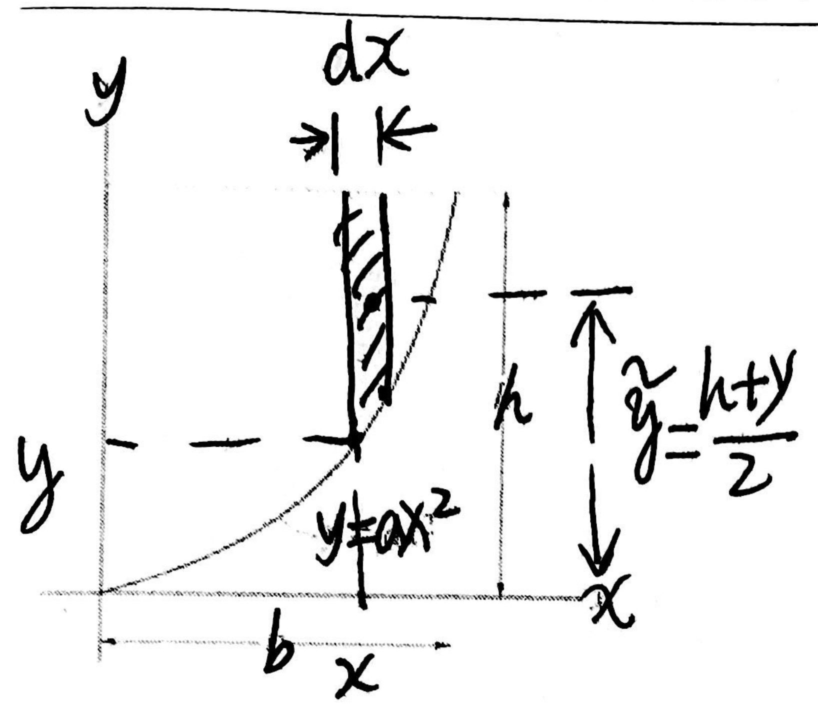
Find x bar of the shaded area. Take a=3 mm, b=2 mm, and h=12 mm. （10pnts）



$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^2 x(h-ax^2)dx}{\int_0^2 (h-ax^2)dx}$$

$$= \frac{\left(\frac{hx^2}{2} - \frac{ax^4}{4}\right)\Big|_0^2}{\left(hx - \frac{a}{3}x^3\right)\Big|_0^2} = \frac{12}{16}\,mm$$

