Fundamental Dynamics–SP24–FINAL REVIEW

1. A particle C, having a mass of 2kg, is lifted from A to B by a rotating rod. If the rod has a constant angular velocity as shown，

   1） determine the velocity of the can at the instant $\theta = 30^\circ$

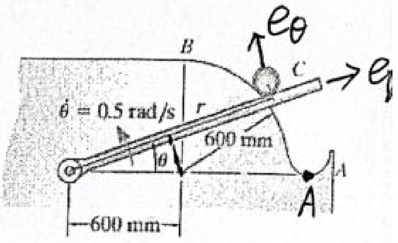
   2） determine the acceleration of the can at the instant $\theta = 30^\circ$

   3） determine the force which the rod exerts on the can at the instant $\theta = 30^\circ$ .Neglect the effects of friction in the calculation and the size of the particle so that trail from A to B is circular, having a radius of 600 mm. 15 POINTS

From geometry $\vec{r} = 2\times 600\cos\theta = 1.2\cos\theta\, e_r$

$\dot{r} = -1.2\sin\theta\cdot\dot{\theta}$

$\ddot{r} = -1.2\cos\theta\cdot\dot{\theta}^2$



$\vec{v}_C = \dot{r}\vec{e}_r + \dot{\theta} r \vec{e}_\theta$

$= -1.2\sin\theta\cdot\dot{\theta}\,\vec{e}_r + \dot{\theta}\cdot r\,\vec{e}_\theta$

$\vec{a}_C = (\ddot{r} - \dot{\theta}^2 r)\vec{e}_r + (r\ddot{\theta} + 2\dot{\theta}\dot{r})\vec{e}_\theta$

$= (-1.2\cos\theta\dot{\theta}^2 - r\dot{\theta}^2)\vec{e}_r + (-2.4\sin\theta)\dot{\theta}^2\vec{e}_\theta$

(1) When $\theta = 30^\circ$ $\quad \vec{v}_C = -1.2\cdot\frac{1}{2}\cdot 0.5\,\vec{e}_r + 0.5\,\vec{e}_\theta \times 1.2\cos 30^\circ$

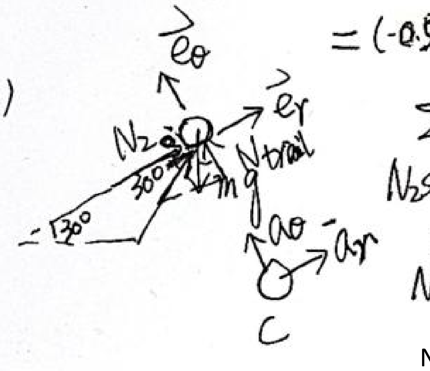
$= (-0.3\,\vec{e}_r + 0.52\,\vec{e}_\theta)$ m/s $\quad v_C = 0.6$ m/s

(2) $\vec{a}_C = (-1.2\cos 30^\circ\cdot 0.5^2 - 1.2\cos 30^\circ\cdot 0.5^2)\vec{e}_r$

$+ 2\cdot 0.5\cdot(-1.2\sin 30^\circ)\cdot 0.5\,e_\theta$

$= (-0.52\,\vec{e}_r - 0.3\,\vec{e}_\theta)$ m/s $\quad a_C = 0.6$ m/s^2

(3)



$\sum F_\theta = m a_\theta$

$N_2\sin 30^\circ + N_{trail} - mg\cos 30^\circ = m a_\theta$

$\sum F_r = m a_r$

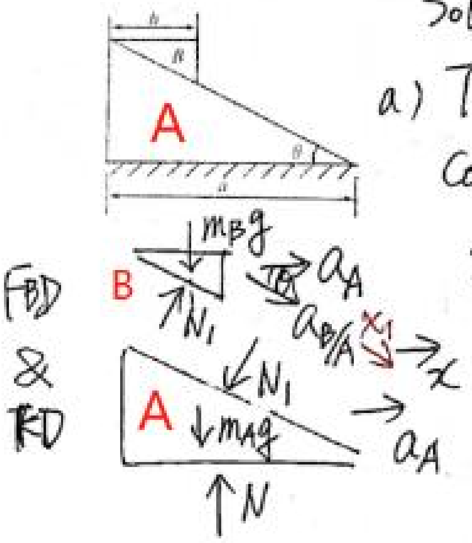
$N_2\cos 30^\circ - mg\sin 30^\circ = m a_r$

$N_2 = 10.13$ N $\qquad N_{trail} = 11.33$ N

2. The wedge *B* with a mass of slides along the wedge *A* with mass of m_A as shown. If the system starts from rest, and all friction forces are neglected, determine:

a) the displacement of the wedge *A* When *B* reaches the horiztonal plane,

b) the acceleration of wedge *A* at the instant.(20pnts)



Solution:

a) The momentum of the system is Conserved in the horizontal dir.

