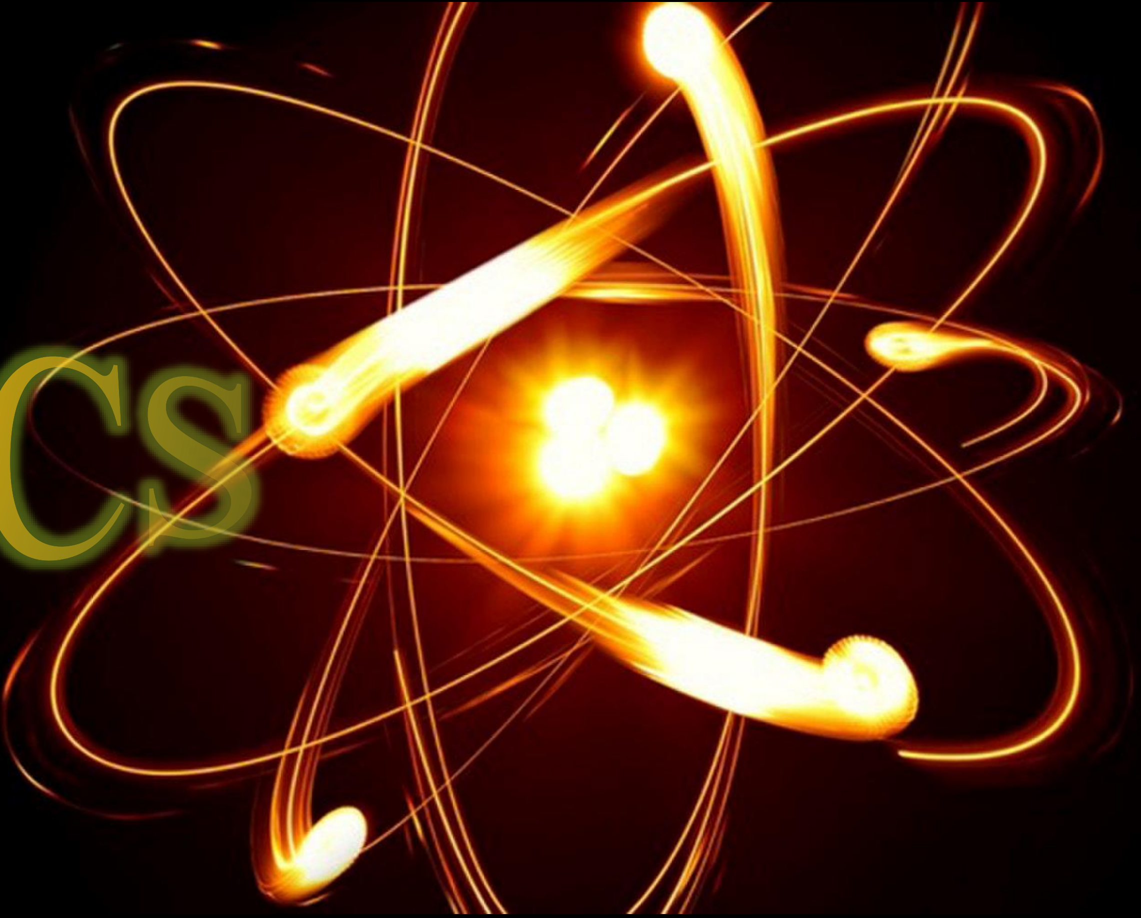
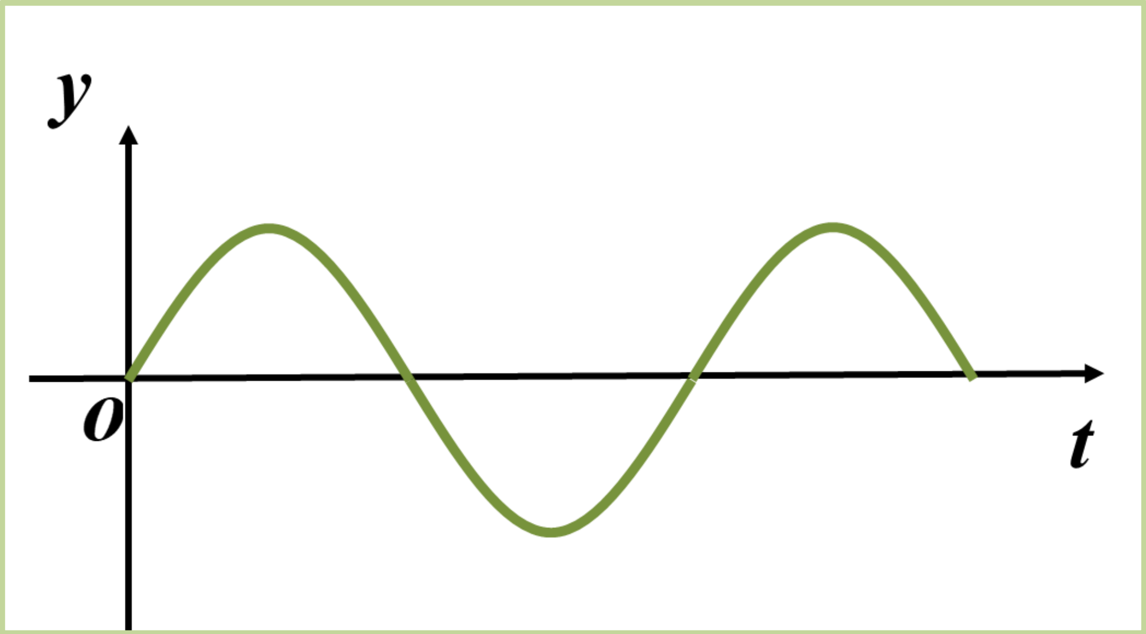
PHYSICS



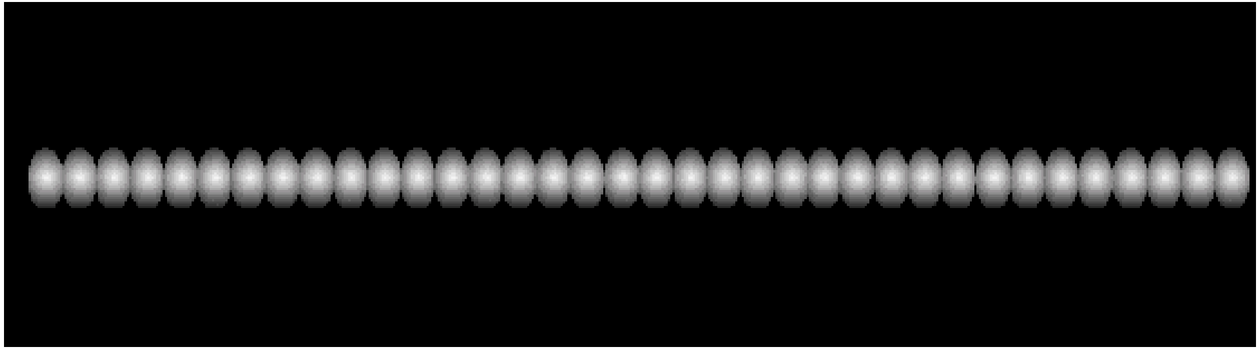
Review

Simple Harmonic Motion



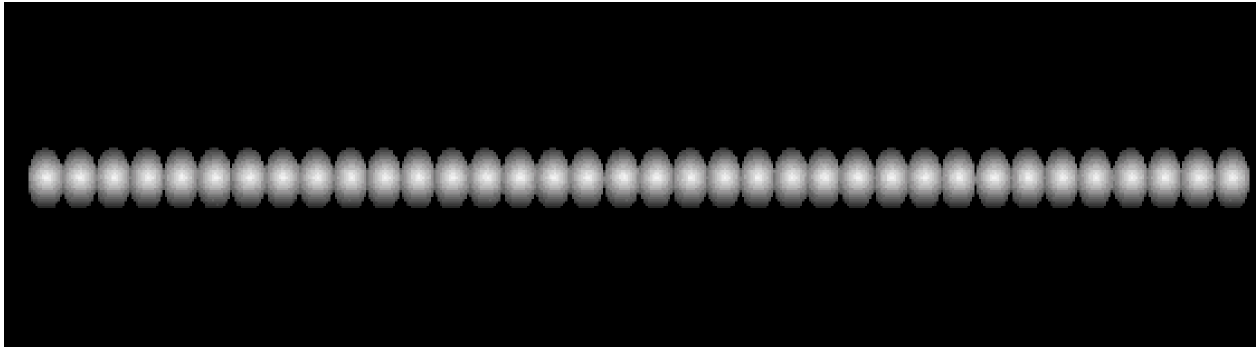
Physics 1: **Mechanics and Waves**

Week 15 – Mechanical Waves



Wave

A **wave** is a traveling disturbance that transports energy



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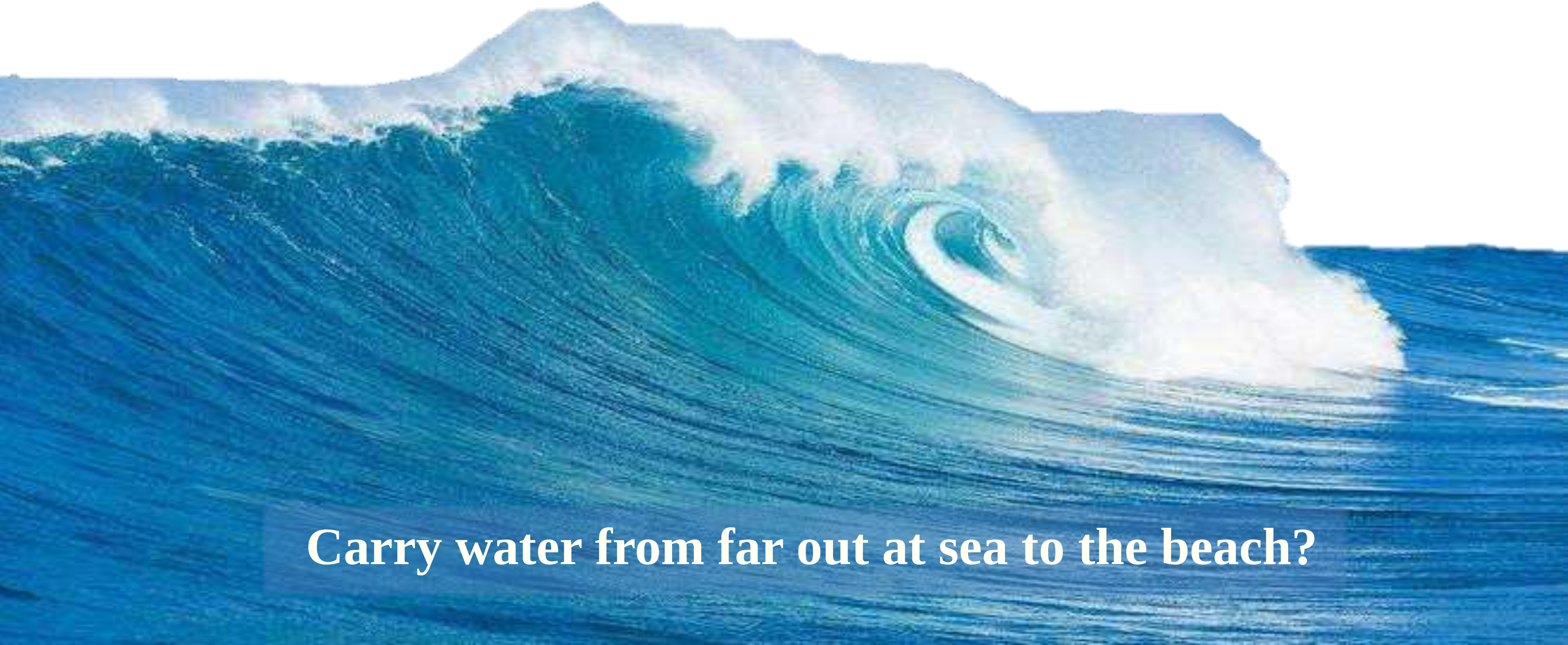


Carry water from far out at sea to the beach?

