IT3011 - Theory & Practices in Statistical Modelling

Probability Distributions

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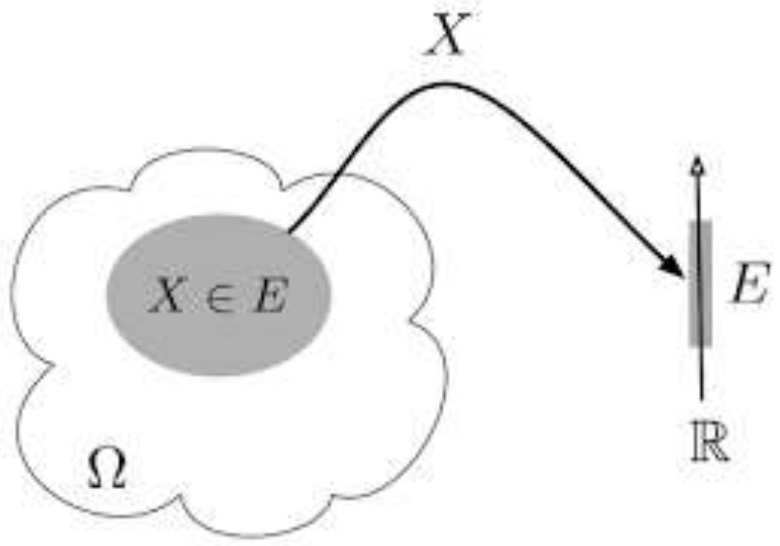
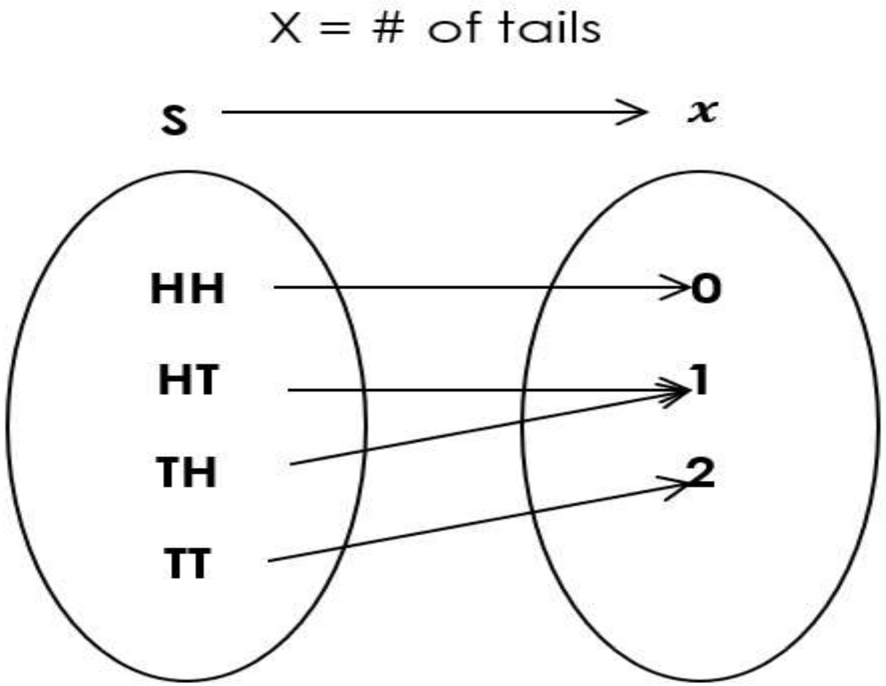
Random Variables

- A random variable is a function that assigns a numerical value to each possible outcome of a random experiment.

