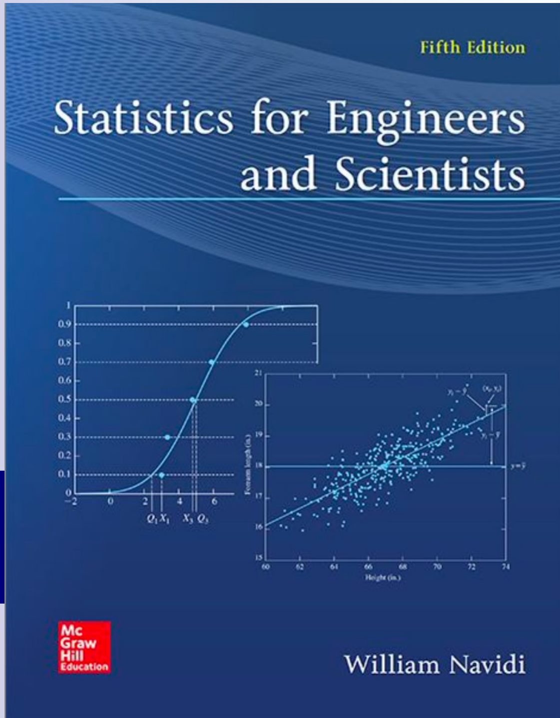


Chapter 5

Confidence Intervals

Ch. 5 Overview (required sections)

- **5-1 Large-Sample Confidence Intervals for a Population Mean**
- **5-2** Confidence Intervals for Proportions
- **5-3** Small-Sample Confidence Intervals for a Population Mean
- **5-4** Confidence Intervals for the Difference Between Two Means
- **5-7** Confidence Intervals with Paired Data

Introduction

- One aspect of inferential statistics is estimation, which is the process of **estimating** the value of a parameter from information obtained from **a sample**.

Introduction…

Example: *"8% of the people surveyed in the United States said that they participate in skiing in the winter time."*

Since the population from which this value was obtained is large, this is an estimate of the true parameter and is derived from data collected from a sample.

Introduction…

- An important question in estimation is that of sample size.
- **How large should the sample be in order to make an accurate estimate?**
- This question is not easy to answer since the size of the sample depends on several factors, such as the accuracy desired and the probability of making a correct estimate.

Confidence Intervals for the Mean When σ Is **Known**

- A **point estimate** is a specific numerical value estimate of a parameter.
- The **best** point estimate of the population mean μ is the sample mean $\bar{X}$.

Example of a Point Estimate

- A college president wishes to estimate the average age of students attending classes this semester.
- The president could select a random sample of 100 students and find the average age of these students, say, 22.3 years.
- From the sample mean, the president could infer that the average age of all the students is 22.3 years.

Three Properties of a Good Estimator

1. The estimator should be an **unbiased estimator**. That is, the expected value or the mean of the estimates obtained from samples of a given size is equal to the parameter being estimated.

Three Properties of a Good Estimator

2. The estimator should be consistent. For a **consistent estimator**, as sample size increases, the value of the estimator approaches the value of the parameter estimated.

Three Properties of a Good Estimator

3. The estimator should be a **relatively efficient estimator**; that is, of all the statistics that can be used to estimate a parameter, the relatively efficient estimator has the smallest variance.

How good is a point estimate?

The answer is that there is no way of knowing how close a particular point estimate is to the population mean.

For this reason, statisticians prefer another type of estimate, called an **interval estimate**.

Confidence Intervals for the Mean When σ Is **Known**

- An **interval estimate** of a parameter is an interval or a range of values used to estimate the parameter.
- This estimate may or may not contain the value of the parameter being estimated.

Example of an Interval Estimate

- For example, an interval estimate for the average age of all students might be

$21.9 < \mu < 22.7$, or **22.3 ± 0.4 years**.

- In an interval estimate, the parameter is specified as being between two values.

Confidence Level of an Interval Estimate

- The **confidence level** of an interval estimate of a parameter is equivalent to the **probability that the interval estimate will contain the parameter**, assuming that **a large number of samples** are selected and that the estimation process on the same parameter is repeated.

Confidence Interval

- A **confidence interval** is a specific interval estimate of a parameter determined by using **data obtained from a sample** and by using the specific confidence level of the estimate.

Formula for the Confidence Interval of the Mean for a Specific α

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Formula for the Confidence Interval of the Mean for a Specific α

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

For a 90% confidence interval: $z_{\alpha/2} = 1.65$

For a 95% confidence interval: $z_{\alpha/2} = 1.96$

For a 99% confidence interval: $z_{\alpha/2} = 2.58$

Summary

Let $X_1, \ldots, X_n$ be a *large* $(n > 30)$ random sample from a population with mean μ and standard deviation σ, so that $\overline{X}$ is approximately normal. Then a level $100(1 - \alpha)\%$ confidence interval for μ is

