Engineering Economic Analysis Homework #3
Fall 2024

Instructions:

Please attempt every problem. You must support every solution with an appropriate amount of work and/or description. Unsupported answers may receive a score of 0. Good luck!

1. (10 pts) Using a 5% annual compound interest rate, what investment today is needed in order to withdraw \$5,000 annually

   (a) for the next 10 years with the first withdrawal at the end of year 1.
   The investment is

   $$P_a = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 5000\left[\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}}\right] = \$38608.675.$$

   +4

   (b) for 10 years if the first withdrawal occurs at the end of year 3.
   The investment is

   $$P_b = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right](1+i)^{-2} = 5000\left[\frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}}\right](1+0.05)^{-3} = \$35019.20.$$

   +6

2. (4 pts) A debt of \$1,000 is incurred at $t = 0$. What is the amount of four equal payments at t = 1, 2, 3, and 4 that will repay the debt if money is worth 10% compounded per period?
   Solution:
   The amount is

   $$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = 1000 \times \left[\frac{0.1(1+0.1)^4}{(1+0.1)^4 - 1}\right] = \$315.471.$$

   +4

3. (8 pts) Five deposits of \$500 each are made a $t = 1, 2, 3, 4$, and 5 into a fund paying 6% compounded per period. How much will be accumulated in the fund at (a) $t = 5$, and (b) $t = 10$?
   Solution:

(a) At $t = 5$, there will be

$$F_5 = 500\left[\frac{(1+0.06)^5 - 1}{0.06}\right] = \$2818.546$$

in the fund. +4

(b) At $t = 10$, the lump sum at $t = 5$ will compound for another 5 years at the same interest rate. Hence, there will be

$$F_{10} = F_5(1+0.06)^5 = \$3771.851$$

+4

4. (8 pts) What equal annual deposits must be made at t = 2, 3, 4, 5, and 6 in order to accumulate \$25,000 at t = 8 if money is worth 10% compounded annually?
Solution:
There are in total $n = 5$ deposits. The worth of these deposits should be of the same worth as the worth of the \$25,000 at $t = 8$ at 10% rate at the same point of time. Hence, we have, at $t = 6$,

$$A\left[\frac{(1+0.1)^5 - 1}{0.1}\right] = 25000(1+0.1)^{-2}$$
$$A = \$3384.245$$

+8

5. (4 pts) Adriana wishes to accumulate \$2,000,000 in 35 years. If 35 end-of-year deposits are made into an account that pays interest at a rate of 7% compounded annually, what size deposit is required each year to meet Adriana's stated objective?
Solution:
The deposit should be

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] = 2,000,000\left[\frac{0.07}{(1+0.07)^{35} - 1}\right] = \$14467.919.$$

+4

6. (6 pts) You take out a loan to buy a new audio system. Your equal annual payments are 20% of the amount you borrowed. The interest rate on the loan is 7% compounded annually. Determine the number of years you will be required

to make payments. (This number may be a nonwhole one such as 4.791, for example.)

Solution:

Assume the equal annual payments is A, then the amount I borrowed is $P = 5A$. Hence we have

$$5A = A\left[\frac{(1+0.07)^n - 1}{0.07(1+0.07)^n}\right] \Rightarrow 5 = \frac{(1+0.07)^n - 1}{0.07(1+0.07)^n}.$$

Solving for n yields

$$n = 6.367$$

+6

7. (8 pts) How much money can be withdrawn at the end of the investment period if \$18,000 is invested at the end of each of 5 years at 9%/year compounded annually, with the lump sum then shifted into an investment paying 7%/year for 8 additional years?

   Solution:

   The future value of the uniform series at the end of the 5 years is

$$F_5 = 18000\left[\frac{(1+0.09)^5 - 1}{0.09}\right] = 107,724.791.$$

   The lump sum then shifted into an investment paying 7%/year for 8 additional years. Hence the amount at the end of the investment period is

$$F_{end} = 107,724.791(1+0.07)^8 = 185091.2471.$$

   +8

8. (10 pts) You decide to open an IRS approved retirement account at your local brokerage firm. Your best estimate is that it will earn 9%. At the end of each year for the next 25 years, you will deposit \$4,000 per year into the account (25 total deposits). Three years after the last deposit, you will begin making annual withdrawals with the first withdrawal right after 3 years of the last deposit.

   (a) How much money is in the account one year before the first withdrawal?

   The future worth of these deposits right after the last deposit is

$$F_{25} = 4000\left[\frac{(1+0.09)^{25} - 1}{0.09}\right] = 338803.5849.$$

   One year before the first withdrawal (that is, two years after the last deposit), there are

$$P_{27} = F_{25}(1+0.09)^2 = 402532.5392.$$

   in the account. +6

(b) If you want to make 30 annual withdrawals, what amount will you be able to withdraw each year?
The amount you can withdraw is

$$A = P_{27}\left[\frac{0.09(1+0.09)^{30}}{(1+0.09)^{30}-1}\right] = 39181.049$$

each year. +4