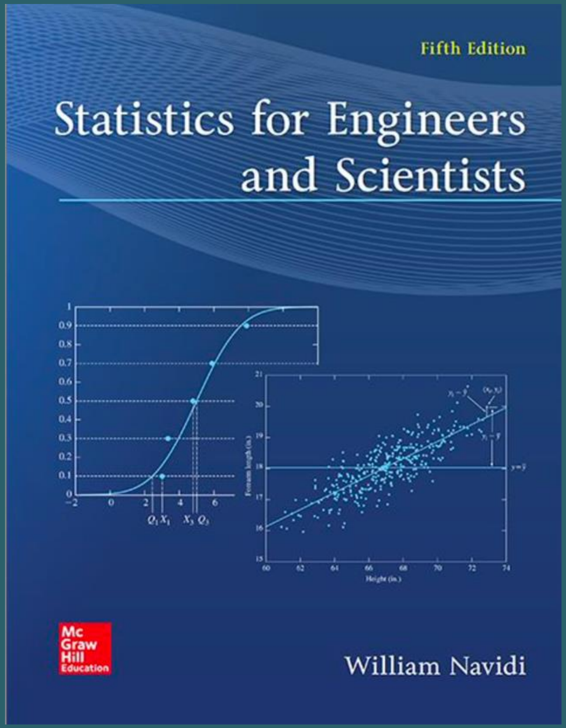

Ch 4: Commonly Used Distributions

(not all sections are required)

Introduction

- Statistical inference involves drawing a sample from a population and analyzing the sample data to learn about the population.
- In many situations, we have an approximate knowledge of the probability mass function or probability density function of the population.
- In these cases, the probability mass or density function can often be well approximated by one of several standard families of curves, or functions.
- In this chapter, we describe some of these standard functions, and for each one we describe some conditions under which it is appropriate.

Ch 4: Overview (Required Sections)

4-1 The Bernoulli Distribution

4-2 The Binomial Distribution

4-3 The Poisson Distribution

4-5 The Normal Distribution

4-9 Some Principles of Point Estimation

4-10 Probability Plots

4-11 Central Limit Theorem

The Bernoulli Trial

Many types of probability problems have only two outcomes or can be reduced to two outcomes.

Examples:

- Toss of a coin (heads or tails).
- Roll of a die (an odd or even number).
- A basketball game (a team either wins or loses).
- A person can be classified as having normal or abnormal blood pressure.
- A multiple-choice question (even though there are 4 or 5 answer choices), can be classified as correct or incorrect.
- The selection of a component from a population of components, some of which are defective.

The Bernoulli Trial…

- If an experiment can result in one of two outcomes.
- One outcome is labeled "success", and the other outcome is labeled "failure".
- The probability of success is denoted by p.
- The probability of failure is therefore $1 - p$.
- Such a trial is called a Bernoulli trial with success probability p.
- Sometimes, it is called a binomial experiment.

The Bernoulli Trial…(p.201)

For any Bernoulli trial, we define a random variable X as follows:

- If the experiment results in 'success', then X = 1.
- Otherwise, X = 0 ('failure').
- It follows that X is a discrete random variable, with probability mass function p(x) defined by

$$p(0) = P(X = 0) = 1 - p$$

$$p(1) = P(X = 1) = p$$

$p(x) = 0$ for any value of x other than 0 or 1

The Bernoulli Distribution (p.201)

- ✓ The random variable *X* is said to have the Bernoulli distribution with parameter p.
- ✓ The notation is X ∼ Bernoulli(p).
- ✓ Figure 4.1 presents probability histograms for the Bernoulli(0.5) and Bernoulli(0.8) probability mass functions.

