ENSC 2113

Engineering Mechanics: Statics

Chapter 4:

Force System Resultants

(Section 4.5)

COLLEGE OF
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Chapter 4 Outline:

4.1 Moment of a Force – Scalar Formulation

4.2 Cross Product

4.3 Moment of a Force – Vector Formulation

4.4 Principle of Moments

4.5 Moment of a Force about a Specified Axis

4.6 Moment of a Couple

4.7 Simplification of a Force and Couple System

4.8 Further Simplification of a Force and Couple System

4.9 Reduction of a Simple Distributed Loading

Chapter 4 Objectives:

- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- To provide a method for finding the moment of a force about a specified axis
- To define the moment of a couple
- To show how to find the resultant effect of a nonconcurrent force system
- To indicate how to reduce a simple distributed loading to a resultant force acting at a specified location

4.5 Moment of a Force About a Specified Axis:

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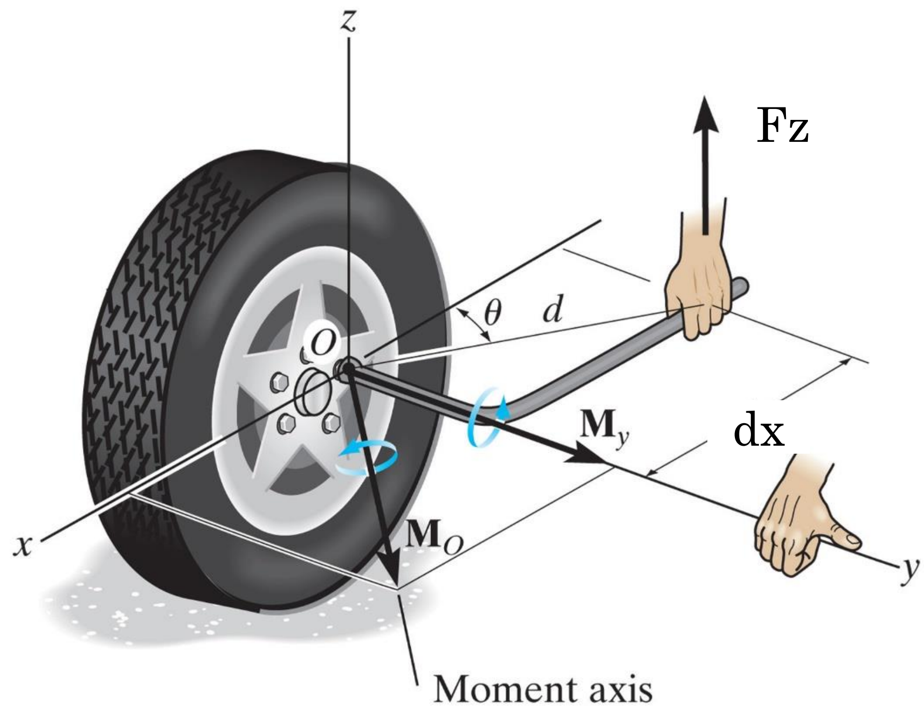


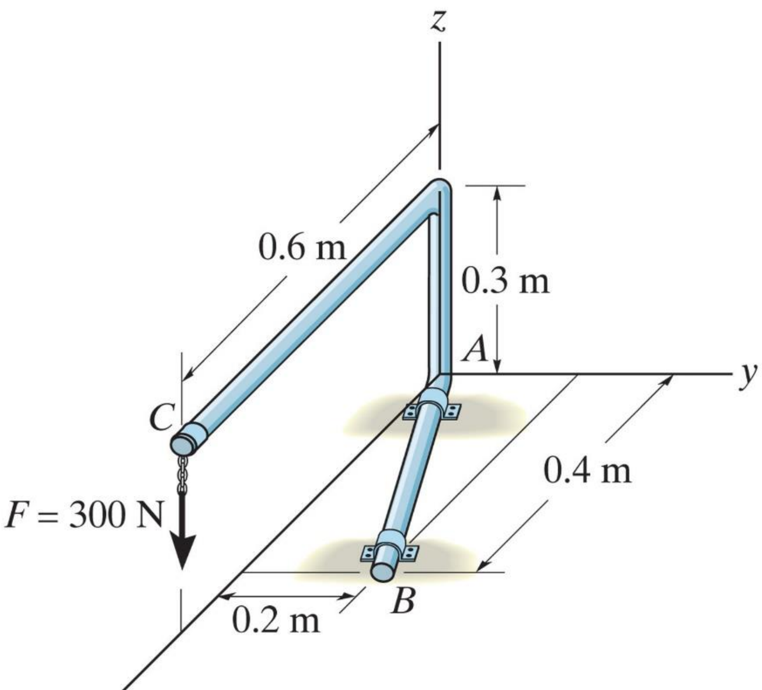
$$|M| = |F|d \qquad \vec{M} = \vec{r} \times \vec{F}$$

