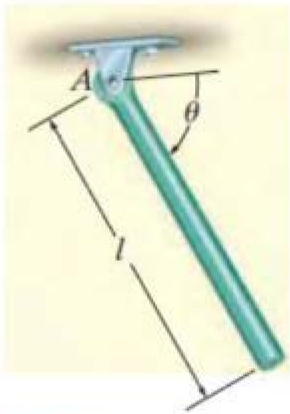
1. The slender rod shownhas a mass m and length l and is released from rest when θ = 0° . Determine the horizontal and vertical components of force which the pin at A exerts on the rod at the instant θ = 90° .(20pnts)



SOLUTION

(a)

Free-Body and Kinetic Diagrams. The free-body diagram for the rod in the general position θ is shown in Fig. 17–17*b*. For convenience, the force components at A are shown acting in the n and t directions. Note that $\boldsymbol{\alpha}$ acts clockwise and so $(\mathbf{a}_G)_t$ acts in the $+t$ direction.

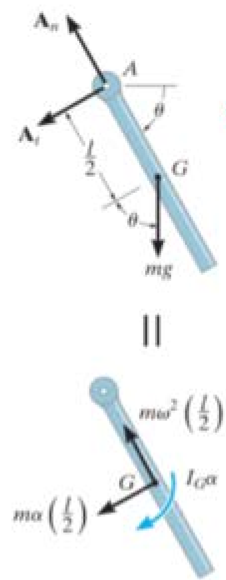
The moment of inertia of the rod about point A is $I_A = \frac{1}{3}ml^2$.

Equations of Motion. Moments will be summed about A in order to eliminate A_n and A_t.

$$+\nwarrow\Sigma F_n = m\omega^2 r_G; \qquad A_n - mg\sin\theta = m\omega^2(l/2) \tag{1}$$

$$+\swarrow\Sigma F_t = m\alpha r_G; \qquad A_t + mg\cos\theta = m\alpha(l/2) \tag{2}$$

$$\circlearrowright + \Sigma M_A = I_A\alpha; \qquad mg\cos\theta(l/2) = \left(\tfrac{1}{3}ml^2\right)\alpha \tag{3}$$



(b)

Fig. 17–17

Kinematics. For a given angle θ there are four unknowns in the above three equations: A_n, A_t, ω, and α. As shown by Eq. 3, α is *not constant;* rather, it depends on the position θ of the rod. The necessary fourth equation is obtained using kinematics, where α and ω can be related to θ by the equation

$$(\circlearrowright +) \qquad \omega\, d\omega = \alpha\, d\theta \tag{4}$$

Note that the positive clockwise direction for this equation *agrees* with that of Eq. 3. This is important since we are seeking a simultaneous solution.

In order to solve for ω at $\theta = 90^\circ$, eliminate α from Eqs. 3 and 4, which yields

$$\omega\, d\omega = (1.5g/l)\cos\theta\, d\theta$$

Since $\omega = 0$ at $\theta = 0^\circ$, we have

$$\int_0^\omega \omega\, d\omega = (1.5g/l)\int_{0^\circ}^{90^\circ}\cos\theta\, d\theta$$

$$\omega^2 = 3g/l$$ → 可以直接使用能量守恒得到，不用积分

Substituting this value into Eq. 1 with $\theta = 90^\circ$ and solving Eqs. 1 to 3 yields

