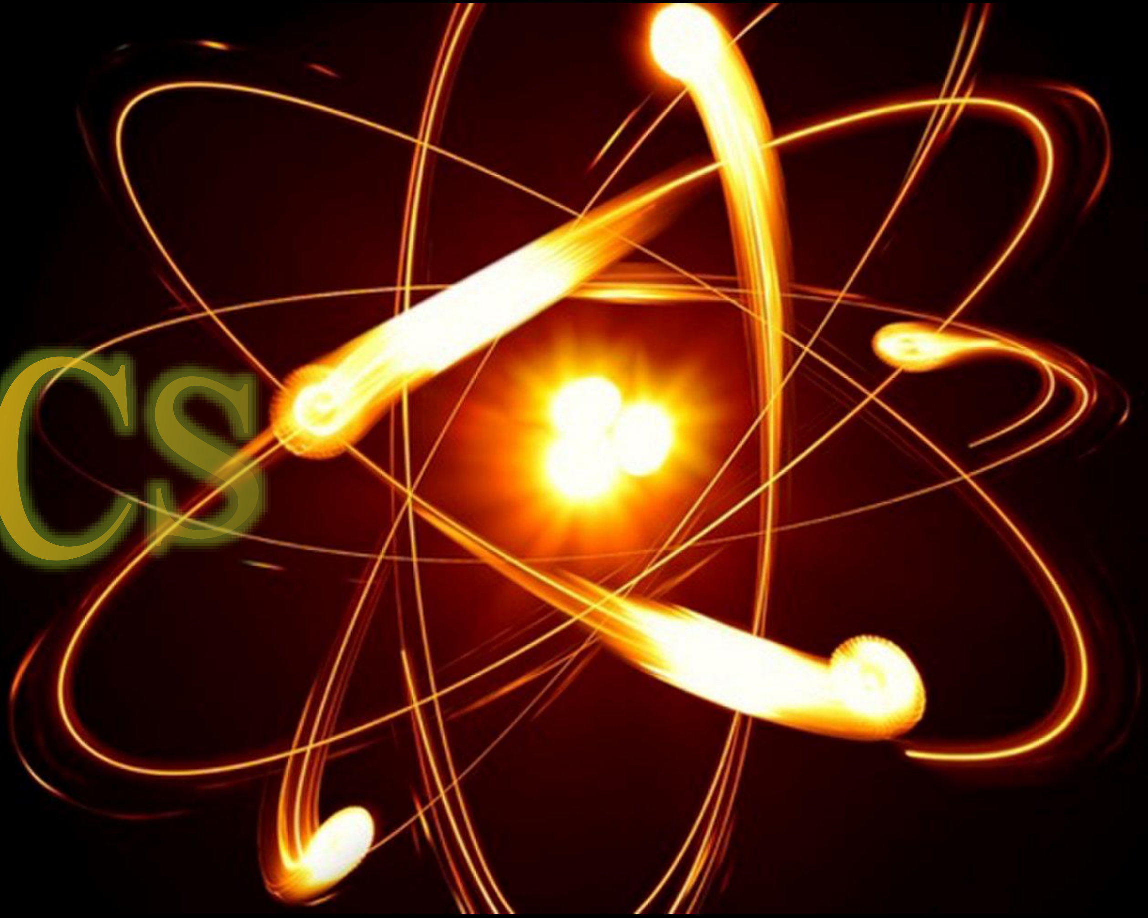
PHYSICS



西南交通大學
Southwest Jiaotong University

Physics 1: **Mechanics and Waves**

Week 10 – Torque and Rotational motion

2023.4

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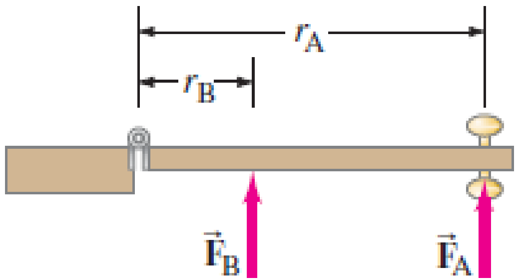
Torque

How to make an object rotate?

Look at the door



‘Effectiveness’ of force depends on

- **amplitude of force**
- **direction of force**
- **distance from pivot (hinge)**

Torque

A **Torque** is also known as the "**Moment of Force**".

It operates at some distance from the pivot point.

It's action **rotates** an object.

$$\boxed{\boldsymbol{\tau} = \vec{\boldsymbol{F}} \times \vec{\boldsymbol{r}}}$$

Torque

$$\boxed{\boldsymbol{\tau} = \vec{\boldsymbol{F}} \times \vec{\boldsymbol{r}}}$$

The directions of torque are “**clockwise**” or “**anti-clockwise**”

The SI unit is N·m, but it is NOT joule (J)!

Torque

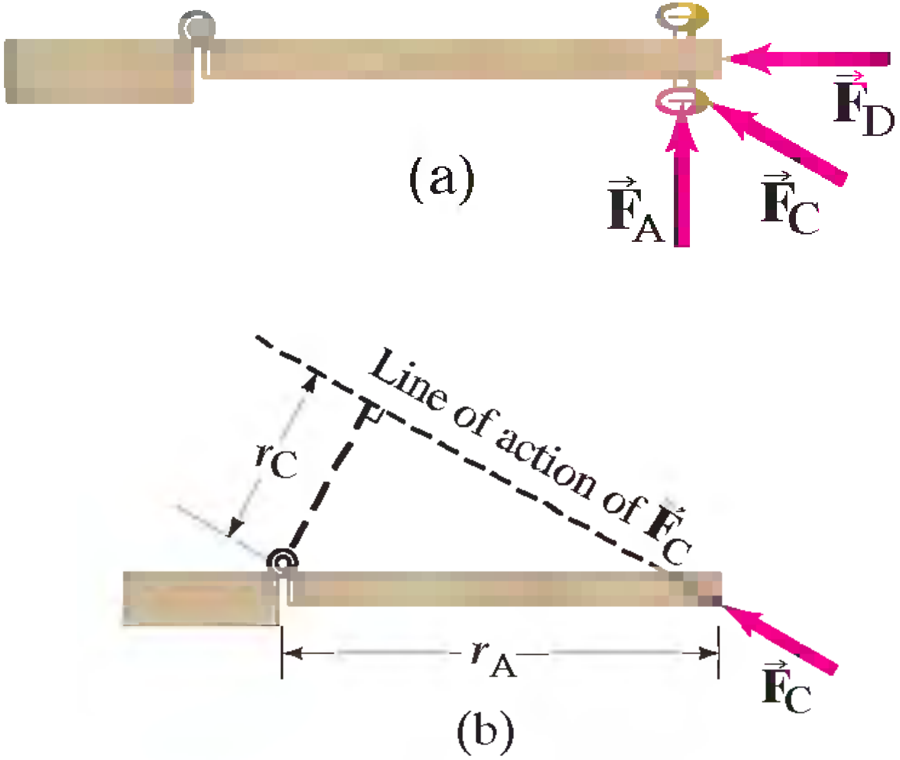
2 ways to find out

torque of F_C

$$\tau = r_\perp F$$

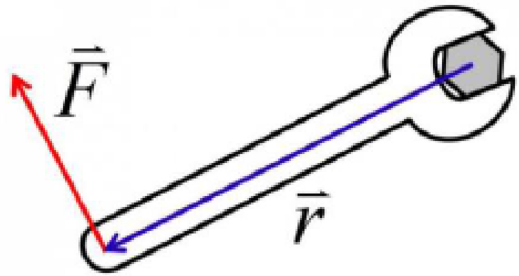
$$\tau = r F_\perp$$

$$\tau = rF\sin\theta$$



Example

A mechanic exerts a force of 100 N perpendicular to the end of a wrench 300 mm long. What is the torque applied to the screw nut ?



