ENSC 2113

Engineering Mechanics: Statics

Chapter 4:

Force System Resultants

(Sections 4.1-4.4)

COLLEGE OF
ENGINEERING, ARCHITECTURE
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Chapter 4 Outline:

4.1 Moment of a Force – Scalar Formulation

4.2 Cross Product

4.3 Moment of a Force – Vector Formulation

4.4 Principle of Moments

4.5 Moment of a Force about a Specified Axis

4.6 Moment of a Couple

4.7 Simplification of a Force and Couple System

4.8 Further Simplification of a Force and Couple System

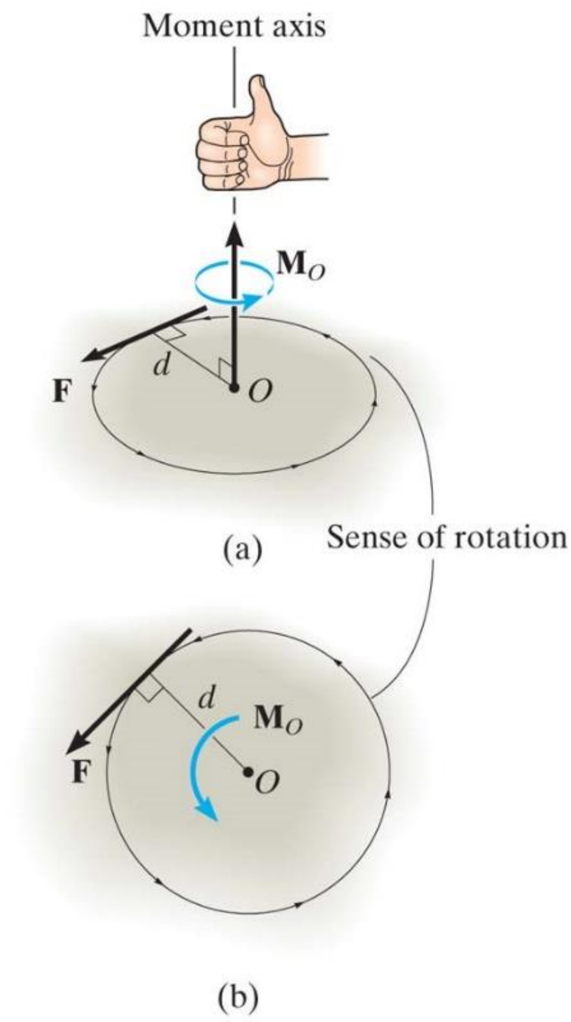
4.9 Reduction of a Simple Distributed Loading

Chapter 4 Objectives:

- To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions
- To provide a method for finding the moment of a force about a specified axis
- To define the moment of a couple
- To show how to find the resultant effect of a nonconcurrent force system
- To indicate how to reduce a simple distributed loading to a resultant force acting at a specified location

Right-Hand Rule:

- Point the thumb of your right hand along the positive axis of rotation.
- Fingers curl in the positive direction
- Sign convention:
  - Counterclockwise is positive, clockwise is negative



4.1: Moment of a Force - Scalar Formulation

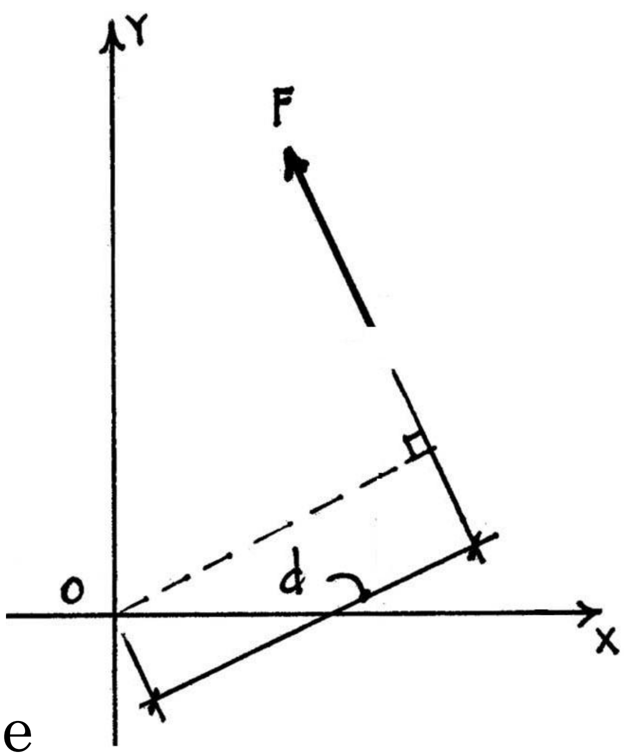
The moment of a force is the tendency of the force to produce rotation about a point or a line.

