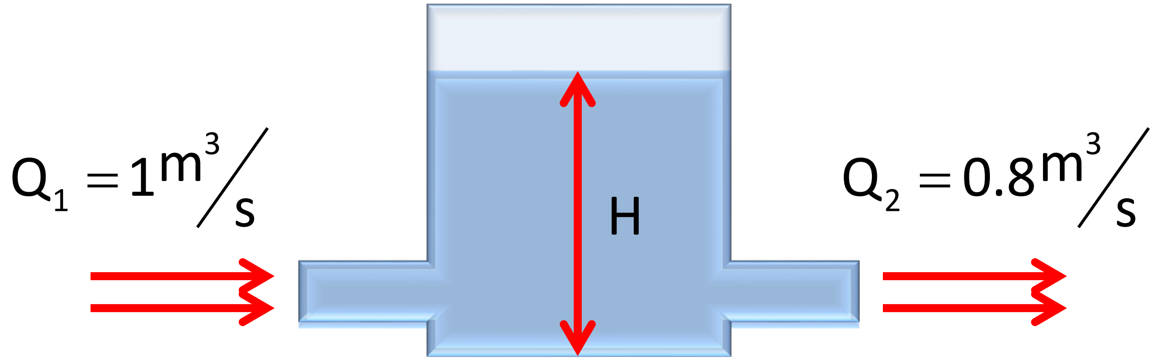
Conservation of Mass…



<u>Question:</u>

If the area of the bottom of the tank is 10m^2, by how much does H increase in 1 minute?

Using the principle of the conservation of mass we state that

$$Q_1 \Delta t = Q_2 \Delta t + \Delta V$$

The starting volume of fluid in the tank = 10H m^3

Hence, we can rewrite

$$Q_1 \Delta t = Q_2 \Delta t + 10 \Delta H$$

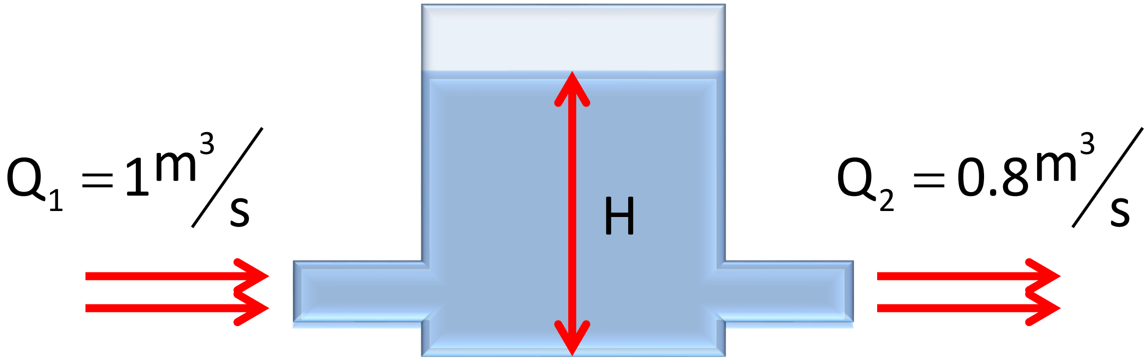
or, $$10 \Delta H = \left(Q_1 - Q_2\right) \Delta t$$

$$\frac{\Delta H}{\Delta t} = \frac{Q_1 - Q_2}{10} = \frac{0.2}{10} = 0.02 \, {}^{m}\!/_{s}$$

Hence, in 1 minute

$$\Delta H = 60 . \frac{\Delta H}{\Delta t} = \left(60\right) . \left(0.02\right) = 1.2 m$$

Conservation of Mass…



<u>Alternatively:</u>

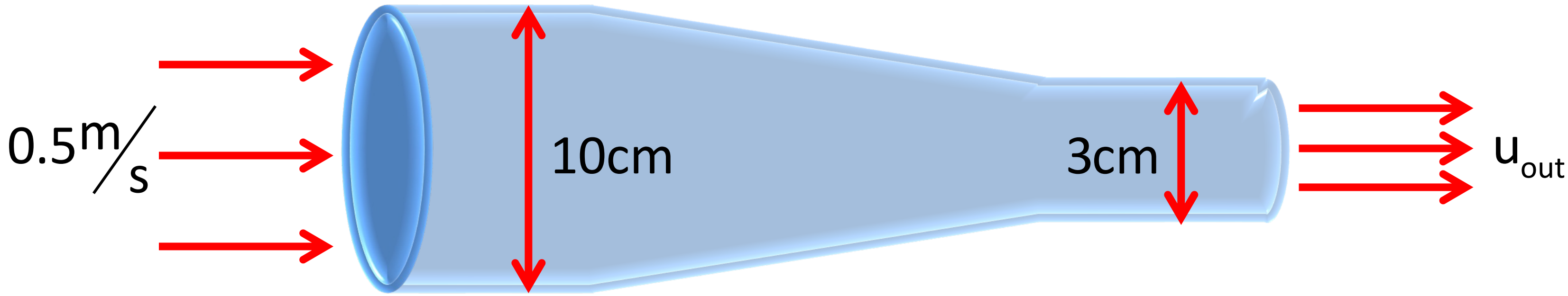
In 1 minute 60m^3 has entered.

In 1 minute 48m^3 leaves.

