STATS/CSE 780
Assignment 3

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2023-11-09

Introduction

Companies rely on employees to carry out operations and achieve business goals. However, when employees are regularly absent from work, carrying out business can become expensive in terms of both finances and time (Araujo et al., 2019). In this study, we apply clustering methods to understand and predict the underlying reasons behind absenteeism at work. The results will provide companies with direction on how to address these reasons and ultimately reduce rates of employee absenteeism.

Methods

The data in this study was sourced from the UC Irvine Machine Learning Repository (Martiniano & Ferreira, 2018). It contains 21 variables and 740 records of absenteeism at work from 2007 to 2010 at a courier company in Brazil (Martiniano & Ferreira, 2018). The data set includes a categorical variable indicating the reason for absenteeism and various variables describing the employee and their working conditions. The categorical variable for absenteeism combined with quantitative variables makes this a suitable data set for hierarchical and k-means clustering. Since we are only interested in clustering using quantitative variables, all categorical variables apart from the reason for absenteeism were removed from the data set, resulting in a total of 12 variables.

Prior to clustering, we first explored the data to check if data transformation was required. There were no missing values, unexpected data types, or unreasonable data types detected in the data. Of the 12 variables, 8 of them had outliers as shown in the box plots in Supp. Materials Figure 3. These were capped by the lower and upper fences of the interquartile range. We also scaled and centered the data.

Once data transformation was done, we then applied clustering methods to the data. We first performed hierarchical clustering. Silhouette plots for cluster sizes 2 to 20 were created and the average silhouette widths were plotted as shown in the left plot of Figure 1 below. A cluster size of 20 resulted in the maximum average silhouette width, however we decided to use 10 clusters instead of 20 because the average widths are quite similar for a cluster size of 20 versus 10 as shown in Figure 1, and there are many negative silhouette widths (i.e. samples placed into the wrong cluster) for a cluster size of 20 as shown in Supp. Materials Figure 4.
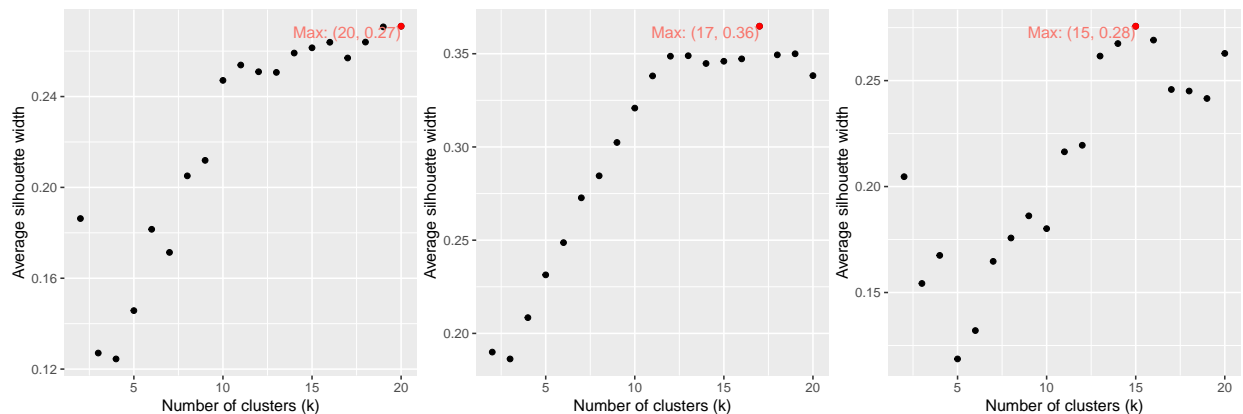
Figure 1: Average silhouette widths for cluster sizes 1 to 20; Left chart is for hierarchical clustering, centre chart is for k-means clustering, and right chart is for PCA followed by hierarchical clustering.

Next, we performed k-means clustering. Similar to the previous method, we created silhouette plots for cluster sizes 2 to 20 and looked for the maximum average silhouette width. In this case, a cluster size of 17 produced the highest average as displayed in the centre plot of Figure 1. We decided to use 17 clusters since other cluster sizes did not show strong improvements in terms of negative silhouette widths as shown in Supp. Materials Figure 5.

