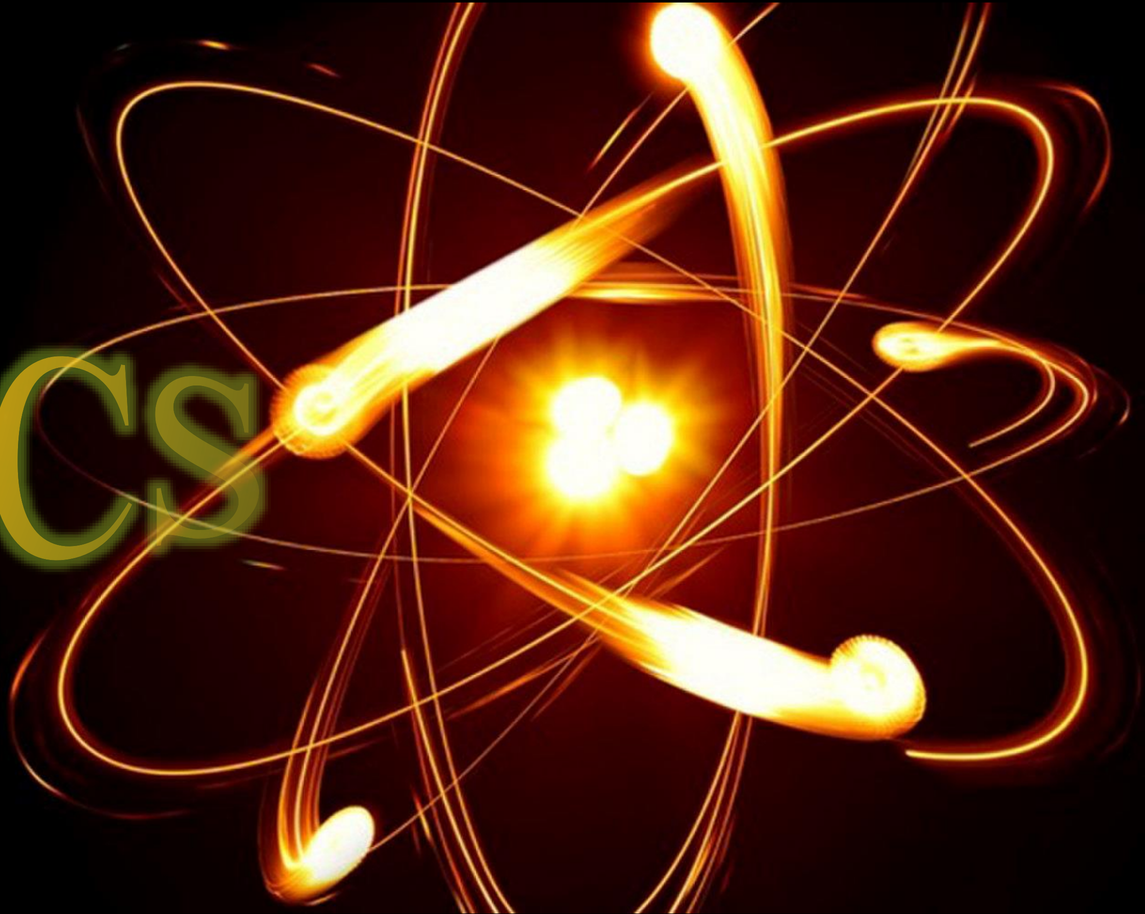
PHYSICS



西南交通大學
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Physics 1: **Mechanics and Waves**

Week 10 – Momentum and Collision

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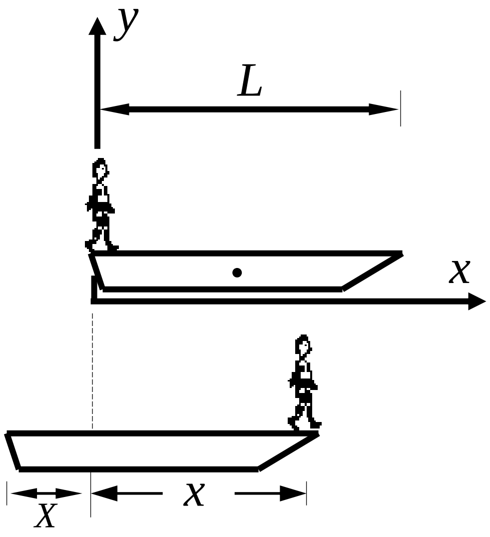
Example

As figure shown, a boat L = 4m, M = 150kg. It is initially rest on the water. When a men, m = 50kg, walks from the head to the tail of the boat, what are the distances of the men and boat relative to the beach, respectively. neglect the friction from water.

