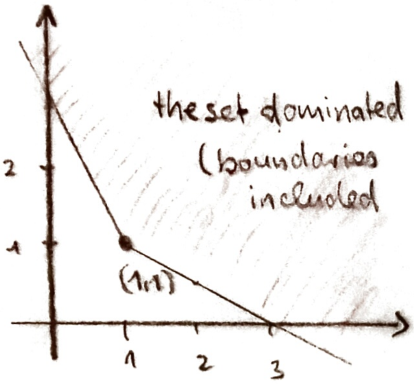
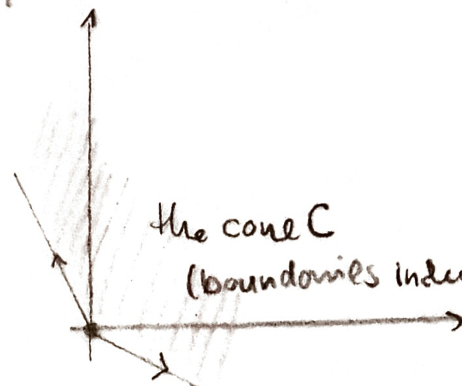
3.1.2019

Solution to task 5, MODA 2017/2018 exam

5a)





5b) Show, that the cone order is a partial order

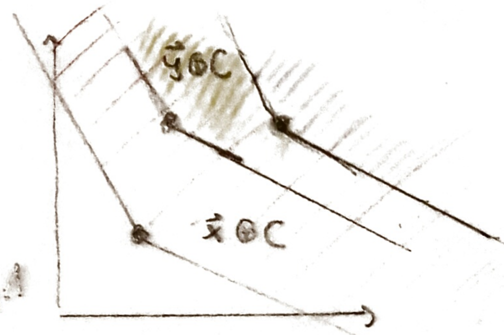
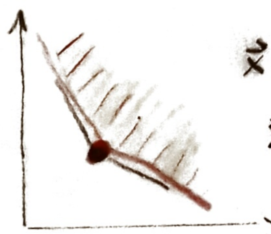


Fig. 1

reflexivity $\vec{x} = \vec{y} \Rightarrow \vec{x} \preceq y$



$\vec{x} = \vec{y} \Rightarrow$

$\vec{x} \oplus C \supset \vec{x} + \{0\} = \{\vec{x}\}$

$\Rightarrow \vec{y} \in \vec{x} \oplus C$

antisymmetry: $\vec{x} \preceq \vec{y} \wedge \vec{y} \preceq \vec{x} \Rightarrow \vec{x} = \vec{y}$

$\vec{x} \preceq \vec{y} \Leftrightarrow \vec{y} \in \vec{x} \oplus C$

$\vec{y} \preceq \vec{x} \Leftrightarrow \vec{x} \in \vec{y} \oplus C \wedge \vec{y} \in \vec{x} \ominus C$

the only element that allows

$\vec{x} \ominus C \cap \vec{x} \oplus C \neq \emptyset$ for a

pointed cone C is $\vec{0}$. Hence

$\vec{x} = \vec{y} \ominus \{\vec{0}\} = \vec{y}$

transitivity: $\underline{\vec{x} \preceq \vec{y} \wedge \vec{y} \preceq \vec{z}} \Rightarrow \vec{x} \preceq \vec{z}$

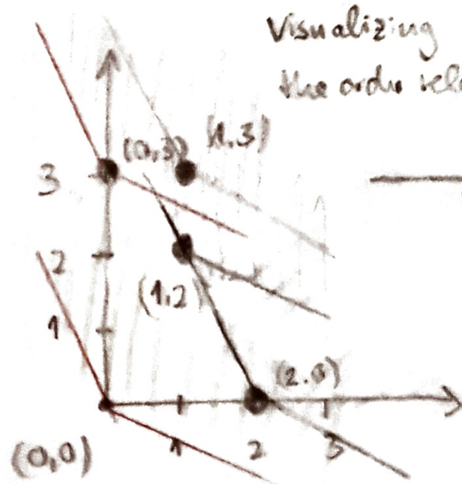
$\underbrace{\vec{x} \oplus C \ni \vec{y}} \wedge \vec{y} \oplus C \ni \vec{z}$

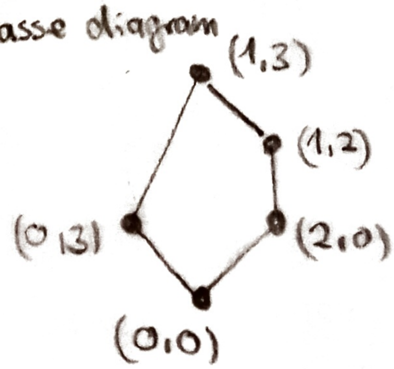
$\vec{x} \oplus C \supseteq \vec{y} \oplus C$ (see Fig 1)

and hence $x \oplus C \ni \vec{z}$

5c)

Visualizing the order relation $\longrightarrow$ Hasse diagram





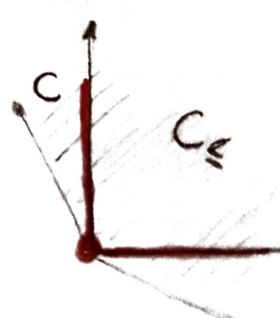


Fig. 2

It holds $\vec{x} \leq \vec{y} \Rightarrow \vec{x} \preceq_C \vec{y}$ ($\preceq_C$ extends $\leq$):

5d) the cone for the weak componentwise order is defined as $C_{\leq}$, a cone generated by (0,1), (1,0), (Fig 2). $C \supseteq C_{\leq}$ and hence $\vec{x} \leq_{\leq} \vec{y} \Rightarrow \vec{y} \in \vec{x} \oplus C_{\leq} \Rightarrow \vec{y} \in \vec{x} \oplus C \Rightarrow \vec{x} \preceq_C \vec{y}$. $\square$