



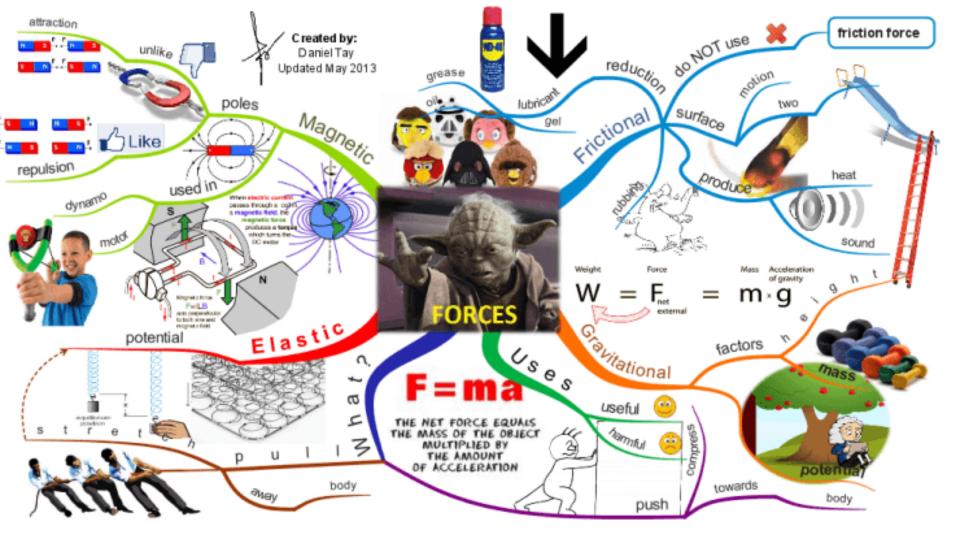
# Physics 1: Mechanics and Waves

Week 2 – Introduction

2023.2

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#### Connections in brains

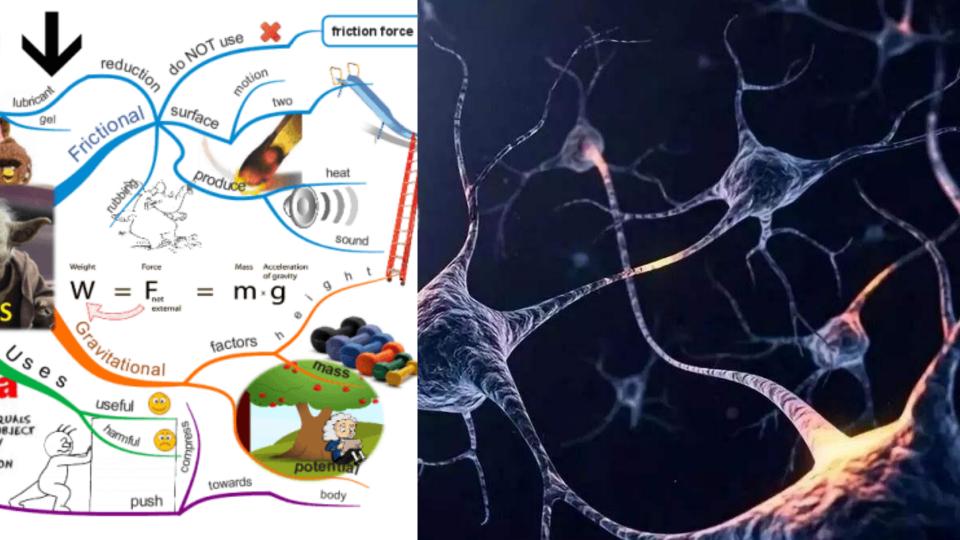
For Knowledge,

If the brain,

Just See or hear Can not be stored permanently

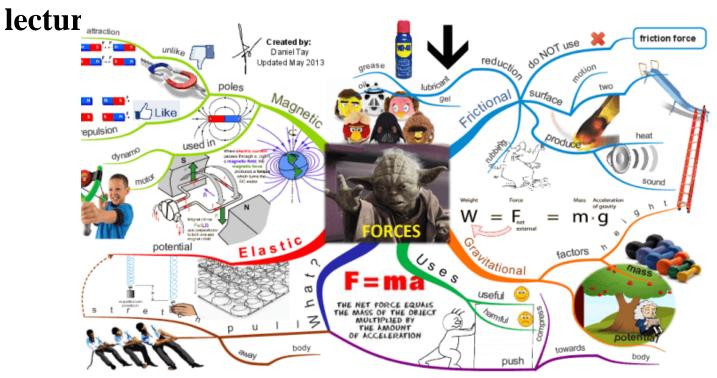
**Build connections** 



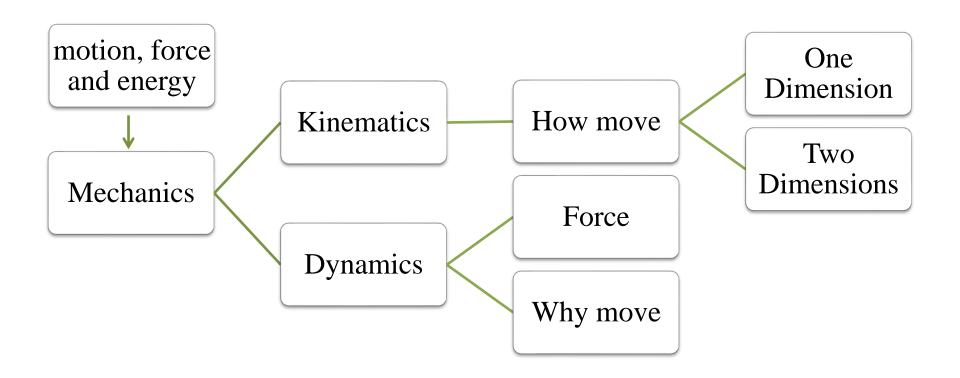


#### Mind map

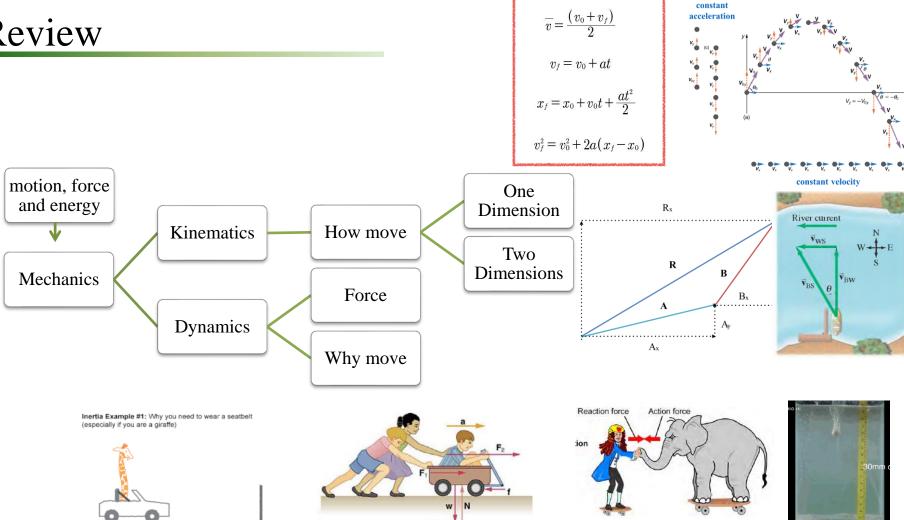
Draw a mind map for what we have learned After our



#### Review



#### Review





#### Physics 1: Mechanics and Waves

Week 8 – Momentum and

the conservation of Momentum

崔雅静 Cathy 2023.4

# What is the physics principle to:

- speed up the rocket?
- get the position of the abandoned part of the rocket?

# Feelings

Do you feel the same, when they are moving towards you?



Rhinoceros: 3000 kg Running at 40 km/h



Dragonfly: 3 g

flying at 40 km/h

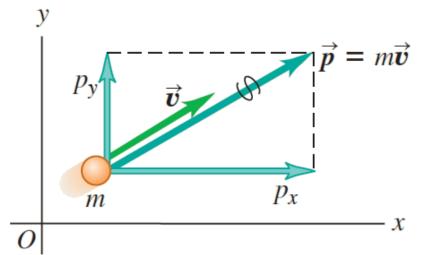


Pistol bullet: 3 g flying at 1200 km/h

#### Momentum

**Momentum of a particle:** The momentum  $\vec{p}$  of a particle is a vector quantity equal to the product of the particle's mass m and velocity  $\vec{v}$ . Newton's second law says that the net force on a particle is equal to the rate of change of the particle's momentum.

$$\vec{p} = m\vec{v}$$



$$\begin{cases} p_x = mv_x \\ p_y = mv_y \\ p_z = mv_z \end{cases}$$

How is the momentum of an object changed?

By the application of a force F!

# The rate of change of momentum

$$\vec{p} = m\vec{v}$$

Differentiate both sides with respect to time

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Newton's second law?

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\stackrel{v \ll c}{\Longrightarrow}$$

$$\vec{F} = m\vec{a}$$

#### Momentum

The total force acting on an object = time rate of change of momentum.

$$\overrightarrow{F}_{net} = \frac{d\overrightarrow{p}}{dt}$$

is more general than

$$\vec{F} = m\vec{a}$$