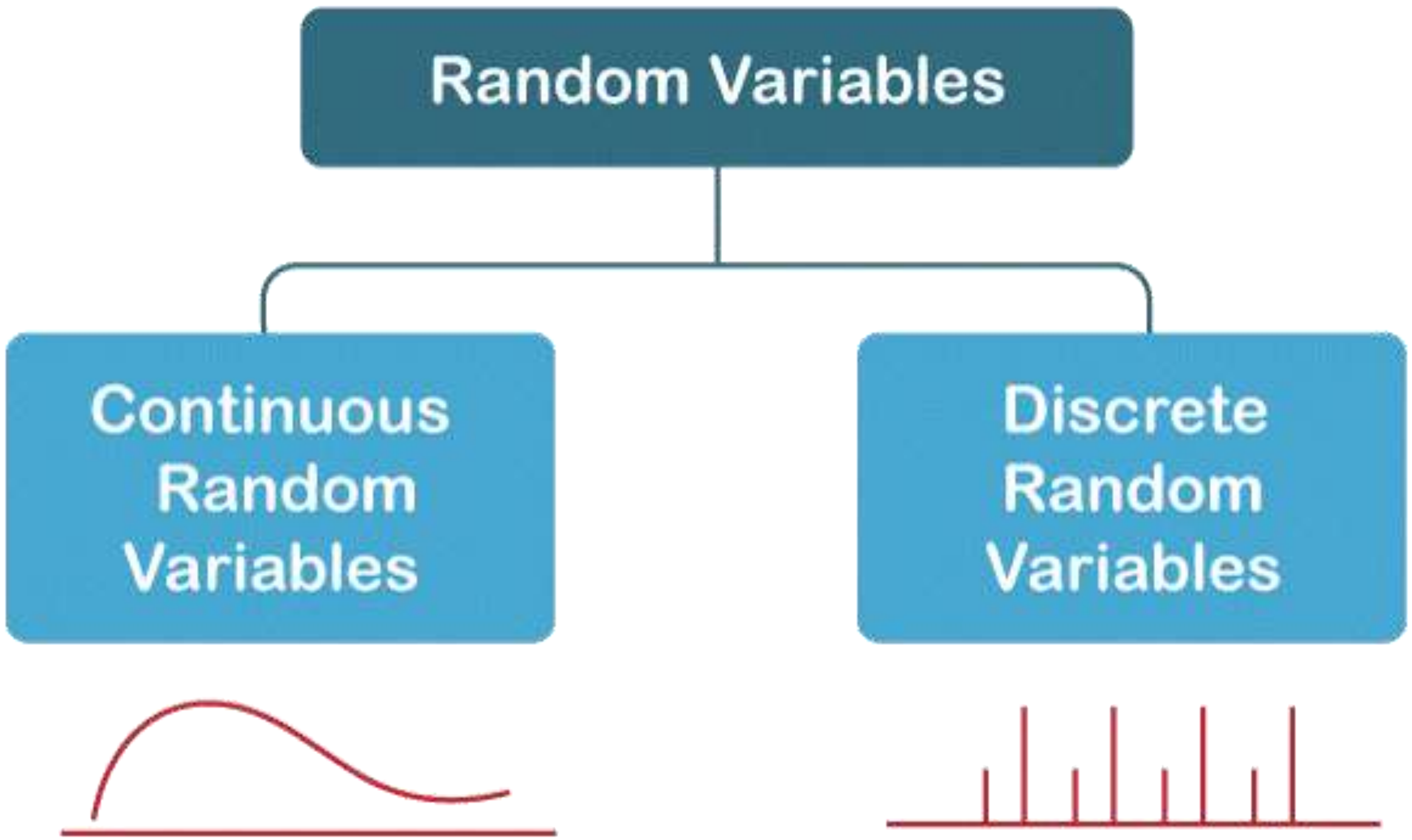


Random Variables

- Ex- X = number of tails obtained when tossing a coin once.



Types of Random Variables



Discrete Random Variables Examples

- Tossing a Coin Three Times
- Sample Space (S):
  - S={$HHH,\ HHT,\ HTH,\ THH,\ HTT,\ THT,\ TTH,\ TTT$}
- Let X= number of heads obtained
- $X\epsilon$ {0,1,2,3}

Discrete Random Variables Examples

Outcome	X
HHH	3
HHT	2
HTH	2
THH	2
HTT	1
THT	1
TTH	1
TTT	0

Discrete Random Variables Examples

- Rolling a Die Once
- Sample Space (S):
  - S={1,2,3,4,5,6}
- Let X= number shown on the die
- $X \in \{1,2,3,4,5,6\}$

Discrete Random Variables Examples

Outcome	X
1	1
2	2
3	3
4	4
5	5
6	6

Discrete Random Variables Examples

- Inspecting 2 Light Bulbs (Defective or Not)
- Sample Space (S):
  - S={DD,DN,ND,NN}
- Let X= number of defective bulbs
- $X \in \{0,1,2\}$

Discrete Random Variables Examples

Outcome	X
DD	2
DN	1
ND	1
NN	0

Continuous Random Variables Examples

- Height of a Student
- Sample Space (S):
  - S={x∈R|140≤x≤190}
- Let X= height of a student (in cm)
- $X \in$ [140,190]

Continuous Random Variables Examples

- Time Taken to Complete an Exam
- Sample Space (S):
  - $S=\{x \in R | 0 < x \leq 180\}$
- Let X= time taken to complete an exam (in minutes)
- $X \epsilon$ (0,180]

Continuous Random Variables Examples

- Weight of a Fruit
- Sample Space (S):
  - $S=\{x \in R | 0.2 \leq x \leq 1.5\}$
- Let X= weight of a mango (in kg)
- $X \epsilon$ [0.2,1.5]

Probability Associated with Random Variables

- Using the values of random variables, associated probabilities can be found.
- Ex:- An experiment consists of tossing two fair coins simultaneously
  - S={(H,H),(H,T),(T,H),(T,T)}
- X= Number of Heads Obtained (0, 1, 2)

- $Ex - \ P(x = 1) = \frac{2}{4} = \frac{1}{2}$

- $Ex - \ P(x \geq 1) = P(x = 1) + P(x = 2) = \frac{3}{4}$

Probability Associated with Random Variables

- Probability of a random variable with any value can be represented in a functional form.

Discrete Random Variables
Probability Mass Function

$$P(X = x) = f(x)$$

Continuous Random Variables
Probability Density Function

Probability Distribution

- Any function has a distribution or an arrangement with its inputs and outputs.

X	f(X)
a	f(a)
b	f(b)
c	f(c)

- In a random variable, this arrangement is called as a Probability Distribution.
- With the nature of the random variable, the distribution can be a **Discrete Probability Distribution** or a **Continuous Probability Distribution**.

Discrete Probability Distributions

- Ex- Toss a fair coin twice
- X = number of heads obtained

X	0	1	2
P(X=x) (Probability)	1/4	2/4	1/4

Discrete Probability Distributions

- Ex- Toss a fair coin twice
- X = number of heads obtained

X	0	1	2
P(X=x) (Probability)	1/4	2/4	1/4
F(X=x) (Cumulative Probability)	1/4	3/4	4/4 = 1

$$P(X \le x) = \sum P(X = x_i)$$

Discrete Probability Distributions

- The probability of a discrete random variable lies between 0 and 1:
  - $0 \le P\ (X = x) \le 1$
- Sum of Probabilities is always equal to 1:
  - $\sum P\ (X = x) = 1$

Continuous Probability Distributions

$$P(a \leq x \leq b) = \int_a^b f(x)dx \geq 0$$

- All properties are common for continuous probability distributions too.

