Multicriteria Optimization and Decision Analysis

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Answer each question on a separate sheet. Put your name, student number and the number of the question you are answering on each and every sheet. Provide full explanations with each of the answers!

Each question is weighted by a number of points, as indicated. The total number of points is 100. Success!

1. **M**ulticriteria Decision Analysis [**20 points**]
   The following table describes different options for a yacht.

Ship (Yacht)	Length $\rightarrow$ max Feet	Fuel $\rightarrow$ min miles per gallon	Price $\rightarrow$ min 1000 Euro	Comfort $\rightarrow$ max Stars
Starship	60	1	600	5
Warmond	40	2	200	3
Albatros	50	1.5	300	3
Beluga	40	2	200	3

1a. [**5 points**] Draw a Parallel Coordinate diagram with all solutions being represented as polylines.

1b. [**5 points**] Identify a solution pair that is indifferent, a solution pair that is incomparable, the efficient set, and the Pareto front.

1c. [**5 points**] Describe briefly the difference between a Derringer Suich and a Harrington desirability function.

1d. [**5 points**] Suppose, a decision maker ranks Albatros higher or equal than Beluga, and Starship higher or equal than Warmond. Formulate a linear program that finds a robust utility function using the Keeney-Raiffa utility form (product with weights in the form of exponents, i.e. $u(x) = f_1(x)^{\alpha_1} f_2(x)^{\alpha_2} \cdots f_m(x)^{\alpha}_m$) that is consistent with this choice.

2. **Mathematical Programming Models [20 points]**
   Formulate the following problems in the language of mathematical programming, using the following form:
   $f(\mathbf{x}) \to \min$, s.t. $g_i(\mathbf{x}) \geq 0, i = 1, ..., q$, $h_j(\mathbf{x}) = 0, j = 1, ..., s$, $\mathbf{x} \in S$
   where S is the decision space, that needs to be specified for the problem. It can be either $\mathbb{R}^n$, $\{0,1\}^n$, or $\mathbb{Z}^n$ or combinations of these.

   2a. [**10 points**] A magic cube has to be designed with integers x_{ij}. It should have 5 rows and 5 columns. The sum of each row and the sum of each column should be the same value, say v. All variables need to be pairwise different, integer, and bigger than 0. Formulate the problem of finding the magic cube with minimal value v.

   2b. [**10 points**] A mountainbike trail should be designed. The length should be as close to 10km as possible and visit a number of nodes, numbered from 1 to n. Each node should be visited exactly once. The distance between nodes is given by a distance table d_{ij}, $i = 1, ..., n$, $j = 1, ..., n$ (in meter). The cyclist starts at node 1 and also ends at node 1. The height of a node is given by the variable h_i, $i = 1, ..., n$ and the absolute height difference between two subsequent nodes of the tour should not exceed $50m$.

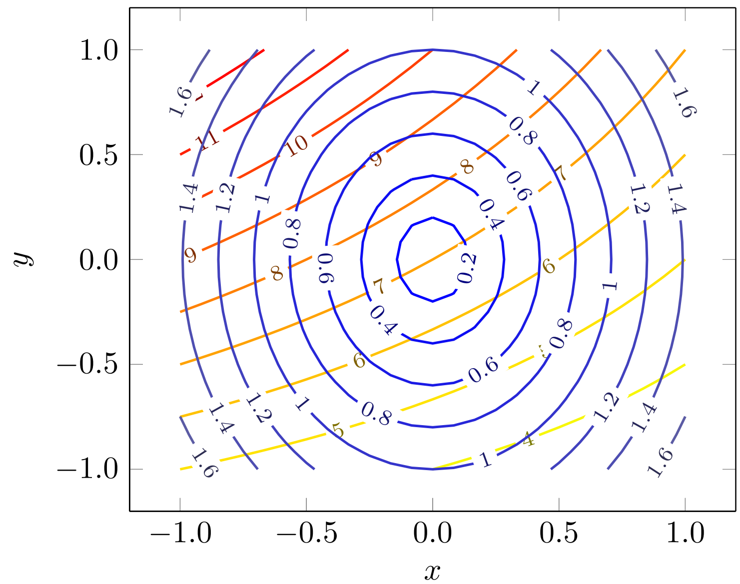
   If you cannot formulate the full problem, try to formulate as many objectives and constraints as possible.

