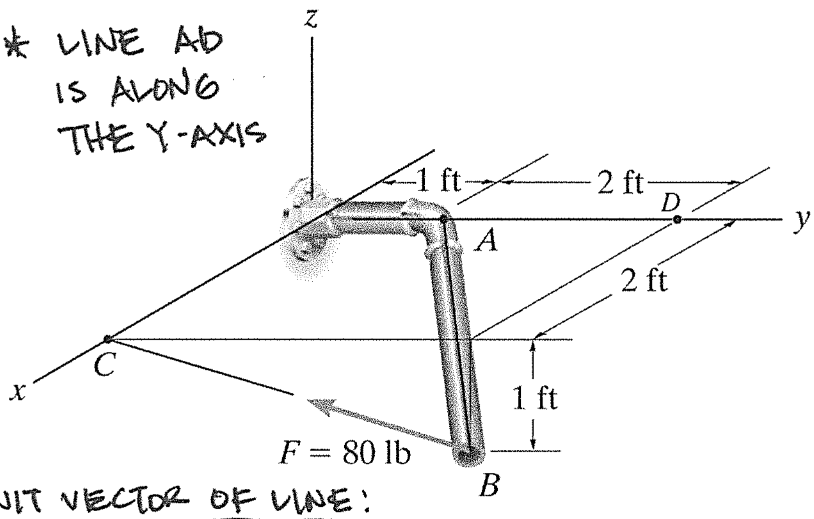
Name: ______________________________ CWID #:________________ Section Number:_______

ENSC 2113 – SPRING 2018 – FINAL EXAM

EACH PROBLEM IS WORTH 25 PTS. BOX YOUR ANSWERS AND PROVIDE PROPER UNITS, WHERE APPLICABLE. CALCULATIONS AND FREE BODY DIAGRAMS MUST BE SHOWN THAT SUPPORT THE ANSWER TO RECEIVE CREDIT.

1. A force with a magnitude of 80 lb follows along a line from B to C. Determine the moment created by the force about a line defined from A to D. Express the result as a Cartesian Vector.

* LINE AD IS ALONG THE Y-AXIS



COORDINATES:

A (0, 1, 0)
B (2, 3, −1)
C (2, 0, 0)
D (0, 3, 0)

POSITION VECTORS:

$\overline{r_{AC}} = \{2\hat{i} - 1\hat{j} + 0\hat{k}\}$
$\overline{r_{DC}} = \{2\hat{i} - 3\hat{j} + 0\hat{k}\}$
$\overline{r_{AB}} = \{2\hat{i} + 2\hat{j} - 1\hat{k}\}$
$\overline{r_{DB}} = \{2\hat{i} + 0\hat{j} - 1\hat{k}\}$
$\overline{r_{OC}} = \{2\hat{i} + 0\hat{j} + 0\hat{k}\}$
$\overline{r_{OB}} = \{2\hat{i} + 3\hat{j} - 1\hat{k}\}$

UNIT VECTOR OF LINE:

$\overline{u_{AD}} = \{0\hat{i} + 1\hat{j} + 0\hat{k}\}$

FORCE VECTOR:

$\overline{r_{BC}} = \{0\hat{i} - 3\hat{j} + 1\hat{k}\}$

$|r_{BC}| = \sqrt{0^2 + (-3)^2 + (1)^2}$
$= \sqrt{10}$

$\overline{u_{BC}} = \{0\hat{i} - \frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}\}$

$\overline{F_{BC}} = |F|\overline{u_{BC}}$
$= 80\{0\hat{i} - \frac{3}{\sqrt{10}}\hat{j} + \frac{1}{\sqrt{10}}\hat{k}\}$
$= \{0\hat{i} - 75.9\hat{j} + 25.3\hat{k}\}$ LB

MOMENT ABOUT A LINE:

$|M| = (\bar{r} \times \bar{F}) \cdot \bar{u}$

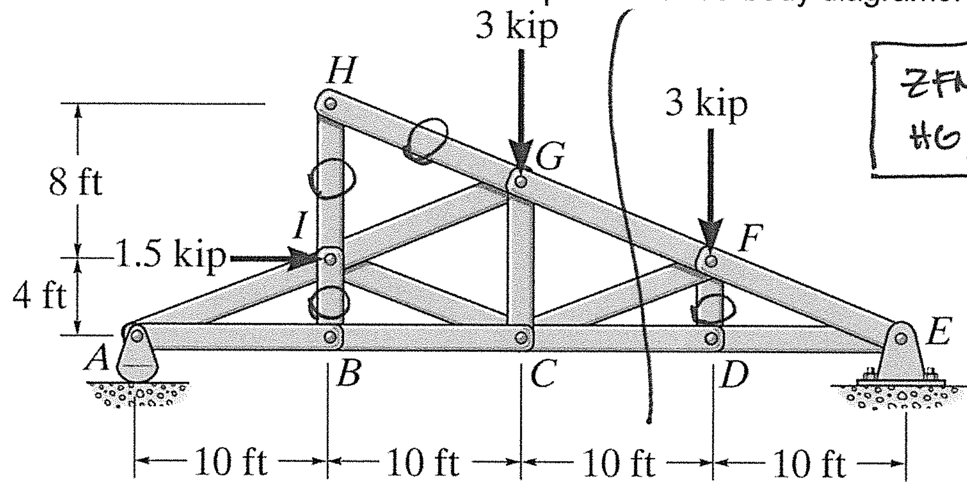
$$= \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & -75.9 & 25.3 \end{vmatrix}$$

$= [(0)(25.3) - (0)(-75.9)](0)$
$- [(2)(25.3) - (0)(0)](1)$
$+ [(2)(-75.9) - (0)(0)](0)$

$= -50.6$

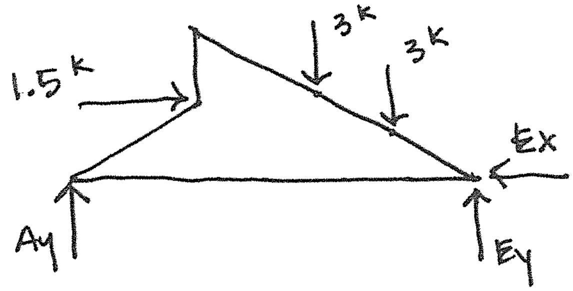
$\overline{M} = -50.6\{0\hat{i} + 1\hat{j} + 0\hat{k}\} = \boxed{\{50.6\hat{j}\} \text{ LB·FT}}$

2. Determine the force in members GF, CF, and CD by the method of sections and the force in member FE by method of joints and indicate if the members are in tension or compression. The truss is supported with a rocker at A and a pin at E. List all zero-force members and draw all pertinent free-body diagrams.



ZFM'S: HI, HG, BI, DF

OVERALL FBD!



$+\circlearrowleft \Sigma M_A = 0 = -1.5(4) - 3(20) - 3(30) + E_y(40)$

$E_y = 3.9^k \uparrow$

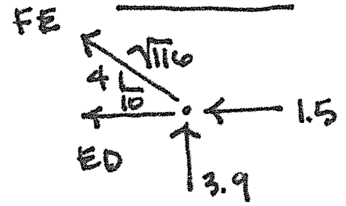
$\uparrow \Sigma F_y = 0 = A_y - 3 - 3 + E_y$

$A_y = 2.1^k \uparrow$

$\xrightarrow{+} \Sigma F_x = 0 = 1.5 - E_x$

$E_x = 1.5^k \leftarrow$

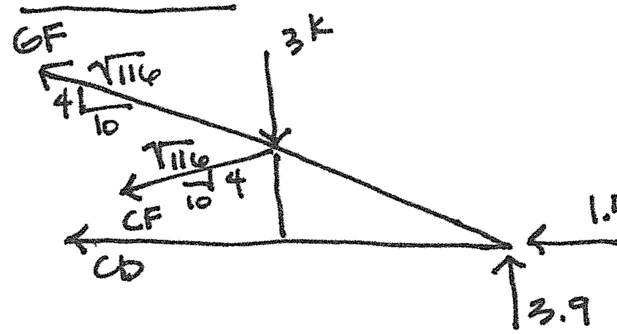
JOINT E!



$\uparrow \Sigma F_y = 0 = 3.9 + \frac{4}{\sqrt{116}} FE$

$\boxed{FE = 10.5^k \text{ (C)}}$

CUT FBD!