Solution 1:

$$\sum F_{ix} = 0 \quad a_{cx} = 0 \rightarrow v_{cx} = 0 \qquad \rightarrow x_c = \text{constant}$$



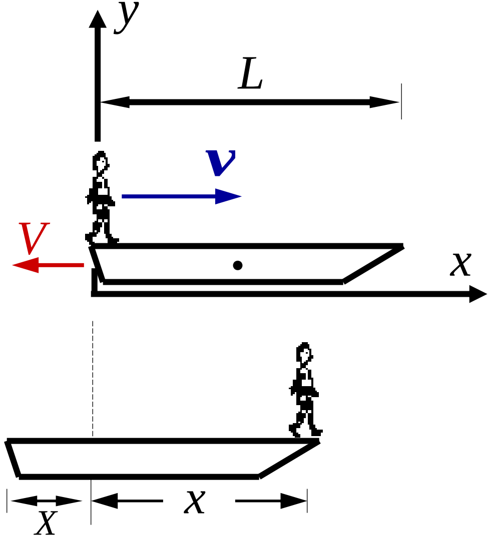
Initial: $$x_c = \frac{m \cdot 0 + M\frac{L}{2}}{m+M}$$ Final: $$x'_c = \frac{m \cdot x + M(x - \frac{L}{2})}{m+M}$$

$$x = \frac{ML}{m+M} = 3\text{m} \quad ; \qquad X = L - x = 1\text{m}$$

Example

Solution 2:

Let $\vec{V}$ and $\vec{v}$ denote the velocity of boat and men relative to the beach at any instant, then



$$mv - MV = 0$$

$$m\int_0^t v\mathrm{d}t - M\int_0^t V\mathrm{d}t = 0$$

$$mx = MX \quad , \quad x + X = L$$

$$X = \frac{m}{M+m}L = 1\mathrm{m} \quad ;$$

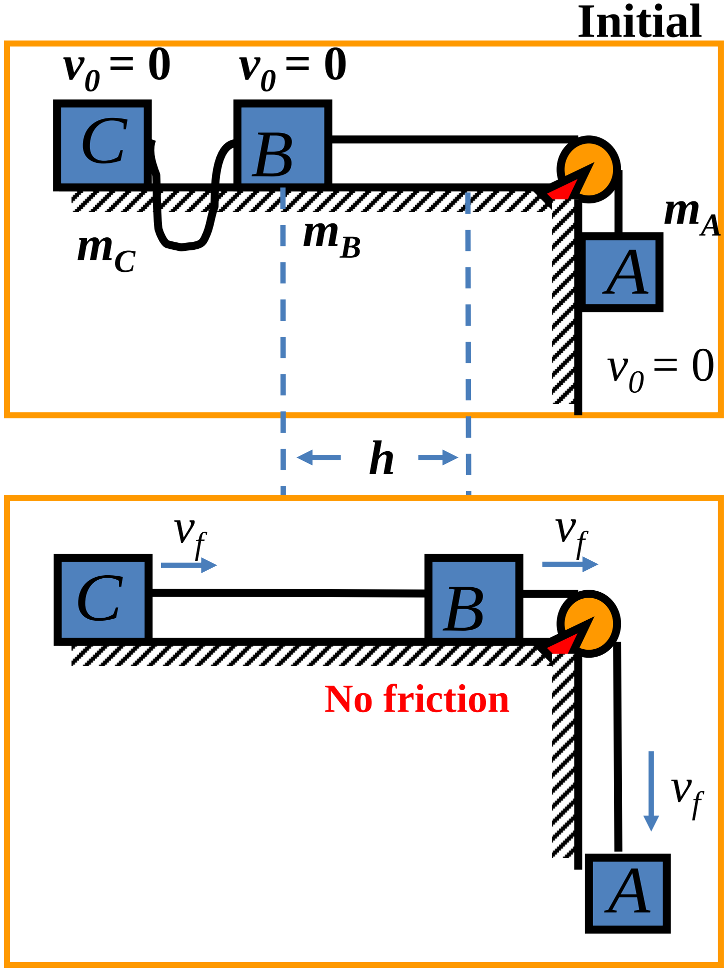
$$x = L - X = 3\mathrm{m}$$

Example – Three-blocks problem

Initially, all three blocks are **at rest**.
The rope between Block B and C is **loose**.
The rope between Block A and B is **tense**.