$$\tau = Fd = 100\ \text{N} \times 0.3\ \text{m}$$

$$\tau = 30\ \text{N}\cdot\text{m}$$

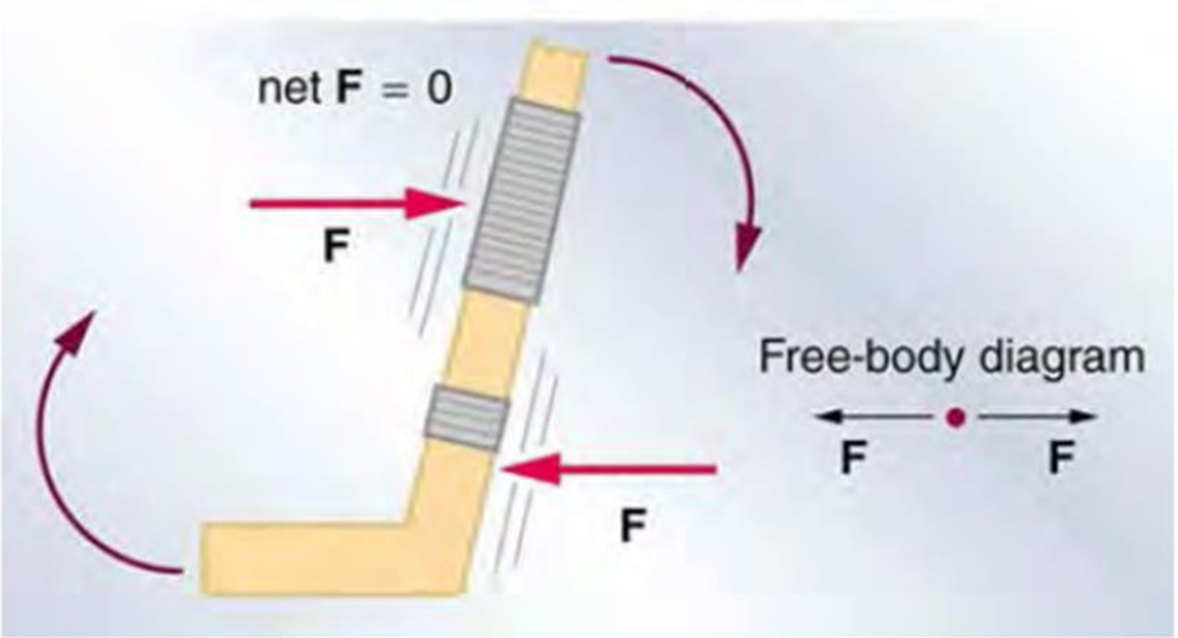
Clockwise – to fasten the screw

Anti-clockwise – to loosen the screw

Force couple

Two **opposite** and **equal** forces that act on **different points** of an object are known as a “**couple**”.

•A **couple** will rotate the object – it exerts a **torque**.

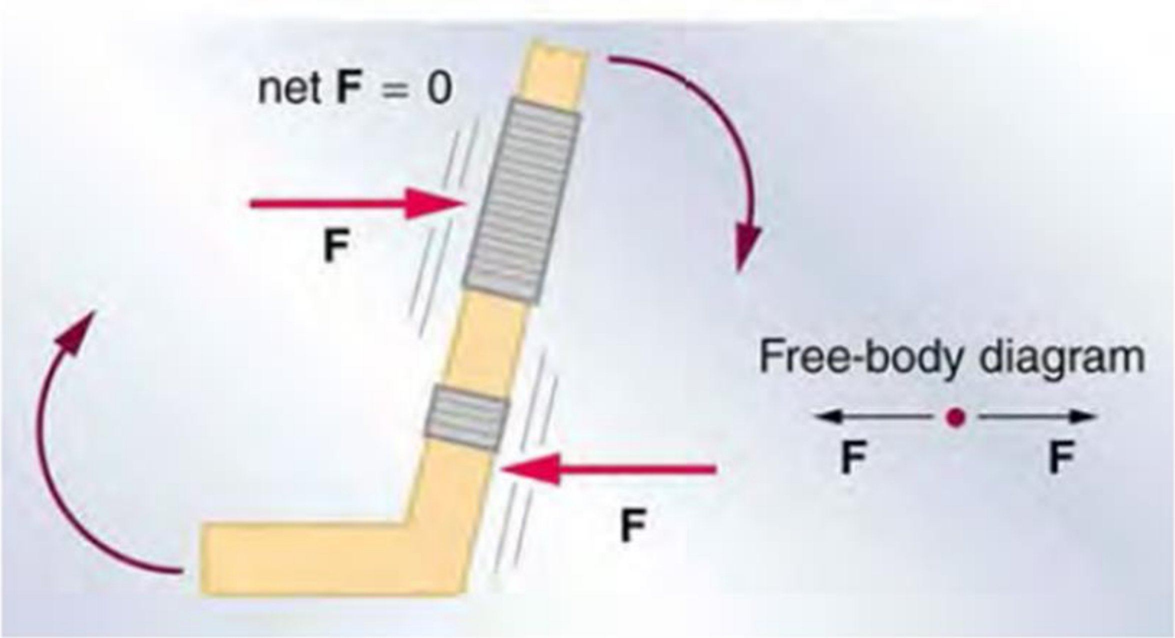


Force couple

Contribute to net force?

Contribute to the torque?

How about
moving
the rotational Center?



Static equilibrium

First condition of equilibrium :

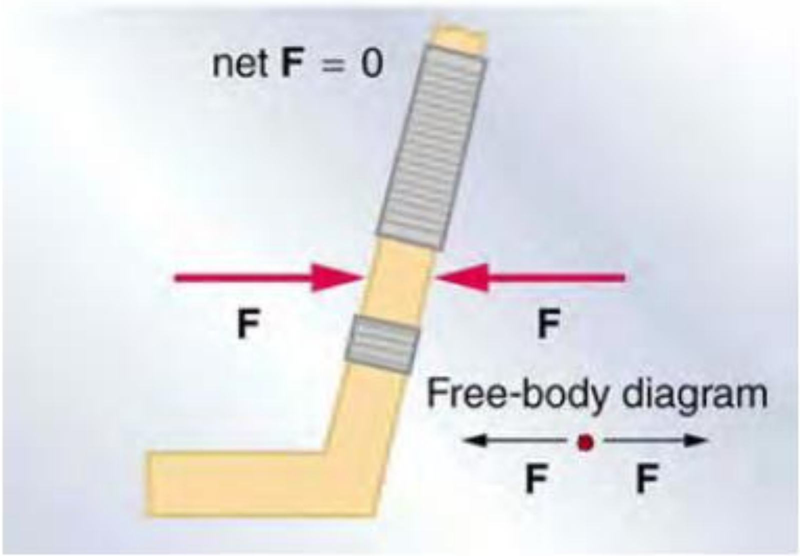
$$\text{net } \mathbf{F} = 0$$

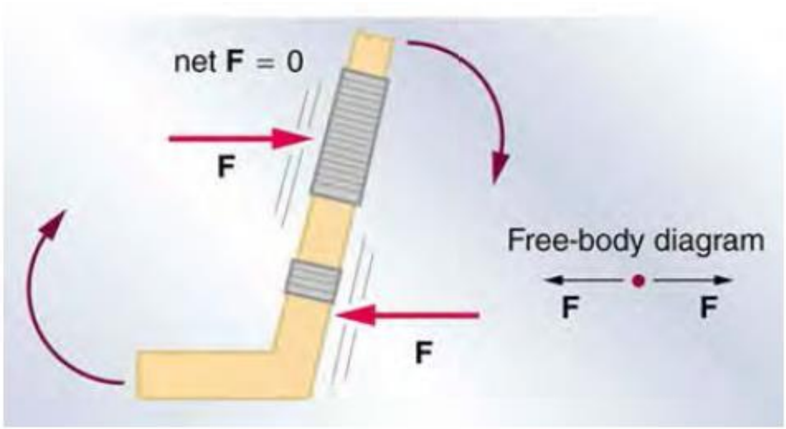
translational equilibrium

Second condition of equilibrium :