because it allows for the mass m to change with time also!

Example: rocket motion! The Rain drop

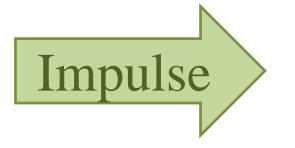
# **Impulse**

$$\vec{F}_{net} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\vec{v})}{\mathrm{d}t}$$

#### **Differential form**

$$\vec{F}_{net} dt = d\vec{p} = d \ (m\vec{v})$$

$$\int_{t_1}^{t_2} \vec{F}_{net} dt = \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1$$



$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt \xrightarrow{\text{Integral form}} \Delta \vec{p}$$

#### Discussion

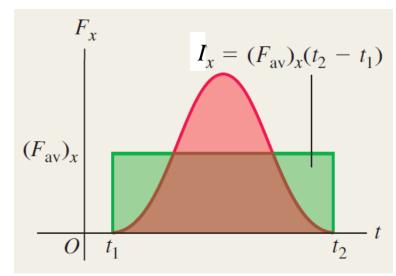
(1) Impulse is a vector, also a process quantity.

#### the impulse - linear momentum theorem

$$|\vec{p}_f - \vec{p}_i| = \Delta \vec{p} = \vec{I}$$

(2) The average force over a time interval

$$\vec{I} = \int_{t}^{t_2} \vec{F} dt = \bar{\vec{F}}(t_2 - t_1) = \bar{\vec{F}} \Delta t$$



#### Impulse-momentum theorem

Net linear impulse delivered to a particle equals the change in linear momentum of the particle  $\vec{I} = \Delta \vec{p} = \vec{p}_{\rm f} - \vec{p}_{\rm i}$ 

#### Descriptions:

- (1) F should be the sum of all external forces. When the force is constant, the direction of I is as same as the force; if F is variable, the direction of I is determined by the integral of  $\int \vec{F} dt$ .
- (2) Both  $\vec{I}$  and  $\vec{P}$  are vectors. They have same units and dimensions. This theorem is only hold for IRF.

#### Impulse-momentum theorem

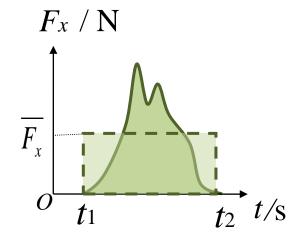
(3) In reality, one often use its component form:

$$\vec{I} = \vec{I}_{x}\vec{i} + \vec{I}_{y}\vec{j}$$

$$\begin{cases}
I_{x} = \int_{t_{i}}^{t_{f}} F_{x} dt = mv_{fx} - mv_{ix} \\
I_{y} = \int_{t_{i}}^{t_{f}} F_{y} dt = mv_{fy} - mv_{iy}
\end{cases}$$

(4) The magnitude of the impulse is equal to the area under the F(t) curve.

$$\overline{F}_x = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} F_x \mathrm{d}t$$



What is the average force on the ball?