Finally, we performed principal component analysis (PCA) followed by hierarchical clustering. 8 principal components (PCs) and 14 clusters were used. Figure 2 shows the scree plot used to select the number of PCs. While the "elbow" of the plot appears to be at approximately 3 PCs, we decided to use 8 PCs instead because the fourth to seventh PCs still explained a large amount of variation (between 5 to 10% each). At the eighth PC, approximately 85% of the variation is explained by all the PCs. To select the cluster size, we created silhouette plots for cluster sizes 2 to 20, as done previously for the other two clustering methods. We selected the cluster size with the maximum average silhouette width as illustrated in the right plot of Figure 1. We ensured that the negative silhouette width was minimal or comparable to other cluster sizes that produced a similar average silhouette width as shown in Supp. Materials Figure 6.
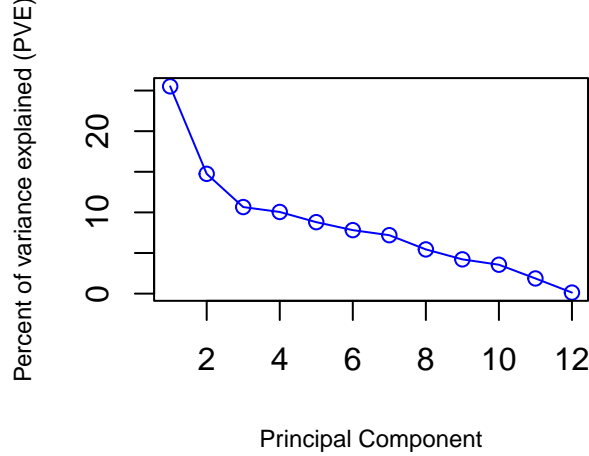
Figure 2: Scree plot

Results

Table 1 shows the results for the three clustering methods using Rand index and adjusted Rand index. All three methods produced a relatively high Rand index between 0.81 and 0.86. K-means clustering had the highest Rand index, followed closely by hierarchical clustering after PCA, and finally hierarchical clustering. Based off the Rand indices alone, it may appear that the clustering models accurately describe and predict absenteeism. However, their adjusted Rand indices indicate that this may not be the case. All three adjusted Rand indices were poor, ranging from 0.09 to 0.18. Hierarchical clustering had the highest adjusted Rand index, followed by hierarchical clustering after PCA, and finally k-means clustering. Overall, hierarchical clustering after PCA resulted in the most balanced results in terms of Rand index and adjusted Rand index, but the poor adjusted Rand index suggests that the clustering results may not be reliable. Further work is required to determine if another method, such as PCA followed by k-means clustering, would better model and predict employee absenteeism.

Table 1: Comparison of clustering methods using Rand index and adjusted Rand index

clustering_method	rand_index	adj_rand_index
hierarchical	0.812	0.176
k-means	0.853	0.092
pca followed by hierarchical	0.847	0.111

4

Supplementary material

Figures

Figure 3: Outliers were detected from the box plots of some variables

n = 740

10 clusters C_j

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

78
308
627
730
219
63
586
514
303
562
223
90
670
120
662
160
564
174
732
425
262

1 : 119 | 0.11
2 : 36 | 0.41
3 : 113 | 0.54
4 : 58 | 0.36
5 : 39 | 0.35
6 : 45 | 0.04
7 : 153 | 0.12
8 : 71 | 0.08
9 : 26 | 0.33
10 : 80 | 0.33

384
500
612
420
443
352
585
336
428
311
714
210
121
85
504
477
146
537
28
300
740

−0.2 0.0 0.2 0.4 0.6 0.8 1.0

Silhouette width s_i

Average silhouette width : 0.25

n = 740

15 clusters C_j

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

1 : 110 | 0.12
2 : 36 | 0.39
3 : 108 | 0.57
4 : 58 | 0.35
5 : 34 | 0.30
6 : 25 | 0.12
7 : 153 | 0.08
8 : 47 | 0.29
9 : 26 | 0.39
10 : 80 | 0.31
11 : 24 | 0.06
12 : 26 | 0.39

