Fundamental Equations of Mechanics of Materials

Axial Load

Normal Stress

$$\sigma = \frac{N}{A}$$

Displacement

$$\delta = \int_0^L \frac{N(x)dx}{A(x)E}$$

$$\delta = \Sigma \frac{NL}{AE}$$

$$\delta_T = \alpha\, \Delta T L$$

Torsion

Shear stress in circular shaft

$$\tau = \frac{T\rho}{J}$$

where

$$J = \frac{\pi}{2} c^4 \quad \text{solid cross section}$$

$$J = \frac{\pi}{2}(c_o^{\,4} - c_i^{\,4}) \quad \text{tubular cross section}$$

Power

$$P = T\omega = 2\pi f T$$

Angle of twist

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma \frac{TL}{JG}$$

Average shear stress in a thin-walled tube

$$\tau_{\text{avg}} = \frac{T}{2tA_m}$$

Shear Flow

$$q = \tau_{\text{avg}} t = \frac{T}{2A_m}$$

Bending

Normal stress

$$\sigma = \frac{My}{I}$$

Unsymmetric bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}, \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$

Shear

Average direct shear stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

Transverse shear stress

$$\tau = \frac{VQ}{It}$$

Shear flow

$$q = \tau t = \frac{VQ}{I}$$

Stress in Thin-Walled Pressure Vessel

Cylinder

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

Sphere

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max}}{2} \quad \text{for } \sigma_{\max}, \sigma_{\min} \text{ same sign}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} \quad \text{for } \sigma_{\max}, \sigma_{\min} \text{ opposite signs}$$

Geometric Properties of Area Elements

Material Property Relations

Poisson's ratio

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy},\ \gamma_{yz} = \frac{1}{G}\tau_{yz},\ \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1+\nu)}$$

Relations Between w, V, M

$$\frac{dV}{dx} = w(x), \qquad \frac{dM}{dx} = V$$

Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI\frac{d^4v}{dx^4} = w(x)$$

$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

Buckling

Critical axial load

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

Critical stress

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{(KL/r)^2},\ r = \sqrt{I/A}$$

Secant formula

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

Energy Methods

Conservation of energy

$$U_e = U_i$$

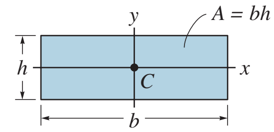
Strain energy

$$U_i = \frac{N^2L}{2AE} \quad \text{constant axial load}$$

$$U_i = \int_0^L \frac{M^2dx}{2EI} \quad \text{bending moment}$$

$$U_i = \int_0^L \frac{f_s V^2dx}{2GA} \quad \text{transverse shear}$$

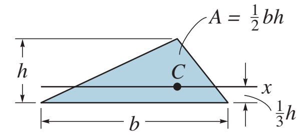
$$U_i = \int_0^L \frac{T^2dx}{2GJ} \quad \text{torsional moment}$$



$I_x = \frac{1}{12}bh^3$

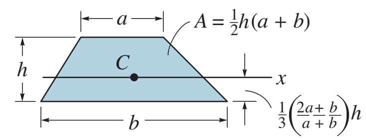
$I_y = \frac{1}{12}hb^3$

Rectangular area

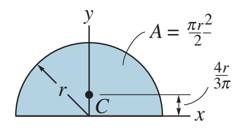


$I_x = \frac{1}{36}bh^3$

Triangular area



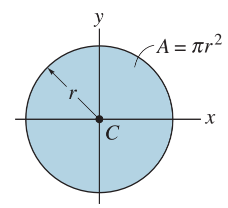
Trapezoidal area



$I_x = \frac{1}{8}\pi r^4$

$I_y = \frac{1}{8}\pi r^4$

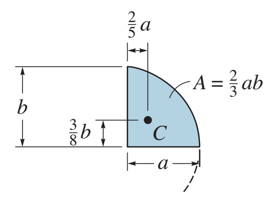
Semicircular area



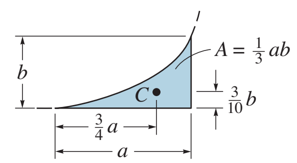
$I_x = \frac{1}{4}\pi r^4$

$I_y = \frac{1}{4}\pi r^4$

Circular area



Semiparabolic area



Exparabolic area

Average Mechanical Properties of Typical Engineering Materials[a]

(SI Units)

