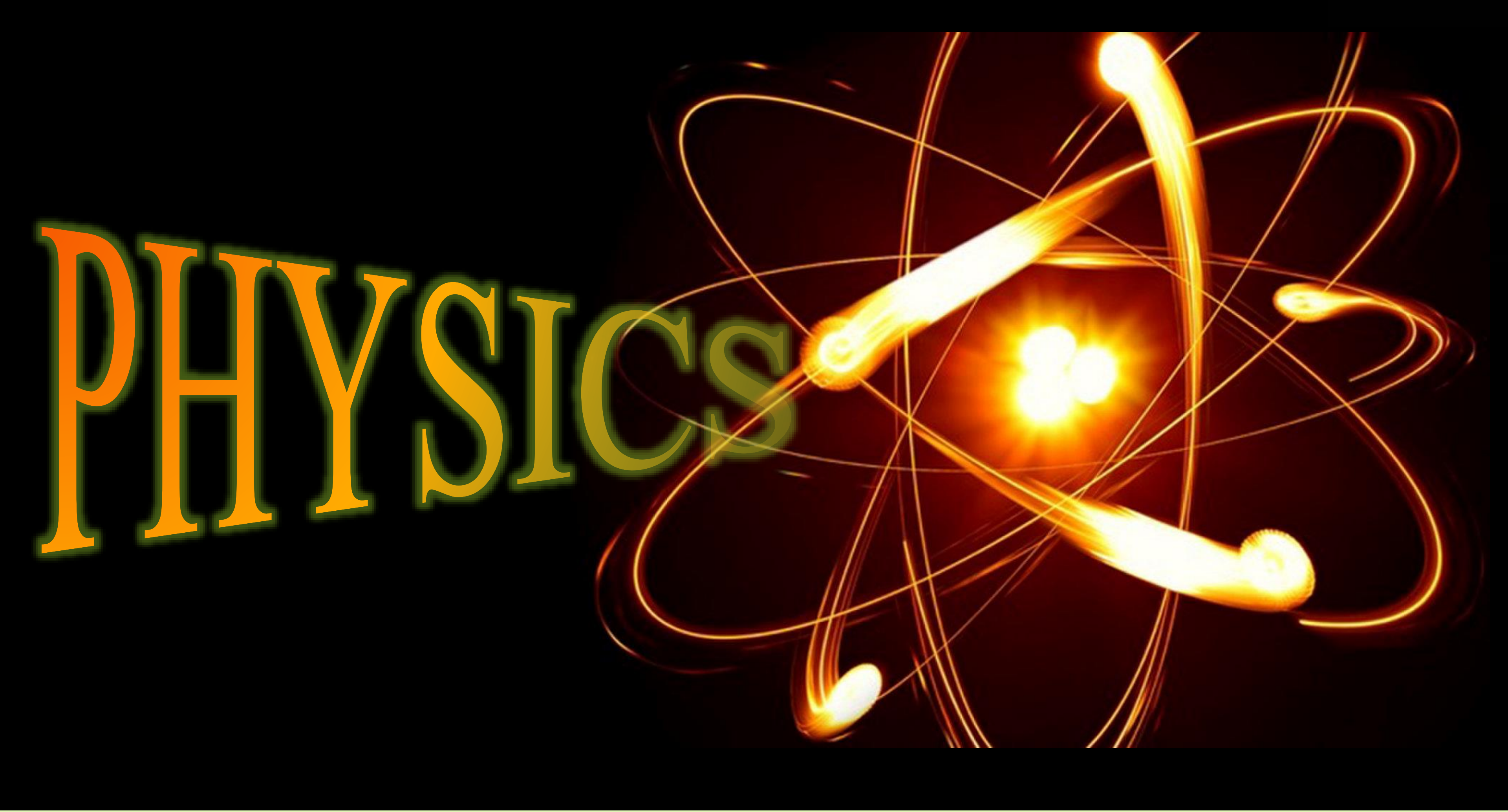
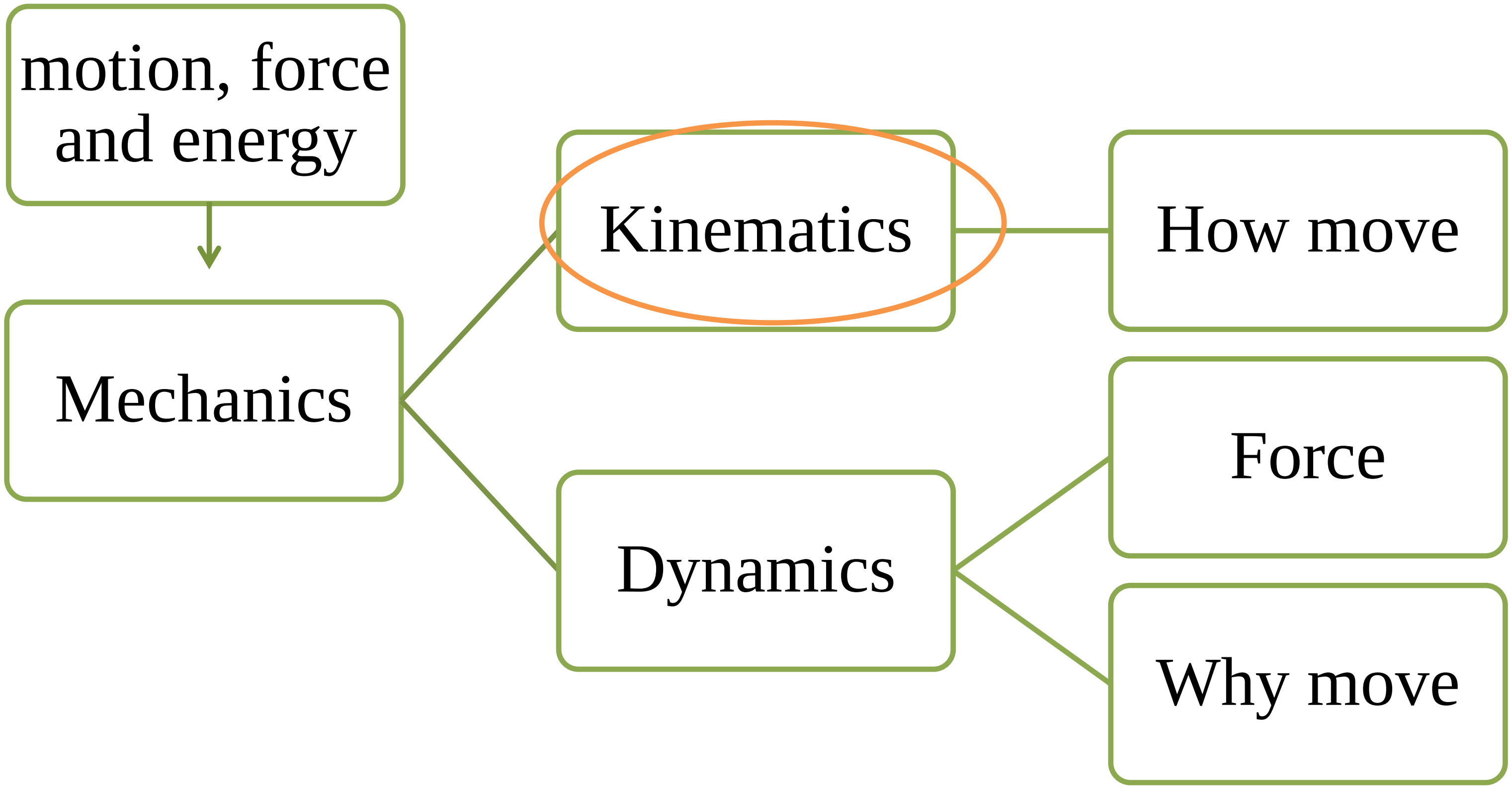

