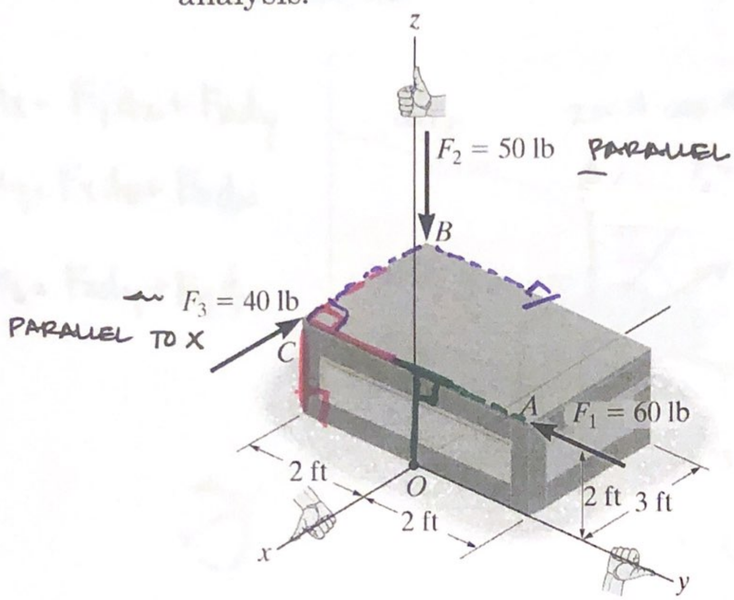
$M_x = F_y d_z + F_z d_y$ $\qquad$ $M_z = F_x d_y + F_y d_x$

$M_y = F_x d_z + F_z d_x$

- Example: Determine the moment about the x, y and z axes using scalar analysis.

$M = F d_\perp$



- PARALLEL ≅ NO MOMENT ABOUT THAT AXIS
- PERPINDICULAR BUT TOUCHING ≅ NO MOMENT
- PERPINDICULAR BUT OFFSET ≅ MOMENT

$F_1 = 60$ lb

x: 2 ft ABOVE $\qquad$ $M_x = F d_\perp = +60\,lb\,(2\,ft) = 120\,lb\,ft$

y: PARALLEL $\qquad$ $M_y = // = 0$

z: d = 0 $\qquad$ M_z = TOUCHING (PUSHING), NOT OFFSET

$F_2 = 50$ lb

x: 2 ft left $\qquad$ $M_x = 50\,lb\,(2\,ft) = 100\,lb\,ft$

y: 3 ft behind $\qquad$ $M_y = -50\,lb\,(3\,ft) = -150\,lb\,ft$

z: PARALLEL $\qquad$ $M_z = 0$

$F_3 = 40$ lb

x: PARALLEL $\qquad$ $M_x = 0$

y: 2 ft above $\qquad$ $M_y = -40\,lb\,(2\,ft) = -80\,lb\,ft$

z: 2 ft left $\qquad$ $M_z = -40\,lb\,(2\,ft) = -80\,lb\,ft$

$M_x = 120\,lb\,ft + 100\,lb + 0 \rightarrow \boxed{M_x = 220\,lb\,ft}$

$M_y = 0 - 150\,lb\,ft - 80\,lb\,ft \rightarrow \boxed{M_y = -230\,lb\,ft}$

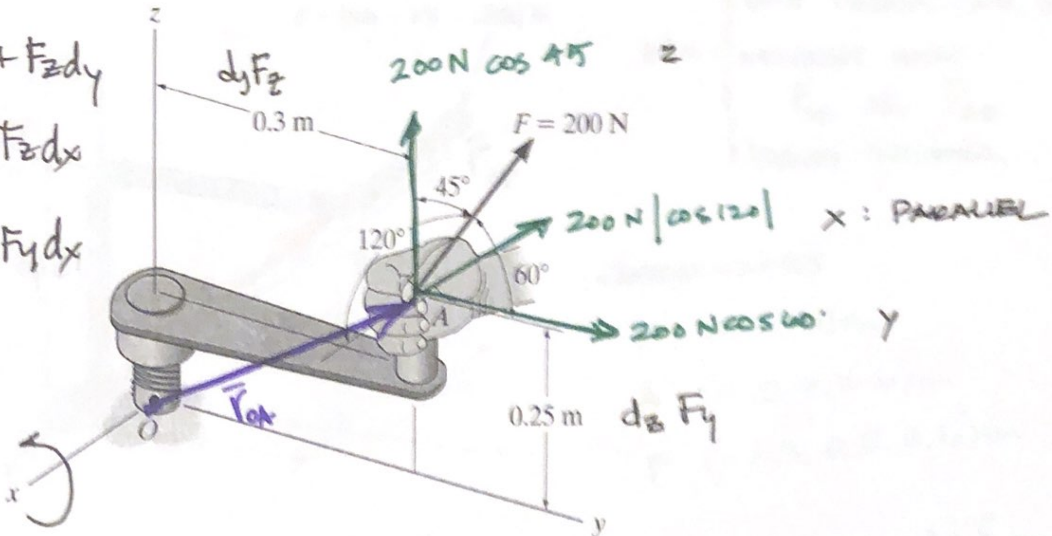
$M_z = 0 + 0 - 80\,lb\,ft \rightarrow \boxed{M_z = -80\,lb\,ft}$

- Example: Determine the moment about the x-axis using both scalar and vector analysis.

