西南交通大學
Southwest Jiaotong University

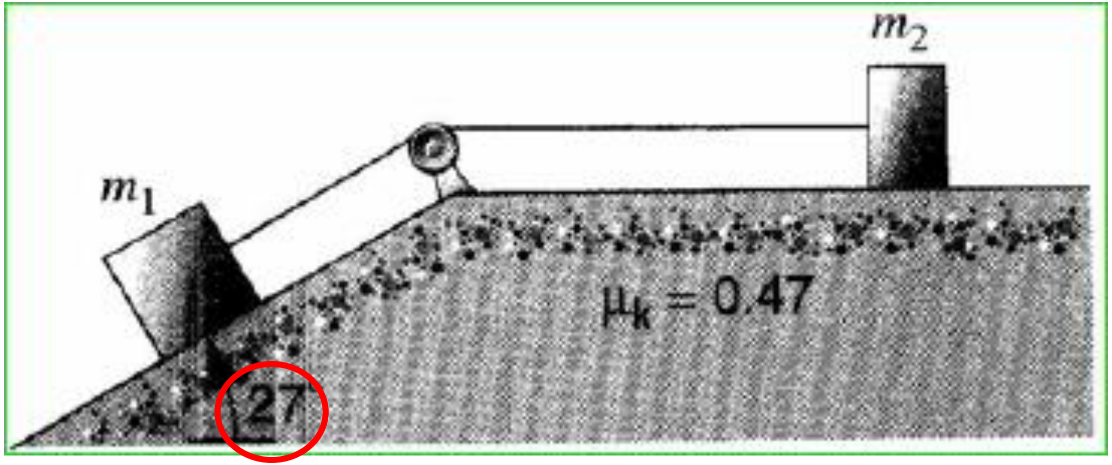
Physics 1: **Mechanics and Waves**

Week 5 – Solution of exercises

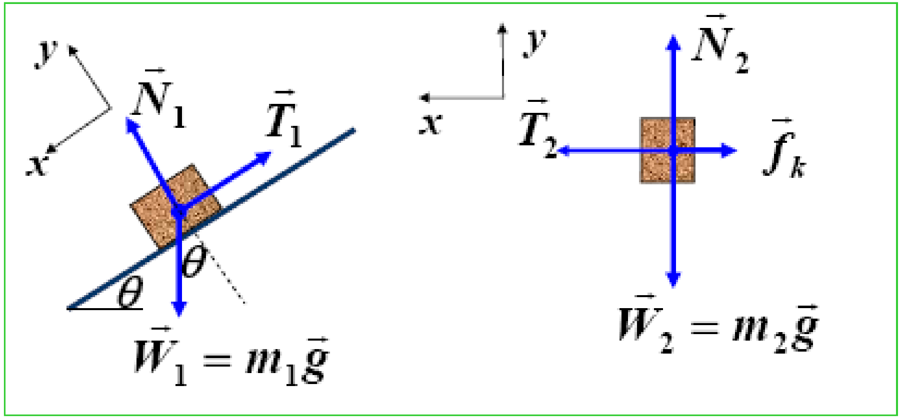
2023.3

Applying Newton's Laws – Example 1

Block $m1$ in the Figure has a mass of 4.20kg and block m_2 has a mass of 2.30kg. The coefficient of kinetic friction between m_2 and the horizontal plane is 0.47. The inclined plane is frictionless. Find (a) the acceleration of the blocks and (b) the tension in the string.