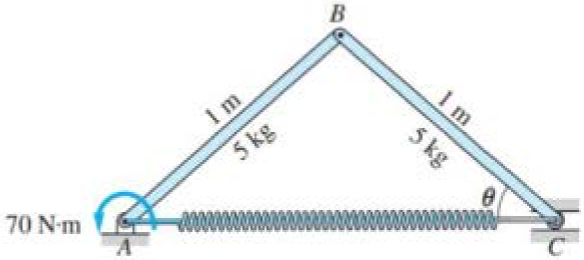
$$\alpha = 0$$

$$A_t = 0 \quad A_n = 2.5\,mg \qquad \textit{Ans.}$$

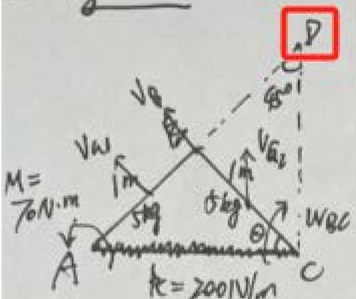
2. The system, which lies in the vertical plane, consists of the 5-kg homogeneous bars AB and BC and the spring AC. The free length of the spring is 1 m, and its stiffness is 200 N/m. A constant 70-N • m couple acts on bar AB. Determine

(a) the work done on the bars as θ changes from 0 to 45°.

(b) The velocity of C at the instant.(30pnts)



3# Dgn 18-3



$V_{G2} = \omega_{BC}\sqrt{1^2+0.5^2} = 1.1180\,\omega_{BC}$

$V_{G1} = \omega_{AB}\cdot\frac{1}{2} = \frac{1}{2}\omega_{BA}$

$V_B = \omega_{BC}\cdot 1 = \omega_{AB}\cdot 1$

$\Rightarrow \omega_{AB} = \omega_{BC}$.

When $\theta = 0°$, $l = 2m$, $\Delta l_1 = 2-1 = 1\,m$

when $\theta = 45°$, $l = \sqrt{2}\,m$, $\Delta l_2 = (\sqrt{2}-1)\,m$.

$U_1 = M(\theta_2-\theta_1) = 70(\frac{\pi}{4}-0) \approx 54.9779\,J$

$U_2 = \frac{1}{2}k\Delta l_1^2 - \frac{1}{2}k\Delta l_2^2 = \frac{1}{2}\times200\times[1^2-(\sqrt{2}-1)^2] \approx 82.8427\,J$.

$U_3 = -2mgh = -2\times5\times9.81\times\frac{1}{2}\times\frac{\sqrt{2}}{2} \approx -34.6836\,J$.

$U_{total} = U_1+U_2+U_3 = 103.1370\,J$.

$T_1 + U_{1\to2} = T_2$

$0 + 103.1370 = \frac{1}{2}mV_{G1}^2 + \frac{1}{2}I_G\omega_{AB}^2 + \frac{1}{2}mV_{G2}^2 + \frac{1}{2}I_G\omega_{BC}^2$

$\frac{1}{2}\times5\times(0.5\omega_{BC})^2 + \frac{1}{2}\times\frac{1}{12}\times5\times1^2\times\omega_{BC}^2 + \frac{1}{2}\times5\times(1.118\omega_{BC})^2 + \frac{1}{2}\times\frac{1}{12}\times5\times1^2\times\omega_{BC}^2$

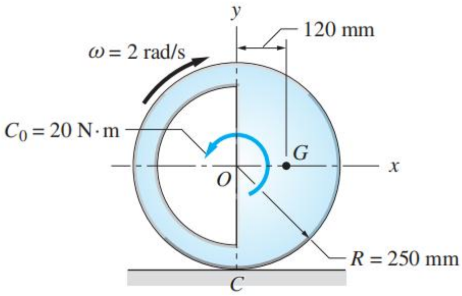
$= 103.1370$

$\Rightarrow \omega_{BC} \approx 4.9753$ rad/s (↻)

图上D是CB杆瞬心

vc=wBC*CD

3.The 40-kg unbalanced wheel is rolling without slipping under the action of a counterclockwise couple C_0 = 20 N • m. When the wheel is in the position shown, its angular velocity is ω = 2 rad/s clockwise. For this position, calculate the angular acceleration α and the forces exerted on the wheel at C by the rough horizontal plane. The radius of gyration of the wheel about its mass center G is k_z = 200 mm.(20pnts)