Continuous Probability Distributions

$$f_X(x) = \begin{cases} cx^2, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Questions:

1. Find the value of c.
2. Find $Pr(0 < X \leq 1)$.

Expected Value

- Discrete

$$\mathbb{E}(X) = \sum_x x f_X(x) = \sum_x x \mathbb{P}(X = x).$$

- Continuous

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

Variance

- Discrete or Continuous

$$\mathrm{Var}(X) = \mathbb{E}\Big((X - \mu_X)^2\Big) = \mathbb{E}(X^2) - \Big(\mathbb{E}(X)\Big)^2.$$

Expected Value and Variance

- Find the E(X) and V(X)

X	0	1	2	3
P_x	1/10	2/10	4/10	3/10

Expected Value and Variance

$$f_X(x) = \begin{cases} cx^2, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Questions:

1. Find the value of c.
2. Find $Pr(0 < X \leq 1)$.
3. Find the expected value $E(X)$ and the variance $Var(X)$.
4. Find the cumulative distribution function (c.d.f.).

Discrete Uniform Distribution

- A random variable is defined to have a discrete uniform distribution, if the probability mass function of X is given by,

$$P_X(x) = \begin{cases} \frac{1}{N}, & \text{for } i = 1, 2, ..., N \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = \frac{N+1}{2}$ and $V(X) = \frac{N^2-1}{12}$

Discrete Uniform Distribution

- Example: Let X represent a random variable taking on the possible values of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and each possible value has equal probability.
- This is a discrete uniform distribution and the probability for each of the 10 possible value is

$$P(X = x_i) = f(x_i) = \frac{1}{10} = 0.10$$

Bernoulli Distribution

- Consider the toss of a coin, which comes up a head with probability p, and a tail with probability 1-p. Define

$$X = \begin{cases} 1, & \text{if } \textit{Head} \\ 0, & \text{if } \textit{Tail} \end{cases}$$

- The random variable X is said to have a Bernoulli distribution and the probability mass function of X is given by,

$$\mathrm{P}_X(x) = \begin{cases} P^x(1-p)^{1-x}, & \text{for x=0 or 1} \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = p$ and $V(X) = p(1\text{-}p)$

Binomial Distribution

- A coin is tossed n times. At each toss, the coin comes up a head with probability p, and a tail with probability 1-p, independently of prior tosses.
- Let X be the number of heads in the n-toss sequence.
- The random variable X is said to have a Binomial distribution with parameters n and p and the probability mass function of X is given by,

$$\mathrm{P}_X(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{(n-x)}, & \text{for } x = 0, 1, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

Also, $\mathrm{E(X)} = \mathrm{np}$ and $\mathrm{V(X)} = \mathrm{np(1\text{-}p)}$

Binomial Distribution

- Example: A die is tossed 3 times. What is the probability of
  - No fives turning up?
  - 1 five?
  - 3 five?

Binomial Distribution

This is a **binomial** distribution because there are only 2 possible outcomes (we get a 5 or we don't).

Now, $n = 3$ for each part. Let X = number of fives appearing.

(a) Here, $x = 0$.

$$P(X=0) = C^n_x p^x q^{n-x} = C^3_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 = \frac{125}{216} = 0.5787$$

(b) Here, $x = 1$.

$$P(X=1) = C^n_x p^x q^{n-x} = C^3_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = \frac{75}{216} = 0.34722$$

(c) Here, $x = 3$.

$$P(X=3) = C^n_x p^x q^{n-x} = C^3_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 = \frac{1}{216} = 4.6296 \times 10^{-3}$$

Hypergeometric Distributions

- Assume that a population of size N which contains M defective items.
- A random sample of size n is drawn and observed that it contains X number of defective items. The random variable X is said to have a hypergeometric distribution and the probability mass function of X is given by

$$P_X(x) = \begin{cases} \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, & \text{for } x = 0, 1, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = n\frac{M}{N}$ and $V(X) = \dfrac{nK}{N}\left(1 - \dfrac{K}{N}\right)\dfrac{N-n}{N-1}$

Hypergeometric Distributions

- Example: Suppose we randomly select 5 cards without replacement from an ordinary deck of playing cards. What is the probability of getting exactly 2 red cards (i.e., hearts or diamonds)?

  - $N = 52$; since there are 52 cards in a deck.
  - $M = 26$; since there are 26 red cards in a deck.
  - $n = 5$; since we randomly select 5 cards from the deck.
  - $x = 2$; since 2 of the cards we select are red.

  We plug these values into the hypergeometric formula

  Thus, the probability of randomly selecting 2 red cards is 0.32513.

Poisson Distributions

- The number of events occurring (X) within a fixed period of time has Poisson distribution and its probability mass function is given by

$$P_X(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & \text{for } x = 0, 1, \ldots, n \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = \lambda$ and $V(X) = \lambda$

- Poisson distribution can be effectively used to approximate Binomial probabilities when the number of trials n is large, and the probability of success p is small.

Poisson Distributions

- Example: A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week he will sell
  - Some policies
  - 2 or more policies but less than 5 policies.

Poisson Distributions

Here, $\mu = 3$

(a) "Some policies" means "1 or more policies". We can work this out by finding 1 minus the "zero policies" probability:

$$P(X > 0) = 1 - P(x_0)$$

Now $P(X) = \dfrac{e^{-\mu}\mu^x}{x!}$ so $P(x_0) = \dfrac{e^{-3}3^0}{0!} = 4.9787 \times 10^{-2}$

Therefore the probability of 1 or more policies is given by:

$$\begin{aligned}\text{Probability} &= P(X \geq 0)\\ &= 1 - P(x_0)\\ &= 1 - 4.9787 \times 10^{-2}\\ &= 0.95021\end{aligned}$$

(b) The probability of selling 2 or more, but less than 5 policies is:

$$P(2 \leq X < 5)$$

$$= P(x_2) + P(x_3) + P(x_4)$$

$$= \frac{e^{-3}3^2}{2!} + \frac{e^{-3}3^3}{3!} + \frac{e^{-3}3^4}{4!}$$

$$= 0.61611$$

Geometric Distribution

- Suppose that we repeatedly and independently toss a coin with probability of a head p.
- The geometric random variable is the number X of tosses needed before a head to come up for the first time. The probability mass function of X is given by,

$$P_X(x) = \begin{cases} p(1-p)^{x-1}; & \text{for } x = 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = \frac{1}{p}$ and $V(X) = \frac{1-p}{p^2}$