The scalar magnitude of the moment of a force about point O is:

$$|M_o| = |F|d$$

where,

- M is the magnitude of the rotation about a point
- d is the perpendicular distance measuring from the point to the force

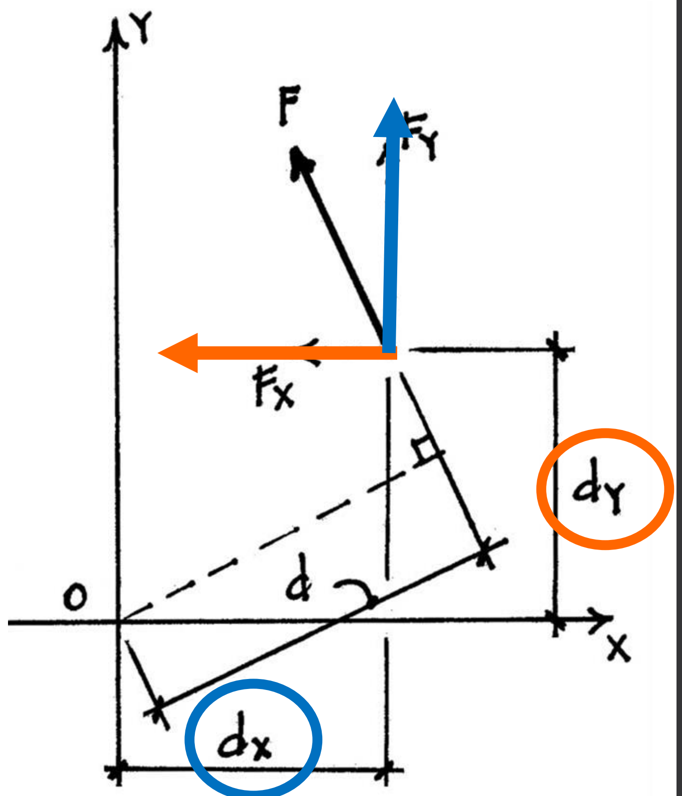


4.1: Moment of a Force - Scalar Formulation

Breaking the force into components may simplify the process.

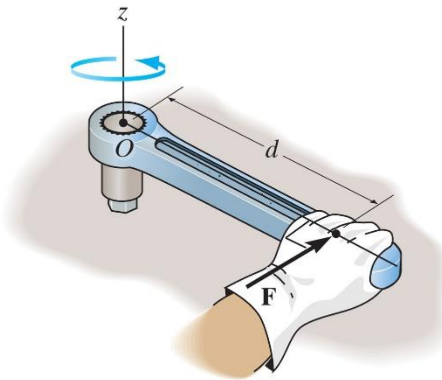
The moment is the sum of **each component** multiplied by its perpendicular distance back to point **O**.

$$|M_o| = |F_x| d_y + |F_y| d_x$$

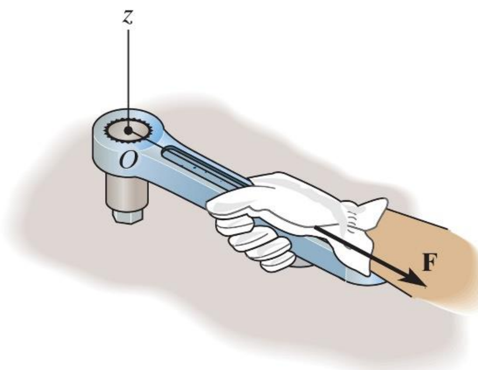


4.1: Moment of a Force - Scalar Formulation

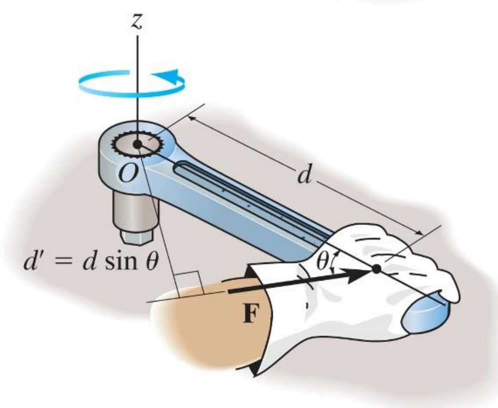
- Maximum rotation occurs when the force is applied perpendicular to the object



- No rotation occurs when the force is applied along the object



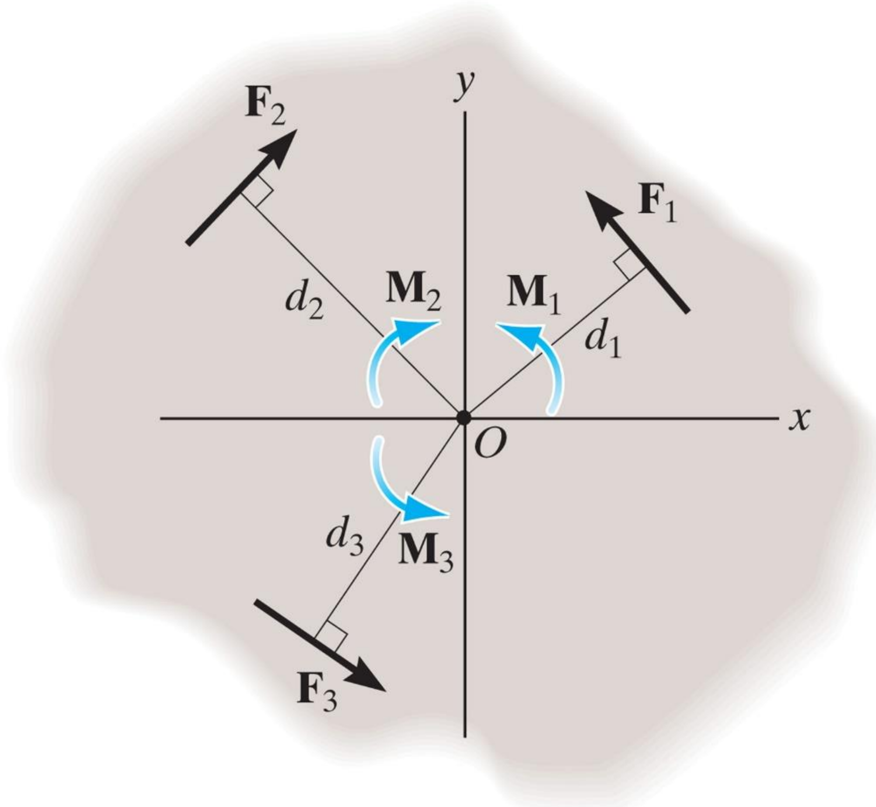
- When the force is applied at an angle other than 90 degrees, a rotation occurs but it is neither the maximum or minimum rotation