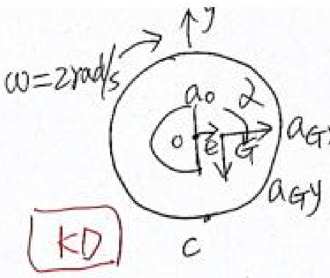


FBD

$C_0 = 20 N\cdot m$ $m = 40kg$

$mg = 40\times 9.81 = 392.4N$

N_c. F_c: unknown



KD

$k_z = 0.2m$

$e = 0.12m$

$R = 0.25m$

$I_G = mk_z^2 = 1.6kgm^2$

α = unknown

C: ICR (瞬心) $v_C = 0$

$a_C \neq 0$ (unknown)

$O: v_O = \vec{\omega}\times\vec{r}_{OC}$

$= -2\vec{k}\times 0.25\vec{j} = 0.5\vec{i}\ m/s$

(顺时针) $(\rightarrow)$

$\vec{a}_O = \vec{\alpha}\times\vec{r}_{OC} = 0.25\alpha\ m/s^2$

(同上步) $(\rightarrow)$

$\vec{a}_G = \vec{a}_O + \vec{\alpha}\times\vec{r}_{OG} - \omega^2\vec{r}_{OG}$

$= 0.25\alpha\vec{i} + (-\alpha\vec{k})(0.12\vec{i}) - 2^2\cdot 0.12\vec{i}$

$= (0.25\alpha - 0.48)\vec{i} - 0.12\alpha\vec{j}$

$\vec{a}_{Gx} = 0.25\alpha - 0.48\ (\rightarrow)\ m/s^2$

$a_{Gy} = 0.12\alpha\ (\downarrow)$

Equs of Motion

$\xrightarrow{+} \Sigma F_x = ma_x$ $F_c = 40(0.25\alpha - 0.48) = 10\alpha - 19.2$ ①

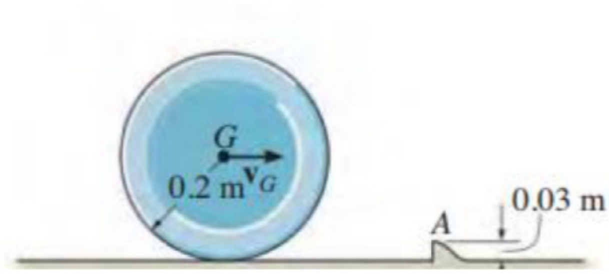
$+\downarrow \Sigma F_y = ma_y$ $mg - N_c = 40\times 0.12\alpha$ $N_c = 392.4 - 4.8\alpha$ ②

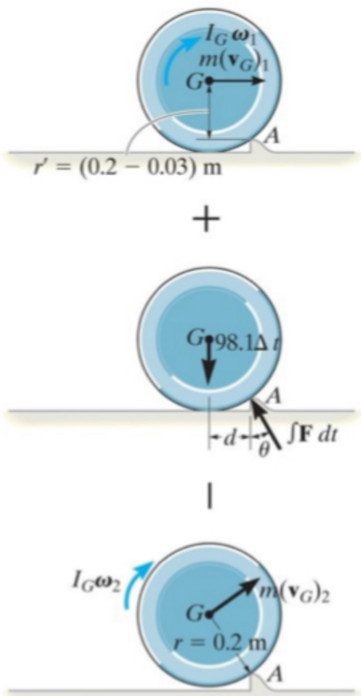
$\circlearrowright + \Sigma M_G(F) = I_G\alpha$ $N_c\cdot 0.12 - F_c\cdot 0.25 - C_0 = 1.6\alpha$ ③