Before it stopped, the time the ball is in contact with the racket: 5 ms

$$m = 0.057 \text{ Kg}$$
  $v = 58 \text{ m/s}$ 



Example: Venus Williams' service in 2007: 209 km/h (58 m/s)

$$I = \Delta p = m\Delta v = 0.057 \ kg \times 58 \ m/s = 3.3 \ kg \cdot m/s$$

$$F = \frac{I}{\Delta t} = \frac{3.3}{0.005} = 660 \text{ N}$$

What's the weight of a 60-kg person?

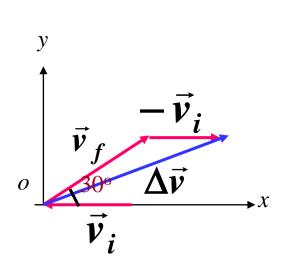
# Less injury



A pitched 140g baseball, in horizontal flight with a speed of 39m/s, is struck by a bat and the ball leaves the bat with a speed of 45m/s, at an upward angle of 30° (a) what impulse act on the ball? (b) If the impact time is 1.2ms, what average force act on the baseball.

(a) 
$$\vec{I} = \vec{p}_{f} - \vec{p}_{i} = m(\vec{v}_{f} - \vec{v}_{i})$$
  
 $\vec{I} = 10.92\vec{i} + 3.15\vec{j} \text{ (kg} \cdot \text{m/s)}$   

$$\begin{cases}
I = \sqrt{I_{x}^{2} + I_{y}^{2}} = 11.4 \text{ (kg} \cdot \text{m/s)} \\
\theta = \tan^{-1} (I_{y}/I_{x}) = 16^{\circ}
\end{cases}$$



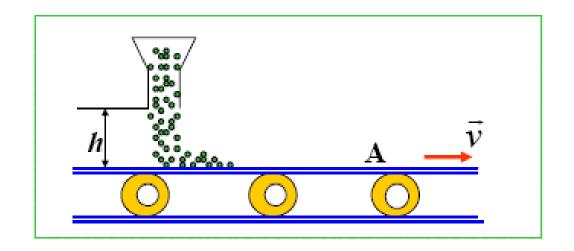
A pitched 140g baseball, in horizontal flight with a speed of 39m/s, is struck by a bat and the ball leaves the bat with a speed of 45m/s, at an upward angle of 30° (a) what impulse act on the ball? (b) If the impact time is 1.2ms, what average force act on the baseball.

(b) : 
$$F_{\text{avg}} \cdot \Delta t = I$$
 :  $F_{\text{avg}} = \frac{I}{\Delta t} = \frac{11.4}{0.0012} = 9500 \, (\text{N})$ 

Discussion: Con

Comparing the  $F_{\text{avg}}$  with the  $F_{\text{ext}}$ .

Coal drops from a stationary hopper of height 2.0 m at rate of 40 kg/s on to conveyer belt moving with a speed of 2.0 m/s, find the average force exerted on the belt by the coal in the process of the transportation. (Ignore the gravity)



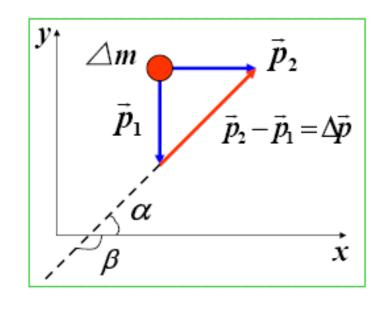
Choose the mass element of the coal as the particle

$$\Delta p_x = p_{2x} - p_{1x} = p_2 = \Delta m v$$

$$\Delta p_y = p_{2y} - p_{1y} = p_1 = \Delta m v'$$

$$= \Delta m \sqrt{2gh}$$

$$v = 2.0 \, m/s$$



According to the impulse-momentum theorem  $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt = \vec{F}_{av} \Delta t = \Delta \vec{p}$ 

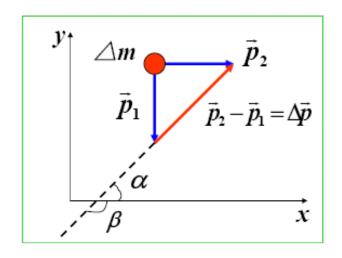
$$F_{xav} = \frac{\Delta p_x}{\Delta t} = \frac{p_2}{\Delta t} = \frac{\Delta mv}{\Delta t_{V'}} = qv = 80 (N)$$

$$F_{yav} = \frac{\Delta p_y}{\Delta t} = \frac{p_1}{\Delta t} = \frac{\Delta m}{\Delta t} = q\sqrt{2gh} = 125.2 (N)$$

$$F_{av} = \sqrt{F_{xav} + F_{yav}} = 149 (N)$$
The angle with respect to the x axis
$$\alpha = \tan^{-1} \frac{F_{yav}}{F_{vav}} = \tan^{-1} \frac{125.2}{80} = 57.4^{\circ}$$

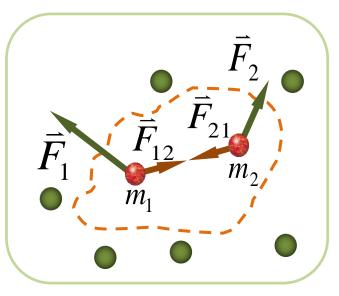
the average force exerted on the belt by the coal

$$F' = 149(N)$$
  
 $\beta = 180^{\circ} - 57.4^{\circ} = 122.6^{\circ}$ 



# Internal and External Force

# **Internal** Objects within the system exert on other objects in the system.



$$\vec{F}_{ext} = \sum_{i} \vec{F}_{ext}^{i}$$
 
$$\vec{F}_{int} = \sum_{i} \vec{F}_{int}^{i} \equiv 0$$

$$\vec{F}_{ext} = \sum_{i=1}^{N} \vec{F}_{ext}^{i} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

For a system of particles, the total momentum changes ...

$$\Delta \vec{p} = \vec{J}_{ext} = \int_{t_1}^{t_2} \vec{F}_{ext} dt$$
 Has nothing to do with internal forces

if 
$$\vec{F}_{ext} = 0$$
, then  $\vec{P} = \sum_{i} \vec{P}_{i} = \text{constant}$ 

When the net external force on a system is zero, the total momentum remains constant

--- the law of conservation of momentum

For a system of particles, the total momentum changes ...