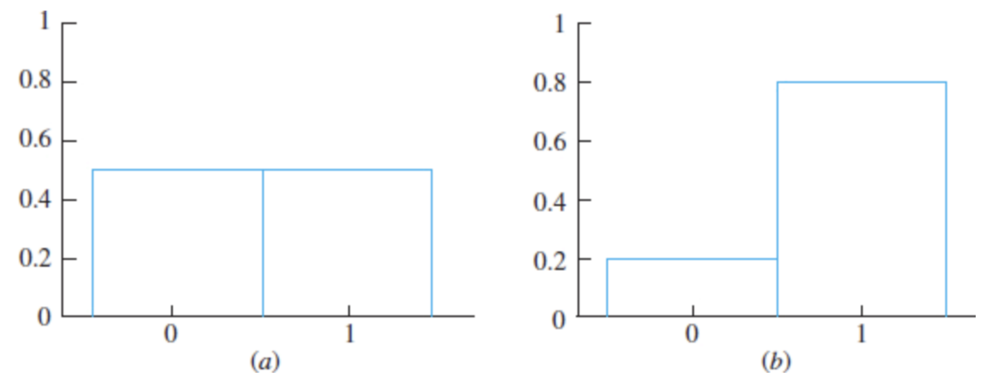

FIGURE 4.1 *(a)* The Bernoulli(0.5) probability histogram. *(b)* The Bernoulli(0.8) probability histogram.

Example 4.1 (p.201)

- A coin has probability 0.5 of landing heads when tossed. Let X = 1 if the coin comes up heads, and X = 0 if the coin comes up tails.
- What is the distribution of X?

Solution:

- ✓ Since X = 1 when heads comes up, heads is the success outcome
- ✓ The success probability, P(X = 1), is equal to 0.5

  Therefore **X ~ Bernoulli(0.5)**

Example 4.2 (p.201)

- A die has probability 1/6 of coming up 6 when rolled.
- Let X = 1 if the die comes up 6, and X = 0 otherwise.
- What is the distribution of X?

Solution:

- ✓ The success probability is $p = P(X = 1) = 1/6$
- ✓ Therefore **$X \sim$ Bernoulli(1/6)**

Example 4.3 (p.201)

- Ten percent of the components manufactured by a certain process are defective.
- A component is chosen at random.
- Let $X = 1$ if the component is defective, and $X = 0$ otherwise.
- What is the distribution of X?

Solution:

- ✓ The success probability is $p = P(X = 1) = 0.1$
- ✓ Therefore **$X \sim$ Bernoulli(0.1)**

Mean and Variance of a Bernoulli Random Variable (p.202)

If $X \sim$ Bernoulli(p), then

$$\mu_X = p \tag{4.1}$$

$$\sigma_X^2 = p(1 - p) \tag{4.2}$$

Example 4.4

Refer to Example 4.3. Find μ_X and σ_X^2.

Solution

Since $X \sim$ Bernoulli(0.1), the success probability p is equal to 0.1. Using Equations (4.1) and (4.2), $\mu_X = 0.1$ and $\sigma_X^2 = 0.1(1 - 0.1) = 0.09$.

The Binomial Distribution (p.204)

- Sampling a single component from a lot and determining whether it is defective is an example of a Bernoulli trial.
- In practice, we might sample several components from a very large lot and count the number of defectives among them.
- This amounts to conducting several independent Bernoulli trials and counting the number of successes.
- The number of successes is then a random variable, which is said to have a **binomial distribution.**

Definition

If a total of *n* Bernoulli trials are conducted, and

- The trials are independent
- Each trial has the same success probability p
- X is the number of successes in the *n* trials

then X has the binomial distribution with parameters n and p, denoted **$X \sim \text{Bin}(n, p)$**

Example 4.5 (p.204)

- A fair coin is tossed 10 times.
- Let X be the number of heads that appear.
- What is the distribution of X?

Solution:

- ✓ There are 10 independent Bernoulli trials, each with success probability $p = 0.5$.
- ✓ The random variable X is equal to the number of successes in the 10 trials.
- ✓ Therefore **$X \sim Bin(10, 0.5)$**

When can we use the binomial distribution?

- Assume that a finite population contains items of two types, successes and failures, and that a simple random sample is drawn from the population.
- If the sample size is **no more than 5%** of the population, the binomial distribution may be used to model the number of successes.