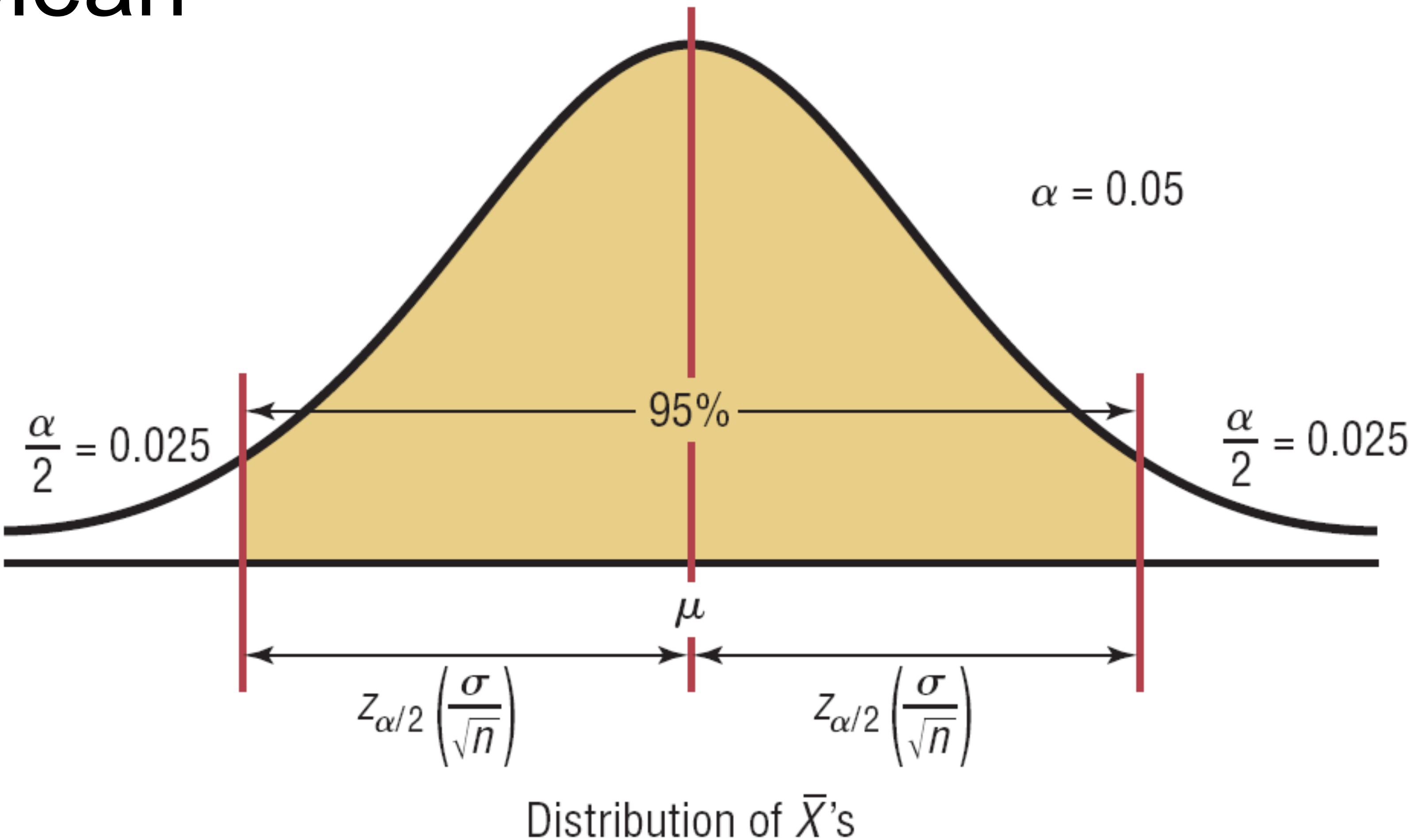
$$\overline{X} \pm z_{\alpha/2}\sigma_{\overline{X}} \tag{5.1}$$

where $\sigma_{\overline{X}} = \sigma/\sqrt{n}$. When the value of σ is unknown, it can be replaced with the sample standard deviation s.

In particular,

- $\overline{X} \pm \dfrac{s}{\sqrt{n}}$ is a 68% confidence interval for μ.
- $\overline{X} \pm 1.645\dfrac{s}{\sqrt{n}}$ is a 90% confidence interval for μ.
- $\overline{X} \pm 1.96\dfrac{s}{\sqrt{n}}$ is a 95% confidence interval for μ.
- $\overline{X} \pm 2.58\dfrac{s}{\sqrt{n}}$ is a 99% confidence interval for μ.
- $\overline{X} \pm 3\dfrac{s}{\sqrt{n}}$ is a 99.7% confidence interval for μ.

95% Confidence Interval of the Mean

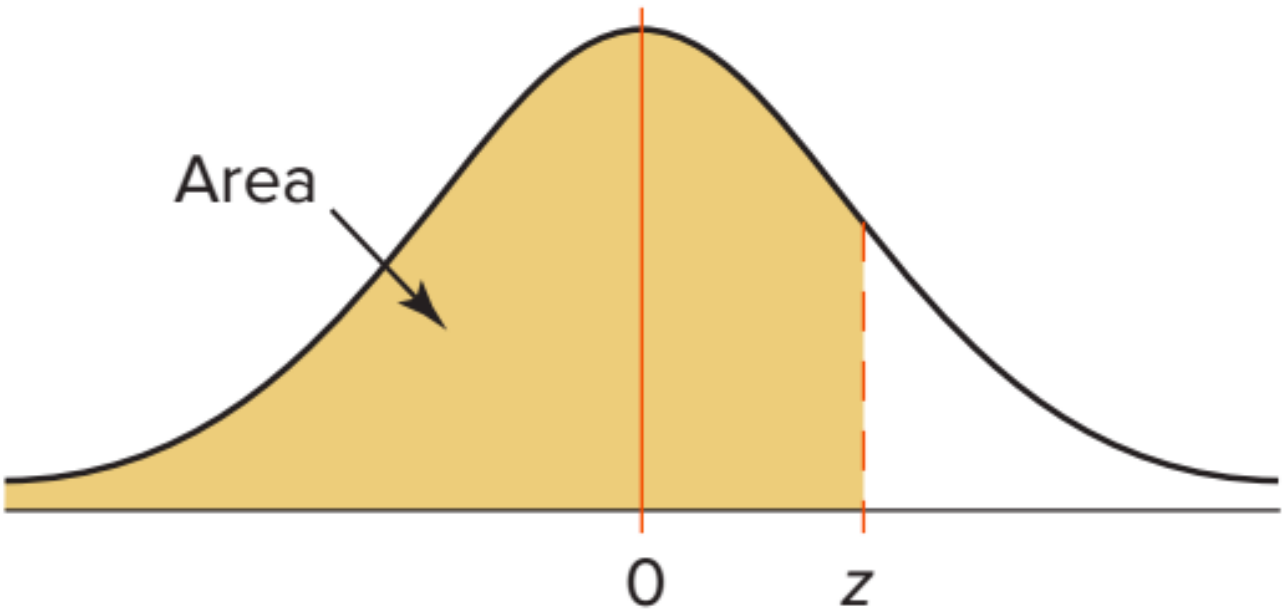


z-distribution (Table A.2, Appendix A)

