ENSC 2143: Strength of Materials

Exam 2 (spring 2020)

Name: Solution by 2π

Section: ______________

$$\sigma = \frac{P}{A_o}$$

$$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$$

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\tau = G\gamma$$

$$\delta = \sum \frac{PL}{AE}$$

$$\tau_{Avg} = \frac{V}{A}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\varepsilon = \frac{\Delta L}{L_o}$$

$$\gamma = \frac{\pi}{2} - \theta'$$

$$\sigma = E\varepsilon$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$u_r = \frac{\sigma_{pl}^2}{2E}$$

Aluminum
- E = 10,600 ksi
- G = 3,700 ksi
- σ_y = 60 ksi
- σ_u = 68 ksi
- τ_y = 25 ksi
- τ_u = 42 ksi

A-36 Steel
- E = 29,000 ksi
- G = 11,000 ksi
- σ_y = 36 ksi
- σ_u = 58 ksi
- τ_y = 21 ksi

Nylon
- E = 400 ksi
- G = 150 ksi
- σ_y = 10 ksi
- τ_y = 8 ksi

Titanium
- E = 17,400 ksi
- σ_y = 134 ksi

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This exam will test your knowledge and skills on strength of materials. You have from 5:30 until 7:30 pm to complete your exam. You may use the supplied equation sheet, a writing utensil, and your calculator. No other external notes or texts are permitted. If you have any questions during the test please raise your hand or approach the instructor or TA and discuss the question quietly.

Problem 1	(25 points)	____________
Problem 2	(25 points)	____________
Problem 3	(25 points)	____________
Problem 4	(25 points)	____________

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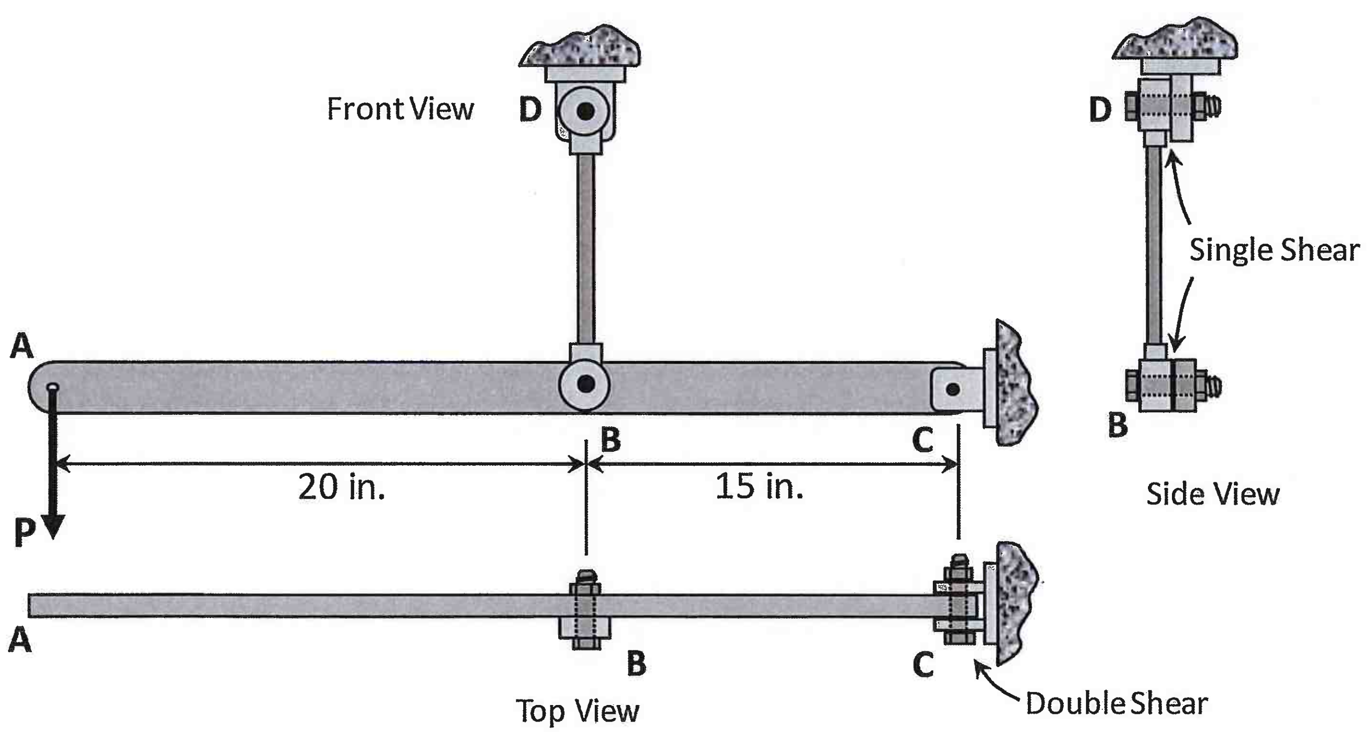
Problem 1