Applying Newton's Laws – Example 1



$$|\vec{T}_1| = |\vec{T}_2| = T$$

$$f_k = \mu_k N_2$$

$$a_{1x} = a_{2x} = a$$

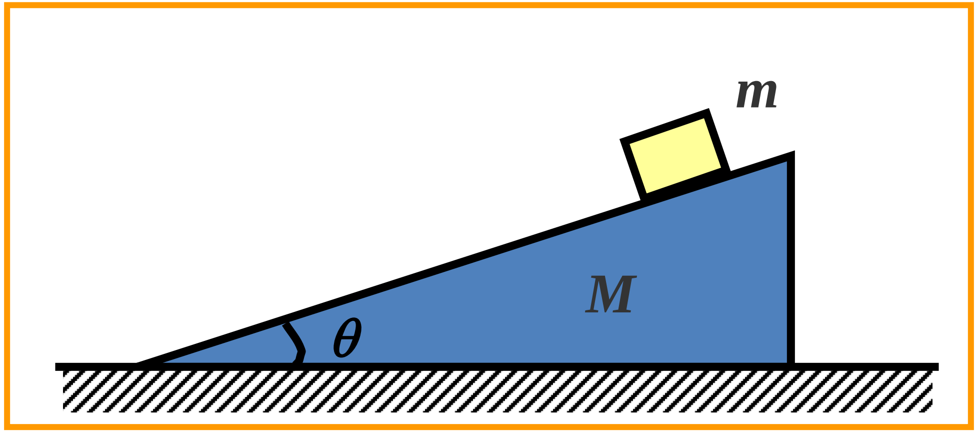
$$\theta = 27°$$

$$\begin{cases} m_1 g \sin\theta - T = m_1 a \\ N_1 - m_1 g \cos\theta = 0 \end{cases}$$

$$\begin{cases} T - \mu_k N_2 = m_2 a \\ N_2 - m_2 g = 0 \end{cases}$$

Applying Newton's Laws – Example 2

A block of mass *m* is placed on a frictionless inclined plane of a triangle block of mass *M* which is on the frictionless horizontal plane , the inclined plane is at the angle θ to the horizontal. Find (a)the magnitude of the normal force of the inclined plane on the block; (b)the acceleration of *m* with respect to *M*.

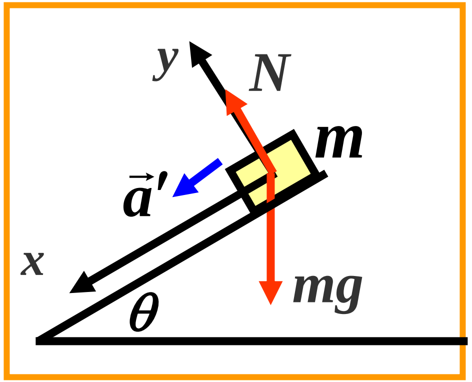


Applying Newton's Laws – Example 2

Solution:

Draw the forces diagram

Choose triangle block as the reference frame



$$F_x = mg\sin\theta = ma' \qquad a' = g\sin\theta$$

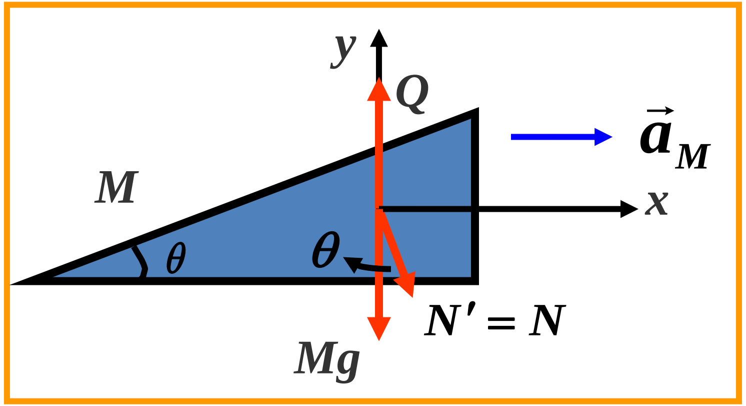
$$F_y = N - mg\cos\theta = 0 \qquad N = mg\cos\theta$$

Are these results right?

Applying Newton's Laws – Example 2

Choose the ground as the reference frame:

For the triangle block



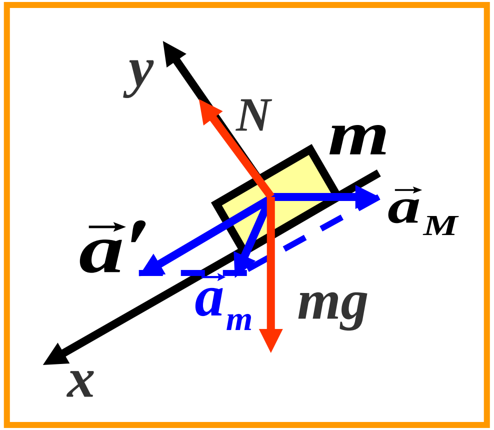
$$\begin{cases} F_x = N\sin\theta = Ma_M & (1) \\ F_y = Q - Mg - N\cos\theta = 0 & (2) \end{cases}$$

