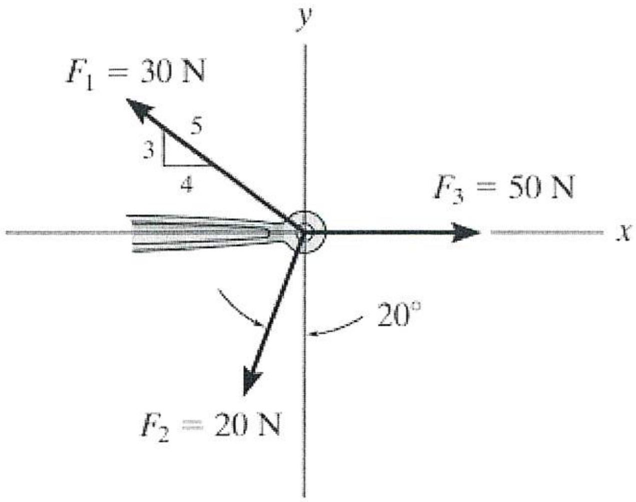
LAW OF COS...

$C^2 = A^2 + B^2 - 2AB\cos C$

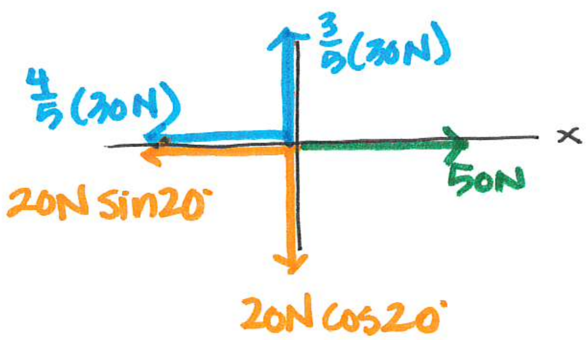
$C^2 = (35N)^2 + (45N)^2 - 2(35N)(45N)\cos 150° = 5977.98 N^2$

$\boxed{C = F_R = 77.3 N}$

Problem #2 (10 pts):



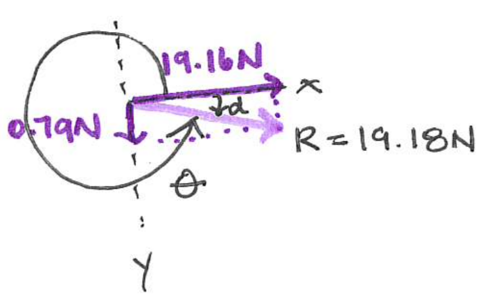
Using **algebraic equations**, calculate the magnitude and angle measured counterclockwise from the positive x-axis of the resultant force. Write the resultant as a Cartesian vector.



$$R_x = \Sigma F_x \rightarrow \; = -\frac{4}{5}(30N) - 20N \sin 20^\circ + 50N = 19.16N \longrightarrow$$

$$R_y = \Sigma F_y \uparrow = \frac{3}{5}(30N) - 20N \cos 20^\circ = -0.79N = 0.79N \downarrow$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(19.16N)^2 + (-0.79N)^2} = 19.18 \text{ N}$$