Hence, 12m^3 remains in the tank.

The area of the base is 10m^2;
therefore, H must increase by 1.2m.

Conservation of Mass…



<u>Question:</u>

Water with a density of 1000 kg m^{-3} flows down the pipe.

Find the flow rate in the pipe. Hence, find the exit velocity of the water.

$$Q_{in} = A.u = \frac{0.1^2 \pi}{4}(0.5) = 0.004\, {}^{m^2}\!/_{s}$$

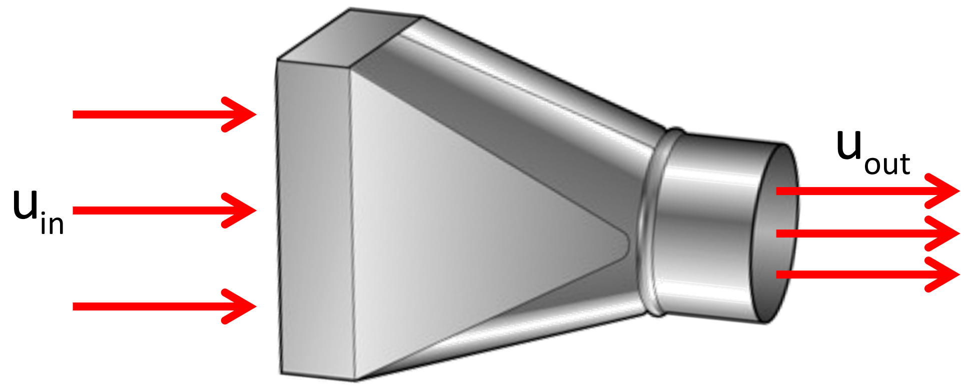
We know that $Q_{in} = Q_{out}$

Hence,

$$u_{out} = \frac{Q_{out}}{A_{out}} = \frac{0.004}{\left(\frac{0.03^2 \pi}{4}\right)} = 5.56\, {}^{m}\!/_{s}$$

Question

Air of density 1.2 kg m^{-3} enters a compressor through a 40mm by 40mm square duct with an average speed of 4m s^{-1}. It is discharged at a speed of 3m s^{-1} in a 25mm diameter pipe. Find the density at the outlet and the mass flow rate.



$$\dot{m}_{in} = \dot{m}_{out}$$

$$(\rho.A.u)_{in} = (\rho.A.u)_{out}$$

$$(1.2).(0.04^2).(4) = \rho.\left(\frac{\pi.0.025^2}{4}\right).(3)$$

$$\therefore \rho = 5.22 \, {}^{kg}/_{m^3}$$

$$\dot{m} = 7.68\left(10^{-3}\right) {}^{kg}/_{s}$$

Conservation of Mass…

<u>Question:</u>

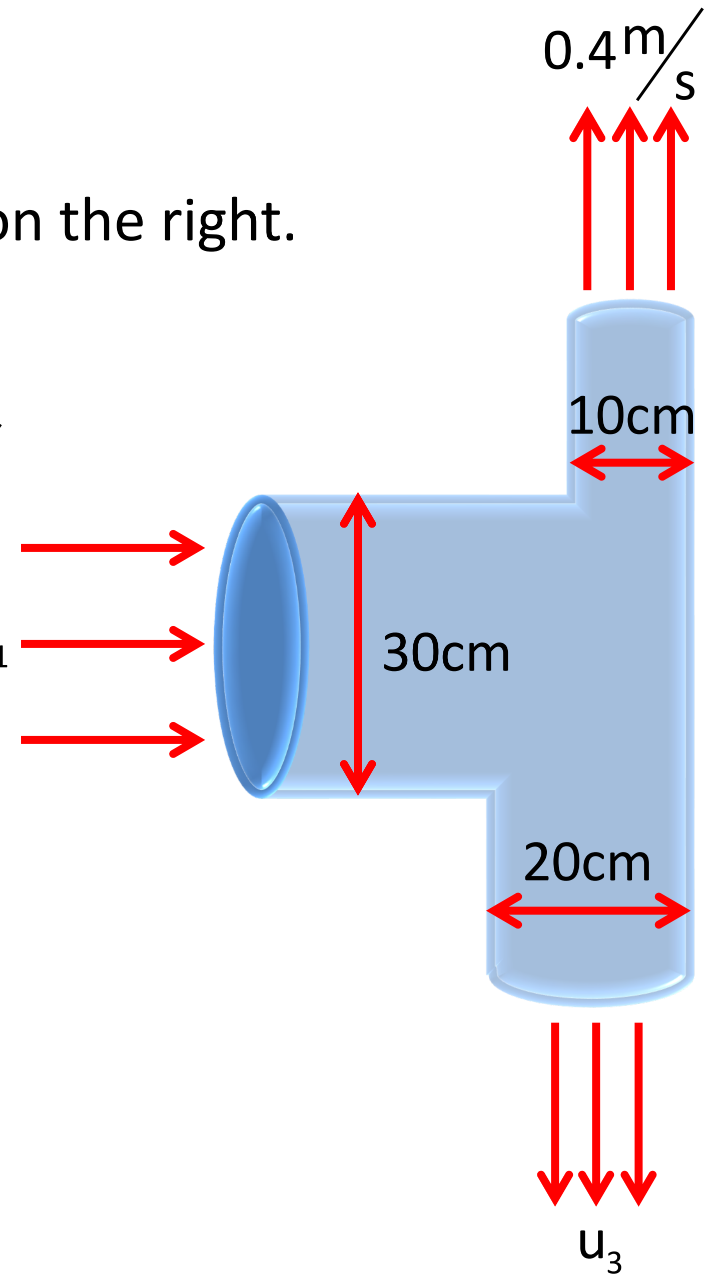
Consider the branching pipe arrangement on the right.
If $Q_1 = 0.01 m^3s^{-1}$ find u_1, u_3, Q_2 & Q_3

