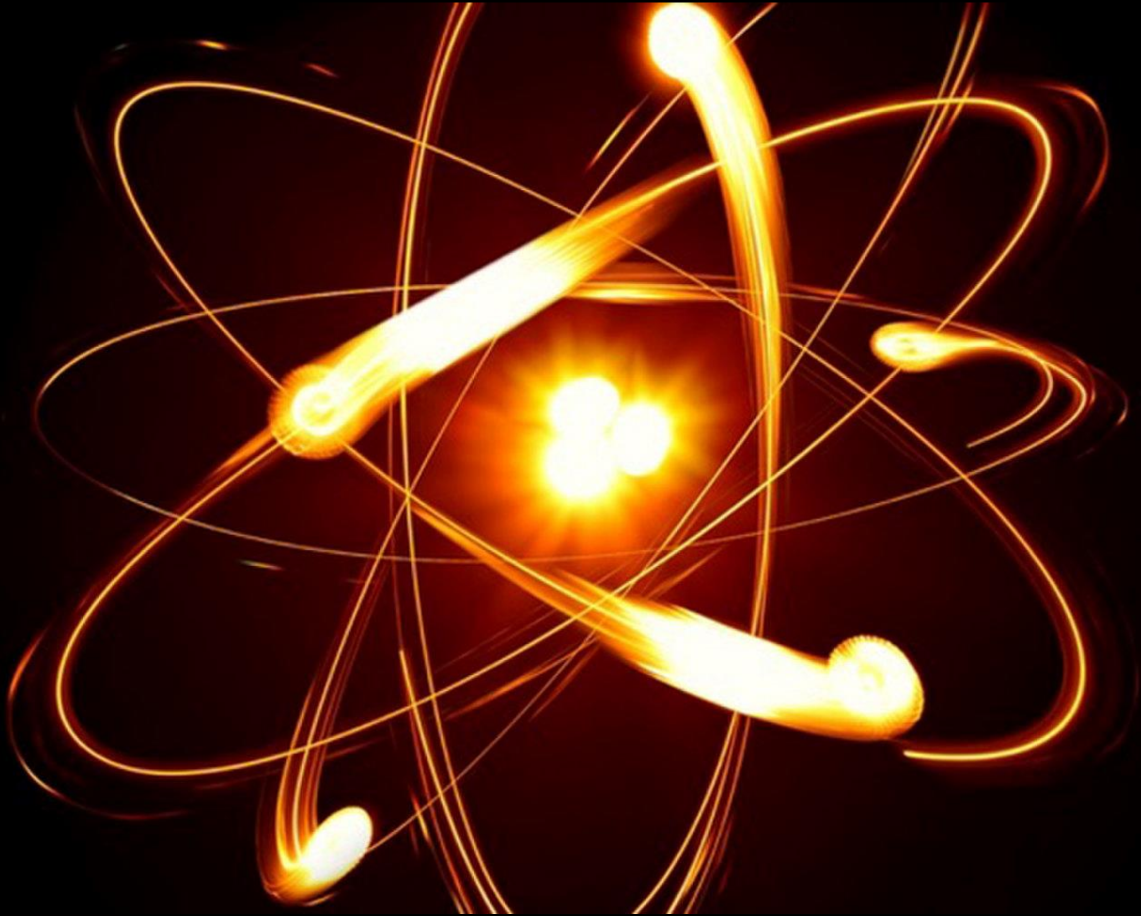


Motion

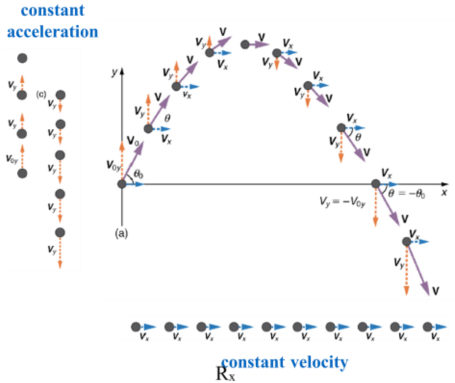
- motion, force and energy
  - Mechanics
    - Kinematics
      - How move
        - One Dimension
        - Two Dimensions
    - Dynamics
      - Force
      - Why move

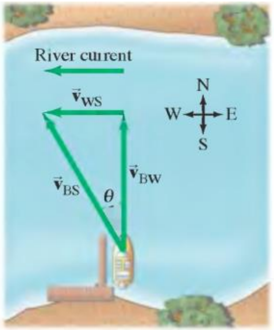
$$\bar{v} = \frac{(v_0 + v_f)}{2}$$

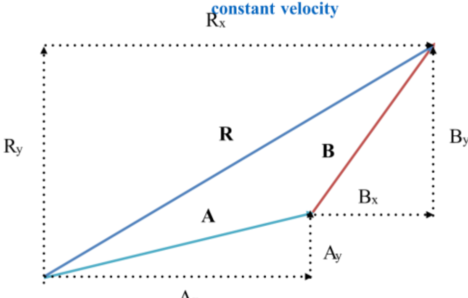
$$v_f = v_0 + at$$

$$x_f = x_0 + v_0 t + \frac{at^2}{2}$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0)$$





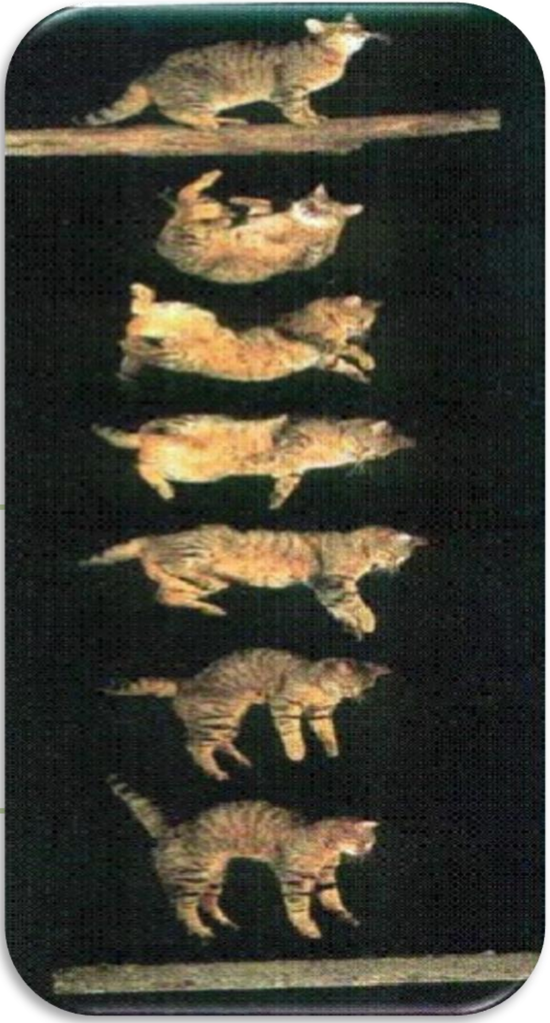


Motion

Motion

Translational

Rotational



Review

Newton’s laws

Drag force

Reference frame

Homework

Parachute	v - t	a - t
With		
Without		

The limitation to applying Newton's law of motion

1. Reference frames

Newton's law can only be used in inertial reference frames (that are not being accelerated).

2. Speed limits

Newtonian mechanics is a good approximation as long as the speed of the system is much less than the speed of light.

3. Quantum mechanics

Newtonian mechanics can not describe or account for many phenomena on the atomic and nuclear scale.

4. Force propagation

5. Chaotic—nonlinear system

The limitation to applying Newton’s law of motion

1. Reference frames

Newton’s law can only be used in

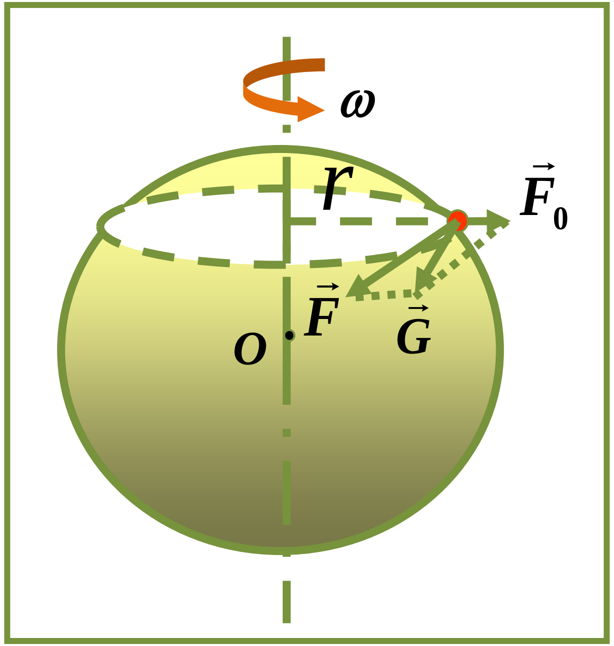
inertial reference frames

Inertial Reference frame

Do inertial frames really exist?

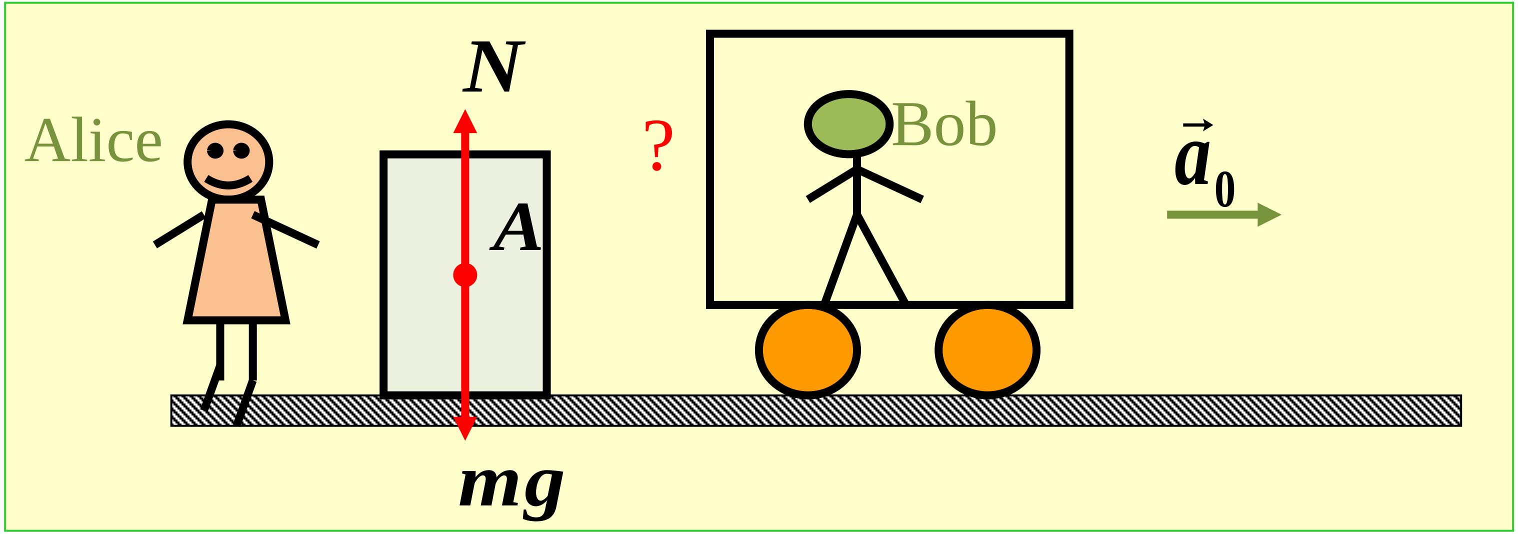
- **Ideal model.**
- **The earth spins on its axis. The centripetal acceleration is less than $3.4\times10^{-2}m/s^2$.**
- **The surface of the earth is a approximate inertial reference frame.**
- **The earth moves around the sun. The centripetal acceleration is $6\times10^{-3}m/s^2$.**

