ENSC 2113
Engineering Mechanics: Statics

Chapter 2:

Force Vectors

(Sections 2.5-2.6)

COLLEGE OF
ENGINEERING, ARCHITECTURE
AND TECHNOLOGY

Chapter 2 Outline:

2.1 Scalars and Vectors

2.2 Vector Operations

2.3 Vector Addition of Forces

2.4 Addition of a System of Coplanar Forces

2.5 Cartesian Vectors

2.6 Addition of Cartesian Vectors

2.7 Position Vectors

2.8 Force Vector Directed Along a Line

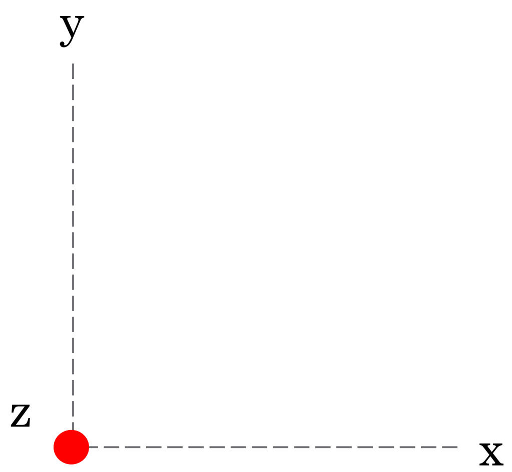
2.9 Dot Product

Chapter 2 Objectives:

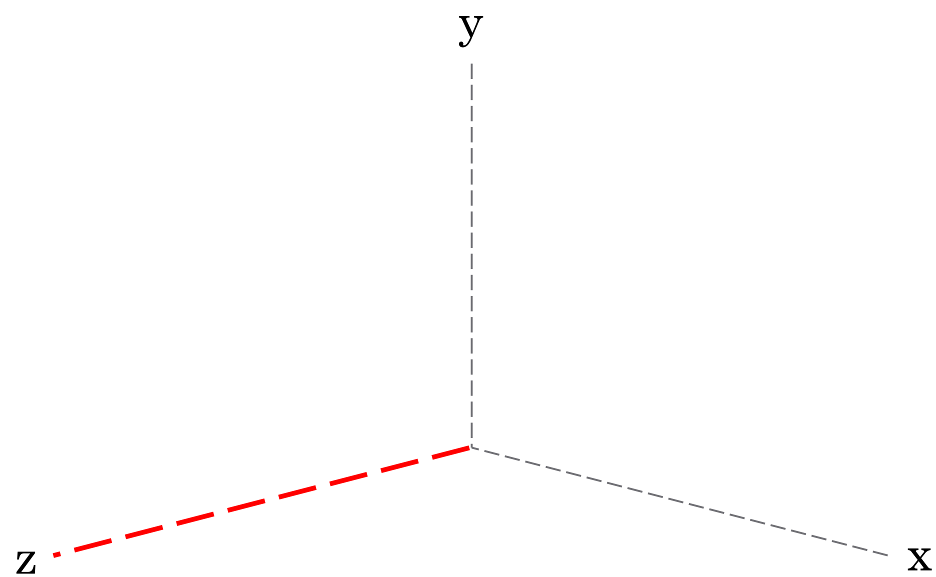
- To show how to add forces and resolve them into components using the Parallelogram Law
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another

2.5 Cartesian Vectors:

- 3-D coordinate system



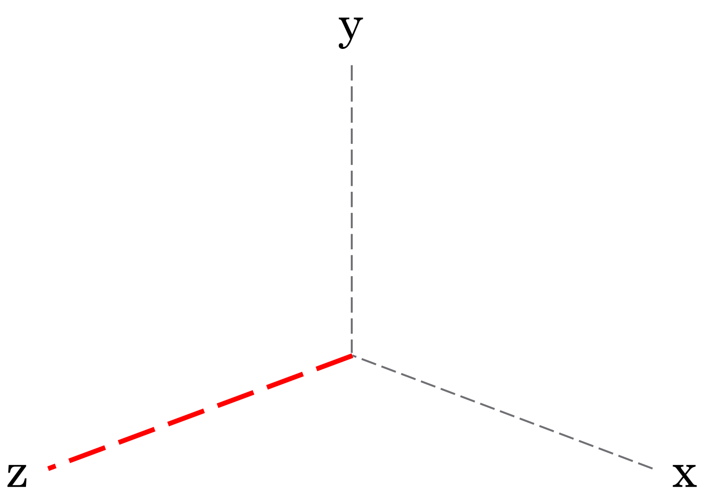
In the 2-D system, the z-axis is coming out of the page