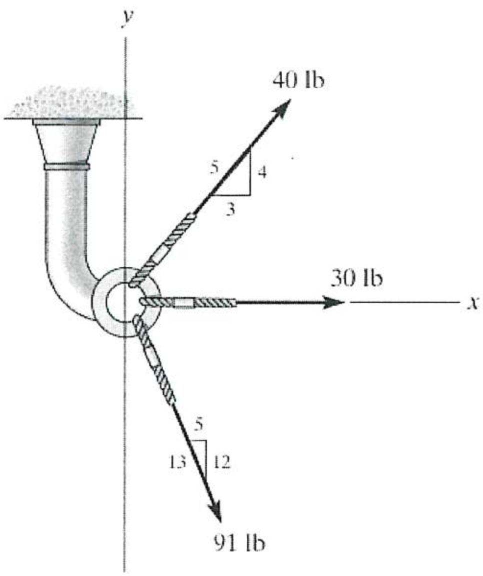
$$\theta = 360^\circ - \alpha$$

$$\alpha = \tan^{-1}\left[\frac{0.79N}{19.16N}\right] = 2.37^\circ$$

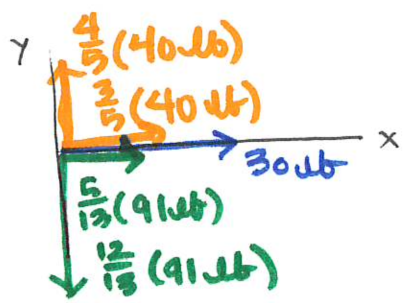
$$\theta = 360^\circ - 2.37^\circ = 357.6^\circ$$

$$\boxed{R = 19.18N \; @ \; \theta = 357.6^\circ}$$

Problem #3 (10 pts):



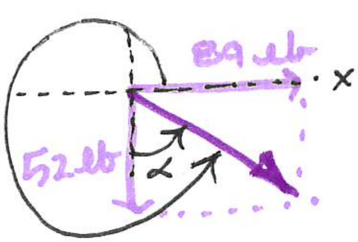
Using **algebraic equations**, calculate the magnitude and angle measured counterclockwise from the positive x-axis of the resultant force. Write the resultant as a Cartesian vector.



$R_x = \Sigma F_x \rightarrow = \frac{3}{5}(40\,lb) + 30\,lb + \frac{5}{13}(91\,lb) = 89\,lb \rightarrow$

$R_y = \Sigma F_y \uparrow = \frac{4}{5}(40\,lb) - \frac{12}{13}(91\,lb) = -52\,lb = 52\,lb \downarrow$

$R = \sqrt{(89\,lb)^2 + (-52\,lb)^2} = 103.1\,lb$



θ

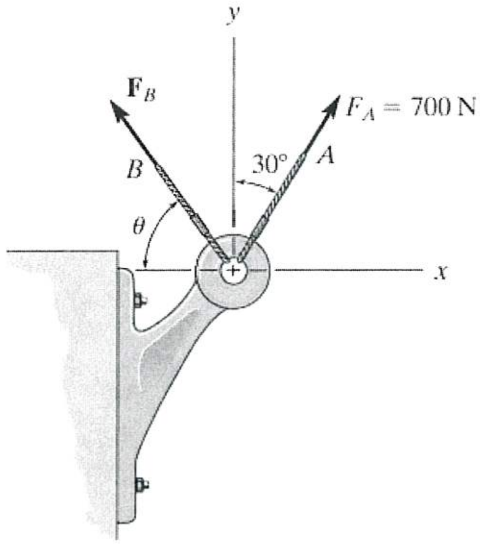
$\theta = 270 + \alpha$

$\alpha = \tan^{-1} \frac{89\,lb}{52\,lb} = 59.7°$

$\theta = 270 + 59.7° = 329.7°$

$\boxed{R = 103.1\,lb \text{ @ } \theta = 329.7°}$

Problem #4 (10 pts):



Calculate the magnitude and angle θ of $\mathbf{F_B}$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.

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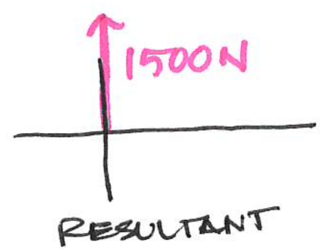