Inertial Reference frame



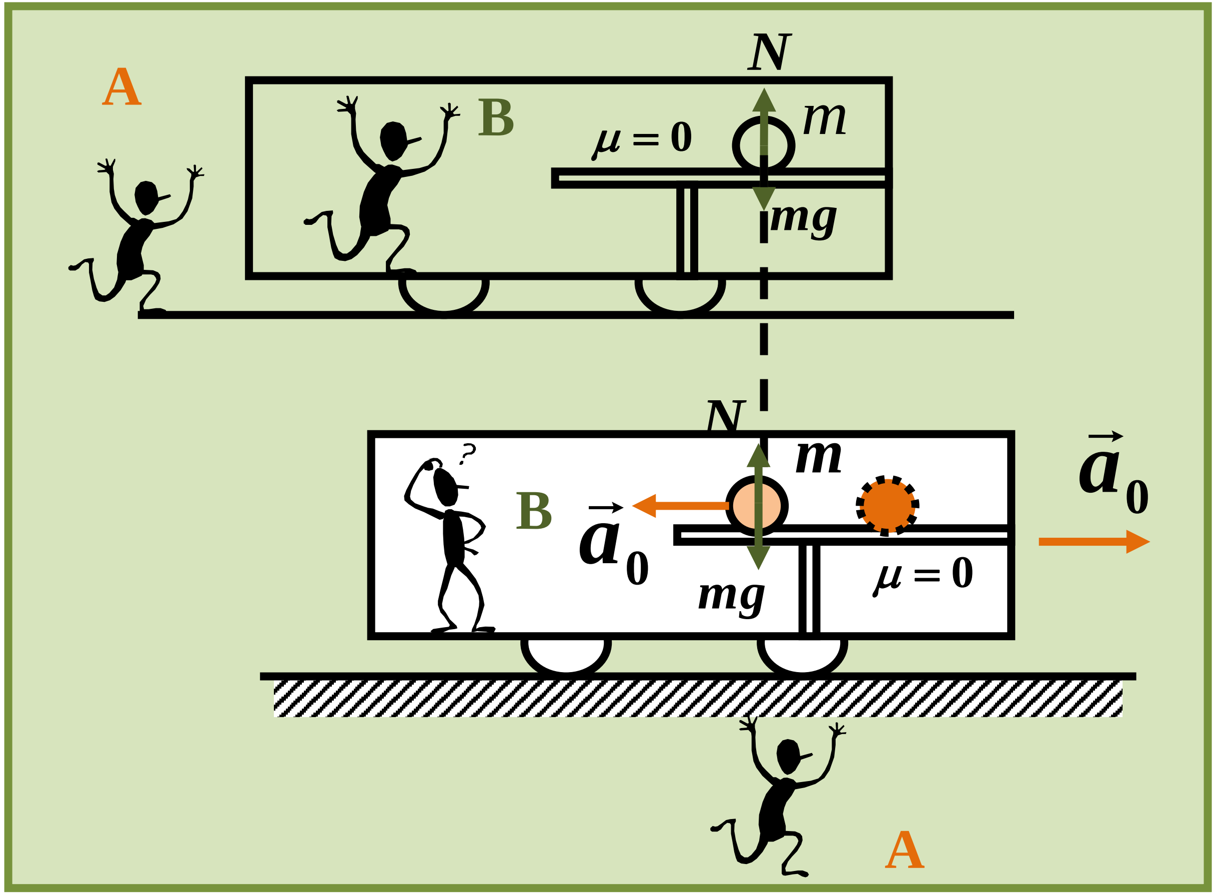
Non-inertial reference frames are accelerated reference frames.

Non-inertial reference frame

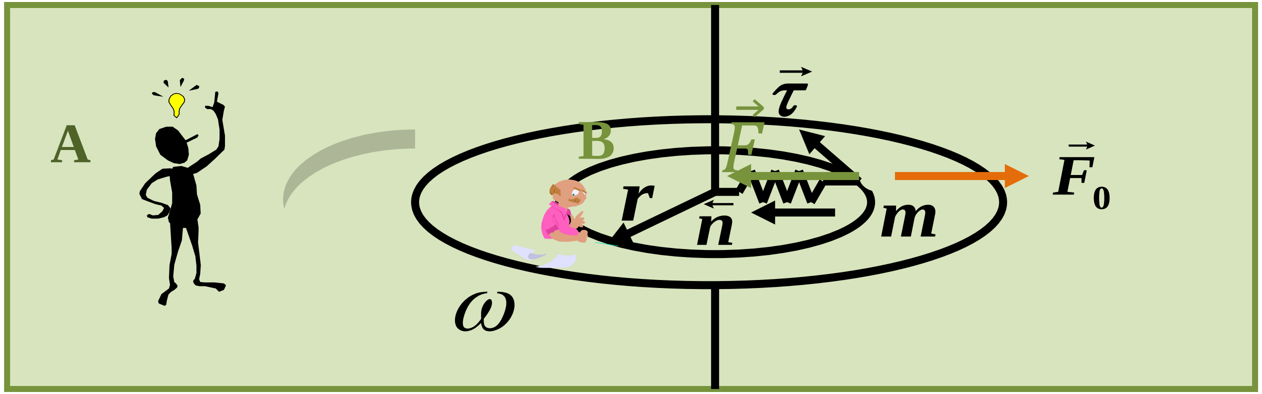


Newton's law is not valid in the car.

Non-inertial reference frame



Non-inertial reference frame



For A： m $\vec{F} = m\omega^2 r\vec{n}$ Circular motion

Why ?

For B： m $\vec{F} = m\omega^2 r\vec{n}$ No motion

西南交通大學
Southwest Jiaotong University

Physics 1: **Mechanics and Waves**

Week 6 – Coordinate system

and Circular motion

2023.3

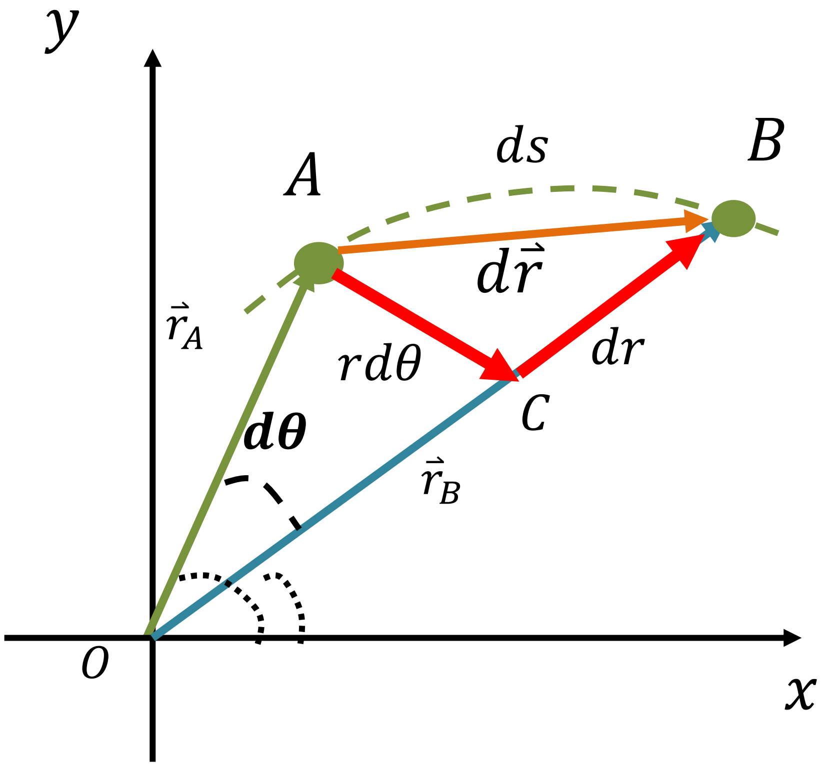
Coordinate system

CARTESIAN

NATURAL

ANGULAR

Cartesian



Coordinate system

CARTESIAN

NATURAL

ANGULAR

Coordinate system

CARTESIAN

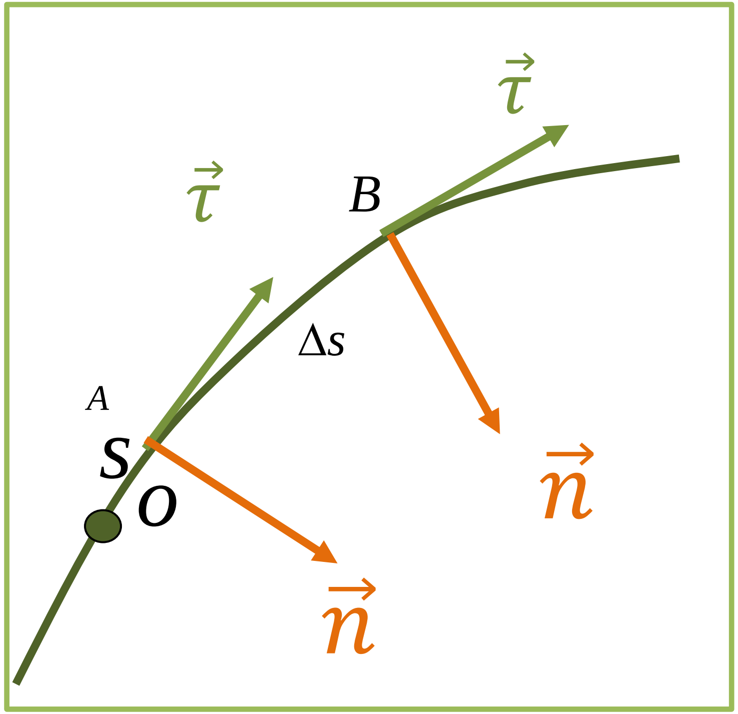
NATURAL

ANGULAR

Natural coordinate description

Trajectory Known

- Choose a point: O
- Location: *s* (*has a direction*)
- Can be decomposed
  - Tangential $\vec{\tau}$
  - normal $\vec{n}$