PHYSICS

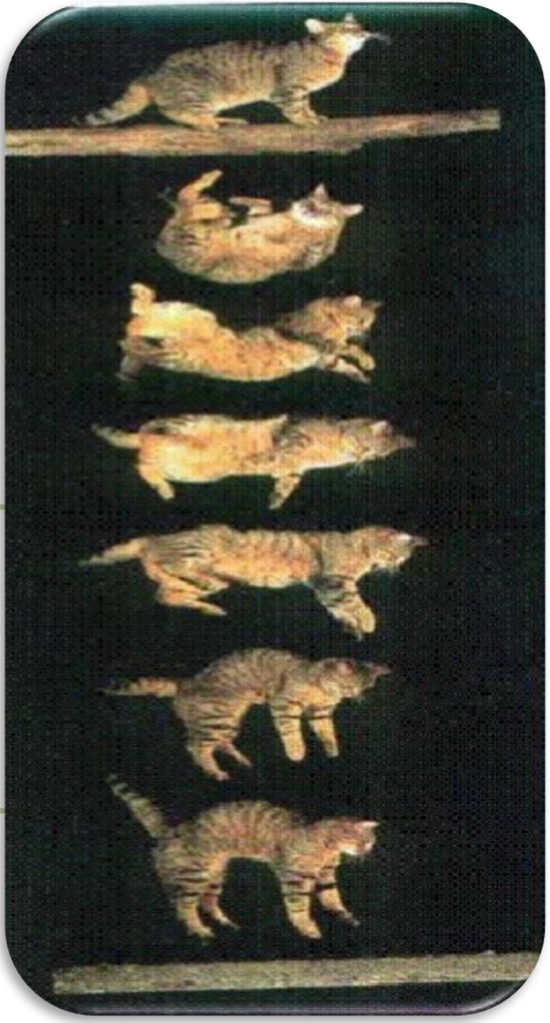
Motion


motion, force and energy
Mechanics
Kinematics
How move
Dynamics
Force
Why move

Motion

Motion

- Translational
- Rotational



Outline

- **Kinematics**
  - Conceptual introduction
    - Reference frame
    - Coordinate system
  - Motion in one dimension
    - Definitions
      - Distance
      - Displacement
      - Velocity
      - Speed
      - Acceleration
    - Free falling (the direction)
  - Motion in two dimension
    - Projectile motion
    - Calculation of 2D motion

Practice

A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is

(*a*) less than 20 m/s.

(*b*) exactly 20 m/s.

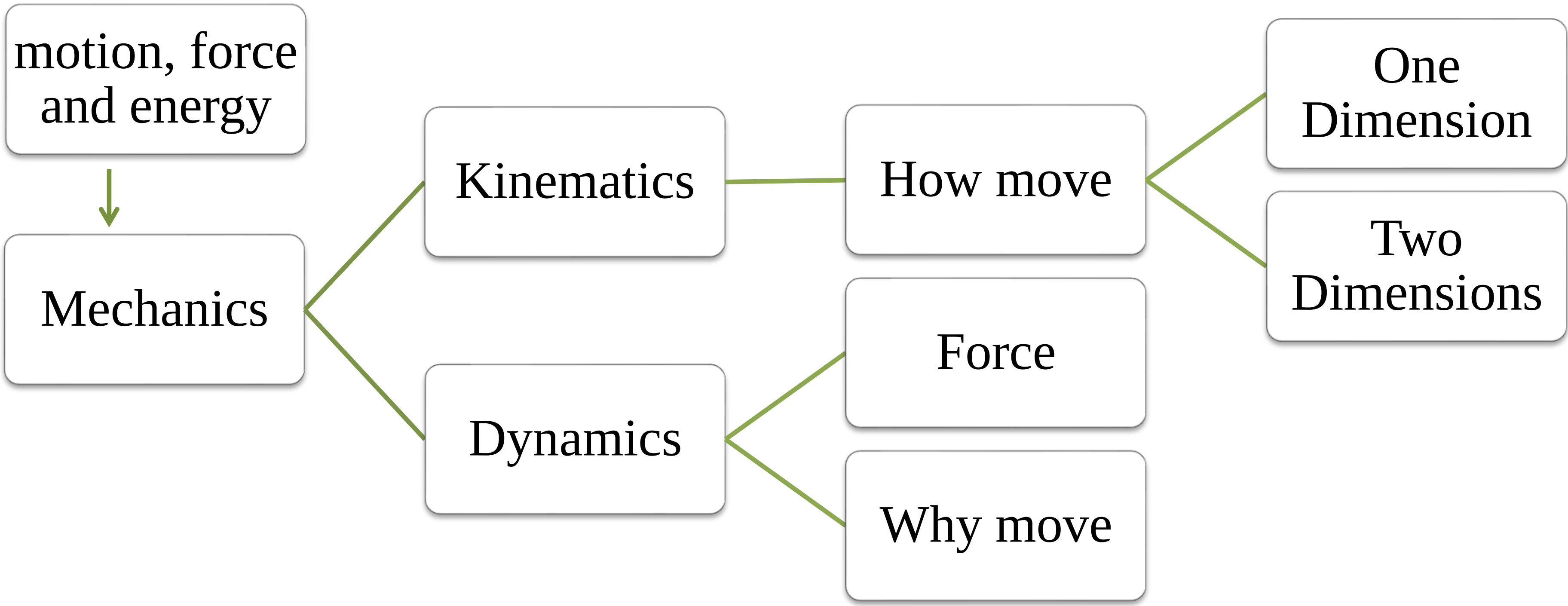
(*c*) more than 20 m/s.

How about equation and the vector diagram?

Summary

Kinematics

Structure


motion, force and energy
Mechanics
Kinematics
Dynamics
How move
Force
Why move
One Dimension
Two Dimensions

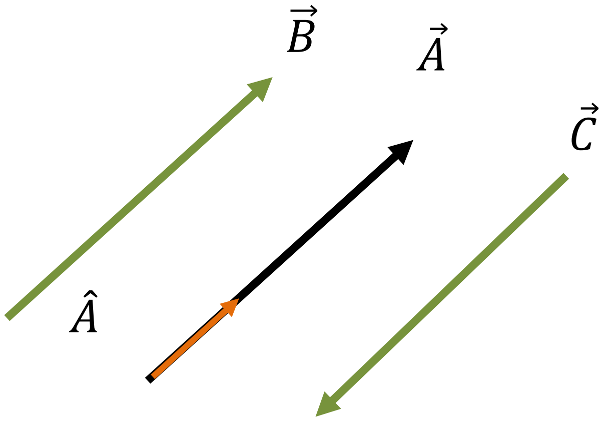
Representation of Vectors

Vector: has both magnitude and a direction, such as $\vec{r},\ \vec{v},\ \vec{a},$