Probability Mass Function of a Binomial Random Variable

The probability of exactly X successes in n trials is

$$\mathbf{P(X)} = \frac{\mathbf{n!}}{X!\,(\mathbf{n} - \mathbf{X})!} \cdot p^{\mathbf{X}} \cdot q^{\mathbf{n-X}}$$

n = number of trials

p = numerical probability of a success

q = 1 – p is the numerical probability of a failure (complement of p)

X = the number of successes

$$P(X) = \underbrace{{}_{n}C_{x}}_{\text{number of possible desired outcomes}} \cdot \underbrace{p^{X} \cdot q^{n-X}}_{\text{probability of a desired outcome}}$$

RECAP - Factorial

- For any counting number ‘n’

  $n! = n*(n-1)*(n-2)...*1$

- For example,

  $5! = 5*4*3*2*1 = 120$

- However, $0! = 1$

Probability Mass Function of a Binomial Random Variable (p.206)

If $X \sim \text{Bin}(n, p)$, the probability mass function of X is

$$p(x) = P(X = x) = \begin{cases} \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x} & x = 0, 1, \ldots, n \\ 0 & \text{otherwise} \end{cases} \tag{4.4}$$

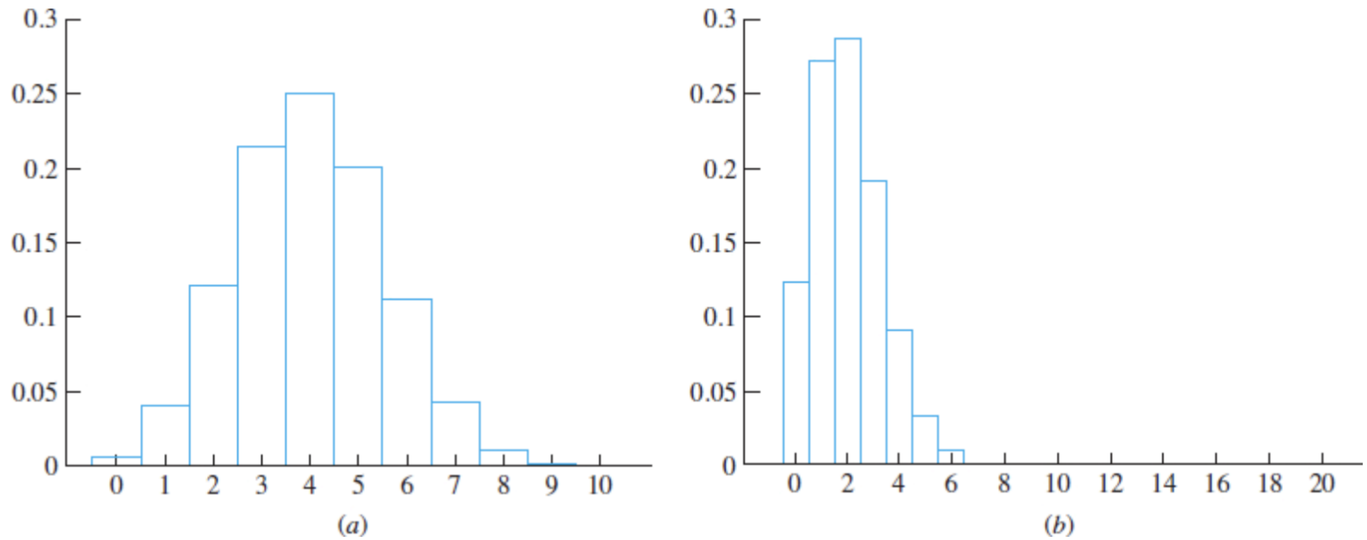

FIGURE 4.2 *(a)* The Bin(10, 0.4) probability histogram. *(b)* The Bin(20, 0.1) probability histogram.

Example 4.8 (p.207)

- A fair die is rolled eight times.
- Find the probability that **no more than 2 sixes** come up.

Solution:

- ✓ Each roll of the die is a Bernoulli trial with success probability 1/6.
- ✓ Let X denote the number of sixes in 8 rolls.
- ✓ Then $X \sim \text{Bin}(8, 1/6)$.

Example 4.8 (p.207)...