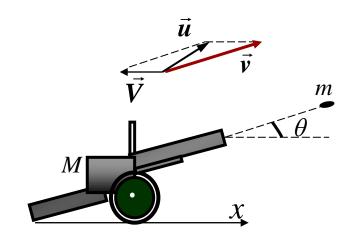
$$\Delta \vec{p} = \vec{J}_{ext} = \int_{t_1}^{t_2} \vec{F}_{ext} dt$$
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if 
$$\vec{F}_{ext} = 0$$
, then  $\vec{P} = \sum_{i} \vec{P}_{i} = \text{constant}$ 

No exceptions to this law have been discovered. So it is considered to be more fundamental than Newton's second law in both macroscopic and microscopic physical worlds.

A gun mounted on wheels of mass M shoots a projectile of mass m with muzzle (出口) velocity of v (对炮车) at an angle  $\theta$  above the horizontal. Find the horizontal recoil (反冲) speed V of the gun (求: 炮车反冲速度V). (neglect friction)

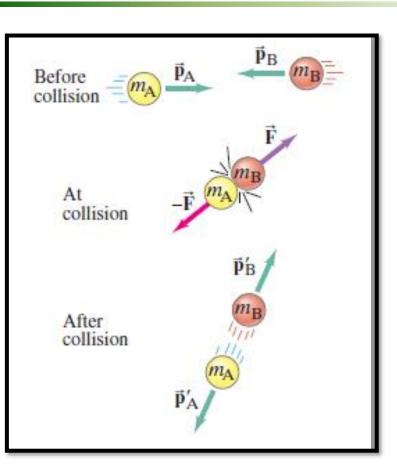
The recoil speed: 
$$V = \frac{m}{m+M} v \cos \theta$$



#### **Explanations**

- (1) Conditions of validity:  $\sum \vec{F}_{\text{ext}} = 0$ , or  $\vec{F}_{\text{ext}} << \vec{F}_{\text{int}}$ Net forces can be neglected in the case of collision, strike, explosion, ..., and so on.
- (2) Conclusion: Only external force can change the state of motion of a system.
- (3) If component of net external force on a system along an axis is zero, the component of *momentum of the system along that axis does not change*.
- (4) It is only suitable for inertial reference frame.

- 20 kg·m/sec Momentum + 20 kg·m/sec No external force 40 kg Momentum 5 kg conserved -0.5 m/sec +4 m/sec Discovered Experimentally Can be derived **BUT ONLY IN 'ISOLATED' SYSTEMS!** 



Before collision 
$$\vec{p}_A$$
  $\vec{p}_A$   $\vec{p}_B$ 

At collision  $\vec{p}_B$ 

After collision  $\vec{p}_A$ 
 $\vec{p}_A$ 

$$\vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}_{\mathrm{B}}}{\Delta t} = \frac{\vec{\mathbf{p}}_{\mathrm{B}}' - \vec{\mathbf{p}}_{\mathrm{B}}}{\Delta t}$$

$$-\vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}_{A}}{\Delta t} = \frac{\vec{\mathbf{p}}_{A}' - \vec{\mathbf{p}}_{A}}{\Delta t}.$$

$$0 = \frac{\Delta \vec{\mathbf{p}}_{\mathrm{B}} + \Delta \vec{\mathbf{p}}_{\mathrm{A}}}{\Delta t} = \frac{\left(\vec{\mathbf{p}}_{\mathrm{B}}' - \vec{\mathbf{p}}_{\mathrm{B}}\right) + \left(\vec{\mathbf{p}}_{\mathrm{A}}' - \vec{\mathbf{p}}_{\mathrm{A}}\right)}{\Delta t}$$

$$\ddot{\mathbf{p}}_{\mathrm{B}}' - \ddot{\mathbf{p}}_{\mathrm{B}} + \ddot{\mathbf{p}}_{\mathrm{A}}' - \ddot{\mathbf{p}}_{\mathrm{A}} = 0,$$

$$\vec{\mathbf{p}}_{\mathrm{A}}' + \vec{\mathbf{p}}_{\mathrm{B}}' = \vec{\mathbf{p}}_{\mathrm{A}} + \vec{\mathbf{p}}_{\mathrm{B}}.$$

# Tell yourself you can

Try to derive on the lecture

# I can

# How can we bounce a ball as high as possible

**IDENTIFY** *the relevant concepts:* Confirm that the vector sum of the external forces acting on the system of particles is zero. If it isn't zero, you can't use conservation of momentum.

#### **SET UP** *the problem* using the following steps:

sketches, including velocity vectors. Assign algebraic symbols to each magnitude, angle, and component. Use letters to label each particle and subscripts 1 and 2 for "before" and "after" quantities. Include any given values such as magnitudes, angles, or components.

1. Treat each body as a particle. Draw "before" and "after"

- 2. Define a coordinate system and show it in your sketches; define the positive direction for each axis.
- 3. Identify the target variables.

#### **EXECUTE** the solution:

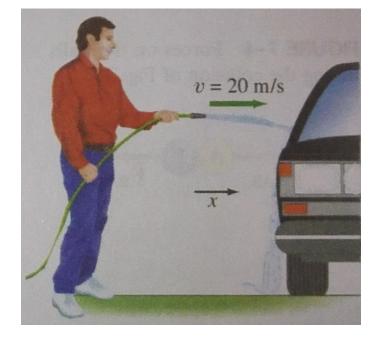
- 1. Write an equation in symbols equating the total initial and final x-components of momentum, using  $p_x = mv_x$  for each particle. Write a corresponding equation for the y-components. Velocity components can be positive or negative, so be careful with signs!
- 2. Solve your equations to find the target variables.

**EVALUATE** *your answer:* Does your answer make physical sense? If your target variable is a certain body's momentum, check that the direction of the momentum is reasonable.

#### Example – washing a car

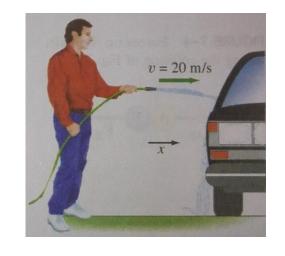
water on the car?

Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it. (We ignore any splashing back.)
What is the force exerted by the



#### Example – washing a car

Water leaves a hose at a rate of  $1.5\ kg/s$  with a speed of  $20\ m/s$  and is aimed at the side of a car, which stops it. (We ignore any splashing back.) What is the **force** exerted by the water on the car?



Force on the car (exerted by the water) = Force on the water (exerted by the car)

$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} \qquad \frac{p_f}{\Delta t} = \frac{\Delta m}{\Delta t} \times v_f = 0$$

$$\frac{p_i}{\Delta t} = \frac{\Delta m}{\Delta t} \times v_i = 1.5 \, kg/s \times 20 \, m/s \qquad F_{car} = 30 \, N$$

#### Reading

The law of conservation of momentum is particularly useful when we are dealing with fairly simple systems such as colliding objects and certain types of "explosions." For example, rocket propulsion, which we saw in Chapter 4 can be understood on the basis of action and reaction, can also be explained on the basis of the conservation of momentum. We can consider the rocket plus its fuel as an isolated system if it is far out in space (no external forces). In the reference frame of the rocket before any fuel is ejected, the total momentum of rocket plus fuel is zero. When the fuel burns, the total momentum remains unchanged: the backward momentum of the expelled gases is just balanced by the forward momentum gained by the rocket itself (see Fig. 7-6). Thus, a rocket can accelerate in empty space. There is no need for the expelled gases to push against the Earth or the air (as is sometimes erroneously thought). Similar examples of (nearly) isolated systems where momentum is conserved are the recoil of a gun when a bullet is fired (Example 7–5), and the movement of a rowboat just after a package is thrown from it.

#### Example – rocket propulsion problem

Mass of rocket: 1000 kg

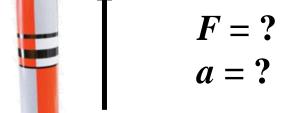
Ejecting gas at a rate of 10 kg/s

The velocity of gas is 3000 m/s

Relative to the rocket

What is the propulsion force on the rocket?

What is its acceleration?



Rate - 10 kg/s

v = 3000 m/s relative to the rocket

# Example – rocket propulsion problem

## Force on gas = Force on rocket

$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} \qquad p_i = \Delta m \times v_i \\ p_f = \Delta m \times v_f$$

$$\Delta p = \Delta m \times (0 - 3000m/s)$$

$$F = \frac{\Delta m \times (-3000m/s)}{\Delta t} = -3 \times 10^4 \, kgm/s^2$$

$$\Delta t = -3 \times 10^{-10} \text{ kg/m/s}$$

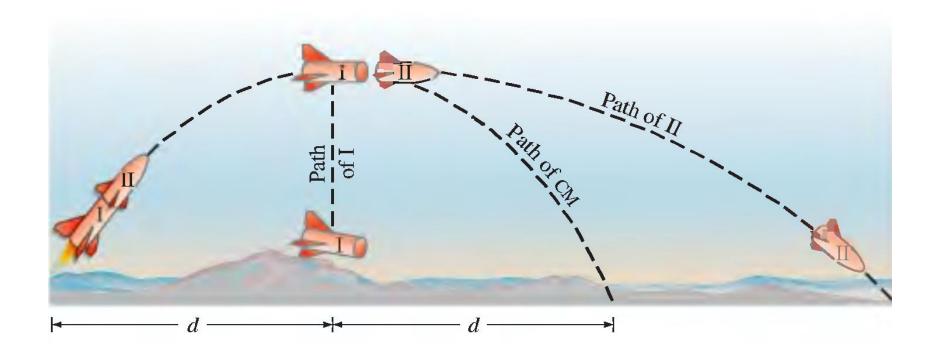
$$F_{propulsion} = 3 \times 10^4 N$$

$$a = \frac{F - Mg}{M} = \frac{3 \times 10^4 - 1000 \times 10}{1000} = 20m/s^2$$



Rate - 10 kg/sv = 3000 m/s relative to the rocket

#### CM and Translational Motion





#### Physics 1: Mechanics and Waves

**Center of mass** 

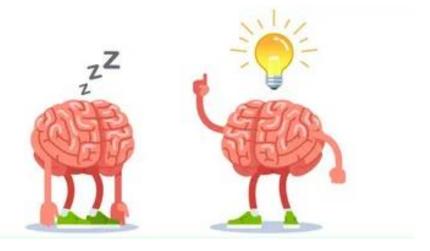
Just have a concept of it

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Cathy

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# Balance a Can?



Fixed mindset

Growth mindset



#### OUTLINE





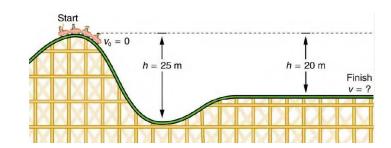


WHAT IS?

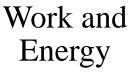
WHY USE? HOW TO USE?

#### How are objects simplified?





Motion of the object









Newton's laws



#### What?

Center of mass

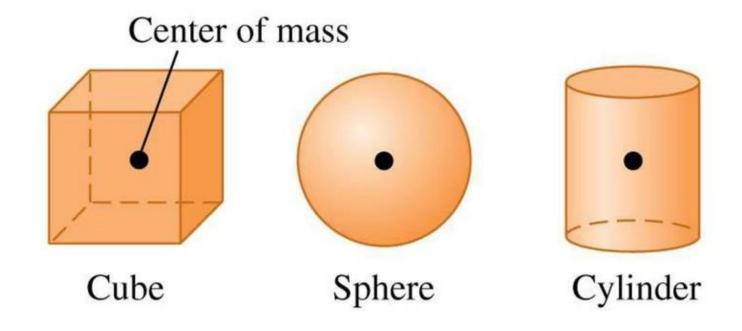
one point that moves in the same path that a **particle** would move