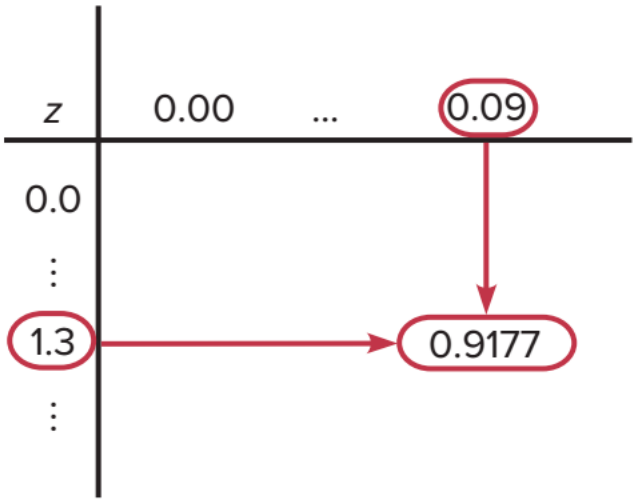
Cumulative Standard Normal Distribution

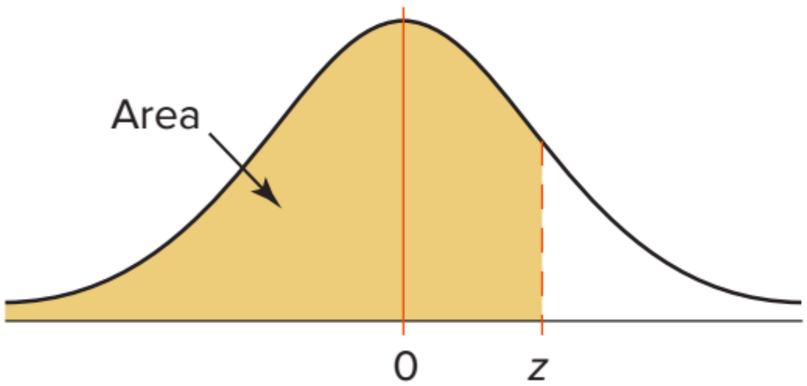
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

z-distribution (Table A.2, Appendix A)



Example: Area value for *z* = *1.39*





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z-distribution (Table A.2, Appendix A)

Cumulative Standard Normal Distribution										
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For a 95% confidence interval: $z_{\alpha/2} = 1.96$

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2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981

For a 99% confidence interval: $z_{\alpha/2} = 2.58$

Maximum Error of the Estimate

The **maximum error of the estimate** (or the **margin of error**) is the maximum likely difference between the point estimate of a parameter and the actual value of the parameter.

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Example: Days to Sell Chevrolet Aveo

A researcher wishes to estimate the number of days it takes an automobile dealer to sell a Chevrolet Aveo. A random sample of 50 cars had a mean time on the dealer's lot of 54 days. Assume the population standard deviation to be 6.0 days. Find the best point estimate of the population mean and the 95% confidence interval of the population mean.

The best point estimate of the mean is 54 days.

$$\bar{X} = 54, \sigma = 6.0, n = 50, 95\% \rightarrow z = 1.96$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Example: Days to Sell Chevrolet Aveo

$$\bar{X} = 54, \sigma = 6.0, n = 50, 95\% \rightarrow z = 1.96$$

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$54 - 1.96\left(\frac{6.0}{\sqrt{50}}\right) < \mu < 54 + 1.96\left(\frac{6.0}{\sqrt{50}}\right)$$

