1) $-x-y \to \min$

$4-2x-y \geq 0$

$2-\frac{1}{2}x-y \geq 0$

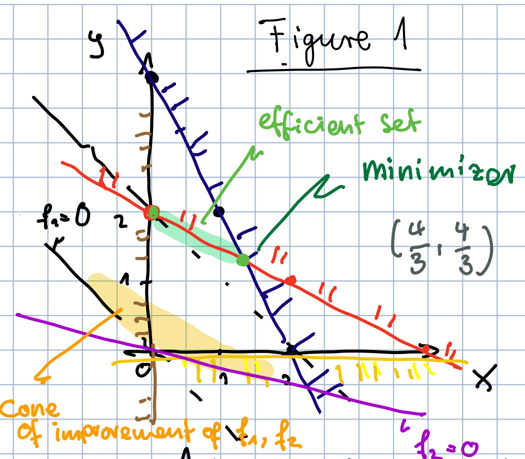
$x \geq 0$

$y \geq 0$

KKT Conditions:

$\exists: \lambda > 0$

Figure 1



$4-2x = 2-\frac{1}{2}x$

$2-2x = -\frac{1}{2}x$

$2 = 1.5x$

$x = 2/1.5 = 4/3$

$\Rightarrow y = 4/3$ symmetry

$$\lambda_0\begin{pmatrix}-1\\-1\end{pmatrix} - \lambda_1\begin{pmatrix}-2\\-1\end{pmatrix} - \lambda_2\begin{pmatrix}-\frac{1}{2}\\-1\end{pmatrix} - \lambda_3\begin{pmatrix}1\\0\end{pmatrix} - \lambda_4\begin{pmatrix}0\\1\end{pmatrix} = 0 \quad (\mathrm{I})$$

$$\lambda_1(4-2x-y)=0 \quad (\mathrm{II}) \qquad \lambda_3 x = 0 \quad (\mathrm{IV})$$

$$\lambda_2\left(2-\frac{1}{2}x-y\right)=0 \quad (\mathrm{III}) \qquad \lambda_4 y = 0 \quad (\mathrm{V})$$

Efficient set: $\quad -x-4y-4 \to \min \qquad y = -\frac{1}{4}x + c$

See: cone of improvement, efficient set (Figure 1)

Fritz John Conditions

$\exists\, \vec{u} \succ 0 : \exists\, \vec{\lambda} \succ 0 :$ ($\succ 0$ stands for $\vec{0}$-dominating)

$$u_1\begin{pmatrix}-1\\-1\end{pmatrix} + u_2\begin{pmatrix}-1\\-4\end{pmatrix} - \lambda_2\begin{pmatrix}-1/2\\-1\end{pmatrix} - \lambda_3\begin{pmatrix}1\\0\end{pmatrix} - \lambda_4\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$g_1(x,y)=0$: $\quad 4-\frac{8}{3}-\frac{4}{3}=0 \Rightarrow$ active

$g_2(x,y)=0$: $\quad 2-\frac{1}{2}\cdot\frac{4}{3}-\frac{4}{3}=0 \Rightarrow$ active

$\leftarrow$ inside cone spanned by active constraints

Task 2.1: $\exists \vec{\lambda} \neq \vec{0}$

$$\lambda_1 \nabla f_1(r,h) + \sum_i \lambda_{i+1} \nabla g_i(r,h) = 0$$

$$2\lambda_1 \pi r h + 2\pi \lambda_2 h + 4\pi r \lambda_2 = 0$$

$$\lambda_1 \pi r^2 + 2\lambda_2 \pi r = 0$$

$$2\pi r h + 2\pi r^2 = C$$

Task 2.2 Case $\lambda_1 = 0$; $\lambda_2 \neq 0$ $\Rightarrow$

$$\lambda_2 (2\pi h + 4\pi r) = \lambda_2 2\pi r$$

$$2\pi h = -2\pi r$$

$$h = -r$$

$\boxed{\text{either radius or height are zero, or at least one negative. } \lightning \Rightarrow \text{not feasible}}$

Case $\lambda_1 = 1$: $\lambda_2 = -2\pi r h / (2\pi h + 4\pi r)$ (I)

$\lambda_2 = -\pi r^2 / 2\pi r = -r/2$ (II)

I $-\frac{r}{2} = -2\pi r h / (2\pi h + 4\pi r)$

$$-2\pi r h - 4\pi r^2 = -4\pi r h$$

$$-4\pi r^2 = -2\pi r h$$

$$2r = h$$

Task 2.3)
$$\left.\begin{aligned} &2\pi r(2r) + 2\pi r^2 = C \\ &4\pi r^2 + 2\pi r^2 = 6\pi r^2 = C \\ &r(c) = \sqrt{\frac{c}{6\pi}}, \quad h(c) = 2\sqrt{\frac{c}{6\pi}} \\ &\text{Efficient set: } \{ (r(c), h(c)) \mid c > 0 \} \end{aligned}\right\}$$ ε-constraint method.