Wave

Mechanical waves

- ➢water, sound and seismic (earthquake) waves.

Electromagnetic waves

- ➢visible and ultraviolet light, radio and television, microwaves, x- rays, radar waves.

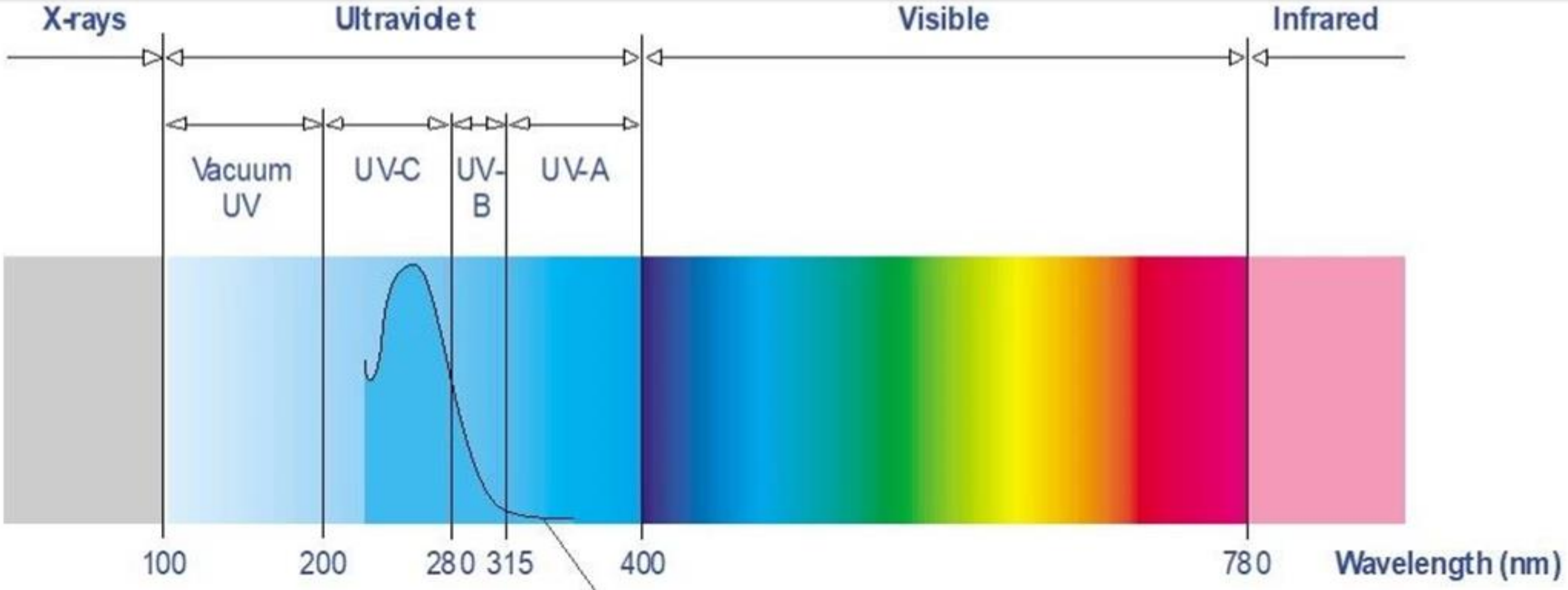
Matter waves

- ➢electrons, protons, and other fundamental particles, and even atoms and molecules.

Wave

Electromagnetic waves

- visible and ultraviolet light, radio and television, microwaves, X-rays, radar waves.



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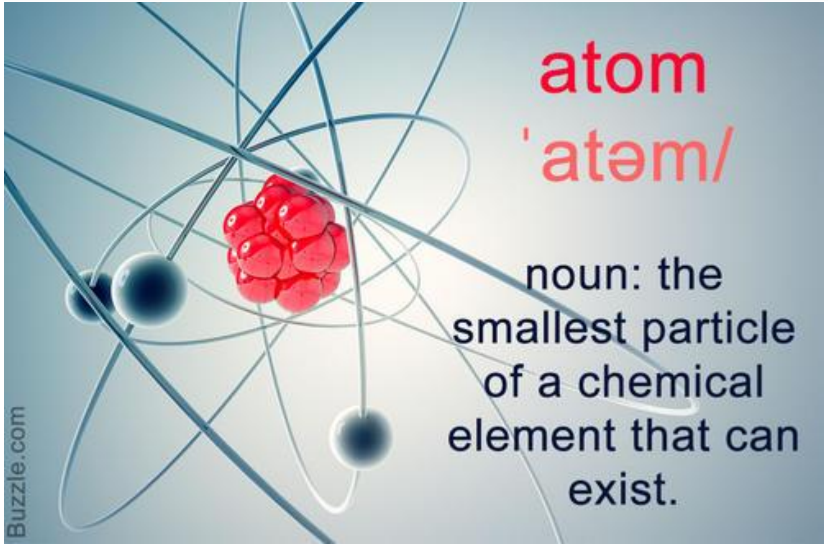

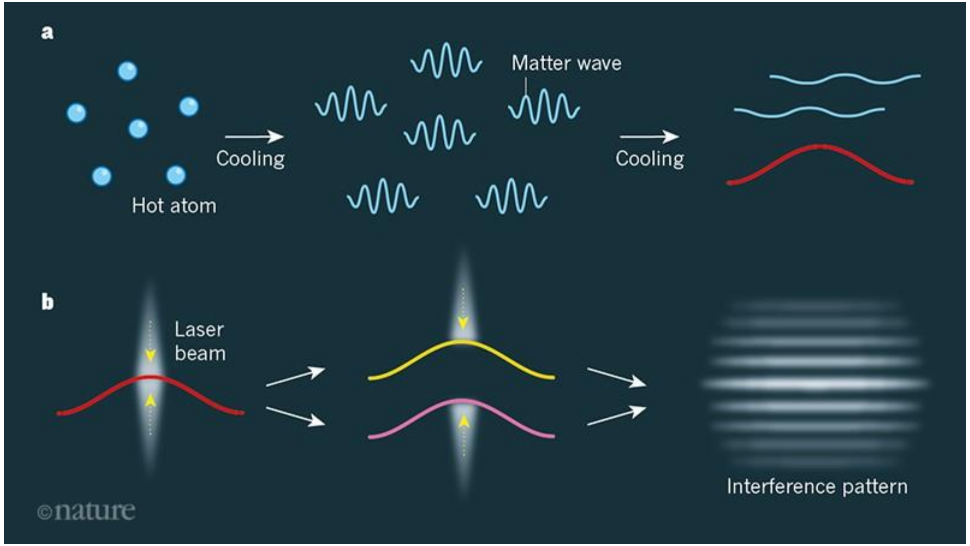

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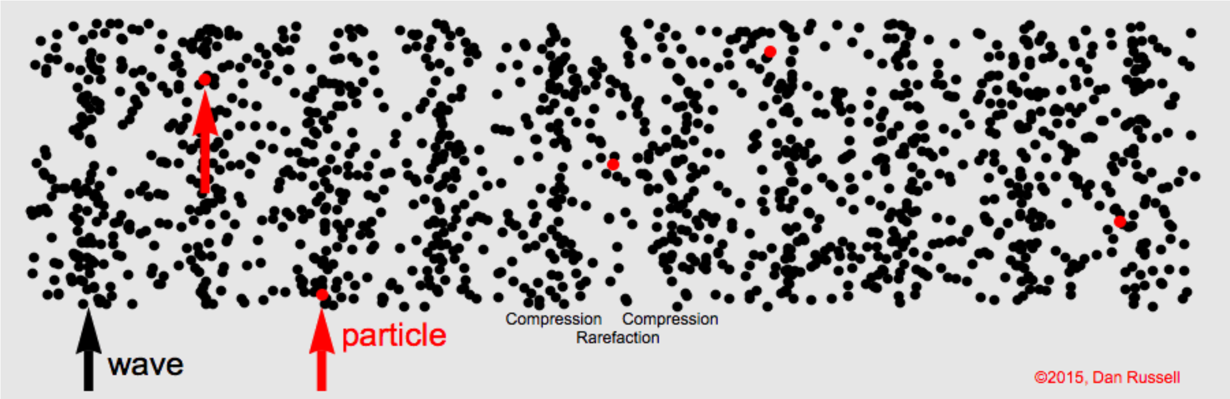

Wave

Mechanical waves

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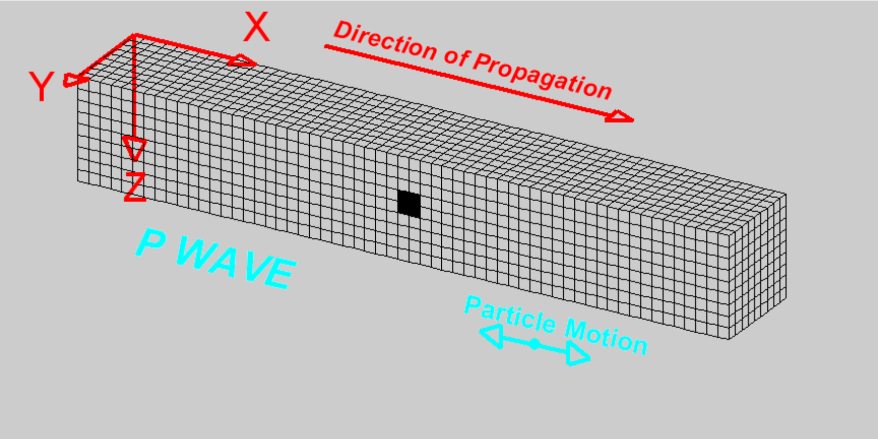
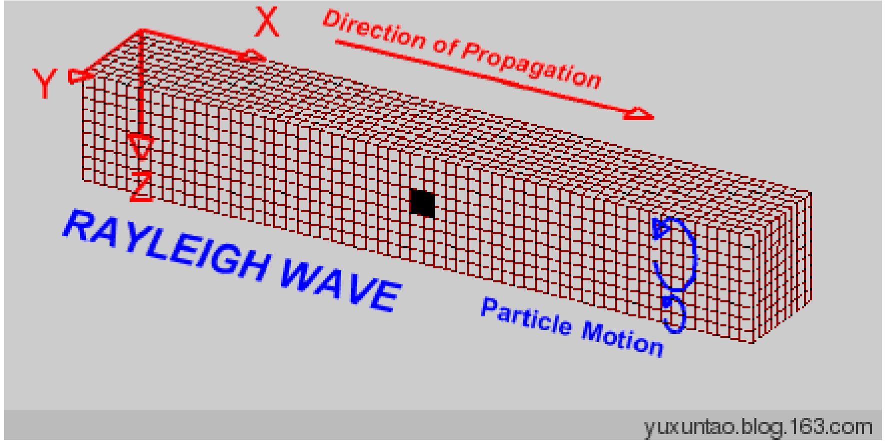




Mechanical waves

Associated with

A source of oscillation

A material which can transmit oscillations (medium)

Physical mechanism through which particles of the medium can influence one another

Mechanical waves

What is transmitting?

