Topological Superconductivity

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I. INTRODUCTION

One of the earliest breakthrews allowed by quantum mechanics is the descritpion of metals and insulators with band theory [2] which saw light in 1930. This theory brought a microscopic understanding of the disctinction between these two classes of materials and led to incredible tecnological advances such as the discovery of the transistor by John Bardeen and Walter Brattain in 1947 [3]. Beyond its increadible success, band theory rapidly came in contact with a myriad of intriguing quantum mechanical effects such as the integer quantum Hall effect discovered in 1980 [4]. In 1982, Thouless et al. [5] figured out the topological nature of the effect and, in term, brought topology closer to band theory. Altough the integer quantum Hall effect (QH) requires a strong external magnetic field, it was theorised in 2005 by Charles Kane and Eugene Mele [6] that similar topological properties could be intrinsicaly realised through the quantum spin Hall effect (QSH) [1]. Experiment then showed in 2007 [7] that HgTe/CdTe quantum wells (mercury telluride heterostructure) could produce a QSH effect. The theory and experiment of QSH effect led to a deeper classification of solids with topological band theory [8]. When applied to insulators, the upgraded band theory creates a seperation between the trivial and the *topological insulators* (TI). The latter is generally caracterised by a metallic boundary and an insulating bulk [9] as opposed to trivial insulators which are insulating everywhere. The present review will focus on basic properties of time reversal symetric topological insulators based on the mercury telluride example. Sec.II presents an overview of important ideas from topological band theory. In sec.III, the main properties of the QSH state are given and compared to the QH effect. Finally, a model of HgTe/CdTe quantum is studied in sec. IV.

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II. ELEMENT OF TOPOLOGICAL BAND THEORY

This sections aims to describe the notion of topological invariant and its consequence on the band structure.

A. Topological equivalence of insulators

In a periodic lattice potential, electrons are described by bloch states $|n, \mathbf{k}\rangle$ where n is a dicrete quantum number and $\mathbf{k}$ is the crystal momentum in the brillouin zone [10]. Each of those states is associated to an energy $E_{n,\mathbf{k}}$. As it varies with $\mathbf{k}$, the energy sweeps a continous range called a band labeled with the number n. Bands are often separated by energy gaps where there are no associated states. Trivial insulators have a gapped ground state meaning that low energy excitations are forbidden by the presence of the gap. On the contrary, topological insulators have metallic (gapless edge states)[11] and different from trivial insulators in a fundamental way.

Slow modifications of the hamiltonian of a trivial insulator will change its band structure while leaving it in its ground state. The *deformation* is said to yield equivalent insulators if the gap doesn't close [11]. This equvalence is topological in the same way the continuous disformation from a torus to a coffee mug is. Just like the coffee mug cannot be continuously disformed into a sphere, a trivial insulator cannot be continuously disformet into a topological insulator.

B. Time reversal symmetry

C. $\mathbb{Z}_2$ invariant

III. HALL EFFECTS

One of the defining properties of topological insulators is the presence of conducting edge states with an insulating bulk. The example considered here in sec. IV is a two dimensionnal material and its edge is therefore one-dimensionnal. The QH and QSH effects both involve one-dimensionnal conduction. In one dimension, electrons can either move forward of backward on the edge of the sample and this restriction is central for Hall effects. [1].

A. QH

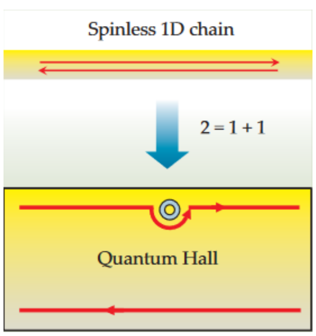

FIG. 1. Schematic representation of the conduction chanels in a spinless quantum Hall system.[1]

B. QSH

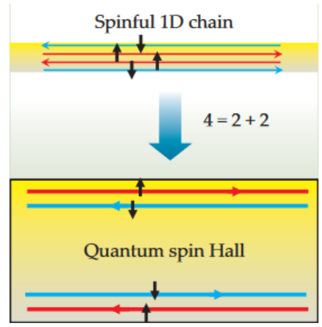

FIG. 2. Schematic representation of the conduction chanels in a spinlful quantum Hall system.

IV. HGTE/CDTE HETEROSTRUCTURE

The first observed topological insulator is a mercury telluride heterostructure consisting of a staking of thin HgTe layers between $\mathrm{Hg_xCd_{1-x}Te}$ [12]. If the thickness of the HgTe layers is right, a spin Hall effect arises. To model this system, the

V. CONCLUSION

1. Opening on other topological systems (topological Superconductivity and charge pumps)

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