4.1: Moment of a Force - Scalar Formulation

- Resultant Moment - Algebraic sum of the moments caused by all of the forces in the system

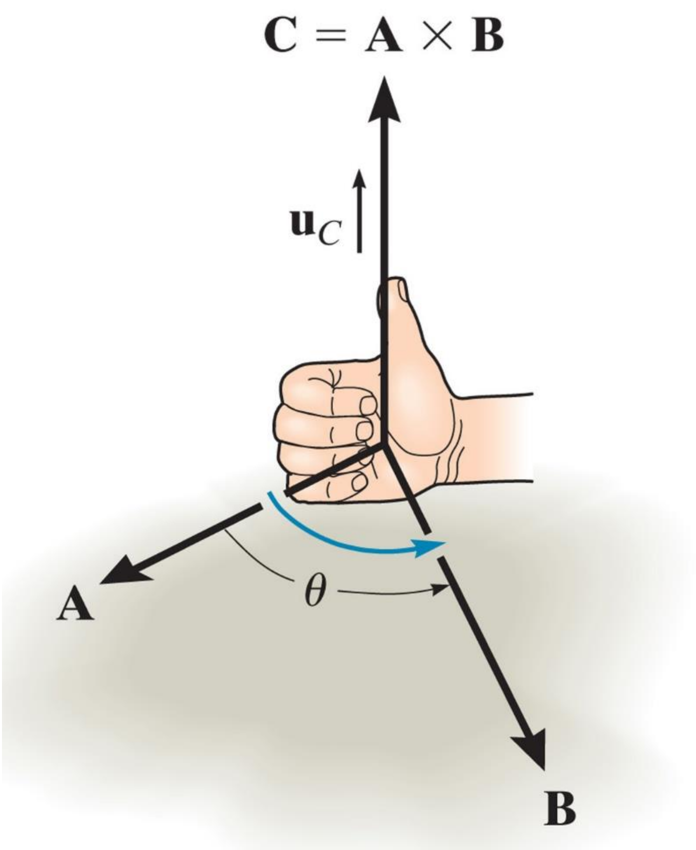
$$\circlearrowleft \Sigma|M_o| = |F|d$$



$$\circlearrowleft \Sigma|M_o| = |F_1|d_1 - |F_2|d_2 + |F_3|d_3$$

4.2: Cross Product

The moment of a force may also be found using the cross product.



- The cross product of vectors A and B yield vector C

$$\vec{C} = \vec{A} \times \vec{B}$$

- Laws of Operation

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \qquad \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