$$Q_1 = A_1.u_1 \therefore 0.01 = \frac{0.3^2\pi}{4}(u_1) \qquad u_1 = 0.141 \, {}^{m}\!/\!_{s}$$

$$Q_2 = A_2.u_2 = \frac{0.1^2\pi}{4}(0.4) = 0.00314 \, {}^{m^3}\!/\!_{s}$$

$$Q_3 = Q_1 - Q_2 = 0.01 - 0.00314 = 0.00686 \, {}^{m^3}\!/\!_{s}$$

$$u_3 = \frac{Q_3}{A_3} = \frac{0.00686}{\left(\frac{0.2^2\pi}{4}\right)} = 0.218 \, {}^{m}\!/\!_{s}$$



Review questions…

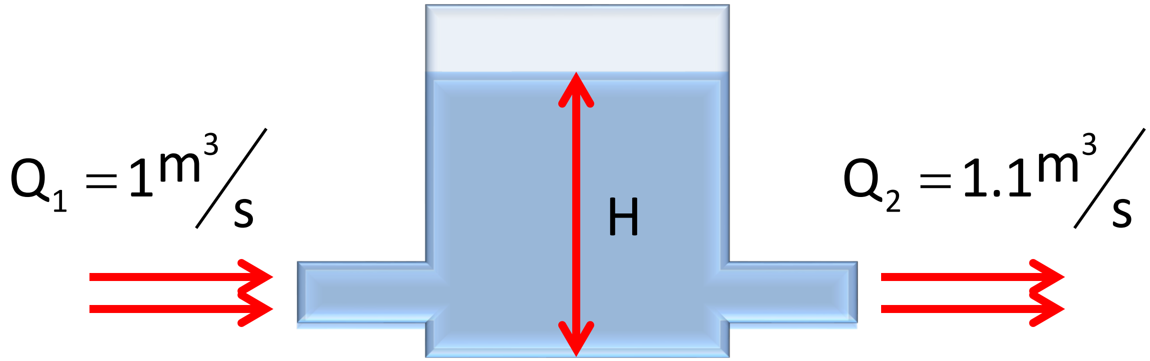
Which of these is not a property of an ideal fluid flow?

(a) Inviscid
(b) Steady
(c) No phase changes
(d) Incompressible

A fluid flows through a pipe with a narrow entrance and a wide exit. Which of the following statements is definitely false.

(a) The flow is steady
(b) The flow is unsteady
(c) The flow is uniform
(d) The flow is not uniform

Question



If the area of the bottom of the tank is 10m^2, by how much does H decrease in 1 minute?

Using the principle of the conservation of mass we state that

$$Q_1\Delta t = Q_2\Delta t + \Delta V$$

The starting volume of fluid in the tank = 10H m^3

Hence we can rewrite

$$Q_1\Delta t = Q_2\Delta t + 10\Delta H$$

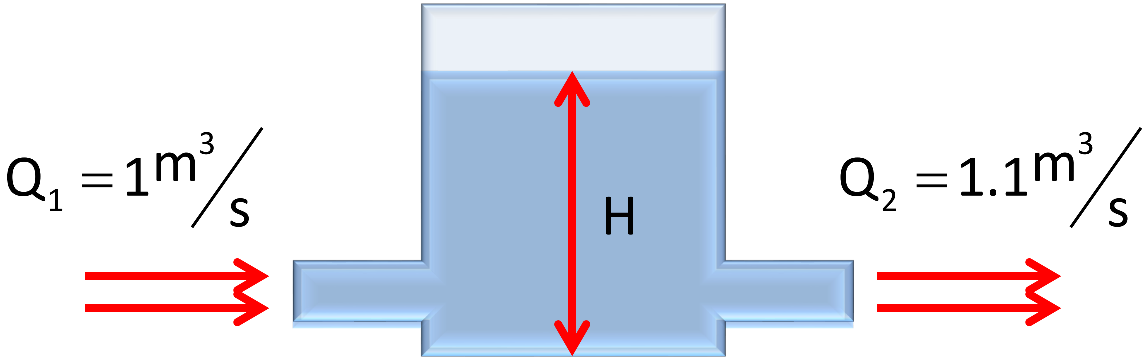
Or, $$10\Delta H = (Q_1 - Q_2)\Delta t$$

$$\frac{\Delta H}{\Delta t} = \frac{Q_1 - Q_2}{10} = -\frac{0.1}{10} = -0.01\,m/s$$

