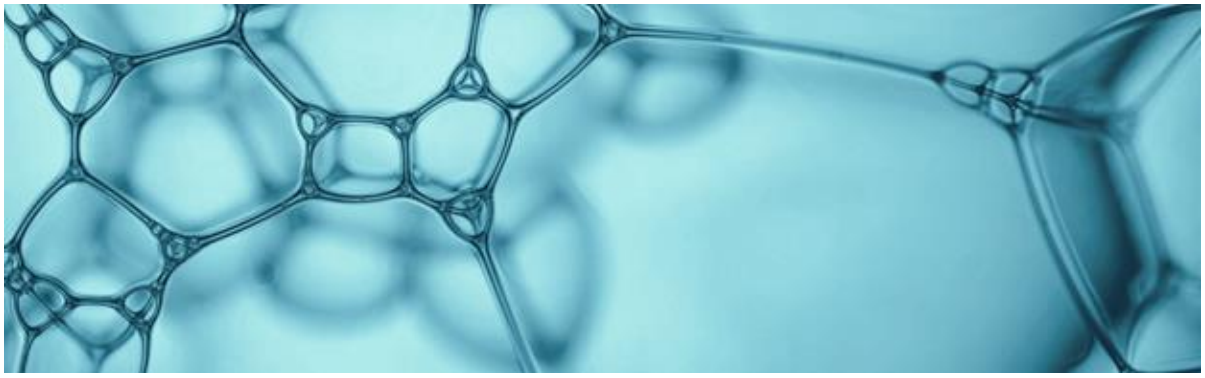

The Art of Mindful Investment

Portfolio Optimization: From Theory to Practice

Introduction

If someone asks us "What is money", we often realize that the concept is abstract, and it can be quite challenging to define it clearly. We often struggle to differentiate between the artifacts from the purpose they serve in our lives. Really, what *is* money?

> *Is it the notes and coins in one's wallet?*
> *Is it the balance in one's bank account?*
> *Rare and precious metals that one owns, maybe?*

These are all just examples of materialized wealth, and not the essence of the concept. If we go deeper, we can make a reasonable case that **money is a way of representing the value of things.** [1] And it is important to know that while money represents value, it is not valuable by itself. Its value derives from the trust, confidence and acceptance given to it by the individuals and the economic system that they are a part of. [2] This means, without the ways to materialize money into something tangible, money itself amounts to nothing. This is why one cannot buy goods and services in a foreign country using the currency of their own country.

To some readers, this whole discussion may sound too hypothetical to be of any practical use. Why go deeper into financial philosophy when we can make decent enough decisions regarding our day-to-day finances? One can use a smartphone simply fine, without thoroughly understanding the inner workings of the device. The authors of this paper are presenting an argument that it is worthwhile to know more about what

money is, what it stands for and how to make it work for one's financial well-being. When it comes to "making your money work for you", one must come across a familiar (and equally nebulous) concept - investments.

Investments are nothing but **acts** of allocating money (or more generally, resources) with the expectation of generating more of it **in the future.** [3]

If one pays attention to the highlighted words in the text above, two things may immediately become apparent -

1. Investments are not the artifacts that you can *have*, they are the actions that you *take* (or more specifically, the financial decisions you make).
2. They cost you in the *present*, but they benefit you (ideally more handsomely) *in the future*.

Hence, the predicament that investments put us into is - **How should I decide on how to act *now*, so that I have more money *later*?** The two features of investments mentioned above make it especially tricky to get them right, because they work against the natural human instincts that millennia of natural evolution have hardwired into our brains.

Firstly, most of us unfortunately prefer to not perform any action which does not produce immediate results. This can be attributed to a cognitive bias called "present bias." We lack the self-discipline to defer our instant gratification for better rewards we might receive later. This cognitive bias (of disregarding our own future self, by assigning more preference to our present self over it) [4], often ends up working against us.

Secondly, our financial plans are often vaguely defined. We seldom define what our better tomorrow is going to look like. We wish we could have "more" money, but we fail to ask ourselves "how much more, *exactly*?". We say "Someday..." when we come across that amazing apartment that we wish to buy one day, but we fail to realize the open-ended nature of our plan. Even the disciplined few among us who have the resolve to make good spending choices often fall into this trap - the lack of well-defined financial goals and structuring one's investments which will achieve that.