$a_M \neq 0$ $\quad$ **M is not inertial reference frame.**

Applying Newton's Laws – Example 2

Choose the ground as the reference frame:

$$\vec{a}_{mG} = \vec{a}_{mM} + \vec{a}_{MG}$$



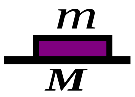
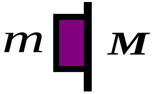
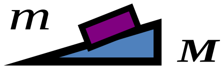
$$\vec{a}_m = \vec{a}' + \vec{a}_M$$

$$\begin{cases} a_{mx} = a' - a_M\cos\theta \\ a_{my} = -a_M\sin\theta \end{cases}$$

$$\begin{cases} F_x = mg\sin\theta = ma_{mx} = m\left(a' - a_M\cos\theta\right) & (3) \\ F_y = N - mg\cos\theta = ma_{my} = -ma_M\sin\theta & (4) \end{cases}$$

Applying Newton's Laws – Example 2 - Check the answer

$$a_M = \frac{mg\cos\theta\sin\theta}{M + m\sin^2\theta} \qquad a' = \frac{(M+m)g\sin\theta}{M+m\sin^2\theta} \qquad N = \frac{Mmg\cos\theta}{M+m\sin^2\theta}$$

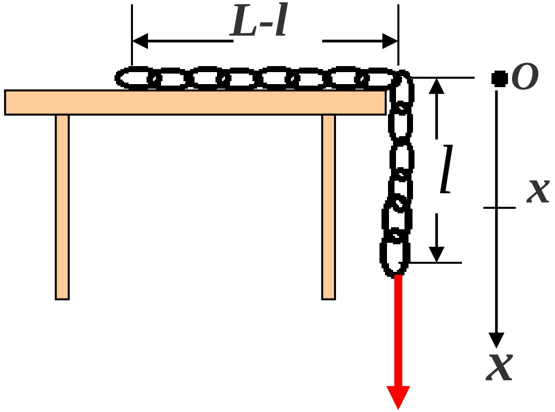
	$\theta = 0$	$\theta = \pi/2$	$M \to \infty$
a_M	 0	 0	0 
a'	0	g	$g\sin\theta$
N	mg	0	$mg\cos\theta$

Applying Newton's Laws – Example 3

A chain has length *L* and mass *M*, and was put on a frictionless table. At *t* = 0, the chain is stationary and length *l* is hanged over the edge.

(a) What is the velocity at the moment that the whole chain leave the table?

(b) The time covered the whole process?



Applying Newton's Laws – Example 3

(a) **Consider the moment of *l*=*x*,**

$$\frac{M}{L}gx = Ma \quad \textbf{(variable);}$$

$$\frac{M}{L}gx = Mv\frac{\mathrm{d}v}{\mathrm{d}x}$$

$$\boldsymbol{F = \frac{M}{L}gx} \quad \textbf{(Variable)}$$

$$\boldsymbol{Lv\mathrm{d}v = gx\mathrm{d}x}$$

$$L\int_0^v v\mathrm{d}v = g\int_l^x x\mathrm{d}x$$

$$\boldsymbol{v(x) = \sqrt{\frac{g}{L}(x^2 - l^2)}} \ ;$$

$$\boldsymbol{v(L) = \sqrt{\frac{g}{L}(L^2 - l^2)}}$$

Applying Newton's Laws – Example 3

(b) $$v(x) = \sqrt{\frac{g}{L}(x^2 - l^2)} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

