ENSC 2113

Engineering Mechanics: Statics

Chapter 2:

Force Vectors

(Sections 2.7-2.8)

COLLEGE OF
ENGINEERING, ARCHITECTURE
AND TECHNOLOGY

Chapter 2 Outline:

2.1 Scalars and Vectors

2.2 Vector Operations

2.3 Vector Addition of Forces

2.4 Addition of a System of Coplanar Forces

2.5 Cartesian Vectors

2.6 Addition of Cartesian Vectors

2.7 Position Vectors

2.8 Force Vector Directed Along a Line

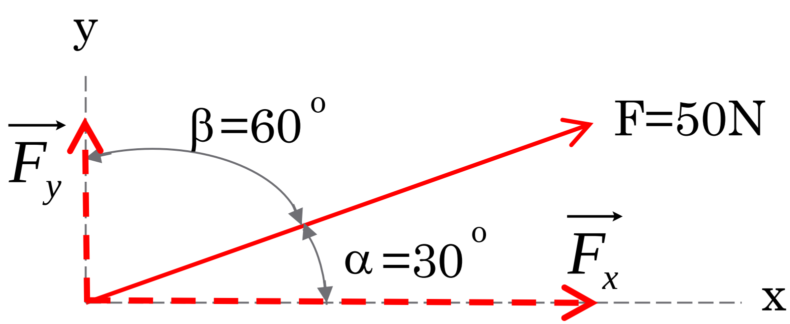
2.9 Dot Product

Chapter 2 Objectives:

- To show how to add forces and resolve them into components using the Parallelogram Law
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction
- To introduce the dot product in order to use it to find the angle between two vectors or the projection of one vector onto another

2D Vector Review:

- Vectors



- Finding the components from the magnitude and direction angles:

$$F_x = F\cos\alpha \; = 50\cos 30 = 43.3N$$

$$F_y = F\cos\beta \; = 50cos60 = 25N$$

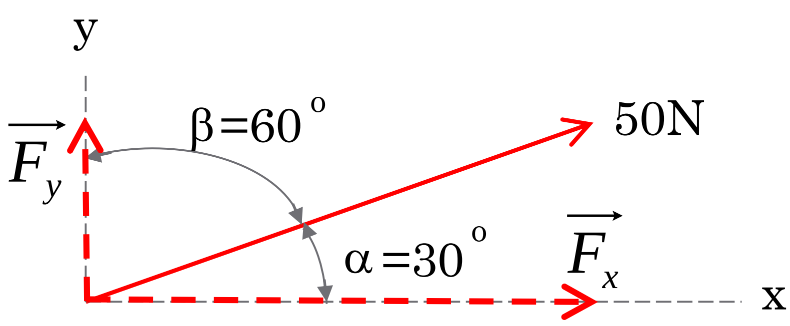
- Finding the direction angles from the magnitude and components:

$$\alpha = \cos^{-1}\frac{43.3}{50} = 30$$

$$\beta = \cos^{-1}\frac{25}{50} = 60$$

2D Vector Review:

- Vectors



  - Unit vector from magnitude and components:

$$\vec{u} = \left\{ \frac{F_x}{F}\hat{i} + \frac{F_y}{F}\hat{j} \right\}$$

$$\vec{u} = \left\{ \frac{43.3}{50}\hat{i} + \frac{25}{50}\hat{j} \right\}$$

$$\vec{u} = \{0.866\hat{i} + 0.5\hat{j}\}$$

- Force magnitude

$$|F| = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

$$|F| = 50\text{N}$$

- Force in Cartesian Vector Form:

$$\vec{F} = \{43.3\hat{i} + 25\hat{j}\}N$$

- Unit vector from angles:

$$\vec{u} = \{\cos 30\hat{i} + \cos 60\hat{j}\}$$

$$\vec{u} = \{0.866\hat{i} + 0.5\hat{j}\}$$

3D Vector Review:

- Vector Components
  - Written in x, y, and z components:

$$A_x = A\cos\alpha$$

$$A_y = A\cos\beta$$

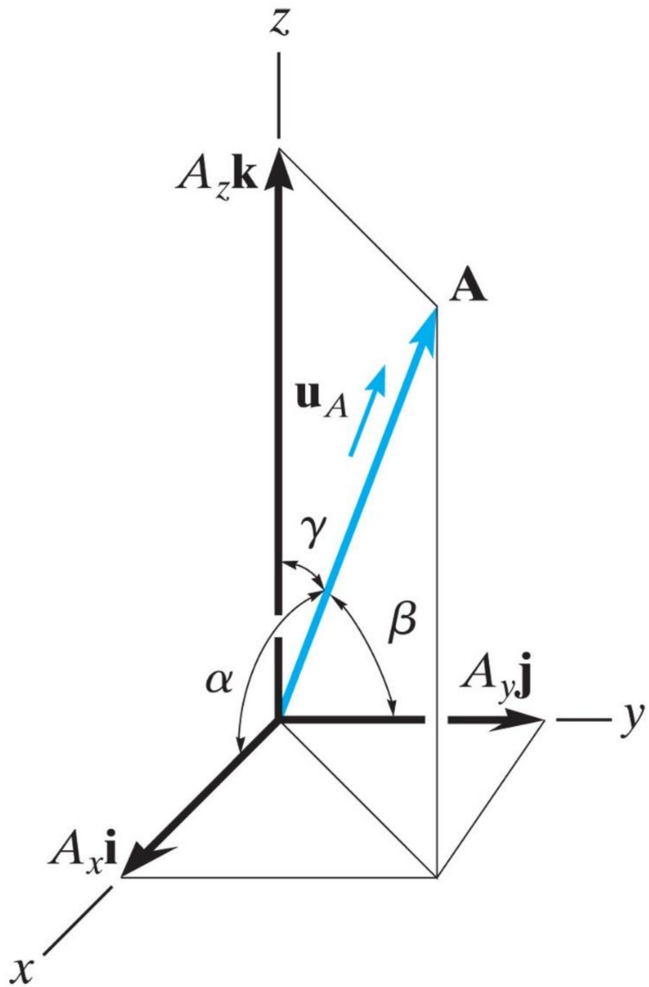
$$A_z = A\cos\gamma$$

  - Written in Cartesian Vector Form:

$$\vec{A} = \left\{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right\}$$

  - Magnitude:

$$|A| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$



3D Vector Review:

- Magnitude and Direction Cosines

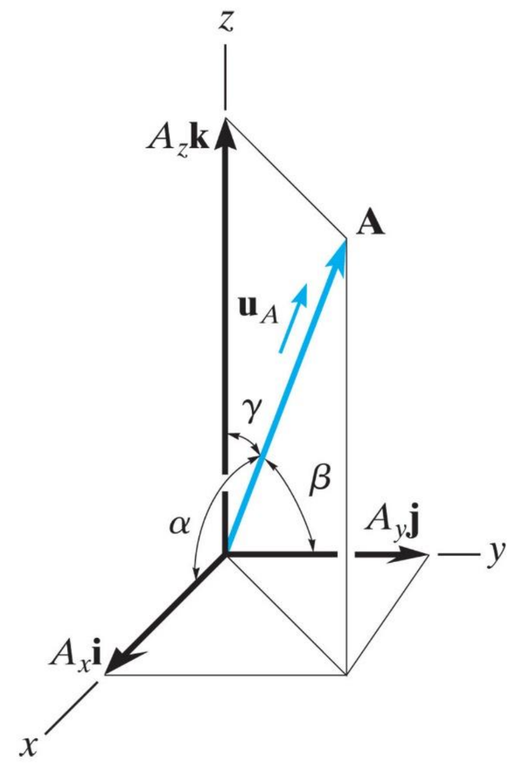
$$\vec{A} = |A|\vec{u}$$

$\vec{u}$ is a unit vector in the direction of A

$$\vec{u} = \left\{\cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}\right\}$$

