Engineering Economic Analysis Homework #5
Fall 2024

Instructions:

Please attempt every problem. You must support every solution with an appropriate amount of work and/or description. Unsupported answers may receive a score of 0. Good luck!

1. (10 pts) An easy payment plan offered by a local electronics store for your new audio system calls for end-of-year payments of \$2,000 at the end of year 1, increasing by 15% each year thereafter through year 4. Your money is well invested and earns a consistent 10% per year.

   (a) What is the present worth of these payments?
   This is a geometric series of cash flows with

$$A_1 = \$2000,\ j = 0.15,\ i = 0.10,\ n = 4.$$

   Hence the present worth of these payments is

$$\begin{aligned} P &= A_1 \left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \\ &= 2000 \times \left[\frac{1-(1+0.15)^4(1+0.10)^{-4}}{0.10-0.15}\right] \\ &= \$7783.792 \end{aligned}$$

+6

   (b) If you prefer to make equal annual payments having the same present worth, how much would they be?
   The equivalent equal payments will be

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = 7783.792 \times \left[\frac{0.1\times(1+0.1)^4}{(1+0.1)^4-1}\right] = \$2455.559.$$

+4

2. (6 pts) You are preparing the business plan for a new company. A net revenue analysis covering the first 6 years is required for obtaining financing. Net revenue in year 1 is expected to be \$50,000 and increase by 15% each year,thereafter. If $i = 12\%$ and the net revenue is assumed to be an end-of-year cash flow, what

is the present value of the cash flow series over the 6 years?
Solution:
This is a geometric series of cash flows with

$$A_1 = \$50,000,\ j = 0.15,\ i = 0.12,\ n = 6.$$

Hence the present worth of these payments is

$$\begin{aligned} P &= A_1 \left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j} \right] \\ &= 50000 \times \left[\frac{1-(1+0.15)^6(1+0.12)^{-6}}{0.12-0.15} \right] \\ &= \$286,447.615 \end{aligned}$$

+6

3. (8 pts) Mario and Claudia deposit \$100 into their joint account at the end of each month. If their account earns 7%/year/quarter (7% per year compounded quarterly), how long will it take them to have a total of \$15,000 in their savings account?
Solution:
The effective interest rate per month is

$$i = (1+0.07/4)^{4/12} - 1 = 0.0058.$$

+3

Assume it takes n months to reach \$15,000. Then

$$F = 100\frac{(1+0.0058)^n - 1}{0.0058} = 15,000.$$

Solving for n yields

$$n = 108.235$$

+5

4. (6 pts) A total of \$10,000 is borrowed and repaid with 20 monthly payments, with the first payment occurring one month after receipt of the \$10,000. The stated interest rate is 4% compounded quarterly. What monthly payment is required?
Solution:
The monthly interest rate is

$$i = (1+0.04/4)^{4/12} - 1 = 0.0033.$$

+3

Hence the monthly payment is

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = 10,000\frac{0.0033 \times (1+0.0033)^{20}}{(1+0.0033)^{20} - 1} = \$517.508$$

+3

5. (15 pts) Annual deposits, beginning with \$500 and increasing by \$100 with each subsequent deposit, are made into a fund paying a nominal 10% per year **compounded continuously**.

   (a) What will the fund amount to after 7 years?
   The effective annual interest rate is

   $$i_{eff} = e^{0.1} - 1 = 0.10517.$$

   +3

   The deposits forms a composite series consisting of a uniform series with $A_t = A_1 = 500$ for $t = 7$ and a gradient series with $G = 100$. Hence the fund amount after 7 years is

   $$\begin{aligned} F_c &= F_u + F_g \\ &= A_1\frac{(1+i_{eff})^n - 1}{i_{eff}} + G\frac{(1+i_{eff})^n - (1+ni_{eff})}{i_{eff}^2} \\ &= 500\frac{(1+0.10517)^7 - 1}{0.10517} + 100\frac{(1+0.10517)^7 - (1+7\times 0.10517)}{0.10517^2} \\ &= \$7328.872 \end{aligned}$$

   +6

   (b) What is the present worth equivalent of the total set of deposits over these 7 years?
   The equivalent present worth is

   $$P_c == F_c(1+i)^{-n} = 7328.872 \times (1+0.10517)^{-7} = \$3639.431$$

   +3

   (c) What is the equal annual equivalent amount of the deposits over these 7 years?
   The equal annual equivalent amount is

   $$A = P_c\frac{i_{eff}(1+i_{eff})^n}{(1+i_{eff})^n - 1} = 3639.431 \times \frac{0.10517(1+0.10517)^7}{(1+0.10517)^7 - 1} = \$760.330.$$

+3

6. (10 pts) A continuous uniform series of deposits totaling \$1,000 per year are made into a fund paying 10% compounded continuously.

   (a) What will the fund amount to after 7 years?
   The uniform annual amount equivalent of the continuous deposits is

   $$A = \bar{A}\frac{e^r - 1}{r} = 1000\frac{e^{0.1} - 1}{0.1} = 1051.7092.$$

   +3

   The effective annual interest rate is

   $$i_{eff} = e^r - 1 = e^{0.1} - 1 = 0.105171.$$

   +3

   Hence the fund amount after 7 years will be

   $$F = A\frac{(1 + i_{eff})^n - 1}{i_{eff}} = 1051.7092 \times \frac{(1 + 0.105171)^7 - 1}{0.105171} = 10137.530.$$

   +2

   (b) What is the present worth equivalent of the total set of deposits over these 7 years?
   The present worth equivalent of the total set of deposits is

   $$P = F(1 + i_{eff})^{-n} = 10137.530(1 + 0.105171)^{-7} = \$5034.146.$$

   +2

7. (10 pts) Consider the following two cash flow series of payments: Series A is a geometric series increasing at a rate of 8% per year. The initial cash payment at the end of year 1 is \$1,000. The payments occur annually for 5 years. Series B is a uniform series with payments of value X occurring annually at the end of years 1 through 5. You must make the payments in either Series A or Series B. Determine the value of X for which these two series are equivalent if your TVOM is $i = 6.5\%$.
   Solution:
   The present value of Series A is

   $$P_A = 1000\frac{1 - (1 + 0.08)^5(1 + 0.065)^{-5}}{0.065 - 0.08} = 4828.960.$$

+4

The present value of Series B is

$$P_B = X\frac{(1+0.065)^5 - 1}{0.065(1+0.065)^5} = 4.15568X.$$

+4

For these two series to be equivalent, we have $P_A = P_B$. That is,

$$4.15568X = 4828.960 \Rightarrow X = \$1162.0144.$$

+2