Determine the maximum load, P, that can be applied in order to maintain a Factor of Safety of 3 for the system. Assume possible failure mechanisms are the shear failure of the bolts at B, C, and D and the tension failure of member BD. A ½" diameter bolt is used at C and ¾" diameter bolts are used at B and D. Member BD has a ¾"x ¾" square cross-section. The ultimate shear strength of the bolts is 75 ksi and the ultimate tensile strength of BD is 60 ksi.



statics

$\circlearrowleft + \Sigma M_c = 0$

$P(35in) - F_{BC}(15in) = 0$

$F_{BD} = \frac{35in}{15in} P$

$F_{BD} = 2.333P$

$+\uparrow \Sigma F_y = 0$

$-P + F_{BD} - C_y = 0$

$C_y = F_{BD} - P = 2.333P - P$

$C_y = 1.333P$

6pts for statics

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Problem 1 Continued

Possible Failure Mechanisms

$$\left\{ \tau_{applied} = \frac{V}{A} \le \left[\tau_{allowed} = \frac{\tau_{ult}}{F.S} = \frac{75\,ksi}{3} \right] = 25\,ksi \right\}$$

Shear of bolts @ B and D

- single shear

- shear force = $F_{BD} = 2.333P$

$$V \rightarrow \frac{2.333P}{\frac{\pi (0.75 in)^2}{4}} \le 25\,ksi \Rightarrow \underline{P \le 4.73\ kips}$$

$A \rightarrow \frac{\pi (0.75 in)^2}{4}$

6pts

Shear of bolt @ C

- Double shear
- shear force = $C_y = 1.333P$

$$V \rightarrow \frac{1.333P}{2\,\frac{\pi (0.5 in)^2}{4}} \le 25\,ksi \Rightarrow \underline{P \le 7.36\ kips}$$

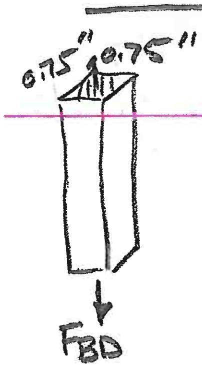
$A \rightarrow 2\,\frac{\pi (0.5 in)^2}{4}$

Double shear

6pts

Tension failure of BD

$$\left[\sigma_{applied} = \frac{P}{A} \right] \le \sigma_{allowed} = \frac{\sigma_{ult}}{F.S.} = \left. \frac{60\,ksi}{3} \right] = 20\,ksi$$



- Tension
- Tensile force = $F_{BD} = 2.333P$

$$\frac{2.333P}{(0.75 in)(0.75 in)} \le 20\,ksi \Rightarrow \underline{P \le 4.82\ kips}$$

6pts

P_{max} is the minimum of the possible failure mechanisms

$$\therefore \boxed{P_{max} = 4.73\ kips}$$

1 pt

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Problem 2 Continued

A) $E = \frac{60\,ksi}{0.002} = \boxed{30{,}000\,ksi}$ 3 pts

B) $G = \frac{43\,ksi}{0.0037} = \boxed{11{,}621.6\,ksi}$ 3 pts

C) $u_r = \frac{1}{2}(60\,ksi)(0.002\,\frac{in}{in}) = \boxed{0.06\,\frac{k\cdot in}{in^3}}$ 3 pts

D) $u_t = 0.06\,\frac{k\cdot in}{in^3} + 60\,ksi(0.025 - 0.002\,\frac{in}{in}) + \left(\frac{80+60\,ksi}{2}\right)(0.0275 - 0.025)$

$+ 80\,ksi\,(0.03 - 0.027\,\frac{in}{in})$

$\boxed{u_t = 1.815\,\frac{k\cdot in}{in^3}}$ 3 pts

E) $P_{yield} = \sigma_y A = (60\,ksi)(4\,in \times 5\,in) = \boxed{1200\,kips}$ 3 pts

F) $P = 1000\,k < 1200\,k \therefore$ elastic

$\Delta = \frac{PL}{AE} = \frac{1000\,k\,(40\,in)}{(20\,in^2)(30{,}000\,ksi)} = 0.0667\,in$