Thirdly, when things seem to work out as per our plans, we tend to think they will continue to do so in the future. When we see a stock's price rising for a couple of weeks in a row, we tend to think that it is going to go up in the following weeks as well. [5] However, the behavior of any complex system is not solely predictable based on its past behavior. This is why one needs strong financial safety nets – just as much as (if not more than) – the retirement funds that one readily invests in.

These and many such errors in human judgements have been a topic of research in behavioral economics and consumer psychology. [6] These are called **cognitive biases** and they affect our lives in more ways than we can imagine. Being cognizant about these unconscious and irrational thought patterns certainly helps us understand the absurdity

behind them; but the only certain way to reduce the potential danger from them is to build robust *systems* which would help us take the right decisions. [7]

In this paper, the authors aim to motivate the use of formal techniques to manage one's personal finance and develop better financial hygiene. Many of these techniques, which have been proposed to help one take better financial decisions in various scenarios, are developed by industry leaders and world-renowned Nobel laureates in the field of economics. Next section provides some introduction to their works. Further sections will provide how these theoretical contributions can be applied to practical scenarios.

Stalwarts of Behavioral Economics

Before we delve deeper into the topic of interest (i.e., personal finance management), let us get to know the people whose works have been the cornerstones of the field of Economics. Some of them have been the absolute pioneers of their respective sub-domains, while others' works have paved the way to some sophisticated research and findings that the authors wish to highlight.

Harry Markowitz

Nobel Prize in Economics and John von Neumann Theory prize recipient and the creator of the Modern Portfolio Theory (MPT), [8] Markowitz's name is an institution in the financial realm. His groundbreaking work revolutionized investment by emphasizing diversification to reduce risk. Markowitz's framework (which is also sometimes called Mean-Variance Theory or MVT) remains a seminal work in the field of portfolio management, providing a systematic approach for constructing well-diversified investment portfolios. His contributions have had a lasting impact on the field and continue to shape the way investors approach portfolio construction.

Jonathan Scheid

MD, Advisory Solutions at Buckingham Wealth Partners and a seasoned professional in Portfolio Management and Behavioral Economics. With a bachelor's degree in business and finance from SCU, he holds designations as a CFA and Accredited Investment Fiduciary®. He has co-authored notable publications, including "Correlation, Return Gaps, and the Benefits of Diversification" with Meir Statman, highlighting the limitations of correlation as an indicator for diversification wherein they emphasize the importance of considering standard deviation and return gaps for a more accurate assessment. [9]

Sanjiv Das

Sanjiv Das, the William and Janice Terry Professor of Finance and Data Science at Santa Clara University's Leavey School of Business, is an accomplished academic with vast

expertise. He held esteemed positions at Harvard Business School and UC Berkeley, and his educational background includes degrees in Finance, Computer Science, and an MBA. With extensive experience in derivatives at Citibank, Das's research spans portfolio theory, machine learning, financial networks, and more. He has authored numerous articles in academic journals, received multiple research and teaching awards, and published the acclaimed book "Derivatives: Principles and Practice." [10]

Meir Statman

Meir Statman, the Glenn Klimek Professor of Finance at Santa Clara University, is a prominent figure in behavioral economics. His research explores how investors and managers make financial decisions and how these decisions affect financial markets. He addresses topics such as investor sentiments, cognitive and emotional biases, and their influence on saving, spending, and portfolio construction. His most notable work is Behavioral Portfolio Theory, a new framework for portfolio construction that incorporates investors' multiple goals and mental accounts, such as safety, potential, and aspiration.

Their work has contributed to the development of sophisticated techniques that we can use to efficiently manage our wealth. However, before getting to know how we can apply these methods and techniques in the context of our personal finance, we should investigate why they are even required. The following section gives a hypothetical account of an unfavorable investment scenario that could have been easily avoided with the use of these concepts.

An (unfortunately) relatable story

Our journey here starts with Mr. X, inspired by the success stories of investors like Warren Buffett and Ray Dalio, eagerly entering the world of stock market investments. In the beginning of his shenanigans, things seemed promising. He invested a sizable portion of his savings in a hot blue-chip stock that had gotten hyped up in the media. Unfortunately, the story does not end well for Mr. X due to some fatal flaws in his investment strategy.

In our example scenario, the stock's price did not continue rising, and Mr. X lost a major portion of his investment corpus in the process. He also did not have an emergency fund to fall back to. He was forced to book his losses to avoid any further depreciation of his investments. Unfortunately, we are all too familiar with such anecdotes around us, and we try to think of the ways to best avoid these pitfalls if we were to be in one of them.