$$\boxed{\bar{x} = \frac{3}{4}\,mm}$$

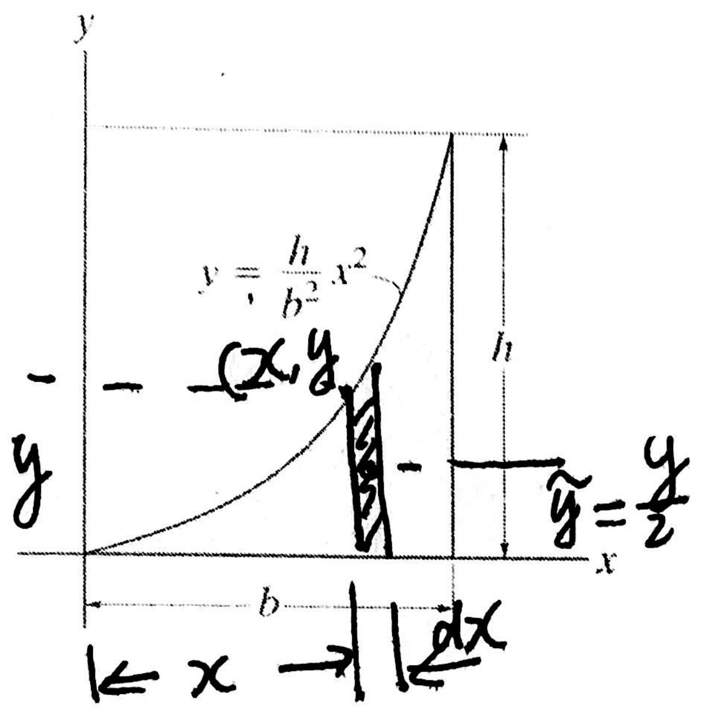
$dA = (h-y)dx$

$= (h-ax^2)dx$

$\tilde{y} = \frac{h+y}{2} = \frac{h+ax^2}{2}$

$\tilde{x} = x$

Calculate x bar of the shaded area. Take b=2 cm and h=3 cm. （10pnts）



$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^2 x \cdot \frac{h}{b^2} x^2 dx}{\int_0^2 \frac{h}{b^2} x^2 dx}$$

$$= \frac{\frac{h}{4b^2} x^4 \Big|_0^2}{\frac{h}{3b^2} x^3 \Big|_0^2}$$

$$= \frac{3}{4} \times 2 = 1.5 \text{ inch}$$

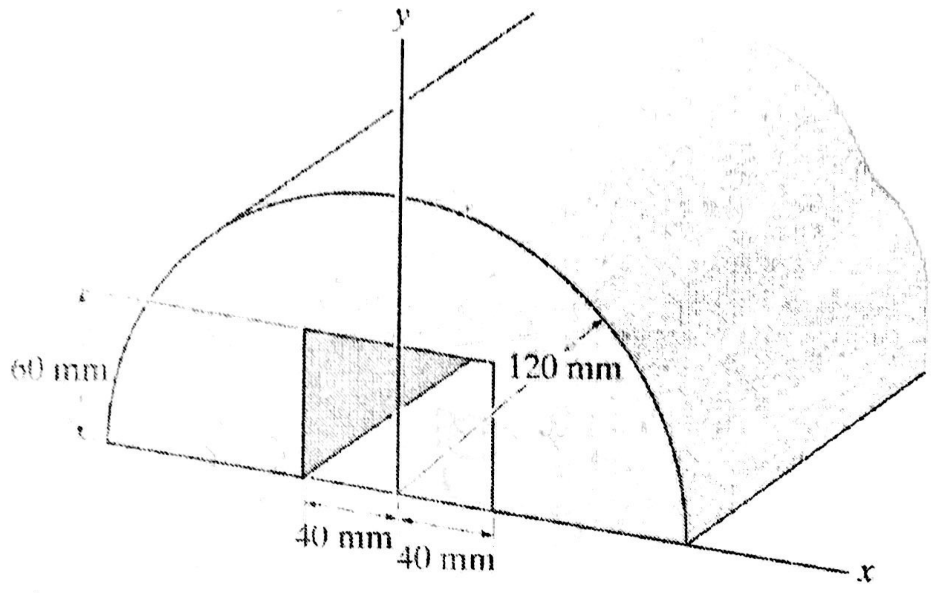
$$\boxed{\bar{x} = 1.5 \text{in}}$$

$dA = y\,dx = \frac{h}{b^2} x^2 \cdot dx$

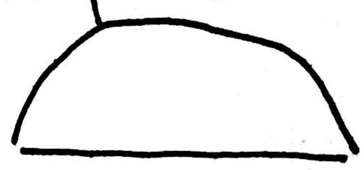
$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

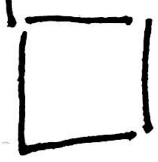
Locate the cetroid, y bar, for the cross-sectional area using the tabular method. （10pnts）



