1) Linear Programming

$$2x_1 + 3x_2 \rightarrow \max \quad (1)$$

$$0.5x_1 + x_2 \le 12 \quad (2)$$

$$2x_1 + x_2 \le 14 \quad (3)$$

$$0.06x_1 + 0.02x_2 \le 0.04(x_1 + x_2) \quad (4)$$

Q 1.1 Solve the problem graphically

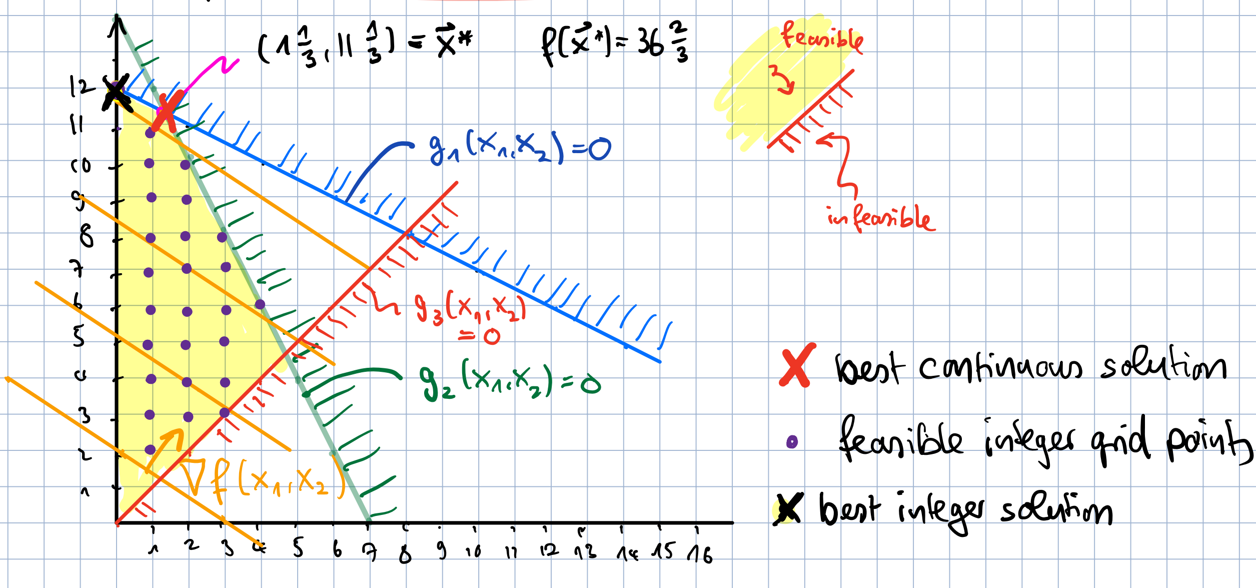
Transform (1) to get iso-heightlines or contours of height C.