$L' = L + \Delta = \boxed{40.0667\,in}$ 3 pts

G) $P = 1300\,k > 1200\,k \therefore$ yielded

$\sigma = \frac{1300\,k}{20\,in^2} = 65\,ksi \Rightarrow \varepsilon = 0.0250 + \left(\frac{65-60}{80-60}\right)(0.0275 - 0.025) = 0.025625$

$\Delta = \varepsilon L = 0.025625(40\,in) = 1.025\,in$

$L' = L + \Delta = \boxed{41.025\,in}$ 3 pts

H) $\varepsilon_{lateral} = -\nu\,\varepsilon_{long}$

from (E)

$\varepsilon_{long} = \frac{\Delta}{L} = \frac{0.0667\,in}{40\,in} = 0.001668$

$\nu = \frac{E}{2G} - 1 = \frac{30{,}000}{2(11{,}621.6)} - 1 = 0.2907$

4 pts

$\varepsilon_{lat} = -0.2907(0.001668)$
$= -0.000485$

$x' = x + \varepsilon_{lat} x$
$= 4\,in + -0.000485\,(4\,in)$
$= 3.998$

$Y' = Y + \varepsilon_{lat} Y$
$= 5\,in + -0.000485(5\,in)$
$= 4.998$

New Dimensions after load $\boxed{3.998\,in \times 4.998\,in}$

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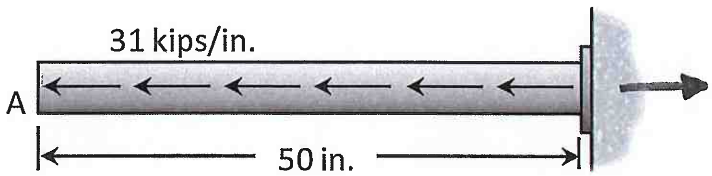
Problem 3

The A-36 steel rod has a 7 in. x 7 in. cross-section and is subjected to the uniformly distributed axial load.

Determine the maximum normal stress in the rod.

Determine the displacement of end A.



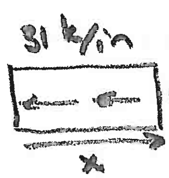
$\rightarrow 31 \frac{k}{in}(50in) = 1550\,k$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{1550\,k}{49\,in^2} = \boxed{31.63\,ksi} < \sigma_y = 36\,ksi$$

$\therefore$ elastic

10 pts

$$\Delta_A = \int_0^L \frac{P(x)}{AE}\,dx$$



$\rightarrow P(x) = 31 \frac{k}{in} x$

$$\Delta_A = \int_0^{50in} \frac{31 \frac{k}{in} x}{AE}\,dx = \frac{31\,k/in}{(49in^2)(29{,}000ksi)} \frac{x^2}{2}\Big|_0^{50in}$$

$$= \frac{31\,k/in}{49in^2\,(29{,}000ksi)} \frac{(50in)^2}{2}$$

$$\boxed{\Delta_A = 0.0273\,in \leftarrow}$$

15pts

If they used $\Delta = \frac{PL}{AE} = \frac{(1550k)(50in)}{49in^2\,(29{,}000ksi)} = 0.0545in$