- ✓ **X ~ Bin(8, 1/6)**
- ✓ We need to find P(X ≤ 2). Using the probability mass function

$$
\begin{aligned}
P(X \le 2) &= P(X = 0 \text{ or } X = 1 \text{ or } X = 2) \\
&= P(X = 0) + P(X = 1) + P(X = 2) \\
&= \frac{8!}{0!(8-0)!}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{8-0} + \frac{8!}{1!(8-1)!}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{8-1} \\
&\quad + \frac{8!}{2!(8-2)!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{8-2} \\
&= 0.2326 + 0.3721 + 0.2605 \\
&= 0.8652
\end{aligned}
$$

Appendix A

Tables

- Table A.1: Cumulative Binomial Distribution
- Table A.2: Cumulative Normal Distribution
- Table A.3: Upper Percentage Points for the Student's *t* Distribution
- Table A.4: Tolerance Factors for the Normal Distribution
- Table A.5: Critical Points for the Wilcoxon Signed-Rank Test
- Table A.6: Critical Points for the Wilcoxon Rank-Sum Test
- Table A.7: Upper Percentage Points for the χ^2 Distribution
- Table A.8: Upper Percentage Points for the *F* Distribution
- Table A.9: Upper Percentage Points for the Studentized Range q_{ν_1,ν_2}
- Table A.10: Control Chart Constants

TABLE A.1 Cumulative binomial distribution

$$F(x) = P(X \le x) = \sum_{k=0}^{x} \frac{n!}{k!(n-k)!} p^k (1-p)^{(n-k)}$$

		p												
n	***x***	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**
2	**0**	0.902	0.810	0.640	0.562	0.490	0.360	0.250	0.160	0.090	0.062	0.040	0.010	0.003
	1	0.997	0.990	0.960	0.938	0.910	0.840	0.750	0.640	0.510	0.438	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	**0**	0.857	0.729	0.512	0.422	0.343	0.216	0.125	0.064	0.027	0.016	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.844	0.784	0.648	0.500	0.352	0.216	0.156	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.984	0.973	0.936	0.875	0.784	0.657	0.578	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	**0**	0.815	0.656	0.410	0.316	0.240	0.130	0.062	0.026	0.008	0.004	0.002	0.000	0.000
	1	0.986	0.948	0.819	0.738	0.652	0.475	0.313	0.179	0.084	0.051	0.027	0.004	0.000
	2	1.000	0.996	0.973	0.949	0.916	0.821	0.688	0.525	0.348	0.262	0.181	0.052	0.014
	3	1.000	1.000	0.998	0.996	0.992	0.974	0.938	0.870	0.760	0.684	0.590	0.344	0.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	**0**	0.774	0.590	0.328	0.237	0.168	0.078	0.031	0.010	0.002	0.001	0.000	0.000	0.000
	1	0.977	0.919	0.737	0.633	0.528	0.337	0.187	0.087	0.031	0.016	0.007	0.000	0.000
	2	0.999	0.991	0.942	0.896	0.837	0.683	0.500	0.317	0.163	0.104	0.058	0.009	0.001
	3	1.000	1.000	0.993	0.984	0.969	0.913	0.812	0.663	0.472	0.367	0.263	0.081	0.023
	4	1.000	1.000	1.000	0.999	0.998	0.990	0.969	0.922	0.832	0.763	0.672	0.410	0.226
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

n	x	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
20	0	0.358	0.122	0.012	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.736	0.392	0.069	0.024	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.925	0.677	0.206	0.091	0.035	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.984	0.867	0.411	0.225	0.107	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.997	0.957	0.630	0.415	0.238	0.051	0.006	0.000	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.989	0.804	0.617	0.416	0.126	0.021	0.002	0.000	0.000	0.000	0.000	0.000
	6	1.000	0.998	0.913	0.786	0.608	0.250	0.058	0.006	0.000	0.000	0.000	0.000	0.000
	7	1.000	1.000	0.968	0.898	0.772	0.416	0.132	0.021	0.001	0.000	0.000	0.000	0.000
	8	1.000	1.000	0.990	0.959	0.887	0.596	0.252	0.057	0.005	0.001	0.000	0.000	0.000
	9	1.000	1.000	0.997	0.986	0.952	0.755	0.412	0.128	0.017	0.004	0.001	0.000	0.000
	10	1.000	1.000	0.999	0.996	0.983	0.872	0.588	0.245	0.048	0.014	0.003	0.000	0.000
	11	1.000	1.000	1.000	0.999	0.995	0.943	0.748	0.404	0.113	0.041	0.010	0.000	0.000
	12	1.000	1.000	1.000	1.000	0.999	0.979	0.868	0.584	0.228	0.102	0.032	0.000	0.000
	13	1.000	1.000	1.000	1.000	1.000	0.994	0.942	0.750	0.392	0.214	0.087	0.002	0.000
	14	1.000	1.000	1.000	1.000	1.000	0.998	0.979	0.874	0.584	0.383	0.196	0.011	0.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.949	0.762	0.585	0.370	0.043	0.003
	16	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.984	0.893	0.775	0.589	0.133	0.016
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.965	0.909	0.794	0.323	0.075
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.992	0.976	0.931	0.608	0.264
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.988	0.878	0.642
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Example 4.9: (p.208)