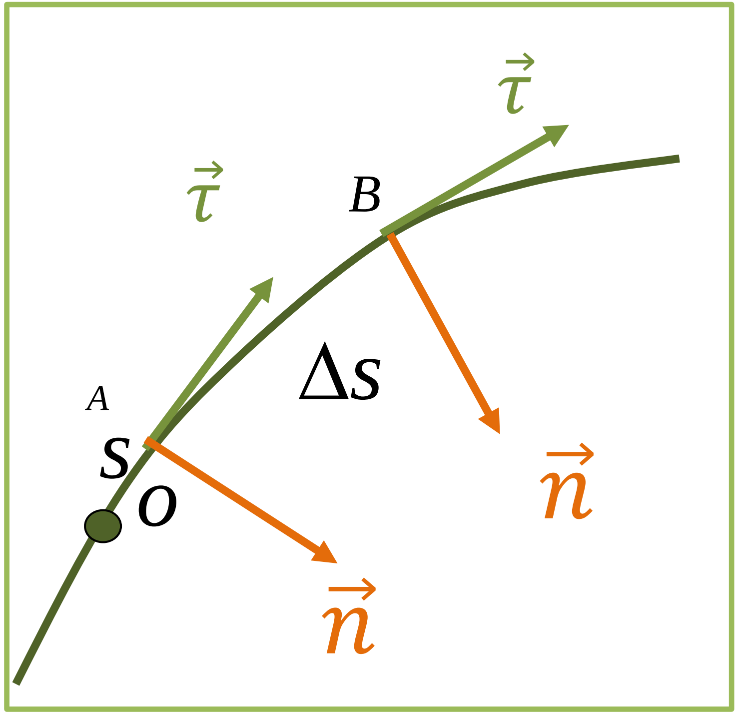


Natural coordinate description

Trajectory Known

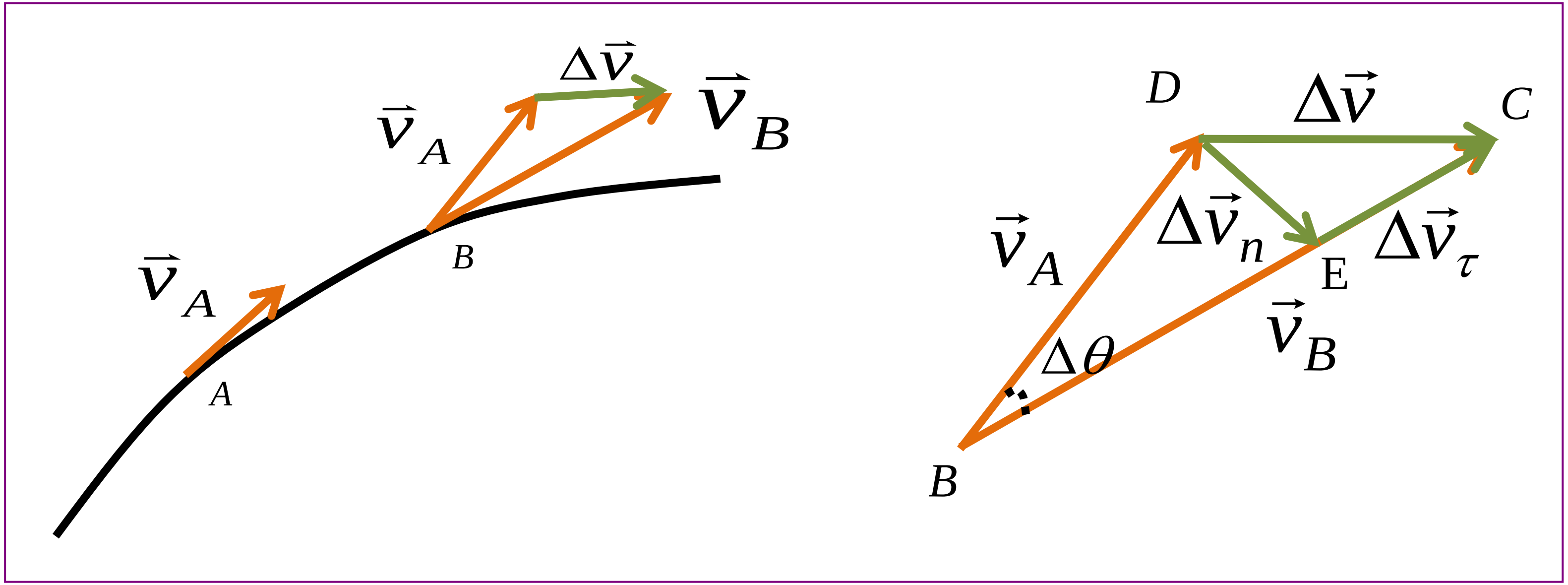
- Location: $s = s(t)$ (*only point*)
- The path Δs
- Velocity: tangential direction

$$\vec{v} = |\vec{v}|\vec{\tau} = \frac{\mathrm{d}s}{\mathrm{d}t}\vec{\tau}$$



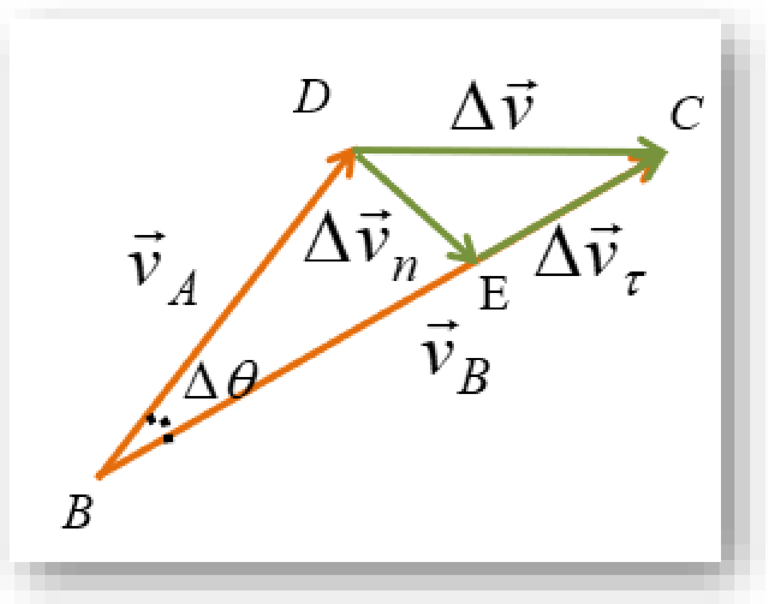
Natural coordinate description

Acceleration $\Delta\vec{v} = \Delta\vec{v}_\tau + \Delta\vec{v}_n$



Natural coordinate description

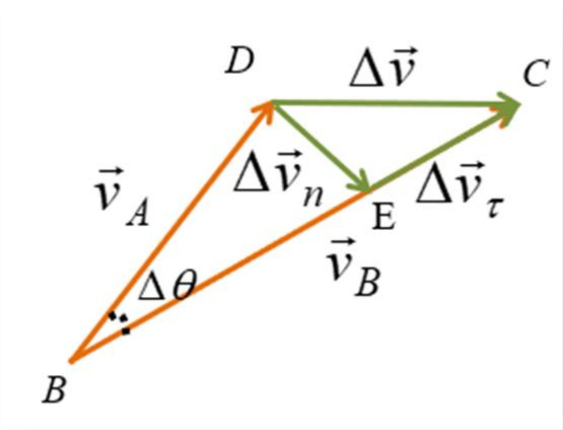
Acceleration $\Delta\vec{v} = \Delta\vec{v}_\tau + \Delta\vec{v}_n$



$$\therefore \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta\vec{v}_\tau}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta\vec{v}_\mathrm{n}}{\Delta t}$$

Natural coordinate description



Acceleration $\Delta\vec{v} = \Delta\vec{v}_\tau + \Delta\vec{v}_n$

The first part:

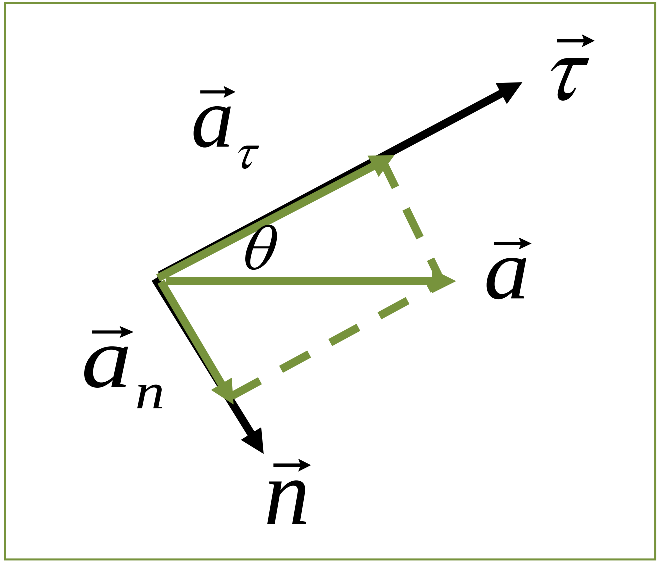
$$\lim_{\Delta t \to 0} \frac{\Delta\vec{v}_\tau}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}\vec{\tau} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau}$$

The second part:

$$\lim_{\Delta t \to 0} \frac{\Delta\vec{v}_\mathrm{n}}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\Delta\theta}{\Delta t}\vec{n} = \frac{v\mathrm{d}\theta}{\mathrm{d}t}\vec{n}$$

$$= v\frac{\mathrm{d}s}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}s}\vec{n} = \frac{v^2}{\rho}\vec{n}$$

Natural coordinate description



$$\therefore \quad \vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau} + \frac{v^2}{\rho}\vec{n} = \vec{a}_\tau + \vec{a}_n$$

Tangential acceleration

$$\vec{a}_\tau = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau}$$

Rate of the change of the magnitude of the velocity. No effect on the direction.

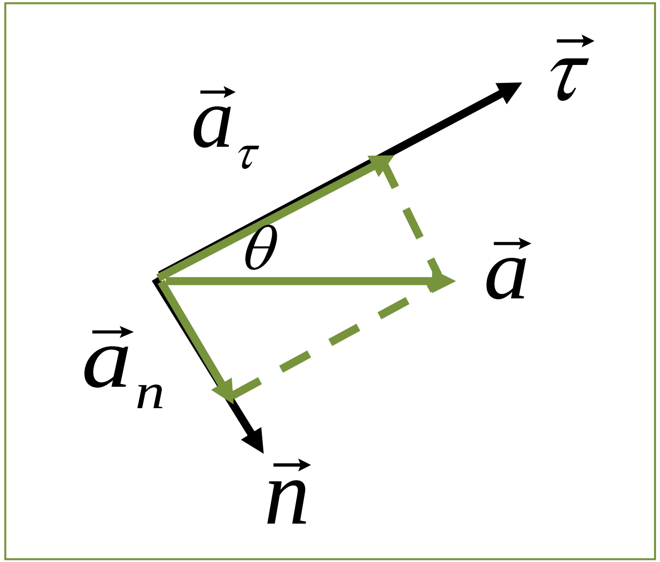
Normal acceleration

$$\vec{a}_n = \frac{v^2}{\rho}\vec{n}$$

Rate of the change of the direction. No effect on the magnitude of the velocity.

Natural coordinate description

$$\vec{a} = a_\tau \vec{\tau} + a_n \vec{n} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau} + \frac{v^2}{\rho}\vec{n}$$



Magnitude: $|\vec{a}| = \sqrt{a_\tau^{\ 2} + a_n^{\ 2}}$

Direction: $\vec{a}$与$\vec{a}_\tau$的夹角 $\theta = \mathrm{arctg}\,\frac{a_n}{a_\tau}$

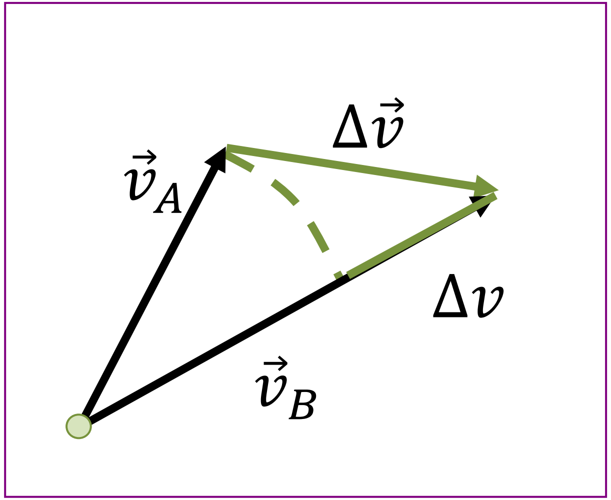
$\vec{a}$ **Pointing to curving inward**

Natural coordinate description

Example:

$$\left|\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right| = \frac{\mathrm{d}v}{\mathrm{d}t} \quad ?$$

$$\left|\vec{a}\right| \neq \left|\vec{a}_\tau\right|$$



$$\boxed{\vec{a} = a_\tau\vec{\tau} + a_n\vec{n} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau} + \frac{v^2}{\rho}\vec{n}}$$

Magnitude: $\left|\vec{a}\right| = \sqrt{a_\tau{}^2 + a_n{}^2}$

Coordinate system

CARTESIAN

NATURAL

ANGULAR

Coordinate system

CARTESIAN

NATURAL

ANGULAR

Circular Motion - Acceleration in curvilinear Motion

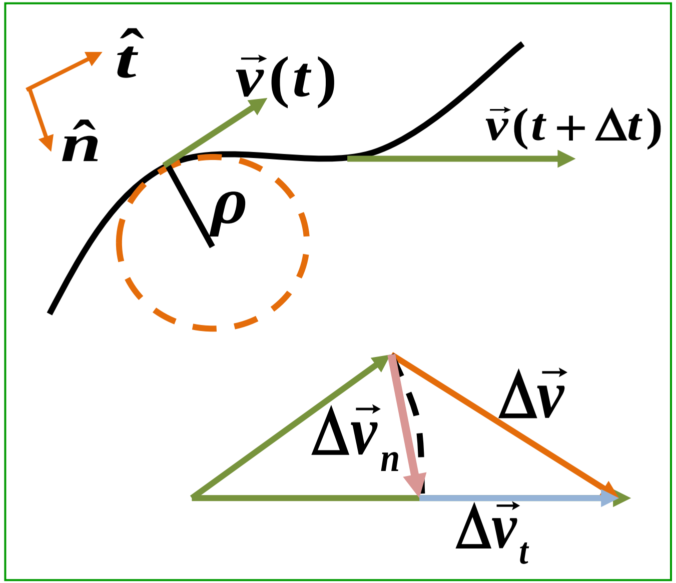
$$\Delta\vec{v} = \Delta\vec{v}_t + \Delta\vec{v}_n$$