−0.2 0.0 0.2 0.4 0.6 0.8 1.0

Silhouette width s_i

Average silhouette width : 0.26

n = 740

20 clusters C_j

$j : n_j \mid \text{ave}_{i \in C_j} s_i$

719
329
373
56
589
339
419
4
446
312
634
496
540
715
677
240
609
89
98
366
740

1 : 110 | 0.06
2 : 36 | 0.39
3 : 108 | 0.54
4 : 16 | 0.30
5 : 42 | 0.48
6 : 25 | 0.09
7 : 25 | 0.09
8 : 91 | 0.12
9 : 47 | 0.27
10 : 23 | 0.41
11 : 80 | 0.29
12 : 62 | 0.13
14 : 16 | 0.64
20 : 9 | 0.38

−0.2 0.0 0.2 0.4 0.6 0.8 1.0
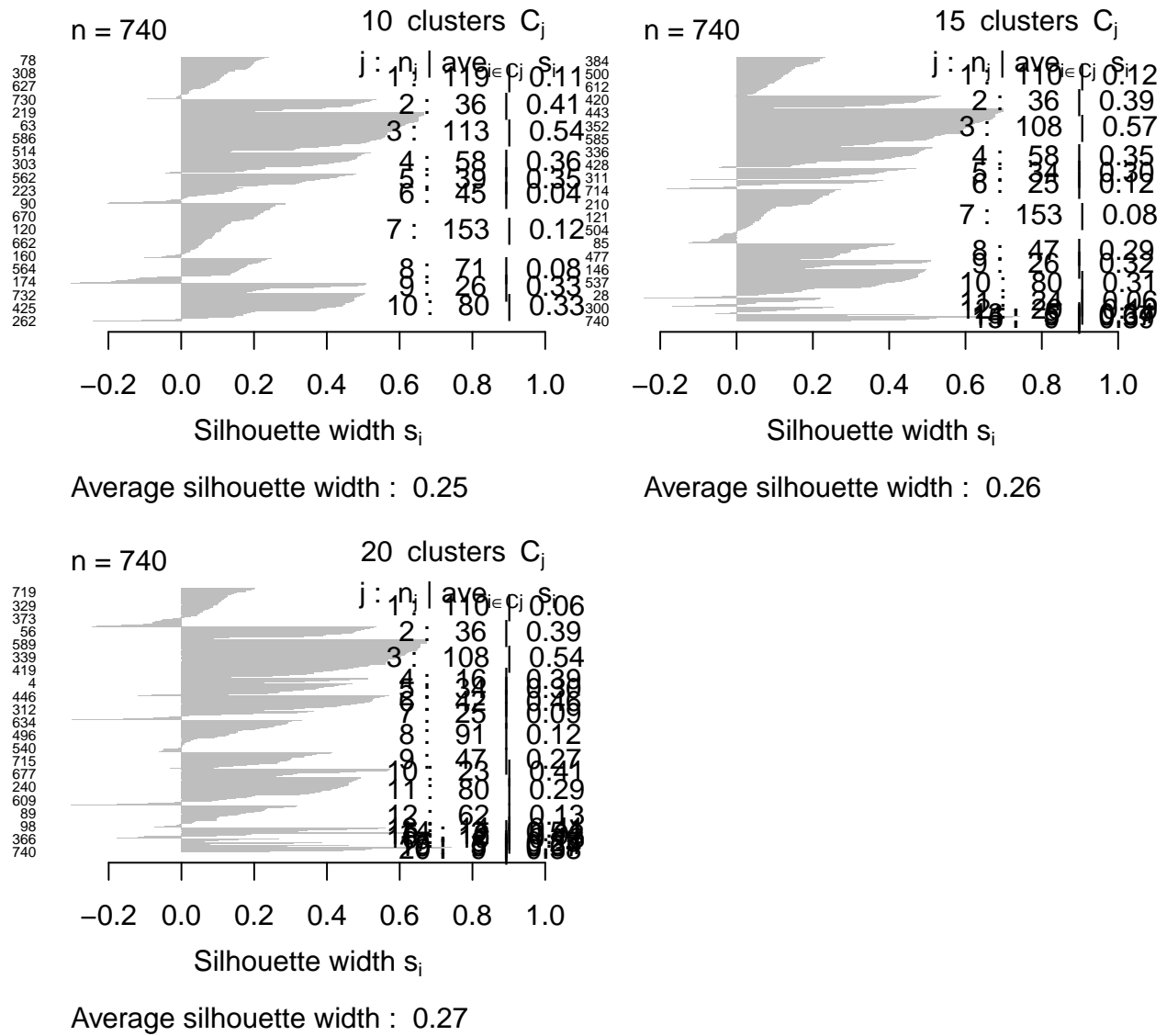
Silhouette width s_i