Block A start to move down due to the weight.

At the moment the rope between Block B and C becomes tense , all blocks are moving at the same speed.
What is this speed v_f?



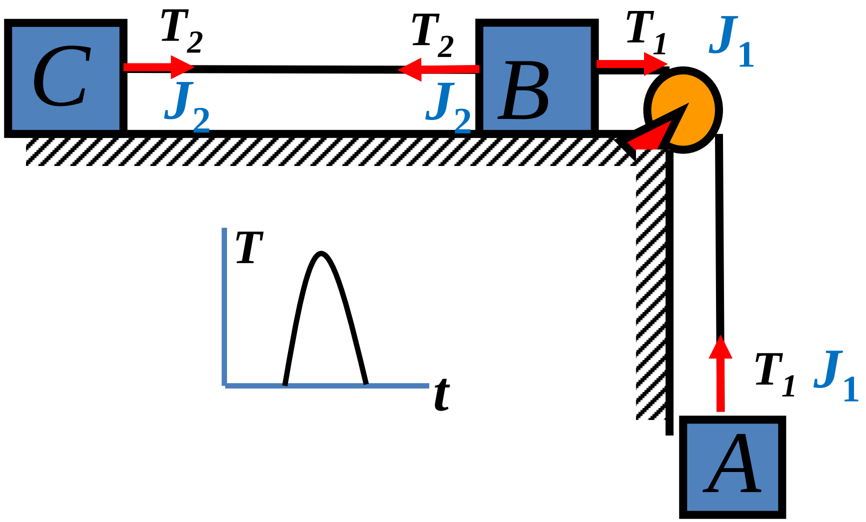
Example – Three-blocks problem

$$\Delta p_A = m_A(v_f - v_i) = -J_1$$

$$\Delta p_B = m_B(v_f - v_i) = J_1 - J_2$$

$$\Delta p_C = m_C(v_f - 0) = J_2$$

$$\Delta p_A + \Delta p_B + \Delta p_C = 0$$



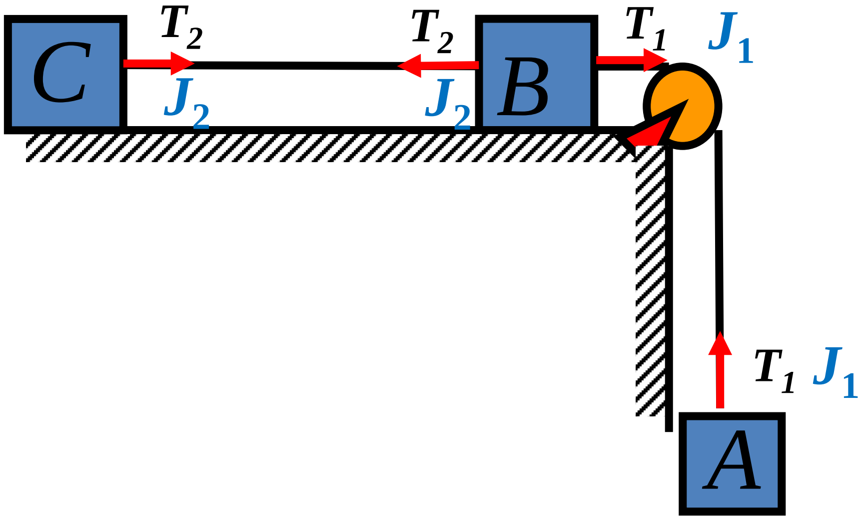
It looks like "conservation of momentum".

But these are only magnitudes, not the momenta themselves!

