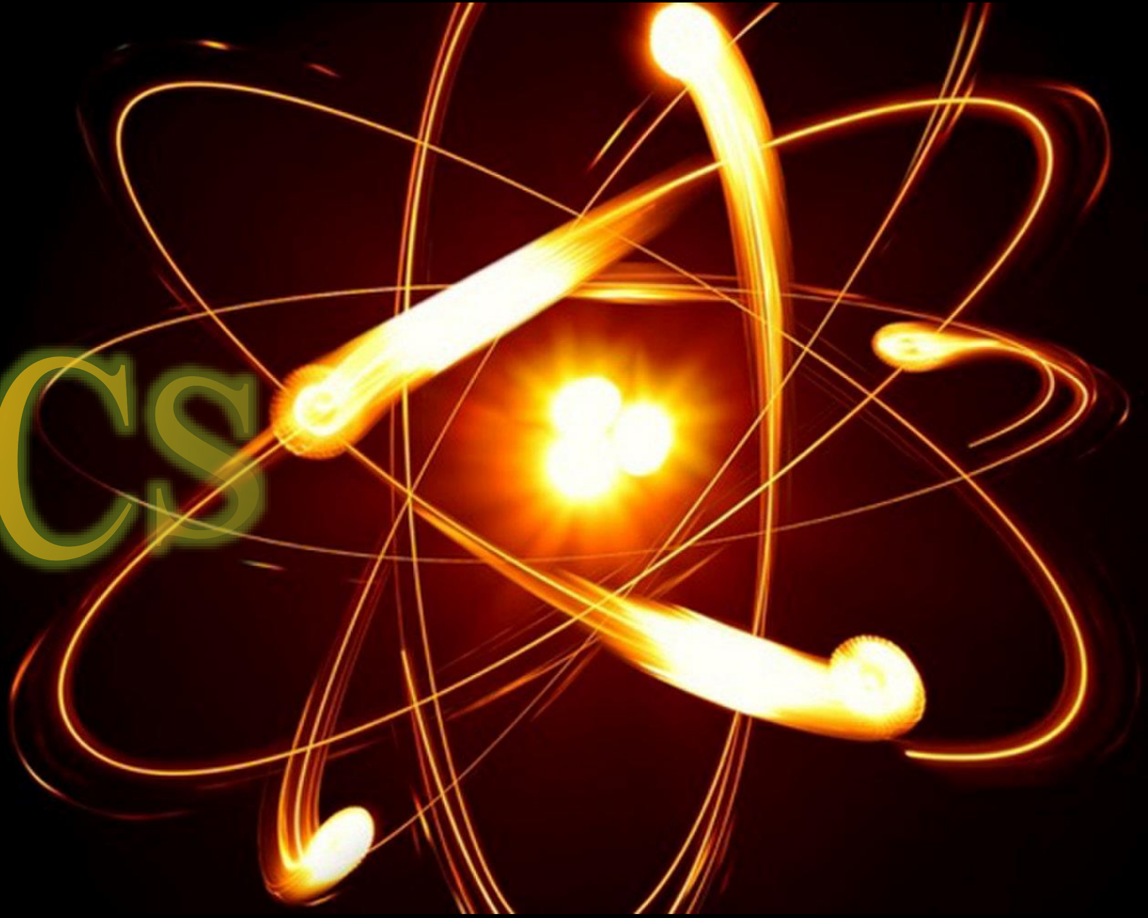
PHYSICS



西南交通大学
Southwest Jiaotong University

Physics 1: **Mechanics and Waves**

Week 2 – Introduction

2023.2

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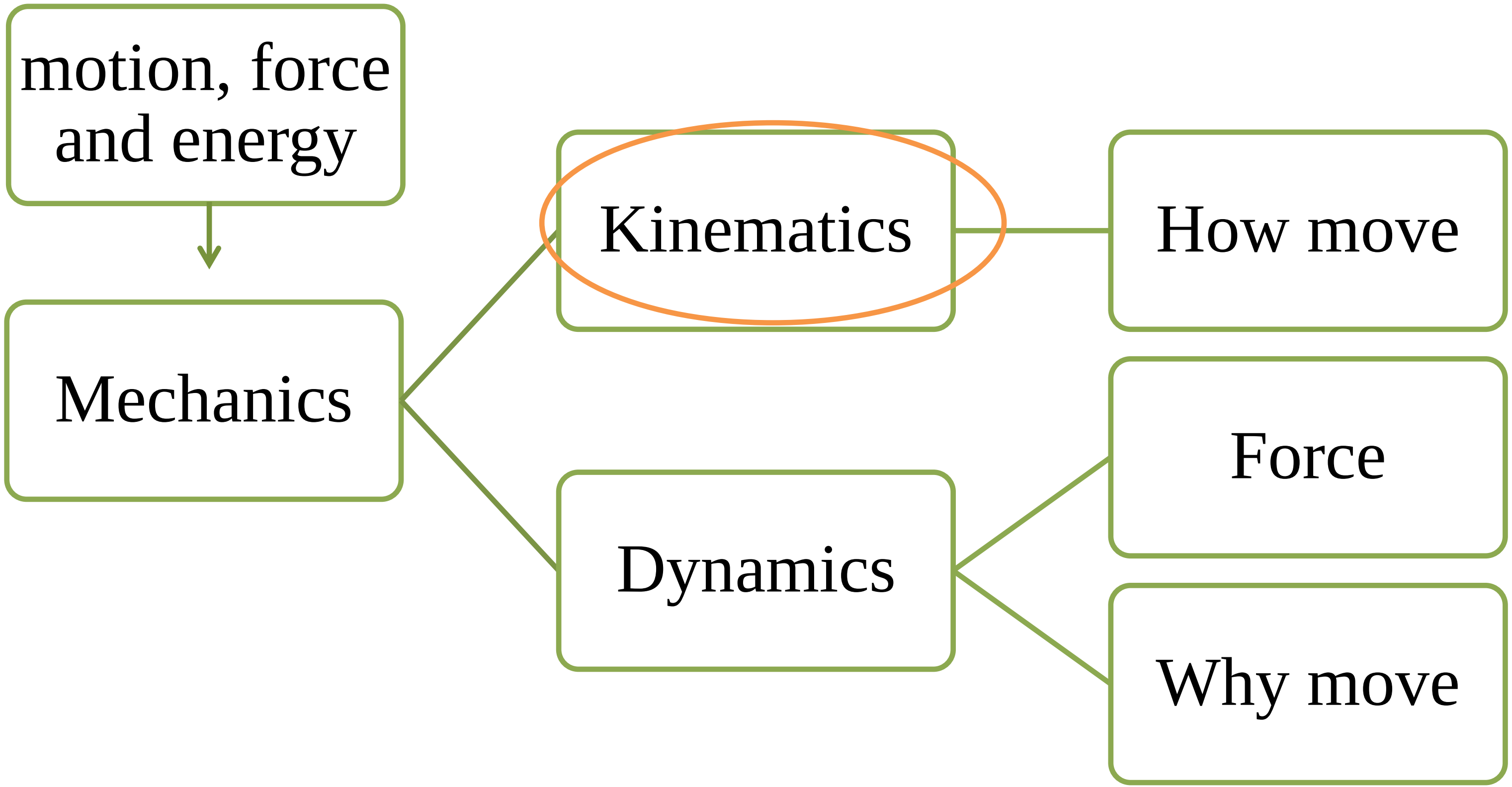
Motion

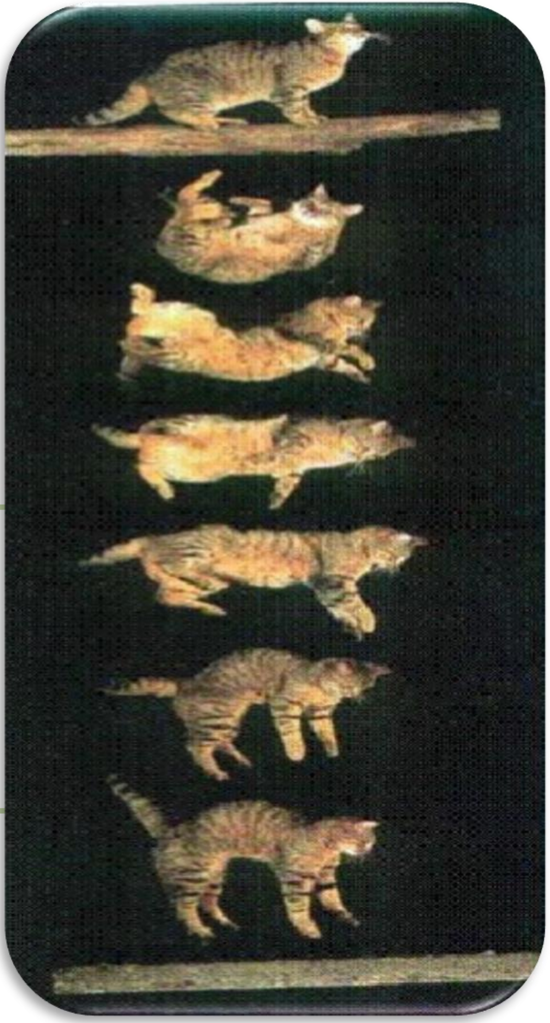

motion, force and energy
Mechanics
Kinematics
Dynamics
How move
Force
Why move

Motion

Motion

Translational

Rotational



Outline

- **Kinematics**
  - Conceptual introduction
    - Reference frame
    - Coordinate system
  - Motion in one dimension
    - Definitions
      - Distance
      - Displacement
      - Velocity
      - Speed
      - Acceleration
    - Free falling (the direction)
  - Motion in two dimension
    - Projectile motion
    - Calculation of 2D motion

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  - Motion in two dimension

Conceptual introduction

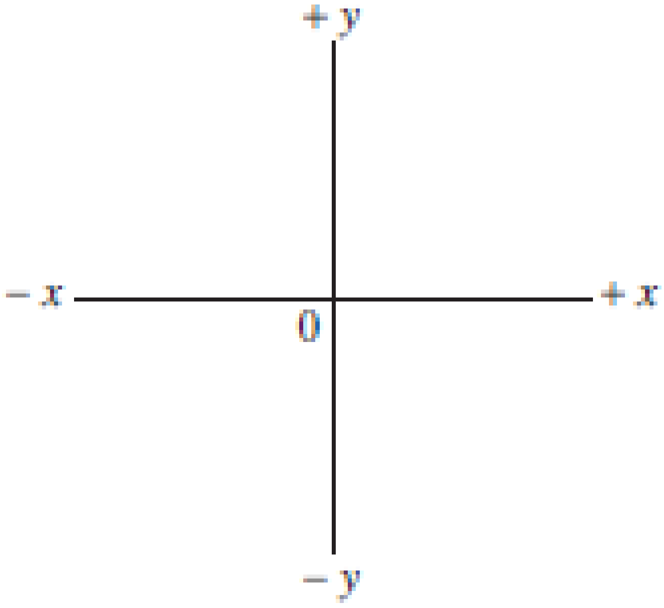
1. Reference frame:

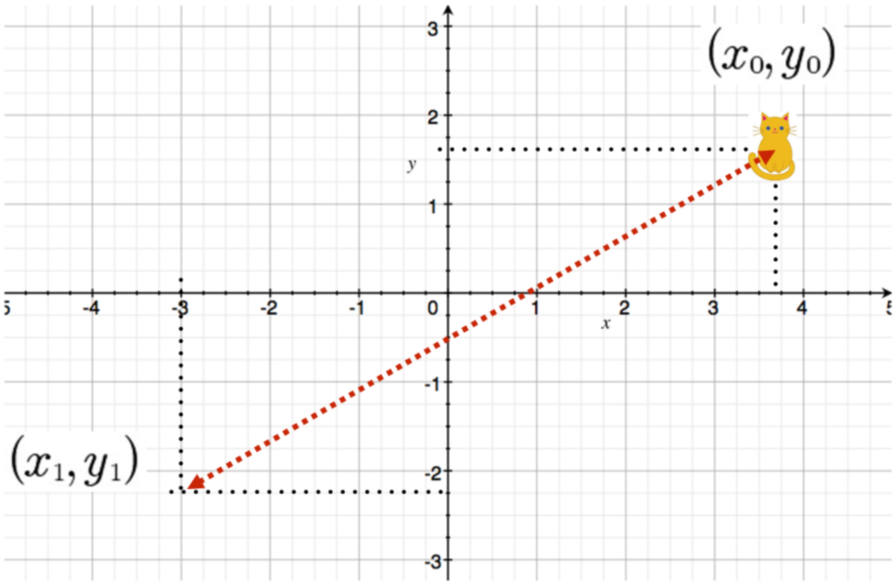
Any measurement of position, distance, or speed must be made respect to reference frame, or frame of reference

1 m/s 300 km/h

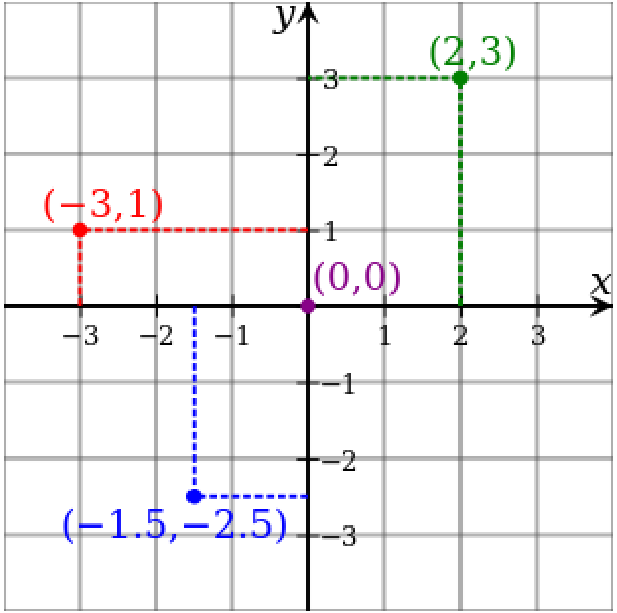
Conceptual introduction

2. Coordinate system (axes): rectangular coordinates



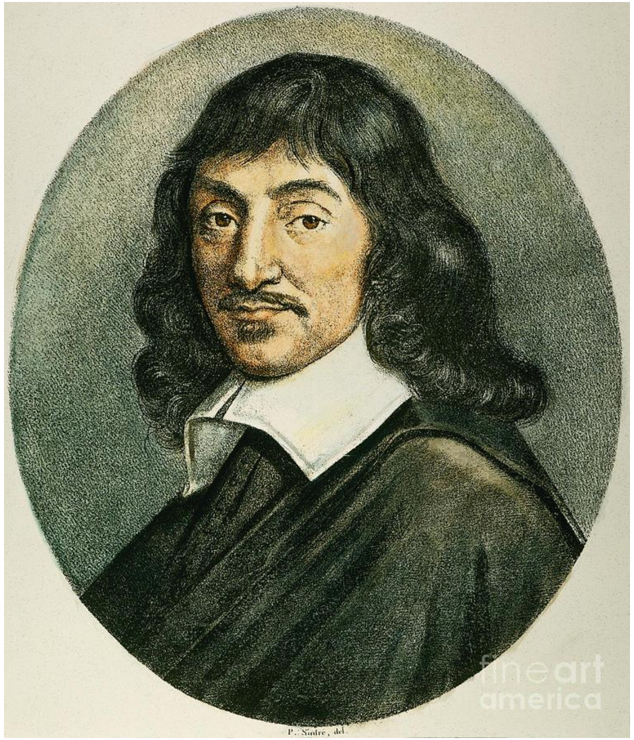


Conceptual introduction



Cartesian coordinate system

(rectangular coordinate system)



René Descartes (1596 –1650)