Average silhouette width : 0.27

Figure 4: Silhouette plots for cluster sizes for hierarchical clustering. k = 20 returned the maximum silhouette width values while k = 10 and 15 are cluster sizes with similar silhouette widths.
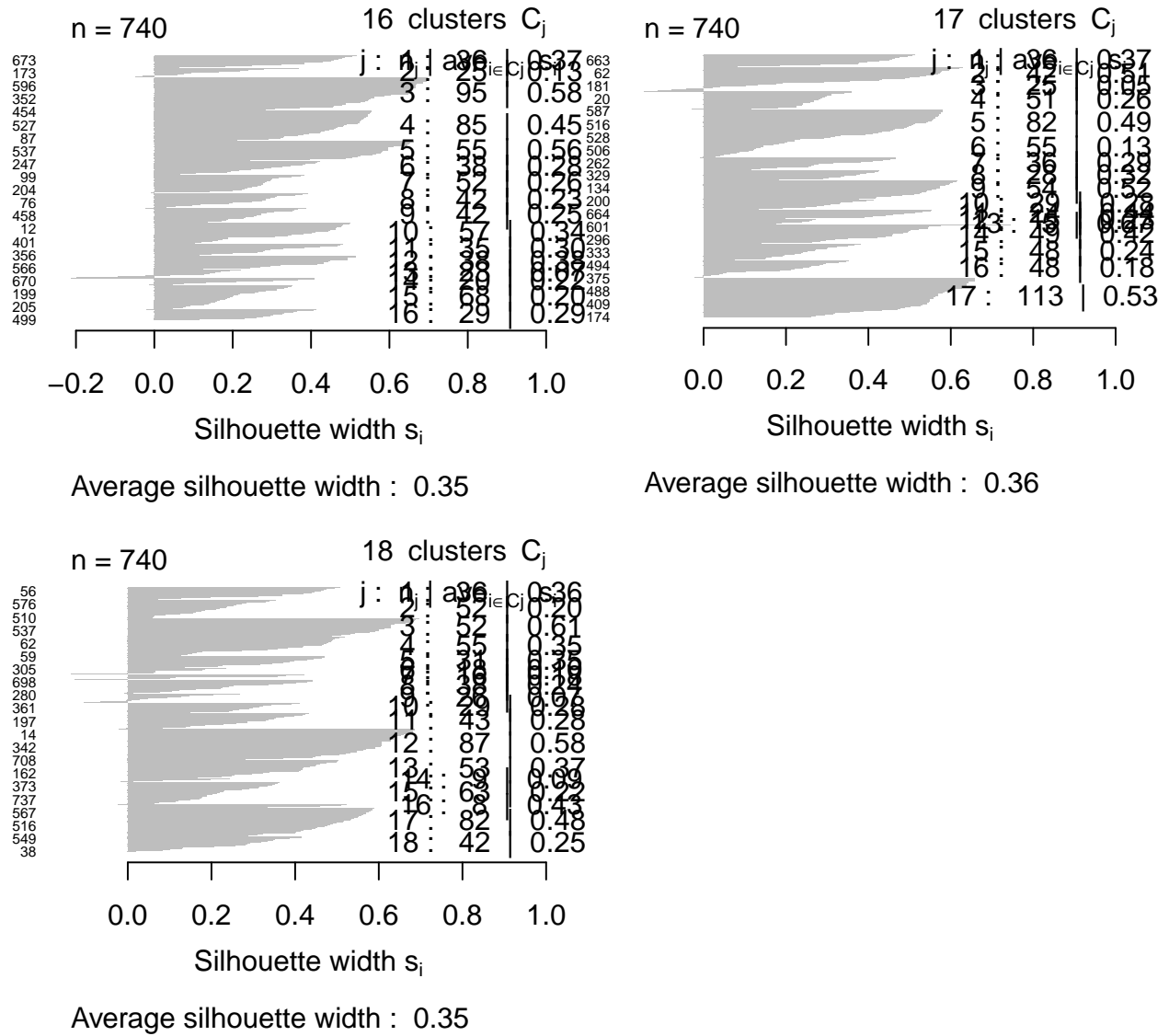
n = 740