$+\circlearrowleft \Sigma M_F = 0 = -CD(4) + 3.9(10) - 1.5(4)$

$\boxed{CD = 8.25^k \text{ (T)}}$

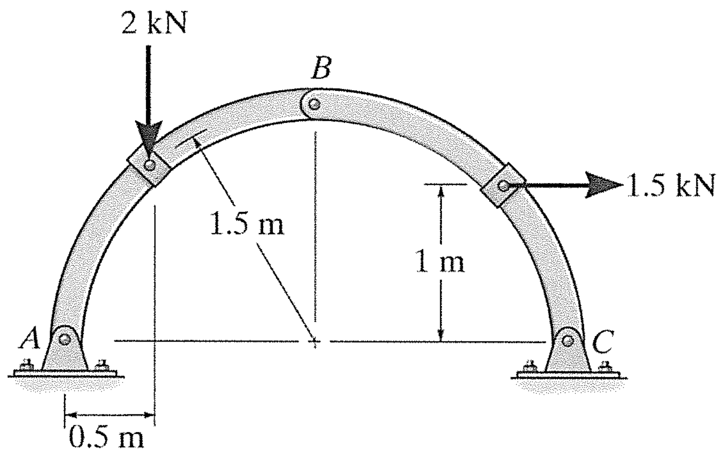
$+\circlearrowleft \Sigma M_C = 0 = \frac{10}{\sqrt{116}} GF(4) + \frac{4}{\sqrt{116}} GF(10) - 3(10) + 3.9(20)$

$\boxed{GF = 6.46^k \text{ (C)}}$

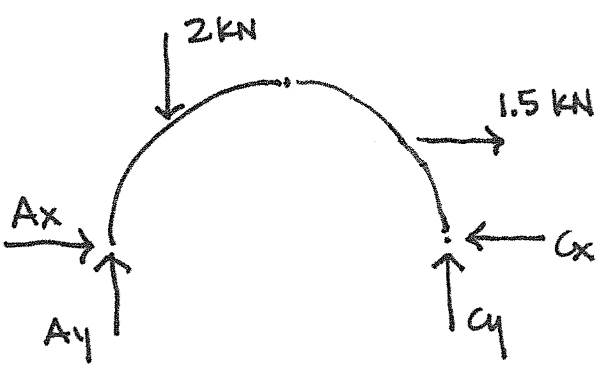
$\uparrow \Sigma F_y = 0 = \frac{4}{\sqrt{116}}(-6.46) - 3 + 3.9 - \frac{4}{\sqrt{116}} CF$

$\boxed{CF = 4.04^k \text{ (C)}}$

3. Given the two-member frame, made up of members AB and BC, calculate the external support reactions for the pins at A and C. Member AB is pinned to member BC with an internal pin at point B. Draw all pertinent free-body diagrams and indicate direction with arrows in your answer. The locations of the applied forces are not pins.



OVERALL FBD:



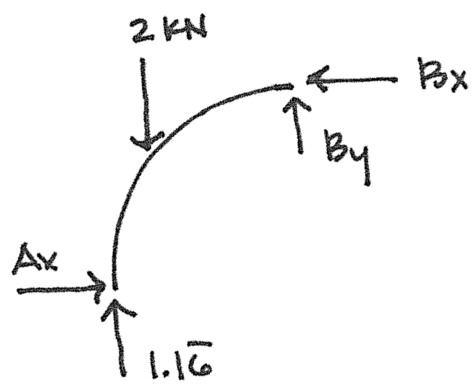
$$+\circlearrowleft \Sigma M_A = 0 = -2(0.5) - 1.5(1) + C_y(3)$$

$$\boxed{C_y = 0.8\bar{3} \text{ KN} \uparrow}$$

$$+\uparrow \Sigma F_y = 0 = A_y - 2 + 0.8\bar{3}$$

$$\boxed{A_y = 1.1\bar{6} \text{ KN} \uparrow}$$

MEMBER AB:



$$+\circlearrowleft \Sigma M_B = 0 = 2(1) - 1.1\bar{6}(1.5) + A_x(1.5)$$

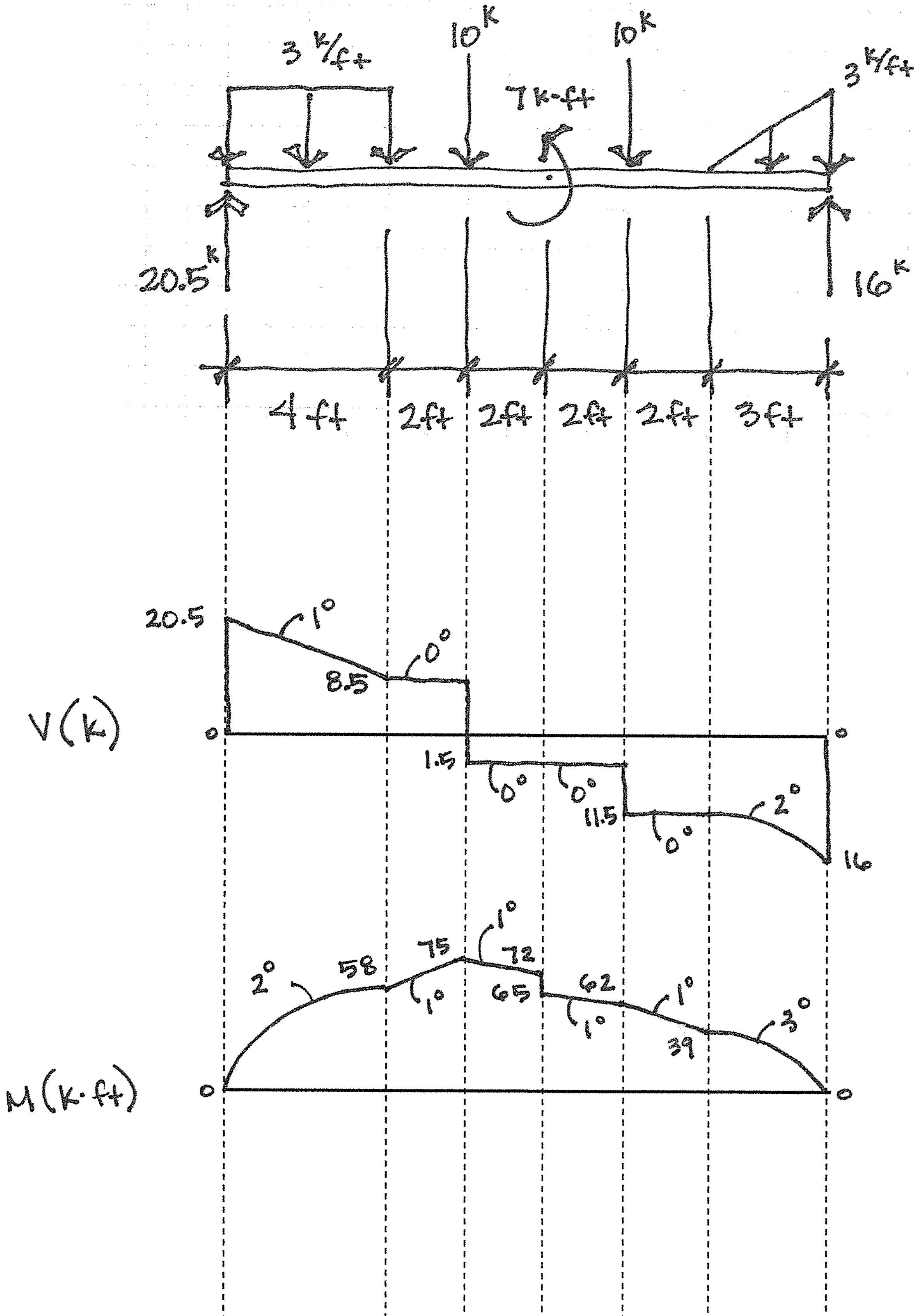
$$\boxed{A_x = 0.1\bar{6} \text{ KN} \leftarrow}$$

FROM OVERALL:

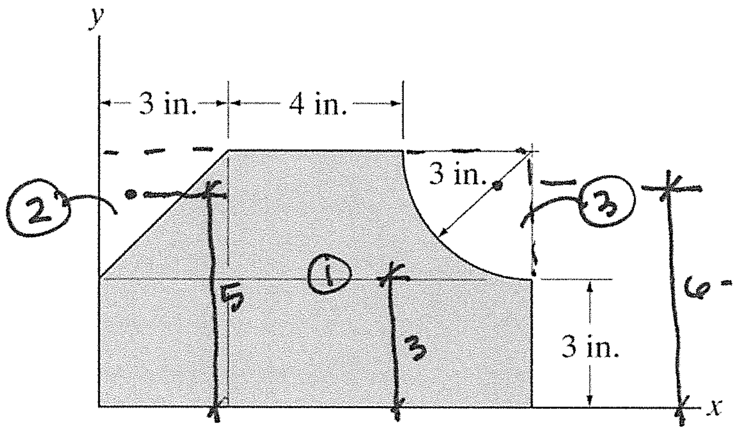
$$+\rightarrow \Sigma F_x = 0 = -0.1\bar{6} + 1.5 - C_x$$

$$\boxed{C_x = 1.3\bar{3} \text{ KN} \leftarrow}$$

4. Draw the shear and bending moment diagrams for the loading condition below. Label all diagrams appropriately.



5. Using tabular form, calculate the moment of inertia about the centroidal x axis, x', for the object below. The coordinates for the overall centroid of the shape are (4.83 in, 2.56 in).



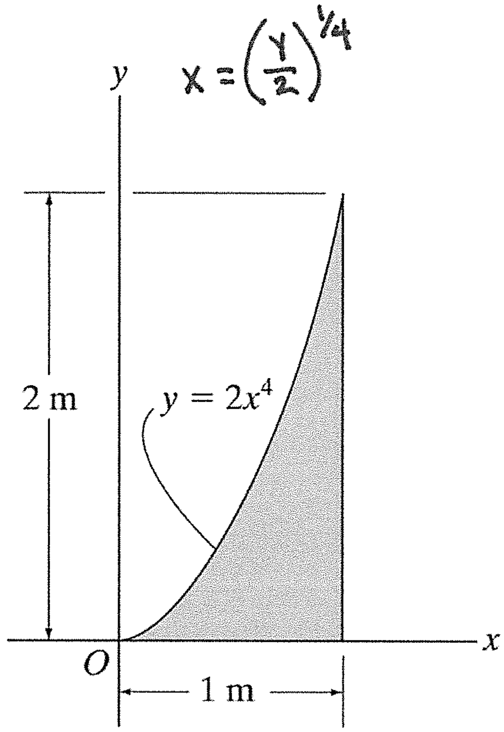
$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$6 - \frac{4r}{3\pi} = 4.73$$

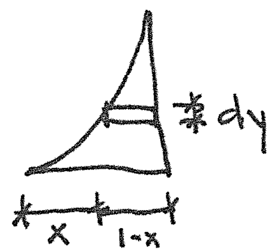
SHAPE	$\bar{I}_{x'}$	AREA	d_y	$A(d_y)^2$
1	$\frac{bh^3}{12} = 180$	$bh = 60$	$2.56 - 3$ -0.44	11.62
2 VOID	$\frac{-bh^3}{36} = -2.25$	$-\frac{1}{2}bh = -4.5$	$2.56 - 5$ -2.44	-26.79
3 VOID	$-0.05488r^4$ $= -4.45$	$-\frac{\pi r^2}{4}$ $= -7.07$	$2.56 - 4.73$ -2.17	-33.29
	173.3			-48.46

$$I_x = 173.3 - 48.46 = \boxed{124.84 \text{ in}^4}$$

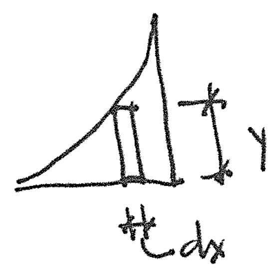
6. Determine the moment of inertia of the shaded area about the x-axis by integration. State which method of integration is used and label the diagram accordingly.