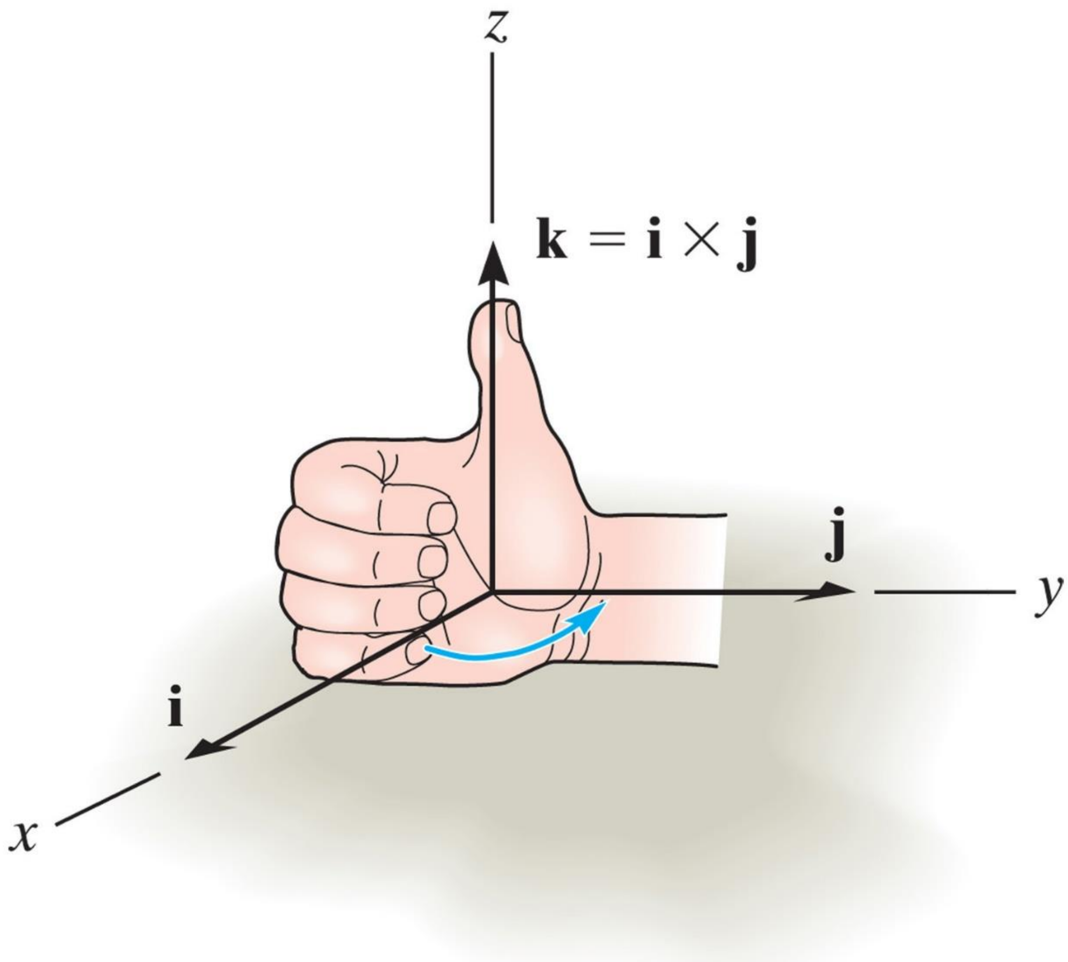
4.2: Cross Product

Cartesian Vector Formulation

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j}$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$$
$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$
$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

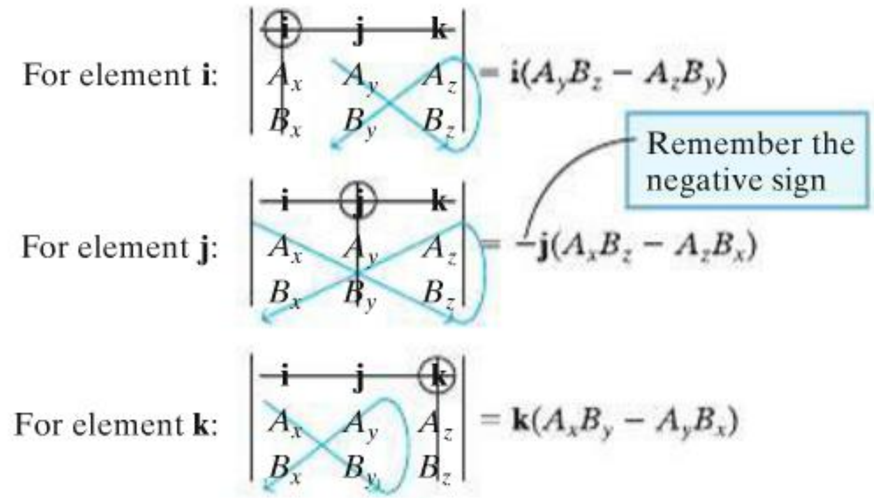
$$\mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{k} = \mathbf{0}$$



4.2: Cross Product

Determinant:

$$\vec{C} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \qquad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \qquad \mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \qquad \mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

$$\vec{C} = \{(A_yB_z - A_zB_y)\boldsymbol{i} - (A_xB_z - A_zB_x)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}\}$$

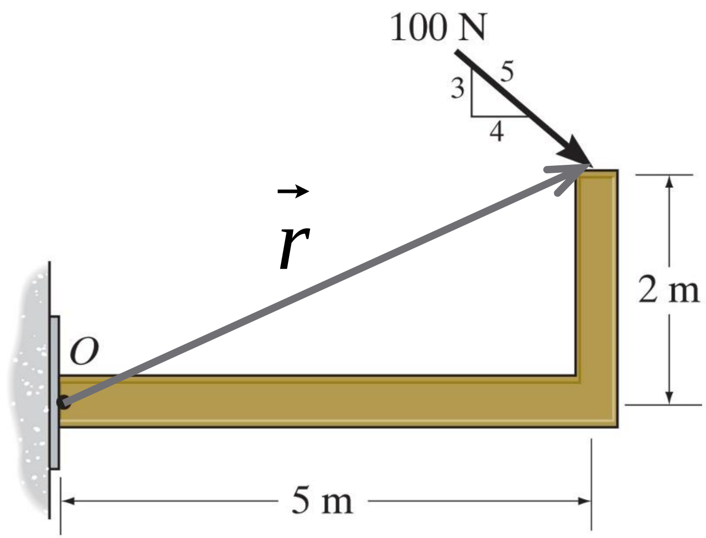
4.3: Moment of a Force - Vector Formulation

- Vector Analysis

$$\overrightarrow{M} = \vec{r} \times \overrightarrow{F}$$

where,

- M is the moment vector about a point
- *r* is the position vector from the point to any location along the force's line of action