$\vec{A}$ $\hat{A}$

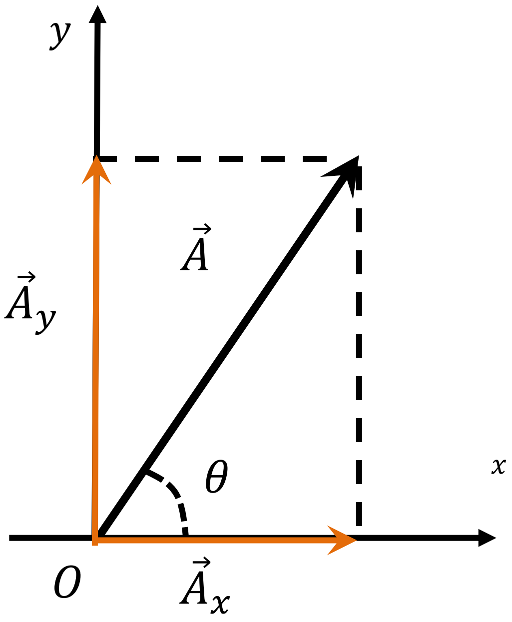
Vector **Unit Vector**

$$\vec{A} = |\vec{A}|\hat{A} = A\hat{A}$$



$$\vec{B} = \vec{A}, \vec{C} = -\vec{A}$$

The Cartesian representation of any vector



$\hat{\imath}$, $\hat{\jmath}$ **represent unit vectors in direction of +*x*-axis or +*y*-axis**

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

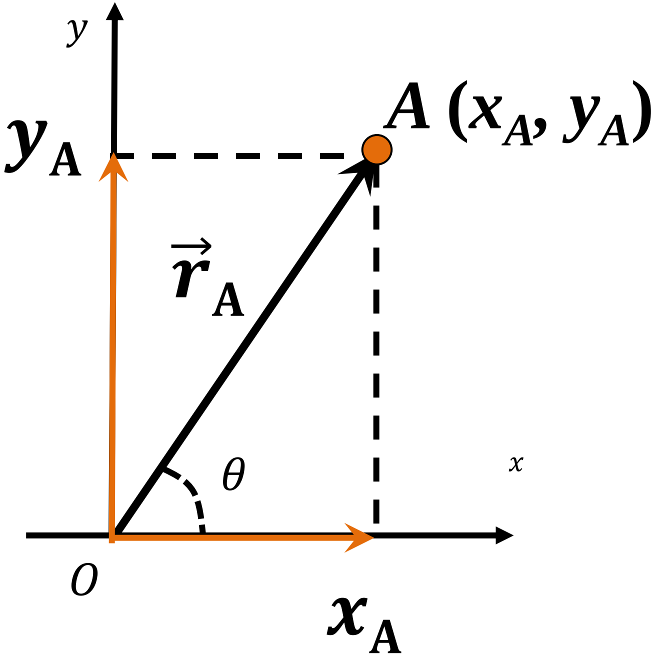
$$\vec{A}_x = A_x\hat{\imath},\ \vec{A}_y = A_y\hat{\jmath}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}$$

$$tg\theta = A_y/A_x$$

Example: Position Vector



Position Vector: $\vec{r}_A$

$$\vec{r}_A = x_A\hat{\imath} + y_A\hat{\jmath}$$

$$|\vec{r}_A| = \sqrt{{x_A}^2 + {y_A}^2}$$

$$tg\theta = y_A/x_A$$

The case of three dimensions

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

We can prove that

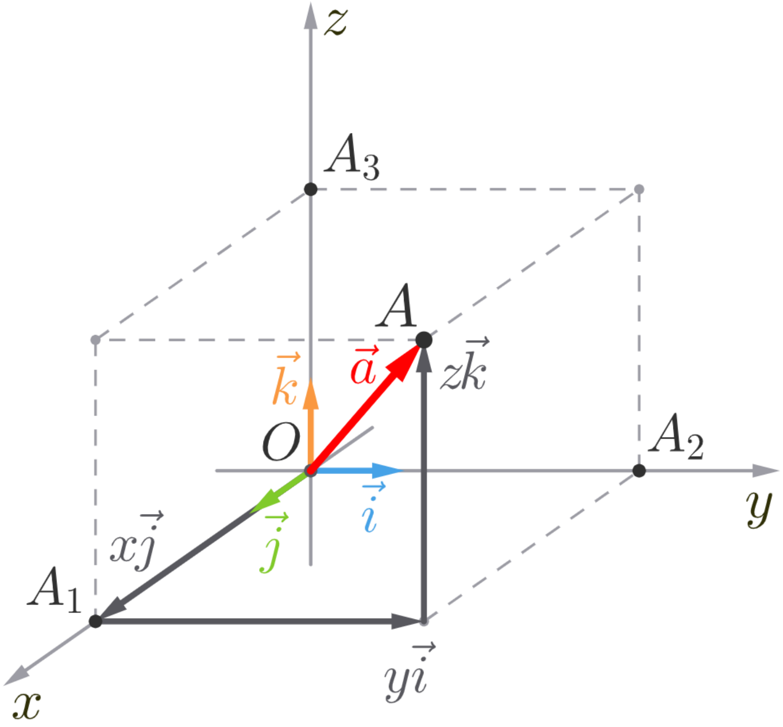
$$A = (\vec{A}\cdot\vec{A})^{1/2} = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

$$\vec{A}\cdot\hat{\imath} = A\cos\alpha = A_x$$

$$\vec{A}\cdot\hat{\jmath} = A\cos\beta = A_y$$

$$\vec{A}\cdot\hat{k} = A\cos\gamma = A_z$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