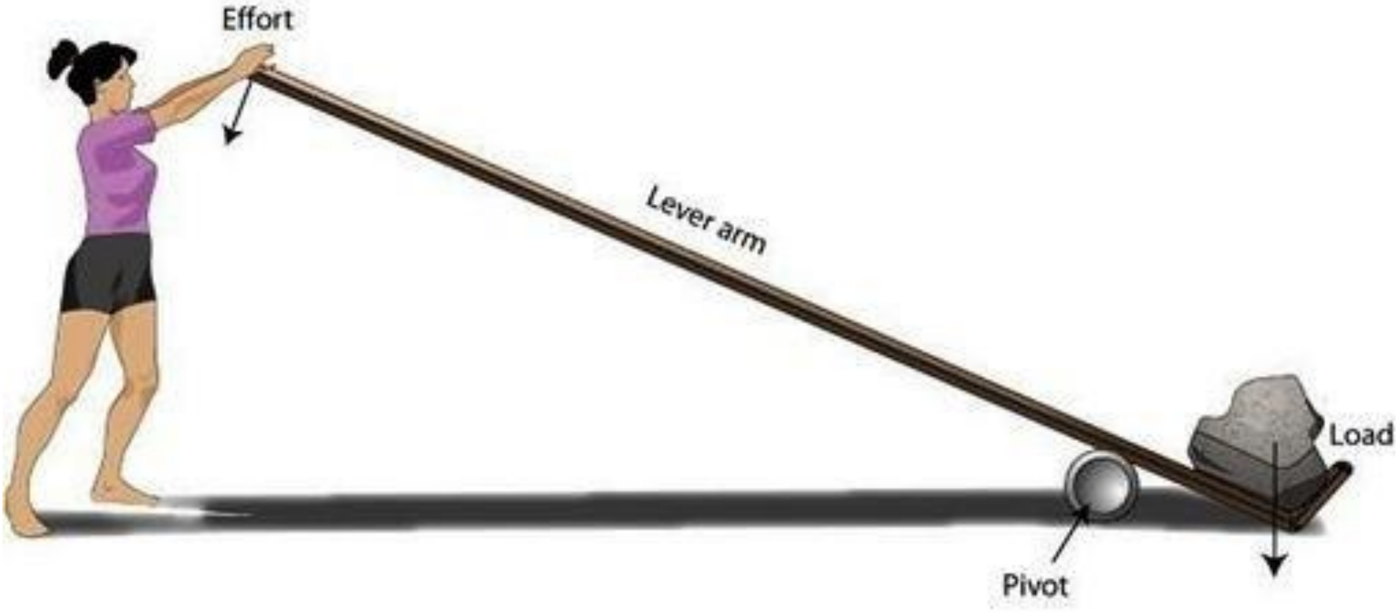
$$\text{net } \tau = 0$$

rotational equilibrium

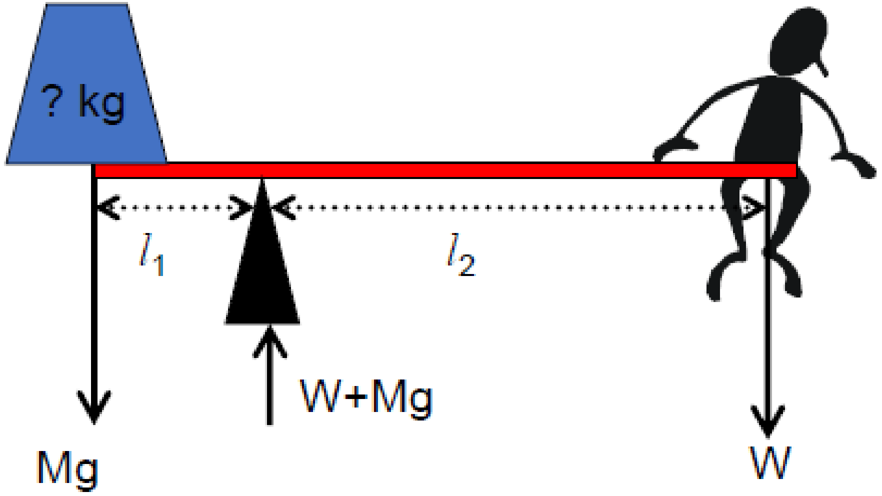




Lever

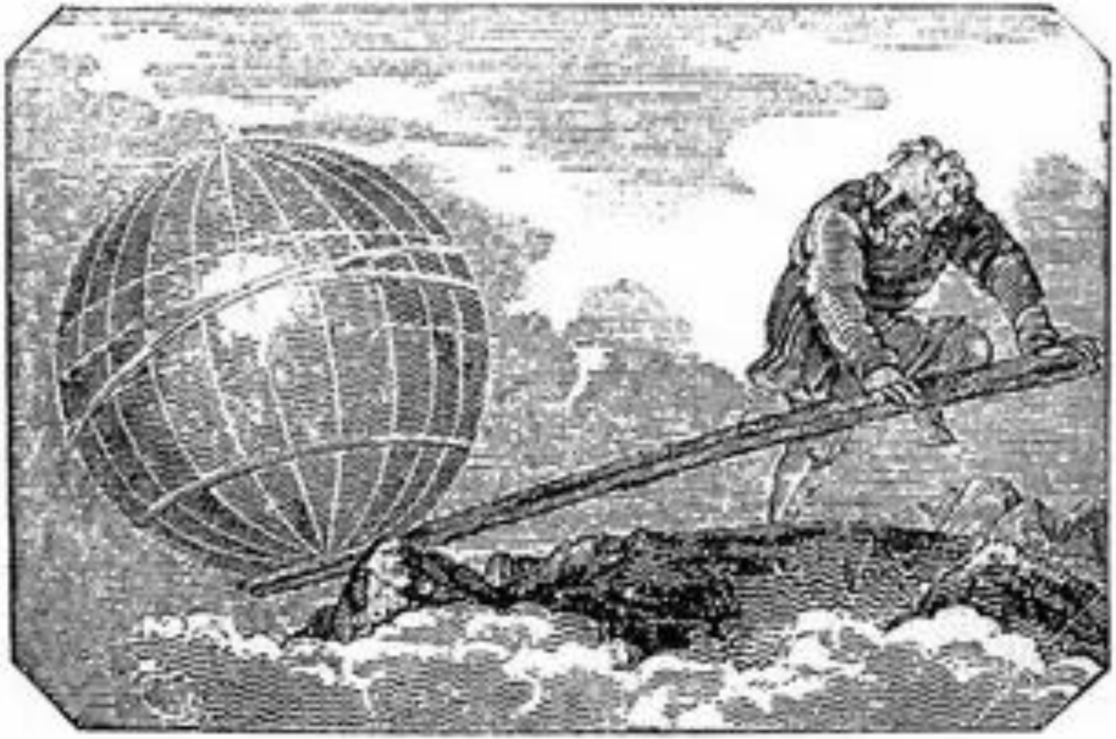


Lever



What is the maximum mass the person can lift ?

Lever



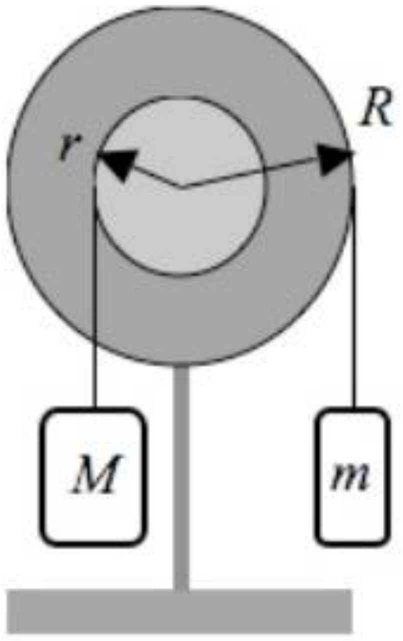
Archimedes: “Give me a lever and I can move the world.”

Example

Under what conditions is the system in static equilibrium?

- → **The sum of the torques has to be 0**

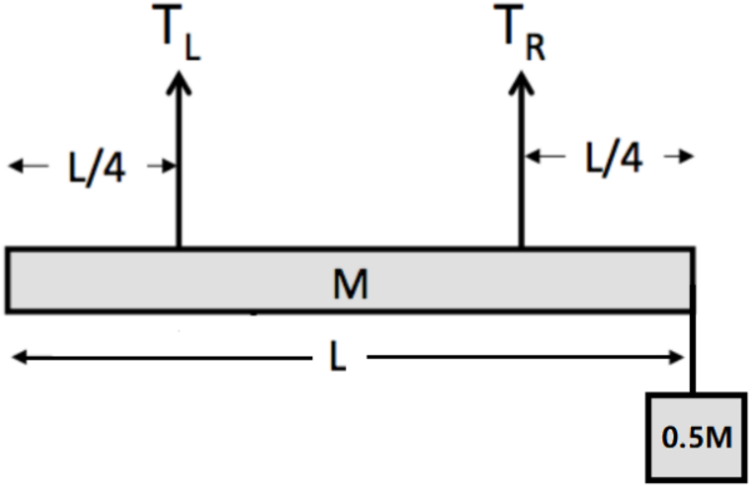
$$\tau_{net} = 0 = \tau_m - \tau_M$$



Example

As shown in the graph, a beam of mass M and length L is suspended by two vertical ropes. A box of mass 0.5 M is suspended by a short rope attached to the right end of the beam. What is the ratio of $T_L : T_R$, if the whole system is in equilibrium?

