Chapter 11 Exam

~~Online~~ Calculus III

Classroom

Fall 2022

Name: KEY

1. Let $\vec{u} = \overrightarrow{PQ}$ and $\vec{v} = \overrightarrow{PR}$ with

$$P(-2,-1,3), Q(0,-2,-5), R(3,-3,0)$$

a) Write $\vec{u}$ and $\vec{v}$ in both component form and as a linear combination of the standard unit vectors $\hat{\imath}, \hat{\jmath}$, and $\hat{k}$.

$\vec{u} = \langle 2,-1,8 \rangle = 2\hat{i} - \hat{j} + 8\hat{k}$ $\quad \vec{v} = \langle 5,-2,-3 \rangle = 5\hat{i} - 2\hat{j} - 3\hat{k}$

b) Find the magnitude of $\vec{u}$.

$$\|\vec{u}\| = \sqrt{4+1+64} = \sqrt{69}$$

c) Find $-23\vec{u} + \vec{v}$. $\quad -23\langle 2,-1,8 \rangle + \langle 5,-2,-3 \rangle$

$\langle -46, 23, 184 \rangle + \langle 5,-2,-3 \rangle$

$\langle -41, 21, 181 \rangle$

d) Find $\vec{u} \cdot \vec{v}$.

$$\vec{u} \cdot \vec{v} = 10 + 2 + 24 = 36$$

e) Is the angle between $\vec{u}$ and $\vec{v}$ obtuse, acute, straight, or right? Explain your reasoning.

Since $\vec{u} \cdot \vec{v} > 0$

$\Rightarrow$ acute

2. Determine whether $\vec{u}$ and $\vec{v}$ are orthogonal, parallel, or neither. Show reasoning.

$$\vec{u} = \langle -8, 2, -3 \rangle$$
$$\vec{v} = \langle -1, 4, 5 \rangle$$

Parallel: $\vec{u} \times \vec{v} = \vec{0} \Rightarrow$ parallel

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & 2 & -3 \\ -1 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} \hat{i} - \cdots$$

$$= (10 + 12)\hat{i} - \cdots$$

$$\neq \vec{0} \Rightarrow \text{Not parallel}$$

Orthogonal: $\vec{u} \cdot \vec{v} = 0 \Rightarrow$ orthogonal

$$\vec{u} \cdot \vec{v} = 8 + 8 - 15 = 1 \neq 0$$

$\Rightarrow$ Not orthogonal.

Neither.

3. Find the projection of $\vec{u}$ onto $\vec{v}$. Draw a picture of this projection.

$$\vec{u} = -3\hat{\imath} + \hat{\jmath} - 2\hat{k}$$
$$\vec{v} = -2\hat{\imath} + 2\hat{k}$$

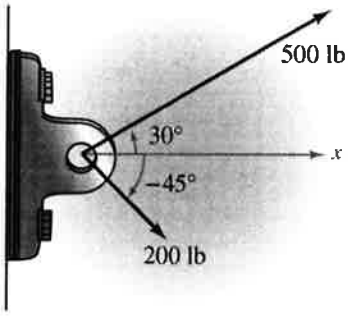
$$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}$$

$$= \left(\frac{6-4}{8}\right)\langle -2, 0, 2\rangle$$

$$= \frac{1}{4}\langle -2, 0, 2\rangle$$

$$= \left\langle -\frac{1}{2}, 0, \frac{1}{2}\right\rangle$$

4. Forces with magnitudes of 500 pounds and 200 pounds act on a machine part at angles of 30° and −45°, respectively, with the x-axis. Find the direction (angle in degrees) and magnitude of the resultant force.



$$\vec{F}_1 = 500\langle \cos 30°, \sin 30° \rangle$$
$$= \langle 250\sqrt{3}, 250 \rangle$$
$$\vec{F}_2 = 200\langle \cos 315°, \sin 315° \rangle$$
$$= \langle 100\sqrt{2}, -100\sqrt{2} \rangle$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = \langle 250\sqrt{3}, 250 \rangle + \langle 100\sqrt{2}, -100\sqrt{2} \rangle$$
$$= \langle 250\sqrt{3} + 100\sqrt{2}, 250 - 100\sqrt{2} \rangle$$
$$\approx \langle 574.4340581, 108.5786438 \rangle$$

$$\|\vec{F}\| \approx \sqrt{341763.809} \approx 584.6057$$

$$\tan\theta = \frac{108.5786438}{574.4340581} \Rightarrow \theta \approx 10.7°$$

5. Find sets of (a) parametric equations and (b) symmetric equations of the line that passes through $P(4,-2,0)$ and $Q(-8,13,5)$.

$\overrightarrow{PQ} = \langle -12, 15, 5 \rangle$ $P(4,-2,0)$

a.) $\begin{cases} x = -12t + 4 \\ y = 15t - 2 \\ z = 5t \end{cases}$

b.) $\frac{x-4}{-12} = \frac{y+2}{15} = \frac{z}{5}$

6. Find the equation of the plane that contains the following two lines

$$L_1: \quad \frac{x-1}{-2} = y-3 = z$$

and

$$L_2: \quad \frac{x+1}{-2} = \frac{y-1}{3} = \frac{z-2}{2}$$

Changed

$P_1(1,3,0)$ $\vec{v}_1 = \langle -2,1,1 \rangle$

$P_2(-1,1,2)$ $\vec{v}_2 = \langle -2,3,2 \rangle$

normal $= \vec{n} = \vec{v}_1 \times \vec{v}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ -2 & 3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}\hat{i} - \begin{vmatrix} -2 & 1 \\ -2 & 2 \end{vmatrix}\hat{j} + \begin{vmatrix} -2 & 1 \\ -2 & 3 \end{vmatrix}\hat{k}$$

$$= (2-3)\hat{i} - (-4+2)\hat{j} + (-6+2)\hat{k}$$

$$= -\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\boxed{-1(x-1) + 2(y-3) - 4z = 0}$$

7. Convert the point from rectangular to cylindrical coordinates.

$(-2, 3, 1)$.

$r^2 = (-2)^2 + (3)^2 = 13 \rightarrow r = \sqrt{13}$

$\tan\theta = \frac{-3}{2} \rightarrow \theta = -56.3099°$ OR $303.69°$

$(\sqrt{13}, 303.69°, 1)$

Convert the point from spherical to rectangular coordinates.

$\left(3, -\frac{\pi}{3}, \frac{\pi}{4}\right)$.

$\rho = 3 \quad \theta = -\frac{\pi}{3} \quad \phi = \frac{\pi}{4}$

$x = 3\cos\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{4}$

$y = 3\sin\left(-\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = -\frac{3\sqrt{6}}{4}$

$z = 3\cos\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2}$

$\left(\frac{3\sqrt{2}}{4}, -\frac{3\sqrt{6}}{4}, \frac{3\sqrt{2}}{2}\right)$

Chapter 11 Exam Grade

Question	Score
1	/15
2	/10
3	/10
4	/15
5	/15
6	/15
7	/20
TOTAL	**/100**