Shape ①



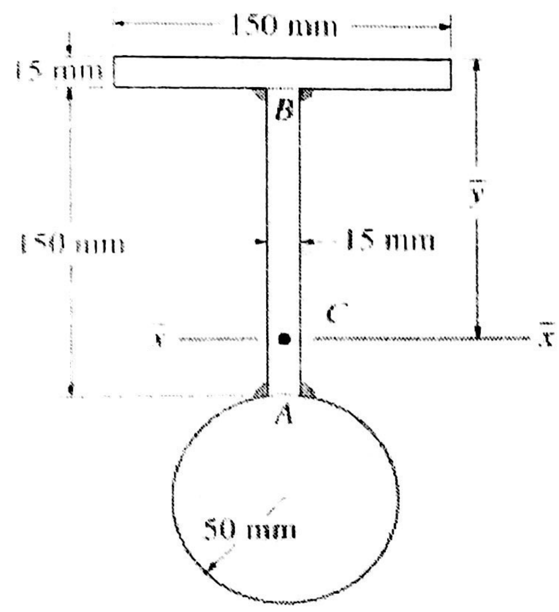
Shape ② Void



Shape	Area (mm^2)	$\tilde{y}$ (mm)	$A\tilde{y}$ (mm^3)
①	$\frac{1}{2}\times\pi\times120^2$	$\frac{4\times120}{3\pi}$	1152000
②	-80×60	30	-144000
Σ	17819.5		1008000

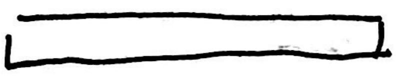
$$\bar{y} = \frac{\Sigma A_i\tilde{y}_i}{\Sigma A_i} = \frac{1008000}{17819.5} = 56.57\,mm$$

Find y bar of the composite shape. Neglect the weld areas at the corners of *A* & *B* in your calculations. Note: y bar is measured from the top of the shape. (15pnts)

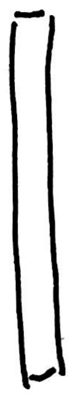


Shape	Area (mm^2)	$\tilde{y}$ (mm)	$A\tilde{y}$ (mm^3)
①	$150\times15 = 2250$	7.5	16875
②	$150\times15 = 2250$	$15+\frac{150}{2} = 90$	202500
③	$\pi\times50^2 = 7854$	$15+150+50 = 215$	1688610
Σ	12354		1907985