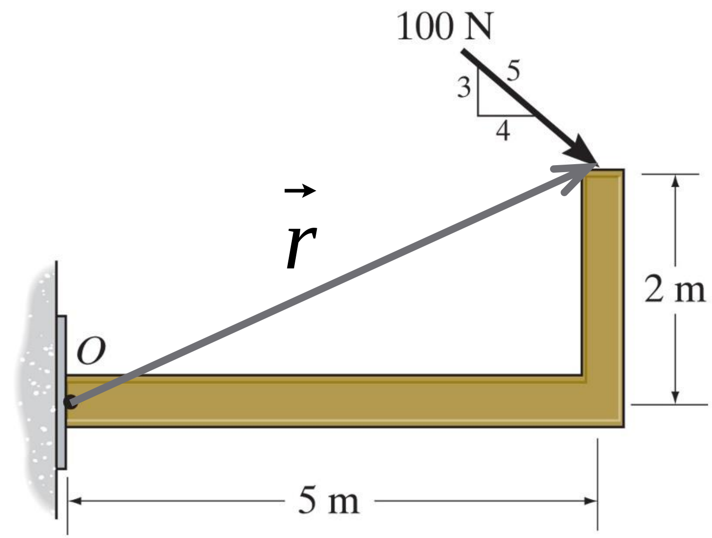


4.3: Moment of a Force - Vector Formulation

- Vector Analysis

$$\overrightarrow{M} = \vec{r} \times \overrightarrow{F}$$

$$\overrightarrow{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\overrightarrow{M} = \{(r_y F_z - r_z F_y)$$

Moment of a Force

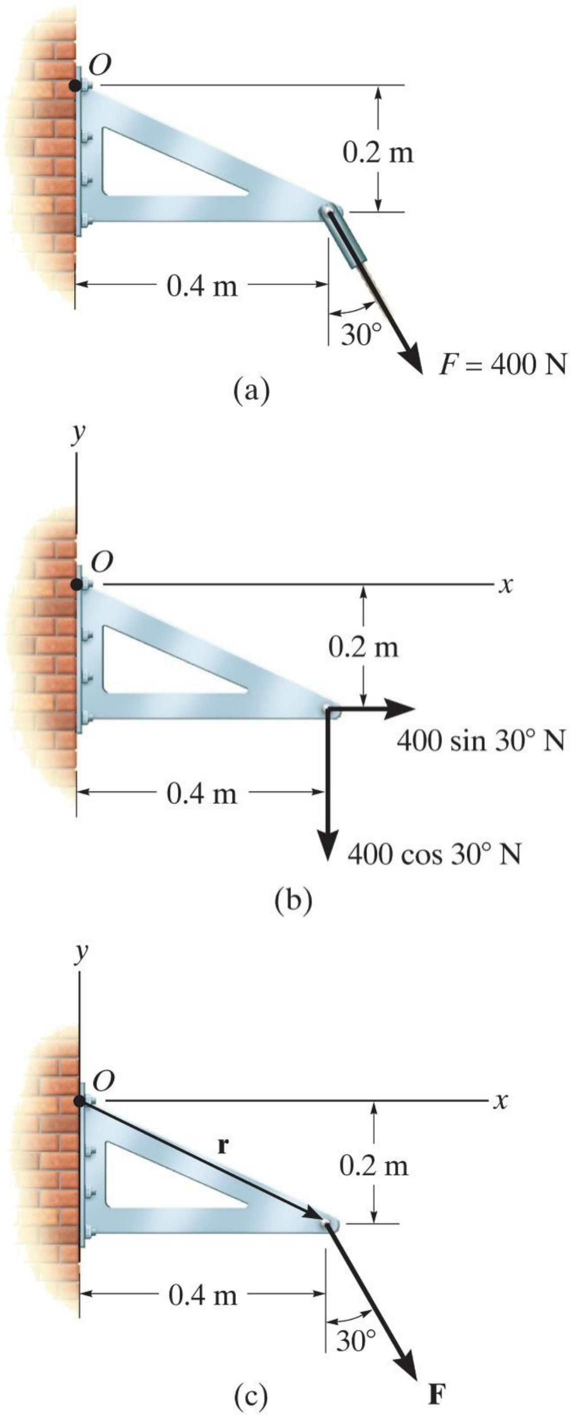
Scalar analysis:

$$|M| = |F|d$$

$$|M| = |F_x|d_y + |F_y|d_x$$

Vector analysis:

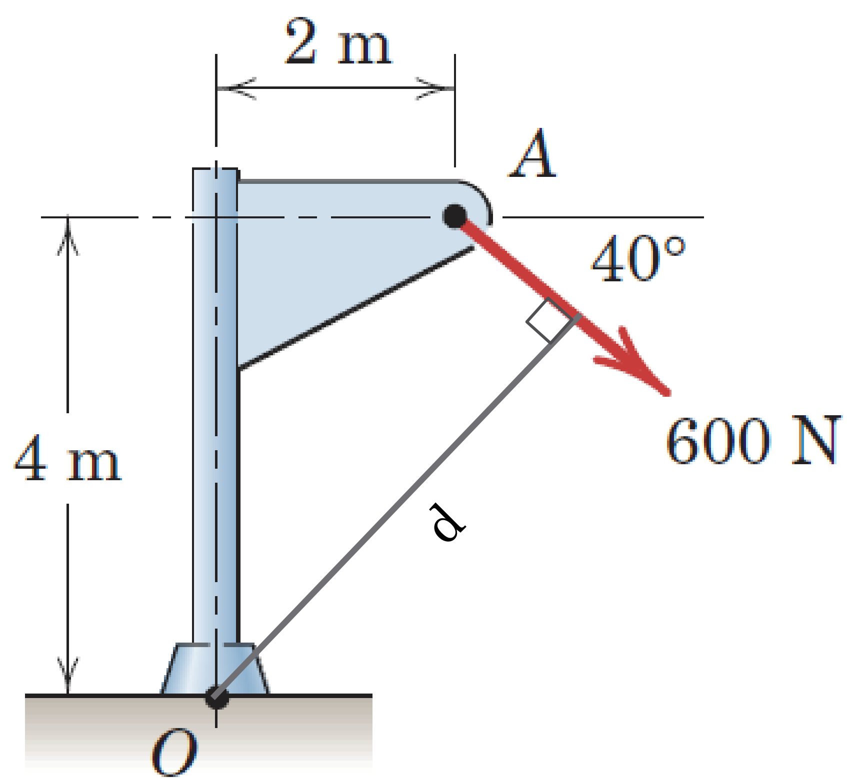
$$\bar{M} = \bar{r} \times \bar{F}$$



Moment:

- The moment can be found in many ways

Determine the moment arm (perpendicular distance) to the force from the point of rotation

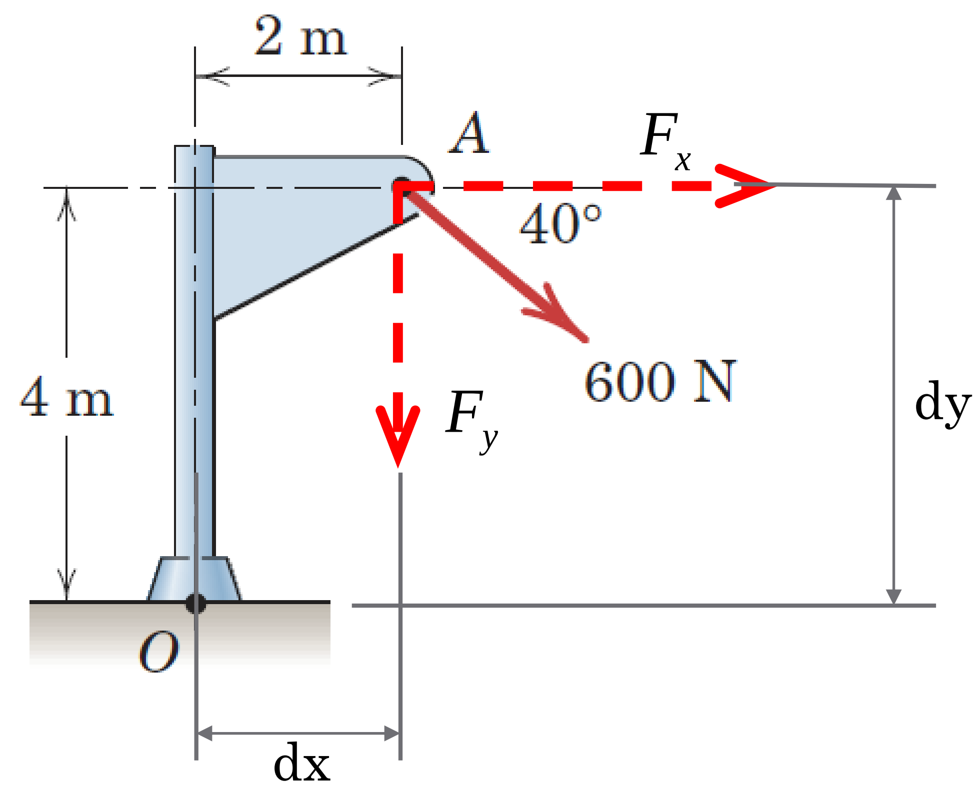


$$|M| = Fd$$

Moment:

- The moment can be found in many ways

Replace the force with rectangular components and perpendicular distances



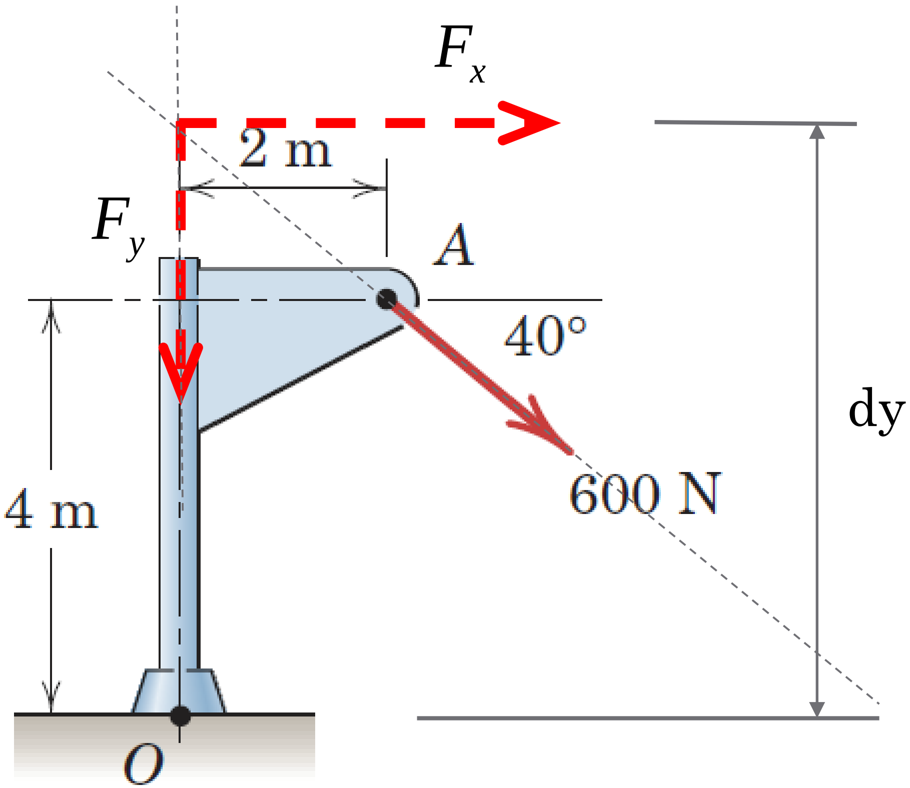
$$|M| = F_x d_y + F_y d_x$$

Moment:

- The moment can be found in many ways
  - By transmissibility

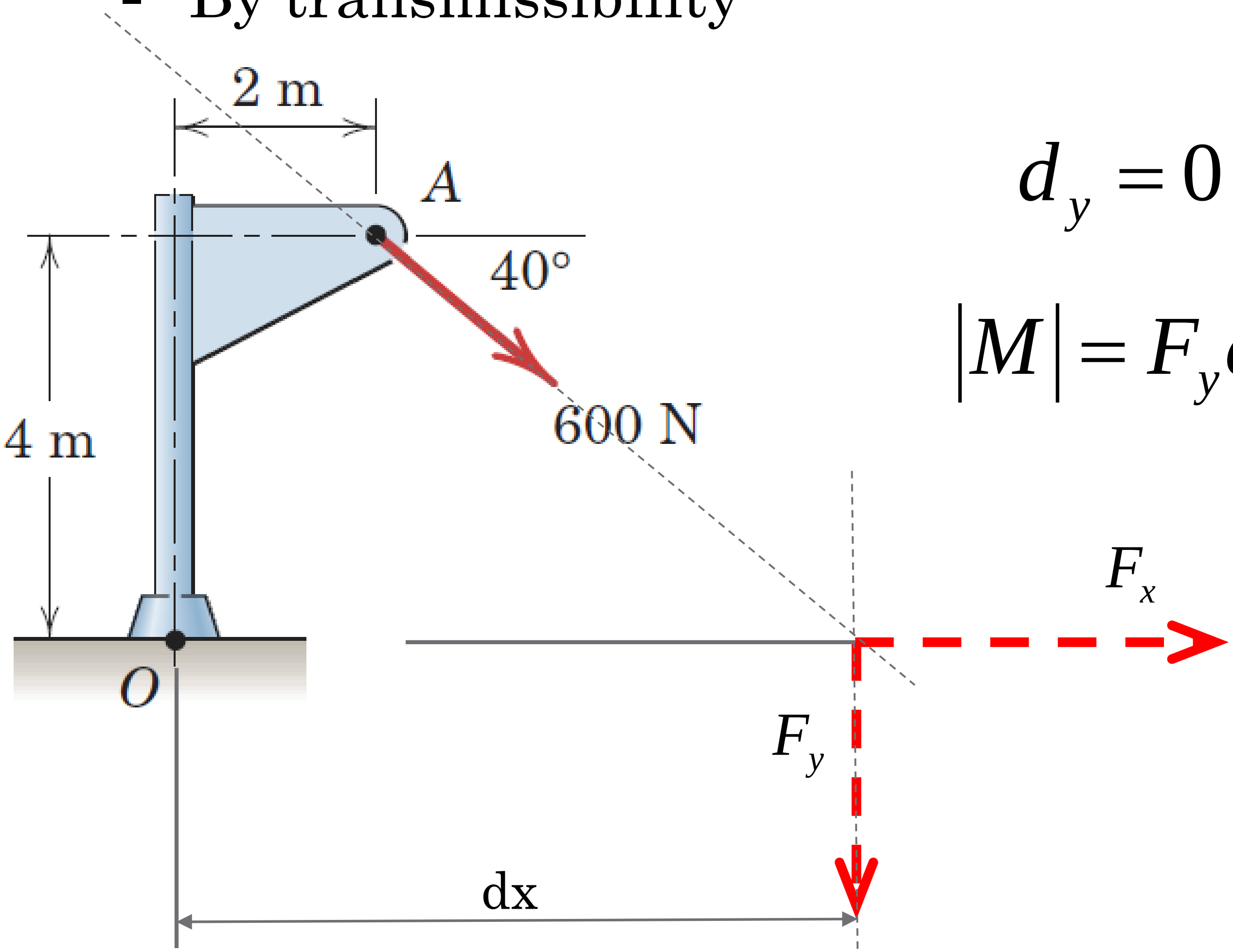
$$d_x = 0$$

$$|M| = F_x d_y$$



Moment:

- The moment can be found in many ways
  - By transmissibility

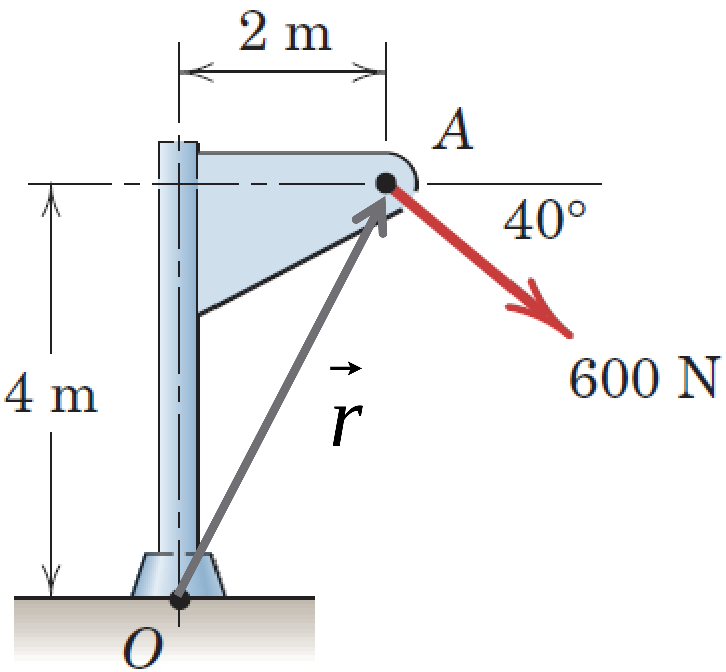


$$d_y = 0$$

$$|M| = F_y d_x$$

Moment:

- The moment can be found in many ways
  - With cross-product



$$\overrightarrow{M} = \vec{r} \times \overrightarrow{F}$$

$$\overrightarrow{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\overrightarrow{M} = \left\{ \left(r_y F_z - r_z F_y \right) \hat{i} - \left(r_x F_z - r_z F_x \right) \hat{j} + \left(r_x F_y - r_y F_x \right) \hat{k} \right\}$$

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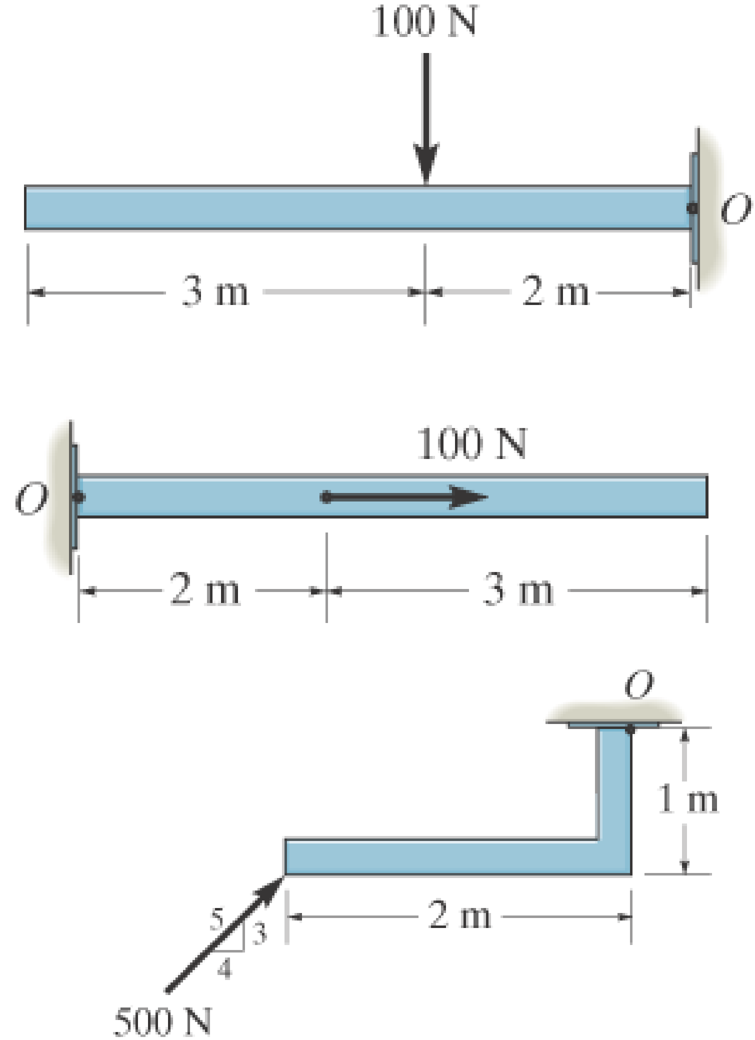
- Example: Determine the moment created about point O by scalar analysis.



- Example: Determine the moment created about point O by scalar and vector analysis.