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}_t + \Delta\vec{v}_n}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\Delta\vec{v}_t}{\Delta t} + \lim_{\Delta t\to 0}\frac{\Delta\vec{v}_n}{\Delta t}$$

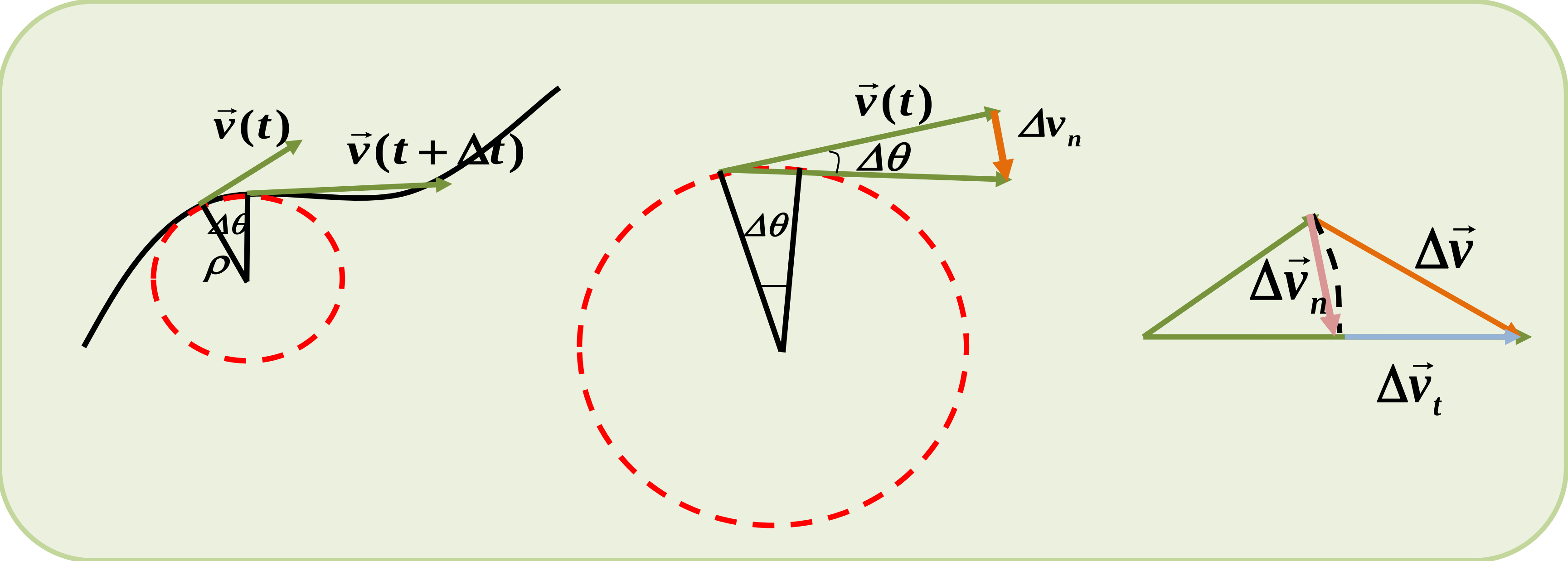
$$= \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{\rho}\hat{n}$$

Natural coordinate system



Special case: nonuniform circular motion $\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{\rho}\hat{n} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{R}\hat{n}$

Circular Motion



Angular position θ

Angular displacement $\Delta\theta$

Angular velocity ω

Angular acceleration α

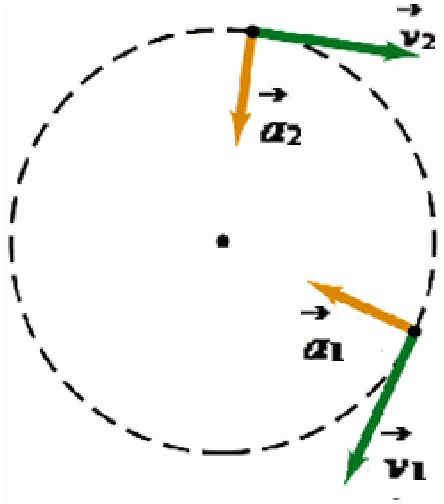
Circular Motion - Uniform

$$\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{r}\hat{n} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{r}\hat{n}$$

A constant speed?

$$\frac{\mathbf{d}\boldsymbol{v}}{\mathbf{d}\boldsymbol{t}} = \mathbf{0}$$

Uniform circular motion



Uniform circular motion

A constant

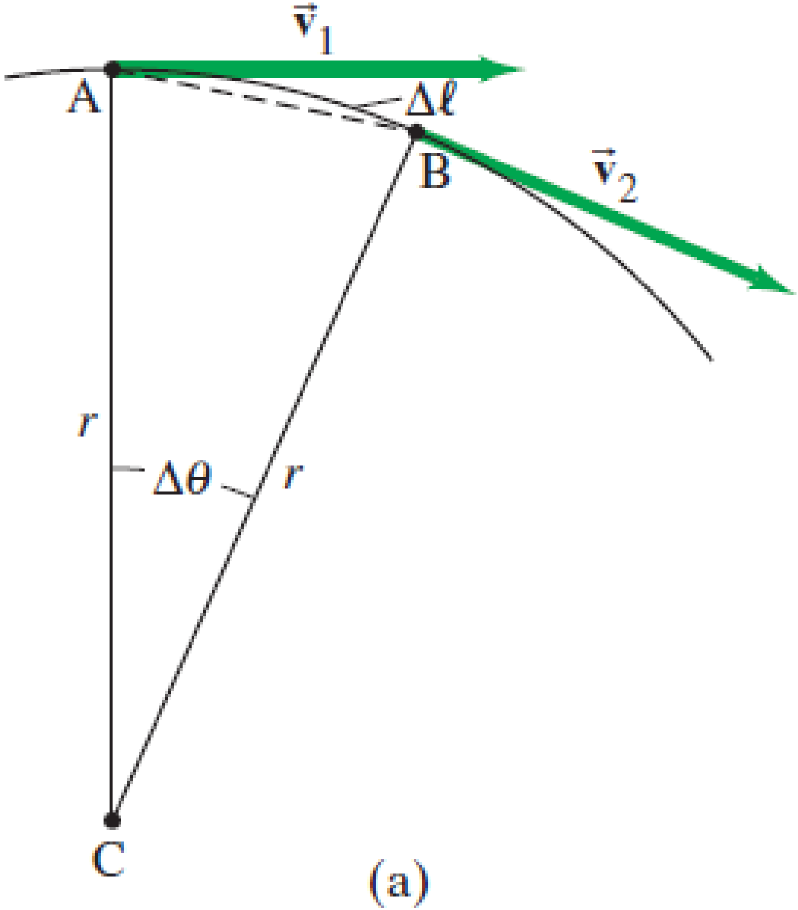
linear tangential speed (*v*) at any instant

magnitude constant but *direction* continuously change

A constant

angular velocity, ω (in rad/s)

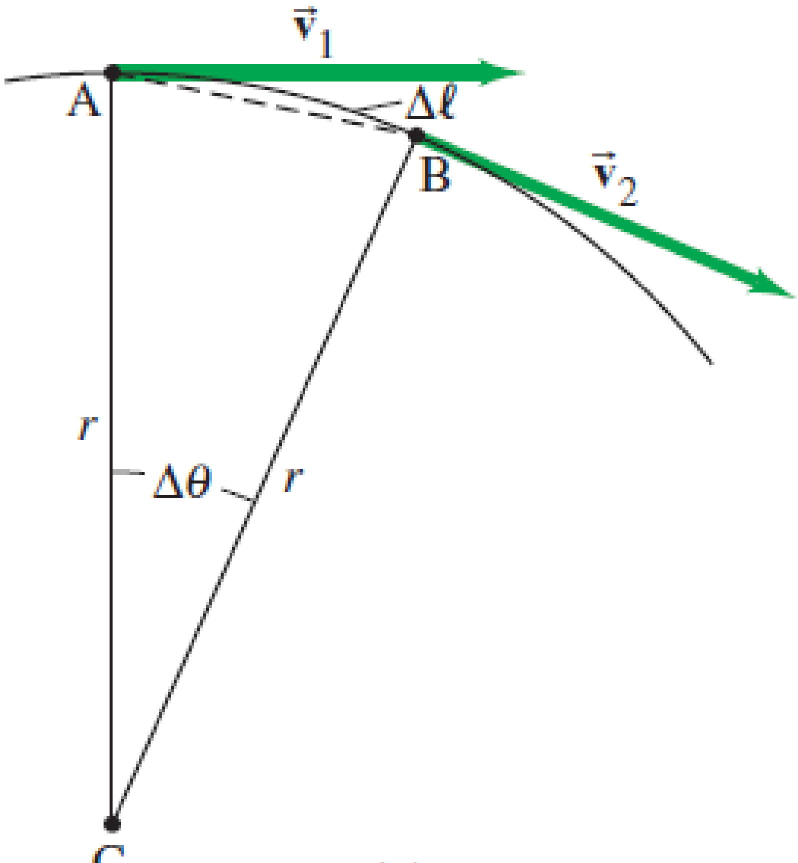
Kinematics of Uniform Circular Motion



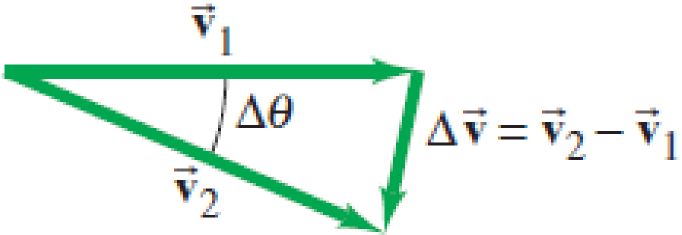
Acceleration ?

Direction and magnitude

Kinematics of Uniform Circular Motion



(a)

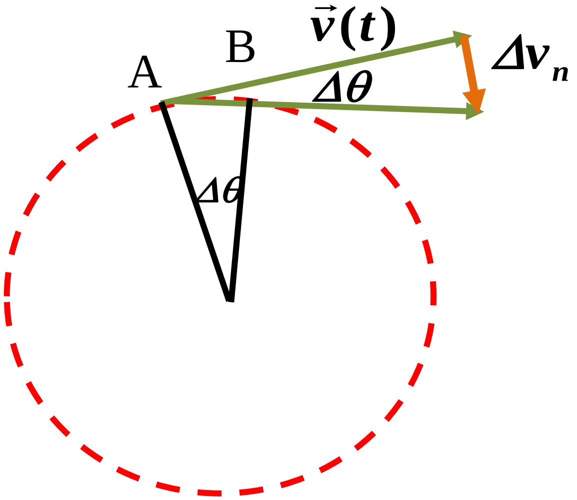


Acceleration ?

$$a_R = \frac{v^2}{r}$$

Derivation

$$\because \frac{\left|(\Delta\vec{v}_n)\right|}{v} = \frac{\overline{A}\,\overline{B}}{R}$$



$$a_n = \lim_{\Delta t\to 0}\frac{\left|(\Delta\vec{v})_n\right|}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\Delta s}{R\Delta t} = \frac{v^2}{R}$$

$$\therefore \left|\vec{a}_n\right| = \frac{v^2}{r}$$

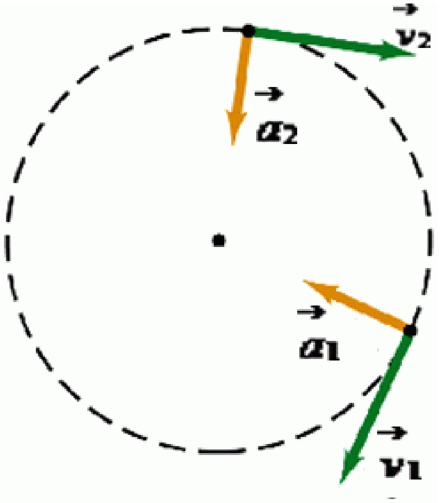
$\vec{n}$: radial direction

a_n, normal acceleration is also called a_r, radial acceleration.