Particles

Energy

State of motion

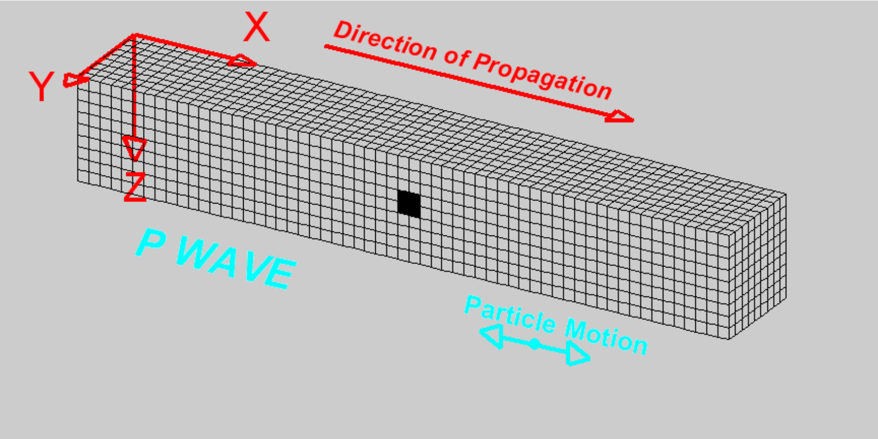
Simple Harmonic Oscillation

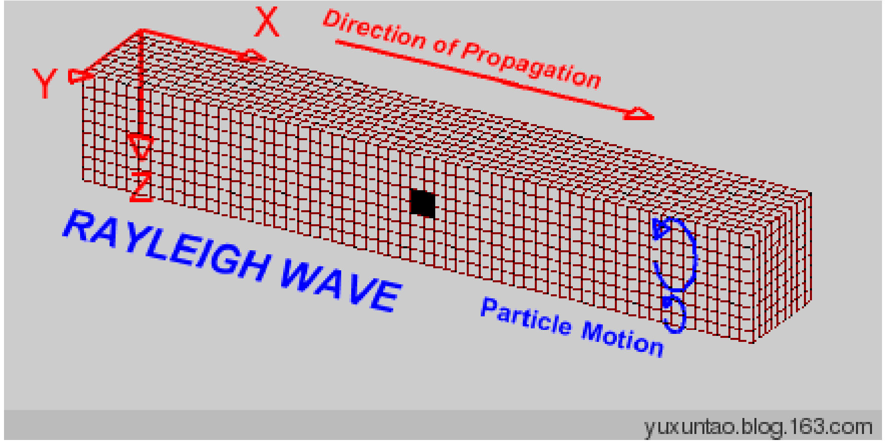
transmitting through the *interactions* between particles

Wave

Mechanical waves

- ➢water, sound and seismic (earthquake) waves.



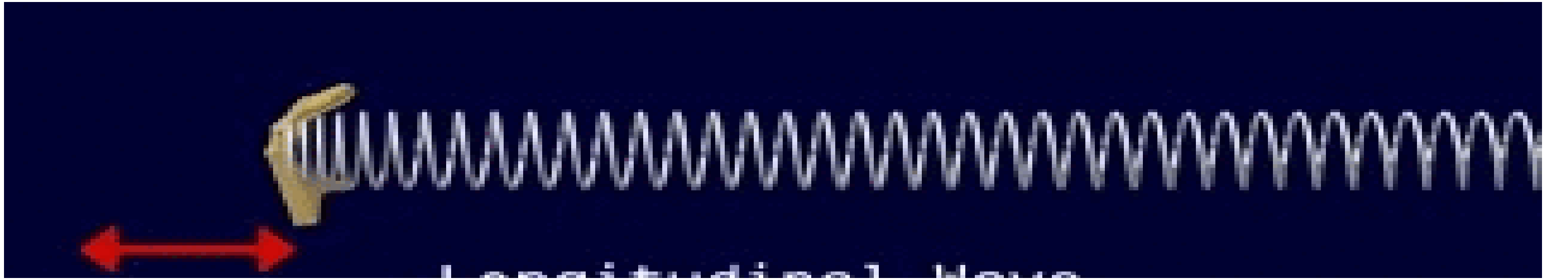


Types of Mechanical Waves

1. Transverse Waves

Transverse waves

2. Longitudinal Waves

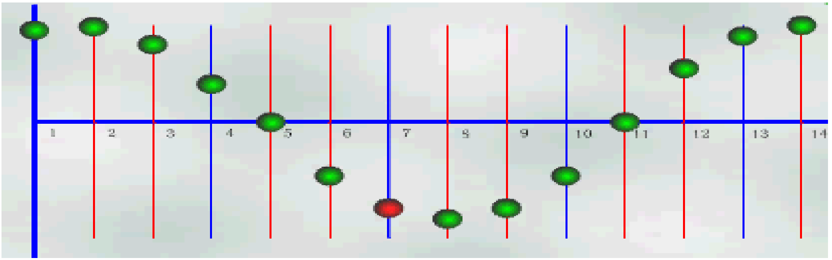


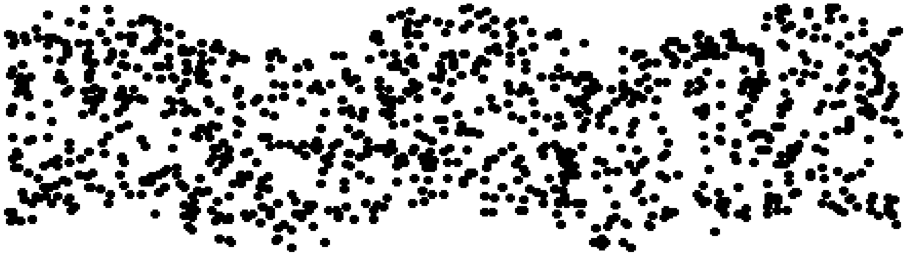
Types of Mechanical Waves

1. Transverse Waves

Transverse waves

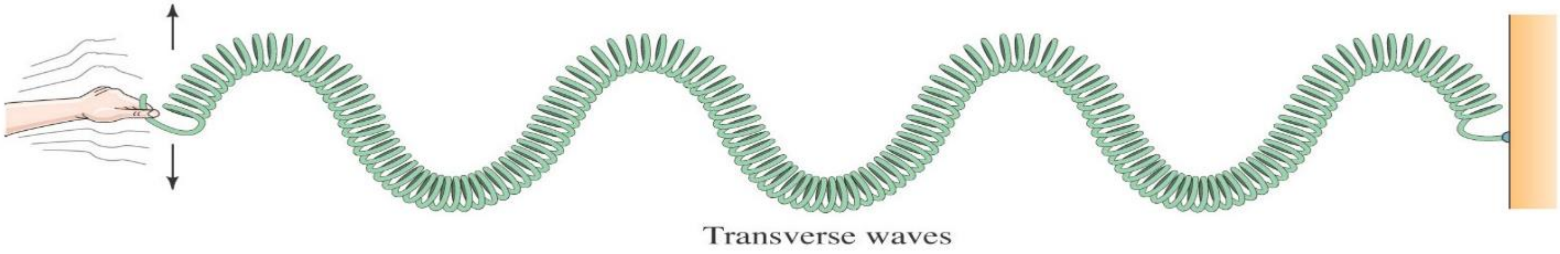
the oscillation of the medium is perpendicular to the direction of the wave