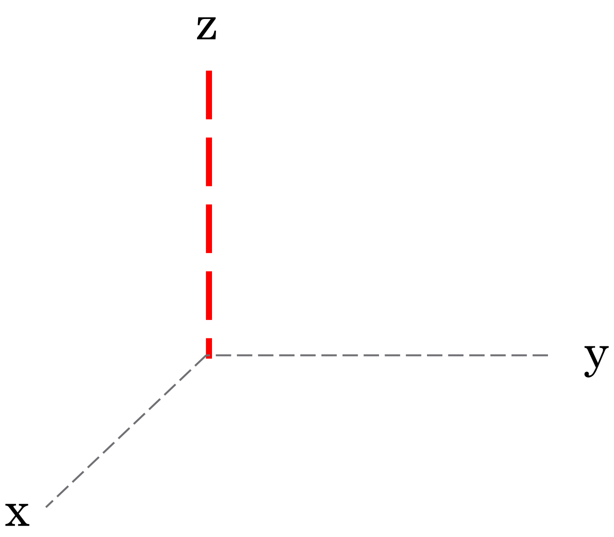


Rotating this system clockwise about the y-axis, the z-axis becomes visible

2.5 Cartesian Vectors:

- 3-D coordinate system

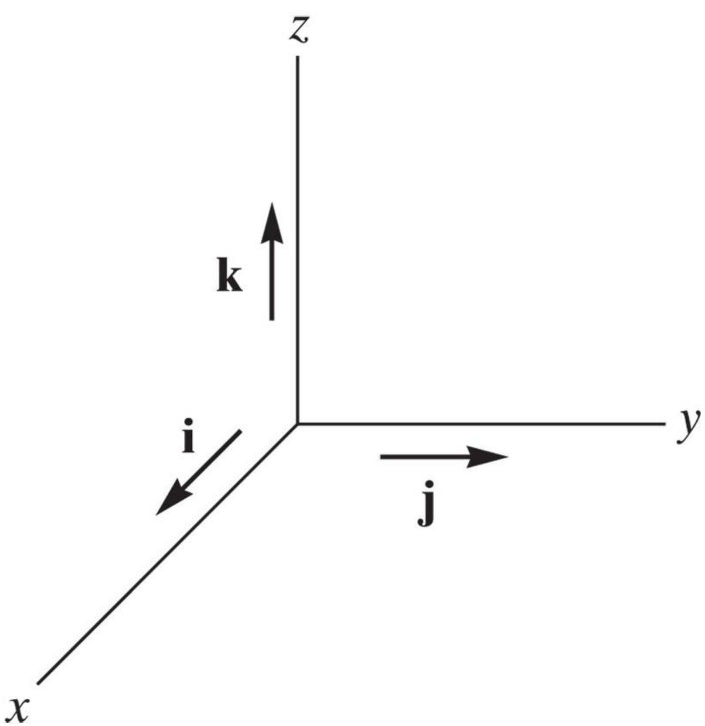


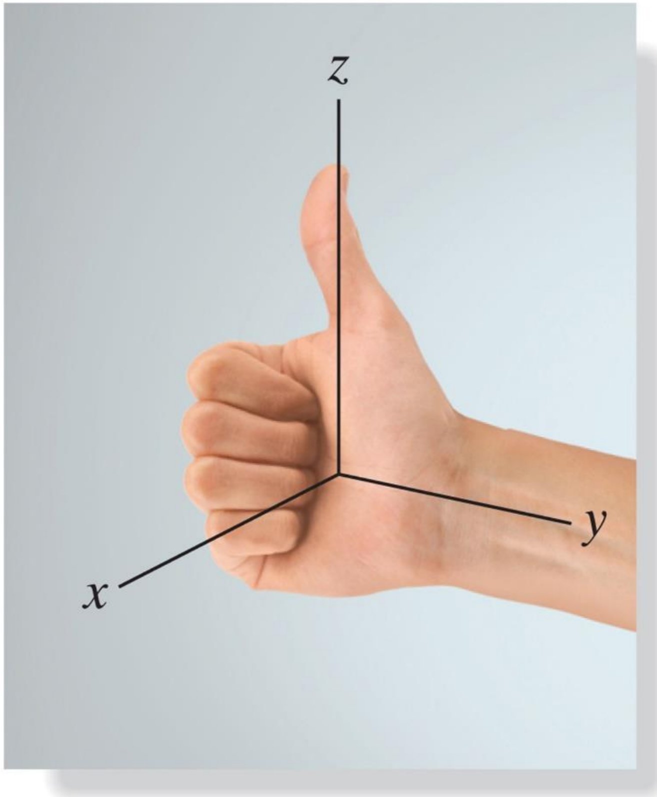


Rotate this system 90 degrees

2.5 Cartesian Vectors:

- 3-D coordinate system