As far as Mr. X is concerned, there are some noticeable drawbacks in his investment strategy -

1. Lack of diversification (invested in only one stock)

2. Inadequate evaluation of risks (relied too heavily on a blue-chip stock's risk-free nature)
3. Inability to hold the position (did not have a financial safety net)
4. Inadequate understanding of markets (placed undue confidence on the bullish hypothesis)
5. Unreliable source of information (relied on the news rather than fundamental understanding of the market)

There are many different methods to solve each of these problems, some of which are covered in the upcoming sections. To avoid making such mistakes, the authors here try to give a bird's-eye view on the methods that have been developed in the last several decades, which have been rigorously evaluated in practice. The reader is encouraged to compare these with their own methodologies which might have proven useful to them. However, it is always better to have more tools in one's toolbox, as the saying goes "If all you have is a hammer, everything is a nail".

The field of Behavioral Economics combines Psychology and Economics, as it examines the ways in which individuals' cognitive biases, emotions, social influences, and other psychological factors impact their choices in the economic context. Let us start with one of the most intuitive and naïve solutions to the problem above, and gradually develop our understanding as to how it can be improved using some sophisticated methods.

A naïve approach

In a nutshell, the solutions to such problems lie in the sub-field of economics called **Portfolio Optimization.** Portfolio optimization is the process of finding the best combination of investments to maximize returns while minimizing risks. The most basic solution to portfolio optimization is also the most intuitive one, and on some level even instinctual.

Take the example of a child, who is walking into a candy store with ₹20 in hand. There she sees two types of candies: A and B. They both cost ₹10 apiece, and she has never tried either of those candies. She really wants to maximally utilize her ₹20 (i.e., have the best possible candy-consuming experience), but what if she buys two of either kind, but does not like the taste of it? It is quite intuitive to think that the kid should go for one of each type - this way even if she does not enjoy one of them, she can still rely on the other one (the chances of finding both candies unenjoyable is quite low). If she enjoys both, she has hit the jackpot! This is the essence of **the 1/N strategy**. In fact, the strategy is robust enough to mitigate the fluctuations in the asset prices. This means the kid should allocate ₹10 to each type of candies, regardless of how much the candies themselves cost apiece!

The idea behind the 1/N strategy is to create a diversified portfolio by allocating the same percentage of funds to each investment option. This approach assumes that all assets in the portfolio have the same return potential as well as the same risks associated with them; and that investors do not have specific information or preferences about individual investments. The main advantage of the 1/N method is its simplicity. Minimal decision making is required, eliminating the need for detailed analysis of individual investments.

In fact, 1/N *is* the optimal investment method when no other information about the assets is known. [11] However, in real life, we *do* know a lot about the assets that we invest in. Investing without prior knowledge about the asset is extremely risky for obvious reasons, and not recommended. As more information becomes available to the investor, the better the quality of decisions one can make based on that information. [12]

Naïve approaches (like the one mentioned above) are simple and intuitive, but often suboptimal. [13] Some of the other naïve methods are equal weighting, inverse volatility weighting, and minimum variance weighting. In the context of stock markets, these methods do not consider the correlations between assets, which can affect the diversification and performance of the portfolio.

There are other investment strategies that have shown significant improvement over the naïve approaches in terms of investor's risk exposure as well as the yield of the selected portfolio. Further sections give a detailed account of their inner workings and the benefits of adding them to one's investment strategy repertoire.

More sophisticated strategies: MVT & BPT

Complex strategies require more data and computation but can potentially offer far better results. They include methods such as Harry Markowitz's Mean-Variance optimization (based on MVT), Black-Litterman optimization (an extension of MVT) and Behavioral Portfolio Theory (BPT). These methods take historical or forecasted (expected) returns, volatilities, and correlations of assets as inputs. They find the optimal portfolio weights that try to maximize (e.g., Sharpe ratio) or minimize (e.g., tracking error) certain key performance indices of the portfolio to be selected.

Mean-Variance Theory (MVT)

The pioneering work of Markowitz, the concept of portfolio selection as an optimization problem was first introduced by Harry Markowitz. His Nobel Prize winning work revolutionized the field by considering the mean and variance of assets as key factors in portfolio optimization.

