Class Test III

1. Calculate the following limits

(1) $\lim_{x\to 0} \frac{e^x - x - 1}{3x^2 + 2x}$

(2) $\lim_{\theta \to \pi/2^-} (\tan\theta - \sec\theta)$

(3) $\lim_{x\to\infty} \frac{e^{3x}}{3e^{3x} + 5}$

(4) $\lim_{x\to\infty} \frac{\sin^4 x}{\sqrt{x}}$

(5) $\lim_{x\to 0^+} \left(\frac{1}{x}\right)^{\tan x}$

2. Suppose the derivative of a function f is $f'(x) = (x+1)^2(x-3)^5(x-6)^4$. On what interval f is increasing?

3. Show that the curve $y = (1+x)/(1+x^2)$ has three points of inflection and they all lie on one straight line.

4. Find f

(1) $f''(x) = \frac{2}{3}x^{2/3}$

(2) $f'(x) = \sqrt{x}(6+5x),\ f(1) = 10$

(3) $f''(t) = 3/\sqrt{t}\,,\quad f(4) = 20, f'(4) = 7$

5. Calculate the definite integrals

(1) $\int_0^{\pi} \sqrt{\sin^3 x - \sin^5 x}\, dx$

(2) $\int_0^4 \frac{x}{\sqrt{1+2x}}\, dx$