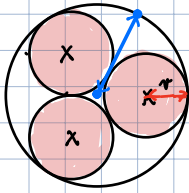


3.1 Place k spheres inside a big sphere of radius 10m .



$$r \rightarrow \max$$

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \geq 2r$$

$$\forall i = 1, \dots, k, \forall j = 1, \dots, k$$

$$\sqrt{x_i^2 + y_i^2 + z_i^2} \geq 10 - r$$

$$\forall i = 1, \dots, k$$

$$b_i \in \{0, 1\} ; i = 1, \dots, k,$$

$$x_i, y_i, z_i \in \mathbb{R}$$

3.2 Place as many spheres as possible

$$\sum_{i=1}^n b_i \rightarrow \max$$

r_B : radius of big sphere

r_b : radius of small sphere

$$r_B = 10\text{m}.$$

$$n = \left\lceil \left(\frac{4}{3} \pi r_B^3 \right) / \left(\frac{4}{3} \pi r_b^3 \right) \right\rceil$$

n is a precomputed upper bound for the number of small spheres.

$$b_i b_j \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \geq 2r_b ; i = 1, \dots, n, j = 1, \dots, n$$

$$b_i \sqrt{x_i^2 + y_i^2 + z_i^2} \leq r_B - r_b$$

$$b_i \in \{0, 1\}, x_i \in \mathbb{R}, y_i \in \mathbb{R}, z_i \in \mathbb{R}$$

$$i = 1, \dots, n, j = 1, \dots, n$$

$$3.3. \sum_{\ell=1}^4 \sum_{i=1}^6 \sum_{j=1}^6 \sum_{k=1}^{L_j} k^2 x_{ijk\ell} + \sum_{\ell=1}^4 \sum_{k=1}^{L_j} \sum_{j=1}^6 (L_j - k)^2 x_{sjk\ell}$$

$$\rightarrow \min$$

Subject to:

2. on day 1 to day 5: 6 hours on day 6: 4 hours

This constraint is reflected in the constants L_j :

$$L_j = \begin{cases} 6 & j=1, \dots, 5 \\ 4 & j=6 \end{cases}$$

$$3. \sum_{j=1}^6 \sum_{k=1}^{L_j} X_{ijk\ell} = 6 \quad i \in \{1, 2, 3\}, \ell = 1, \dots, 4$$

$$\sum_{j=1}^6 \sum_{k=1}^{L_j} X_{ijk\ell} = 2 \quad i \in \{4, \dots, 10\}, \ell = 1, \dots, 4$$

$$4. \sum_{j=1}^6 \sum_{k=1}^{L_j} \sum_{\ell=1}^4 X_{1j k \ell} \leq 1$$

$$5. \sum_{j=1}^6 \sum_{k=1}^{L_j} X_{ijk\ell} \leq 2 \quad : \ell = 1, \dots, 4, i = 1, \dots, 10$$

here I interpret the constraint as follows:

each subject is at most 2 times taught per class on a single day.

$$6. X_{ijk\ell} + X_{i(j+1)k\ell} \leq 1 \quad : \forall i \in \{1, \dots, 10\}:$$

$$\forall j \in \{1, \dots, 5\}$$

$$\forall k \in \{1, \dots, L_j\}$$

$$\forall \ell \in \{1, 4\}$$

Domain:

$$X_{ijk\ell} \in \{0, 1\} \quad : \forall i \in \{1, \dots, 10\}:$$

$$\forall j \in \{1, \dots, 5\}$$

$$\forall k \in \{1, \dots, L_j\}$$

$$\forall \ell \in \{1, 4\}$$

□