So x_c won't move. $x_{B/A}$

$$m_A x_A + m_B(x_A + \underline{a-b}) = 0 \quad (1)$$

$$\boxed{x_A = -\frac{m_B(a-b)}{m_A+m_B} \ (\leftarrow)}$$

b) Take derivatives abt (1) twice

$$m_A a_A + m_B(a_A + a_{B/A}\cos\theta) = 0 \quad (2)$$

Eqn of B along x_1:

$$\Sigma F_{x_1} = m_B g\sin\theta = m_B(a_A\cos\theta + a_{B/A}) \quad (3)$$

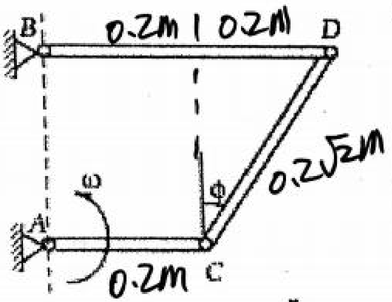
$$(2)+(3) \rightarrow \ a_{B/A} = g\sin\theta - a_A\cos\theta$$

$$\boxed{a_A = -\frac{m_B g\sin\theta\cos\theta}{m_A + m_B(\cos^2\theta+1)}} \ (\leftarrow) \text{ constant}$$

You can also try the conservation of Energy.

3.Rod AC rotates about point A with a constant angular speed $\omega_1 = 2\mathrm{rad/s}$, AC and BD are horizontal at the instant as shown in the figure. If the length of AC and BD are 0.2m and 0.4m, respectively; $\phi = 45°$,

a) determine the angular speed of rod BD and CD at the instant shown,
b) the angular acceleration of rod BD and CD at the instant shown,
c) the speed and acceleration of point D. 20 POINTS

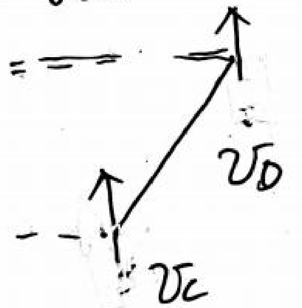


$v_C = \omega \cdot AC = 2 \times 0.2 = 0.4 m/s$

$v_D = v_C = 0.4 m/s$ (momentarily translation)

$\boxed{\omega_{BD} = \frac{v_D}{BD} = \frac{0.4}{0.4} = 1\ rad/s}$

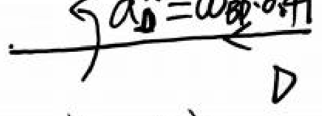
$\boxed{\omega_{CD} = 0}$ (momentarily translation)



Accelerations :

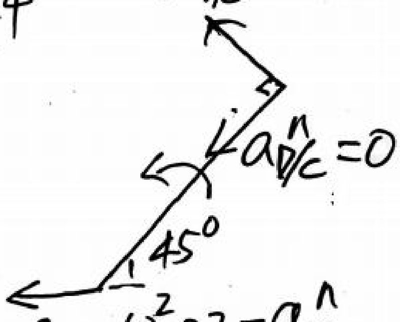
$a_D^t = \alpha_{BD} \cdot 0.4$

$a_D^n = \omega_{BD}^2 \cdot 0.4$



$a_{D/C}^t = \alpha_{CD} \cdot 0.2\sqrt{2}$

$a_{D/C}^n = 0$



$a_C = \omega^2 \cdot 0.2 = a_C^n$

$\vec{a}_D = \vec{a}_D^n + \vec{a}_D^t = \vec{a}_C + \vec{a}_{D/C}^t + \vec{a}_{D/C}^n$

$$\begin{cases} \omega_{BD}^2 \cdot 0.4 = a_C + a_{CD}^t \frac{\sqrt{2}}{2} = \omega^2 \cdot 0.2 + 0.2\alpha_{CD} \\ \alpha_{BD} \cdot 0.4 = \alpha_{CD} \cdot 0.2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 0.2\alpha_{CD} \end{cases}$$