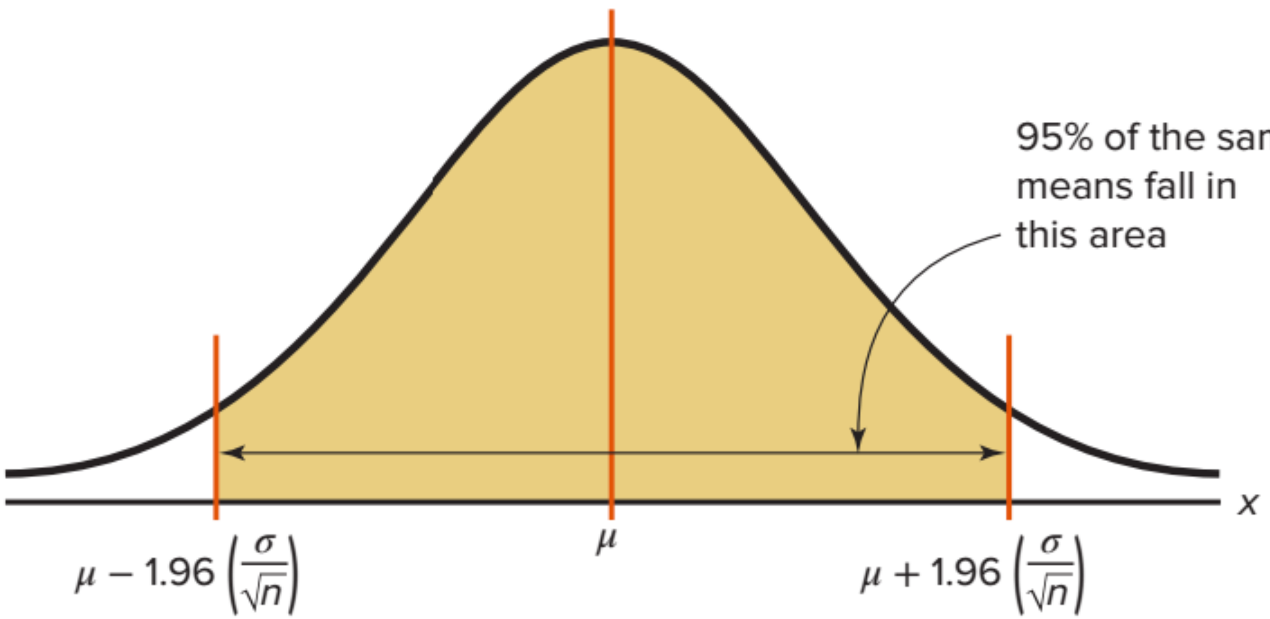
$$54 - 1.7 < \mu < 54 + 1.7$$

$$52.3 < \mu < 55.7$$

$$52 < \mu < 56$$

One can say with **95% confidence** that the interval between **52 and 56 days** contains the population mean, based on a sample of 50 automobiles.