Geometric Distribution

- Example: A farmer tests milk quality from cows one by one. Each test has a probability of passing equal to 0.2. Tests are independent. Probability that the first success occurs on the 4th test

$$P(X = 4) = (1 - 0.2)^3 \times 0.2$$
$$= (0.8)^3 \times 0.2$$
$$= 0.1024$$

Negative Binomial Distribution

- Let X denote the number of trials until the r^{th} success.
- Then, the probability mass function of X is

$$\mathrm{P}_X(x) = \begin{cases} \binom{x-1}{r-1}(1-p)^{x-r}p^r, & \text{for } x = r, r+1, r+2, \ldots.. \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = \frac{r}{p}$ and $V(X) = \frac{r(1-p)}{p^2}$

Negative Binomial Distribution

- Example: A dairy quality inspector tests milk samples one by one. Each test has a probability of being acceptable equal to 0.3.Tests are independent. Probability that the 3rd acceptable sample occurs on the 7th test

$$P(X=7) = \binom{6}{2}(0.3)^3(0.7)^4$$

$$= 15 \times 0.027 \times 0.2401$$

$$= 0.0972 \text{ (approximately)}$$

Continuous Uniform Distribution

- A random variable is defined to have a uniform distribution, if the probability density function of X is given by

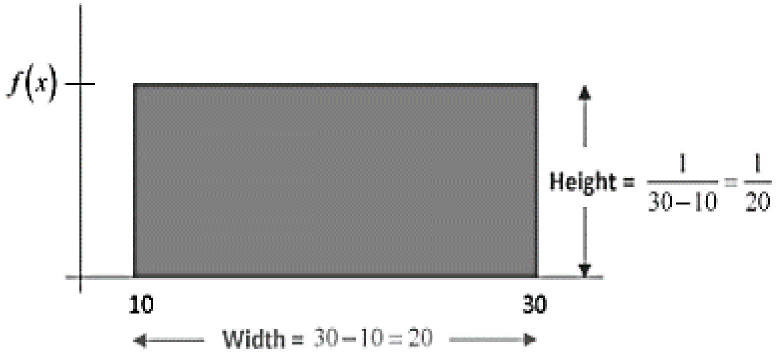
$$f_X(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Also, $E(X) = \frac{a+b}{2}$ and $V(X) = \frac{(b-a)^2}{12}$

Uniform Distribution

- Imagine you live in a building that has an elevator that will take you to your floor.
- From experience, once you push the button to call the elevator, it takes between ten and thirty seconds for you to arrive at your floor.
- This means the elevator arrival is uniformly distributed between 10 and 30 seconds once you hit the button.
- So, if X is a continuous uniform random variable has probability density function mean, and variance is as follows.

Uniform Distribution



Probability Density Function

$$f(x)=\begin{cases}\frac{1}{b-a} & a \le x \le b \\ 0 & elsewhere\end{cases} \rightarrow f(x)=\begin{cases}\frac{1}{20} & 10 \le x \le 30 \\ 0 & elsewhere\end{cases}$$

Mean

$$\mu = E(X) = \int x f(x)\,dx \rightarrow \int_{10}^{30} x\left(\frac{1}{20}\right)dx = \int_{10}^{30}\left(\frac{1}{20}x\right)dx = \frac{1}{40}x^2\Big|_{10}^{30} = 20$$

or

$$\mu = E(X) = \frac{b+a}{2} \rightarrow E(X) = \frac{30+10}{2} = 20$$

Variance

$$\sigma^2 = V(X) = \int (x-\mu)^2 f(x)\,dx \rightarrow \int_{10}^{30}(x-20)^2\left(\frac{1}{20}\right)dx = \int_{10}^{30}\left(\frac{1}{20}x^2 - 2x + 20\right)dx = \frac{100}{3} \approx 33.33$$

or

$$\sigma^2 = V(X) = \frac{(b-a)^2}{12} \rightarrow V(X) = \frac{(30-10)^2}{12} = \frac{100}{3} \approx 33.33$$

Standard Deviation

$$\sigma = \sqrt{\sigma^2} = \sqrt{33.33} = 5.774$$

Normal Distribution

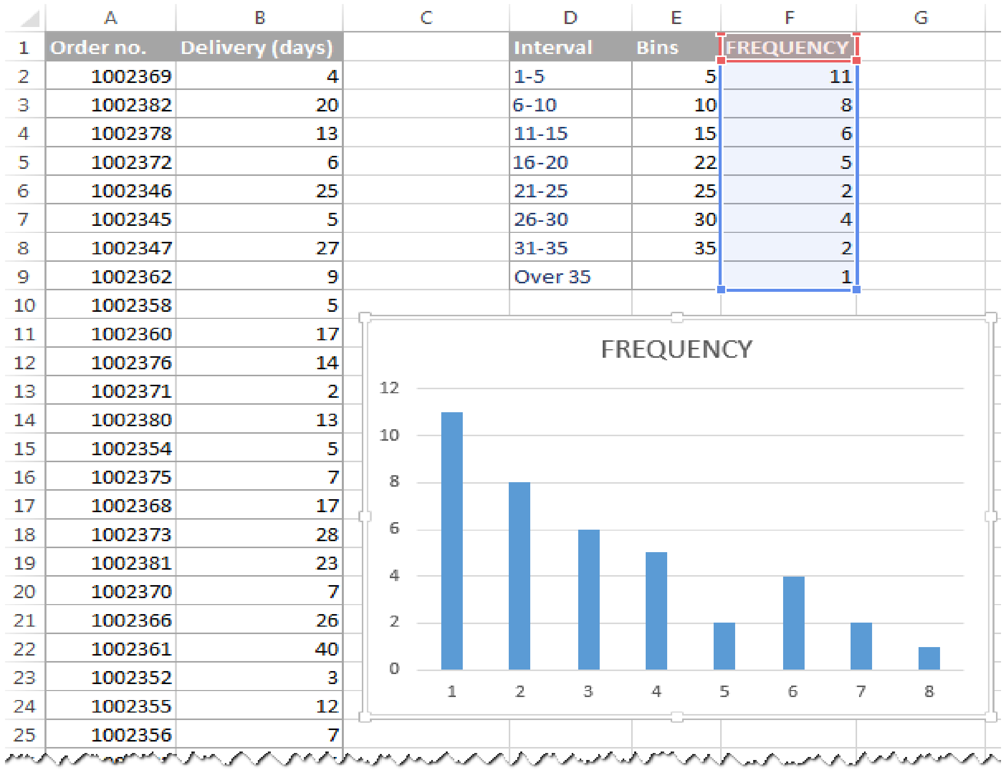
- A random variable is defined to have a normal distribution with mean μ and variance σ^2 , if the probability density function of X is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for} \quad -\infty \leq x \leq \infty$$