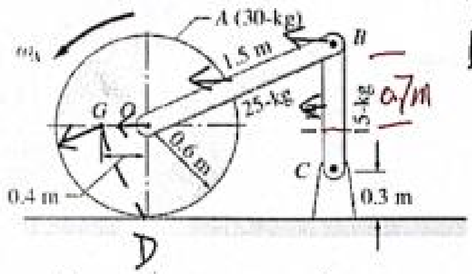
$\alpha_{CD} = -2\ rad/s^2\ (\circlearrowleft)$ $\alpha_{BD} = -1\ rad/s^2\ (\circlearrowleft)$

$\vec{a}_D = -\omega_{BD}^2 \cdot 0.4\vec{i} + \alpha_{BD} \cdot 0.4\vec{j} = (-0.4\vec{i} - 0.4\vec{j})\ m/s^2$

4. The radius of gyration of disk A about its mass center G is $k_z = 0.5$ m. In the position shown, the disk rolls without slipping with the angular velocity $\omega_A = 4$ rad/s. a) Determine the kinetic energy of the system in this position.

b) Determine the linear momentum of the system in this position.

c) Determine the angular momentum Hz of the system about the axis passing through C in this position.(15pnts)



$\vec{r}_{G/D} = \vec{r}_{DG}$ $\quad \vec{r}_{O/D} = \vec{r}_{DO}$

Disk A: $I_{AG} = m k_z^2 = 30 \cdot 0.5^2 = 7.5\,kg\cdot m^2$

D: ICR, $v_D = 0$

$\vec{v}_O = \vec{\omega}_A \times \vec{r}_{O/D} = 4\vec{k}\times(0.6\vec{j})$

$= -2.4\vec{i}$ m/s (←)

$\vec{v}_G = \vec{\omega}_A \times \vec{r}_{G/D} = 4\vec{k}\times(-0.4\vec{i}+0.6\vec{j})$

$= (-2.4\vec{i} - 1.6\vec{j})$ m/s

$v_{Gx} = 2.4$ m/s (←) $\quad v_{Gy} = 1.6$ m/s (↓)

OB: momentorily traslate

$\vec{v}_B = \vec{v}_O = \vec{v}_{OBG} = 2.4$ m/s (←) $= -2.4\vec{i}$ m/s

CB: rotates abt pnt C $\quad \vec{v}_B = \omega_{CB}\vec{k}\times\vec{r}_{CB} = -\omega_{CB}\, l\, \vec{i}$

$I_C = \frac{1}{3} m_{CB} l^2 = \frac{15}{3}\cdot 1^2 = 5\,kgm^2$ $\quad \omega_{CB} = 2.4$ rad/s

$\vec{v}_{BCG} = \frac{1}{2}\vec{v}_B = 1.2$ m/s (←). $(-1.2\vec{i})$

a) $T = \frac{1}{2} I_{AG}\omega_A^2 + \frac{1}{2} m_A v_G^2 + \frac{1}{2} m_{OB}\cdot v_{OBG}^2 + \frac{1}{2}\cdot I_C\omega_{CB}^2$

$= \frac{1}{2}\times 7.5\times 4^2 + \frac{1}{2}\times 30\times(2.4^2+1.6^2) + \frac{1}{2}\times 25\times 2.4^2 + \frac{1}{2}\times 5\times 2.4^2$

$\boxed{T = 271.2\,J}$

b) $\vec{L} = m_A \vec{v}_G + m_{OB}\vec{v}_{OBG} + m_{BC}\cdot\vec{v}_{CBG}$

$= 30(-2.4\vec{i} - 1.6\vec{j}) + 25(-2.4\vec{i}) + 15(-1.2\vec{i})$

$\boxed{L = (-150\vec{i} - 48\vec{j})\,kgm/s}$ $\qquad \boxed{H_z(C) = 185.5\,kgm^2/s}$

c) $H_z = I_{AG}\omega_A + \vec{r}_{CG}\times\vec{v}_G m_A + m_{OB} v_{OBG}(1-0.35) + I_C\omega_{BC}$ (↺)

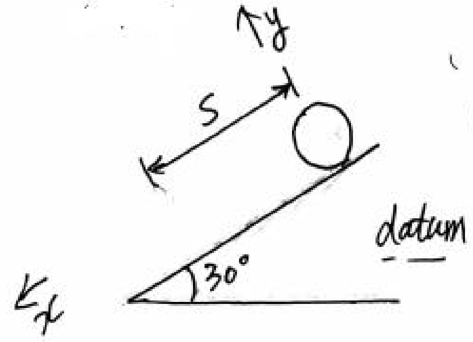
$= 7.5\times 4 + 30\times 2.4(0.6-0.3) + 30\times 1.6\times(0.4+\sqrt{1.5^2-0.7^2}) + 25\times 2.4\times(0.3+0.35) + 5\times 2.4$

5. The homogeneous disk A of mass m and radius r is released from rest. Assume that the disk rolls without slipping, determine

a) The velocity and acceleration of the center of the wheel when it travels down a distance s;

b) The friction between the wheel and the inclined surface at the instant.

