1) $-x - y \to \min$

$4 - 2x - y \geq 0$

$2 - \frac{1}{2}x - y \geq 0$

$x \geq 0$

$y \geq 0$

KKT Conditions: $\exists: \lambda > 0$

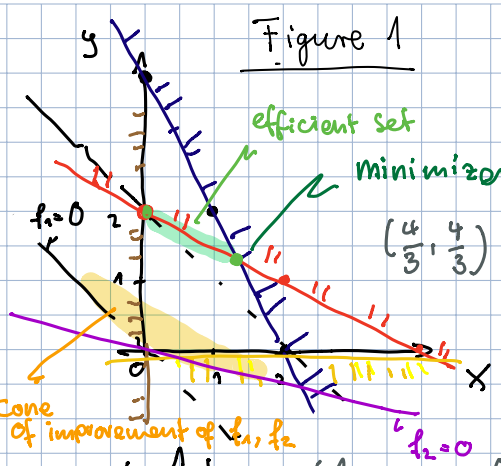


Figure 1

$4 - 2x = 2 - \frac{1}{2}x$

$2 - 2x = -\frac{1}{2}x$

$2 = 1.5x$

$x = 2/1.5 = 4/3$

$\Rightarrow y = 4/3$ symmetry

$$\lambda_0 \begin{pmatrix} -1 \\ -1 \end{pmatrix} - \lambda_1 \begin{pmatrix} -2 \\ -1 \end{pmatrix} - \lambda_2 \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix} - \lambda_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \lambda_4 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 \quad (I)$$

$\lambda_1 (4 - 2x - y) = 0$ (II)

$\lambda_2 (2 - \frac{1}{2}x - y) = 0$ (III)

$\lambda_3 x = 0$ (IV)

$\lambda_4 y = 0$ (V)

Efficient set: $-x - 4y - 4 \to \min$ $\quad y = -\frac{1}{4}x + c$

Fritz John Conditions

See: cone of improvement, efficient set (Figure 1)

$\exists \vec{u} > 0: \exists \vec{\lambda} > 0:$ (> 0 stands for $\vec{0}$-dominating)

$$u_1 \begin{pmatrix} -1 \\ -1 \end{pmatrix} + u_2 \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \lambda_2 \begin{pmatrix} -1/2 \\ -1 \end{pmatrix} - \lambda_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \lambda_4 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$g_1(x, y) = 0: \quad 4 - \frac{8}{3} - \frac{4}{3} = 0 \Rightarrow$ active

$g_2(x, y) = 0: \quad 2 - \frac{1}{2}\cdot\frac{4}{3} - \frac{4}{3} = 0 \Rightarrow$ active

← inside cone spanned by active constraints

Task 2.1: $\exists \vec{\lambda} \neq \vec{0}$

$$\lambda_1 \nabla f_1(r,h) + \sum_i \lambda_{i+1} \nabla g_i(r,h) = 0$$

$$2\lambda_1 \pi r h + 2\pi \lambda_2 h + 4\pi r \lambda_2 = 0$$

$$\lambda_1 \pi r^2 + 2\lambda_2 \pi r = 0$$

$$2\pi r h + 2\pi r^2 = C$$

Task 2.2 Case $\lambda_1 = 0$; $\lambda_2 \neq 0$ $\Rightarrow$

$$\lambda_2 (2\pi h + 4\pi r) = \lambda_2 2\pi r$$

$$2\pi h = -2\pi r$$

$$h = -r$$

either radius or height are zero, or at least one negative. $\lightning$ $\Rightarrow$ not feasible

Case $\lambda_1 = 1$:

$$\lambda_2 = -2\pi r h / (2\pi h + 4\pi r) \quad \text{(I)}$$

$$\lambda_2 = -\pi r^2 / 2\pi r = -r/2 \quad \text{(II)}$$

I $-\frac{r}{2} = -2\pi r h / (2\pi h + 4\pi r)$

$$-2\pi r h - 4\pi r^2 = -4\pi r h$$

$$-4\pi r^2 = -2\pi r h$$

$$2r = h$$

Task 2.3) $2\pi r (2r) + 2\pi r^2 = C$

$$4\pi r^2 + 2\pi r^2 = 6\pi r^2 = C$$

$$r(C) = \sqrt{\frac{C}{6\pi}}, \qquad h(C) = 2\sqrt{\frac{C}{6\pi}}$$

Efficient set: $\{ (r(c), h(c)) \mid c > 0 \}$

ε-constraint method.