Circular Motion - Uniform

A circular path: ***constant speed v***

Acceleration: ***radially inward, centripetal*** $\vec{a}$



$$\vec{a} = \frac{v^2}{r}\hat{r}$$ -- **Constant speed**

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$ -- **Radial Unit Vector**

Time to complete a circle $$T = \frac{2\pi r}{v}$$ -- **Period**

Circular Motion - Describe the circular motion

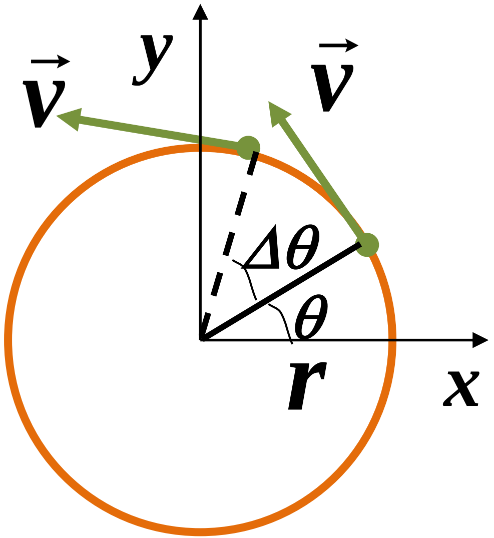
(1) Angular position (coordinate)θ (*t*)

(2) Angular displacement $\Delta\theta$ (*t*)

Arc length $$\Delta s = r\Delta\theta$$

(3) Speed and position of the particle

$$v = \frac{ds(t)}{dt} = \lim_{\Delta t \to 0s} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0s} \frac{r\Delta\theta}{\Delta t} = r\frac{d\theta(t)}{dt}$$



$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$$
$$= r\cos\theta\hat{i} + r\sin\theta\hat{j}$$

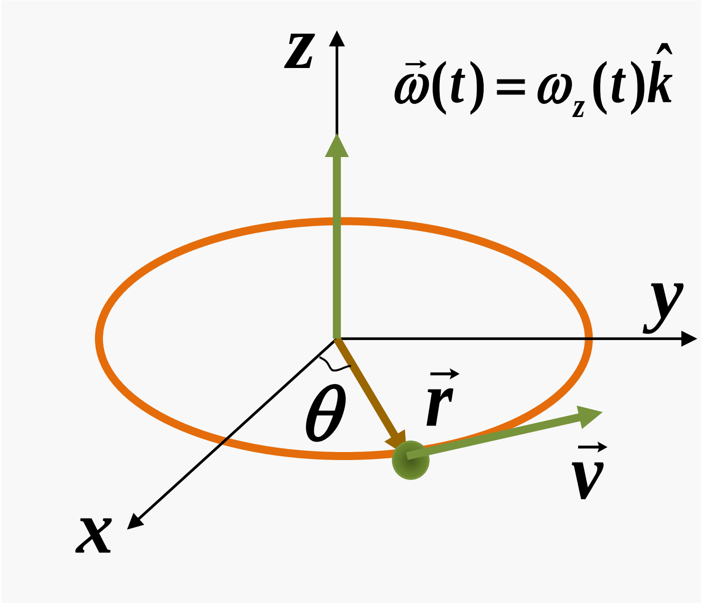
Circular Motion - Describe the circular motion

(4) Angular speed and angular velocity vector

Right-hand rule for $\hat{k}$

Angular speed: **the time rate of change of the angular coordinate**

$$\omega_z(t) = \frac{d\theta(t)}{dt}$$



Angular velocity vector

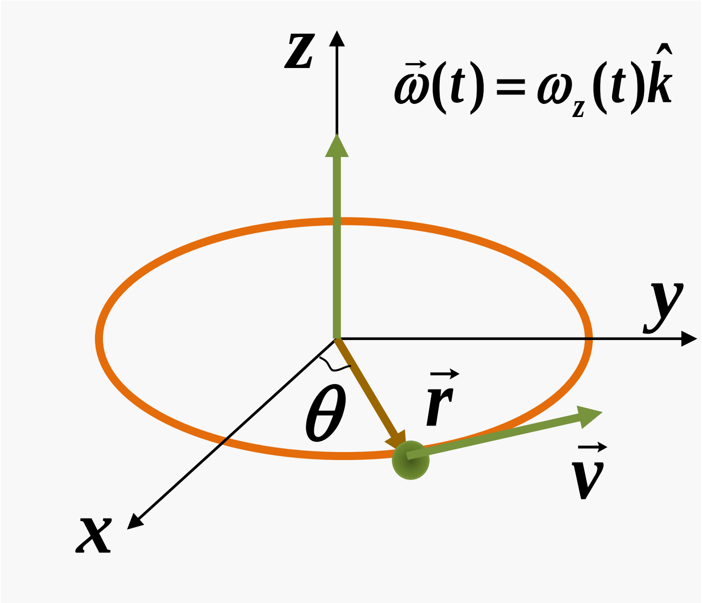
$$\vec{\omega}(t) = \omega_z(t)\hat{k} = \frac{d\theta(t)}{dt}\hat{k}$$

Circular Motion - Describe the circular motion

(5) The relation of $\vec{v}(t)$, $\vec{\omega}(t)$ and $\vec{r}(t)$

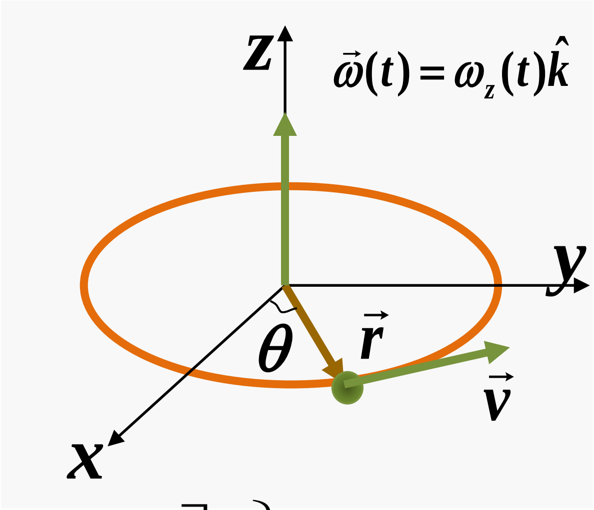
$$\vec{v}(t) = \vec{\omega}(t) \times \vec{r}(t)$$

$$v(t) = r\omega(t)$$



Circular Motion - Describe the circular motion

(6) Centripetal acceleration and the angular speed of uniform circular motion



$$\vec{r}(t) = r\cos\theta\,(t)\hat{\imath} + r\sin\theta\,(t)\hat{\jmath}$$

$$\vec{a}_c(t) = \frac{d^2\vec{r}(t)}{dt^2} = \frac{d}{dt}\left[\frac{d\vec{r}(t)}{dt}\right]$$

$$= \frac{d}{dt}\left\{\left[-r\frac{d\theta}{dt}\sin\theta(t)\right]\hat{i} + \left[r\frac{d\theta}{dt}\cos\theta(t)\right]\hat{j}\right\}$$

$$= -\left(\frac{d\theta}{dt}\right)^2\left\{[r\cos\theta(t)]\hat{i} + [r\sin\theta(t)]\hat{j}\right\} \quad = -\omega_z^2\vec{r}(t)$$

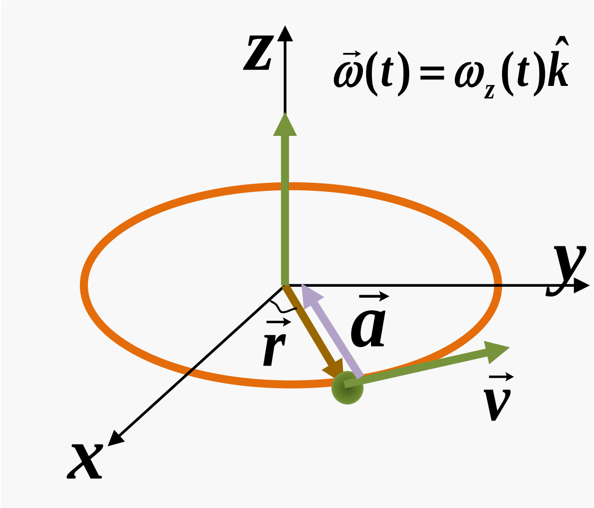
Circular Motion - Describe the circular motion

(6) Centripetal acceleration and the angular speed of uniform circular motion

$$\vec{r}(t) = r\cos\theta\,(t)\hat{\imath} + r\sin\theta\,(t)\hat{\jmath}$$

$$\vec{a}_c(t) = \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\left[\vec{\omega}(t)\times\vec{r}(t)\right]$$

$$= \vec{\omega}(t)\times\frac{d\vec{r}(t)}{dt} = \vec{\omega}(t)\times\vec{v}(t)$$

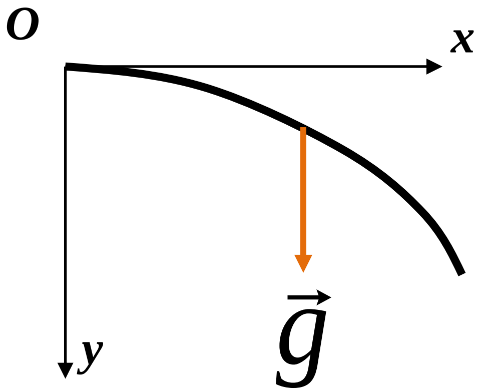


Direction: **Opposite to** $\vec{r}$ **Magnitude:** $a_c = r\omega_z^2$

Example

A stone is shot with speed of 30m/s and in horizontal direction. At *t*= 5.0s, what are its acceleration in tangent and normal direction?

$$\vec{v}_0 = 30\vec{i}\,,\ \vec{a} = \vec{g} = g\vec{j}$$

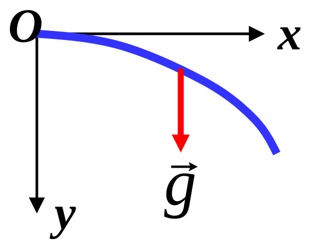


$a_n(5) = ?$ $\qquad a_t(5) = ?$

Example

A stone is shot with speed of 30m/s and in horizontal direction. At *t*= 5.0s, what are its acceleration in tangent and normal direction?

$$\vec{v}_0 = 30\vec{i}\,,\ \vec{a} = \vec{g} = g\vec{j} \qquad a_n(5) = ? \quad a_t(5) = ?$$



Solution: $\because \vec{v}(t) = v_0\vec{i} + gt\vec{j} \qquad v = |\vec{v}| = \sqrt{v_0^2 + (gt)^2}$

$$\therefore a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{g^2 t}{\sqrt{v_0^2 + (gt)^2}}\Big|_{t=5} = 8.58\mathrm{m/s^2}$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{g^2 - a_t^2} \approx 5.14\mathrm{m/s^2}$$

Non-uniform Circular Motion - angular acceleration

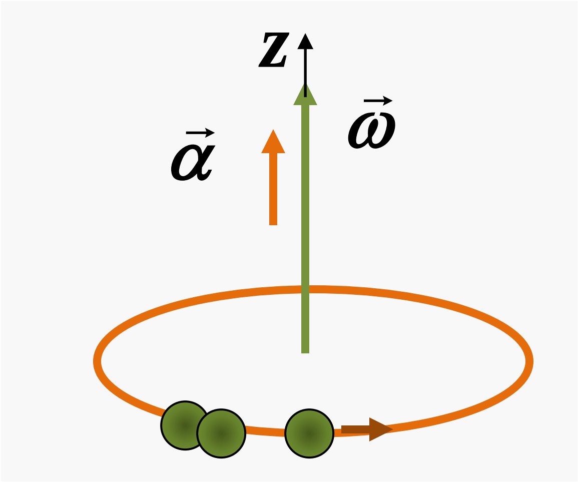
$$\vec{a}(t)=\frac{\mathrm{d}\vec{v}(t)}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}[\vec{\omega}(t)\times\vec{r}(t)]$$

$$=\frac{\mathrm{d}\vec{\omega}(t)}{\mathrm{d}t}\times\vec{r}(t)+\vec{\omega}(t)\times\frac{\mathrm{d}\vec{r}(t)}{\mathrm{d}t}$$

$$=\boxed{\vec{\alpha}(t)}\times\vec{r}(t)+\boxed{\vec{\omega}(t)\times\vec{v}(t)}$$