- According to a 2016 report by J.D. Power and Associates, approximately **20%** of registered automobiles have been recalled for a manufacturer's defect but have not been repaired.
- **Twelve** automobiles are sampled at random.
- What is the probability that **fewer than four** of them have been recalled but not repaired?

Example 4.9: (p.208) - SOLUTION

- Let X represent the number of automobiles in the sample that have been recalled but not repaired.
- Then $X \sim \text{Bin}(12, 0.2)$.
- The probability that fewer than four have been recalled but not repaired is $P(X \leq 3)$.
- We consult Table A.1 with $n = 12$, $p = 0.2$, and $x = 3$
- We find that $\mathbf{P(X \leq 3) = 0.795}$

		p												
n	*x*	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
12	0	0.540	0.282	0.069	0.032	0.014	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.882	0.659	0.275	0.158	0.085	0.020	0.003	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.980	0.889	0.558	0.391	0.253	0.083	0.019	0.003	0.000	0.000	0.000	0.000	0.000
	3	0.998	0.974	0.795	0.649	0.493	0.225	0.073	0.015	0.002	0.000	0.000	0.000	0.000
	4	1.000	0.996	0.927	0.842	0.724	0.438	0.194	0.057	0.009	0.003	0.001	0.000	0.000
	5	1.000	0.999	0.981	0.946	0.882	0.665	0.387	0.158	0.039	0.014	0.004	0.000	0.000
	6	1.000	1.000	0.996	0.986	0.961	0.842	0.613	0.335	0.118	0.054	0.019	0.001	0.000
	7	1.000	1.000	0.999	0.997	0.991	0.943	0.806	0.562	0.276	0.158	0.073	0.004	0.000
	8	1.000	1.000	1.000	1.000	0.998	0.985	0.927	0.775	0.507	0.351	0.205	0.026	0.002
	9	1.000	1.000	1.000	1.000	1.000	0.997	0.981	0.917	0.747	0.609	0.442	0.111	0.020
	10	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.980	0.915	0.842	0.725	0.341	0.118
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.986	0.968	0.931	0.718	0.460
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Example 4.10: (p.208)

- Refer to Example 4.9
- What is the probability that **more than 1 of the 12 cars** has been recalled but not repaired?

SOLUTION:

- ✓ Sometimes the best way to compute the probability of an event is to compute the probability that the event does not occur, and then subtract from 1

Example 4.10: (p.208)

<u>SOLUTION:</u>

- ✓ Let X represent the number of cars that have been recalled but not repaired.
- ✓ We wish to compute $P(X > 1)$.
- ✓ Table A.1 presents probabilities of the form $P(X \leq x)$.
- ✓ Therefore, we note that $P(X > 1) = 1 - P(X \leq 1)$.
- ✓ Consulting the table with $n = 12$, $p = 0.2$, and $x = 1$, we find that $P(X \leq 1) = 0.275$.