$$|M_y| = F_x d_z + F_z d_x \qquad |M_y| = \vec{u} \cdot [\vec{r} \times \vec{F}]$$

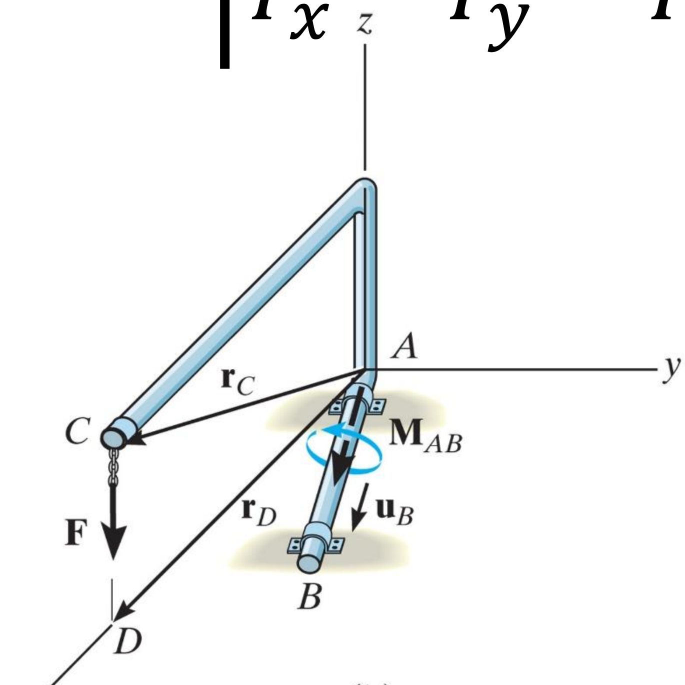
4.5: Moment of a Force - Vector Formulation

- Vector Analysis

$$|M| = \vec{u} \cdot \left[\vec{r} \times \vec{F}\right] \qquad |M| = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



(a)



(b)

4.5 Moment of a Force About a Specified Axis:

In Matrix form, the ***magnitude*** of the matrix is:

$$|M| = \vec{u} \cdot \left(\vec{r} \times \vec{F}\right) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where,

u = unit vector of the line (or axis) of interest

r = position vector from point on line to point on force

F = force vector

4.5: Moment of a Force - Vector Formulation

- Vector Analysis

$$|M| = \vec{u} \cdot \left[\vec{r} \times \vec{F}\right]$$

$$|M| = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$|M| = \left\{\left(r_y F_z - r_z F_y\right)u_x - \left(r_x F_z - r_z F_x\right)u_y + \left(r_x F_y - r_y F_x\right)u_z\right\}$$

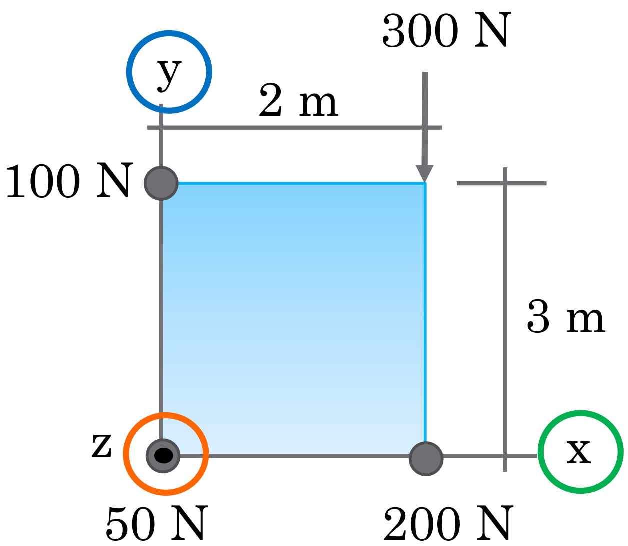
4.5: Moment of a Force - Vector Formulation