Task 3.1: Mixed Integer programming formulation (MINLP)

$$g_1(\vec{x}, \vec{y}, \vec{b}) = b_i \left(\sqrt{x_i^2 + y_i^2} - 8.5 \right) \leq 0$$

$$g_2(\vec{x}, \vec{y}, \vec{b}) = b_i b_j \left(\sqrt{x_i^2 + y_i^2} - 3 \right) \geq 0$$

w.l.o.g. the center of big disc is $(0,0)^T$

$$\vec{x} \in \mathbb{R}^N, \ \vec{y} \in \mathbb{R}^N, \ \vec{b} \in \mathbb{B}^N = \{0,1\}^N \subset \mathbb{Z}^N, \quad N = 44$$

Task 3.2: Integer Linear Programming formulation

Observation: Due to symmetry it holds $\sum_{j=1}^{n} a_{ij}$ is the degree of v_i

Total degree $\sum_{i=1}^{n} b_i \sum_{j=1}^{n} a_{ij} \rightarrow \max$

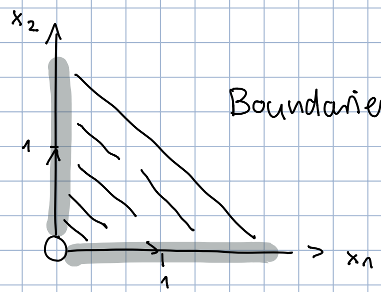
s.t. $\sum_{i=1}^{n} b_i = k, \qquad b_i \in \{0,1\}, \quad i = 1, \ldots, n$

4.1 Cone generators of the Pareto dominance order in 2-D are

$$\vec{d}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{d}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

We formulate the dominance cone as:

$$C = \{ \vec{y} \in \mathbb{R}^2 \mid \exists \lambda_1, \lambda_2 : \vec{y} = \lambda_1 \vec{d}_1 + \lambda_2 \vec{d}_2 , \vec{\lambda} \neq 0 \}$$



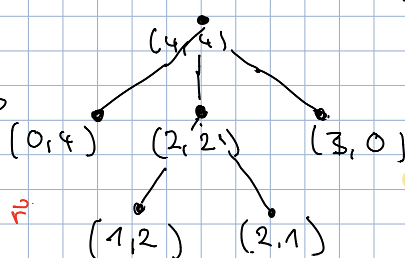
Boundaries are included, excepting (0,0)

4.2 Non-dominated points in S are (0,4), (1,2), (3,0), (2,1)

Incomparable to (2,2) are (0,4) and (3,0)

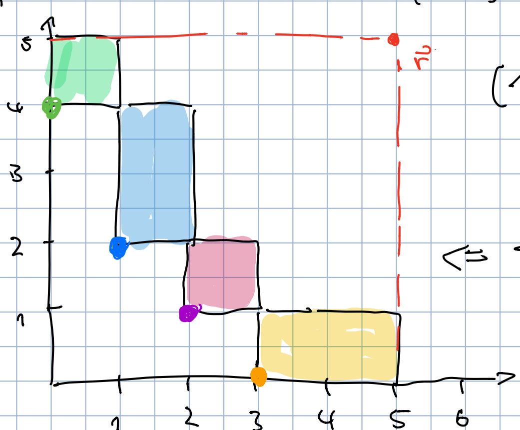
4.3 Hasse Diagram:

(Figure 2) $\Longrightarrow$



Remark: It can be drawn differently but it matters which links and in case of comparability, which node on top which other node. □

4.4



$\Leftarrow$ Fig 2.:

Contributions to Hypervolume are as follows (see Fig 2)

$\Delta HV((0,4)) = 1$, $\Delta HV((1,2)) = 2$, $\Delta HV((2,1)) = 1$, $\Delta HV((3,0)) = 2$

Non dominance rank 1: (best) : $\{(0,4), (1,2), (2,1), (3,0)\} =: R_1$

" 2 : $\{(2,2)\} =: R_2 \rightsquigarrow$ non dominated after removing R_1

" 3 : $\{(4,4)\} =: R_3 \rightsquigarrow$ non-dominated after removing R_1, R_2 from S

– end of exam solution – June 21, M.E. □