Southwest Jiaotong University

Physics 2: **Electricity , Optics and Quanta**

Week 4 - Capacitor

2023.9

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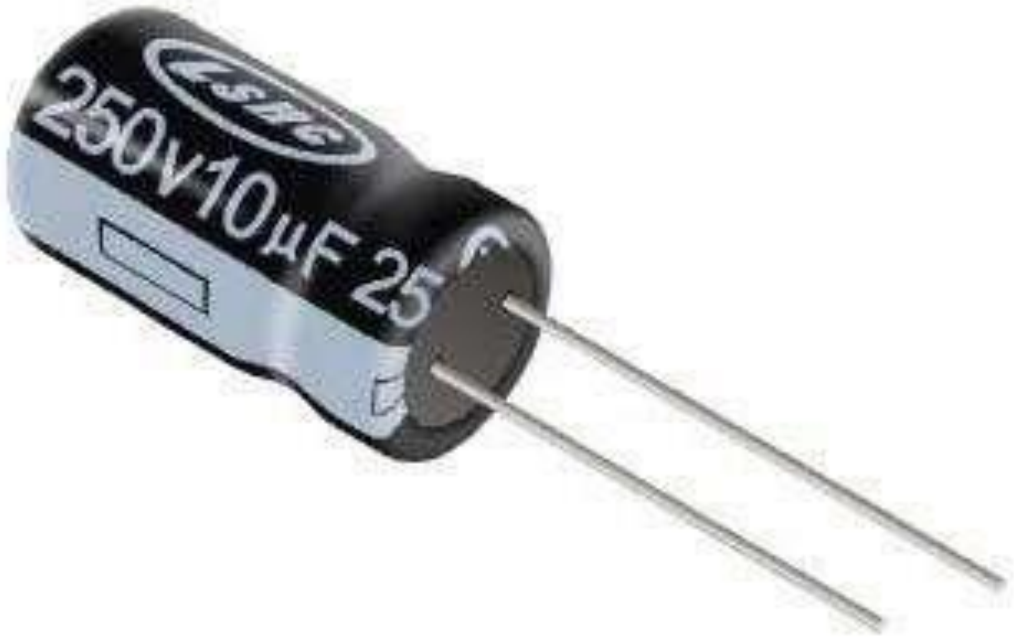
Outline

Basic properties of capacitor

- Connection in circuit

Charging or discharging a capacitor

Capacitors



$$\boxed{Q = CV}$$

$$\boxed{Q = \frac{A\varepsilon_0}{D}\Delta V}$$

capacitance

$$E \rightarrow \sigma \rightarrow Q \rightarrow V \Rightarrow C - \varepsilon_r$$

Capacitors

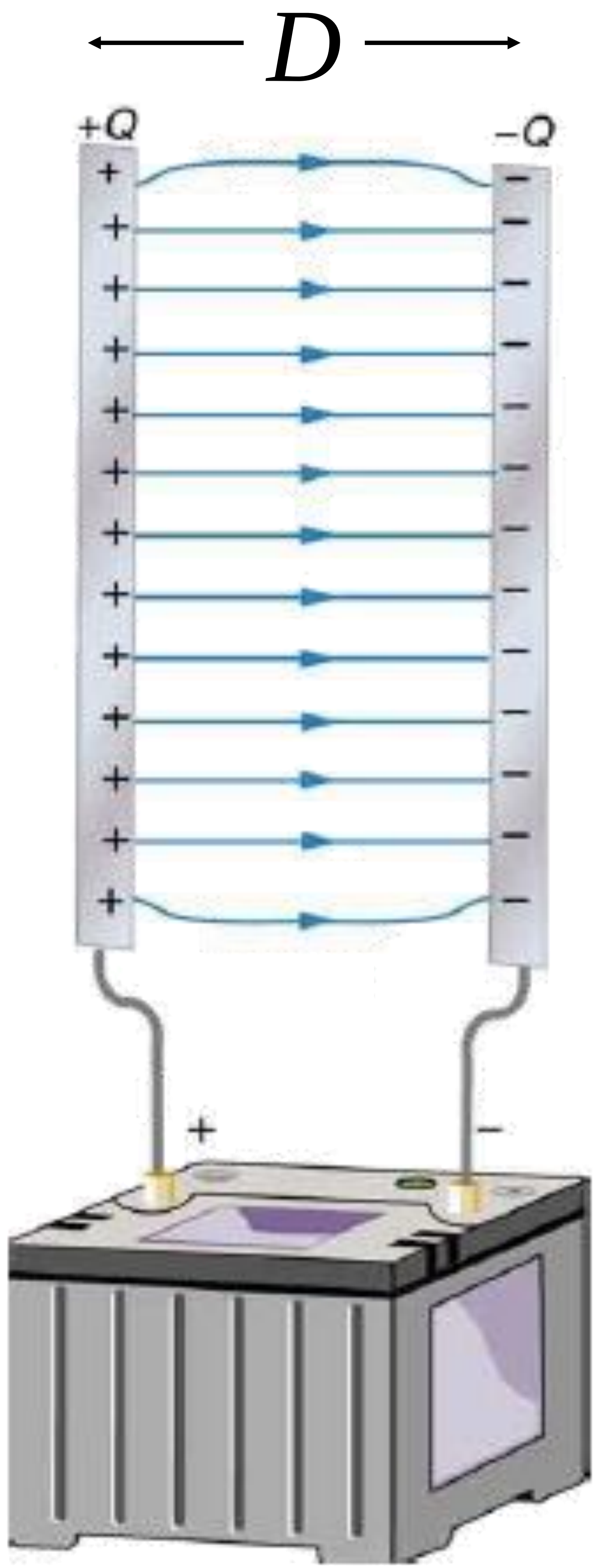
$$Q = \frac{A\varepsilon_0}{D}\Delta V$$

the charge on the plate(s) is **proportional** to the voltage

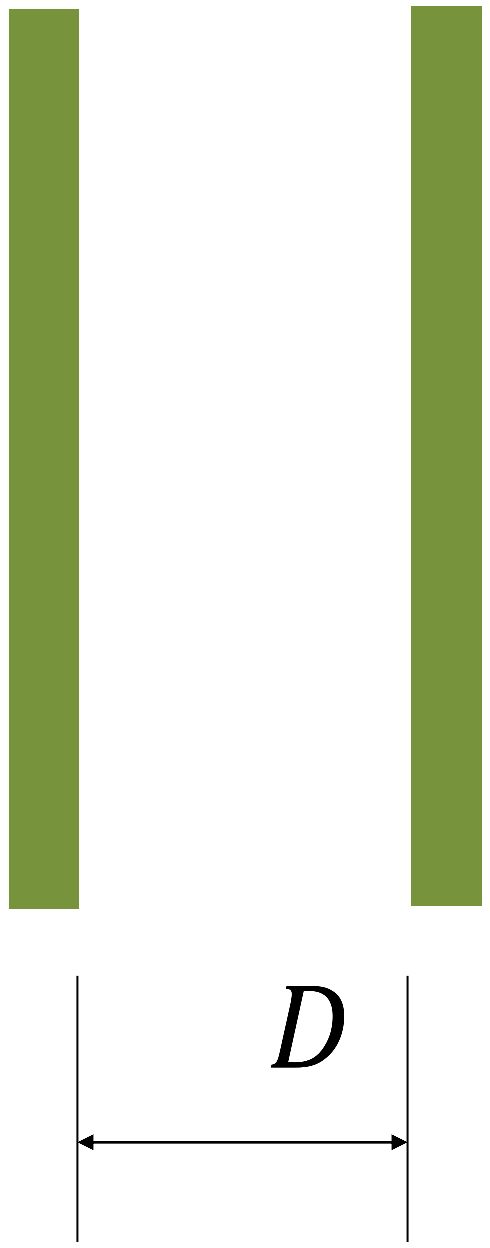
The coefficient is called the "**capacitance**"

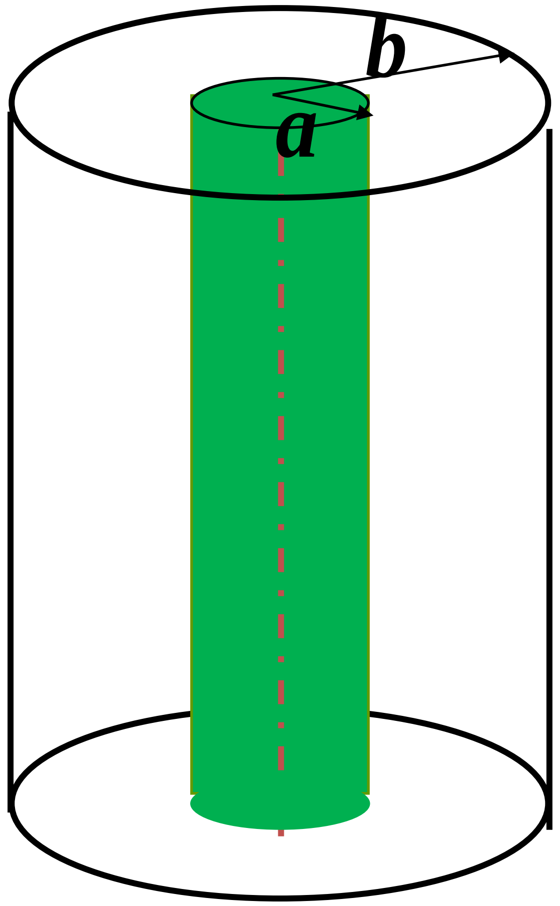
$$C = \frac{Q}{\Delta V}$$

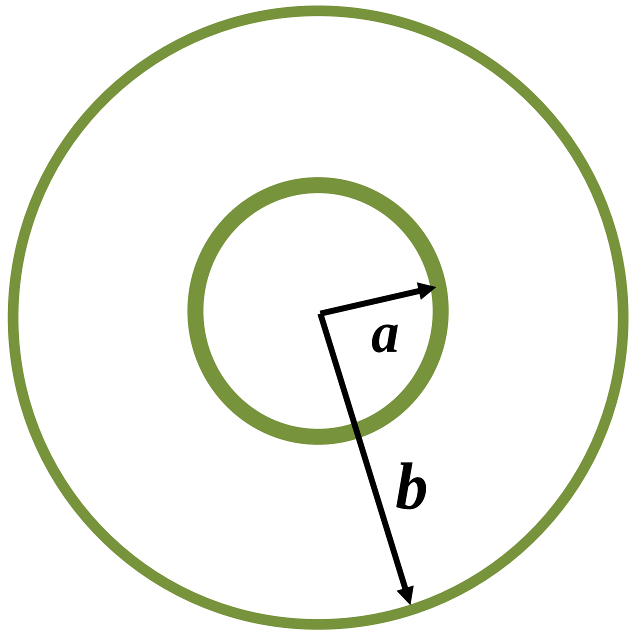
For parallel-plates $$C = \frac{A\varepsilon_0}{D}$$



Different shapes of capacitors







C depends on?

$$C = \frac{Q}{V}$$

Units – farad (F)

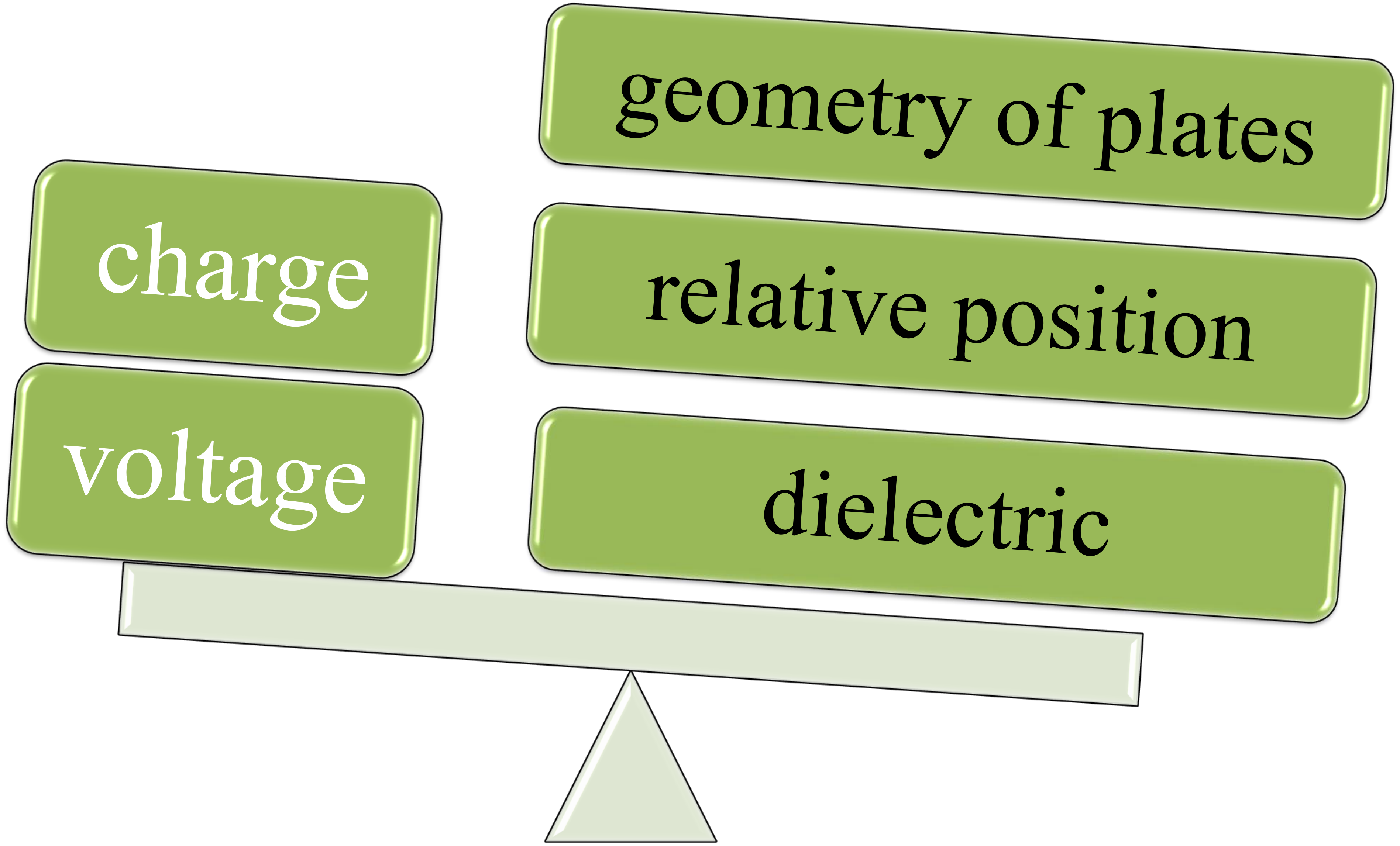
$$1\,F = \frac{1\ \text{Coulomb}}{1\ \text{Volt}}$$