Latinized: ***Renatus Cartesius***

Philosophy, Math, Physics, physiology

Outline

Kinematics

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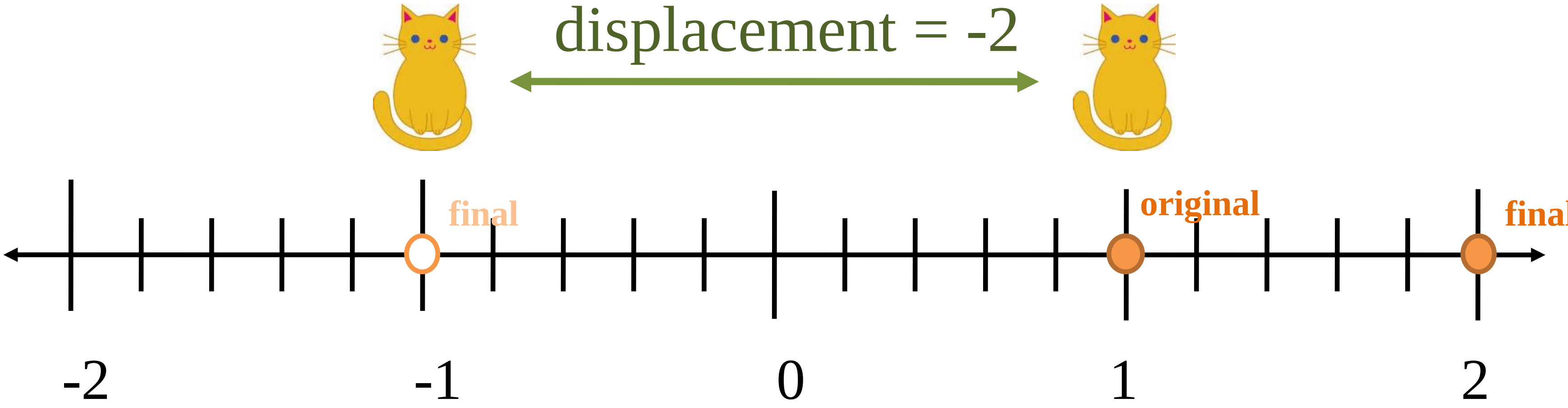
Displacement



displacement a = -2

displacement b = 3

total displacement = **1**

distance travelled = 2+3 = 5

Displacement

Distance: an objects has travelled

Displacement: change in position

Vector
Or
Scalar

Displacement =
final position – original position

Vectors and Scalars

Vectors

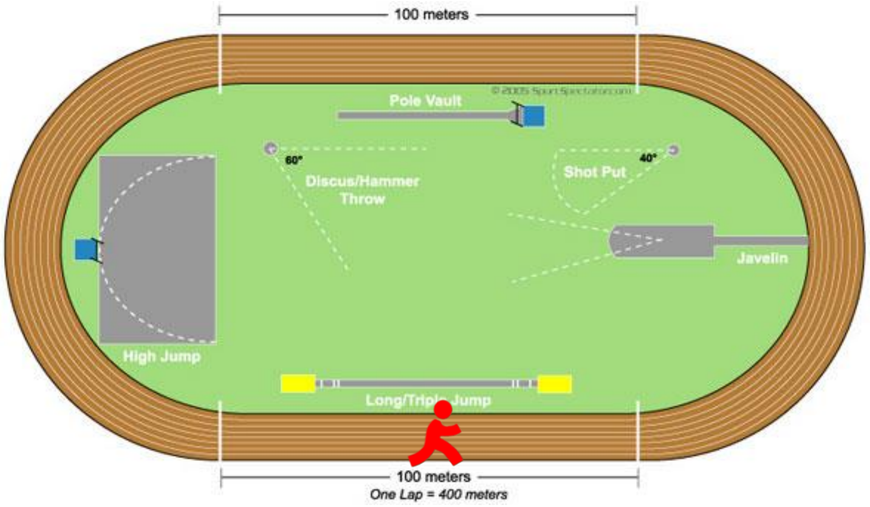
- **magnitude + direction**
- displacement
- velocity
- acceleration
- positive /negative direction

Scalars

- **magnitude only**
- distance
- speed
- temperature
- height
- calories

Displacement

Example： In a track-and-field event, an athlete runs exactly **once around** an oval track, a total distance of **400 m**.
Find the runner’s **displacement** for the race.



Total distance :

400 m

Total displacement :

0 m

Velocity and Speed

Average Velocity $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{displacement}{change\ in\ time}$

Instantaneous Velocity $\boldsymbol{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$

Average speed $average\ speed = \frac{distance}{change\ in\ time}$

SI unit : m/s

Velocity and Speed

Example: 400 m

Total distance : 400 m

Total displacement : 0 m

Average velocity: 0 m/s

Average speed: 1.33 m/s

Velocity and Speed

EXAMPLE 2–3 **Car changes speed.** A car travels at a constant 50 km/h for 100 km. It then speeds up to 100 km/h and is driven another 100 km. What is the car's average speed for the 200-km trip?

APPROACH At 50 km/h, the car takes 2.0 h to travel 100 km. At 100 km/h it takes only 1.0 h to travel 100 km. We use the defintion of average velocity, Eq. 2–2.

SOLUTION Average velocity (Eq. 2–2) is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100\text{ km} + 100\text{ km}}{2.0\text{ h} + 1.0\text{ h}} = 67\text{ km/h}.$$

NOTE Averaging the two speeds, $(50\text{ km/h} + 100\text{ km/h})/2 = 75\text{ km/h}$, gives a wrong answer. Can you see why? You must use the definition of $\bar{v}$, Eq. 2–2.

Acceleration

Average acceleration $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{change\ in\ velocity}{change\ in\ time}$

Instantaneous acceleration $\vec{a} = \frac{d\vec{v}}{dt}$

SI unit : m/s^2

Vector
Or
Scalar

Changes of *v*? $\longrightarrow$ $a \neq 0$

change in **speed**

change in **direction**

Summary

position x $\xrightarrow{\text{change}}$ displacement Δx / time Δt

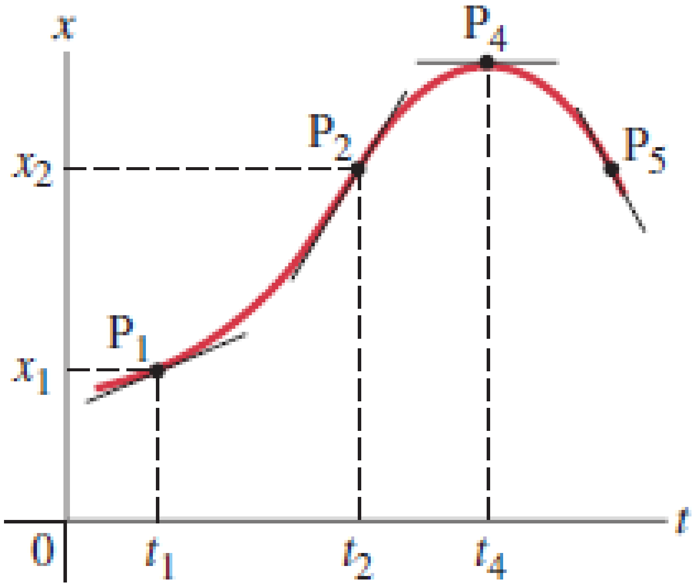
$\Big\downarrow$ $\dfrac{dx}{dt}$ (Calculus)

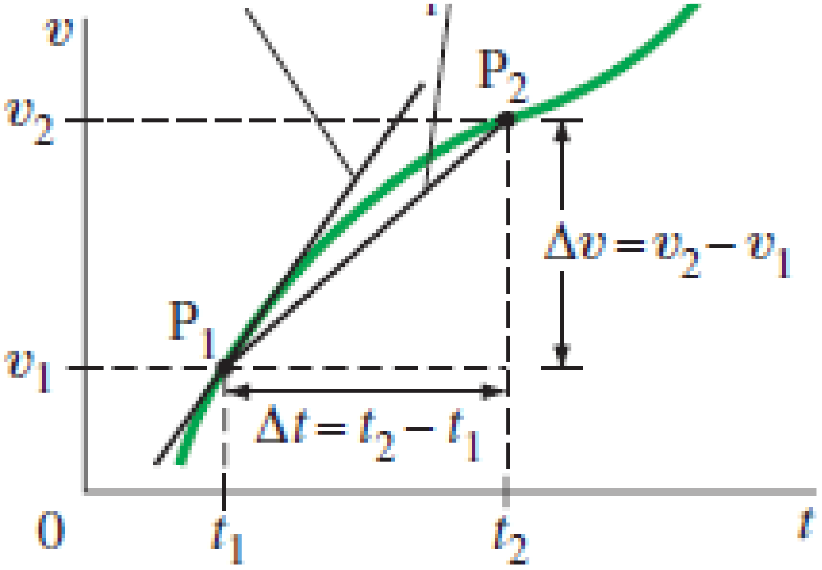
Δv / time Δt $\xleftarrow{\text{change}}$ velocity v

$\Big\downarrow$ $\dfrac{dv}{dt}$ (Calculus)

Acceleration a

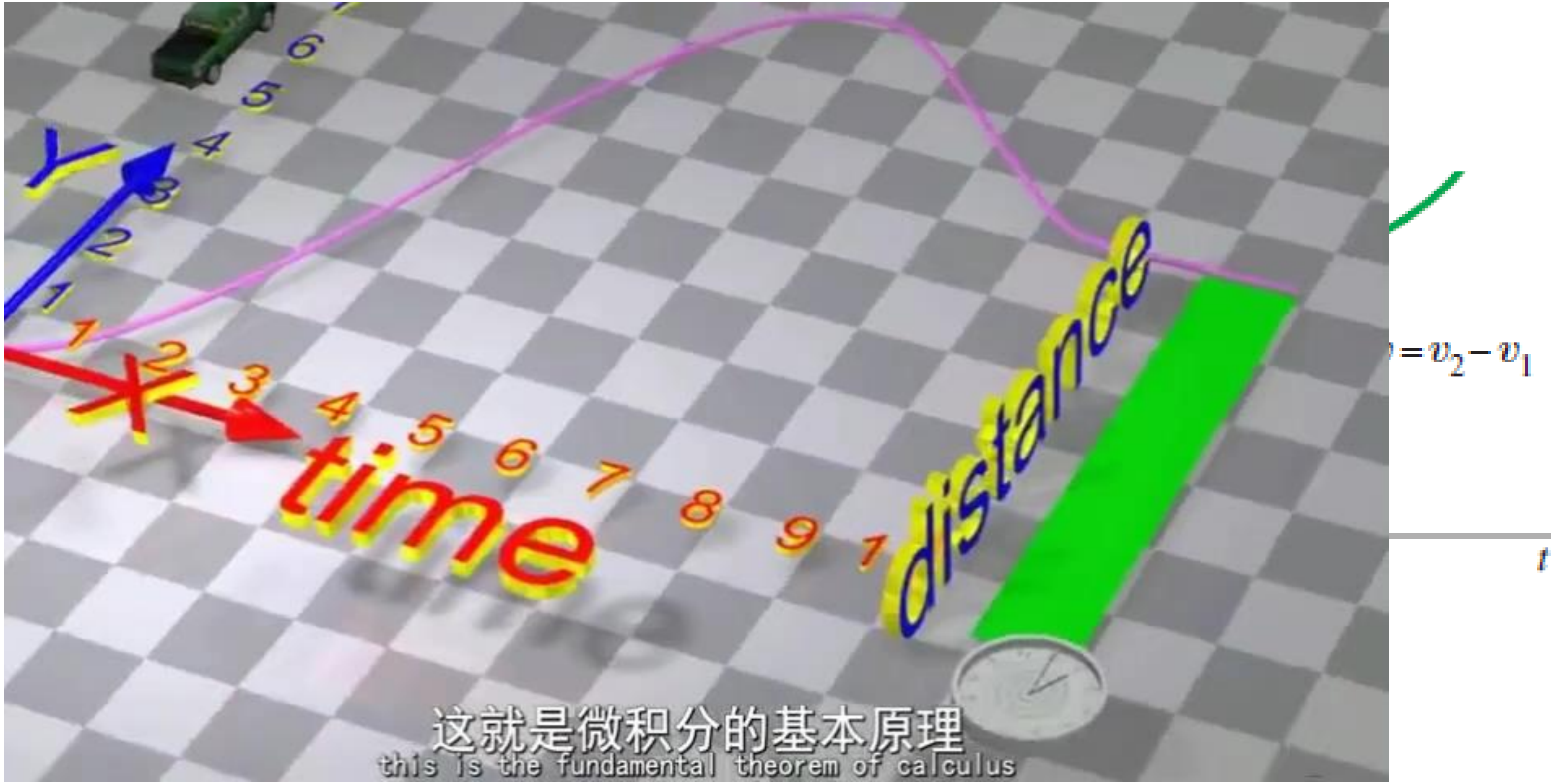
Slope of the curve





$$v = \frac{dx}{dt}$$

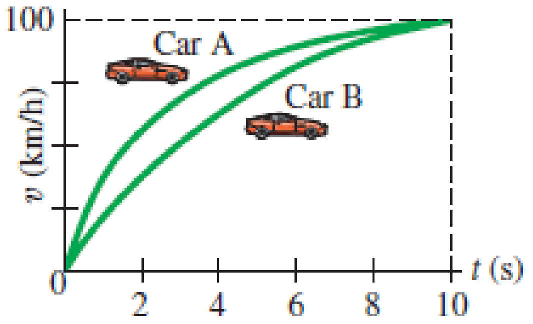
Slope of the curve



Examples

CONCEPTUAL EXAMPLE 2–17 **Analyzing with graphs.** Figure 2–31 shows the velocity as a function of time for two cars accelerating from 0 to 100 km/h in a time of 10.0 s. Compare (*a*) the average acceleration; (*b*) the instantaneous acceleration; and (*c*) the total distance traveled for the two cars.

FIGURE 2–31 (below) Example 2–17.



RESPONSE (*a*) Average acceleration is $\Delta v/\Delta t$. Both cars have the same Δv (100 km/h) over the same time interval $\Delta t = 10.0$ s, so the average acceleration is the same for both cars. (*b*) Instantaneous acceleration is the slope of the tangent to the v vs. t curve. For the first 4 s or so, the top curve (car A) is steeper than the bottom curve, so car A has a greater acceleration during this interval. The bottom curve is steeper during the last 6 s, so car B has the larger acceleration for this period. (*c*) Except at $t = 0$ and $t = 10.0$ s, car A is always going faster than car B. Since it is going faster, it will go farther in the same time.

Motion at constant *a*

Average velocity: $$\bar{v} = \frac{(v_0 + v_f)}{2}$$

$$v_f = v_0 + at$$

Final position: $$x_f = x_0 + \bar{v}t$$

$$\boxed{x_f = ?\ ?\ ?\ ?}$$

Motion at constant *a*

Average velocity: $$\bar{v} = \frac{(v_0 + v_f)}{2}$$

Final position:

$$\left.\begin{aligned} x_f &= x_0 + \bar{v}t \\ v_f &= v_0 + at \end{aligned}\right\}$$

How about time is not known?

$$\boxed{\Longrightarrow v_f^2 = v_0^2 + 2a(x_f - x_0)}$$

Motion at constant *a*

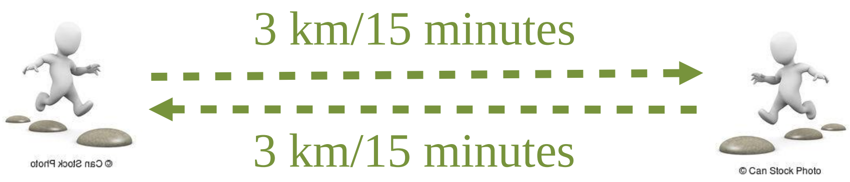
Go backward

$$a = \frac{dv}{dt} \longrightarrow v = \frac{dx}{dt} \longrightarrow x$$

Final velocity: $$v_f = v_0 + at$$

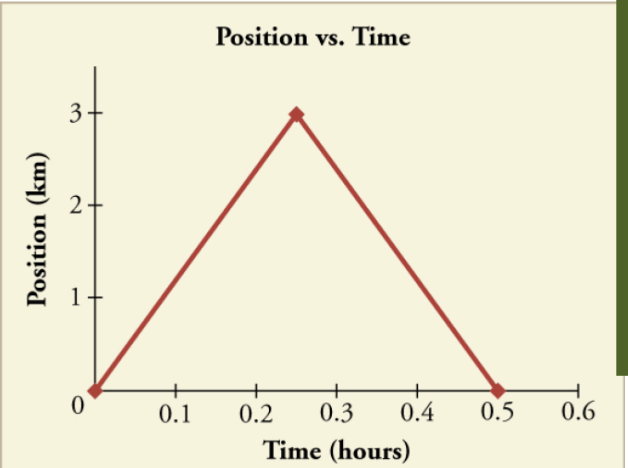
Final Position: $$x_f = x_0 + v_0 t + \frac{at^2}{2}$$

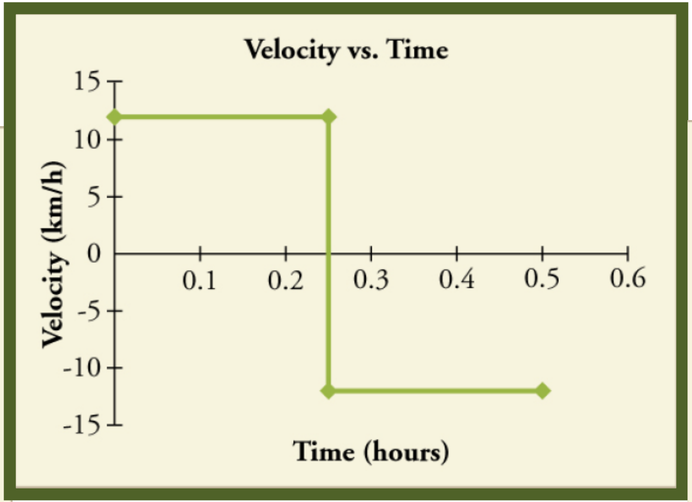
Using graphs to describe motion

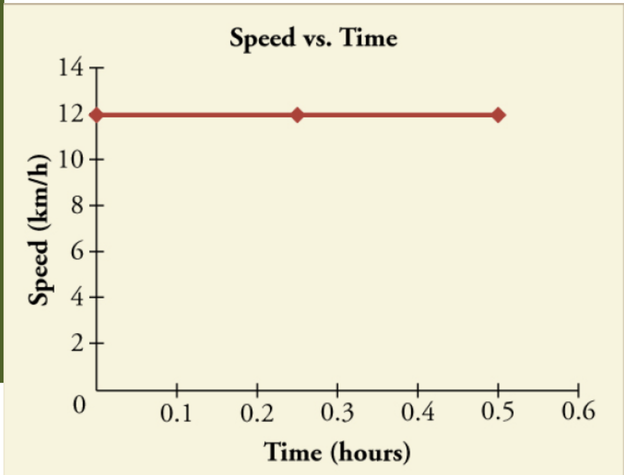


Draw:

- Position - time
- Velocity - time
- Speed - time



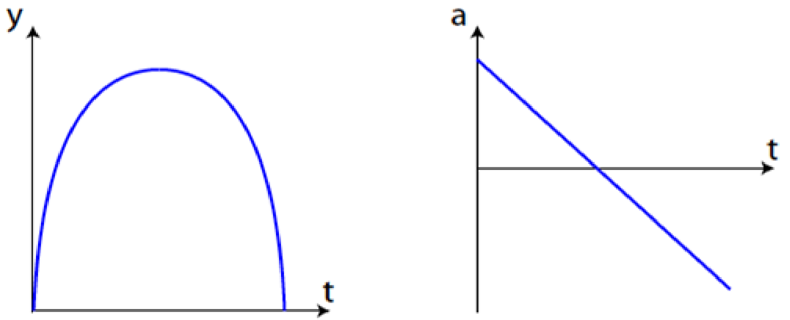




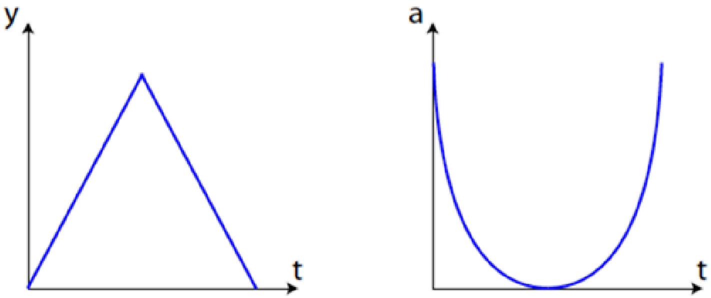
Example

A ball is thrown **vertically upward** from the ground. Which pair of graphs best describes the **motion of the ball** as a function of time while it is in the air? Neglect air resistance

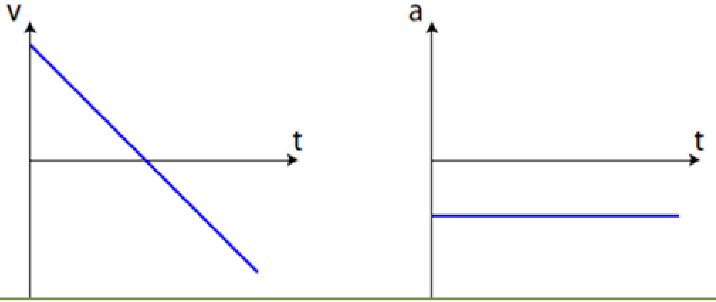
(A)



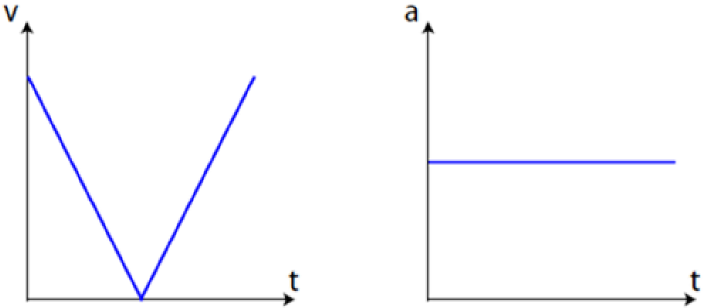
(B)



(C)



(D)



Example

Airplane landing at 250 km/h, decelerating at -1.5 m/s^2

What distance to stop completely?

6 steps for solving a Physics problem

- **Step 1 : Examine the situation to determine which physical principles are involved**
- **Step 2 : Make a list of what is given or can be inferred from the problem as stated (identify the knowns)**
- **Step 3 : Identify exactly what needs to be determined in the problem (identify the unknowns)**
- **Step 4: Find an equation of set of equations that can help you solve the problem**
- **Step 5 : Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units**
- **Step 6 : Check the answer to see if it is reasonable. Does it make sense?**

6 steps for solving a Physics problem

- **Step 1** : Principles
- **Step 2** : Knowns (Picture, positive directions)
- **Step 3** : Unknowns
- **Step 4**: Equations
- **Step 5** : Substitute
- **Step 6** : Check the answer to see if it is reasonable. Does it **make sense**?

Does it make sense?

Airplane landing at 250 km/h, decelerating at -1.5 m/s^2

What distance to stop completely?

6 steps for solving a Physics problem

- **Step 1** : Principles

kinematics

- **Step 2** : Knowns (Picture, positive directions)

$$v_0 = 250\,\mathrm{km/h}$$
$$x_0 = 0$$
$$a = -1.5\,\mathrm{m/s^2}$$
$$v_f = 0\,\mathrm{m/s}$$

- **Step 3** : Unknowns

x_f

- **Step 4**: Equations

$$v_f = v_0 + at \quad x_f = x_0 + \bar{v}t$$

- **Step 5** : Substitute

numbers in the equation

- **Step 6** : Check the answer to see if it is reasonable. Does it **make sense**?

Does it make sense?

Airplane landing at 250 km/h, decelerating at -1.5 m/s^2

What distance to stop completely?

$$x_0 = 0$$

$$v_0 = 250\,\mathrm{km/h} = \frac{250,000\,\mathrm{m}}{3600\,\mathrm{s}} = 69.4\,\mathrm{m/s}$$

$$\Longrightarrow \bar{v} = \frac{(v_0 + v_f)}{2} = 34.7\ \mathrm{m/s}$$

$$v_f = 0\,\mathrm{m/s}$$

$$a = -1.5\,\mathrm{m/s^2}$$

$$v_f = v_0 + at \quad \Longrightarrow t = \frac{(v_f - v_0)}{a} = \frac{-69.4\,\mathrm{m/s}}{-1.5\,\mathrm{m/s^2}} = 46.3\,\mathrm{s}$$

$$x_f = x_0 + \bar{v}t \quad = 0 + (34.7\,\mathrm{m/s} \times 46.3\,\mathrm{s}) = 1607.5\,\mathrm{m}$$

Summary of Kinematic Equations (constant *a*)

$$\bar{v} = \frac{(v_0 + v_f)}{2}$$

$$v_f = v_0 + at$$

$$x_f = x_0 + v_0 t + \frac{at^2}{2}$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0)$$

Why focus on constant *a* ?

Freely Falling objects:

***a* = ?**

Gravity

All objects fall towards the ground with the same acceleration

***g* = 9.80 m/s²…**

… neglecting **air resistance** and **friction**!

Gravity

$V_f = V_0 + g t$

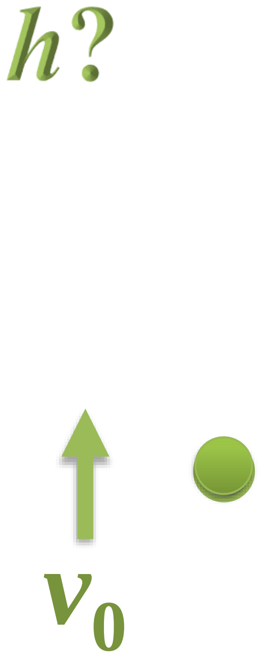
$0 = 15.5 + (9.8) t$

$t = 1.6$ sec #

v_0

$a = -g = -9.80 \text{ m/s}^2$

Gravity



$$\boxed{x_f = x_0 + v_0 t + \frac{at^2}{2}}$$

$$\boxed{v_f^2 = v_0^2 + 2a(x_f - x_0)}$$

$$\Downarrow$$

$$h_{\max} = \frac{v_0^2}{2g}$$

Gravity

$$\bar{v} = \frac{(v_0 + v_f)}{2}$$

$$v_f = v_0 + at$$

$$x_f = x_0 + \bar{v}t$$

$$\Longrightarrow$$

$$x_f = x_0 + v_0 t + \frac{at^2}{2}$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0)$$

$$\Big\Downarrow \; v_f = 0$$

$$\boxed{h_{\max} = \frac{v_0^2}{2g}}$$

Logical thinking

Not just **recite**

Throw a ball up in the air

Position vs time

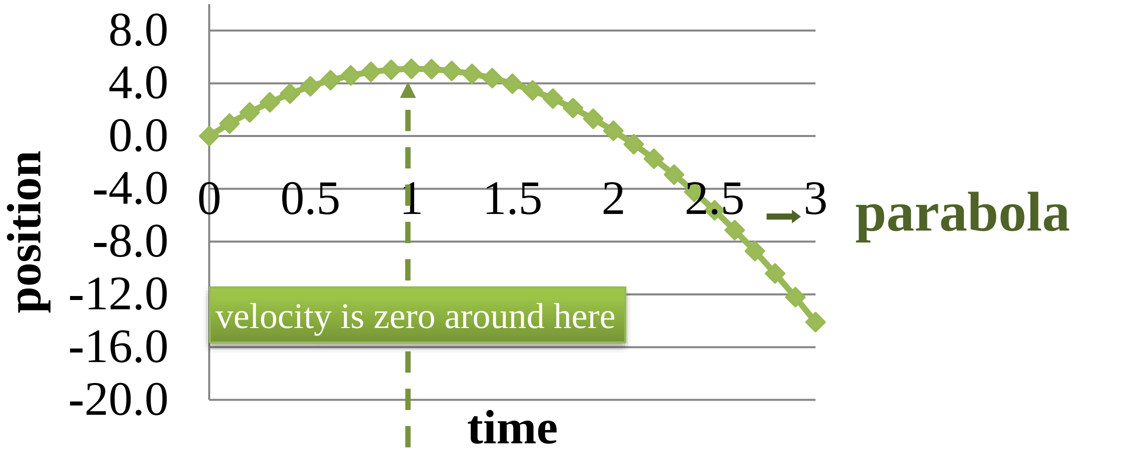
$$y = y_0 + v_0 t - \frac{gt^2}{2}$$

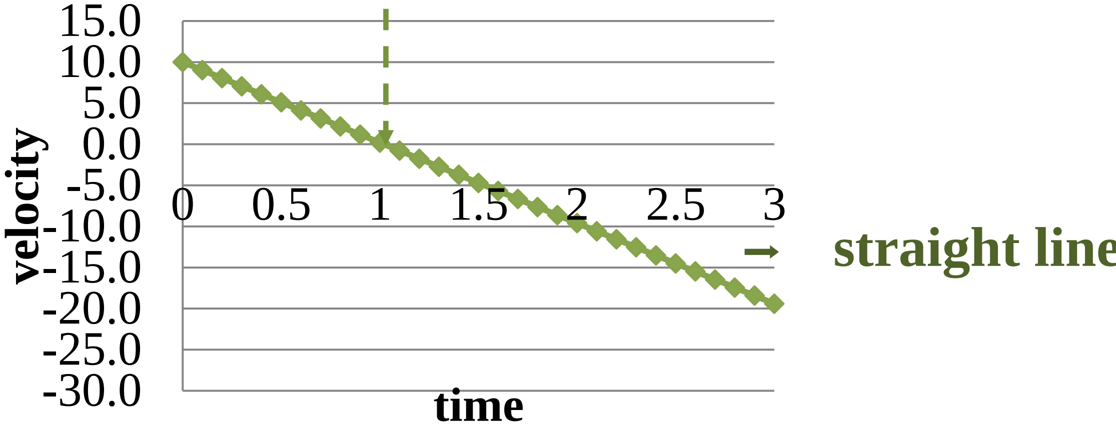
quadratic

Velocity vs time

$$v = v_0 - gt$$

linear





Practice

58. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\ \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

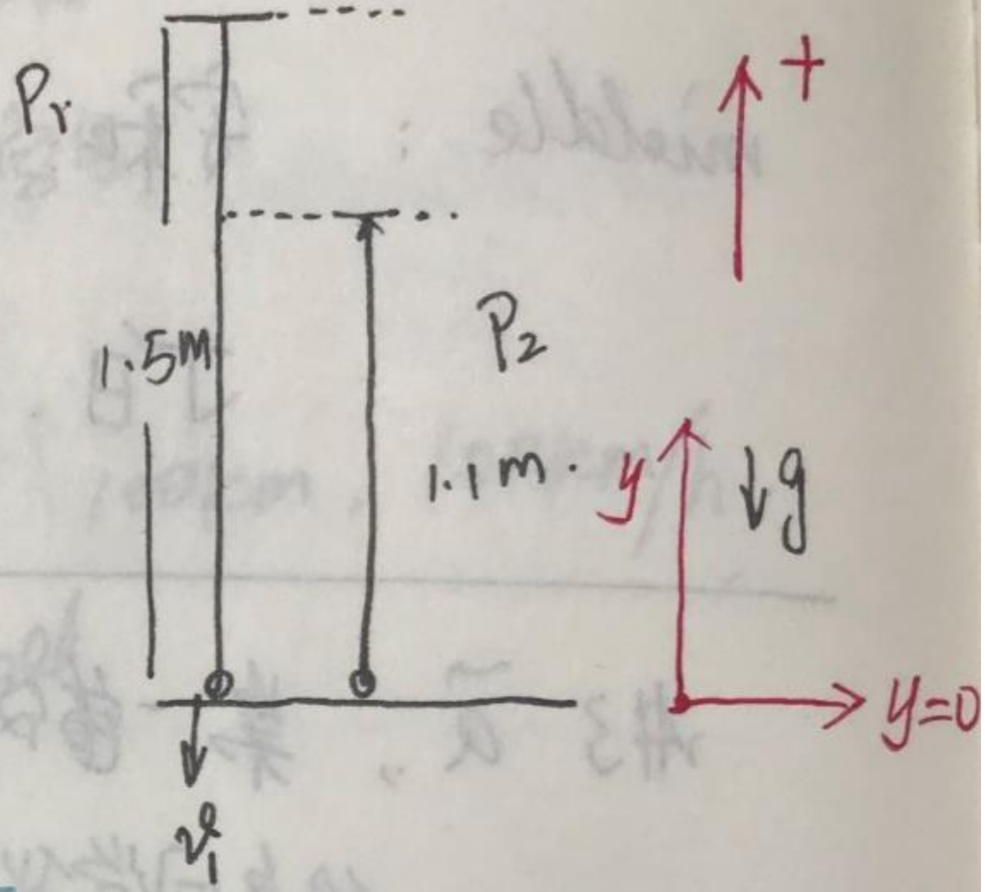
$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

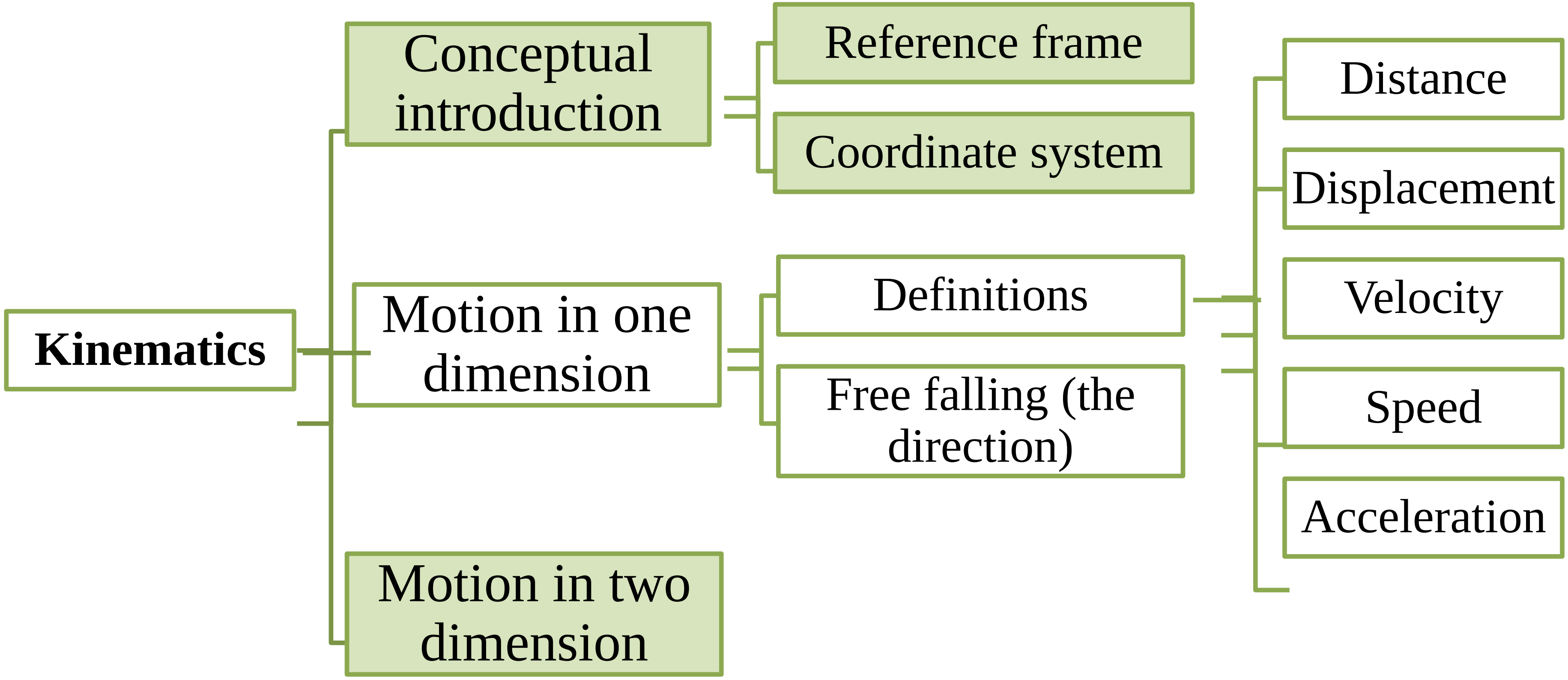
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$