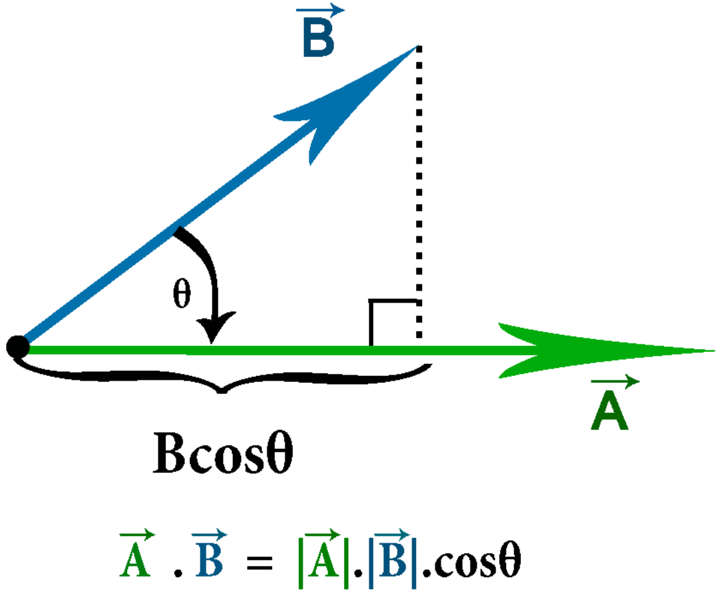


Multiplying a vector by a vector

(1) The scalar (dot) product

$$\vec{A} \cdot \vec{B} = C$$

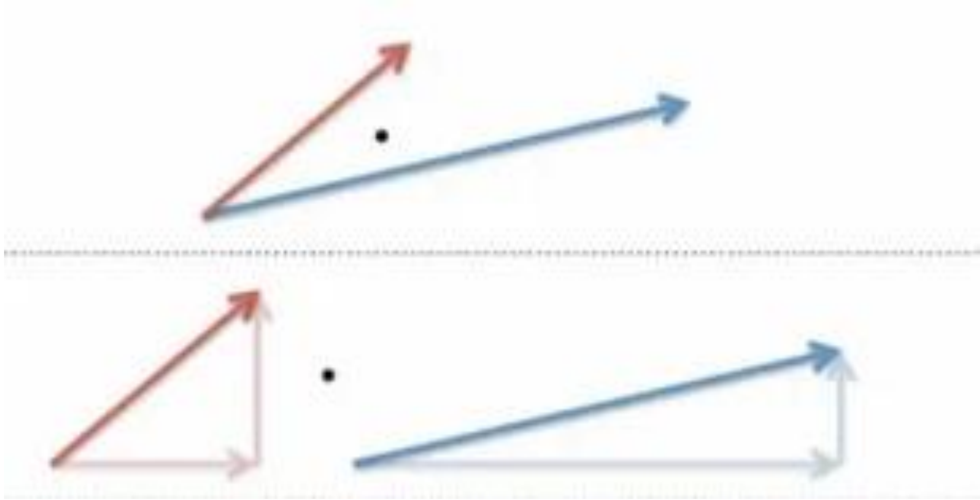
$$C = AB \cos\theta$$

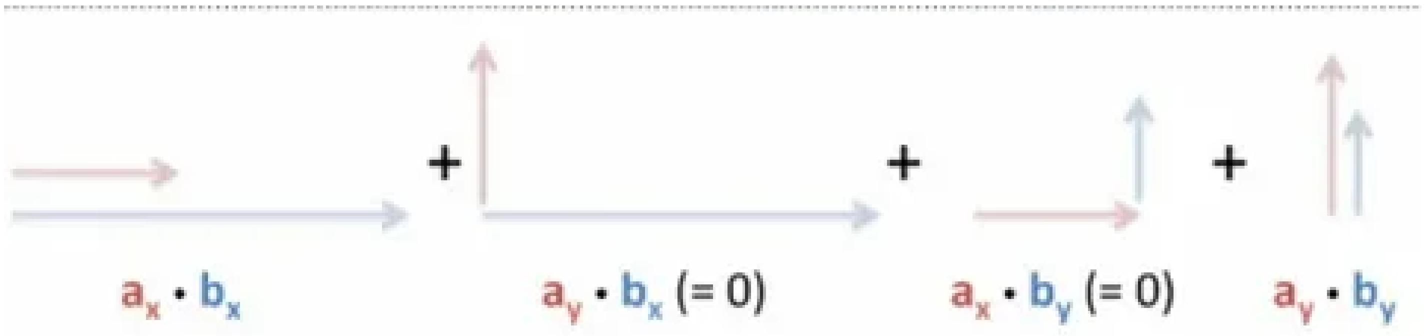


Multiplying a vector by a vector

(1) The scalar (dot) product

Decompose of each one





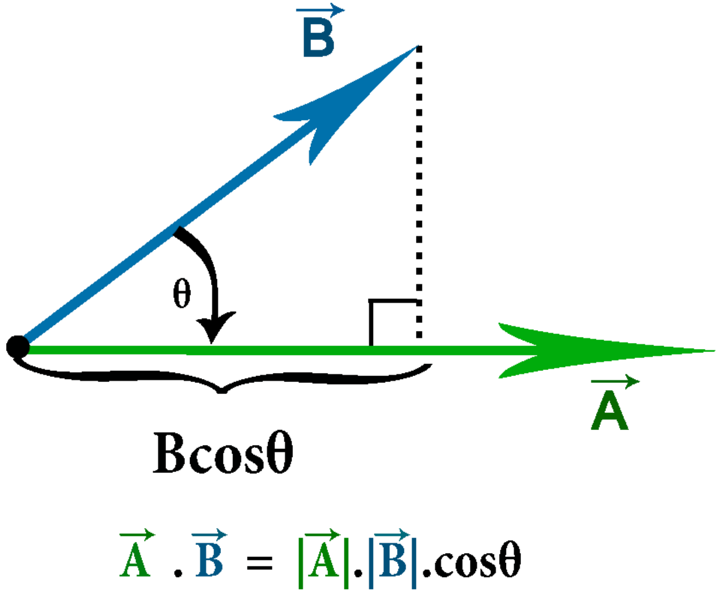
Multiplying a vector by a vector

(1) The scalar (dot) product

$$\vec{A} \cdot \vec{B} = C$$

$$C = AB \cos\theta$$

$$C = A_x B_x + A_y B_y + A_z B_z$$

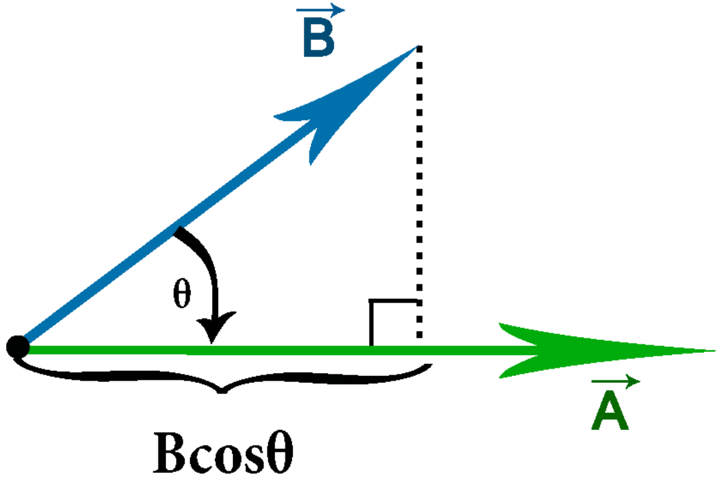


Multiplying a vector by a vector

(1) The scalar (dot) product

The magnitude of $\vec{A}$

$$\boldsymbol{A} = |\vec{\boldsymbol{A}}| = (\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{A}})^{1/2}$$



$$\vec{A} \,.\, \vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta$$

Multiplying a vector by a vector

(1) The scalar (dot) product

A, B are always positive

$\alpha < \pi/2$

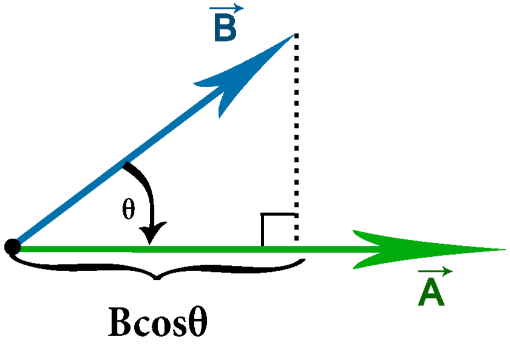
$$\vec{A} \cdot \vec{B} = AB\cos\alpha > 0$$

$\alpha > \pi/2$

$$\vec{A} \cdot \vec{B} = AB\cos\alpha < 0$$

$\alpha = \pi/2$

$$\vec{A} \cdot \vec{B} = AB\cos\alpha = 0$$



$\vec{A} \cdot \vec{B} = |\vec{A}|.|\vec{B}|.\cos\theta$

$$\vec{A} \cdot \vec{B} = C$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A}.\vec{C}$$