In a gain frame, humans are risk averse. For example, given the choice between (1) rolling a 6-sided die for the chance at winning ₹1500 or (2) receiving ₹200 with certainty, most of us would overwhelmingly prefer the safe offer of ₹200; even though the expected gain

from the first scenario is higher than the second one. This risk aversion is the primary justification behind MVT's central tenet that the investors want to minimize the risk. To mathematically formulate the concept of risk and rewards, MVT models the mean value of an asset as the returns and the variance (the spread of the value of the asset over a specific period) as the risk.

MVT posits that the investor wants to minimize risk while maximizing returns. This follows that one portfolio (say, Portfolio A) is considered superior to another (say, Portfolio B), if Portfolio A produces better returns for the same amount of risk undertaken, or, conversely, reduces the risk undertaken for the same expected return. As one can expect this leads to a trade-off between the risk undertaken and expected returns.

MVT solves this by defining an "efficient frontier" for a set of assets. An "efficient" portfolio is the portfolio that for a given level of risk undertaken produces the maximum returns possible. Naturally, for a given set of assets, there exist multiple such portfolios which are theoretically equally (minimally) risky while generating equal (maximal) returns in expectation. The efficient frontier is a collection of all such efficient portfolios.

Mean-Variance Theory (MVT) also formalizes the concept of diversification by using correlation to model how closely together the prices of two assets 'move'. This is where the correlation matrix of assets comes into picture, which captures how the price of each asset correlates with every other asset. Each cell of the matrix corresponds to a pair of assets, and the more positive the value of the cell the more the correlation between the corresponding asset pair (and vice versa).

Behavioral Portfolio Theory (BPT)

For all its merits, MVT makes a subtle yet fundamental assumption -

1. *All investors want to minimize risk.*
2. *All investors are rational.*

The first assumption, one might argue, does hold some merit. Any rational person would want their portfolio to increase in size while being exposed to the minimum amount of risk possible. However, introspectively this assumption might prove to be overly simplistic. We observe that investors have diverse goals and aims when it comes to their portfolios.

For example, suppose a couple wants to save money for two main purposes: (1) their children's college tuitions and (2) retirement. It is reasonable for them to assign different risk preferences for each of these goals. They may be more risk-tolerant for the college fund, seeking higher annual returns of 12-15% or more; while being more risk-averse with their retirement savings, settling for 7-9% annual returns. The time horizon and criticality of each goal can also influence their risk-taking behavior.

The second assumption has been repeatedly proven incorrect in real-world situations. Recently we have witnessed bizarre scenarios where public opinion was swindled to influence the stock markets, which resulted in some extremely unpredictable market swings – cases in point: Bitcoin crash, Tesla debacle in 2022 and the historical GameStop short squeeze.

Behavioral Portfolio Theory (BPT) takes a different approach at attempting to describe investor behavior. BPT posits that investors behave like Maslow's hierarchy of needs. It is important to note that risk seeking can be optimal for BPT investors, while MVT investors are always risk averse.

Limitations of MVT and BPT

MVT and BPT can capture the trade-off between return and risk more accurately than the naïve methods and can incorporate investors' preferences and beliefs more flexibly. However, MVT and BPT also have some limitations, such as sensitivity to input parameters, estimation errors, model assumptions, and computational complexity.

MVT assumes that investors are rational and only care about the statistical properties of the prices of the assets (such as price mean and variation). However, this assumption is often weak, as investors may have other preferences (rational or otherwise). Moreover, MVT relies on historical data to estimate the expected returns and covariances of the assets, which may not be stable or accurate over time. MVT also suffers from estimation errors, which can lead to inefficient or unrealistic portfolios.

BPT is an extension of MVT that incorporates investors' views on the expected returns of the assets and a prior distribution based on the market equilibrium. BPT aims to overcome some drawbacks of MVT by allowing investors to express their subjective beliefs and adjust their portfolios accordingly. However, BPT also has some limitations, such as the difficulty of eliciting and quantifying investors' views, the sensitivity of the results to the choice of prior distribution and confidence level, and the complexity of the model and its implementation.

Investors are attracted to MVT by its logic and practical application, whereas BPT captures the inherent subjectivity of the investors that the financial choices are made with. Researchers have proposed several frameworks where both can be combined to mitigate the disadvantages that each of them poses individually. One such framework is called Mental Accounting framework for portfolio optimization, which is discussed in detail in the following section.