Example – Three-blocks problem

Final result (dynamic method):

$$v_i = \sqrt{\frac{2m_A gh}{(m_A + m_B)}}$$



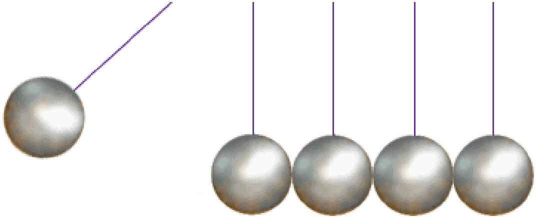
Collision

- Collision
  - 1D Head-on
    - Elastic
    - Inelastic
  - 2D Not head on
    - Components

Collision- Elastic and Inelastic

Inelastic !

Elastic !



What happens in collision

– sudden change of velocity

– very short time

– very large *internal forces*

– very small impulses from *external forces*

– usually *external forces* can be neglected

Conservation of Momentum in collision

If the impulse force is much larger than any external forces (such as gravitational force, friction, ……), as is the case in most of the collision, we can neglect the external forces entirely and treat the system as an isolated system. Then the momentum is conserved in the collision.

The individual momenta of the particles do change, but the total momenta of the system of colliding particles does not.

Difference in Elastic and Inelastic collisions

	Elastic	Inelastic
Kinetic energy	conserved	not conserved
Momentum	conserved	conserved

(both parties movable)

Perfectly inelastic collisions

Example

For one dimensional motion, show that two particles with relative speed $\vec{v}_{A1} - \vec{v}_{B1}$ will leave each other with same relative speed after an elastic collision, that means

$$\vec{v}_{A1} - \vec{v}_{B1} = -(\vec{v}_{A2} - \vec{v}_{B2})$$

Example

Solution:

The momentum and mechanical energy is conserved in an elastic collision

$$\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2$$

$$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = m_A \vec{v}_{A2} + m_B \vec{v}_{B2}$$

Example

Solving the equations, one can get

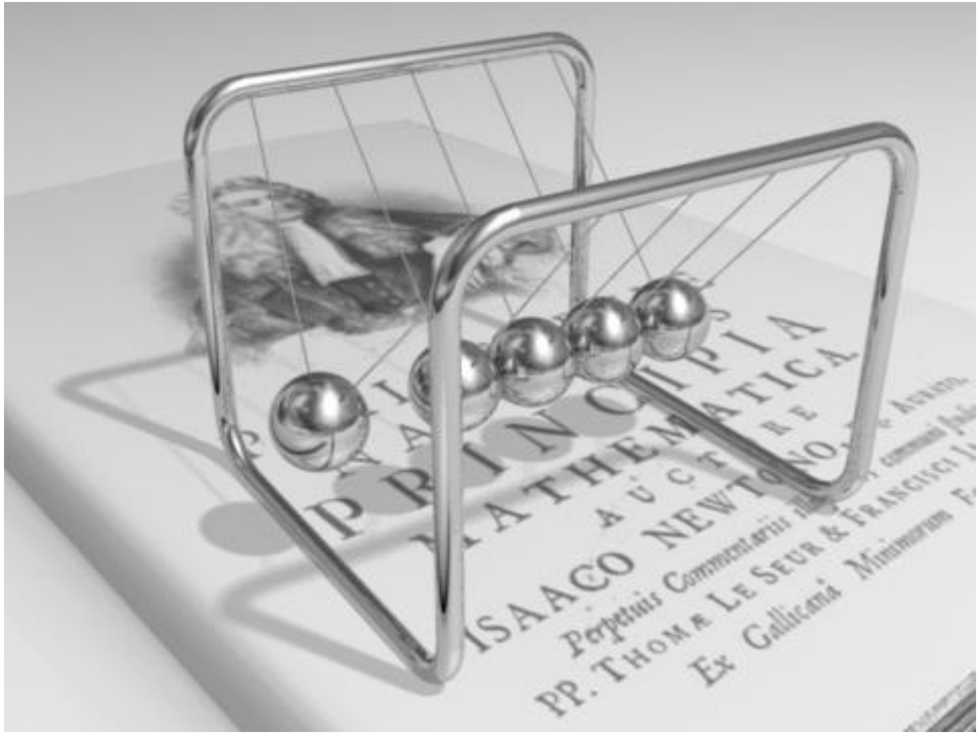
$$\vec{v}_{A2} = \frac{m_A - m_B}{m_A + m_B}\vec{v}_{A1} + \frac{2m_B}{m_A + m_B}\vec{v}_{B1}$$

$$\vec{v}_{B2} = \frac{m_B - m_A}{m_A + m_B}\vec{v}_{B1} + \frac{2m_A}{m_A + m_B}\vec{v}_{A1}$$

