Math 135 Alexander Maguire
Assignment 10 20396195

1) Prove $(1+i^{2n})(1+i^{n})=0$ or 4 for $n\in\mathbb{N}, n>0$:
Since *i* oscillates with four states, proving this for 1..4 through modulus applies to all values of n in the specified range.

$(1+i^{2n})(1+i^{n})$
$=1+i^{n}+i^{2n}+i^{3n}$
$=1+i^{n}+i^{2n}+i^{3n}$

Case 1:
$=1+i^{1}+i^{2}+i^{3}$
$=1+i-1-i$
$=0$

Case 2:
$=1+i^{2}+i^{4}+i^{6}$
$=1+-1+1-i$
$=0$

Case 3:
$=1+i^{3}+i^{6}+i^{9}$
$=1-i-1--i$
$=0$

Case 4:
$=1+i^{4}+i^{8}+i^{12}$
$=1+1+1+1$.
$=4$

Thus $(1+i^{2n})(1+i^{n})=0$ or 4, as we have covered all cases and those were our only outputs.

2)
$$z^{2}=2\bar{z}$$
$$(x+iy)(x+iy)=2(x-iy)$$
$$x^{2}+2\text{ixy}-y^{2}=2\text{x}-2\text{iy}$$
$$(x^{2}-y^{2})+(2xy)i=2\text{x}-2\text{iy}$$
$$x^{2}-y^{2}=2x$$
$$2xy=-2\text{y}$$
$$x=-1$$
$$(-1)^{2}-y^{2}=2(-1)$$
$$1-y^{2}=-2$$
$$y^{2}=3$$
$$y=\pm\sqrt{3}$$

And so, $z=-1\pm i\sqrt{3}$.

3) a)
$$z+i\,w=2$$
$$z=2-i\,w$$
$$i\,z+2\mathrm{w}=3$$
$$i(2-i\,w)+2\mathrm{w}=3$$
$$2\mathrm{i}+w+2\mathrm{w}=3$$
$$3\mathrm{w}=3-2\mathrm{i}$$
$$w=\frac{1}{3}(3-2\mathrm{i})$$

b)
$$iz+(1+i)w=-3+i$$
$$(1+i)w=-3+i(1-z)$$
$$w=\frac{-3+i(1-z)}{(1+i)}$$
$$(2+i)z+(3-2\mathrm{i})w=4\mathrm{i}$$
$$(2+i)z+\frac{(3-2\mathrm{i})(-3+i(1-z))}{1+i}=4\mathrm{i}$$
$$2\mathrm{z}+iz+\frac{(3-2\mathrm{i})(-3+i-iz)}{1+i}=4\mathrm{i}$$
$$2\mathrm{z}+iz+\frac{-9+3\mathrm{i}-3\mathrm{iz}+6\mathrm{i}+2-2\mathrm{z}}{1+i}=4\mathrm{i}$$
$$2\mathrm{z}+iz+\frac{9\mathrm{i}-2\mathrm{z}-3\mathrm{iz}-7}{1+i}=4\mathrm{i}$$
$$2\mathrm{z}+iz+\frac{9\mathrm{i}-2\mathrm{z}-3\mathrm{iz}-7}{1+i}\cdot\frac{1-i}{1-i}=4\mathrm{i}$$
$$2\mathrm{z}+iz+\frac{9\mathrm{i}-2\mathrm{z}-3\mathrm{iz}-7-9+2\mathrm{iz}-3\mathrm{z}+7\mathrm{i}}{2}=4\mathrm{i}$$
$$4\mathrm{z}+2\mathrm{iz}+16\mathrm{i}-5\mathrm{z}-iz-16=8\mathrm{i}$$
$$iz+16\mathrm{i}-z-16=8\mathrm{i}$$
$$-z-16=i(8-z-16)$$
$$-z-16=i(-8-z)$$
$$z+16=8\mathrm{z}+iz$$
$$z(1-8-i)=-16$$
$$z=\frac{16}{7+i}\cdot\frac{7-i}{7-i}=\frac{1}{50}(112-16\mathrm{i})$$

4)
$$|z+w|^2+|z-w|^2=2|z|^2+2|w|^2$$
$$(z+w)\overline{(z+w)}+(z-w)\overline{(z-w)}=2\,z\,\overline{z}+2\,w\,\overline{w}$$
$$(z+w)(\overline{z}+\overline{w})+(z-w)(\overline{z}-\overline{w})=2\,z\,\overline{z}+2\,w\,\overline{w}$$
$$(z\,\overline{z}+z\,\overline{w}+w\,\overline{z}+w\,\overline{w})+(z\,\overline{z}-z\,\overline{w}-w\,\overline{z}+w\,\overline{w})=2\,z\,\overline{z}+2\,w\,\overline{w}$$
$$z\,\overline{z}+z\,\overline{z}+w\,\overline{w}+w\,\overline{w}=2\,z\,\overline{z}+2\,w\,\overline{w}$$
$$2\,z\,\overline{z}+2\,w\,\overline{w}=2\,z\,\overline{z}+2\mathrm{w}\,\overline{w}$$
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5) If $z_n = x_n + y_n i$,

$$|z_1 - z_2| \geqslant |z_1| - |z_2|$$
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \geqslant \sqrt{x_1^2 + y_1^2} - \sqrt{x_2^2 + y_2^2}$$
$$\sqrt{x_1^2 - 2x_1 x_2 + x_2^2 + y_1^2 - 2y_1 y_2 + y_2^2} \geqslant \sqrt{x_1^2 + y_1^2} - \sqrt{x_2^2 + y_2^2}$$
$$x_1^2 - 2x_1 x_2 + x_2^2 + y_1^2 - 2y_1 y_2 + y_2^2 \geqslant x_1^2 + y_1^2 - 2\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2} + x_2^2 + y_2^2$$
$$x_1 x_2 + y_1 y_2 \leqslant \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}$$
$$x_1 x_2 + y_1 y_2 \leqslant \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}$$
$$x_1 x_2 + y_1 y_2 \leqslant \sqrt{x_1^2 x_2^2 + x_1^2 y_2^2 + x_2^2 y_1^2 + y_1^2 y_2^2}$$

This is obviously true, as the outside terms are inside (squared, then rooted) with other terms. The geometric interpretation of this inequality is that the length of a different in vectors is longer than or equal to the different of the individual vector lenghts.