Pr
1.5M
P2
1.1m.
+
y
g
y=0
v1

Outline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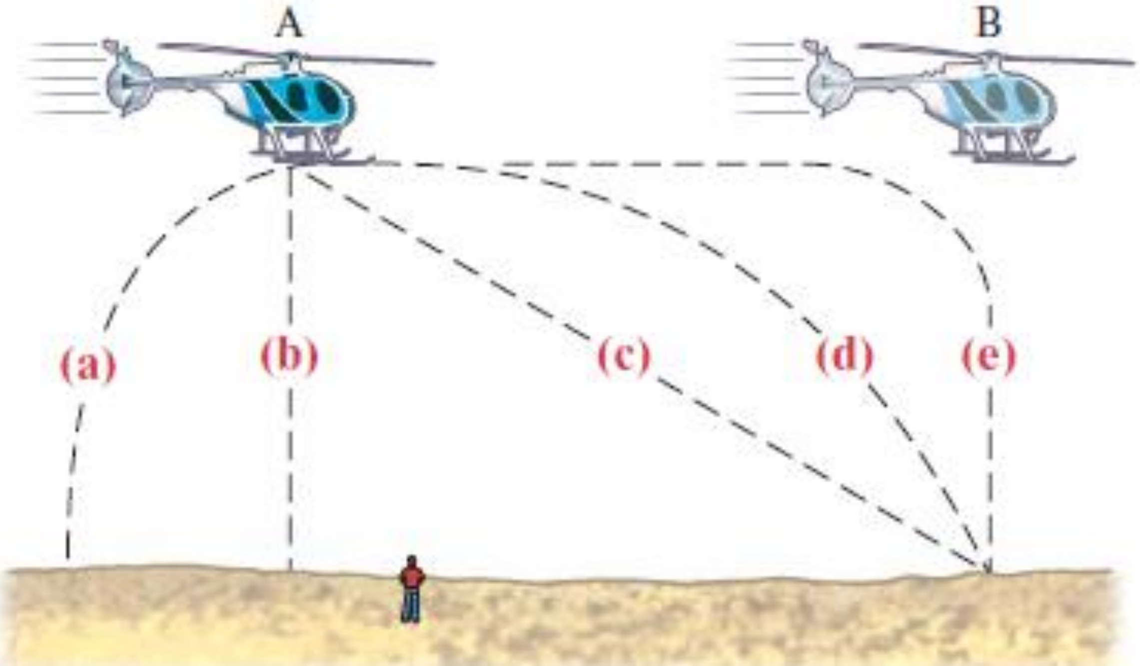
Motion in two dimensions

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



Motion in two dimensions

Displacement

Velocity

Acceleration

The motions are dependent?

The motions in directions x and y are **independent**

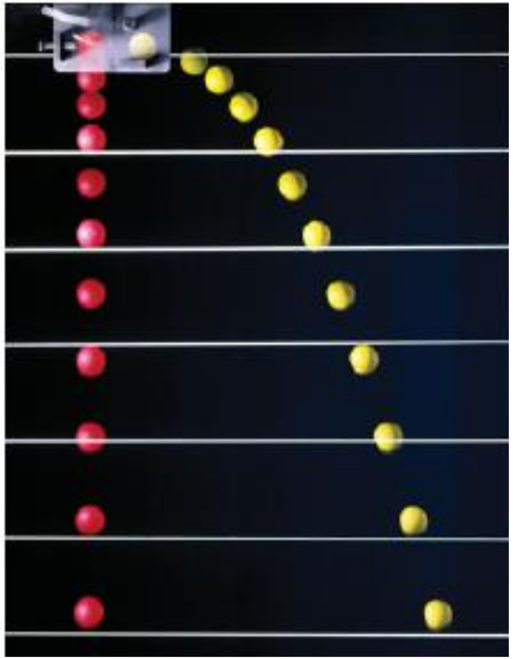
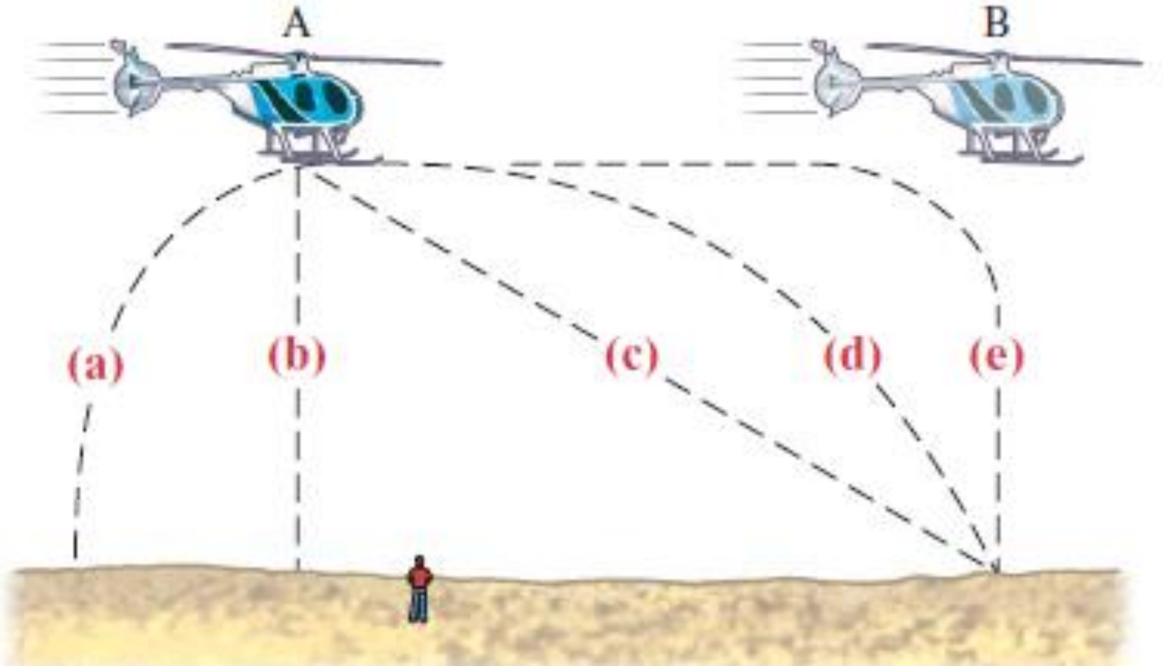


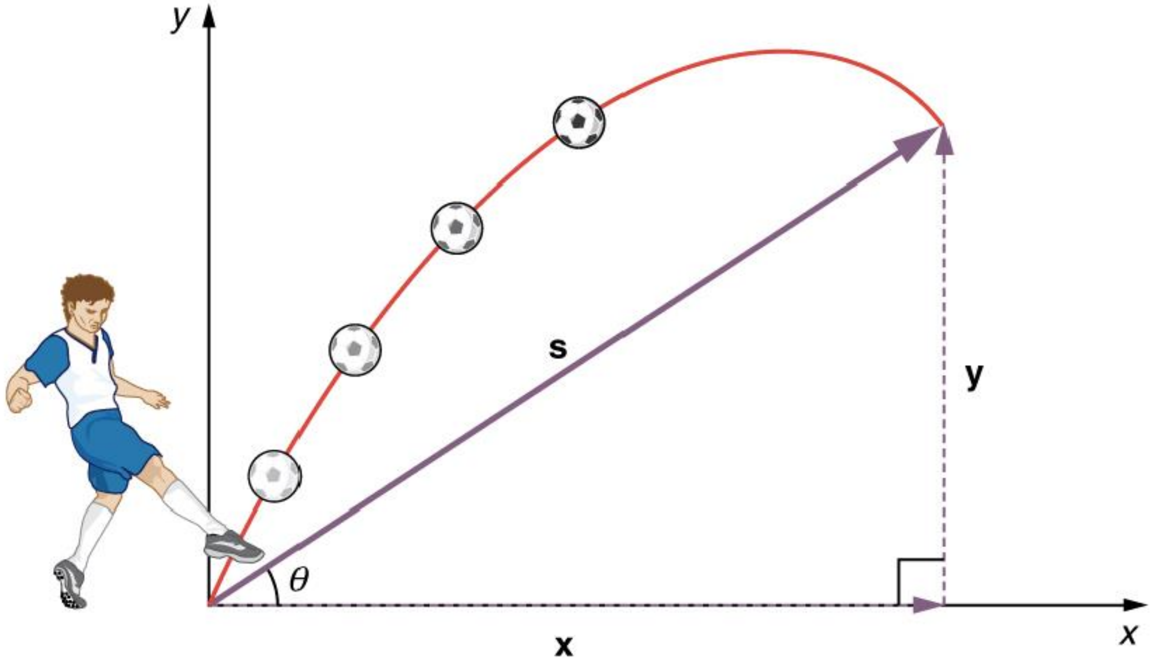
FIGURE 3–19 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

Motion in two dimensions

A small heavy box of emergency supplies is dropped from a moving helicopter at point A as it flies at constant speed in a horizontal direction. Which path in the drawing below best describes the path of the box (neglecting air resistance) as seen by a person standing on the ground?



Projectile Motion



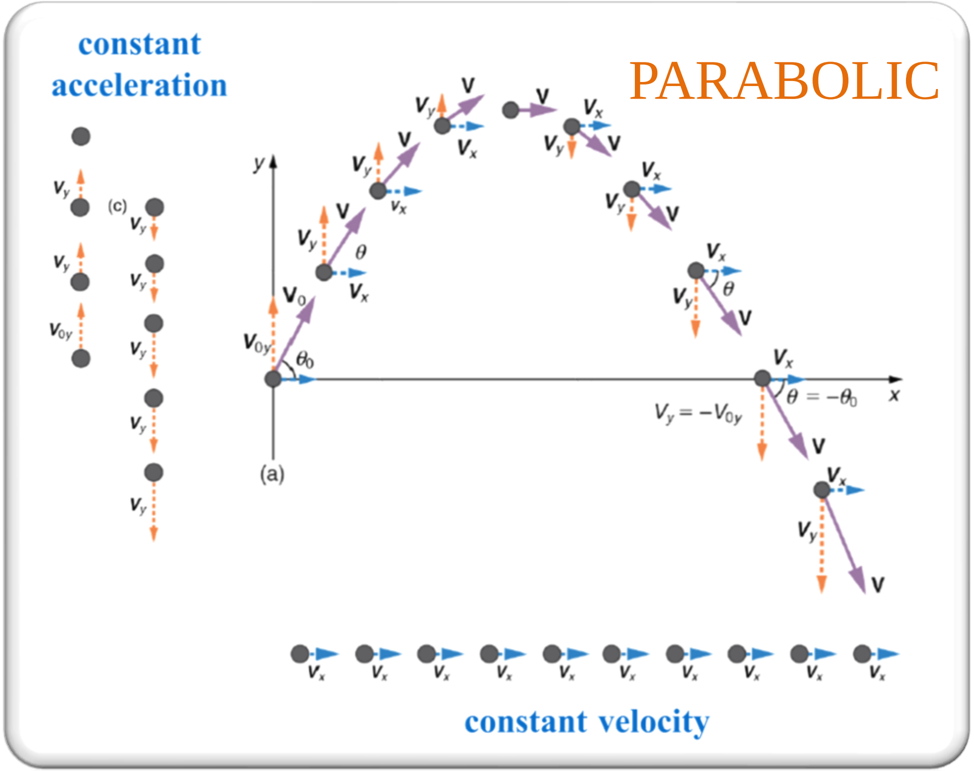
Projectile Motion

3 steps to solve

Separate

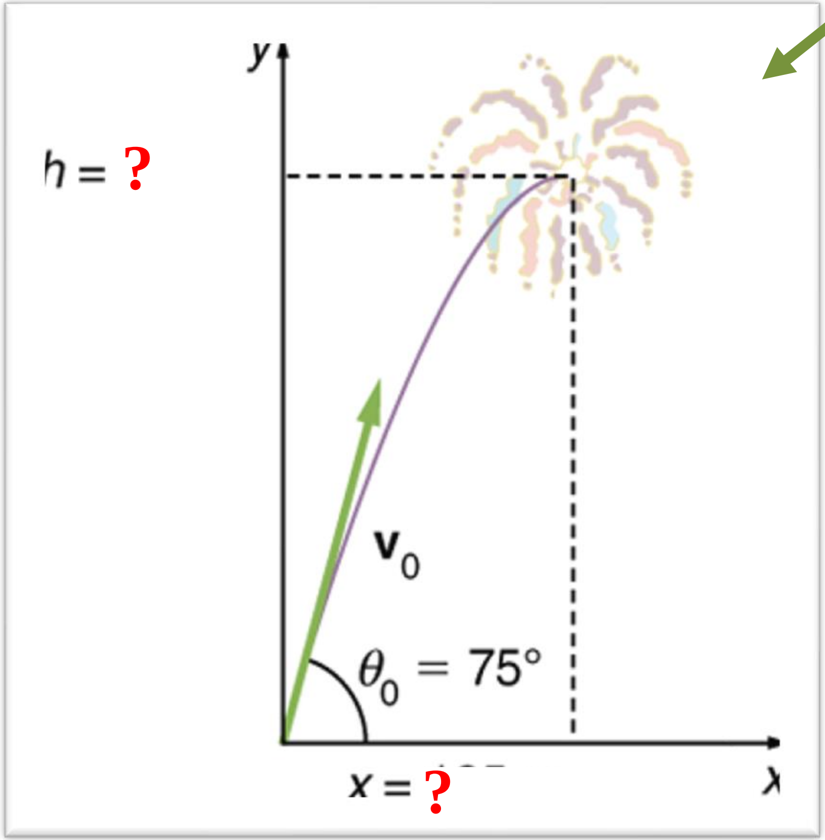
Analyze separately

Recombine



How to place the fireworks?

Example 1: Firework



Firework set to explode when it reaches its maximum height

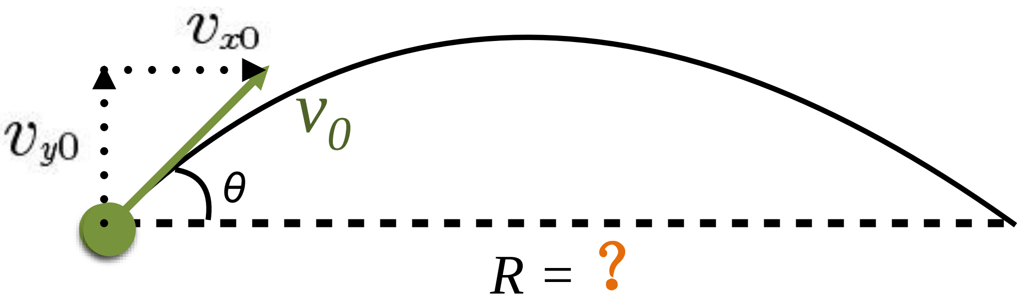
initial speed = 70 m/s

angle = 75 degrees

Approach:

1. Height
2. Time of the explosion
3. What is the horizontal distance

Example 2



$v_{y0} = v_0 \sin\theta$

$v_{x0} = v_0 \cos\theta$

Vertical

$$y = y_0 = 0$$

$$y = y_0 + v_{y0}t - \frac{1}{2}gt^2 = y_0$$

$$\Longrightarrow t = \frac{2v_{y0}}{g}$$

Horizontal distance covered :

$$R = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Trigonometric

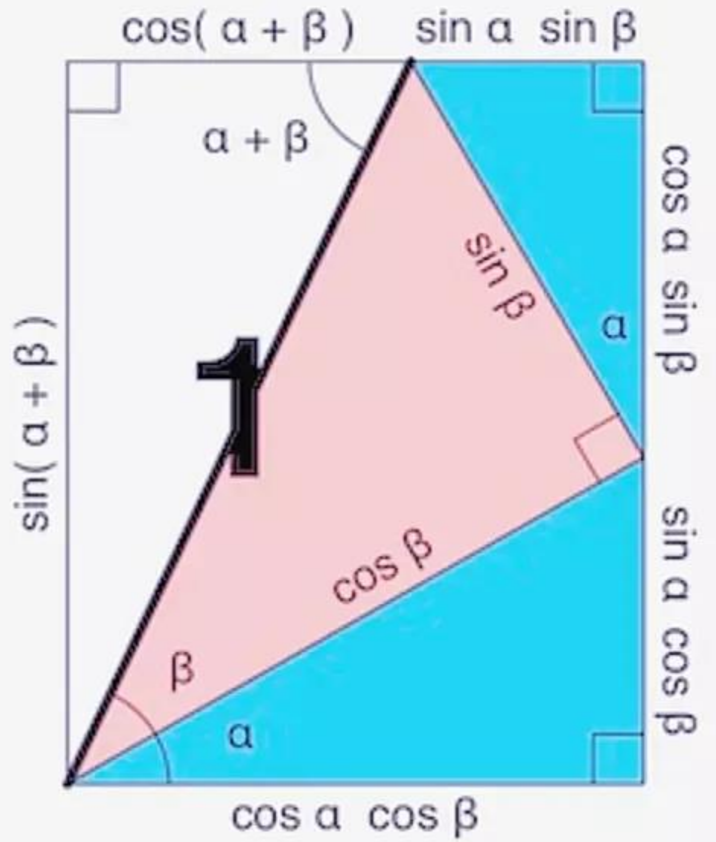
Example 2

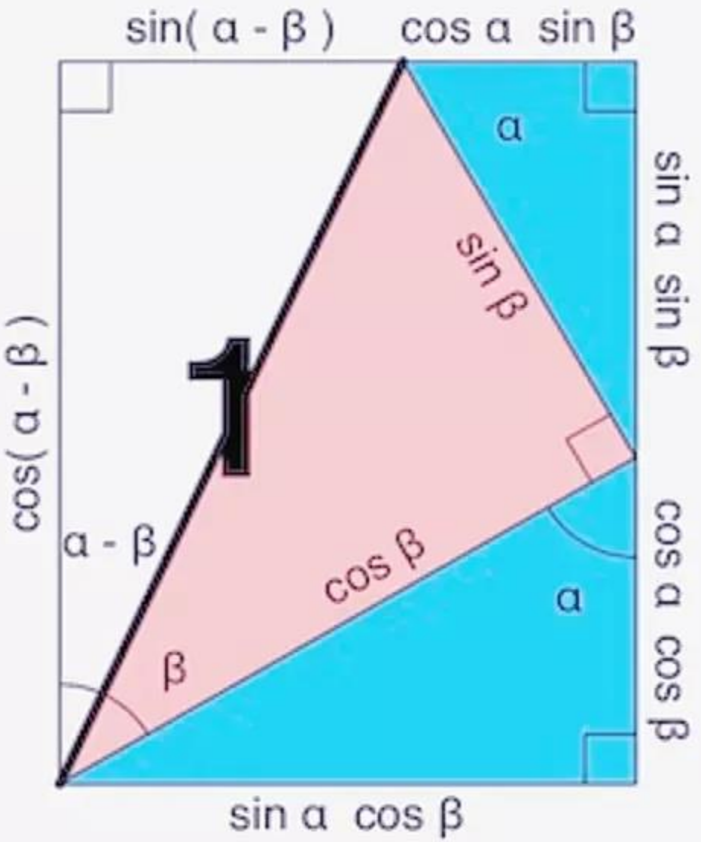
$$R = v_{x0} t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

Trigonometric identity $\quad 2\sin\theta\cos\theta = \sin 2\theta$

So the range can be expressed as: $\quad R = \frac{v_0^2 \sin(2\theta)}{g}$

Maximum range? $\quad \sin 2\theta = 1 \quad \theta = 45^o$





$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta)$

Motion in 2 dimensions

Displacement : $\vec{d} = \overrightarrow{d_x} + \overrightarrow{d_y}$

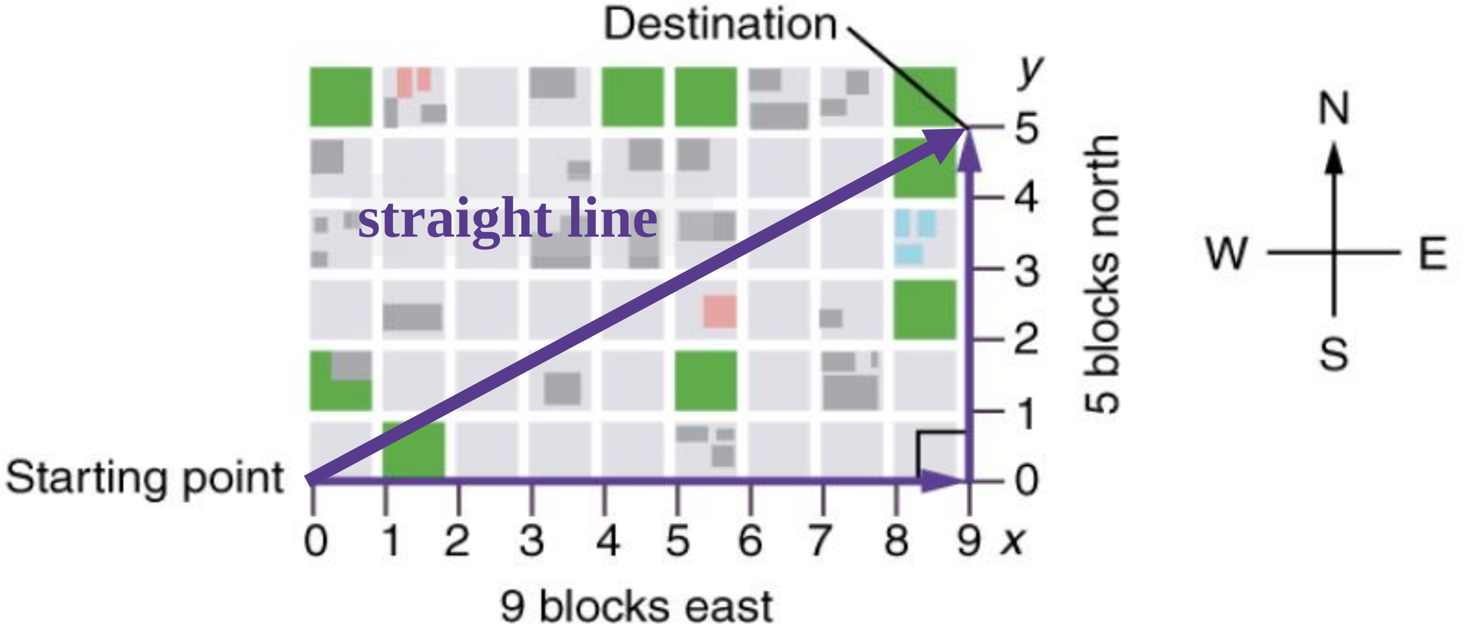
Velocity : $\vec{v} = \overrightarrow{v_x} + \overrightarrow{v_y}$

Acceleration : $\vec{a} = \overrightarrow{a_x} + \overrightarrow{a_y}$

The motions in directions *x* and *y* are independent

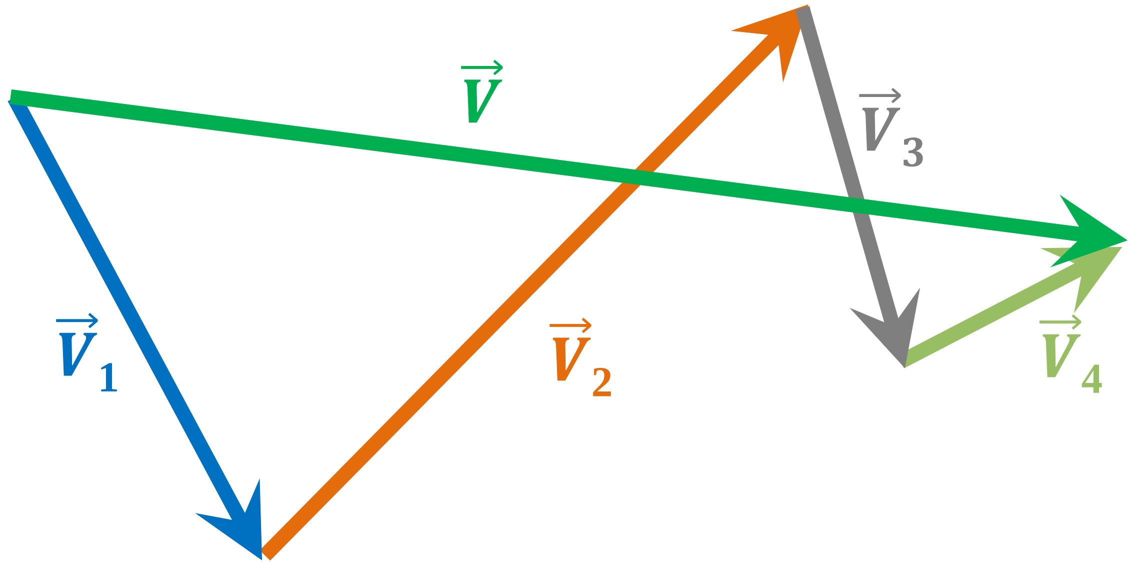
Decompose of Vectors

The displacement can be described by a vector and **decomposed into** East-component and North-component



Sum of vectors

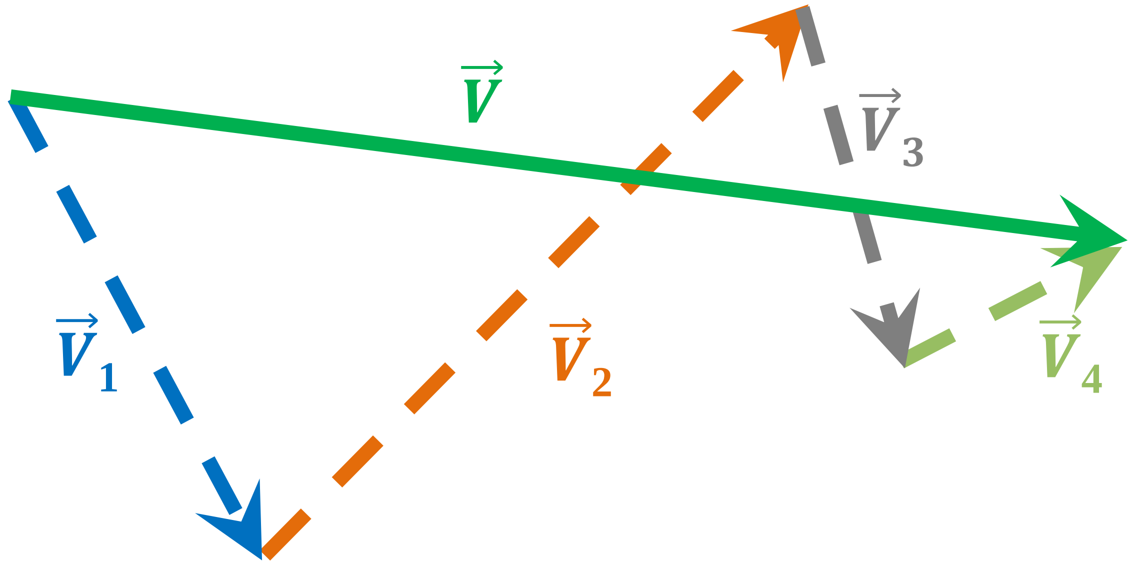
Tail-to-tip” method (1-2-3-4 Vectors)



Order?

Sum of vectors

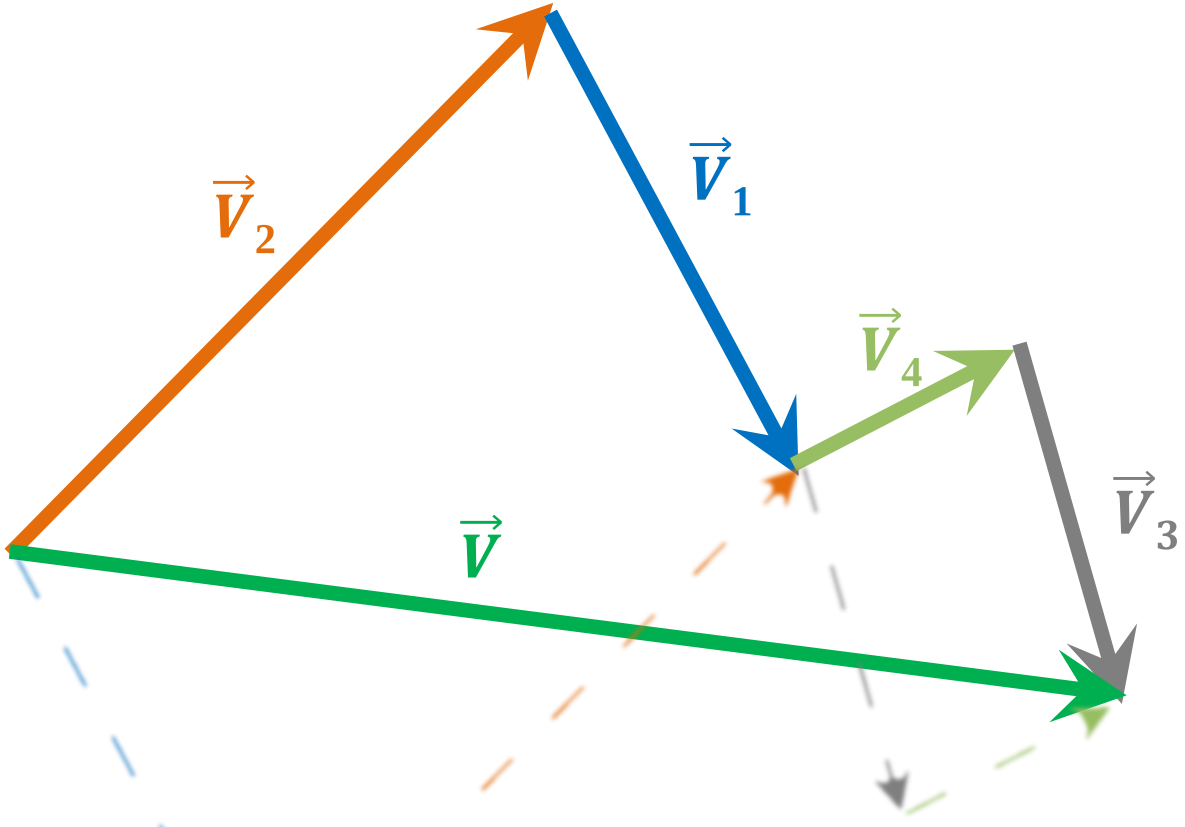
Tail-to-tip” method (1-2-3-4 Vectors)



Order?

Sum of vectors

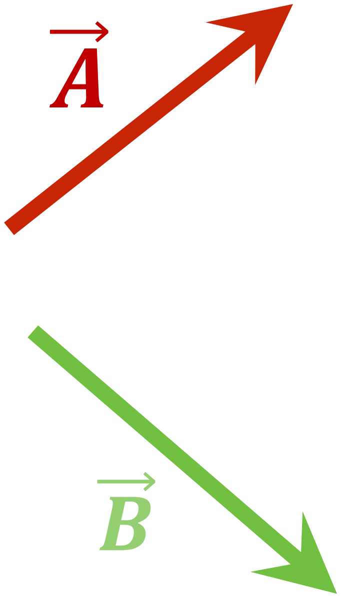
Tail-to-tip” method (2-1-4-3 Vectors)