Different shapes of capacitors

$$C = \frac{Q}{V}$$

$$C = \frac{A\varepsilon_0}{D}$$

C depends on

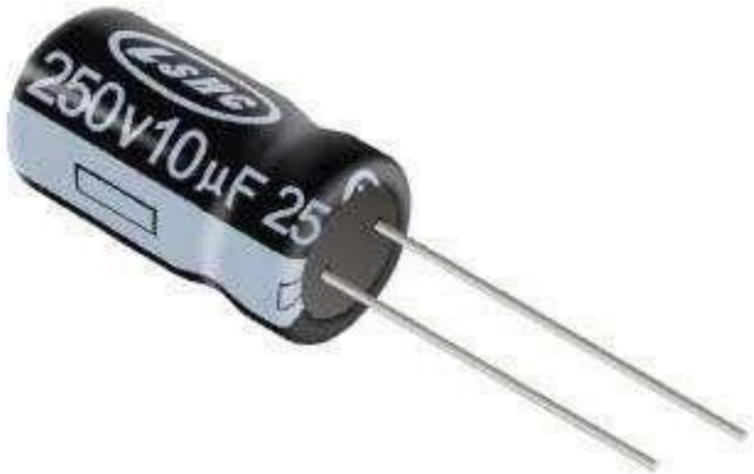


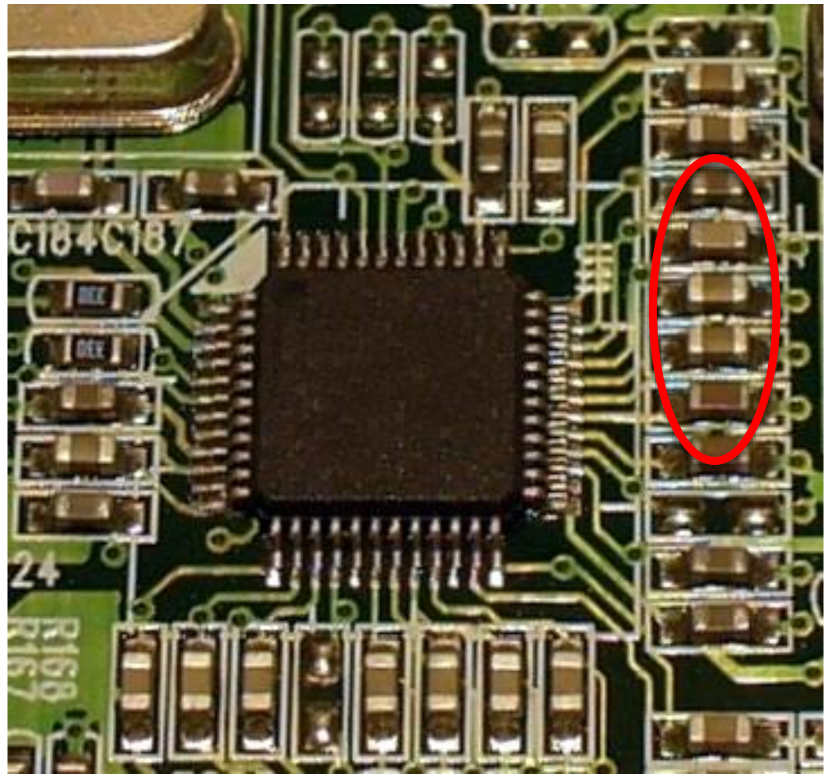
Capacitors

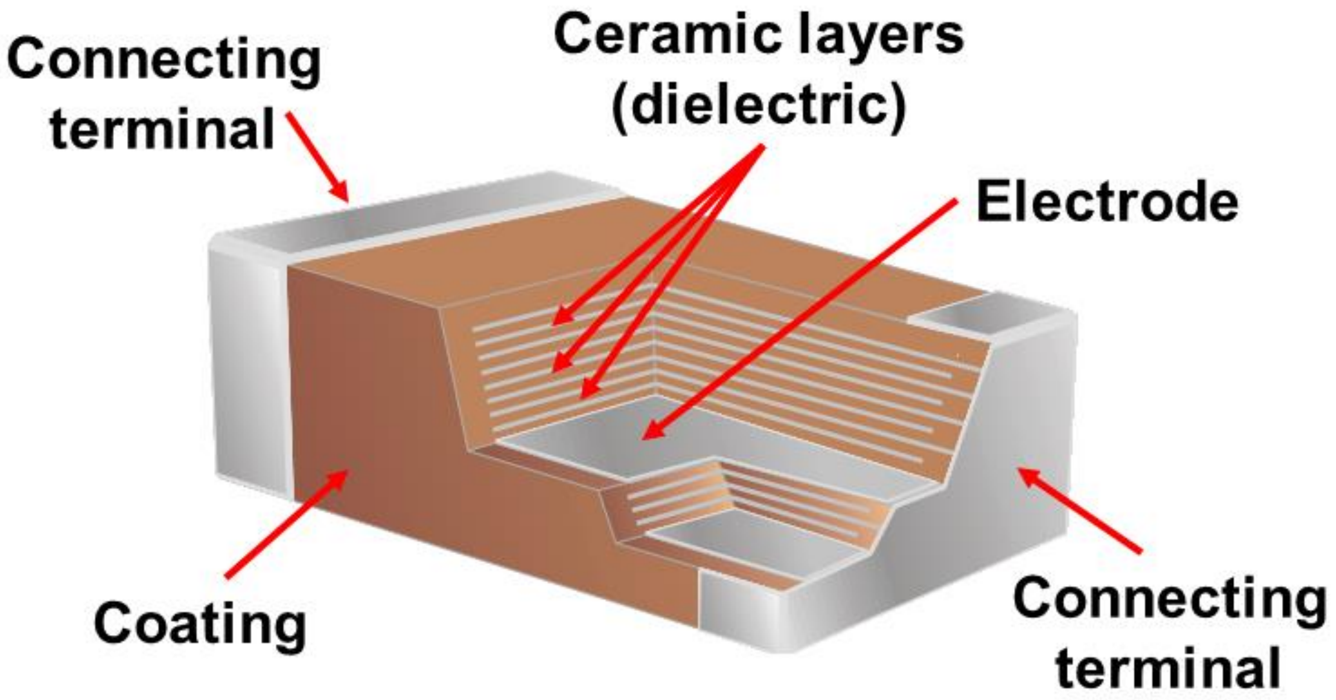
Electrolytic capacitors

non-solid (wet)

Ceramic capacitors



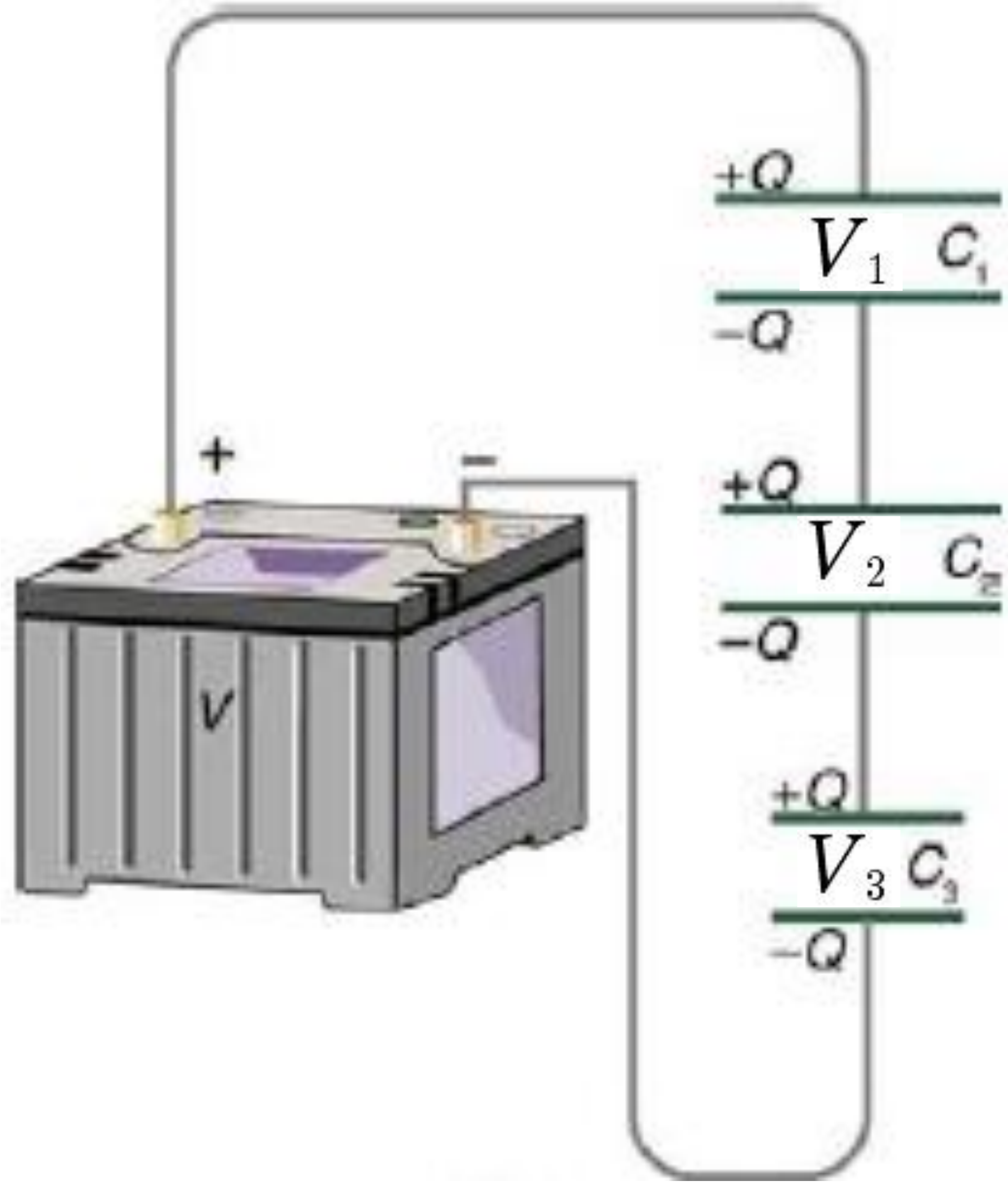




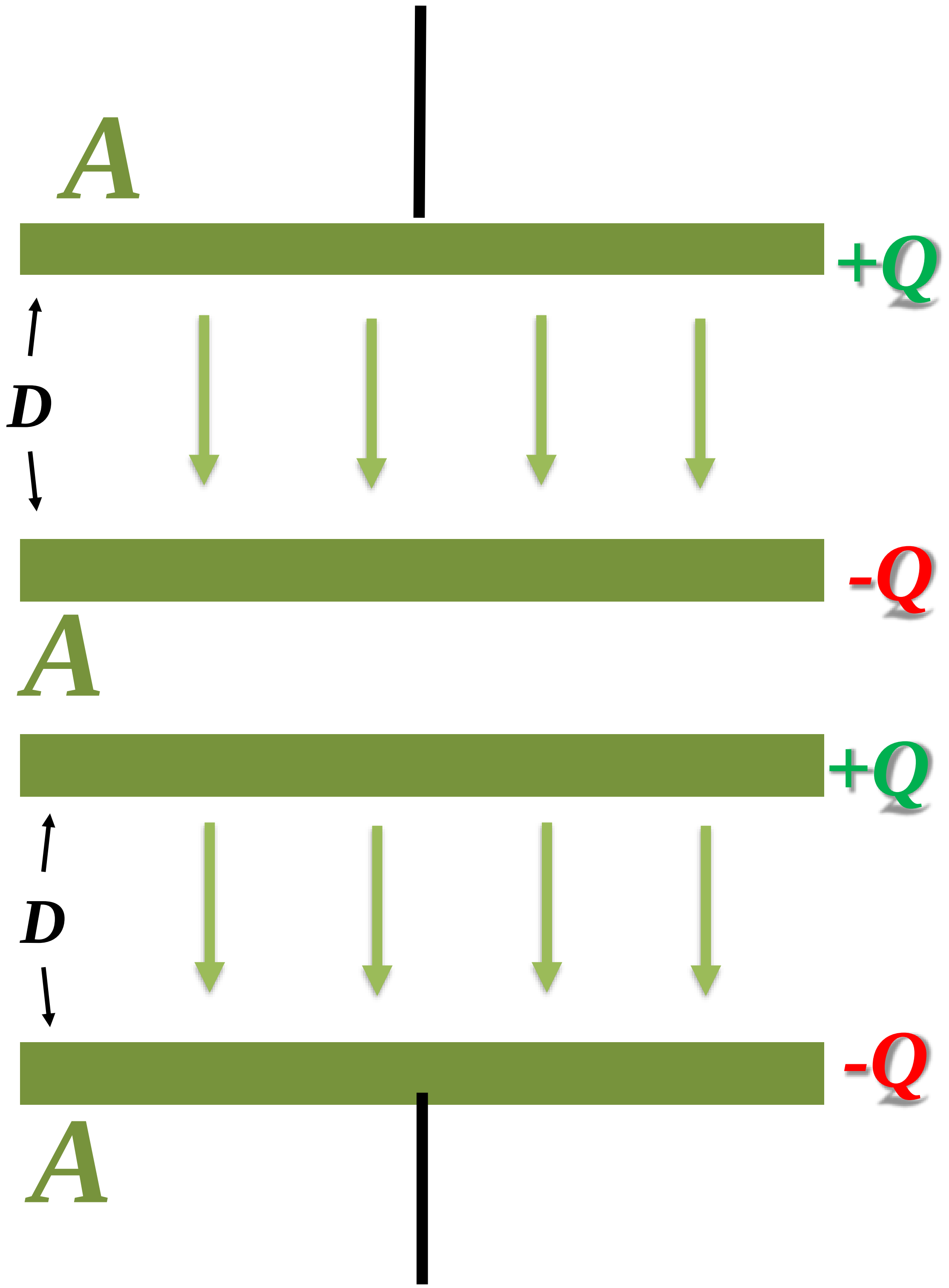
solid

Connecting capacitors “in series”

What is the “equivalent” capacitance ?