95% Confidence Interval of the Mean



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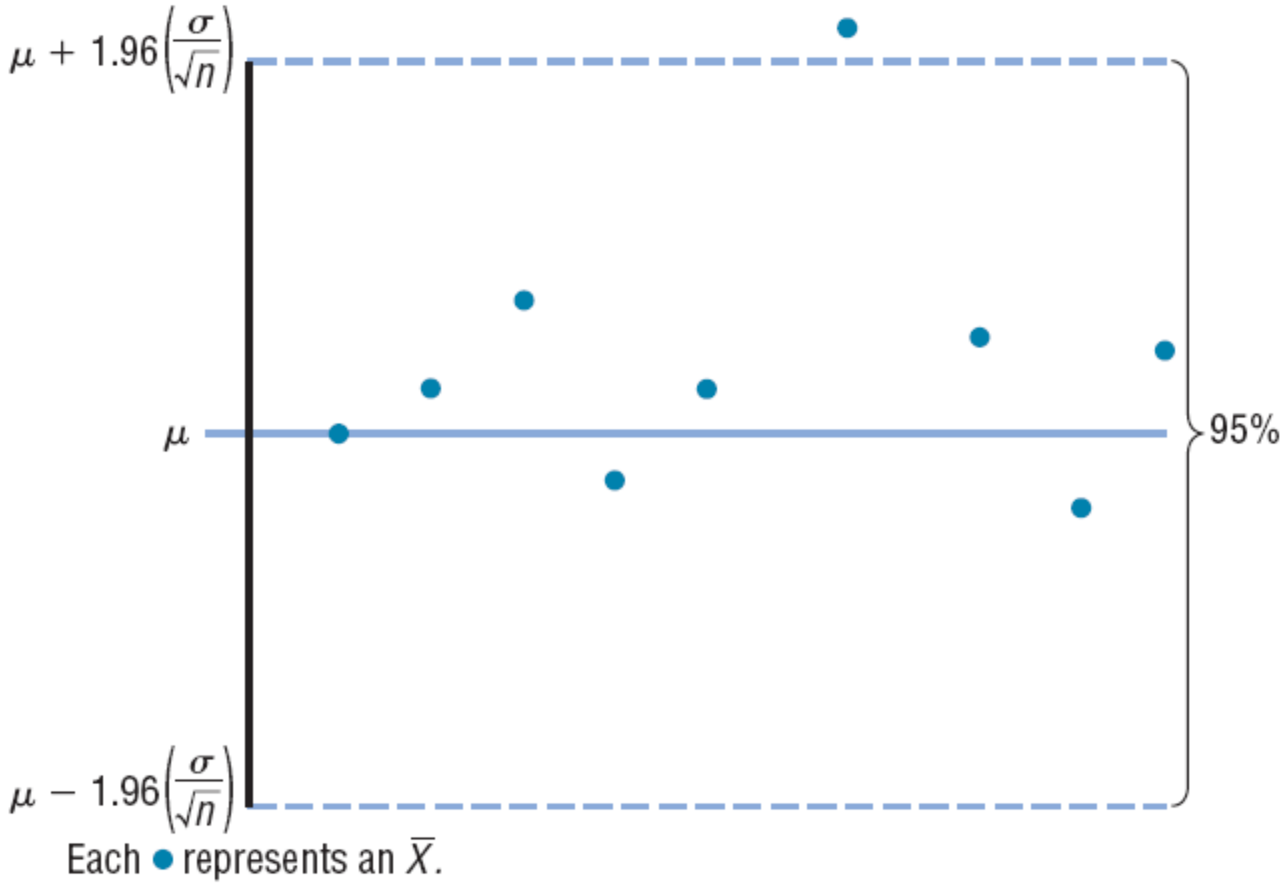
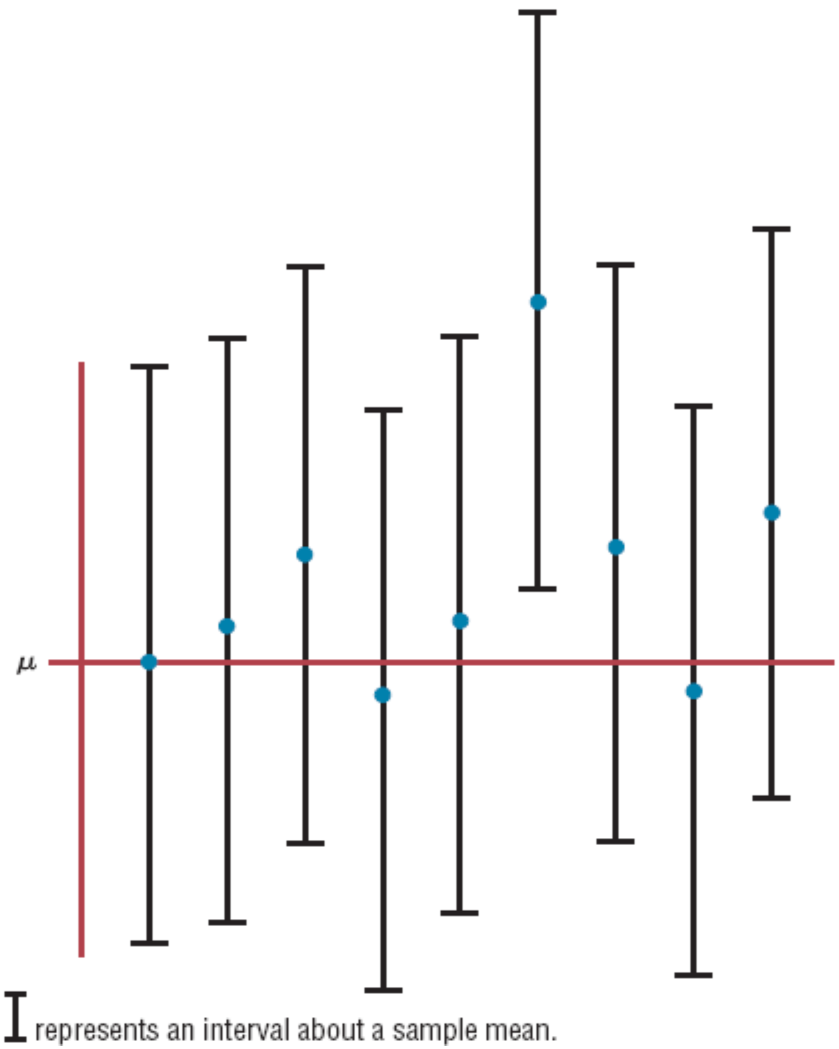
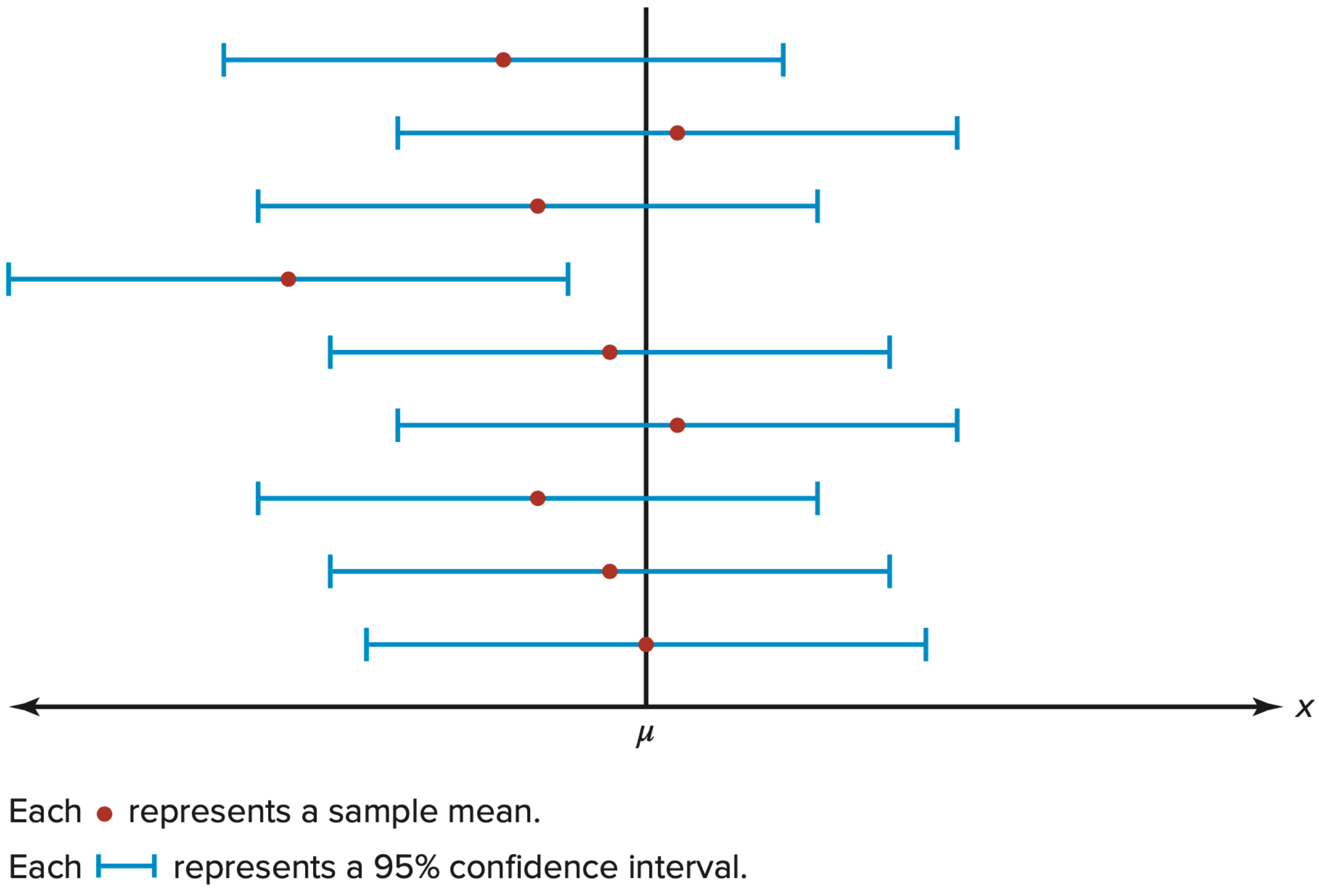