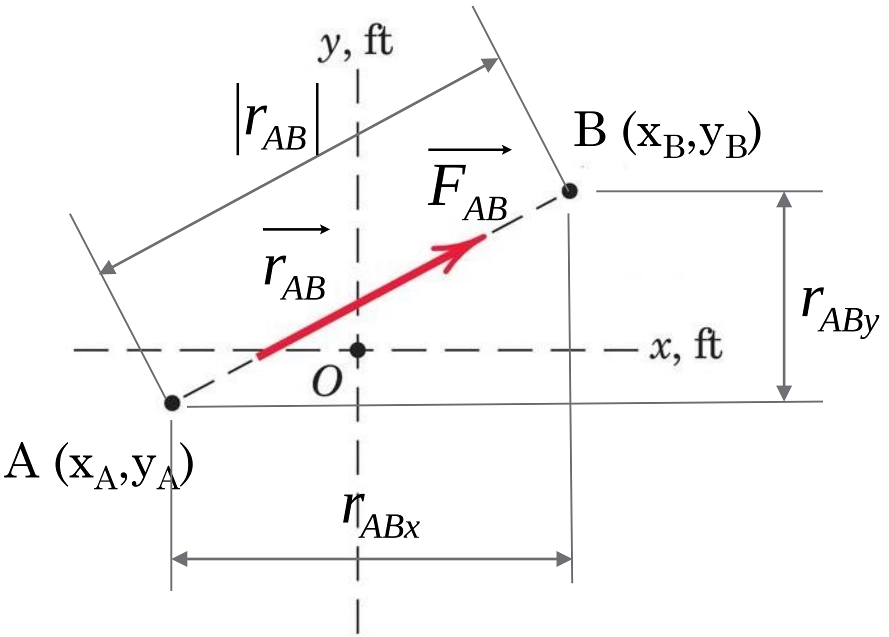
$$\vec{A} = |A|\left\{\cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}\right\}$$

$$\vec{A} = \left\{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right\}$$



2.7 Position Vectors:

- Position Vectors and Unit Vectors
  - Utilized to find rectangular components of force
  - A position vector measures the distance between two points using the coordinates



$$r_{ABx} = (x_B - x_A)\,ft$$

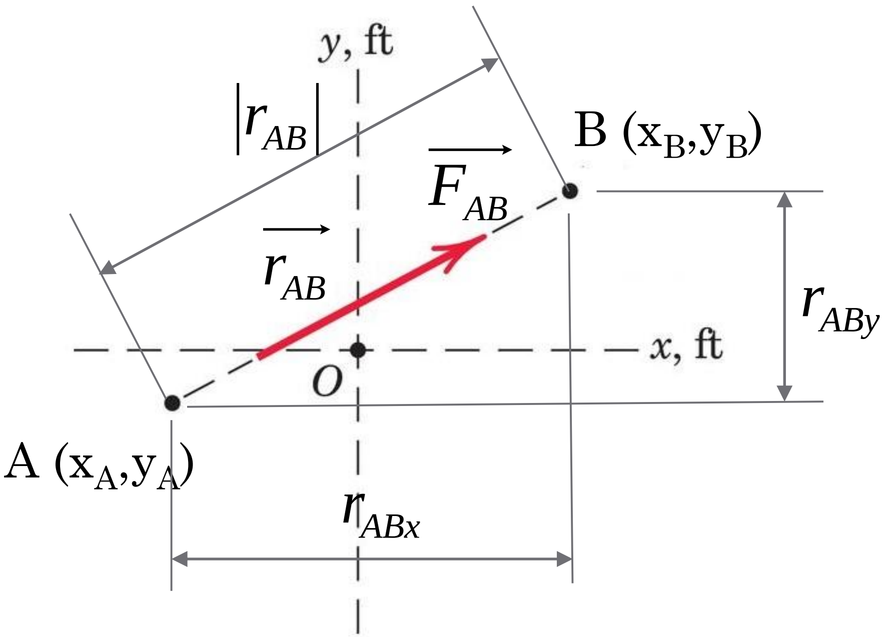
$$r_{ABy} = (y_B - y_A)\,ft$$

$$\overrightarrow{r_{AB}} = \{r_{ABx}\hat{i} + r_{ABy}\hat{j}\}\,ft$$

$$|r_{AB}| = \sqrt{(r_{ABx})^2 + (r_{ABy})^2}\,ft$$

2.7 Position Vectors:

- Position Vectors and Unit Vectors
  - Recall that a unit vector is any vector divided by its magnitude
  - Apply to the determined position vector