Connecting capacitors "in series"



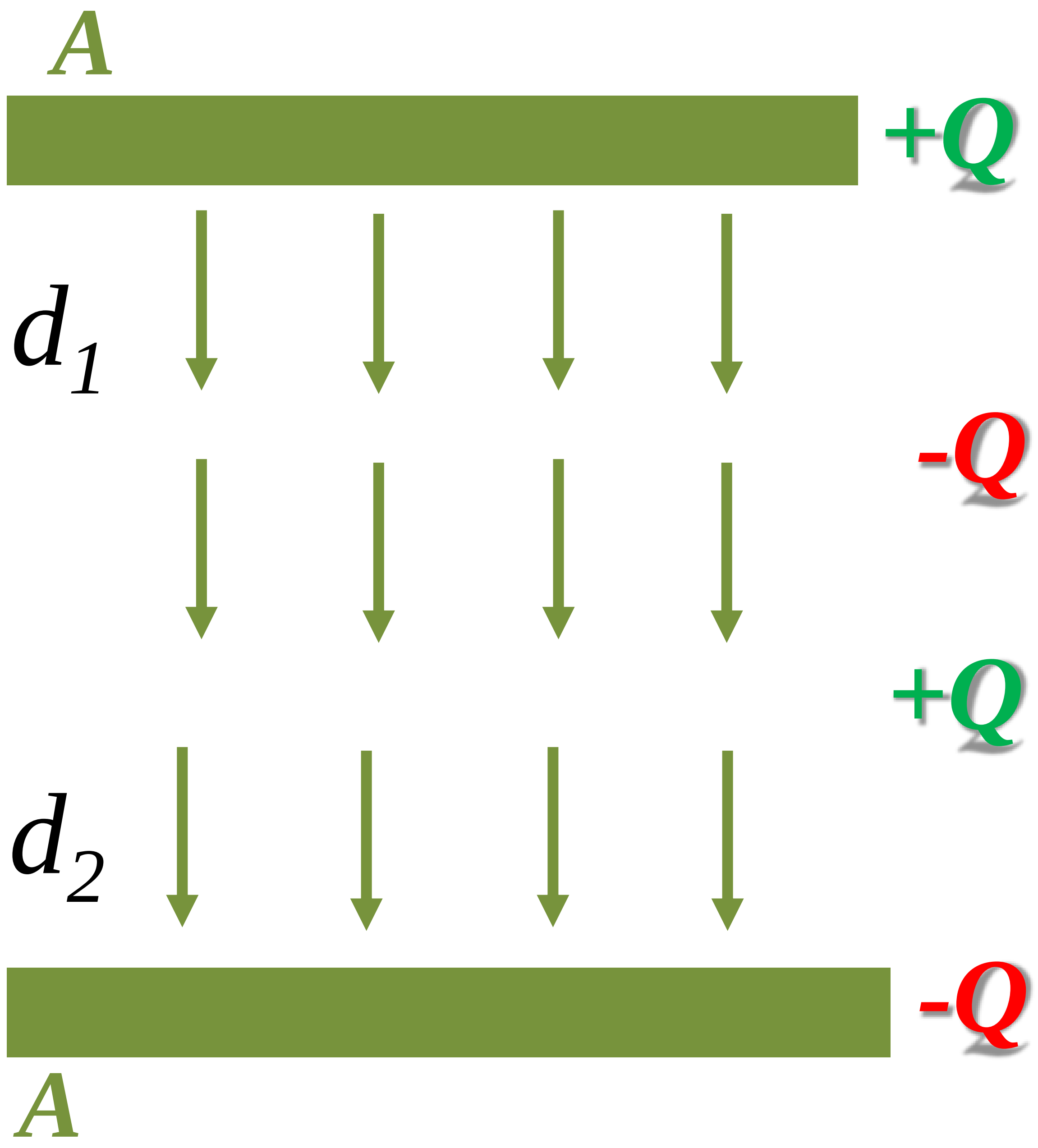
$$C = \frac{A\varepsilon_0}{D}$$

$$D \longrightarrow 2D$$

$$C \longrightarrow C/2$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C}$$

Capacitance change



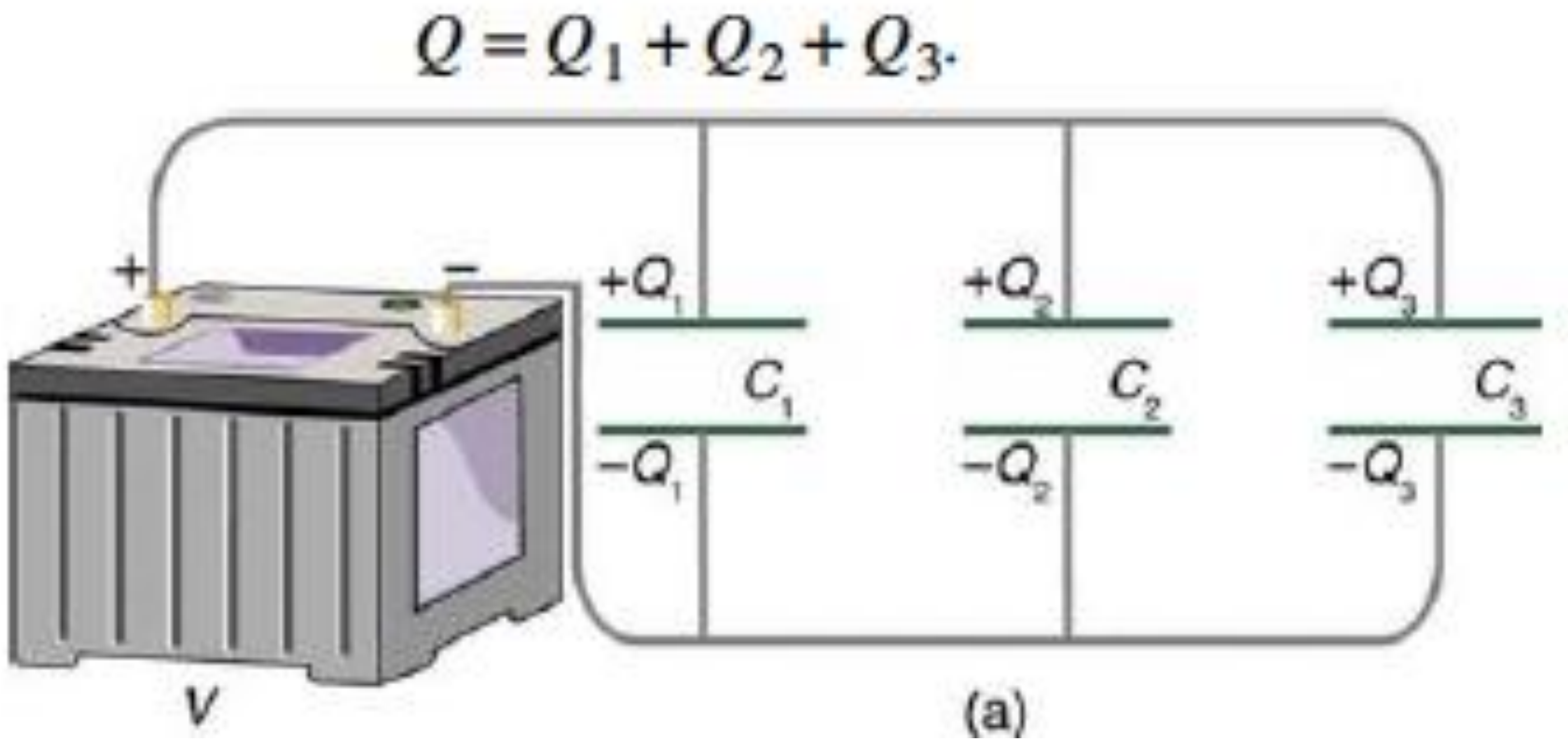
How will the capacitance change after the insertion of a piece of metal?

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1 + d_2}{A\varepsilon_0}$$

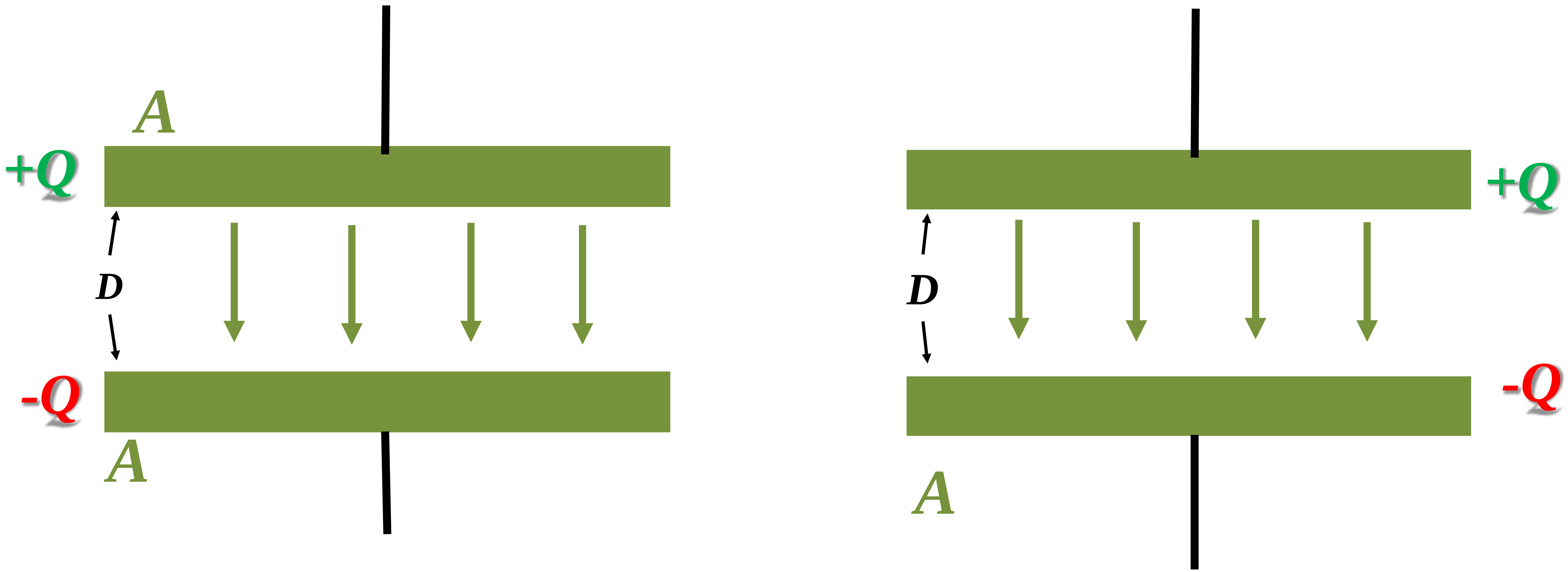
$$C_{eq} = \frac{A\varepsilon_0}{d_1 + d_2} > C = \frac{A\varepsilon_0}{D}$$

Connecting capacitors “in parallel”

What is the “equivalent” capacitance ?



Connecting capacitors "in parallel"