(15pnts)



a) The conservation of Energy

$T_1 = 0 \quad V_1 = mgs\sin 30^\circ$

$T_2 = \frac{1}{2} I_G \omega^2 + \frac{1}{2} m v_G^2$

$= \frac{1}{2}\left[\frac{1}{2} m r^2 \cdot \omega^2 + m(\omega r)^2\right]$

$= \frac{3}{4} m \omega^2 r^2$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$\frac{3}{4} m\omega^2 r^2 = mg \cdot s \cdot \sin 30^\circ$

$\boxed{v_G = \omega \cdot r = \sqrt{\frac{2gs}{3}}}$

$v_G = \omega \cdot r$

$a_G = \alpha \cdot r$

FBD & KD

b) Equs of Motion of A

$\Sigma F_y = 0 = N - mg\cos 30^\circ \quad N = mg\cos 30^\circ$ （可不写）

$\Sigma F_x = m a_G = F_f + mg\sin 30^\circ \quad F_f = m \cdot \alpha \cdot r - mg\sin 30^\circ$

$\Sigma M_G(F) = I_G \alpha = -F_f \cdot r = \frac{1}{2} m r^2 \alpha$

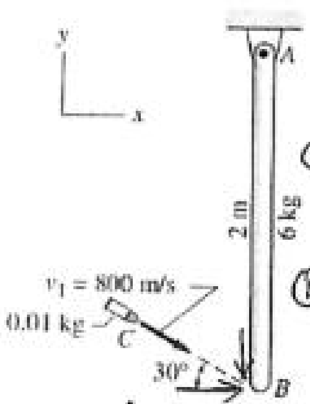
$a_G = \alpha \cdot r = \frac{g}{3} \quad F_f = -\frac{1}{6} mg \ (\nearrow)$

（与图示相反）

6. A 0.01 kg bullet *C* is fired at end *B* of the 6-kg homogeneous slender bar AB. The bar is initially at rest, and the initial velocity of the bullet is v_1 = 800 m/s, directed as shown. Assuming that the bullet becomes embedded in the bar, calculate

(a) the angular velocity ω_2 of the bar immediately after the impact;

(b) the maximum height the center of AB will reach after the impact. (15pnts)



$v_{1x} = 800\cos 30^\circ = 400\sqrt{3}$ m/s (→)

$v_{1y} = 400$ m/s (↓) $I_A = \frac{1}{3} m_b \cdot 2^2$

(a) The angular Momentum of the system

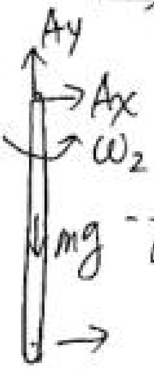
① abt pnt A is conserved

$H_{A1} = H_{A2}$ ↻

$m_C v_{1x} \cdot 2 = I_A \omega_2 + m_C(\omega_2 \cdot 2) \cdot 2$

$0.01 \times 400\sqrt{3} \times 2 = \frac{\omega_2}{3} \times 6 \times 2^2 + 0.01 \times \omega_2 \cdot 2 \cdot 2$

②



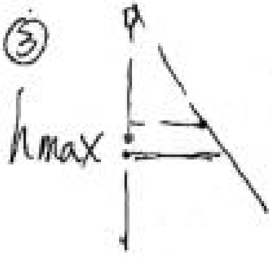
Datum

$\boxed{\omega_2 = 1.723 \text{ rad/s}}$

(b) Apply the conservation of energy

$T_2 = \frac{1}{2} I_A \omega_2^2 + \frac{1}{2} m_C v_{C2}^2$ $V_2 = 0$

③



$T_3 = 0$ $V_3 = m_b g h_{max} + m_c g h_{max}$

$\frac{1}{2} \times \left(\frac{1}{3} \times 6 \times 2^2\right) \times \omega_2^2 + \frac{1}{2} \times 0.01 \times (2\omega_2)^2$

$= (6 + 0.01) g h_{max}$

$\boxed{h_{max} = 0.20 \text{ m}}$