- ✓ Therefore **$P(X > 1) = 1 - 0.275 = 0.725$**

		p												
n	***x***	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**
12	**0**	0.540	0.282	0.069	0.032	0.014	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.882	0.659	0.275	0.158	0.085	0.020	0.003	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.980	0.889	0.558	0.391	0.253	0.083	0.019	0.003	0.000	0.000	0.000	0.000	0.000
	3	0.998	0.974	0.795	0.649	0.493	0.225	0.073	0.015	0.002	0.000	0.000	0.000	0.000
	4	1.000	0.996	0.927	0.842	0.724	0.438	0.194	0.057	0.009	0.003	0.001	0.000	0.000
	5	1.000	0.999	0.981	0.946	0.882	0.665	0.387	0.158	0.039	0.014	0.004	0.000	0.000
	6	1.000	1.000	0.996	0.986	0.961	0.842	0.613	0.335	0.118	0.054	0.019	0.001	0.000
	7	1.000	1.000	0.999	0.997	0.991	0.943	0.806	0.562	0.276	0.158	0.073	0.004	0.000
	8	1.000	1.000	1.000	1.000	0.998	0.985	0.927	0.775	0.507	0.351	0.205	0.026	0.002
	9	1.000	1.000	1.000	1.000	1.000	0.997	0.981	0.917	0.747	0.609	0.442	0.111	0.020
	10	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.980	0.915	0.842	0.725	0.341	0.118
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.986	0.968	0.931	0.718	0.460
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Example: Tossing 3 Coins

A coin is tossed 3 times. Find the probability of getting exactly two heads

Solution 1 - using the sample space

This problem can be solved by looking at the sample space. There are three ways to get two heads.

HHH, **HHT, HTH, THH**, TTH, THT, HTT, TTT

The answer is 3/8, or 0.375

Example: Tossing 3 Coins…

A coin is tossed 3 times. Find the probability of getting exactly two heads

Solution 2 – using a binomial distribution

1. There are a fixed number of trials (three).
2. There are only two outcomes for each trial, heads or tails.
3. The outcomes are independent of one another (the outcome of one toss in no way affects the outcome of another toss).
4. The probability of a success (heads) is ½ in each case.

Example: Tossing 3 Coins…

The probability of exactly X successes in n trials is

$$\mathbf{P(X)} = \frac{\mathbf{n!}}{X!\,(\mathbf{n}-\mathbf{X})!} \cdot p^{\mathbf{X}} \cdot q^{\mathbf{n-X}}$$

In this case:

n = 3 (number of trials)
X = 2 (the number of successes)
p = ½ (numerical probability of a success)
q = ½ (numerical probability of a failure)

$$\mathbf{P(2\ \boldsymbol{heads})} = \frac{3!}{2!(3-2)!} \cdot \frac{1}{2}^{2} \cdot \frac{1}{2}^{1} = \frac{3}{8} = \mathbf{0.375}$$

Example: Tossing 3 Coins…

A coin is tossed 3 times. Find the probability of getting exactly two heads

<u>Solution 3 – using Table A.1 (Appendix A)</u>

$n = 3, x = 2, p = 0.50 \rightarrow P(X=2) = 0.875 - 0.500 = 0.375$

		p												
n	***x***	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**
2	**0**	0.902	0.810	0.640	0.562	0.490	0.360	0.250	0.160	0.090	0.062	0.040	0.010	0.003
	1	0.997	0.990	0.960	0.938	0.910	0.840	0.750	0.640	0.510	0.438	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	**0**	0.857	0.729	0.512	0.422	0.343	0.216	0.125	0.064	0.027	0.016	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.844	0.784	0.648	0.500	0.352	0.216	0.156	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.984	0.973	0.936	0.875	0.784	0.657	0.578	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Example: Tossing 3 Coins…

A coin is tossed 3 times. Find the probability of getting exactly two heads

Solution 3 – using a table from a different source.

$$n=3,\ p=\tfrac{1}{2}=0.5,\ X=2\ \rightarrow P(2)=\boxed{0.375}$$

TABLE B The Binomial Distribution

		p												
n	*x*	0.05	0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	0.95
2	0	0.902	0.810	0.640	0.563	0.490	0.360	0.250	0.160	0.090	0.063	0.040	0.010	0.002
	1	0.095	0.180	0.320	0.375	0.420	0.480	0.500	0.480	0.420	0.375	0.320	0.180	0.095
	2	0.002	0.010	0.040	0.063	0.090	0.160	0.250	0.360	0.490	0.563	0.640	0.810	0.902
3	0	0.857	0.729	0.512	0.422	0.343	0.216	0.125	0.064	0.027	0.016	0.008	0.001	
	1	0.135	0.243	0.384	0.422	0.441	0.432	0.375	0.288	0.189	0.141	0.096	0.027	0.007
	2	0.007	0.027	0.096	0.141	0.189	0.288	0.375	0.432	0.441	0.422	0.384	0.243	0.135
	3		0.001	0.008	0.016	0.027	0.064	0.125	0.216	0.343	0.422	0.512	0.729	0.857
4	0	0.815	0.656	0.410	0.316	0.240	0.130	0.062	0.026	0.008	0.004	0.002		

Exercise

According to the Bureau of Labor Statistics, 23% of the US workforce in 2015 was employed in the wholesale and retail trade industry.