16 clusters C_j

673
173
596
352
454
527
87
537
247
99
204
76
458
12
401
356
566
670
199
205
499

j : n_j | $ave_{i \in C_j}$ s_i

1 : 86 | 0.37
2 : 25 | 0.13
3 : 95 | 0.58
4 : 85 | 0.45
5 : 55 | 0.56
6 : 38 | 0.28
7 : 52 | 0.26
8 : 42 | 0.23
9 : 42 | 0.25
10 : 57 | 0.34
11 : 35 | 0.30
12 : 38 | 0.36
13 : 28 | 0.07
14 : 28 | 0.22
15 : 68 | 0.20
16 : 29 | 0.29

663
62
181
20
587
516
528
506
262
329
134
200
664
601
296
333
494
375
488
409
174

−0.2   0.0   0.2   0.4   0.6   0.8   1.0

Silhouette width s_i

Average silhouette width : 0.35

n = 740

17 clusters C_j

j : n_j | $ave_{i \in C_j}$ s_i

1 : 86 | 0.37
2 : 45 | 0.01
3 : 25 | 0.12
4 : 51 | 0.26
5 : 82 | 0.49
6 : 55 | 0.13
7 : 36 | 0.39
8 : 52 | 0.52
9 : 30 | 0.56
10 : 22 | 0.46
11 : 23 | 0.42
12 : 48 | 0.47
13 : 43 | 0.47
14 : 48 | 0.24
15 : 48 | 0.43
16 : 48 | 0.18
17 : 113 | 0.53

0.0   0.2   0.4   0.6   0.8   1.0

Silhouette width s_i

Average silhouette width : 0.36

n = 740

18 clusters C_j

56
576
510
537
62
59
305
698
280
361
197
14
342
708
162
373
737
567
516
549
38

j : n_j | $ave_{i \in C_j}$ s_i

1 : 86 | 0.36
2 : 55 | 0.20
3 : 52 | 0.61
4 : 55 | 0.35
5 : 31 | 0.28
6 : 18 | 0.18
7 : 18 | 0.12
8 : 20 | 0.56
9 : 20 | 0.66
10 : 43 | 0.26
11 : 43 | 0.28
12 : 87 | 0.58
13 : 53 | 0.37
14 : 50 | 0.09
15 : 63 | 0.22
16 : 89 | 0.43
17 : 82 | 0.48
18 : 42 | 0.25