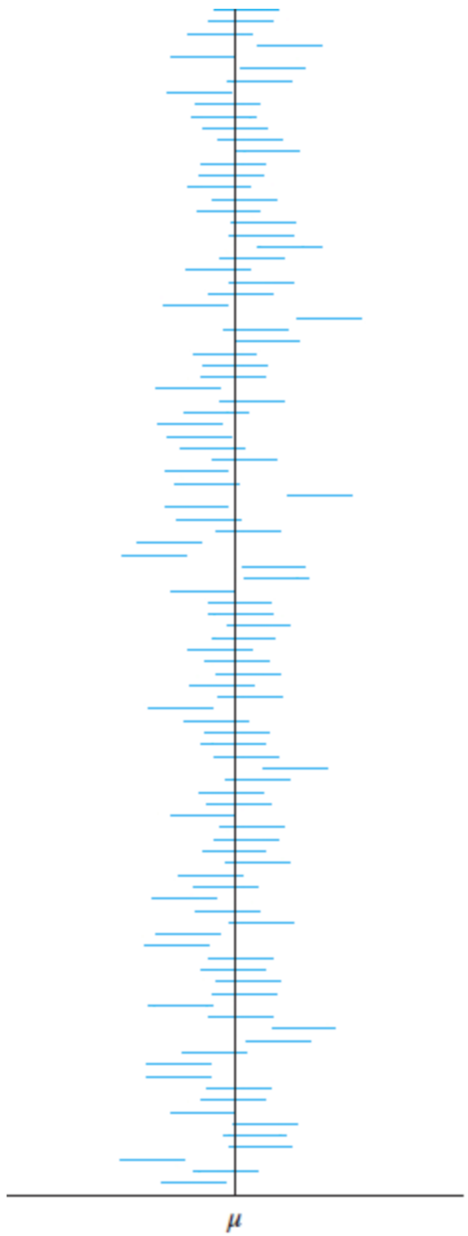
95% Confidence Interval of the Mean



One can be 95% confident that an interval built around a specific sample mean would contain the population mean.

95% Confidence Interval of the Mean





(a)

FIGURE 5.4(a)

- ✓ One hundred 68% confidence intervals for a population mean, each computed from a different sample.
- ✓ Although precise, they fail to cover the population mean 32% of the time.
- ✓ This high failure rate makes the 68% confidence interval unacceptable for practical purposes.

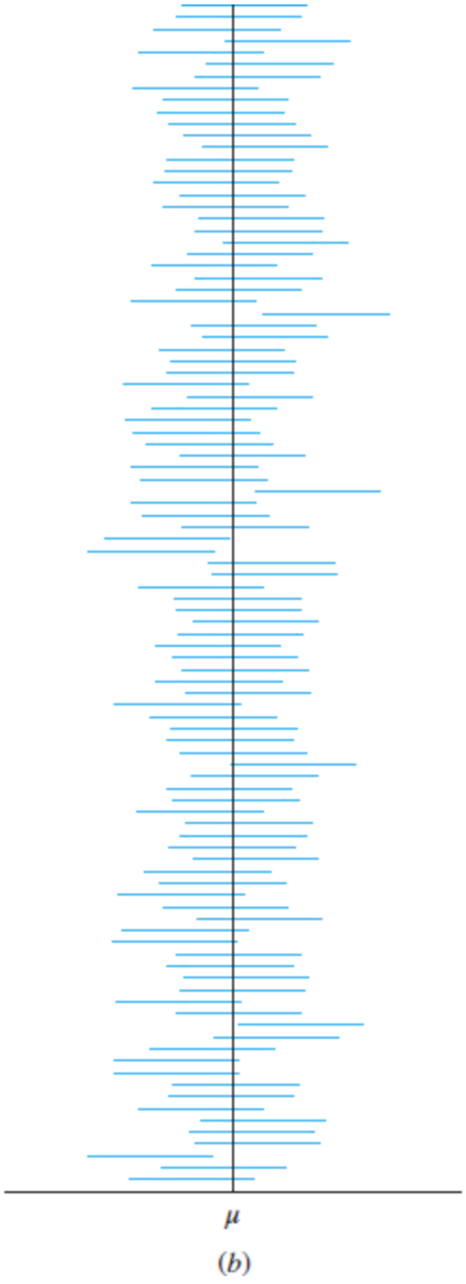


FIGURE 5.4(b)

- ✓ One hundred 95% confidence intervals computed from these samples.
- ✓ This represents a good compromise between reliability and precision for many purposes.