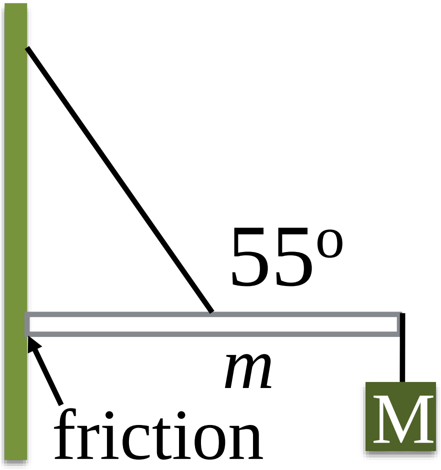
A 1: 2
B 1: 1.33
C 2: 7
D 1: 4
E 1: 5



Static equilibrium

$M = 12 \text{ kg}$

$m = 8 \text{ kg}$



What is the tension in the wire?

What is the strength of the force exerted by the wall on the bar?

Steps to solve a statics problem

1. **Make sure it IS a statics problem (no acceleration)**

2. **Choose a pivot point (where are the most unknowns)**

3. **Draw the forces (free body diagram)**

4. **Write equations for forces (*x* and *y*) and torque**

5. **Solve the equations**

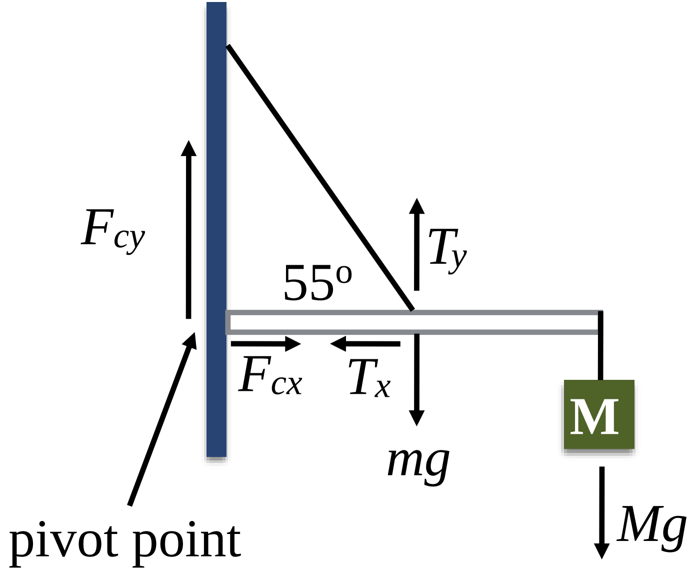
Static equilibrium

First condition of equilibrium:

Direction *x*: $F_{cx} - T_x = 0$

Direction *y*: $F_{cy} + T_y - mg - Mg = 0$

$$T_x = T\cos 55^o \qquad T_y = T\sin 55^o$$

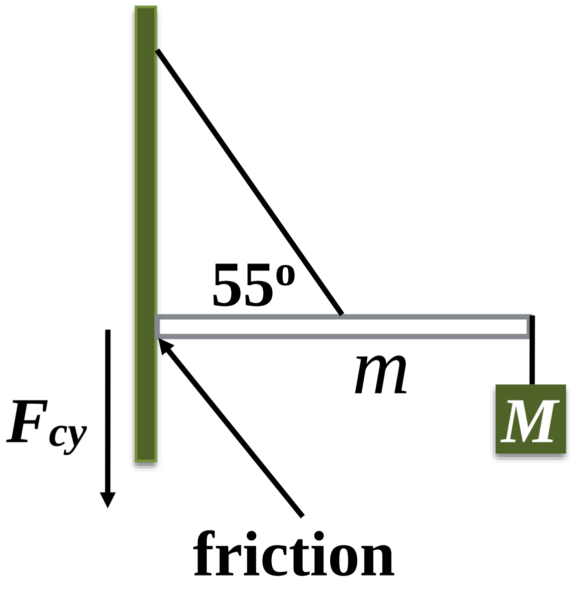


Second condition of equilibrium: $\sum \tau = 0$

$$\frac{L}{2} r T_y - \frac{L}{2} mgr - LMgr = 0 \longrightarrow T_y = (m + 2M)\, g$$

The ‘center of mass’ is at *L*/2

Static equilibrium



$$T = \frac{(m+2M)\,g}{\sin 55^o} = \frac{(8+12)\,(10)}{\sin 55^o} = 390\,\mathrm{N}$$

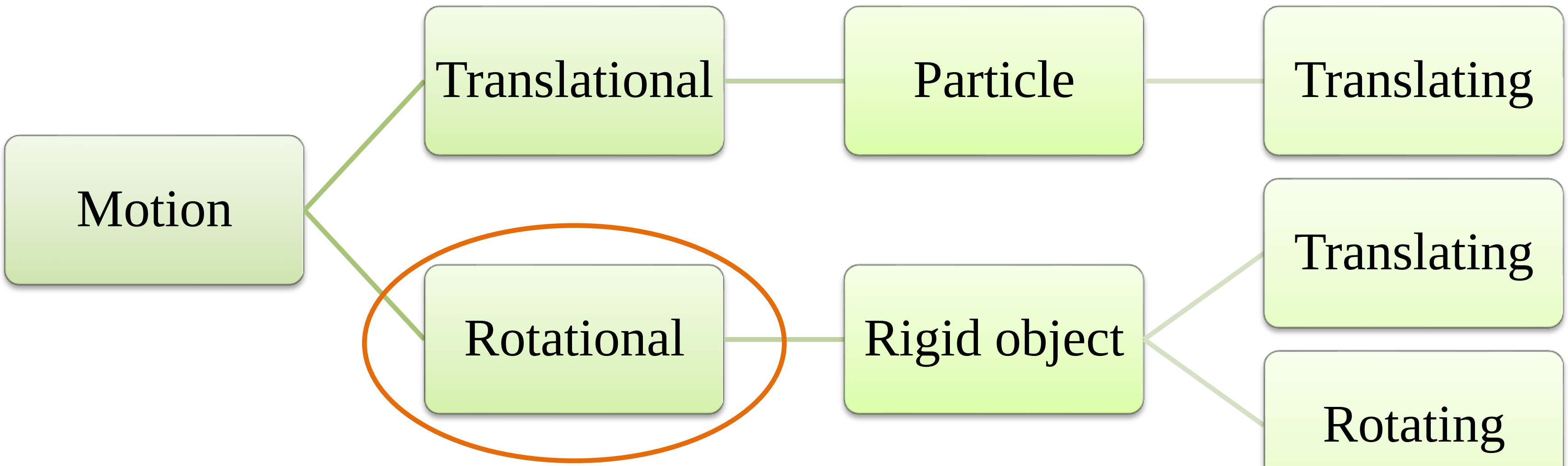
$$F_{cx} = T_x = T\cos 55^o = 220\,\mathrm{N}$$

$$F_{cy} + T_y - mg - Mg = 0$$

$$F_{cy} = -(m+2M)\,g + mg + Mg = -Mg = -120\,\mathrm{N}$$

F_{cy} is in the negative direction

Motion


Motion
Translational
Rotational
Particle
Rigid object
Translating
Translating
Rotating

Question:

CHAPTER-OPENING QUESTION—Guess now!

A solid ball and a solid cylinder roll down a ramp. They both start from rest at the same time and place. Which gets to the bottom first?

(a) They get there at the same time.
(b) They get there at almost exactly the same time except for frictional differences.
(c) The ball gets there first.
(d) The cylinder gets there first.
(e) Can't tell without knowing the mass and radius of each.