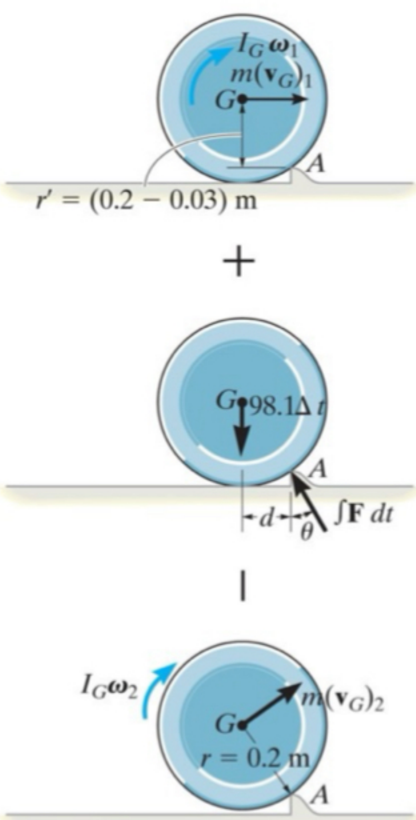
①②③ $\rightarrow$ $\boxed{\alpha = 6.82 rad/s}$ $= 31.888/4.676$

$\boxed{N_c = 359.7N(\uparrow)}$ $\boxed{F_c = 49.0N(\rightarrow)}$

4. The 10-kg wheel shown has a moment of inertia $I_G = 0.156\ \text{kg}\cdot\text{m}^2$. Assuming that the wheel does not slip or rebound, determine the minimum velocity v_G it must have to just roll over the obstruction at A. (30pnts)







Modeling and Analysis:

- Draw the FBD and Impulse and Momentum Diagrams for the wheel and apply the consevation of the angular momentum about A.

$$(H_A)_1 = (H_A)_2$$

$$mv_{G1}r' + I_G\omega_1 = mv_{G2}r + I_G\omega_2$$

$$mV_{G_1} r' + I_{G_1}\frac{V_{G_1}}{r} = mV_{G_2}r + I_{G_2}\frac{V_{G_2}}{r}$$

$$V_{G_1}\left(mr' + \frac{I_G}{r}\right) = V_{G_2}\left(mr + \frac{I_{G_2}}{r}\right)$$

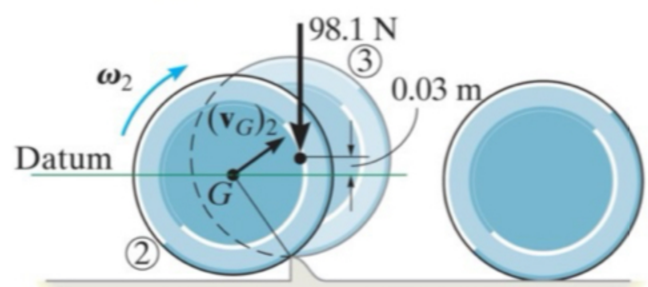
- Kinematics $\quad v_{G1} = \omega_1 r \quad v_{G2} = \omega_2 r$

- The expression of final angular velocity.

$$v_{G2} = v_{G1}\frac{mr' + I_G/r}{mr + I_G/r}$$

$$= v_{G1}\frac{10\times(0.2\quad 0.03) + 0.156/0.2}{10\times 0.2 + 0.156/0.2}$$

$$= 0.892 v_{G1}$$



- Apply the Conservation of Energy, determine the minimum kinetic energy to roll over the obstruction.

$$T_2 + V_2 = T_3 + V_3$$

$$\tfrac{1}{2}mv_{G1}^2 + \tfrac{1}{2}I_G\omega_1^2 + 0 = 0 + mg\times 0.03$$

$$v_{G2,\min}^2 = \frac{2g\times 0.03}{1 + I_G/mr^2} = \frac{2\times 9.81\times 0.03}{1 + 0.156/(10\times 0.2^2)} = 0.423 (\text{m/s})^2$$

$$v_{G2,\min} = 0.651\ \text{m/s}$$

$$\boxed{v_{G1,\min} = v_{G2,\min}/0.892 = 0.730\ \text{m/s}\ (\rightarrow)}$$