The relative speed after collision:

$$\begin{aligned}\vec{v}_{A2} - \vec{v}_{B2} &= \left(\frac{2m_B}{m_A + m_B} - \frac{m_B - m_A}{m_A + m_B}\right)\vec{v}_{B1} \\ &\quad + \left(\frac{m_A - m_B}{m_A + m_B} - \frac{2m_A}{m_A + m_B}\right)\vec{v}_{A1} \\ &= -(\vec{v}_{A1} - \vec{v}_{B1})\end{aligned}$$

Elastic collision with same mass



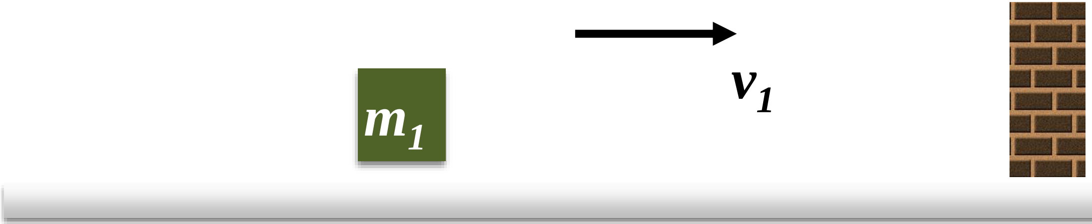
Transferring momentum and energy

Collision Example

$$m_1 v_1 = m_1 v_{1f} + m_2 v_{2f} \qquad \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2} = \frac{m_1 v_1^2}{2}$$

$$v_{1f} = \frac{m_1 v_1 \pm m_2 v_1}{(m_2 + m_1)} = \begin{cases} v_1 \\ \left(\dfrac{m_1 - m_2}{m_2 + m_1}\right) v_1 \end{cases}$$

Elastic Collison with fixed objects (ground/wall)



$$\boldsymbol{m_2 \to \infty}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_2 + m_1}\right) v_1$$

$$v_{2f} = \frac{m_1 v_1 - m_1 v_{1f}}{m_2}$$

$$m_2 \gg m_1$$

$$v_{1f} = -v_1$$

$$v_{2f} = 0$$

Inelastic Collision in 1 Dimension

The **internal kinetic energy changes**

Perfectly/Completely inelastic collision:

→ the two objects stick together

A perfectly inelastic collision **removes more kinetic energy than any other collision**

Energy lost

$m_1 = 0.4\ kg$

$v_1 = 100\ m/s$

$\boldsymbol{m_2 = 80\ kg}$

Goal keeper stopping a football

- no friction in air
- but kinetic energy is lost in the friction between the ball and the glove
- But how much energy is lost?

almost all lost!

Energy lost

Goal keeper stopping a football

- no friction in air
- but kinetic energy is lost in the friction between the ball and the glove

$m_1 = 0.4\, kg$

$v_1 = 100\, m/s$

$\boldsymbol{m_2 = 80\, kg}$

$$m_1 v_1 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = v_{2f}$$

$$\Longrightarrow v_{1f} = \frac{m_1 v_1}{(m_1 + m_2)} = 0.50\, m/s$$

9-3 Collisions

4. Inelastic collisions

If the total energy of the particles involved in a collision is not conserved, the collision is called an inelastic collision. If the particles stick together after collision, the collision is called a completely inelastic collision.

From $$W_{total} = \Delta KE_{total}$$

For a collision which there are no external forces, we have

$$W_{total} = 0 \quad \Delta KE_{total} \neq 0$$

Why?

Where does the lost energy go?