Also, $E(X) = \mu$ and $V(X) = \sigma^2$

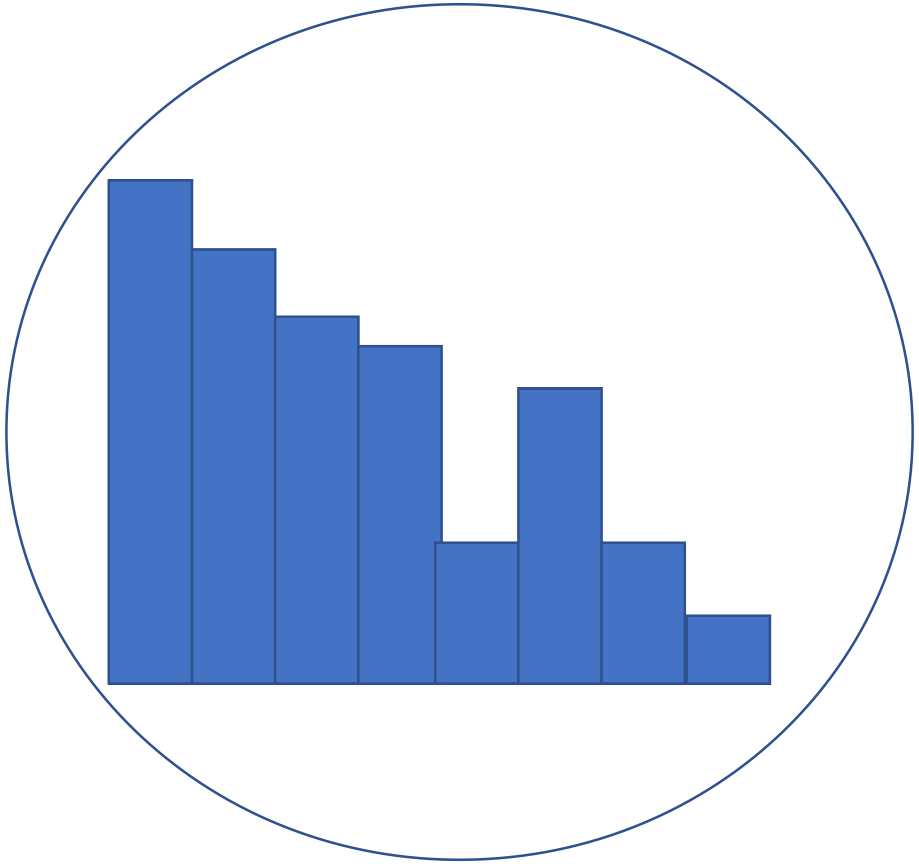
Normal Distribution

The distribution of a statistical data set (or a population) is a listing or function showing all the possible values (or intervals) of the data and how often they occur. Consider the following example



