PHYS 234 - A3

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Problem 1 i:

$$z = -3 + 4i$$

$$r = ||z||$$
$$= 5$$

$$\theta = \arctan \frac{\Im}{\Re}(z)\Big|_{z=-3+4i}$$
$$= 2.214\,\text{rad}$$

$$z = 5(\cos 2.214 + i\sin 2.214)$$
$$= 5e^{2.214i}$$

Problem 1 ii:

$$z = 7 - 1i$$

$$r = ||z||$$
$$= 7.07$$

$$\theta = \arctan \frac{\Im}{\Re}(z)\bigg|_{z=7-1i}$$
$$= -0.142 \,\mathrm{rad}$$

$$z = 7.07(\cos -0.142 + i \sin -0.142)$$
$$= 7.07e^{-0.142i}$$

Problem 1 iii:

$$z = -2 - 5i$$

$$r = ||z||$$
$$= 5.39$$

$$\theta = \arctan \frac{\Im}{\Re}(z)\bigg|_{z=-2-5i}$$
$$= -1.951 \,\mathrm{rad}$$

$$z = 5.39(\cos -1.951 + i \sin -1.951)$$
$$= 5.39e^{-1.951i}$$

Problem 2 i:

$$r = |3|^2 + |4|^2$$
$$= 5$$

$$|\psi_{1,norm}\rangle = \frac{|\psi_1\rangle}{r^2}$$
$$= \frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle$$

$$P_z = |\langle + | \psi_{1,norm}\rangle|^2$$
$$= \left|\frac{3}{5}\right|^2$$
$$= 0.36$$

$$P_x = \left|\frac{1}{\sqrt{2}}(\langle +| + \langle -|)|\psi_{1,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle + | \psi_{1,norm}\rangle + \langle - | \psi_{1,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(3 + 4)\right|^2$$
$$= 0.98$$

$$P_y = \left|\frac{1}{\sqrt{2}}(\langle +| + i\langle -|)|\psi_{1,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle + | \psi_{1,norm}\rangle + i\langle - | \psi_{1,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(3 + i(4))\right|^2$$
$$= 0.5$$

Problem 2 ii:

$$r = |1|^2 + |2i|^2$$
$$= 2.236$$

$$|\psi_{2,norm}\rangle = \frac{|\psi_2\rangle}{r^2}$$
$$= \frac{1}{2.236}|+\rangle + \frac{2i}{2.236}|-\rangle$$

$$P_z = |\langle +|\psi_{2,norm}\rangle|^2$$
$$= \left|\frac{1}{2.236}\right|^2$$
$$= 0.2$$

$$P_x = \left|\frac{1}{\sqrt{2}}(\langle +|+\langle -|)|\psi_{2,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle +|\psi_{2,norm}\rangle + \langle -|\psi_{2,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(1+2i)\right|^2$$
$$= 0.5$$

$$P_y = \left|\frac{1}{\sqrt{2}}(\langle +|+i\langle -|)|\psi_{2,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle +|\psi_{2,norm}\rangle + i\langle -|\psi_{2,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(1+i(2i))\right|^2$$
$$= 0.9$$

Problem 2 iii:

$$e^{ipi/3} = \frac{1/+i\sqrt{3}}{2}$$

$$r = |3|^2 + |0.5 + 0.866i|^2$$
$$= 3.162$$

$$|\psi_{3,norm}\rangle = \frac{|\psi_3\rangle}{r^2}$$
$$= \frac{3}{3.162}|+\rangle + \frac{0.5 + 0.866i}{3.162}|-\rangle$$

$$P_z = |\langle + | \psi_{3,norm}\rangle|^2$$
$$= \left|\frac{3}{3.162}\right|^2$$
$$= 0.9$$

$$P_x = \left|\frac{1}{\sqrt{2}}(\langle+| + \langle-|)|\psi_{3,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle+|\psi_{3,norm}\rangle + \langle-|\psi_{3,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(3 + 0.5 + 0.866i)\right|^2$$
$$= 0.65$$

$$P_y = \left|\frac{1}{\sqrt{2}}(\langle+| + i\langle-|)|\psi_{3,norm}\rangle\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(\langle+|\psi_{3,norm}\rangle + i\langle-|\psi_{3,norm}\rangle)\right|^2$$
$$= \left|\frac{1}{\sqrt{2}}(3 + i(0.5 + 0.866i))\right|^2$$
$$= 0.76$$

Problem 3:

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}\,|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}\,|-\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{3}}\,|+\rangle - i\frac{\sqrt{2}}{\sqrt{3}}\,|-\rangle$$

$$|\psi_3\rangle = -\frac{1}{\sqrt{3}}\,|+\rangle - i\frac{\sqrt{2}}{\sqrt{3}}\,|-\rangle$$

P	$|\phi_1\rangle$	$|\phi_2\rangle$	$|\phi_3\rangle$
$_z\langle+|$	0.333	0.333	0.333
$_z\langle-|$	0.667	0.667	0.667
$_x\langle+|$	0.5	0.5	0.5
$_x\langle-|$	0.5	0.5	0.5
$_y\langle+|$	0.971	0.029	0.971
$_y\langle-|$	0.029	0.971	0.029