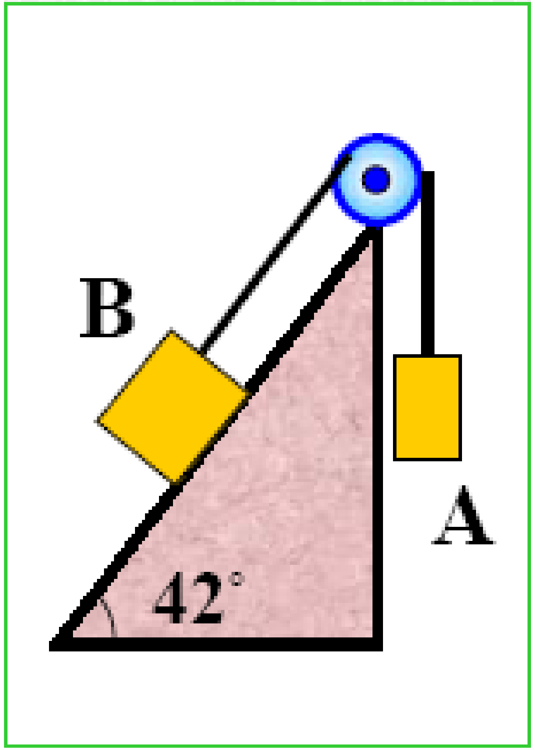
$$\int_l^L \frac{\mathrm{d}x}{\sqrt{x^2 - l^2}} = \int_0^t \sqrt{\frac{g}{L}}\,\mathrm{d}t$$

so $$t = \sqrt{\frac{L}{g}} \ln \frac{L + \sqrt{L^2 - l^2}}{l}$$

Applying Newton's Laws – Exercise 1

As shown in figure, the block B weighs 94kg and block A weighs 29 kg. Between block B and the plane the coefficient of static friction is 0.56 and the coefficient of the kinetic friction is 0.25.

(a) find the acceleration if B is moving up the plane.
(b) what is the acceleration if B is moving down the plane?

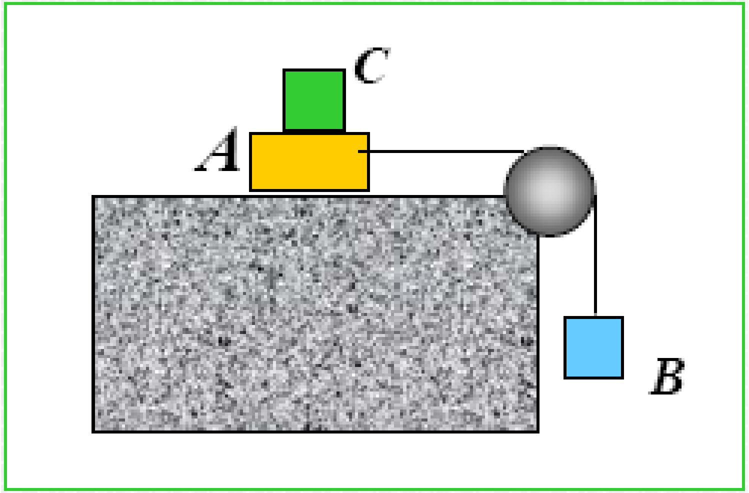


Applying Newton's Laws – Exercise 2

In the figure, A is a 4.4 kg block and B is a 2.6 kg block. The coefficients of static and kinetic friction between A and the table are 0.18 and 0.15.

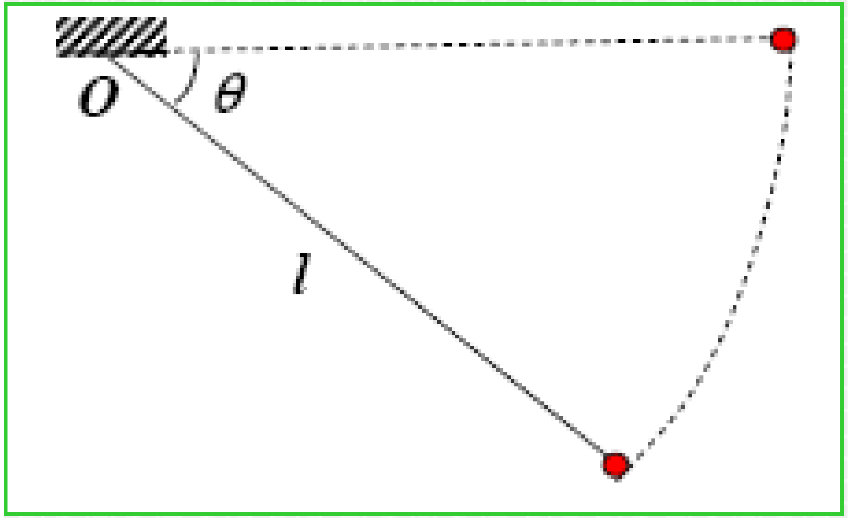
(a) Determine the minimum mass of the block C that must be placed on A to keep it from sliding.