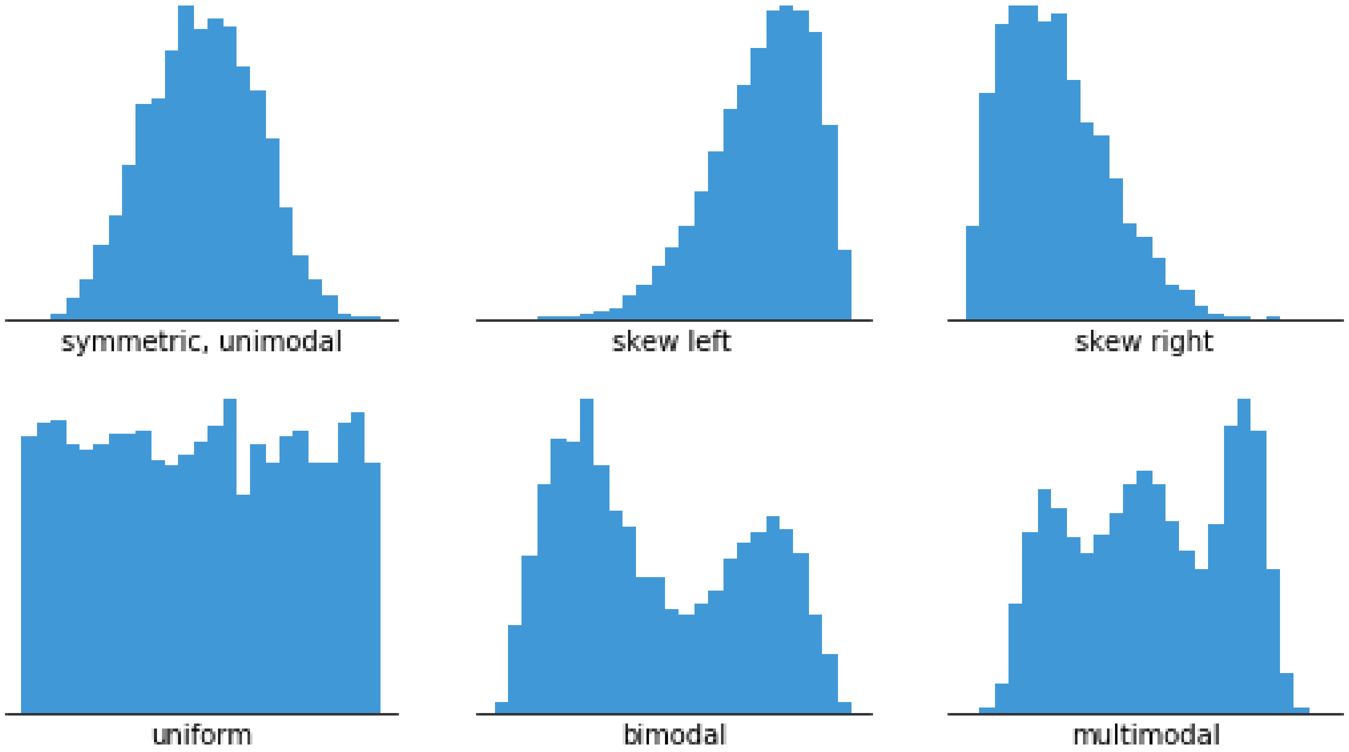
	A	B	C	D	E	F	G
1	Order no.	Delivery (days)		Interval	Bins	FREQUENCY	
2	1002369	4		1-5	5	11	
3	1002382	20		6-10	10	8	
4	1002378	13		11-15	15	6	
5	1002372	6		16-20	22	5	
6	1002346	25		21-25	25	2	
7	1002345	5		26-30	30	4	
8	1002347	27		31-35	35	2	
9	1002362	9		Over 35		1	
10	1002358	5					
11	1002360	17					
12	1002376	14					
13	1002371	2					
14	1002380	13					
15	1002354	5					
16	1002375	7					
17	1002368	17					
18	1002373	28					
19	1002381	23					
20	1002370	7					
21	1002366	26					
22	1002361	40					
23	1002352	3					
24	1002355	12					
25	1002356	7					

Distribution

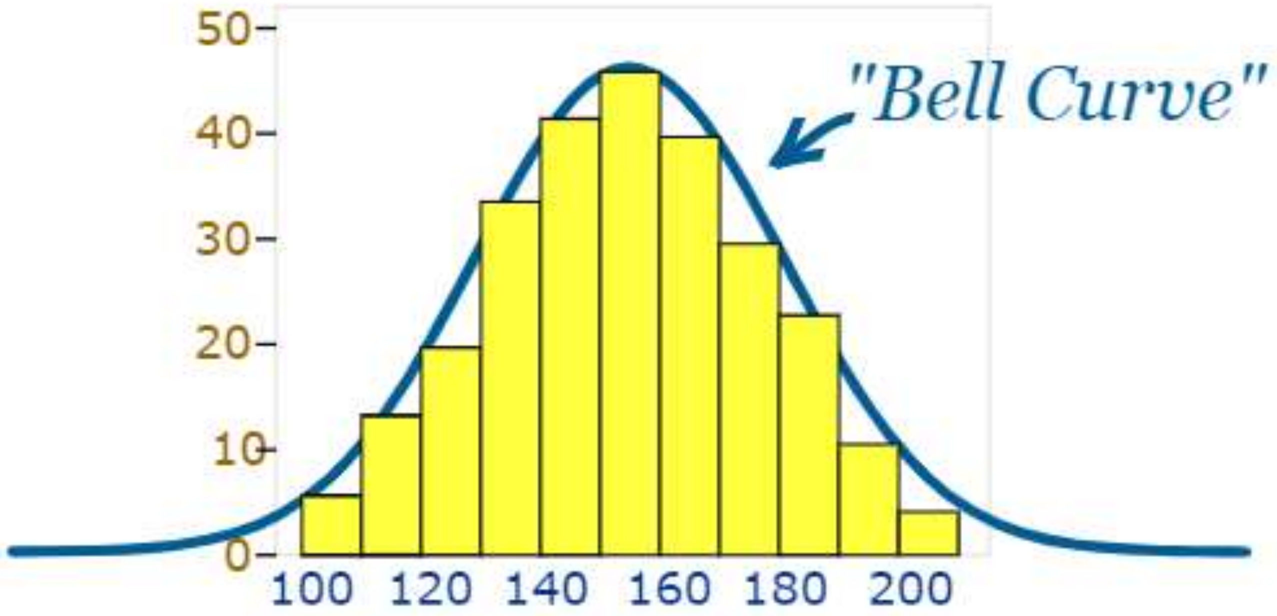


Normal Distribution

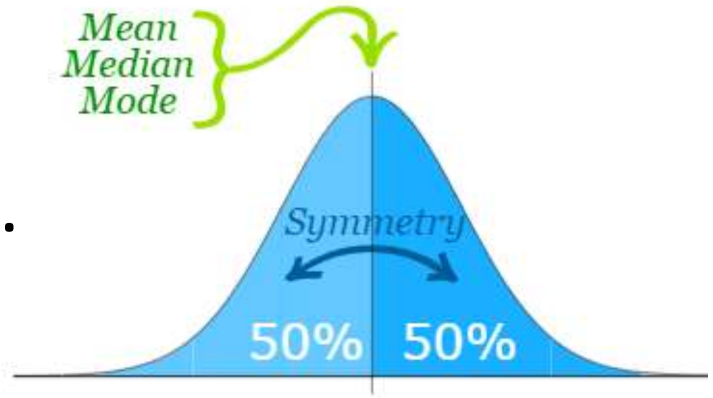
There are several shapes of these distributions.