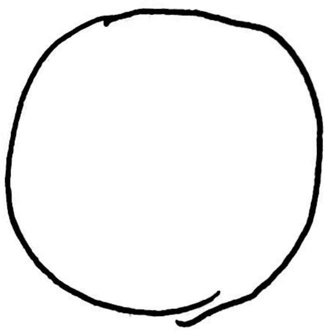
①



②



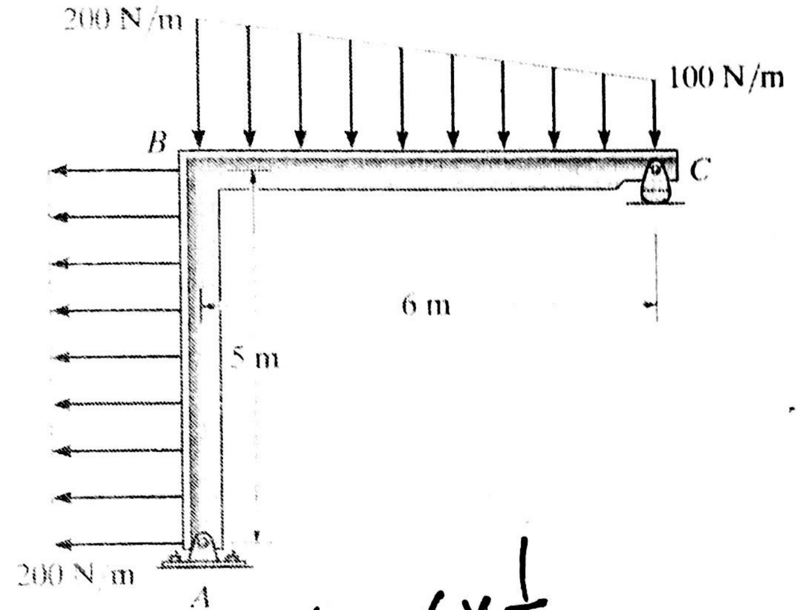
③



$$\bar{y} = \frac{\Sigma \tilde{y}_i A_i}{\Sigma A_i} = \frac{1907985}{12354}$$

$$\boxed{\bar{y} = 154.4\ mm}$$

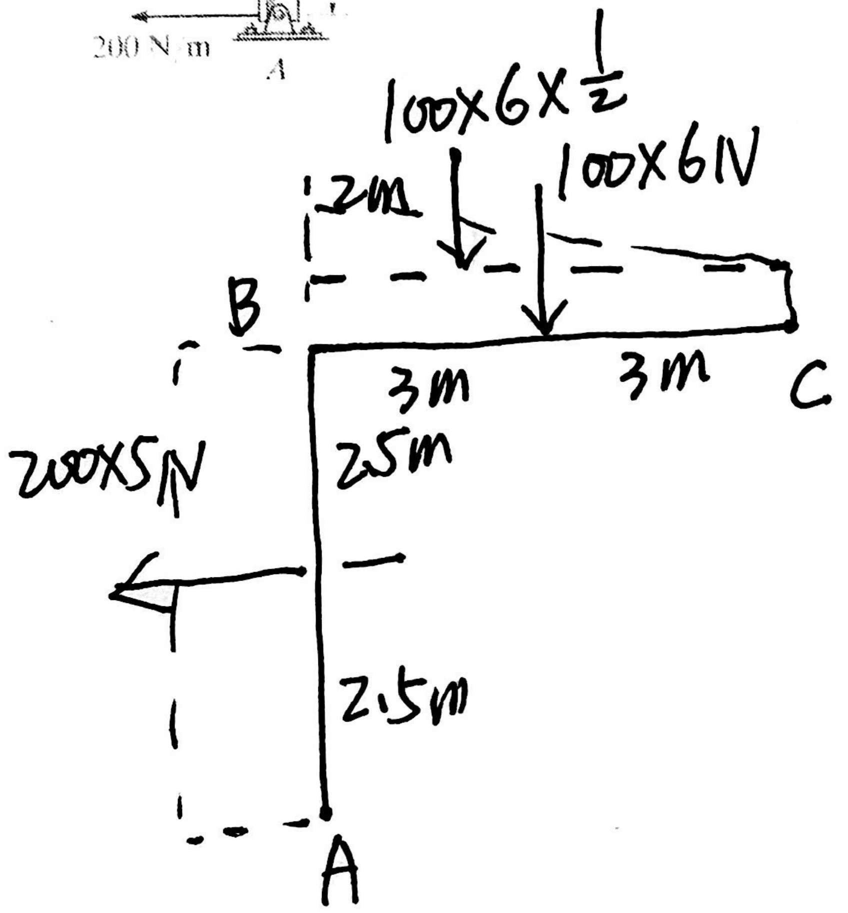
Replace the loading system with an equivalent resultant force and specify where its line of action intersects member BC measured from C. （15pnts）



① $F_{RX} = -200\times5 = -1000N$ (←)

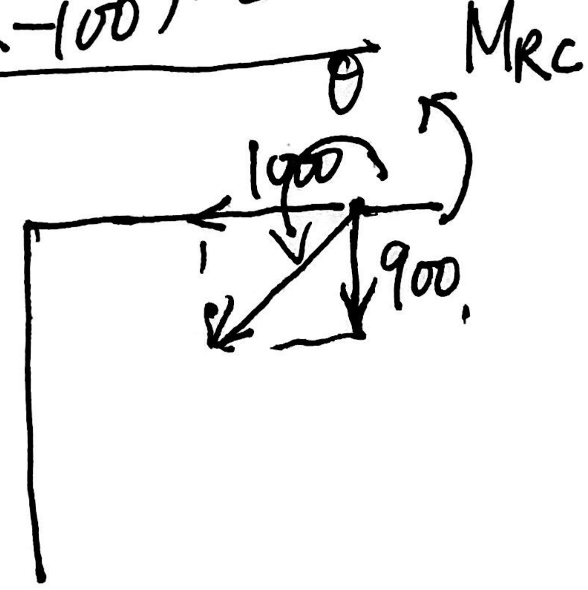
$F_{Ry} = -100\times6\times\frac{1}{2} - 100\times6$
$= -900N$ (↓)