Materials		Density ρ (Mg/m^3)	Moduls of Elasticity E (GPa)	Modulus of Rigidity G (GPa)	Yield Strength (MPa) σ_Y Tens.	Comp.[b]	Shear	Ultimate Strength (MPa) σ_u Tens.	Comp.[b]	Shear	%Elongation in 50 mm specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°C$
Metallic													
Aluminum Wrought Alloys	2014-T6	2.79	73.1	27	414	414	172	469	469	290	10	0.35	23
	6061-T6	2.71	68.9	26	255	255	131	290	290	186	12	0.35	24
Cast Iron Alloys	Gray ASTM 20	7.19	67.0	27	–	–	–	179	669	–	0.6	0.28	12
	Malleable ASTM A-197	7.28	172	68	–	–	–	276	572	–	5	0.28	12
Copper Alloys	Red Brass C83400	8.74	101	37	70.0	70.0	–	241	241	–	35	0.35	18
	Bronze C86100	8.83	103	38	345	345	–	655	655	–	20	0.34	17
Magnesium Alloy	[Am 1004-T61]	1.83	44.7	18	152	152	–	276	276	152	1	0.30	26
Steel Alloys	Structural A-36	7.85	200	75	250	250	–	400	400	–	30	0.32	12
	Structural A992	7.85	200	75	345	345	–	450	450	–	30	0.32	12
	Stainless 304	7.86	193	75	207	207	–	517	517	–	40	0.27	17
	Tool L2	8.16	200	75	703	703	–	800	800	–	22	0.32	12
Titanium Alloy	[Ti-6Al-4V]	4.43	120	44	924	924	–	1,000	1,000	–	16	0.36	9.4
Nonmetallic													
Concrete	Low Strength	2.38	22.1	–	–	–	12	–	–	–	–	0.15	11
	High Strength	2.37	29.0	–	–	–	38	–	–	–	–	0.15	11
Plastic Reinforced	Kevlar 49	1.45	131	–	–	–	–	717	483	20.3	2.8	0.34	–
	30% Glass	1.45	72.4	–	–	–	–	90	131	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.47	13.1	–	–	–	–	2.1[c]	26[d]	6.2[d]	–	0.29[e]	–
	White Spruce	3.60	9.65	–	–	–	–	2.5[c]	36[d]	6.7[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition,mechanical working of the specimen,or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

Average Mechanical Properties of Typical Engineering Materials[a]

(U.S. Customary Units)

Materials		Specific Weight (lb/in^3)	Moduls of Elasticity E (10^3) ksi	Modulus of Rigidity G (10^3) ksi	Yield Strength (ksi) σ_Y			Ultimate Strength (ksi) σ_u			%Elongation in 2 in. specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α (10^{-6})/°F
					Tens.	Comp.[b]	Shear	Tens.	Comp.[b]	Shear			
Metallic													
Aluminum Wrought Alloys	2014-T6	0.101	10.6	3.9	60	60	25	68	68	42	10	0.35	12.8
	6061-T6	0.098	10.0	3.7	37	37	19	42	42	27	12	0.35	13.1
Cast Iron Alloys	Gray ASTM 20	0.260	10.0	3.9	–	–	–	26	96	–	0.6	0.28	6.70
	Malleable ASTM A-19	0.263	25.0	9.8	–	–	–	40	83	–	5	0.28	6.60
Copper Alloys	Red Brass C83400	0.316	14.6	5.4	11.4	11.4	–	35	35	–	35	0.35	9.80
	Bronze C86100	0.319	15.0	5.6	50	50	–	35	35	–	20	0.34	9.60
Magnesium Alloy	[Am 1004-T61]	0.066	6.48	2.5	22	22	–	40	40	22	1	0.30	14.3
Steel Alloys	Structural A-36	0.284	29.0	11.0	36	36	–	58	58	–	30	0.32	6.60
	Structural A992	0.284	29.0	11.0	50	50	–	65	65	–	30	0.32	6.60
	Stainless 304	0.284	28.0	11.0	30	30	–	75	75	–	40	0.27	9.60
	Tool L2	0.295	29.0	11.0	102	102	–	116	116	–	22	0.32	6.50
Titanium Alloy	[Ti-6Al-4V]	0.160	17.4	6.4	134	134	–	145	145	–	16	0.36	5.20
Nonmetallic													
Concrete	Low Strength	0.086	3.20	–	–	–	1.8	–	–	–	–	0.15	6.0
	High Strength	0.086	4.20	–	–	–	5.5	–	–	–	–	0.15	6.0
Plastic Reinforced	Kevlar 49	0.0524	19.0	–	–	–	–	104	70	10.2	2.8	0.34	–
	30% Glass	0.0524	10.5	–	–	–	–	13	19	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.017	1.90	–	–	–	–	0.30[c]	3.78[d]	0.90[d]	–	0.29[e]	–
	White Spruce	0.130	1.40	–	–	–	–	0.36[c]	5.18[d]	0.97[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition,mechanical working of the specimen,or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

[c] Measured perpendicular to the grain.

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