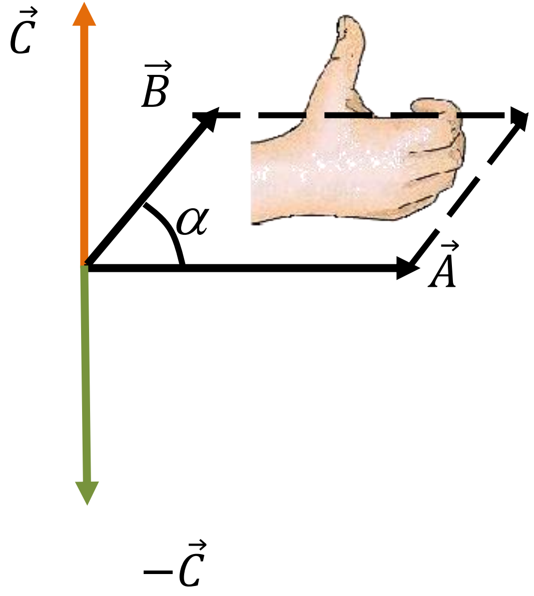
Multiplying a vector by a vector

(2) The vector (cross) product

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{B} \times \vec{A} = -\vec{C}$$

$$\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



Right-hand rule

Multiplying a vector by a vector

(2) The vector (cross) product

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{B} \times \vec{A} = -\vec{C}$$

$$\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

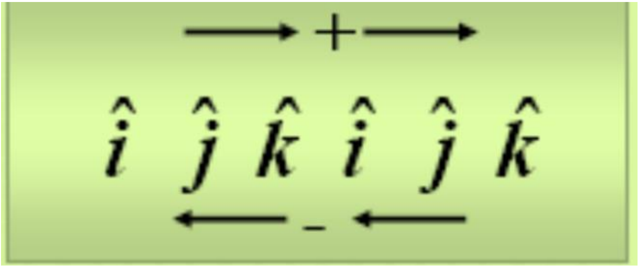
$$\vec{C} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = AB\sin\alpha$, direction is as the figure showing.

Multiplying a vector by a vector

(2) The vector (cross) product

Mnemonic:



$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{B} \times \vec{A} = -\vec{C}$$

where

$\hat{\imath} \times \hat{\imath} = 0$	$\hat{\imath} \times \hat{\jmath} = \hat{k}$	$\hat{\imath} \times \hat{k} = -\hat{\jmath}$
$\hat{\jmath} \times \hat{\imath} = -\hat{k}$	$\hat{\jmath} \times \hat{\jmath} = 0$	$\hat{\jmath} \times \hat{k} = \hat{\imath}$
$\hat{k} \times \hat{\imath} = \hat{\jmath}$	$\hat{k} \times \hat{\jmath} = -\hat{\imath}$	$\hat{k} \times \hat{k} = 0$

Multiplying a vector by a vector

(2) The vector (cross) product

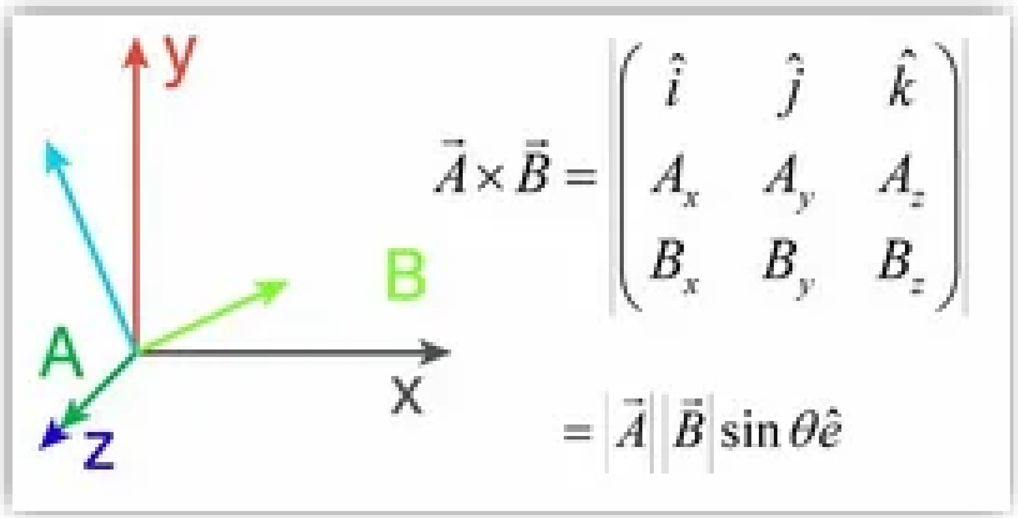
α is always less than π

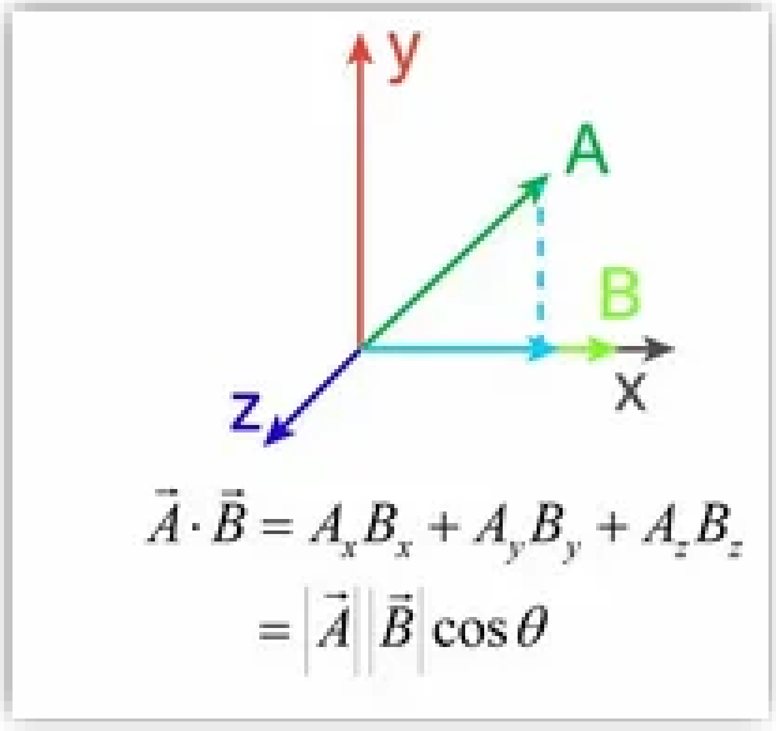
$$\vec{A}\times\vec{A}=0 \quad \text{or} \quad \vec{B}\times\vec{B}=0$$

$$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A} \qquad \vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