Then Take off 10pts

So Give them 5pts of the 15

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Problem 4

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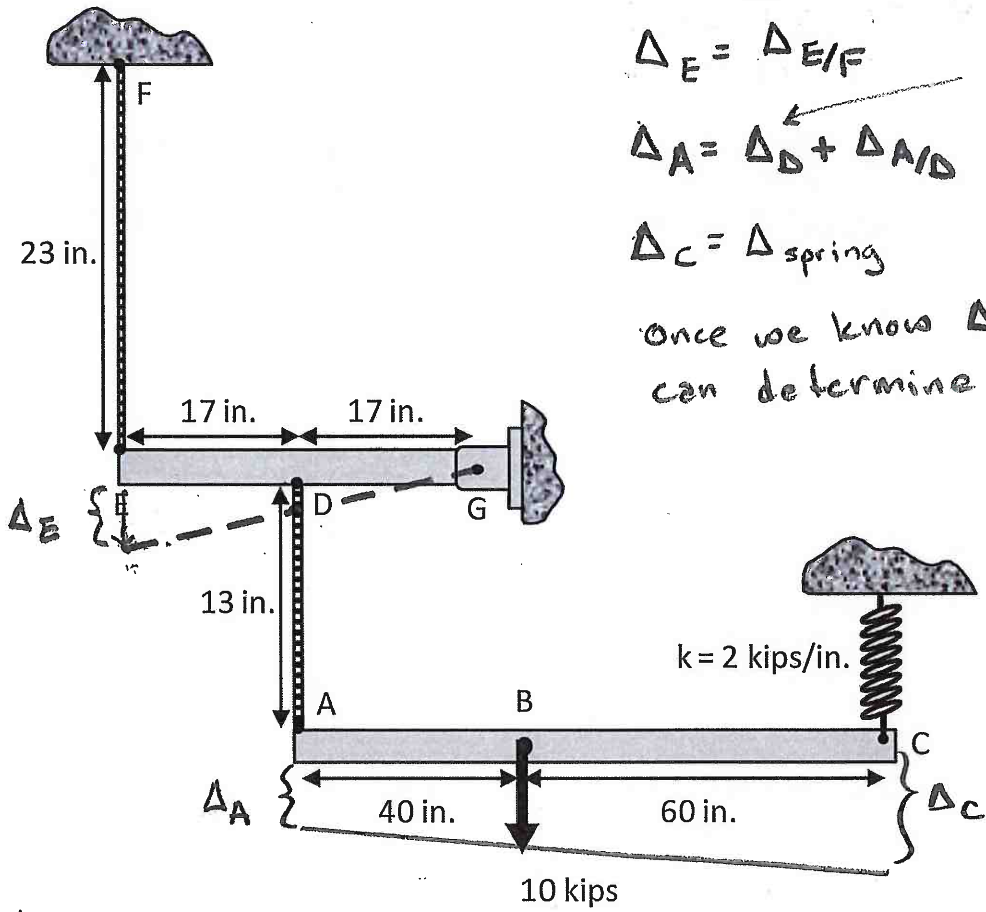
A 10 kips load is applied to a rigid bar AC that is connected to A-36 steel cable AD at point B. The steel cable AD attaches to rigid bar EG at point D. Rigid bar EG is supported by A-36 steel cable EF. The steel cables have a cross-sectional area = 0.10 in². Determine the vertical displacement of points A, B, C, D, and E.



$\Delta_E = \Delta_{E/F}$

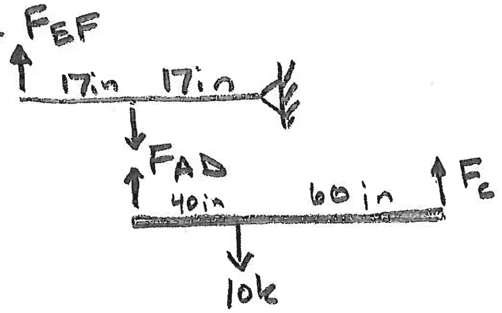
$\dfrac{\Delta_D}{17\text{in}} = \dfrac{\Delta_E}{34\text{in}}$

$\Delta_A = \Delta_D + \Delta_{A/D}$

$\Delta_C = \Delta_{spring}$

Once we know Δ_C + Δ_A we can determine Δ_B

Statics



6 pts for statics

Lower bar

$\circlearrowleft + \Sigma M_c = 0$ $10k(60in) - F_{AD}(100in) = 0$

$\boxed{F_{AD} = 6k}$ tension

$+\uparrow \Sigma F_y = 0$ $F_{AD} + F_c - 10k = 0$

$\boxed{F_c = 4k \text{ tension}}$

upper bar

$\circlearrowleft + \Sigma M_G = 0$

$F_{AD}(17in) - F_{EF}(34in) = 0$

$\boxed{F_{EF} = 3k \text{ tension}}$

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Problem 4 Continued

3pts for σ check

$$\Delta_E = \Delta_{E/F} = \frac{PL}{AE} \text{ if elastic}$$

check $\sigma = \frac{3k}{0.1 in^2} = 30 ksi < 36 ksi$ $\therefore$ elastic

$$= \frac{3k(23in)}{(0.1in^2)(29{,}000ksi)} = 0.023793in$$

$\boxed{\Delta_E = 0.0238in \downarrow}$ 2pts

$$\frac{\Delta_D}{17in} = \frac{\Delta_E}{34in} \Rightarrow \Delta_D = \frac{17}{34}(0.023793in) = 0.011897in$$

$\boxed{\Delta_D = 0.0119in \downarrow}$ 2pts

$\Delta_A = \Delta_D + \Delta_{A/D}$

check to see if AD is elastic

3pts for σ_{check}

$\sigma = \frac{6k}{0.1in^2} = 60ksi > \sigma_y = 36ksi$

$\therefore$ yielded and $\Delta_{A/D} = \varepsilon L$

use σ-ε diagram

@60ksi $\varepsilon \approx 0.14$

$$\Delta_A = \underbrace{0.011897in}_{\Delta_D} + \overbrace{0.14(13in)}^{2 \text{ pts}}_{\varepsilon_{AD} L_{AD}} = 1.831897in$$