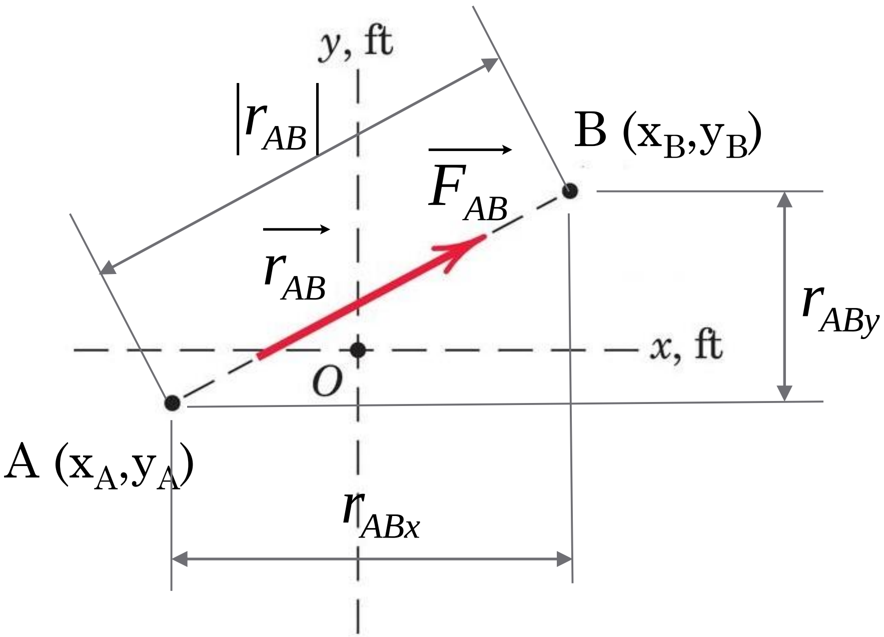
$$\overrightarrow{r_{AB}} = \{r_{ABx}\hat{i} + r_{ABy}\hat{j}\}ft$$

$$|r_{AB}| = \sqrt{(r_{ABx})^2 + (r_{ABy})^2}\ ft$$

$$\overrightarrow{u_{AB}} = \left\{\frac{\overrightarrow{r_{AB}}}{|r_{AB}|}\right\}$$

2.7 Position Vectors:

- Position Vectors and Unit Vectors
  - Force components are found by multiplying the force magnitude by the unit vector



$$\overrightarrow{u_{AB}} = \left\{ \frac{\overrightarrow{r_{AB}}}{|r_{AB}|} \right\}$$

$$\overrightarrow{F_{AB}} = |F| \left\{ \frac{\overrightarrow{r_{AB}}}{|r_{AB}|} \right\}$$

