ENSC 2113
Engineering Mechanics: Statics

Chapter 4:

Force System Resultants

(Sections 4.1-4.4)

COLLEGE OF
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AND TECHNOLOGY

Chapter 4 Outline:

4.1 Moment of a Force – Scalar Formulation

4.2 Cross Product

4.3 Moment of a Force – Vector Formulation

4.4 Principle of Moments

4.5 Moment of a Force about a Specified Axis

4.6 Moment of a Couple

4.7 Simplification of a Force and Couple System

4.8 Further Simplification of a Force and Couple System

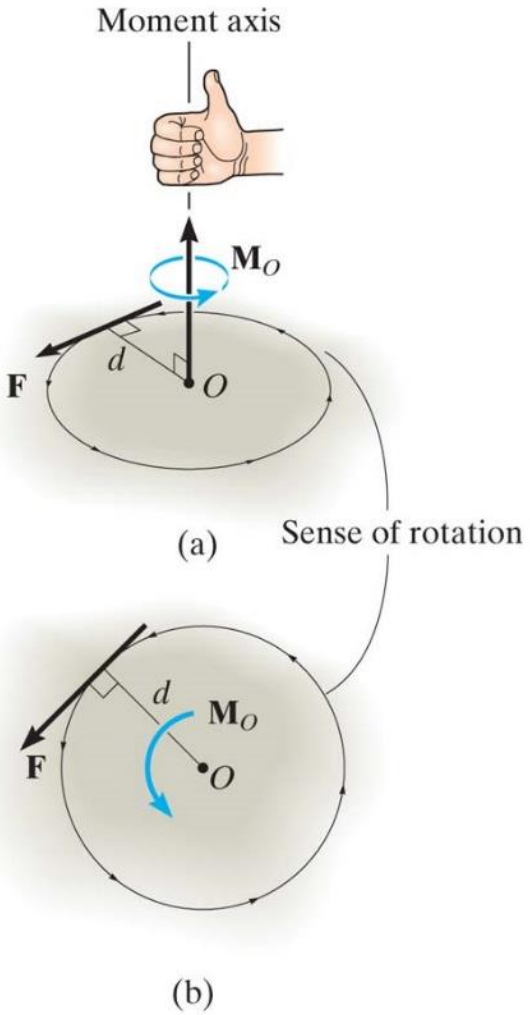
4.9 Reduction of a Simple Distributed Loading

Chapter 4 Objectives:

- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- To provide a method for finding the moment of a force about a specified axis
- To define the moment of a couple
- To show how to find the resultant effect of a nonconcurrent force system
- To indicate how to reduce a simple distributed loading to a resultant force acting at a specified location

Right-Hand Rule:

- Point the thumb of your right hand along the positive axis of rotation.
- Fingers curl in the positive direction
- Sign convention:
  - Counterclockwise is positive, clockwise is negative

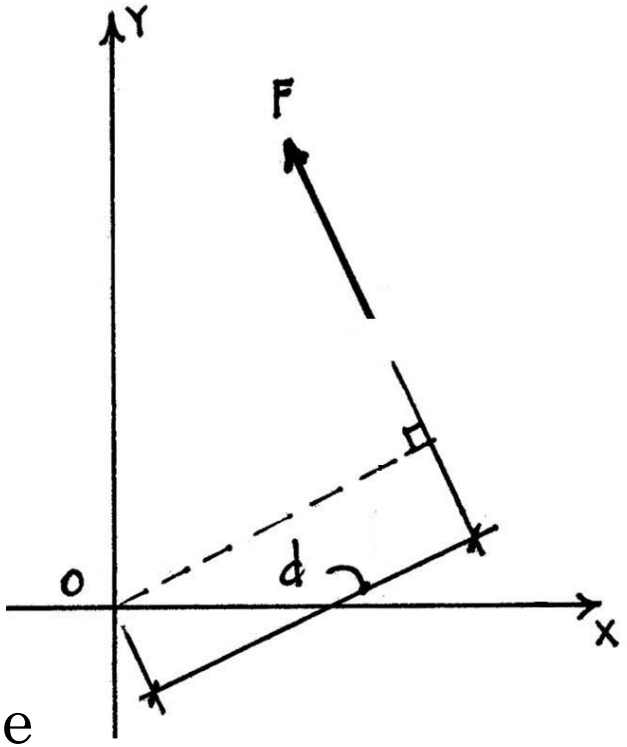


4.1: Moment of a Force - Scalar Formulation

The moment of a force is the tendency of the force to produce rotation about a point or a line.