(b) Block C is suddenly lifted off A. What is the acceleration of block A?

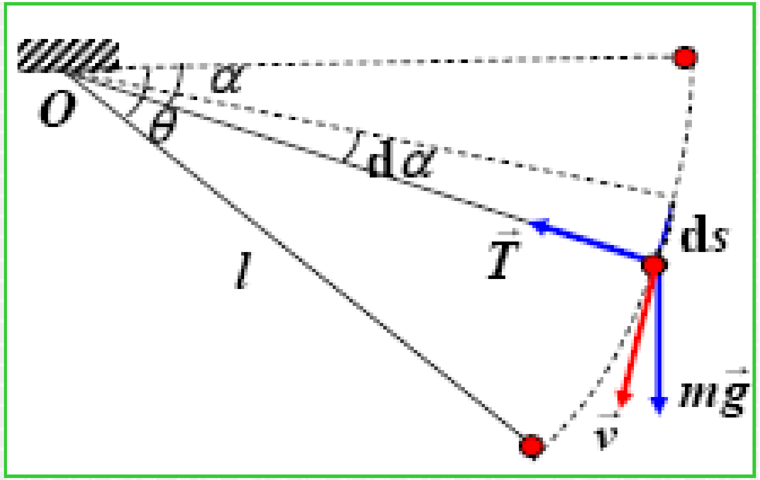


Applying Newton's Laws – Exercise 3

An ideal pendulum with a pear of mass *m* and an inflexible, massless thread of length *l*. Keep the pendulum at horizontal initially, and then drop it from rest. When the angle between the thread and the horizontal is *θ*, find the speed of the pear and the tension in the thread.



Applying Newton's Laws – Exerc



Solution:

$$mg\cos\alpha = ma_\tau = m\frac{dv}{dt}$$

$$mg\cos\alpha ds = m\frac{dv}{dt}ds = m\frac{ds}{dt}dv$$

$$ds = ld\alpha \qquad \frac{ds}{dt} = v$$

$$gl\cos\alpha d\alpha = vdv$$

$$\int_0^\theta gl\cos\alpha d\alpha = \int_0^{v_\theta} vdv$$

$$v_\theta = \sqrt{2gl\sin\theta}$$

$$T - mg\sin\theta = ma_n = m\frac{v_\theta^2}{l}$$

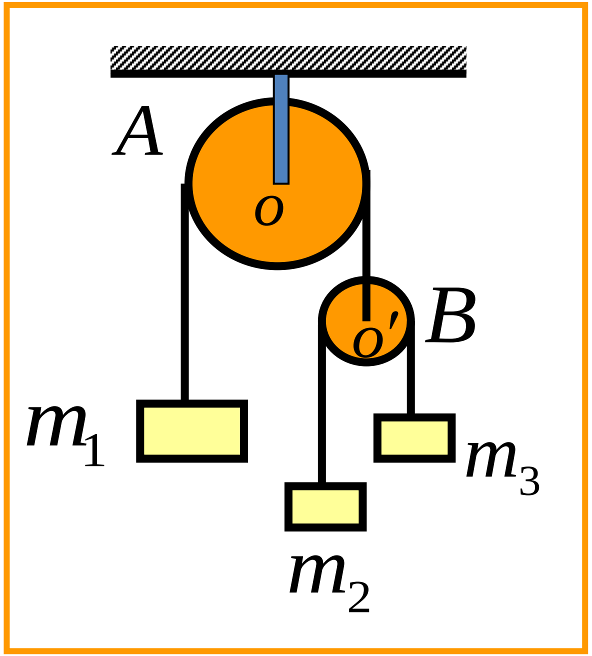
$$T_\theta = 3mg\sin\theta$$

Applying Newton's Laws – Exercise 4

There are three blocks of mass m_1, m_2, m_3 respectively, and $m_1>m_2+m_3$. Pulleys and strings are massless, the kinetic friction forces between the pulley and the strings are ignored.