Angular acceleration

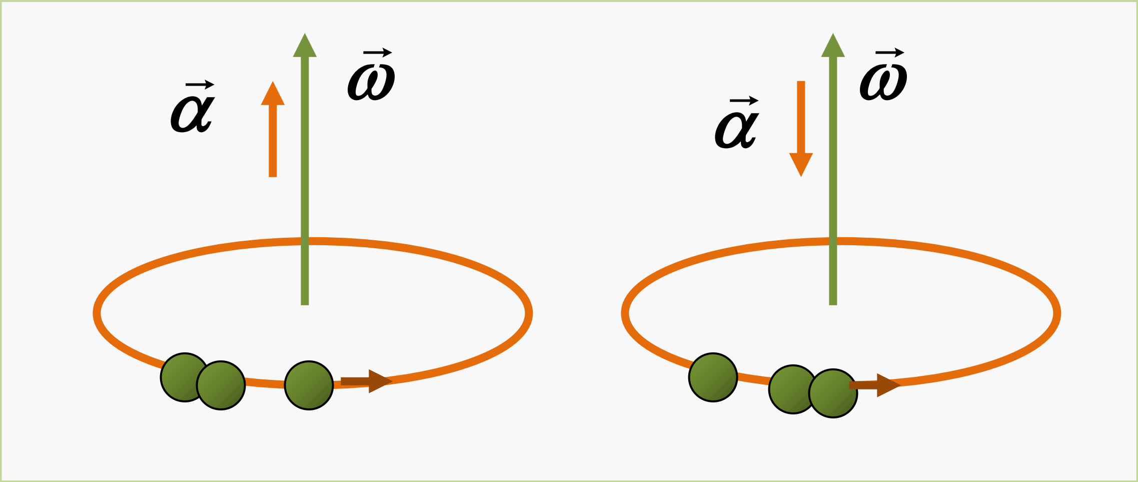
centripetal acceleration



$$\vec{\alpha}(t)=\frac{\mathrm{d}\vec{\omega}(t)}{\mathrm{d}t}=\alpha_z(t)\hat{k}$$

SI： rad/s^2

Non-uniform Circular Motion – direction of angular acceleration



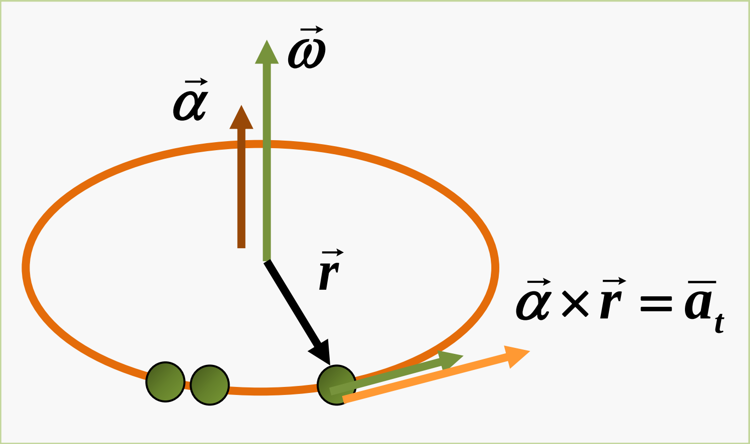
Speeding up: $\vec{\alpha}(t)$ is parallel to $\vec{\omega}(t)$.

Slowing down: $\vec{\alpha}(t)$ is antiparallel to $\vec{\omega}(t)$.

Non-uniform Circular Motion – tangential acceleration

$$\vec{a}(t) = \vec{\alpha}(t) \times \vec{r}(t) + \vec{\omega}(t) \times \vec{v}(t)$$

$$= r\alpha_z \hat{t} + \omega v \hat{n} = \frac{\mathrm{d}\omega}{\mathrm{d}t} r\hat{t} + \frac{v^2}{r}\hat{n}$$



$$\vec{a}_t(t) = \vec{\alpha}(t) \times \vec{r}(t)$$

$$\vec{a}_t(t) = r\alpha_z \hat{t} = \frac{\mathrm{d}\omega}{\mathrm{d}t} r\hat{t} = \frac{\mathrm{d}(r\omega)}{\mathrm{d}t}\hat{t} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t}$$

Review

1. Acceleration in curvilinear Motion

$$\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{\rho}\hat{n} = \vec{a}_t + \vec{a}_n$$

2. Nonuniform circular motion

$$\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{r}\hat{n} = \vec{a}_t + \vec{a}_c$$

3. Acceleration in uniform circular motion

$$\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\hat{t} + \frac{v^2}{r}\hat{n} = \frac{v^2}{r}\hat{n}$$

Review

4. Description of the circular motion with angular coordinates

①Angular position (coordinate) $\theta(t)$

② Angular displacement $\Delta\theta(t)$

③ angular speed and angular velocity vector

$$\vec{\omega}(t) = \omega_z(t)\hat{k} = \frac{\mathrm{d}\theta(t)}{\mathrm{d}t}\hat{k}$$

SI： rad/s

④ angular acceleration

$$\vec{\alpha}(t) = \frac{\mathrm{d}\vec{\omega}(t)}{\mathrm{d}t} = \alpha_z(t)\hat{k}$$

SI： rad/s^2

Review

5. The relationship between angular and linear quantities in circular motion

$$\vec{r}(t) = r\cos\theta\hat{i} + r\sin\theta\hat{j}$$

$$\Delta s = r\Delta\theta$$

$$v = \frac{\mathrm{d}s(t)}{\mathrm{d}t} = r\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = r\omega$$

$$\vec{v}(t) = \vec{\omega}(t)\times\vec{r}(t)$$

$$\begin{aligned}\vec{a}(t) &= \vec{\alpha}(t)\times\vec{r}(t) + \vec{\omega}(t)\times\vec{v}(t)\\ &= r\alpha_z\hat{t} + r\omega^2\hat{n}\\ &= \vec{a}_t + \vec{a}_c\end{aligned}$$

$$a_t = r\alpha_z$$

$$a_n = r\omega^2$$

Nonuniform circular motion - with a constant angular acceleration

Initial time: $\alpha_z = \textbf{constant}, \quad \omega(t_i) = \omega_{z0}, \quad \theta(t_i) = \theta_0$

From $\alpha_z = \dfrac{d\omega_z}{dt} = \textbf{costant}$

Integrate in both sides $\displaystyle\int_{\omega_{z0}}^{\omega_z} d\omega_z = \int_0^t \alpha_z dt, \quad \omega_z(t) = \omega_{z0} + \alpha_z t$

From $\omega_z(t) = \dfrac{d\theta(t)}{dt}$

Integrate in both sides $\displaystyle\int_{\theta_0}^{\theta} d\theta = \int_0^t \omega_z dt = \int_0^t (\omega_{z0} + \alpha_z t) dt$

$$\theta(t) = \theta_0 + \omega_{z0} t + \frac{1}{2}\alpha_z t^2$$

Nonuniform circular motion - with a constant angular acceleration

Comparison : nonuniform circular motion with a constant angular acceleration and the rectilinear motion with a constant acceleration

$$\begin{cases} \alpha_z = \dfrac{\mathrm{d}\omega_z}{\mathrm{d}t} = \text{costant} \\ \omega_z(t) = \omega_{z0} + \alpha_z t \\ \theta(t) = \theta_0 + \omega_{z0} t + \dfrac{1}{2}\alpha_z t^2 \\ \omega_z{}^2 - \omega_{z0}^2 = 2\alpha_z \Delta\theta \end{cases}$$

$$\begin{cases} a_x = \dfrac{\mathrm{d}v_x}{\mathrm{d}t} = \text{costant} \\ v_x(t) = v_{x0} + a_x t \\ x(t) = x_0 + v_{x0} t + \dfrac{1}{2} a_x t^2 \\ v^2 - v_0^2 = 2a\Delta x \end{cases}$$

The circular motion - Exercise 1

Your antique stereo turntable of radius 13.7 cm, initially spinning at 33.0 revolutions per minute, is shut off. The turntable coasts to a stop after 120 s. Assume a constant angular acceleration. Calculate the angular acceleration of the turntable and the number of revolutions through which it spins as it stops.

The circular motion - Exercise 2

A particle begins at rest on a circular track and is subjected to a constant angular acceleration of magnitude α beginning when t = 0 s.

(a) Show that the magnitudes of the tangential and centripetal accelerations of the particle are equal when $t = (1/\alpha)^{1/2}$ independent of the radius of the circular track.

(b) What is the angle that the total acceleration vector makes with the radial direction at this time?

The circular motion - Exercise 3

A point P on the edge of a rotational disk with radius R, the distance traveling by the point P is

$$\boldsymbol{s = v_0 t + \frac{1}{2} b t^2}$$

where v_0 and b are constants, what are the speed and the magnitude of the acceleration of the point P at any instant t?

Why

How to make sure the speed limit when a car rounds a curve?

Why a people in a satellite orbit close to the Earth will experience “weightlessness”

There is no gravity in space?

What is the net force on an astronaut at rest inside the space station?

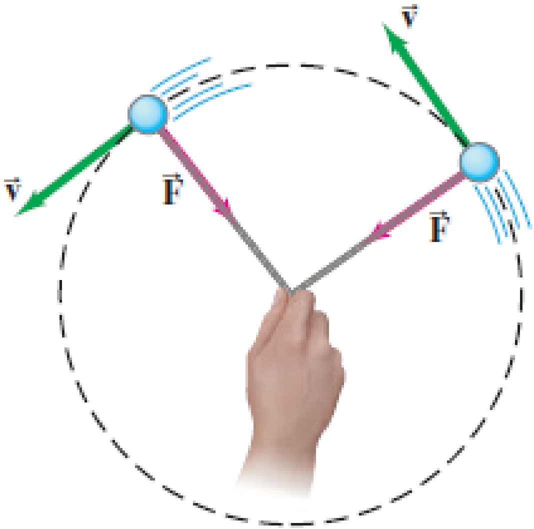
Dynamics of Uniform Circular Motion

Centripetal force :

If the particle has a mass *m* and is being accelerated it must be subject to a force.

$$F = ma$$

This force is acting towards the center of the circle – it is centripetal (meaning = center seeking)



$$F = ma_{cent} = \frac{mv^2}{r}$$

Dynamics of Uniform Circular Motion

What is the normal force at the top and at the bottom ?

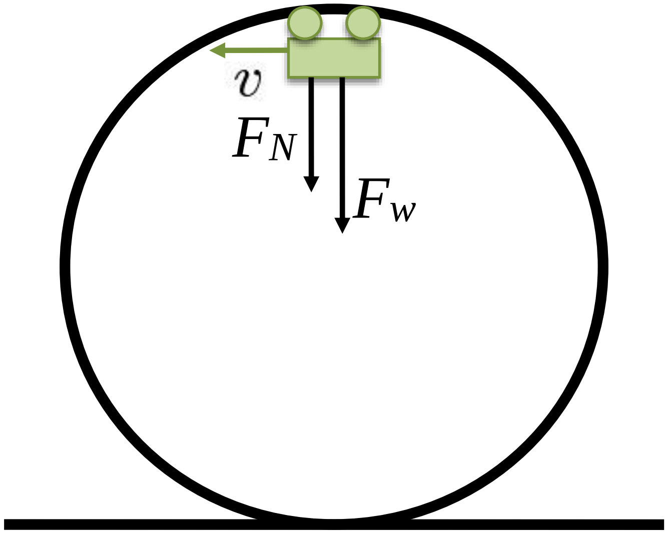
Roller Coaster

speed = 15 m/s

diameter = 40 m

total mass = 1200 kg

Example



On top of the loop

$$F_N + F_w = \frac{mv^2}{r}$$

$$F_N = \frac{mv^2}{r} - mg$$

$$F_N = 1500\text{N}$$

At the bottom

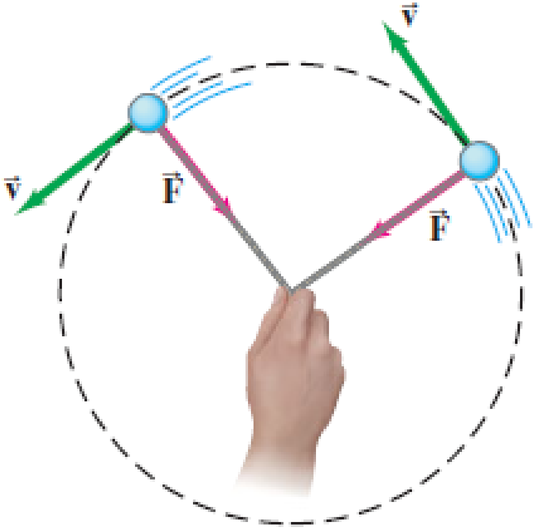
$$F_N - F_w = \frac{mv^2}{r}$$

$$F_N = \frac{mv^2}{r} + mg$$

$$F_N = 25,500\text{ N}$$

Centripetal Force

What will happen without this force?