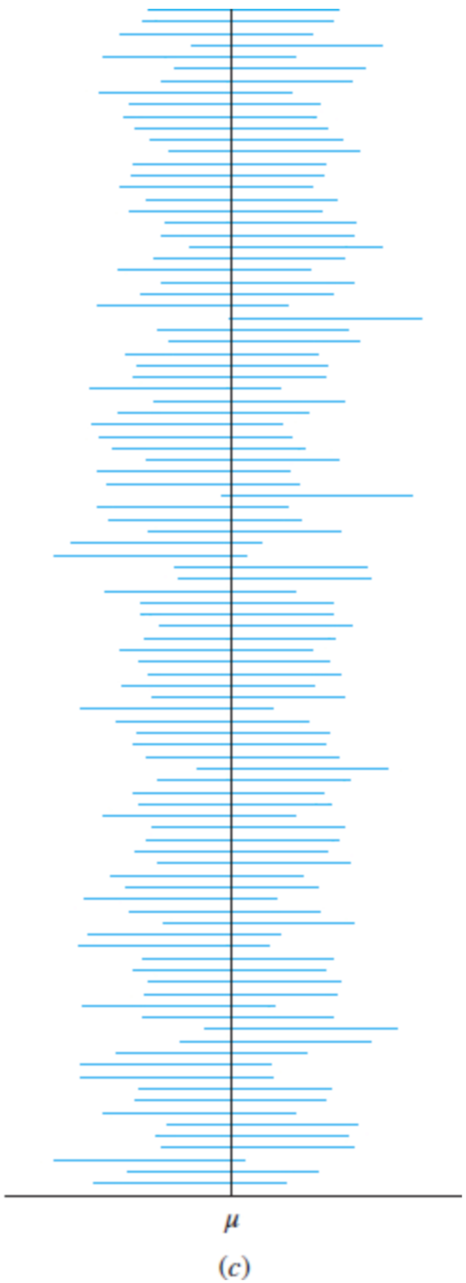
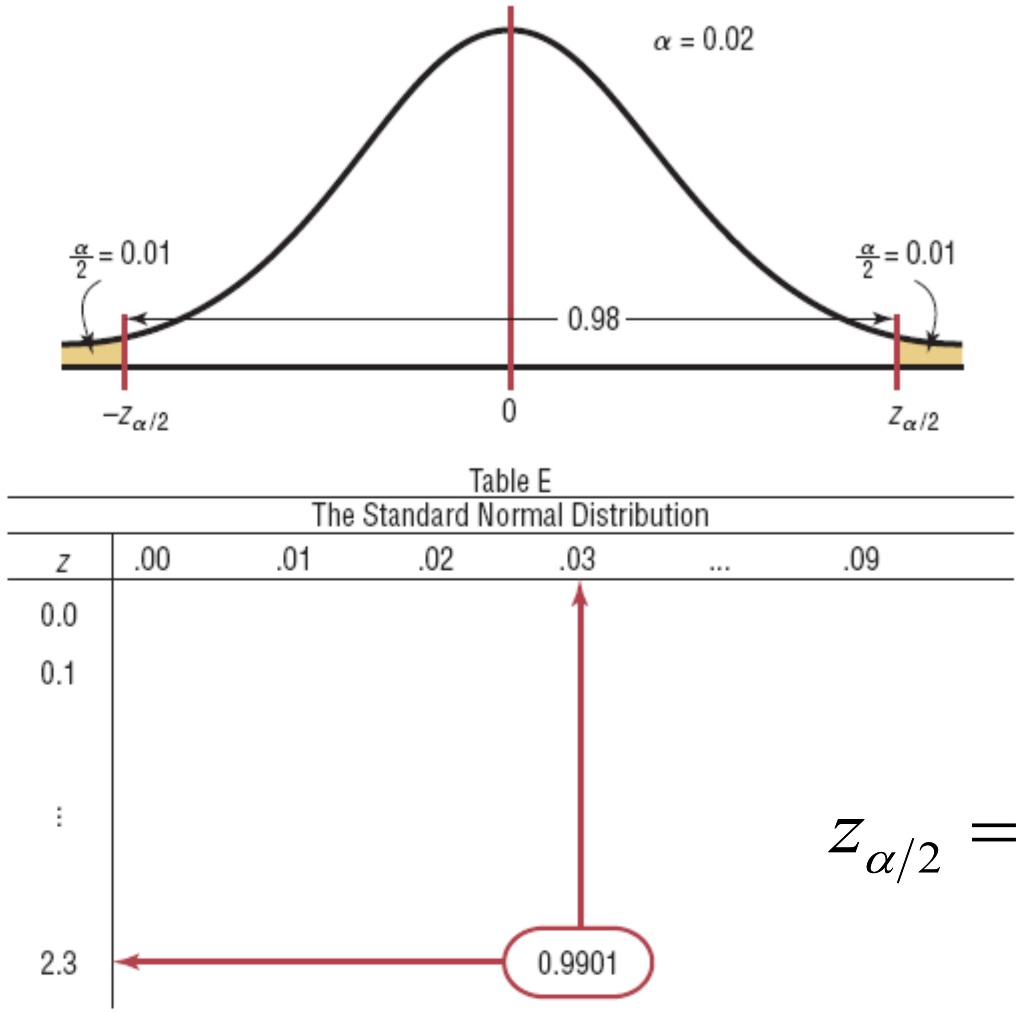


FIGURE 5.4(c)

- ✓ One hundred 99.7% confidence intervals computed from these samples.
- ✓ These intervals fail to cover the population mean only three times in 1000.
- ✓ They are extremely reliable, but imprecise.

Finding $z_{\alpha/2}$ for 98% Confidence Level



$$z_{\alpha/2} = 2.33$$

z-distribution (Table A.2, Appendix A)

Cumulative Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

Example: Credit Union Assets

The following data represent a random sample of the assets (in millions of dollars) of 30 credit unions in southwestern Pennsylvania. Assume $\sigma = 14.405$.

Find the 90% confidence interval of the mean.