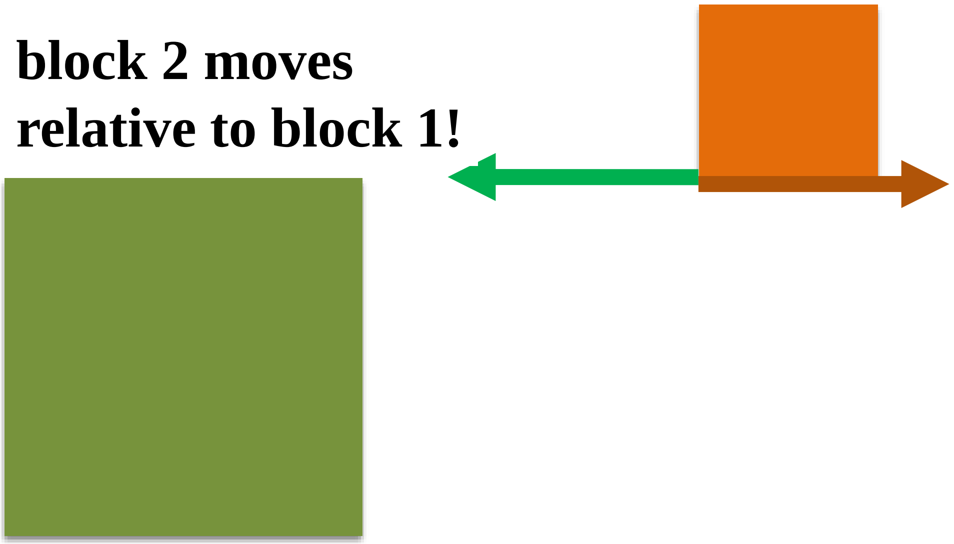
Kinetic energy → **??? energy**

Where does the lost energy go?

Friction accelerates block 2

both blocks move at the same speed

block 2 moves relative to block 1!

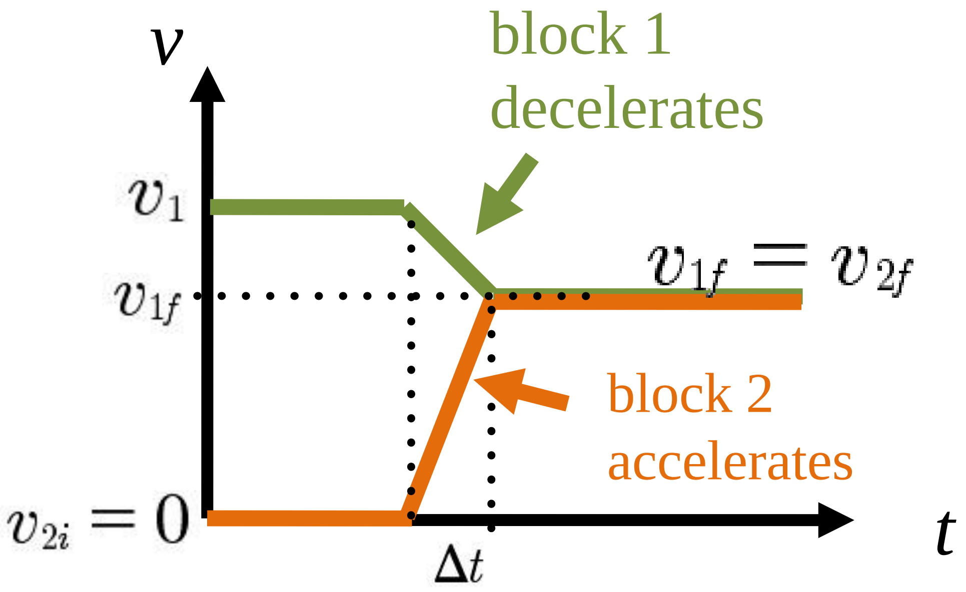


reaction force decelerates block 1

Where does the lost energy go?

Positive work + Negative work = Total work = 0 ?

Where does the lost energy go?



action = reaction

$$F_f = m_2 a_2 = -m_1 a_1$$

Work on block 1 :

$$W_1 = -m_1 a_1 d_1$$

Work on block 2 :

$$W_2 = m_2 a_2 d_2$$

$W_1 \neq W_2$ **because** $d_1 \neq d_2$

Solving collision problems

	Elastic	Inelastic
Kinetic energy	conserved	Same final velocity
Momentum	conserved	conserved