$x_2 = C/3 - 2/3\, x_1$

Transform (2): $x_2 \le 12 - 0.5x_1$

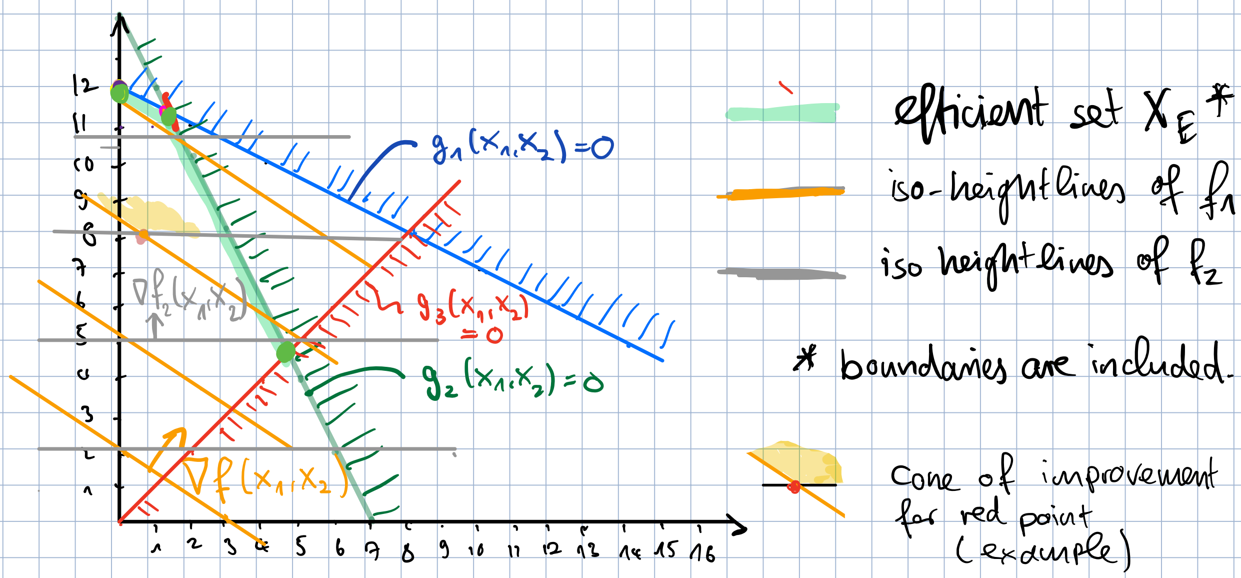
Transform (3): $x_2 \le 14 - 2x_1$

Transform (4): $0.02x_1 \le 0.02x_2 \Rightarrow x_1 \le x_2$



Q 1.2 The best integer solution is the highest feasible point on the nodes of the integer lattice $\mathbb{Z} \times \mathbb{Z}$. In this problem it is $z^* = (0, 12)$.

>> Q.1.3: The second objective to be maximized is x_2



Q1.4 In the KKT conditions we investigate $\lambda_1 = 1$ and show that it leads to a non-negative solution for $\lambda_2, \lambda_3, \lambda_4$, which shows that the KKT conditions hold in $\bar{x}^*$.

Step 1) State the KKT conditions for the Linear program

$$f_1(x_1,x_2) = 2x_1 + 3x_2$$

$$g_1(x_1,x_2) = 0.5\,x_1 + x_2 - 12 \leq 0$$

$$g_2(x_1,x_2) = 2x_1 + x_2 - 14 \leq 0$$

$$g_3(x_1,x_2) = 0.02\,x_1 - 0.02\,x_2 \leq 0$$

$$g_4(x_1,x_2) = -x_1 \leq 0 \qquad (x_1 \geq 0)$$

$$g_5(x_1,x_2) = -x_2 \leq 0 \qquad (x_2 \geq 0)$$

$$-\lambda_1 \begin{pmatrix} \frac{\partial f(x_1,x_2)}{\partial x_1} \\ \frac{\partial f(x_1,x_2)}{\partial x_2} \end{pmatrix} + \sum_{i=1}^{5} \lambda_{i+1} \begin{pmatrix} \frac{\partial g_i(x_1,x_2)}{\partial x_1} \\ \frac{\partial g_i(x_1,x_2)}{\partial x_2} \end{pmatrix} = 0$$

$$\lambda_{i+1}\, g_i(x_1,x_2) = 0, \quad i = 1,\dots,5$$

Q1.4

>> Solving the artial derivatives we obtain:

$$\lambda_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \lambda_4 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_5 \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \lambda_6 \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\lambda_2 (0.5 x_1 + x_2 - 12) = 0$$
$$\lambda_3 (2 x_1 + x_2 - 14) = 0$$
$$\lambda_4 (0.02 x_1 - 0.02 x_2) = 0$$
$$-\lambda_5 x_1 = 0$$
$$-\lambda_6 x_2 = 0$$

If we now insert $x_1^* = 1\frac{1}{3}$ $x_2^* = 11\frac{1}{3}$

we will find that $\lambda_4 = \lambda_5 = \lambda_6 = 0$

and with $\lambda_1 = 1$ we obtain $\lambda_2 = 2\frac{2}{3}$, $\lambda_3 = \frac{1}{3}$.

Hence the KKT condition that there exists $\vec{\lambda} = (\lambda_1, \ldots, \lambda_6) \succ \vec{0}$ with the KKT equations satisfied, is given. Note, that $\succ \vec{0}$ stands for dominating $(0,0,0,0,0,0)$ in this case. That is, all values of λ_i must be non-negative (≥ 0) and at least one value of λ_i positive (>0), for $i = 1, \ldots, 6$.

The Karush Kuhn Tucker condition thus holds.

$\square$