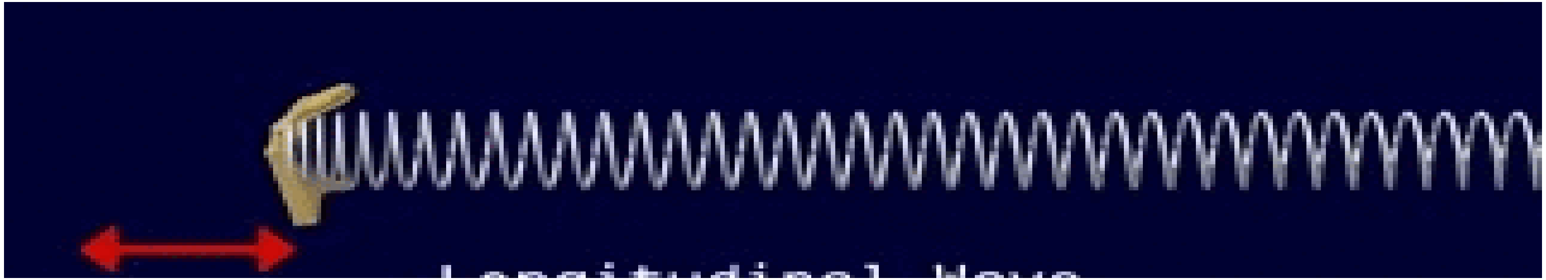


Types of Mechanical Waves

1. Transverse Waves

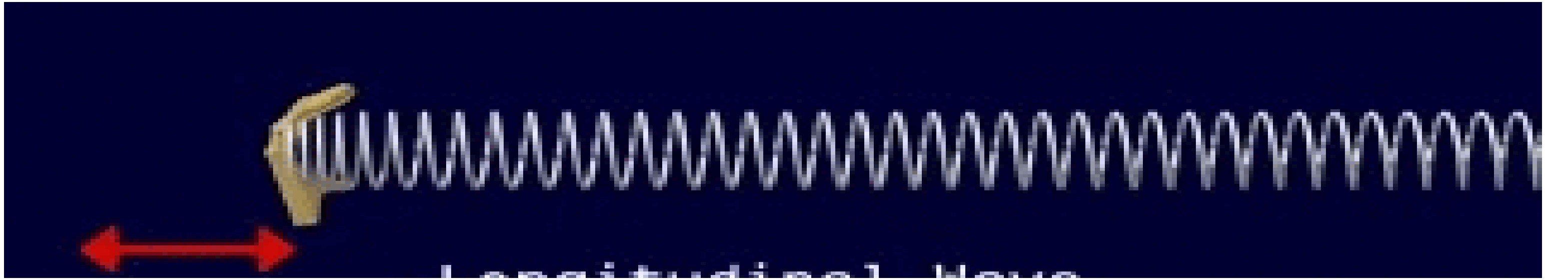


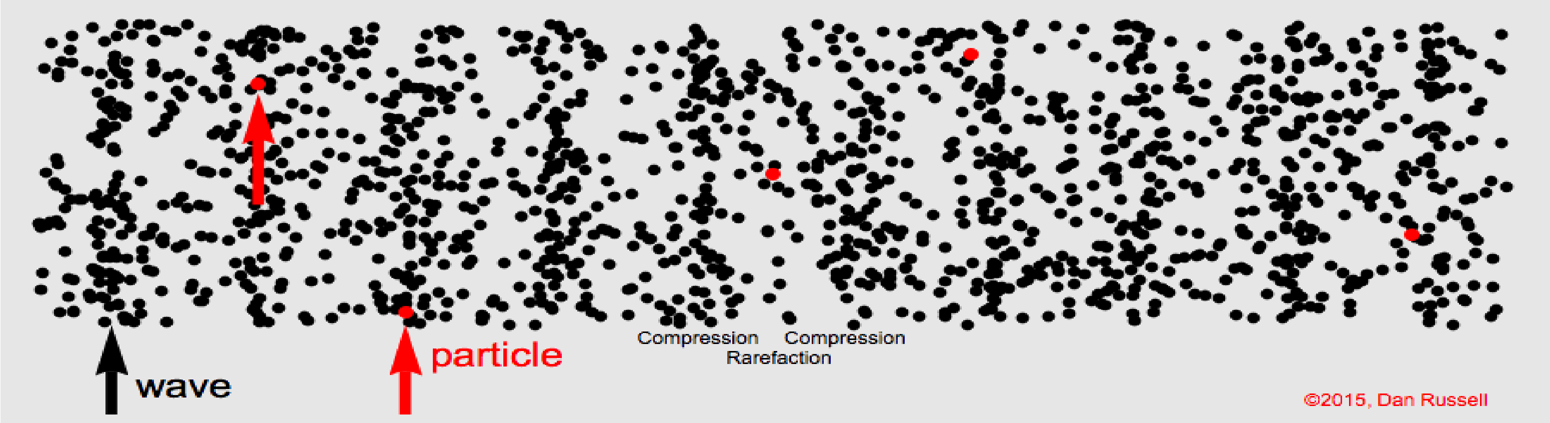
2. Longitudinal Waves



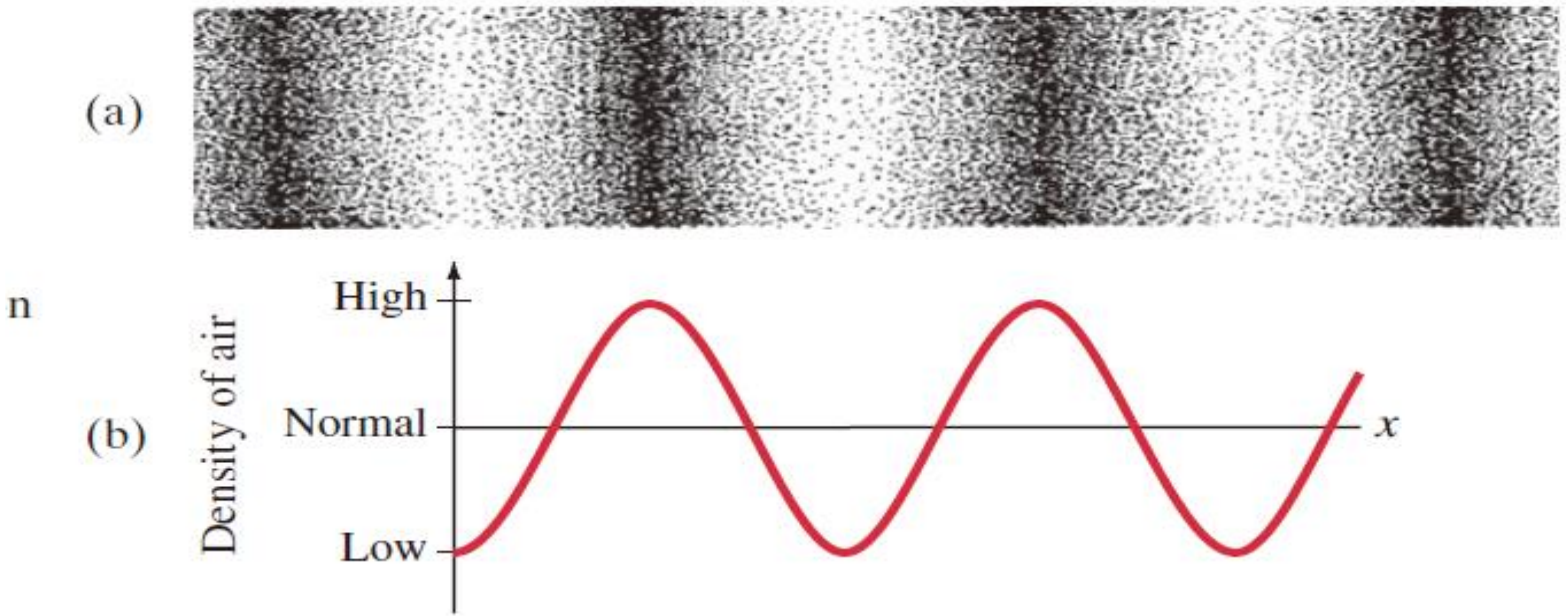
Types of Mechanical Waves

2. Longitudinal Waves



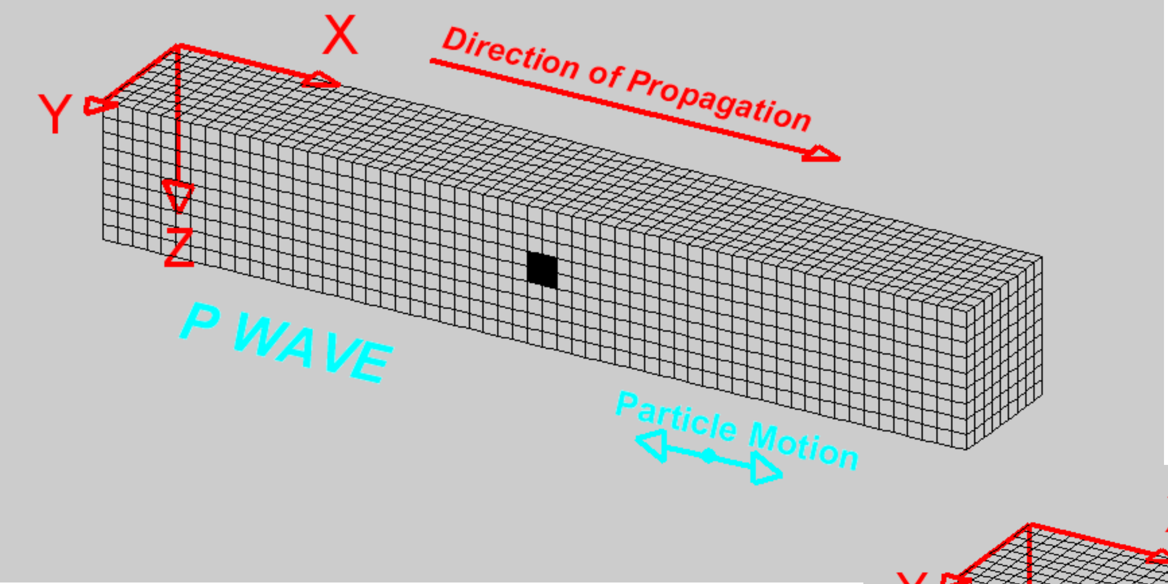


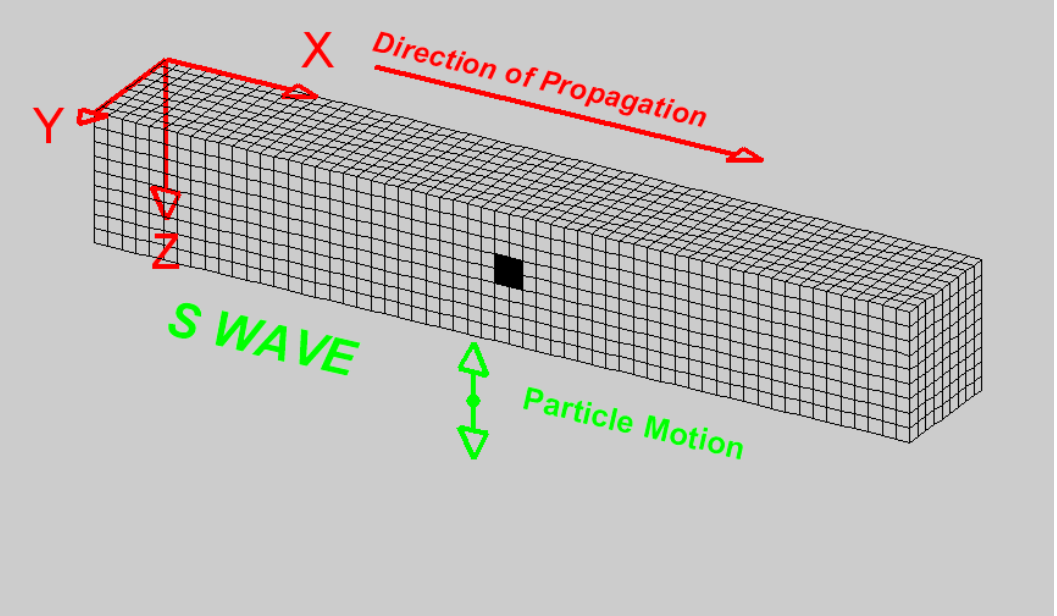
Longitudinal Waves



A longitudinal wave can be represented graphically by plotting the density of air molecules (or coils of slinky) versus position at a given instant.

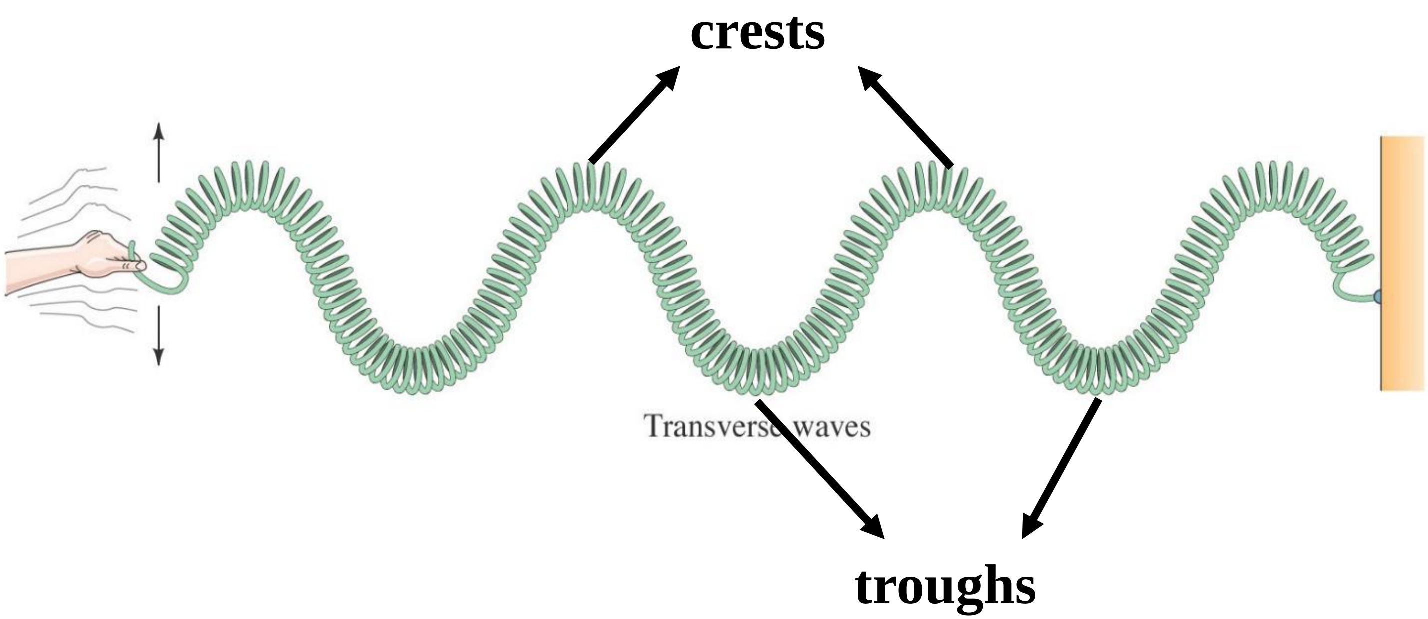
Note that the graph looks much like a transverse wave.