Elastic collisions in 2 dimensions

Elastic collisions in 2 dimensions

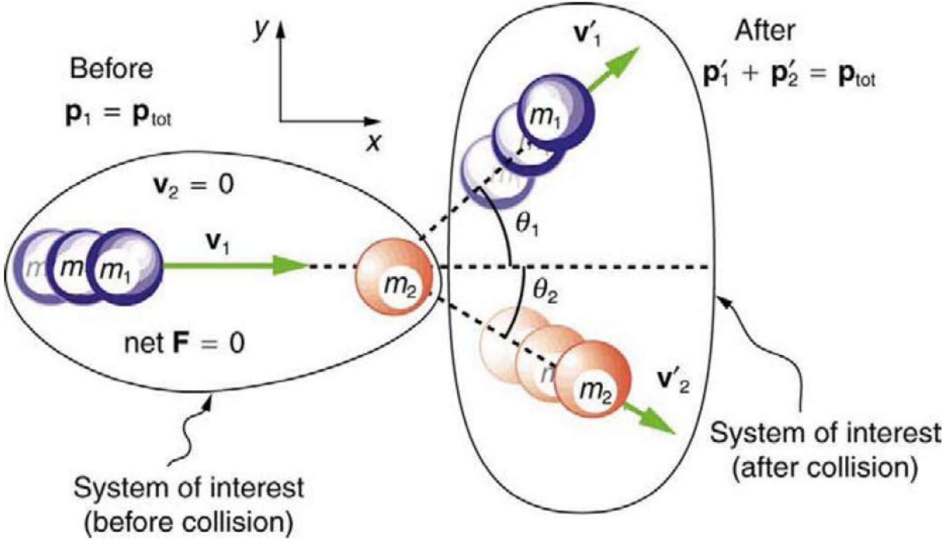
Momentum is conserved for all the vector components

Chose coordinate system with an axis in the direction of the velocity of the moving ball

We do not consider **rotation**!

Assume **“point”** masses

Elastic collisions in 2 dimensions



➞ conservation of momentum in the *x*-direction:

$$m_1 v_{1x}^{in} = m_1 v_{1x}^{f} + m_2 v_{2x}^{f}$$

$$\Longrightarrow m_1 v_1^{in} = m_1 v_1^f \cos\theta_1 + m_2 v_2^f \cos\theta_2$$

➞ conservation of momentum in the *y*-direction:

$$0 = m_1 v_{1y}^{f} + m_2 v_{2y}^{f}$$

$$\Longrightarrow m_1 v_1^f \sin\theta_1 + m_2 v_2^f \sin\theta_2 = 0$$

2 equations, 4 unknowns !

Elastic collisions in 2 dimensions

2 equations, 4 unknowns !

More information!

Conservation of kinetic energy

$$\frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2} = \frac{m_1 v_1^2}{2}$$

→ **the angle? the final velocities?**

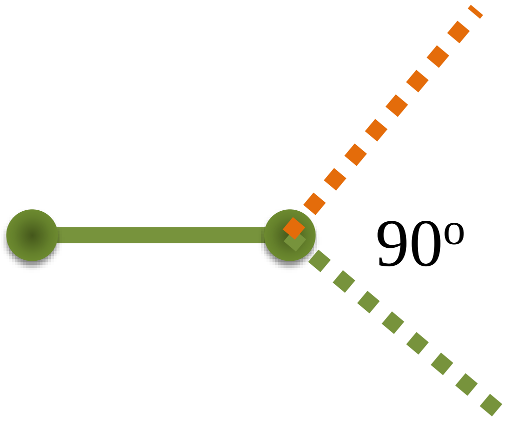
One of $\boldsymbol{v_{1f}}$, $\boldsymbol{v_{2f}}$, $\boldsymbol{\theta_1}$ **and** $\boldsymbol{\theta_2}$, or any relation/restriction between them.

If knowing more than one of ***them***, we may avoid dealing with kinetic energy

Collision

- **Collision**
  - **1D Head-on**
    - **Elastic**
      - $E_k = E_k'$
    - **Inelastic**
      - $E_k \neq E_k'$
      - **Total energy conserved**
  - **2D Not head on**
    - **Components**
      - **coordinate system**

Collision



Two elastic balls collide on a frictionless horizontal table top.

One ball was originally at rest.

The other ball has a mass of 3 kg, and an initial speed of 15 m/s.

After collision the two balls are moving in directions that are perpendicular to each other ($\theta_1 - \theta_2 = 90^o$).

Find the mass of the ball initially at rest.