# Center of Mass Trajectory

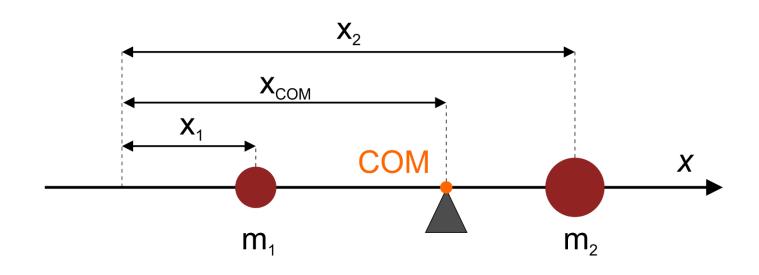
MIT Department of Physics Technical Services Group

#### Center of mass of symmetrical objects (uniform)



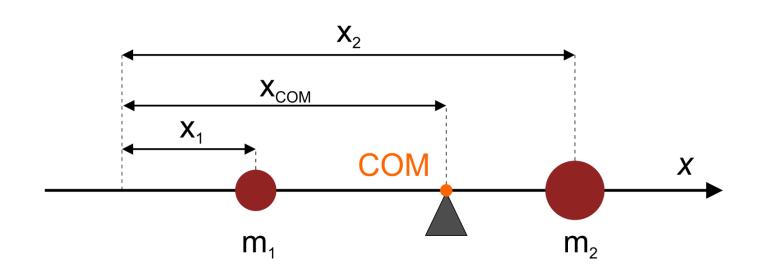


#### Center of mass



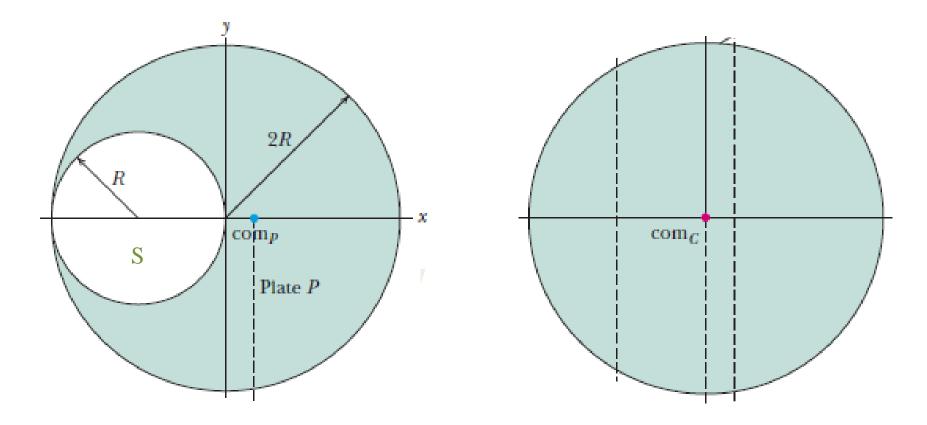
$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

#### Center of mass

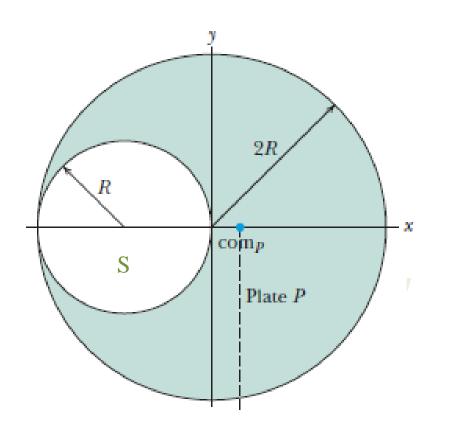


$$x_{CM} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$

### Example of Center of mass



#### Example of Center of mass



$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P}$$

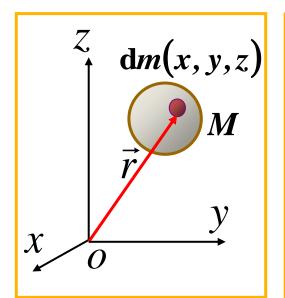
$$x_{S+P} = x_C = 0$$

$$x_P = -x_S \, \frac{m_S}{m_P}$$

$$x_P = \frac{1}{3}R$$

#### The center of mass

For the common objects as essentially continuous distribution of matter

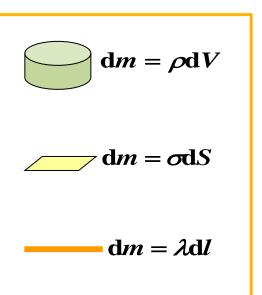


$$\vec{r}_c = \frac{\int \vec{r} \, dm}{M}$$

$$\vec{r}_c = \frac{\int \vec{r} \, dm}{M}$$

$$y_c = \frac{\int y \, dm}{M}$$

$$z_c = \frac{\int z \, dm}{M}$$



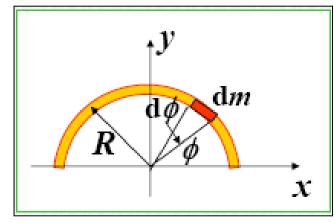
#### Example

A thin strip of material is bent into the shape of a semicircle of radius *R*. Find its center of mass.

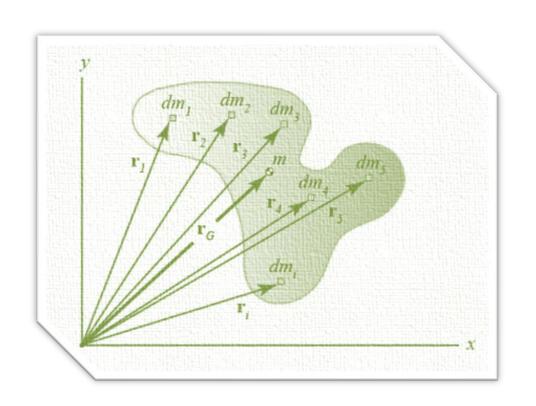
#### Solution:

since the symmetry, we obtained  $x_{CM} = 0$ 

$$y_{CM} = \frac{1}{M} \int y dm = \frac{1}{M} \int_0^{\pi} (R \sin \phi) \frac{M}{\pi} d\phi$$
$$= \frac{R}{\pi} \int_0^{\pi} \sin \phi d\phi = \frac{2R}{\pi} = 0.637R$$



