Exam 2017/2018 MODA Task 4

A building with a box shape ...

$(x_1)^2 + 4x_2x_1 \to \min$ s.t. $(x_1)^2 x_2 = 32$ (or $x_1^2 x_2 - 32 = 0$)

Lagrange multiplier rule (general): $\exists \vec{\lambda} \neq \vec{0}: \lambda_1 \nabla f(\vec{x}) + \sum_{i=1}^{m} \lambda_{i+1} \nabla g_i(\vec{x}) = 0$
and $\lambda_1 = 1 \vee \lambda_1 = 0$

Now, for special case:

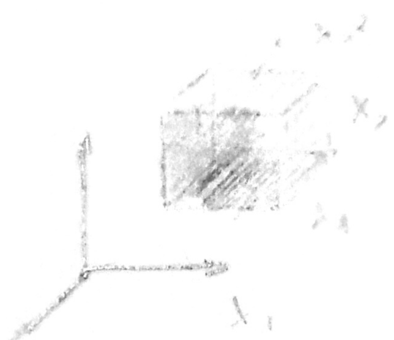
(1) $2\lambda_1 x_1 + 4\lambda_1 x_2 + 2\lambda_2 x_1 x_2 = 0$ $\left(\frac{\partial}{\partial x_1}\right)$

(2) $\lambda_1 x_1 + \lambda_2 x_1^2 = 0$ $\left(\frac{\partial}{\partial x_2}\right)$

(3) $x_1^2 x_2 - 32 = 0$

Now, we solve for $\lambda_1 = 0$. $2\lambda_2 x_1 x_2 \overset{(1')}{=} 0$, $\lambda_2 x_1^2 \overset{(2')}{=} 0$,

$x_1^2 x_2 - 32 \overset{(3')}{=} 0 \Rightarrow x_1^2 = 32/x_2$



(2') $\Rightarrow 32\lambda_2 / x_2 = 0 \Rightarrow \lambda_2 = 0, x_2 \neq 0$

$\Rightarrow \{(x_1, x_2) \mid x_1^2 = 32/x_2\}$ are satisfying Lagrange conditions (1)-(3)

However: $\vec{\lambda} \neq 0$ is not satisfied $\Rightarrow \lambda_1 = 0$ no sol.

Next, try $\lambda_1 = 1$:

$2x_1 + 4x_2 + 2\lambda_2 x_1 x_2 = 0$

$x_1 + \lambda_2 x_1^2 = 0$ (2'')

$x_1^2 x_2 - 32 = 0$ (3'')

(3'') $\Rightarrow x_2 = 32/x_1^2$ (2'') $\lambda_2 = -\frac{1}{x_1} \vee \lambda_2 = \frac{1}{x_1}$

(1'') $2x_1 + 128/x_1^2 - 64/x_1 = 0$ $\mid \cdot x_1^2$

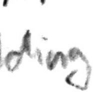
$2x_1 + 64/x_1^2 = 0 \Leftrightarrow x_1^3 = -64$. no sol.

this might be due to non convexity

Insertion

<u>School method:</u> $x_1^2 + 4x_1 \cdot 32/x_1^2 \to \min$

$x_2 = 32/x_1$ insert in $f(x_1, x_2)$ $\to$ $2x_1 - 128/x_1^2 = 0$ $64 = x_1^3 \Rightarrow x_1 = 4$

$x_2 = 32/4 = 8$. ( tall building) □