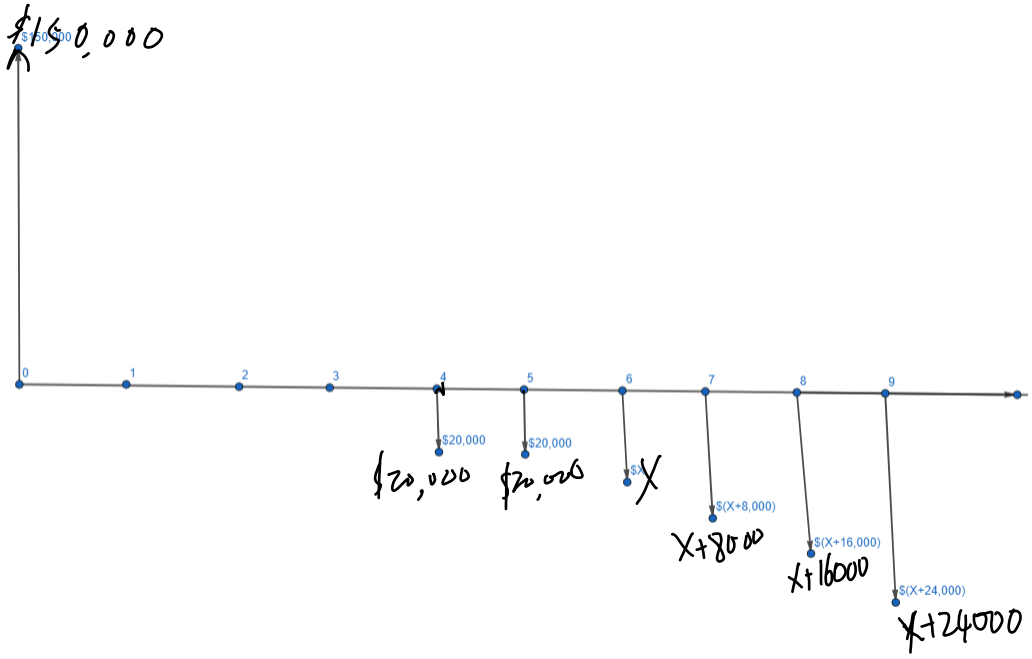
Instructions:

Please attempt every problem. You must support every solution with an appropriate amount of work and/or description. Unsupported answers may receive a score of 0. Good luck!

1. (10 pts) David is borrowing \$150,000 from Hartford Bank to open Road and Off-Road Bicycle Shop. David expects it to take a few years before the shop earns a sizeable profit, so he has arranged for no payments on the loan until the end of the fourth year. The first and second payments are due 4 and 5 years, respectively, from today in the amounts of \$20,000 each. Starting at the end of year 6, a series of 4 annual end-of-year payments will be made. The first of these is \$X. Each subsequent payment is \$8,000 greater than the previous payment. Draw the cash flow diagram from David's perspective.
Solution:

+10



2. (6 pts) If you want to triple your money at an interest rate of 6% per year compounded annually, for how many years would you have to leave the money

in the account?
Solution:
To triple the money, we need

$$F = P(1+i)^n = 3P$$

That is,

$$(1+0.06)^n = 3$$
$$n = \frac{ln3}{ln1.06} = 18.854 \text{ years.}$$

+6

3. (10 pts) For your 21st birthday, your grandfather offers you a gift of $1,000 today. However, you have the choice of waiting 3 years and receiving $1,500 or waiting 5 years and receiving $3,000. If your money grows at a rate of 8% compounded annually, which alternative should you choose?
Solution:
To make comparison between these alternatives, we need to convert the money to a value at a common time point. For instance, we can compare their present value at $t = 0$. +2

- The present value of the 1st alternative is $PV_1 = \$1000.$
- The present value of the 2nd alternative is

$$PV_2 = 1500(1+0.08)^{-3} = \$1190.748.$$

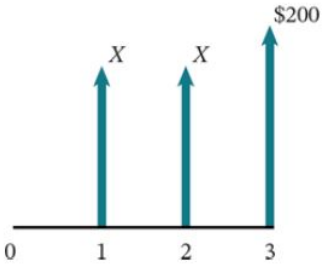
- The present value of the 3rd alternative is

$$PV_3 = 3000(1+0.08)^{-5} = \$2041.750.$$

+6
Since the 3rd alternative have the largest present value, so I would choose $3000 in 5 years. +2

4. (6 pts) Consider the following cash flow diagram. What is the value of X if the present worth of the diagram is $400 and the interest rate is 15% compounded annually?
Solution:
Since the present worth of the cash flows is $400, we have

$$P = X(1+0.15)^{-1} + X(1+0.15)^{-2} + 200(1+0.15)^{-3} = 400$$



Solving for X yields

$$X = \$165.157.$$

+6

5. (6 pts)Ben deposits \$5,000 now into an account that earns 7.5% interest compounded annually. He then deposits \$1,000 per year at the end of the 1st and 2nd years. How much will the account contain 10 years after the initial deposit?
Solution:
The account will contain

$$\begin{aligned} F &= 5000(1+0.075)^{10-0} + 1000(1+0.075)^{10-1} + 1000(1+0.075)^{10-2} \\ &= \$14,005.874 \end{aligned}$$

10 years after the initial deposit. +6

6. (12 pts) Maria deposits \$1,200, \$500, and \$2,000 at t = 1, 2, and 3, respectively. If the fund pays 8% compounded per period, what sum will be accumulated in the fund at (a) $t = 3$, and (b) $t = 6$? Solution:

a) At $t = 3$,

$$\begin{aligned} F_3 &= 1200(1+0.08)^{3-1} + 500(1+0.08)^{3-2} + 2000 \\ &= \$3939.68 \end{aligned}$$

+6

b) At $t = 6$,

$$\begin{aligned} F_3 &= 1200(1+0.08)^{6-1} + 500(1+0.08)^{6-2} + 2000(1+0.08)^{6-3} \\ &= \$4962.862 \end{aligned}$$

+6