3. **Linear Programming [10 points]**
   A gardener owns $10m^2$ land. She grows two types of crops – savoy and carrots. The seed for savoy costs $1€/m^2$, the seed for carrots $4€/m^2$. The amount of carrots grown on $1m^2$ can be sold for 6€ and the amount of savoy harvested from $1m^2$ for 2€. The maximal amount of money to invest in seeds is 12€.

   3a. [**5 points**] Formulate the linear program and solve the problem of maximizing the profit under the given constraints graphically. Determine the minimizer and indicate the feasible and infeasible regions.

   3b. [**5 points**] Due to a plague of the savoy-mold, the amount of herbicide required per m^2 savoy is $5g$ whereas the carrots do not require herbicide. To protect the environment, the gardener wants to minimize the use of herbicide while at the same time being profitable. Find the efficient set for this multiobjective problem. You can neglect the cost of the herbicide.

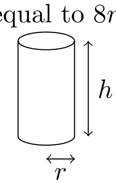
4. **Karush Kuhn Tucker Conditions [10 points]**: Consider the multi-objective optimization problem $f_1(x,y) = 7 - 2x + 3y - xy \rightarrow \max$, and $f_2(x,y) = \sqrt{2x^2 + y^2} \rightarrow \min$, $x, y \in [-1, 1]$.



4a. [**5 points**] Write up the KKT conditions for this problem.

4b. [**5 points**] Identify the efficient set in the contour-graph. You can do this on the exam paper. Please add your name and hand in the plot with the efficient set indicated.

5. **Constrained Optimization [10 points]**
A storage tank with a cylinder shape needs to be optimized. The steel surface area – to be minimized – is $S(r,h) = 2\pi rh + \pi r^2$ and the volume is $V(r,h) = \pi r^2 h$ and should be equal to $8m^3$.



5a. [**5 points**] State the equation system of the Lagrange multiplier rule.

5b. [**5 points**] Find the optimal solution. Express it in terms of a number. You can compose the number out of constants, such as π and $\sqrt{2}$, but the term should not include variables anymore.

6. **Mathematical Programming Models** [**15 points**] A cone order $\mathbf{x}^1 \preceq_{\mathcal{C}} \mathbf{x}^2 \Leftrightarrow \mathbf{x}^2 \in \mathbf{x}^1 \oplus \mathcal{C}$ is given by the acute polyhedral cone $\mathcal{C}$ with basis $\mathbf{u} = (1, 0.5)$ and $\mathbf{v} = (0.5, 1)$, that is $\mathcal{C} = \{\mathbf{y} \in \mathbb{R}^2 | \exists \lambda_1 \geq 0, \lambda_2 \geq 0 : \mathbf{y} = \lambda_1 \mathbf{u} + \lambda_2 \mathbf{v}\}$

   6a. [**5 points**] Describe graphically the cone and the set of points that is dominating and that is domiated by $\mathbf{p} = (1, 1)$ in the cone-order.

   6b. [**5 points**] Is the cone order $\prec_{\mathcal{C}}$ a partial order and does it extend the Pareto dominance order? Provide reasons for your answer.

   6c. [**5 points**] Draw the Hasse diagram for the set $\{(1, 1), (2, 2), (3, 3), (2, 0), (0, 1)\}$. What are the minimal and the maximal elements?

7. **Metaheuristics** [**15 points**] In evolutionary multiobjective optimization it is the goal to find an approximation to the Pareto front.

   7a. [**5 points**] Determine the Hypervolume indicator of the population $P = \{(9, 0), (8, 1), (7, 5), (5, 8), (7, 6), (9, 9)\}$ for reference point $(10, 10)$. What are the hypervolume contributions of the points? (consider minimization)

   7b. [**5 points**] Describe the sorting of NSGA-II for the population P (defined in 7a.).

   7c. [**5 points**] Describe the difference between the classes NP and NP complete, supposing that $NP \neq P$. Given a problem in NP hard would provably require exponential time. What would this say about the conjecture $NP \neq P$?

Wishing you success!