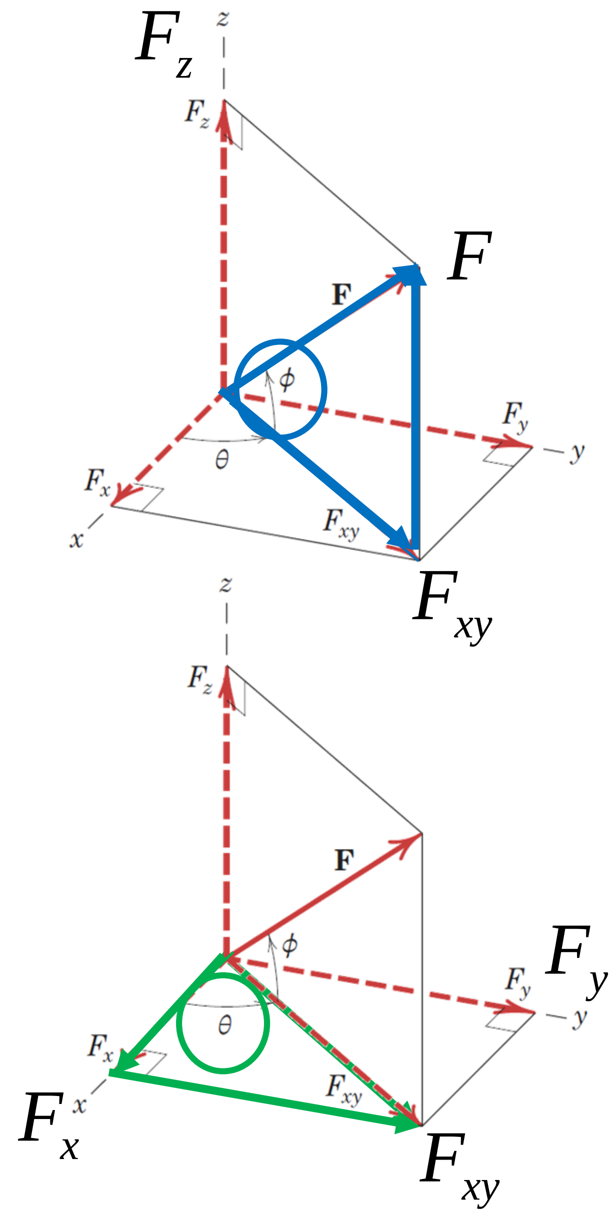
$$\overrightarrow{F_{AB}} = \left\{ F_{ABx}\hat{i} + F_{ABy}\hat{j} \right\}$$

2.8 Force Vector Directed Along a Line

1. Specification by two angles

- Step 1: Horizontal and Vertical Components (z-xy plane)

$$F_{xy} = F\cos\phi \qquad F_z = F\sin\phi$$



- Step 2: Components in x-y plane

$$F_x = F_{xy}\cos\theta$$

$$F_y = F_{xy}\sin\theta$$

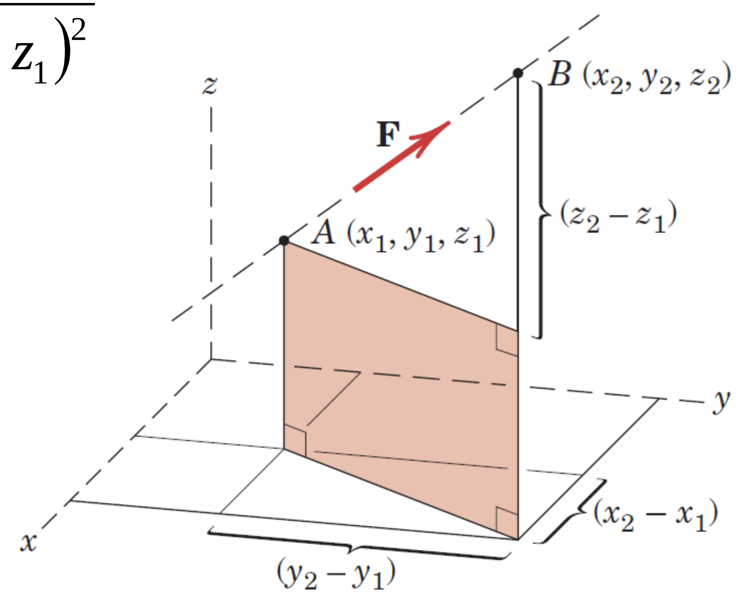
2.8 Force Vector Directed Along a Line

2. Specification by two points on line of action

- $\vec{F} = |F|\vec{u}\ ,\quad \vec{u} = \dfrac{Position\ vector\ of\ Force}{Magnitude\ of\ Position\ Vector\ of\ Force}$

$$\overrightarrow{F_{AB}} = |F|\vec{u} = |F|\frac{\overrightarrow{r_{AB}}}{|r_{AB}|}$$

$$|F|\frac{\{(x_2 - x_1) + (y_2 - y_1) + (z_2 - z_1)\}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$



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