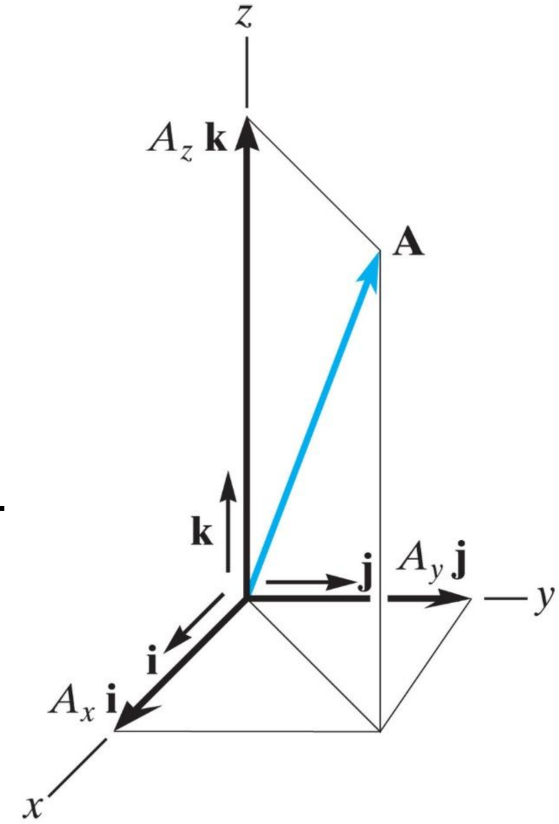
2.5 Cartesian Vectors:

- Cartesian Vector Representation

$$\vec{A} = \{A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\}$$

- Magnitude of a Vector

$$A = \sqrt{(A_x)^2 + \left(A_y\right)^2 + (A_z)^2}$$



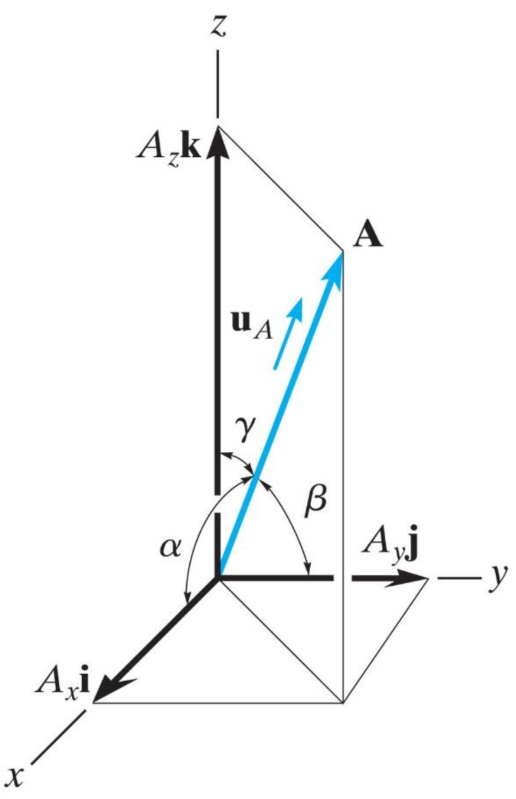
2.5 Cartesian Vectors:

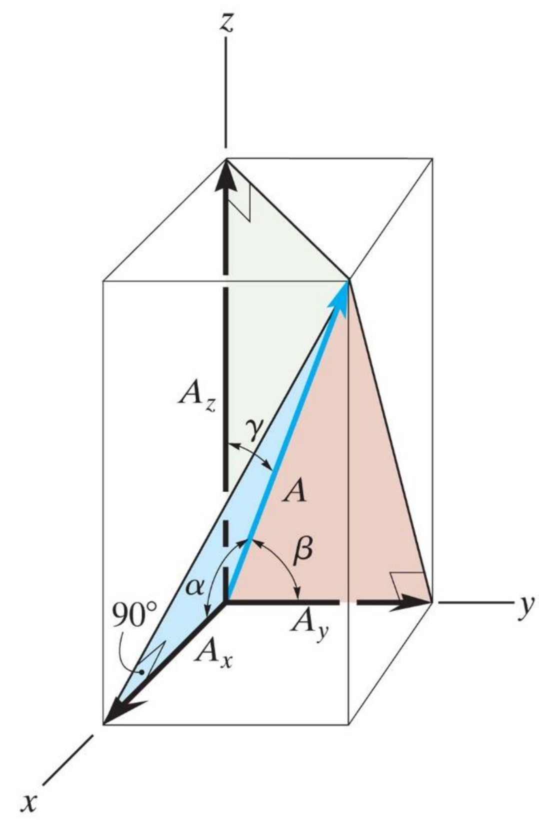
- Direction Angles or *Direction Cosines*

α = angle between the vector and the positive x-axis

β = angle between the vector and the positive y-axis

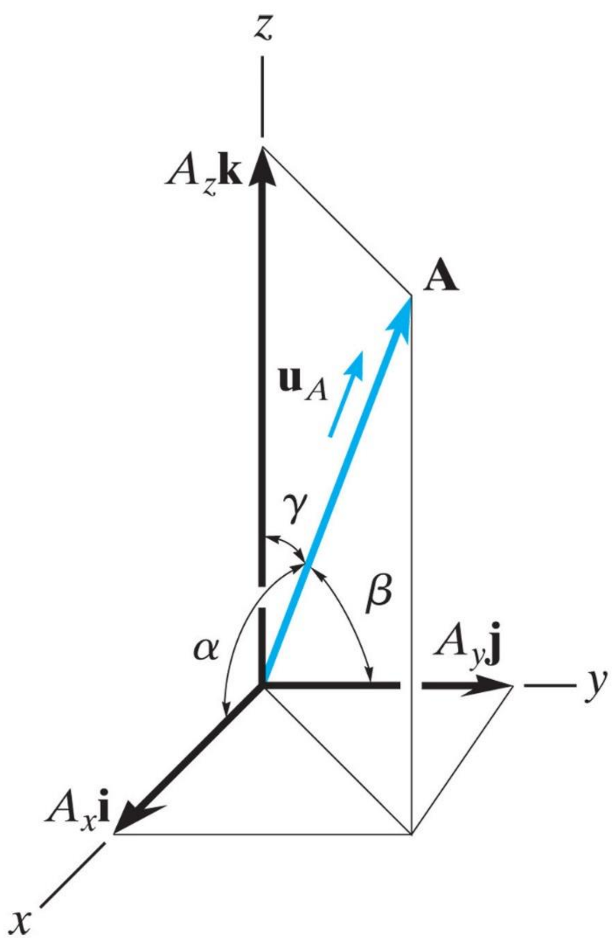
γ = angle between the vector and the positive z-axis





2.5 Cartesian Vectors:

- Direction Angles or *Direction Cosines*



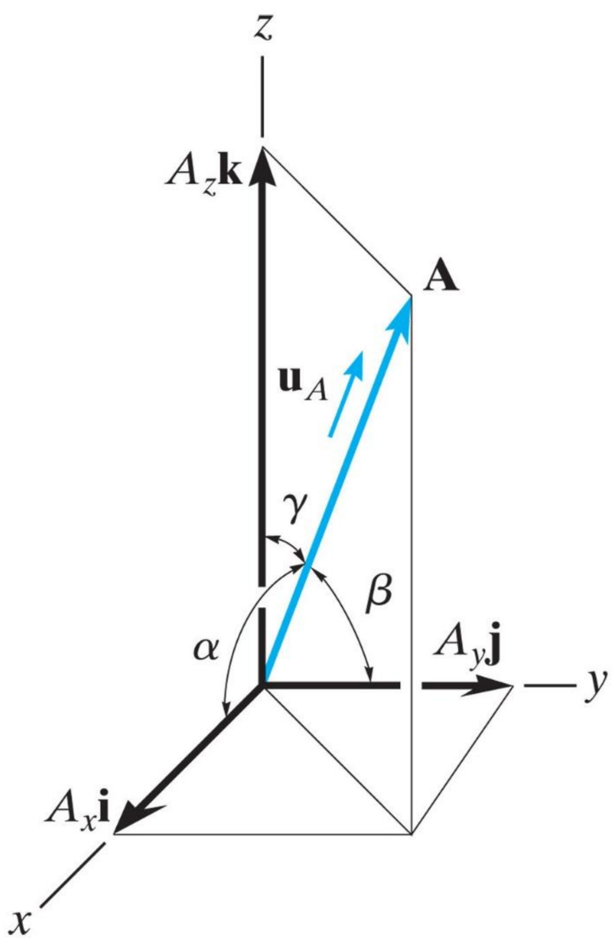
- Finding the components from the magnitude and direction angles:

$$A_x = A\cos\alpha$$
$$A_y = A\cos\beta$$
$$A_z = A\cos\gamma$$

Note: angles >90 degrees produce negative components

2.5 Cartesian Vectors:

- Solving for the direction angles



- Finding the direction angles from the magnitude and components

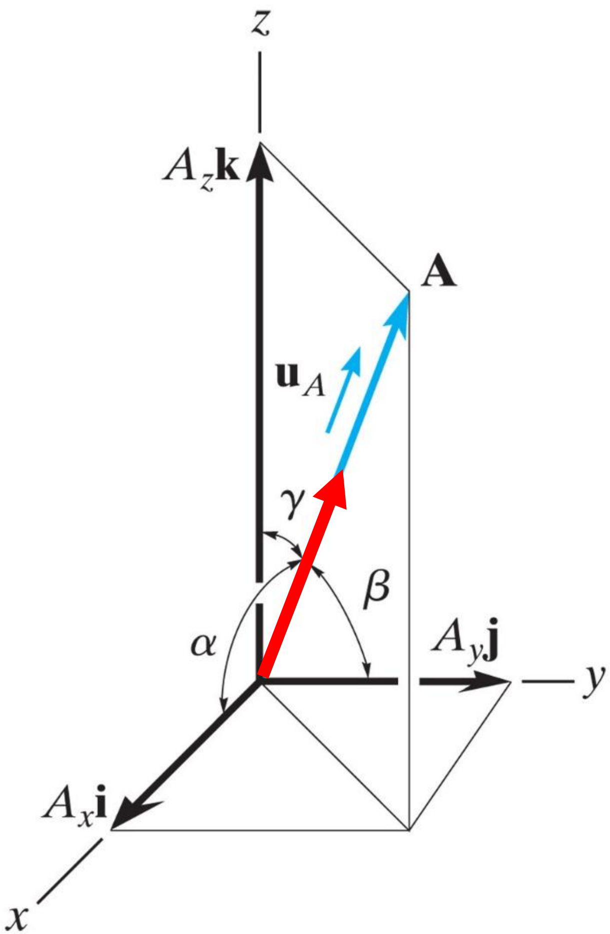
$$\alpha = \cos^{-1}\frac{A_x}{A}$$

$$\beta = \cos^{-1}\frac{A_y}{A}$$

$$\gamma = \cos^{-1}\frac{A_z}{A}$$

2.5 Cartesian Vectors:

- Unit vector – using direction angles



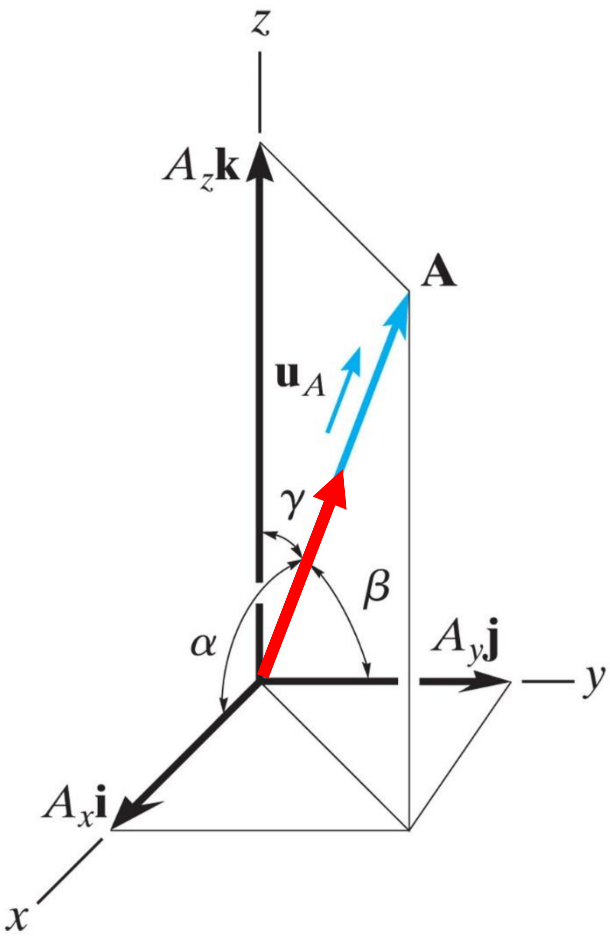
$$\vec{A} = \{A\cos\alpha\,\hat{\imath} + A\cos\beta\,\hat{\jmath} + A\cos\gamma\,\hat{k}\}$$

