MATH 2233 Differential Equations

Chapter 3 Mathematical Models and Numerical Methods Involving First-Order Equations

Section 3.2 Math Modeling - Compartmental Analysis

Goal of this section

1. Use differential equations to model a fluid mixing problem in a compartment.
2. Use differential equations to model temperature change based on Newton's law of cooling.

1. Mixing Problems

Example 1. *A large tank initially holds* 1000 *liters of pure water. A brine solution of salt begins to flow at a constant rate of* 6 *L/min. The concentration of brine solution in this inflow is* 0.1 *kg/L. The solution inside the tank is kept well-stirred and is flowing out of the tank at a rate of* 6 *L/min. Determine when the concentration of the salt in the tank will reach* 0.05 *kg/L.*

Example 2. ***(Different in-flow and out-flow rates)***
For the mixing problem described in Example 1, assume now that the brine leaves the tank at a rate of 5 *L/min instead of* 6 *L/min, with all else being the same. Determine the concentration of salt in the tank as a function of time.*

2. Newton's Law of Cooling

The rate of heat loss of a body is proportional to

Notations

- $T(t)$:
- T_m:
- $\dfrac{dT}{dt}$:

Newton's Law of Cooling:

Example 3. *When a cake is removed from an oven, its temperature is $300^o F$. After 3 minutes, the temperature is $200^o F$. How long will it take for the cake to cool off to the room temperature of $70^o F$?*

Example 4. ***(Newton's Law of Warming)***
A thermometer reading 70^oF *is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads* 110^oF *after* $\frac{1}{2}$ *minute and* 145^oF *after 1 minute. How hot is the oven?*

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Section 3.4 Modeling - Newtonian Mechanics

Goal of this section

1. use first-order differential equation to model Newtonian Mechanics.

Mechanics is the study of the motion of objects and the effect of forces acting on force objects.

Newtonian Mechanics deals with the motion of ordinary objects—that is, objects that are large compared to an atom and slow moving compared with the speed of light.

1. Newton's Second Law of Motion

Assume that m is the mass of the object, $x(t)$ is the displacement, $v(t) = x'(t)$ is the velocity, $a(t) = x''(t) = v'(t)$ is the acceleration. Also assume that the net force F depends only on the time t and the velocity v, but not on the position x. Then by the Newton's second law of motion

Systems of Units

There are two systems of units: the **US Customary System** and the **Meter-Kilogram-Second (MKS) System**:

Unit	US System	MKS System
Distance		
Mass		
Time		
Force		
g **(Earth)**		

Example 1. *An object of mass m is given an initial downward velocity v_0 and allowed to fall under the influence of gravity. Assuming the gravitational force is constant and the force due to air resistance is proportional to the velocity of the object, determine the equation of motion for this object.*

Example 2. *An object of mass* 3 *kg is released from rest* $500m$ *above the ground and allowed to fall under the influence of gravity. Assume the gravitational force is constant, with* $g = 9.81m/s^2$*, and the force due to air resistance is proportional to the velocity of the object with proportionality constant* $b = 3N \cdot s/m$*. Determine when the object will strike the ground.*

Example 3. *A parachutist whose mass is $75kg$ drops from a helicopter hovering 4000 meters above the ground and falls toward the earth under the influence of gravity. Assume the gravitational force is constant. Assume also that the force due to air resistance is proportional to the velocity of the parachutist, with the proportionality constant $b_1 = 15N \cdot s/m$ when the chute is closed and with constant $b_2 = 105N \cdot s/m$ when the chute is open. If the chute does not open until 1 minute after the parachutist leaves the helicopter, after how many seconds will she hit the ground?*