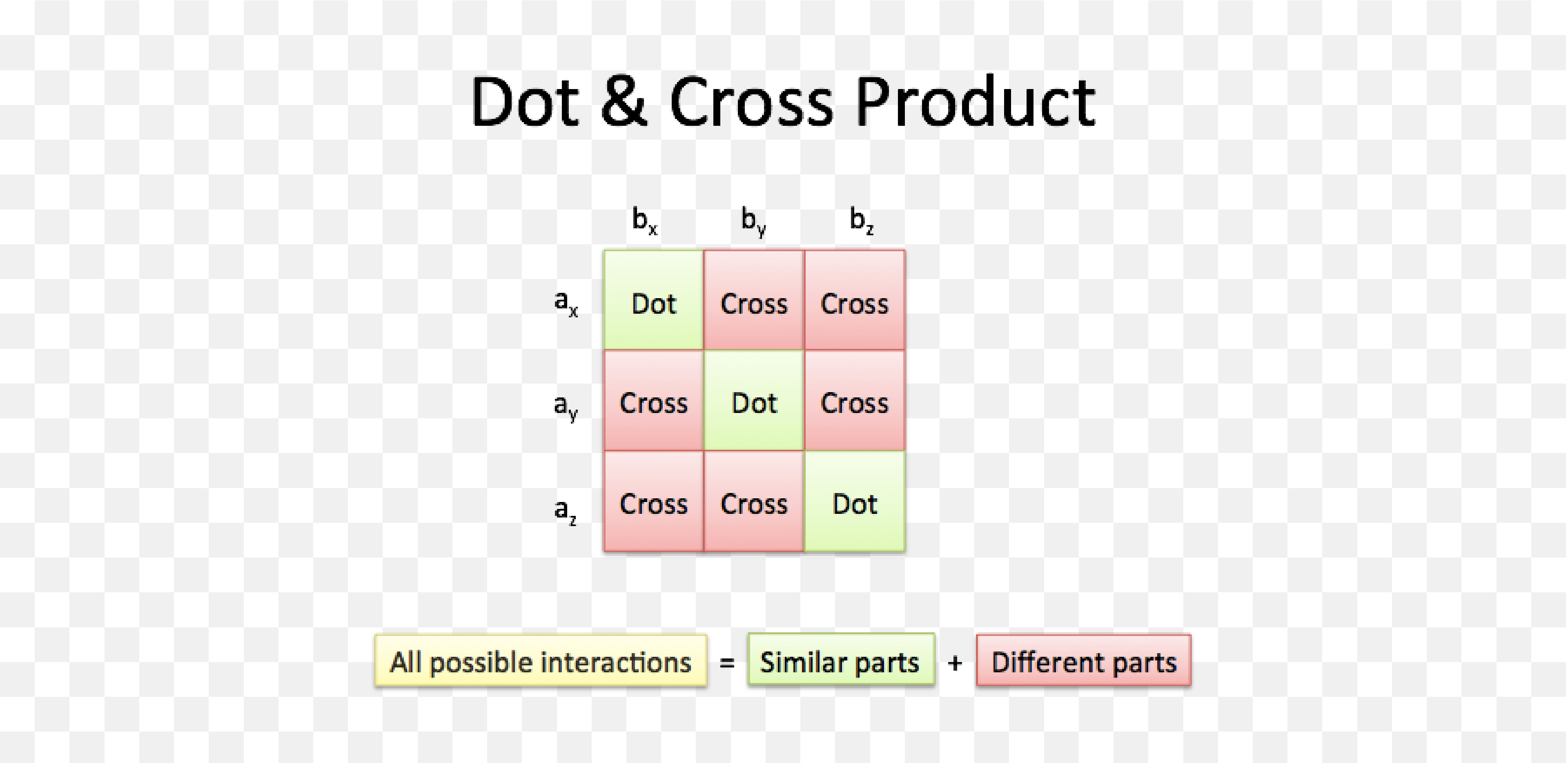
$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

Vector review





Vector review



Variation of a vector

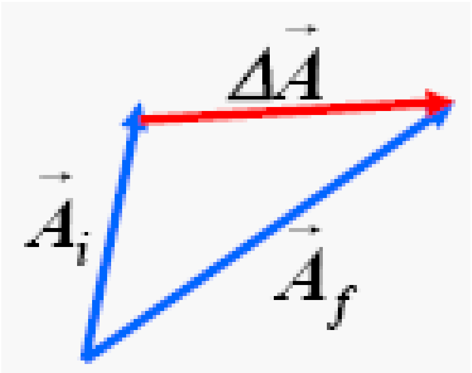
(1).The Magnitude changes, the direction is preserved;

(2).The direction changes, the magnitude is preserved;

(3). Both the magnitude and direction change.

$$\Delta\vec{A} = \vec{A}_f - \vec{A}_i$$

$$\vec{A}_f = \vec{A}_i + \Delta\vec{A}$$



Differentiation of a vector

$$\frac{d\vec{A}}{dt} = \frac{d}{dt}(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) = \frac{dA_x}{dt}\hat{\imath} + \frac{dA_y}{dt}\hat{\jmath} + \frac{dA_z}{dt}\hat{k}$$

$$\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

$$\frac{d}{dt}(\vec{A} - \vec{B}) = \frac{d\vec{A}}{dt} - \frac{d\vec{B}}{dt}$$

$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$$

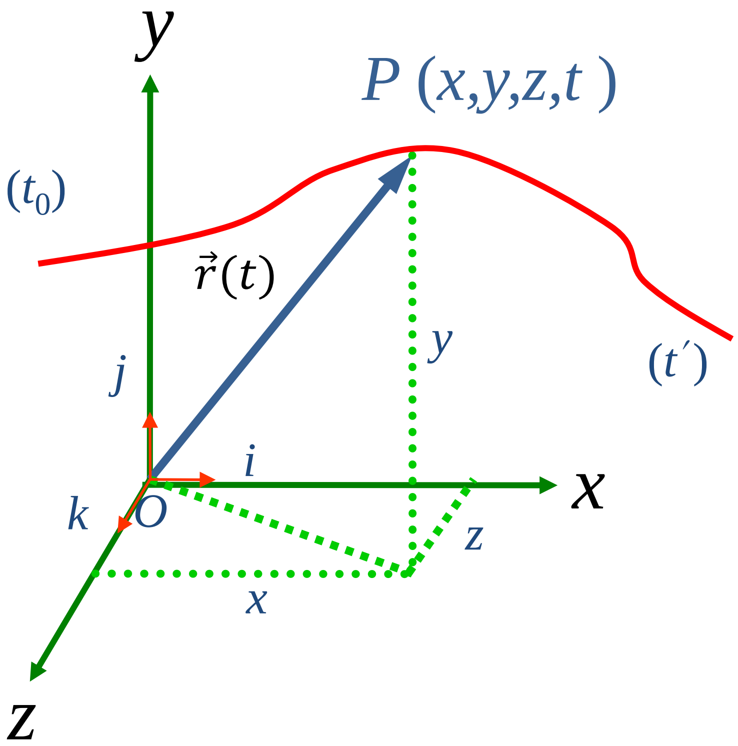
$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

Position and displacement

Position vector –The location of a particle relative to the origin of a coordinate system.

For a Cartesian system:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$



Position and displacement

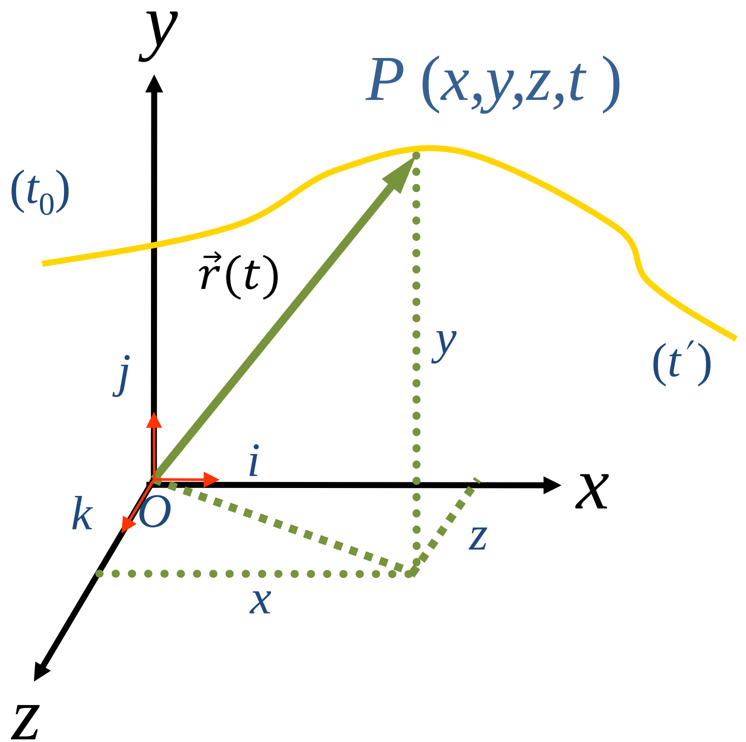
$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