Hence, in 1 minute

$$\Delta H = 60.\frac{\Delta H}{\Delta t} = (60).(-0.01) = -0.6m$$

Question…



<u>Alternatively:</u>

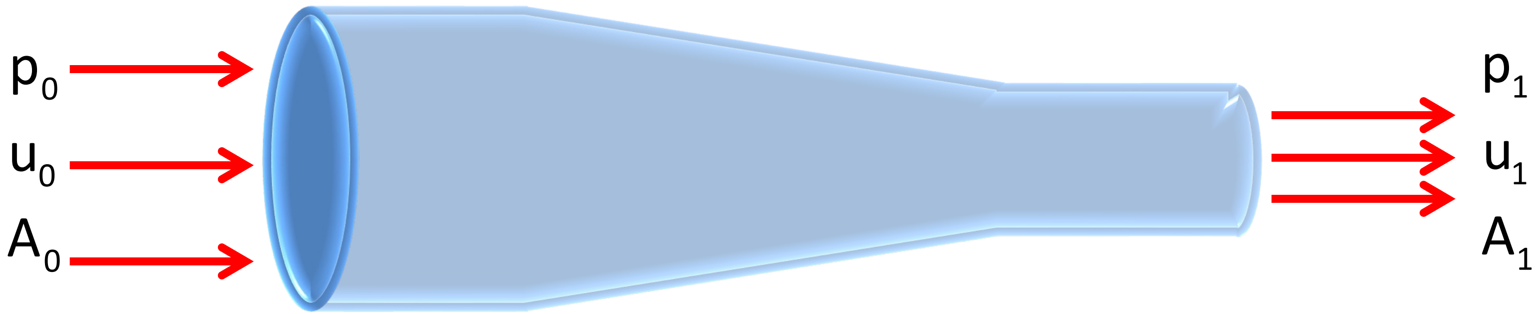
In 1 minute $60m^3$ has entered.

In 1 minute $66m^3$ leaves.

Hence, $6m^3$ has left the tank.

The area of the base is $10m^2$;
therefore, H must fall by 0.6m.

Bernoulli's Equation…

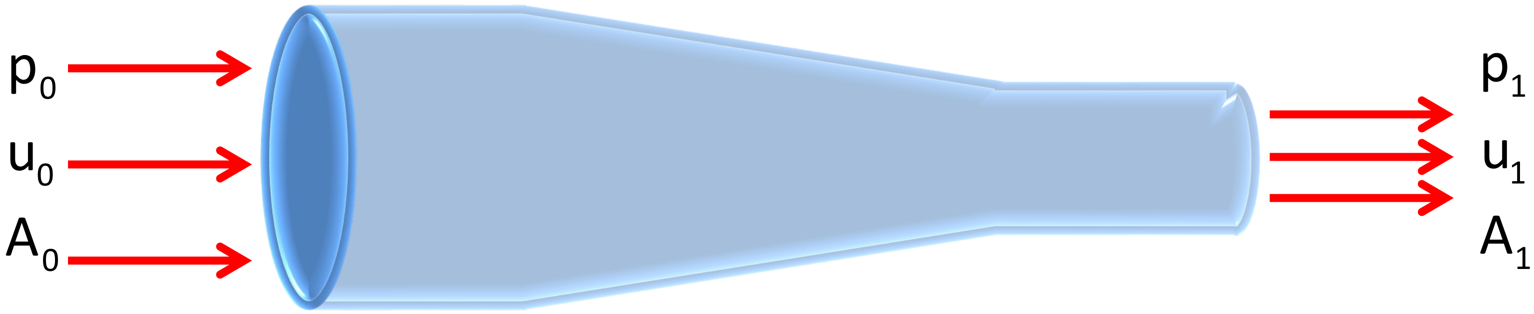


Question

Using Bernoulli's equation, determine if the pressure is higher or lower at point 1.

a. The pressure is higher at point 1
b. The pressure is lower at point 1
c. There is not enough information to answer the question
d. I don't know

Bernoulli's Equation…



<u>Question</u>

Using Bernoulli's equation, determine if the pressure is higher or lower at point 1.

By conservation of mass

$$\rho.A_0.u_0 = \rho.A_1.u_1$$

We are only dealing with incompressible flows therefore if

$$A_0 > A_1 \qquad u_0 < u_1$$

Bernoulli's equation gives us

$$\frac{p_0}{\rho.g} + z_0 + \frac{u_0^2}{2g} = \frac{p_1}{\rho.g} + z_1 + \frac{u_1^2}{2g}$$

z is constant hence

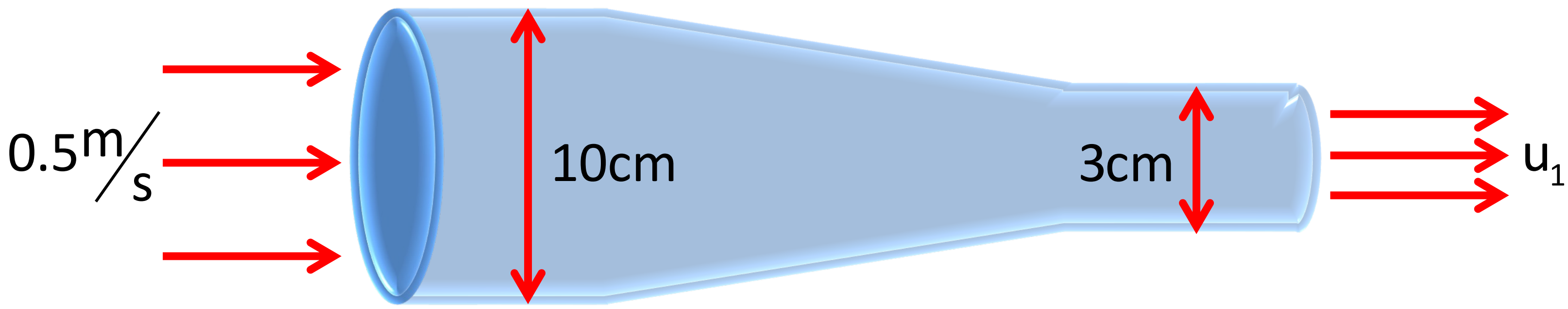
$$p_0 + \frac{\rho.u_0^2}{2} = p_1 + \frac{\rho.u_1^2}{2}$$

