MaA 2017/2018 Task 3 (Exam Solution)

Let us consider an LP ...

First objective is transformed:

$-x_1 - 2x_2 = c$, chose, e.g., $c=0 \Rightarrow x_2 = -\frac{1}{2}x_1$

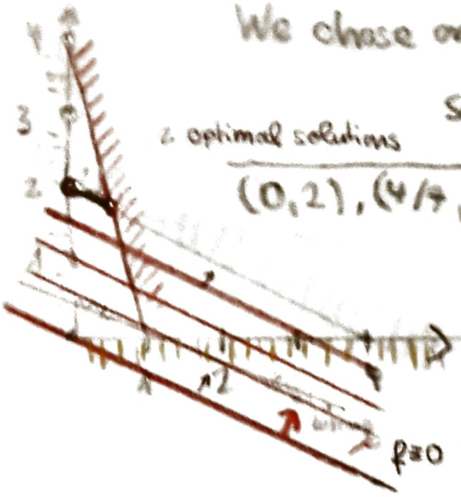
We will add parallel indifference / level curves to the diagram.

Secondly the constraints are transformed by solving for x_2, $g_i(\cdot) \leq 0$

$4x_1 + x_2 - 4 \leq 0 \Leftrightarrow x_2 \leq 4 - 4x_1$

$\frac{1}{2}x_1 + x_2 - 2 \leq 0 \Leftrightarrow x_2 \leq 2 - \frac{1}{2}x_1$

We also have $x_1 \geq 0$ and $x_2 \geq 0$



We chose one optimal solution (minimizer), for the sake of simplicity $(x_1, x_2) = (0,2)$.

↳ optimal solutions

$(0,2), (4/7, 12/7)$

↳ line segment

3 b) $\exists \vec{\lambda} > 0 : \lambda_1 \nabla f(\vec{x}) + \sum_{i=1}^{m} \lambda_i \nabla g_i(\vec{x}) = 0$

$\forall j = 1, \ldots, m: \lambda_{j+1} g_j(\vec{x}) = 0$

Next: Make specific for 3a:

$\lambda_2 (4x_1 + x_2 - 4) = 0$

$\lambda_3 (\frac{1}{2}x_1 + x_2 - 2) = 0$

$\lambda_4 (-x_1) = 0 \Rightarrow \lambda_4$ free

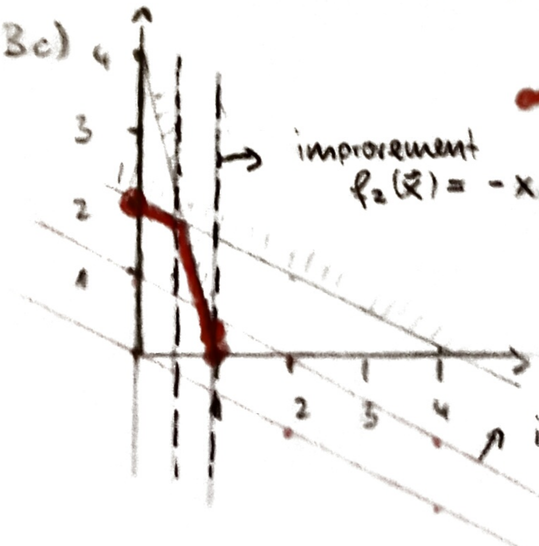
$\lambda_5 (-x_2) = 0 \Rightarrow \lambda_5 = 0$

$-\lambda_1 + 4\lambda_2 + \frac{1}{2}\lambda_3 - \lambda_4 = 0$

$-2\lambda_1 + \lambda_2 + \lambda_3 - \lambda_5 = 0$

choose $\lambda_1 = \lambda_2 = \lambda_3 = 1$ $\lambda_4 = 3.5$, $\vec{\lambda} > 0$ q.e.d.

3c)



efficient set

In these points it is no longer possible to improve f_1 or f_2 while not getting worse in the other objective or leaving feasible subspace.

□