$R_x = \sum F_x \rightarrow \qquad -F_B\cos\theta + 700N\sin 30 = 0$ ①

$R_y = \sum F_y \uparrow \qquad F_B\sin\theta + 700N\cos 30 = 1500N$ ②

① $F_B\cos\theta = 700N\sin 30 = 350\,N$

② $F_B\sin\theta = 1500N - 700N\cos 30 = 893.78\,N$

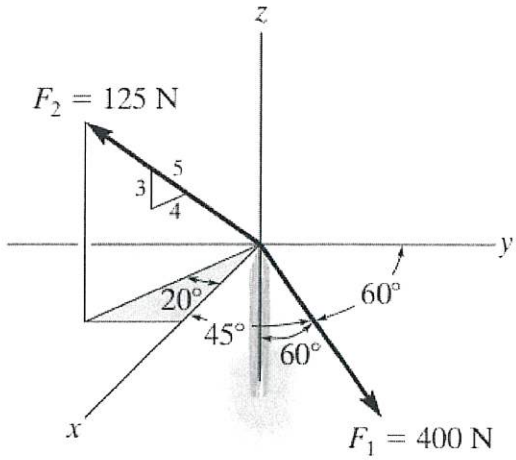
$\tan\theta = \frac{\sin\theta}{\cos\theta} \rightsquigarrow \frac{②}{①} \rightsquigarrow \frac{\cancel{F_B}\sin\theta}{\cancel{F_B}\cos\theta} = \frac{893.78N}{350\,N}$

$\tan\theta = 2.554 \rightsquigarrow \boxed{\theta = 68.6°}$

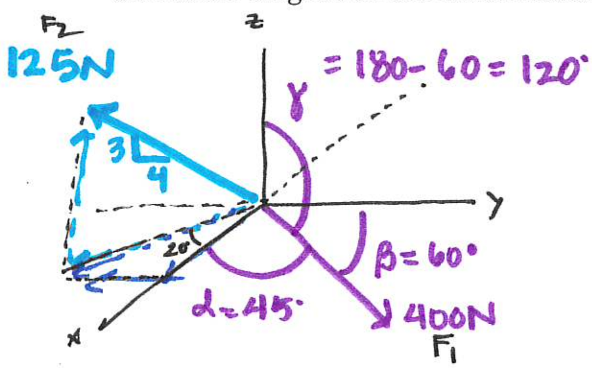
$F_B\cos 68.6° = 350N$

$\boxed{F_B = 960N}$

Problem #5 (15 pts):



Write each force in **Cartesian Vector** form then calculate the magnitude and direction angles of the resultant force.



$$\vec{F}_1 = 400N\ \{\cos 45\ \bar{i} + \cos 60\ \bar{j} + \cos 120\ \bar{k}\}$$

$$= \{400 \cos 45\ \bar{i} + 400 \cos 60\ \bar{j} + 400 \cos 120\ \bar{k}\} N$$

$$\boxed{\vec{F}_1 = \{282.8\ \bar{i} + 200\ \bar{j} - 200\ \bar{k}\} N}$$

$$\vec{F}_2 = \{125N\left(\tfrac{4}{5}\right)(\cos 20)\ \bar{i} - 125N\left(\tfrac{4}{5}\right)\sin 20\ \bar{j} + 125N\left(\tfrac{3}{5}\right)\bar{k}\}$$

$$\boxed{\vec{F}_2 = \{93.97\ \bar{i} - 34.2\ \bar{j} + 75\ \bar{k}\} N}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = \{(282.8 + 93.97)\bar{i} + (200 - 34.2)\bar{j} + (-200 + 75)\bar{k}\} N$$

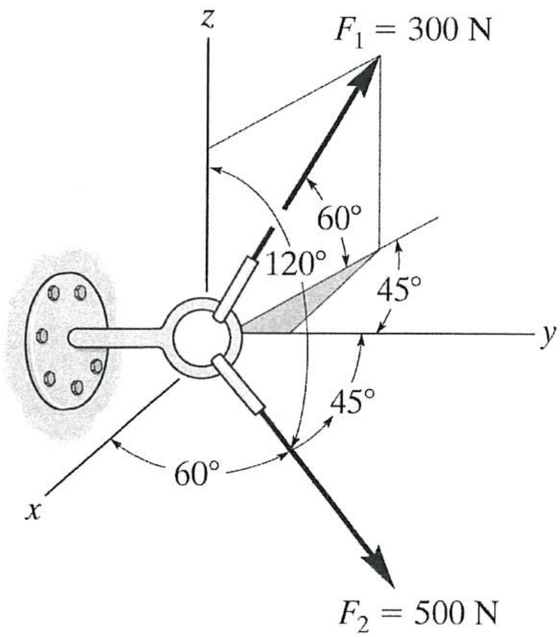
$$\vec{R} = \{376.8\ \bar{i} + 165.8\ \bar{j} - 125\ \bar{k}\} N$$