$$p_0 - p_1 = \frac{\rho}{2}\left(u_1^2 - u_0^2\right)$$

We can easily determine that

$$u_1^2 > u_0^2 \qquad \therefore p_0 > p_1$$

Bernoulli's Equation...



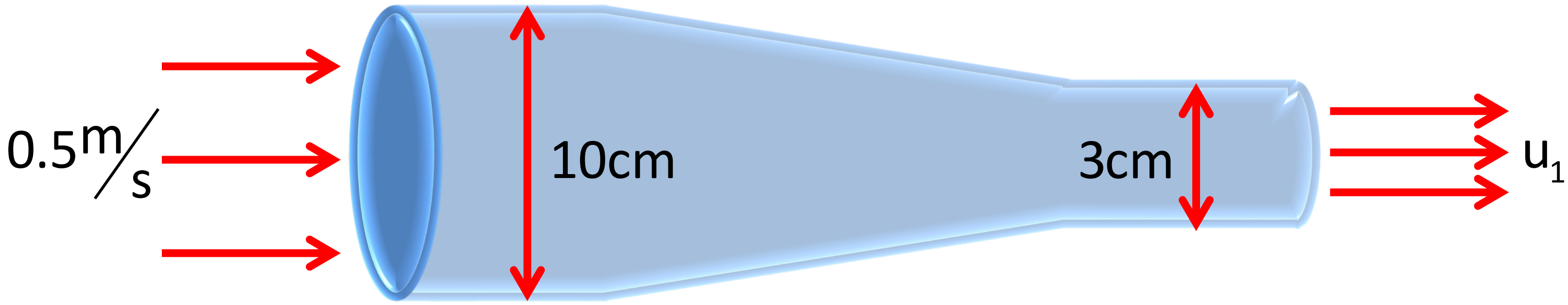
<u>Question</u>

The incompressible fluid at the exit of the nozzle above is at atmospheric pressure.

Find the gauge pressure at the entrance of the nozzle if the density of the fluid is 1000kg/m^3

a. 12.8 kPa
b. 15.3 kPa
c. 21.6 kPa
d. None of these

Bernoulli's Equation…



<u>Question</u>

The incompressible fluid at the exit of the nozzle above is at atmospheric pressure. Find the gauge pressure at the entrance of the nozzle if the density of the fluid is 1000kg/m^3

The fluid is incompressible, hence by conservation of mass

$$A_0.u_0 = A_1.u_1$$

$$u_1 = 5.56 \, m/s$$

Using Bernoulli's equation

$$\frac{p_0}{\rho.g} + z_0 + \frac{u_0^2}{2g} = \frac{p_1}{\rho.g} + z_1 + \frac{u_1^2}{2g}$$

p_1 is at atmospheric pressure, which we can set at zero. Hence

$$\frac{p_0}{\rho.g} + \frac{u_0^2}{2g} = \frac{u_1^2}{2g} \qquad p_0 = \frac{\rho}{2}\left(u_1^2 - u_0^2\right)$$

