PHYS 234 - A5

Alexander Maguire

amaguire@uwaterloo.ca

20396195

June 11, 2014

Problem 1 i:

$$P_{z+} = |_z \langle + | \rangle|^2$$
$$= \frac{1}{6} |(1)(i+1) + 2()|^2$$
$$= 0.333$$

$$P_{z-} = 1 - P_{z+}$$
$$= 0.667$$

Problem 1 ii:

$$P_{x+} = |_x \langle + | \rangle|^2$$
$$= \frac{1}{6} |(0.707)(i+1) + 2(-0.707)|^2$$
$$= 0.167$$

$$P_{x-} = 1 - P_{x+}$$
$$= 0.833$$

Problem 1 iii:

$$\langle S_z \rangle_{|\rangle} = \langle \,|\, S_z \,|\, \rangle$$
$$= \frac{\begin{bmatrix} 1-i & 2 \end{bmatrix}}{\sqrt{6}} \frac{\hbar}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{\sqrt{6}} \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$$
$$= -0.167\hbar$$

Problem 1 iv:

$$\langle S_x \rangle_{|\rangle} = \langle \,|\, S_x \,|\, \rangle$$
$$= \frac{\begin{bmatrix} 1-i & 2 \end{bmatrix}}{\sqrt{6}} \frac{\hbar}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{1}{\sqrt{6}} \begin{bmatrix} 1+i \\ 2 \end{bmatrix}$$
$$= 0.333\hbar$$

Problem 2:

$$\psi = \sqrt{\frac{3}{5}} \,|+\rangle + \sqrt{\frac{2}{5}} \,|-\rangle$$

$$\langle S_x \rangle_\psi = \langle \psi \,|\, S_x \,|\, \psi \rangle$$
$$= \frac{1}{5} \begin{bmatrix} \sqrt{3} & \sqrt{2} \end{bmatrix} \frac{\hbar}{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{3} \\ \sqrt{2} \end{bmatrix}$$
$$= 0.98\hbar$$

Problem 3:

Given 20% $|-\rangle_x$, 75% $|+\rangle_y$, 5% $|+\rangle_z$:

$$\rho = \sum_i p_i \, |\psi_i\rangle \langle\psi_i|$$

$$= 0.2\left(|-\rangle_{x\ x}\langle-|\right) + 0.75\left(|+\rangle_{y\ y}\langle+|\right) + 0.05\left(|+\rangle_{z\ z}\langle+|\right)$$

$$= 0.2{-}\mathbf{S_x} + 0.75\mathbf{S_y} + 0.05\mathbf{S_z}$$

$$= \frac{\hbar}{2}\begin{bmatrix} 0.005 & -0.2 + 0.75i \\ 0.2 - 0.75i & -0.05 \end{bmatrix}$$

$$\langle\mathbf{S_y}\rho\rangle_{|\psi\rangle} = \frac{\hbar^2}{4}Tr\left\{ \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0.005 & -0.2 + 0.75i \\ 0.2 - 0.75i & -0.05 \end{bmatrix} \right\}$$

$$= \frac{\hbar^2}{4}(0.75 + 0.2i + -0.75 - 0.2i)$$

$$= 0$$

Problem 4:

$$|\pm\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle \pm |-\rangle\right)$$

$$\mathbf{S_x}|\pm\rangle_x = \pm\frac{\hbar}{2}|\pm\rangle_x$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}|\pm\rangle_x = \pm\frac{\hbar}{2}|\pm\rangle_x$$

$$a\,|+\rangle + b\,|-\rangle = +\frac{\hbar}{2}\left(|+\rangle + |-\rangle\right)$$

$$c\,|+\rangle + d\,|-\rangle = -\frac{\hbar}{2}\left(|+\rangle - |-\rangle\right)$$

$$\rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$