$$R = \sqrt{(376.8N)^2 + (165.8N)^2 + (-125N)^2} \rightarrow \boxed{R = 430 N}$$

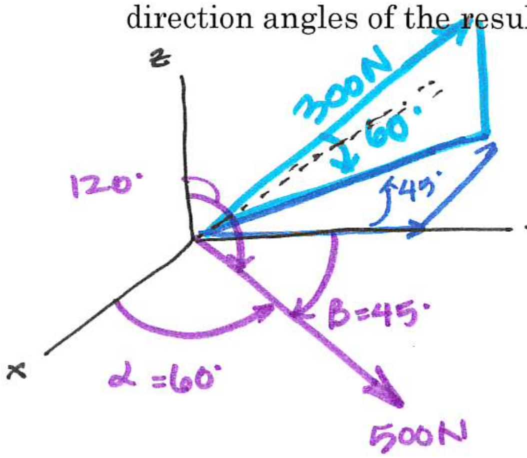
$$\alpha = \tan^{-1}\frac{R_x}{R} = \tan^{-1}\frac{376.8N}{430N} \qquad \beta = \tan^{-1}\frac{165.8N}{430N} \qquad \gamma = \tan^{-1}\frac{-125N}{430N}$$

$$\boxed{\alpha = 41.2°} \qquad \boxed{\beta = 21.1°} \qquad \boxed{\gamma = -16.2°}$$

Problem #6 (15 pts):



Write each force in **Cartesian Vector** form then calculate the magnitude and direction angles of the resultant force.



$$\vec{F}_1 = \{-300\cos 60 \sin 45° \,\vec{i} + 300\cos 60 \cos 45 \,\vec{j} + 300 \sin 60 \,\vec{k}\}$$

$$\boxed{\vec{F}_1 = \{-106.1\,\vec{i} + 106.1\,\vec{j} + 259.8\,\vec{k}\}\text{N}}$$

$$\vec{F}_2 = \{500\cos 60 \,\vec{i} + 500 \cos 45 \,\vec{j} + 500 \cos 120 \,\vec{k}\}\text{N}$$

$$\boxed{\vec{F}_2 = \{250\,\vec{i} + 353.6\,\vec{j} - 250\,\vec{k}\}\text{N}}$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = \{(-106.1+250)\vec{i} + (106.1+353.6)\vec{j} + (259.8-250)\vec{k}\}\text{N}$$

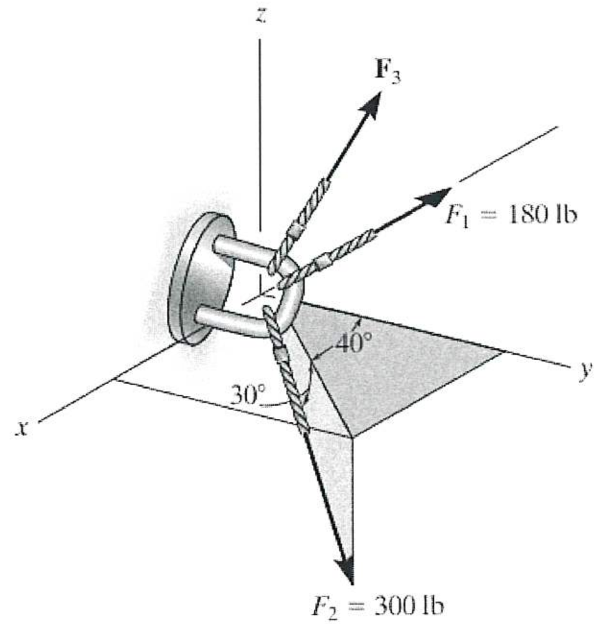
$$= \{143.9\,\vec{i} + 459.7\,\vec{j} + 9.8\,\vec{k}\}\text{N}$$