$$p_0 = \frac{1000}{2}\left(5.56^2 - 0.5^2\right) = 15310\text{Pa}$$

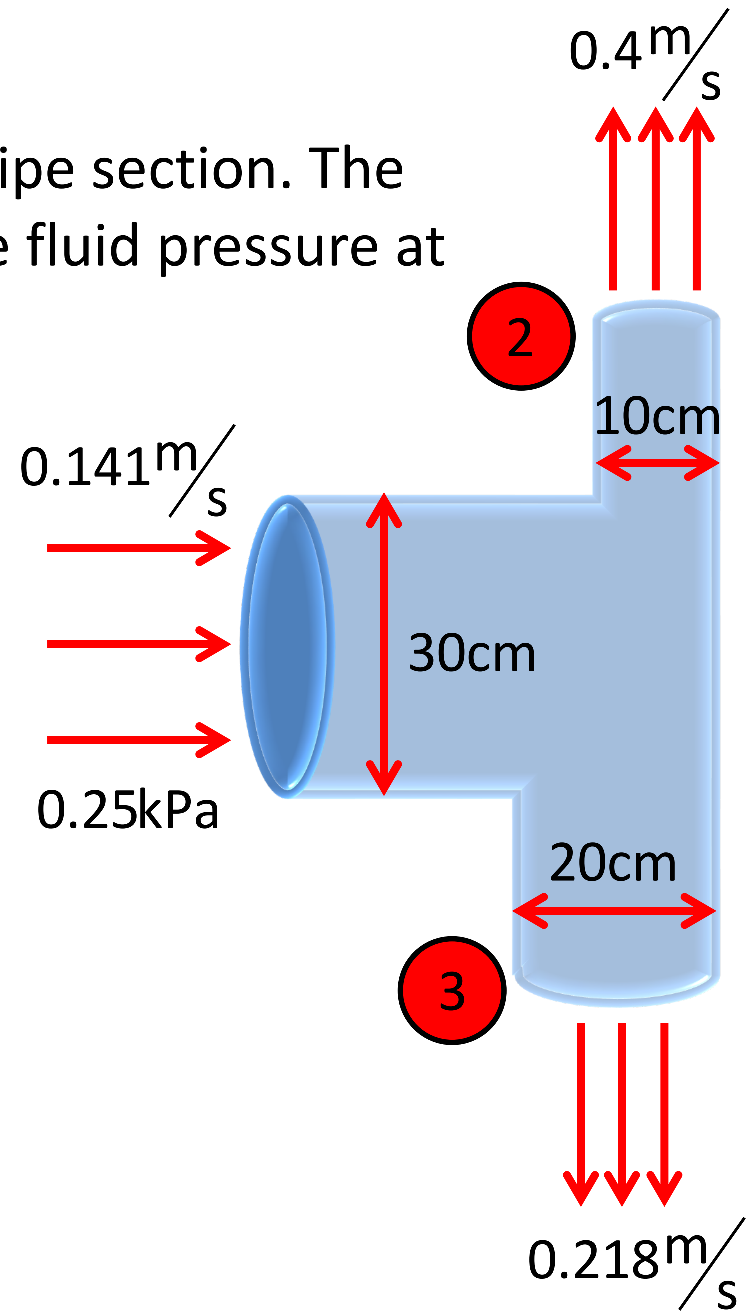
This is a gauge pressure of 15.3kPa

Bernoulli's Equation…

<u>Question</u>

Consider the plan view of this branching pipe section. The density of the fluid is 1000 kg.m^{-3}. Find the fluid pressure at points 2 and 3.

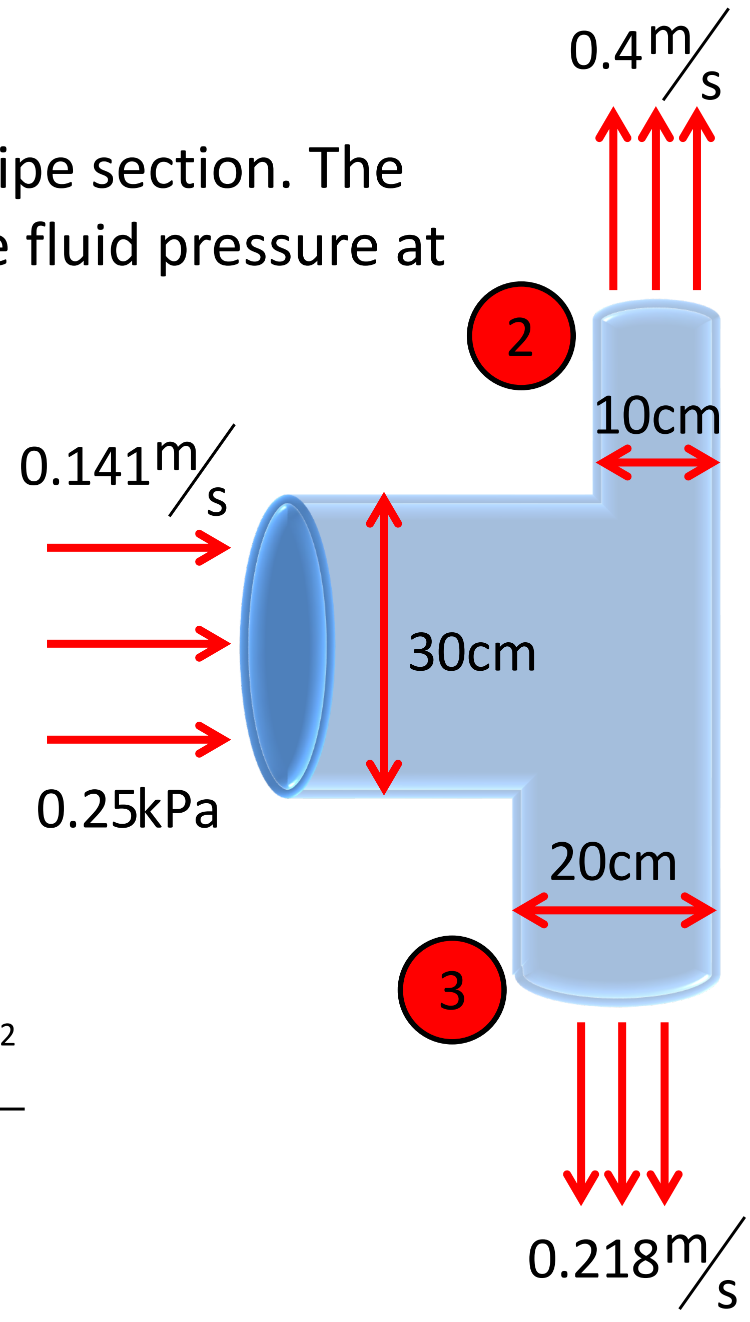
a. **p_2 = 0.18kPa p_3 = 0.236kPa**
b. p_2 = 0.236kPa p_3 = 0.18kPa
c. None of these
d. There are too many unknowns to answer the question



Bernoulli's Equation...