Once the magnitude of the moment is found, the components may be determined by multiplying by the unit vector of the line.

$$|M| = \vec{u} \cdot (\vec{r} \times \vec{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M} = |M|\vec{u}$$

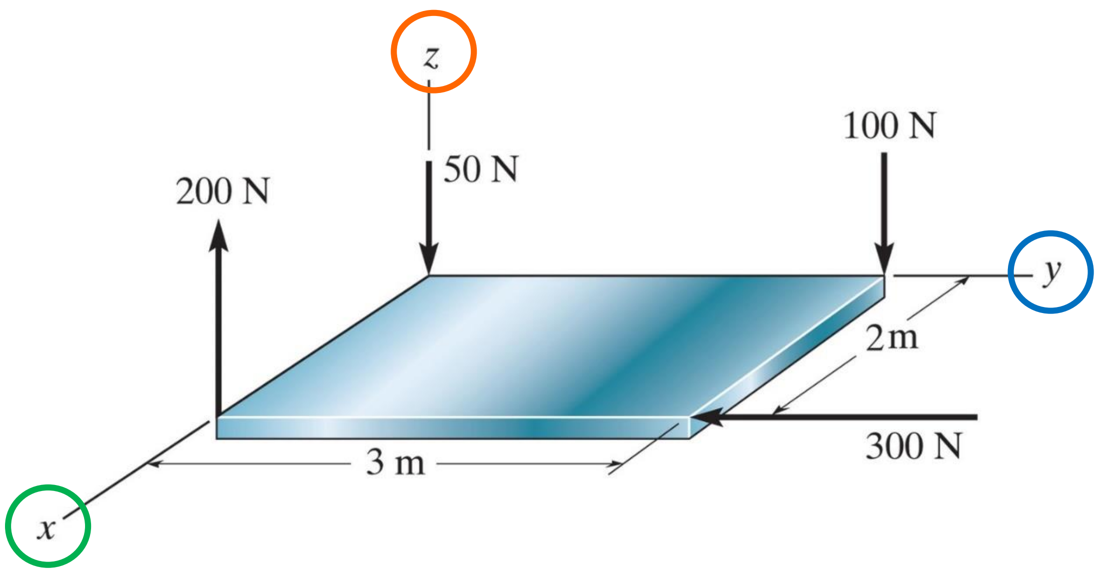
4.5: Moment of a Force - Scalar Formulation