$$C = \frac{A\varepsilon_0}{D}$$

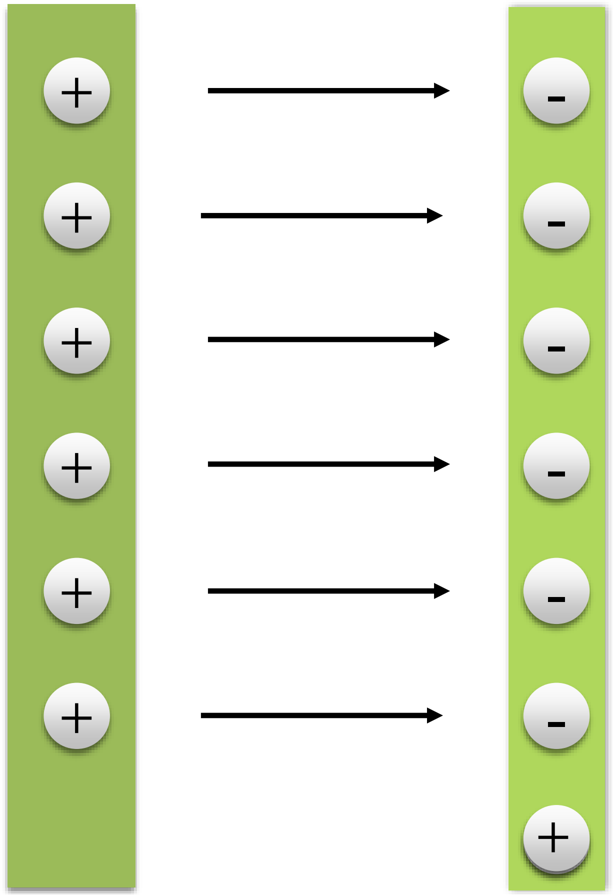
$A \rightarrow 2A$

$C \rightarrow 2C$

$$C_{eq} = C + C$$

Energy of the Capacitor

$Q = ne \qquad -Q = -ne$



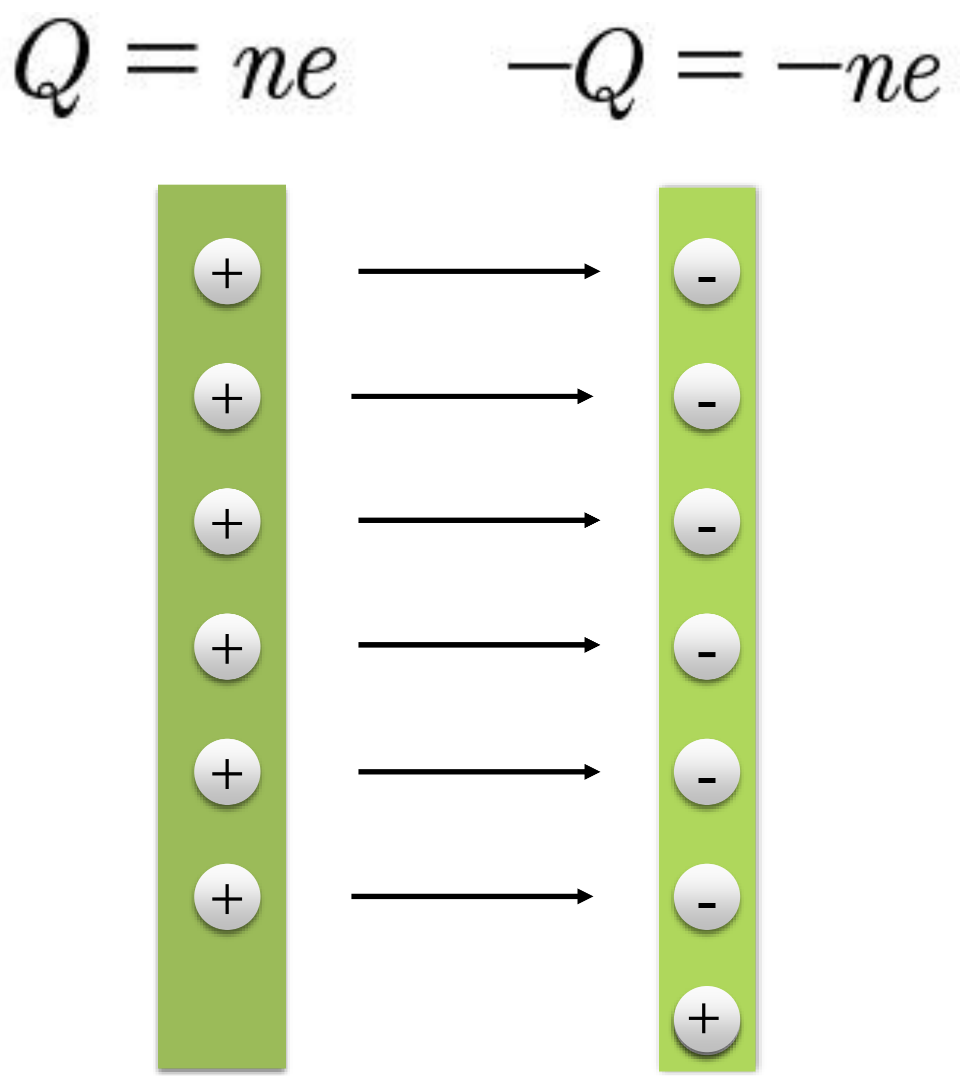
An energy is needed!

What is the energy stored in the Capacitor?

- Energy is required to bring a charge from one plate to the other, due to the electric force doing (negative) work

- The energy stored is the sum of the energy required to move all of the charges.

Energy of the Capacitor



*An energy of **e**V_{tot} is needed!*

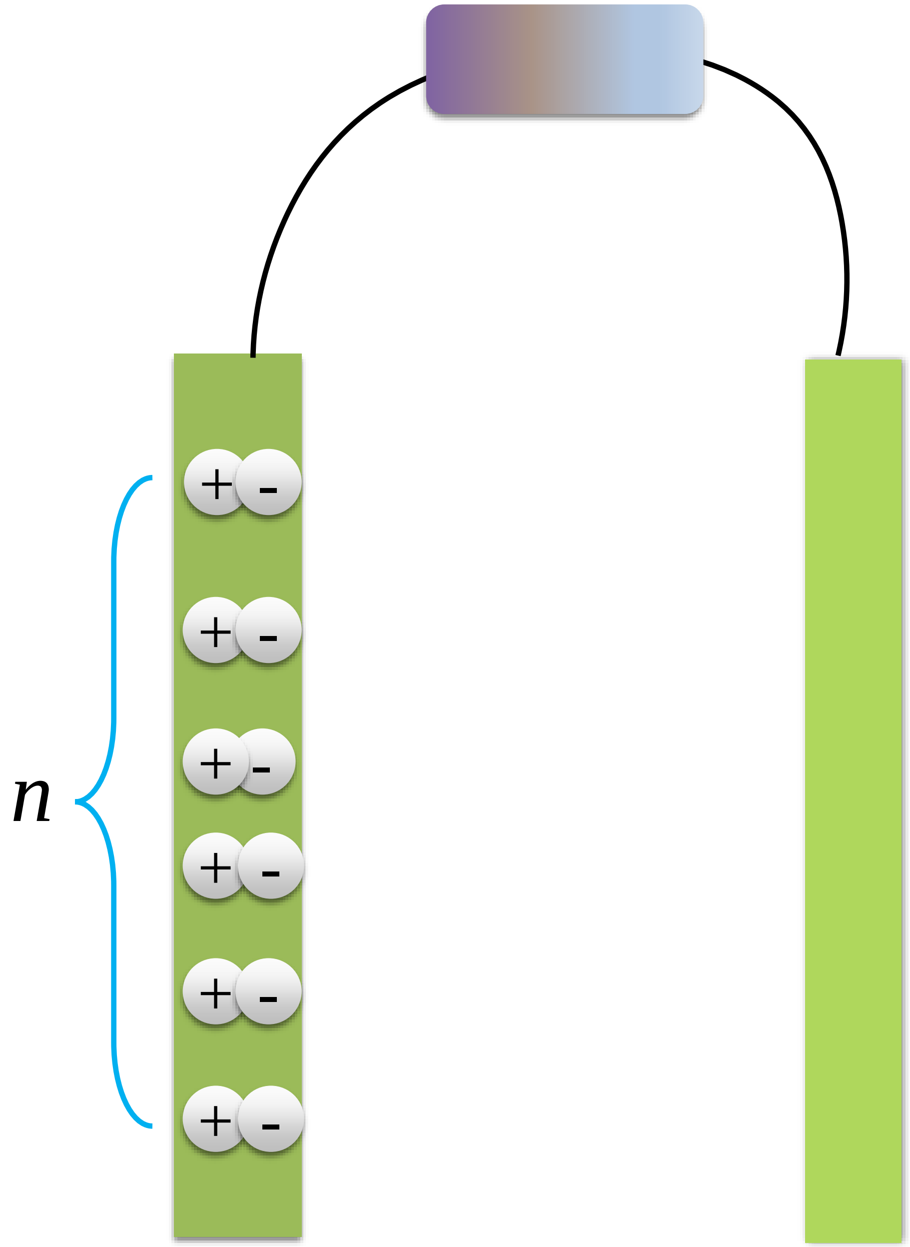
Capacitor with charge Q = *ne*

e = elementary charge

$$V_{tot} = \frac{Q}{C} = \frac{ne}{C}$$

- More energy is required when voltage is higher
- As you add charges, you need more and more energy to move a charge

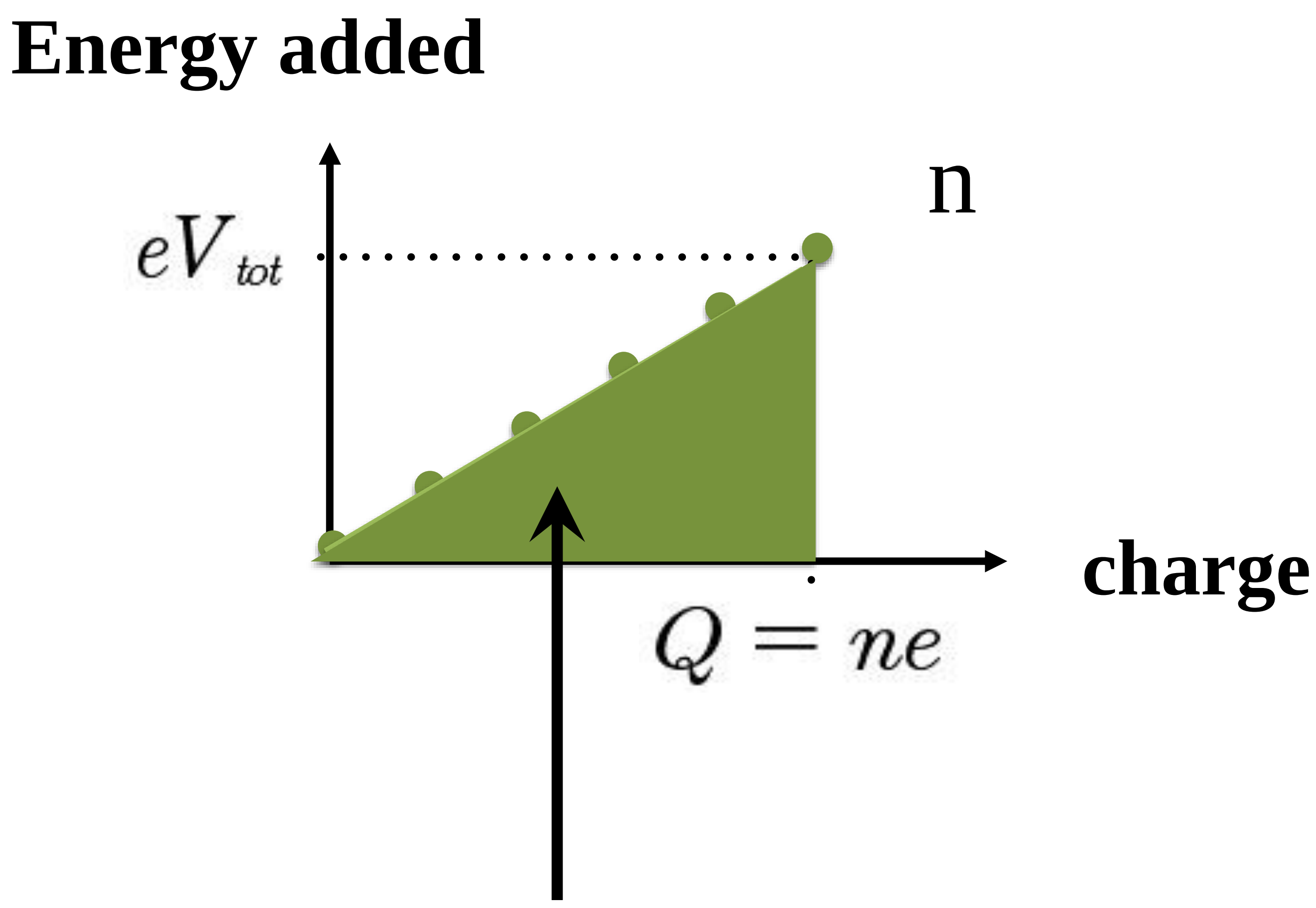
Energy of the Capacitor



charge number	voltage across plates	energy required
1	$V=0$	$eV=0$
2	$V_1=\frac{e}{C}=\frac{Q}{nC}=\frac{V_{tot}}{n}$	$eV_1=\frac{eV_{tot}}{n}$
3	$V_2=\frac{2V_{tot}}{n}$	$eV_2=\frac{2eV_{tot}}{n}$
	$\vdots$	$\vdots$
n	$V_n=\frac{(n-1)V_{tot}}{n}\approx V_{tot}$	$eV_n=\frac{(n-1)eV_{tot}}{n}$ $\approx eV_{tot}$

Energy of the Capacitor

charge number	voltage across plates	energy required
1	$V = 0$	$eV = 0$
2	$V_1 = \frac{e}{C} = \frac{Q}{nC} = \frac{V_{tot}}{n}$	$eV_1 = \frac{eV_{tot}}{n}$
3	$V_2 = \frac{2V_{tot}}{n}$	$eV_2 = \frac{2eV_{tot}}{n}$
	$\vdots$	$\vdots$
	$V_n = \frac{(n-1)V_{tot}}{n} \approx V_{tot}$	$eV_n = \frac{(n-1)eV_{tot}}{n} \approx eV_{tot}$



Total energy added

Energy of the Capacitor

The energy needed to add a charge *dq* is:

$$dW = V\,dq = \frac{q}{C}\,dq$$

The total energy added is:

$$\int dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

Total energy stored in a capacitor:

$$\boxed{E_{cap} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}}$$

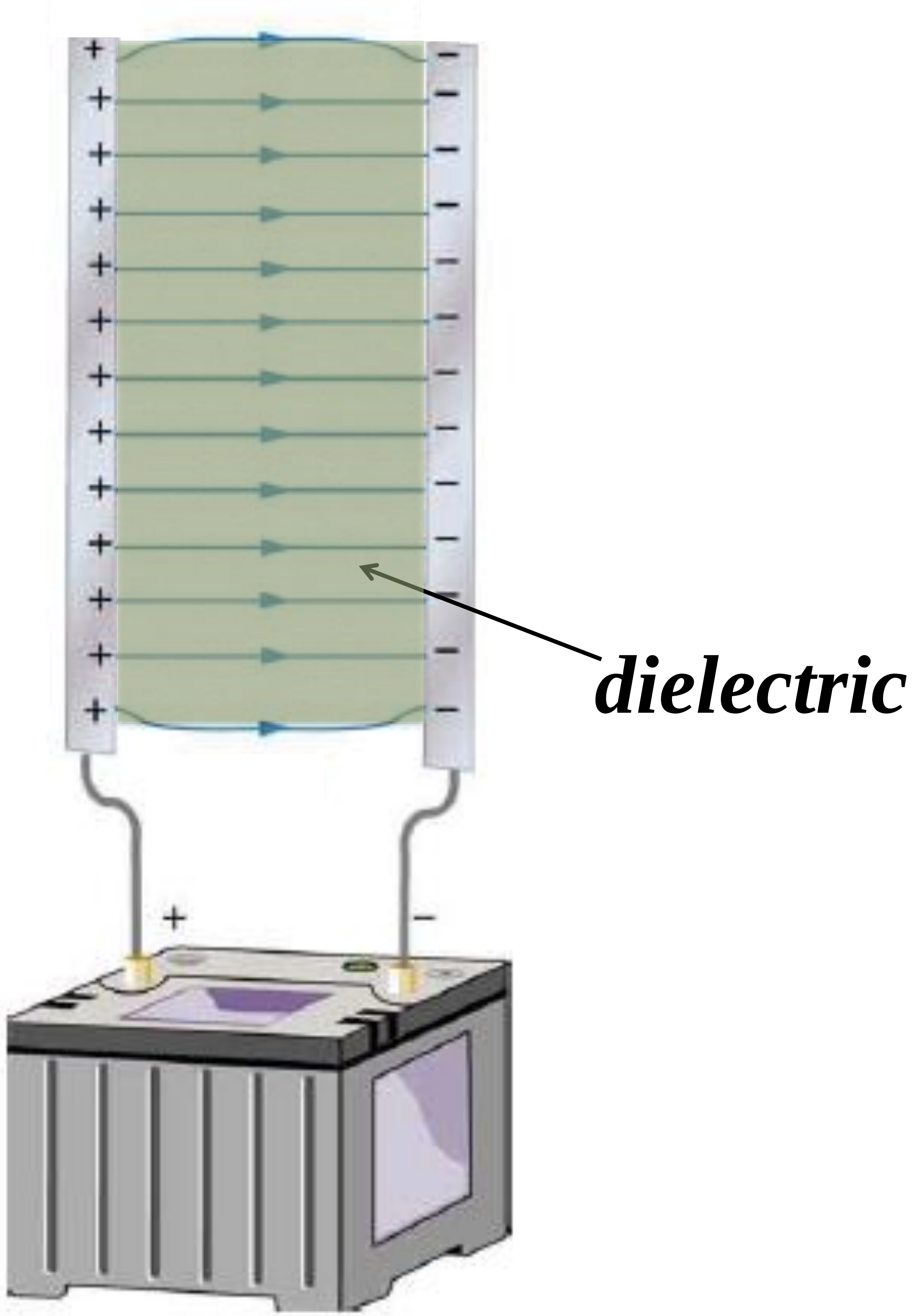
It is also the electric energy of a capacitor