12.23	16.56	4.39
2.89	1.24	2.17
13.19	9.16	1.42
73.25	1.91	14.64
11.59	6.69	1.06
8.74	3.17	18.13
7.92	4.78	16.85
40.22	2.42	21.58
5.01	1.47	12.24
2.27	12.77	2.76

Example: Credit Union Assets…

Step 1: Find the mean. We calculate $\bar{X}$ = 11.091

Step 2: Find $\alpha/2$. 90% CL → $\alpha/2 = 0.05$

Step 3: Find $z_{\alpha/2}$. 90% CL → $\alpha/2 = 0.05$ → $z_{.05} = 1.65$

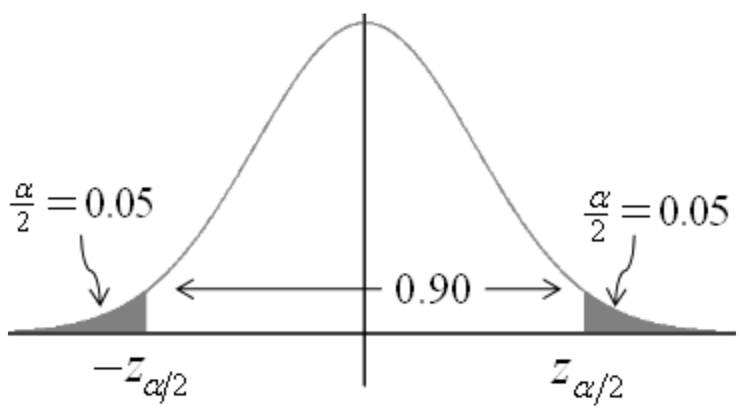


Table E

The Standard Normal Distribution

z	.00	…	.04	.05	…	.09
0.0						
0.1						
⋮						
1.6			0.9495	$\boxed{0.9505}$		

z-distribution (Table A.2, Appendix A)

TABLE E *(continued)*

Cumulative Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
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0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
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1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Example: Credit Union Assets…

Step 4: Substitute in the formula.

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$11.091 - 1.65\left(\frac{14.405}{\sqrt{30}}\right) < \mu < 11.091 + 1.65\left(\frac{14.405}{\sqrt{30}}\right)$$

$$11.091 - 4.339 < \mu < 11.091 + 4.339$$

$$6.752 < \mu < 15.430$$

One can be 90% confident that the population mean of the assets of all credit unions is between \$6.752 million and \$15.430 million, based on a sample of 30 credit unions.

Note on Using Technology

- Many examples in this course use raw data. If you are using computer or calculator programs to find the solutions, the answers you get may vary somewhat from the ones given in the textbook or in the lectures.

- This is so because computers and calculators do not round the answers in the intermediate steps and can use 12 or more decimal places for computation. Also, they use more exact values than those given in the tables in the back of this book.

- **These discrepancies are part of statistics.**

Formula for Minimum Sample Size Needed for an Interval Estimate of the Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

where ***E*** is the **maximum error of estimate (*or* margin of error)**. If necessary, round the answer up to obtain a whole number. That is, if there is any fraction or decimal portion in the answer, use the next whole number for sample size *n.*

Example: Automobile Thefts

A sociologist wishes to estimate the average number of automobile thefts in a large city per day within 2 automobiles. He wishes to be 99% confident, and from a previous study the standard deviation was found to be 4.2. How many days should he select to survey?

Example: Automobile Thefts…

A sociologist wishes to estimate the average number of automobile thefts in a large city per day within 2 automobiles. He wishes to be 99% confident, and from a previous study the standard deviation was found to be 4.2. How many days should he select to survey?

SOLUTION

Since $\alpha = 0.01$ (or $1 - 0.99$), $z_{\alpha/2} = 2.58$ and $E = 2$. Substitute in the formula

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left[\frac{(2.58)(4.2)}{2}\right]^2 = 29.35$$

Round the value up to 30. Therefore, to be 99% confident that the estimate is within 2 automobiles of the true mean, the sociologist needs to sample the thefts for at least 30 days.

In most cases in statistics, we round off; however, when determining sample size, we always round up to the next whole number.

Probability versus Confidence

- If a 95% confidence interval for the population mean μ was computed to be (12.304, 12.696), it is **tempting to say** that the probability is 95% that μ is between 12.304 and 12.696.

- This, however, is **not correct**.

- The term probability refers to random events, which can come out differently when experiments are repeated. The numbers 12.304 and 12.696 are fixed, not random. The population mean is also fixed. There is no randomness involved.

- Therefore, we say that we have **95% confidence** (**not probability**) that the population mean is in this interval.

Probability versus Confidence…

- On the other hand, let's say that we are discussing a method used to compute a 95% confidence interval.
- The method will succeed in covering the population mean 95% of the time, and fail the other 5% of the time.
- In this case, whether the population mean is covered or not is a random event, because it can vary from experiment to experiment.
- Therefore, it is **correct to say** that **a method** for computing a 95% confidence interval **has probability 95%** of covering the population mean.

Probability versus Confidence…

Example 5.5

A 90% confidence interval for the mean diameter (in cm) of steel rods manufactured on a certain extrusion machine is computed to be (14.73, 14.91). True or false: The probability that the mean diameter of rods manufactured by this process is between 14.73 and 14.91 is 90%.

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Solution

False. A specific confidence interval is given. The mean is either in the interval or it isn't. We are 90% confident that the population mean is between 14.73 and 14.91. The term *probability* is inappropriate.

Probability versus Confidence…

Example **5.6**

An engineer plans to compute a 90% confidence interval for the mean diameter of steel rods. She will measure the diameters of a large sample of rods, compute $\overline{X}$ and s, and then compute the interval $\overline{X} \pm 1.645s/\sqrt{n}$. True or false: The probability that the population mean diameter will be in this interval is 90%.

Solution

True. What is described here is a method for computing a confidence interval, rather than a specific numerical value. It is correct to say that a method for computing a 90% confidence interval has probability 90% of covering the population mean.

End of Section 5-1