0.0   0.2   0.4   0.6   0.8   1.0

Silhouette width s_i

Average silhouette width : 0.35

Figure 5: Silhouette plots for k-means clustering. k = 17 returned the maximum silhouette width values while k = 16 and 18 are cluster sizes with similar silhouette widths.
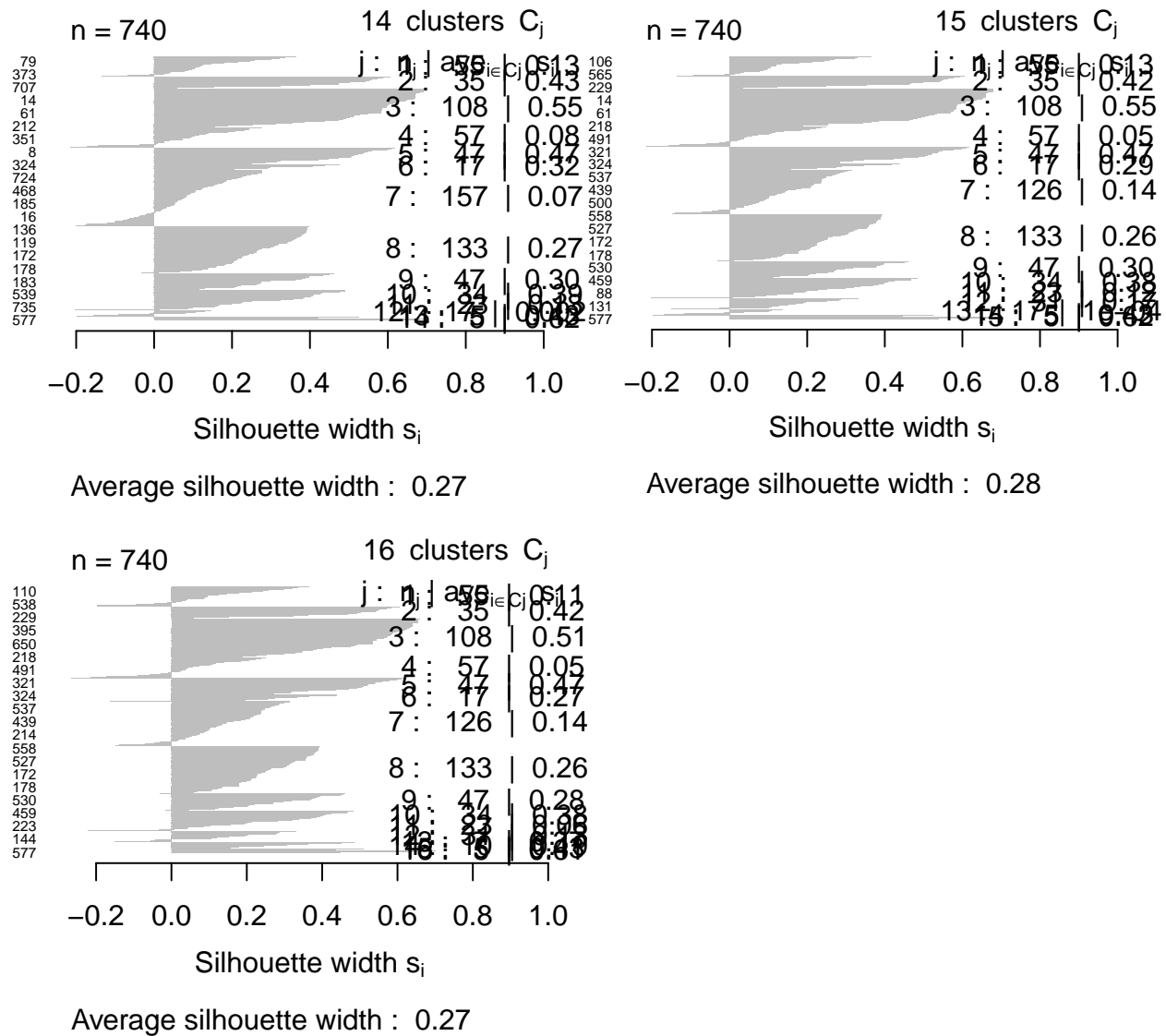
Figure 6: Silhouette plots for hierarchical clustering after PCA. k = 15 returned the maximum silhouette width values while the k = 14 and 16 are cluster sizes with similar silhouette widths.