<u>Question</u>

Consider the plan view of this branching pipe section. The density of the fluid is 1000 $kg.m^{-3}$. Find the fluid pressure at points 2 and 3.



Using Bernoulli's equation

$$\frac{p_0}{\rho.g}+\frac{u_0^2}{2g}=\frac{p_2}{\rho.g}+\frac{u_2^2}{2g}=\frac{p_3}{\rho.g}+\frac{u_3^2}{2g}$$

or $$p_0+\frac{\rho.u_0^2}{2}=p_2+\frac{\rho.u_2^2}{2}=p_3+\frac{\rho.u_3^2}{2}$$

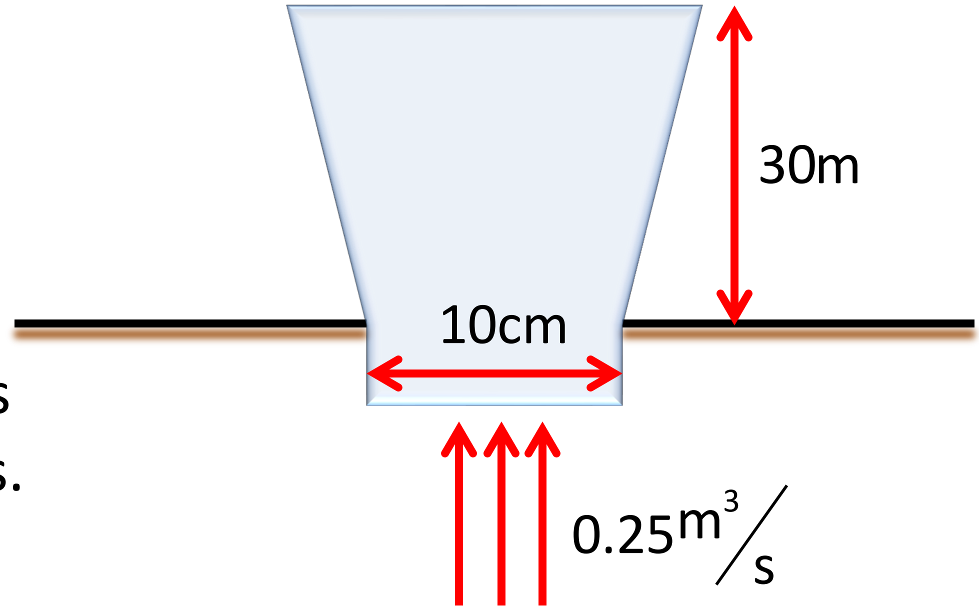
Hence

$$250+\frac{(1000)(0.141)^2}{2}$$

$$=p_2+\frac{(1000)(0.4)^2}{2}=p_3+\frac{(1000)(0.218)^2}{2}$$

$$p_2=0.18\text{kPa} \qquad p_3=0.236\text{kPa}$$

Bernoulli's Equation…



Question

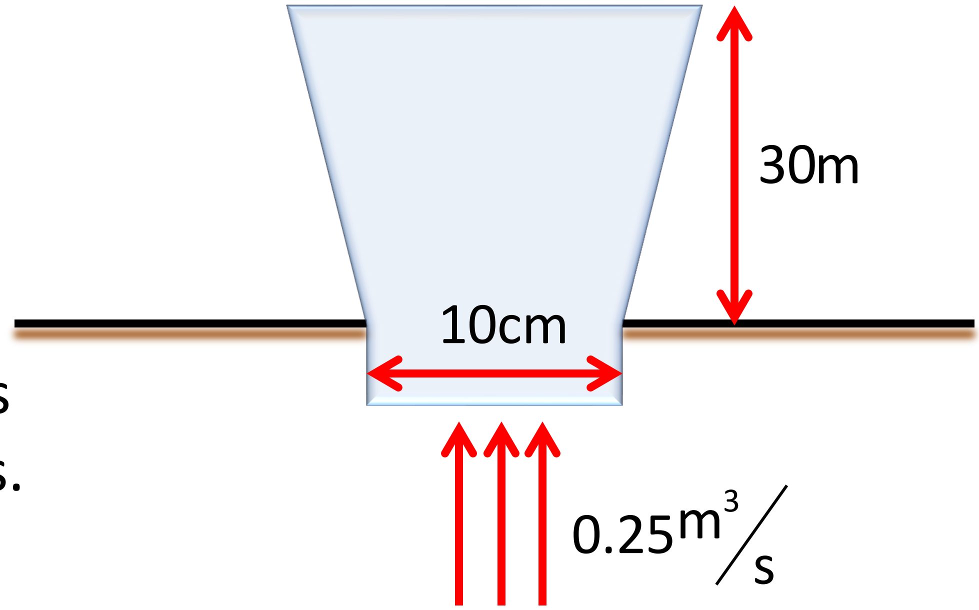
Consider this vertical jet to the right. Use Bernoulli's equation to determine the velocity and radius of the jet at a height of 30 metres.