Energy with dielectric

How will the energy stored in the capacitor change after the dielectric being removed?

$$E = \frac{1}{2}CV^2 \qquad C = \frac{A\varepsilon}{D}$$

$$E = \frac{A\varepsilon}{2D}V^2 \qquad \boxed{\varepsilon \downarrow \;\Rightarrow\; E \downarrow}$$



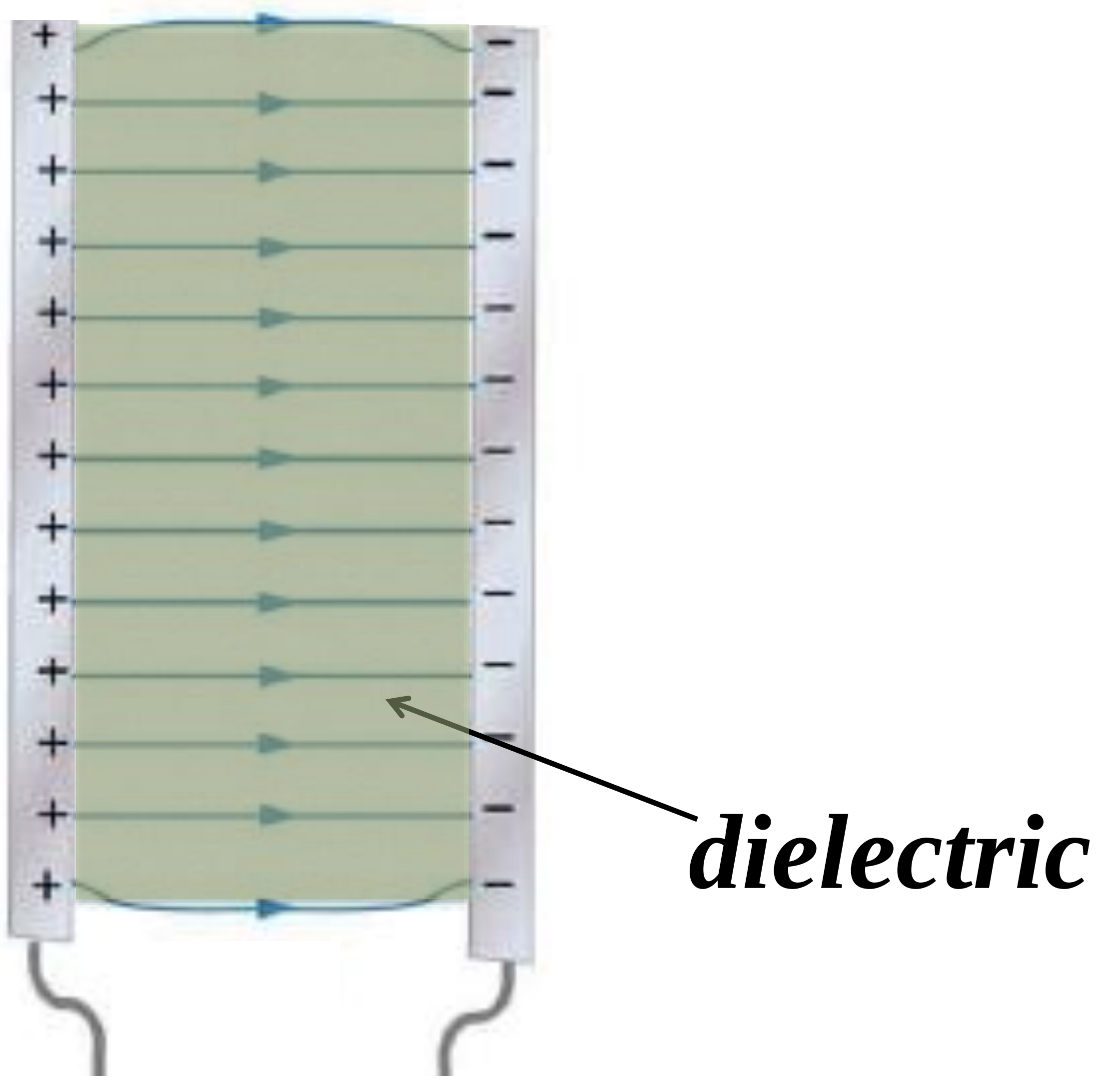
Energy with dielectric

How will the energy stored in the capacitor change after the dielectric being removed?

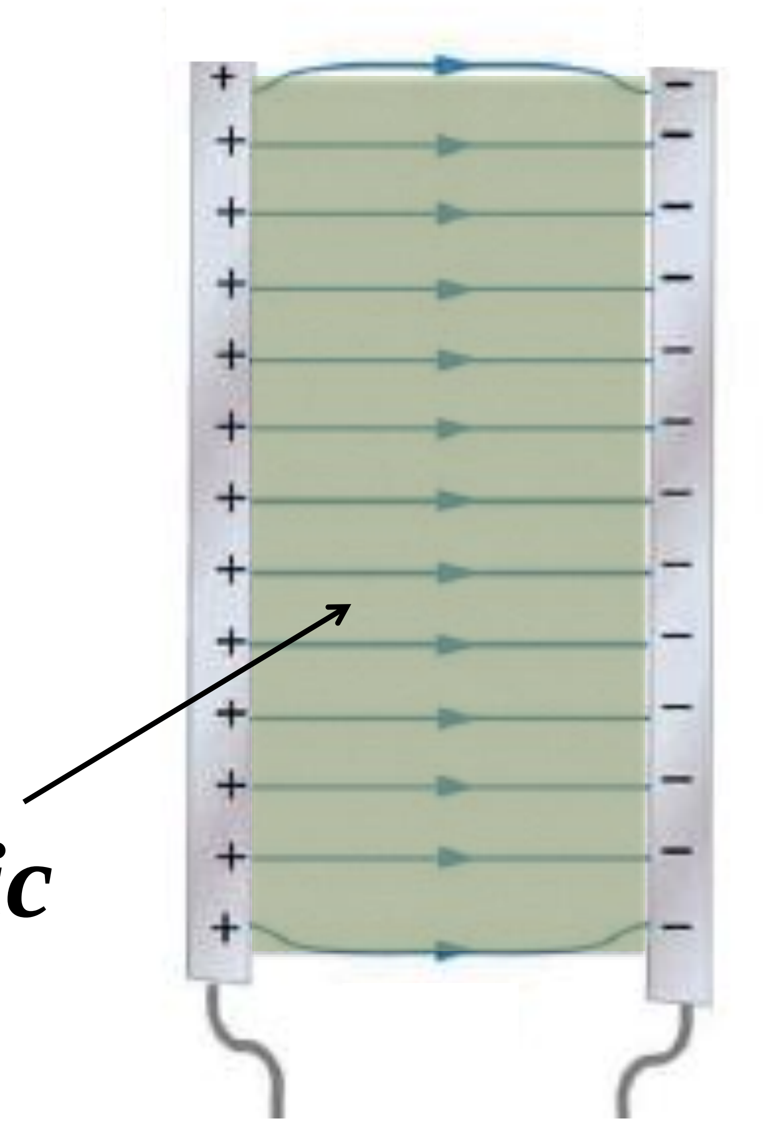
$$E = \frac{Q^2}{2C} \qquad C = \frac{A\varepsilon}{D}$$

$$E = \frac{DQ^2}{2A\varepsilon}$$

$$\boxed{\varepsilon \downarrow \;\Rightarrow\; E \uparrow}$$



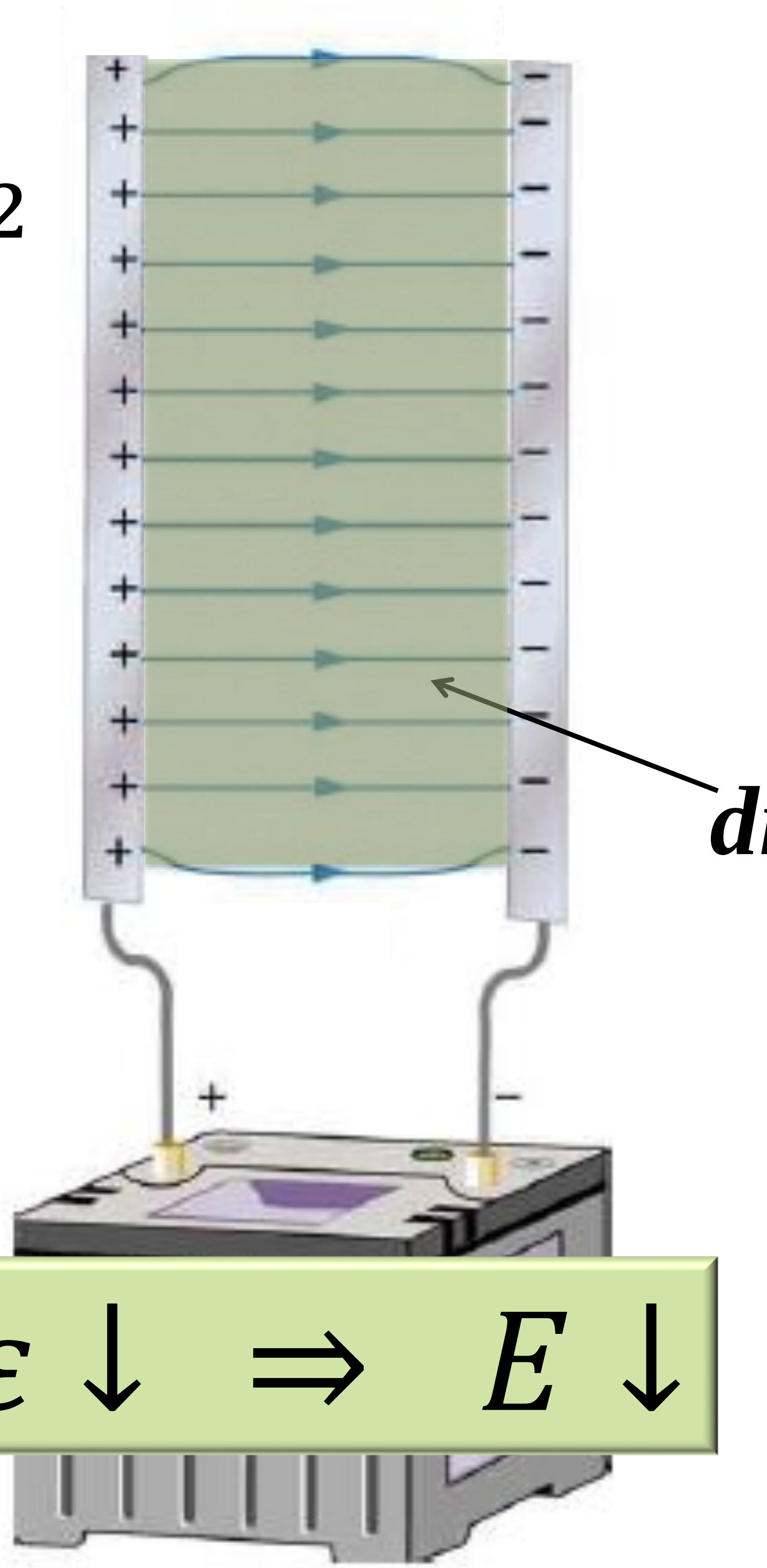
Energy with dielectric



$$E = \frac{DQ^2}{2A\varepsilon}$$

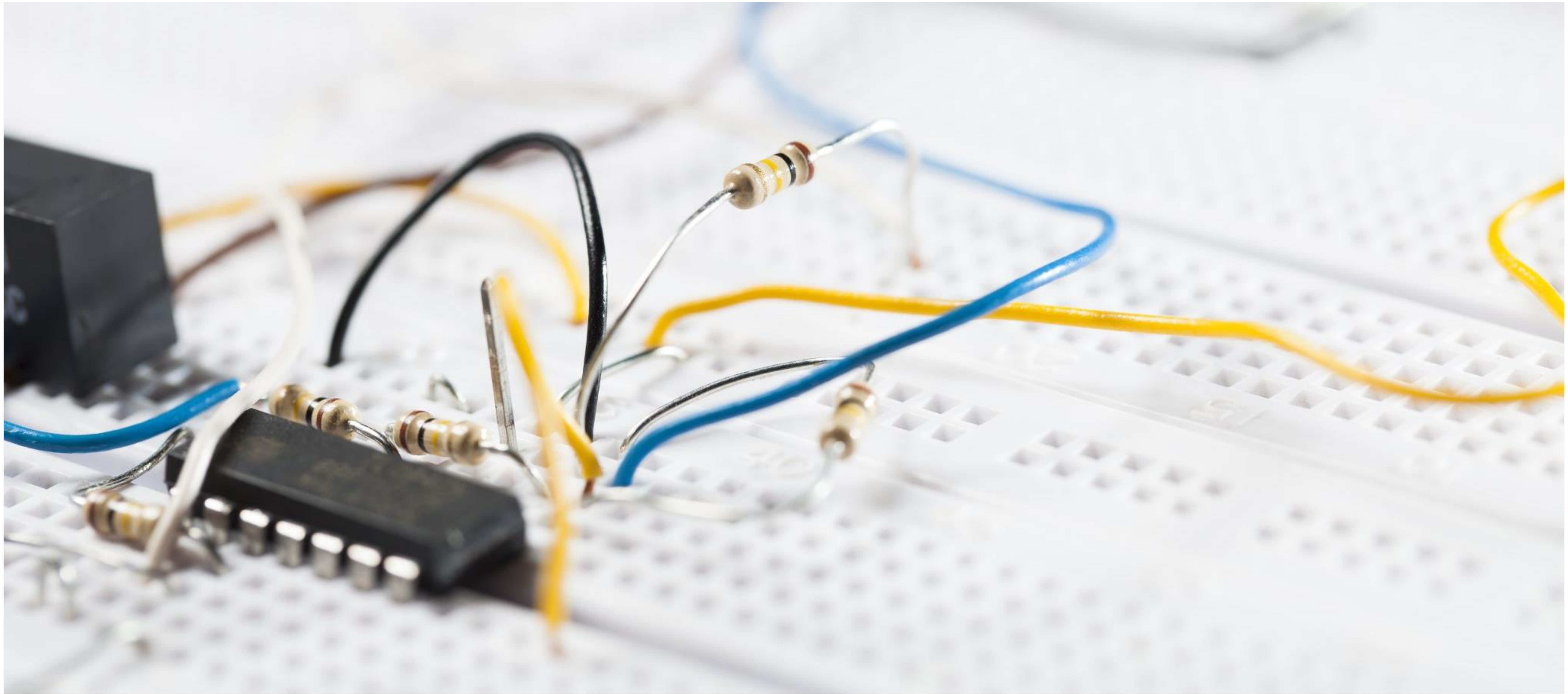
$$\boxed{\varepsilon \downarrow \;\Rightarrow\; E \uparrow}$$