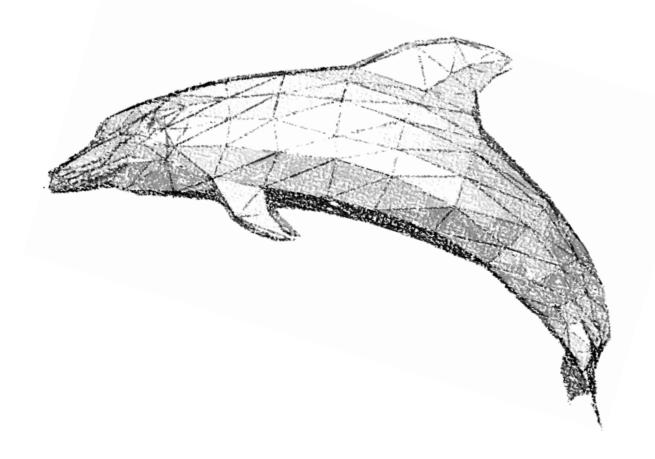
#### Determine Center of Mass



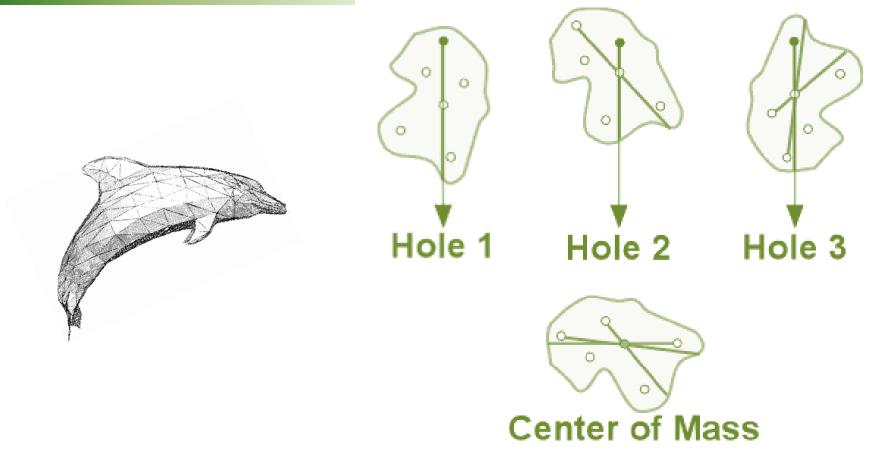
Have to

Calculate?

#### **Extended object?**



#### **Determine Center of Mass**



#### Center of our body



15 cm above the public symphysis

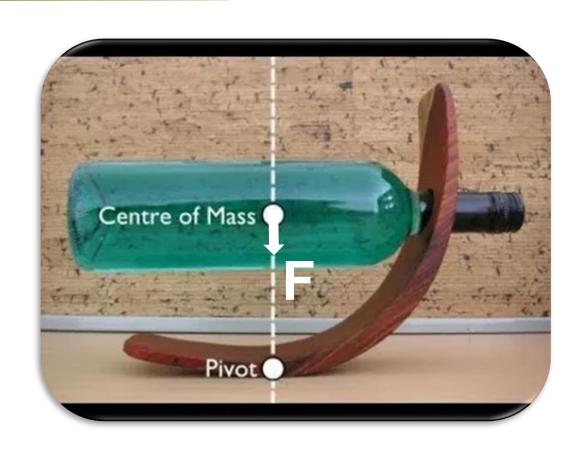
55% of standing height in **female** 

57% of standing height in **male** 

#### Balance



#### Balance



#### Balance







#### More

Experiments of center of mass

https://video.tudou.com/v/XMjc4O
TgyODgyNA==.html?f=50513641

#### Homework:

Any problem of the demonstration about the CM of the Cola Can?



#### N-II Law for the motion of the center of mass

$$\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt} = \frac{\sum_{i=1}^{N} m_i \frac{d\vec{r}_i}{dt}}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{m} \qquad m \vec{v}_{CM} = \sum_i m_i \vec{v}_i = \sum_i \vec{p}_i = \vec{p}$$

$$\vec{F} = \frac{d\vec{P}}{dt} \longrightarrow \vec{F} = m\vec{a}_c$$

The sum of all forces acting on a system = total mass of the system times  $\vec{a}$  of its center of mass.

If a particle system exerted by zero force, the velocity of mass center keep constant.

#### Example

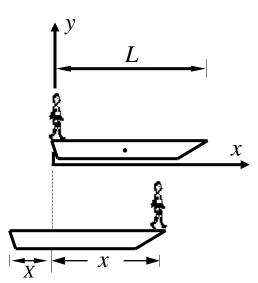
As figure shown, a boat L=4m, M=150kg. It is initially rest on the water. When a men, m = 50kg, walks from the head to the tail of the boat, what are the distances of the men and boat relative to the beach, respectively. neglect the friction from water.

#### Solution 1:

$$\sum F_{ix} = 0$$
  $a_{cx} = 0$   $\rightarrow v_{cx} = 0$   $\rightarrow x_c$  =constant

Initial: 
$$x_c = \frac{m \cdot 0 + M \frac{L}{2}}{m + M}$$
 Final:  $x'_c = \frac{m \cdot x + M(x - \frac{L}{2})}{m + M}$ 

$$x = \frac{ML}{m+M} = 3m \quad ; \qquad X = L - x = 1m$$



#### Example

#### Solution 2:

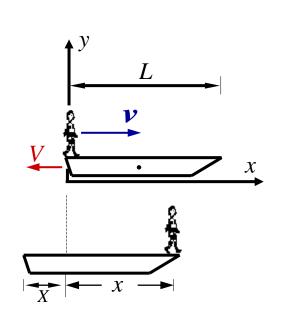
Let Wind de wote the velocity of boat and men relative to the beach at any instant, then