Task 3.1: Mixed Integer programming formulation (MINLP)

$$g_1(\vec{x}, \vec{y}, \vec{b}) = b_i \left(\sqrt{x_i^2 + y_i^2} - 8.5 \right) \leq 0$$

$$g_2(\vec{x}, \vec{y}, \vec{b}) = b_i b_j \left(\sqrt{x_i^2 + y_i^2} - 3 \right) \geq 0$$

} w.l.o.g. the center of big disc is $(0,0)^T$

$$\vec{x} \in \mathbb{R}^N, \ \vec{y} \in \mathbb{R}^N, \ \vec{b} \in \mathbb{B}^N = \{0,1\}^N \subset \mathbb{Z}^N, \quad N = 44$$

Task 3.2: Integer Linear Programming formulation

Observation: Due to symmetry it holds $\sum_{j=1}^{n} a_{ij}$ is the degree of v_i

Total degree $\sum_{i=1}^{n} b_i \sum_{j=1}^{n} a_{ij} \rightarrow \max$

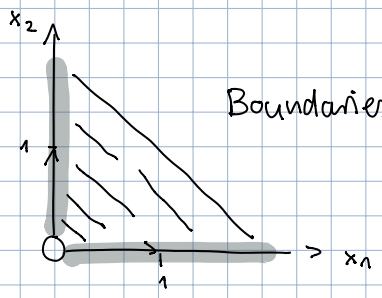
s.t. $\sum_{i=1}^{n} b_i = k, \qquad b_i \in \{0,1\}, \ i = 1, \dots, n$

4.1 Cone generators of the Pareto dominance order in 2-D are

$$\vec{d}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad \vec{d}_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

We formulate the dominance cone as:

$$C = \{\vec{y} \in \mathbb{R}^2 \mid \exists \lambda_1, \lambda_2 : \vec{y} = \lambda_1 \vec{d}_1 + \lambda_2 \vec{d}_2, \vec{\lambda} \neq 0\}$$



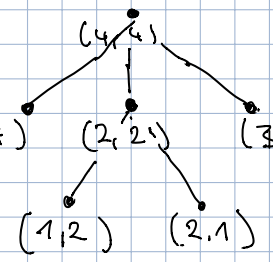
Boundaries are included, excepting (0,0)

4.2 Non-dominated points in S are (0,4), (1,2), (3,0), (2,1)

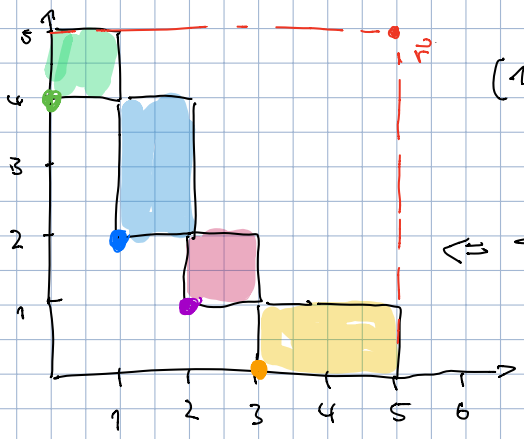
Incomparable to (2,2) are (0,4) and (3,0)

4.3 Hasse Diagram:

(Figure 2) ⟹



4.4



⟺ Fig 2.:

Remark:
It can be drawn differently but it matters which links and in case of comparability, which node on top which other node. □

Contributions to Hypervolume are as follows (see Fig 2)

$\Delta HV((0,4)) = 1$, $\Delta HV((1,2)) = 2$, $\Delta HV((2,1)) = 1$, $\Delta HV((3,0)) = 2$

Non dominance rank 1: (best) : $\{(0,4), (1,2), (2,1), (3,0)\} =: R_1$

" 2: $\{(2,2)\} =: R_2$ ~ non dominated after removing R_1

" 3: $\{(4,4)\} =: R_3$ ~ non-dominated after removing R_1, R_2 from S

– end of exam solution – June 21. M.E. □