$$E = \frac{A\varepsilon}{2D} V^2$$

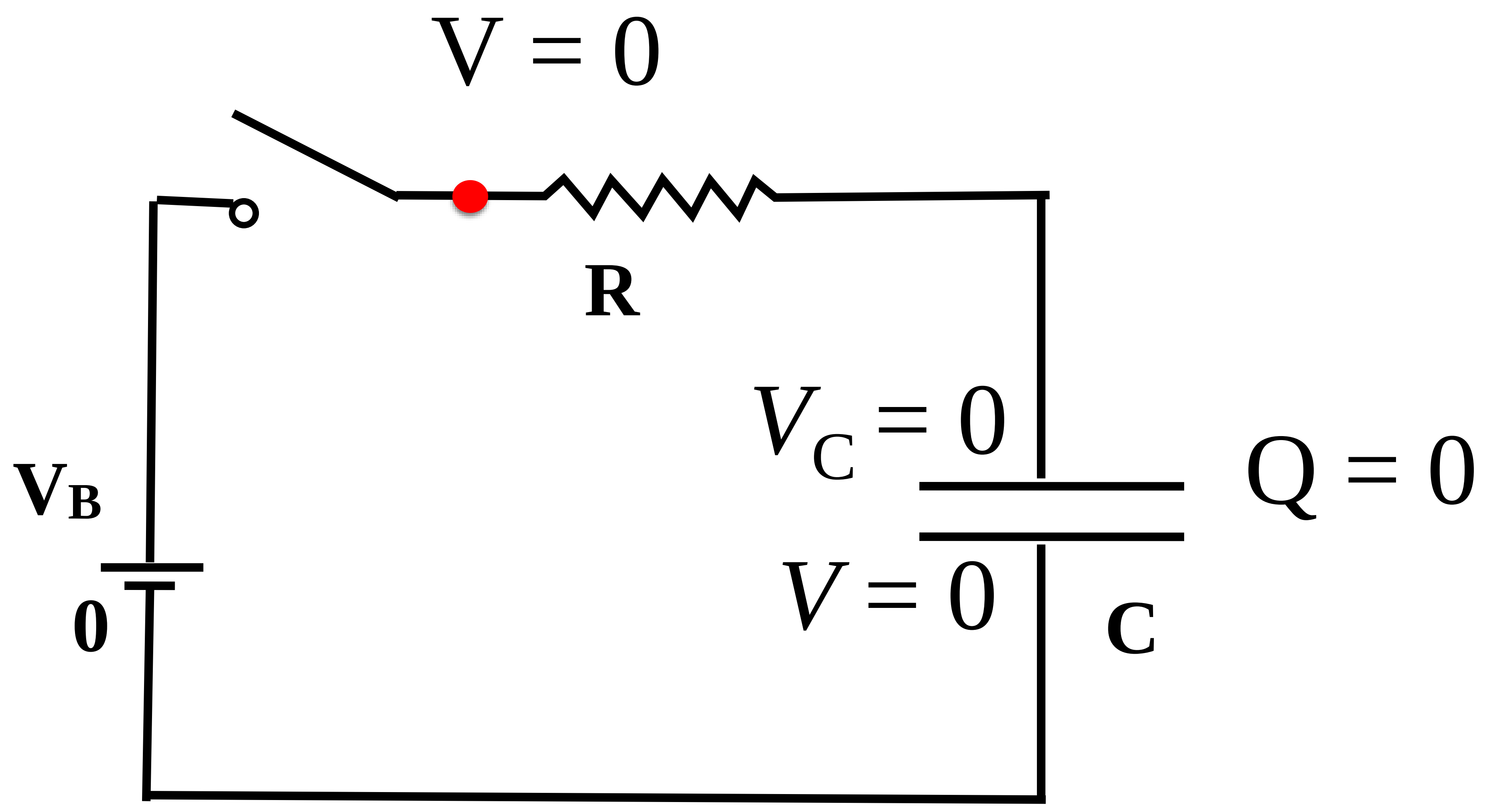


$$\boxed{\varepsilon \downarrow \;\Rightarrow\; E \downarrow}$$

The RC circuit & charging/discharging a capacitor

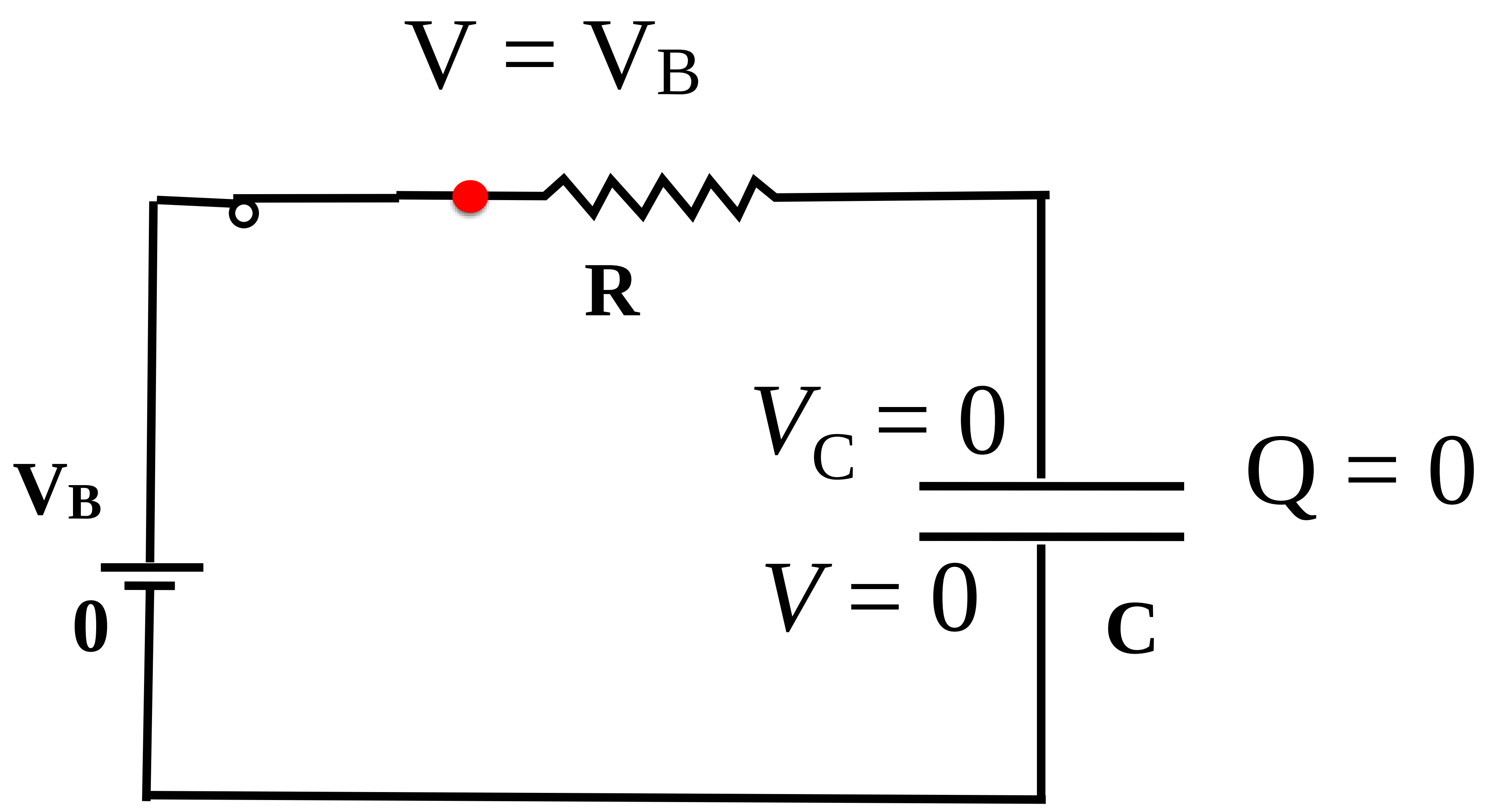


The RC circuit



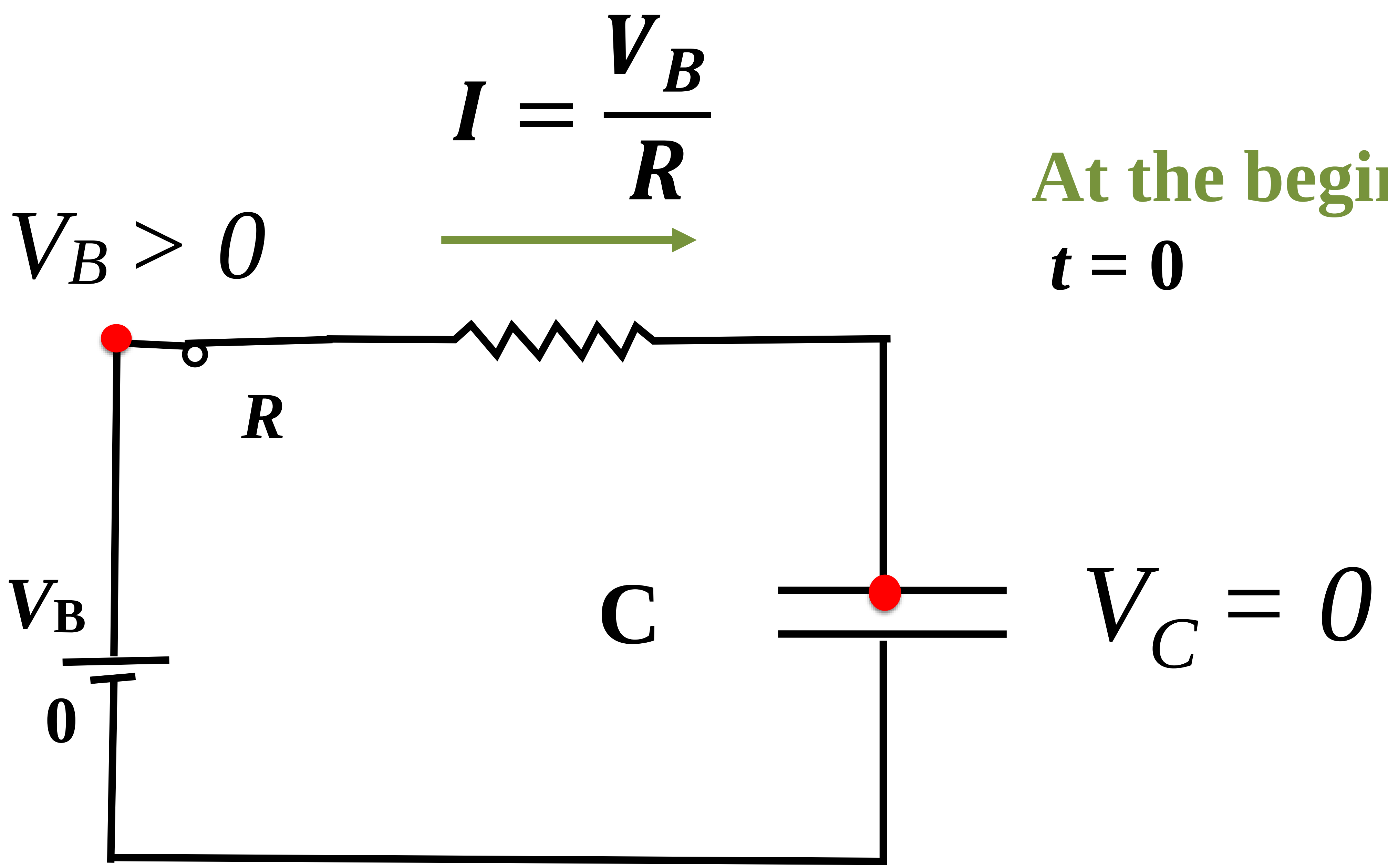
What happens when we close the switch?

The RC circuit



What happens when we close the switch?