(a) find the accelerations of m_1, m_2 and m_3.

(b) (b) what are the tensions of the two strings.

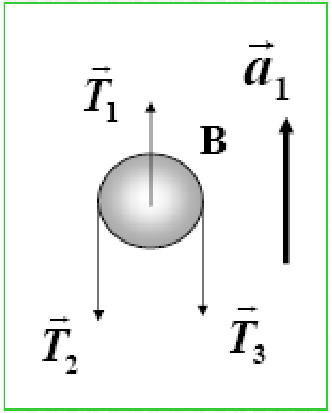


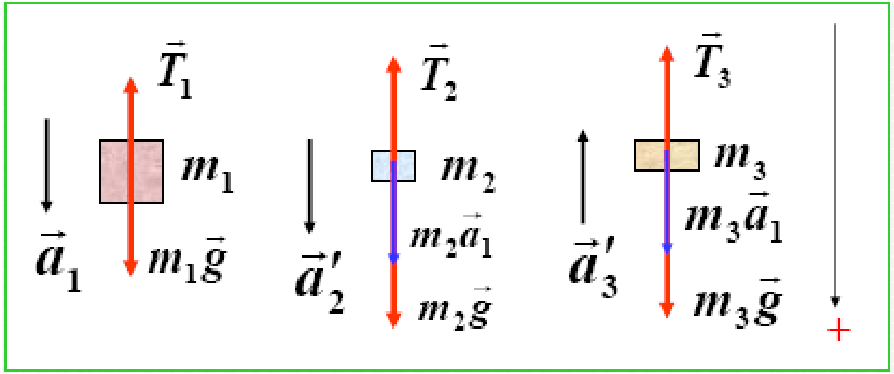
B is not a inertial reference frame!

Applying Newton's Laws – Exercise 4

Solution:

(a) The free-body diagrams





$$m_1 g - T_1 = m_1 a_1$$

$$m_2 g + m_2 a_1 - T_2 = m_2 a'_2$$

$$m_3 g + m_3 a_1 - T_3 = -m_3 a'_2$$

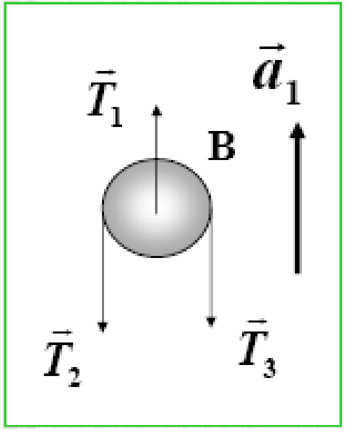
$$T_1 = T_2 + T_3$$

$$T_2 = T_3$$

$$a'_2 = a'_3$$

Applying Newton's Laws – Exercise 4

(b)



because $$\vec{a}_{m_2G} = \vec{a}'_{m_2P} + \vec{a}_{PG}$$

$$\vec{a}_2 = \vec{a}'_2 + \vec{a}_1$$

If the direction of $\vec{a}_2$ is downward

Then $$a_2 = a'_2 - a_1 \qquad \textbf{(1)}$$

because $$\vec{a}_3 = \vec{a}'_3 + \vec{a}_1$$

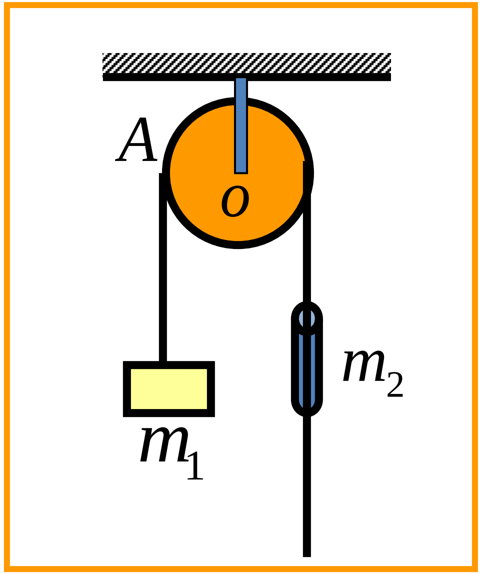
Assume $\vec{a}_3$ is upward

then $$-a_3 = -a'_3 - a_1 \qquad \textbf{(2)}$$

From equations (1) and (2) ,we can get a_2 an a_3.