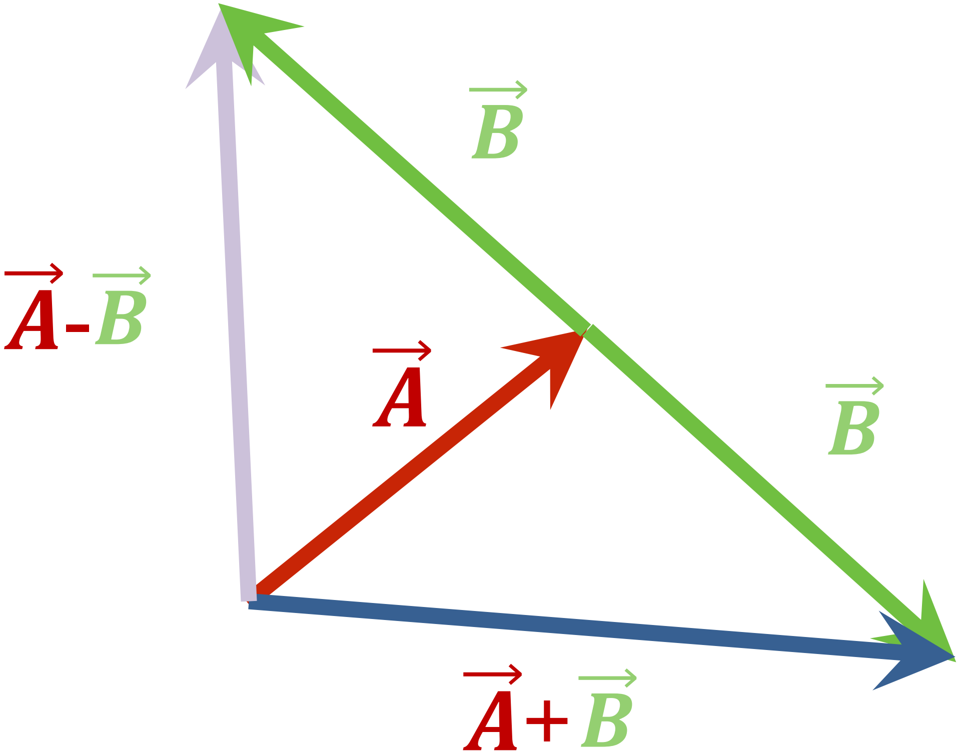


→ independen t of the order

Subtraction of the vectors

Add a vector of opposite direction

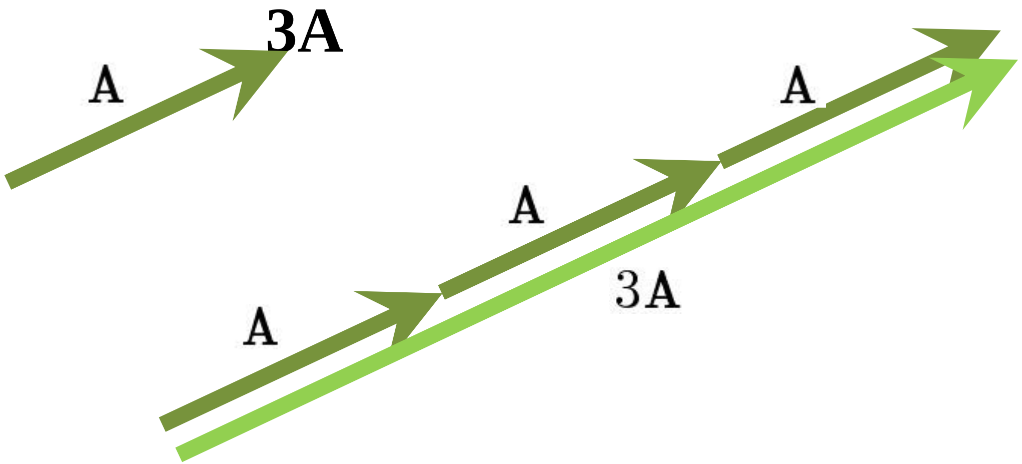




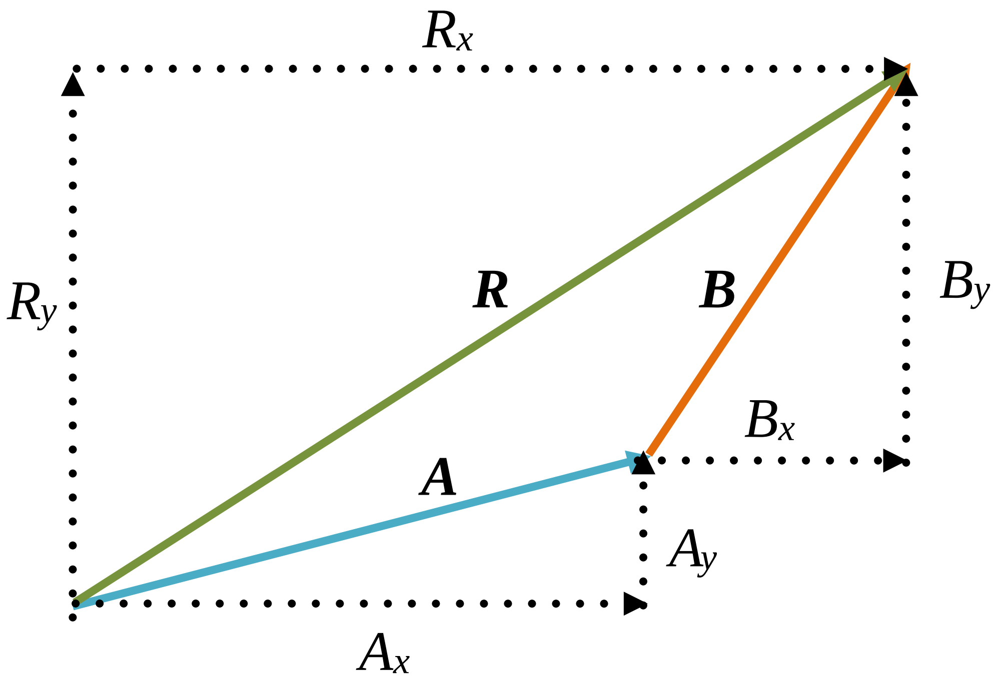
Vector multiplication by a scalar

same direction

3 times longer



Sum of vectors: sum by components



$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

Conceptual introduction

1. Reference frame:

Any measurement of position, distance, or speed must be made respect to reference frame, or frame of reference

How about velocities are not along the same line?

Relative velocity

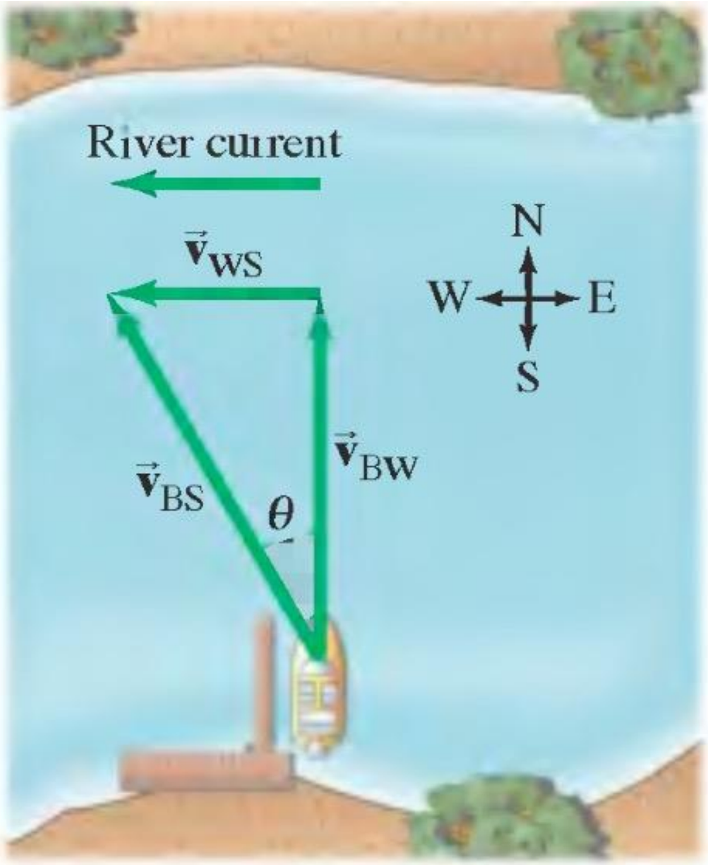


FIGURE 3–28 A boat heads north directly across a river which flows west. Velocity vectors are shown as green arrows:

$\vec{v}_{BS}$ = velocity of **B**oat with respect to the **S**hore,

$\vec{v}_{BW}$ = velocity of **B**oat with respect to the **W**ater,

$\vec{v}_{WS}$ = velocity of **W**ater with respect to the **S**hore (river current).

As it crosses the river, the boat is dragged downstream by the current.

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}.$$

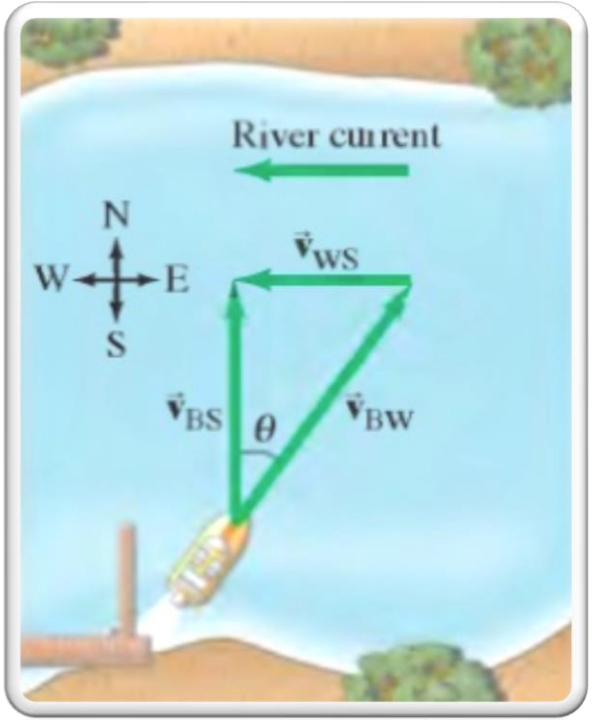
Example

EXAMPLE 3–10 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85$ m/s. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20$ m/s, at what upstream angle must the boat head? (See Fig. 3–29.)

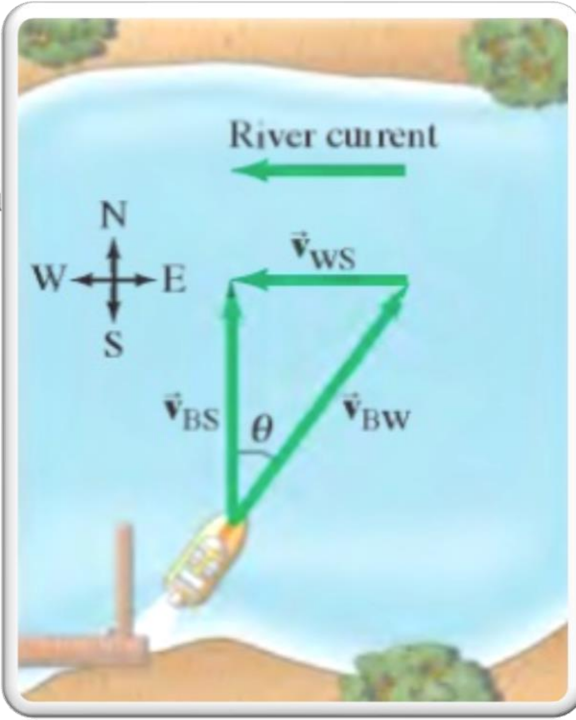
$v_{BW} = 1.85$ m/s

$v_{WS} = 1.20$ m/s

Boat travel north directly



EXAMPLE 3–10 **Heading upstream.** A boat's speed in still water is $v_{BW} = 1.85\text{ m/s}$. If the boat is to travel north directly across a river whose westward current has speed $v_{WS} = 1.20\text{ m/s}$, at what upstream angle must the boat head? (See Fig. 3–29.)



SOLUTION Vector $\vec{\mathbf{v}}_{BW}$ points upstream at angle θ as shown. From th

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\text{ m/s}}{1.85\text{ m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

Practice

A car travels 10 m/s east. Another car travels 10 m/s north. The relative speed of the first car with respect to the second is

(*a*) less than 20 m/s.

(*b*) exactly 20 m/s.

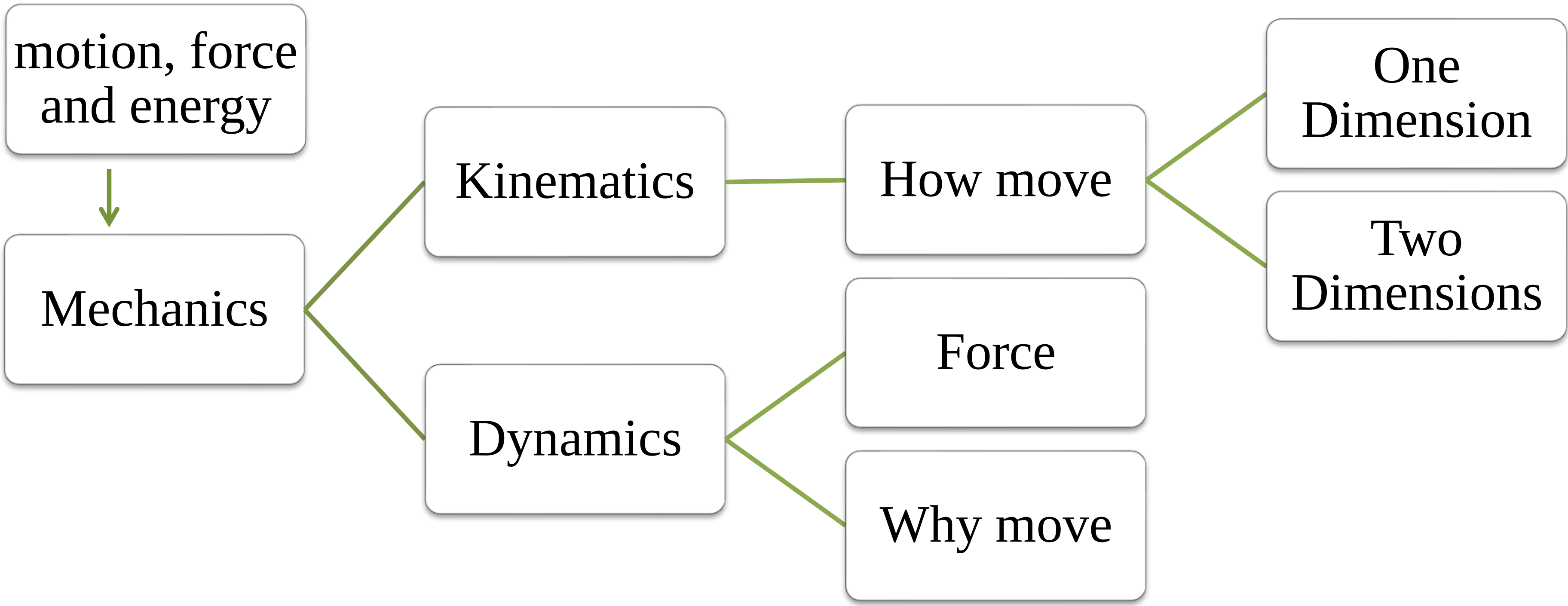
(*c*) more than 20 m/s.

How about equation and the vector diagram?

Summary

Kinematics

Structure


motion, force and energy
Mechanics
Kinematics
Dynamics
How move
Force
Why move
One Dimension
Two Dimensions

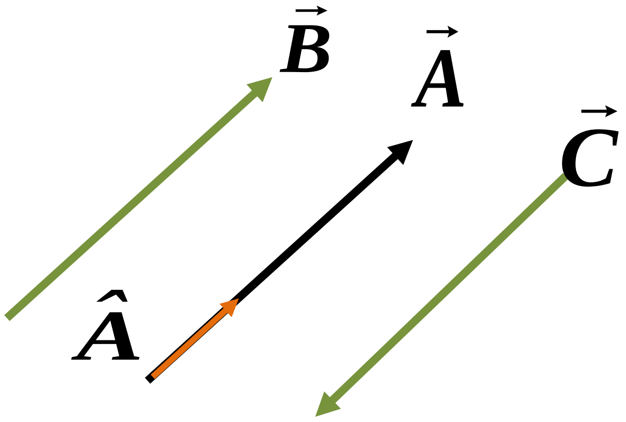
Representation of Vectors

Vector: has both magnitude and a direction, such as $\vec{r}$, $\vec{v}$, $\vec{a}$,

$$\vec{\mathbf{A}} \qquad \hat{\mathbf{A}}$$

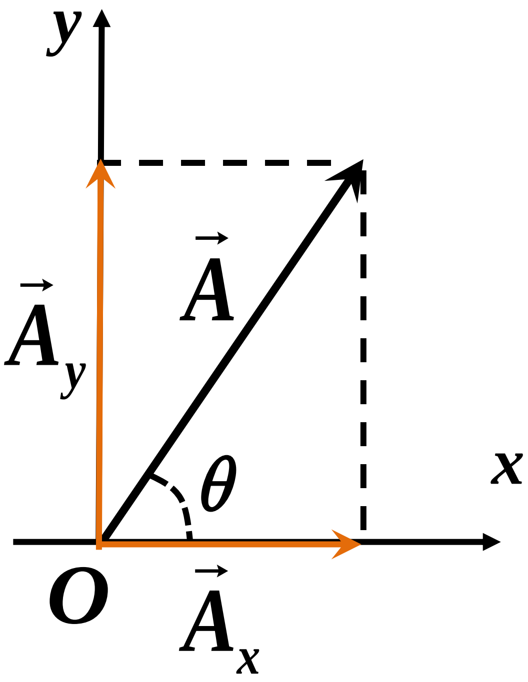
Vector **Unit Vector**

$$\vec{\mathbf{A}} = \left|\vec{\mathbf{A}}\right|\hat{\mathbf{A}} = A\hat{\mathbf{A}}$$



$$\vec{B} = \vec{A}, \vec{C} = -\vec{A}$$

The Cartesian representation of any vector



$\hat{i}, \hat{j}$ **represent unit vectors in direction of +*x*-axis or +*y*-axis**

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\vec{A}_x = A_x\hat{\imath}, \ \vec{A}_y = A_y\hat{\jmath}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\left|\vec{A}\right| = A = \sqrt{A_x^2 + A_y^2} \qquad tg\theta = A_y / A_x$$