X
Y
Z
Direction of Propagation
P WAVE
Particle Motion

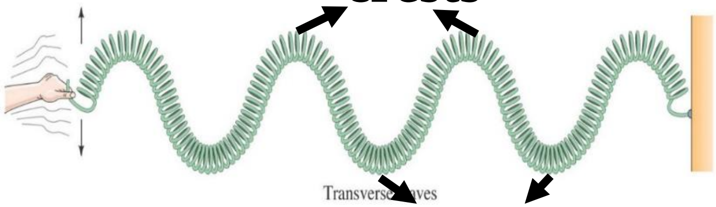

X
Y
Z
Direction of Propagation
S WAVE
Particle Motion

Types of Mechanical Waves

Travelling Wave



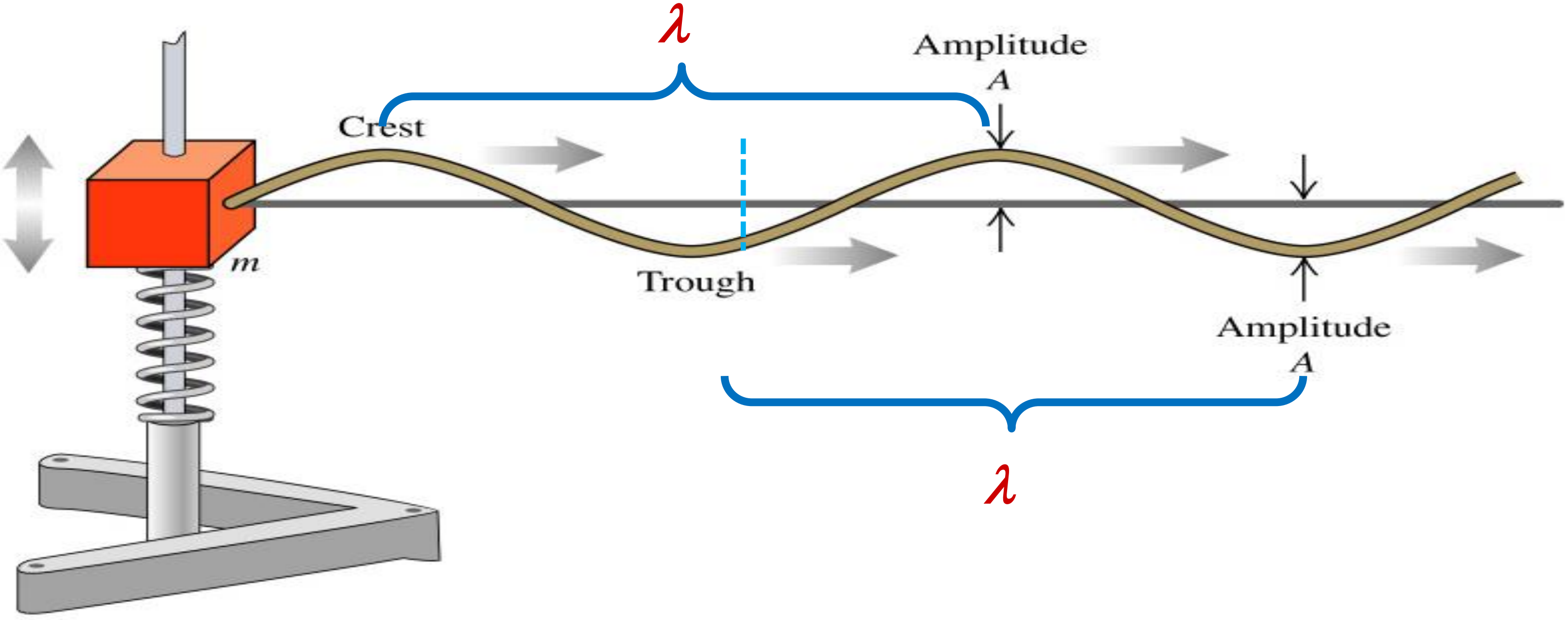
Travelling wave



CREST	TROUGH
➢ the point in the medium which exhibits the maximum amount of positive or upwards displacement from the equilibrium position	➢ the point in the medium which exhibits the maximum amount of negative or downwards displacement from the equilibrium position.

Travelling wave

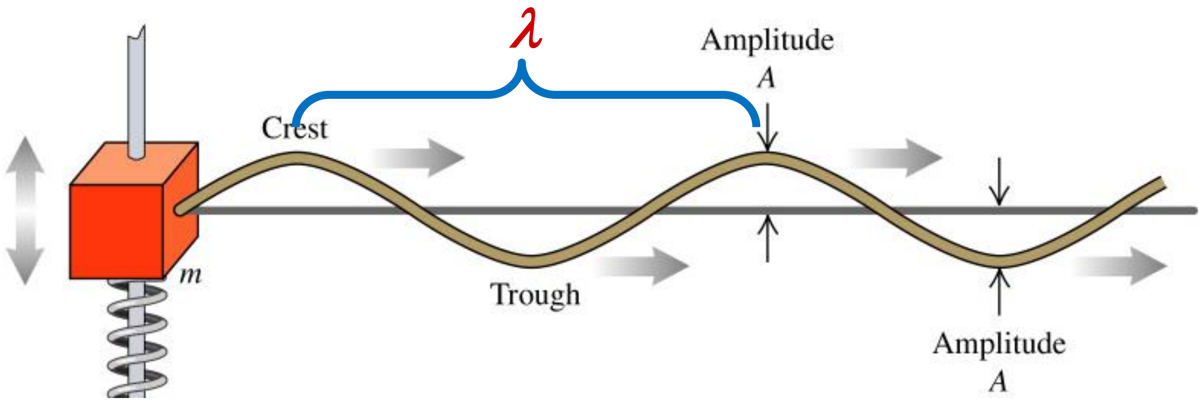
Wavelength λ: **The distance between *identical* *points* on the wave (a crest and the adjacent crest or trough to adjacent trough)**



Travelling wave - period

Frequency f $\quad T = \frac{1}{f}$

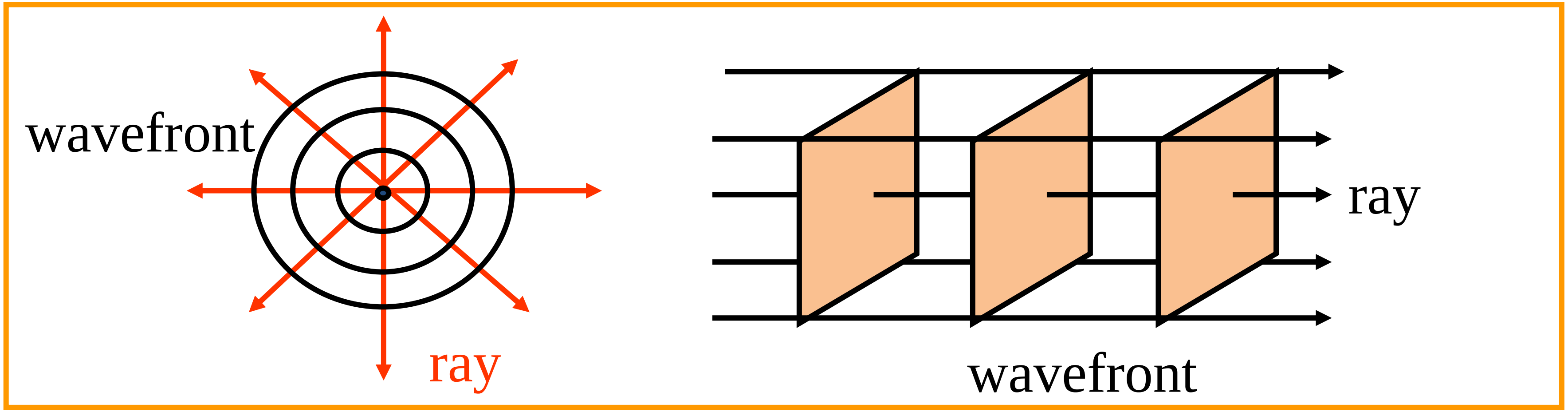
Period T	
SHM: the period of the harmonic oscillation of the particle in medium	Travelling wave: the time needed for the wave to move for one wavelength λ



Wavefronts and wave ray

Wavefronts - the surfaces on them all the points are in the same state of motion.

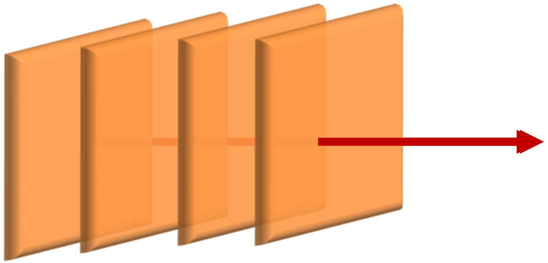
Wave ray - the direction which are perpendicular to the wavefronts or parallel to the velocity of the waves



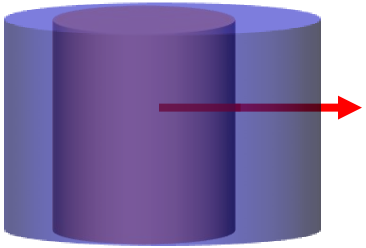
Wavefronts

According to the shape of the wavefronts

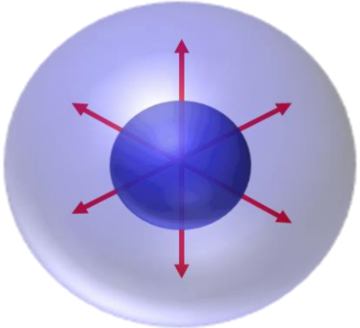
Plane waves: the wavefronts are plane



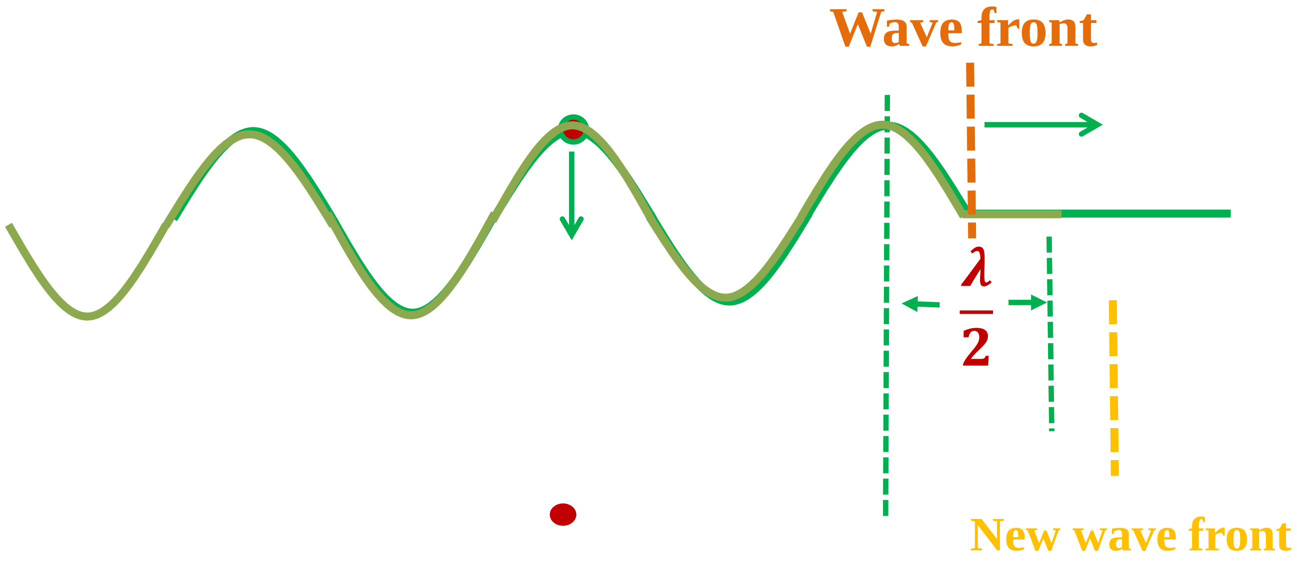
Cylindrical wave: the wavefronts are cylindrical