Centripetal Force

EXERCISE D To negotiate a flat (unbanked) curve at a *faster* speed, a driver puts a couple of sand bags in his van aiming to increase the force of friction between the tires and the road. Will the sand bags help?

Increasing the friction force to increase a and v

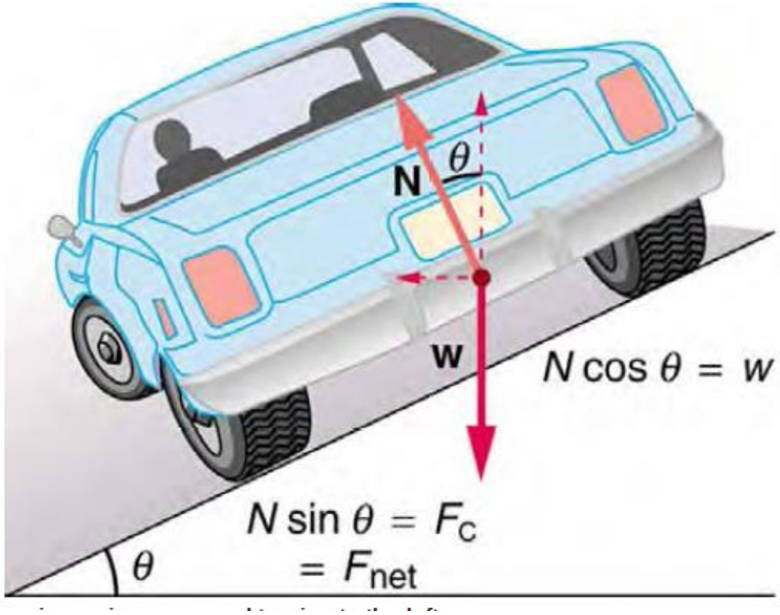
For a faster speed, will the sand bags help?

What will help?

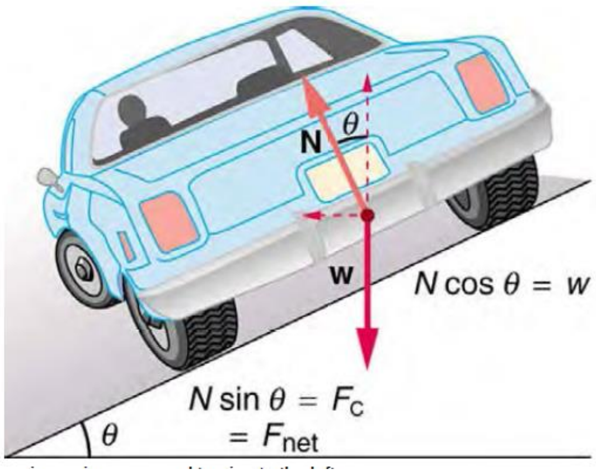
Banked curves

To reduce the chance of skidding, banking road

What’s the ideal angle for *v*



Banked curves



We want: $N\sin\theta = \dfrac{mv^2}{r}.$

But also: $N\cos\theta = mg.$

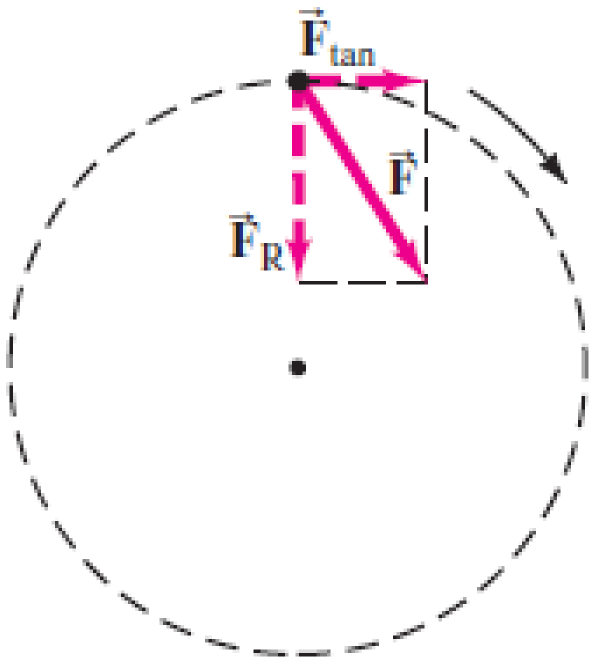
$$mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

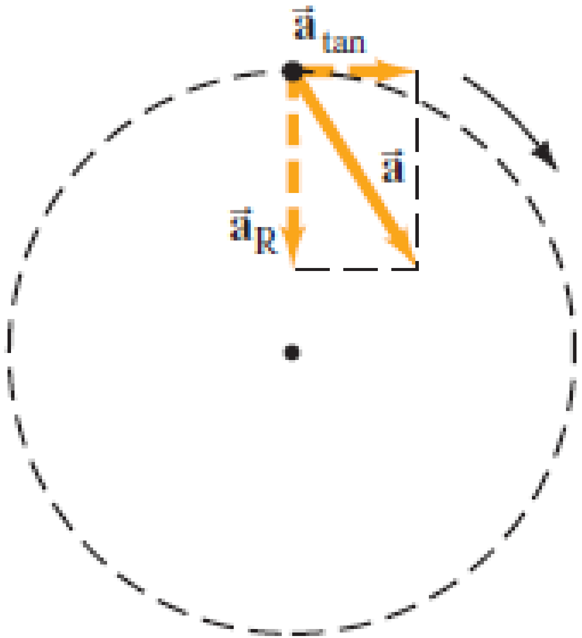
$$mg\tan(\theta) = \frac{mv^2}{r}$$

Does not depend on the mass

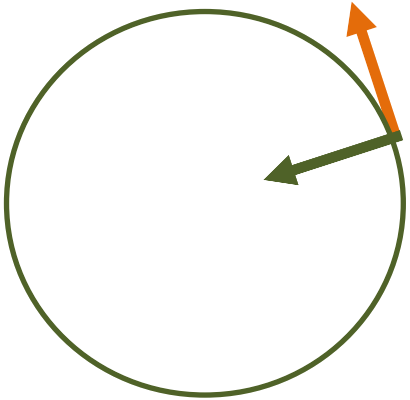
'Ideal' angle: $\tan\theta = \dfrac{v^2}{rg.}$

Nonuniform Circular Motion





Nonuniform Circular Motion - Angular acceleration

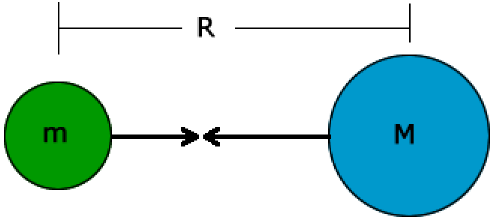


$$\vec{a} = \vec{a}_{\tan} + \vec{a}_R$$

Angular acceleration

Tangential and centripetal acceleration

Newton's law of Universal Gravitation



Every particle in the universe ***attracts*** every other particle

with a force that is

proportional to the product of their masses and

inversely proportional to the square of the distance between them.

This force acts ***along the line*** joining the two particles.

$$F_G = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \mathrm{N\,m^2/kg^2}$$

Universal Gravitational Constant ($m^3/kg \cdot s^2$)

Example

Estimate the gravitational force of attraction between two students sitting next to each other.

Gravitational constant: $G = 6.67\times10^{-11}$ N·m^2·kg^{-2})

Assume the mass of each student to be m = 60 kg ($m_1 = m_2$)

Assume two students are sitting 0.5 m apart in classroom

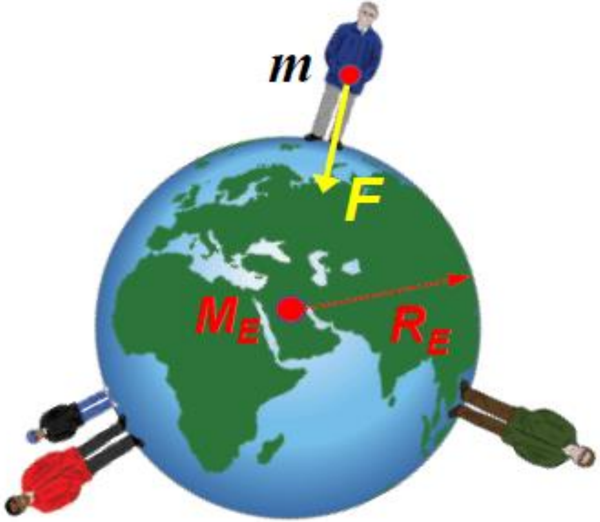
$$\boxed{F_G = \frac{Gm_1 m_2}{r^2}}$$

$$F = 9.6\times10^{-7}N \approx 1\mu N$$

Small but measurable

The gravity of the earth

For a body with ***spherical symmetry***, such as the Earth, all the mass can be regarded to be at the ***center of mass***, when calculating the gravitational force.



$M_E = 5.97 \times 10^{24}$ kg
$m = 60$ kg
$R_E = 6.37 \times 10^{6}$ m
$G = 6.67 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$

$$F_G = \frac{G m_1 m_2}{r^2}$$

$$F = 588.8N$$

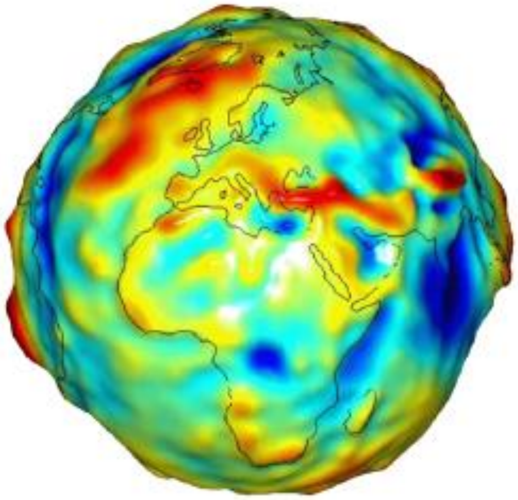
Gravity Near the Earth's Surface

$$F_G = \frac{Gm_1 m_2}{r^2}$$

$$F_G = mg$$

$$g = G\frac{m_E}{{r_E}^2}$$

Map of Map of g



Gravity not exactly the same on the surface of Earth

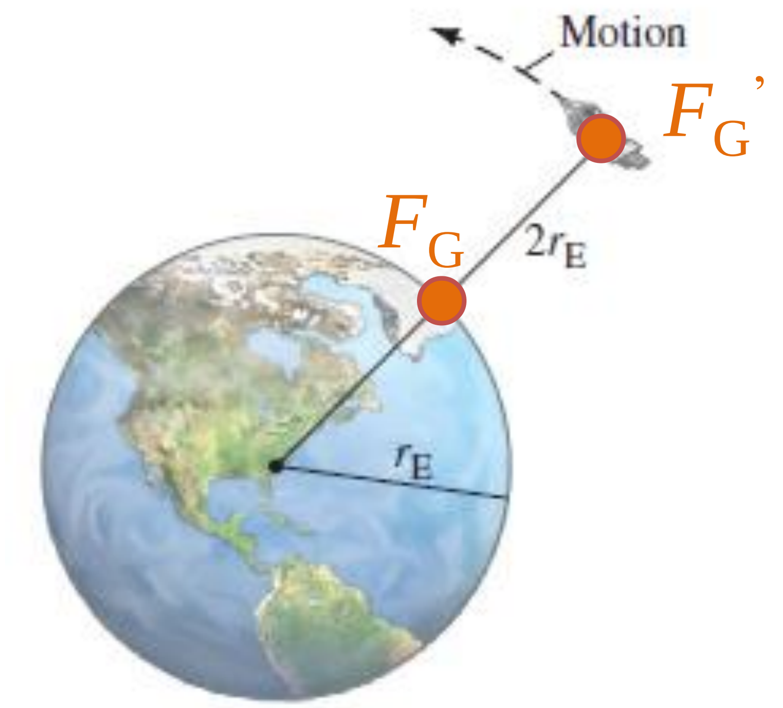
The gravity of the earth

$$F_G = \frac{G m_1 m_2}{r^2}$$

For a body with ***spherical symmetry***, such as the Earth, all the mass can be regarded to be at the ***center of mass***, when calculating the gravitational force.