The scalar magnitude of the moment of a force about point O is:

$$|M_o| = |F|d$$



where,

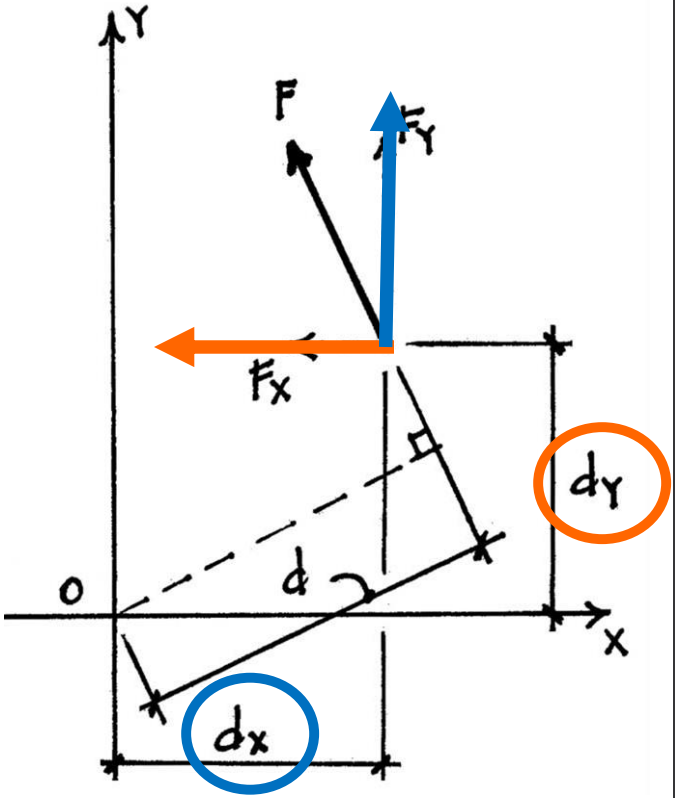
- M is the magnitude of the rotation about a point
- d is the perpendicular distance measuring from the point to the force

4.1: Moment of a Force - Scalar Formulation

Breaking the force into components may simplify the process.

The moment is the sum of **each component** multiplied by its perpendicular distance back to point **O**.

$$|M_o| = |F_x|d_y + |F_y|d_x$$



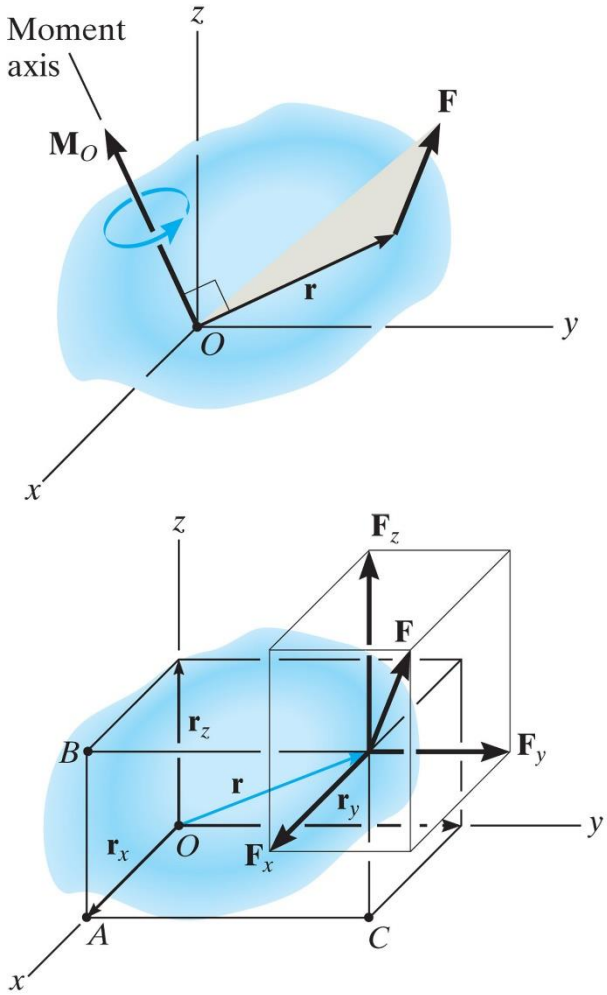
4.3: Moment of a Force - Vector Formulation