$M_x = F_y d_z + F_z d_y$

$M_y = F_x d_z + F_z d_x$

$M_z = F_x d_y + F_y d_x$



SCALAR

$$M_x = \underbrace{200\,N \cos 120}_{0 \text{ IN X-DIRECTION}} - (200\,N \cos 60)(0.25\,m) + (200\,N \cos 45)(0.3\,m)$$

$$M_x = 0 - 25\,Nm + 42.43\,Nm$$

$$\boxed{M_x = 17.43\,Nm}$$

VECTOR

$$M = \begin{vmatrix} \text{UNIT VECTOR–LINE} \\ \text{POSITION VECTOR MOMENT ARM} \\ \text{FORCE VECTOR} \end{vmatrix} \begin{matrix} \\ m \\ N \end{matrix}$$

COORDINATES

POINT ON X-AXIS (0, 0, 0) m

A (0, 0.3, 0.25) m

$u_{x\text{-axis}} = \{1\bar{i} + 0\bar{j} + 0\bar{k}\}$

$\bar{r}_{OA} = \{0\bar{i} + 0.3\bar{j} + 0.25\bar{k}\}$ m

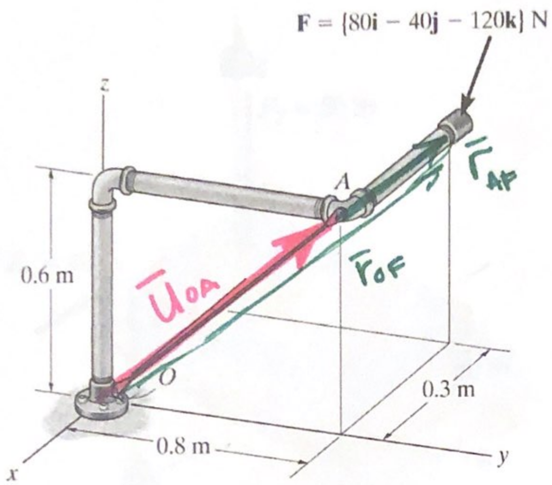
$\bar{F} = \{200 \cos 120\, \bar{i} + 200 \cos 60\, \bar{j} + 200 \cos 45\, \bar{k}\}$ N

$$M = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ 200\cos 120 & 200\cos 60 & 200\cos 45 \end{vmatrix} \begin{matrix} \\ m \\ N \end{matrix} = \begin{matrix} (0.3(200\cos 45) - 0.25(200\cos 60))1 \\ -(0(200\cos 45) - .25(200\cos 120))0 \\ +(0(200\cos 60) - 0.3(200\cos 120))0 \end{matrix}$$

$$= (17.43 + 0 + 0)\,Nm$$

$$\boxed{M = 17.43\,Nm}$$

- Example: Determine the moment about a line going from O to A using vector analysis.



$$M = \begin{vmatrix} \text{UNIT VECTOR LINE, } \bar{u}_{OA} \\ \text{MOMENT ARM } \bar{r}_{OF} \text{ OR } \bar{r}_{AF} \\ \text{FORCE VECTOR} \end{vmatrix}$$

COORDINATES

O $(0, 0, 0)$ m

A $(0, 0.8, 0.6)$ m

F $(-0.3, 0.8, 0.6)$ m

UNIT VECTOR, $\bar{u}_{OA}$

$$\bar{r}_{OA} = \{(0-0)i + (0.8-0)j + (0.6-0)k\} \text{ m}$$

$$\bar{r}_{OA} = \{0i + 0.8j + 0.6k\} \text{ m}$$

$$|r|_{OA} = \sqrt{(0m)^2 + (0.8m)^2 + (0.6m)^2} = 1 \text{ m}$$

$$\bar{u}_{OA} = \frac{\bar{r}_{OA}}{|r|} = \{0i + 0.8j + 0.6k\} \quad \times \text{ NO UNIT}$$

POSITION $\bar{r}_{OF}$ $\qquad \bar{r}_{OF} = \{-0.3i + 0.8j + 0.6k\}$ m ← NEED

FORCE $\bar{F}$ $\qquad \bar{F} = \{80i - 40j - 120k\}$ N

$$M_{OA} = \begin{vmatrix} 0 & 0.8 & 0.6 \\ -0.3 & 0.8 & 0.6 \\ 80 & -40 & -120 \end{vmatrix} \begin{matrix} \\ \text{m} \\ \text{N} \end{matrix} = \left[0 - [-0.3(-120) - (0.6)(80)](0.8) + [-0.3(-40) - 0.8(80)](0.6)\right]$$

$$= [0 + 9.6 - 31.2] = \boxed{-21.6 \text{ Nm} = M_{OA}}$$

CHECK $\bar{r}_{AF}$

$$M_{OA} = \begin{vmatrix} 0 & 0.8 & 0.6 \\ -0.3 & 0 & 0 \\ 80 & -40 & -120 \end{vmatrix} = (0) - (-0.3(-120) - 0)0.8 + (-0.3(-40) - 0)0.6$$

$$0 - 28.8 + 7.2 = \boxed{-21.6 \text{ NM} = M_{OA}}$$

$$\bar{M}_{OA} = M_{OA}\bar{u}_{OA} = -21.6 \text{ Nm} \{0i + 0.8j + 0.6k\}$$

$$\boxed{\bar{M}_{OA} = \{0i - 17.28j - 12.96k\} \text{ Nm}}$$