The RC circuit

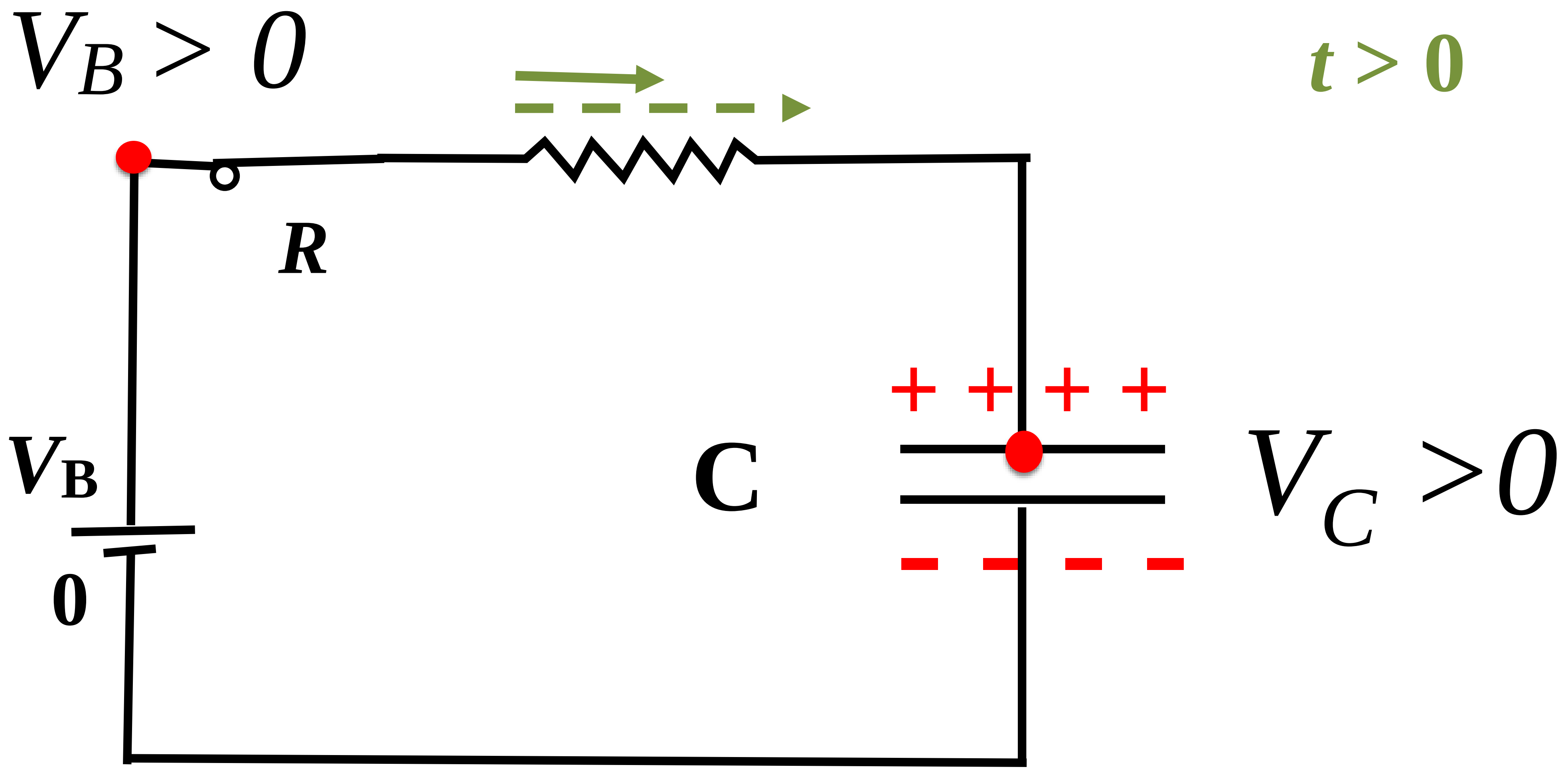


At the beginning

$t = 0$

The RC circuit

$$I = \frac{V_B - VC}{R} < \frac{V_B}{R}$$



The current decreases from the beginning!

Charge a capacitor

voltage of battery (constant)

voltage of capacitor (a function of time)

$$I = \frac{V_B - V_C}{R} = \frac{\Delta Q}{\Delta t}$$

increase of the charge on the capacitor

$$V_C = \frac{Q}{C}$$

a function of time

$$\frac{\Delta Q}{\Delta t} = \frac{V_B - (Q/C)}{R} = \frac{-Q}{RC} + \frac{V_B}{R}$$

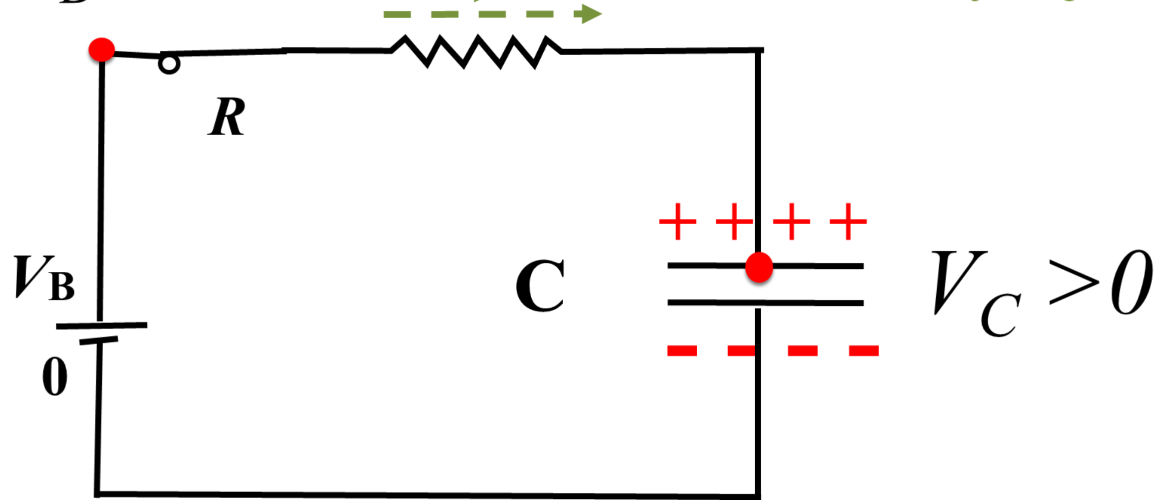
$$\boxed{\tau = RC}$$

Time constant of RC circuit

$$\Longrightarrow \frac{dQ}{dt} = \frac{-Q}{\tau} + \frac{V_B}{R}$$

differential equation!

$V_B > 0$ $\quad \boldsymbol{I = \frac{V_B - V_C}{R} < \frac{V_B}{R}}$ $\quad$ **later $t > 0$**



solution: $Q = \frac{\tau V_B}{R} - \frac{\tau V_B}{R} e^{-t/\tau}$

$$\boxed{V_C = V_B\left(1 - e^{-t/RC}\right)}$$

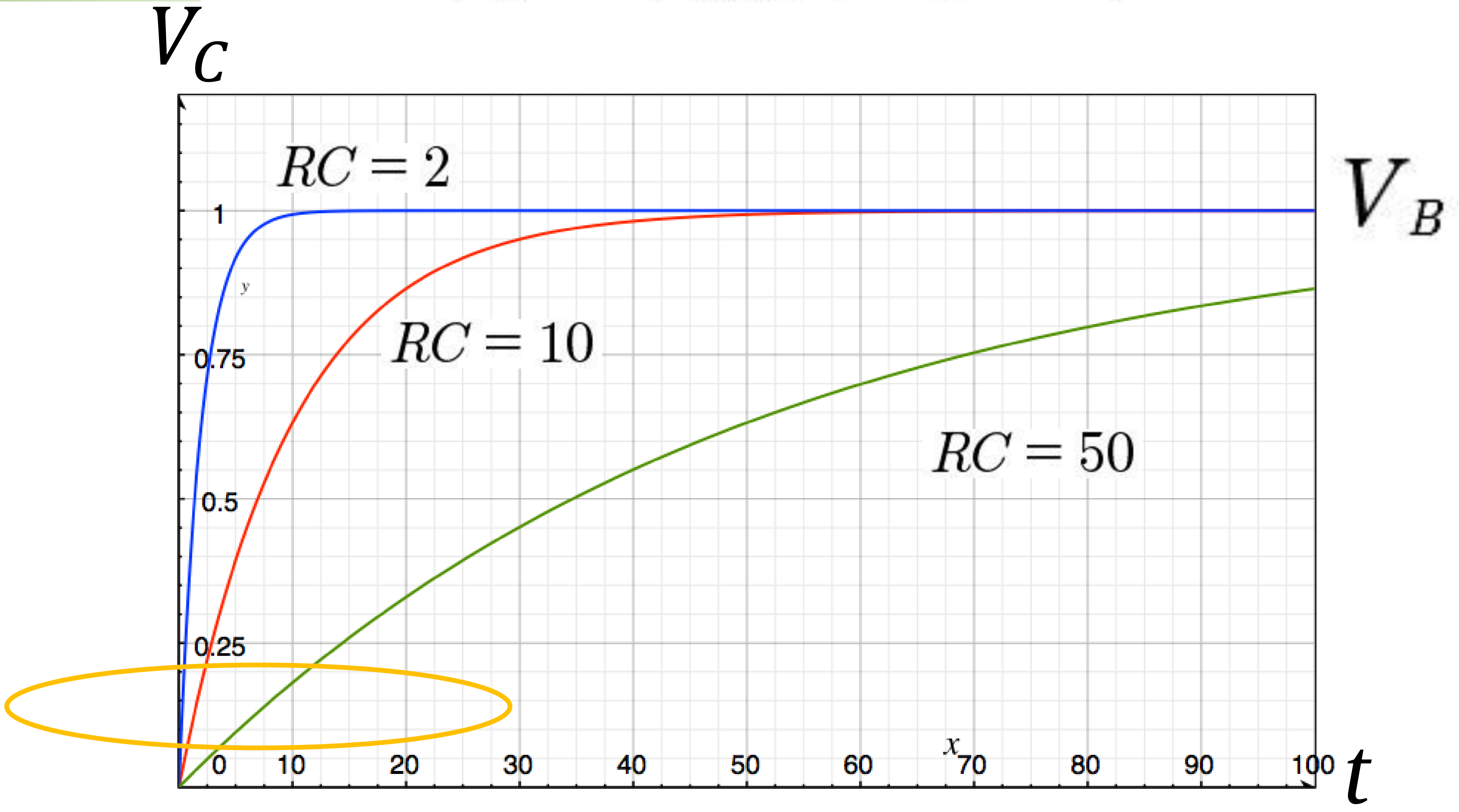
Base for the natural logarithm

Charge a capacitor

$$V_C = V_B\left(1 - e^{-t/RC}\right)$$

$$\boxed{\tau = RC}$$

Time constant
of RC circuit



For small *t*:

$$e^{-t/RC} \approx 1 - (t/RC) \quad \Longrightarrow V_C \approx V_B \frac{t}{RC}$$

Current when charging

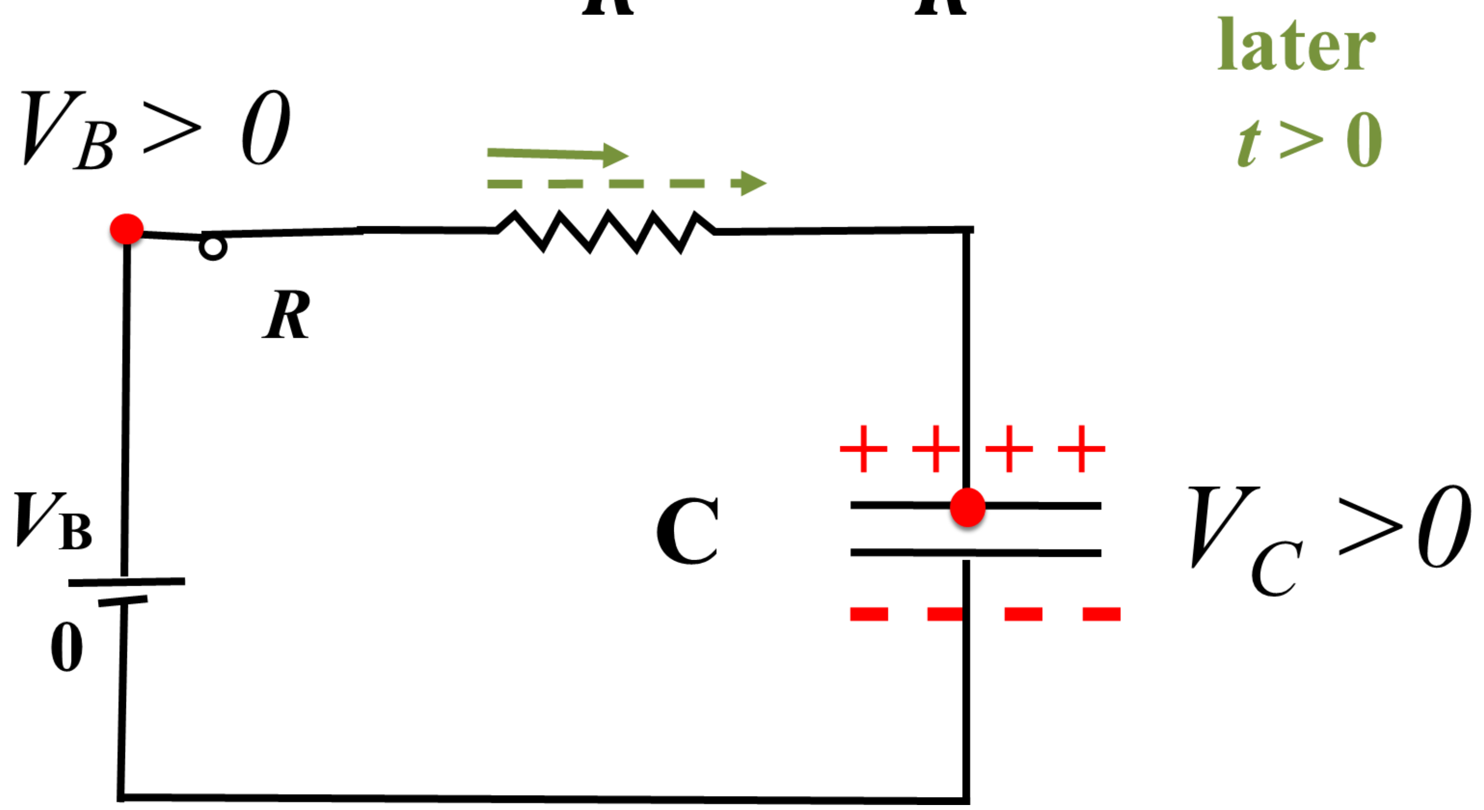
What is the current in *R* ?