Spherical wave: the wavefronts are spherical



Travelling wave – wave speed



$$\Delta t = \frac{T}{2}$$

$$\Delta x = \frac{\lambda}{2}$$

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$$

$$\boxed{v = \frac{\lambda}{T} = \lambda f}$$

It depends solely on the mechanical properties of the medium

Wave speed

It depends solely on the mechanical properties of the medium

sound wave

in air
340 *m/s*

in water
1500 *m/s*

in steel
5000 *m/s*

Wave speed

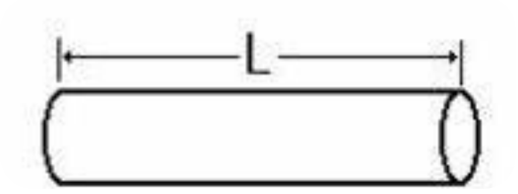
- Longitudinal wave
  - $v = \sqrt{\dfrac{E}{\rho}}$
    - Young's modulus
    - Density
- Transverse wave
  - $v = \sqrt{\dfrac{G}{\rho}}$
    - Shear modulus
    - Density
- Wave on a string
  - $v = \sqrt{\dfrac{T}{\mu}}$
    - Tension
    - Mass per unit length

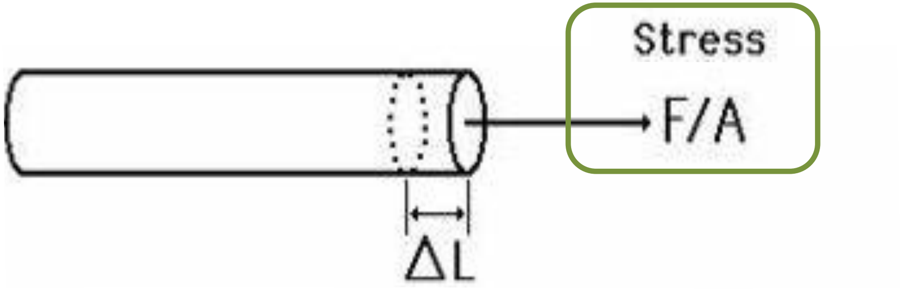
Wave speed - L – Yong's modulus

Longitudinal wave

$$v = \sqrt{\frac{E}{\rho}}$$

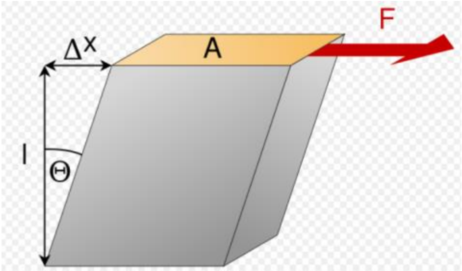
$$E = \frac{stress}{strain} = \frac{F/A}{\Delta l / l_o}$$

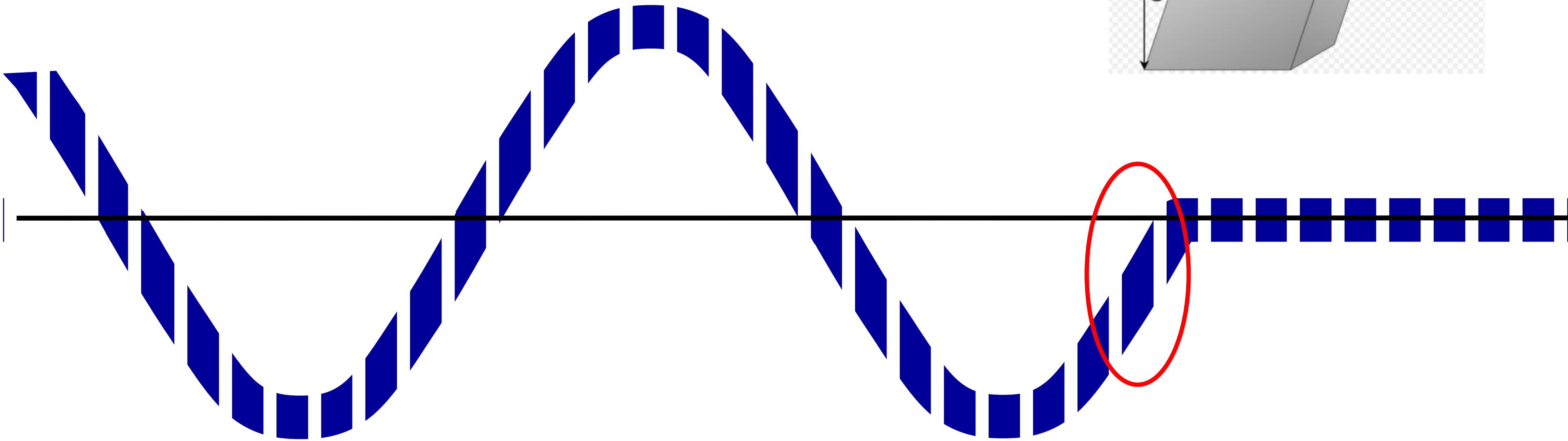




Strain
ΔL/L

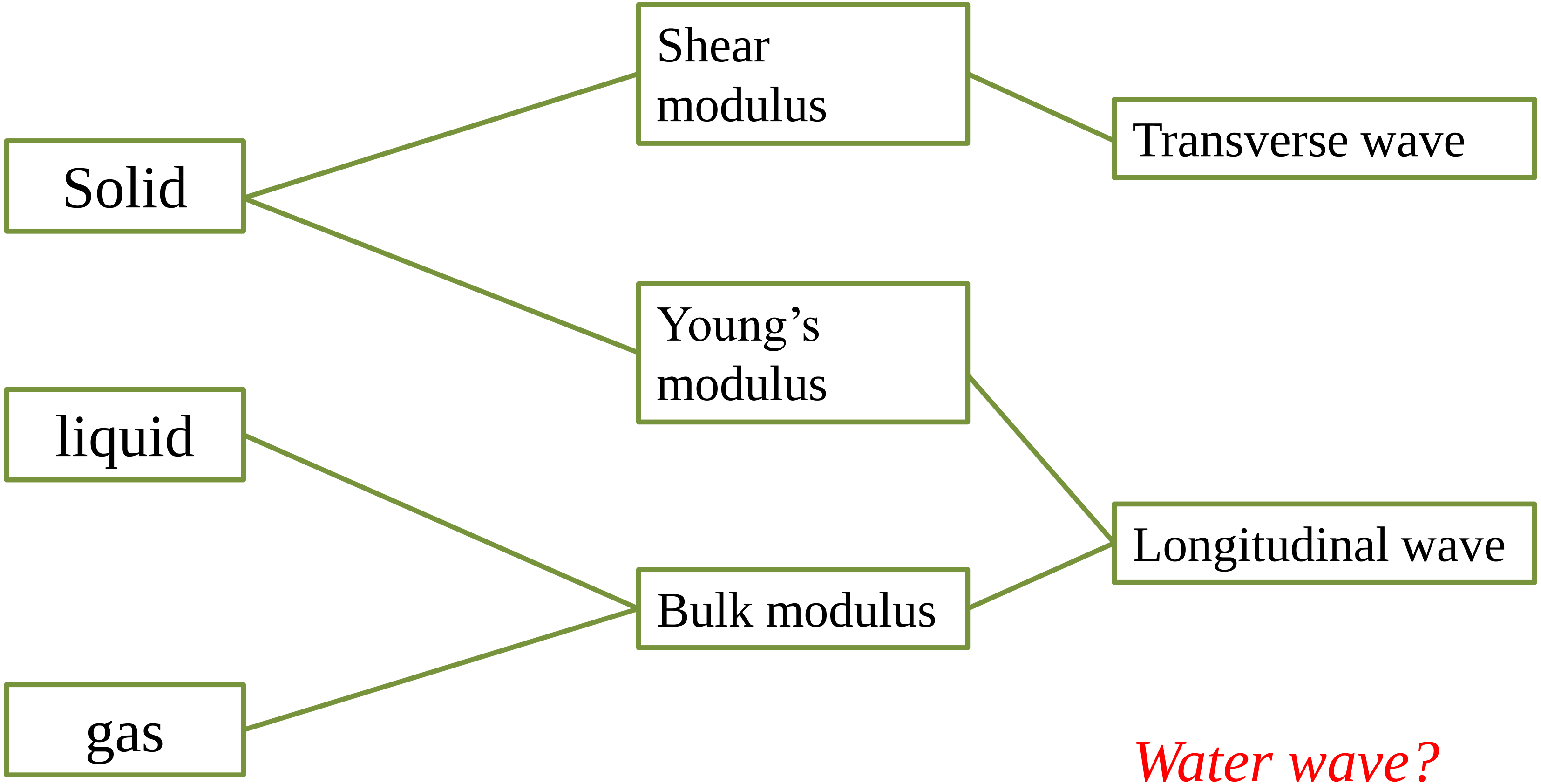
Wave speed - T - Shear modulus





$$G \stackrel{\text{def}}{=} \frac{\tau_{xy}}{\gamma_{xy}} = \frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$$

Summary



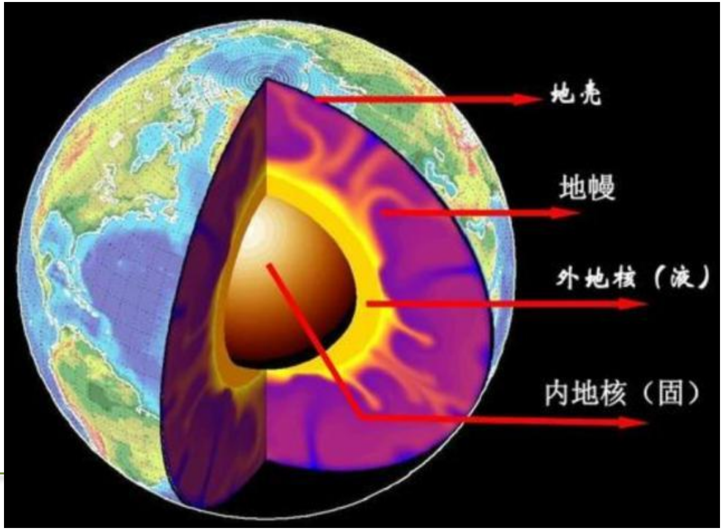
Water wave?