BASIC EQN:



PARALLEL-AXIS:



BASIC EQUATION:

$$I_x = \int y^2\,dA$$

$$dA = (1-x)\,dy$$

$$I_x = \int_0^2 y^2(1-x)\,dy$$

$$= \int_0^2 y^2\left(1-\left(\frac{y}{2}\right)^{1/4}\right)dy$$

$$= \int_0^2 y^2 - \frac{1}{(2)^{1/4}}\,y^{9/4}\,dy$$

$$= \frac{y^3}{3} - 0.841\,\frac{y^{13/4}}{13/4}\Bigg|_0^2$$

$$= 2.667 - 2.46$$

$$= \boxed{0.205\,m^4}$$

PARALLEL-AXIS THEOREM:

$$I_x = \int dI_x = \int d\bar{I}_x' + dA\,\tilde{y}^2$$

$$dA\,\tilde{y}^2 = \frac{y^3}{4}\,dx$$

$$d\bar{I}_x' = \frac{y^3}{12}\,dx$$

$$I_x = \int_0^1 \frac{y^3}{12} + \frac{y^3}{4}\,dx = \int_0^1 \frac{y^3}{3}\,dx$$

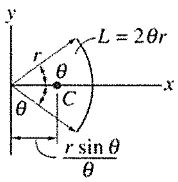
$$= \int_0^1 \frac{(2x^4)^3}{3}\,dx = \int_0^1 \frac{8x^{12}}{3}\,dx$$

$$= \frac{8}{3}\int_0^1 x^{12}\,dx = \frac{8}{3}\,\frac{x^{13}}{13}\Bigg|_0^1$$

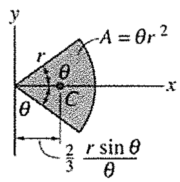
$$= \boxed{0.205\,m^4}$$

Geometric Properties of Line and Area Elements

Centroid Location	Centroid Location	Area Moment of Inertia



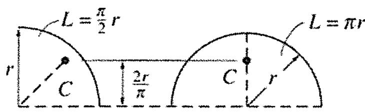
Circular arc segment



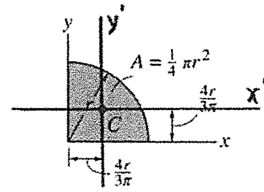
Circular sector area

$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2}\sin 2\theta)$

$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2}\sin 2\theta)$



Quarter and semicircle arcs

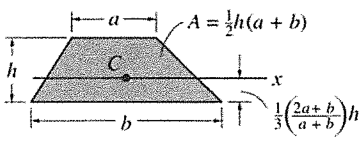


Quarter circle area

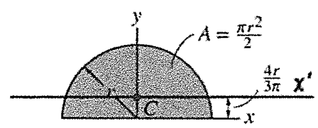
$I_{x'} = I_{y'} = .05488 r^4$

$I_x = \frac{1}{16}\pi r^4$

$I_y = \frac{1}{16}\pi r^4$



Trapezoidal area

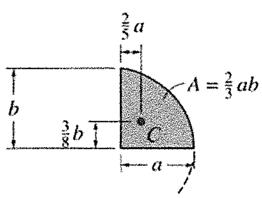


Semicircular area

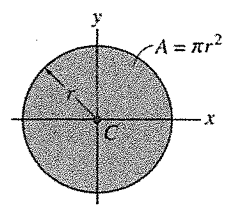
$I_{x'} = .1098 r^4$

$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$



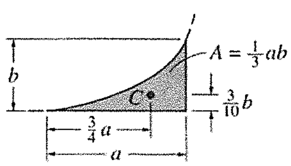
Semiparabolic area



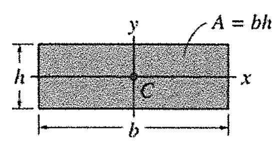
Circular area

$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$



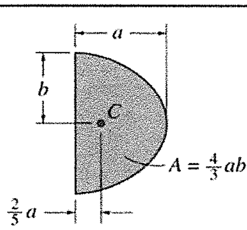
Exparabolic area



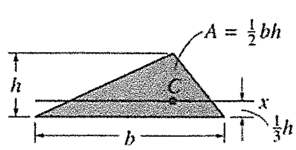
Rectangular area

$I_x = \frac{1}{12}bh^3$

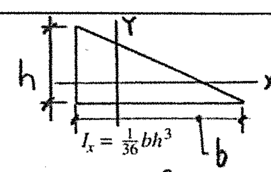
$I_y = \frac{1}{12}hb^3$



Parabolic area



Triangular area



$I_x = \frac{1}{36}bh^3$

$I_Y = \frac{hb^3}{36}$