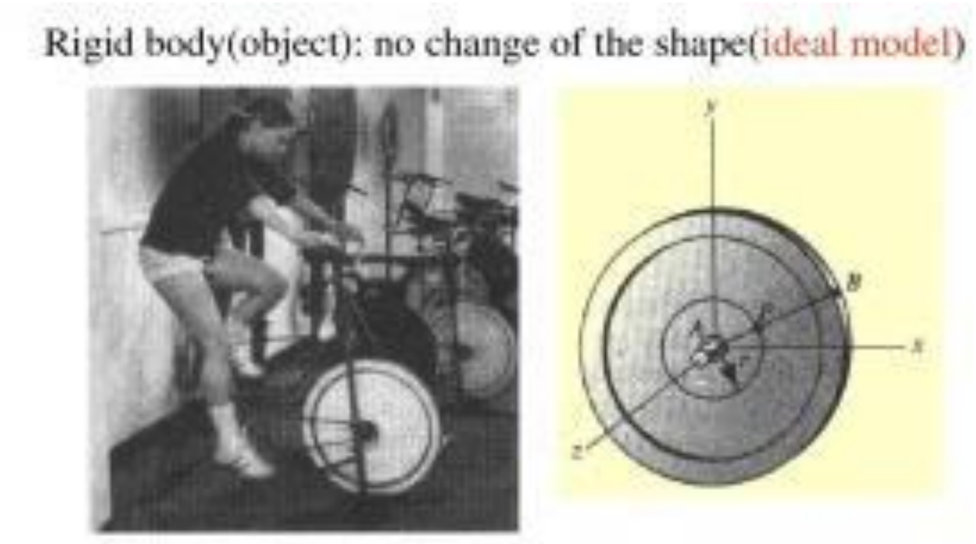
Dynamics of rotational motion

Rigid objects

Definite shape : Particles stay in fixed position

No vibrating and deforming

A good approximation



Angular Quantities

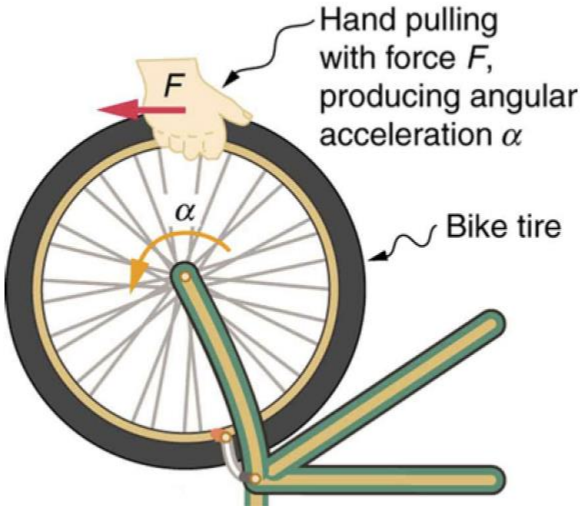
Here is the correspondence between linear and rotational quantities:

TABLE 8–1 Linear and Rotational Quantities

Linear	Type	Rotational	Relation[‡]
x	displacement	θ	$x = r\theta$
v	velocity	ω	$v = r\omega$
a_{tan}	acceleration	α	$a_{tan} = r\alpha$

[‡] You must use radians.

How to relate torque to angular acceleration?



$$F = ma$$

$$\rightarrow F = mr\alpha$$

$$\tau = rF = mr^2\alpha$$

$$\boxed{\tau = I\alpha}$$

equivalent to $F = ma$

$I = mr^2$ is the moment of inertia

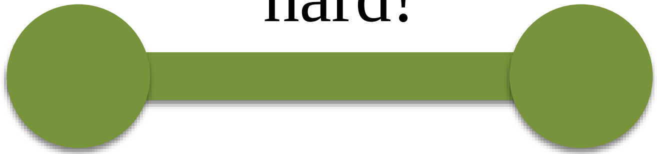
α is the angular acceleration

Moment of Inertia

Moment of Inertia depends on where the mass is located!

hard!

easy!

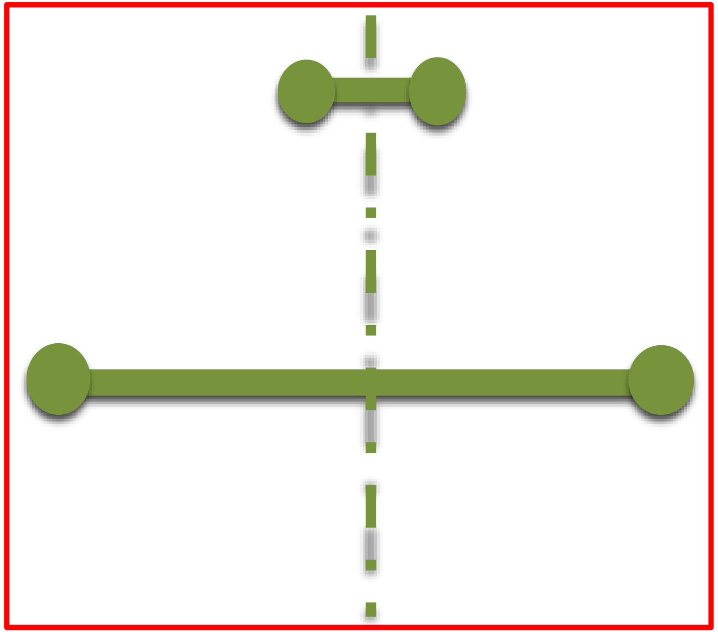


Total inertia should be calculated as : $I = \sum mr^2$ …or integrated

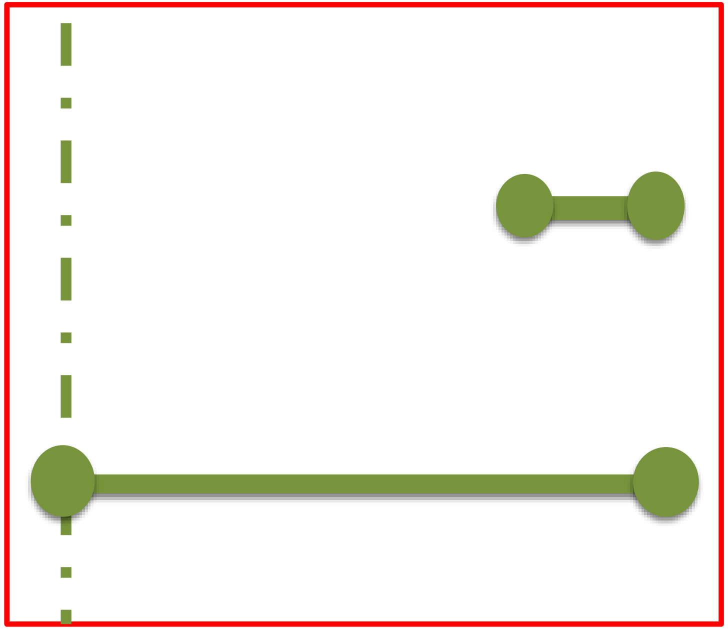
Moment of Inertia

$$m_1 = m_2 = 5 \text{ Kg}$$

Different location



Different Axis



Moment of Inertia

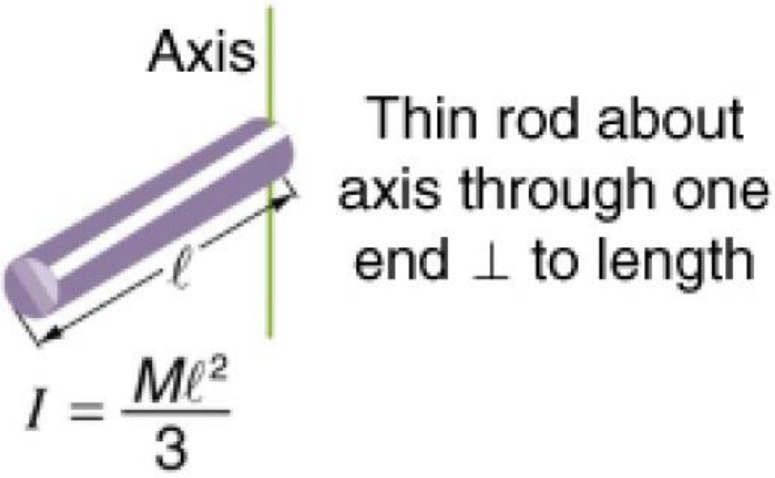
For a continuous rigid body, the summation in the rotational inertia can be substituted by an integral ,

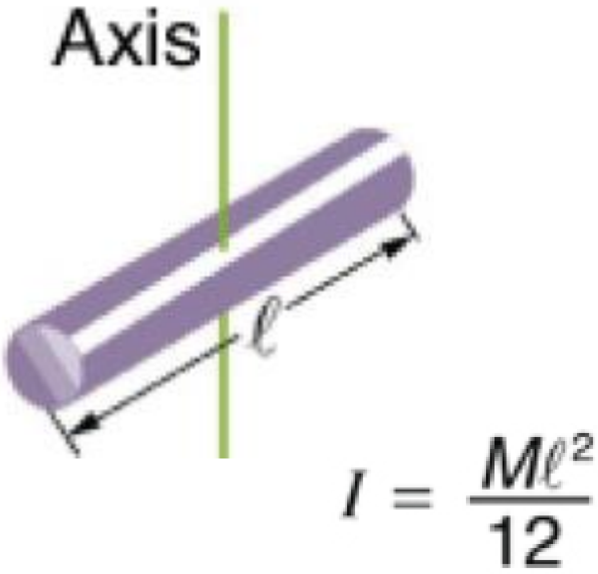
$$\boxed{I = \int r^2 \mathrm{d}m}$$

The moment of inertia I is a measure of the rotational inertia of a body, which plays the same role for rotational motion that mass does for translational motion.