$$mv - MV = 0$$

$$m\int_0^t v dt - M\int_0^t V dt = 0$$

$$mx = MX \quad , \quad x + X = L$$

$$X = \frac{m}{M+m}L = 1m \quad ;$$



$$x = L - X = 3$$
m

#### CM and Translational Motion

$$v = \frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$

$$Mv_{CM} = m_1 x_1 + m_2 x_2 + m_3 x_3$$

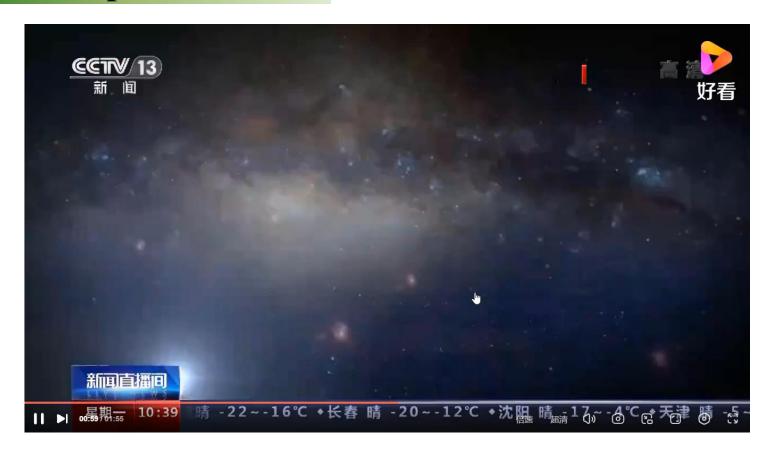
$$a = \frac{dv}{dt}$$

$$Mv_{CM} = m_1 v_1 + m_2 v_2 + m_3 v_3$$

$$Ma_{CM} = m_1 a_1 + m_2 a_2 + m_3 a_3$$

 $Ma_{CM} = F_{net}$ 

#### **Rocket Propulsion**



# **Rocket Propulsion**

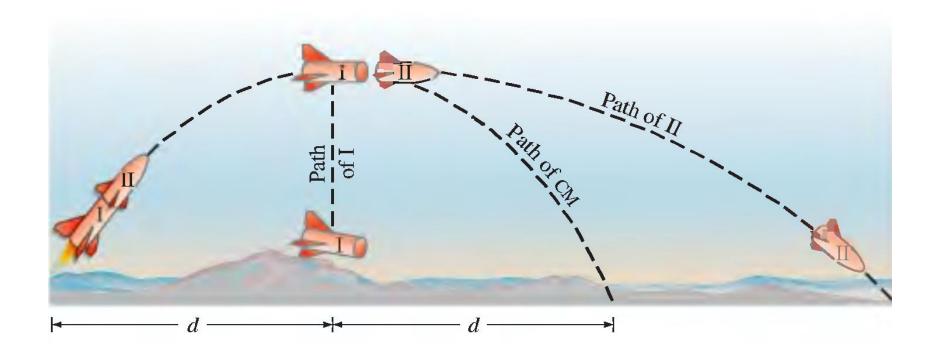


What is the physics principle to:

Speed up the rocket?

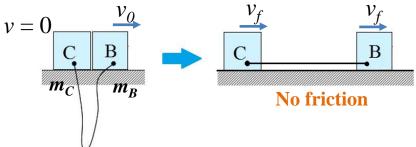
Get the position of the abandoned part of the rocket?

#### CM and Translational Motion



#### Example – Two blocks connected by a rope

Initially, Block C is **at rest**, Block B is moving to the right at the velocity of  $v_0$ , and the rope between them is **loose**.



yes

After the rope becomes tense, Block B and C are moving at the same velocity. What is this velocity  $v_f$ ?

Is the momentum conserved?

What are the initial and final momenta?

$$v_f = \frac{m_B v_0}{m_B + m_0}$$

#### Example – Two blocks connected by a rope

rope becoming tense = effective collision

During collision 
$$\begin{cases} \Delta t \to 0 \\ F_{int} \gg F_{ext} \end{cases} J_{ext} = F_{ext} \Delta t \to 0$$

external forces can be neglected! -- total momentum is conserved

What about if an external force is also very strong, comparable with that large internal force?

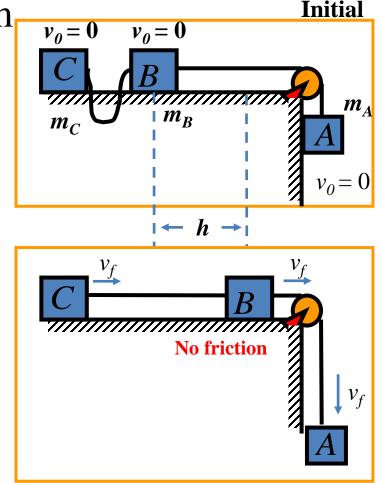
#### Example – Three-blocks problem

Initially, all three blocks are **at rest**. The rope between Block B and C is **loose**.

The rope between Block A and B is **tense**.

Block A start to move down due to the weight.

At the moment the rope between Block B and C becomes tense, all blocks are moving at the same speed. What is this speed  $v_f$ ?



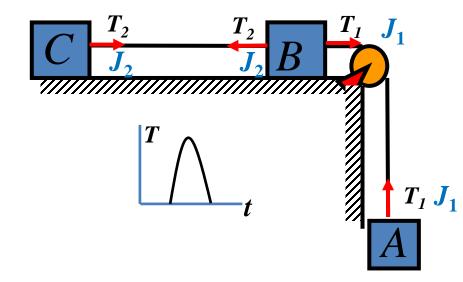
#### Example – Three-blocks problem

$$\Delta p_A = m_A(v_f - v_i) = -J_1$$

$$\Delta p_B = m_B(v_f - v_i) = J_1 - J_2$$

$$\Delta p_C = m_C(v_f - 0) = J_2$$

$$\Delta p_A + \Delta p_B + \Delta p_C = 0$$



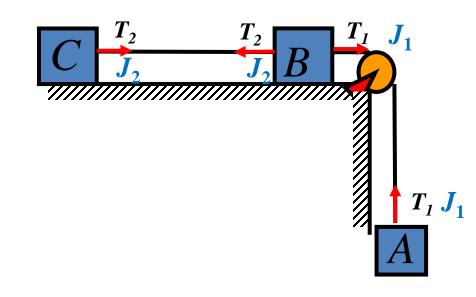
It looks like "conservation of momentum".

But these are only magnitudes, not the momenta themselves!

#### Example – Three-blocks problem

# Final result (dynamic method):

$$v_i = \sqrt{\frac{2m_A gh}{(m_A + m_B)}}$$



#### Summary

Mindmap