Motion function：

$$\vec{r} = \vec{r}(t) \quad = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Trajectory equation：

$$\boldsymbol{f\,(x, y, z) = 0}$$



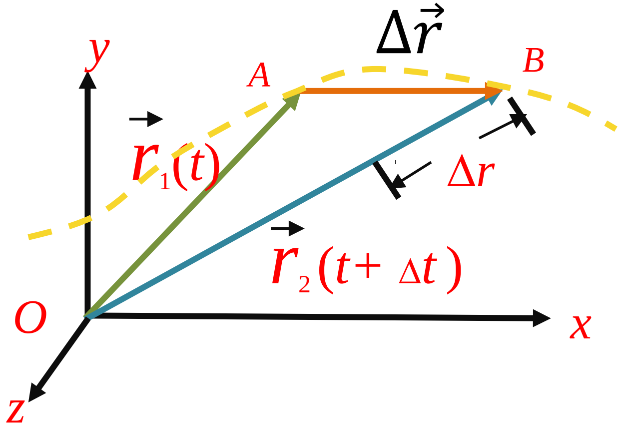
Position and displacement

Displacement $\Delta\vec{r}$ — A particle is changing in its position

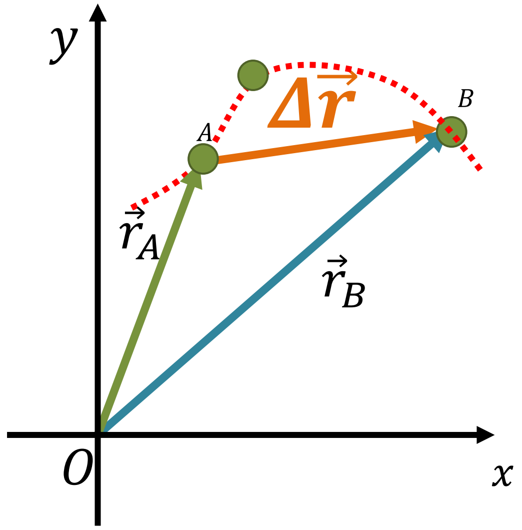
$$\Delta\vec{r} = \vec{r}_2(t+\Delta t) - \vec{r}_1(t) = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$$



Displacement Vector



$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

$$|\Delta\vec{r}| = \overline{AB}$$

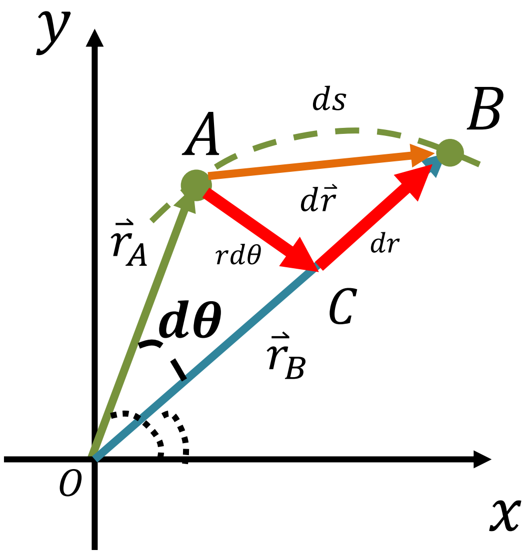
$$|\Delta\vec{r}| \neq \overset{\frown}{AB}$$

$$\Delta s = \overset{\frown}{AB}$$

Displacement is different from distance.

Displacement Vector

A very small displacement during a very small time interval $|d\vec{r}| = \overline{AB}$



A very small distance: $ds = \widehat{AB}$

$t \to 0$ $\quad |d\vec{r}| = ds$

Let $\overline{OA} = \overline{OC}$

$\overline{AC} = rd\theta \qquad \overline{CB} = dr$

$\because \overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$

$\therefore d\vec{r} = \overrightarrow{AC} + \overrightarrow{CB} = rd\theta\hat{\theta} + dr\hat{r}$

$$\boxed{|d\vec{r}| \neq dr}$$

Example 1

Chose the correct equation:

$$(1) v = \frac{d\vec{r}}{dt}$$

$$(2) v = \frac{d|\vec{r}|}{dt}$$

$$(3) \vec{v} = \frac{d\vec{r}}{dt}$$

$$(4) v = \frac{dr}{dt}$$

Example 2

Chose the correct equation:

$$(1)a = \frac{dv}{dt} \qquad (2)a = \frac{d\vec{v}}{dt}$$

$$(3)\vec{a} = \frac{d^2\vec{r}}{dt^2} \qquad (4)a = \frac{d|\vec{v}|}{dt}$$

Review

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \vec{v}(t)$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\Delta\vec{r} = \vec{r}_2(t + \Delta t) - \vec{r}_1(t) = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\vec{i} \quad +(y_2 - y_1)\vec{j} \quad +(z_2 - z_1)\vec{k}$$

$$= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{\overrightarrow{\mathrm{d}\, r}}{\mathrm{d}\, t} = \frac{\mathrm{d}\left[x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}\right]}{\mathrm{d}\, t} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \vec{v}(t)$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right) = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

Example

If we know the position vector of a particle

$$\vec{r} = 2t\hat{\imath} + (2 - t^2)\hat{\jmath} \qquad \text{(SI)}$$

Find the trajectory of the particle; the position vector at t = 0s and t = 2s; the velocity and the acceleration of the particle at instant t = 2s.

Solution: (1)trajectory

$$\begin{cases} x = 2t \\ y = 2 - t^2 \end{cases}$$

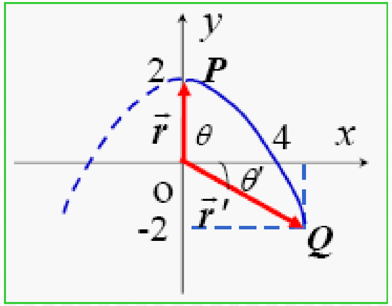
Eliminate t, we can get $y = 2 - \frac{x^2}{4}$ —parabola

(2) position vector:

t = 2s， x = 4 y = -2 $\vec{r}' = 4\vec{i} - 2\vec{j}$

t = 0s， x = 0 y= 2 $\vec{r} = 2\vec{j}$

Example



$$y = 2 - \frac{x^2}{4}$$

$$\vec{r} = 2\vec{j}$$

$$\vec{r}' = 4\vec{i} - 2\vec{j}$$

The magnitude：

$$r = |\vec{r}| = 2(m)$$

$$r' = |\vec{r}'| = \sqrt{4^2 + (-2)^2} = 4.47(m)$$

The direction：

The angle between $\vec{r}$ and x-axis $\theta = arctg\frac{2}{0} = 90°$

The angle between $\vec{r}'$ and x-axis $\theta' = arctg\frac{-2}{4} = -26°32'$

Example

(3)The velocity: $\vec{r} = 2t\vec{i} + (2 - t^2)\vec{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\vec{i} - 2t\vec{j}$$

Magnitude: $v_x = 2 \qquad v_y = -2t$

$$v = \sqrt{v_x^2 + v_y^2} = 2\sqrt{1 + t^2}$$

$$t = 2 \qquad v_2 = 2\sqrt{5} = 4.47m \cdot s^{-1}$$

The angle between velocity and *x*-axis:

$$\alpha = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-4}{2}$$

Example

(3)The acceleration: $\vec{r} = 2t\hat{\imath} + (2 - t^2)\hat{\jmath}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\imath} - 2t\hat{\jmath}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = -2\hat{\jmath}$$

The magnitude: $|\vec{a}| = a = 2$

Direction of the acceleration: $-\hat{\jmath}$

Exercise: The position of a small bumper car in an amusement park ride is described as a function of time by the coordinates

$$x = 0.2t^2 + 5.0t + 0.5 \qquad \text{(SI)}$$

$$y = -1.0t^2 + 10.0t + 2.0 \qquad \text{(SI)}$$

Find (a) the position vector at t=1.0 s and t=3.0 s.