Mental Accounting

Mental Accounting is a concept in the field of Behavioral Economics which discusses the different values a consumer places to the same amount of money depending on the

circumstances. The central tenet of Mental Accounting is that money is a fungible resource. Richard Thaler introduced the concept in his landmark paper published in 1985 to explain some typical but irrational consumer behavior. [14] Mental Accounting refers to the set of irrational cognitive thumb-rules used by individuals and households to organize, evaluate, and track their finances. [15] As a classic example of these cognitive biases, you can think of someone saving up money in the money jar for some anticipating expense, while accruing large credit card debt. In the portfolio optimization context, Mental Accounting takes a slightly different meaning. Mental Accounting here refers to the practice of creating and maintaining different portfolios for different goals. Depending on the end goal, a portfolio takes on different forms.

Mental Accounting Framework for Portfolio Optimization

Das, Scheid, Markowitz and Statman collectively proposed Mental Accounting (MA) framework for portfolio optimization in a paper published in 2010. [16] MA emerges as a synthesis of the best features of both MVT and BPT. It combines the behavioral science approach of BPT to help investors make better decisions with the pragmatism, rationalism, and structure that MVT offers.

MVT's simplistic approach of assigning risk aversion coefficients to investors fails to capture the nuanced nature of risk preferences. Risk aversion is context-dependent, and investors may exhibit varying attitudes toward risk based on different goals.

Take for example a couple who want to invest for two foreseen expenses, a college fund for their child and a retirement fund. These two goals are fundamentally different. The college fund is an investment for the shorter term and with lower consequences if the investment fails to reach the threshold level due to widespread availability of student loans. Thus, it naturally follows that the risk aversion for such an investment will be lower. On the other hand, a retirement fund is a long-term investment, and the consequences are more dire. Thus, the couple is much more risk-averse in this scenario.

MA acknowledges these complexities and provides a more comprehensive framework for portfolio optimization. MA addresses unanswered questions posed by MVT, such as creating portfolios that best align with one's goals, determining their position on the efficient frontier, understanding one's attitude toward risk, and applying MVT with varying risk attitudes across different goals.

How Does it Work?

The central idea of this MA framework is to create **goal-based sub-portfolios** which have different subjective levels of risk associated with them. For example, now Mr. X wants to invest some amount of money for three goals –

1. ₹1,00,000 for a vacation (a luxury, can afford to take an elevated risk)

2. ₹1,00,000 to get a bike for his daughter (somewhat of a necessity, so a lower-risk goal)
3. ₹3,00,000 as the college fund for his son (absolute necessity, hence a no-risk goal)

As discussed before, Mr. X recognizes that his propensity to take risks varies across these three investments. Under MVT, Mr. X would have been asked to conjure a singular "risk aversion coefficient," for all his investment goals; which is not only tedious and difficult to predict, but often also vastly at odds with his actual risk-affinity.

MA, however, takes a different approach. For each of these goals, Mr. X would be asked to estimate two numbers –

1. The return the portfolios should provide in the worst-case scenario (H).
2. The probability of failing to reach H which he would be comfortable with (α).

In our example's context, Mr. X should be able to state something of the nature of "For my vacation goals, I want to invest ₹1,00,000. I want to grow this amount, but I am willing to afford the worst-case scenario where I make a 15% loss on my investment. I will be fine if I experience this worst-case scenario one out of five times (i.e., with 20% probability). However, I cannot afford any losses with my son's college funds! I will be ok to bear some losses one out of five times, but that is where I draw the line!"

Notice that the conversation is now happening in terms of defining the worst-case scenario and stating the ability to experience the worst-case scenario while still moving forward. This is a much more nuanced formulation of the problem than how MVT treats the investment scenario to be a simple optimization problem with a global minimum. The coefficients H and α are borne BPT-esque questions wherein investors are asked to predict more relatable and meaningful values, rather than to state indecipherable coefficients for MVT. Hence, the accuracy of estimating these values is much higher for the everyday investor.