② $\circlearrowleft + \Sigma M_C = 600\times3 + 300\times4 - 1000\times2.5$
$= 500 N\cdot m$ (↺)



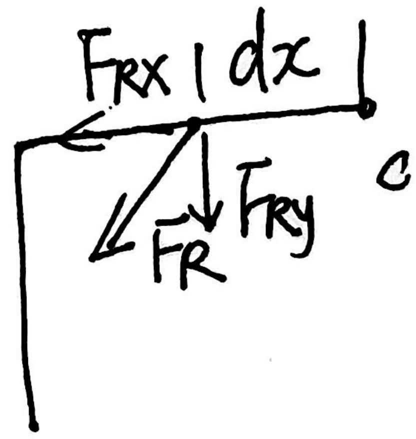
$F_R = \sqrt{(-1000)^2 + (-900)^2} = 1345.4N$

$\theta = \arctan\left(\frac{-900}{-100}\right) = 222°$

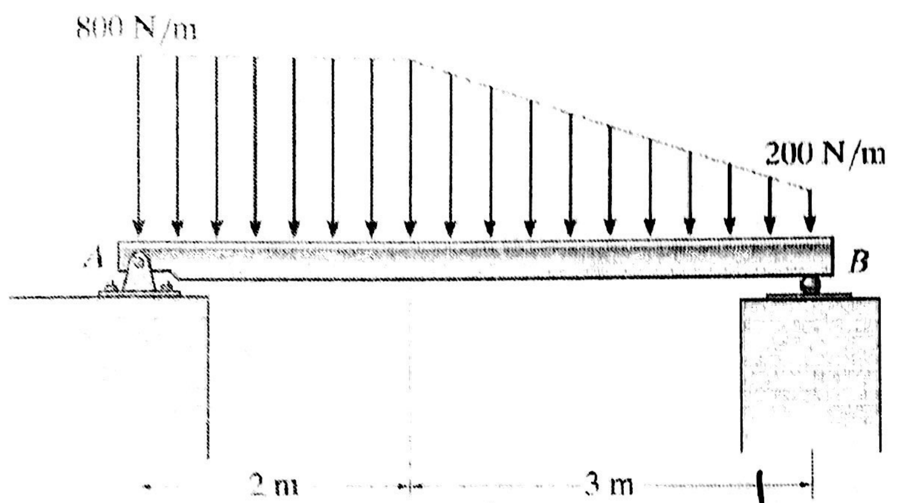


$$\boxed{\begin{aligned} dx &= \left|\frac{\Sigma M_{RC}}{F_{Ry}}\right| \\ &= \frac{500}{900} = 0.56m \end{aligned}}$$

to the left of C.



Replace the loading system with an equivalent resultant force and specify its location measured from A. （10pnts）