$$|M_z| = F_x d_y + F_y d_x$$

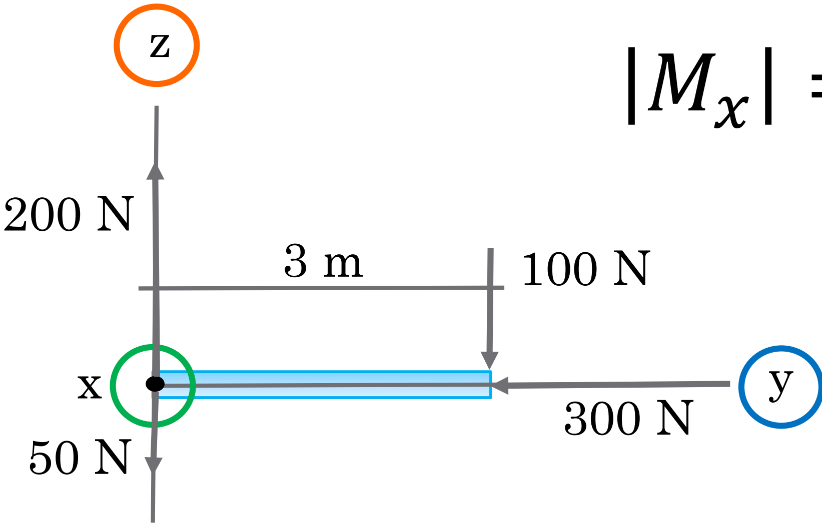
$$|M_z| = (0)(0) + (-300)(2)$$

$$|M_z| = -600\ N \cdot m$$



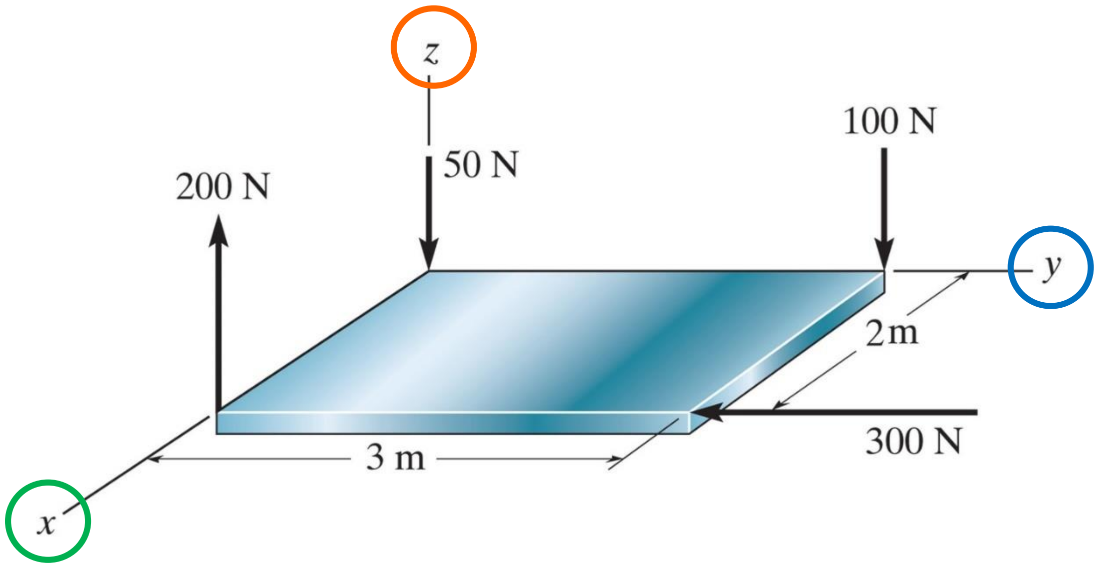
4.5: Moment of a Force - Scalar Formulation

