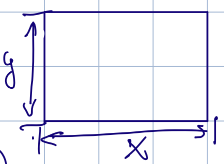
Example ε-constraint method:

Consider optimization of rectangle, circumference vs. surface.

$f_1(x,y) = C(x,y) = 2x + 2y \rightarrow \min$



$f_2(x,y) = S(x,y) = xy \rightarrow \max, \quad (x,y \geq 0)$

Lagrange Multiplier rule, setting constraint $f_2(x,y) = S_\varepsilon$ (1)

$f_1(x,y) = 2x + 2y \rightarrow \min, \quad \text{s.t.} \quad xy = S_\varepsilon$

$$\frac{\partial L}{\partial x}(x,y) = \lambda_0 \cdot 2 + \lambda_1 y = 0 \qquad (2)$$

$$\frac{\partial L}{\partial y}(x,y) = \lambda_0 2 + \lambda_1 x = 0 \qquad (3)$$

$\lambda_0 = 0$: $\lambda_1 y = 0, \quad \lambda_1 x = 0, \quad xy = S_\varepsilon, \ \lambda_1 \neq 0 \Rightarrow x = y = 0$

$\lambda_0 = 1$: $\quad 2 + \lambda_1 y = 0 \Rightarrow y = -2/\lambda_1$; likewise $x = -2/\lambda_1$

with (1) we get $\frac{4}{\lambda_1^2} = S_\varepsilon \Rightarrow \lambda_1 = \sqrt{4/S_\varepsilon} = 2/\sqrt{S_\varepsilon}$

Now, the efficient set is given by

parameterized 'curve'

$$X_E = \left\{ (x,y) \in (\mathbb{R}_0^+)^2 \mid x = \sqrt{S_\varepsilon}, y = \sqrt{S_\varepsilon}, S_\varepsilon > 0 \right\} \cup \{(0,0)\}$$

To formulate Pareto Front, e.g. express S_ε in terms of C_ε:

$C_\varepsilon = 2x + 2y = 2\sqrt{S_\varepsilon} + 2\sqrt{S_\varepsilon} = 4\sqrt{S_\varepsilon}, \ x,y > 0, \ C_\varepsilon(0) = 0$

$S_\varepsilon(C_\varepsilon) = (C_\varepsilon/4)^2$, this is the Pareto front $(C_\varepsilon \geq 0)$ $\square$