Normal Distribution



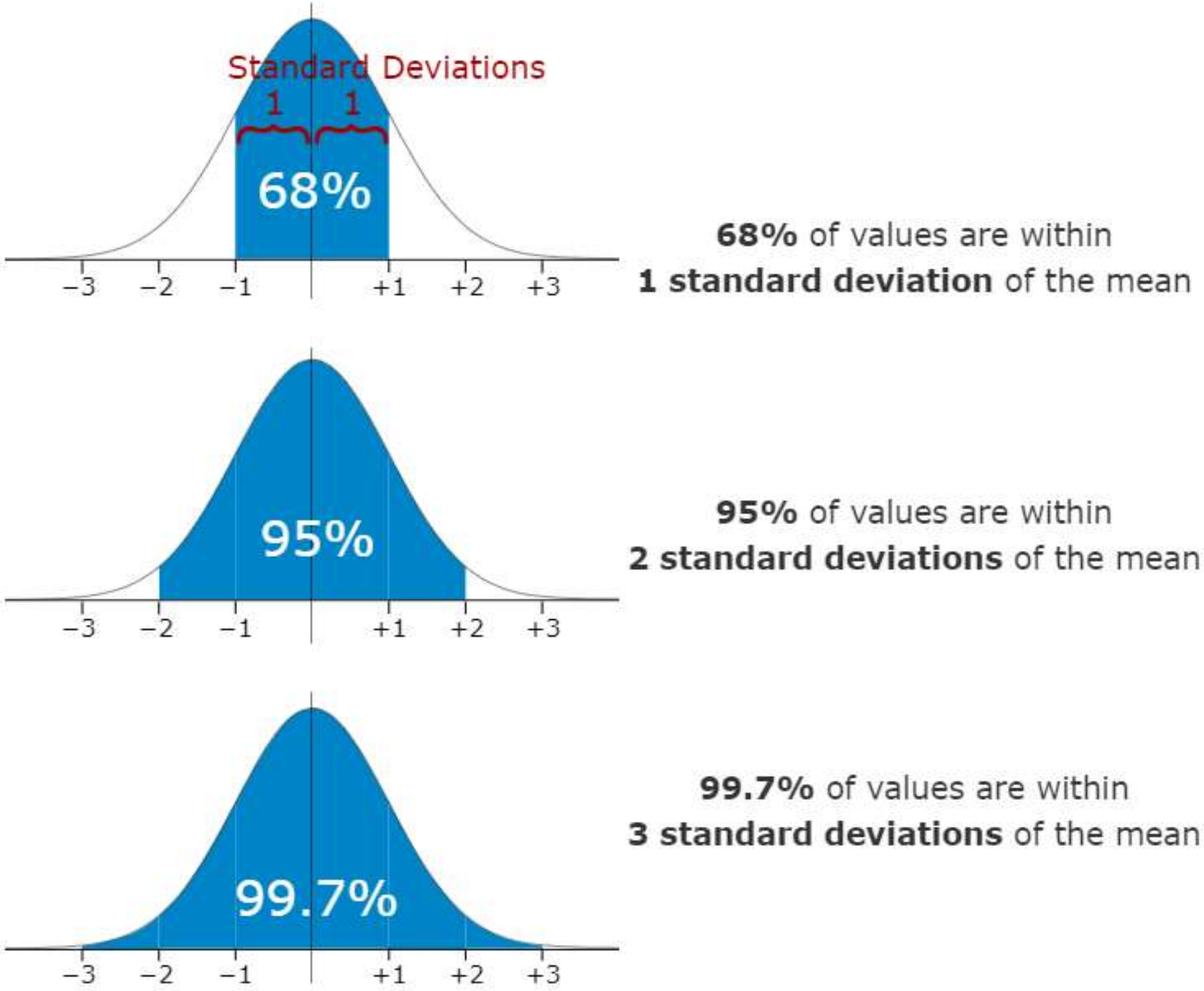
$X \sim N(\mu, \sigma^2)$



Important parameters are the **mean**$(\boldsymbol{\mu})$ and the **variance** $(\boldsymbol{\sigma}^{\mathbf{2}})$**.**

Standard deviation $(\boldsymbol{\sigma})$ is the square root of variance.

Normal Distribution – Empirical Rule



Normal Distribution

Ex- Mean age of a population is 50. If the standard deviation is 5, what proportion of people are in the population from age 40 to age 60.