Water wave

Surface wave

T + L wave

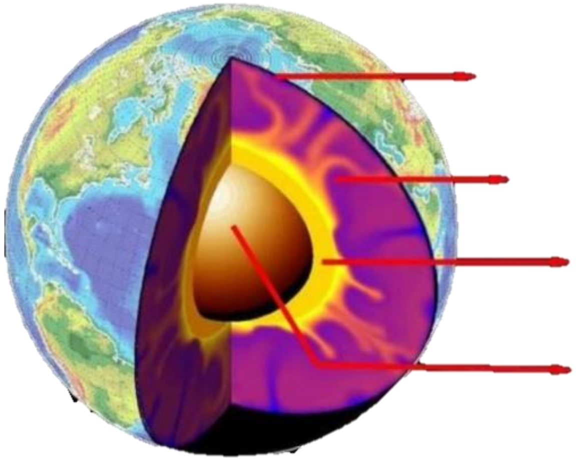
Wave in an Earthquake

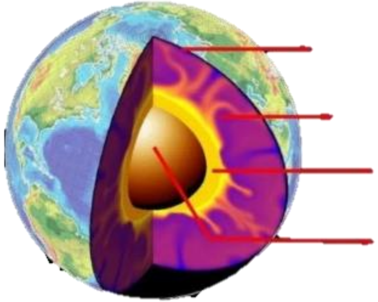


Vibrating of the ground

Wave	Transverse	S – wave (Shear)
	Longitudinal	P – wave (Pressure)

Wave in an Earthquake





Both longitudinal and transverse waves can travel through a solid since the atoms or molecules can vibrate about their relatively fixed position in any direction. But only longitudinal waves can propagate through a fluid because any transverse motion would not experience any restoring force since a fluid is readily deformable. This fact was used by geophysicists to infer that a portion of the earth core must be liquid. After an earthquake longitudinal waves are detected diametrically across the earth but not transverse waves.

Example

Sound waves are longitudinal waves in air.

The speed of sound depends on the temperature.

At 20 °C it is 344 m/s.

What is the wavelength of a sound wave in air at 20 °C if the frequency is 262 Hz?

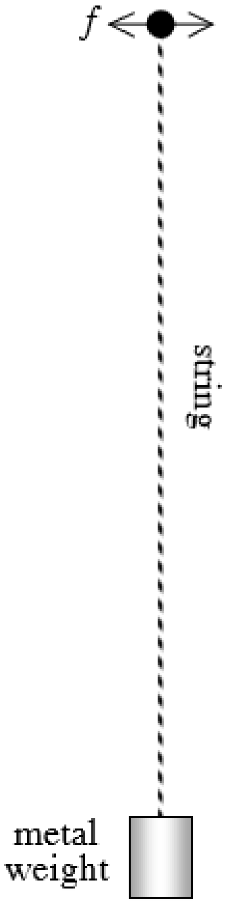
Solution

$$\lambda = \frac{v}{f} = \frac{344}{262} = 1.31m$$

Example

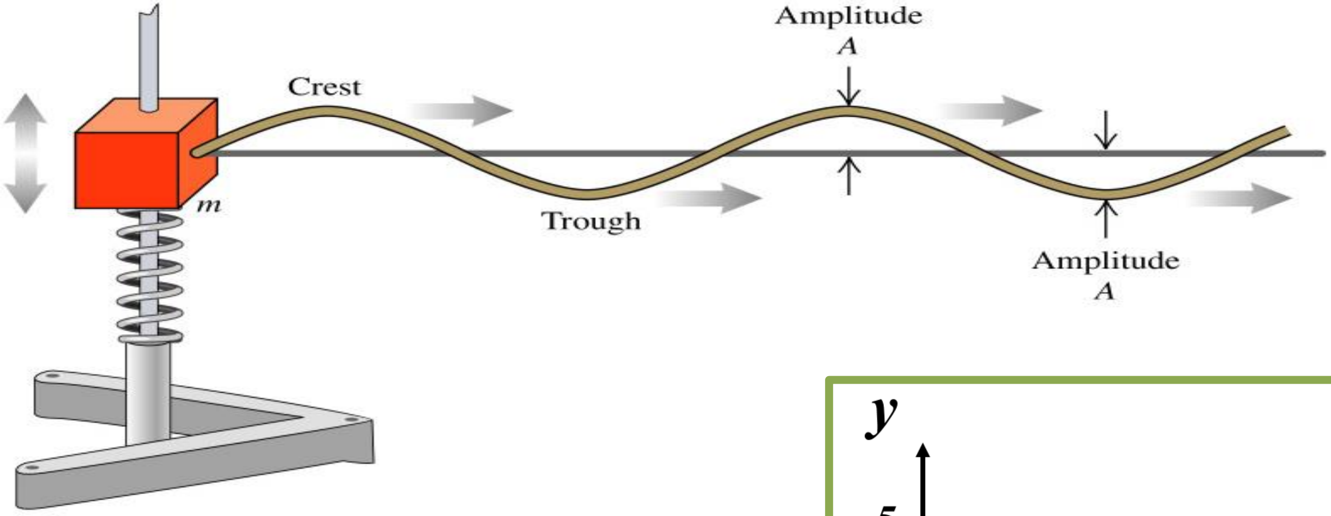
A metal weight is hung at the end of a string, creating a uniform tension in the string. The string's mass is 50. g and its length is 1.5 m. The top end of the string is fixed to a motor that can vibrate horizontally with a small amplitude at the frequency $f = 100$ Hz

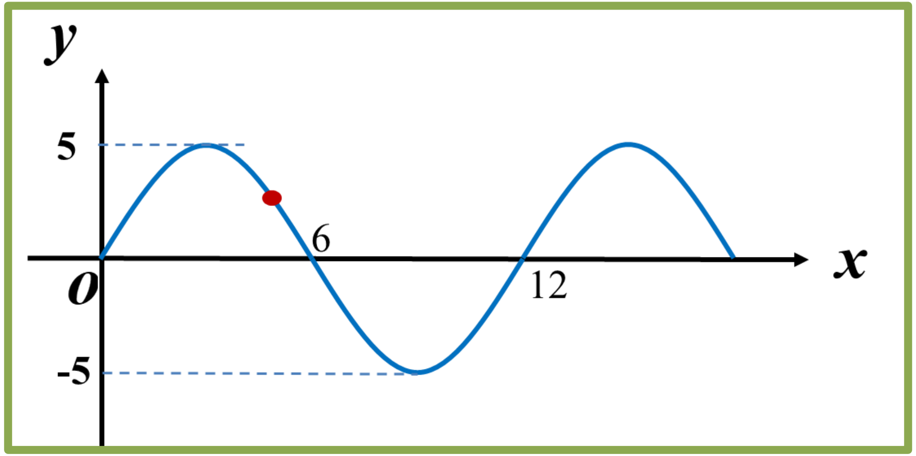
If the wave length along the string is 15 cm, calculate the **mass** of the metal weight at the end of the string.



$$v = \sqrt{\frac{T}{\mu}}$$

Continuous periodic waves





Continuous periodic waves

1. The particles of medium do not move forward with the wave. They only oscillate around their equilibrium positions.

$$\boxed{y(x,t) = A\cos(\omega t + \varphi)}$$

2. Each particle is doing the SAME *simple harmonic motion* with different initial phases (neglecting the energy dissipation)

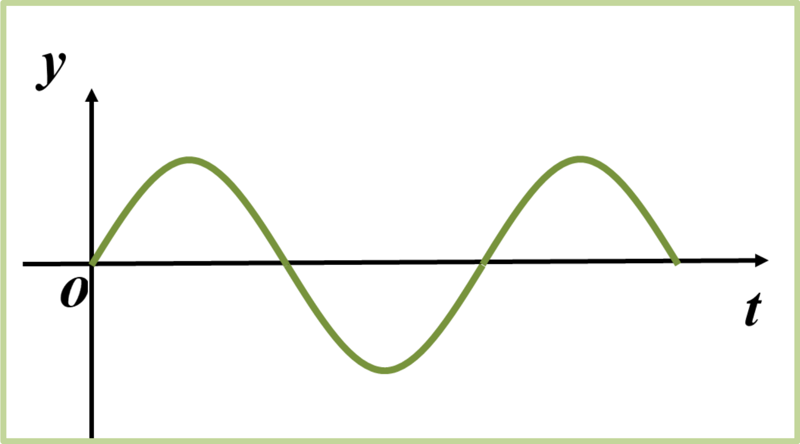
$$y(x,t) = A\cos(\omega t + \varphi)$$

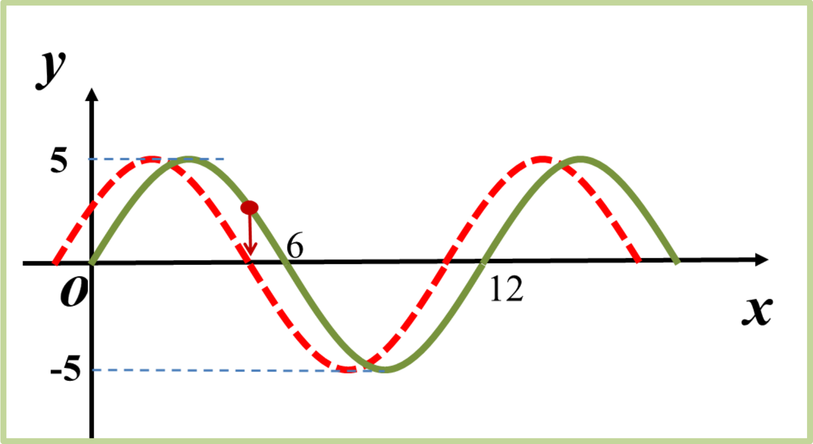
φ depends on x

Wave function

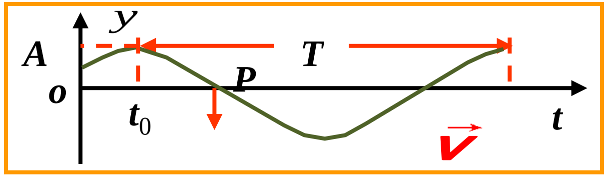
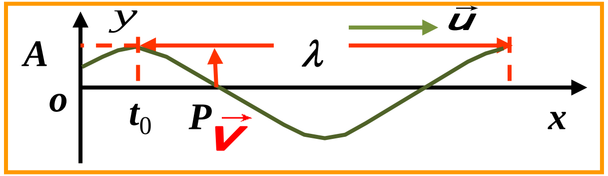
SHM

Continuous wave





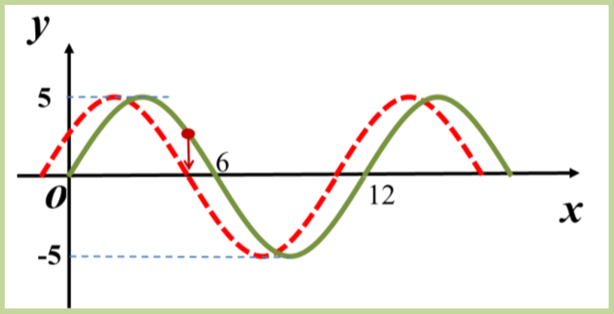
SHM and Continuous wave

	振动曲线	波形曲线
图形		
研究对象	某质点位移随时间变化规律	某时刻，波线上各质点位移随位置变化规律
物理意义	由振动曲线可知 周期T. 振幅A 初相 φ_0 某时刻 $\vec{v}$ 其方向参看下一时刻状况	由波形曲线可知 该时刻各质点位移 波长λ, 振幅A 只有t=0时刻波形才能提供初相 某质点$\vec{v}$ 方向参看前一质点
特征	对确定质点曲线形状一定	曲线形状随t 向前平移

Wave Function

The motion of particles in the medium can be described by the wave function:

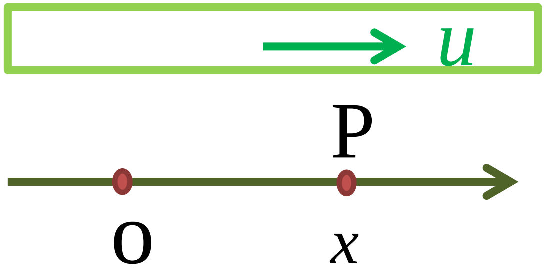
$$y = y(x,t)$$



Displacement of **a particle**

X	$y(x_0,t)$	The motion of the particle at position x_0
t	$y(x,t_0)$	Positions of the particles in medium at t_0
	$y = y(x_0,t_0)$	The displacement of the particle at position x_0 and the instance t_0.