- Vector Analysis

$$\overrightarrow{M} = \vec{r} \times \overrightarrow{F}$$

$$\overrightarrow{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

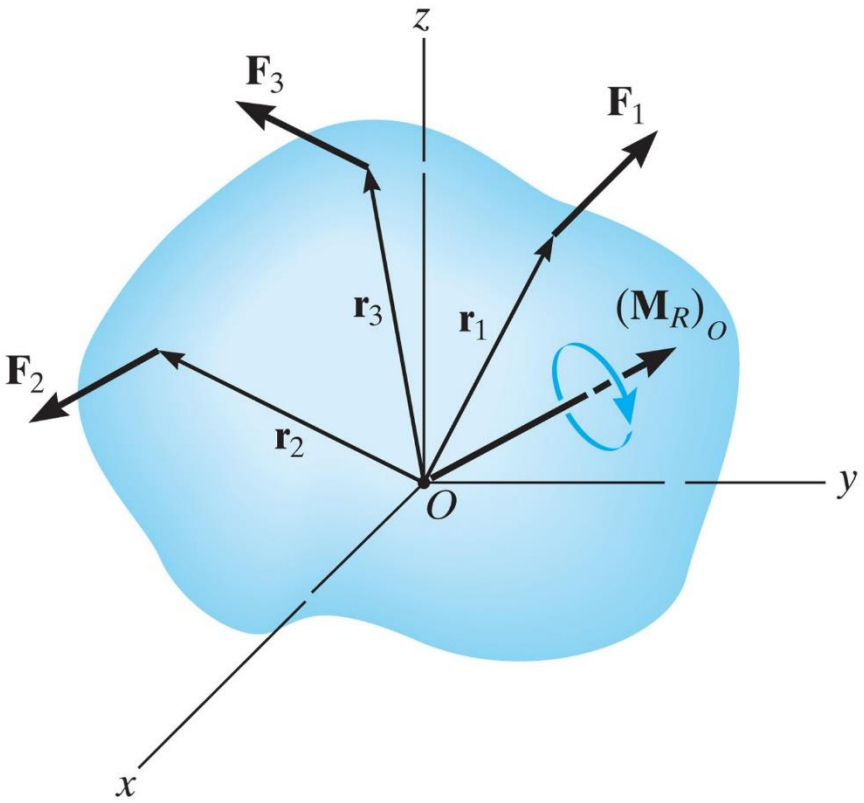
$$\overrightarrow{M} = \left(r_y F_z - r_z F_y\right)\hat{i}$$



Resultant Moment

Scalar analysis:

$$|M_R| = \Sigma |F| d$$



Vector analysis:

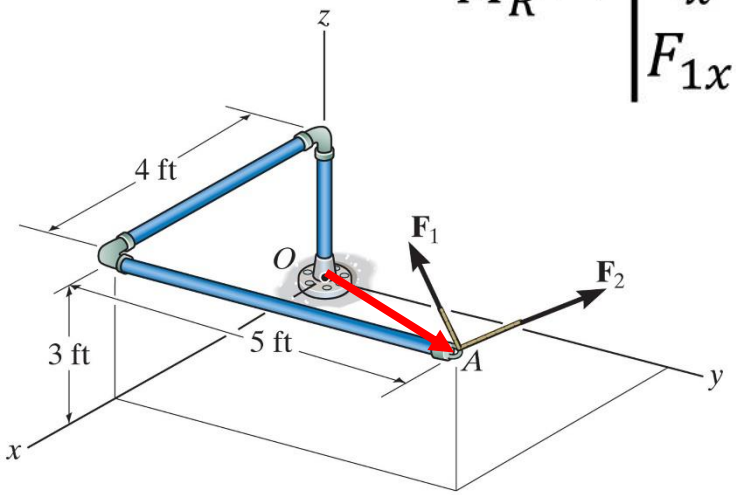
$$\overrightarrow{M_R} = \Sigma (r \times F)$$

$$\overrightarrow{M_R} = (r_1 \times F_1) + (r_2 \times F_2) + (r_3 \times F_3)$$