If 8 US workers are randomly selected, what is the probability that exactly 3 are employed in the wholesale and retail trade industry?

$$\mathbf{P(3)} = \frac{8!}{3!\,(8-3)!} \cdot (0.23)^3 \cdot (0.77)^{8-3} = \mathbf{0.184}$$

Criteria (1)

Check and make sure that all of the criteria for the binomial experiment are satisfied.

1. There must be a fixed number of trials.

Eight workers will be randomly selected

n= 8

2. Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. These outcomes can be considered as either success or failure.

There are only 2 possible outcomes. Either the selected worker is employed in the wholesale and retail trade industry or not.

Success = selected worker is employed in wholesale and retail trade industry

Criteria (2)

3. The outcomes of each trial must be independent of one another.

The employment industry of one randomly selected employee is not influenced by the industry of employment of another. So the outcomes of each trial would be independent of one another.

4. The probability of a success must remain the same for each trial.

The Bureau of Labor Statistics reports that 23% of the US workforce in 2015 was employed in the wholesale and retail industry.

$p = 0.23 \qquad q = 1 - 0.23 = 0.77$

Criteria are Satisfied

We have determined that the criteria for a binomial experiment are satisfied, we can use the formula for calculating a binomial probability to find this probability.

Binomial Calculation

According to the Bureau of Labor Statistics, 23% of the US workforce in 2015 was employed in the wholesale and retail trade industry. If 8 US workers are randomly selected, what it the probability that exactly 3 are employed in the wholesale and retail trade industry?

$X = 3$ $n = 8$ $p = 0.23$ $q = 1 - 0.23 = 0.77$

$$\mathbf{P(3)} = \frac{8!}{3!\,(8-3)!} \cdot (0.23)^3 \cdot (0.77)^{8-3} = \mathbf{0.184}$$

Answer

The probability that exactly 3 out of 8 randomly selected US workers in 2015 would be employed in the wholesale and retail trade industry would be **0.184**.

The Mean and Standard Deviation of a Binomial Random Variable

The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard Deviation: $\sigma = \sqrt{npq}$

The Mean and Standard Deviation of a Binomial Random Variable (p.209)

Summary

If $X \sim \text{Bin}(n, p)$, then the mean and variance of X are given by

$$\mu_X = np \tag{4.5}$$

$$\sigma_X^2 = np(1-p) \tag{4.6}$$

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard Deviation: $\sigma = \sqrt{npq}$

Example: Rolling an 8-sided die

An 8-sided die (with the numbers 1 through 8 on the faces) is rolled 560 times. Find the mean, variance, and standard deviation of the number of 7s that will be rolled.

Example: Rolling an 8-sided die…

An 8-sided die (with the numbers 1 through 8 on the faces) is rolled 560 times. Find the mean, variance, and standard deviation of the number of 7s that will be rolled.

SOLUTION

This is a binomial experiment with $n = 560$, $p = \frac{1}{8}$, and $q = \frac{7}{8}$ so that

$$\mu = n \cdot p = 560 \cdot \frac{1}{8} = 70$$

$$\sigma^2 = n \cdot p \cdot q = 560 \cdot \frac{1}{8} \cdot \frac{7}{8} = 61\tfrac{1}{4} = 61.25$$

$$\sigma = \sqrt{61.25} = 7.826$$

In this case, the mean of the number of 7s obtained is 70. The variance is 61.25, and the standard deviation is 7.826.

Interim Summary

A **binomial experiment** has four requirements.

1) There must be a fixed number of trials.
2) Each trial can have only two outcomes.
3) The outcomes are independent of each other, and the probability of a success must remain the same for each trial.
4) The probabilities of the outcomes can be found by using the binomial formula or the binomial distribution table.

End of Sections 4.1 and 4.2