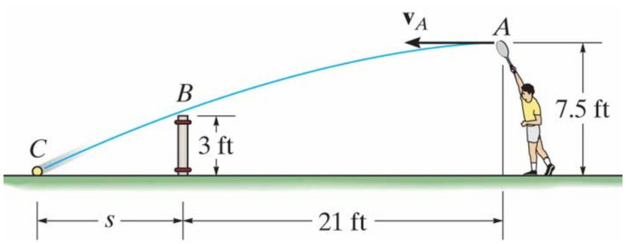
Name and Discussion Section Number ________________________________________

**ENSC 2123** **Exam 1** **Fall 2011**

Question 1 (30 Points): Determine the **horizontal velocity $\boldsymbol{v_A}$** of a tennis ball at $A$ so that it just clears the net at $B$. Also, find the **distance $\boldsymbol{s}$** where the ball strikes the ground.




$$x_0 = 0ft, x_B = 21ft, x_C = 21ft + s, y_0 = 7.5ft, y_B = 3ft, y_C = 0ft$$

y-direction

$$3ft = 7.5ft + 0t + \frac{1}{2}\left(-32.2\frac{ft}{s^2}\right)t^2 \rightarrow t_B = 0.529s$$

x-direction

$$21ft = 0ft + v_A t_B \rightarrow \boldsymbol{v_A} = \frac{\mathbf{21}\boldsymbol{ft}}{\mathbf{0.529}\boldsymbol{s}} = \mathbf{39.72}\frac{\boldsymbol{ft}}{\boldsymbol{s}}$$

y-direction

$$0ft = 7.5ft + 0t + \frac{1}{2}\left(-32.2\frac{ft}{s^2}\right)t^2 \rightarrow t_C = 0.683s$$

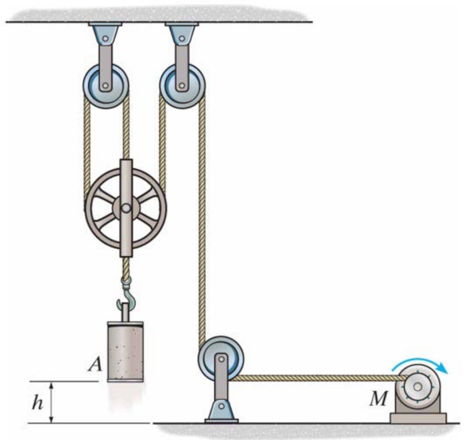
x-direction

$$21ft + s = 0ft + \left(39.72\frac{ft}{s}\right)(0.683s)$$

$$\boldsymbol{s = 6.11ft}$$

Name and Discussion Section Number ___________________________________________

Question 2 (20 points): If the rope is drawn towards the motor $M$ at a speed of $v_M = 5\left(t^{\frac{3}{2}}\right)\frac{m}{s}$, where $t$ is in seconds, determine the **speed of cylinder $\boldsymbol{A}$** when $t = 5s$. Please mark where you set the datum line.




$$3s_A + s_M = l_T$$

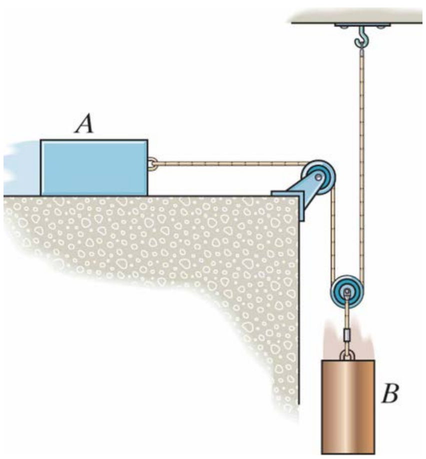
$$3v_A = -v_M = -5\left(t^{\frac{3}{2}}\right)$$

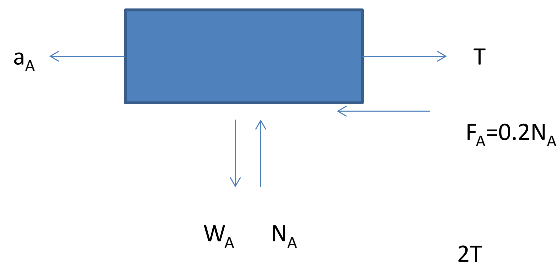
$$v_A(t = 5s) = -\frac{5}{3}\left(5^{\frac{3}{2}}\right) = -18.63\frac{m}{s}$$

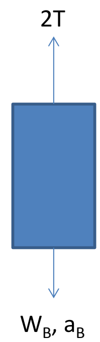
$$\boldsymbol{v_A = 18.63\frac{m}{s}} \uparrow$$

Name and Discussion Section Number ______________________________

Question 3 (30 points): The 10lb block $A$ travels to the right at $v_A = 2\frac{ft}{s}$ at the instant shown. If the coefficient of kinetic friction is $\mu_k = 0.2$ between the surface and $A$, determine the **velocity of $A$** when it has moved 4ft. Block $B$ has a weight of 20lb. Hint: Remember rope lengths!




