

PHYS 234 - A3

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Problem 1 i:

$$z = -3 + 4i$$

$$\begin{aligned} r &= ||z|| \\ &= 5 \end{aligned}$$

$$\begin{aligned} \theta &= \arctan \frac{\Im}{\Re}(z) \Big|_{z=-3+4i} \\ &= 2.214 \text{ rad} \end{aligned}$$

$$\begin{aligned} z &= 5(\cos 2.214 + i \sin 2.214) \\ &= 5e^{2.214i} \end{aligned}$$

Problem 1 ii:

$$z = 7 - 1i$$

$$\begin{aligned} r &= ||z|| \\ &= 7.07 \end{aligned}$$

$$\begin{aligned} \theta &= \arctan \frac{\Im}{\Re}(z) \Big|_{z=7-1i} \\ &= -0.142 \text{ rad} \end{aligned}$$

$$\begin{aligned} z &= 7.07(\cos -0.142 + i \sin -0.142) \\ &= 7.07e^{-0.142i} \end{aligned}$$

Problem 1 iii:

$$z = -2 - 5i$$

$$\begin{aligned} r &= ||z|| \\ &= 5.39 \end{aligned}$$

$$\begin{aligned} \theta &= \arctan \frac{\Im}{\Re}(z) \Big|_{z=-2-5i} \\ &= -1.951 \text{ rad} \end{aligned}$$

$$\begin{aligned} z &= 5.39(\cos -1.951 + i \sin -1.951) \\ &= 5.39e^{-1.951i} \end{aligned}$$

Problem 2 i:

$$\begin{aligned}
 r &= |3|^2 + |4|^2 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 |\psi_{1,norm}\rangle &= \frac{|\psi_1\rangle}{r^2} \\
 &= \frac{3}{5}|+\rangle + \frac{4}{5}|-\rangle
 \end{aligned}$$

$$\begin{aligned}
 P_z &= |\langle + | \psi_{1,norm} \rangle|^2 \\
 &= \left| \frac{3}{5} \right|^2 \\
 &= 0.36
 \end{aligned}$$

$$\begin{aligned}
 P_x &= \left| \frac{1}{\sqrt{2}} (\langle + | + \langle - |) |\psi_{1,norm}\rangle \right|^2 \\
 &= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{1,norm}\rangle + \langle - | \psi_{1,norm}\rangle) \right|^2 \\
 &= \left| \frac{1}{\sqrt{2}} (3 + 4) \right|^2 \\
 &= 0.98
 \end{aligned}$$

$$\begin{aligned}
 P_y &= \left| \frac{1}{\sqrt{2}} (\langle + | + i \langle - |) |\psi_{1,norm}\rangle \right|^2 \\
 &= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{1,norm}\rangle + i \langle - | \psi_{1,norm}\rangle) \right|^2 \\
 &= \left| \frac{1}{\sqrt{2}} (3 + i(4)) \right|^2 \\
 &= 0.5
 \end{aligned}$$

Problem 2 ii:

$$\begin{aligned}
r &= |1|^2 + |2i|^2 \\
&= 2.236
\end{aligned}$$

$$\begin{aligned}
|\psi_{2,norm}\rangle &= \frac{|\psi_2\rangle}{r^2} \\
&= \frac{1}{2.236} |+\rangle + \frac{2i}{2.236} |-\rangle
\end{aligned}$$

$$\begin{aligned}
P_z &= |\langle + | \psi_{2,norm} \rangle|^2 \\
&= \left| \frac{1}{2.236} \right|^2 \\
&= 0.2
\end{aligned}$$

$$\begin{aligned}
P_x &= \left| \frac{1}{\sqrt{2}} (\langle + | + \langle - |) |\psi_{2,norm}\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{2,norm}\rangle + \langle - | \psi_{2,norm}\rangle) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (1 + 2i) \right|^2 \\
&= 0.5
\end{aligned}$$

$$\begin{aligned}
P_y &= \left| \frac{1}{\sqrt{2}} (\langle + | + i \langle - |) |\psi_{2,norm}\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{2,norm}\rangle + i \langle - | \psi_{2,norm}\rangle) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (1 + i(2i)) \right|^2 \\
&= 0.9
\end{aligned}$$

Problem 2 iii:

$$e^{ipi/3} = \frac{1 + i\sqrt{3}}{2}$$

$$\begin{aligned}
r &= |3|^2 + |0.5 + 0.866i|^2 \\
&= 3.162
\end{aligned}$$

$$\begin{aligned}
|\psi_{3,norm}\rangle &= \frac{|\psi_3\rangle}{r^2} \\
&= \frac{3}{3.162} |+\rangle + \frac{0.5 + 0.866i}{3.162} |-\rangle
\end{aligned}$$

$$\begin{aligned}
P_z &= |\langle + | \psi_{3,norm} \rangle|^2 \\
&= \left| \frac{3}{3.162} \right|^2 \\
&= 0.9
\end{aligned}$$

$$\begin{aligned}
P_x &= \left| \frac{1}{\sqrt{2}} (\langle + | + \langle - |) |\psi_{3,norm}\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{3,norm}\rangle + \langle - | \psi_{3,norm}\rangle) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (3 + 0.5 + 0.866i) \right|^2 \\
&= 0.65
\end{aligned}$$

$$\begin{aligned}
P_y &= \left| \frac{1}{\sqrt{2}} (\langle + | + i \langle - |) |\psi_{3,norm}\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (\langle + | \psi_{3,norm}\rangle + i \langle - | \psi_{3,norm}\rangle) \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} (3 + i(0.5 + 0.866i)) \right|^2 \\
&= 0.76
\end{aligned}$$

Problem 3:

$$\begin{aligned}
|\psi_1\rangle &= \frac{1}{\sqrt{3}}|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle \\
|\psi_2\rangle &= \frac{1}{\sqrt{3}}|+\rangle - i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle \\
|\psi_3\rangle &= -\frac{1}{\sqrt{3}}|+\rangle - i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle
\end{aligned}$$

P	$ \phi_1\rangle$	$ \phi_2\rangle$	$ \phi_3\rangle$
${}_z\langle+ $	0.333	0.333	0.333
${}_z\langle- $	0.667	0.667	0.667
${}_x\langle+ $	0.5	0.5	0.5
${}_x\langle- $	0.5	0.5	0.5
${}_y\langle+ $	0.971	0.029	0.971
${}_y\langle- $	0.029	0.971	0.029