PHYS 234 - A2

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Problem 1:

Historically, the relevant length scale of the Davisson-Gerner experiment was approximately 1 Å, but is closer to 1×10^{-4} [$\mathring{A}$] with modern breakthroughs in aperture design.

It is so hard to detect wave-like phenomena for macroscopic objects because by the de Broglie wavelength equation, wavelength is inversely proportional to momentum. Macroscopic objects have a large mass, which implies they have a large momentum, which means their wavelengths are far too large to be measured by the Davisson-Gerner experimental apparatus.

Problem 2:

a) Since additive interference happens when $2d \sin\theta = n\lambda$ for $n \in \mathbb{Z}$, in order to maximize our wavelength $\theta = \pi/2$ when measured from the plane of refraction. For a first-order maximum, $n = 1$. is We are looking at a diagonal plane of atoms, each 0.91 Å from one another measured on the axis. Because d measures the distance between the planes, $d = 0.91\,\text{Å}/\sqrt{2} = 0.64\,\text{Å}$.

$$\lambda = 2d \sin\pi/2 = 1.3\,\text{Å}$$

b) Assuming our electrons are traveling at non-relativistic speeds, $K = \frac{1}{2}mv^2 = 300\,\text{eV}$. Solving this gives $v = 1.03 \times 10^7\,\text{m/s}$, or about $0.0343c$, justifying our use of the classical equation.

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = 0.71\,\text{Å}$$

$$\theta = \arcsin\frac{\lambda}{2d} = 0.59\,\text{rad}$$

Problem 3:

a)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = 0.5\,\text{g}$$
$$k = \mu(2\pi\nu)^2 = 197\,\text{kg/s}^2$$

$$U = \frac{1}{2}kx^2 = 9.9 \times 10^{-3}\,\text{J}$$
$$= 6.20 \times 10^{16}\,\text{eV}$$

$$n = \frac{U}{h\nu} = 1.49 \times 10^{29}$$

b)

$$\frac{E_{1.49\times10^{29}+1}}{E_{1.49\times10^{29}}} - 1 \approx 0.000\%$$

No change is detectable whatsoever.

c)

$$E_1 = h\nu = 4.14 \times 10^{-1}\,\text{eV}$$
$$E_2 = 0.827\,\text{eV}$$
$$E_3 = 1.24\,\text{eV}$$

d)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = 8.5 \times 10^{-25}\,\mathrm{g}$$
$$k = \mu(2\pi\nu)^2 = 336\,\mathrm{kg/s^2}$$

$$x_1 = \sqrt{2\frac{E_1}{k}} = 0.21\,\text{Å}$$
$$x_2 = \sqrt{2\frac{E_2}{k}} = 0.29\,\text{Å}$$
$$x_3 = \sqrt{2\frac{E_3}{k}} = 0.36\,\text{Å}$$

Problem 4i:

$$\text{let } \hat{A} = \begin{pmatrix} 2 & 7 \\ 7 & 2 \end{pmatrix}$$
$$\begin{aligned} 0 &= \det \hat{A} - \lambda \hat{I} \\ &= (2-\lambda)^2 - 7^2 \\ &= 4 - 4\lambda + \lambda^2 - 49 \\ &= (\lambda - 9)(\lambda + 5) \end{aligned}$$

$$\begin{aligned} \hat{A}\begin{pmatrix} a \\ b \end{pmatrix} &= \lambda_1 \begin{pmatrix} a \\ b \end{pmatrix} \\ \begin{pmatrix} 2a + 7b \\ 7a + 2b \end{pmatrix} &= 9 \begin{pmatrix} a \\ b \end{pmatrix} \\ v_1 &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \hat{A}\begin{pmatrix} a \\ b \end{pmatrix} &= \lambda_2 \begin{pmatrix} a \\ b \end{pmatrix} \\ \begin{pmatrix} 2a + 7b \\ 7a + 2b \end{pmatrix} &= -5 \begin{pmatrix} a \\ b \end{pmatrix} \\ v_2 &= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \end{aligned}$$

Problem 4ii:

$$\begin{aligned}
\text{let } \hat{B} &= \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \\
0 &= \det \hat{B} - \lambda \hat{I} \\
&= (1-\lambda)(3-\lambda) + 2 \\
&= (\lambda - (2-i))(\lambda + (2+i))
\end{aligned}$$

$$\begin{aligned}
\hat{B} \begin{pmatrix} a \\ b \end{pmatrix} &= \lambda_1 \begin{pmatrix} a \\ b \end{pmatrix} \\
\begin{pmatrix} a - 2b \\ a + 3b \end{pmatrix} &= (2-i) \begin{pmatrix} a \\ b \end{pmatrix} \\
v_1 &= \begin{pmatrix} -1-i \\ 1 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\hat{B} \begin{pmatrix} a \\ b \end{pmatrix} &= \lambda_2 \begin{pmatrix} a \\ b \end{pmatrix} \\
\begin{pmatrix} a - 2b \\ a + 3b \end{pmatrix} &= (2+i) \begin{pmatrix} a \\ b \end{pmatrix} \\
v_2 &= \begin{pmatrix} -1+i \\ 1 \end{pmatrix}
\end{aligned}$$

Problem 4iii:

$$\begin{aligned}
\text{let } k &= \frac{\hbar}{2} \\
\text{let } \hat{C} &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
0 &= \det \hat{C} - \lambda \hat{I} \\
&= \lambda^2 - 1 \\
&= (k\lambda - 1)(k\lambda + 1)
\end{aligned}$$

$$\begin{aligned}
k\hat{C} \begin{pmatrix} a \\ b \end{pmatrix} &= \lambda_1 \begin{pmatrix} a \\ b \end{pmatrix} \\
k \begin{pmatrix} -a - ib \\ ia - b \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\
v_1 &= k \begin{pmatrix} -i \\ 1 \end{pmatrix}
\end{aligned}$$

$$k\hat{C}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda_2 \begin{pmatrix} a \\ b \end{pmatrix}$$

$$k\begin{pmatrix} a - ib \\ ia + b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$v_2 = k\begin{pmatrix} i \\ 1 \end{pmatrix}$$

Problem 5:

$$\hat{A} = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix}$$

a)

$$\hat{A} + \hat{B} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 3 \\ 3i & 3-2i & 4 \end{pmatrix}$$

b)

$$\hat{A}\hat{B} = \begin{pmatrix} -3 & 1+3i & 3i \\ 4+3i & 9 & 6-2i \\ 6i & 6-2i & 6 \end{pmatrix}$$

c)

$$\left[\hat{A}, \hat{B}\right] = \begin{pmatrix} -1 & 1 & i & 2 & 0 & -i \\ 2 & 0 & 3 & 0 & 1 & 0 \\ 2i & -2i & 2 & i & 3 & 2 \end{pmatrix}$$

d)

$$\bar{\hat{A}} = \begin{pmatrix} -1 & 2 & 2i \\ 1 & 0 & -21 \\ i & 3 & 2 \end{pmatrix}$$

e)

$$\hat{A}^* = \begin{pmatrix} -1 & 1 & -i \\ 2 & 0 & 3 \\ -2i & 2i & 2 \end{pmatrix}$$

f)

$$\hat{A}^\dagger = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix}$$

g) $Tr(\hat{B}) = 2 + 1 + 2 = 5$