The case of three dimensions

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

We can prove that

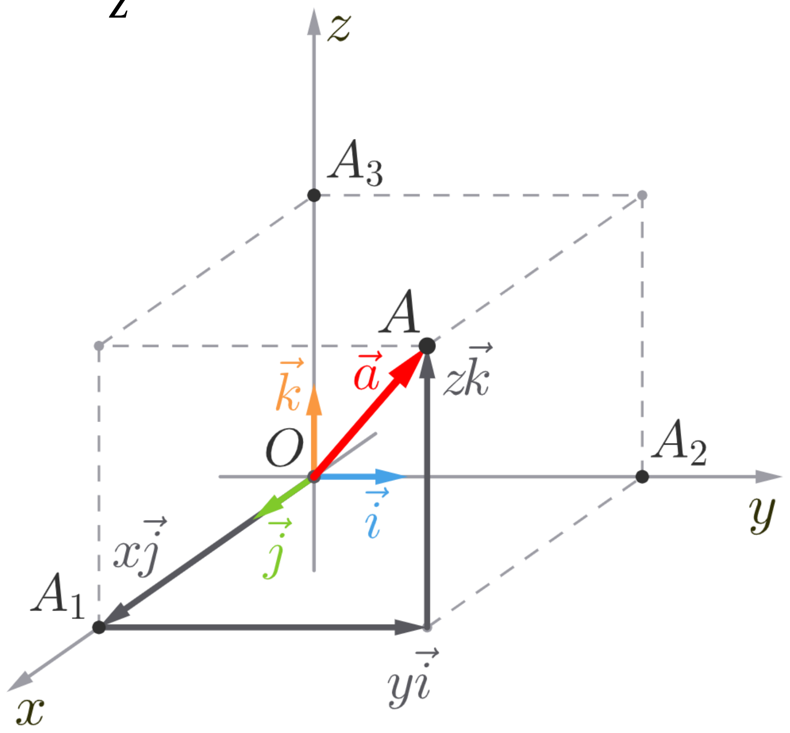
$$A = (\vec{A}\cdot\vec{A})^{1/2} = (A_x^2 + A_y^2 + A_z^2)^{1/2}$$

$$\vec{A}\cdot\hat{i} = A\cos\alpha = A_x$$

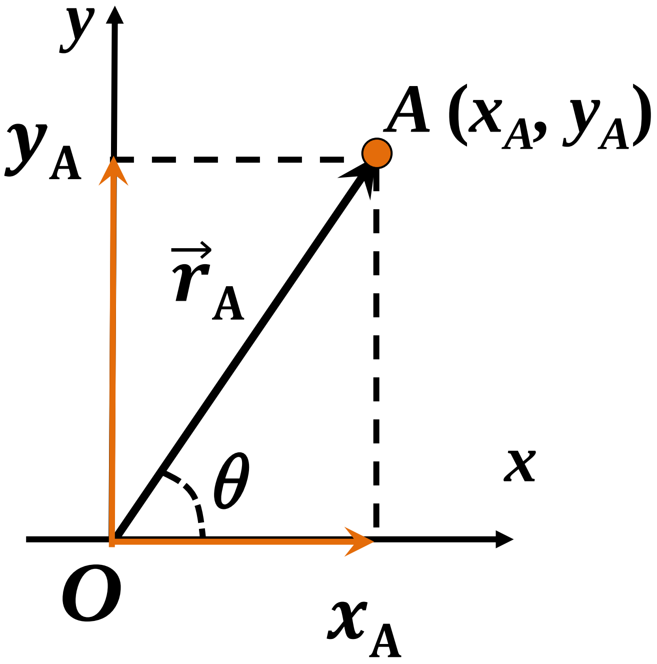
$$\vec{A}\cdot\hat{j} = A\cos\beta = A_y$$

$$\vec{A}\cdot\hat{k} = A\cos\gamma = A_z$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



Example: Position Vector



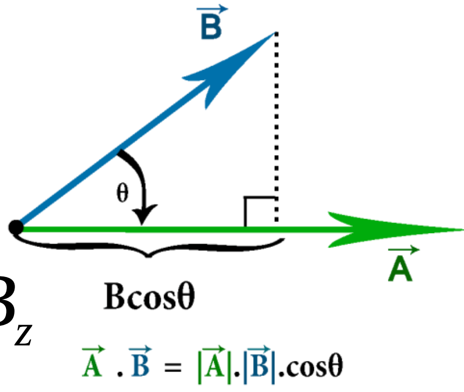
Position Vector: $\vec{r}_A$

$$\vec{r}_A = x_A \hat{i} + y_A \hat{j}$$

$$|\vec{r}_A| = \sqrt{x_A^{\ 2} + y_A^{\ 2}}$$

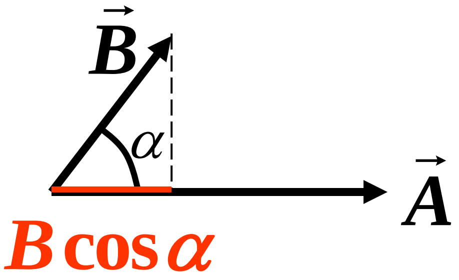
$$tg\theta = y_A / x_A$$

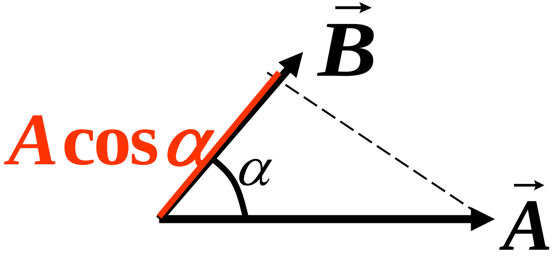
Multiplying a vector by a vector



(1) The scalar (dot) product

$$\vec{A}\cdot\vec{B}=C \quad \begin{cases} C = A_x B_x + A_y B_y + A_z B_z \\ C = AB\cos\alpha \end{cases}$$



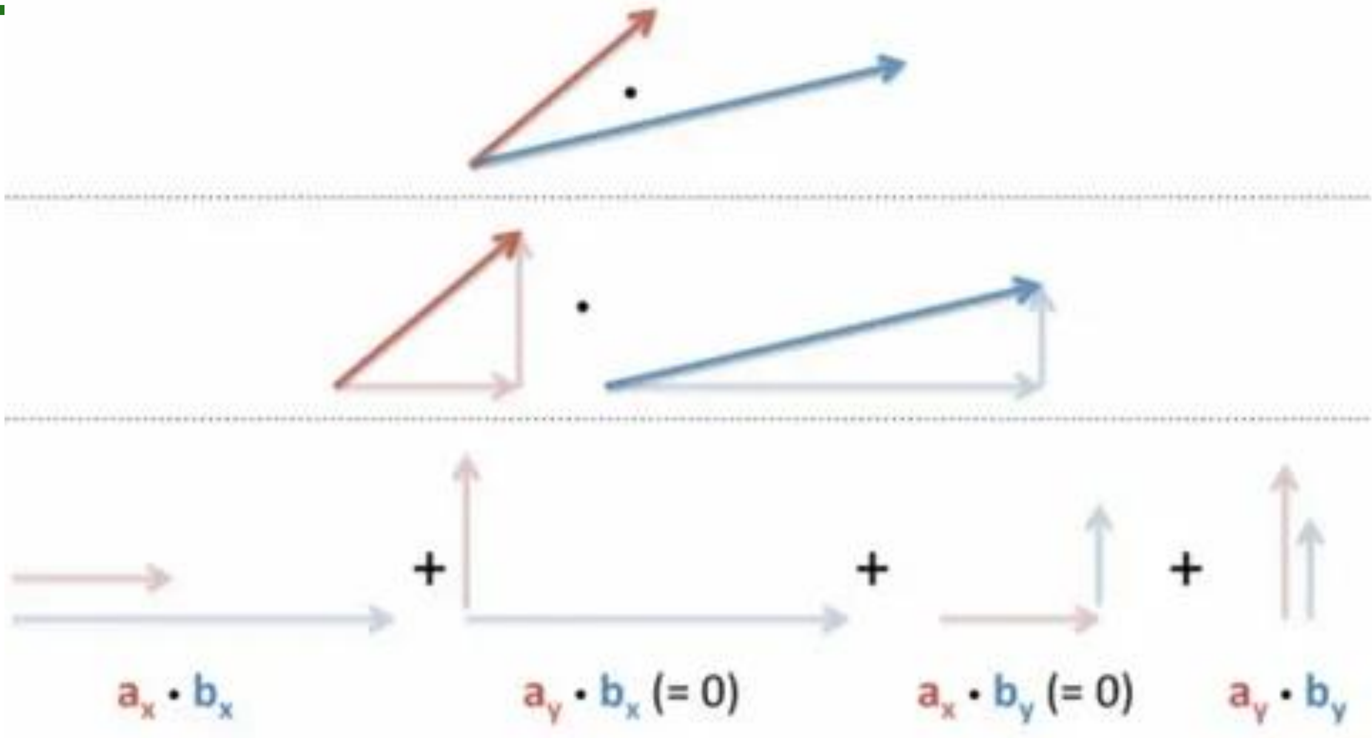


The magnitude of $\vec{A}$ $\quad A = |\vec{A}| = (\vec{A}\cdot\vec{A})^{1/2}$

Dot & Cross Product

	b_x	b_y	b_z
a_x	Dot	Cross	Cross
a_y	Cross	Dot	Cross
a_z	Cross	Cross	Dot

All possible interactions = Similar parts + Different parts



$a_x \cdot b_x$
+
$a_y \cdot b_x$ (= 0)
+
$a_x \cdot b_y$ (= 0)
+
$a_y \cdot b_y$

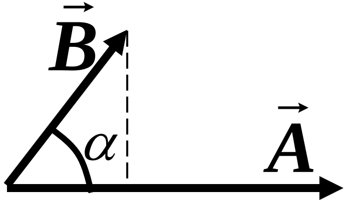
Multiplying a vector by a vector

(1) The scalar (dot) product

Caution!

$$\vec{A}\cdot\vec{B}=C \qquad C=AB\cos\alpha$$

A, B are always positive



$$\alpha<\pi/2 \quad \vec{A}\cdot\vec{B}=AB\cos\alpha>0$$

$$\alpha>\pi/2 \quad \vec{A}\cdot\vec{B}=AB\cos\alpha<0$$

$$\alpha=\pi/2 \quad \vec{A}\cdot\vec{B}=AB\cos\alpha=0$$

$$\vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

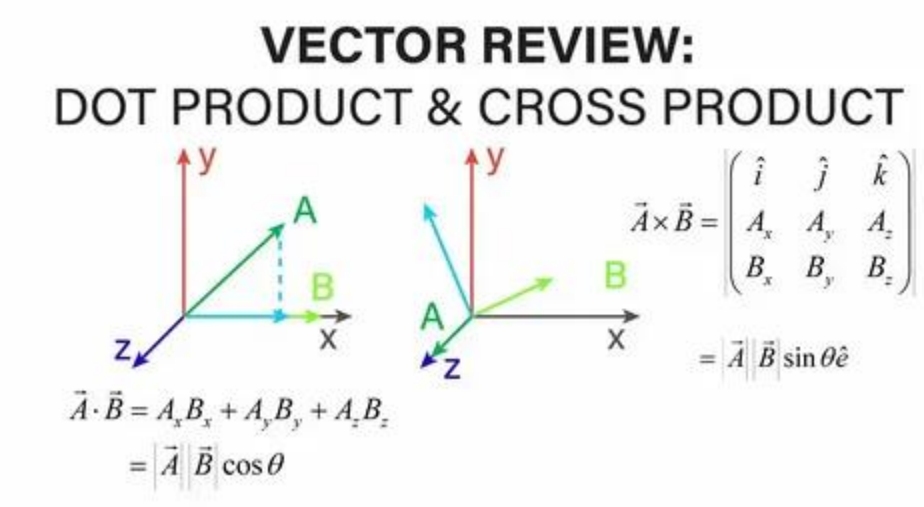
Multiplying a vector by a ve

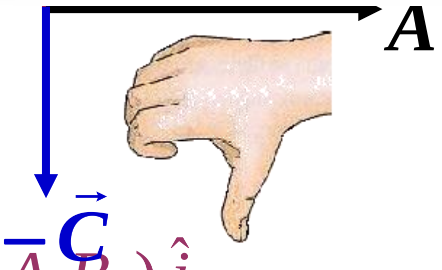
(2) The vector (cross) product

$$\vec{A}\times\vec{B}=\vec{C} \qquad \vec{B}\times\vec{A}=-$$

$$\vec{C}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

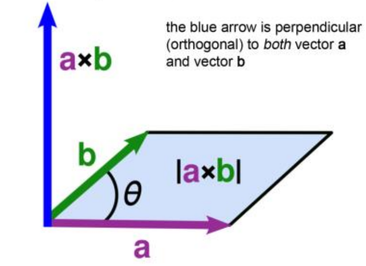
$$\vec{C}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$





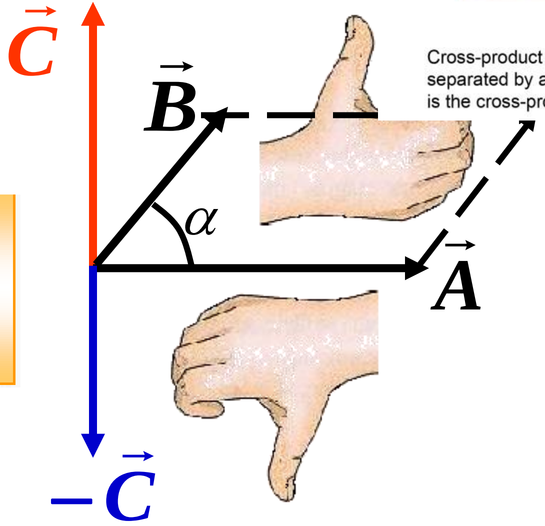
Multiplying a vector by a vector

(2) The vector (cross) product



Cross-product of two vectors a and b separated by angle θ. Vertical vector is the cross-product value & direction

$$\vec{A}\times\vec{B}=\vec{C} \qquad \vec{B}\times\vec{A}=-\vec{C}$$



Mnemonic: $\overset{\longrightarrow + \longrightarrow}{\hat{i}\ \hat{j}\ \hat{k}\ \hat{i}\ \hat{j}\ \hat{k}}$ with $\longleftarrow - \longleftarrow$ below

where

$\hat{i}\times\hat{i}=0$	$\hat{i}\times\hat{j}=\hat{k}$	$\hat{i}\times\hat{k}=-\hat{j}$
$\hat{j}\times\hat{i}=-\hat{k}$	$\hat{j}\times\hat{j}=0$	$\hat{j}\times\hat{k}=\hat{i}$

Multiplying a vector by a vector

(2) The vector (cross) product

Note:

α [illegible]ways less than π

$$\vec{A}\times\vec{A}=0 \quad \text{or} \quad \vec{B}\times\vec{B}=0$$

$$\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

Variation of a vector

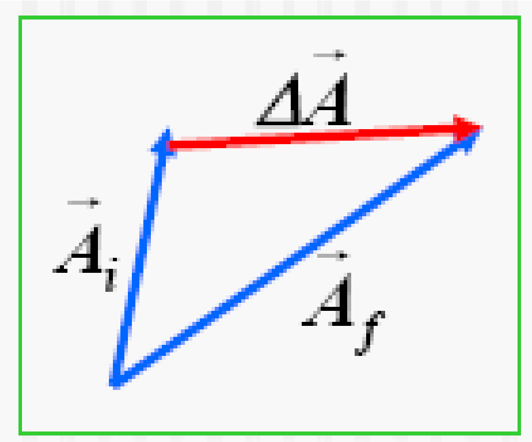
(1).The Magnitude changes, the direction is preserved;

(2).The direction changes, the magnitude is preserved;

(3). Both the magnitude and direction change.

$$\Delta\vec{A} = \vec{A}_f - \vec{A}_i$$

$$\vec{A}_f = \vec{A}_i + \Delta\vec{A}$$



Differentiation of a vector

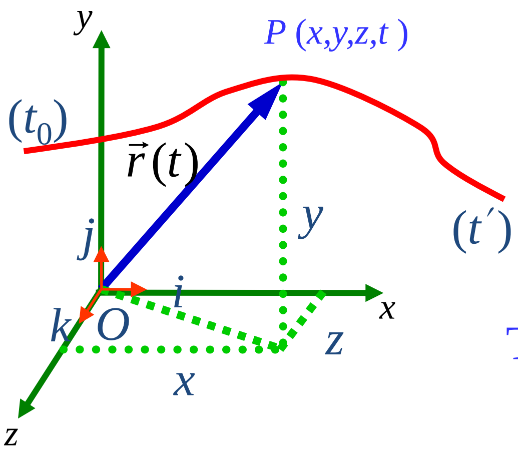
$$\frac{d\vec{A}}{dt} = \frac{d}{dt}(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) = \frac{dA_x}{dt}\hat{i} + \frac{dA_y}{dt}\hat{j} + \frac{dA_z}{dt}\hat{k}$$

$$\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt}$$

$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$$

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

4. Position vector（位矢）——The location of a particle relative to the origin of a coordinate system.



For a Cartesian system:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

Motion function (运动方程):

$$\vec{r} = \vec{r}(t)$$

$$= x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Trajectory equation ——轨道方程:

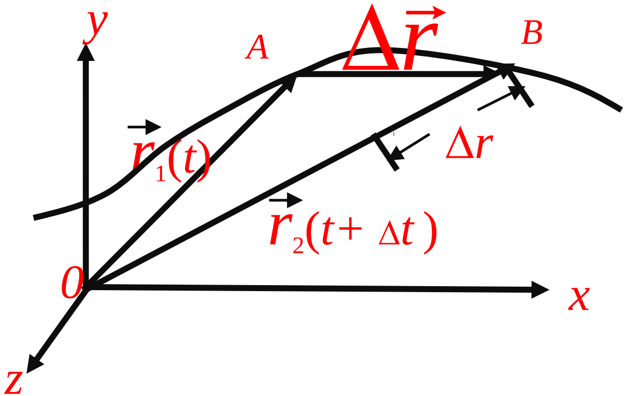
$$f(x, y, z) = 0$$

5. Displacement $\Delta\vec{r}$ (位移矢量)——— A particle is changing in its position(描述质点空间位置变化的矢量):

$$\Delta\vec{r} = \vec{r}_2(t+\Delta t) - \vec{r}_1(t) = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$$



The displacement vector extends from the head of the initial position vector to the head of the later position vector.