Moment of Inertia

Use integral method to calculate I of these two: linear density





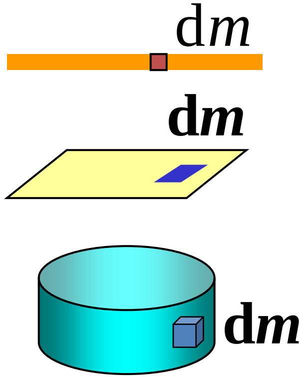
Moment of Inertia

$$\mathbf{d}\boldsymbol{m} = \begin{cases} \lambda\,\mathrm{d}l \\ \sigma\,\mathrm{d}S \\ \rho\,\mathrm{d}V \end{cases}$$

Linear density： λ

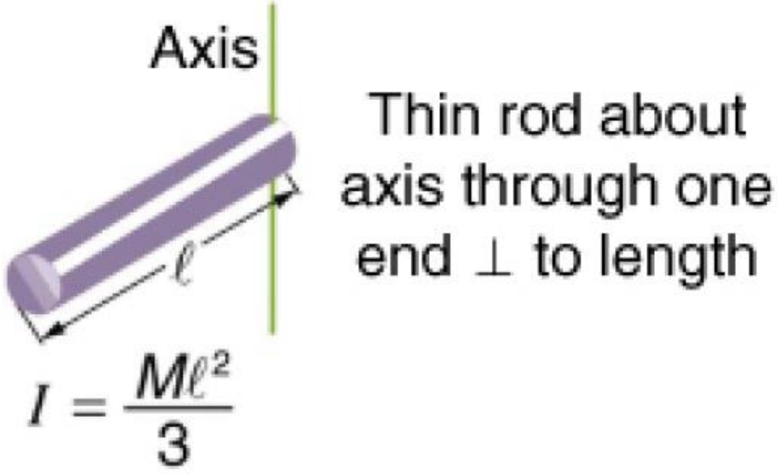
Surface density： σ

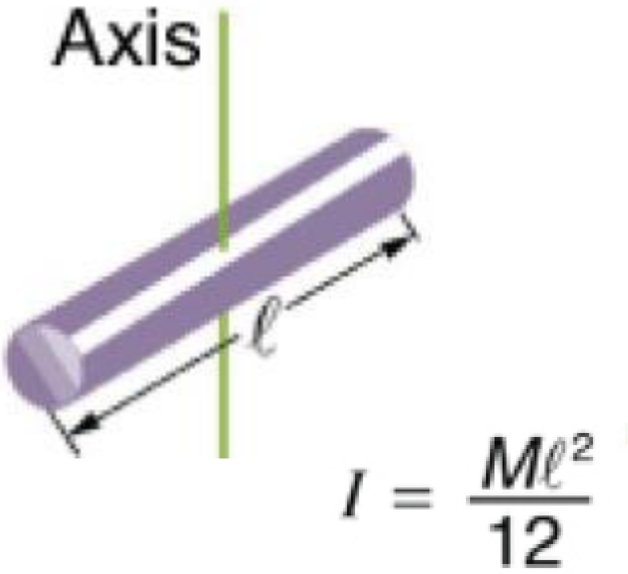
Volume density： ρ



Moment of Inertia

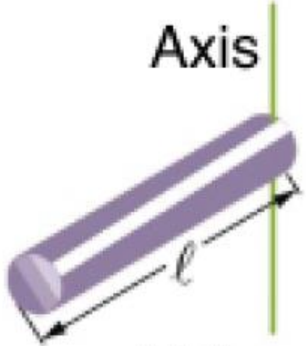
Use integral method to calculate I of these two: linear density



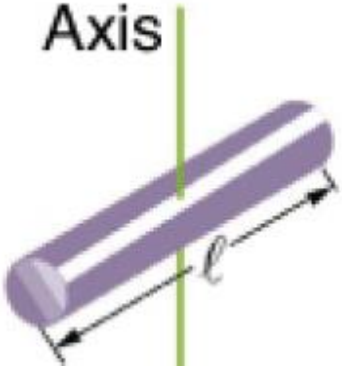


Parallel-axis theorem

$$\boxed{I_o = I_c + mh^2 = \frac{1}{12}mL^2 + m(\frac{1}{2}L)^2 = \frac{1}{3}mL^2}$$



$$I = \frac{M\ell^2}{3}$$



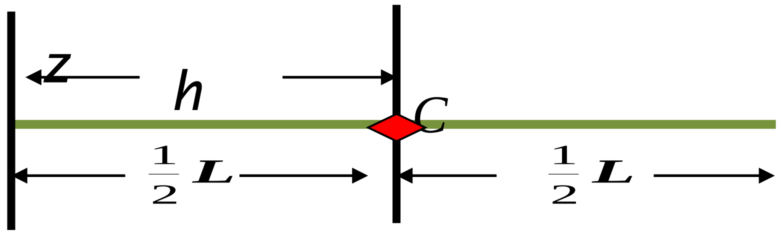
$$I = \frac{M\ell^2}{12}$$

Parallel-axis theorem

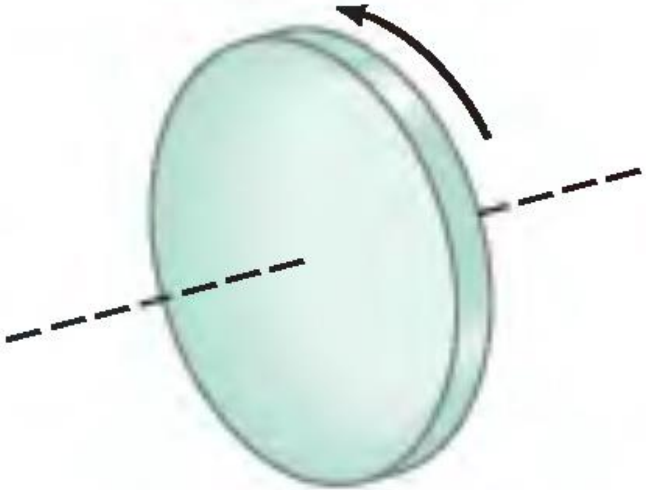
Let I_c be the rotational inertia of body with respect to the center of mass, and h be the perpendicular distance between given axis and the axis through center of mass (these two axes must be parallel).

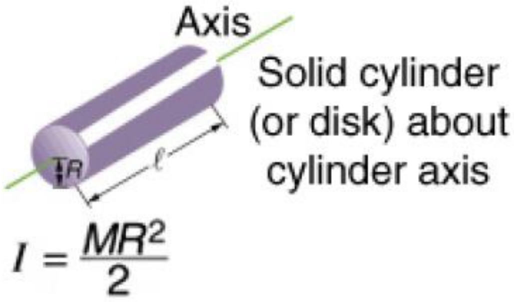
The rotational inertia I about the given axis is:

$$\boxed{I = I_C + Mh^2}$$



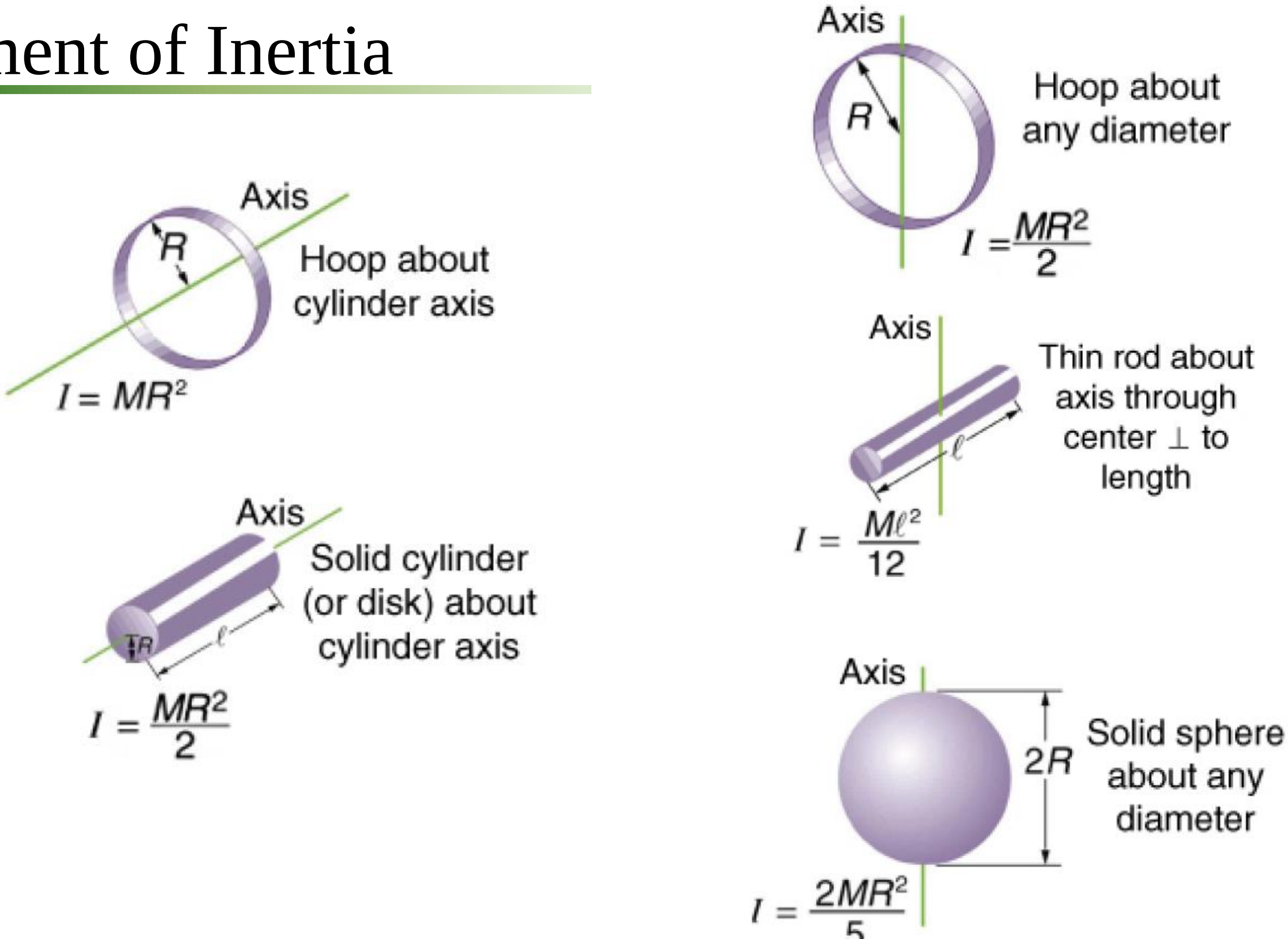
Solid cylinder



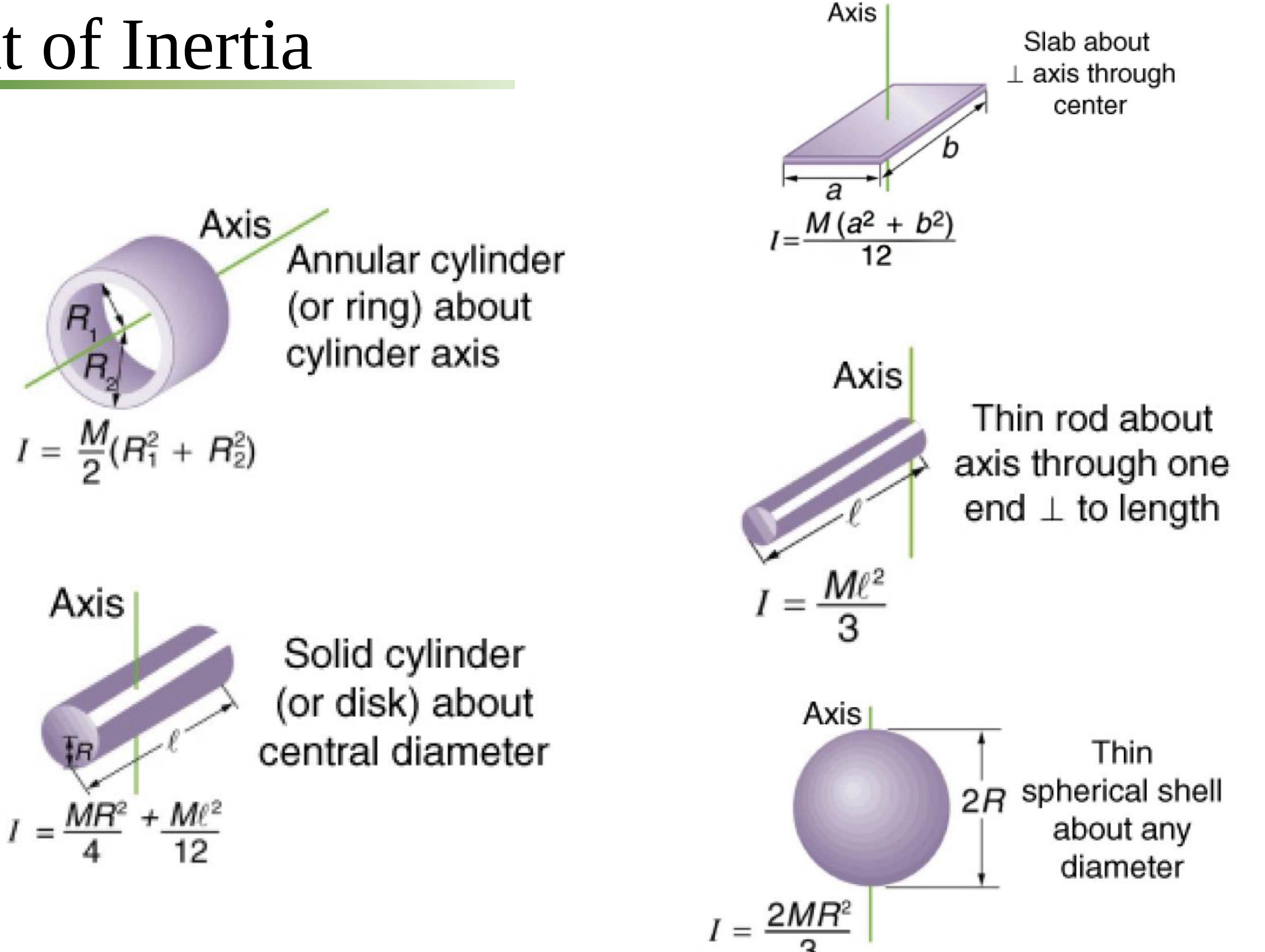


Area density

Moment of Inertia



Moment of Inertia

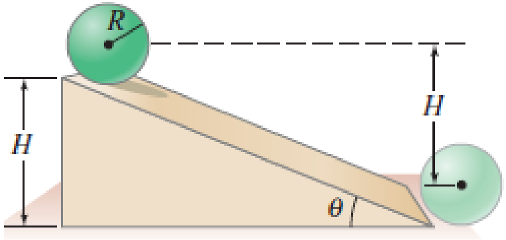


Rotational Kinetic Energy

Linear kinetic Energy $$KE = \frac{1}{2}mv^2$$

Rotational kinetic Energy $$KE = \frac{1}{2}I\omega^2$$

How about rotates while its center of mass undergoes translational motion?