① $F_{Rx}=0$

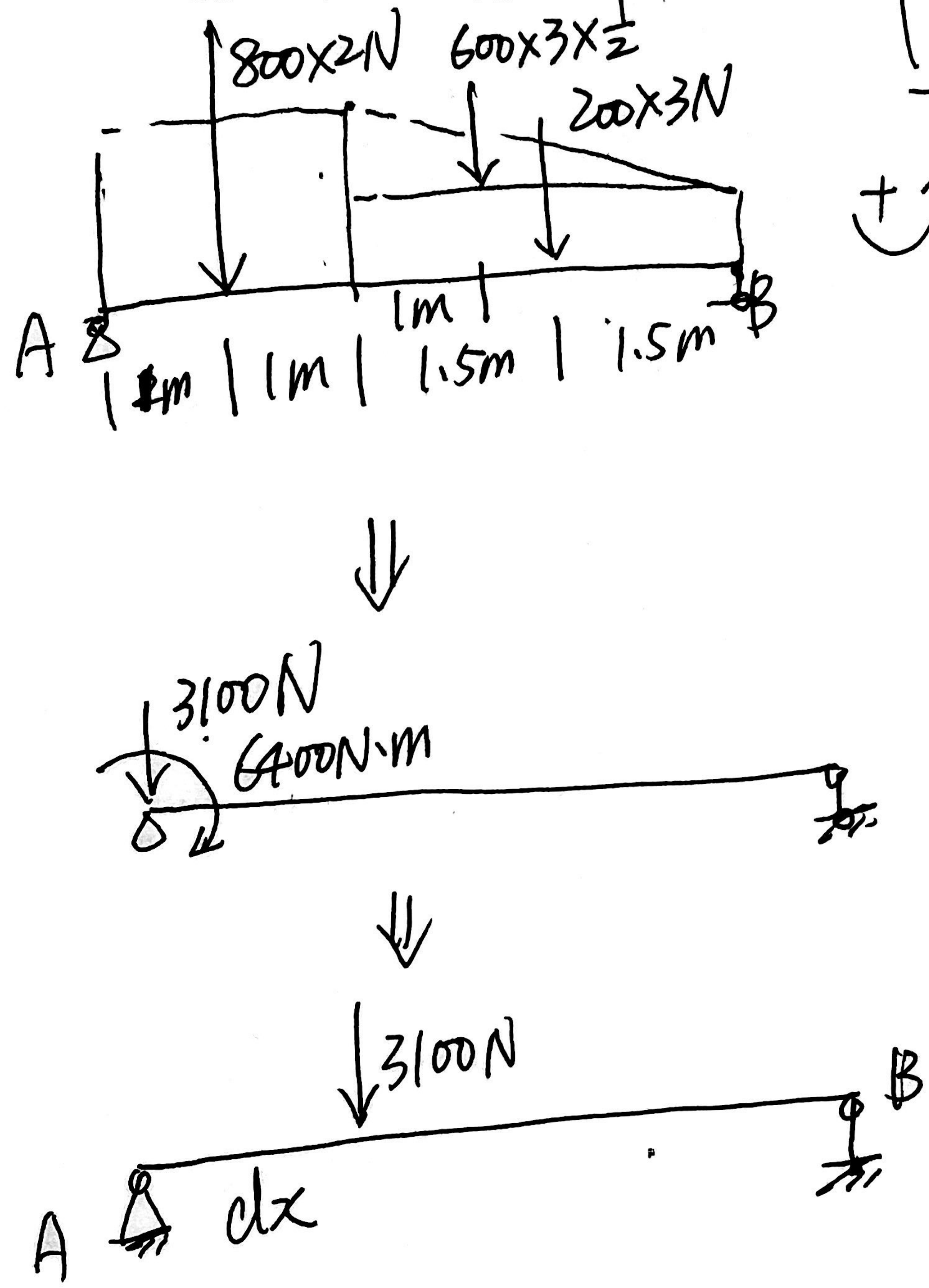
$$F_{Ry}=-800\times2-600\times3\times\frac{1}{2}-200\times3=-3100N(\downarrow)$$

$$\boxed{F_R=3100N(\downarrow)}$$

$$+\circlearrowleft\sum M_A=-800\times2\times1-600\times3\times\frac{1}{2}\times3-200\times3\times3.5$$

$$=-6400N\cdot m$$

$$=6400N\cdot m(\circlearrowright)$$



$$d_x=\frac{6400}{3100}=2.06m$$

to the right of A..