Let us try to formulate the scenario in the example above with some concrete numbers. The following are Mr. X's parameters for his goals –

Goal	Worst-case returns (H)	Prob [returns < H] (α)
Vacation	-15%	20%
Bike	-5%	15%
College fund	0%	20%

These values are then converted into risk-aversion coefficients (γ). The risk aversion coefficient (γ) captures the expected risk-return tradeoff. It is the rate at which an investor will forego expected return for less variance (which is a canonical metric for risk within MVT). [17] In MA, we arrive at γ values using certain standard methods, discussion of which falls outside of the scope of this paper. For the given example –

Goal	Worst-case returns (H)	Prob [returns < H] (α)	Risk-aversion coefficient (γ)
Vacation	-15%	20%	0.8773
Bike	-5%	15%	2.7063
College fund	0%	20%	3.7950

Risk aversion coefficients can be used to find efficient portfolios in MVT. Using the risk aversion coefficients, optimum MVT portfolios are then calculated, and portfolios with the following risk and expected returns (mean and variance) are created for the user.

Goal	Worst-case returns (H)	Prob [returns < H] (α)	Risk-aversion coefficient (γ)	Exp. Returns	Risk
Vacation	-15%	20%	0.8773	26.35%	49.13%
Bike	-5%	15%	2.7063	12.18%	16.57%
College fund	0%	20%	3.7950	10.23%	12.30%

Eventually these optimal sub-portfolios are combined into a MVT optimal portfolio depending on the size of the investment.

Goal	Exp. Returns	Risk	Percentage of the portfolio
Vacation	26.35%	49.13%	20%
Bike	12.18%	16.57%	20%
College fund	10.23%	12.30%	60%
COMBINED	100%	13.84%	20.32%

In summary, the MA approach offers a personalized and intuitive method for investors to optimize their portfolios based on individual goals and risk preferences, enhancing accuracy and alignment in investment decisions.

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[1] Mankiw, N. G. (2018). Principles of economics (8th ed.). Cengage Learning.
[2] Zhou, X. (2009). Evolution of the international monetary system. In BIS Papers No 49: The international financial crisis and policy challenges in Asia and the Pacific (pp. 29-44). Bank for International Settlements.
[3] Bodie, Z., Kane, A., & Marcus, A. J. (2014). Investments (10th ed.). McGraw-Hill Education.
[4] Mischel, Walter, Ebbe B. Ebbesen, and Antoinette Raskoff Zeiss. "Cognitive and attentional mechanisms in delay of gratification." Journal of personality and social psychology 21.2 (1972): 204.
http://www.kokdemir.info/courses/psk101/docs/Mischel-DelayofGrat.pdf
[5] Gilovich, Thomas, Robert Vallone, and Amos Tversky. "The hot hand in basketball: On the misperception of random sequences." Cognitive psychology 17.3 (1985): 295-314.
https://labs.la.utexas.edu/gilden/files/2016/04/Gilo.Vallone.Tversky.pdf
[6] Kahneman, Daniel. Thinking, fast and slow. Macmillan, 2011.
http://dspace.vnbrims.org:13000/jspui/bitstream/123456789/2224/1/Daniel-Kahneman-Thinking-Fast-and-Slow-.pdf
[7] Thaler, Richard H., and Cass R. Sunstein. "Nudge: Improving decisions about health, wealth, and happiness." (2009).
[8] Markowitz, Harry M. "Foundations of portfolio theory." *The journal of finance* 46.2 (1991): 469-477.
[9] Statman, Meir, and Jonathan Scheid. "Correlation, return gaps, and the benefits of diversification." *Journal of Portfolio Management* 34.3 (2008): 132.
[10] Sundaram, Rangarajan K., and Sanjiv Ranjan Das. *Derivatives: principles and practice*. New York, NY: McGraw-Hill Irwin, 2011.
[11] DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal. "Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy?." *The review of Financial studies* 22.5 (2009): 1915-1953.
[12] Kritzman, Mark, Sébastien Page, and David Turkington. "In defense of optimization: the fallacy of 1/N." *Financial Analysts Journal* 66.2 (2010): 31-39.
[13] Hwang, Inchang, Simon Xu, and Francis In. "Naive versus optimal diversification: Tail risk and performance." *European Journal of Operational Research* 265.1 (2018): 372-388.
[14] Thaler, Richard. "Mental accounting and consumer choice." *Marketing science* 4.3 (1985): 199-214.
[15] Thaler, Richard H. "Mental accounting matters." *Journal of Behavioral decision making* 12.3 (1999): 183-206.
[16] Das, Sanjiv, et al. "Portfolio optimization with mental accounts." *Journal of financial and quantitative analysis* 45.2 (2010): 311-334.
[17] Idzorek, Thomas. "A step-by-step guide to the Black-Litterman model: Incorporating user-specified confidence levels." Forecasting expected returns in the financial markets. Academic Press, 2007. 17-38.