$$50 \pm 2 \times 5 = [40,60]$$

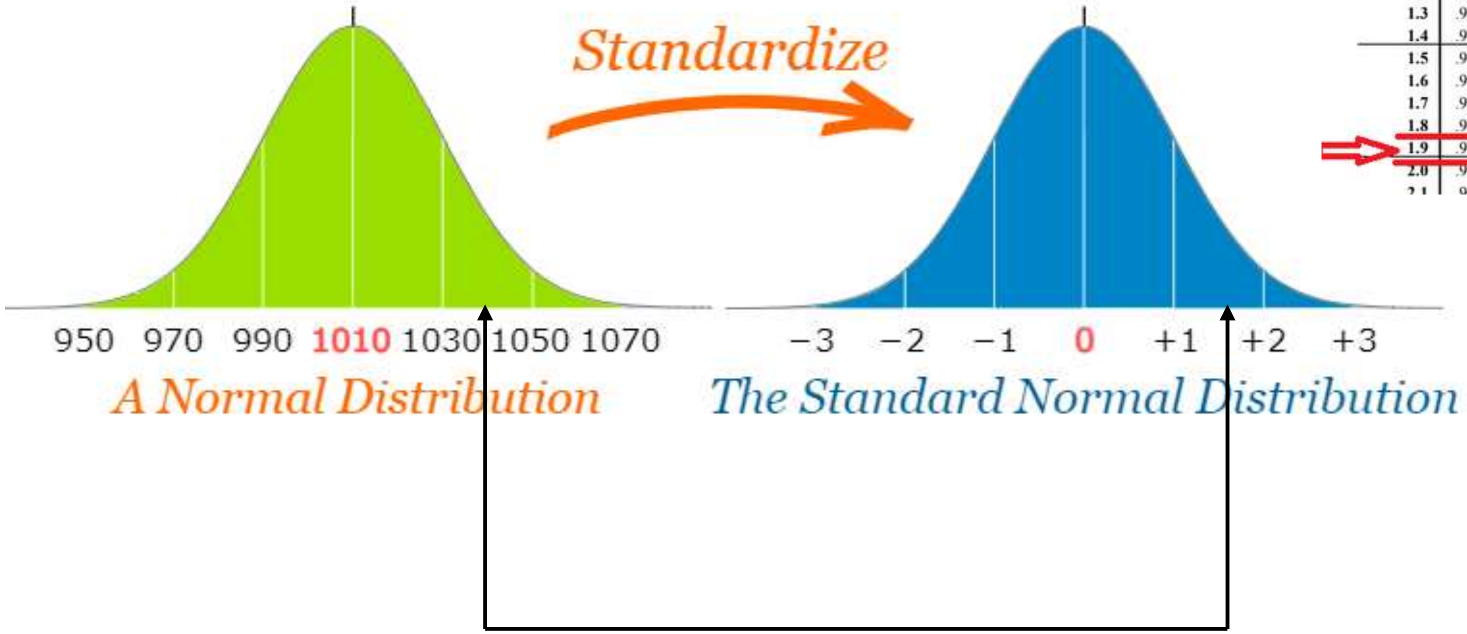
So Empirical Rule says that 95% of the population are in age 40 to 60 range.

What proportion of people are in the population from age 42 to age 58?

Normal Distribution

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

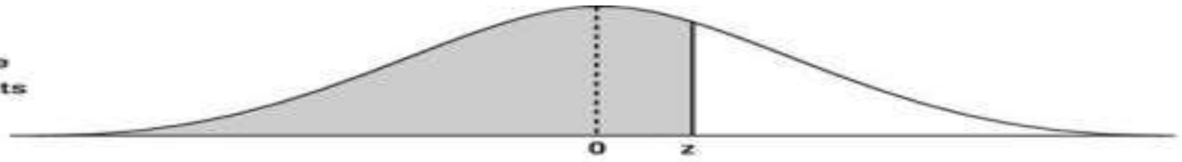
Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574



$$z = \frac{x - \mu}{\sigma}$$

Normal Distribution – Z Table

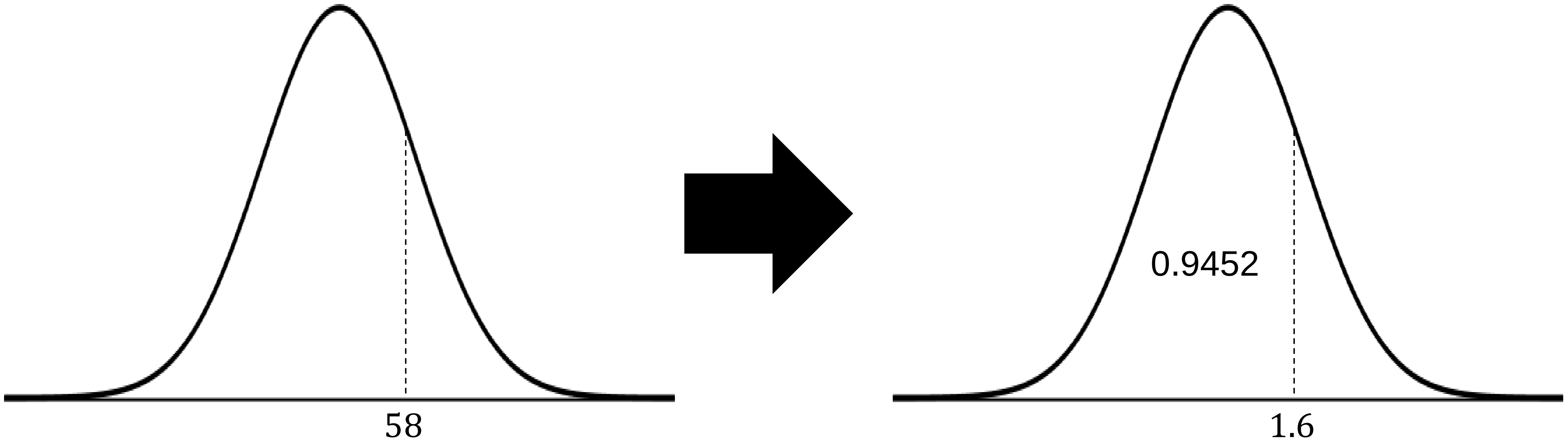
Number in the table represents $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Normal Distribution

Assume that the mean is 50 and the standard deviation is 5.



Normal Distribution

What proportion of people are in the population from age 42 to age 58?

- Find the proportion of people below age 58
- Find the proportion of people below age 42
- Get the difference between these proportions

What proportion of people in the population who are older than 58?

Exponential Distribution

- A random variable is defined to have a exponential distribution, if the probability density function of X is given by

$$\mathrm{f}_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } 0 \leq x \leq \infty \end{cases}$$

Also, $E(X) = \frac{1}{\lambda}$ and $V(X) = \frac{1}{\lambda^2}$

Exponential Distribution

- Example: At a certain location on a highway, the number of cars exceeding the allowed speed limit per half hours is 8.4. What is the probability of waiting time is 5 minutes?

$$f(X = 5\ minutes) = 0.28e^{-0.28\times 5} = 0.07$$

- What is the probability of waiting time less than 5 minutes?

$$f(X < 5\ minutes) = \int_0^5 0.28e^{-0.28x}\, dx$$