$\boxed{\Delta_A = 1.8319 in \downarrow}$ 2pts

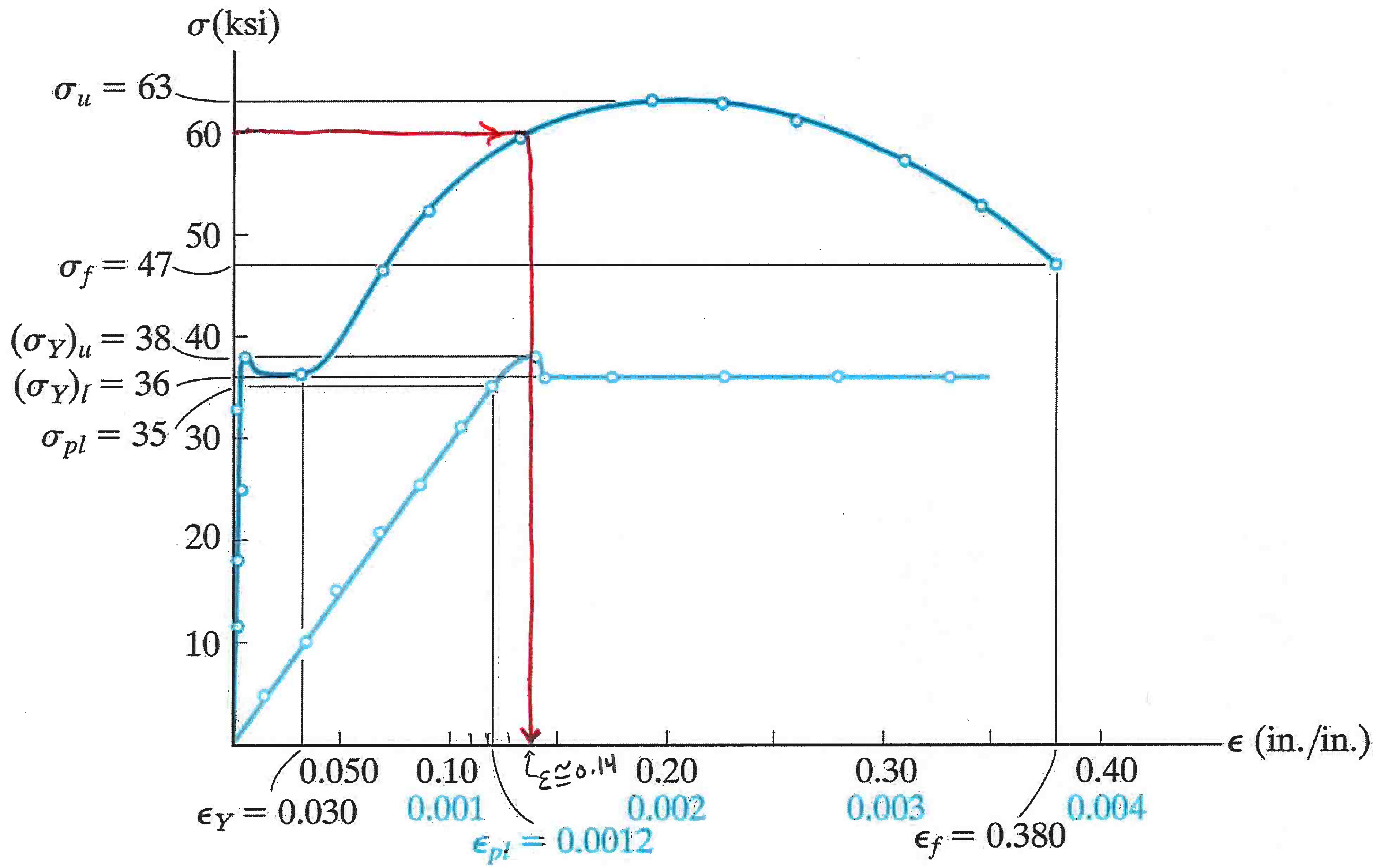
$$\Delta_C = \frac{P_{spring}}{k} = \frac{4k}{2 k/in} = 2in$$

$\boxed{\Delta_c = 2in \downarrow}$ 2pts

$$\Delta_B = 1.8319in + (2 - 1.8319in)\left(\frac{40in}{100in}\right) = 1.89914in$$

2 pts

$\boxed{\Delta_B = 1.8991 in \downarrow}$



Stress Strain Diagram for A-36 Steel