(b) the displacement vector between these time.

(c) average velocity over the period from 1.0 s to 3.0 s, and the velocity at t=3.0 s.

(d) the magnitude and direction of the acceleration at t=1.0 s and t=3.0 s.

3-4. Rectilinear motion with a constant acceleration

1. Some rules

(1). ignore the effects of air resistance

(2). the origin of the coordinate could be chosen discretionarily

(3).write the constant acceleration as $\vec{a} = a_x \hat{\imath}$

(4). choose the initial time instant to be

$$t_i = 0 \qquad \Delta t = t_f - t_i = t$$

(5). let $x(t_i) = x(0) = x_0$ $\qquad v_x(t_i) = v_x(0) = v_{x0}$

Rectilinear motion with a constant acceleration

From $$\frac{dv_x(t)}{dt} = a_x$$

We have $$\int_{v_{x0}}^{v_x(t)} dv_x\,(t) = \int_0^t a_x\,dt$$

$$v_x(t) = v_{x0} + a_x t$$

Likewise, from $$\frac{dx(t)}{dt} = v_x(t)$$

We have $$\int_{x_0}^{x(t)} dx(t) = \int_0^t v_x dt = \int_0^t (v_{x0} + a_x t) dt$$

$$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

Rectilinear motion with a constant acceleration

$$a_x = con\tan t \quad \text{where} \quad \vec{a} = a_x \hat{\imath}$$

$$\begin{cases} v_x(t) = v_{x0} + a_x t & \text{where} \quad \vec{v}(t) = v_x(t)\hat{\imath} \\ x(t) = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 & \text{where} \quad \vec{r}(t) = x(t)\hat{\imath} \end{cases}$$

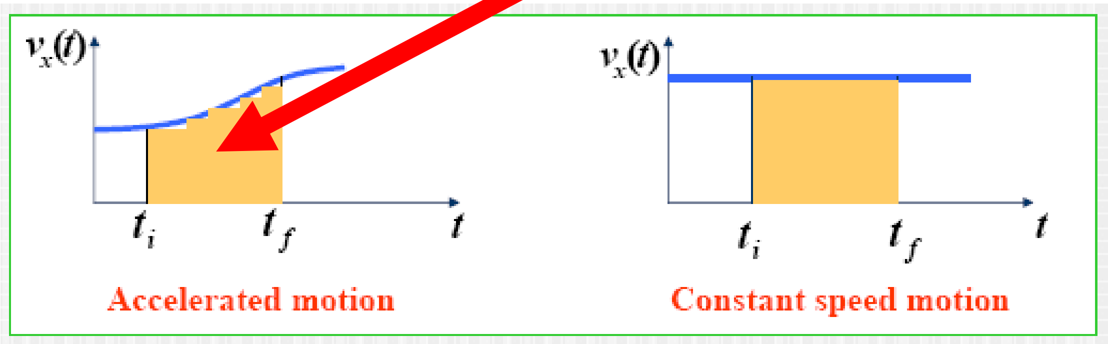
Eliminate t in equations about v_x and x

$$v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$$

3-5. Geometric interpretations

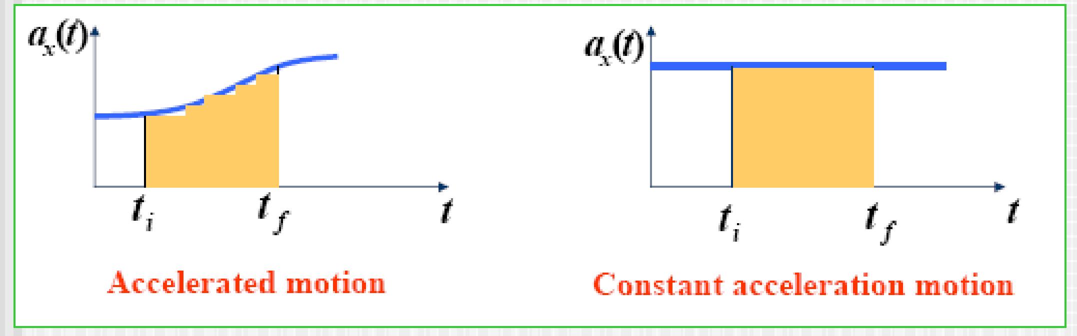
1. The change in the position vector component

$$v_x = \frac{dx}{dt} \qquad dx = v_x(t)dt \qquad \Delta x = \int_{x(t_i)}^{x(t_f)} dx = \int_{t_i}^{t_f} v_x(t)\, dt$$

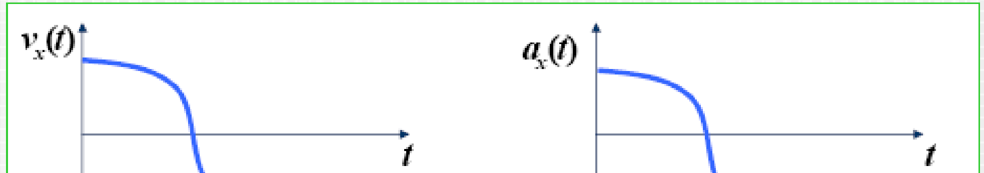


2. The change in the velocity component

$$a_x = \frac{dv_x}{dt} \qquad dv_x = a_x dt \qquad \Delta v_x = \int_{v_x(t_i)}^{v_x(t_f)} dv_x = \int_{t_i}^{t_f} a_x(t)\, dt$$



3. What does the negative areas mean?



Example:

A particle moves along x direction, $a = 2 + 6x^2$. At t =0, x_0=0, v_0=0. What is its $v(x)$?

Solution:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad ; \quad \mathrm{d}v = (2 + 6x^2)\mathrm{d}t$$

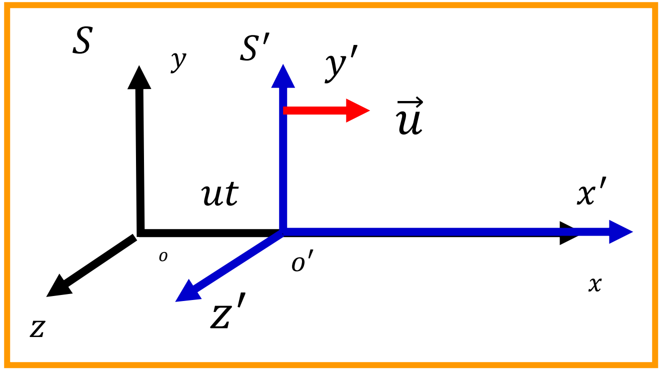
$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

$$a\mathrm{d}x = v\mathrm{d}v$$

$$\int_0^x (2 + 6x^2)\mathrm{d}x = \int_0^v v\mathrm{d}v$$

$$2(x + x^3) = \frac{1}{2}v^2$$

$y \;//\; y'$ $\quad z \;//\; z'$



$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

$$\begin{cases} v'_x = v_x - u \\ v'_y = v_y \\ v'_z = v_z \end{cases}$$