$$I(t) = \frac{V_B - V_C(t)}{R}$$

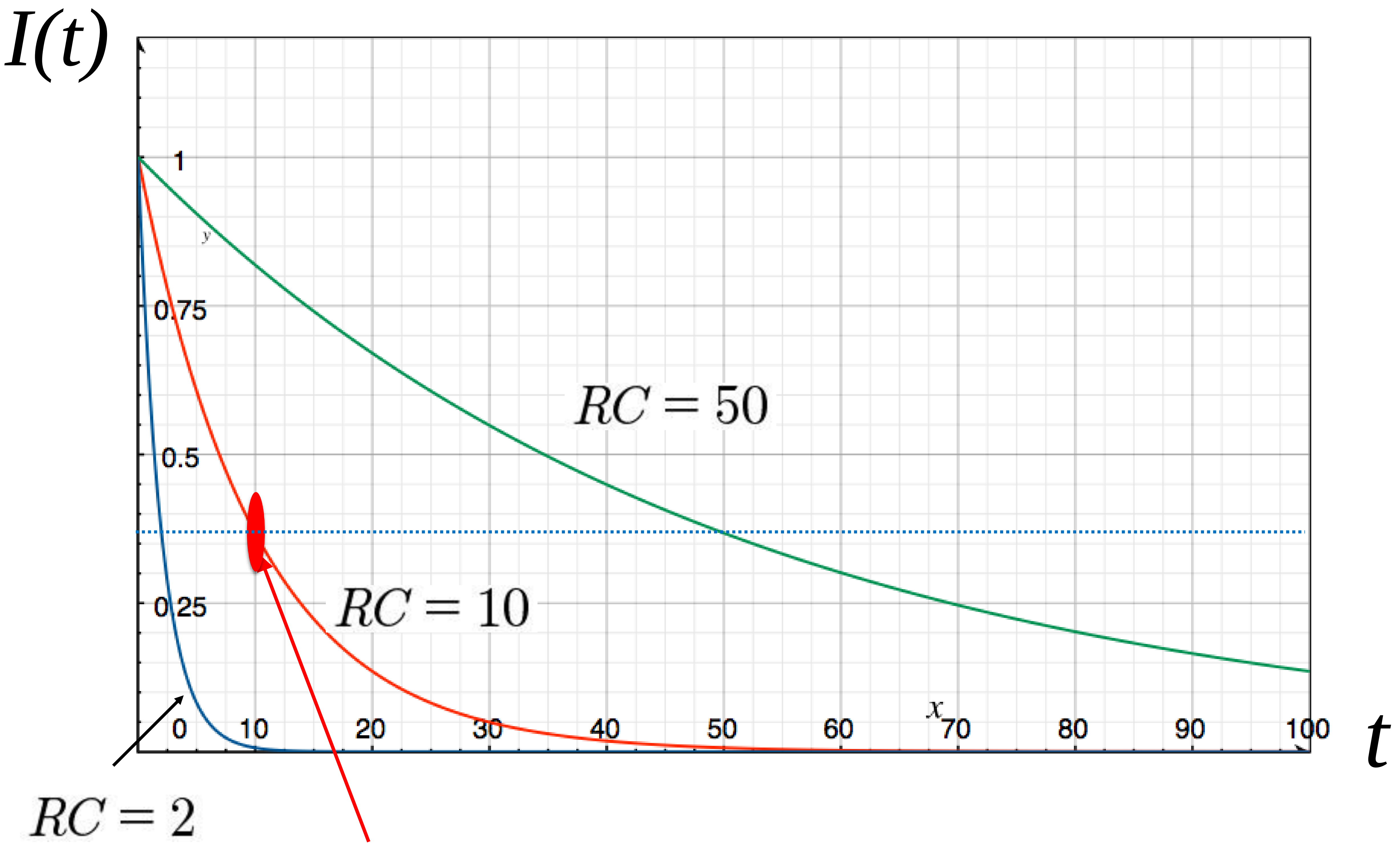
$$V_C = V_B\left(1 - e^{-t/RC}\right)$$

$$\boxed{I(t) = \frac{V_B}{R} e^{-t/RC}}$$

$$I = \frac{V_B - VC}{R} < \frac{V_B}{R}$$



Current when charging



$$I(t) = \frac{V_B}{R} e^{-t/RC}$$

Exponential decay

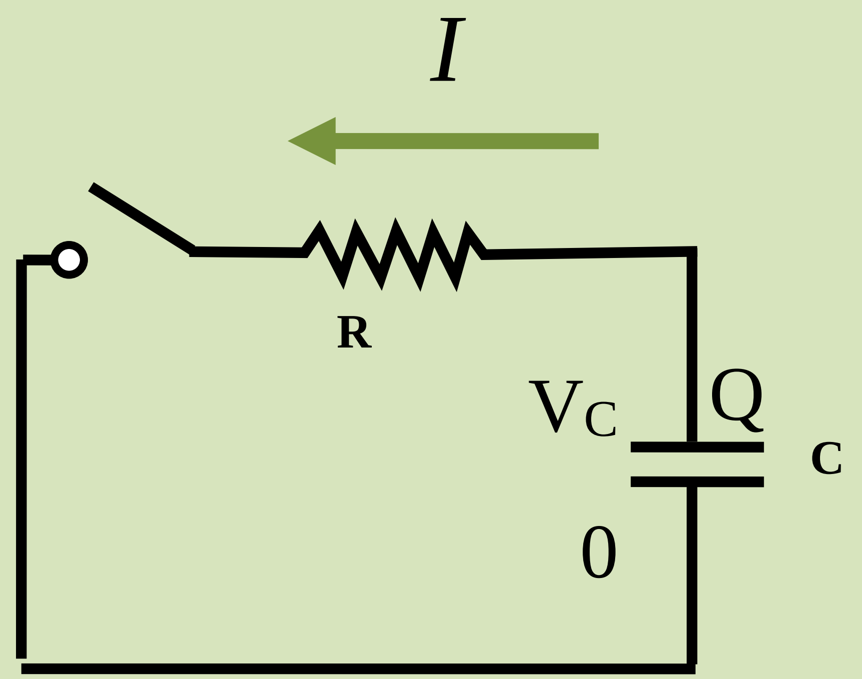
At the ***t*** = *time constant*, the function decays to *1/e* = 37% of its original value

Discharging a capacitor

$$I = \frac{\Delta Q}{\Delta t} = \frac{-V_C}{R} = \frac{-Q}{RC}$$

negative sign because the current goes in the opposite direction (removing charges on the capacitor)

$$\Longrightarrow \frac{dQ}{dt} = \frac{-Q}{\tau}$$

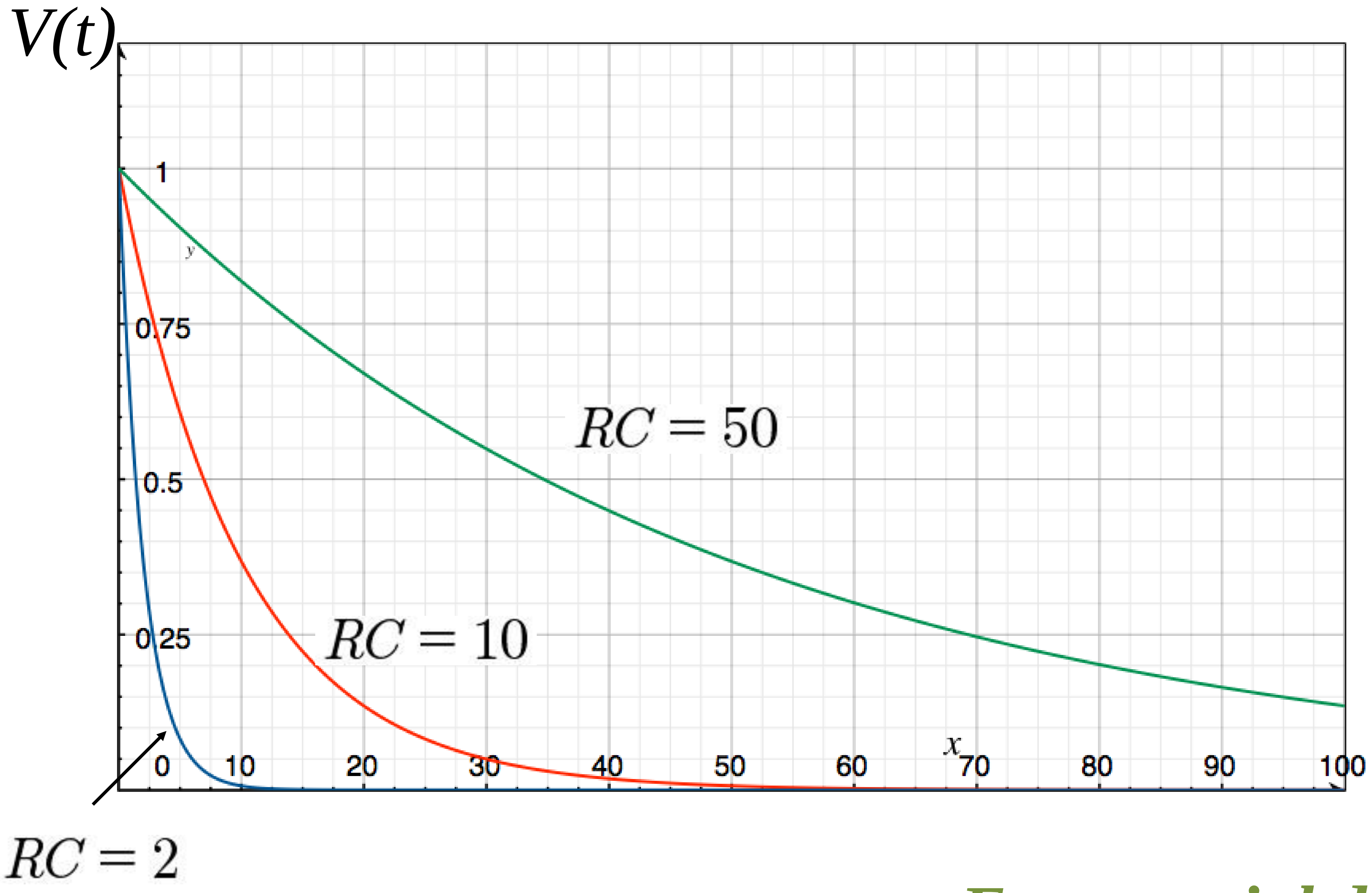


solution: $Q\left(t\right) = Q_0 e^{-t/RC}$

$$Q_0 = Q\left(0\right) = C\,V_C$$

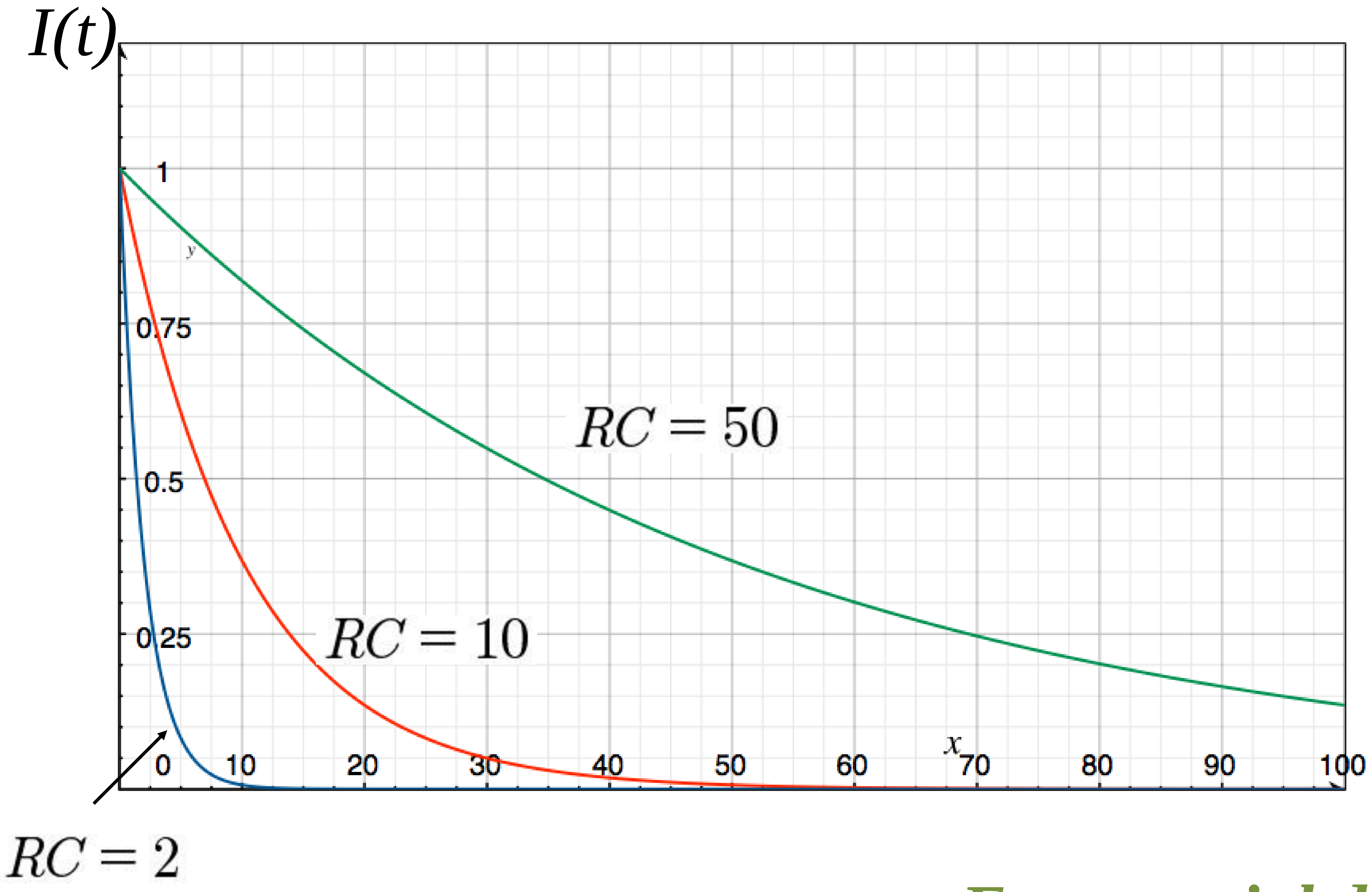
$$\boxed{V\left(t\right) = \frac{Q\left(t\right)}{C} = V_C e^{-t/RC}}$$

Discharging a capacitor



Exponential decay

Discharging a capacitor



Exponential decay

Capacitors' application