Block A:

$$N_A = W_A = 10lbf$$

$$T - 0.2(10)lbf = \frac{10lbf}{32.2\frac{ft}{s^2}}(-a_A)$$

Block B:

$$2T - 20lbf = \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

Rope Length:

$$a_A = -2a_B$$

$$T = \frac{10lbf}{32.2\frac{ft}{s^2}}(-a_A) + 2lbf$$

$$= \frac{10lbf}{32.2\frac{ft}{s^2}}(2a_B) + 2lbf$$

$$2\left(\frac{10lbf}{32.2\frac{ft}{s^2}}(2a_B) + 2lbf\right) - 20lbf$$

$$= \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

$$\frac{40lbf}{32.2\frac{ft}{s^2}}a_B + 4lbf - 20lbf = \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

**Without Rope Length ($a_A$) to the right**

Block A:

$$N_A = W_A = 10lbf$$

$$T - 0.2(10)lbf = \frac{10lbf}{32.2\frac{ft}{s^2}}(a_A)$$

Block B:

$$2T - 20lbf = \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

Relate Accelerations

$$a_A = 2a_B$$

$$T = \frac{10lbf}{32.2\frac{ft}{s^2}}(a_A) + 2lbf = \frac{10lbf}{32.2\frac{ft}{s^2}}(2a_B) + 2lbf$$

$$2\left(\frac{10lbf}{32.2\frac{ft}{s^2}}(2a_B) + 2lbf\right) - 20lbf$$

$$= \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

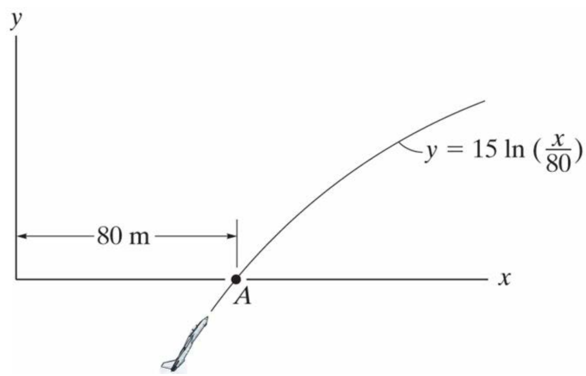
$$\frac{40lbf}{32.2\frac{ft}{s^2}}a_B + 4lbf - 20lbf = \frac{20lbf}{32.2\frac{ft}{s^2}}(-a_B)$$

$$a_B = 8.58\frac{ft}{s^2} \downarrow$$
$$a_A = \frac{17.17ft}{s^2} \rightarrow$$
$$v^2 = \left(2\frac{ft}{s}\right)^2 + 2\left(17.17\frac{ft}{s^2}\right)(4ft) \rightarrow v = \mathbf{11.9\frac{ft}{s}}$$

$$a_B = 8.58\frac{ft}{s^2} \downarrow$$
$$a_A = \frac{17.17ft}{s^2} \rightarrow$$
$$v^2 = \left(2\frac{ft}{s}\right)^2 + 2\left(17.17\frac{ft}{s^2}\right)(4ft) \rightarrow v = \mathbf{11.9\frac{ft}{s}}$$

Name and Discussion Section Number ____________________________________

Question 4 (20 points): The jet plane is traveling with a constant speed of 110m/s along the curved path. Determine the **magnitude of the acceleration** of the plane at the instant it reaches point $A$ $(y = 0)$.




$$v_T = 110\frac{m}{s}, a_T = 0\frac{m}{s^2}$$

$$y = 15\ln\left(\frac{x}{80}\right), \frac{dy}{dx} = \frac{15}{x}, \frac{d^2y}{dx^2} = -\frac{15}{x^2}$$

$$\rho = \left|\frac{\left[1 + \left(\frac{15}{80}\right)^2\right]^{\frac{3}{2}}}{-\frac{15}{80^2}}\right| = 449.4m$$

$$a_n = \frac{\left(110\frac{m}{s}\right)^2}{449.4m} = \mathbf{26.9\frac{m}{s^2}}$$

# Oklahoma State University

## Engineering Science 2123 - Elementary Dynamics

## Equation Sheet

### General Definitions

$$v = \frac{ds}{dt}$$
$$a = \frac{dv}{dt}$$
$$ads = vdv$$

### Cylindrical Coordinates

$$\mathbf{r} = r\,\mathbf{e}_r + z\,\mathbf{e}_z$$
$$\dot{\mathbf{r}} = \dot{r}\,\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta + \dot{z}\,\mathbf{e}_z$$
$$\ddot{\mathbf{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{e}_\theta + \ddot{z}\,\mathbf{e}_z$$

$$\tan\psi = \frac{r}{dr/d\theta}$$ (angle from radial to tangent)

### Constant Acceleration

$$s = s_0 + v_0(t - t_0) + \frac{1}{2}a_c(t - t_0)^2$$
$$v = v_0 + a_c(t - t_0)$$
$$v^2 = v_0^2 + 2a_c(s - s_0)$$

### Relative Motion (Translating Axes)

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

### Kinetics of a Particle

$$\Sigma\mathbf{F} = \frac{d(m\mathbf{v})}{dt}$$

$$\Sigma\mathbf{F} = m\mathbf{a}$$ (constant mass)

### Cartesian Coordinates

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
$$\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$
$$\mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$

### Normal and Tangential Components

$$\mathbf{v} = v(t)\,\mathbf{e}_t$$
$$\mathbf{a} = a(t)\,\mathbf{e}_t + \left(\frac{v^2}{\rho}\right)\mathbf{e}_n$$
$$\frac{1}{\rho} = \left|\frac{y''}{\left[1 + (y')^2\right]^{3/2}}\right| = \left|\frac{r^2 + 2(r')^2 - rr''}{\left[r^2 + (r')^2\right]^{3/2}}\right|$$

$$y = f(x), \qquad y' = \frac{dy}{dx}, \qquad y'' = \frac{d^2y}{dx^2}$$

$$r = g(\theta), \qquad r' = \frac{dr}{d\theta}, \qquad r'' = \frac{d^2r}{d\theta^2}$$