a. $d_1 = 0.056m$
b. $d_1 = 0.111m$
c. **$d_1 = 0.124m$**
d. $d_1 = 0.596m$

Bernoulli's Equation…

<u>Question</u>

Consider this vertical jet to the right. Use Bernoulli's equation to determine the velocity and radius of the jet at a height of 30 metres.



$$Q = A_0.u_0 = \frac{0.1^2\pi}{4}.u_0 = 0.25$$

$$\therefore u_0 = 31.8\,{}^{m}\!/_{s}$$

Using Bernoulli's equation

$$\frac{p_0}{\rho.g} + z_0 + \frac{u_0^2}{2g} = \frac{p_1}{\rho.g} + z_1 + \frac{u_1^2}{2g}$$

We know that $p_0 = p_1$

And $z_0 = 0$

Hence, $$\frac{u_0^2}{2g} = z_1 + \frac{u_1^2}{2g}$$

Or, $$\frac{31.8^2}{2g} = 30 + \frac{u_1^2}{2g} \quad \therefore u_1 = 20.6\,{}^{m}\!/_{s}$$

$$Q = A_1.u_1 = \frac{d_1{}^2\pi}{4}(20.6) = 0.25$$

$$\therefore d_1 = 0.124\text{m}$$

Question…

A pipe carries water with a density of 1000kg m^{-3}. The pipe contracts from a cross sectional area of 0.3m^2 at A to 0.2m^2 at B. The flow has a velocity of 1.8m s^{-1} at A, and point B is 20.25cm below point A. Assume (for this question only) that g = 10m s^{-2}, which of the following statements is correct?

A) The pressures at A and B are equal

B) The pressure at A is less than the pressure at B

C) The pressure at B is less than that at A

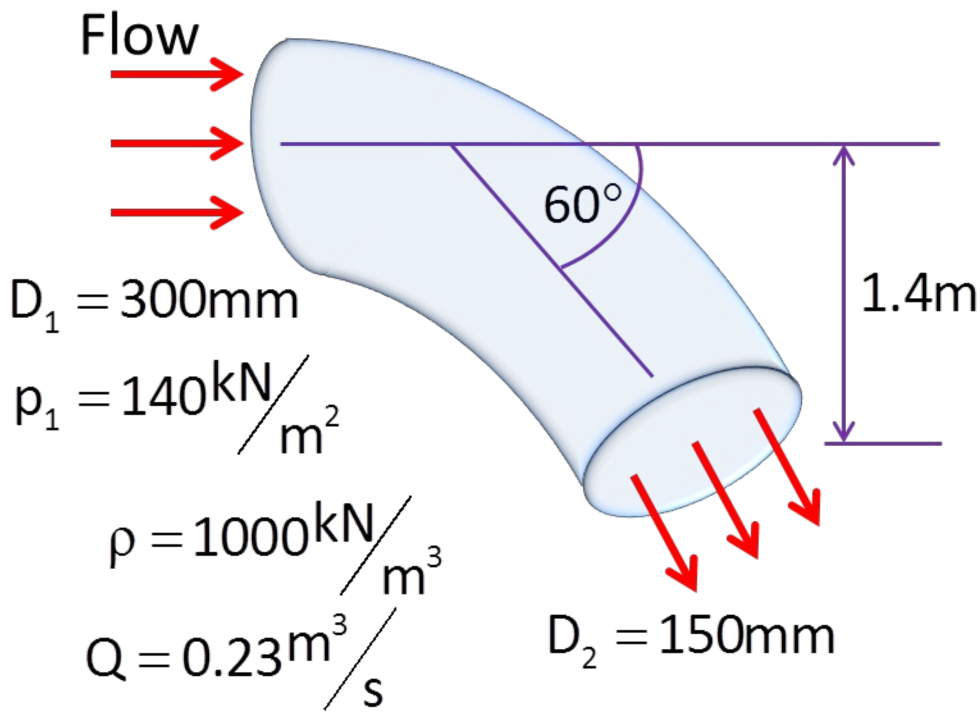
D) There is not enough information to answer this question.

Exercise 1

Consider the pipe bend shown below. The pipe is vertical and rotates through 60^{o} so that the outlet has an elevation 1.4m below the inlet. The mass of water in the pipe section is 85kg.

a) Using the conservation of mass, calculate the inlet and outlet velocities.

b) Using Bernoulli's equation, calculate the pressure at the outlet.



Exercise 1 - Solution

a) Using the conservation of mass, calculate the inlet and outlet velocities.

$$u_1 = \frac{Q}{\pi . r_1^2} = \frac{0.23}{\pi . \left(0.15^2\right)} = 3.254 \, {}^{m}\!/\!{}_{s}$$

$$u_2 = \frac{Q}{\pi . r_2^2} = \frac{0.23}{\pi . \left(0.075^2\right)} = 13.015 \, {}^{m}\!/\!{}_{s}$$

b) Using Bernoulli's equation, calculate the pressure at the outlet.

$$\frac{p_1}{\rho . g} + z_1 + \frac{v_1^2}{2.g} = \frac{p_2}{\rho . g} + z_2 + \frac{v_2^2}{2.g}$$

$$\frac{140{,}000}{(9.81).(1000)} + 1.4 + \frac{3.254^2}{2.(9.81)} = \frac{p_2}{\rho . g} + 0 + \frac{13.015^2}{2.(9.81)}$$

$$\therefore p_2 = 74{,}333 P_a$$