Harmonic Wave



Simple Harmonic Wave

- the propagation of simple harmonic motion in medium

The wave function of the harmonic wave: $\boxed{y(x,t) = A\cos(\omega t + \varphi)}$

$$\boxed{y = A\cos\left[\omega\left(t - \frac{x}{u}\right) + \phi\right]}$$

amplitude

angular frequency

phase constant

Wave speed

Alternative Expressions of Wave Function

$$* \ y = A\cos\left[\ \omega\left(t-\frac{x}{u}\right)+\phi\ \right]$$

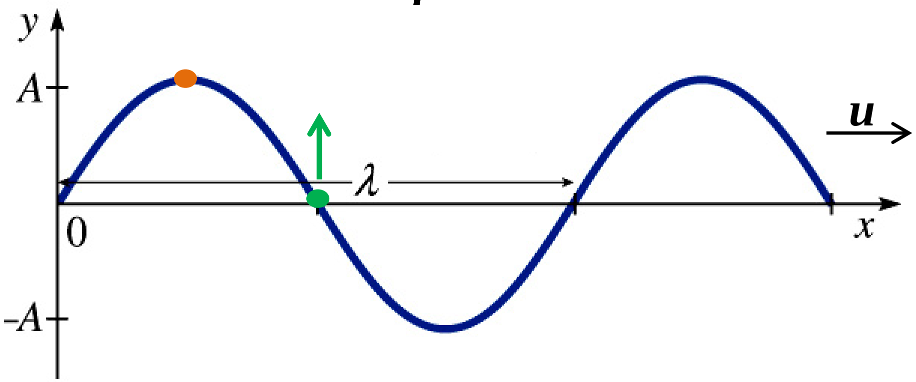
$$y(x,t) = A\cos[2\pi(\frac{t}{T}-\frac{x}{\lambda})+\varphi_0]$$

$$y(x,t) = A\cos[\frac{2\pi}{\lambda}(vt-x)+\varphi_0]$$

Comparing with SHM: $y(t) = A\cos(\omega t+\varphi)$

SHM of two particles-change in the phase angle

***waveform* of *t* = 0**



$$y_1 = A\cos(\omega t + \varphi_1)$$

$$\varphi_1 = 0$$

$$y_2 = A\cos(\omega t + \varphi_2)$$

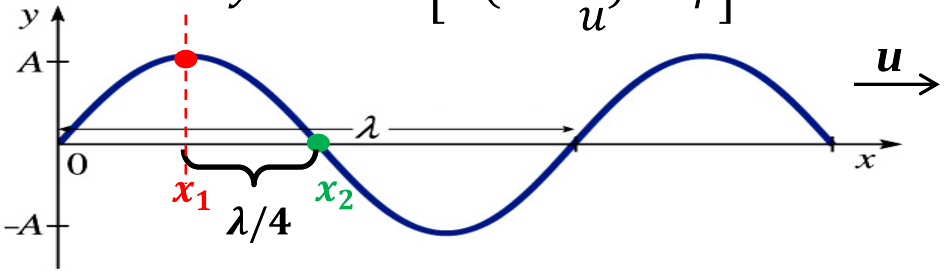
$$\varphi_2 = -\frac{1}{2}\pi$$

The phase change

$$\Delta\varphi = \varphi_2 - \varphi_1 = -\frac{1}{2}\pi$$

Think about the wave function

$$y = A\cos\left[\omega\left(t - \frac{x}{u}\right) + \phi\right]$$



$$y_1 = A\cos\left(\omega t - \frac{\omega x_1}{u} + \phi\right)$$

$$\varphi_1 = -\frac{\omega x_1}{u} + \phi$$

$$y_2 = A\cos\left(\omega t - \frac{\omega x_2}{u} + \phi\right)$$

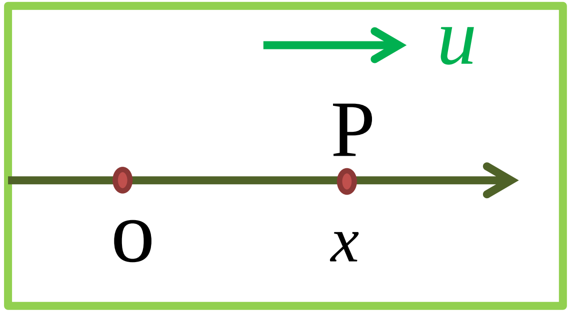
$$\varphi_2 = -\frac{\omega x_2}{u} + \phi$$

$$\Delta\varphi = \varphi_2 - \varphi_1 = -\left(\frac{\omega x_2}{u} - \frac{\omega x_1}{u}\right)$$

$$\Delta\varphi = -\frac{2\pi}{\lambda}(x_2 - x_1) = -\frac{1}{2}\pi$$

Consistent!

Wave function



$$\Delta\varphi = -\frac{2\pi}{\lambda}\Delta x$$

Along the direction of propagation of the wave, the phase falls behind

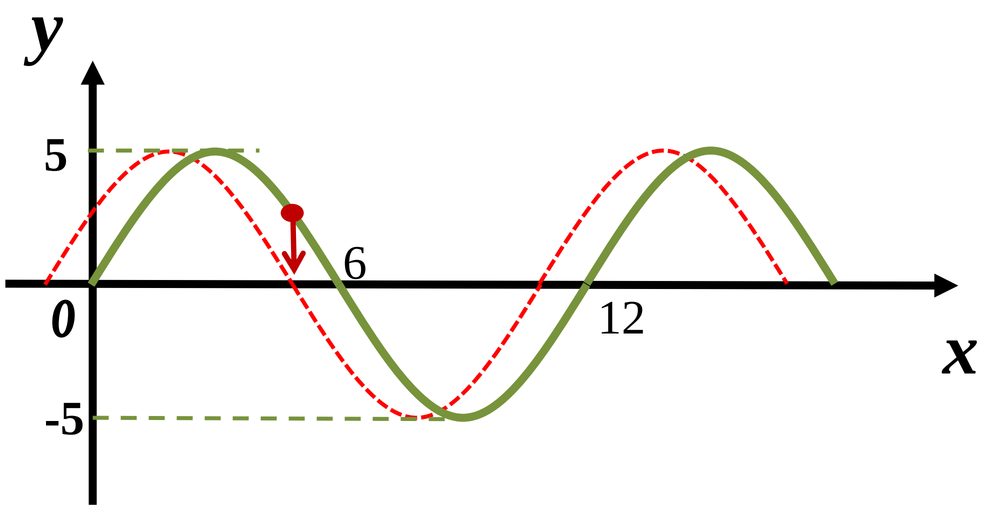
So – the phase of particles "*in front*" is caused by the particles "*behind*" it.

Wave propagates to the negative direction?

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x$$

$$y = A\cos\left[\omega\left(t + \frac{x}{u}\right) + \phi\right]$$

Example



The waveform at $t = 0$ is shown in the graph.

The wave speed is **18**.

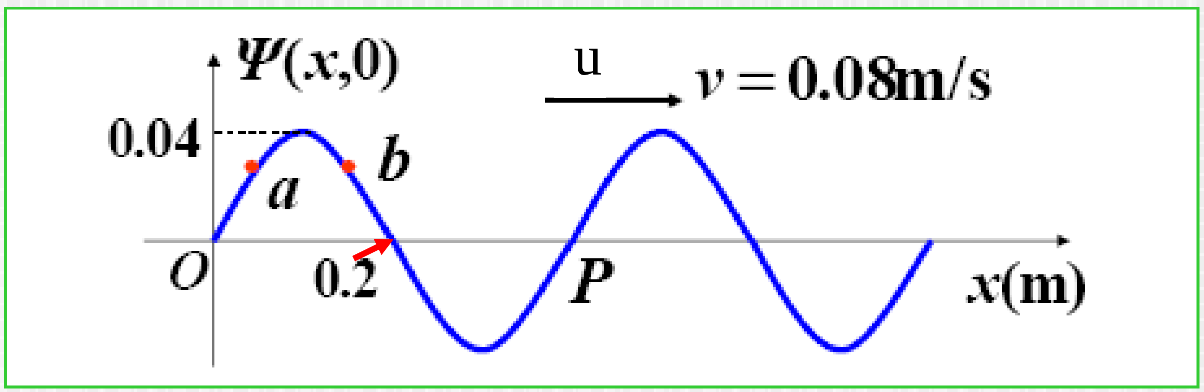
The direction of motion of a particle in the medium is given.

Find the wave function.
(all quantities are in SI units)

$$y = 5cos\left[3\pi\left(t + \frac{x}{18}\right) - \frac{\pi}{2}\right]$$

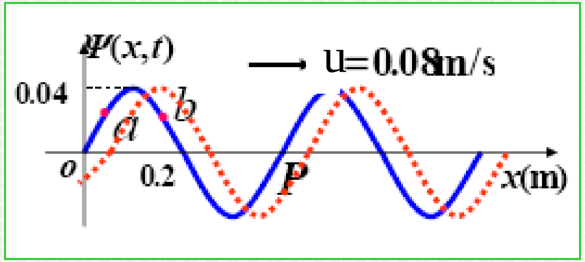
Example

The figure depicts the waveform of a traveling cosine wave at instants t = 0 s, find (a) the oscillatory equation of point o;
(b) the wave function ;
(c) the oscillatory equation of point P;
(d) the moving directions of points a and b.



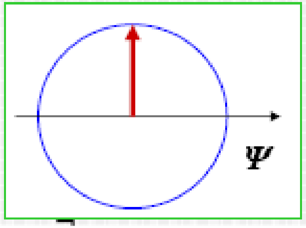
Solution:

(a) $A = 0.04(m) \quad \lambda = 0.4(m)$



$$\omega = 2\pi \frac{u}{\lambda} = 0.4\pi(rad.s^{-1})$$

The initial phase angle of point *o*:



$$\varphi_0 = \frac{\pi}{2}$$

$$\psi(0,t) = 0.04\cos(0.4\pi t + \frac{\pi}{2})$$

(b) the wave function

$$\psi(x,t) = 0.04\cos\left[0.4\pi(t - \frac{x}{0.08}) + \frac{\pi}{2}\right]$$

(c) the oscillatory equation of point P

$$\psi(x_p, t) = 0.04\cos\left[0.4\pi(t - \frac{0.4}{0.08}) + \frac{\pi}{2}\right]$$

$$= 0.04\cos\left[0.4\pi t + \frac{\pi}{2}\right]$$

(d) the moving directions of points *a* and *b*.