Code

```
# ----- SETUP ----- #
packages <- c("knitr", "tidyverse", "ggplot2", "cluster", "fossil")
lapply(packages, library, character.only = TRUE)
# Read data, extract labels, and keep only quantitative data
absentData_full <- read.csv("Absenteeism_at_work.csv", sep = ";")
```

```r
absentData_lab <- absentData_full$`Reason.for.absence`
absentData_notclean <- absentData_full %>%
  select(-c("Reason.for.absence","ID","Month.of.absence","Day.of.the.week","Seasons",
            "Disciplinary.failure","Education","Social.drinker","Social.smoker"))
# ----- DATA EXPLORATION ----- #
# Check data types, min, max, and missing data
data_type <- sapply(absentData_notclean,class)
min <- sapply(absentData_notclean, function(col){min(col,na.rm=TRUE)})
max <- sapply(absentData_notclean, function(col){max(col,na.rm=TRUE)})
nulls <- sapply(absentData_notclean, function(col){sum(is.na(col))})
blanks <- sapply(absentData_notclean,
                 function(col){ifelse(is.na(sum(col == "")), 0, sum(col == ""))})
data.frame(row.names = names(nulls), data_type=data_type, min=min, max=max,
           nulls_blanks=nulls+blanks)
# Create box plots to check for outliers
b01 <- boxplot(absentData_notclean$Transportation.expense, ylab = "Transportation expense")
b02 <- boxplot(absentData_notclean$Distance.from.Residence.to.Work,
               ylab = "Distance from residence to work")
b03 <- boxplot(absentData_notclean$Service.time, ylab = "Service time")
b04 <- boxplot(absentData_notclean$Age, ylab = "Age")
b05 <- boxplot(absentData_notclean$Work.load.Average.day,
               ylab = "Work load average per day")
b06 <- boxplot(absentData_notclean$Hit.target, ylab = "Hit target")
b07 <- boxplot(absentData_notclean$Son, ylab = "Son")
b08 <- boxplot(absentData_notclean$Pet, ylab = "Pet")
b09 <- boxplot(absentData_notclean$Weight, ylab = "Weight")
b10 <- boxplot(absentData_notclean$Height, ylab = "Height")
b11 <- boxplot(absentData_notclean$Body.mass.index, ylab = "Body mass index")
b12 <- boxplot(absentData_notclean$Absenteeism.time.in.hours,
               ylab = "Absenteeism time in hours")
# ----- DATA CLEANSING -----
```

```r
# Handle outliers by capping them using interquartile range
cap <- function(val, bplot) {
  lower_fence <- bplot$stats[2]-(1.5*(bplot$stats[4]-bplot$stats[2])) #Q1-1.5*IQR
  upper_fence <- bplot$stats[4]+(1.5*(bplot$stats[4]-bplot$stats[2])) #Q3+1.5*IQR
  val <- ifelse(val < lower_fence, lower_fence, val)
  val <- ifelse(val > upper_fence, upper_fence, val)
  val
}
absentData <- absentData_notclean %>%
  mutate(Transportation.expense = cap(val=Transportation.expense, bplot=b01),
         Service.time = cap(val=Service.time, bplot=b03),
         Age = cap(val=Age, bplot=b04),
         Work.load.Average.day = cap(val=Work.load.Average.day, bplot=b05),
         Hit.target = cap(val=Hit.target, bplot=b06),
         Pet = cap(val=Pet, bplot=b08),
         Height = cap(val=Height, bplot=b10),
         Absenteeism.time.in.hours = cap(val=Absenteeism.time.in.hours, bplot=b12))
# Scale the data
absentData_sd <- scale(absentData)
# ----- CLUSTERING FUNCTIONS ----- #
# Get silhouette for k-means clustering
kmcSilK <- function(k, data){
  x_k <- kmeans(data, k, nstart = 20)
  silhouette(x_k$cluster, dist(data))
}
# Get silhouette for hierarchical clustering
hcSilK <- function(k, data, method = "complete"){
  hc_out <- hclust(dist(data), method = method)
  hc_clusters <- cutree(hc_out, k)
  silhouette(hc_clusters, dist(data))
}
```

```r
# Plot silhouette
plotSil <- function(sil){
  plot(sil, nmax= 800, cex.names=0.5, main = "", border=NA)
}
# Choose k using goodness-of-clustering
# k = the k values to test
# silFun = the silhouette function
# data = the data used in silFun
chooseK <- function(k, silFun, data) {
  # Get silhouettes and their widths
  sil_k <- lapply(k, silFun, data=data)
  sil_score <- sapply(sil_k, function(x) {mean(x[,"sil_width"])})
  # Find the k with the max width
  sil_max <- max(sil_score)