$$|R| = \sqrt{(143.9)^2 + (459.7)^2 + (9.8)^2} = 481.8\text{N}$$

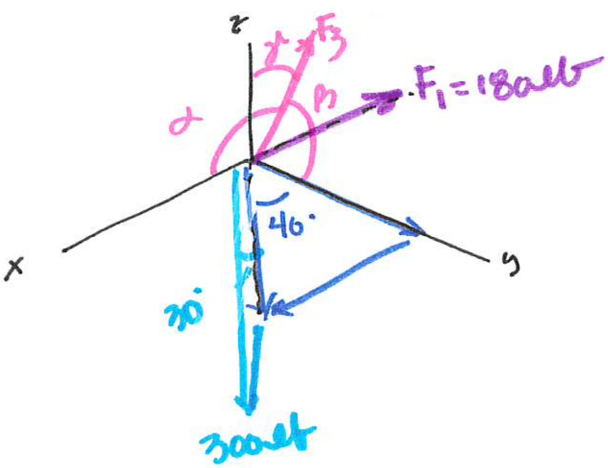
$$\alpha = \tan^{-1}\frac{143.9\text{N}}{481.8\text{N}} \qquad \beta = \tan^{-1}\frac{459.7\text{N}}{481.8\text{N}} \qquad \gamma = \tan^{-1}\frac{9.8\text{N}}{481.8\text{N}}$$

$$\boxed{\alpha = 16.6°} \qquad \boxed{\beta = 43.7°} \qquad \boxed{\gamma = 1.2°}$$

Problem #7 (15 pts):



Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero. $\boxed{\text{Hint: } \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$



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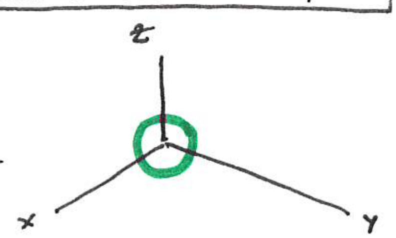


$R = \{0\,i + 0\,j + 0\,k\}$

$\vec{F}_1 = \{-180\,i + 0\,j + 0\,k\}$

$\vec{F}_2 = \{300\cos 30 \sin 40\,i + 300 \cos 30 \cos 40\,j - 300 \sin 30\,k\}$ lb

$= \{167\,i + 199\,j - 150\,k\}$ lb

$\vec{F}_3 = \{F\cos\alpha\,i + F\cos\beta\,j + F\cos\gamma\,k\}$ lb

$\Sigma F_x = R_x \qquad -180 + 167 + F\cos\alpha = 0 \qquad F\cos\alpha = 13$ lb $\rightarrow \cos\alpha = \frac{13}{F}$

$\Sigma F_y = R_y \qquad 199 + F\cos\beta = 0 \qquad F\cos\beta = -199$ lb $\rightarrow \cos\beta = \frac{-199}{F}$

$\Sigma F_z \uparrow = R_z \qquad -150 + F\cos\gamma = 0 \qquad F\cos\gamma = 150$ lb $\rightarrow \cos\gamma = \frac{150}{F}$

$\left(\frac{13}{F}\right)^2 + \left(\frac{-199}{F}\right)^2 + \left(\frac{150}{F}\right)^2 = 1 \qquad \boxed{F = 249.5 \text{ lb}}$

$62270 = F^2$

PROBLEM #7 (con't)

$$\boxed{F = 249.5\,lb}$$

$F\cos\alpha = 13\,lb$

$F\cos\beta = -199\,lb$

$F\cos\gamma = 150\,lb$

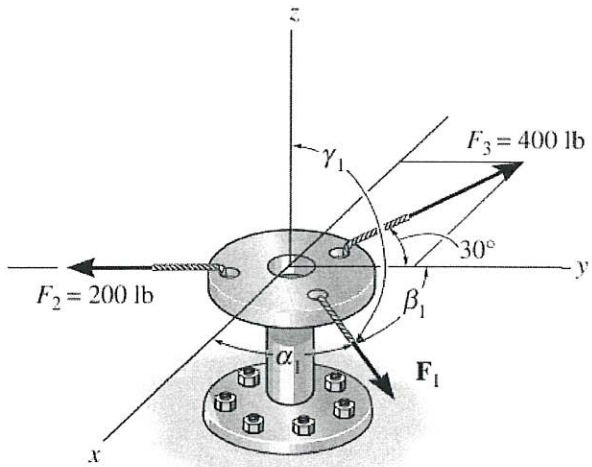
$\alpha = \cos^{-1}\frac{13\,lb}{249.5\,lb}$

