Class Test II

1. Calculate $\frac{dy}{dx}$

(1) $y = \frac{\sec 2x}{1+\tan 2x}$

(2) $y = \sin\left(\tan\sqrt{1+x^3}\right)$

(3) $y = \log_5(xe^x)$

(4) $y = \tan\sqrt{1-x}$

(5) $y = e^{-2x}\cos 4x$

(6) $y = \ln\left(x+\sqrt{x^2-1}\right)$

(7) $y = \sin^{-1}\sqrt{\sin x}$

(8) $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$

(9) $\tan^{-1}(x^2y) = x + xy^2$

(10) $\sin(xy) = x^2 - y$

2. (a) If $f(x) = x\sqrt{5-x}$, find $f'(x)$.

(b) Find equations of the tangent lines to the curve $y = x\sqrt{5-x}$ at the points $(1,2)$ and $(4,4)$.

3. Use logarithmic differentiation to find the derivative of the function.

(1) $y = (\ln x)^{\cos x}$

(2) $y = \sqrt{x}e^{x^2-x}(x+1)^{2/3}$

(3) $y = \frac{(x-1)(x-4)}{(x-2)(x-3)}$

(4) $y = \frac{e^{-x}\cos^2 x}{x^2+x+1}$

4. Determine whether $f'(0)$ exists or not.

$$f(x) = \begin{cases} x^2 \sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

5. At what numbers is the following function differentiable?

$$f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \geq 2 \end{cases}$$

6. Find the absolute maximum and absolute minimum values of on the given interval.

(1) $f(x) = \frac{x}{x^2 - x + 1}$, $[0,3]$ (2) $f(x) = 2\cos x + \sin 2x$, $[0, \pi/2]$

7. Use the Mean Value Theorem to prove the in equality

$$|\sin a - \sin b| \leq |a - b|$$