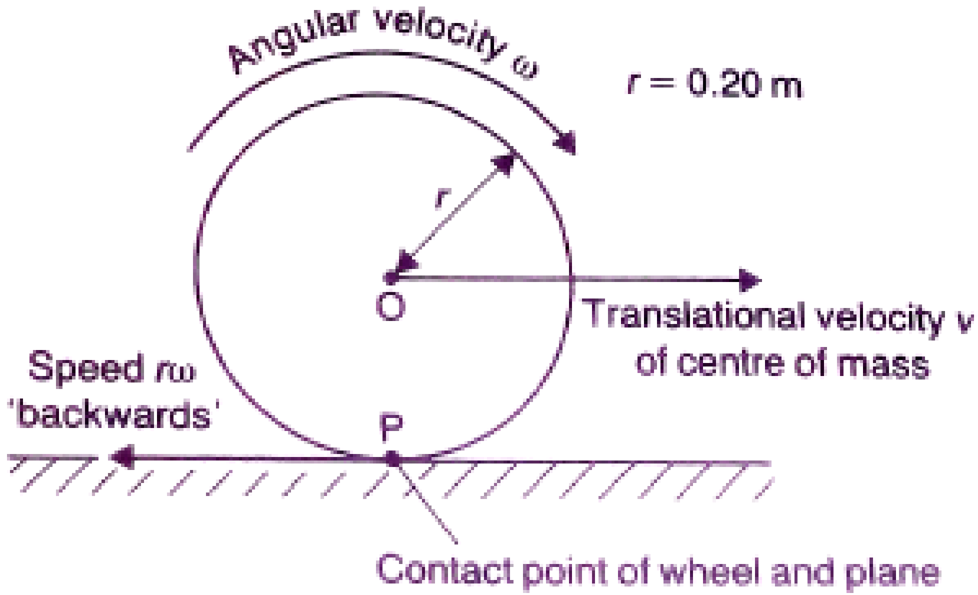


$$KE = \frac{1}{2}mv_{\mathrm{CM}}{}^2 + \frac{1}{2}I_{\mathrm{CM}}\omega^2$$

Example

Calculate the kinetic energy of a cylinder of mass 12 kg and radius 0.20 m, rolling along a horizontal plane with translational velocity of 0.30 ms^{-1}.

The moment of inertia of the cylinder is 0.24 kgm^2.



Example

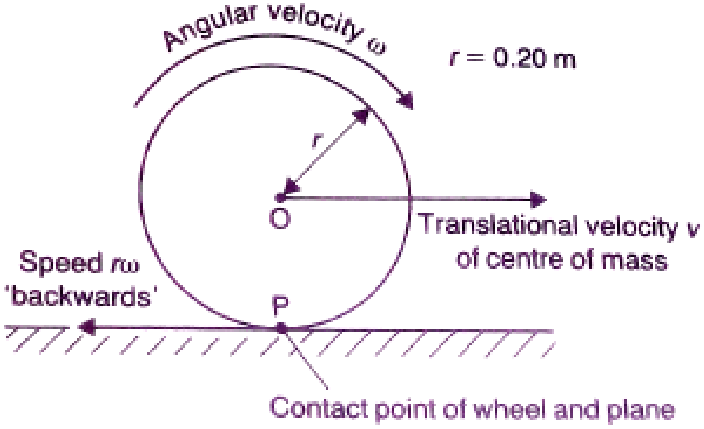
No sliding because it is rolling, so P must be stationary at instant of contact, hence forwards velocity must be cancelled by the velocity $v = \omega r$ in the opposite direction, so $\omega = v/r$.

$KE_{TOTAL} = KE_{TRANS} + KE_{ROT}$

$= ½\, mv^2 + ½\, I\omega^2$

$= ½ \times 12 \text{ x } 0.3^2 + ½ \times 0.24 \times 1.5^2$

$= 0.81$ J



Example

CHAPTER-OPENING QUESTION—Guess now!

A solid ball and a solid cylinder roll down a ramp. They both start from rest at the same time and place. Which gets to the bottom first?

(a) They get there at the same time.
(b) They get there at almost exactly the same time except for frictional differences.
(c) The ball gets there first.
(d) The cylinder gets there first.
(e) Can't tell without knowing the mass and radius of each.

How about a ball and a box

Example

47. (III) An *Atwood machine* consists of two masses, $m_A = 65$ kg and $m_B = 75$ kg, connected by a massless inelastic cord that passes over a pulley free to rotate, Fig. 8–52. The pulley is a solid cylinder of radius $R = 0.45$ m and mass 6.0 kg. (*a*) Determine the acceleration of each mass. (*b*) What % error would be made if the moment of inertia of the pulley is ignored? [*Hint*: The tensions F_{TA} and F_{TB} are not equal. We discussed the Atwood machine in Example 4–13, assuming $I = 0$ for the pulley.]

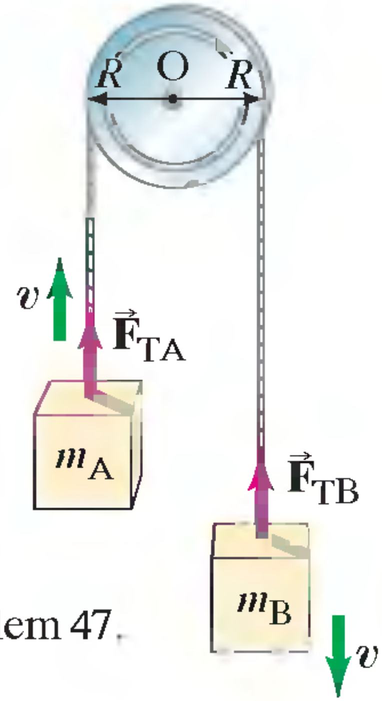


FIGURE 8–52 Problem 47. Atwood machine.

Solution:

Draw the free-body diagram for each mass

-

$$\sum F_{y\mathrm{A}} = F_{\mathrm{TA}} - m_{\mathrm{A}}g = m_{\mathrm{A}}a \quad \rightarrow \quad F_{\mathrm{TA}} = m_{\mathrm{A}}g + m_{\mathrm{A}}a$$

$$\sum F_{y\mathrm{B}} = m_{\mathrm{B}}g - F_{\mathrm{TB}} = m_{\mathrm{B}}a \quad \rightarrow \quad F_{\mathrm{TB}} = m_{\mathrm{B}}g - m_{\mathrm{B}}a$$

$$\sum \tau = F_{\mathrm{TB}}r - F_{\mathrm{TA}}r = I\alpha = I\frac{a}{R}$$

Angular Momentum

Linear momentum $p = mv$

Angular momentum $L = I\omega$

Force and Torque $F = \frac{\Delta p}{\Delta t}$ $\text{net}\,\tau = \frac{\Delta L}{\Delta t}$

Conservation **If** $\text{net}\,\tau = 0$ **then angular momentum is conserved**

Conservation of Angular Momentum

How could you spin faster?

If $\text{net}\,\tau = 0$ **then angular momentum is conserved**

$L = I\omega$ remains the same

If $I \searrow$ then $\omega \nearrow$

$I = mr^2$ is the rotational inertia

What if you change the radius r ?

CONCEPTUAL

Physics

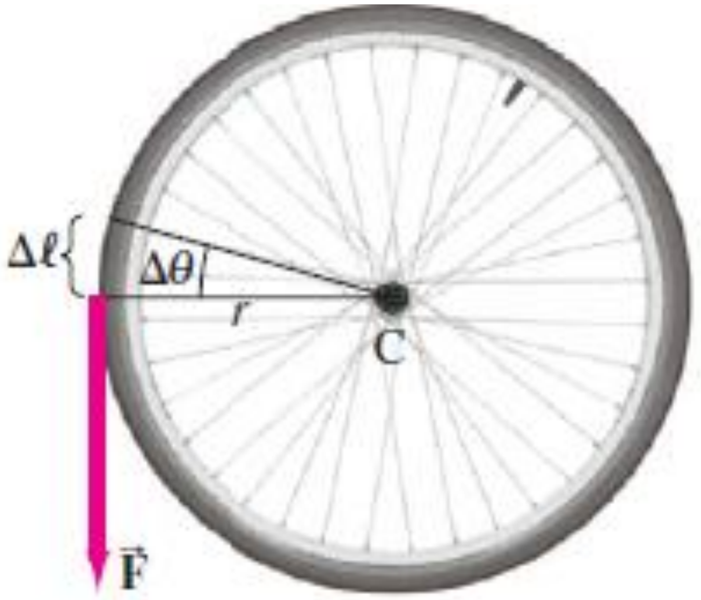
with Paul G. Hewitt

Work done by Torque

$$W = F\,\Delta\ell = Fr\,\Delta\theta.$$

Because $\tau = rF$, then

$$W = \tau\,\Delta\theta$$



The conservations in the linear system also works in the rotational system.

Conclusion

Group

Translation	Rotation	Connection
x		
v		
a		
m		
F		
$\mathrm{KE} = \frac{1}{2}mv^2$		
$p = mv$		
$W = Fd$		
$\Sigma F = ma$		
$\Sigma F = \frac{\Delta p}{\Delta t}$		

Rotational motion