What is F_G for an object at the surface of the earth?

How about F_G' for two Earth radii?



Example

The mass of the Moon is 7.35×10^{22} kg and the radius of the Moon is 1740 km. What is ***g*** on the surface of the Moon, $\boldsymbol{g}_{\text{Moon}}$?

$$g_{Moon} = \frac{GM_{Moon}}{r_{Moon}^2} = \frac{6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{\left(1740 \times 10^3\right)^2} = 1.62\,ms^{-2}$$

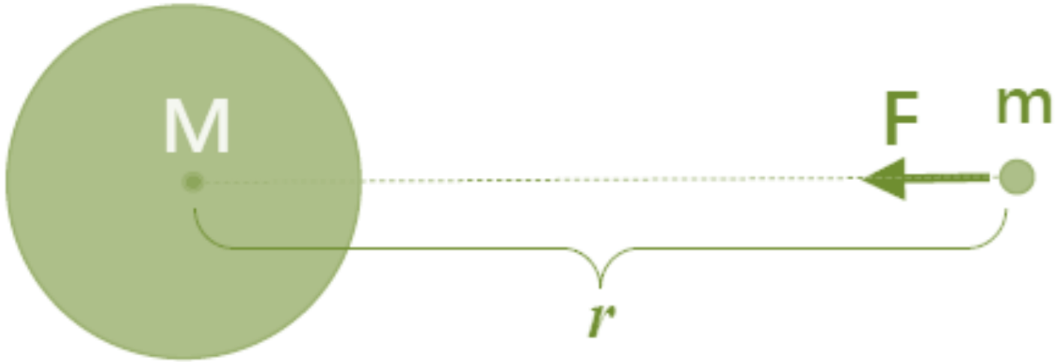
$$\frac{g_{Earth}}{g_{Moon}} = \frac{9.80}{1.62} = 6.0$$

- How does your mass change on the moon?
- How does your weight change on the moon?

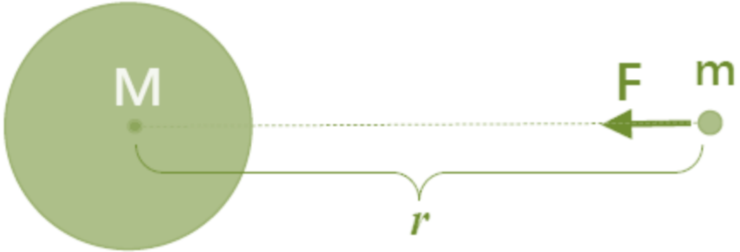
Gravitational potential energy

Suppose the Earth is stationary.

What is the potential energy of an object a distance *r* away from the center of Earth?



Gravitational potential energy



Let’s move the object to

infinitely far away .

The work done by the gravitational force is :

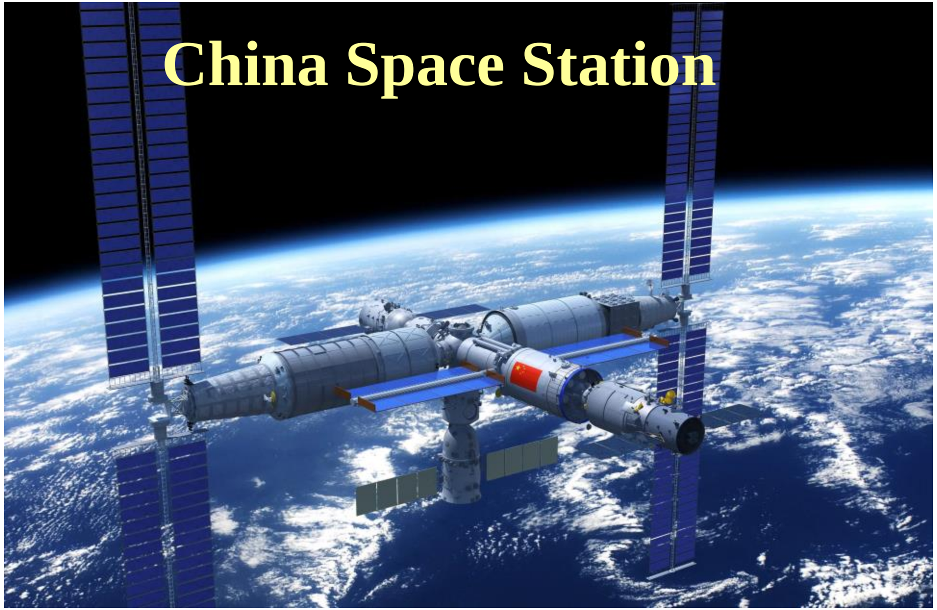
$$W = \int_r^\infty \vec{F} \cdot d\vec{r} \; = -\int_r^\infty F\,dr \; = -\int_r^\infty G\frac{Mm}{r^2}dr$$

$$\mathrm{W} = \frac{-GMm}{r}$$

Negative work = increase of potential energy

Gravity in orbit

380 km above the ground



r = 6751 km

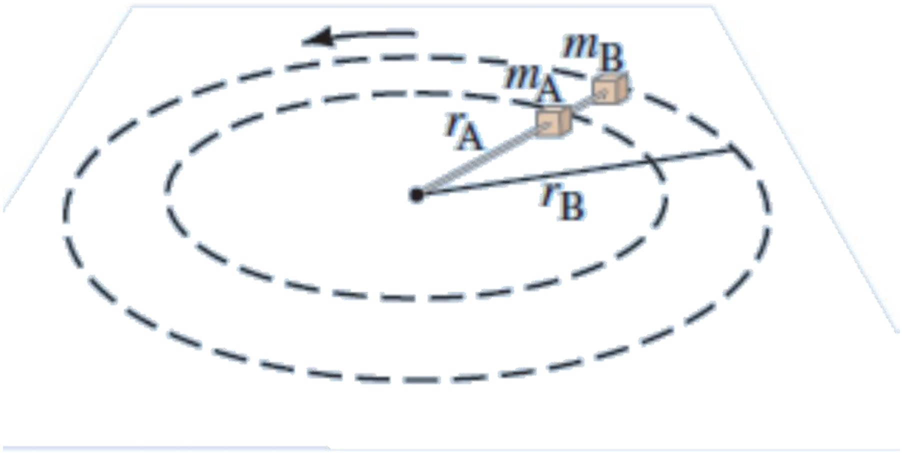
g = 8.75 ms -2

Satellites

Which needs a faster speed?

Close to the earth ?

Far away from the earth?



Satellites

$$F_G = F_{cent}$$

$$\frac{GM_{earth}m_{moon}}{r^2} = \frac{m_{moon}v^2}{r}$$

$$\Rightarrow \quad v^2 = G(M_{earth}/r)$$

Orbital speed: $v = \sqrt{\dfrac{GM}{r}} = \sqrt{gr}$

Does not depend on its own mass!

All orbiting objects at the same distance must have the same ***Speed and period***

Period of an orbit

$$T = \frac{4\pi^2 r^3}{GM}$$

Kepler's Third Law of planetary motion


near-Earth orbits
geostationary orbit

Near-Earth orbits

Low near-Earth orbits are usually from **160 to 2000 km** above earth.

With a very rough approximation, we assume **the orbit radius is the radius of the Earth** (which could be true for planets without atmosphere)

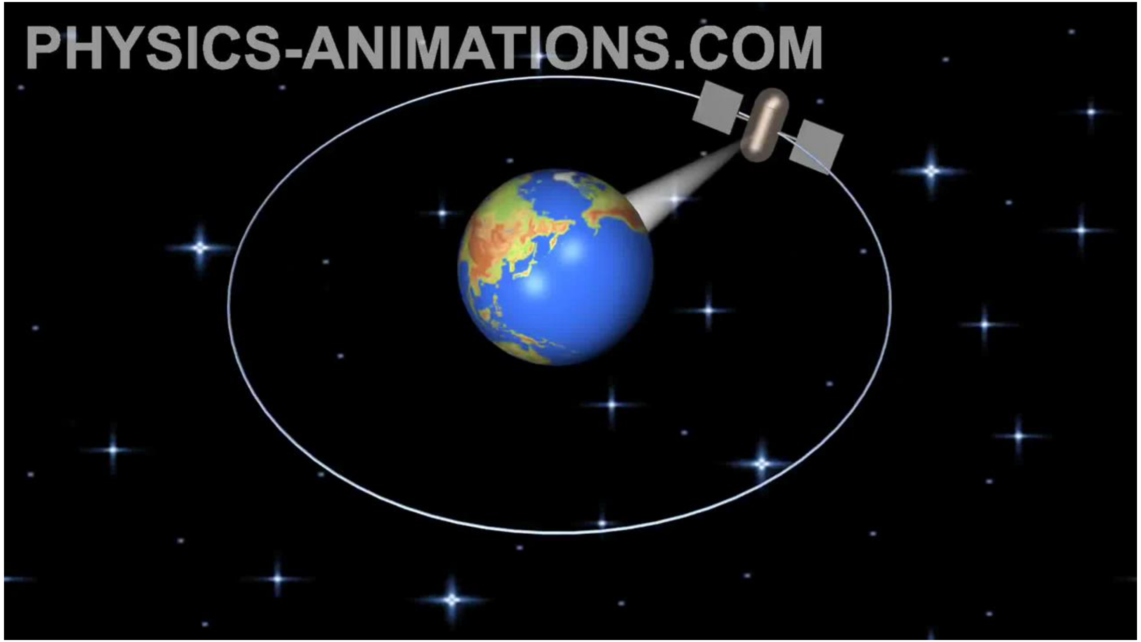
Orbital speed: $v = \sqrt{\boldsymbol{gr}} = \sqrt{\mathbf{9.8 \times 6.37 \times 106}} = \mathbf{7910}\ \boldsymbol{m/s}$

The first cosmic speed

$$\boldsymbol{The\ orbital\ period\ is\ about\ 84\ mins}$$

Geostationary satellite (Geosynchronous)

The orbit's period is equal to the Earth's rotation period



Geostationary satellite

The period of the satellite equals the spinning period of the Earth (24 h)

What is the height of this kind of satellite?

$$\frac{GM_{earth}m_{sat}}{r^2} = \frac{m_{sat}v^2}{r}$$

$$\frac{GM_{earth}}{r} = v^2$$

$$v = \frac{2\pi r}{T}$$

$$\frac{GM_{earth}}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$4\pi^2 r^3 = GM_{earth}T^2$$

$$r = \sqrt[3]{\frac{GM_{earth}T^2}{4\pi^2}}$$

$$h \cong 6r_E = 36{,}000 \text{ km}$$

How about the gravity at the satellite?

Orbit of the satellite

Example

If a satellite is travelling at 7 km/s, what is the radius of its orbit?

Orbital Speed: $v = \sqrt{\frac{GM}{r}}$ $\quad r = \frac{GM}{v^2}$

$$r = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(7000)^2} = 8.14 \times 10^6 m$$

(1760 km above the Earth's surface)

What is the total energy of the satellite?

$$E_k = \frac{1}{2} m v^2 \qquad E_p = -G\frac{Mm}{r} = -m\frac{GM}{r} = -mv^2$$

$$E_k + E_p = -\frac{1}{2} m v^2 = -\frac{GMm}{2r}$$

Negative and inversely proportional to the radius

Weightlessness

Why a people in a satellite orbit close to the Earth will experience “weightlessness”?

There is no gravity in space?

What is the net force on an astronaut at rest inside the space station?

Apparent weightlessness

A person on a scale in an elevator.

If the elevator fall freely, the ***reading of the scale would be 0***

Apparent **(not real)**

1. Don’t seem to have weight
2. Gravity doesn’t disappear

Weightlessness

Apparent weightlessness?