Applying Newton's Laws – Exercise 5

A small metal tube is sliding down along the string with the acceleration a_2 with respect to the string as shown in Figure. The mass of the block is m_1, and the mass of the tube is m_2, the string is massless, the kinetic friction force between the pulley and the string is ignored. Find the accelerations of the block and the tube with respect to the ground, the tension of the string and the friction force between the string and the tube.



The string is not a inertial reference frame!

Applying Newton’s Laws – Exercise 5

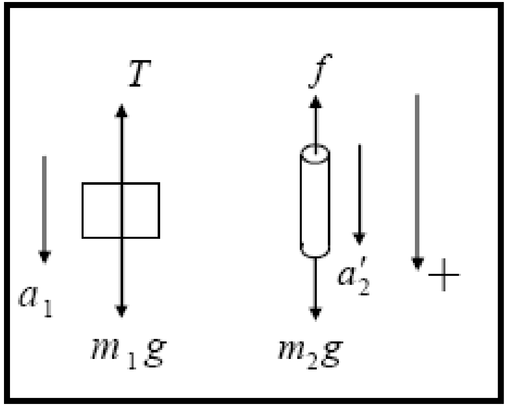
Solution:

$$m_1 g - T = m_1 a_1$$

$$m_2 g - f = m_2 a_2'$$

$$f = T$$

$$a_2' = a_2 - a_1$$



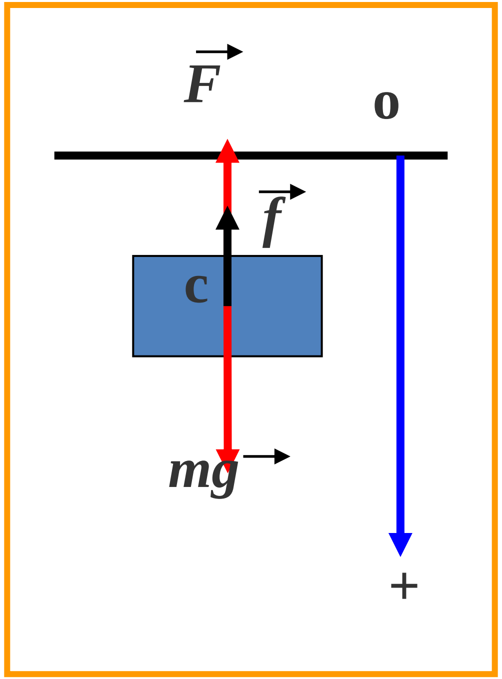
$$a_1 = \frac{(m_1 - m_2)g + m_2 a_2}{m_1 + m_2}$$

$$a_2' = \frac{m_1 a_2 - (m_1 - m_2)g}{m_1 + m_2}$$

$$T = f = \frac{(2g - a_2)m_1 m_2}{m_1 + m_2}$$

Applying Newton's Laws – Exercise 6

A submarine is sinking in the sea. If the buoyancy is F, the kinetic friction is $f = kAv$, where A is the area of the cross section of the submarine. Find the function of the speed of the submarine with respect to the time.



Applying Newton's Laws – Exercise 6

Solution:

$$mg - F - kAv = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\int_0^v \frac{m\mathrm{d}v}{mg - F - kAv} = \int_0^t \mathrm{d}t$$

$$-\frac{m}{kA}\ln\frac{mg\text{-}F\text{-}kAv}{mg - F} = t$$

$$\frac{mg - F - kAv}{mg - F} = e^{-\frac{kA}{m}t}$$

$$v = \frac{mg - F}{kA}\left(1 - e^{-\frac{kA}{m}t}\right)$$

Applying Newton's Laws – Exercise 6

Discussion

$$t = 0 \quad v = 0, \qquad t \uparrow \quad v \uparrow \quad \frac{\mathrm{d}v}{\mathrm{d}t} \downarrow,$$

$$t \to \infty \quad v = v_{\max} = \frac{mg - F}{kA} = Constant$$

terminal speed