- The unit vector is one unit in the same direction as the vector and can be found using the direction angles:

$$\vec{u} = \{\cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k}\}$$

2.5 Cartesian Vectors:

- Unit vector – using magnitude and components



The unit vector is one unit in the same direction as the vector and can be found by dividing the vector by the magnitude:

$$\vec{u} = \left\{\frac{A_x}{A}\hat{\imath} + \frac{A_y}{A}\hat{\jmath} + \frac{A_z}{A}\hat{k}\right\}$$

2.5 Cartesian Vectors

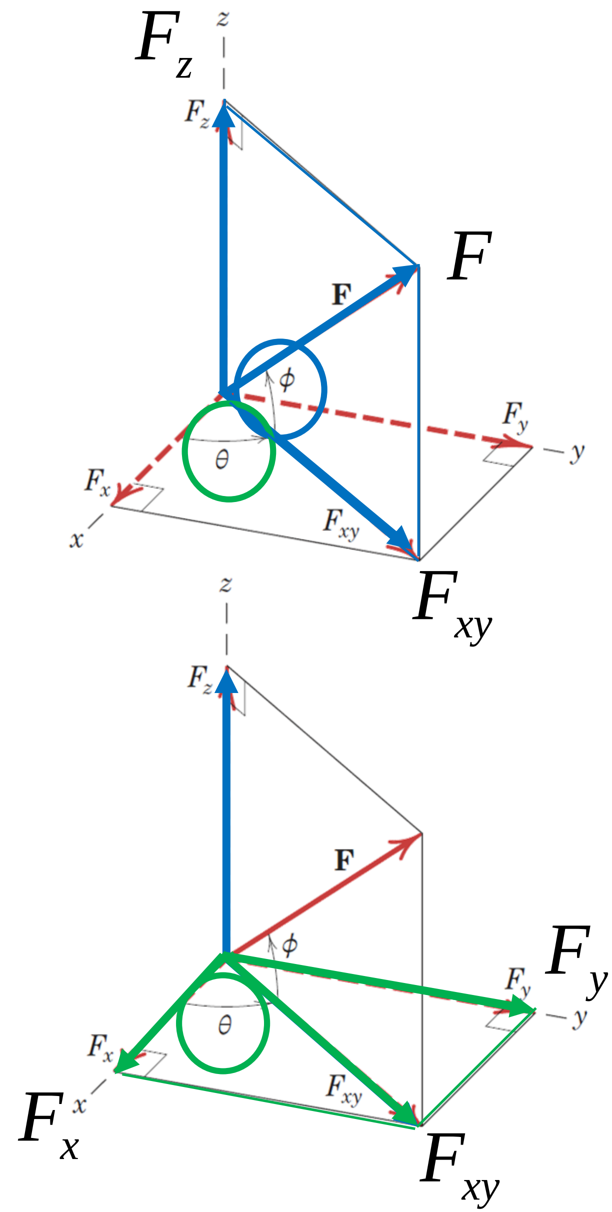
- Specification by two angles
  - Step 1: Horizontal and Vertical Components (z-xy plane)

$$\boxed{F_{xy}} = \boxed{F\cos\phi} \qquad F_z = F\sin\phi$$

  - Step 2: Components in x-y plane

$$F_x = F_{xy}\cos\theta = \boxed{F\cos\phi\cos\theta}$$

$$F_y = F_{xy}\sin\theta = \boxed{F\cos\phi\sin\theta}$$



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