  sil_max_k <- match(sil_max, sil_score)+min(k)-1
  # Plot the silhouette widths and label the maximum
  silData <- tibble(k, sil_score)
  max_point <-tibble(k=sil_max_k,sil_score=sil_max)
  max_lab <- paste0("Max: (",sil_max_k,", ",round(sil_max,2), ")")
  plot <- ggplot(silData, aes(x=k, y=sil_score)) + geom_point() +
    labs(x="Number of clusters (k)", y="Average silhouette width") +
    geom_point(data=max_point, colour="red") +
    geom_text(data=max_point, aes(label=ifelse(k==sil_max_k,max_lab,""),color="red"),
              hjust=1,vjust=1) +
    theme(legend.position="none")
  # Return plot, silhouettes, and max k
  list(plot=plot, sil_k=sil_k, max_k=sil_max_k)
}
# ----- AGGLOMERATIVE HIERARCHICAL CLUSTERING ----- #
# Get silhouette scores for multiple k values and plot them
set.seed(3)
```

```r
k <- c(2:20)
hc_good_of_cluster <- chooseK(k, hcSilK, absentData_sd) # uses complete linkage
hc_good_of_cluster$plot
plotSil(hc_good_of_cluster$sil_k[[9]])
plotSil(hc_good_of_cluster$sil_k[[14]])
plotSil(hc_good_of_cluster$sil_k[[19]])
k <- 10
# Perform hierarchical clustering using best k
set.seed(3)
hc_out <- hclust(dist(absentData_sd))
ri_hc <- rand.index(cutree(hc_out, k), as.numeric(as.factor(absentData_lab)))
ari_hc <- adj.rand.index(cutree(hc_out, k), as.numeric(as.factor(absentData_lab)))
# ----- K-MEANS CLUSTERING ----- #
# Get silhouette scores for multiple k values and plot them
set.seed(3)
k <- c(2:20)
kmc_good_of_cluster <- chooseK(k, kmcSilK, absentData_sd)
kmc_good_of_cluster$plot
# Based off silhouette plots, choose the best k
plotSil(kmc_good_of_cluster$sil_k[[15]])
plotSil(kmc_good_of_cluster$sil_k[[16]])
plotSil(kmc_good_of_cluster$sil_k[[17]])
k <- kmc_good_of_cluster$max_k
# Perform k-means clustering with best k and compute the rand indices
set.seed(3)
km_out <- kmeans(absentData, k, nstart = 20)
km_clusters <- km_out$cluster
ri_kmc <- rand.index(km_clusters, as.numeric(as.factor(absentData_lab)))
ari_kmc <- adj.rand.index(km_clusters, as.numeric(as.factor(absentData_lab)))
# ----- HIERARCHICAL CLUSTERING AFTER PCA ----- #
# Proportion of variance explained
```

```r
set.seed(3)
pr_out <- prcomp(absentData, scale = TRUE)
# Scree plot
pve <- 100 * pr_out$sdev^2 / sum(pr_out$sdev^2)
plot(pve, type = "o", cex.lab=0.75,
  xlab = "Principal Component", col = "blue",
  ylab = "Percent of variance explained (PVE)")
# Get silhouette scores for multiple k values and plot them
set.seed(3)
k <- c(2:20)
pcahc_good_of_cluster <- chooseK(k, hcSilK, pr_out$x[, 1:8])
pcahc_good_of_cluster$plot
# Based off silhouette plots, choose the best k
plotSil(pcahc_good_of_cluster$sil_k[[13]])
plotSil(pcahc_good_of_cluster$sil_k[[14]])
plotSil(pcahc_good_of_cluster$sil_k[[15]])
k <- pcahc_good_of_cluster$max_k
set.seed(3)
hc_out <- hclust(dist(dist(pr_out$x[, 1:8])))
hc_clusters <- cutree(hc_out, k)
ri_pcahc <- rand.index(hc_clusters, as.numeric(as.factor(absentData_lab)))
ari_pcahc <- adj.rand.index(hc_clusters, as.numeric(as.factor(absentData_lab)))
# ----- COMPARISON ----- #
clustering_method <- c("hierarchical", "k-means", "pca followed by hierarchical")
rand_index <- round(c(ri_hc, ri_kmc, ri_pcahc),3)
adj_rand_index <- round(c(ari_hc, ari_kmc, ari_pcahc),3)
comparison <- data.frame(clustering_method = clustering_method,
                    rand_index=rand_index, adj_rand_index=adj_rand_index)
kable(comparison)
```

References

Araujo, V. S., Rezende, T. S., Guimarães, A. J., Araujo, V. J. S., & Campos Souza, P. V. de. (2019). A hybrid approach of intelligent systems to help predict absenteeism at work in companies. In *SN applied sciences.* Springer. https://doi.org/10.1007/s42452-019-0536-y

Martiniano, A., & Ferreira, R. (2018). *Absenteeism at work.* UC Irvine Machine Learning Repository. https://doi.org/10.24432/C5X882

R Core Team. (2023). *R: A language and environment for statistical computing.* R Foundation for Statistical Computing. https://www.R-project.org/