$\beta = \cos^{-1}\frac{-199\,lb}{249.5\,lb}$

$\gamma = \cos^{-1}\frac{150\,lb}{249.5\,lb}$

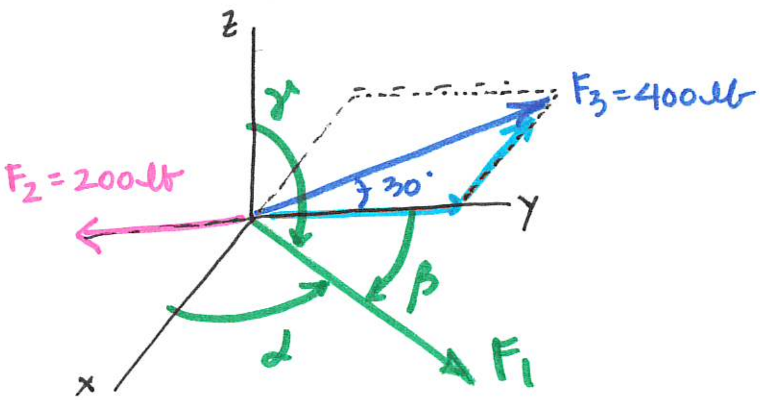
$$\boxed{\begin{aligned}\alpha &= 87^\circ\\ \beta &= 142.9^\circ\\ \gamma &= 53^\circ\end{aligned}}$$

Problem #8 (15 pts):



Specify the magnitude and coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb. Note that $\mathbf{F}_3$ lies in the x-y plane.

↳ z axis



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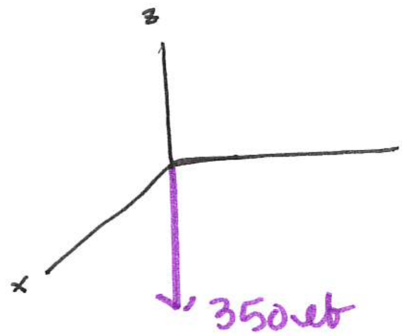

$\vec{F}_R = \{0\bar{i} + 0\bar{j} - 350\bar{k}\}$ lb

$\vec{F}_1 = \{F_1\cos\alpha\,\bar{i} + F_1\cos\beta\,\bar{j} + F_1\cos\gamma\,\bar{k}\}$

$\vec{F}_2 = \{0\bar{i} - 200\bar{j} + 0\bar{k}\}$ lb

$\vec{F}_3 = \{-400\sin 30\,\bar{i} + 400\cos 30\,\bar{j} + 0k\}$ lb

$\Sigma F_x = R_x$ $\qquad F_1\cos\alpha - 400\sin 30 = 0$ $\qquad \cos\alpha = \dfrac{400\sin 30}{F_1}$

$\Sigma F_y = \vec{R_y}$ $\qquad F_1\cos\beta - 200 + 400\cos 30 = 0$ $\qquad \cos\beta = -\dfrac{146.4}{F_1}$

$\Sigma F_z = R_z$ $\qquad F_1\cos\gamma = -350$ lb $\qquad \cos\gamma = \dfrac{-350}{F_1}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\left(\dfrac{200}{F_1}\right)^2 + \left(-\dfrac{146.4}{F_1}\right)^2 + \left(\dfrac{-350}{F_1}\right)^2 = 1$

$$\boxed{\begin{aligned} F_1 &= 428.9 \text{ lb} \\ \alpha &= 62.2° \\ \beta &= 110° \\ \gamma &= 144.7° \end{aligned}}$$