$$|M_x| = F_y d_z + F_z d_y$$

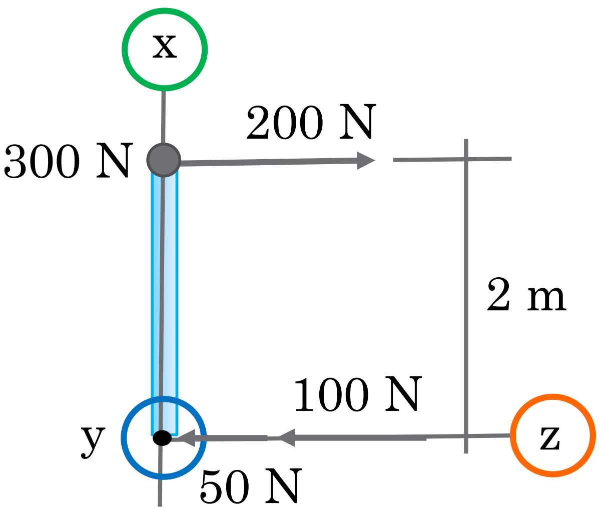


$$|M_x| = (300)(0) + (-100)(3)$$

$$|M_x| = -300\ N \cdot m$$



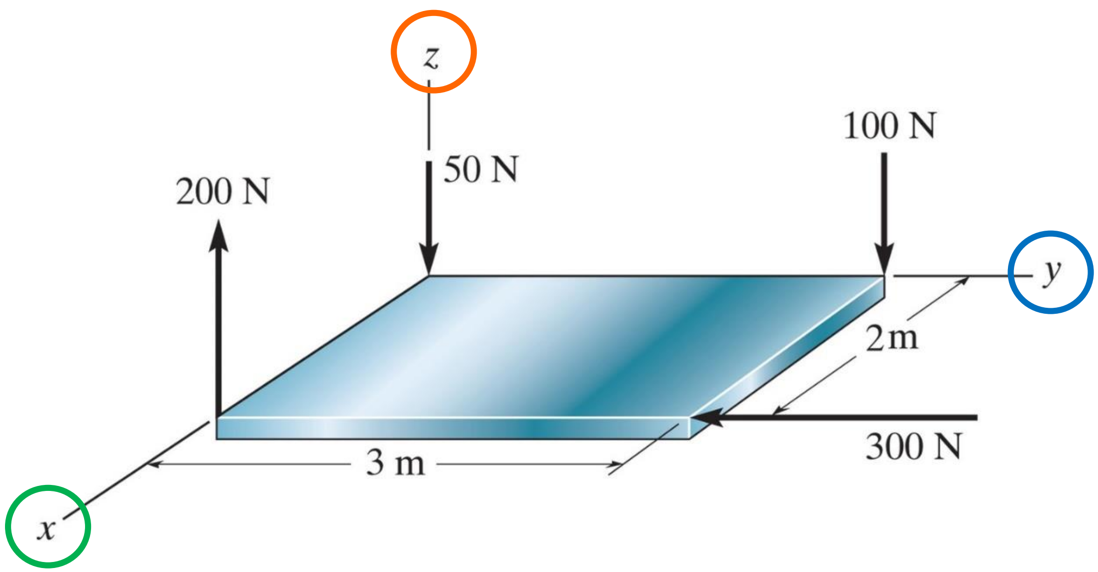
4.5: Moment of a Force - Scalar Formulation



$$|M_y| = F_x d_z + F_z d_x$$

$$|M_y| = (300)(0) + (-200)(2)$$

$$|M_y| = -400\ N \cdot m$$

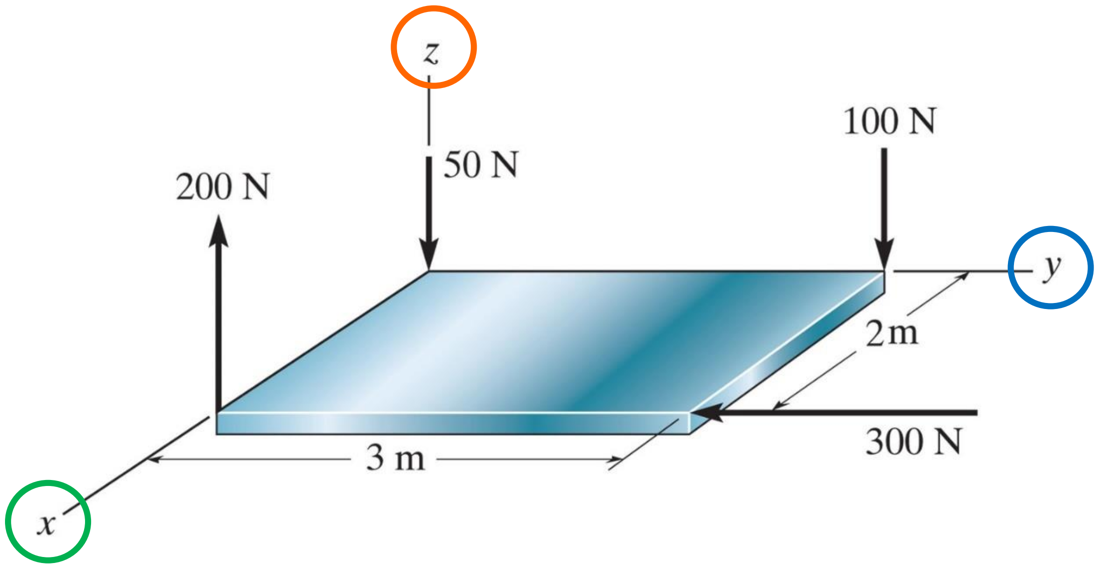


4.5: Moment of a Force - Scalar Formulation

$$|M_x| = F_y d_z + F_z d_y = -100(3) = -300\ N \cdot m$$

$$|M_y| = F_x d_z + F_z d_x = -200(2) = -400\ N \cdot m$$

$$|M_z| = F_x d_y + F_y d_x = -300(2) = -600 N \cdot m$$



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