Notes about displacement:

(i) Vector —— The magnitude of vector should be the length of this vector, i.e.

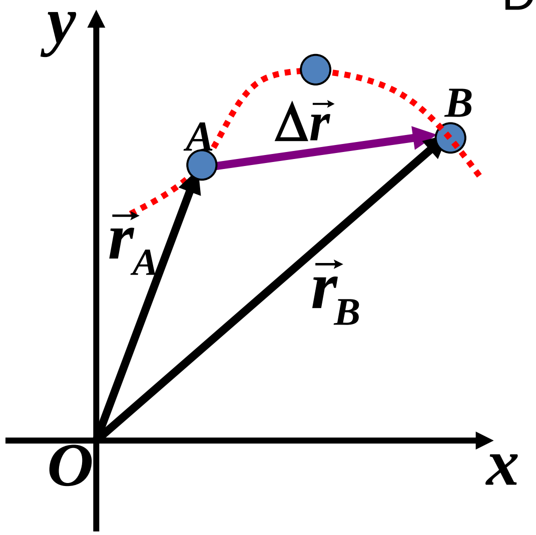
$$|\Delta\vec{r}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(ii) Difference from the path (路程)

Displacement depends only on the position of head to tail, independent with the real path (与具体路径无关)，状态量;

Path is the total lengths of the path

Example: Displacement Vector



Displacement Vector: $\Delta\vec{r}$

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

Caution!

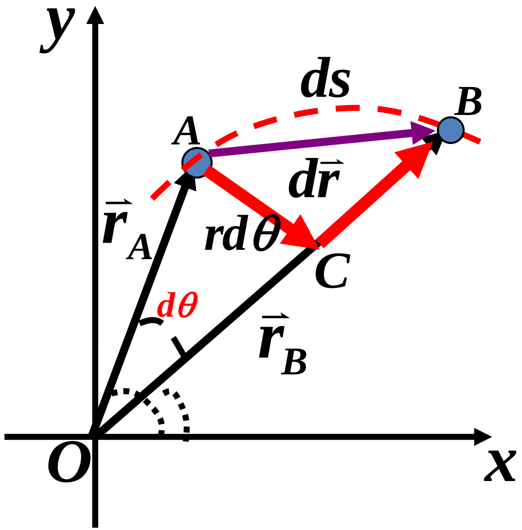
$$|\Delta\vec{r}| = \overline{AB}$$

$$|\Delta\vec{r}| \neq \overset{\frown}{AB}$$

$$\Delta s = \overset{\frown}{AB}$$

Discussion: A very small displacement during a small time interval

A very small displacement: $|d\vec{r}| = \overline{AB}$



A very small distance:

$$ds = \widehat{AB}$$

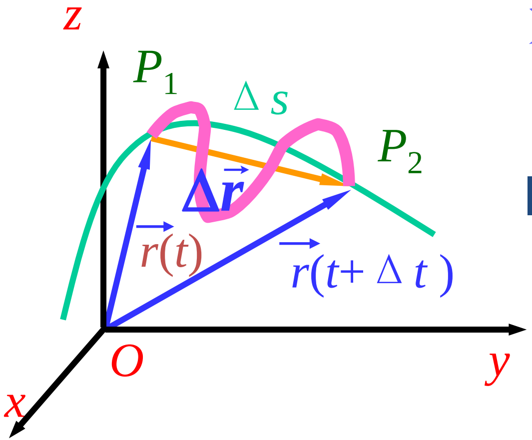
When time interval approaches to 0: $\boxed{|d\vec{r}| = ds}$

Le t $\overline{OA} = \overline{OC}$

$\overline{AC} = rd\theta \quad \overline{CB} = dr$

$$\because \overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$$

Cautio $\boxed{|d\vec{r}| \neq dr}$

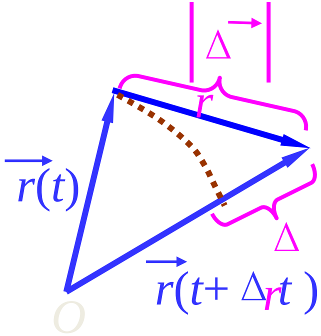


Hey!

$$|\Delta\vec{r}| \neq \Delta s$$

In the limit as $\Delta t \to 0$, $\boxed{|\mathrm{d}\,\vec{r}| = |\mathrm{d}\,s|}$

$$|\Delta\vec{r}| \overset{?}{=} \Delta r$$



(iii) The displacement is independent on the choice of origin.

3-2. Average Velocity and Instantaneous Velocity

1. Average velocity

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\begin{cases} \textbf{Direction is along } \Delta\vec{r} \\ \textbf{Magnitude is } |\bar{\vec{v}}| \end{cases}$$

2. Instantaneous velocity (瞬时速度)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}\right]$$

$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \vec{v}(t)$$

Direction is along tangent line

3. Average and Instantaneous speed (速率)

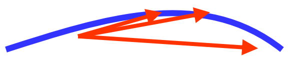
Average speed: $\bar{v} = \dfrac{\Delta s}{\Delta t}$

Instantaneous speed: $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{\mathrm{d}s}{\mathrm{d}t}$

4. The relation between the Instantaneous velocity and instantaneous speed

The magnitude of instantaneous velocity equals to instantaneous speed (瞬时速度的大小等于瞬时速率).

Proof:



$$\because \Delta t \,?\, 0 \quad ds = \lim_{\Delta t \quad 0} \Delta s \quad d\vec{r} \quad \lim_{\Delta t \quad 0} \Delta\vec{r} \quad |d\vec{r}| = ds$$

$$\therefore v = |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta s} \cdot \frac{\Delta s}{\Delta t}$$

$$= \lim_{\Delta t \quad 0} \frac{|\Delta \vec{r}|}{\Delta s} \,?\, \lim_{\Delta t \quad 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{\mathrm{d}s}{\mathrm{d}t}$$

Note:

Speed: $v = \frac{\mathrm{d}s}{\ }$ and $v \neq \frac{\mathrm{d}r}{\ }$

Example 1:

Chose the correct equation:

$$(1)\boldsymbol{v} = \frac{\boldsymbol{d\vec{r}}}{\boldsymbol{dt}}$$

$$(2)\mathbf{v} = \frac{\boldsymbol{d|\vec{r}|}}{\boldsymbol{dt}}$$

$$(3)\vec{v} = \frac{d\vec{r}}{dt}$$

$$(4)\mathbf{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}}$$

Example 2:

Chose the correct equation:

$$(1)\ a = \frac{dv}{dt} \qquad (2)\ a = \frac{d\vec{v}}{dt}$$

$$(3)\ \vec{a} = \frac{d^2\vec{r}}{dt^2} \qquad (4)\ a = \frac{d|\vec{v}|}{dt}$$

Review

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\Delta\vec{r} = \vec{r}_2(t+\Delta t) - \vec{r}_1(t) = \vec{r}_2 - \vec{r}_1$$

$$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$= \Delta x\vec{i} + \Delta y\vec{j} + \Delta z\vec{k}$$

$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}\right]$$

$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k} = \vec{v}(t)$$

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right) = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$$

Example

If we know the position vector of a particle

$$\vec{r} = 2t\hat{i} + (2 - t^2)\hat{j} \qquad \text{(SI)}$$

Find the trajectory of the particle; the position vector at t=0s and t=2s; the velocity and the acceleration of the particle at instant t=2s.

Solution: (1)trajectory

$$\begin{cases} x = 2t \\ y = 2 - t^2 \end{cases}$$

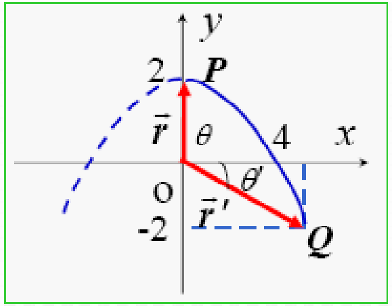
Eliminate t, we can get $y = 2 - \frac{x^2}{4}$ —parabola

(2) position vector:

t = 2s， x = 4 y = -2 $\vec{r}\,' = 4\vec{i} - 2\vec{j}$

t = 0s， x = 0 y= 2 $\vec{r} = 2\vec{j}$

Example



$$y = 2 - \frac{x^2}{4}$$

$$\vec{r} = 2\vec{j}$$

$$\vec{r}\,' = 4\vec{i} - 2\vec{j}$$

The magnitude：

$$r = |\vec{r}| = 2(m)$$

$$r' = |\vec{r}\,'| = \sqrt{4^2 + (-2)^2} = 4.47(m)$$

The direction：

The angle between $\vec{r}$ and x-axis $\qquad \theta = arctg\frac{2}{0} = 90°$

The angle between $\vec{r}\,'$ and x-axis $\qquad \theta' = arctg\frac{-2}{4} = -26°32'$

Example

(3)The velocity: $\vec{r} = 2t\vec{i} + (2 - t^2)\vec{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\vec{i} - 2t\vec{j}$$

Magnitude: $v_x = 2 \quad v_y = -2t$

$$v = \sqrt{v_x^2 + v_y^2} = 2\sqrt{1 + t^2}$$

$$t = 2 \quad v_2 = 2\sqrt{5} = 4.47 m \cdot s^{-1}$$

The angle between velocity and *x*-axis:

$$\alpha = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-4}{2}$$

Example

(3)The acceleration: $\vec{r} = 2t\hat{i} + (2 - t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 2t\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = -2\hat{j}$$

The magnitude: $|\vec{a}| = a = 2$

Direction of the acceleration: $-\ \hat{j}$

Exercise: The position of a small bumper car in an amusement park ride is described as a function of time by the coordinates

$$x = 0.2t^2 + 5.0t + 0.5 \qquad \text{(SI)}$$

$$y = -1.0t^2 + 10.0t + 2.0 \qquad \text{(SI)}$$

Find (a) the position vector at t=1.0 s and t=3.0 s.

(b) the displacement vector between these time.

(c) average velocity over the period from 1.0 s to 3.0 s, and the velocity at t=3.0 s.

(d) the magnitude and direction of the acceleration at t=1.0 s and t=3.0 s.

3-4. Rectilinear motion with a constant acceleration

1. Some rules

(1). ignore the effects of air resistance

(2). the origin of the coordinate could be chosen discretionarily

(3).write the constant acceleration as $\vec{a} = a_x \hat{i}$

(4). choose the initial time instant to be

$$t_i = 0 \quad \Delta t = t_f - t_i = t$$

(5). let $x(t_i) = x(0) = x_0 \quad v_x(t_i) = v_x(0) = v_{x0}$

Rectilinear motion with a constant acceleration

From $$\frac{dv_x(t)}{dt} = a_x$$

We have $$\int_{v_{x0}}^{v_x(t)} dv_x(t) = \int_0^t a_x dt$$

$$v_x(t) = v_{x0} + a_x t$$

Likewise, from $$\frac{dx(t)}{dt} = v_x(t)$$

We have $$\int_{x_0}^{x(t)} dx(t) = \int_0^t v_x dt = \int_0^t (v_{x0} + a_x t) dt$$

$$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

Rectilinear motion with a constant acceleration

$$\begin{cases} a_x = \text{constant} & \text{where} \quad \vec{a} = a_x \hat{i} \\ v_x(t) = v_{x0} + a_x t & \text{where} \quad \vec{v}(t) = v_x(t)\hat{i} \\ x(t) = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 & \text{where} \quad \vec{r}(t) = x(t)\hat{i} \end{cases}$$

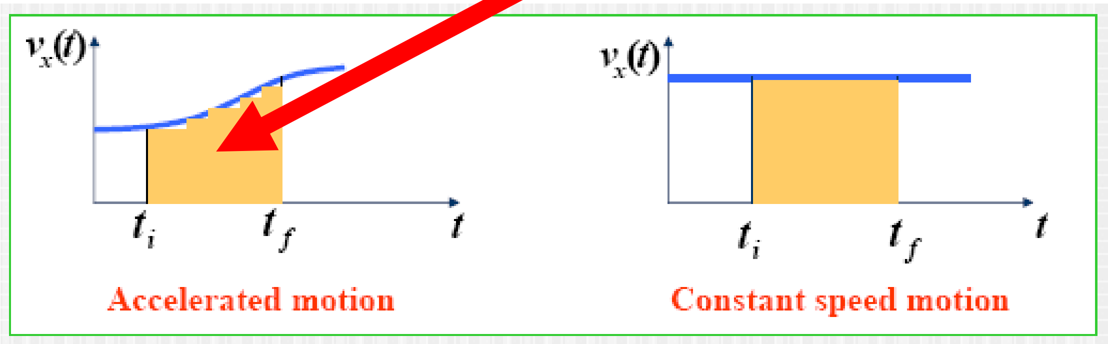
Eliminate t in equations about v_x and x

$$v_x^2 - v_{x0}^2 = 2a_x(x - x_0)$$

3-5. Geometric interpretations

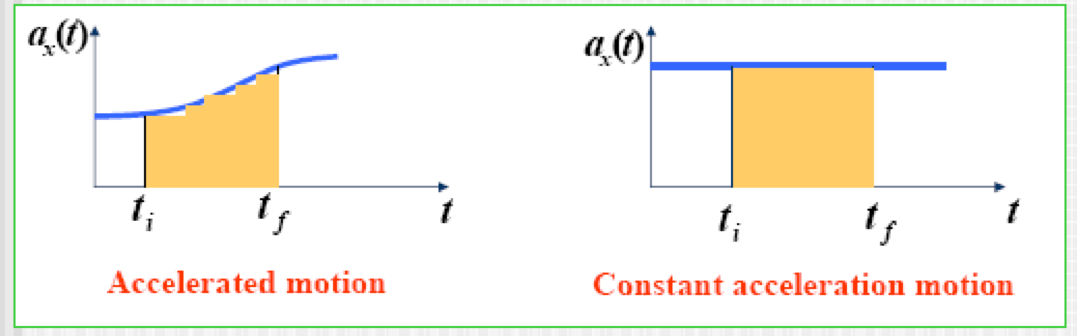
1. The change in the position vector component

$$v_x = \frac{dx}{dt} \quad dx = v_x(t)dt \quad \Delta x = \int_{x(t_i)}^{x(t_f)} dx = \int_{t_i}^{t_f} v_x(t)dt$$

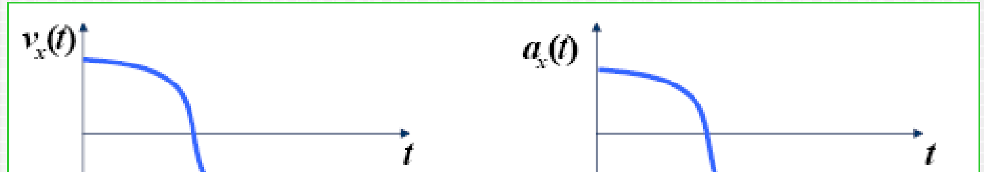


2. The change in the velocity component

$$a_x = \frac{dv_x}{dt} \quad dv_x = a_x dt \quad \Delta v_x = \int_{v_x(t_i)}^{v_x(t_f)} dv_x = \int_{t_i}^{t_f} a_x(t)dt$$



3. What does the negative areas mean?



Example:

A particle moves along x direction, $a = 2+6x^2$. At t =0, x_0=0， v_0=0. What is its $v(x)$?

Solution:

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad ; \quad \mathrm{d}v = (2+6x^2)\mathrm{d}t$$

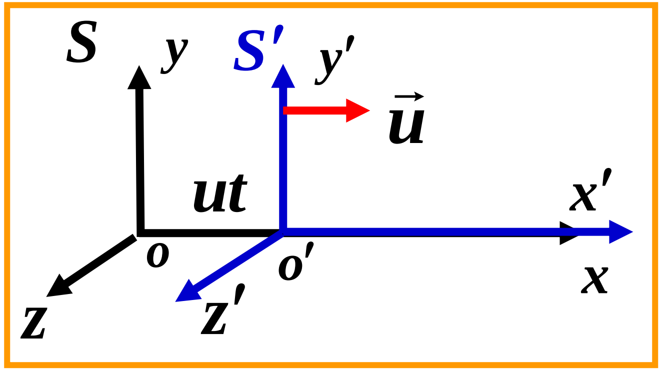
$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

$$a\mathrm{d}x = v\mathrm{d}v$$

$$\int_0^x (2+6x^2)\mathrm{d}x = \int_0^v v\mathrm{d}v$$

$$2(x+x^3) = \frac{1}{2}v^2$$

$$y \, // \, y' \qquad z \, // \, z'$$



$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

$$\begin{cases} v'_x = v_x - u \\ v'_y = v_y \\ v'_z = v_z \end{cases}$$