MATH 2233 Differential Equations

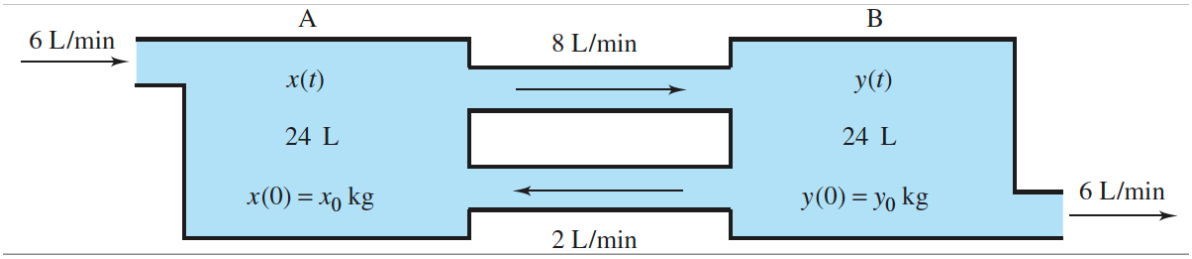
Chapter 5 Introduction to Systems and Phase Plane Analysis

Section 5.1 Interconnected Fluid Tanks

Goal of this section

- Study the double mixing model using a system of linear differential equations.

Example 1. *Two large tanks, each holding 24 liters of a brine solution, are interconnected by pipes. See Figure below. Fresh water flows into tank A at a rate of 6 L/min, and fluid is drained out of tank B at the same rate; also 8 L/min of fluid are pumped from tank A to tank B, and 2 L/min from tank B to tank A. The liquids inside each tank are kept well stirred so that each mixture is homogeneous. Initially, the brine solution in tank A contains x_0 kg of salt and that in tank B initially contains y_0 kg of salt. Formulate differential equation(s) to govern this mixing problem.*



We have two unknown functions $x(t)$ and $y(t)$, denoting the amount of salt in the tank A and B, respectively.

Example 2. *Solve the system of differential equations from Example 1*

$$\begin{cases} x' = -\frac{1}{3}x + \frac{1}{12}y \\ y' = \frac{1}{3}x - \frac{1}{3}y. \end{cases}$$

Section 5.2 Differential Operators and Elimination Method

Goal of this section

- solve a system of differential equations with constant coefficients using differential operators.

Differential Operator

Recall the derivative notation $y'(t) = \dfrac{dy}{dt} = \dfrac{d}{dt}y$.

- We introduce another symbol D called differential operator

- Using this notation, the differential equation $y'' + 4y' + 3y = 0$ can be represented by

Example 1. *Show that the operator* $(D+3)(D+1)$ *is the same as* $(D+1)(D+3)$ *and also the same as* $D^2 + 4D + 3$.

Example 2. *Show that the operator* $(D+3t)D$ *is not the same as* $D(D+3t)$.

In this example, we will see how to solve a first-order system of two linear differential equations.

Example 3. *Solve the system of differential equations*

$$\begin{cases} x' = 3x - 4y + 1 \\ y' = 4x - 7y + 10t \end{cases}$$

We generalize the elimination method for the system of higher-order linear differential equations.

Example 4 (**system of higher-order equations).** *Find a general solution for*

$$\begin{cases} x'' + y' - x + y = -1 \\ x' + y' - x = t^2 \end{cases}$$