Resultant Moment

If the forces are concurrent (share a common point),

$$\overline{M_R} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_{1x} & F_{1y} & F_{1z} \end{vmatrix} + \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_{2x} & F_{2y} & F_{2z} \end{vmatrix}$$



Or,

$$\overline{M_R} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_{Rx} & F_{Ry} & F_{Rz} \end{vmatrix}$$

Where,

$$\overline{F_R} = (F_{1x} + F_{2x})\hat{\imath} + (F_{1y} + F_{2y})\hat{\jmath} + (F_{1z} + F_{2z})\hat{k}$$

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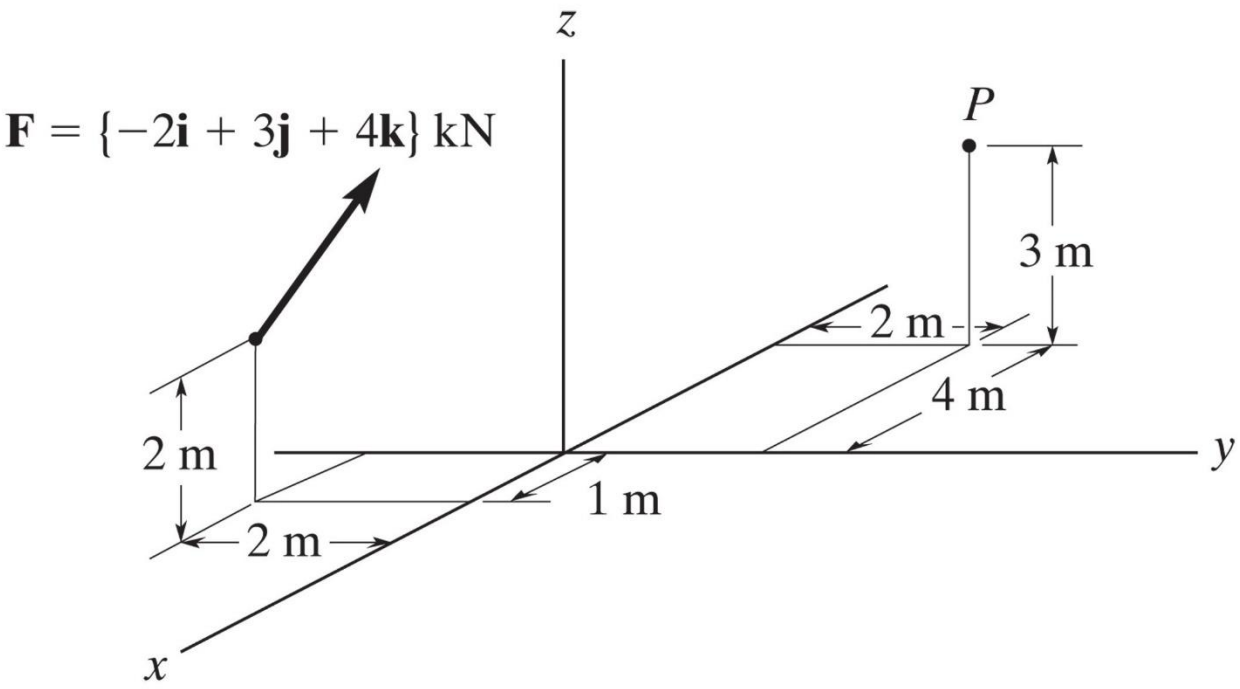
- Example: Determine the moment created by the force about point *P*.



- Example: Determine the resultant moment created by the forces about point *O*.